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ABSTRACT

Heralded single photon sources (HSPSs) from spontaneous parametric down-conversion are widely used as single photon sources. We study the photon number statistics of an HSPS carrying orbital angular momentum in our laboratory and observe the sub-Poissonian statistics using only photodetectors and an oscilloscope.

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I. INTRODUCTION

The photon is a prominent qubit candidate in quantum information processing (QIP). Most of these applications require a single photon source (SPS) which will provide an “on-demand,” deterministic supply of photons. Such ideal SPSs can be realized in crystal color centers,¹ quantum dots,² single atoms,³ single ions,⁴ and single molecules.⁵ The photon being an indivisible quantum of energy and the consequent impossibility of copying associated information from an unknown arbitrary state makes single photon sources an ideal choice for quantum information processing, secure communication, and metrology. Spontaneous parametric down-conversion (SPDC) is one of the most widely used processes to generate single photon sources as it provides a robust and bright source at room temperature.⁶ In this process, a pump photon is down-converted in frequency in a $\chi^{(2)}$ nonlinear crystal to a pair of photons, conventionally called the signal (*s*) and the idler (*i*). The detection of one photon from the pair heralds the presence of another, and this conditioned detection serves as a technique for generating single photon sources. SPDC is also important in QIP as a bright source of entangled photon pairs⁷ and thus finds applications in quantum teleportation,⁸ quantum cryptography,⁹ and quantum dense coding.¹⁰

A single photon source shows nonclassical properties such as sub-Poissonian photon number distribution and antibunching. The inherent thermal nature of the photon distribution in the individual arms (i.e., signal or idler) evokes the possibility of occurrences of multiple photons and hence would qualify the source as less quantum than it is expected to be. The occurrence of multiple photon pairs and consequent photon bunching threatens the security in cryptography related applications. Thus, it is important to study and quantify the nonclassicality of the single photon sources being used. A general classification puts the sources into three categories, super-Poissonian, Poissonian, and sub-Poissonian, according to the photon number distribution. Classical sources, such as thermal and coherent radiation, show super-Poissonian and Poissonian statistics, respectively, while certain nonclassical sources, such as a single photon source, show sub-Poissonian distribution. Violations of bounds defined for the number statistics of classical sources imply the nonclassical behavior of single photon sources.^{11–14} Photon antibunching is another manifestation of the nonclassical behavior of the source and can be observed through the measurement of the degree of second-order coherence, $g^{(2)}(\tau)$. Single photon sources show antibunching, and hence, $g^{(2)}(0)$ falls to zero. Although antibunching and sub-Poissonian statistics are completely nonclassical

effects and tend to occur together in many systems, they are distinct effects and need not necessarily be associated with one another.^{15,16} For a heralded SPS, antibunching can be observed using a heralded Hanbury Brown-Twiss (HBT) type experiment.^{17,18} Twisted single photons carrying orbital angular momentum (OAM) are gaining interest in quantum information processing as they provide infinite number of orthogonal states to encode information.^{19,20} The statistical second order correlation of classical beams^{21,22} as well as photons²³ carrying OAM has already been studied. In the present work, we verify the sub-Poissonian nature of heralded single photons generated in parametric down-conversion having Gaussian as well as higher order OAM using a simple setup consisting of photodetectors and an oscilloscope.

II. THEORY

One can define an ideal single photon source as a Fock state for which the number statistics gives mean, $\mu = 1$, and variance, $\sigma^2 = 0$. Treatment of the electromagnetic field as a quantum harmonic oscillator allows us to define quadrature field operators, $\hat{X}_1$ and $\hat{X}_2$, which can be shown to follow the uncertainty relation,²⁴ $\langle \Delta \hat{X}_1^2 \rangle \langle \Delta \hat{X}_2^2 \rangle \geq 1/16$. The minimum uncertainty state, with $\langle \Delta \hat{X}_1^2 \rangle = \langle \Delta \hat{X}_2^2 \rangle$, is the coherent state,

$$|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \quad (1)$$

which defines a Poissonian distribution of photon number,²⁵ n , with $\langle (\Delta \hat{n})^2 \rangle = \langle \hat{n} \rangle$. Squeezing these states in terms of number or phase is achieved when the respective variance is less than that corresponding to the coherent state. The photon number variance can be written as

$$\langle \Delta \hat{n}^2 \rangle = \langle \hat{n} \rangle + \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} - \langle \hat{a}^\dagger \hat{a} \rangle^2 \rangle, \quad (2)$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators. Based on the variance, the Mandel Q-parameter²⁶ is defined by

$$Q \equiv \frac{\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} - \langle \hat{a}^\dagger \hat{a} \rangle^2 \rangle}{\langle \hat{a}^\dagger \hat{a} \rangle} = \frac{\langle (\Delta \hat{n})^2 \rangle - \langle \hat{n} \rangle}{\langle \hat{n} \rangle}. \quad (3)$$

It classifies light sources on the basis of photon number fluctuations. $Q \geq 0$ for coherent and all other classical sources of light, while sub-Poissonian light is identified with $Q < 0$.^{26,27}

The detected photoelectron statistics and the statistics of the incident photons can be different depending upon various factors such as the coherence time of the incident light, detector dead-time, and efficiency of detection. The first two are taken care of while setting the binning window and pump intensity, respectively. This is discussed in detail in Sec. III. Correcting for the efficiency, the mean and standard deviation for the detected-photon distribution can be written as²⁸

$$\langle m \rangle = \eta \langle n \rangle, \quad (4)$$

$$\sigma^2(m) = \eta^2 \sigma^2(n) + \eta(1 - \eta) \langle n \rangle, \quad (5)$$

where m is the detected photocounts, n is the actual photocounts, η is the detector quantum efficiency, and σ^2 is the variance. Hence, the expression for the Fano factor will be

$$F_n = (F_m - (1 - \eta)) / \eta. \quad (6)$$

Correcting for the efficiency of detection, the Q-parameter will be

$$Q_n = Q_m / \eta. \quad (7)$$

III. EXPERIMENT

The experimental setup used for building the statistics of the heralded single photon source is given in Fig. 1.²⁹ A 405 nm diode laser (Toptica TopMode) is used as a pump to generate photon pairs from a Type-I BiBO crystal (Crestech, 5 mm thickness). The polarization of the pump is oriented along the crystal axis using a half-wave plate. The output pairs are spectrally filtered using interference filters of passband of 810 ± 5 nm to rule out the nondegenerate pairs. Two diametrically opposite regions are selected from the cone of correlated pairs of photons generated in the Type-I SPDC. These individual arms (signal and idler) are then collimated and coupled to a single mode fiber (SMF, idler) and a multimode fiber (MMF, signal) through a fiber coupler (CFC-5X-B).

The individual signal and idler photons are detected using single photon detectors. We have used two single photon counting modules (SPCMs, Excelitas SPCM-AQRH-16-FC) placed in both signal and idler arms for the detection of the photon pairs. Photomultiplier tubes (PMTs), which will be more common in laboratories than an SPCM can also be used for the detection. The coincidences are maximized by optimizing the coupling to the fiber. The

FIG. 1. Experimental setup to determine the photon number statistics of the heralded PDC source. HWP—half wave plate, BiBO—nonlinear crystal, BD—beam dump, IF—interference filters, FC—fiber couplers, and SPCMs—single photon counting modules. Different OAM orders can be imparted to the heralded single photon by placing SPPs of different orders in pump and projecting the idler photon in a single mode fiber.

FIG. 3. Detected photon number distribution for a coherent source. Here, n corresponds to the number of detected photons within a binning window, and $P(n)$ is the probability of obtaining n detection events per bin. The mean photon number is approximately equal to the variance.

of detection is determined to be $\eta = 0.3$. Incorporating this value, the Q -parameter for the heralded photon source is determined as -0.33 for the heralded single photons. A set of repeated measurements reveals the average value, $Q = -0.331 \pm 0.004$, for a heralded SPS generated using a Gaussian pump profile. $Q = -1$ implies an ideal single photon source where photons come out in order with a variance that equals to zero in their temporal distribution. Intuitively, Q is expected to be less negative (i.e., reduced nonclassicality) for the case of heralded SPS as the pair generation is a spontaneous process and it would result in a nonzero variance. In addition to this, various losses randomize the detection of a HSPS and hence result in a

FIG. 4. Detected photon number distribution for a heralded single photon source. Here, n corresponds to the number of coincidence detection within a binning window, and $P(n)$ is the probability of obtaining n coincidence events per bin.

reduced nonclassicality in measurement. While the intention of this work is not to formulate a rigorous theoretical model for the number statistics of heralded photons, we matched the experimentally observed data with the statistics of a simulated coincidence series. A coincidence series is generated from two correlated time series consisting only 0 and 1 which correspond to the individual idler and signal series. The correlated time series can be generated by randomly distributing 1's in a time series (such that the sum of 1's is matched with the experimentally observed single counts) and duplicating it. The random distribution of 1's replicates the spontaneous nature of the pair generation process. However, the actual scenario would differ from this as losses in the system would individually randomize these series further. After incorporating a loss of $\eta = 0.3$ in each time series separately by randomly erasing 30% of 1's, we build the coincidence series the same way we did during postprocessing of the experimental data 2. For parameters such as efficiency, binning window, coincidence window, total counts, and time series length

FIG. 5. Detected photon number distribution for thermal photons in the individual idler (top) and signal (bottom) arms of the downconverted pairs. Here, n corresponds to the number of single arm (idler/signal) photons within a binning window, and $P(n)$ is the probability of obtaining n detection events per bin.

matched with the corresponding experimentally observed values, the Q -parameter turns out to be -0.3 . Hence, the experimentally observed value of Q for HSPS is reasonable with the predictions from a lossy model for spontaneously generated correlated pairs of photons.

In the same way, photon number histograms are generated for the individual arms of the down-converted pairs of photons (Fig. 5). For an average photon number $\mu = 0.1$, the variances were 0.126 and 0.128, respectively, for the signal and idler arms which are significantly apart from that corresponds to a Poissonian distribution. With $\eta = 0.3$, the Q -parameter is calculated to be 0.87 and 0.93 for the idler and signal arms, respectively, with statistical errors of the order of 10^{-3} . This super-Poissonian behavior is expected since the individual signal and idler photons from the down-conversion process are in a thermal state.^{33,36,37} Being thermal in nature, the single arm statistics is expected to be such that $Q = 1$.

The Q -parameter for the heralded single photons having different OAM, obtained by pumping with an optical vortex and projecting the idler arm to $l = 0$, are given in Table I. The time series of photon detection events of 20.5 ms length are recorded using the oscilloscope. 10 such iterations are recorded to determine the Q -parameter within statistical errors. For heralded single photons, it is found to violate the classical bounds of number statistics (i.e., $Q \geq 0$) by ~ 10 standard deviations for photons carrying different OAM. The Q -parameter is observed to decrease with the increasing order of twisted photon OAM. This can be attributed to the reduction in the number of coincidence events with higher order pump as the singles in the idler arm (with $l = 0$) decrease with pump order. The effects on the probability amplitude of the two photon coincidences and spectral bandwidth when pumped with different Laguerre-Gaussian modes is already discussed in the literature.^{38,39}

The background counts due to ambient stray light (<100 cps) has been measured and subtracted from the detected counts. After-pulsing probability for the detectors that are used is about 0.5% and is negligible. Ringing is taken care of by appropriately terminating the output pulse into a $50\ \Omega$ load at the oscilloscope. Triggering from noise signals is also taken care of during postprocessing. This is done by keeping a threshold of 1.0 V while doing the conversion of the time series to binary series since the output TTL levels after BNC couplers are >1.5 V.

TABLE I. Mandel Q -parameter obtained for the photon number distributions of twisted single photons having different orders generated in parametric down conversion.

Twisted photon OAM, l	Q -parameter
0	-0.331 ± 0.002
1	-0.336 ± 0.003
2	-0.343 ± 0.005
3	-0.353 ± 0.008

V. CONCLUSIONS

Conventional quantum optics experiments to determine the nonclassical statistical behavior of single photon sources involve time-to-digital converters along with associated postprocessing. Here, we introduce a simple technique to build the number statistics of a heralded single photon source using a digital oscilloscope. The signal and idler in the SPDC output are detected and registered in the oscilloscope. From the recorded data, a time series corresponding to the coincident detection and the corresponding number distribution are obtained. In this way, one can observe the sub-Poissonian behavior of the photon number statistics of the source using a set of detectors and an oscilloscope available in almost all the optics laboratories. We also show that in this way we can determine the number statistics of heralded single photons carrying OAM and thus can characterize them for further applications involving OAM of single photons.

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