Yong Yuan • Junzhi Cui • Herbert A. Mang

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Editors

Yong Yuan
Tongji University
Shanghai
China

Junzhi Cui
Academy of Mathematics
and System Sciences
Beijing, China

Herbert A. Mang
Vienna University of Technology
Austria

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Computational Simulation Methods for Composites Reinforced by Fibres

Vladimír Kompiš^{1*}, Zuzana Murčinková², Sergey Rjasanow³,
Richards Grzhibovskis³ and Qing-Hua Qin⁴

¹DSSI, j.s.c., Wolkrova 4, 82105 Bratislava, Slovakia

²Technical University Košice, Faculty of Manufacturing Technologies, Štúrova 31,
080 01 Prešov, Slovakia

³Department of mathematics and Computer Science, University of Saarland, Germany

⁴Department of Engineering, Australian National University, ACT 0200, Australia

Abstract. Trefftz-FEM (T-FEM), Adaptive Cross Approximation BEM (ACA BEM) and Method of Continuous Source Functions (MCSF) are presented for simulation of Composites Reinforced by Short Fibres (CRSF) with the aim to show possibilities of reduction the problem of complicated and important interactions in such composite materials.

Keywords: Trefftz-FEM, adaptive cross approximation BEM, method of continuous source functions, composite materials, short fibres

1 Introduction

Fibres are the most effective reinforcing material. Outstanding mechanical, thermal and electro-mechanical properties of Carbon Nano-Tubes (CNT), carbon fibres and some other fibres are well known. Composites Reinforced by Short Fibres/tubes (CRSF) are often defined to be materials of future with excellent electro-thermo-mechanical (ETM) properties. Understanding the behaviour of such composite materials is essential for structural design. Computational simulations play an important role in this process. Usually, strength, stiffness, thermal and electrical conduction of fibres are much larger than those of the matrix material. Very large is also the aspect ratio of the short fibres. Because of these properties very large gradients are localized in all ETM fields along the fibres and in the matrix. The fields define the interaction of the fibres with the matrix, with the other fibres, with the boundaries of the domain/structure. Accurate computational simulation of the fields is important for correct assessment of the material behaviour.

* Corresponding author, e-mail: vladimir.kompis@aos.sk

In this paper FEM, BEM and the new developed MCSF are compared for simulation of CRSF. The problems with in-homogeneities can be reduced when the FEM using special (Trefftz-type) functions are used for the formulation. The BEM reduces the original 3D formulation to the problem on the fibre-matrix boundary. Although, the resulting Galerkin system is fully populated, novel techniques can be applied to approximate it by a data-sparse structure. In this study the Adaptive Cross Approximation (ACA) procedure combined with the H-matrix technique is used to achieve this reduction of complexity and storage requirements. Drastic reduction of the problem enables the Fast Multi-pole (FM) BEM (Liu et al., 2005) by which the kernel functions of boundary integrals are substituted by corresponding truncated Taylor series expansion and resulting discrete dipoles and moments are substituted for the continuum in far field interaction. However, the near field interaction is solved by classical BEM formulation. Boundary meshless methods usually need neither the discretization of the domain, nor the boundary. In the special problem like CRSF, 1D distributed source functions by the MCSF enable to reduce drastically the problem comparing to the FMBEM also for the near field interaction. All the three methods are presented in simulation of the CRSF problems.

2 Description of the Methods Used in Computation Models

Trefftz- (T-) type formulations were developed with the aim to enable formulation of domain field by boundary parameters using shape functions which satisfy the governing equations in corresponding domain/subdomain. Commercial FEM package PROCISION (A Guide to Procision 3.5, 1999) contains two types of T-elements: polynomial and nonpolynomial, stress concentration elements. The last one element properly reflects asymptotic behaviour of exact solution in the region of stress concentration. The solution is obtained by adaptive procedure by comparing the strain energy in computational steps and it is stopped when the change in the last two steps is smaller than 1%. The T-FE's can be much larger than classical FE's also for complicated shape of the element. The global and local (element) errors are evaluated from the sub-domain (element) boundary conditions. The accuracy of the element is increased by increasing the order of the T-polynomials of the shape function and it can increase up to 12th order in the models.

The second approach used in this study is the Boundary Element Method (BEM) for equations of linear elastostatics (Lamè system of equations). More precisely, we implement a Galerkin formulation of BEM with piecewise linear basis functions to interpolate the displacement field and piecewise constant basis functions for the tractions on the boundary. The input data consist of the phase geometry, material parameters, and the conditions at infinity. Hence, an interface problem is formulated, and the solution on the matrix-fibre boundary is found by inverting the Steklov-Poincaré operator. The operator is expressed through the

hyper-singular, the double layer, the adjoint double layer, and the single layer operators and its Galerkin discretization turns out to be symmetric and positive definite matrix. This enables us to use the conjugate gradient iterative method to find the solution. Moreover, we use a hierarchical clustering of boundary elements to partition each of the Galerkin matrices into blocks. The blocks that represent interactions of well separated clusters are replaced by their low rank approximants. The approximants are found by means of the ACA procedure (Rjasanow and Steinbach, 2007; Bebendorf and Grzhibovskis, 2006). The overall complexity and memory requirement of the described Galerkin BEM is, up to a log factor, linear in terms of the number of mesh nodes. The accuracy, however, is of order two for the displacement field and the stress field inside either phase or matrix material. Mathematical properties of the method were extensively explored and its approximation errors were rigorously estimated (Rjasanow and Steinbach, 2007). We underline the mathematically justified second order accuracy of the stress field inside the domains as main advantage of this technique in our study.

The third approach is a boundary meshless method, the MCSF (Kompiš et al., 2008), which uses 1D continuous forces (the fundamental or Kelvin solution well known from the BEM), dipoles and couples along the fibre axis to simulate the fibre-matrix interaction. The interaction is simulated by satisfying continuity of displacements and strains along the fibre-matrix interface in collocation points located on the interface. The continuous distribution of the source functions is approximated by non-uniform rational B-splines (NURBS). Cubic B-splines have been used in the models. The aspect ratio of the fibres is usually very large and the stiffness of the fibres is often much larger than the stiffness of the matrix. Then the axial stiffness of the fibre is also much larger than its bending and tangential stiffness and it is possible to assume that the strain is constant in each cross section of the fibre in the fibre direction. If the fibres are symmetrically distributed around a fibre than fictive forces or dipoles in fibre direction and other dipoles perpendicular to fibre direction continuously distributed along the fibre axis can correctly simulate the interaction of the fibre with the matrix and with other fibres. Moreover, discrete dipoles are added in the fibre ends in order to improve inter-domain continuity in the end parts of the fibre. However, if the fibres are irregularly distributed in the matrix and/or they are curved, then additional couples with their vectors perpendicular to the fibre axis are necessary to obtain resulting cross sectional force acting approximately in the fibre axis and so, to simulate the stiffening effect with good accuracy. The integrals giving resulting action of the source functions in the collocation points are quasi-singular or quasi-hyper-singular and are evaluated numerically or analytically by symbolic manipulation. Heat sources and heat dipoles are used as source functions to simulate the temperature field interactions of fibres with matrix and with other fibres (Kompiš and Murčinková, 2009). Number of collocation points is usually larger than the number of parameters defining the intensities of source functions and the problem is solved in the least square sense. The matrix is full and so restricted for computations of large problems by smaller computers. Technique similar to those used for ACABEM

(Rjasanow and Steinbach, 2007; Bebendorf and Grzhibovskis, 2006) and FMBEM as mentioned above can contribute to further reduction of the models based on the MCSF.

Figure 1 T-FEM mesh and detail on fibre end

Figure 2 BEM meshes

Table 1. Comparison of results for one fibre with aspect ratio 1:10

R=5, L=100	T-FEM	ACA BEM	MCSF
Number of equations	-	1266	36
Number of elements	88	840	-
Stress ZZ [MPa]	262	265	249

Figure 3 Force distribution along a single fibre in the matrix with aspect ratios 1:10 (solid line) and 1:50 (dash line) by MCSF

Figure 4 Stress ZZ in the fibre direction for a patch of 3 x 3 x 3 fibres by T-FEM

Figure 5 Heat flow through the fibre surface in perpendicular direction along fibre axis of R=1, L=1000

Figure 6 Heat flow through the cross-section along fibre axis of R=1, L=1000

Figure 7 Heat flow through the fibre surface in perpendicular direction along fibre axis of $R=1$, $L=100$

Figure 8 Heat flow through the cross-section along fibre axis of $R=1$, $L=100$

3 Computational Results and Conclusions

Computations were performed for several problems in order to compare all FEM, BEM and meshless formulations for linear elasticity and stationary heat flow in solids. Only some results are included due to restrictions of the paper. More details and results will be published later. Figures 1 and 2 contain the mesh for Trefftz FEM and for BEM used for one fibre with aspect ratio 1:10. The results obtained by all methods are compared in Table 1. Because of linear problems, all quantities are dimensionless. Material modules of matrix and fibre are 10^4 and 10^6 , respectively, for one fibre and 1900 and 1.9×10^6 , respectively, for patch $3 \times 3 \times 3$ fibres without overlap. Figure 3 gives force distribution along a single fibre in the matrix with aspect ratios 1:10 and 1:50, respectively. Stresses in the fibre direction for a patch of $3 \times 3 \times 3$ fibres by T-FEM are presented in Figure 4. Figures 5 to 8 present heat flow in the fibre both in fibre direction (through the fibre/matrix interface in Figures 5 and 7 and resulting flow through the fibre cross-section in Figures 6 and 8). In all cases the fibres with of radius $R = 1$ and length $L = 1000$ (Figures 5 and 6) and $L = 100$ (Figures 7 and 8) are regularly distributed in layers with or without overlap. The fibre axes are spaced by 16 from each other in perpendicular direction and gaps of 16 and 160, respectively in the fibre direction in the examples given in Figures 5 and 6 and by 2 and with gap of 2 in examples shown in Figures 7 and 8. In Figures 5 and 6, dot line represents results of model without and solid line with overlap and gap 16, dash line with overlap and gap 160. In Figures 7 and 8, solid line represents results of model without and dash line with overlap.

The results show that both elasticity and temperature fields have similar behaviour. The fibres, which are much stiffer and have much better conductivity than the matrix, can considerably increase the stiffness and conductivity of the composite material in the fibre direction. There is very strong interaction between all fibres and matrix and neighbour fibres and the topology of the fibres can strongly influence the effect. The stress in fibre direction can exceed the stresses in the matrix and in the fibre-matrix interface by several orders and so the short fibres are the most effective reinforcing material. All three methods imply considerable problem reduction the largest one is achieved by the MCSF.

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Preface

Following the great progress made in computing technology, both in computer and programming technology, computation has become one of the most powerful tools for researchers and practicing engineers. It has led to tremendous achievements in computer-based structural engineering and there is evidence that current developments will even accelerate in the near future. To acknowledge this trend, Tongji University, Vienna University of Technology, and Chinese Academy of Engineering, co-organized the International Symposium on Computational Structural Engineering 2009 in Shanghai (CSE'09).

CSE'09 aimed at providing a forum for presentation and discussion of state-of-the-art development in scientific computing applied to engineering sciences. Emphasis was given to basic methodologies, scientific development and engineering applications. Therefore, it became a central academic activity of the International Association for Computational Mechanics (IACM), the European Community on Computational Methods in Applied Sciences (ECCOMAS), The Chinese Society of Theoretical and Applied Mechanic, the China Civil Engineering Society, and the Architectural Society of China.

A total of 10 invited papers, and around 140 contributed papers were presented in the proceedings of the symposium. Contributors of papers came from 20 countries around the world and covered a wide spectrum related to the computational structural engineering.

As Chair of the Organizing Committee of CSE'09, I would like to thank all the participants and the authors for their contributions. We would also like to gratefully acknowledge the guidance and cooperation provided by the International Advisory Committee and the Scientific Committee as well as the support provided by the members of the Local Organizing Committee. In particular, we appreciate the financial support provided by the National Natural Science Foundation of China, the Eurasia-Pacific Uninet, and the Ministry of Science and Technology of the People's Republic of China.

Zuyan Shen
Academician of Chinese Academy of Engineering
Professor of Tongji University
Shanghai, China