A Reduced-State-Space Markov Chain Monte Carlo Method for Iterative Spatial Multiplexing MIMO

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Abstract—Markov Chain Monte Carlo (MCMC) method applied as Multiple-Input-Multiple-Output (MIMO) detector has shown near capacity performance. However, the conventional MCMC method suffers from an error floor in the high signal-to-noise (SNR) region. This paper proposes a novel robust reduced-state-space MCMC (RSS-MCMC) method, which utilizes the *a priori* information for the first time to qualify the reliable decoded bits from the entire signal space. The new robust MCMC method is developed to deal with the unreliable bits by using the reliably decoded bit information to cancel the interference that they generate. The performance comparison shows that the new technique has improved performance compared to the conventional approach, and further complexity reduction can be obtained with the assistance of the *a priori* information. Furthermore, the complexity and performance tradeoff of the new method can be optimized for practical realizations.

I. INTRODUCTION

Communication system employing multiple antennas at transmitter and receiver, known as Multiple-Input-Multiple-Output (MIMO) spatial multiplexing system, becomes popular in recent years. Similar to a multiuser system, each transmit antenna may be viewed as a user and the channel gains between one transmit antenna and multiple receive antennas can be viewed as spreading code for the corresponding user.

Recently, a statistical method called the Markov Chain Monte Carlo (MCMC) method [1] has been applied to MIMO detection. In the MCMC method, statistical inferences are developed by simulating the underlying process through Markov chain, which searches the state space defined by the transmitted signal. The basic idea is to draw random samples of unknown transmitted signal from their conditional posterior distribution and then to calculate the marginal *a posteriori* distribution by averaging over the random samples. Hence, MCMC starts from uniformly distributed samples and walks through the transmitted signal space in a stochastic manner to look for the important/significant samples close to the transmitted signal. In practical implementation, Gibbs sampler (GS) [2]–[5] is a particular realization of the Markov chain process.

Authors in [6] made the comparison between the MCMC detector and the sphere decoder (SD) detector [7]. The results in [6] show that the MCMC detector outperforms the SD detector in low signal-to-noise (SNR) region. However, it has

been found that the MCMC detector has degraded performance as SNR increases and suffers from error floor at high SNR [5], [8]. Authors in [5] proposed two solutions: (1) run parallel independent Markov chains; (2) assume a higher noise variance. These solutions show performance improvement in the medium SNRs. Authors in [8] investigated the source of the undesired behavior and proposed a number of ad-hoc methods, such as using ZF or MMSE solution to initialize the Markov chain, and run more iterations with additional samples if error occurs after Cyclic Redundancy Check (CRC). These methods are effective, however, the complexity is inevitably increased.

This paper proposes a novel robust reduced-state-space MCMC (RSS-MCMC) method to improve the performance at high SNR. In the modified MCMC method, the *a priori* information is utilized to qualify the reliably decoded bits from the entire signal space by developing two reliability constraints. Then the fully loaded MIMO system is transformed into a partially loaded system by using the reliably decoded bit information to cancel the interference that they generate. Finally, the Gibbs sampler is performed on the reduced signal space. Meanwhile, the new MCMC method has another benefit, that is, the complexity can be reduced because the reliably decoded bits are processed by the interference canceler rather than the complex Gibbs sampler.

This paper is organized as follows. In Section II, the MIMO spatial multiplexing system is described and the iterative detection and decoding are introduced. In Section III, the RSS-MCMC method is proposed. Simulation results are shown in Section IV. Section V concludes the paper. Notations used in this paper are as follows. Matrices and vectors are denoted by symbols in bold face and $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ represent complex conjugate, transpose and Hermitian transpose. $E\{\cdot\}$ denotes the statistical expectation. $[\mathbf{X}]_{i,j}$ indicates the $(i,j)^{th}$ elements of a matrix $\mathbf{X}$, and similarly, $[\mathbf{x}]_i$ indicates the element i in a vector $\mathbf{x}$. Finally, $\{x\}$ represents the sequence x .

II. SYSTEM MODEL

A. MIMO Spatial Multiplexing System

The MIMO spatial multiplexing system considered in this paper is shown in Fig. 1. There are N_T transmitting antennas and N_R receiving antennas. The information bits $\{\mathbf{b}\}$ are first encoded and interleaved to coded bits sequences $\{\mathbf{d}\}$. These

coded bits are mapped into a sequence of M -ary complex symbols $\{\mathbf{x}\}$ represented by a vector $\mathbf{x} = [x_0, x_1, \dots, x_{N_T-1}]^T$.

Assuming that each transmitting and receiving antenna link undergoes independent flat fading, the system model can be expressed as:

$$\mathbf{y} = \mathbf{h}\mathbf{x} + \mathbf{n}, \quad (1)$$

where $\mathbf{y}$ is the received signal arranged as $N_R \times 1$ vector $\mathbf{y} = [y_0, y_1, \dots, y_{N_R-1}]^T$. $\mathbf{h}$ is $N_R \times N_T$ channel matrix. $\mathbf{n}$ is $N_R \times 1$ additive white Gaussian noise (AWGN) vector with covariance $\sigma^2 \mathbf{I}_{N_R}$. It is convenient to reformulate the system model from a complex value domain to a real value domain by defining the $2N_R \times 1$ vector $\mathbf{Y}$, $2N_T \times 1$ vector $\mathbf{X}$, $2N_R \times 1$ vector $\mathbf{N}$ as:

$$\begin{aligned} \mathbf{Y} &= \begin{bmatrix} \Re(\mathbf{y})^T & \Im(\mathbf{y})^T \end{bmatrix}^T \\ \mathbf{X} &= \begin{bmatrix} \Re(\mathbf{x})^T & \Im(\mathbf{x})^T \end{bmatrix}^T \\ \mathbf{N} &= \begin{bmatrix} \Re(\mathbf{n})^T & \Im(\mathbf{n})^T \end{bmatrix}^T \end{aligned}$$

and $2N_R \times 2N_T$ matrix $\mathbf{H}$ as:

$$\mathbf{H} = \begin{bmatrix} \Re(\mathbf{h}) & -\Im(\mathbf{h}) \\ \Im(\mathbf{h}) & \Re(\mathbf{h}) \end{bmatrix},$$

where $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts of the argument. Then the system can be expressed in as:

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N}. \quad (2)$$

We can divide the transmitted signal sequence $\mathbf{X}$ into two sets, namely the reliable signal set $\mathbf{X}_R$ and unreliable signal set $\mathbf{X}_U$. The reliable signal set $\mathbf{X}_R$ contains the bits that satisfy the reliability constraint, which is proposed in Section III, and are considered as correctly detected in the previous iteration. Similarly, the unreliable signal set $\mathbf{X}_U$ contains the bits that not satisfy the reliability constraint and are considered as unknown. Therefore, the system can be expressed in as:

$$\begin{aligned} \mathbf{Y} &= \mathbf{H} \cdot \text{diag}(\mathbf{e}) \cdot \mathbf{X} + \mathbf{H}(\mathbf{I}_{2N_T} - \text{diag}(\mathbf{e}))\mathbf{X} + \mathbf{N} \\ &= \mathbf{H}\mathbf{X}_U + \mathbf{H}\mathbf{X}_R + \mathbf{N}, \end{aligned} \quad (3)$$

where $\mathbf{X}_U = \text{diag}(\mathbf{e})\mathbf{X}$, $\mathbf{X}_R = (\mathbf{I}_{2N_T} - \text{diag}(\mathbf{e}))\mathbf{X}$, $\text{diag}(\cdot)$ is the diagonal function, and $\mathbf{e}$ is the $2N_T \times 1$ vector which contains the position of unreliable bits. In this paper, the new MCMC method treats the reliable signal set $\mathbf{X}_R$ as the interference, which can be deterministically removed.

B. Iterative Detection and Decoding

In the conventional non-iterative receiver, the MIMO detector tries to maximize the likelihood of the transmitted signal, that is, to minimize the Euclidian Distance to the received signal given that the channel is known, which is given by:

$$\hat{\mathbf{X}} = \arg \max_{\mathbf{X} \in \mathcal{A}} p(\mathbf{Y}|\mathbf{X}) = \arg \min_{\mathbf{X} \in \mathcal{A}} \|\mathbf{Y} - \mathbf{H}\mathbf{X}\|^2, \quad (4)$$

where $\mathcal{A}$ is the signal set of dimension $2N_T$. On the other hand, in the iterative decoding, the MIMO detector aim to

Fig. 1. MIMO spatial multiplexing transmitter with iterative receiver

maximize a posteriori(MAP) probability of transmitted signal sequence, which is given by:

$$\hat{\mathbf{X}} = \arg \max_{\mathbf{X} \in \mathcal{A}} p(\mathbf{X}|\mathbf{Y}) = \arg \max_{\mathbf{X} \in \mathcal{A}} \frac{p(\mathbf{Y}|\mathbf{X})p(\mathbf{X})}{p(\mathbf{Y})}. \quad (5)$$

In equation (5), $p(\mathbf{X})$ is considered as the *a priori* information fed back from channel decoder. In the first iteration, the *a priori* information is not available, the ML metric (4) is employed in the MIMO detector in Fig. 1. From the second iteration onwards, MAP detection metric (5) is employed. In each iteration, the MIMO detector selects points $\{\hat{\mathbf{X}}\}$ from the completed signal constellation set to form a list $\mathcal{U}$ to compute the extrinsic log likelihood ratio (LLR) of the coded bit as:

$$\begin{aligned} \lambda_1^e(d_k(\hat{\mathbf{X}})) &= \frac{1}{2} \sum_{\hat{\mathbf{X}} \in \mathcal{U}_k^+} \left(-\frac{1}{\sigma^2} \|\mathbf{Y} - \mathbf{H}\hat{\mathbf{X}}\|^2 + \mathbf{d}_{-k}^T \lambda_{2,-k}^e \right) \\ &\quad - \frac{1}{2} \sum_{\hat{\mathbf{X}} \in \mathcal{U}_k^-} \left(-\frac{1}{\sigma^2} \|\mathbf{Y} - \mathbf{H}\hat{\mathbf{X}}\|^2 + \mathbf{d}_{-k}^T \lambda_{2,-k}^e \right), \end{aligned} \quad (6)$$

where $d_k(\hat{\mathbf{X}})$ is the k^{th} coded bit in sequence $\{\mathbf{d}\}$ representing the Markov chain enumerated symbol vector $\hat{\mathbf{X}}$. $\mathcal{U}_k^+$ and $\mathcal{U}_k^-$ denote the subset of $\mathcal{U}$ for which $d_k(\hat{\mathbf{X}})$ is $+1$ and -1 respectively. $\mathbf{d}_{-k}$ is obtained from sequence $\mathbf{d}$ by removing the k^{th} coded bit. $\lambda_{2,-k}^e$ is the extrinsic LLR of sequence $\mathbf{d}_{-k}$ from the channel decoder. After MIMO detection, the sequence of extrinsic LLR $\{\lambda_1^e\}$ of coded bits is deinterleaved and passed on to the channel decoder to complete one iteration.

III. MIMO DETECTOR WITH RSS-MCMC METHOD

Studies in [8] revealed that at high SNR, a high portion of the coded bits can be detected in the first few iterations. The LLRs of these bits have large value such that some of the transition probabilities in the underlying Markov chain may become very small. As a result, the Markov chain may be divided into a number of of nearly disjoint chains because the high probability associated with large LLR value prevent the transitions among the disjoint chains. Hence, the Markov chain has less chance to visit sufficient states. This phenomenon is undesired for the stochastic approach of the MCMC method, which requires a large amount of distinct samples in order to cover the whole state space defined by the transmitted signal. In the RSS-MCMC method, our approach is to first transform the fully loaded MIMO system into a partially loaded system by canceling the reliably decoded bits, which are qualified

by the proposed novel reliability constraints derived from the *a priori* information. Then we run Markov chains only for a subset of the signal space (unreliable bits) in the resultant sub-system.

A. Reliability Constraints

We first utilize the *a priori* information to propose two reliability constraints to construct the reliable signal set. Intuitively, the LLRs fed back from channel decoder (*a priori* information) are the direct measure of the reliability of the coded bits. Unfortunately, the analytical treatment of the soft output of channel decoder is difficult. Nevertheless, we make use of the Gaussian consistent assumption based on the empirical observations from simulations in [9]. The Gaussian consistent assumption states as follows: 1) For large interleavers the *a priori* information remain fairly uncorrelated from channel observation over many iterations. 2) The probability density function of the extrinsic information of channel decoder (*a priori* information of detector respectively) approach Gaussian-like distribution with increasing number of iterations. Gaussian consistent assumption suggests that the *a priori* information as input of MIMO detector can be modeled by applying an independent Gaussian random variable η with variance σ_η^2 and zero mean in conjunction with the known coded bit $d_k \in \{-1, 1\}$ as follows:

$$\lambda_1^e(d_k) = \mu \cdot d_k + \eta, \quad (7)$$

where the mean value μ satisfies the relation [9]

$$\mu = \frac{\sigma_\eta^2}{2}. \quad (8)$$

To compute μ and σ_η^2 , we can compute the 2nd order statistics of LLRs, which is given by:

$$E\{\|\lambda_1^e(d_k)\|^2\} = \mu^2 + \sigma_\eta^2 = \mu^2 + 2\mu. \quad (9)$$

The solution of μ can be obtained by taking the positive root of equation (9):

$$\mu = -1 + \sqrt{1 + E\{\|\lambda_1^e(d_k)\|^2\}} \approx \sqrt{E\{\|\lambda_1^e(d_k)\|^2\}}, \quad (10)$$

if $E\{\|\lambda_1^e(d_k)\|^2\}$ is significant larger than one over iterations. And the solution of σ_η^2 is obtained as:

$$\sigma_\eta^2 = 2\mu \approx 2\sqrt{E\{\|\lambda_1^e(d_k)\|^2\}}. \quad (11)$$

Therefore, the conditional distribution of the *a priori* information given that $d_k = 1$ and $d_k = -1$ is Gaussian with $\mathcal{N}(\mu, \sigma_\eta^2)$ and $\mathcal{N}(-\mu, \sigma_\eta^2)$ respectively. Then we can obtain a threshold ρ from the error probability of the *a priori* information as follows:

Case $d_k = -1$:

$$\begin{aligned} P(\text{error}|d_k = -1) &= \int_{\rho}^{\infty} \frac{1}{\sqrt{2\pi\sigma_\eta^2}} \exp\left(-\frac{(\lambda_1^e + \mu)^2}{2\sigma_\eta^2}\right) d\lambda_1^e \\ &= \int_{\frac{\rho+\mu}{\sigma_\eta}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\gamma^2}{2}\right) d\gamma \\ &= Q\left(\frac{\rho + \mu}{\sigma_\eta}\right), \end{aligned} \quad (12)$$

where $Q(\cdot)$ is the Q-function. Then we have

$$\frac{\rho + \mu}{\sigma_\eta} = Q^{-1}(P(\text{error}|d_k = -1)), \quad (13)$$

where $Q^{-1}(\cdot)$ is the inverse of Q-function. Hence, we can obtain the threshold ρ as:

$$\rho = \sigma_\eta \cdot Q^{-1}(P(\text{error}|d_k = -1)) - \mu. \quad (14)$$

Case $d_k = 1$:

$$\begin{aligned} P(\text{error}|d_k = 1) &= \int_{-\infty}^{\rho} \frac{1}{\sqrt{2\pi\sigma_\eta^2}} \exp\left(-\frac{(\lambda_1^e - \mu)^2}{2\sigma_\eta^2}\right) d\lambda_1^e \\ &= \int_{\frac{-\rho+\mu}{\sigma_\eta}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\gamma^2}{2}\right) d\gamma \\ &= Q\left(\frac{-\rho + \mu}{\sigma_\eta}\right). \end{aligned} \quad (15)$$

Then we have

$$\frac{-\rho + \mu}{\sigma_\eta} = Q^{-1}(P(\text{error}|d_k = 1)). \quad (16)$$

Hence, we can obtain the threshold ρ as:

$$\rho = -\sigma_\eta \cdot Q^{-1}(P(\text{error}|d_k = 1)) + \mu. \quad (17)$$

Combine the cases of $d_k = -1$ and $d_k = 1$, we propose to use this threshold ρ as first reliability constraint, which can be obtain as:

$$\rho = |\sigma_\eta \cdot Q^{-1}(P(\text{error}|d_k = \pm 1)) - \mu|. \quad (18)$$

It can be seen that this threshold ρ is adaptive to the pre-defined error probability, and first order and second order statistics of the *a priori* information over iterations. The coded bits with absolute LLR values great than this threshold will be considered as qualified from this constraint.

Furthermore, as discussed before, LLRs with large values result in disjoint Markov chains with less chance to visit the significant samples. The insufficient number of distinct samples may generate ill-conditioned LLRs [7], which has large value but sign flipped. This can be explained by equation (6). We can rewrite the LLR in (6) as the summation of the ML LLR and the *a priori* LLR:

$$\lambda_1^e(d_k(\hat{\mathbf{X}})) = \lambda_{1,ML}^e(d_k(\hat{\mathbf{X}})) + \lambda_{1,AP}^e(d_k(\hat{\mathbf{X}})), \quad (19)$$

where

$$\lambda_{1,ML}^e(d_k(\hat{\mathbf{X}})) = \frac{1}{2} \left(\sum_{\hat{\mathbf{X}} \in \mathcal{U}_k^+} -\frac{1}{\sigma^2} \|\mathbf{Y} - \mathbf{H}\hat{\mathbf{X}}\|^2 + \sum_{\hat{\mathbf{X}} \in \mathcal{U}_k^-} \frac{1}{\sigma^2} \|\mathbf{Y} - \mathbf{H}\hat{\mathbf{X}}\|^2 \right), \quad (20)$$

and

$$\lambda_{1,AP}^e(d_k(\hat{\mathbf{X}})) = \frac{1}{2} \left(\sum_{\hat{\mathbf{X}} \in \mathcal{U}_k^+} \mathbf{d}_{-k}^T \lambda_{2,-k}^e - \sum_{\hat{\mathbf{X}} \in \mathcal{U}_k^-} \mathbf{d}_{-k}^T \lambda_{2,-k}^e \right). \quad (21)$$

The ML LLR is the measure of the Euclidian Distance between the enumerated coded bits and the actual transmitted coded bits, and the *a priori* LLR is the measure of likelihood of the coded bit of interest as seen by other interfering bits. The computation of LLR in (6) relies on ML LLR initially, then is joint determined by ML LLR and the *a priori* LLR over iterations, and finally dominated by the *a priori* LLR. If we purely rely on the threshold as unique reliability constraint, once LLR error occurs in the reliable signal set, the large but sign flipped LLR value may dominate the computation of the *a priori* LLR for current bit of interest, even though the ML LLR has the correct sign. This phenomenon is especially undesirable in MCMC, because the later drawn samples are influenced by the earlier drawn samples. Our investigation shows that the receiver suffers from error propagation as in the hard decision DFE. Therefore, we propose the second reliability constraint that the sign of the *a priori* LLR should be the same as the ML LLR as:

$$\lambda_{1,ML}^e(d_k(\hat{\mathbf{X}})) \cdot \lambda_{1,AP}^e(d_k(\hat{\mathbf{X}})) > 0. \quad (22)$$

This second reliability constraint ensures that only the coded bits with the *a priori* LLR enhancing the ML LLR over iterations are considered to be reliable. In practical implementation, the ML LLR from first iteration can be stored and doesn't have to be computed in every iteration. The *a priori* LLR in each iteration can be obtained by subtracting the ML LLR from first iteration in equation (6).

B. RSS-MCMC Method

In the RSS-MCMC method, the interference from the bits in the reliable signal set $\mathbf{X}_R = \{d_{r,1}, d_{r,2} \dots\}$ as constructed by the two reliability constraints is first canceled from received signal. Extended from equation (3), the output of interference canceler can be obtain as:

$$\begin{aligned} \mathbf{Y}' &= \mathbf{Y} - \mathbf{H}\hat{\mathbf{X}}_R \\ &= \mathbf{H}\mathbf{X}_U + \underbrace{\mathbf{H}(\mathbf{X}_R - \hat{\mathbf{X}}_R)}_{\mathcal{N}(0, \sigma^2 \mathbf{I}_{2N_R})} + \mathbf{N}. \end{aligned} \quad (23)$$

The residue interference and noise have zero mean and covariance matrix $\sigma^2 \mathbf{I}_{2N_R}$ because the LLRs of the bits in the reliable signal set are large.

The next step of the RSS-MCMC method is to run Markov chain for the coded bits in the unreliable signal set $\mathbf{X}_U =$

$\{d_{u,1}, d_{u,2} \dots\}$. Given $\mathbf{Y}'$ and the *a priori* information λ_2^e , the *a posteriori* probability is evaluated as follows:

$$\begin{aligned} P(d_{u,k} = \pm 1 | \mathbf{Y}', \lambda_2^e) &= \sum_{\mathbf{X}_{U,-k}} P(d_{u,k} = \pm 1, \mathbf{X}_{U,-k} | \mathbf{Y}', \lambda_2^e) \\ &= \sum_{\mathbf{X}_{U,-k}} P(d_{u,k} = \pm 1 | \mathbf{Y}', \mathbf{X}_{U,-k}, \lambda_2^e) P(\mathbf{X}_{U,-k} | \mathbf{Y}', \lambda_2^e). \end{aligned} \quad (24)$$

The Gibbs sampler initializes the samples in $\mathbf{X}_U$ with equal probability of 0.5, and proceeds with drawing one sample in $\mathbf{X}_U$ at a time. The procedure is summarized as follows:

- Initialize $\mathbf{X}_U$ randomly with equal probability
- for $n = 1$ to N
 - draw $d_{u,1}^{(n)}$ from $P(d_{u,1} | d_{u,2}^{(n-1)}, d_{u,3}^{(n-1)}, \dots, \mathbf{Y}', \lambda_2^e)$
 - draw $d_{u,2}^{(n)}$ from $P(d_{u,2} | d_{u,1}^{(n)}, d_{u,3}^{(n-1)}, \dots, \mathbf{Y}', \lambda_2^e)$
 - draw $d_{u,3}^{(n)}$ from $P(d_{u,3} | d_{u,1}^{(n)}, d_{u,2}^{(n)}, d_{u,4}^{(n-1)}, \dots, \mathbf{Y}', \lambda_2^e)$
 - $\vdots$
 - draw $d_{u,k}^{(n)}$ from $P(d_{u,k} | d_{u,1}^{(n)}, \dots, d_{u,k+1}^{(n-1)}, \dots, \mathbf{Y}', \lambda_2^e)$
 - $\vdots$

It can be seen that in the n^{th} loop, the k^{th} sample is drawn based on the probability $P(d_{u,k} = \pm 1 | \mathbf{Y}', \mathbf{X}_{U,-k}^{(n)}, \lambda_2^e)$ that partially depends on the $(k-1)$ samples drawn in the n^{th} loop, and partially depends on the rest samples drawn in the $(n-1)^{th}$ loop. That is $\mathbf{X}_{U,-k}^{(n)} = \{d_{u,1}^{(n)}, d_{u,2}^{(n)}, \dots, d_{u,k+1}^{(n-1)}, \dots\}$. This probability is obtained by first computing the *a posteriori* LLR:

$$\begin{aligned} \lambda_1^{(n)}(d_{u,k}) &= \ln \frac{P(d_{u,k} = 1 | \mathbf{Y}', \mathbf{X}_{U,-k}^{(n)}, \lambda_2^e)}{P(d_{u,k} = -1 | \mathbf{Y}', \mathbf{X}_{U,-k}^{(n)}, \lambda_2^e)} \\ &= \ln \frac{P(\mathbf{Y}' | \mathbf{X}_{U,-k}^{(n)}, d_{u,k} = 1)}{P(\mathbf{Y}' | \mathbf{X}_{U,-k}^{(n)}, d_{u,k} = -1)} + \lambda_2^e(d_{u,k}). \end{aligned} \quad (25)$$

The computation of $P(\mathbf{Y}' | \mathbf{X}_{U,-k}^{(n)}, d_{u,k} = \pm 1)$ follows the equation (23) as follows:

$$P(\mathbf{Y}' | \mathbf{X}_{U,-k}^{(n)}, d_{u,k} = \pm 1) = \mathcal{K} \exp\left(-\frac{\|\mathbf{Y}' - \mathbf{H}\mathbf{X}_{U,-k}^{(n)}\|^2}{2\sigma^2}\right), \quad (26)$$

where $\mathcal{K}$ is a constant. Once $\lambda_1^{(n)}(d_{u,k})$ is obtained, we have

$$P(d_{u,k} = 1 | \mathbf{Y}', \mathbf{X}_{U,-k}^{(n)}, \lambda_2^e) = \frac{1}{1 + \exp(-\lambda_1^{(n)}(d_{u,k}))}. \quad (27)$$

After going through the above procedure, important samples [5] are drawn for the bits in the unreliable set and they form the list $\mathcal{U}$ together with the bits in the reliable set. The final step is to compute the extrinsic LLR for each bit by applying equation (6).

The most significant differences between the RSS-MCMC method and conventional MCMC methods can be summarized as follows. Firstly, the statistically significant samples can be drawn in an interference reduced system rather than a fully loaded system. In the RSS-MCMC method, interference from reliable bits are removed in (23), which results in a MIMO system with less interference, and the Gibbs sampler

performance can be improved. Secondly, the method operates in a reduced -state space. If all the bits are reliably decoded, the RSS-MCMC method is an interference canceler. On the other hand, if all the bits are unreliably decoded, the RSS-MCMC method is the same as conventional MCMC method. Otherwise, the RSS-MCMC detector is the hybrid conventional MCMC method and interference canceler.

IV. SIMULATION RESULTS

A. Simulation Setup

We consider a 4×4 MIMO spatial multiplexing system and compare the Bit Error Rate (BER) for the iterative receivers with the conventional MCMC method [5] and the iterative receiver with the RSS-MCMC method. We also show the complexity reduction by considering the computation load of interference canceler against that for Gibbs samplers. The complexity is measured by the number of the drawn samples in the RSS-MCMC method over the total number of samples drawn in the conventional MCMC method. We run four parallel Markov chains, and within a Markov chain, there are five samples for each coded bit. The channel model for each transmit and receive antenna are independent flat Rayleigh fading channel. A rate-1/2 $(171, 133)_8$ convolutional code is used for channel coding. The modulation includes QPSK and 16QAM. Unless otherwise stated, the threshold ρ is selected to achieve 10^{-4} coded bit error rate. We refer to the conventional MCMC GS method in [5] as “Conventional MCMC” and the RSS-MCMC method as “RSS-MCMC”. “itr 2”, “itr 3”, and “itr 4” denote the 2^{nd} , 3^{rd} and 4^{th} iteration.

B. Numerical Results

Fig. 2 shows the BER performance for the iterative receivers with the MCMC methods over 4 iterations. It can be seen that in both QPSK and 16QAM modulations, the RSS-MCMC method improves the performance over the conventional MCMC method. At high SNR, the conventional MCMC detector shows degraded performance in the 3^{rd} and 4^{th} iteration, such undesired phenomenon has been observed in [5], [8]. Nevertheless, the RSS-MCMC does not show the error floor.

Fig. 3 shows the complexity reduction per SNR for the RSS-MCMC method over the conventional MCMC method. In both QPSK and 16QAM modulation, the complexity reduction increases as the E_b/N_0 increases. There is no complexity reduction in the first iteration as the interference cancellation has not been employed because there is no *a priori* information available. In the 2^{nd} iteration, the complexity reduction reaches 30% for QPSK and 8% for 16QAM at 6dB. As iteration goes, more bits satisfy the reliability constraints. This can be observed in the 4^{th} iteration that more than 50% and 30% bits in QPSK and 16QAM respectively at 6dB are considered as reliably decoded bits and canceled in the RSS-MCMC method rather than continuously drawn samples in the conventional MCMC method.

Fig. 4 shows the complexity reduction for the RSS-MCMC method over iterations for QPSK modulation. There is a

Fig. 2. BER performance for the iterative receivers with the conventional MCMC detector, and with the RSS-MCMC detector in a 4×4 MIMO spatial multiplexing system with QPSK and 16QAM modulation

Fig. 3. Complexity reduction for the RSS-MCMC detector over SNR with QPSK and 16QAM modulation

significant increase of the complexity reduction from the 2^{nd} and the 3^{th} iteration, while the complexity reduction saturates in 4^{th} iteration. This observation indicates the evolution of the reliability of LLRs over iterations. It also indicates that after four iterations, the LLRs converges to equilibrium and the further complexity reduction is marginal. Similar results can be observed for 16QAM modulation, which is not shown here. Table I summarize the total complexity reductions. The total complexity reduction is the summation of complexity in each iteration including the first iteration. Generally, the complexity reduction in QPSK is more than that in 16QAM. It can be seen that 35% computation power can be saved in QPSK at 6dB, while 26% computation power can be saved in 16QAM at 12dB.

Finally, there exists a performance and complexity tradeoff at different SNRs as we move the threshold ρ . If the threshold is high, the RSS-MCMC method becomes the conventional MCMC method. If the threshold is low, the RSS-MCMC

Fig. 4. Complexity reduction for the RSS-MCMC detector over iterations with QPSK modulation

TABLE I
TOTAL COMPLEXITY REDUCTIONS

SNR	Reduction in QPSK	SNR	Reduction in 16QAM
0dB	12.5%	6dB	18%
2dB	22%	8dB	21.5%
4dB	31.5%	10dB	24.5%
6dB	35%	12dB	26%

method tends towards the interference canceler. In other words, optimal ρ values exist at different SNRs to provide the best BER performance with maximum complexity reduction. Fig. 5 shows the complexity vs. performance for the RSS-MCMC method with 16QAM modulation if we move the threshold to target different LLR reliability, i.e. to achieve different complexity reductions. We can observe the trend of movement of the optimal point for different SNR regions, that is the optimal point moves towards the right lower corner of the figure. This trend means that in the low SNR region, the extrinsic LLRs are less reliable, hence it's better for the Gibbs sampler to draw more samples rather than less because the MCMC method is good at detection at low SNR. On the other hand, in the high SNR region, the extrinsic LLRs are so reliable that interference canceler can provide a system with less interference, the RSS-MCMC method can have better performance compared to the conventional MCMC method. Consequently, in practical realization, we can either select the optimal ρ at different SNRs, or we can select suboptimal ρ for all SNRs so that the good BER performance and reasonable complexity reduction can be obtained.

V. CONCLUSION

In this paper, a novel RSS-MCMC method is proposed. We first utilize the *a priori* information to develop two reliability constraints to construct the reliably decoded signal set, then we propose the RSS-MCMC method which cancels the interference from the reliably decoded bits and runs Markov chain

Fig. 5. Complexity and performance tradeoff for the RSS-MCMC detector over iterations with 16QAM modulation at 8dB and 10dB

only on the unreliable bits. Performance comparison shows that the RSS-MCMC methods has better performance, especially in high SNR region. And further complexity reduction can be achieved from the interference canceler rather than the complex Gibbs sampler.

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REFERENCES

- [1] C. R. Robert and G. Casella, *Monte Carlo Statistical Methods*. New York: Springer, 1999.
- [2] V. Buchoux, O. Cappe, and E. Moulines, "Turbo multiuser detection for coded DS-CDMA systems: a gibbs sampling approach," in *Proc. Thirty-Fourth Asilomar Conf. Signals, Systems and Computers*, 29 Oct.-1 Nov 2000, pp. 1426–1430.
- [3] B. Lu and X. Wang, "Bayesian blind turbo receiver for coded OFDM systems with frequency offset and frequency selective fading," vol. 12, no. 19, pp. 2516–2527, Dec 2001.
- [4] R. Chen, J. S. Liu, and X. Wang, "Convergence analysis and comparisons of markov chain monte carlo algorithms in digital communications," *IEEE Trans. Signal Processing*, vol. 50, no. 2, pp. 255–270, Feb 2002.
- [5] B. Farhang-Boroujeny, H. Zhu, and Z. Shi, "Markov chain monte carlo algorithm for CDMA and MIMO communication systems," *IEEE Trans. Signal Processing*, vol. 54, no. 5, pp. 1896–1909, May 2006.
- [6] H. Zhu, B. Farhang-Boroujeny, and R.-R. Chen, "On performance of sphere decoding and markov chain monte carlo detection methods," *IEEE Signal Processing Lett.*, vol. 12, no. 10, pp. 669–672, Oct 2005.
- [7] B. Hochwald and S. ten Brink, "Achieving near capacity on a multiple-antenna channel," *IEEE Trans. Commun.*, vol. 51, no. 3, pp. 389–399, Mar 2003.
- [8] X. Mao, P. Amini, and B. Farhang-Boroujeny, "Markov chain monte carlo MIMO detection methods for high signal-to-noise ratio regimes," in *Proc. IEEE GLOBECOM*, Washington, DC, 26-30 Nov 2007, pp. 3979–3983.
- [9] S. ten Brink, "Convergence behavior of iteratively decoded parallel concatenated codes," *IEEE Trans. Commun.*, vol. 49, no. 10, pp. 1727–1737, Oct 2001.