Improved integrated nucleus-nucleus inelastic cross sections for light nuclides in Geant4

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Abstract
We propose a new root-mean-square radius parameterization for light nuclei $A \leq 30$ suitable for use in Geant4 calculations of nucleus-nucleus total hadronic inelastic scattering cross sections. The new approach takes into account the proton-neutron asymmetry of the reactants, and was fit to 360 measured total inelastic cross sections from the EXFOR database. Measured nuclear radii are better described in the new approach than the current Geant4 implementation, particularly for unstable nuclides, and there is better agreement with measured cross sections for both stable and unstable nuclides. The improved parameterization should help in carbon-ion therapy applications in particular.

Keywords: Nucleus-Nucleus cross section, Geant4, Nuclear radii

The Monte Carlo simulation toolkit Geant4 [1, 2] models particle transport in matter, and is used in a wide range of research fields including space science, radiation protection, and medical physics. One key factor in simulations of ion transport is the total inelastic (or reaction) cross section. For example, in hadron therapy applications, at energies of 400 MeV/nucleon, up to 70% of $^{12}\text{C}$ beam nuclei may undergo some nuclear reaction before reaching the tumour site [3]. Precise treatment planning therefore requires reliable predictions for inelastic cross sections of $^{12}\text{C}$, and any lighter (possibly unstable) secondary fragments, interacting with any isotopes found in the body.

Geant4 uses a model based on the Glauber approximation [4] to calculate integrated inelastic cross sections for nucleus-nucleus interactions (GG model) [5, 6]. The approach assumes the colliding nuclei have Gaussian shaped density distributions, allowing analytic evaluation of the density convolutions required. A Gribov screening correction is included [7], as well as a phenomenological Coulomb repulsion correction, which reduces the cross section at low energies. Nuclear radii enter as a key parameter in the model. In the current Geant4 implementation, the radii are parameterized in terms of the mass number $A$. However, it takes no account of the proton-neutron asymmetry of the nuclides, making it unreliable for unstable nuclei.

In this paper, we propose an improved radius model for use with the Geant4 GG cross sections. Our primary aim is to improve the predictive power of the model for hadron therapy and space radiation applications, and we therefore focus on light nuclides with $A \leq 30$. The model includes a term dependent on the proton-neutron asymmetry, designed to improve predictions for unstable isotopes. The new radius model is fit to reproduce measured inelastic cross sections from the EXFOR database [8], and then is compared to the existing Geant4 implementation (Throughout the present work, we compare to Geant4 version 10.5.) and experimental measurements of radii [9, 10, 11, 12, 13]. The cross sections from the GG model, using the new radii, are then compared to experimental data [14, 15, 16, 17, 18, 19, 20] from the EXFOR database [8].

We first briefly present the Glauber-Gribov model that is implemented (and widely used) within Geant4. Several publications by Grichine provide more detail on the Geant4 implementation [21, 22, 23]. The model assumes the two colliding nuclei have Gaussian shaped density distributions, with radius parameters $R_p$ and $R_t$. The corre-
The values of the parameter center of mass kinetic energy, and number of protons in the projectile (target), the approximate Coulomb barrier. Here, range as 1.26 (is the isospin-averaged nucleon-nucleon cross section for corresponding that the number of neutrons \( N \) neutron asymmetry of the nuclides, with respect to the ber dependence, and match the forms from the current those at stability, even where \( A \) is the same. To address this deficiency, we propose an alternative form, given by:

\[
R(A) = \begin{cases} 
  r_0(1 - A^{-2/3})A^{1/3} & (10 < A \leq 30) \\
  r_2A^{1/3} & (30 < A < 50) \\
  r_0A^{0.27} & (A \geq 50).
\end{cases}
\] (3)

The parameters of the value \( r_0 \) have been chosen for each range as 1.26 (10 < \( A \leq 15 \)), 1.19 (15 < \( A \leq 20 \)), 1.12 (20 < \( A \leq 30 \)), 1.10 (30 < \( A < 50 \)) and 1.00 (\( A \geq 50 \)). For light stable nuclides with \( A < 10 \), the radii are taken from electron scattering measurements of charge radii. Specific values are given for \( ^1\text{H} \) and \( ^3\text{He} \), and a single value is used for all Lithium isotopes (2.4 fm), and all Beryllium isotopes (2.51 fm). Other isotopes with \( A < 10 \) (\( ^6\text{He} \), \( ^8\text{He} \), and \( ^8\text{B} \)) default to the 30 < \( A < 50 \) parameterization (1.1A^{1/3}).

The primary limitation of the present approach is that the radii are dependent only on the mass number \( A \) and not the relative numbers of protons and neutrons. Unstable nuclides, with asymmetric numbers of protons and neutrons, can have dramatically different nuclear radii from those at stability, even where \( A \) is the same. To address this deficiency, we propose an alternative form, given by:

\[
R(A, Z) = c_1A^{1/3} + c_2A^{1/3} + c_3\Delta(A, Z)^2.
\] (4)

The first and second terms account for the gross mass number dependence, and match the forms from the current Geant4 model. The third term accounts for the proton-neutron asymmetry of the nuclides, with respect to the valley of \( \beta \)-stability, and is defined as follows. We assume that the number of neutrons \( N = A - Z \) for nuclides at stability follows approximately:

\[
N_e(A) = 0.5A + (0.028A)^2 - (0.011A)^3.
\] (5)
reasonably well. For $^{12}$C+$^{27}$Al, the alternative model is considerably better, with the GG version exhibiting a significant systematic overestimation of the data. Though our new model is specifically focused on improving the results for unstable situations, the $^{27}$Al results suggest that more careful benchmarking of stable systems is urgently required. The other reactions, $^{15}$C+$^{12}$C, $^{19}$O+$^{28}$Si, and $^{16}$N+$^{28}$Si, highlight the sparsity of experimental data for unstable systems, but the data are generally better described with the new radius model.

In summary, we have proposed an improved radius model suitable for calculations of nuclear inelastic cross sections of light nuclides with $A \leq 30$. The model shows better agreement with the experimental measurements of radii, particularly for unstable light isotopes when compared to the current model used in Geant4. As a result, there is improved agreement with experimental inelastic cross sections. The disagreement between the existing Geant4 model and the data for $^{12}$C+$^{27}$Al indicate more comprehensive benchmarking is required.

A number of further improvements may be made. The expression used for the barrier energy $V_B$ could be improved by careful analysis of available low-energy inelastic cross sections (see e.g., [24]) including comparisons for unstable nuclides where available. The parameterizations of $\sigma_{pp}$ and $\sigma_{np}$ could also be improved to remove the slight discontinuities near 300 and 500 MeV/nucleon. Though not the focus of this paper, improvements must also be made for heavier systems. This will require a different approach to the one taken here, since very little experimental data is available, and the density distributions for $A > 30$ nuclides are non-Gaussian.

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References


Figure 1: Nuclear radii as a function of number of neutrons for different light elements. The grey bands indicate the proton (left) and neutron (right) driplines where the nuclides are no longer bound.
Figure 2: Integrated GG cross sections as a function of kinetic energy of the projectile. The existing GG model is shown in blue, and the version presented here shown in red.