Ubiquitous Rogue Waves

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This talk is devoted to rogue waves. This novel scientific term was born in nautical mythology, entered the science of ocean waves [1-3] and gradually moved into other fields: optics [4-7], matter waves [8], superfluids [9], capillary waves [10], plasmas and many others [11]. There is a multiplicity of mathematical descriptions of rogue waves. Variety of ideas how the waves gain higher amplitudes than expected is also large. Linear theories exist on the same right as nonlinear ones. The most common approach is based on the nonlinear Schrödinger equation [1]. The basic phenomenon related to this description is Benjamin-Fair or Bespalov-Talanov instability or more generally speaking modulation instability. Peregrine first noticed [12] that such instability can be responsible for a quick increase in the wave amplitude in the ocean. A set of special rational solutions localized both in space and time is one of the models of rogue waves that is well developed by now [13,14]. In optical fibers, the major phenomenon that results in high amplitude pulses is modulation instability and inelastic interaction of solitons [5-7]. The latter is caused by third order dispersion and Raman effect and leads to a multiple amplification of stronger solitons [11,15].

There are at least two fundamental reasons for great interest in generating rogue waves in laboratory conditions. First, this opens possibilities for detailed studies of their properties as well as testing applicability of the mathematical models developed for their descriptions - something unthinkable in the natural conditions. Second, being an essentially nonlinear phenomenon, rogue waves allow us to understand deeply the nature and the dynamics of instabilities in nonlinear systems. One of the main features of the rogue waves until now was their chaotic nature; their appearance in the open ocean is defined by the chaotic nature of the waves in general. Even in optics, their experimental observation is based on detection of random events [4], despite the fact that laboratory conditions would allow us to make experiments more deterministically.

Rogue waves may exist in Bose-Einstein condensates either loaded into a parabolic trap or embedded in an optical lattice [8]. In the latter case, rogue waves can be observed in condensates with positive scattering length. They are immensely enhanced by the lattice. Local atomic density may increase up to tens times. Discrete systems can also support rogue waves [17]. Depending on a particular model, these may have similarities with continuous rogue waves but can also have special features. One of the physical examples is an array of optical waveguides. In this case, the rogue waves can be observed as energy concentration in one of the channels [18]. Advantage of integrable models here is in the possibility to present analytical solutions similar to the case of NLSE.

An important issue in the theory of rogue waves is the possibility for their forecasting and early warning. The latter can be done based on the spectra of a rogue wave that is emerging in generally chaotic field [19]. In optics, spectral measurements are quite common while for the ocean waves this could be a challenge. A good example of application of spectral measurements for water waves in a small scale is the case of capillary waves [10]. Perhaps, some ideas can be transferred from here to the case of ocean waves.

A special challenge is financial rogue waves [20]. This area is significantly less developed than rogue waves in the ocean and in optics. Prediction and early warning for these can save billions and billions of dollars. Thus, concentration of efforts in this direction could be quite rewarding.

References

- [1] A. R. Osborne, Nonlinear ocean waves, Academic Press, 2009.
- [2] P.A.E.M. Janssen J. Phys. Oceanography, **33**, 2001 (2003); C. Kharif and E. Pelinovsky, Eur. J. Mech. B/Fluids **22**, 603, 2003.
- [3] M. Onorato et al. Phys. Fluids **17**, 078101 (2005).
- [4] D. R. Solli et al. Nature **450**, 1054-1058 (2007); D. R. Solli et al., Phys. Rev. Lett. **101**, 233902 (2008).
- [5] J. M. Dudley, G. Genty, F. Dias, B. Kibler, and N. Akhmediev, Optics Express, **17**, 21497-21508 (2009).
- [6] G. Genty et al. Appl. Phys. B **94**, 187-194 (2009).
- [7] G. Genty, C. M. DeSterke, O. Bang, F. Dias, N. Akhmediev, J. M. Dudley, Physics Letters A **373**, 989-996 (2010).
- [8] Yu.V. Bludov, V.V. Konotop and N. Akhmediev, "Matter rogue waves", Physical Review A, **80**, 033610, (2009).
- [9] A. N. Ganshin, V.B. Efimov, G.V. Kolmakov, L.P. Mezhev-Deglin, P.V.E. McClintock, Phys. Rev. Lett., **101**, 065303 (2008).
- [10] M. Shats, H. Punzmann, and H. Xia, Capillary Rogue Waves, Phys. Rev. Lett., **104**, 104503 (2010).
- [11] N. Akhmediev and E. Pelinovsky (Eds.), "Rogue waves - towards a unifying concept?: Discussions and debates", Special Issue of Eur. Phys. J., Special Topics, **185** (2010).
- [12] D. H. Peregrine, J. Austral. Math. Soc. Ser. B **25**, 16 (1983).
- [13] N. Akhmediev, A. Ankiewicz and M. Taki, Phys. Lett. A **373**, 675 (2009).
- [14] N. Akhmediev, A. Ankiewicz and J. M. Soto-Crespo, Phys. Rev. E, **80**, 026601 (2009).
- [15] M. Taki, A. Mussot, A. Kudlinski, E. Louvergneaux, M. Kolobov, M. Douay, Phys. Lett. A **374**, 691 (2010).
- [16] B. Kibler, J. Fatome, C. Finot, G. Millot, F. Dias, G. Genty, et al, Nature Physics, doi:10.1038/nphys1740 (2010).
- [17] A. Ankiewicz, N. Akhmediev, and J. M. Soto-Crespo, Physical Review E **82**, 026602 (2010).
- [18] Yu.V. Bludov, V.V. Konotop and N. Akhmediev, Optics Letters, **34**, No 19, 3015-3017 (2009).
- [19] N. Akhmediev, A. Ankiewicz, J. M. Soto-Crespo, J. M. Dudley, Physics Letters A, (2011): doi:10.1016/j.physleta.2010.12.027.
- [20] Zhenya Yan, Financial rogue waves, Communications in Theoretical Physics, **54**, 947 (2010).