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Citation: [Phys. Plasmas](#) **19**, 083509 (2012); doi: 10.1063/1.4747701

View online: <http://dx.doi.org/10.1063/1.4747701>

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Axial force imparted by a current-free magnetically expanding plasma

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(Received 8 March 2012; accepted 9 August 2012; published online 21 August 2012)

The axial force imparted from a magnetically expanding, current-free, radiofrequency plasma is directly measured. For an argon gas flow rate of 25 sccm and an effective rf input power of ~ 800 W, a maximum force of ~ 6 mN is obtained; ~ 3 mN of which is transmitted via the expanding magnetic field. The measured forces are reasonably compared with a simple fluid model associated with the measured electron pressure. The model suggests that the total force is the sum of an electron pressure inside the source and a Lorentz force due to the electron diamagnetic drift current and the applied radial magnetic field. It is shown that the Lorentz force is greatest near the magnetic nozzle surface where the radial pressure gradient is largest. © 2012 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4747701>]

I. INTRODUCTION

The momentum delivered by charged particles in plasmas has been an important topic associated with astrophysical jets,¹ magnetospheric physics,² thermonuclear fusion devices,³ and electric propulsion systems.⁴ It is essential to understand and model the gain and loss mechanisms of the momentum for clarifying the acceleration and confinement processes of charged particles in naturally occurring and in laboratory plasmas. Application of plasma acceleration in divergent magnetic fields to electric propulsion devices is also of interest, and these processes have been investigated in current-driven plasmas such as applied-field magnetoplasma-dynamic arcjet plasmas.^{5,6} In this type of system, the Lorentz force due to the discharge current and applied or self-induced magnetic field increases the plasma momentum and the resultant thrust. Recently, the utilization of electrodeless, current-free, magnetically expanding plasmas, e.g., helicon sources, is being considered as a propulsion device.^{7–9} A number of experiments and particle-in-cell simulations of current-free magnetically expanding plasmas have shown that supersonic ion acceleration occurs simultaneously with the formation of an electric double layer (DL).^{10–13}

In the current-free DLs, studies of the electron energy distribution or probability functions have shown that only the high energy electron population can overcome the potential drop of the DLs and neutralize the accelerated group of ions.^{14–17} Hence, it is hypothesized that current-free propulsion system does not require a neutralizer as the electron and ion fluxes emitted from the thruster are equal. On the other hand, theoretical studies involving the spontaneous formation of the DLs in magnetically expanding plasmas have shown that no additional net momentum is delivered by the DL.^{18,19} Instead, one- and two-dimensional theories have shown enhancement of the thrust from an expanding plasma flow cross-section or an expanding magnetic field.^{18,20} These

theoretical works are now being extended to the study of plasma detachment from the magnetic field lines.²¹ Direct measurement of the thrust from magnetically expanding or geometrically expanding rf plasmas is presently under investigation.^{22–26} Some experiments have demonstrated that the plasma pressure force, which is mainly from the plasma electrons inside the source tube, is converted into ion momentum through a DL or ambipolar electric field^{22,24} as predicted theoretically.¹⁸ More recently, direct measurement of the axial force component solely resulting from the expanding field has been briefly reported and compared with a simple fluid model.²⁵ It has also been shown that the magnetic nozzle can be treated as the physical wall of the mechanical nozzle.²⁷

In this paper, the detailed measurement and analysis of the axial force from the magnetically expanding plasma are presented. Here the axial force onto the magnetic field is identified as a Lorentz force due to the azimuthal electron diamagnetic current and the radial component of the applied magnetic field. In the model, the radial ion inertia is neglected and a 1D-approximated magnetic nozzle is incorporated for simplifying the analysis. These subjects still remain further problems.

II. EXPERIMENTAL SETUP

Experiments are performed in the *Irukandji* vacuum chamber at the Australian National University, which is 100 cm in diameter and 140 cm long, and is pumped down to a base pressure of about 10^{-6} Torr with a turbomolecular/rotary pumping system. The base pressure is measured with an ionization gauge connected to the side port of the chamber. The rf plasma source immersed in *Irukandji* and shown in Fig. 1(a) is presently used. The source has a 25 cm long, 9 cm inner diameter Pyrex glass tube with a glass back wall. An expanding magnetic field is applied by using a combination of two axial solenoids located at $z = -5.5$ cm (downstream solenoid current, I_{Bdown}) and -18.5 cm (upstream solenoid current, I_{Bup}), where $z = 0$ is defined as the open exit of the source tube. Two types of magnetic field configurations corresponding to $(I_{Bup}, I_{Bdown}) = (6 \text{ A}, 6 \text{ A})$, and $(0 \text{ A}, 6 \text{ A})$ are

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FIG. 1. (a) Schematic diagram of the experimental setup. (b) Typical thrust balance calibration curve relating the applied force to the displacement measured by the laser displacement sensor.

used in the present experiments, where the calculated axial profile of B_z , and x - z contour plots of B_z and B_r are shown in Fig. 2, together with the magnetic field lines [solid lines in Figs. 2(b)–2(f)]. These magnetic field configurations are labelled as “A mode” [Figs. 2(a)–2(c)] and “B mode” [Figs. 2(d)–2(f)], respectively. Argon gas (at 25 sccm) is fed into the source tube via a small ceramic tube which is mechanically isolated from the source tube and passes through a small center hole in the glass back wall. The gas pressure in the vacuum chamber is about 1 mTorr, which is measured using a baratron gauge connected to the side port of the chamber. An 11 cm inner diameter, 2-turn rf loop antenna, mechanically isolated from the plasma source tube, is located at $z = -12$ cm, and powered from a 13.56 MHz rf power generator (the maximum rf output power is about 900 W) through an impedance matching circuit.²⁸ When in operation, the plasma produced inside the source tube flows downstream along the expanding magnetic field.

The measurement of the thrust force imparted by the plasma is performed with a grounded thrust balance, which consists of four aluminum support columns [not shown in Fig. 1(a)] and a double pendulum system. The pendulums are made of 0.1 mm thick stainless steel flexible plates. The displacement of the pendulum, which is moved by the force imparted from the plasma, is measured using a high-resolution rf-shielded laser displacement sensor (resolution, 0.1 μm ;

sampling rate, 312.5 Hz) mounted onto the aluminum support structure of the balance. The solenoids of the rf plasma source are attached to the thrust balance, which is immersed within the *Irukandji* vacuum chamber. The laser system is initially operated with the flowing argon gas and no plasma for a period of about 20 s; the rf power and argon plasma are subsequently turned on for 10 s, and after the plasma is turned off, additional data are taken for another 30 s. Here no displacement is detected when the argon gas is introduced; hence the force by the neutral gas is negligible in the present experimental configuration. It is observed that the thrust balance moves toward the negative z direction during operation, and returns to the initial position when the plasma is turned off. Because the forces applied to the balance result in an oscillation of the pendulum with a frequency of about 1 Hz, which is determined by the total weight of the source system and length of the pendulum arms, the measured data are filtered to remove the oscillating component. The displacement due to the force originating from the plasma corresponds to the difference between the positions measured during the “plasma off” and “plasma on” periods.

The axial force imparted by the plasma can be calculated from the measured displacement using a calibration coefficient relating the displacement to the force, which is obtained as follows. A calibration basket is tied to the back of the source tube with a thin horizontal thread and a second thread attached to the support structure of the thrust balance [see Fig. 1(a)]. Before pumping down the chamber, a small mass of 0.32 g is put on the basket in 20 s intervals, and the equilibrium positions are measured with the laser displacement sensor. As the horizontal force by the small mass pieces can be calculated from simple mechanics, the measured displacements for the calculated forces can be obtained. Typical results of the displacement as a function of the applied force are plotted in Fig. 1(b) as open circles together with the fitted line derived by a least-squares fit. The line gives a calibration coefficient of ~ 0.5 mN/ μm .

The thrust is equivalent to the force imparted to the whole plasma source structure consisting of the glass tube and the expanding magnetic field provided by the solenoids. Only the force imparted to these components, which are connected to the pendulum thrust balance, can be transmitted to the balance and the resulting displacement measured by the laser sensor as described above. In the present experiment, the measurements of the force are performed when both the glass source tube and solenoids are attached to the balance, and also when only the solenoids are attached. These two distinct measurement configurations allow for direct measurements of the total force T_{total} resulting from both the source tube and magnetic field, and of the force T_B resulting solely from the magnetic field.

The electron temperature T_e and the local plasma potential V_p are measured with a radially and axially movable Langmuir probe (LP), assuming a Maxwellian electron energy distribution. The local plasma density ($n_i \simeq n_e = n$) is estimated from the ion saturation current of the LP, together with V_p and T_e , and using Sheridan’s sheath expansion model.²⁹ For all of the measurements, the rf antenna current is simultaneously measured with a Rogowski coil,

FIG. 2. Axial profile of the magnetic field B_z and x - z profiles of B_z and B_r for ((a)-(c)) A and ((d)-(f)) B modes. The magnetic field lines are shown by the solid lines in the x - z profiles. The color bars show the absolute values of the magnetic field strength in Gauss.

and the effective rf power absorbed into the plasma is estimated using the measured antenna vacuum resistance of 0.71Ω as in Ref. 30. The rf power transfer efficiency is typically in the range of 68%-78% for the A mode, and 79%-88% for the B mode.

III. EXPERIMENTAL RESULTS

A. Axial force measurements

The measured total force T_{total} and the force T_B resulting solely from the expanding magnetic field are plotted as a function of the effective rf power for the A and B modes in Fig. 3, as open squares and open circles, respectively. The total force T_{total} increases with the effective rf power, and reaches a maximum of about 2 mN and 6 mN for the A and B modes, respectively. Although the plasma density and electron temperature measured inside the source tube for these modes are similar for the same effective rf power (described in Fig. 6), a significant difference in T_{total} is observed. The force T_B resulting from the magnetic field increases from about 0.6 mN for the A mode, to about 3 mN for the B mode; indicating that the larger value of T_{total} measured for the B mode results from the larger magnetic force component T_B . The experimental results in Fig. 3 clearly demonstrate the direct measurement of the axial force onto the magnetic field in current-free magnetically expanding plasmas, and the subsequent contribution of the force T_B

to the total force T_{total} . The origin of the force T_B will be derived in the theoretical section below and the experimental results will be compared with the theoretical results.

B. Plasma parameter measurements

Measurements of the basic plasma properties are performed using the LP for both the A and B modes. Figure 4(a) shows the normalized electron pressure ($p_e = n_p k_B T_e$)

FIG. 3. Directly measured forces T_{total} (open squares) and T_B (open circles) as a function of the effective rf power for the (a) A mode and (b) B mode, respectively. The forces T_s (filled triangles), T_B (filled circles), and T_{total} (filled squares) calculated using Eq. (12) are also plotted together with the fitted curves (dotted, dashed, and solid lines) added as visual guide.

FIG. 4. Axial profiles of the (a) normalized electron pressure $p_{en}(0, z) = p_e(0, z)/p_e(0, z_0)$ and (b) the local plasma potential for the A mode (open triangles, 470 W effective rf power) and the B mode (open circles, 540 W effective rf power), respectively.

derived from the plasma density n_p and the electron temperature T_e measured along the z axis, where k_B is the Boltzmann constant and the measured effective powers absorbed into the plasma are 470 W and 540 W for the A and B modes, respectively. The axial positions z_0 having the maximum electron pressure along the z axis are $z_0 = -17$ and -7 cm for the A and B modes, respectively. The strong plasma pressure drop of 2–3 orders of magnitude is observed near the source exit ($z \sim 0$ cm) for the A mode, while a gradual plasma pressure decrease downstream of the source exit is seen for the B mode. It should be mentioned that the pressure change along the z axis mainly originates from the axially decreasing density, since the electron temperature remains fairly constant.

The local plasma potential estimated from the I-V characteristics of the LP is plotted in Fig. 4(b) as open triangles and squares for the A and B modes, respectively; it is found that a localized potential drop over several centimeters is formed near the source exit for the A mode, while the gradual decrease of the plasma potential is observed for the B mode. In steady state plasmas having the particle source on the high-potential side, the low-energy electrons are confined electrostatically in the upstream plasma and only the high-energy electrons can overcome the potential drop and flow into the downstream side. Hence, the observed potential structures in Fig. 4(b) are consistent with the pressure profiles shown in Fig. 4(a).

The detailed radial measurements are performed at the axial position z_0 having the maximum electron pressure for both modes. The normalized electron pressure derived from n_p and T_e measured along the r axis is plotted in Fig. 5 as open triangles and open circles for the A and B modes,

FIG. 5. Radial profiles of the normalized electron pressure measured at z_0 for the A mode (open triangles, 470 W effective rf power, $z_0 = -17$ cm) and the B mode (open circles, 540 W effective rf power, $z_0 = -7$ cm). The dashed and solid lines are the fitting curves corresponding to Eqs. (15) and (16), respectively.

respectively. These radial profiles are required for modelling of the two-dimensional plasma pressure profiles, used for the calculations in the two-dimensional thrust theory described below.

The absolute values of the electron temperature T_e and the plasma density n_p are also important parameters to calculate the thrust, the T_e and n_p at $(r, z) = (0, z_0)$ for the A and B modes are plotted in Fig. 6 as a function of the effective rf power, where the measurements are done at the positions having the maximum electron pressure. It is found that both the T_e and n_p inside the source tube for the A and B modes are similar for the same effective rf power and used for the calculations in the thrust theory.

IV. THEORY AND MODELING

A. Thrust derivation

In this section, the thrust from the magnetically expanding plasmas is theoretically derived from a fluid model. The momentum equation for a charged particle species j in a steady-state collisionless plasma is

$$m_j \nabla \cdot (n_j \mathbf{v}_j \mathbf{v}_j) = q_j n_j (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}) - \nabla p_j, \quad (1)$$

FIG. 6. (a) Electron temperature and (b) plasma density measured by the LP at $(r, z) = (0 \text{ cm}, z_0)$ for the A (open triangles) and B (open circles) modes, as a function of the effective rf power absorbed into the plasma.

where m_j , n_j , $\mathbf{v}_j$, q_j , p_j are the mass, density, velocity, charge, and pressure of the particle species j . $\mathbf{E}$ and $\mathbf{B}$ are the electric-field and magnetic-field vectors. When the continuity equation $\nabla \cdot (n_j \mathbf{v}_j) = 0$ is satisfied, the left-hand side (lhs) in Eq. (1) corresponds to the inertial term $m_j n_j (\mathbf{v}_j \cdot \nabla) \mathbf{v}_j$. Assuming quasi-neutrality ($n_e \sim n_i = n$), negligible electron inertia, isotropic electron temperature, and cold ions ($T_i \sim 0$), Eq. (1) for electrons and ions can, respectively, be written as

$$en(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) = -\nabla p_e, \quad (2)$$

$$m_i \nabla \cdot (n \mathbf{v}_i \mathbf{v}_i) = en(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}), \quad (3)$$

where $p_e = nk_B T_e$ is the electron pressure. When we consider an axisymmetric system, the radial and axial components of Eqs. (2) and (3) in cylindrical coordinates (r, θ, z) can be written as

$$-en(E_r + v_\theta B_z) = \frac{\partial p_e}{\partial r}, \quad (4)$$

$$-en(E_z - v_\theta B_r) = \frac{\partial p_e}{\partial z}, \quad (5)$$

$$en(E_r + u_\theta B_z) = 0, \quad (6)$$

$$en(E_z - u_\theta B_r) = \frac{1}{r} \frac{\partial}{\partial r} (rmnu_r u_z) + \frac{\partial}{\partial z} (mnu_z^2), \quad (7)$$

where (E_r, E_θ, E_z) , (B_r, B_θ, B_z) , (v_r, v_θ, v_z) , (u_r, u_θ, u_z) , and n are the electric field, magnetic field, electron velocity, ion velocity, and plasma density, respectively, which are functions of (r, z) . The radial component of the ion inertia is neglected in Eq. (6) as the ions are mainly accelerated along the z axis in this type of plasma system whether by the DL or by the ambipolar axial electric fields.^{9,31} It is well known that this term includes an effect of plasma ion rotation, which would mainly be produced by an $\mathbf{E} \times \mathbf{B}$ drift in collisionless plasmas; proper treatment of this term still remains as a theoretical problem. A net axial momentum flux τ per unit cross section can be given by

$$\tau = mnu_z^2 + p_e, \quad (8)$$

where the thrust is given by the integration of τ in a cross section. By eliminating E_z combining Eqs. (5) and (7), τ can be expressed as

$$\frac{\partial \tau}{\partial z} = en(v_\theta - u_\theta)B_r - \frac{1}{r} \frac{\partial}{\partial r} (rmnu_r u_z), \quad (9)$$

where the first term of the right-hand side (rhs) is the Lorentz force due to the net azimuthal current and the radial component of the magnetic field. The electron and ion azimuthal current can be obtained from Eqs. (4) and (6) as

$$env_\theta = -en \frac{E_r}{B_z} - \frac{1}{B_z} \frac{\partial p_e}{\partial r}, \quad (10)$$

$$enu_\theta = -en \frac{E_r}{B_z}. \quad (11)$$

As seen in Eqs. (10) and (11), the azimuthal electron and ion currents originate from the electron $\mathbf{E} \times \mathbf{B}$ drift, the electron

diamagnetic drift, and the ion $\mathbf{E} \times \mathbf{B}$ drift, respectively. Hence the net current can be expressed by only the electron diamagnetic current as the $\mathbf{E} \times \mathbf{B}$ drift cannot drive a net current in the present model. When the plasma expands with an axially varying plasma radius $r_p(z)$ and has zero density at $r \geq r_p(z)$, the total axial force $T_{total}(z)$ is given by the volume integration of Eq. (9) as

$$T_{total}(z) = T_s - 2\pi \int_{z_0}^z \int_0^{r_p(z)} r \frac{B_r}{B_z} \frac{\partial p_e}{\partial r} dr dz - 2\pi \int_{z_0}^z \int_0^{r_p(z)} \frac{\partial}{\partial r} (rmnu_r u_z) dr dz, \quad (12)$$

where $T_s = 2\pi \int_0^{r_s} r p_e(r, z_0) dr$ corresponds to the thrust due to the maximum electron pressure^{18,24} and to the constant of integration along z when integrating from z_0 , where r_s is the source tube radius. This term is conserved along z in the absence of a magnetic field even for collisional plasmas as suggested by Fruchtman,³² because the electron pressure is converted into the ion momentum through the DL or ambipolar electric fields, and total momentum of the ions and neutrals is conserved even if the momentum transfer occurs through ion-neutral charge exchange collision. For simplicity, the contributions from the magnetic field and the radial source wall upstream of z_0 are not included in Eq. (12).

The integration from z_0 to the source exit in the third term of the rhs in Eq. (12) presents an axial momentum delivered by the ions flowing into the radial source wall; it vanishes when the plasma density is assumed to be zero at the plasma edge. This assumption would be fairly valid for the thrust analysis because ions colliding with the radial wall are mostly axially slower ions; hence the momentum loss to the radial wall would be small.³² As previously demonstrated, the integration from the source exit ($z = 0$) to the downstream side in the third term of the rhs in Eq. (12) also vanishes as there is no mechanical wall into which the axial momentum of ions can be lost.²⁴

The second term of the rhs in Eq. (12) shows the axial force imparted from the Lorentz force and corresponds to the measured force T_B onto the magnetic field in Fig. 3. It results from the azimuthal current $[B_z^{-1}(\partial p_e / \partial r)]$ of the electron diamagnetic drift and the radial component B_r of the applied magnetic field. Hence, the present theory expresses the gain (loss) of the net axial force for negative (positive) radial gradient of the electron pressure in an expanding magnetic field in addition to the electron pressure term T_s inside the source tube. Recently, it has mathematically been demonstrated that the force due to the electron diamagnetic current is equivalent to the physical nozzle whose cross section expands along z .²⁷ The physical nozzle converts the radial pressure force into an axial force to the nozzle wall.²⁷ In this sense, the role of the magnetic nozzle would also be a conversion of radial pressure into axial pressure resulting in thrust, since the electron diamagnetic drift is driven by the radial gradient of the electron pressure as indicated in Eq. (10). The advantage of the magnetic nozzle would be that this can be equivalent to the semi-infinite physical nozzle, although the issue of plasma detachment from the magnetic field lines is still under investigation.

B. Plasma modeling

To theoretically calculate the momentum gain given by Eq. (9), and the thrust given by Eq. (12), modelling of the two-dimensional electron pressure $p_e(r, z)$ and of the two-dimensional magnetic field (B_r, B_z) shown in Fig. 2 is incorporated with an one-dimensional approximation of the magnetic nozzle as below. The plasma radius $r_p(z)$ along z is assumed to be determined by the radial source wall inside the source tube ($z \leq 0$), and by the expanding magnetic field downstream of the source exit ($z \geq 0$) as³³

$$r_p(z) = \begin{cases} r_s & \text{for } z \leq 0 \\ r_s \sqrt{B_z(0, 0)/B_z(0, z)} & \text{for } z > 0. \end{cases} \quad (13)$$

The two-dimensional r - z profile of the electron pressure, normalized by the maximum pressure $p_e(0, z_0)$, which can be experimentally obtained from the electron temperature and plasma density measurements in Fig. 6, is modelled as

$$\frac{p_e(r, z)}{p_e(0, z_0)} = p_{en}(0, z)f(r, z), \quad (14)$$

where $p_{en}(0, z)$ is the normalized electron pressure along z and corresponds to Fig. 4(a), and $f(r, z)$ is a function giving a radial profile of the normalized electron pressure at z . To fit the function $f(r, z_0)$ to the measured radial profile of the electron pressure at z_0 shown in Fig. 5, $f(r, z)$ for $r \leq r_p(z)$ for the A and B modes is chosen as

$$f(r, z) = 1 - \left\{ \frac{r}{r_p(z)} \right\}^{a_1}, \quad (15)$$

$$f(r, z) = a_2 \left[1 - \left\{ \frac{r}{r_p(z)} \right\}^{a_1} \right]^{a_3} + (1 - a_2) \left[1 - \left\{ \frac{r}{r_p(z)} \right\}^{a_4} \right]^{a_5}, \quad (16)$$

respectively, and $f(r, z)$ is assumed to be zero for $r > r_p(z)$. a_1 - a_5 are fitting parameters of the electron pressure profiles along r at z_0 for the A and B modes, respectively, and are assumed to be conserved along z . The fitting curves of the measured electron pressure along r for the A and B modes are plotted in Fig. 5 as dashed and solid lines, respectively, where the values of a_1 - a_5 are $a_1 = 5.37$ for the A mode and a_1 - $a_5 = 2.07, 3.72, 2.16, 2.89$, and 4.83 for the B mode. As is seen in Fig. 5, the fitting functions approximate the measured profiles well. By using Eqs. (13)–(16) and the normalized electron pressure measured along z , the two-dimensional r - z profiles of the logarithm of the normalized electron pressure $\log[p_e(r, z)/p_e(0, z_0)]$ are calculated and shown in Fig. 7 for both modes. Subsequently, the theoretical momentum gain profile $\partial\tau/\partial z$ and the thrust T_s , T_B , and T_{total} can be calculated from Eqs. (9)–(12), and from the data in Figs. 2, 6, and 7, and the results are shown in Figs. 8 and 3.

C. Comparison between theory and experiments

The calculated results are discussed and compared to the directly measured forces here. Figures 8(a) and 8(b) show the r - z profiles of the calculated momentum gain normalized by the maximum electron pressure, i.e., $(\partial\tau/\partial z)/p_e(0, z_0)$

FIG. 7. r - z profiles of the logarithm of the normalized electron pressure modeled by Eq. (14) for (a) A and (b) B modes. The pressure outside the plasma radius r_p is assumed to be zero.

for the A and B modes, respectively. Here the momentum gain is calculated in the range from z_0 having the maximum pressure, which is used for the integration in Eq. (12). For both cases, the momentum gain occurs near the plasma edge, because the pressure gradient causing the electron diamagnetic drift is predominant there. Further, the momentum gain downstream of the source for the B mode appears to be much greater than that for the A mode, as the electron pressure in the expanding field region downstream of the source exit for the B mode is greater than that for the A mode, where the electron diamagnetic drift current and the resultant Lorentz force is proportional to the absolute value of the electron pressure.

Equation (12) is equivalent to the volume integration of $\partial\tau/\partial z$, where the integration along z is performed from z_0 to the most downstream position of the electron pressure measurement ($z = 40$ cm) for both modes. The calculated T_B , T_s , and T_{total} are plotted in Fig. 3 as filled circles, filled triangles, and filled squares, respectively, where the dashed, dotted, and solid lines are fitted curves added as a visual guide. The measured forces T_B (open circles) and T_{total} (open squares) are in fairly good agreement with the calculated thrusts (filled circles and filled squares) for the two cases tested in the present experiments. This demonstrates that the axial force corresponding to the thrust can be enhanced by the

FIG. 8. r - z profiles of $\partial\tau/\partial z$ normalized by the maximum electron pressure, i.e., the gain of the thrust per unit cross section for (a) A and (b) B modes.

presence of the expanding magnetic field, as a result of the momentum gain due to the Lorentz force, which is produced by the electron diamagnetic current $[B_z^{-1}(\partial p_e/\partial r)]$ and the radial component B_r of the applied field.

Since the downstream electron pressure for mode B is larger than that for mode A, the contribution from the electron diamagnetic current for mode B is greater and gives the larger T_B and hence the larger resultant T_{total} as measured in Fig. 3. However, the discrepancy of about 20% between the theoretical and measured T_{total} and T_B is observed for the B mode, which could result from the following assumptions. In the present simple fluid model, the radial component of the ion inertial term and the axial force delivered onto the radial source wall are neglected. The calculation by Ahedo and Merino has suggested that the azimuthal current due to the radial ion inertial term decreases the thrust,²⁰ and according to their calculation, it is considered that the present model estimates an upper limit to the thrust component T_B . Actually the calculated T_B for the B mode is overestimated when compared with the measured value of T_B , thus, consistent with this view. As the radial ion inertial term includes the radial and azimuthal ion velocities, measurements of these velocity components are required to include the effect of this term. Also, the two-dimensional profile of the electron pressure used is simplified. In addition, the axial integration range

from z_0 to $z = 40$ cm in Eq. (12) could relate to the issue of plasma detachment from the magnetic field, which remains an open area of research.

V. SUMMARY

Direct measurements of the axial thrust force imparted from a magnetically expanding current-free plasma operating in two modes have been performed and compared with a two-dimensional fluid theory. The thrust increase due to the electron diamagnetic current interacting with the radial magnetic field is clearly demonstrated by a direct measurement of the force component transmitted via the expanding field. The main contribution to this force is shown to occur near the plasma edge where the largest radial pressure gradient and the resultant electron diamagnetic current are present.

ACKNOWLEDGMENTS

The authors would like to thank Professor A. Fruchtman of the Holon Institute of Technology for many useful discussions. A part of this work is financially supported by an Australian Research Council Discovery grant (DP 1096653), an Australian Research Council Linkage grant (LP 0883456), a Grant-in-Aid for Young Scientists (A 22684031) from MEXT in Japan, and the TEPCO Research Foundation.

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