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THE GENERATION MODEL AND THE ELECTROWEAK CONNECTION

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The "electroweak connection", which forms one of the cornerstones of the Standard Model of particle physics, is formulated within the framework of the Generation Model. It is shown that the electroweak connection can be derived assuming that the weak interactions are effective interactions rather than fundamental interactions, arising from a U(1) × SU(2) local gauge invariance, which is spontaneously broken by a Higgs field.

Keywords: Standard model; generation model; electroweak interaction; U(1) × SU(2) gauge invariance; Higgs field.

1. Introduction

One of the cornerstones of the Standard Model[1] (SM) of particle physics is its treatment of the electromagnetic and weak interactions in terms of a U(1) × SU(2) local gauge theory, known as the Glashow, Weinberg and Salam (GWS) model.[2–4] This model leads to a relation between the electromagnetic and weak interactions, which has been called "the electroweak connection"[1]: *the charge-preserving weak interaction is completely fixed by the electromagnetic interaction and the charge-changing weak interaction.* In this sense, the electromagnetic interaction involving the neutral photons and the weak interactions involving the charged W bosons and the neutral Z bosons are related but are not strictly unified, since the relationship involves two independent coupling constants, an electric charge (Q) and a weak "charge" (g_w).

In recent years an alternative model, the Generation Model (GM),[5–8] has been developed. This model is based upon several concepts which differ from those employed by the SM. Thus it is of interest to ascertain whether these different concepts are compatible with the electroweak connection of the SM, which is in excellent agreement with the experimental data.

Firstly, the GM employs a *unified* classification scheme,[5] involving only three independent additive quantum numbers (charge Q, particle number p and generation quantum number g) for both the leptons and quarks, rather than the different quantum numbers used by the SM: (i) charge Q, lepton number L, electron lepton number L_e, muon lepton number L_μ and tau lepton number L_τ for leptons; (ii) charge Q, baryon number A, strangeness S, charm C, bottomness B and topness T for quarks. In this unified system, the particle number p replaces both lepton number L and baryon number A, while the generation quantum number g replaces the remaining quantum numbers, L_e, L_μ, L_τ, S, C, B and T of the SM. Furthermore, the generation quantum number, unlike S, C, B and T of the SM, is conserved in all weak interaction processes.

The unified classification scheme leads to new isospin relations.[6] For strong isospin, its third component I_3 is related to charge by the equation:

$$Q = I_3 + \frac{1}{2}(p + g)\,. \tag{1}$$

For weak isospin, its third component i_3 is related to charge by the equation:

$$Q = i_3 + \frac{1}{2}p\,, \tag{2}$$

which implies the relation:

$$i_3 = I_3 + \frac{1}{2}g\,. \tag{3}$$

Since Q, p and g are strictly conserved in all interactions, these equations imply that both I_3 and i_3 are also conserved in all interactions. The equations also suggest an underlying flavor SU(3) symmetry for both leptons and quarks and led to the development of a composite model, the Composite Generation Model (CGM),[7] of these fundamental particles of the SM.

Secondly, the GM assumes[8] that hadrons are composed of mixed-quark states. This differs from the SM in which hadrons are assumed to be composed of pure-quark states, i.e. single flavor quarks. Thus in the GM, the proton is considered to consist essentially of two up (u) quarks and one mixed (d') quark, which is a linear superposition of the down (d) quark, the strange (s) quark and the bottom (b) quark:

$$d' = V_{ud}d + V_{us}s + V_{ub}b\,, \tag{4}$$

where V_{ij} are the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix,[9,10] rather than two up quarks and one down quark as in the SM. The different treatment of quark mixing in the GM allows for a conserved additive quantum number, g, to be allotted to the physical quarks. This leads to selection rules which permit the quarks to be classified into weak isospin doublets, analogous to the leptons, so that one has complete lepton-quark universality. Thus, in the GM, the quark weak isospin doublets are (u, d), (c, s) and (t, b), where c and t denote the charm and the top quark, respectively, rather than the mixed-quark doublets (u, d'), (c, s') and (t, b') of the SM. The pure-quark weak isospin doublets, (u, d), (c, s) and (t, b), are analogous to the three lepton doublets, (ν_e, e^-), (ν_μ, μ^-) and (ν_τ, τ^-), associated with the conserved lepton numbers, L_e, L_μ and L_τ of the SM, respectively. In the GM both the pure-quark and lepton doublets are associated with the conservation of the generation quantum number g.

In the CGM the mediating massive vector bosons, W^+, Z^0 and W^- are considered to be composite particles, consisting of colorless sets of three rishons or three antirishons, so that they are *not* fundamental particles, associated with a $\mathrm{U}(1) \times \mathrm{SU}(2)$ *local* gauge theory as in the SM. The weak interactions are simply residual interactions of the CGM super-strong "color" force, which binds rishons together, analogous to the strong nuclear interactions, mediated by massive mesons, being residual interactions of the strong color force of the SM, which binds quarks together. In the CGM both the strong color force and the strong nuclear force of the SM are residual interactions of the super-strong color force. Since the weak interactions are not considered to be fundamental interactions, there is no requirement for the existence of a Higgs field[11] within the framework of the GM.

The GM also differs from the SM in demanding that electroweak processes can be described using quantum numbers which do *not* depend upon the handedness of a particle. This requirement is an essential feature of the CGM in which both leptons and quarks are composed of rishons, and the conservation of the three independent additive quantum numbers, Q, p and g, corresponds to the conservation of the three kinds of rishons. In the usual development (see Sec. 2) of the SM, it seemed natural to treat the upper and lower components of the Dirac spinors differently. For massless particles, these components correspond to particles with left-handed and right-handed helicity, respectively, depending upon the representation employed. This approach led to the requirement that the upper and lower components of the Dirac spinors, loosely referred to as left-handed and right-handed particles, respectively, are associated with *different* additive quantum numbers for weak isospin (i_3) and weak hypercharge (y) satisfying an equation, analogous to the usual Gell-Mann–Nishijima relation[12,13]:

$$Q = i_3 + \frac{1}{2} y \,. \tag{5}$$

In the SM, the left-handed fermions are assumed to be weak isospin doublets ($i = \frac{1}{2}$) while the right-handed fermions are assumed to be weak isospin singlets ($i = 0$). This rather bizarre notion, together with the requirement of the "Higgs mechanism"[11,14] to spontaneously break the $\mathrm{U}(1) \times \mathrm{SU}(2)$ local gauge symmetry to generate the masses of the gauge bosons W^+, Z^0 and W^-, led to the derivation of the electroweak connection.

The main purpose of this paper is to show that the electroweak connection can also be obtained within the framework of the GM. In Sec. 2 we shall describe the development of the GSW model in more detail in order to understand the essential differences between the GSW model and the GM. In Sec. 3 we shall formulate the electroweak connection within the framework of the GM. Section 4 states the conclusions.

2. The Standard Model and the Electroweak Connection

The SM derivation of the electroweak connection is based upon two main ideas: (i) nature exhibits an SU(2) symmetry associated with the weak interaction, as indicated by experimental observations of weak interaction processes; (ii) the weak interaction is a *fundamental* interaction on a par with the electromagnetic interaction. This led to the notion that the weak interaction is a consequence of an SU(2) *local* gauge transformation, analogous to the electromagnetic interaction, which obeys a U(1) local gauge transformation.

The concept of gauge invariance as a physical principal governing the fundamental interactions between elementary particles was first proposed by Weyl[15] in an attempt to extend ideas employed by Einstein's General Theory of Relativity, involving the gravitational force, to the case of the electromagnetic interaction. This initial attempt by Weyl, involving a "scale invariance" of spacetime, failed. However, with the development of quantum mechanics, it was realized[16–18] that Weyl's original gauge theory could be given a new interpretation: a gauge transformation corresponds to a change in the phase of the wavefunction describing a particle, rather than a change of scale.

In the case of quantum electrodynamics (QED), which describes the interaction of photons, electrons and positrons, the Lagrangian density is

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu \,, \tag{6}$$

where the electromagnetic field-strength tensor is given as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,. \tag{7}$$

Here, A_μ is the electromagnetic field, ψ is the electron field, $\bar{\psi} = \psi^\dagger\gamma^0$, m and e are the mass and charge of the electron, respectively, and the γ^μ are the Dirac matrices:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma^k = \begin{pmatrix} 0 & -\sigma_k \\ \sigma_k & 0 \end{pmatrix}, \tag{8}$$

where σ_k $(k = 1, 2, 3)$ are the Pauli matrices.

The Lagrangian (6) is invariant under a U(1) local gauge transformation of the electron field $\psi(x)$ given by

$$\psi(x) \to \psi'(x) = e^{i\lambda(x)}\psi(x) \,, \tag{9}$$

where $\lambda(x)$ is an arbitrary real function of position, provided that simultaneously

$$A_\mu \to A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\lambda(x) \,. \tag{10}$$

In this case, the requirement of a U(1) local gauge invariance is said to lead to the occurrence of a *fundamental* interaction, the gauge field A_μ, acting between the particles of the electron field $\psi(x)$. The electromagnetic current of electrons, given by

$$j^\mu_{em} = \bar{\psi}\gamma^\mu\psi \,, \tag{11}$$

is conserved, independent of whether the U(1) transformation is local or global (i.e. λ constant).

In the SM derivation of the electroweak connection, the symmetry involved is more complex, namely U(1) $\times$ SU(2), so that the local gauge principle associated with QED has to be extended to the non-Abelian case of SU(2), as studied by Yang and Mills[19] in 1954. This extension is made more difficult since, in general, the gauge principle does not provide a *unique* fundamental interaction: the nature of the interaction and also the symmetry involved have essentially to be determined by experiment.

In 1938, Klein[20] suggested that the weak interactions could be mediated by massive charged bosons, now called W^+ and W^- bosons, which had properties similar to those of photons. He termed them "electrically charged photons" but unlike photons they were massive in order to satisfy the very short-range nature of the weak interactions. These charged bosons were analogous to the massive mesons predicted by Yukawa[21] as the mediators of the short-range strong nuclear interaction.

During the late 1940s it was found that the weak interactions possessed a property called "universality". Analysis of experiments revealed that the coupling constants for μ-decay and μ-capture were of the same order of magnitude as those for nuclear β-decay. This led to the hypothesis of a *universal weak interaction.*[22–26]

In 1957 Schwinger,[27] following the ideas of Klein, suggested a weak isospin triplet (W^+, γ, W^-) of vector fields, whose universal couplings generated both the charge-changing weak interactions and the electromagnetic interaction: the two oppositely charged W fields mediating the charge-changing weak interactions while the neutral field (γ) mediated the electromagnetic interaction. This suggestion was based upon the notion that the weak interactions are *fundamental* interactions like the electromagnetic interaction and that these interactions arose from an SU(2) local gauge theory. This endeavor by Schwinger was the first attempt to unify the electromagnetic and weak interactions. However, it suffered from the fact that the large masses of the W bosons, required to account for the very short-range nature of the weak interactions, had to be inserted into the theory "by hand" in conflict with the gauge invariance requirements of the theory.

In 1958, it was shown[28,29] that the so-called "V–A" theory of weak interactions described the observed[30–32] parity violations which had been predicted by Lee and Yang[33] in 1956. Although the V–A theory only contained charge-changing weak interactions, it was quickly realized that the corresponding weak currents involved what came to be known as weak isospin doublets: e.g. $(\nu_e,\ e^-)$, which had weak isospin $i = \frac{1}{2}$ with ν_e and e^- having third component $i_3 = +\frac{1}{2}$ and $i_3 = -\frac{1}{2}$, respectively.

Indeed, Bludman[34] in 1958 suggested that many aspects of the weak interactions could be described by an SU(2) Yang–Mills gauge theory with a triplet of three vector bosons, W^+, W^0 and W^-, in a "weak isospin" space. Moreover, Bludman showed that the "V–A" interaction was invariant under a *global* gauge transformation given by

$$\psi \to \psi' = \exp[i\mathbf{\Lambda} \cdot \boldsymbol{\tau}\,\Gamma]\psi\,, \tag{12}$$

where ψ is a "doublet" of Dirac spinors describing the weak isospin doublet fields, $\mathbf{\Lambda}$ is an arbitrary constant vector, $\boldsymbol{\tau}$ is the weak isospin vector with the Pauli matrices divided by two as components:

$$\tau_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \tau_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \tau_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{13}$$

and defining

$$\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{14}$$

$$\Gamma = \frac{1}{2}(1 - \gamma^5)\,, \tag{15}$$

is the usual operator, which projects out left-handed particle states. In addition, Bludman showed that other kinds of possible weak interactions, scalar, tensor and pseudoscalar, are not invariant under the transformation (12). This formulation of Bludman indicated the possibility of weak *neutral* currents, which are distinct from the usual electromagnetic currents.

In 1961, Glashow proposed[2] that the weak interaction as suggested by Bludman could be associated with the electromagnetic interaction if the overall symmetry was extended to a U(1) × SU(2) local gauge theory. This was a major step in the development of the electroweak connection.

Glashow's model involved both a triplet ($i = 1$) of vector bosons (W^+, W^0, W^-) and a singlet ($i = 0$) vector boson B^0. The two neutral bosons "mixed" in such a way that they produced a massive Z^0 boson and the massless photon (γ):

$$\gamma = B^0 \cos\theta_W + W^0 \sin\theta_W\,, \tag{16}$$

$$Z^0 = -B^0 \sin\theta_W + W^0 \cos\theta_W\,, \tag{17}$$

where θ_W is the electroweak mixing angle. Experiment requires the masses of the weak gauge bosons, W and Z, to be heavy so that the weak interactions are very short-ranged. On the other hand, Glashow's proposal, based upon the concept of a non-Abelian SU(2) Yang–Mills gauge theory, requires the mediators of the weak interactions to be massless like the photon. In 1961, Glashow simply inserted the masses of the weak bosons by hand.

In order to ensure that the electromagnetic interactions mediated by the photon conserved parity, while the weak interactions mediated by W^+, Z^0 and W^- bosons did not, Glashow introduced the notion of "weak hypercharge" (y), related to the charge (Q) and the third component of weak isospin by the analogue of the Gell-Mann–Nishijima relation (Eq. (5)).

In addition it was found to be convenient to explicitly separate the left- and right-handed helicity states of the Dirac spinors. Thus the left-handed helicity states of the electron neutrino ν_e and the electron e^- were considered to form a weak isospin doublet:

$$\chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L = \Gamma \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} . \tag{18}$$

On the other hand, the right-handed helicity states of the Dirac spinors were assumed to be weak isospin singlets, e.g. $(\nu_e)_R$ and $(e^-)_R$ have $i = i_3 = 0$. At the time the electron neutrino was assumed to be massless and $(\nu_e)_R$ non-existent.

Glashow also assumed that the weak hypercharge current was related to the electromagnetic current and the third component of the weak isospin current by a relation analogous to the Gell-Mann–Nishijima relation:

$$\frac{1}{2} j_y^\mu = j_{em}^\mu - j_3^\mu , \tag{19}$$

where

$$j_y^\mu = \bar{\chi}\gamma^\mu \tilde{y}\chi, \qquad j_{em}^\mu = \bar{\chi}\gamma^\mu \tilde{Q}\chi, \qquad j_3^\mu = \bar{\chi}\gamma^\mu \Gamma \tilde{i}_3 \chi . \tag{20}$$

Here the wave function χ represents either a left-handed doublet or a right-handed singlet wave function, and $\tilde{y}$, $\tilde{Q}$ and $\tilde{i}_3$ are the corresponding weak hypercharge, charge and weak isospin matrix operators, respectively.

This implied that the underlying symmetry was $\mathrm{SU}(2)_L \times \mathrm{U}(1)_y$, where $\mathrm{SU}(2)_L$ refers to weak isospin, involving only left-handed particles, while $\mathrm{U}(1)_y$ refers to weak hypercharge, involving both left-handed and right-handed particles. In order to accommodate these ideas, different additive quantum numbers are required for left-handed and right-handed particles. This arises as a direct consequence of Eq. (19): for left-handed particles, $Q = i_3 + \frac{1}{2}y$, but for right-handed particles, $Q = \frac{1}{2}y$, since $i_3 = 0$.

In practice we have both left-handed and right-handed currents for each pair of leptons associated with a given weak interaction. Thus there are currents corresponding to the three generations of leptons: (ν_e, e^-), (ν_μ, μ^-) and (ν_τ, τ^-). In addition, following the development of the quark model,[35–37] the hadronic currents of the V–A theory can be interpreted as quark currents. In the quark model of the SM, it is customary to hypothesize that the quark which couples to the up quark via the weak interactions is the mixed-quark state given by Eq. (4). This "sharing" of the weak interaction between the components of the mixed-quark (d') state in the case of the pair (u, d') is required to describe the experimental data in terms of a universal weak interaction. Thus in the SM there are currents corresponding to the three generations of quarks: (u, d'), (c, s') and (t, b'), where s' and b' are CKM mixed-quark states of the d, s and b quarks, analogous to the d' mixed-quark state of Eq. (4). However, in the following, we shall only need to consider one such pair of fermions in order to establish the electroweak connection, since each pair contributes additively and independently to each total current. For simplicity, we shall consider the lepton pair (ν_e, e^-).

Glashow[2] proposed that the electroweak interaction was of the form:

$$H_{ew} = g_w \mathbf{j}^\mu \cdot \mathbf{W}_\mu + g'_w j_0^\mu B_\mu \,, \tag{21}$$

where the first term corresponds to an $SU(2)_L$ symmetry and the second term with weak charge g'_w corresponds to an independent U(1) symmetry. The current associated with the second term, j_0^μ, is required to be invariant in the weak isospin space possessing $SU(2)_L$ symmetry. Glashow assumed that the current j_0^μ was identical with the weak hypercharge current $\frac{1}{2}j_y^\mu$. In fact, the appropriate current, as demonstrated by Gottfried and Weisskopf,[38] is $(j_{em}^\mu - j_3^\mu)$, which is not equal to $\frac{1}{2}j_y^\mu$ unless different quantum numbers are employed for left-handed and right-handed particles, as assumed by Glashow. Thus the complete electroweak interaction is

$$H_{ew} = g_w \mathbf{j}^\mu \cdot \mathbf{W}_\mu + g'_w (j_{em}^\mu - j_3^\mu) B_\mu \,. \tag{22}$$

It should be noted that the currents $\mathbf{j}^\mu$ and j_{em}^μ in Eq. (22) can be defined to be

$$\mathbf{j}^\mu = \bar{\chi}\gamma^\mu \Gamma \boldsymbol{\tau} \chi, \qquad j_{em}^\mu = \bar{\chi}\gamma^\mu \tilde{Q} \chi \,, \tag{23}$$

where the wave function χ now represents a doublet of *complete* Dirac spinors, since the right-handed singlet states do not contribute to $\mathbf{j}^\mu$ and the charge matrix operator is diagonal. This implies that Eq. (22) can in principle be obtained without the separation of the Dirac spinors into left- and right-handed helicity states.

From this interaction it is quite straightforward to derive the electroweak connection. The total neutral (charge-preserving) interaction is given by

$$H_{ew}^0 = g'_w B_\mu j_{em}^\mu + (g_w W_\mu^3 - g'_w B_\mu) j_3^\mu \,. \tag{24}$$

For the neutral electron neutrino, only the second term involving j_3^μ contributes so that this must be associated with the weak boson Z^0, i.e.

$$Z_\mu = (g_w W_\mu^3 - g'_w B_\mu)/(g_w^2 + g_w'^2)^{\frac{1}{2}} \,, \tag{25}$$

where we have inserted a normalization factor $(g_w^2 + g_w'^2)^{-\frac{1}{2}}$ so that the states created and destroyed by Z_μ (and A_μ) are normalized in the same manner as those of B_μ and W_μ^3. From Eqs. (17) and (25) it is seen that the electroweak mixing angle is given by

$$\sin\theta_W = g'_w/(g_w^2 + g_w'^2)^{\frac{1}{2}}, \qquad \cos\theta_W = g_w/(g_w^2 + g_w'^2)^{\frac{1}{2}} \,. \tag{26}$$

The orthogonal linearly independent combination, corresponding to Eq. (25), is the electromagnetic interaction:

$$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W \,. \tag{27}$$

From Eq. (27) we have, equating the associated currents and coupling constants,

$$e j_{em}^\mu = g'_w (j_{em}^\mu - j_3^\mu) \cos\theta_W + g_w j_3^\mu \sin\theta_W \,, \tag{28}$$

so that

$$e = g'_w \cos\theta_W = g_w \sin\theta_W \,. \tag{29}$$

Furthermore, if we write the weak neutral interaction as $g_Z j^\mu_{NC} Z_\mu$, then from Eq. (25) we have, equating the associated currents and coupling constants,

$$g_Z j^\mu_{NC} = g_w j^\mu_3 \cos\theta_W - g'_w (j^\mu_{em} - j^\mu_3) \sin\theta_W \,, \tag{30}$$

so that

$$g_Z j^\mu_{NC} = g_w (j^\mu_3 - j^\mu_{em} \sin^2\theta_W)/\cos\theta_W \,. \tag{31}$$

Thus the complete electroweak interaction has the form:

$$H_{ew} = e j^\mu_{em} A_\mu + g_w (j^\mu_1 W^1_\mu + j^\mu_2 W^2_\mu) + g_w (j^\mu_3 - j^\mu_{em} \sin^2\theta_W) Z_\mu / \cos\theta_W \,, \tag{32}$$

which gives the electroweak connection: the neutral weak interaction mediated by the Z^0 bosons is completely determined by the electromagnetic and charge-changing interactions, and their coupling constants e and g_w.

The boson mass problem was resolved by Weinberg[3] and Salam,[4] who independently employed the idea of spontaneous symmetry breaking, involving the Higgs mechanism. In this way, the W and Z bosons acquire mass and the photon remains massless. Indeed the GWS model gives the relative masses of the W and Z bosons in terms of the electroweak mixing angle:

$$M_W = M_Z \cos\theta_W \,. \tag{33}$$

The Higgs mechanism was also able to cure the associated fermion mass problem[39]: the finite masses of the leptons and quarks cause the Lagrangian describing the system to violate the $SU(2)_L$ gauge invariance. By coupling the originally massless fermions to the scalar Higgs field, it is possible to produce the observed physical fermion masses without violating the gauge invariance. However, the GWS model requires the existence of a new massive spin zero boson, the Higgs boson, which to date remains to be detected. In addition, the fermion-Higgs coupling strength is dependent upon the mass of the fermion, so that a new parameter is required for each fermion mass in the theory.

In 1971, 't Hooft showed[40] that the GWS model of the electroweak interactions was renormalizable and this self-consistency of the theory led to its general acceptance. In 1973, events corresponding to the predicted neutral currents mediated by the Z^0 boson were observed,[41,42] while bosons, with approximately the expected masses, were discovered[43,44] in 1983, thereby confirming the GWS model.

3. The Generation Model and the Electroweak Connection

The GM adopts the point of view that the weak interactions, associated with the weak isospin symmetry, are *not* fundamental interactions arising from an SU(2) local gauge theory as in the SM. Rather, the weak interactions are residual interactions of a super-strong force, responsible for binding the constituents of the leptons and quarks together.[7] This latter force is assumed to be a super-strong color force, analogous to the strong color force of the SM, which binds quarks together to form baryons or mesons, and is associated with a local SU(3) gauge field mediated by massless particles (hypergluons). In the CGM, the strong color force of the SM is a different residual interaction of the super-strong color force.

Thus in the GM, the weak interactions are assumed to be "effective" interactions, i.e. they are approximate interactions that contain the appropriate degrees of freedom to describe the experimental data occurring at sufficiently low energies for which any substructure and its associated degrees of freedom may be ignored. In the CGM, leptons, quarks and the W and Z bosons are all considered to be composite particles, built out of rishons or antirishons, held together by the super-strong color force. The massive vector bosons, which mediate the effective weak interactions, are analogous to the massive mesons, which mediate the effective nuclear interactions between neutrons and protons.

The non-fundamental nature of the weak interactions in the GM (and the CGM) means that the question of renormalizability does not arise. Thus the mediating particles may be massive since this does not destroy any SU(2) local gauge invariance giving rise to a fundamental interaction. In the CGM the fundamental interaction is the super-strong color interaction, which in principle leads to a renormalizable theory for the electroweak interactions, provided the substructure of leptons, quarks and the W and Z bosons is taken into account.

The appropriate effective weak interaction is required to be obtained from experiment by analyzing weak interaction processes at the relevant energies. In this section, we shall follow the development of the phenomenological approach to the weak interaction inherent in the V–A theory in order to determine the effective weak interaction.

The first weak interaction process, neutron β-decay,

$$n^0 \to p^+ + e^- + \bar{\nu}_e \,, \tag{34}$$

was discovered[45] in 1896. However, the first successful theory of β-decay was not published until 1934 by Fermi.[46,47] This was partly due to such weak interactions being complicated by the involvement of strongly interacting particles. Taking into account Pauli's neutrino hypothesis of 1930,[48] Fermi assumed, by analogy with the electromagnetic decay of an excited atom involving the emission of a photon, that β-decay was also described by a vector interaction, with an electron-antineutrino pair being emitted at a single spacetime point.

Fermi described the β-decay process in terms of two interacting currents, analogous to the Dirac electromagnetic current, j^μ_{em}, so that the matrix element describing the process could be written as

$$M = \frac{G}{\sqrt{2}} j_1^\mu j_2^\mu \,, \tag{35}$$

where G is the Fermi weak coupling constant and j_1^μ and j_2^μ are given by

$$j_1^\mu = \bar{\psi}_p \gamma^\mu \psi_n , \qquad j_2^\mu = \bar{\psi}_e \gamma^\mu \psi_\nu \, . \tag{36}$$

Unfortunately, this four-fermion point contact model failed to describe later experimental data of weak interaction processes. This led to a generalization of the currents in the Fermi model. In addition to the vector currents involving γ^μ, scalar, tensor, axial vector and pseudoscalar currents were introduced into the matrix elements describing the weak interaction processes. This generalization allowed all the available β-decay data at the time to be described. However, there still remained some outstanding problems and later more data, which could not be understood using the generalized Fermi model.

Firstly, pion decay such as

$$\pi^- \to \mu^- + \bar{\nu}_\mu \, , \tag{37}$$

could not be interpreted within the model until the quark structure of hadrons was proposed,[35–37] so that the above pion decay could be understood as the four-fermion transition:

$$\pi^- \equiv (d + \bar{u}) \to \mu^- + \bar{\nu}_\mu \, . \tag{38}$$

Secondly, it was noted that for sufficiently high energies ($\approx$ 300 GeV) the matrix element M of Eq. (35) leads to a cross section for the scattering of two fermions, which violates the unitarity condition associated with the partial wave amplitudes. This led to the introduction of a propagator, corresponding to a massive intermediate vector boson (the W boson) mediating the charge-changing weak interaction, into the matrix element M. For low energies, this gives

$$M = \frac{g_w^2}{8M_W^2} j_1^\mu j_2^\mu \, , \tag{39}$$

where M_W is the mass of the W boson so that equating Eqs. (35) and (39) we can relate M_W to the Fermi weak coupling constant:

$$\frac{G}{\sqrt{2}} = \frac{g_w^2}{8M_W^2} \, . \tag{40}$$

Thus, although the insertion of a massive boson propagator into the matrix element M did not fully resolve the unitarity problem,[39] it does allow the boson mass to be estimated from Eq. (40) using values of G and g_w obtained from experiment: the estimated mass $M_W \approx 80$ GeV.

Thirdly, the 1957 discovery[30–32] of parity violations in weak interaction processes was in contradiction with the Fermi model, which only involved vector currents. This led to two new hypotheses: (i) the two-component neutrino;[49–51] (ii) the weak interaction involves only left-handed fermions.[28,29]

The two-component neutrino hypothesis requires the neutrino to be massless. In this case the neutrino will exist in a state of definite helicity. In 1958, the helicity of the neutrino participating in a weak interaction was measured[52] and was found to be negative and the neutrino left-handed. At the time this was taken as confirmation of the two-component hypothesis. However, in recent years, evidence has been found[53] that neutrinos have mass, albeit very small. Thus the left-handed nature of neutrinos must be attributed to the weak interactions rather than to the neutrinos themselves, i.e. it arises as a consequence of the second hypothesis above.

If the second hypothesis is adopted, the matrix element describing the β-decay weak interaction processes may be written as in Eq. (35) but now the interacting currents become

$$j_1^\mu = \bar{\psi}_p \bar{\Gamma} \gamma^\mu \Gamma \psi_n = \bar{\psi}_p \gamma^\mu \Gamma \psi_n \,, \tag{41}$$

and

$$j_2^\mu = \bar{\psi}_e \gamma^\mu \Gamma \psi_\nu \,, \tag{42}$$

since

$$\bar{\Gamma} = \frac{1}{2}(1+\gamma^5), \qquad \bar{\Gamma}\gamma^\mu = \gamma^\mu \Gamma, \qquad \Gamma^2 = \Gamma \,. \tag{43}$$

Here, the presence of the projection operator Γ ensures that only the left-handed components of the fermion fields are involved and since $\bar{\Gamma}\Gamma = 0$, that any scalar, tensor and pseudoscalar interactions are forbidden.

The universality of the charge-changing weak interaction processes, mediated by the W^+ and W^- bosons, and the observation of parity nonconservation in such processes, led to the discovery of an $\mathrm{SU}(2)_L$ symmetry in nature. While this symmetry appears to be *exact* for leptons, so that the weak charge (g_w) of leptons is conserved, in the case of quarks, only the vector (V) part of the weak charge is conserved,[28] while the axial vector (A) part is not.[54] It seems that for those processes in which strong interactions are also involved, the axial vector currents are *not* conserved. The origin of the axial vector parts of the weak interactions, which lead to parity violating processes, is still not understood. Thus for the purposes of this paper, we shall assume that in the *absence* of strong quark–quark interactions, that the axial vector part of the weak charge is conserved. Furthermore, as in the SM case, for simplicity we shall consider the lepton pair (ν_e, e^-) as representative of the set of fermion pairs, which contribute additively and independently to each total current.

In the GM the weak isospin states are not separated into left-handed doublets and right-handed singlets: we have pure weak isospin doublets so that the particles do *not* have quantum numbers, i_3 and y, which depend upon their handedness. Furthermore, the weak hypercharge quantum number y is replaced by the particle number p [see Eqs. (2) and (5)]. Thus, the weak isospin doublet (ν_e, e^-) can be written as a doublet of complete Dirac spinors:

$$\chi = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} . \tag{44}$$

In the GM it is expected that the electroweak interaction, H_{ew} [see Eq. (22)], as deduced primarily from experiment, will be invariant under a $\mathrm{U}(1)_p \times \mathrm{SU}(2)_L$ *global* gauge transformation, corresponding to the conservation of both particle number p and weak charge g_w. Thus, the interaction H_{ew} should be invariant under the global gauge transformation

$$\chi \to \chi' = \exp[i\lambda p] \exp[i\boldsymbol{\Lambda} \cdot \boldsymbol{\tau}\Gamma]\chi\,, \tag{45}$$

where λ and $\boldsymbol{\Lambda}$ is an arbitrary constant and an arbitrary constant vector, respectively.

We have from Eq. (22)

$$H_{ew} = g_w \bar{\chi}\gamma^\mu \Gamma(\boldsymbol{\tau} \cdot \mathbf{W}_\mu)\chi + g'_w \bar{\chi}\gamma^\mu(\tilde{Q} - \Gamma\tau_3)\chi B_\mu\,. \tag{46}$$

For an infinitesimal $\boldsymbol{\Lambda}$ global gauge transformation (45)

$$\begin{aligned} H_{ew} \to H'_{ew} = H_{ew} &- ig_w \bar{\chi}\gamma^\mu \Gamma[(\boldsymbol{\Lambda} \cdot \boldsymbol{\tau})(\boldsymbol{\tau} \cdot \mathbf{W}_\mu) - (\boldsymbol{\tau} \cdot \mathbf{W}_\mu)(\boldsymbol{\Lambda} \cdot \boldsymbol{\tau})]\chi \\ &- ig'_w \bar{\chi}\gamma^\mu \Gamma[(\boldsymbol{\Lambda} \cdot \boldsymbol{\tau})(\tilde{Q} - \tau_3\Gamma) - (\tilde{Q} - \tau_3\Gamma)(\boldsymbol{\Lambda} \cdot \boldsymbol{\tau})]\chi B_\mu\,, \end{aligned} \tag{47}$$

so that

$$H'_{ew} = H_{ew} + g_w \bar{\chi}\gamma^\mu \Gamma[\boldsymbol{\tau} \cdot (\boldsymbol{\Lambda} \times \mathbf{W}_\mu)]\chi - g'_w \bar{\chi}\gamma^\mu \Gamma\bar{\Gamma}(\boldsymbol{\Lambda} \times \boldsymbol{\tau})_3 \chi B_\mu\,. \tag{48}$$

The term involving g'_w is invariant under the global gauge transformation since

$$i[(\boldsymbol{\Lambda} \cdot \boldsymbol{\tau})(\tilde{Q} - \tau_3\Gamma) - (\tilde{Q} - \tau_3\Gamma)(\boldsymbol{\Lambda} \cdot \boldsymbol{\tau})] = \bar{\Gamma}(\boldsymbol{\Lambda} \times \boldsymbol{\tau})_3\,, \tag{49}$$

$\bar{\Gamma} = 1 - \Gamma$ and the matrix charge operator $\tilde{Q}$ is of the form

$$\tilde{Q} = \begin{pmatrix} Q & 0 \\ 0 & Q - 1 \end{pmatrix}, \tag{50}$$

which is true for each lepton and quark doublet. Thus, since $\Gamma\bar{\Gamma} = 0$

$$H'_{ew} = H_{ew} + g_w \bar{\chi}\gamma^\mu \Gamma[\boldsymbol{\tau} \cdot (\boldsymbol{\Lambda} \times \mathbf{W}_\mu)]\chi\,. \tag{51}$$

The term involving g_w is invariant under the global gauge transformation provided that simultaneously, either

$$\mathbf{W}_\mu \to \mathbf{W}'_\mu = \mathbf{W}_\mu - (\boldsymbol{\Lambda} \times \mathbf{W}_\mu)\,, \tag{52}$$

or

$$\boldsymbol{\tau} \to \boldsymbol{\tau}' = \boldsymbol{\tau} + \Gamma(\boldsymbol{\Lambda} \times \boldsymbol{\tau})\,. \tag{53}$$

The latter compensating transformation arises from the relation

$$\boldsymbol{\tau} \cdot (\boldsymbol{\Lambda} \times \mathbf{W}_\mu) = -\mathbf{W}_\mu \cdot (\boldsymbol{\Lambda} \times \boldsymbol{\tau}) \tag{54}$$

and the requirement that under the $\mathrm{SU}(2)_L$ global gauge transformation the doublet weak isospin basis states remain weak isospin eigenstates. Similar transformations are required for other operators such as $\tilde{Q}$ in the weak isospin space. Thus, for the matrix operator $\tilde{Q}$ we have

$$\tilde{Q} \to \tilde{Q}' = \tilde{Q} + \Gamma(\mathbf{\Lambda} \times \boldsymbol{\tau})_3 \tag{55}$$

so that the combined operator $(\tilde{Q} - \Gamma\tau_3)$ transforms to

$$\tilde{Q} + \Gamma(\mathbf{\Lambda} \times \boldsymbol{\tau})_3 - \Gamma\tau_3 - \Gamma^2(\mathbf{\Lambda} \times \boldsymbol{\tau})_3 = \tilde{Q} - \Gamma\tau_3\,. \tag{56}$$

It is concluded that Eq. (53) is more natural and appropriate than Eq. (52) as the compensating transformation to maintain global gauge invariance. Furthermore, the electroweak interaction given by Eq. (22) has the appropriate properties to qualify as the effective electroweak interaction within the framework of the GM. In this case, the derivation of the electroweak connection (Eq. (32)) follows essentially the same path as that given in Sec. 2 for the SM. Moreover, the relation (Eq. (33)) between the masses of the W and Z bosons is given by Eq. (32) if it is assumed that the strengths of the charge-changing and the neutral currents are about the same.

4. Conclusion and Discussion

The relationship between the electromagnetic and weak interactions, known as the electroweak connection, was first derived within the framework of the SM. It gives the electroweak interaction in terms of two independent coupling constants, an electric charge and a weak charge. This electroweak interaction is in excellent agreement with the experimental data. However, its derivation within the framework of the SM suffers from a number of problems.

Firstly, the SM requires the existence of a scalar Higgs field to spontaneously break the assumed U(1) $\times$ SU(2) local gauge symmetry. This implies the existence of a new massive spin zero boson, the Higgs boson, which to date remains to be detected.

Secondly, the SM requires the Higgs field to couple to the originally massless fermions, the leptons and quarks, to produce their finite masses, in a manner which does not violate the assumed gauge invariance. In the SM this requires the fermion-Higgs coupling strength to be dependent upon the mass of the fermion, so that a new parameter is required for each fermion mass in the theory.

In addition, the requirement of a Higgs field, which fills the whole of space, leads to a cosmological term in the General Theory of Relativity, which is much larger than is allowed by observations.[55] Thus, either the Higgs field does not exist, or its energy density is canceled by some, as yet unknown, contribution.

In the GM, all the above problems inherent in the SM are avoided by assuming that the weak interactions are not fundamental interactions arising from a local gauge invariance. The weak interactions are treated as effective interactions, arising from residual interactions of the super-strong force, which binds the constituents of leptons and quarks together. It is found that a global U(1) x SU(2) gauge invariance, corresponding to the conservation of particle number p and weak charge g_w, is sufficient to determine the electroweak connection.

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