Characteristic Triangular Spectra of Extreme Localised Structures: Insight from Optics into Rogue Wave Early Warning

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The field of “optical rogue wave physics” began in 2007 and has since become a major international research effort involving many international groups [1]. A central challenge in ongoing studies in the field is to determine whether the emergence of extreme localized states is associated with particular early-time field properties that can be used as an ``early-warning'' indicator. In an optical context such indicators are important for identifying regimes of noise-driven dynamics in supercontinuum generation whilst, in the oceanic context, there are important consequences for developing wave forecasting and early-warning methods. Some progress has already been made in this regard, and when considering modulation instability seeded by weak perturbations, the presence of particular modulation frequencies in the input field has been shown to be strongly correlated with an increased likelihood of rogue wave emergence [2]. In this paper, we discuss a complementary indicator of rogue wave presence and show that information on the presence of rogue wave structures can in fact also be contained in the *shape* of the spectrum of the propagating field. In particular, we show that large classes of rational localized soliton solutions to the nonlinear Schrodinger equation (NLSE) are associated with a characteristic triangular spectrum when measured (as is commonly the case) on a logarithmic scale. The presence of such log-triangular structure has already been associated with Akhmediev breather localization in the early stage of supercontinuum generation [3], and the results here generalize this to a wider class of solution.

A variety of different approaches can be used to construct rational solutions to the NLSE, but until recently, the spectral characteristics of these solutions have attracted little attention. However, we find that the presence of a triangular spectral characteristic appears as a universal property of both limiting rational solitons as well as newly-reported higher-order rogue wave structures [4]. Fig. 1(a) for example shows the recently-observed Peregrine soliton [5] and, although it undergoes dramatic growth and decay (associated with corresponding temporal compression), Fig. 1(b) shows that it (remarkably) exhibits a triangular spectrum at all points in its evolution. A second order rogue wave solution is shown in Fig.1(c). This solution can be considered as a nonlinear superposition of two Peregrine solitons and Fig. 1(d) shows its corresponding spectrum. Although in this case, the spectrum at maximum amplitude is more complex than a simple log-triangular structure, it is significant that its initial shape as it evolves (e.g. $z = -4$) is indeed near-triangular.

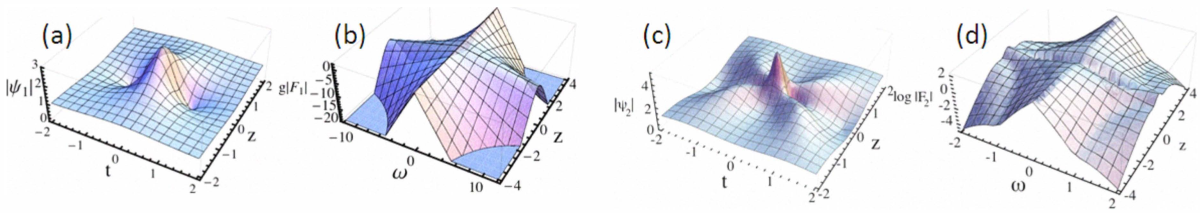

Fig. 1 *(a) Fundamental (j=1) rogue wave (Peregrine soliton). (b) The sp*ectrum of the fundamental rogue wave, plotted on a log scale i.e. $\log|F_1(\omega;\, z)|$. The always-present delta function at $\omega = 0$ associated with the finite background is not shown. (c) Second-order (j =2) rogue wave. (d) Spectrum of second order (j=2) rogue wave on log scale, $\log|F_2(\omega;\, z)|$, during evolution along the z-axis.

Further calculations of different-order solutions and numerical studies in the presence of noise over a wider parameter range confirm our analytic findings presented here. These results thus suggest that triangular shapes on logarithmic spectral plots can be considered as characteristic and universal features associated with early-time emergence of rational soliton rogue waves. Our paper will discuss likely experimental scenarios where these effects can be seen in nonlinear fiber optics, and also consider the intriguing possibility of using results from optics to propose an equivalent early warning technique for the oceanic case.

References

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