Techniques for high-fidelity X-ray micro-tomography of metal components

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ABSTRACT

Additive manufacturing is revolutionising the production of complex-shaped components and finding widespread application in the industries where production volumes are low and part geometries are complex. High-fidelity 3D imaging of additively-manufactured metal parts, as the most effective non-destructive evaluation method, is crucial to maintaining the quality of safety-critical components during manufacturing. However, 3D X-ray imaging of large metallic parts is complicated by strong, nonlinear interactions between the sample and the beam; phenomena such as beam hardening, photon starvation and X-ray scatter are difficult to account for and cause artifacts in the computed 3D tomographic volume. These artifacts can obscure real defects and cause severe inaccuracies in analysis and evaluation. Here we present several techniques developed in the CTLab at the Australian National University that enhance the micro-tomography instruments' ability to image large, complex metallic components.

KEYWORDS

scanning trajectories; x-ray beam-hardening; x-ray metal artifacts; x-ray scatter

1. Introduction

Additive manufacturing (AM) can create 3-dimensional (3D) metallic components with complicated geometries that cannot be made with conventional manufacturing processes, and offers lower costs for low-volume production runs. AM is therefore an appealing method for the manufacture of components (including safety-critical components) in low-volume industries such as aerospace.

In safety-critical scenarios, high-quality non-destructive evaluation is essential, and the most suitable 3D imaging technique is high-resolution X-ray computed tomography (CT). X-ray CT has many advantages: (i) it can be performed on-site (in contrast to Neutron-based imaging), (ii) it produces 3D images (tomograms) that reproduce faithfully the interior structure of the sample, and (iii) it is ideally suited for the length scales involved. A recent review article on X-ray CT of AM components by Du Plessis *et al* [1] outlines the capabilities and limitations of X-ray CT in AM inspection.

They observed that, given current technology, the size of metal parts that can be analysed is limited due to image degradation from reduced X-ray penetration, X-ray beam-hardening, and other effects. Here we investigate methods to reduce these limitations. Although these methods can be applied outside AM, the work presented here is motivated by contemporary challenges in validation of AM techniques including the identification of micro-cavities and miscellaneous micro-defects.

A 3D X-ray CT image is computed from a set of 2D radiographs (or projection images) that are collected from various viewing angles. These viewing angles are achieved by moving the sample along a prescribed scanning trajectory relative to the X-ray source and detector. Each radiograph records the intensity of the X-ray beam, attenuated by transmission through the sample, that can be compared with the non-attenuated beam. However, X-ray attenuation is a complicated, energy-dependent process that encompasses photoelectric absorption, Rayleigh (elastic) scattering, Compton (inelastic) scattering, and electron-positron pair production. Lab-based X-ray sources produce polychromatic (or broad-spectrum) beams which interact strongly and non-linearly with large metallic components. These interactions, when modelled incorrectly as they often are, cause artifacts in the reconstructed tomograms. These artifacts (e.g. streaks across the image, non-uniform grey-scale values) can obscure critical defects and significantly hinder analysis. Three of the most problematic issues are:

- **X-ray photon starvation**, where regions of the detector behind highly attenuating aspects of the object receive few X-rays. In these regions the recorded X-ray levels are comparable to background noise and the high noise levels cause a severe instability in standard X-ray reconstruction algorithms that implicitly treat noise as a uniform Gaussian rather than the correct Poisson model.
- **X-ray beam-hardening**, where the spectrum of the X-ray beam changes significantly as it passes through the sample. As the X-ray spectrum changes, so too does its interactions with the sample. This behaviour is not captured in X-ray CT reconstruction models, thus causing artifacts.
- **X-ray scatter**, where scattered photons corrupt the radiographs. X-ray CT reconstruction assumes that X-ray photons travel along straight lines from the source to the detector. Large metal parts need to be imaged using highly penetrating X-rays with energies in the 100-500 keV range where the dominant interaction is Compton scattering. These scattered X-ray photons emanate in a wide cone from the object and some hit the detector and corrupt the radiographs.

Here we present several techniques in development which deal with these three issues for the custom-built micro-CT instruments at the ANU CTLab. These techniques are built on a novel scanning trajectory that makes tomography more robust to artifacts and enables rapidly-converging iterative tomographic reconstruction. Iterative reconstruction incorporates a forward step that can model more complex aspects of the X-ray imaging system, with the potential to account for non-ideal X-ray interactions and thereby remove artifacts. In what follows, we first outline the suite of proposed techniques in Section 2 and demonstrate the performance of these techniques in various simulations and experiments in Section 3. We then discuss the efficacy of these techniques, envision future directions of research, and conclude with some remarks in Section 4.

Figure 1. Four types of X-ray source scanning trajectories for a cylindrical object: (a) circular trajectory; (b) helical trajectory; (c) double helical trajectory; (d) space-filling trajectory. Note that (a–c) are implemented as discrete sampling of continuous line trajectories, while (d) is a discrete trajectory by design.

2. Materials and Methods

Our approach to imaging AM components has five main aspects. Sub-sections 2.1 and 2.2 outline an optimised scanning trajectory and the associated rapidly-converging iterative tomographic reconstruction (ITR) algorithm suitable for large data-sets. Straightforward enhancements to ITR that improve modelling of the physical processes causing photon starvation and beam hardening are discussed in sections 2.3 and 2.4. Finally, section 2.5 describes hardware and software techniques being developed to reject scattered X-rays or to remove their detrimental contributions.

2.1. Optimised scanning trajectory

A hypothetical tomographic scanning trajectory that covers every possible source/detector position would include a great deal of redundant data. Selecting the subset of positions that make up our scanning trajectory is one of the main choices of a tomographic imaging system.

Traditionally, “line” trajectories are developed in continuous space, then implemented by discrete sampling along these lines (see e.g., Fig. 1a–c). Circular scans are ubiquitous (Fig. 1a), but provide an insufficient set of views for reconstruction [2], causing artifacts at high cone angles that have the greatest impact on sharp, flat features perpendicular to the rotation axis. Cone-beam effects are typically mitigated by careful sample positioning, but the complex geometries of AM components present a particular challenge since it may be impossible to predict defect locations and orientations. A helical trajectory contains sufficient data for accurate reconstruction at high cone angles (Fig. 1b) [3], but the resolution varies spatially within the tomogram and it is difficult to co-align the set of measurements [4]. These issues can be mitigated using redundancy via a double-helix trajectory (Fig. 1c) or a low-pitch helix at the expense of increased experiment time.

We have developed a *Space-Filling* trajectory (SFT, Fig. 1d) that departs from a finely-sampled continuous trajectory [5]. It uses a widely-separated set of source positions that are isotropically distributed in the space of possible viewing directions, and provide maximally independent information about a component of unknown geometry.

SFT satisfies a discrete form of Tuy data sufficiency [2]; its isotropic nature maximises the probability of viewing through thin regions of the AM component, where beam-hardening and photon starvation are minimal. This ensures that at least some of the measurements collected are (relatively) free of these artifacts, so uncorrected artifacts in the reconstructed tomograms tend to be dispersed evenly through angle space, reducing their impact. SFT also enables rapid batch-scanning of samples by providing data that is more easily co-aligned to correct for instrument misalignment

and motion instability [6,7].

Discrete trajectories like the SFT cannot be reconstructed using an analytical (filtered back projection type) algorithm; instead an ITR method is required. We have leveraged the properties of this trajectory to develop a rapidly converging ITR, as outlined in the next section.

2.2. Iterative tomographic reconstruction

Conventional reconstruction is performed using a one-shot analytical method such as filtered back-projection. While one-shot methods are fast, they are extremely restrictive: the experimental data collected must be manipulated to fit idealised geometric and attenuation conditions. One-shot methods also provide no feedback to the validity of these manipulations. An iterative tomographic reconstruction (ITR) algorithm includes a forward-projection operation where the experiment is simulated; this enables validation and tuning of any manipulations applied. An ITR algorithm may also be further extended to reduce artifacts, as shown in the next section. The main drawback of ITR algorithms is that they are computationally intensive. In the following we describe a fast ITR method for the SFT.

The set of X-ray paths passing through each point in the reconstructed volume from measured data collected using an SFT has an isotropic nature. In fact, the point-spread-function (PSF) from back-projection can be well approximated by a shift-invariant blur. Thus a sharpening convolution, that is an approximate inverse to this blur, can be used as a pre-conditioner to achieve rapid convergence of ITR [5]. This property, combined with the multi-grid scheme outlined in [8], enables the ITR algorithm to converge in only two iterations. At CTLab we have been routinely reconstructing data sets containing over 100 gigavoxel data sets with this ITR algorithm since 2016. Within the reconstruction scheme one can apply regularisation based on compressed sensing concepts such as sparsity in volume and/or gradient space, and we have also integrated motion correction into the ITR algorithm [6].

The ITR algorithm provides the platform to deal with the non-ideal X-ray interactions encountered when scanning AM components. Low-resolution modelling of CT imaging is computationally cheap, so a multigrid iterative framework that performs calculations at multiple resolution (“grid”) levels can cheaply incorporate complex models of low-resolution phenomena such as beam-hardening and scatter. It also enables us to naturally deal with X-ray beam-blocking devices such as collimators in reconstruction: blocked regions can be ignored. In contrast, when using one-shot reconstruction methods, unknown data in blocked regions must be interpolated, reducing the final tomogram resolution. One can also potentially use the ITR algorithm to bias the tomographic result towards properties known *a priori*, e.g., an object geometry matching a CAD model. However, this approach will only be effective once non-ideal X-ray interactions are understood and accounted for. This is explored in the remainder of this section.

2.3. Maximum-likelihood for photon starvation

When using an analytical reconstruction algorithm, photon-starvation artifacts are typically minimised through “metal artifact reduction” algorithms that identify and remove highly-attenuated regions from radiographs by interpolation or in-painting [9,10]. These methods prevent reliable defect identification when imaging metal AM

components, as the highly attenuating object itself is of interest. ITR algorithms, by contrast, can incorporate a physically-accurate noise model that implicitly handles photon-starved regions.

Many algorithmically- and computationally-complex ITR algorithms have been developed under the physically-accurate assumption of Poisson-distributed photon shot noise, e.g. via the expectation-maximisation method. In practice, simpler and faster L_2 minimisation algorithms (i.e. least-squares) are preferred, such as those from *Thibault et al.* [11]. Note that L_2 -norm minimisation finds the maximum-likelihood (ML) solution assuming uniform additive Gaussian noise in attenuation measurements.

Minimisation of an arbitrary inner-product-norm is amenable to the same ITR algorithms, and allows approximate likelihood maximisation under a broad range of noise models. Most notably, the Poisson noise inherent in detection events is incorporated very easily via a pre-back-projection weighting by the measured photon counts [11,12]. This technique combines the simplicity and computational performance of least-squares methods with the accuracy of previous statistical methods. Advantages of statistical reconstruction extend beyond noise reduction— for example, automatically dealing with detector regions blocked by a collimator employed for scatter rejection.

A general framework for adapting least-squares ITR to any noise model was outlined in [12], which has since been extended to Poisson-Gaussian noise convolutions that are known to model the physics of imaging more accurately [13]. An effective technical implementation of general inner-product-norm minimisation is given by *Thibault et al.* [11].

2.4. Polychromatic model for beam-hardening correction

X-ray “beam hardening” refers to the change in the spectrum of an X-ray beam as it transmits through material: lower energy (soft) photons are attenuated preferentially over high energy (hard) photons, resulting in an increase in the mean energy of the spectrum (hardening). The attenuation characteristics of the beam is therefore constantly changing as it passes through matter, which causes problems for the CT reconstruction algorithm. Beam hardening is a widespread challenge in X-ray CT that does not admit any general solution, and has been the subject of extensive research for over four decades [14].

Conventional reconstruction algorithms assume monochromatic X-rays; in the presence of beam hardening one observes *cupping* and *streaking* artifacts in the tomogram. These artifacts are reduced experimentally with physical filtering that *pre-hardens* the X-ray beam, but this reduces beam flux and is not a complete solution for severe beam hardening which occurs for dense (e.g. AM metal) components. One-shot analytical reconstruction methods implicitly assume monochromatic attenuation of the beam, so any correction of beam hardening must be done through pre- or post-processing. One widely-used approach is to *linearise* the radiographs to approximate the attenuation that would have occurred if monochromatic illumination had been used[15]. This technique is only accurate for single-material objects, and requires independent experimental calibration for each experimental condition.

The principal limitation of all above methods is that they have insufficient information to solve the problem. On the other hand, within ITR methods beam hardening correction can be incorporated into the model itself. More complex ITR algorithms such as [16] perform well by incorporating knowledge of the X-ray beam spectrum and including a realistic physical model for how X-ray attenuation varies with energy.

Figure 2. Overall (left) and close-up (right) views CAD diagram of anti-scatter collimator design for cone-beam radiation, $400 \times 400 \times 100$ mm in size, proposed for fabrication in Ni-alloy. Note the tapered design to accommodate diverging radiation; structures like this can only be fabricated with additive manufacturing.

However, they are extremely computationally intensive.

To reduce the computational demands, we have developed a suite of beam-hardening models [17] based on simplifications of the Alvarez-Macovski X-ray attenuation model [18]. Here attenuation is modelled as a combination of photoelectric absorption and Compton scatter, each of which is a function of material properties A , Z , and ρ (atomic weight, number and density) as well as X-ray energy, E . These simplified models of X-ray attenuation directly account for (rather than correct for) beam hardening, and can be incorporated into the forward-projection operation of our multi-grid ITR scheme with minimal computational cost. Experimental results from employing this method are presented in Section 3. We have found that the most useful and widely applicable simplifications are: (1) assuming attenuation is due to photoelectric absorption only, or (2) assuming material density is proportional to material atomic number. An automated technique to select the appropriate model and parameter values was proposed in [17] and is under development. When the simplified attenuation model is a good approximation, and the beam spectrum and detector response are well characterised, we have observed that the resulting tomogram is quantitative, i.e. contains the physical attenuation of the material at each location within the object.

2.5. X-ray scatter rejection/estimation

For a primary beam with X-ray energies less than 1 MeV, the principal X-ray attenuation mechanisms of materials are photoelectric absorption, Rayleigh (elastic) scatter, and Compton (inelastic) scatter. At energies above about 80 keV, Compton scatter becomes dominant, causing the object to become an X-ray source of secondary photons that are directed in a broad cone. For weakly attenuating objects, the scattered photons incident on the detector are typically insignificant compared with the transmitted primary beam. For highly attenuating samples where resolution is weighed against signal to noise ratio (SNR), the scattered signal can become significant. This scattered signal isn't accounted for in standard X-ray CT reconstruction, leading to structured artifacts. Scatter effects can be mitigated by moving the detector away from the primary source to reduce the cone-angle (solid angle captured by the detector), although this comes at the expense of reduced signal to noise ratio (SNR). Aside from this, there are fundamentally three approaches to scatter correction, in increasing complexity: rejection through collimation; measurement via a beam-stop-array followed by compensation; and, most generally, incorporating scatter within the forward model of ITR (see [19] and [20] for a review).

Collimation is the natural first step in scatter reduction, particularly at CTLab where the scattered signal at the detector is high due to the use of extremely high

cone angles ($> 60^\circ$) to maximise SNR. With a physical collimator (referred to as an "anti-scatter grid" in medical imaging) only photons that appear to originate from very near the X-ray source point will pass through the collimator and reach the detector. We have designed and constructed a collimator dedicated to high cone-angle geometries. Incidentally, it was possible for us to construct this collimator because of AM. (See Fig. 2). The effectiveness of collimation depends on the collimator geometry: the collimation holes need a high depth:diameter ratio and need to be much smaller than the scanned object. The collimator is also more effective when placed near the object. For high cone angles these constraints mean that the collimator dimensions scales with the sample, so that a given collimator will only be effective for a small range of object sizes. Collimation alone is therefore not a general solution.

We have also used a beam-stop array (BSA) to estimate and correct for scatter with some success (see section 3). The BSA is constructed from 100 5mm diameter Tungsten-carbide bearings arranged in a hexagonal lattice. Due to the high magnification and cone angle in CTLab instruments, the BSA must be placed after the object, making analysis somewhat more complex since (1) scatter from the bearings must be subtracted from the estimate, but varies depending on the transmission through the object; (2) the scattered photons captured behind each bearing are from different regions of the object so an overall average must be estimated, and (3) like the collimator, the bearing size used must be a function of the object size.

3. Results

Figure 3. Vertical slices through tomographic reconstruction of a 1-inch diameter pewter pendant using (a) double-helix trajectory (DHT) and one-shot reconstruction [3]; (b) space-filling trajectory (SFT) with multi-grid ITR. Grey levels windows in both images are identical; (c) Performance of BH correction schemes given SFT data and the polychromatic spectrum: (c-i) no BH correction, (c-ii); BH correction assuming attenuation is modelled by photoelectric absorption only; (c-iii) BH correction assuming attenuation is modelled by the Alvarez-Macovski equation [18] with density proportional to Z .

Figures 3 and 4 demonstrate the improved performance, i.e., lower levels of noise and fewer artifacts, that result from employing SFT (section 2.1) combined with ITR (section 2.2). In Fig. 5 we demonstrate a reduction in photon-starvation artifacts from adapting the ITR to find a ML solution given a physically-accurate noise model (section 2.3). Fig. 3 similarly demonstrates a reduction in beam-hardening artifacts that results from the incorporation of the simplified Alvarez-Macovski attenuation model (section 2.4).

Figure 6 shows the result from using a beam-stop array at a subset of viewing angles to directly measure and interpolate the wide-angle X-ray scatter [19]. The beam-stop array blocks all non-scatter signal at various points on the detector, so by interpolating between these points we are able to separate out the scattered and primary beam components of the measured signal. The parameters of a simple scatter model,

Figure 4. Subsets of the central vertical slice of reconstructed data of a combined 5mm sandstone-limestone sample. (a) Data collected with a DH trajectory, reconstructed by Katsevich filtered backprojection [3]. (b) Data collected with SFT, reconstructed with two iterations of the multi-grid ITR described here. The reconstructed data sets are 2640 voxels across with a $2.0\mu\text{m}$ voxel pitch. (a-i) and (b-i) provide a zoom-in of a small central region of (a) and (b) as indicated by the squares.

Figure 5. Tomographic reconstruction of simulated data with Poisson noise showing the improvement in quality when using the appropriate noise model in the ITR algorithm. On the right the conventional least-squares conjugates gradient method (CGM) was modified for a Poisson distribution as described in [12].

(convolution of the intensity projection with a Gaussian function plus a constant), were determined by fitting to the BSA scatter measurements. The modelled scatter was then removed from each measured projection before tomographic reconstruction.

4. Discussion

Grey levels in figure 3 are identical in both of the leftmost slices, so the effects of the SFT can be identified by visual comparison of the empty areas surrounding the pendant. Imaging with the double-helix trajectory results in streaks (characteristic of beam-hardening) through the empty areas, which are not present in the SFT image. The maximally-isotropic nature of SFT has spread these artifacts throughout the reconstructed volume, benefiting subsequent analysis [5]. Note that neither image contains explicit artifact correction and the interior of the pewter cap appears less dense in both images. Figure 4 better shows the effect of SFT (and subsequent ITR) on image resolution and SNR: two iterations of ITR produce a reconstruction qualitatively similar to analytic reconstruction from double-helix data. Comparison of the inset slices shows the ITR produces an image that is slightly sharper and noisier. This has been observed consistently in our use of SFT, and suggests some over-sharpening by the preconditioning filter. This is the subject of ongoing research.

Figure 5 shows the the performance of a simple conjugate-gradient (CG) ITR method adapted to use the Poisson noise model. This simulation is designed to measure resolution as the minimum resolvable line separation under conditions of extreme

Figure 6. Vertical slice through tomographic reconstruction of 7.2mm square-cross-section steel rod, imaged with X-rays generated as 150keV-peak with a 3mm stainless-steel filter. The grey-scale window is $[-0.045, 0.45]$ attenuation/cm (used to accentuate scatter effects in the air). A beam-stop array was employed to determined a model for wide-angle X-ray scatter. (a) result with beam-hardening correction (BHC); (b) results with scatter subtraction and BHC.

photon starvation. The resolution is substantially enhanced by using the correct noise model (rightmost panel) compared to standard CG ITR or one-shot reconstruction.

The two rightmost panels of figure 3 further demonstrates the beneficial effects of physically-accurate modelling in an ITR. In this case assuming that the pewter pendant has density proportional to atomic number (rightmost panel) yields the best correction for the beam-hardening artifacts visible within the cap at the top of the pendant. In the right-most panel of figure 3, the cap is correctly shown to be of near-constant density. Neglecting the Compton effect in the attenuation model leads to incomplete correction of beam-hardening artifacts (mid-right panel, figure 3), demonstrating the Compton effect’s significance at the X-ray energies used to image the pendant. Note that no scatter correction is applied in these models.

Figure 6 demonstrates the reduction in the “halo” artifacts by subtracting an estimate of the wide-angle X-ray scatter (calibrated by measurements using a BSA) in the radiographs. The level of detail in the top of the post has improved. In this preliminary work, features within the object are not as dramatically improved — further research into more sophisticated scatter models is underway.

We have presented a summary of the methods under development at CTLab to deal with non-ideal X-ray interactions such as photon-starvation, beam-hardening, and scatter. The results show the ability to reduce artifacts by application of each of these methods. In combination, we expect significant improvements in imaging, (and thus analysis, defect detection, etc.), of large highly-attenuating objects such as AM components. Future work includes: developing a generalised trajectory (and corresponding reconstruction algorithm) to tailor the specific set of viewing angles to the geometry of the component; automating the selection of the appropriate beam-hardening model and corresponding parameter values; and developing scatter models that can be incorporated into the iterative tomographic reconstruction algorithm.

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