Contents lists available at ScienceDirect

Journal of The Japanese and International Economies

journal homepage: www.elsevier.com/locate/jjie

Decline in the persistence of real exchange rates, but not sufficient for purchasing power parity

Tatsuyoshi Okimoto^{a,*}, Katsumi Shimotsu^{b,c}

^a Graduate School of International Corporate Strategy (ICS), Hitotsubashi University, 2-2-1 Hitotsubashi, Chiyoda-ku, Tokyo 101-8439, Japan

^b Department of Economics, Hitotsubashi University, 2-1, Naka, Kunitachi, Tokyo, 186-8601 Japan

^c Department of Economics, Queen's University, Kingston, Ontario, K7L 3N6 Canada

ARTICLE INFO

Article history:

Received 11 July 2009

Revised 10 June 2010

Available online 25 June 2010

JEL classification:

C14

C22

F31

F36

Keywords:

Deviations from PPP

Economic stabilization

Financial integration

ABSTRACT

Okimoto, Tatsuyoshi, and Shimotsu, Katsumi—Decline in the persistence of real exchange rates, but not sufficient for purchasing power parity

The paper investigates the possibility of decline in the persistence of real exchange rates, or deviations from PPP. To this end, we test the null hypothesis of no decline in the PPP deviation persistence between two subsamples using a fractional integration framework. The test rejects the null at the 10% significance level for nine out of 17 countries, providing solid evidence for a decline in the persistence of real exchange rates. However, the decline is not sufficient for PPP, meaning we fail to reject the unit root hypothesis even in the latter period for all 17 countries. In addition, our rolling-window estimates show that the real exchange rate of many countries have experienced a sharp drop in their persistence once we use samples starting from the mid-1980s. Finally, we examine the relationship between the dynamics of PPP deviation persistence and several economic variables and confirm that the speed of convergence of PPP deviations is highly related to economic/financial integration and world economic stabilization. *J. Japanese Int. Economies* **24** (3) (2010) 395–411. a Graduate School of International Corporate Strategy (ICS), Hitotsubashi University, 2-2-1 Hitotsubashi, Chiyoda-ku, Tokyo 101-8439, Japan; bDepartment of Economics, Hitotsubashi University, 2-1, Naka, Kunitachi, Tokyo, 186-8601 Japan; cDepartment of Economics, Queen's University, Kingston, Ontario, K7L 3N6 Canada.

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* Corresponding author. Fax: +81 3 4212 3020.

E-mail address: tokimoto@ics.hit-u.ac.jp (T. Okimoto).

1. Introduction

Purchasing power parity (PPP) is one of the most important, and empirically controversial, theories in international macroeconomics. PPP simply advocates that the equilibrium exchange rate of two currencies should equalize their purchasing power. The idea behind PPP is very intuitive: once converted to a common currency, national price levels should be equal. Although many researchers believe that some variant of PPP holds in the long-run, there are diverse empirical results regarding the PPP hypothesis, in particular for the recent floating rate period.

In this paper, we examine the PPP hypothesis from a different point of view than previous studies. Specifically, we investigate the possibility of decline in the persistence of real exchange rates, or deviations from PPP, by testing the null hypothesis of no decline in the persistence of PPP deviations in the last 30 years. Furthermore, we examine the dynamics of the persistence of PPP deviations during the last three decades. To our best knowledge, none of the previous research investigates changes in the persistence of real exchange rates systematically. There are, however, several interests to examine the dynamics of the persistence of PPP deviations. The first relates to financial market integration. According to IMF's *Annual Report on Exchange Arrangements and Exchange Restrictions* (AREAER), many industrial countries experienced a rapid increase in the degree of financial openness since mid-1980s.¹ Likewise, the *de facto* measures recently constructed by Lane and Milesi-Ferretti (2006) indicate that financial integration in industrial countries was promoted gradually in the 1970s and 1980s, and accelerated in the mid-1990s.² From these observations, we can imagine that PPP should hold more naturally in recent periods. It is, therefore, instructive to examine whether we can find a stronger evidence of PPP in more recent integrated real exchange rates.

Another interest comes from the US and world economic stabilization. Following Kim and Nelson (1999) and McConnell and Perez-Quiros (2000), who point out a sharp decline in the variance of the US economic growth rate in the mid-1980s, several studies provide evidence of commensurate changes toward US/world economic stabilizations. For instance, Clarida et al. (2000) estimate a forward-looking monetary policy function, and show that the US monetary policy has been more stabilizing after 1980. Stock and Watson (2002) and Sensier and van Dijk (2004) find declines in the volatility in a number of US economic time series around the mid-1980s, including series such as employment growth, consumption growth, wage, and price inflation. Following these studies, Kim et al. (2004) and Herrera and Pesavento (2005) provide further supports for the US economic stabilization by identifying possible explanations for the reduction of the variance in US GDP growth. Regarding world economic stabilization, Stock and Watson (2005) find a reduction in the magnitude of the common international shocks contributing to a substantial moderation in the volatility of the GDP growth rates over the past 40 years in the G7 countries (except for Japan).

In addition, recent literature finds a corresponding decline in inflation persistence in the US and other industrial countries. For instance, Kumar and Okimoto (2007) find a marked decline in the US inflation persistence around the early 1980s.³ Furthermore, they find similar declines in the inflation persistence of other G7 countries, except for Italy, suggesting the possibility of world economic stabilization. A natural question raised from these studies is whether we can observe commensurate changes toward world economic stabilization for other economic variables. This paper provides an answer to this question for real exchange rates, or deviations from PPP. If there is a decline in the persistence of real exchange rates, as we will show in this paper, this indicates new evidence of world economic stabilization toward PPP.

The null hypothesis to be investigated formally in this paper is that there has been no significant decline in the persistence of deviations from PPP over the past three decades for industrial countries. This hypothesis is tested against the alternative that there has indeed been a marked and sustained decline in the persistence of PPP deviations. To this end, we employ a fractional integration framework, which provides a powerful tool to detect changes in the persistence for highly persistent time series, here real exchange rates. In the fractional integration framework, our null hypothesis is formu-

¹ AREAER reports a set of *de jure* measures of legal restrictions on cross-border capital flows, and is widely used to measure financial openness.

² See Kose et al. (2006) for details of financial integration and related measures.

³ See Taylor (2000) and Cogley and Sargent (2001, 2005) for other studies which find a similar decline in US inflation persistence.

lated as no change in the order of fractional integration, d , and alternative as a decline in d . This paper conducts two analyses to examine this hypothesis for major industrial countries using US dollar-based real exchange rates.

First, we conduct a formal statistical test of the null of no change in d using two evenly divided subsamples. In this analysis, we do not try to specify the correct timing nor transition process of possible declines in PPP deviation persistence, since it is almost a formidable task. Rather, we simply use two subsamples of the data, and test the difference in d between the two subsamples. This may not be the most powerful way to detect a decline of persistence, since it does not specify the possible timing and type of structural changes. However, if there has been a significant decline, the test should detect it. In fact, this is the case. The tests of the null hypothesis $d_1 = d_2$ against the alternative $d_1 > d_2$, where d_1 and d_2 are orders of integration of real exchange rates for the first and second subsample, rejects the null for nine out of 17 countries at the 10% significance level. This result provides solid evidence for a decline in the persistence of deviations from PPP in recent years. We, however, fail to find the mean-reverting behavior in real exchange rates even in the latter period for all 17 countries. Thus, our analysis detects some changes in the characteristics of real exchange rates toward PPP, but they are not sufficient for PPP.

Second, we employ a 15-year rolling-window estimation to examine the dynamics of persistence of real exchange rates. This rolling-window estimation is simple, but can still provide very useful information regarding the timing and transition process of declines in the persistence of PPP deviations. Our 15-year rolling window estimates indicate that many real exchange rates have experienced a sharp drop in persistence once we begin to use samples starting from the mid-1980s. Interestingly, this timing almost coincides with the timing of US/world economic stabilization reported by other studies including Kim and Nelson (1999) and Kumar and Okimoto (2007). We also examine the relationship between the persistence of PPP deviations and several economic variables. Our analysis demonstrates that convergence speed of PPP deviations is highly related to two financial integration measures by Lane and Milesi-Ferretti (2006), trade openness, and inflation, but not to productivity growth, providing new evidence of economic/financial integration and world economic stabilization toward PPP.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 discusses the methodology we use to obtain estimates for order of fractional integration, or a measure of persistence. Section 4 presents our main estimation results and their implications. Section 5 concludes.

2. Review of the related literature

In this section, we review related research, and clarify our contribution. More comprehensive review can be found in Sarno and Taylor (2002).

Most empirical studies employ unit root tests or cointegration analysis, and fail to find evidence in favor of PPP; see Corbae and Ouliaris (1988), Enders (1988), Meese and Rogoff (1988), Mark (1990), Patel (1990), and Edison and Pauls (1993), among many others. Notwithstanding, many researchers consider that these negative results reflect poor performance of their econometric methodologies rather than evidence against PPP. In particular, the low power of unit root and cointegration tests has been often pointed out; for example, see Hakkio (1986).

To overcome this problem, several approaches have been developed. The first approach uses a more stable PPP relationship over a longer time horizon to find stronger evidence for PPP. Those examples include Abuaf and Jorion (1990), Kim (1990), Ardeni and Lubian (1991), Glen (1992), and Lothian and Taylor (1996). As indicated by Engel (2000), however, using longer-span data may not completely solve the problems associated with testing PPP. In addition, it is questionable whether the exact same PPP relationship holds in such a long period. Even if PPP remains true for the entire period, the convergence speed of PPP deviations can be very different under different exchange rate regimes, such as the Bretton Woods era and the flexible exchange rate period.

An alternative approach employs panel unit root tests to improve the power of standard unit root tests. Along this vein, studies such as Wei and Parsley (1995), Oh (1996), Wu (1996), and Papell (1997)

apply panel unit root tests to real exchange rate data of several countries in the flexible exchange rate period, and find evidence in favor of PPP. One concern with these panel studies is their ignorance of cross-sectional dependence, as emphasized by O'Connell (1998). Another concern is their use of the null of joint nonstationarity. As indicated by Taylor and Sarno (1998), it is possible that joint nonstationarity of a group of real exchange rates may be rejected when only one of these series is stationary.

Another approach that has been considered is the fractional integration approach, which extends the standard unit root framework. Diebold et al. (1991) and Cheung and Lai (1993) find evidence of long-memory, but mean reversion, in long historical series of real exchange rates, while Cheung and Lai (2001) and Achy (2003) find similar results in the recent floating rate period. On the other hand, Baum et al. (1999) fail to reject the unit root hypothesis against the fractional integration alternative for the post-Bretton Woods era.

In sum, there is growing evidence supporting PPP, but the evidence is not sufficient to conclude that PPP holds. In particular, none of above studies considers the possibility of a movement toward PPP in recent periods, even though there are several reasons to expect such a trend, as emphasized in the introduction. It is, therefore, worth investigating whether we can find empirical evidence for this possibility, which is the main purpose of this paper. To this end, we semiparametrically estimate the order of fractional integration in real exchange rate, and use it to measure the persistence of real exchange rate. This framework has several advantages over the standard unit root tests or ARFIMA framework, as discussed in detail in the next section.

3. Methodology

In this paper, we propose to use fractionally integrated ($I(d)$) processes to assess the persistence of real exchange rate. Offering a generalization of the classical dichotomy between $I(0)$ and $I(1)$ processes, fractionally integrated processes can provide a more powerful framework to detect mean reversion than the standard unit root tests. In this section, we discuss our measure of persistence and its estimation methodology.

3.1. Fractionally integrated processes and measures of persistence

A process X_t is said to be an $I(d)$ process if its fractional difference, $(1 - L)^d X_t$, is an $I(0)$ process. The fractional difference operator $(1 - L)^d$ is defined by means of the gamma function

$$(1 - L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k - d)L^k}{\Gamma(-d)\Gamma(k + 1)},$$

where the parameter d is allowed to take any real value. When d is a nonnegative integer, the infinite-order summation terminates, giving the standard integrated processes. An $I(d)$ process is stationary and invertible when $-\frac{1}{2} < d < \frac{1}{2}$. An $I(d)$ process with $d \geq 1/2$ is nonstationary, but is still mean reverting if $1/2 \leq d < 1$. Importantly, an $I(d)$ process with $0 < d < 1$ can accommodate slowly decaying autocorrelations (when stationary) and slowly decaying impulse response function that are inconsistent with either an $I(0)$ or an $I(1)$ process.

The long-run dynamics of an $I(d)$ process is governed by the parameter d , which is our measure of persistence. Using the value of d as a measure of persistence has several attractive features for the purpose of this paper. First, $I(d)$ processes allow us to compare persistence of highly persistent series more powerfully as discussed above. Second, the integration parameter d has little to do with the short-run dynamics of the data. The largest autoregressive root, which is commonly used as a measure of long-run dynamics, is intimately related with the first-order autocorrelation of the data. As such, it is affected by both short and long-run dynamics. Third, unlike the local-to-unity parameter in the local-to-unity model, d can be estimated consistently from the data. See Kumar and Okimoto (2007) for further discussion on the attraction of using the value of d as a measure of persistence.

3.2. Estimation of order of fractional integration

We use the two-step exact local Whittle (2ELW) estimator by Shimotsu (2010) that extends the exact local Whittle (ELW) estimator by Shimotsu and Phillips (2005). The 2ELW estimator is a semiparametric estimator, which is robust to misspecification of the short-run dynamics of the process. This feature is attractive for the paper, because our interest is in the long-run dynamics of real exchange rate, and we want to impose as little assumptions as possible on the short-run dynamics. Another useful feature of the 2ELW estimator is that it accommodates both stationary ($d < 1/2$) and nonstationary ($d \geq 1/2$) fractionally integrated processes. We do not want to impose *a priori* restrictions on whether $d \geq 1/2$, because the theory of PPP itself implies no restriction on the stationarity of the real exchange rate.

The ELW estimator assumes that the fractionally integrated process X_t is generated by the model

$$(1-L)^d X_t = u_t \mathbf{1}\{t \geq 1\}, \quad t = 0, \pm 1, \dots, \quad (3.1)$$

where $\mathbf{1}\{\cdot\}$ denotes the indicator function. u_t is a mean-zero $I(0)$ process with spectral density $f_u(\lambda)$ satisfying $f_u(\lambda) \sim G$ for $\lambda \sim 0$.

Shimotsu and Phillips (2005) define the ELW estimator of d as

$$\hat{d} = \arg \min_{d \in [\Delta_1, \Delta_2]} R(d), \quad (3.2)$$

where $R(d) = \log \hat{G}(d) - 2dm^{-1} \sum_{j=1}^m \log \hat{\lambda}_j$, $\hat{G}(d) = m^{-1} \sum_{j=1}^m I_{(1-L)^d X}(\lambda_j)$, and $I_{(1-L)^d X}(\lambda_j)$ is the periodogram of $(1-L)^d X_t$ evaluated at $\lambda_j = 2\pi j/n$. In what follows, we distinguish the true value of d by d_0 . Shimotsu and Phillips (2005) show that, under some conditions including $d_0 \in (\Delta_1, \Delta_2)$ with $\Delta_2 - \Delta_1 \leq \frac{9}{2}$,

$$m^{1/2}(\hat{d} - d_0) \rightarrow_d N\left(0, \frac{1}{4}\right), \quad \text{as } n \rightarrow \infty,$$

where m is chosen so that $1/m + m^{1+2\beta}(\log m)^2 n^{-2\beta} + m^{-\gamma} \log n \rightarrow 0$ for any $\gamma > 0$. Here β represents the degree of approximation of the spectral density of u_t around the origin by G .

Shimotsu (2010) develops the two-step ELW (2ELW) estimator that extends the ELW estimator to accommodate an unknown mean, so that the model that generates the data is

$$X_t = (1-L)^{-d} u_t \mathbf{1}\{t \geq 1\} + \mu_0, \quad t = 0, \pm 1, \dots \quad (3.3)$$

The 2ELW estimator estimates the unknown mean, μ_0 , by a weighted average of the sample mean and the initial observation. Shimotsu (2010) shows that the 2ELW estimator has the same asymptotic distribution as the ELW estimator.

The value of m is chosen by the researcher. The choice involves a bias-variance tradeoff; using a too small m increases the variance of the estimator, while using a too large m induces bias in estimation because of the effect from short-run dynamics. The value of β is known to be two for many probable models of u_t . Hence, the largest possible choice of m is slightly smaller than $n^{4/5}$. In practice, more conservative choices such as $n^{0.65}$ or $n^{0.75}$ are often used.

Several studies on world economic stabilization, such as Stock and Watson (2002) and Sensier and van Dijk (2004), find evidence of heteroskedasticity in many economic series. Robinson and Henry (1999) show that conditional heteroskedasticity in u_t does not affect the asymptotic distribution of the local Whittle estimator (Robinson, 1995), a related semiparametric estimator. In light of this result, we conjecture that the asymptotic distribution of the ELW estimator is not affected by conditional heteroskedasticity.

4. Empirical analysis

We use monthly US dollar-based real exchange rates for 17 industrial countries with the sample period from January 1974 to December 2006. We set the beginning of the sample period to be the first year following the shift to the current floating exchange rate regime. The data are collected from IMF's International Financial Statistics (IFS). We use the CPI (IFS line 64) as the measure of prices, and the

Table 1Estimates of d : $m = 396^{0.65} \approx 48$.

Country	2ELW	95% CI	Z_t	Half-life	LW
Austria	1.042	[0.900, 1.183]	−1.842	[529, ∞]	1.039
Belgium	1.052	[0.910, 1.193]	−2.005	[1175, ∞]	1.050
Canada	0.981	[0.840, 1.123]	−1.390	[37, ∞]	0.977
Denmark	1.034	[0.893, 1.176]	−1.830	[330, ∞]	1.031
Finland	1.017	[0.875, 1.158]	−2.354	[129, ∞]	1.016
France	1.075	[0.933, 1.216]	−2.112	[17434, ∞]	1.072
Germany	1.038	[0.896, 1.179]	−1.852	[412, ∞]	1.033
Greece	0.985	[0.844, 1.127]	−1.076	[41, ∞]	0.977
Italy	1.022	[0.880, 1.163]	−1.806	[164, ∞]	1.019
Japan	0.999	[0.858, 1.141]	−1.916	[65, ∞]	0.988
Netherlands	1.030	[0.888, 1.171]	−2.038	[249, ∞]	1.028
Norway	0.967	[0.825, 1.108]	−2.111	[25, ∞]	0.965
Portugal	0.977	[0.835, 1.118]	−1.237	[33, ∞]	0.976
Spain	1.087	[0.945, 1.228]	−1.685	[168,917, ∞]	1.085
Sweden	1.033	[0.891, 1.174]	−2.128	[301, ∞]	1.030
Switzerland	0.991	[0.850, 1.133]	−2.184	[49, ∞]	0.985
United Kingdom	0.916	[0.774, 1.057]	−1.587	[10, ∞]	0.925

Note: The sample period is from 1974:1 to 2006:12. 2ELW is the two-step ELW estimate (Shimotsu, 2010). Phillips-Perron Z_t -statistic is computed using 10 lags. The 5% and 10% critical values of the Z_t -statistic is -2.874 and -2.570 , respectively. LW is one plus the local Whittle estimate (Robinson, 1995) from the differenced data.

end-of-period domestic currency units per US dollar (IFS line ae) as the exchange rate. We follow Papell (1997) in selecting countries whose exchange rate is examined. These countries consists of those classified as industrial by the IMF not including Australia, Iceland, Ireland, Luxembourg, and New Zealand. We exclude Luxembourg because it maintained a currency union with Belgium. Australia, Iceland, Ireland, and New Zealand do not have monthly CPI data for the entire sample period. For Euro-countries, their exchange rate after 1997 is calculated from the U.S. dollar-Euro exchange rate and the conversion rate between Euro and each national currency.⁴

We hypothesize that there has been no significant decline in the persistence of these real exchange rates, or deviations from PPP, over the past three decades. To examine the hypothesis, we use two methods; the first compares two equally divided subsamples, and the second is a 15-year rolling-window estimation. While the former provides us a way to test the hypothesis statistically, the later allows us to investigate the dynamics of the persistence of PPP deviations more informatively. In what follows, we first present the estimates of d from the whole sample to justify the use of the fractional integration framework, and then discuss the outcome of the two analyses.

4.1. Whole sample analysis

For the first analysis, we report the estimates of the orders of fractional integration for real exchange rates, or deviations from PPP, for 17 industrial countries using the whole sample. Throughout this subsection, we do not consider the possibility of changes in the persistence of real exchange rates. This is because we want to confirm that the order of fractional integration is a suitable measure of persistence before conducting formal tests of declines in the persistence of PPP deviations. The results from the whole sample analysis support the nonstationarity of PPP deviations, and give us a solid reason to use the fractional integration framework to detect declines in their persistence.

The second column of Table 1 reports the 2ELW estimates of the orders of fractional integration for real exchange rates. We set the bandwidth to $m \approx n^{0.65}$, namely $m = 48$ for this analysis and $m = 31$ for the subsample analysis, respectively. The asymptotic standard error of each estimate is $1/\sqrt{4 \times 48} = 0.072$, and the asymptotic 95% confidence interval is shown in the third column of Ta-

⁴ Our empirical results are not affected significantly by this use of US dollar-Euro exchange rate after 1997, since the results of ten Euro-countries are not necessarily similar as we will show below.

ble 1. As can be seen, all estimates are close to one. From the 95% confidence interval, we reject the stationarity hypothesis, i.e., $d < 1/2$, at the 5% significance level. The p -value for the tests of the hypothesis $d < 1/2$ (not reported here) is smaller than 0.1%, providing strong evidence of nonstationarity of PPP deviations for all countries. The fourth column of Table 1 reports the Phillips–Perron Z_t -statistic for the null hypothesis that each real exchange rate has a unit root. The lag length is chosen to be 10. The 5% and 10% critical values of the Z_t -statistic is -2.874 and -2.570 , respectively. Corroborating most previous studies, we cannot reject the null of unit root for any of the real exchange rate series at the usual significance level, indicating the nonstationarity of PPP deviations.

The fifth column of Table 1 reports the 95% confidence interval of the half-life of deviations from PPP. These intervals are computed from the 95% confidence interval of d using the relation

$$\frac{\partial X_{t+k}}{\partial u_t} \sim \frac{k^{d-1}}{\Gamma(d)} \quad \text{as } k \rightarrow \infty.$$

Since all the 95% upper bounds of d are larger than one, the 95% upper bound of the PPP deviation half-life is infinity for all countries. This finding is consistent with the conclusion from the previous studies such as Murray and Papell (2002) and Rossi (2005): the data are not sufficiently informative to pin down the half-life. The lower bound of the half-life is larger than the typical estimates based on Dickey–Fuller type regressions (Murray and Papell, 2002; Rossi, 2005). This is due to the shape of the impulse response function of fractionally integrated models. The impulse response function of the autoregressive model has an exponential decay, whereas that of the fractionally integrated model has a geometric decay. Consequently, fractionally integrated models produce larger half-life estimates, in particular when it involves long-run dynamics.

The half-life is not an informative measure to investigate changes in persistence, since an unbounded confidence interval does not allow us to conduct formal hypothesis tests of changes in persistence. To the contrary, the confidence intervals of the order of fractional integration are sufficiently tight, and we can use d as a measure of persistence to test the null hypothesis of no decline in the persistence of PPP deviations.

We also estimate d using the local Whittle estimator (Robinson, 1995) to check the robustness of our results. Note that the differenced series of an $I(d)$ process is $I(d-1)$. These estimates are calculated as follows. First, we take the difference of a real exchange rate series. Then, we estimate the order of integration of the differenced series by the local Whittle estimator. Finally, we add one to the estimate to get the estimate of d of the original series. Since the local Whittle estimator has a normal asymptotic distribution only when $-1/2 < d < 3/4$, this procedure implicitly assumes $d-1$ is larger than $-1/2$, namely $d > 1/2$. The last column of Table 1 reports the estimates. Not surprisingly, the estimates are very close to the 2ELW estimates based on the original series.

To sum, the results of the whole sample analysis clearly indicate the nonstationarity of real exchange rates and the usefulness of fractional integration framework to detect possible declines in the persistence of PPP deviations toward PPP.

4.2. Results of subsample analysis

In this section, we conduct formal statistical tests using two equally divided subsamples. The first subsample starts from January 1974, and ends in June 1990, while the second subsample is from July 1990 to December 2006. In this analysis, we do not pursue identifying the probable timing, nor the type of declines in the persistence of PPP deviations. Ideally, we can increase the power of the tests by correctly specifying the timing and type of the transition process. However, it is very difficult to identify the type of structural changes, such as instantaneous breaks or gradual changes, and using a misspecified model may lead to erroneous conclusions. Therefore, we simply use two equally lengthed subsamples, suggesting our tests are conservative in the sense that they may not detect declines in PPP deviation persistence most powerfully. If we can reject the null of no decline with these conservative tests, this constitutes strong evidence for a decline in the persistence of real exchange rates toward PPP.

To conduct a formal test, we need to derive the joint distribution of the two estimates of the integration parameter from the two subsamples. This can be done as follows. Suppose X_t , $t = 1, \dots, n_1$ is generated by model (3.3) with the integration parameter d_1 , and suppose X_t , $t = n_2, \dots, n$ with $n_1 < n_2$, is generated by model (3.3) with the integration parameter d_2 . Let $\hat{d}_1$ be the 2ELW estimator of d from X_t , $t = 1, \dots, n_1$ with the bandwidth parameter m_1 , and define $\hat{d}_2$ analogously using X_t , $t = n_2, \dots, n$ and m_2 . Then, we have

$$\begin{pmatrix} m_1^{1/2}(\hat{d}_1 - d_1) \\ m_2^{1/2}(\hat{d}_2 - d_2) \end{pmatrix} \rightarrow_d N\left(0, \begin{bmatrix} 1/4 & 0 \\ 0 & 1/4 \end{bmatrix}\right), \quad \text{as } n \rightarrow \infty. \quad (4.1)$$

A sketch of the proof can be found in the Appendix. Thus, the two estimates of the integration parameter from the two subsamples are asymptotically independent. Based on this result, we can formally test the hypothesis that there has been no significant decline in the persistence of the deviations from PPP over the past three decades. This amounts to testing the null hypothesis of $d_1 = d_2$ against the alternative hypothesis of $d_1 > d_2$, where d_1 and d_2 are orders of fractional integration of the first and second subsample, respectively.

The second and third columns of Table 2 report the 2ELW estimates of the orders of fractional integration (or persistence parameter values) of the US dollar-based real exchange rate for each country and subsample. The asymptotic standard error of each estimate is $1/\sqrt{4 \times 31} = 0.090$. As can be seen, all the estimates of d_1 are >1 , indicating highly persistent behavior of deviations from PPP. On the other hand, all the estimates from the second subsample are smaller than those from the first subsample except for Portugal. Furthermore, many of the estimates are <1 , although they are not significantly different from 1.

These results suggest that deviations from PPP are less persistent in the second subsample. To examine this point more rigorously, we test the null hypothesis of $d_1 = d_2$ against the alternative hypothesis of $d_1 > d_2$. The last two columns of Table 2 show the difference between two estimates and the p -values of the test using the asymptotic distribution (4.1). The null hypothesis is rejected at the 5% significance level for France, Japan, and Spain, and at the 10% level for Austria, Belgium, Denmark, Italy, Netherlands, and Switzerland. Thus, the equality of the persistence of PPP deviations between two subsamples are rejected at the 10% significance level for nine out of 17 countries. For Canada, Germany, Norway, and Sweden, the results are only marginally insignificant with less than 20% p -values. For Finland, Greece, Norway, and the United Kingdom, the estimates of d for the first

Table 2

Estimates of d from two subsamples.

Country	d_1	d_2	$d_1 - d_2$	p -value (%)
Austria	1.138	0.969	0.169	9.2
Belgium	1.153	0.986	0.167	9.5
Canada	1.134	1.007	0.127	16.0
Denmark	1.186	0.979	0.207	5.1
Finland	1.087	1.068	0.019	44.2
France	1.210	0.999	0.211	4.9
Germany	1.131	0.974	0.157	10.8
Greece	1.004	0.987	0.016	44.9
Italy	1.162	0.983	0.178	8.0
Japan	1.228	0.984	0.243	2.8
Netherlands	1.150	0.959	0.192	6.6
Norway	1.052	0.933	0.119	17.5
Portugal	1.010	1.026	-0.016	54.9
Spain	1.225	0.955	0.270	1.7
Sweden	1.169	1.030	0.139	13.7
Switzerland	1.109	0.919	0.190	6.8
United Kingdom	1.069	0.986	0.083	25.6

Note: d_1 is the 2ELW estimate of d from the subsample 1974:1 to 1990:6 and d_2 is the 2ELW estimate of d from the subsample 1990:7 to 2006:12. The p -value is for the test of the null of $d_1 = d_2$ against the alternative of $d_1 > d_2$.

Table 3Robustness check for estimates of $d_1 - d_2$ with respect to bandwidth and subsamples.

First period	1974:1–1990:6		1976:1–1991:6		1974:1–1984:12		1974:1–1988:12	
Second period	1990:7–2006:12		1991:7–2006:12		1988:1–2006:12		1992:1–2006:12	
Bandwidth	$m = n^{0.75}$		$m = n^{0.65}$		$m = n^{0.65}$		$m = n^{0.65}$	
Country	$d_1 - d_2$	p -value (%)	$d_1 - d_2$	p -value (%)	$d_1 - d_2$	p -value (%)	$d_1 - d_2$	p -value (%)
Austria	0.152	6.1	0.200	6.4	0.148	12.9	0.193	7.1
Belgium	0.146	6.9	0.166	10.3	0.197	6.6	0.172	9.6
Canada	−0.007	52.7	0.080	27.3	0.018	44.5	0.184	8.0
Denmark	0.155	5.7	0.219	4.8	0.179	8.7	0.224	4.4
Finland	0.039	34.7	0.055	33.7	0.047	36.0	0.028	41.7
France	0.141	7.6	0.220	4.7	0.191	7.3	0.235	3.7
Germany	0.160	5.2	0.177	8.9	0.137	14.9	0.188	7.7
Greece	0.036	35.8	0.016	45.1	−0.006	51.9	0.023	43.1
Italy	0.102	15.0	0.190	7.4	0.162	10.8	0.179	8.7
Japan	0.048	31.3	0.260	2.4	0.218	4.8	0.298	1.2
Netherlands	0.171	4.1	0.182	8.3	0.157	11.6	0.205	5.9
Norway	0.140	7.6	0.149	12.8	0.160	11.2	0.119	18.2
Portugal	0.017	43.1	0.012	46.3	0.013	46.1	0.019	44.1
Spain	0.130	9.3	0.269	2.0	0.208	5.7	0.252	2.8
Sweden	0.112	12.7	0.158	11.4	0.132	15.7	0.096	23.3
Switzerland	0.203	1.9	0.203	6.1	0.206	5.9	0.212	5.4
United Kingdom	0.067	24.7	0.145	13.5	0.191	7.3	0.062	32.0

Note: d_1 is the 2ELW estimate of d from the first period, and d_2 is the 2ELW estimate of d from the second period. The p -value is for the test of the null of $d_1 = d_2$ against the alternative of $d_1 > d_2$.

subsample are relatively low, which is the main reason why the test cannot reject the null of no decline in PPP deviation persistence. These results provide solid evidence for the decline in the persistence of the deviations from PPP, suggesting that the behavior of real exchange rates has become more consistent with the PPP hypothesis in more recent years.

Note, however, that our estimates of d_2 are not significantly different from 1 for all 17 countries. Since an $I(d)$ time series is not mean-reverting if $d \geq 1$, there is no indication of PPP even in the less persistent subsample. Thus, our results show that the decline in PPP deviation persistence is sufficiently large to reject the null of $d_1 = d_2$ for many countries, but not large enough to reject the null of $d_2 = 1$, providing no supportive evidence for PPP even in recent years.⁵

We also examine the robustness of the results in Table 2 with respect to the bandwidth and sample period used for the subsample analysis. The second and third columns of Table 3 report the difference of the estimates, $d_1 - d_2$, and the asymptotic p -value of the test of the null hypothesis of $d_1 = d_2$ against the alternative of $d_1 > d_2$ for $m = n^{0.75} \approx 52$. In general, the results are very similar as those for $m = n^{0.65}$, in particular 13 out of 17 countries share the same significant/insignificant results. The results for Germany and Norway are significant instead of insignificant, while the results for Italy and Japan are insignificant. As a consequence, the results for $m = n^{0.65}$ and $m = n^{0.75}$ have the same nine significant results.

In Table 2, we split the sample in the middle, each sample (1974:1–1990:6 and 1990:7–2006:12) having 198 observations. We also examine how the results in Table 2 are affected by the sample period used in the analysis. Considering the fact that most countries experienced high inflation around 1974, we change the beginning of sample to 1976, and report estimates of $d_1 - d_2$ and their associated p -values in the fifth and sixth columns of Table 3. In addition, since the US dollar depreciated dramatically between 1985 and 1987 due to the Plaza Accord, the results with excluding this period are documented in the seventh and eighth columns of Table 3. Lastly, we provide the results using the first

⁵ This could be because of our ignorance of the nonlinear behavior in real exchange rates. As many studies suggest, the existence of transaction costs including transportation cost and trade barriers implies nonlinear real exchange rate adjustment toward PPP. Once this nonlinearity is considered, we may find mean-reverting behavior in PPP deviations, as Michael et al. (1997) and Taylor et al. (2001). Modeling the nonlinearity in semiparametric fractional integration framework is, however, beyond the scope of the present paper.

Fig. 1. Dynamics of PPP deviation persistence for G7 countries. This figure plots the 15-year rolling-window 2ELW estimates of the real exchange rate against the end year of the sample period.

and last 15-year subsamples to get a flavor of the results based on a 15-year rolling-window estimation given in the next subsection. As can be seen, when we use different subsamples, the evidence in favor of the decline in d becomes stronger for some countries and weaker for other countries with 7–11 significant results. Some point estimates in Table 3 are negative, but they are highly insignificant. Overall, the results in Table 3 are similar to Table 2, suggesting the decline in PPP deviation persistence.

In sum, our results provide solid evidence of decline in the persistence of real exchange rates toward PPP. The decline, however, is not sufficient for PPP, meaning we fail to find mean-reverting behavior in real exchange rates for all countries, even if we use the less persistent second subsample.

4.3. Rolling-window estimation

To obtain additional insight, and further support for our empirical findings of declines in the persistence of the deviations from PPP, we apply a 15-year rolling-window estimation to the entire sample. First, we estimate the order of fractional integration, d , or the persistence parameter, using the first 15 years of the data (specifically, from January 1974 to December 1988). The data are then updated by 1 year increments, and d is re-estimated for the updated window (that is, for the period from January 1975 to December 1989). This procedure is repeated until the end of the sample period. Thus, the last estimate of d is based on the period from January 1992 to December 2006. The rolling-window estimation is easy to implement, and provides a significant amount of information about the underlying dynamics of the persistence of PPP deviations. In particular, this analysis can help highlight the periods over which there would likely have been a pronounced decline in the persistence of PPP deviations. Further, it gives useful observations about whether an instantaneous break, or a gradual change, better describes the transition process of d .

Fig. 2. Dynamics of PPP deviation persistence for non-G7 countries with a significant decline. This figure plots the 15-year rolling-window 2ELW estimates of the real exchange rate against the end year of the sample period.

Fig. 1 depicts the 15-year rolling-window estimates of the persistence parameter of the real exchange rate, along with the end year of the sample period, for the G7 countries.⁶ The figure shows remarkable similarities among the dynamics of the persistence of the G7 real exchange rates. For the first decade ending in 1998, the persistence of each real exchange rate decreased only slightly. Then, all the countries experienced a rapid decline in the persistence of PPP deviations between 1999 and 2002. Note that Fig. 1 is drawn against the end year of estimated samples. In other words, the persistence of PPP deviations for the G7 countries declined notably once we start using samples starting mid-1980s. Interestingly, this period roughly coincides with previous studies' findings on the timing of a possible structural change toward stability in the US/world economy, such as Kim and Nelson (1999) and Kumar and Okimoto (2007). The persistence estimates for the G7 countries rebounded a little in 2003, and after that remained almost unchanged until 2006.

Fig. 2 plots the 15-year rolling-window estimates for non-G7 countries, which have a significantly different PPP deviation persistence between two subsamples. The results are quite striking; all graphs behave practically same. In addition, they share analogous patterns with the G7 countries. In particular, all countries underwent sharp declines in PPP deviation persistence between 1999 and 2002.

Fig. 3 shows the 15-year rolling-window estimates for non-G7 countries whose differences in PPP deviation persistence between two subsamples are not significant. Despite the fact that the changes in the persistence of PPP deviations are less remarkable in these countries, their dynamics are still similar to those observed in Figs. 1 and 2. In particular, the commensurate decline in PPP deviation persistence between 1999 and 2002 can also be seen in Fig. 3, although the magnitude is not as striking as that of Figs. 1 and 2.

⁶ Note that the asymptotic standard error of each estimate is $1/\sqrt{4 \times 29} = 0.093$.

Fig. 3. Dynamics of PPP deviation persistence for non-G7 countries with an insignificant decline. This figure plots the 15-year rolling-window 2ELW estimates of the real exchange rate against the end year of the sample period.

4.4. Relationship between PPP deviation persistence and economic variables

The above-mentioned empirical findings raise the obvious questions regarding the factors behind the decline in the persistence of PPP deviations. In this subsection, we investigate the relationship between the estimated PPP deviation persistence and several economic variables, including inflation, and the financial integration measures recently constructed by Lane and Milesi-Ferretti (2006).

A number of possible factors have contributed to the decline in the persistence of PPP deviations: the increase of world economic relations, competition and globalization, development of the world transportation system, reduction of trade barriers, evolution in information technology, and the improvement of monetary policy design and implementation. Along this vein, Cheung and Lai (2000) examine four possible determinants of PPP deviation persistence: inflation experience, productivity growth, trade openness, and government spending. They find that inflation and government spending exert significant effects on PPP deviation persistence, while productivity growth and trade openness are not significant determinants for the persistence of real exchange rates. More importantly, they find that a considerable portion of the variations in PPP deviation persistence cannot be explained by these four factors. Thus, which factors are the most important determinants remains an important open question.

Although fully identifying the factors responsible for a decline in PPP deviation persistence is beyond the scope of this paper, it is very informative to examine the relationship between several economic variables and the persistence of deviations from PPP based on our empirical results. To this end, we calculate the correlation between the estimated PPP deviation persistence and financial integration, inflation, productivity growth, and trade openness.

For financial integration, we employ two quantitative measures recently constructed by Lane and Milesi-Ferretti (2006), and recommended by Kose et al. (2006). The first measure, IFIDGP, is the ratio of the sum of gross stocks of foreign assets and liabilities to the GDP:

Table 4

Correlation between financial integration measures and rolling-window estimates.

Country	IFIGDP	GEQGD	Inflation	Productivity growth	Trade openness
Austria	−0.882	−0.881	0.339	0.378	−0.870
Belgium	−0.854	−0.876	0.276	0.282	−0.881
Canada	−0.794	−0.737	0.440	−0.160	−0.834
Denmark	−0.869	−0.918	0.427	−0.057	−0.890
Finland	−0.519	−0.405	0.005	0.067	−0.459
France	−0.886	−0.841	0.444	0.185	−0.904
Germany	−0.871	−0.855	0.498	0.526	−0.844
Greece	−0.606	−0.673	0.841	−0.445	−0.688
Italy	−0.838	−0.821	0.653	0.277	−0.748
Japan	−0.470	−0.815	0.791	0.383	−0.898
Netherlands	−0.875	−0.870	−0.103	0.524	−0.881
Norway	−0.835	−0.855	0.540	0.057	−0.755
Portugal	−0.587	−0.559	0.110	0.277	−0.461
Spain	−0.953	−0.944	0.727	−0.082	−0.958
Sweden	−0.862	−0.819	0.789	−0.486	−0.883
Switzerland	−0.837	−0.834	0.596	0.007	−0.882
United Kingdom	−0.882	−0.917	0.607	−0.271	−0.785
Average	−0.789	−0.801	0.469	0.086	−0.801

Note: IFIGDP and GEQGD are quantitative measures of financial integration constructed by Lane and Milesi-Ferretti (2006). Their definition is provided in (4.2) and (4.3), respectively.

$$\text{IFIGDP} = \frac{\text{FA} + \text{FL}}{\text{GDP}}, \quad (4.2)$$

where FA (FL) denotes the stock of external assets (liabilities). The second measure, GEQGD, focuses exclusively on portfolio equity and FDI holdings:

$$\text{GEQGD} = \frac{\text{PEQA} + \text{FDIA} + \text{PEQL} + \text{FDIL}}{\text{GDP}}, \quad (4.3)$$

where PEQA (PEQL) denotes the stock of portfolio equity assets (liabilities) and FDIA (FDIL) denotes the stock of direct investment assets (liabilities). According to these measures, financial integration in industrial countries was promoted gradually in the 1970s and 1980s, and accelerated in the mid-1990s, as reported by Lane and Milesi-Ferretti (2006).

The remaining three variables are adopted from Cheung and Lai (2000) and are calculated as follows. Inflation is the annual CPI-based inflation, while productivity growth is the annual growth rate of the per capita real GDP. To indicate trade openness, we use total trade expressed as a percentage of the GDP, in other words, exports plus imports divided by the GDP and multiplied by 100.

Table 4 reports the correlation between the five variables and the rolling-window estimates for each country based on the sample period from 1986 to 2006.⁷ As can be seen, PPP deviation persistence is clearly negatively correlated with financial integration and trade openness, and the correlation coefficient is smaller than −0.5, in most cases, and about −0.8 on average. Although correlation does not necessarily imply causation, this result indicates that a decline in the persistence of PPP deviations occurred concurrently with the increase in financial integration and trade openness. This is relevant because the basic idea underlying the PPP is that goods market arbitrage can induce parity in prices. Accordingly, PPP deviations are corrected over time through adjustments in financial and trade flows.

Table 4 also shows a considerable positive correlation between inflation and persistence in PPP deviations. The correlation is estimated to be positive for all countries, except for the Netherlands, at on average of 0.47. Since controlling inflation is one of the most important elements for economic stabilization, as emphasized by Clarida et al. (2000), the result suggests a strong relationship between economic stabilization and the speed of convergence of PPP deviations.

⁷ Both IFIGDP and GEQGD are available only up to 2004.

Lastly, correlation between productivity growth and PPP deviation persistence is provided in the fifth column of Table 4. As can be seen, there is little tendency in correlation with 11 positive and six negative results of 0.09 on average. Thus, our analysis reveals that high productivity growth does not necessarily accompany faster convergence of PPP deviations. We also construct alternate productivity growth measure using labor productivity data from OECD.stat and examine its correlation with PPP deviation persistence. The results are similar (12 positive and five negative results of 0.15 on average), and the overall conclusion remains unchanged.

In summary, we find that our estimates of PPP deviation persistence are significantly negatively correlated with financial integration and trade openness, significantly positively correlated with inflation, but insignificantly correlated with productivity growth. In other words, our correlation analysis demonstrates a strong relationship between PPP deviation persistence and economic/financial integration and world economic stabilization, but no relationship with productivity growth. Some of these results are not consistent with the findings of Cheung and Lai (2000), who find the significant negative correlation between their PPP deviation persistence measure and inflation, but insignificant correlation between PPP deviation persistence and trade openness. We are not able to ascertain, however, if our results compare with theirs for the following reasons. First, Cheung and Lai (2000) investigate persistence of real exchange rates in both industrial and developing countries and find more, rather than less, parity reversion for developing countries than for industrial countries. Our analysis focuses on industrial countries. Second, Cheung and Lai (2000) compute half-lives for many developing countries using models with a linear trend, a mean shift, or a linear trend with a break, whereas we analyze persistence of real exchange rates around a constant mean. Therefore, it is premature to conclude that our results contradict those of Cheung and Lai (2000). Further detailed analysis and comparison would be interesting but is left for future research.

5. Conclusions

Purchasing power parity (PPP) is one of the most important, and empirically controversial, theories in international macroeconomics. A number of empirical studies regarding the PPP hypothesis have reached diverse results and been unable to find decisive evidence, in particular for the recent floating rate period.

In this paper, we examined the PPP hypothesis from a different point of view than previous studies to provide new evidence supporting PPP. Specifically, this paper investigated the possibility of a decline in the persistence of real exchange rates, or deviations from PPP, by testing the null hypothesis of no decline in the persistence of 17 industrial countries' US dollar-based real exchange rates in the last 30 years. To this end, we employed a fractional integration framework, and used the order of fractional integration as a measure of persistence of deviations from PPP. Confirming the appropriateness of our method by whole sample analysis, we conducted formal statistical tests by comparing estimates of the persistence parameter for two subsamples. We found marked and significant declines in PPP deviation persistence for nine out of 17 countries. However, we failed to find mean-reverting behavior in real exchange rates, even in the latter period, for all countries. Thus, we conclude that there have been declines in the persistence of real exchange rates toward PPP, but they are not sufficient for PPP.

To obtain additional insight on declines in the persistence of real exchange rates, we provided the dynamics of PPP deviation persistence by applying a 15-year rolling-window estimation. The results demonstrated remarkable similarities in the dynamics of each real exchange rate's persistence. In particular, most countries experienced a rapid decline in the persistence of PPP deviations once we began to use samples starting from the mid-1980s. Interestingly, this period roughly coincides with previous studies' findings on the timing of a possible structural change toward stability in the US/world economy. We also confirmed that the persistence of PPP deviations are strongly related to two financial integration measures by Lane and Milesi-Ferretti (2006), trade openness and inflation, but not to productivity growth, providing new evidence of economic/financial integration and world economic stabilization toward PPP.

As a final contribution, the paper opens up an interesting econometric issue. If the conclusions of this study are regarded as robust, and we believe they are, investigating the dynamics of PPP deviation persistence more carefully would be a conceivable agenda for further research. Our results strongly suggest that the order of fractional integration is changing over time. Therefore, examining which model can best describe the dynamics could be a fruitful endeavor. Obviously, one-time permanent structural change is one way, while gradual change could be another possibility. Accommodating both models, smooth transition parameter model by [Lin and Teräsvirta \(1994\)](#) may be one attractive approach.

Acknowledgments

We thank two anonymous referees for helpful comments and suggestions. Financial support from SSHRC and the JSPS Postdoctoral Fellowships for Research Abroad are gratefully acknowledged.

Appendix A. sketch of the proof of (4.1)

We show that (4.1) holds for the ELW estimator of [Shimotsu and Phillips \(2005\)](#) when the data are generated by

$$\begin{aligned}(1-L)^{d_1}X_t &= u_t \mathbf{1}\{t \geq 1\}, \quad t = 1, \dots, n_1, \\ (1-L)^{d_2}X_t &= u_t \mathbf{1}\{t \geq 1\}, \quad t = n_2, \dots, n, \quad n_2 > n_1,\end{aligned}$$

i.e., the initial value of the processes is zero. Then, the asymptotic distribution of the 2ELW estimator follows from repeating the argument of [Shimotsu \(2010\)](#).⁸

Let $R_1(d)$ and $R_2(d)$ be the objective function defined analogously to $R(d)$ in (3.2) but using $X_1, \dots, X_{n_1}$ and $X_{n_2}, \dots, X_n$, respectively. It follows from a Taylor expansion

$$m_1^{1/2}(\hat{d}_1 - d_1) = - \left[\frac{\partial^2}{\partial d^2} R_1(\bar{d}) \right]^{-1} m_1^{1/2} \frac{\partial}{\partial d} R_1(d_1), \quad \bar{d} \in [d_1, \hat{d}_1].$$

It follows from [Shimotsu and Phillips \(2005, p. 1916 and p. 1918\)](#), that $(\partial^2 / \partial d^2) R_1(\bar{d}) = 4 + o_p(1)$ and

$$m_1^{1/2} \frac{\partial}{\partial d} R_1(d_1) = \frac{2m_1^{-1/2} \sum_{j=1}^{m_1} v_{1j} [2\pi I_{1\varepsilon}(\lambda_j) - 1] + o_p(1)}{1 + o_p(1)} \rightarrow_d N(0, 4),$$

where $I_{1\varepsilon}(\lambda_j)$ is the periodogram of $\varepsilon_1, \dots, \varepsilon_{n_1}$, and $v_{1j} = \log j - m_1^{-1} \sum_{j=1}^{m_1} \log j$. Therefore,

$$m_1^{1/2}(\hat{d}_1 - d_1) = -\frac{1}{2} m_1^{-1/2} \sum_{j=1}^{m_1} v_{1j} [2\pi I_{1\varepsilon}(\lambda_j) - 1] + o_p(1).$$

Similarly, we obtain

$$m_2^{1/2}(\hat{d}_2 - d_2) = -\frac{1}{2} m_2^{-1/2} \sum_{j=1}^{m_2} v_{2j} [2\pi I_{2\varepsilon}(\lambda_j) - 1] + o_p(1),$$

where $I_{2\varepsilon}(\lambda_j)$ is the periodogram of $\varepsilon_{n_2}, \dots, \varepsilon_n$.

First, consider a special case in which ε_t is iid. Then, $\hat{d}_1$ and $\hat{d}_2$ are asymptotically independent because $\sum_{j=1}^{m_1} v_{1j} [2\pi I_{1\varepsilon}(\lambda_j) - 1]$ and $\sum_{j=1}^{m_2} v_{2j} [2\pi I_{2\varepsilon}(\lambda_j) - 1]$ are independent from the independence between $\varepsilon_1, \dots, \varepsilon_{n_1}$ and $\varepsilon_{n_2}, \dots, \varepsilon_n$. Thus (4.1) follows.

⁸ [Shimotsu \(2010\)](#) shows that the 2ELW estimator accommodates non-zero initial condition, and has the same asymptotic distribution as the ELW estimator. See [Shimotsu \(2010\)](#).

For a general case in which ε_t is a martingale difference sequence, as assumed in Shimotsu and Phillips (2005), a more tedious argument is required. We only provide an outline of the proof. The required result follows if we show

$$\begin{pmatrix} m_1^{-1/2} \sum_{j=1}^{m_1} v_{1j} [2\pi I_{1\varepsilon}(\lambda_j) - 1] \\ m_2^{-1/2} \sum_{j=1}^{m_2} v_{2j} [2\pi I_{2\varepsilon}(\lambda_j) - 1] \end{pmatrix} \rightarrow_d N(0, I_2),$$

where I_2 is a 2×2 identity matrix. As in Robinson (1995, p. 1644), write down the left hand side as

$$\begin{pmatrix} \sum_{t=1}^{n_1} z_{1t} \\ \sum_{t=n_2}^n z_{2t} \end{pmatrix},$$

where z_{1t} and z_{2t} are martingale difference sequences, and defined analogously to z_t in Robinson (1995, p. 1644). Then, applying a martingale CLT to this, as in Robinson (1995, pp. 1644–1647), shows that this converges to $N(0, I_2)$ in distribution. $\square$

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