Finite Horizon H^∞ Control for a Class of Sampled-Data Linear Quantum Systems

Aline I. Maalouf * *Member, IEEE*, Ian R. Petersen * *Fellow, IEEE*

*SEIT, UNSW@ADFA, Canberra, ACT 2600

Email: alinemaalouf@gmail.com, i.r.petersen@gmail.com

Abstract

In this paper, a finite horizon H^∞ control problem is solved for a class of linear quantum systems using a dynamic game approach for the case of sampled-data measurements. The methodology adopted involves an equivalence between the quantum problem and two auxiliary classical problems.

Index Terms

Quantum control, finite horizon H^∞ control, sampled-data measurements.

I. INTRODUCTION

The control of quantum systems is a rapidly growing and evolving field whose applications include quantum computing, control of molecular dynamics, control of semiconductor nano-devices, control of charged particles in beam accelerators; etc., see [1].

In practical applications, it is unavoidable that quantum systems are subject to all kinds of disturbances and uncertainties. Note that incomplete knowledge and unknown errors can also be treated as uncertainties. Thus, robustness has been identified as an important aspect for the practical application of quantum technology.

Recently, several robust control methods in classical control theory have been extended into the quantum domain. For instance, [2] extended J.C. Willems theory of dissipative systems to the quantum domain. This general theory, which combines perspectives from the quantum physics and control engineering communities, provides useful methods for analysis and design

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of dissipative quantum systems. The authors describe the interaction of the plant and a class of exosystems in general quantum feedback network terms. The results include an infinitesimal characterization of the dissipation property, which generalizes the well-known Positive Real and Bounded Real Lemmas, and is used to study some properties of quantum dissipative systems. The authors also show how to formulate control design problems using quantum network models, which implements Willems control by interconnection for open quantum systems. This control design formulation includes, for example, standard problems of stabilization, regulation, and robust control. On the other hand, the small gain theorem has been applied to the stability analysis of quantum feedback networks [3]. A transfer function approach has been applied to the feedback and robust control for single input single output quantum systems [4], [5].

An H^∞ controller synthesis problem has been formulated for a class of linear quantum stochastic systems in [6]. In fact, H^∞ methods are used in control theory to synthesize controllers in order to achieve robust performance and/or stabilization. To use H^∞ methods, the control system designer expresses the control problem as a mathematical optimization problem and then finds the controller that solves it. H^∞ techniques have the advantage over classical control techniques in that they are readily applicable to problems involving multivariable systems with cross-coupling between channels. However, their disadvantages include the level of mathematical understanding required to apply them successfully and the need for a reasonably good model of the system to be controlled. Problem formulation is important since any synthesized controller will only be optimal in the formulated sense: optimizing the wrong thing often makes things worse rather than better. Also, non-linear constraints such as saturation are generally not well-handled.

One of the major concentrated activities in the control theory of the past three decades has been the development of the H^∞ -optimal control theory which addresses the issue of the worst-case controller design for classical linear plants subject to unknown additive disturbances and plant uncertainties, including problems of disturbance attenuation, model matching, and tracking. Formulated originally in the frequency domain, the main tools used during the early phases of research on this classical class of problems have been operator and approximation theory, spectral factorization, and Youla parametrization. Further work has shown that in a time-domain characterization, Riccati equations of the type that arise in linear quadratic differential games play a key role. Among the different time-domain approaches to this class of worst-case design problems, the one that uses the framework of dynamic differential game theory seems to be the

most natural. This is because the original H^∞ -optimal control problem (in its equivalent time-domain formulation) is in fact a minimax optimization problem and, hence, a zero-sum game in which the controller can be viewed as the minimizing player and disturbance as the maximizing player. In the recent paper [6], the problem of systematic robust control system design for a class of quantum linear systems is tackled via an H^∞ approach. The results of [6] have also been implemented experimentally as described in [7]. In [6], the quantum H^∞ control problem was addressed by considering real and imaginary quadratures of the quantum system variables. This involved converting the equations involving the complex annihilation and creation operators into a form involving the real quadratures.

Another development is at the level of the H^∞ control theory for a special class of quantum systems. Although the development of the H^∞ control theory for a more general class of linear quantum systems has been investigated in [6], the authors in [8] considers a special class of linear quantum systems (a special case of the one considered in [6]) and simplifies the design of the H^∞ controller for the optical cavity in [6] as the dimension of the complex Riccati equations to be solved to construct the controller is half the dimension of the corresponding real Riccati equations required in [6] to construct the controller. Furthermore, the authors in [9] and [10] extends the theory of H^∞ control to the finite horizon case for continuous time quantum systems and quantum systems with delayed measurements respectively.

In this paper, following the approach of [11], we solve a finite horizon H^∞ control problem for a class of linear quantum systems for the case of sampled-data measurements. In that case, the continuous time quantum output is connected to an array of homodyne detectors and converted to a classical signal that is fed into a classical anti-aliasing filter and then sampled. In the case in which two quadratures of a quantum variable are to be measured, it is assumed that a beam splitter has been introduced before the two corresponding homodyne detectors. The resulting closed-loop system is a hybrid quantum/classical system with continuous time and discrete time states.

The main contribution of the paper is that we consider hybrid linear quantum systems involving classical/quantum variables which are mainly encountered in the field of quantum optics. The authors are the first to consider such type of quantum systems. The second contribution is that we apply to them a classical controller derived from the classical theory through an auxiliary classical system behaving in the same manner as the hybrid quantum/classical system. This approach is novel and proper to the authors.

The theory of H^∞ control for sampled data quantum systems developed in this paper can be applied mainly to systems in the field of quantum optics as illustrated by the example provided at the end of our paper. Such type of systems can be modeled using linear quantum stochastic differential equations of the form given by equation (1) Which is a combination of mixed classical/quantum variables. In this paper, the controller is not a quantum system by itself but a classical system which is easier to implement than a quantum system controller. Therefore, in quantum experiments involving quantum optics systems, such as optical cavities, it is possible to apply the theory developed in this paper which shows that the classical H^∞ control theory involving sampled data controller of the form given by equation (3) can still be applicable to hybrid quantum systems combining quantum/ classical variables of the form as given in equation (1).

II. PROBLEM FORMULATION

Consider the mixed quantum classical linear given by

$$\begin{aligned} dx(t) &= A(t)x(t)dt + B(t)\beta_u(t) + D(t)\beta_{w_1}(t)dt + \bar{K}(t)d\bar{v}(t); \\ y(t_k) &= C_d(t_k)x(t_k) + N_d(t_k)\beta_{w_2}(t_k); \quad z(t) = H(t)x(t) + G(t)\beta_u(t); \end{aligned} \quad (1)$$

where $x(t)$ is the quantum variable, $A(t) \in \mathbb{R}^{n \times n}$, $B(t) \in \mathbb{R}^{n \times n_u}$, $D(t) \in \mathbb{R}^{n \times n_{w_1}}$, $\bar{K}(t) \in \mathbb{R}^{n_K \times n_{\bar{v}}}$, $\beta_{w_1}(t)$ is a square integrable classical disturbance signal. The set of all such $\beta_{w_1}(t)$ is denoted by $\mathcal{W}$. $\beta_u(t)$ is a self-adjoint adapted process. The vector $d\bar{v}(t)$ represents any additional quantum noise in the plant. $\beta_{w_2}(t_k)$ is a classical disturbance signal. $y(t_k)$ is the sampled data output and $z(t)$ is the continuous time controlled output. n , n_{w_1} , n_u and $n_{\bar{v}}$ are positive integers. $C_d(t_k) \in \mathbb{R}^{n_y \times n}$, $N_d(t_k) \in \mathbb{R}^{n_y \times n_{w_2}}$ and (n_k is a positive integer) for all $k \in [0, N]$. Furthermore, $H(t) \in \mathbb{R}^{n_z \times n}$ and $G(t) \in \mathbb{R}^{n_z \times n_u}$ where n_z is a positive integer.

We assume that the state of the quantum system is a Gaussian state with mean $\check{x}_0 \in \mathbb{R}^n$ and covariance matrix Y_0 ; e.g., see [12]. Then $\langle x_0 \rangle = \check{x}_0$ and

$$Y_0 = \frac{1}{2} \langle (x_0 - \check{x}_0)(x_0 - \check{x}_0)^T + ((x_0 - \check{x}_0)(x_0 - \check{x}_0)^T)^T \rangle. \quad (2)$$

Here, $\langle \cdot \rangle$ denotes classical/quantum expectation; e.g., see [13]. We will fix Y_0 but $\check{x}_0$ will be taken as part of the disturbance.

System (1) satisfies the mixed quantum-classical physical realizability conditions of Lemma 9 of [14].

Consider a classical sampled-data controller $\mathcal{K}$ defined by a differential equation with jumps:

$$\begin{aligned} dx_c(t) &= F_c(t)x_c(t)dt; \quad x_c(0) = x_{c0}; \quad x_c(t_k^+) = F_{cd}(t_k)x_c(t_k^-) + G_{cd}(t_k)y(t_k); \\ \beta_u(t) &= H_c(t)x_c(t) \end{aligned} \quad (3)$$

where $x_c(t) \in \mathbb{R}^{n_c}$ is the controller state. For all $t \in [0, t_f]$, $F_c(t) \in \mathbb{R}^{n_c \times n_c}$ and $H_c(t) \in \mathbb{R}^{n_u \times n_c}$. Also, $F_{cd}(t_k) \in \mathbb{R}^{n_c \times n_c}$, $G_{cd}(t_k) \in \mathbb{R}^{n_c \times n_{yp}}$. The closed-loop system obtained by making the identification $\beta_u(t) = H_c(t)x_c(t)$ and interconnecting equations (??) and (3) is described by the following quantum-classical stochastic differential equations with jumps:

$$\begin{aligned} d\eta(t) &= \tilde{A}(t)\eta(t)dt + \tilde{D}(t)\beta_{w_1}(t)dt + \tilde{K}(t)d\bar{v}(t); \quad \eta(0) = \eta_0; \\ \eta(t_k^+) &= \tilde{A}_d(t_k)\eta(t_k^-) + \tilde{D}_d(t_k)\beta_{w_2}(t_k); \quad z(t) = \tilde{H}(t)\eta(t) \end{aligned} \quad (4)$$

where $\eta(t) = \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix}$, $\eta(t_k^-) = \begin{bmatrix} x(t_k) \\ x_c(t_k^-) \end{bmatrix}$, $\eta(t_k^+) = \begin{bmatrix} x(t_k) \\ x_c(t_k^+) \end{bmatrix}$, $\tilde{A}(t) = \begin{bmatrix} A(t) & B(t)H_c(t) \\ 0 & F_c(t) \end{bmatrix}$,
 $\tilde{D}(t) = \begin{bmatrix} D(t) \\ 0 \end{bmatrix}$, $\tilde{K}(t) = \begin{bmatrix} \bar{K}(t) \\ 0 \end{bmatrix}$, $\tilde{A}_d(t_k) = \begin{bmatrix} I & 0 \\ G_{cd}(t_k)C_d(t_k) & F_{cd}(t_k) \end{bmatrix}$,
 $\tilde{D}_d(t_k) = \begin{bmatrix} 0 \\ G_{cd}(t_k)N_d(t_k) \end{bmatrix}$, $\tilde{H}(t) = \begin{bmatrix} H(t) & G(t)H_c(t) \end{bmatrix}$ and $\eta_0 = \begin{bmatrix} x_0 \\ x_{c0} \end{bmatrix}$.

The existence of solutions to (4) follows along similar lines to the case of classical stochastic differential equations with jumps [15] since the jumps only occur in the classical controller state.

We take the overall disturbance as $\bar{\beta}_w(t) = (\check{x}_0, \beta_{w_1}(t), \beta_{w_2}(t_k))$. Therefore, we consider, whether under the given measurement scheme, the upper value of the game with cost function

$$\begin{aligned} L_\gamma(\mathcal{K}, \bar{\beta}_w) &= \langle x(t_f)^T Q_f x(t_f) \rangle + \int_0^{t_f} \langle z(t)^T z(t) \rangle dt - \gamma^2 \check{x}_0^T Q_0 \check{x}_0 \\ &\quad - \gamma^2 \left\{ \int_0^{t_f} \beta_{w_1}(t)^T \beta_{w_1}(t) dt + \sum_{k=0}^{N-1} (\beta_{w_2}(t_k)^T \beta_{w_2}(t_k)) \right\} \\ &= \langle x(t_f)^T Q_f x(t_f) \rangle + \int_0^{t_f} \langle x(t)^T H(t)^T H(t) x(t) \rangle dt + \int_0^{t_f} \langle \beta_u(t)^T \beta_u(t) \rangle dt \\ &\quad - \gamma^2 \check{x}_0^T Q_0 \check{x}_0 - \gamma^2 \left\{ \int_0^{t_f} \beta_{w_1}(t)^T \beta_{w_1}(t) dt + \sum_{k=0}^{N-1} (\beta_{w_2}(t_k)^T \beta_{w_2}(t_k)) \right\} \end{aligned} \quad (5)$$

is bounded, and if in the affirmative, obtain a corresponding min-sup controller. Here, $Q_f = Q_f^T \geq 0$ and $Q_0 = Q_0^T \geq 0$ are given weighting matrices.

For the quantum-classical closed-loop system (4), we define the covariance matrix $P(t)$ by $P(t) = \frac{1}{2} \langle \eta(t)\eta(t)^T + (\eta(t)\eta(t)^T)^T \rangle$. Note that $P_0 = P(0) = \begin{bmatrix} Y_0 + \tilde{x}_0\tilde{x}_0^T & \tilde{x}_0x_{c_0}^T \\ x_{c_0}\tilde{x}_0^T & x_{c_0}x_{c_0}^T \end{bmatrix}$. Now,

$$dP(t) = \frac{1}{2} \left\{ \langle d\eta(t)\eta(t)^T \rangle + \langle (d\eta(t)\eta(t)^T)^T \rangle + \langle \eta(t)d\eta(t)^T \rangle + \langle (\eta(t)d\eta(t)^T)^T \rangle + \langle d\eta(t)d\eta(t)^T \rangle + \langle (d\eta(t)d\eta(t)^T)^T \rangle \right\}.$$

An expression for $dP(t)$ obtained using the quantum Ito rule is given by

$$dP(t) = \tilde{A}(t)P(t)dt + P(t)\tilde{A}(t)^Tdt + \tilde{D}(t)\beta_{w_1}(t)\langle\eta(t)^T\rangle dt + \langle\eta(t)\rangle\beta_{w_1}(t)^T\tilde{D}(t)^Tdt + \tilde{K}(t)S_v(t)\tilde{K}(t)^Tdt \quad (6)$$

where $S_v(t)dt = \frac{1}{2} \langle d\bar{v}(t)d\bar{v}(t)^T + (d\bar{v}(t)d\bar{v}(t)^T)^T \rangle$. Thus, we obtain

$$\dot{P}(t) = \tilde{A}(t)P(t) + P(t)\tilde{A}(t)^T + \tilde{D}(t)\beta_{w_1}(t)\langle\eta(t)^T\rangle + \langle\eta(t)\rangle\beta_{w_1}(t)^T\tilde{D}(t)^T + \tilde{K}(t)S_v(t)\tilde{K}(t)^T; \quad (7)$$

$$P(t_k^+) = \tilde{A}_d(t_k)P(t_k^-)\tilde{A}_d(t_k)^T + \tilde{D}_d(t_k)\beta_{w_2}(t_k)\beta_{w_2}(t_k)^T\tilde{D}_d(t_k)^T + \tilde{A}_d(t_k)\langle\eta(t_k^-)\rangle\beta_{w_2}(t_k)^T\tilde{D}_d(t_k)^T + \tilde{D}_d(t_k)\beta_{w_2}(t_k)\langle\eta(t_k^-)^T\rangle\tilde{A}_d(t_k)^T. \quad (8)$$

Now $\eta(t_f) = \begin{bmatrix} x(t_f) \\ x_c(t_f) \end{bmatrix}$ so that $Q_f^{1/2}x(t_f) = \begin{bmatrix} Q_f^{1/2} & 0 \end{bmatrix} \begin{bmatrix} x(t_f) \\ x_c(t_f) \end{bmatrix} = \begin{bmatrix} Q_f^{1/2} & 0 \end{bmatrix} \eta(t_f)$ and $x(t_f)^T Q_f^{1/2} = \eta(t_f)^T \begin{bmatrix} Q_f^{1/2} \\ 0 \end{bmatrix}$. Hence, $\langle x(t_f)^T Q_f x(t_f) \rangle = \langle x(t_f)^T Q_f^{1/2} Q_f^{1/2} x(t_f) \rangle = \text{tr}(\hat{Q}_f P(t_f))$ where $\hat{Q}_f = \begin{bmatrix} Q_f & 0 \\ 0 & 0 \end{bmatrix}$. Also, $\langle x(t)^T H(t)^T H(t) x(t) \rangle = \text{tr}(\hat{Q}(t)P(t))$ where $\hat{Q}(t) = \begin{bmatrix} H(t)^T H(t) & 0 \\ 0 & 0 \end{bmatrix}$ and $\langle \beta_u(t)^T \beta_u(t) \rangle = \text{tr}(\bar{Q}(t)P(t))$ where $\bar{Q}(t) = \begin{bmatrix} 0 & 0 \\ 0 & H_c(t)^T H_c(t) \end{bmatrix}$.

Note that $\tilde{H}(t)^T \tilde{H}(t) = \hat{Q}(t) + \bar{Q}(t) = \begin{bmatrix} H(t)^T H(t) & 0 \\ 0 & H_c(t)^T H_c(t) \end{bmatrix}$. Hence,

$$L_\gamma(\mathcal{K}, \bar{\beta}_w) = \text{tr}(\hat{Q}_f P(t_f)) + \int_0^{t_f} \text{tr}(\tilde{H}(t)^T \tilde{H}(t) P(t)) dt - \gamma^2 \left[\tilde{\eta}_0^T \hat{Q}_0 \tilde{\eta}_0 + \int_0^{t_f} \beta_{w_1}(t)^T \beta_{w_1}(t) dt \right] - \gamma^2 \left[\sum_{k=0}^{N-1} (\beta_{w_2}(t_k)^T \beta_{w_2}(t_k)) \right] \quad (9)$$

where $\hat{Q}_0 = \begin{bmatrix} Q_0 & 0 \\ 0 & 0 \end{bmatrix}$ and $\check{\eta}_0 = \langle \eta_0 \rangle$.

Suppose $\bar{\beta}_w(\cdot) \in \Omega_q = \mathbb{R}^n \times \mathcal{W} \times \mathcal{W}_d$. $\mathfrak{M}$ is the set of admissible controllers $\mathcal{K}$. Admissible controllers are controllers which are of the form given by (3) and under which the closed-loop system defined by (??) and (3) has a solution for every $\bar{\beta}_w(\cdot) \in \Omega_q$. Also, $L_\gamma(\mathcal{K}, \bar{\beta}_w)$ defined in (5) can be written in terms of the closed-loop system variable $\eta(t)$ as

$$\begin{aligned} L_\gamma(\mathcal{K}, \bar{\beta}_w) &= \left\langle \eta(t_f)^T \hat{Q}_f \eta(t_f) \right\rangle + \left\langle \int_0^{t_f} \eta(t)^T \tilde{H}(t)^T \tilde{H}(t) \eta(t) dt \right\rangle \\ &\quad - \gamma^2 \left(\check{\eta}_0^T \hat{Q}_0 \check{\eta}_0 + \int_0^{t_f} \beta_{w_1}(t)^T \beta_{w_2}(t) dt + \sum_{k=0}^{N-1} (\beta_{w_2}(t_k)^T \beta_{w_2}(t_k)) \right) \\ &= \text{tr} \left(\hat{Q}_f P(t_f) \right) + \int_0^{t_f} \text{tr} \left(\tilde{H}(t)^T \tilde{H}(t) P(t) \right) dt \\ &\quad - \gamma^2 \left(\check{\eta}_0^T \hat{Q}_0 \check{\eta}_0 + \int_0^{t_f} \beta_{w_1}(t)^T \beta_{w_1}(t) dt + \sum_{k=0}^{N-1} (\beta_{w_2}(t_k)^T \beta_{w_2}(t_k)) \right). \end{aligned}$$

Problem $\mathcal{P}_\gamma$. Determine necessary and sufficient conditions on γ such that the quantity $\inf_{\mathcal{K} \in \mathfrak{M}} \sup_{\bar{\beta}_w \in \Omega_q} L_\gamma(\mathcal{K}, \bar{\beta}_w)$ is finite, and for each such γ find a controller $\mathcal{K}$ that achieves the minimum. The infimum of all γ 's that satisfy these conditions will be denoted by γ_q^* .

III. AUXILIARY CLASSICAL STOCHASTIC AND DETERMINISTIC SYSTEMS

Our solution to the problem $\mathcal{P}_\gamma$ involves defining classical stochastic and deterministic systems corresponding to the quantum-classical system (1). Then the solution is obtained by solving a disturbance attenuation problem for the classical deterministic system. We define the following classical linear stochastic system with sampled data measurements

$$\begin{aligned} d\xi(t) &= A(t)\xi(t)dt + B(t)\beta_u(t)dt + D(t)\beta_{w_1}(t)dt + \bar{K}(t)S_v^{1/2}(t)d\tilde{v}(t); \quad t \geq 0; \\ y(t_k) &= C_d(t_k)\xi(t_k) + N_d(t_k)\beta_{w_2}(t_k); z(t) = H(t)\xi(t) + G(t)\beta_u(t) \end{aligned} \quad (10)$$

where $\xi(0) = \xi_0$ is a Gaussian random variable with mean $\check{x}_0$ and covariance matrix Y_0 . $\tilde{v}(t)$ is a standard Wiener process.

Using the controller $\mathcal{K}$ defined by (3), the corresponding closed-loop system is obtained by making the identification $\beta_u(t) = H_c(t)x_c(t)$ and interconnecting equations (10) and (3) to give:

$$\begin{aligned} d\mu(t) &= \tilde{A}(t)\mu(t)dt + \tilde{D}(t)\beta_{w_1}(t)dt + \tilde{K}_s(t)d\tilde{v}(t); \quad \mu(0) = \mu_0; \\ \mu(t_k^+) &= \tilde{A}_d(t_k)\mu(t_k^-) + \tilde{D}_d(t_k)\beta_{w_2}(t_k); \quad z(t) = \tilde{H}(t)\mu(t) \end{aligned} \quad (11)$$

where $\mu(t) = \begin{bmatrix} \xi(t) \\ x_c(t) \end{bmatrix}$, $\mu(t_k^+) = \begin{bmatrix} \xi(t_k) \\ x_c(t_k^+) \end{bmatrix}$, $\mu(t_k^-) = \begin{bmatrix} \xi(t_k) \\ x_c(t_k^-) \end{bmatrix}$, $\mu_0 = \begin{bmatrix} \xi_0 \\ x_{c0} \end{bmatrix}$ and $\tilde{K}_s(t) = \tilde{K}(t)S_v^{1/2}(t)$. We assume that the random variable $\mu(0) = \mu_0$ is normal with mean $m = \begin{bmatrix} \check{\xi}_0 \\ x_{c0} \end{bmatrix}$ where $\check{\xi}_0 = E(\xi_0)$ and covariance $R_0 = E((\mu_0 - m)(\mu_0 - m)^T) = \text{diag}(\Pi_0, 0)$. Then, $\Pi_0 = E((\xi_0 - \check{\xi}_0)(\xi_0 - \check{\xi}_0)^T)$. Here, $E(\cdot)$ denotes the stochastic expectation. Also, we define

$$\begin{aligned} \hat{L}_\gamma(\mathcal{K}, \bar{\beta}_w) &= E(\xi(t_f)^T Q_f \xi(t_f)) + \int_0^{t_f} E(z(t)^T z(t)) dt \\ &\quad - \gamma^2 \left\{ E(\check{x}_0^T Q_0 \check{x}_0) + \int_0^{t_f} \beta_{w_1}(t)^T \beta_{w_1}(t) dt + \sum_{k=0}^{N-1} (\beta_{w_2}(t_k)^T \beta_{w_2}(t_k)) \right\}. \end{aligned} \quad (12)$$

For the stochastic closed-loop system (11), we define the covariance matrix $\hat{P}(t) = E(\mu(t)\mu(t)^T)$. Note that $\hat{P}(0) = \hat{P}_0 = P(0) = \begin{bmatrix} Y_0 + \check{x}_0 \check{x}_0^T & \check{x}_0 x_{c0}^T \\ x_{c0} \check{x}_0^T & x_{c0} x_{c0}^T \end{bmatrix}$. Using the classical Ito rule, we can write $d\hat{P}(t) = E(d\mu(t)\mu(t)^T) + E(\mu(t)d\mu(t)^T) + E(d\mu(t)d\mu(t)^T)$. Also, we have $Q_f^{1/2}\xi(t_f) = \begin{bmatrix} Q_f^{1/2} & 0 \end{bmatrix} \mu(t_f)$ and $\xi(t_f)^T Q_f^{1/2} = \mu(t_f)^T \begin{bmatrix} Q_f^{1/2} \\ 0 \end{bmatrix}$.

Hence, $E(\xi(t_f)^T Q_f \xi(t_f)) = E(\xi(t_f)^T Q_f^{1/2} Q_f^{1/2} \xi(t_f)) = \text{tr}(\hat{Q}_f \hat{P}(t_f))$. Also, $E(z(t)^T z(t)) = \text{tr}(\tilde{H}(t)^T \tilde{H}(t) \hat{P}(t))$. Thus, $\hat{L}(\mathcal{K}, \bar{\beta}_w) = \text{tr}(\hat{Q}_f \hat{P}(t_f)) + \int_0^{t_f} \text{tr}(\tilde{H}(t)^T \tilde{H}(t) \hat{P}(t)) dt$ and

$$\begin{aligned} \hat{L}_\gamma(\mathcal{K}, \bar{\beta}_w) &= \text{tr}(\hat{Q}_f \hat{P}(t_f)) + \int_0^{t_f} \text{tr}(\tilde{H}(t)^T \tilde{H}(t) \hat{P}(t)) dt \\ &\quad - \gamma^2 \left[\check{\mu}_0^T \hat{Q}_0 \check{\mu}_0 + \int_0^{t_f} \beta_{w_1}(t)^T \beta_{w_1}(t) dt \right] - \gamma^2 \sum_{k=0}^{N-1} (\beta_{w_2}(t_k)^T \beta_{w_2}(t_k)) \end{aligned} \quad (13)$$

where $\check{\mu}_0 = E(\mu_0) = \check{\eta}_0 = \langle \eta_0 \rangle$.

Theorem 3.1: Given any admissible controller $\mathcal{K} \in \mathfrak{M}$ and any $\beta_{w_1}(\cdot) \in \mathcal{W}$ and any $\beta_{w_2}(\cdot) \in \mathcal{W}_d$, the covariance matrices $P(t)$ and $\hat{P}(t)$ are equal for all $t \in [0, t_f]$.

As a consequence of Theorem 3.1, $L_\gamma(\mathcal{K}, \bar{\beta}_w) \equiv \hat{L}_\gamma(\mathcal{K}, \bar{\beta}_w)$ for all classical linear controllers $\mathcal{K}$ of the form (3) and for all disturbance inputs $\beta_{w_1}(t) \in \mathcal{W}$ and $\beta_{w_2}(t_k) \in \mathcal{W}_d$. We can rewrite the performance index for the closed-loop classical stochastic system as:

$$\begin{aligned} \hat{L}_\gamma(\mathcal{K}, \bar{\beta}_w) &= E(\mu(t_f)^T \hat{Q}_f \mu(t_f)) + \int_0^{t_f} E(\mu(t)^T \tilde{H}(t) \tilde{H}(t)^T \mu(t)) dt \\ &\quad - \gamma^2 \left\{ \check{\mu}_0^T \hat{Q}_0 \check{\mu}_0 + \int_0^{t_f} \beta_{w_1}(t)^T \beta_{w_1}(t) dt + \sum_{k=0}^{N-1} (\beta_{w_2}(t_k)^T \beta_{w_2}(t_k)) \right\}. \end{aligned} \quad (14)$$

This follows since $E\left(\mu(t_f)^T \hat{Q}_f \mu(t_f)\right) = \text{tr}\left(\hat{Q}_f \hat{P}(t_f)\right)$ where $\hat{P}(t) = E\left(\mu(t)\mu(t)^T\right)$ and $E\left(\mu(t)^T \tilde{H}(t)^T \tilde{H}(t) \mu(t)\right) = \text{tr}\left(\tilde{H}(t) \tilde{H}(t)^T \hat{P}(t)\right)$. We want to maximize $\hat{L}_\gamma(\mathcal{K}, \bar{\beta}_w)$ over $\bar{\beta}_w(\cdot)$. Using Theorem 3.1, maximizing $\hat{L}_\gamma(\mathcal{K}, \bar{\beta}_w)$ over $\bar{\beta}_w(\cdot)$ is equivalent to maximizing $L_\gamma(\mathcal{K}, \bar{\beta}_w)$ over $\bar{\beta}_w(\cdot)$. By taking $\tilde{x}_0$ as a part of the unknown disturbance, the quantum cost function $L_\gamma(\mathcal{K}, \bar{\beta}_w)$ defined in Problem P_γ is equal $\hat{L}_\gamma(\mathcal{K}, \bar{\beta}_w)$ since $P(\cdot)$ and $\hat{P}(\cdot)$ are equal.

We now consider a deterministic classical system:

$$\begin{aligned}\dot{\xi}(t) &= A(t)\xi(t) + B(t)\beta_u(t) + D(t)\beta_{w_1}(t); \quad t \geq 0; \quad \xi_0 = \xi(0) = \tilde{x}_0; \\ y(t_k) &= C_d(t_k)\xi(t_k) + N_d(t_k)\beta_{w_2}(t_k); \quad z(t) = H(t)\xi(t) + G(t)\beta_u(t)\end{aligned}\quad (15)$$

The closed-loop system corresponding to (15) and (3) is given by:

$$\begin{aligned}\dot{\mu}(t) &= \tilde{A}(t)\mu(t) + \tilde{D}(t)\beta_{w_1}(t); \quad \mu_0 = m; \\ \mu(t_k^+) &= \tilde{A}_d(t_k)\mu(t_k^-) + \tilde{D}_d(t_k)\beta_{w_2}(t_k); \quad z(t) = \tilde{H}(t)\mu(t)\end{aligned}$$

where $m = \begin{bmatrix} \tilde{x}_0 \\ x_{c_0} \end{bmatrix}$. For $\gamma > 0$, we also introduce the following performance index

$$\begin{aligned}\tilde{L}_\gamma(\mathcal{K}, \bar{\beta}_w) &= \xi(t_f)^T Q_f \xi(t_f) + \int_0^{t_f} z(t)^T z(t) dt \\ &\quad - \gamma^2 \left(\int_0^{t_f} \beta_{w_1}(t)^T \beta_{w_1}(t) dt + \tilde{x}_0^T Q_0 \tilde{x}_0 + \sum_{k=0}^{N-1} \beta_{w_2}(t_k)^T \beta_{w_2}(t_k) \right) \\ &= \xi(t_f)^T Q_f \xi(t_f) + \int_0^{t_f} (\xi(t)^T H(t)^T H(t) \xi(t) + \beta_u(t)^T \beta_u(t)) dt \\ &\quad - \gamma^2 \left(\int_0^{t_f} \beta_{w_1}(t)^T \beta_{w_1}(t) dt + \tilde{x}_0^T Q_0 \tilde{x}_0 + \sum_{k=0}^{N-1} \beta_{w_2}(t_k)^T \beta_{w_2}(t_k) \right).\end{aligned}$$

Also, $\tilde{L}_\gamma(\mathcal{K}, \bar{\beta}_w)$ can be rewritten in terms of the closed-loop state $\mu(t)$ as

$$\begin{aligned}\tilde{L}_\gamma(\mathcal{K}, \bar{\beta}_w) &= \mu(t_f)^T \hat{Q}_f \mu(t_f) + \int_0^{t_f} \mu(t)^T \tilde{H}(t)^T \tilde{H}(t) \mu(t) dt \\ &\quad - \gamma^2 \left(\int_0^{t_f} \beta_{w_1}(t)^T \beta_{w_1}(t) dt + \tilde{\mu}_0^T \hat{Q}_0 \tilde{\mu}_0 + \sum_{k=0}^{N-1} \beta_{w_2}(t_k)^T \beta_{w_2}(t_k) \right) \\ &= \text{tr}\left(\hat{Q}_f \hat{P}(t_f)\right) + \int_0^{t_f} \text{tr}\left(\tilde{H}(t)^T \tilde{H}(t) \hat{P}(t)\right) dt \\ &\quad - \gamma^2 \left(\int_0^{t_f} (\beta_{w_1}(t)^T \beta_{w_1}(t)) dt + \tilde{\mu}_0^T \hat{Q}_0 \tilde{\mu}_0 \right) - \gamma^2 \left(\sum_{k=0}^{N-1} \beta_{w_2}(t_k)^T \beta_{w_2}(t_k) \right)\end{aligned}\quad (16)$$

The corresponding disturbance attenuation problem to be solved is the following:

Problem $\tilde{\mathcal{P}}_\gamma$. Determine necessary and sufficient conditions on γ such that the quantity $\inf_{\mathcal{K} \in \mathfrak{M}} \sup_{\bar{\beta}_w \in \Omega_q} \tilde{L}_\gamma(\mathcal{K}, \bar{\beta}_w)$ is finite, and for each such γ find a controller $\mathcal{K}$ that achieves the minimum. The infimum of all γ 's that satisfy these conditions will be denoted by γ_c^* .

Let

$$\hat{V}(m) = \sup_{\bar{\beta}_w \in \Omega_q} \hat{L}_\gamma(\mathcal{K}, \bar{\beta}_w), \quad (17)$$

$$\tilde{V}(m) = \sup_{\bar{\beta}_w \in \Omega_q} \tilde{L}_\gamma(\mathcal{K}, \bar{\beta}_w). \quad (18)$$

Theorem 3.2: Given any $m = \begin{bmatrix} \check{x}_0 \\ x_{c0} \end{bmatrix} \in \mathbb{R}^{(n+n_c)}$, the supremum $\hat{V}(m)$ in the stochastic case is related to the corresponding supremum $\tilde{V}(m)$ in the deterministic case by

$$\hat{V}(m) = \tilde{V}(m) + \alpha \quad (19)$$

where α is independent of m and depends on the variances of the noises.

Theorem 3.3: $\hat{L}_\gamma(\mathcal{K}, \bar{\beta}_w)$ has a finite supremum over $\bar{\beta}_w \in \Omega_q$ for all $m \in \mathbb{R}^{(n+n_c)}$ if and only if $\tilde{L}_\gamma(\mathcal{K}, \bar{\beta}_w)$ has a finite supremum in $\bar{\beta}_w \in \Omega_q$ for all $m \in \mathbb{R}^{(n+n_c)}$.

Proof: The proof follows from Theorem 3.2. ■

Theorem 3.4: In the quantum case, $L_\gamma(\mathcal{K}, \bar{\beta}_w)$ has a finite supremum over $\bar{\beta}_w \in \Omega_q$ for all $m \in \mathbb{R}^{(n+n_c)}$ if and only if $\tilde{L}_\gamma(\mathcal{K}, \bar{\beta}_w)$ has a finite supremum over $\bar{\beta}_w \in \Omega_q$ for all $m \in \mathbb{R}^{(n+n_c)}$.

Proof: The proof follows from Theorems 3.3 and 3.1. ■

In order to solve the sampled-data finite horizon quantum H^∞ problem P_γ for the system (15), we now introduce a Generalized Riccati Differential Equation (GRDE) with jumps:

$$\dot{\Sigma}(t) = A(t)\Sigma(t) + \Sigma(t)A(t)^T + \gamma^{-2}\Sigma(t)H(t)^TH(t)\Sigma(t) + D(t)D(t)^T; \quad (20)$$

$$\Sigma(t_k^+) = \Sigma(t_k^-)[I + C_d(t_k)^T\tilde{N}_d(t_k)^{-1}C_d(t_k)\Sigma(t_k^-)]^{-1} \quad (21)$$

where $\tilde{N}_d(t_k) = N_d(t_k)N_d(t_k)^T$ and $\Sigma(0) = Q_0^{-1}$. We also introduce the GRDE:

$$\begin{aligned} \dot{Z}(t) + Z(t)A(t) + A(t)^TZ(t) - Z(t)(B(t)B(t)^T - \gamma^{-2}D(t)D(t)^T)Z(t) + H(t)^TH(t) &= 0, \\ Z(t_f) &= Q_f. \end{aligned} \quad (22)$$

In addition, with $\rho(\cdot)$ denoting the spectral radius, we introduce the condition

$$\rho(\Sigma(t)Z(t)) < \gamma^2, \quad \forall t \in [0, t_f]. \quad (23)$$

Also, we consider the following sampled data controller of the form (3)

$$\dot{\check{\xi}}(t) = (A(t) + \gamma^{-2}\Sigma(t)H(t)^TH(t))\check{\xi}(t) + B(t)\hat{\beta}_{u_1}(t); \quad \check{\xi}(0) = 0; \quad (24)$$

$$\check{\xi}(t_k^+) = \check{\xi}(t_k^-) + \Sigma(t_k^+)C_d(t_k)^T\tilde{N}_d(t_k)^{-1}(y(t_k) - C_d(t_k)\check{\xi}(t_k^-)) \quad (25)$$

where

$$\hat{\beta}_{u_1}(t) = -B(t)^TZ(t)(I - \gamma^{-2}\Sigma(t)Z(t))^{-1}\check{\xi}(t). \quad (26)$$

Theorem 3.5: (See Theorem 5.4 on page 217 in [16]) Consider the problem $\tilde{\mathcal{P}}_\gamma$ with sampled-data state measurements, and let the corresponding optimum attenuation level be γ_c^* .

- (i) For a given $\gamma > 0$, if the jump Riccati differential equations (20)- (21) and (22) have solutions over $[0, t_f]$, and if the condition (23) is satisfied, then necessarily $\gamma \geq \gamma_c^*$.
- (ii) For each such γ , there exists an optimal controller, given by (24) (25) and (26).
- (iii) If either (20)-(21) or (22) has a conjugate point in $[0, t_f]$, or if (23) fails to hold, then $\gamma \leq \gamma_c^*$; i.e., for any smaller γ (and possibly for the one considered) the supremum in problem $\tilde{\mathcal{P}}_\gamma$ is infinite for any admissible controller.

Combining Theorems 3.4 and 3.5 leads to the main result of the paper as follows.

Theorem 3.6: Consider the disturbance attenuation problem for the quantum system (1) $\mathcal{P}_\gamma$ and let the corresponding optimum attenuation level be γ_q^* .

- (i) For a given $\gamma > 0$, if the jump Riccati differential equations (20)- (21) and (22) have solutions over $[0, t_f]$, and if the condition (23) is satisfied, then necessarily $\gamma \geq \gamma_q^*$.
- (ii) For each such γ , there exists an optimal controller, given by (24) (25) and (26).
- (iii) If either (20)-(21) or (22) has a conjugate point in $[0, t_f]$, or if (23) fails to hold, then $\gamma \leq \gamma_q^*$; i.e., for any smaller γ (and possibly for the one considered) the supremum in problem $\mathcal{P}_\gamma$ is infinite for any admissible controller.

As a consequence of Theorems 3.5 and 3.6, we can conclude that $\gamma_q^* = \gamma_c^*$.

IV. NUMERICAL EXAMPLE

We consider an application of the sampled-data quantum H^∞ control theory developed above to a physical quantum system consisting of two cascaded optical squeezer cavities as shown in Figure 2 (a). In this figure, the HD block refers to a homodyne detector and the MD block refers to a modulator.

Fig. 1. (a) Schematic for the control of two cascaded optical squeezer cavities. (b) Open and closed loop frequency responses.

In the quantum H^∞ control problem under consideration, the aim is to attenuate the effect of disturbance signal $w_1(t)$ on the controlled output signal $z_1(t)$. We regard the signal $v_1(t)$ as a desired signal to be detected, which we will later estimate from measurements of $z_1(t)$. Hence, we do not wish to significantly attenuate the effect of $v_1(t)$ on $z_1(t)$. The dynamics of the optical squeezer system can be described as in [17] and for the chosen values of the squeezer parameters leads to a model of the form below where

$$\begin{aligned}
 A_p &= \begin{bmatrix} -13.7 & 0 & 0.2 & 0 \\ 0 & -15.3 & 0 & 0.2 \\ -0.2 & 0 & -1450 & 0 \\ 0 & -0.2 & 0 & -1450 \end{bmatrix}; \quad B_p = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 30.8221 \end{bmatrix}; \quad D_p = \begin{bmatrix} 3.0822 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \\
 K_p &= \begin{bmatrix} 4.3589 & 0 & 0 & 3.1623 & 0 & 0 & 0 \\ 0 & 4.3589 & 0 & 0 & 3.1623 & 0 & 0 \\ 0 & 0 & 43.5890 & 0 & 0 & 31.6228 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 31.6228 \end{bmatrix}; \\
 C_p &= \begin{bmatrix} 13.4350 & 0 & 0 & 0 \end{bmatrix}; \quad N_p = -10^{-6}; \quad L_p = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
 \end{aligned}$$

$$H_p = \begin{bmatrix} 44.8336 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } G_p = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \end{bmatrix}.$$

We combine the squeezer system plant with an anti-aliasing filter with transfer function $F(s) = \frac{10}{s+137}$ to form a system of the form (1) and we sample the output of the resulting system at a sampling frequency of 2 KHz. A controller of the form (3) is designed by solving the Riccati equations (20), (21) and (22) for the case of one hundred samples in the interval $[0, 0.1]$ occurring at evenly spaced times t_k . For $\gamma = 1$, the corresponding GRDE's had suitable solutions defining the controller (24) (25) and (26) which solves the finite horizon quantum H^∞ problem for $\gamma = 1$. For values of $\gamma < 1$, the condition (23) was found to be violated. The Bode plot in Figure 2 (b) shows the frequency responses from input disturbance $w_1(t)$ to the output $z(t) = z_2(t)$ and also from the signal input $v_1(t)$ to the output $z(t) = z_2(t)$ in both the open-loop and closed-loop cases. Based on these Bode plots, we can conclude that the disturbance signal has been significantly attenuated after applying the sampled-data classical controller whereas the input signal $v_1(t)$ has been only slightly attenuated. Thus, the use of the sample-data H^∞ controller would significantly improve the prospect of detecting the signal $v_1(t)$.

V. CONCLUSION

This paper shows that a sampled-data finite horizon quantum H^∞ control problem can be solved by solving a corresponding deterministic H^∞ problem. From this, the solution to the sampled-data finite horizon quantum H^∞ control problem can be obtained in terms of a pair of GRDEs.

APPENDIX A

PROOF OF THEOREM 3.1

Proof: Using the classical Ito rule, we can derive an expression for $d\hat{P}(t)$ as follows:

$$\begin{aligned} d\hat{P}(t) = & \tilde{A}(t)\hat{P}(t)dt + \hat{P}(t)\tilde{A}(t)^T dt + \tilde{D}(t)\beta_{w_1}(t)E(\mu(t)^T)dt \\ & + E(\mu(t))\beta_{w_1}(t)^T \tilde{D}(t)^T dt + \tilde{K}(t)S_v(t)\tilde{K}(t)^T dt. \end{aligned}$$

On the other hand, let $\bar{\mu}(t) = E(\mu(t))$ and $\bar{\eta}(t) = \langle \eta(t) \rangle$. Taking the expectation of $d\mu(t)$ in equation (11) and the quantum expectation of $d\eta(t)$ in equation (4), we get

$$\frac{d\bar{\mu}(t)}{dt} = \tilde{A}(t)\bar{\mu}(t) + \tilde{D}(t)\beta_{w_1}(t); \quad \frac{d\bar{\eta}(t)}{dt} = \tilde{A}(t)\bar{\eta}(t) + \tilde{D}(t)\beta_{w_1}(t).$$

Note that $\bar{\mu}(0) = E(\mu(0)) = \bar{\mu}_0 = \check{\mu}_0$ and $\bar{\eta}(0) = \langle \eta(0) \rangle = \check{\eta}_0 = \check{\mu}_0$. Thus, $\bar{\mu}(t) = E(\mu(t)) = \bar{\eta}(t) = \langle \eta(t) \rangle$. Hence, $\hat{P}(\cdot)$ satisfies the differential equation (7). Moreover, $\hat{P}(0) = P(0)$. Hence, $\hat{P}(\cdot) \equiv P(\cdot)$. Now, let $\bar{\mu}(t_k^+) = E(\mu(t_k^+))$ and $\bar{\eta}(t_k^+) = \langle \eta(t_k^+) \rangle$. Taking the expectation of $\mu(t_k^+)$ in equation (11) and the quantum expectation of $\eta(t_k^+)$ in equation (4), we get

$$\bar{\mu}(t_k^+) = \tilde{A}_d(t_k)\bar{\mu}(t_k^-) + \tilde{D}_d(t_k)\beta_{w_2}(t_k); \quad \bar{\eta}(t_k^+) = \tilde{A}_d(t_k)\bar{\eta}(t_k^-) + \tilde{D}_d(t_k)\beta_{w_2}(t_k).$$

For $t_k = 0$, we have that $\bar{\mu}(0) = E(\mu(0)) = \check{\mu}_0$ and $\bar{\eta}(0) = \langle \eta(0) \rangle = \check{\eta}_0 = \check{\mu}_0$. Thus, $\bar{\mu}(t_k^+) = E(\mu(t_k^+)) = \bar{\eta}(t_k^+) = \langle \eta(t_k^+) \rangle$. Hence, $P(t_k^+) \equiv \hat{P}(t_k^+)$. Moreover, $\hat{P}(0) = P(0)$. Consequently, $\hat{P}(\cdot) \equiv P(\cdot)$. This completes the proof of the theorem. ■

PROOF OF THEOREM 3.2

Proof: The proof of this theorem requires lengthy straightforward but tedious algebraic derivations. These derivations are omitted from this paper (see [18] for complete details). ■

PROOF OF THEOREM 3.6

Proof: Assume that the Riccati differential equations (20)-(21) and (22) have solutions over $[0, t_f]$ for a given $\gamma > 0$ and that the condition (23) is satisfied. Using Theorem 3.5, it follows that $\gamma \geq \gamma_c^*$ for the problem $\tilde{P}_\gamma$ and that $\tilde{L}_\gamma(\mathcal{K}, \bar{\beta}_w)$ has a finite supremum over $\bar{\beta}_w \in \Omega_q$ for all $m \in \mathbb{R}^{(n+n_c)}$. Using Theorem 3.4, it follows that $L_\gamma(\mathcal{K}, \bar{\beta}_w)$ has a finite supremum over $\bar{\beta}_w \in \Omega_q$ for all $m \in \mathbb{R}^{(n+n_c)}$. Thus, the quantity $\inf_{\mathcal{K} \in \mathfrak{M}} \sup_{\bar{\beta}_w \in \Omega_q} \tilde{L}_\gamma(\mathcal{K}, \bar{\beta}_w)$ is finite if and only if the quantity $\inf_{\mathcal{K} \in \mathfrak{M}} \sup_{\bar{\beta}_w \in \Omega_q} L_\gamma(\mathcal{K}, \bar{\beta}_w)$ is finite. Hence, the problem P_γ is equivalent to the problem $\tilde{P}_\gamma$. Therefore, using Theorem 3.5, it follows that $\gamma \geq \gamma_q^*$ for the problem P_γ . Also, using Theorem 3.5, it follows that for each such γ , there exists an optimal controller, given by (24)-(25) and (26). Now, if either (20)-(21) or (22) has a conjugate point in $[0, t_f]$ or if (23) fails to hold, then using Theorem 3.5, it follows that $\gamma \leq \gamma_c^*$ for the deterministic problem $\tilde{P}_\gamma$; i.e., for any smaller γ (and possibly for the one considered) the supremum in problem $\tilde{P}_\gamma$ is infinite for any admissible $\mathcal{K} \in \mathfrak{M}$. Using the equivalence between the problems $\tilde{P}_\gamma$ and P_γ , it follows that $L_\gamma(\mathcal{K}, \bar{\beta}_w)$ for the quantum problem P_γ has an infinite supremum over $\bar{\beta}_w \in \Omega_q$ for all $m \in \mathbb{R}^{(n+n_c)}$. Hence, $\gamma \leq \gamma_q^*$ for the quantum problem P_γ . This completes the proof of the theorem. ■

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