Insurance: Mathematics and Economics 64 (2015) 385–395

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

The effect of objective formulation on retirement decision making

Adam Butt [a,*], Gaurav Khemka [b]

[a] Research School of Finance, Actuarial Studies and Applied Statistics, Australian National University, Canberra, ACT, 2601, Australia
[b] Faculty of Business, Bond University, Gold Coast, Queensland, 4229, Australia

ARTICLE INFO

Article history:
Received November 2014
Received in revised form
May 2015
Accepted 12 July 2015
Available online 26 July 2015

JEL classification:
C150
C610
D140

Keywords:
Bootstrap method
Simulation
Dynamic programming
Optimisation
Retirement

ABSTRACT

For a retiree who must maintain both investment and longevity risks, we consider the impact on decision making of focusing on an objective relating to the terminal wealth at retirement, instead of a more correct objective relating to a retirement income. Both a shortfall and a utility objective are considered; we argue that shortfall objectives may be inappropriate due to distortion in results with non-monotonically correlated economic factors. The modelling undertaken uses a dynamic programming approach in conjunction with Monte-Carlo simulations of future experience of an individual to make optimal choices. We find that the type of objective targetted can have a significant impact on the optimal choices made, with optimal equity allocations being up to 30% higher and contribution amounts also being significantly higher under a retirement income objective as compared to a terminal wealth objective. The result of these differences can have a significant impact on retirement outcomes.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

The trend away from defined benefits towards defined contributions as the dominant way of privately funding retirement income needs is part of a greater trend towards providing individuals with greater choice in the financing of their retirement. However, a side effect of this trend is the transference of significant amounts of risks to individuals in retirement. The traditional picture of an individual retiring with a significant pension provided by their employer (whom they likely worked for over the greater part of their career), perhaps supplemented by a government-provided, age-based pension, is becoming largely absent from the retirement plans of many individuals across the world. This has been replaced with an environment where individuals are responsible for their own decisions in financing any retirement income above that of the age-based pension. Many countries, such as Australia, Chile, Mexico, Norway and Switzerland require individuals (or employers on behalf of individuals) to make compulsory defined contributions to a retirement savings account, which provide individuals with a significant pot of money with which to generate an income in retirement. Other countries such as Canada, China, Japan, New Zealand, the United Kingdom and the United States, whilst not having compulsory occupational systems, have a significant number of employers sponsoring defined contribution arrangements. Finke and Wolf (2013) provide a brief summary of country-specific retirement income systems. However, in many countries, legislation and regulation provides little guidance as to what decisions need to be made by individuals in order to manage this pot to meet their needs and wants in retirement. For example, in Australia, products which allow almost complete flexibility of withdrawal and investment decisions, but correspondingly leave individuals with no protection against the many risks they face in retirement, have become the dominant product in generating an income in retirement (Retirement Incomes Working Group, 2013). Therefore, the myriad of choices that individuals make both before, at, and during retirement will have a significant impact on their retirement experience.

The purpose of this paper is to investigate the impact on optimal choices of the structure of the financial objective of the individual in retirement, the effect of which has been largely unexplored in the literature. We are particularly interested in objectives that measure wealth at retirement (which we call "terminal wealth" objectives) compared to what we believe are more correct objectives that measure income in retirement (which we call "retirement income" objectives), and their impact on decisions. The structure of the paper is as follows. In Section 2, a selection of

* Corresponding author.
E-mail addresses: adam.butt@anu.edu.au (A. Butt), gkhemka@bond.edu.au (G. Khemka).

http://dx.doi.org/10.1016/j.insmatheco.2015.07.004
0167-6687/© 2015 Elsevier B.V. All rights reserved.

the literature on decision making related to retirement will be reviewed. In addition to this, in Section 3, a general model that can be used to make these required choices is presented. This model uses a dynamic programming approach in conjunction with non-parametric Monte-Carlo simulations of future experience of an individual to make optimal choices. Section 4 outlines the objective functions considered in the paper, whilst Section 5 presents the results. Section 6 concludes the paper and discusses potential future research.

2. A brief summary of the literature on individual decisions relating to retirement

The literature on individual decisions relating to retirement is extensive; a brief and non-exhaustive overview is provided in this section.

2.1. Overall methodologies and assumptions for modelling individual decisions in retirement

Initially, a researcher must consider whether the model will allow decisions to be made on a dynamic basis (relating to changing environmental factors) or on a static basis (with decisions being made only at the time of commencement of the modelling). Clearly, a model that allows decisions to be made on a dynamic basis is superior, although the formation of this sort of model is far more complex compared to a static model. While the majority of recent research employs a dynamic approach in the modelling of financial outcomes in retirement, there are a few exceptions. Amongst the exceptions, some (for example Butt and Deng, 2012) use a static approach, while others (for example Antolín et al., 2010; and Basu et al., 2011) allow choices to change over time but only by formulae pre-defined at the start of the projection period.

Amongst the users of the dynamic approach, the most common methodology applied is called dynamic programming. This approach, in its discrete form, expresses the overall problem of optimising a financial objective of an individual over their entire retirement in terms of the financial objective over a series of time periods (typically annually). In each of the time periods, the model makes the decisions recursively, whilst assuming that all future decisions (already made due to the recursive structure) are also optimal. Some studies, such as Gerrard et al. (2004, 2005), Steinorth and Mitchell (2012), and Kremer et al. (2014), focus on decisions made whilst retired only. Other studies, such as Vigna and Haberman (2001), Haberman and Vigna (2002), Basu et al. (2011), Blake et al. (2011) and Korn et al. (2011), only consider the working stage of life in preparation for retirement. Many of these "working stage only" studies assume an individual will purchase a life annuity upon retirement, and hence it is not necessary to model decisions in retirement. Studies which consider all stages of life often take a "life-cycle" approach to modelling decisions; where retirement decisions are integrated with uncertainty in labour market experience (for example, Horneff et al., 2013).

Within each of the time frame options modelled above, a number of approaches can be used in generating the outputs required. Much of the previous literature uses an analytical approach, where stochastic equations are used to generate the full distribution of the outputs for analysis, with pioneering work in solving retirement decision problems done by Vigna and Haberman (2001) and Haberman and Vigna (2002) in pre-retirement and Gerrard et al. (2004, 2005) in post-retirement. Analytical approaches require the use of relatively simple assumptions in order to ensure the outputs can be mathematically expressed. For example, Gerrard et al. (2005) require an assumption of constant mortality in order to generate a closed-form solution to post-retirement decisions on investment, withdrawal and annuitisation. In relation to economic variables, Pirvu and Zhang (2012) assume risky asset prices follow geometric Brownian motion with drift in optimising investment, consumption and life insurance decisions over the whole life-cycle.

An alternative to an analytical approach is a Monte-Carlo simulation approach. The significant advantage of using a simulation approach is that the component stochastic environmental assumptions and financial objectives can take virtually any form and can still be used to generate the required outputs. This is in contrast to an analytical approach which requires assumptions that are simple enough to allow outputs that can be mathematically solved. An example of a study that uses a simulation approach is by Steinorth and Mitchell (2012). Due to the computational burdens of complex models, much of the literature has used a quadrature approach, where the elements of the model that would otherwise require the use of simulations are discretised into nodes that are weighted according to their likelihood of occurring given the stochastic assumptions selected. Examples of studies that have used a quadrature approach are Blake et al. (2011) and Horneff et al. (2013).

A great deal of the literature has been focused on investment strategy decisions for retirement. Many previous studies have considered investment strategy in addition to another decision such as withdrawal amount (for example Gerrard et al., 2005) or choice of retirement product (for example Horneff et al., 2013; and Steinorth and Mitchell, 2012, allow the purchase of a variable annuity product with guarantees, whilst Pang and Warshawsky, 2013, investigate the timing of purchase of annuities). The amount to save towards generating a retirement income has tended to receive less attention in the literature, although studies such as Kopcke et al. (2013) have used consumption as a decision variable, which when combined with a model of labour income, becomes effectively a decision on both savings and withdrawal.

The modelling to be undertaken in this paper uses a dynamic programming approach with Monte-Carlo simulations. We consider investment strategy and consumption decisions only. Further information on this approach, including a mathematical description, can be found in Section 3.3. The assumptions made about the environment in the modelling in this paper are outlined in Section 3.2.

2.2. Financial objectives in retirement

Any optimisation model of this type must target a specific financial objective. One approach to the measurement of the achievement of financial objectives is a shortfall approach. Under this approach, a baseline objective is set and, where this objective is met, shortfall is not considered to have occurred. No benefit is considered to be gained by exceeding the baseline objective. Where the objective is not met then a shortfall is said to have occurred and a penalty is incurred. The penalty incurred may or may not be related to the size of the shortfall. An early example of a shortfall approach is by Haberman and Vigna (2002), who investigate both the probability of shortfall, and expected shortfall in relation to replacement rates generated by terminal wealth. A later example of this is in Butt and Deng (2012), where decision making was undertaken by minimising the number of years that the individual was expected to live after running out of money. In this case, the amount withdrawn from the retirement savings account was fixed and a shortfall occurred when the account balance reduced to zero. In both of the above examples, the penalty incurred is linearly related to the size of the shortfall (e.g. equal to the number of years of life after the account reduced to zero) although a non-linear approach could also be used.

An alternative approach to the question of adequacy is using a utility approach. Utility functions can take different forms, although they tend to be rather limited in their structure under analytic approaches. This is due to the need for the utility functions

to be able to be expressed in a distributional form as a function of the environmental assumptions. Examples of utility functions used in the literature include constant relative risk aversion utility (Horneff et al., 2013; Kopcke et al., 2013), prospect theory (Blake et al., 2011), log utility (Korn et al., 2011), Epstein–Zin utility (Blake et al., 2014), and piecewise linear approaches (Dempster and Medova, 2011).

Assuming that at least one stochastic assumption is being used, the value of the financial objective output will have a distributional form rather than a fixed value. The most common approach for decision-making is to optimise some form of point estimate from the distribution, typically minimising or maximising the expected value or perhaps some percentile of the (function of the) output random variable as appropriate. More rarely, a risk-return profile will be presented that does not give a single decision; for example Antolín et al. (2010), present a return (median of the replacement rate) versus risk (5th percentile of the replacement rate) plot of various investment strategies, whilst Yao et al. (2014) use a similar approach in investigating terminal wealth. Bernstein et al. (2013) use a stochastic dominance framework to investigate replacement rates for various investment strategies.

One significant concern we have with the literature is the plethora of measurement approaches that are not consistent with the underlying goal of generating an income for retirement (see Van Wyk, 2012). Many papers focus on measures of wealth at retirement rather than measures of retirement income itself. Examples of these (amongst studies previously discussed in Section 2) are Vigna and Haberman (2001), Haberman and Vigna (2002), Basu et al. (2011), Korn et al. (2011), and Yao et al. (2014). Whilst some of these papers define their objective functions as a replacement rate, these replacement rates are simply the terminal wealth divided by a deterministic annuity function and so are effectively terminal wealth measures. Clearly, in a world where individuals have flexibility of investment and withdrawal in retirement, there is a lot of "experience" yet to occur after the act of retirement. We are concerned that failing to appropriately define objectives will distort the decisions being made in planning for retirement. Whilst it may be more justifiable using a wealth at retirement measure ("terminal wealth") for an individual purchasing a life annuity, even then there are factors such as interest rates and mortality rates that mean that a terminal wealth measure may not perfectly correlate with the associated income generated in retirement. We would argue that income in retirement measures ("retirement income") should always be used when modelling retirement decision making.

Hence, the focus of the modelling in this paper will be on a comparison of the impact of terminal wealth compared to retirement income approaches to objective measurement on the decisions made. The approach taken is perhaps most closely linked to Antolín et al. (2010), where the effect on investment strategy of the choice of product for retirement income generation is investigated. In our paper, the effect of the objective structure on the choice of investment strategy and consumption is investigated. We choose a shortfall and a utility objective for comparison; these are described in more detail (including a mathematical formulation) in Section 4.

3. A dynamic programming approach to decision making in retirement

This section describes the dynamic programming model used for analysis in the paper. Initially, in Section 3.1, the basic outputs of the simulation model are presented. Section 3.2 outlines the specific assumptions made in order to generate these model outputs. Section 3.3 presents the dynamic programming model that will be used for decision making in this paper.

3.1. Projecting outcomes in retirement

Before presenting a model of how decisions are made, it is necessary to define basic mathematical relationships between the key variables being considered. Since, the dynamic programming model makes decisions on an annual basis, all relationships will also be defined on an annual basis. We start with calculating the retirement savings account, $A(x+1)$ at exact age $x+1$.

$$A(x+1) = A(x) \times (1+i(x)) + (C(x) - W(x)) \times (1+i(x))^{0.5}; \quad (1)$$

where $i(x)$ is the investment return percentage for the year whilst age x, $C(x)$ are the contributions made to the account whilst age x and $W(x)$ are the withdrawals made from the account whilst age x. In an environment of complete flexibility of withdrawal and investment decisions in retirement, the retirement savings account can be expressed similarly in all stages of life.

The use and/or calculation of all component elements of Eq. (1) will be defined in Sections 3.2 and 3.3.

3.2. Assumptions made in the analysis

In order to generate the model output described in Section 3.1, the assumptions about "environmental" factors are explained in this section. In general, these assumptions are as simple as possible whilst still meeting the purpose of this paper to investigate the impact of different financial objectives on optimal decision making. Future work could investigate the impact of using more realistic assumptions, although some basic analysis we have undertaken of the effect of some assumptions (available on request) demonstrates the robustness of the key results of the paper to the assumptions. Where specific numerical assumptions are necessary, Australian data has been used to generate the assumptions, although this does not unduly impact the generality of the analysis.

(a) *Products available and tax/regulatory environment*

Retirees are assumed to maintain all investment, longevity and other risks in retirement. Hence, income will be generated in retirement by withdrawals made from the retirement savings account. Once the retirement savings account is depleted no further withdrawals can be made. No fees are allowed for as this will not materially impact the comparison between objectives. No tax is allowed for. In addition, there is no allowance for a government-provided age-based pension. This is done to keep the analysis independent of any country-specific regulatory and tax environment.

(b) *General economic assumptions*

For simplicity, individuals are assumed to have a choice between two asset classes: equities and cash. An assumption about the distribution of the return on these asset classes is required for the simulations. A non-parametric approach is used, with historic returns being bootstrapped (see Efron and Tibshirani, 1994). The advantage of a bootstrap approach is that no formal distributional structure is imposed upon the variables, although it does assume that historical returns are representative of the expected distribution of future returns. An example of a bootstrapping approach in the retirement decisions literature can be found in Basu et al. (2011), although as far as we are aware this approach has not been used in conjunction with dynamic programming in the literature.

The data used for bootstrapping is the daily, rolling annual returns on the UBS AU Bank Bill All Maturities Index (cash) and S&P/ASX200 Accumulation Index (equities) from 1 January 1993 to 31 December 2012. For each simulation a single year of returns is generated at random with replacement. For example, a single simulation might generate the return on cash and equities for the year from 27 May 1998 to 26 May 1999.

The asset class returns above are expressed on a real basis, with the inflation measure chosen being Average Weekly Earnings

(AWE; calculated by the Australian Bureau of Statistics, Catalogue No. 6302.0). An earnings inflation measure is chosen rather than a price inflation measure to reflect the desire of individuals to maintain the living standards of the population rather than simply maintaining purchasing power. It also ensures no indexation of income is required in the model (see (d) below). Since AWE are only calculated on a quarterly basis, the daily increase in AWE are assumed to be constant between quarters in order to generate the daily, rolling annual increases in AWE required by the bootstrapping process described above.

Finally, it is necessary to obtain information about long-term interest rates for the purpose of pricing equivalent life annuities in different years (see Section 4 and the Appendix). These real rates are bootstrapped as per above using the 10 year zero coupon yield on government bonds at the end of the previous month, as calculated by the Reserve Bank of Australia.

(c) *Individual wealth*

The individual is assumed to hold all wealth in the retirement savings account, with no other assets being used to generate a retirement income. Since the retirement savings account is a state variable (see Section 3.3), a wide variety of balances are to be tested, and hence a single assumption for the current wealth of the individual is unnecessary.

(d) *Individual earnings*

The individual is assumed to work full-time up until exact age 65, where work will cease completely and retirement will occur permanently.

Since the projection is being performed on a real (salary inflation) basis (see (b) above), it is not necessary to make any assumption about future salary increases of the individual due to general salary inflation. In addition, no assumption is made about future salary increases due to promotion. As a result of these assumptions, as long as the financial objectives are appropriately scaled, the model outputs are independent of the income of the individual. However, for clarity of the results, it is assumed that before retirement the individual earns an overall income from labour of \$72,800 per annum, with the total consumption $P(x)$ of the individual whilst age x being expressed as follows:

$$P(x) = \begin{matrix} S(x) - C(x) & \leftarrow x < 65 \\ W(x) & \leftarrow x \geq 65; \end{matrix} \tag{2}$$

where $S(x)$ is the labour income earned by the individual whilst age x and is equal to \$72,800, and $C(x)$ and $W(x)$ are defined as per Section 3.1.

(e) *Mortality*

The probabilities of death, $q(x)$, of individuals whilst aged x are determined by reference to the male rates in the Australian Life Tables 2005–07 (Australian Government Actuary, 2009). Deaths are assumed to occur uniformly over the year aged x. However $q(x)$ is assumed to be zero for $x < 65$ to reflect the fact that individuals planning for retirement will typically do so on the assumption that they will live until retirement age. Mortality improvement is not allowed for.

3.3. The dynamic programming model

A tabular and basic mathematical description of the dynamic programming process is now provided. First the individual planning for retirement must have a financial objective in retirement. In this paper, we will assume that decisions will be made by optimising some form of point estimate of this objective. This point estimate of the objective is called the "value function" $V(x, y)$, where x is the age of the individual and y is the "state" of the individual aged x. The state simply refers to all of the factors in addition to being aged x (which can also be considered to be a state) that are assumed to affect the decisions made by the individual. Hence, y may be a vector if more than one "state variable" is chosen.

In this study $V(x, y)$ is simply the average of all individual simulated values $v_i(x, y)$, where $i = 1 \ldots 10{,}000$ (i.e. 10,000 simulations are run). The calculation of $v_i(x, y)$ is dependent on the value function chosen, but will simply be some function of the model outputs described in Section 3.1. For the purposes of this section the value function V is assumed to be unknown, although it must be able to be broken down into annual components (i.e. time separable) for each of the discrete steps of the dynamic programming process. The value functions tested in this paper are presented in Section 4.

The decision(s) in optimising $V(x, y)$ for any x and y is defined as $D(x, y)$. The number of items in D will depend on the number of "decision variables" being used in the optimisation procedure.

Since a simulation approach is to be used, a full distribution for V is not available and hence V and D cannot be mathematically determined as a function of y for any value of x. Thus a representative range of options of y is picked (i.e. the state space has been discretised) so that, where necessary, the value of V and D can be estimated for intermediate values of y by interpolation. The range of V and D values determined for each combination of y and x are represented by Table 1.

First the values of $D(x_m, y_k(x_m))$ and respective optimised $V(x_m, y_k(x_m))$ are calculated for $k = 1 \ldots n$ by simulating outcomes for the year beginning age x_m. Since this is the final year (i.e. the individual is assumed to die at exact age $x_m + 1$ if they have not already died by that age), V is calculated as the value function for that year only.

Once $V(x_m, y_k(x_m))$ has been calculated for $k = 1 \ldots n$, then the iterative procedure can proceed to calculating $D(x_m - 1, y_k(x_m - 1))$ and respective optimised $V(x_m - 1, y_k(x_m - 1))$ for $k = 1 \ldots n$ for the year beginning age $x_m - 1$. This is done by simulating value function outcomes for the year beginning age $x_m - 1$ and optimising $V(x_m - 1, y_k(x_m - 1))$, where individual simulation outcomes $v_i(x_j - 1, y_k(x_j - 1))$ for $j = 1 \ldots m$ are calculated using the following Bellman equation:

$$\begin{aligned} v_i\left(x_j - 1, y_k(x_j - 1)\right) &= v_i'\left(x_j - 1, y_k(x_j - 1)\right) \\ &\quad + V\left(x_j, y_i\left(x_j, y_k(x_j - 1)\right)\right)\left(1 - q(x_j)\right) df_i(x_j). \end{aligned} \tag{3}$$

In this equation $v_i'(x_j - 1, y_k(x_j - 1))$ is the value function output given the initial state variables $y_k(x_j - 1)$ and the experience for the simulated year only. $V(x_j, y_i(x_j, y_k(x_j - 1)))$ is the value function output for all future years based on the simulated state variables $y_i(x_j, y_k(x_j - 1))$ at the end of the simulation year and assuming optimal decisions from age x_j onwards. Note that $y_k(x_j - 1)$ can only take the discrete values set in the procedure, whilst $y_i(x_j, y_k(x_j - 1))$ can take any value as it is based on the simulated experience from age $x_j - 1$ to age x_j. $V(x_j, y_i(x_j, y_k(x_j - 1)))$ can be obtained from previous iterations of the procedure using interpolation as required (see the Appendix). $q(x_j)$ is the mortality rate of individuals whilst aged x_j (as defined in Section 3.2(e)) and $df_i(x_j)$ is a discount factor, which is calculated by reference to the real rate of return on cash in simulation i (see the Appendix).

This iterative procedure continues until finishing with $D(x_1, y_k(x_1))$ and respective optimised $V(x_1, y_k(x_1))$ for $k = 1 \ldots n$.

The above discussion has been expressed in general terms for the decision and state variables. In the analysis in this paper the decision variables are the allocation percentage to the equities and cash asset classes described in Section 3.2(b) (note that short selling or margin borrowing is not allowed) and the contribution amounts to and withdrawal amounts from the retirement savings account (see Section 3.1). Since the contribution amount to the retirement savings account is just the difference between the income earned from labour and the amount consumed, and the withdrawal amount is also equal to the amount consumed, then

Table 1
Output from the dynamic programming process.

$y(x_1)$	$D(x_1, y(x_1)); V(x_1, y(x_1))$	$y(x_2)$	$D(x_2, y(x_2)); V(x_2, y(x_2))$	…	$y(x_m)$	$D(x_m, y(x_m)); V(x_m, y(x_m))$
$y_1(x_1)$	$D(x_1, y_1(x_1)); V(x_1, y_1(x_1))$	$y_1(x_2)$	$D(x_2, y_1(x_2)); V(x_2, y_1(x_2))$	…	$y_1(x_m)$	$D(x_m, y_1(x_m)); V(x_m, y_1(x_m))$
$y_2(x_1)$	$D(x_1, y_2(x_1)); V(x_1, y_2(x_1))$	$y_2(x_2)$	$D(x_2, y_2(x_2)); V(x_2, y_2(x_2))$	…	$y_2(x_m)$	$D(x_m, y_2(x_m)); V(x_m, y_2(x_m))$
$y_3(x_1)$	$D(x_1, y_3(x_1)); V(x_1, y_3(x_1))$	$y_3(x_2)$	$D(x_2, y_3(x_2)); V(x_2, y_3(x_2))$	…	$y_3(x_m)$	$D(x_m, y_3(x_m)); V(x_m, y_3(x_m))$
…	…	…	…	…	…	…
$y_n(x_1)$	$D(x_1, y_n(x_1)); V(x_1, y_n(x_1))$	$y_n(x_2)$	$D(x_2, y_n(x_2)); V(x_2, y_n(x_2))$	…	$y_n(x_m)$	$D(x_m, y_n(x_m)); V(x_m, y_n(x_m))$

the decision on contribution amounts and withdrawal amounts are equivalent to a decision on consumption.

These are a relatively simple set of decision variables. However, this could be expanded for future research to take into account different products and flexible transition to retirement. Due to the relatively simple assumptions made in Section 3.2, the only relevant state variable that will be used in this paper is the balance in the retirement savings account (hence y in this section is equivalent to A from Section 3.1).

We now explain how the values of x_j and $y_k(x_j)$ are determined in this study. The youngest age tested is $x_1 = 25$, with all other x_j's increasing in one year increments. The oldest age tested is $x_m = 109$ (which is the oldest age at which mortality rates are available for), with individuals assumed to die at exact age 110 if they have not already died by that age. The values of $y_k(x_j)$ are determined separately for each x_j. Firstly, the value of y_1 is set to zero to reflect an individual who has no retirement savings. The value of y_n is set at a high enough value so that the value of V or the decision variable(s) (as appropriate) is very close to its upper or lower possible bounds (as appropriate). The value of n is set to be 21, with $y_2 \ldots y_{20}$ linearly increasing between zero and y_{21}.

The simulation required by this section was performed in SAS, with the optimisation required using the PROC OPTMODEL procedure. Analysis of the actual output of the modelling in Section 5 was performed in R.

4. Objective functions analysed in this paper

As discussed in Section 2.2, we are concerned that objective functions that measure terminal wealth rather than retirement income itself may distort retirement decision making. Hence, we analyse the impact on decisions of "equivalent" retirement income (RI) and terminal wealth (TW) functions for one each of a shortfall approach (Expected Shortfall) and utility function approach (Constant Relative Risk Aversion). The objective functions considered are outlined briefly as follows; a full mathematical description is provided in the Appendix.

Expected Shortfall (ES)

This approach is based on Haberman and Vigna (2002). Note that Haberman and Vigna (2002) do not actually use this shortfall approach for decision making, but as a simulation investigation subsequent to an analytic approach which uses a quadratic objective function.

The objective under the ES approach is to minimise the expected amount by which the goals of the retiree fail to be achieved; i.e. the expected difference between desired retirement income and actual retirement income. The ES function attaches no "positive experience" to any amount exceeding the retirement goal.

For the RI objective, the withdrawal amount (i.e. $W(x)$) is defined as the December 2013 ASFA Retirement Standard (Association of Superannuation Funds of Australia, 2014) comfortable lifestyle (single) budget of $42,158 and is assumed to be withdrawn until the retirement savings account is exhausted. After this occurs, ES amounts will start being accrued whilst the individual is still alive. Given this, $W(x)$ is not a decision variable. We also assume that during the working phase, the base labour income consumed by an individual (i.e. $P(x)$) is $65,000, while the contributions (i.e. $C(x)$) made to the retirement savings account are equal to 12% of this base labour income (i.e. $7,800). This implies that the overall labour income (i.e. $S(x)$) is equal to $72,800. Under this formulation, where both $W(x)$ and $C(x)$ are constants, the investment strategy is the only decision variable for the ES approach.

We also need an equivalent TW objective for comparison. This is based on a failure to achieve a sufficient retirement savings account at age 65 to be able to purchase a life annuity which generates an annual income of $42,158 for the life of the individual. The price of this life annuity is calculated as the expected present value of the $42,158 annual payments using the real long-term interest rate and mortality rates described in Section 3.2.

Constant Relative Risk Aversion (CRRA) utility

Utility approaches share similarities with shortfall approaches, in that they are functions of the outputs of interest that are to be optimised. However, utility approaches tend to be functions of consumption, and hence lend themselves to having consumption as a decision variable. Hence, both investment strategy and consumption are used as decision variables. Under this approach, the labour income ($S(x)$) equals $72,800, with $C(x)$ and $W(x)$ being the decision variables that determine consumption pre and post retirement respectively. Overall utility can be thought of as the sum of the utilities of the consumption in each year, with the objective being to maximise expected utility. The RI objective allows individuals to continue choosing investment strategy and consumption in retirement, whilst the equivalent TW objective assumes the individual purchases a life annuity at retirement age and therefore enjoys a known utility after retirement from the income generated by the life annuity.

Under CRRA utility the risk aversion of the individual is a constant proportion of the consumption of the individual. The approach used is consistent with that of Horneff et al. (2013) and Kopcke et al. (2013). As per Horneff et al. (2013) a coefficient of relative risk aversion of 5 is tested.

Mathematical formulations of the objective functions analysed can be found in the Appendix.

5. Results

5.1. Optimal decisions

We first present the results of the optimal allocation to equities under the ES objective for RI and TW versions in Fig. 1. Note that decisions for the TW version only go until age 64. For greater clarity, the post-65 optimal allocations under the RI version are presented in a separate graph. Note that, for this and all future results, underlying data and SAS code can be obtained from the authors upon request.

Ignoring, for the moment, the unusual shape of the Age 64 TW line, the results presented in Fig. 1 are not unexpected. The allocation to equities tends to decrease as balance increases for a given age. This implies that where an individual has a target retirement income, the greater the balance currently built up to meet that target, the lesser the need for the individual to increase that balance through the use of risky investments to meet the target retirement income. For similar reasons, prior to retirement for a given balance the allocation to equities increases with age, reflecting a greater

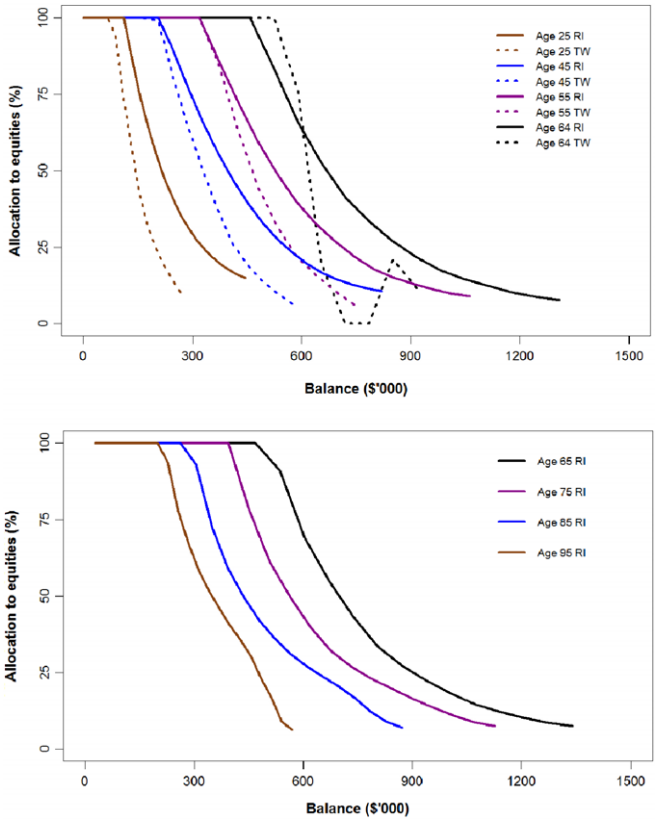

Fig. 1. Asset allocation under the ES objective.

need to invest in risky assets to meet the target retirement income. The opposite occurs post retirement, as the balance needs to fund income for a shorter period as age increases, thus reducing the need for investment in risky assets.

In most cases the allocation to risky assets is lower under the TW version than the RI version, reflecting the fact that the target of TW at a single time (via an annuity purchase at retirement) makes equity investment more risky than the target of RI which requires withdrawals over a period of time. This implies that individuals can take more investment risk prior to retirement if they are intending to maintain investment risk over their retirement, compared to if they are planning on transferring investment risk to a counterparty upon retirement. These differences become extremely large at relatively higher balance levels; for example the optimal equity allocation for a 25 year old with a balance of $178,160 is 64% under RI and 31% under TW. The exception to this is where the balance is relatively low close to retirement; in this instance the upside risk of equity investment encourages the TW investor to allocate a higher amount to equities in seeking a high return to quickly increase the retirement savings account to a level that will purchase the desired annuity. For example the optimal equity allocation for a 64 year old with a balance of $523,440 is 83% under RI and 100% under TW.

We now comment on the unusual shape of the Age 64 TW line. First we note that the very nature of the shortfall objective means that some simulations will be contributing to the expected shortfall (i.e. the average shortfall across all simulations) and some will not. All other things remaining equal, the lower the initial balance the greater the proportion of simulations that contribute to shortfall. Second we note that under TW, the shortfall at age 64 is primarily a function of two key stochastic economic components; the returns on the asset classes invested in, and the long-term interest rate used to price the annuity purchased at the end of the year.

Fig. 2 presents the evolution across selected balances of the average equity return and the average long-term interest rate from the simulations that contribute to shortfall at age 64.

There is a clear correlation between the proportion of simulations that experience a shortfall, and the equity returns and long-term interest rates of those simulations. The correlation is most consistent with long-term interest rates (97%) compared to equity returns (70%), as many balances above $700,000 do not have any equity investment at all and hence in these cases equity returns do not impact shortfall. However, there is an unusual downward spike in the equity returns between $700,000 and $800,000 balances. Whilst the simulations that contribute to shortfall above a balance of $800,000 have an average real equity return of around 0% (compared to an average across all simulations of 6.2%), for those above a balance of $700,000 but below $800,000 the average real equity return drops to as low as −10.7%. These equity returns are a function of the extremely low equity returns which occurred during the global financial crisis, but they did not occur when long-term interest rates were at their lowest. For this reason, some equity investment is tolerated when long-term interest rates are at their lowest (and shortfall affects even the highest balances). However the non-monotonic relationship between equity returns and long-term interest rates means that equity investment is not tolerated when simulations with slightly higher long-term interest rates, but very poor equity performance, start to experience shortfall.

This issue is not present at ages younger than 64, as under the ES TW dynamic programming procedure shortfall is directly linked to long-term interest rates in only the year prior to retirement (and autocorrelation of interest rates is not allowed for). Hence the results prior to age 64 in Fig. 1 are relatively smooth.

We believe this issue reveals a key inadequacy/feature in expected shortfall measures, and for that matter any objective where a "zero" measure is possible, in that non-monotonic correlations between factors affecting shortfall can result in non-monotonic decision making. This is not an issue for utility measures such as CRRA as all simulations contribute to the objective measure for all balances. Additionally, this would not be an issue were the economic model not to allow non-monotonic correlations between key economic variables.

Moving on, we note that we cannot directly use the results in Fig. 1 as evidence or otherwise of the appropriateness of a "lifestyle" investment strategy (see Haberman and Vigna, 2002), as the choice of equity allocation is determined by the balance pathway that an individual takes as they age. To investigate this further we perform 10,000 simulations on a forward looking basis starting for an individual aged 25 with a starting balance of $0. Decisions made are based on the optimal decisions outlined above, with linear interpolation performed to determine optimal asset allocations. Objective values are calculated on the same basis as described in Section 4 and the Appendix. We investigate the median balance and optimal equity allocation across lifetime for these simulations; the results are shown in Fig. 3. Note that for TW, decisions are made using the TW objective version at ages up to 64 and then the RI objective version after that (since the TW objective version makes decisions only up until age 64).

The results in Fig. 3 demonstrate that the lifecycle strategy seems appropriate given the expected shortfall objective function and the underlying assumptions in the model. Individuals will expect to decrease their allocation to equities as they get closer to retirement, as the effect of increasing balance over time is expected to be significant in reducing equity allocation. The opposite occurs post retirement; as an individual ages they will expect to increase their allocation to equities because this ageing increases the likelihood that their balance will be insufficient to provide a retirement income for the whole of life, thus necessitating an attempt to increase the retirement savings account through investment in risky assets. The lower TW allocation to equities is demonstrated up until age 64, with the jump at age 65 being a result of switching to the RI objective after retirement and having a lower balance than

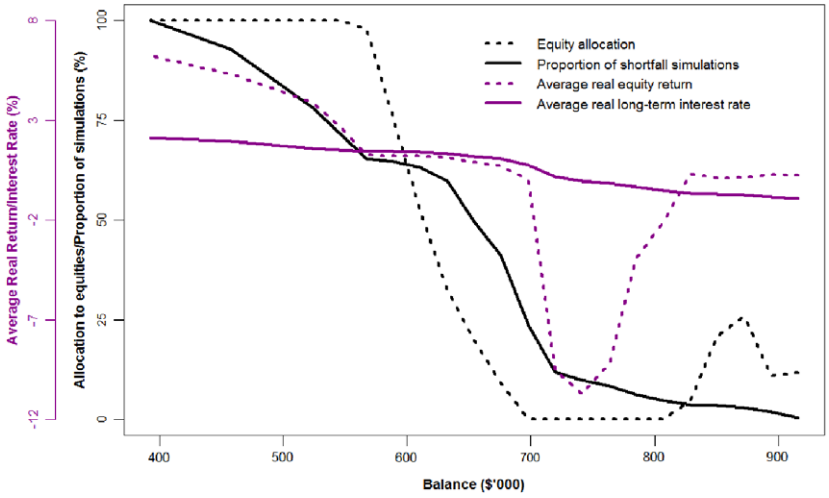

Fig. 2. Average equity return for ES TW results at age 64.

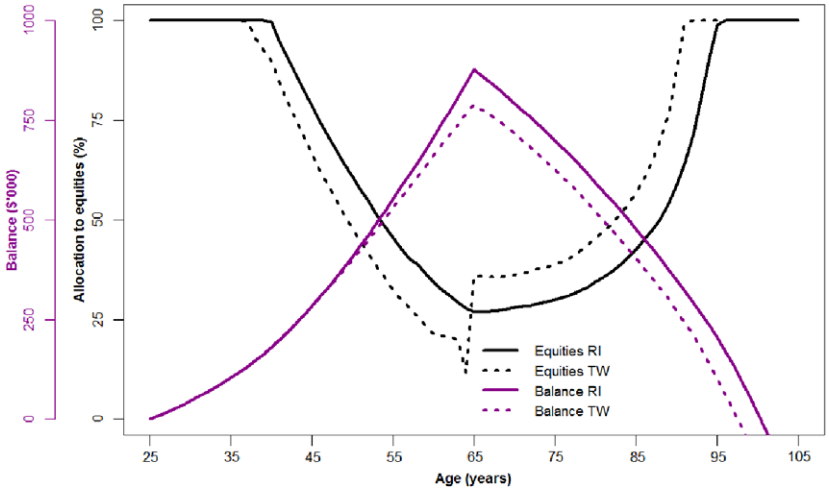

Fig. 3. Median balance and asset allocations for the ES objective.

an individual who used the RI version before retirement. The drop in allocation to equities at age 64 is due to the distortion in results discussed in relation to Fig. 2.

We now present the results of the optimal allocation to equities and consumption amounts under the CRRA objective for RI and TW versions in Fig. 4 (asset allocation) and Fig. 5 (consumption).

The results in Figs. 4 and 5 reflect that under CRRA, the asset allocation decision has a much different structure than under ES. After retirement, the allocation to equities is not impacted by the balance amount or age, and the consumption amount is linearly related to balance both pre and post retirement (as demonstrated by Samuelson, 1969). The amount of consumption as a percentage of balance (i.e. the slope of the straight lines) increases with age as the future expected life decreases, at rates of 4.5%/5.6%/7.7%/11.4% for ages 65/75/85/95.

Before retirement there is a clear decrease in the allocation to equities with age, which is consistent with the argument of Bodie et al. (1992) that investors should reduce their allocation to equities as their "human capital" (i.e. their future ability to earn an income through labour) decreases. The reduction in equity as balance increases can be understood in the same context; as the balance increases the proportion of wealth in human capital decreases and hence results in a decrease in equity allocation. However at extremely high balances this is offset by the boundary condition that consumption can only increase to the value of income earned by labour.

Comparing the RI and TW results shows that RI requires a higher allocation to equities in almost all circumstances and a higher contribution amount (lower consumption) pre-retirement. The higher allocation to equities under RI can be explained in the same way it was for the ES objective; that the target of TW at a single time (via an annuity purchase at retirement) makes equity investment more risky than the target of RI which requires withdrawals over a period of time.

The higher contribution decision of RI is a little harder to understand intuitively, although it may be related to the fact that as the RI version requires a sufficient balance to manage potential longevity uncertainties that it therefore leads to a higher buildup of assets pre-retirement. To assist in this analysis we now, like we did for the ES objective, investigate the median balance, optimal equity allocation and optimal consumption amounts across lifetime for the 10,000 simulations for an individual aged 25 with a starting balance of $0; the results are shown in Fig. 6. Again linear interpolation is performed to determine optimal asset allocations and consumption amounts. As per Fig. 3, TW decisions are made using the TW objective version at ages up to 64 and then the RI objective version after that.

The results in Fig. 6 demonstrate that the lifecycle strategy seems appropriate given the CRRA utility objective function and the underlying assumptions in the model. Interestingly, the allocation to equities under TW is slightly higher than under RI—as the lower balance under TW has a more significant impact than

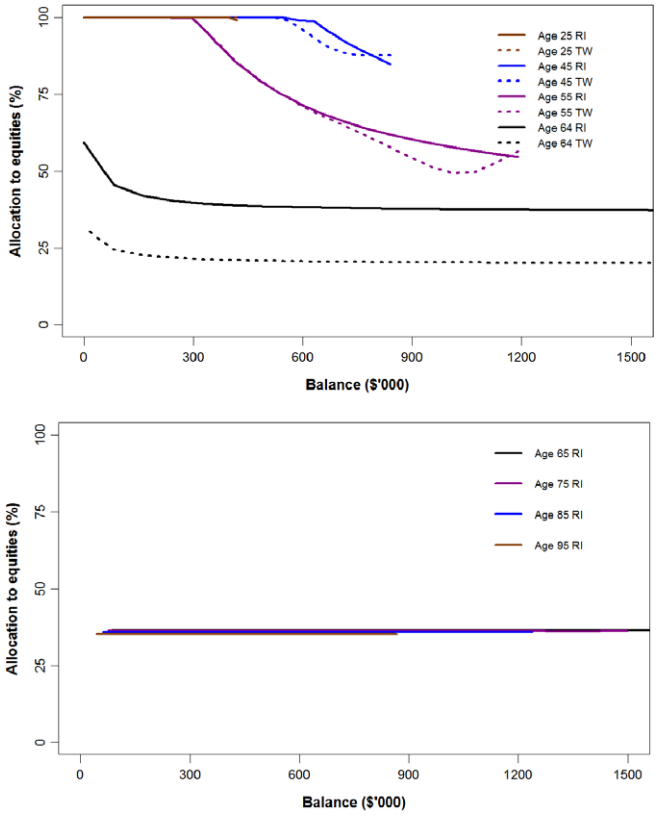

Fig. 4. Asset allocation under the CRRA utility objective.

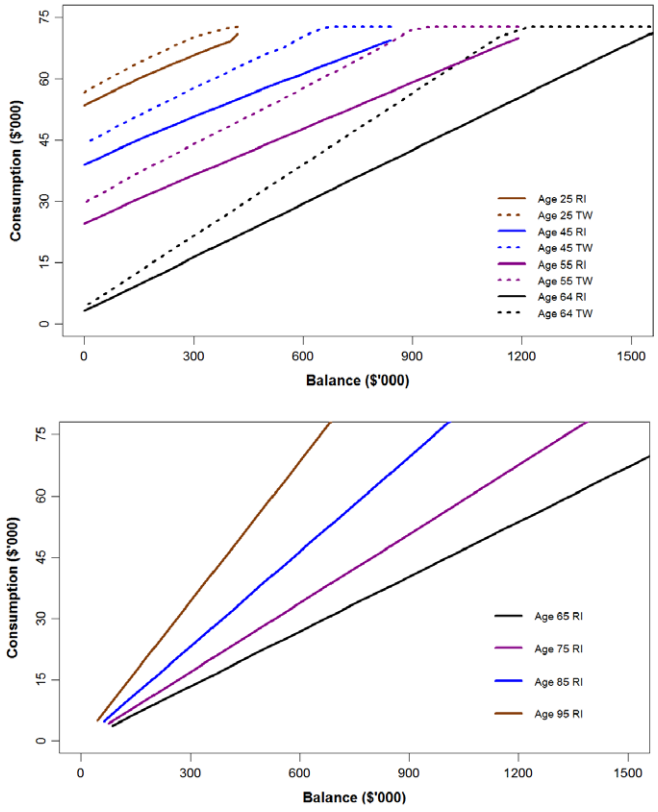

Fig. 5. Consumption under the CRRA utility objective.

the fact that the equity allocation tends to be higher under RI than TW ceteris paribus. The exception to this is a sharp drop in the equity allocation under TW at age 64; this is a reflection of the protection element in the final year of decision making under TW. As observed in Fig. 4 there is little impact of age or balance on the post-retirement equity allocation.

The upward sloping consumption curves indicate that individuals will expect to contribute more in earlier years to take advantage of the compounding of returns, although this is also a function of the choice of discount rate in the utility function. As observed in Fig. 5, individuals are expected to contribute more under RI than under TW, leading to the higher balance observed for RI compared to TW. The consumption decision of RI is broadly consistent upon the transfer from pre-retirement to post-retirement, as the underlying objective is consistent pre and post retirement. The lower balance under TW leads to a significant drop in consumption after retirement, although the median annuity that would be purchased at age 65, were the annuity under TW available to be purchased (see Section 4 and the Appendix), is $80,198, which is actually greater than the consumption amount at any time in Fig. 6. Hence it appears that purchasing an annuity provides a higher utility than maintaining risk and flexibility, although again this is a function of the choice of discount rate in the utility function and other assumptions made in the analysis (in particular that no profit loading is applied to the annuity). It is important to remember that what we are interested in here is not the choice between annuities and flexible withdrawals; but instead the effect of objective definition on individuals who are assumed to use flexible withdrawals.

5.2. Effect of decisions

Having established the optimal decisions in Section 5.1, we now turn our attention to estimating the impact on retirement income outcomes of these optimal decisions. For both objective functions tested we will compare two scenarios. The first and optimal scenario is to make decisions using the RI objection version at all ages. The second and sub-optimal scenario is to make decisions using the TW objective version at ages up to 64 and then the RI objective version after that, as was done in Figs. 3 and 6. Our aim is to establish the significance of making decisions on sub-optimal TW basis before retirement on the retirement income outcomes. Results are presented in Fig. 7 (ES) and Fig. 8 (CRRA).

In making this comparison, we run 10,000 simulations on a forward looking basis in the same way as described in Section 5.1, but now starting from ages 25, 35, 45 and 55 and balances starting from zero and increasing in $10,000 increments.

As expected, Fig. 7 shows that results under RI give a lower expected shortfall than under TW. The gap between RI and TW is relatively consistent at all ages, with the gap being biggest at higher balances, where the differences between the optimal equity allocations are largest (as shown in Section 5.1). The largest absolute gap between the RI and TW lines is at age 55 and a balance of $510,000 where the expected shortfall for RI is $25,604 and for TW is $28,255.

Again the results in Fig. 8 are as expected, with the utility results under RI being clearly superior compared to TW. Comparing numerical values of the utility function are not as useful as they are for expected shortfall as the utility values do not have an intuitive interpretation. However, there is a fairly clear 20%–25% difference between the RI and TW lines at lower balance levels for all ages, which tends to decrease a little as balance increases.

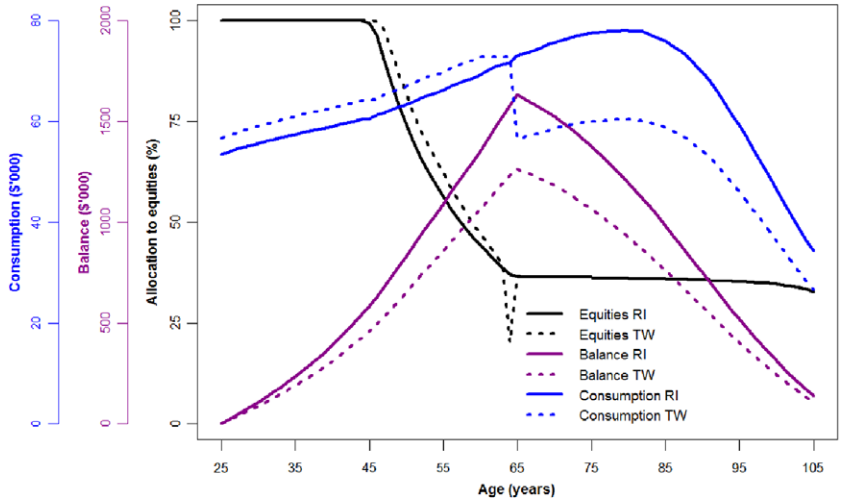

Fig. 6. Median balance, asset allocations and consumption choices for the CRRA objective.

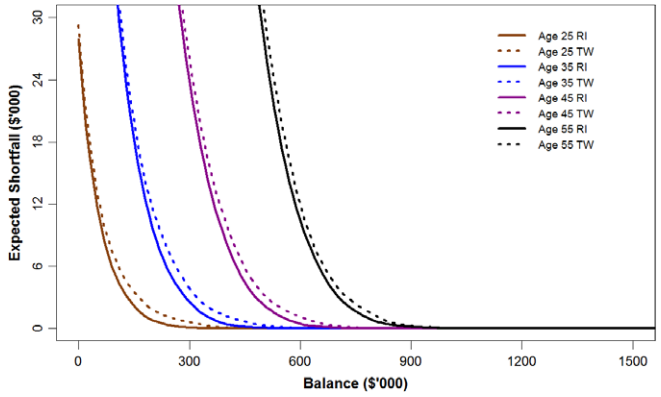

Fig. 7. Expected Shortfall (ES) comparisons.

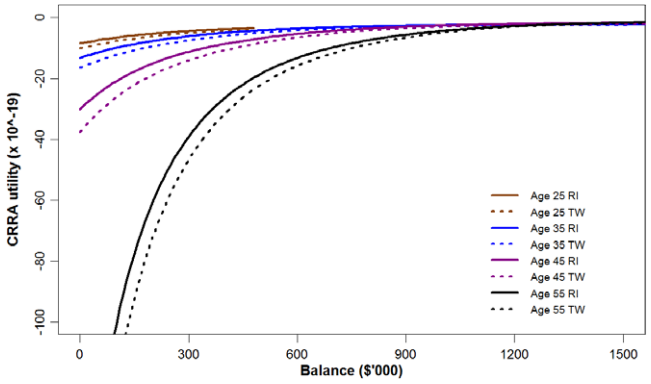

Fig. 8. CRRA utility comparisons.

6. Conclusions and future research

This paper has investigated optimal decision making in retirement. In particular we are interested in understanding the impact of a retiree focusing on an objective relating to the retirement savings account balance at retirement (TW), when we would argue a more correct measure is a retirement income objective (RI). The modelling undertaken uses a dynamic programming approach in conjunction with Monte-Carlo simulations of future experience of an individual to make optimal choices.

We have found that the objective focused on can have a significant impact on the optimal choices made, with optimal equity allocations being up to 30% higher under RI as compared to TW, due to the fact that individuals can take more investment risk prior to retirement if they are intending to maintain investment risk over their retirement, compared to if they are planning on transferring investment risk to a counterparty upon retirement. Individuals also contribute more under RI as compared to TW, due to the need to provide some protection from the possibility of living longer than expected.

The result of these differences can have a significant impact on retirement outcomes, with an incorrect formulation of objective function potentially leading to an increased shortfall up to $2,500 or a consumption utility of up to 25% lower. In addition, we have found that for objectives where a "zero" measure is possible (such as Expected Shortfall), that non-monotonic correlations between factors affecting the objective can result in non-monotonic decision making. We would hence argue that objective measures where all scenarios contribute to the objective are more appropriate for decision making.

There is clearly much potential for future research in this area. In a world where many countries are facing an ageing population, and governments are increasingly passing the burden of planning for retirement onto individuals, having a coherent approach to making retirement decisions is vital in ensuring the best possible retirement outcomes. Clearly it is not possible for most individuals to understand the processes being followed in undertaking this work. The first step is to more clearly understand individuals' objectives in retirement (including bequest motives) so that they can be more adequately captured in the model. This can then be combined with making the model elements outlined in Section 3.2 more sophisticated. For example, additions to the model could include the introduction of elements that are specific to the legislative and tax environments of individual countries, as well as introduction additional decision variables (e.g. retirement age, additional products, etc.) and state variables (e.g. health status including the impact of mortality improvement, family status, current economic climate, etc.) that can more adequately model retirement experience.

This research demonstrates the importance of defining financial objectives in a coherent way and the impacts on decisions made and outcomes experienced when this is not the case.

Acknowledgements

The authors would like to thank attendees at seminars at Australian National University and Griffith University for their

helpful comments on earlier drafts of the work, as well as two anonymous reviewers. This work was funded by an internal grant R62860 71B4 from the Research School of Finance, Actuarial Studies and Statistics at the Australian National University.

Appendix. Objective functions considered

This appendix provides mathematical notation for the objective functions described in Section 4. The notation defined in Section 3 will be used. Any new notation will be defined the first time only and then be used consistently from there onwards.

Expected shortfall (ES)

For the **retirement income approach**, the ES can be defined as follows for a single simulation i and retirement savings account state variable $A_k(x)$:

$$\begin{aligned} v_i\left(x, A_k(x)\right) &= 42{,}158\left[x+1-R_i\left(x, A_k(x)\right)\right] \\ &\times\left(\begin{array}{c}(1-q(x))\left(df_i(x)^{\frac{R_i(x,A_k(x))+1-x}{2}}\right) \\ +\dfrac{q(x)}{2}\left[x+1-R_i\left(x, A_k(x)\right)\right]\left(df_i^{\frac{3R_i(x,A_k(x))+1-3x}{4}}\right)\end{array}\right) \\ &+V\left(x+1, A_i\left(x+1, A_k(x)\right)\right) df_i(x)\left(1-q(x)\right) \\ &\text{where } V\left(110, A_i\left(110, A_k(109)\right)\right)=0 \end{aligned} \tag{A.1a}$$

$R_i(x, A_k(x))$ is the timing of ruin of the individual (i.e. the exact time at which $A_i = 0$) where $A_i(x+1, A_k(x)) \leq 0$, and is calculated as $x + W(x)/[A_k(x) - A_i(x+1, A_k(x))]$. Where $A_i(x+1, A_k(x)) > 0$ then $R_i(x, A_k(x))$ can simply be defined as $x+1$. $s_i(x)$ is the general increase in labour income over the year aged x, and $c_i(x)$ is the cash rate applying over the year aged x, with the discount factor $df_i(x)$ being equal to $[1 + s_i(x)]/[1 + c_i(x)]$. Note that the effect of $df_i(x)$, for this and all future equations, is to assume that nominal future expected shortfalls are discounted at the actual cash rate of the simulation.

For the **terminal wealth approach**, it is not necessary to include mortality as we ignore the possibility of mortality before retirement (see Section 3.2(g)). Hence, the ES can be defined as follows for a single simulation i and retirement savings account state variable $A_k(x)$:

$$\begin{aligned} v_i\left(x, A_k(x)\right) &= V\left(x+1, A_i\left(x+1, A_k(x)\right)\right) df_i(x) \\ &\text{where } v_i\left(64, A_k(64)\right) \\ &= \max\left[0, \left[LP_i(65) - A_i\left(65, A_k(64)\right)\right]\right] df_i(64) \end{aligned} \tag{A.1b}$$

$LP_i(65)$ is the price of the life annuity purchased at age 65, for a retirement income of \$42,158 per annum and is calculated as follows:

$$\begin{aligned} &LP_i\left(65\right) \\ &= 42{,}158\sum_{t=0}^{44}\left\{\begin{array}{l}\left[\dfrac{(1+s)}{(1+il_i(65))}\right]^t p(65,t) \\ \times\left\{\begin{array}{l}(1-q(65+t))\left[\dfrac{1+s}{(1+il_i(65))}\right]^{0.5} \\ +\dfrac{q(65+t)}{2}\left[\dfrac{1+s}{(1+il_i(65))}\right]^{0.25}\end{array}\right.\end{array}\right. \end{aligned} \tag{A.1c}$$

s is the long-term expected increase in labour income and is equal to 3.849% p.a., $il_i(65)$ is the long-term interest rate at age 65, $p(65, t)$ is the probability that an individual aged 65 will survive to age $65 + t$, and $q(65 + t)$ is the probability that an individual aged $65 + t$ will die over the next year.

For both of Eqs. (A.1a) and (A.1b), in order to calculate $V(x + 1, A_i(x + 1, A_k(x)))$ for all $A_i(x + 1)$ values, linear interpolation of the log values of $V(x + 1, A_k(x + 1))$ is performed.

Constant Relative Risk Aversion (CRRA)

For the **retirement income approach**, the CRRA can be defined as follows for a single simulation i and retirement savings account state variable $A_k(x)$:

$$\begin{aligned} v_i\left(x, A_k(x)\right) &= \frac{P\left(x, A_k(x)\right)^{1-\rho}}{1-\rho}\left[\begin{array}{c}(1-q(x))\left(df_i(x)^{0.5}\right) \\ +\dfrac{q(x)}{2}\left(df_i(x)^{0.25}\right)\end{array}\right] \\ &+V\left(x+1, A_i\left(x+1, A_k(x)\right)\right) df_i(x)\left(1-q(x)\right) \\ &\text{where } V\left(110, A_i\left(110, A_k(109)\right)\right)=0 \\ &\text{and } P\left(109, A_k(109)\right) = A_k(109)\times\left(1+i_i(109)\right)^{0.5} \end{aligned} \tag{A.2a}$$

ρ is the coefficient of risk aversion and $P(x, A_k(x))$ is the consumption amount for an individual aged x with retirement savings account state variable $A_k(x)$.

For the **terminal wealth approach**, the CRRA can be defined as follows for a single simulation i and retirement savings account state variable $A_k(x)$:

$$\begin{aligned} v_i\left(x, A_k(x)\right) &= \frac{P\left(x, A_k(x)\right)^{1-\rho}}{1-\rho}\left(df_i(x)^{0.5}\right) \\ &+V\left(x+1, A_i\left(x+1, A_k(x)\right)\right) df_i(x) \\ &\text{where } V\left(65, A_i\left(65, A_k(64)\right)\right) \\ &= \left(\frac{\left(LI_i(65)\right)^{1-\rho}}{1-\rho}\right)\left(\frac{LP_i\left(65\right)}{42{,}158}\right) \end{aligned} \tag{A.2b}$$

$LI_i(65)$ is the annual income generated from a life annuity purchased at age 65 with retirement savings account balance $A_i(65, A_k(64))$, and is equal to $42{,}158 \times A_i(65, A_k(64))/LP_i(65)$. Again the future component of the objective function is discounted at the actual cash rate of the simulation.

For both Eqs. (A.2a) and (A.2b), $P(x, A_k(x))$ is constrained to a maximum of labour income (i.e. \$72,800) before age 65 and from age 65 onwards is constrained so that the withdrawal amount cannot cause the retirement savings account to drop below zero in any simulation. $P(x, A_k(x))$ is also constrained so that it cannot be less than zero. In order to calculate $V(x+1, A_i(x+1, A_k(x)))$ for all $A_i(x + 1)$ values, linear interpolation of $(V(x + 1, A_k(x + 1)) \times (1 - \rho))^{(\rho-1)}$ is performed.

References

Antolín, P., Payet, S., Yermo, J., 2010. Assessing default investment strategies in defined contribution pension plans. In: OECD Working Papers on Finance, Insurance and Private Pensions. No. 2. OECD Publishing.

Association of Superannuation Funds of Australia, 2014. The ASFA Retirement Standard. December 2013.

Australian Government Actuary, 2009. Australian Life Tables 2005-07.

Basu, A.K., Byrne, A., Drew, M.E., 2011. Dynamic lifecycle strategies for target date. J. Portf. Manag. 37 (2), 83–96.

Bernstein, S., Fuentes, O., Villatoro, F., 2013. Default investment strategies in a defined contribution pension system: a pension risk model application for the Chilean case. J. Pension Econ. Finance 12 (4), 379–414.

Blake, D., Wright, D., Zhang, Y., 2011. Target-driven investing: Optimal investment strategies in defined contribution pension plans under loss aversion. J. Econom. Dynam. Control 37 (1), 195–209.

Blake, D., Wright, D., Zhang, Y., 2014. Age-dependent investing: Optimal funding and investment strategies in defined contribution pension plans when members are rational life cycle financial planners. J. Econom. Dynam. Control 38, 105–124.

Bodie, Z., Merton, R.C., Samuelson, W.F., 1992. Labor supply flexibility and portfolio choice in a life-cycle model. J. Econom. Dynam. Control 16, 427–449.

Butt, A., Deng, Z., 2012. Investment strategies in retirement: in the presence of a means-tested government pension. J. Pension Econ. Finance 11 (2), 151–181.

Dempster, M.A.H., Medova, E., 2011. Planning for retirement: Asset liability management for individuals. In: Asset and Liability Management Handbook. pp. 409–432.

Efron, B., Tibshirani, R.J., 1994. An Introduction to the Bootstrap. Chapman & Hall/CRC.

Finke, R., Wolf, R. (Eds.), 2013. Allianz International Pensions Country Factsheets 2013. Alliance SE.
Gerrard, R., Haberman, S., Vigna, E., 2004. Optimal investment choices postretirement in a defined contribution pension scheme. Insurance Math. Econom. 35, 321–342.
Gerrard, R., Haberman, S., Vigna, E., 2005. The management of De-cumulation risks in a defined contribution pension scheme. N. Am. Actuar. J. 10 (1), 84–110.
Haberman, S., Vigna, E., 2002. Optimal investment strategies and risk measures in defined contribution pension schemes. Insurance Math. Econom. 31, 35–69.
Horneff, V., Mitchell, O.S., Maurer, R., Rogalla, R., 2013. Optimal life cycle portfolio choice with variable annuities offering liquidity and investment downside protection. Pension Research Council, Working Paper 2013-04.
Kopcke, R.W., Webb, A., Hurwitz, J., Li, Z., 2013. Rethinking optimal wealth accumulation and decumulation in the wake of the financial crisis. Centre for Retirement Research Working Paper 2013-1.
Korn, R., Siu, T.K., Zhang, A., 2011. Asset allocation for a DC pension fund under regime switching environment. Eur. Actuar. J. 1 (2), S362–S377.
Kremer, A., Liese, F., Homölle, S., Clausen, J., 2014. Optimal consumption and portfolio choice of retirees with longevity risk. J. Pension Econ. Finance 13 (3), 227–249.
Pang, G., Warshawsky, M., 2013. Retirement income strategies in expected utility and loss aversion frameworks. Unpublished Paper.
Pirvu, T.A., Zhang, H., 2012. Optimal investment, consumption and life insurance under mean-reverting returns: The complete market solution. Insurance Math. Econom. 51, 303–309.
Retirement Incomes Working Group, 2013. Post retirement funding in Australia. Presented to the Actuaries Summit, 20–21 May, 2013.
Samuelson, P.A., 1969. Lifetime portfolio selection by dynamic stochastic programming. Rev. Econ. Stat. 51 (3), 239–246.
Steinorth, P., Mitchell, O.S., 2012. Valuing variable annuities with guaranteed minimum lifetime withdrawal benefits. Pension Research Council, Working Paper 2012-03.
Van Wyk, B., 2012. Introducing a retirement benefit target. Presented to the Joint IACA, IAAHS and PBSS Colloquium, Hong Kong, 2012.
Vigna, E., Haberman, S., 2001. Optimal investment strategy for defined contribution pension schemes. Insurance Math. Econom. 28, 233–262.
Yao, H., Lai, Y., Ma, Q., Jian, M., 2014. Asset allocation for a DC pension fund with stochastic income and mortality risk: A multi-period mean–variance framework. Insurance Math. Econom. 54, 84–92.