SOME PROBLEMS OF MUSICAL ACOUSTICS

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Introduction

To many people the subject of musical acoustics has a peculiarly nineteenth century sound about it – surely Rayleigh and Helmholtz between them solved all the important problems! But physics and music seem to have had a particular affinity since the time of Archimedes and a surprisingly large number of physicists play musical instruments and are interested in the study of their acoustical properties.

For some the wave of the future lies with electronics and the synthesis of entirely new sounds, and I would not decry their enthusiasm, but my purpose here is to look briefly at the analytical approach to traditional musical instruments which has made such great progress in the past few decades. In this field at any rate we must recognise the human ear as superior to the wave analyser, the insight of the master craftsman as superior to the products of computer aided design. But we can use these modern tools to understand the acoustical bases for traditional design solutions and, in this way, both renew our respect for the subtlety of the intuitive approach and extend traditional designs for new purposes.

Strings and Pipes

From ancient times to the present day most musical instruments have been based on one or other of two basic resonant systems – the stretched string or the air column. How simple they seem and with what completeness we expound them to our classes! The transverse eigenfunctions of the string are sine functions and their frequencies are integral multiples $n\nu_0$ of the fundamental frequency ν_0. The sound given out by the string when it is plucked is rich in harmonics, with the particular mixture being determined by the plucking point, and each overtone decays exponentially as it loses energy. The organ pipe is nearly as simple, with the open and closed varieties to add interest, and the sound spectrum is $n\nu_0$ or $(2n - 1)\nu_0$ for the two cases. The end correction seems a little mysterious, but it is just 0.6 times the radius so there is obviously some straightforward calculation somewhere. How simple it all is!

Actually, of course, we are deluding ourselves. The overtones of a real string are not harmonic – the stiffness of the string requires that the n^{th} overtone has frequency $n\nu_0\,(1 + \alpha n^2)$, where the inharmonicity parameter α increases as the square of the radius for a string of given length tuned to a given pitch: α for a given string also varies inversely with the tension, which explains the miserable twang emitted by a lightly loaded sonometer in our first year laboratories! The eigenfunctions are not really quite sine functions either, since the stiffness makes the differential equation of fourth order, and they are all coupled together non-linearly through their effect on the tension, or more properly through the longitudinal modes of the string.

We could, perhaps, neglect all this as theoretical sophistry were it not for its practical implications. A piano is tuned by first setting a tempered scale over a central octave and then extending this to the full compass by tuning successive octaves above and below. But an octave is tuned by eliminating beats between the fundamental of the upper note and the first overtone of the lower note. If this first overtone is sharper than a true second harmonic, then the octave will be stretched.

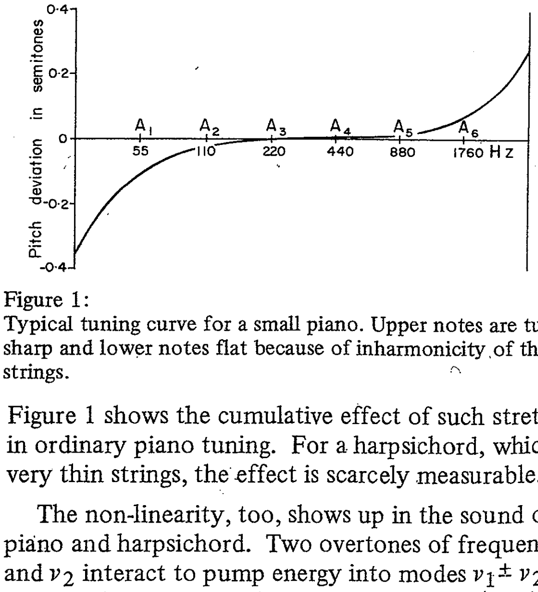

Figure 1:
Typical tuning curve for a small piano. Upper notes are tuned sharp and lower notes flat because of inharmonicity of the strings.

Figure 1 shows the cumulative effect of such stretching in ordinary piano tuning. For a harpsichord, which has very thin strings, the effect is scarcely measurable.

The non-linearity, too, shows up in the sound of both piano and harpsichord. Two overtones of frequencies ν_1 and ν_2 interact to pump energy into modes $\nu_1 \pm \nu_2$ but, due to inharmonicity, this is not exactly a normal mode for the string and slow beat phenomena are produced among the upper partials even for a single string. It is these variations which give 'life' to the sound.

Having gone this far, however, we have barely started, for the string is attached to non-rigid supports which couple it to a more-or-less resonant soundboard structure loaded by its own internal damping and its radiation resistance to the air. It is hard to know where to begin an analysis, even if all we wish to do is to specify the wire gauges and overspinning of bass strings to be used to give even volume and properly graded decay time over the whole compass. A classical harpsichord and a modern piano both behave superbly in their own fashion but it is not yet because physicists understand exactly what is going on!

The situation with a simple organ pipe is even more complex, since we must deal not only with the pipe but also with the air jet which excites it. Even the pipe is bad enough, for the simple end-correction turns out to decrease as the frequency increases so that all the higher resonances of a simple pipe are sharp of true harmonics of the fundamental. Of course we never really have a 'simple pipe' in an instrument, for it is always connected to the blowing mechanism, so that the individual resonances may be either sharp or flat in particular cases.

What happens, though, when we blow the pipe? Are the upper partials harmonics or do they coincide with the pipe resonances, and what determines their amplitudes?

The answer is that the blowing mechanism is generally strongly non-linear, whether it is an air jet or a vibrating reed, and this non-linearity couples together all the upper partials and forces them into harmonic relation to the fundamental. The amplitude of each harmonic then depends upon the characteristics of the non-linear excitation mechanism and the extent to which the harmonic frequency agrees with the frequency of one of the pipe resonances. For a narrow pipe the end correction is small and the resonances are nearly harmonic so that the sound is rich in overtones, while a wide pipe has a dull simple sound. Because there is no inharmonicity, the organ scale is true, rather than being stretched like the piano scale.

A major problem for an organ designer is to construct each rank of pipes so that it is tonally coherent and, somewhat surprisingly, this does not mean that each pipe has the same amount of harmonic development. Rather, the bass pipes must be relatively narrower in scale than the treble pipes so that their harmonic development is greater. Organ builders have good empirical rules to solve this problem, and physics is only now beginning to understand their basis.

For the designer of instruments like the flute, oboe or trombone the problem of balance is different, since the same pipe must serve as the resonator over the whole compass. Once again traditional designs are often excellent and theory is only now beginning to be able to understand their success and to suggest minor modifications.

The Violin Family

Instruments of the violin family have, for a long time, been the basis of orchestral music and to many musicians a fine violin represents the greatest achievement of all time in the field of instrument making. How strange then that the modern violin developed to its present stage during the lifetime of just one group of Italian craftsmen and that the instruments which they produced three hundred years ago are still regarded as the pinnacle of perfection.

In the past twenty years the reasons for the quality of these instruments have been painstakingly analysed and it is now possible to produce modern instruments which nearly equal them. More significantly, perhaps, a whole new string family has been developed based on the violin as prototype but replacing viola, cello and double-bass by more compatible designs and adding a further four instruments to the series to make eight in all. The importance of this development is hard to judge as yet, but may prove to be very great.

Performance Technique

There are, of course, many other instruments to which individual physicists have given attention but, instead of discussing these, let us look briefly at an entirely different aspect of musical acoustics. Given an instrument, how does one play it and, in particular, how do distinguished performers play it?

Some instruments, like the organ, are fairly easy to analyse since the action is largely mechanical and interest then focusses on the purely musical aspects of performance. Even the piano comes into this class, since the possible range of variation in touch is relatively small. At the other end of the scale, if we regard the human voice as an instrument, then performance is such an individual and physiological matter that most physicists would hesitate to undertake an investigation. Between these two extremes come the 'personal' instruments like the violin, flute and oboe which are readily accessible to physical measurement, and here modern studies are beginning to disclose information of considerable importance.

Generally speaking the studies follow two lines – to analyse the sound produced and to relate it to actual performance technique through measurement of parameters such as bow pressure and speed for violinists or blowing pressure and lip configuration for wind players. Once we are armed with a sufficient understanding of the acoustical principles underlying the sounding of the instrument itself, these observations can yield information which is both interesting for its own sake and important for those who wish to improve their performance technique of that of their students.

At this stage, of course, we begin to move away from the field of simple physics into neighbouring considerations of psychology, physiology and aesthetics. We must ask questions about the nature of human perception and attempt to make value judgments based on experience and intuition rather than following the established general principles which guide most physical research. It is something of a leap from the equation of motion of an open pipe to the niceties of vibrato in flute playing and our foothold on the other side is not yet secure, but physics has, I am sure, something to contribute.

References

For those who would like to read about some of these things in rather greater detail, the following are suggested as a suitable staring point:

J. Backus. *'The Acoustical Foundations of Music' London:* John Murray, 1970.

A.H. Benade. *'The Physics of Brasses' Scientific American,* July 1973, pp. 24-35.

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