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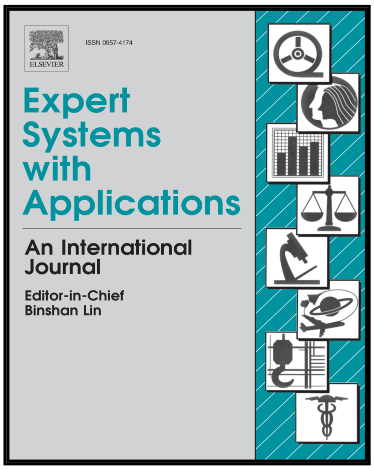

ZBWM: The Z-number Extension of Best Worst Method and its Application for Supplier Development

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Highlights

- Proposing a novel integration of Z numbers and Best Worst Method.
- The method results in lower inconsistency.
- The uncertainty of the real word decisions is considered in the proposed method.

ZBWM: The Z-number Extension of Best Worst Method and its Application for Supplier Development

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Abstract:

Best Worst Method (BWM) has recently been proposed as a method for Multi Criteria Decision Making (MCDM). Studies show that BWM compared with other methods such as Analytic Hierarchy Process (AHP), leads to lower inconsistency of the results while reducing the number of required pairwise comparisons. MCDM methods such as BWM require accurate information. However, it often happens in practice that a level of uncertainty accompanies the information. The main aim of this paper is to address this problem and provide an integration of BWM and Z-numbers, namely ZBWM. Providing BWM with Z-numbers enables the BWM method to handle the uncertainty of information of a multi-criteria decision. Additionally, the capabilities of the proposed method in the process of utilizing the linguistic information dealing with big data are highlighted. The proposed method is examined to address a supplier development problem. By experimental results, we show that ZBWM results lower inconsistency when compared with BWM. A Z-number contains subjectivity in its fuzzy part, which can be addressed in future applications of ZBWM.

Keywords: Z-numbers; BWM; ZBWM; Fuzzy sets theory; Unstructured data

1. Introduction

Best Worst Method (BWM) is a Multi Criteria Decision Making (MCDM) method that was recently proposed by Rezaei (2015). This method leads to lower inconsistency of the results and reduces the number of required pairwise comparisons relative to other methods such as Analytic Hierarchy Process (AHP). It has received significant attention since its proposal. Several researchers have applied the method to address different problems (Gupta and Barua, 2016; Mou, Xu, and Liao, 2016; Rezaei, 2016; van de Kaa, Kamp, and Rezaei, 2017). As proposed by Zadeh, this method is more powerful when combined with Z-numbers (Zadeh, 2011). A Z-number includes two fuzzy variables and is shown as $Z = (A, B)$. The first component, A, is a restriction on a real value of the variable, X. The second component, B, is the first components degree of certainty.

MCDM methods such as BWM require decision makers to assign numbers such as weights of criteria and scores of alternatives (Guo and Zhao, 2017). Generally, an MCDM method works as follows. Assume that the following matrix shows the scores $(a_{11} \ldots, a_{nm})$ that the alternatives $(a_1 \ldots, a_n)$ receive with respect to the criteria $(\mathrm{c}_1 \ \ldots, \mathrm{c}_\mathrm{m})$.

$$\begin{array}{c c} & \begin{array}{cccc} c_1 & c_2 & \cdots & c_m \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_n \end{array} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \end{array} \quad (1)$$

A set of normalized weights, $\mathrm{w}_1 \ \ldots, \mathrm{w}_\mathrm{m}$, are assigned to $\mathrm{c}_1 \ \ldots, \mathrm{c}_\mathrm{m}$, and the value of alternative *'i'* is as follows.

$$V_{ai} = \sum_{\mathrm{j}=1}^{\mathrm{m}} \mathrm{a}_{\mathrm{ij}} w_j \quad (2)$$

When the number is being assigned, a question may arise as to how certain decisions are made when assigning those crisp numbers (Azadeh, Saberi, Atashbar, Chang, and Pazhoheshfar, 2013; Asadabadi, 2016, and 2017). This question plays an important role in the process of making a multiple criteria decision and has motivated the current study. It should be noted that although in reality, many numbers that are used in the field of decision analysis are in Z-numbers they are not treated as such because it is much easier to calculate with ordinary numbers.

Fundamentally, the implication of Z-numbers is a step towards formalization of the significant human uncertainty to make particular decisions in an unpredictable environment (Kang, Wei, Li, and Deng, 2012). Utilizing the mentioned uncertainty is a difficult challenge (Zadeh, 1968). In comparison with fuzzy numbers, Z-numbers have more capability to describe human knowledge. They can be used to describe both restraint and reliability (Deng and Chan, 2011). Particularly with regard to the current presentation of big data, which is mostly unstructured text (Asadabadi, Saberi and Chang, 2017), MCDM methods should be modified to handle such data. Z-numbers are a candidate to handle the uncertainty of information in the comparison matrix by adding a reliability level to fuzzy linguistic values. Although Z-numbers are relatively new (Zadeh, 2011), there has been considerable research on the application of Z-numbers in different domains (Bakar and Gegov, 2015; Yaakob and Gegov, 2016; Yager, 2012). There are few studies applying Z-numbers when dealing with multi-criteria decision-making problems (Bakar and Gegov, 2015; Kang, Wei, Li, and Deng, 2012a; Yaakob and Gegov, 2016). Considering the importance of information accuracy in the domain of MCDM methods (Yaakob and Gegov, 2016), further studies are required to examine the capabilities of Z-numbers when applied in combination with MCDM methods (Kang et al., 2012a). To address that, this paper proposes a combination of Z-numbers and a new MCDM method, namely BWM, and applies the combined method for supplier development purpose.

In this study, the concept of Z-numbers is combined with BWM, and a framework is proposed. The proposed approach is used to address the supplier development problem discussed by Rezaei, Wang and Tavasszy (2015). ZBWM results in less inconsistency when compared with other approaches. From an expert systems point of view, ZBWM suits the current era of big data by considering the large amount of unstructured data that can be used for empowering any expert decision-making system. About 80 percent of the data is unstructured which is predominantly text. This figure shows how important it is to devise the model that is able to quantify the text for using in MCDM techniques. Z-numbers have this ability that has added more modeling capability to normal fuzzy numbers in handling text data.

This paper is firstly organized by presenting a review of literature applying BWM and Z-numbers. Then explained is an integrated approach utilizing a combination of Z-numbers and BWM. Subsequently, the method is validated through a case study to calculate the weights of all the important criteria for a supplier development process. A brief discussion and conclusion follow.

2. Literature Review

In this section, a brief review of BWM is submitted. Following that, the concept of Z-numbers is explained, and related work is discussed.

2.1. BWM

Based on pairwise comparison, BWM (Rezaei, 2015) can obtain the weight of criteria and alternatives with respect to different criteria. In comparison with methods such as AHP, this reduces the number of comparisons between alternatives (Hafezalkotob and Hafezalkotob, 2017). According to BWM, the best and the worst criterion are firstly identified by decision makers. Then, with respect to each criterion, best and worst alternatives are selected. After that, other alternatives are compared with the best and worst options separately against the criterion. The final scores of the alternatives are derived by collecting the weights of different sets of criteria and alternatives. To check the reliability of the comparisons, a consistency ratio has been developed by Rezaei (2015). The consistency ratio presents the reliability of the results. As it does not involve many comparisons, the final weights derived from BWM are relatively reliable in comparison with other MCDM methods such as AHP (Rezaei, 2015).

BWM, since its original proposal in 2015 (Rezaei, 2015), has received considerable attention. It has been applied within a food supply chain context to find the best supplier among qualified suppliers (Rezaei, Nispeling, Sarkis, and Tavasszy, 2016). Gupta, Anand and Gupta (2017) apply the method to find the most important enablers of technological innovation. Ahmad, Rezaei, Sadaghiani and Tavasszy (2017) identify the collective importance of the external forces affecting the sustainability of oil and gas supply chains. Salimi and Rezaei (2016) measure the weights of inputs and outputs of collaborative Ph.D. projects between university and industry. Ren, Liang and Chan (2017) determine the weights of the criteria and the priority of the technologies for sustainability assessment of the technologies for the treatment of urban sewage sludge. Salimi and Rezaei (2017) calculated the weights of a firm's R&D performance. de-Magistris and Gracia (2017) find the most valued European food label. Ghimire, Boyer, Chung and Moss (2016) compare the consumers' preference shares of turfgrass attributes. Shojaei, Haeri and Mohammadi (2017) propose a model to evaluate and rank airports. Praditya and Janssen (2017) derive factors of information sharing arrangements by assessing the importance of factors in shaping information sharing from the outlook of both public and private organizations. Ahmadi, Kusi-Sarpong and Rezaei. (2017) propose a framework to determine and rank the social sustainability of supply chains in manufacturing firms. Gupta et al. (2017) identify and list different barriers to energy efficiency in buildings and address issues in a range of areas. Chitsaz and Azarnivand (2017) investigate the sources of water management problems, formulate strategies, and prioritize the alternatives using a combination of BWM and the Strength-Weakness-Opportunity-Threat (SWOT) technique. Pamučar, Gigovic, Bajic and Janosevic (2017) find sites for wind farms' installment by proposing an integration of the Geographical Information Systems (GIS) and BWM. Stević, Pamucar, Kazimieras Zavadskas, Cirovic and Prentkovskis (2017) use rough BWMs to formulate the problem of the internal transport of a paper manufacturing company as a multi-criteria decision-model.

The variety of applications of this method abound in popularity given the short time since its inception. The fuzzy set theory has been proposed by Zadeh to solve the practical problems under an uncertain environment (Zadeh, 1965). A fuzzy set is a class of objects with a continuum of grades of membership (Babbar and Amin, 2018; Nazari, Fallah, Kazemipoor, and Salehipour, 2018). In order to solve real life decision-making problems, considering the uncertainty arising from lack of complete information and qualitative judgment of decision makers, BWM was extended to consider fuzzy environments (Guo and Zhao, 2017). Fuzzy BWM has been used to evaluate the weights of the severity of pulmonary emphysema criteria, and then propose consistency ratio to check the reliability of the derived results (Mou et al., 2016). For example, Hafezalkotob et al. (2017) propose a novel group decision making approach to combine the opinion of senior decision makers and the opinion of the experts to help senior decision makers undertake a remarkable exchange between democratic and autocratic decision making methods. Recent studies on Fuzzy BWM show that the fuzzy set theory is more efficient in dealing with human judgment than a classical BWM method (Guo et al., 2017; Hafezalkotob et al., 2017). Furthermore, it has been shown that the inconsistency level of Fuzzy BWM is significantly lower than the classical BWM (Guo et al., 2017). Pamučar et al. (2017) provide an improvement in the steps of original BWM and Multi Attributive Border Approximation Area Comparison (MABAC) based on the Interval Valued Fuzzy Rough Number (IVFRN) approach. Therefore, the subjectivity that exists when defining the borders of fuzzy sets and showed a high degree of stability is removed.

Fuzzy numbers, however, fail to take into account the uncertainty of human judgment. This study combines Z-numbers with BWM to address the drawback.

2.2. Z-numbers

The concept of Z-numbers is intended to provide a basis for computation with numbers that are not totally reliable. Z-numbers has been proposed by Zadeh (2011) as a generalized version of the theory of uncertainty (Zadeh, 2006). Z-numbers have been used in combination with AHP to identify the criteria for reliable evaluation of the best universities under uncertain environmental conditions (Azadeh et al., 2013). Mohamad, Shasharani and Kamis (2014) proposed a method of ranking fuzzy number based on Z-numbers to solve a risk analysis problem. Peng and Wang (2017) introduce hesitant uncertain linguistic Z-numbers using linguistic models and solve an enterprise resource planning problem for proof of the validity of the proposed method. Sahrom and Dom (2015) extent the AHP-Fuzzy Data Envelopment Analysis Method using Z-numbers through integrating reliability and fuzzy numbers and ranking the priority of 20 bridge structures. Yaakob and Gegov (2016) proposed a modification of TOPSIS method to solve MCDM problems based on the concept of Z-numbers (Z-TOPSIS) and solve a stock selection problem for demonstration.

The probability and reliability of fuzzy events can be measured (Zadeh, Fu, Tanaka, and Shimura 2008) and Z-numbers can be converted to classical fuzzy numbers (Kang, Wei, Li, and Deng, 2012). For further illustration, Z-numbers and the associated concepts are explained (Azadeh et al., 2013).

Zadeh (2011) states that a Z-number is affiliated with an uncertain variable Z, which is an ordered pair of fuzzy numbers, namely 'A' and 'B'. The first component, 'A', is a fuzzy subset of the domain 'X' of the variable 'Z' while the second one, 'B', is a fuzzy subset of the unit interval. Examples of these Z-valuations are presented below.

The time of the trip: (Approximately 1 hour, usually)
The temperature in summer: (High, Surely)

The restriction $R(X)$ is as presented below.

$$R(X): X\ is\ A \quad (3)$$

This restriction is referred to as a possibilistic restriction, with 'A' playing the role of the possible distribution of 'X'. More specifically it can be stated as follows.

$$R(X): X\ is\ A\ \rightarrow Poss\ (X = u) = \mu_A(u) \quad (4)$$

In the above equation, μ_A is the membership function of A, and 'U' is a generic value of 'X'. μ_A can be viewed as a constraint associated with $R(X)$. This means that $\mu_A(u)$ is the degree to which 'u' satisfies the constraint. In the case that 'X' is a random variable, the probability distribution of 'X' plays the role of a probabilistic restriction on 'X'. The probabilistic restriction is expressed as follows.

$$R(X): X\ is\ p \quad (5)$$

The probability density function of 'X' is as follows.

$$R(X): X\ is\ p\ \rightarrow\ Prob(u\ \leq\ X\ \leq\ u\ +\ du)\ =\ p(u)du \quad (6)$$

2.2.1. Z-number Suitability for BWM

This paper provides an integrated approach combining Z-numbers and BWM. The main reason for this integration is its capability to describe human knowledge; the capability that is greater than fuzzy numbers. Similar uncertainty management techniques such as rough numbers, interval rough numbers and interval valued fuzzy-rough numbers, which have been used recently to improve BWM, do not have the probabilistic feature of Z-numbers (Pamučar, Petrovic and Cirovic, 2018, Gigović et al., 2017, Pamučar et.al, 2017). This integration can also be useful in handling big data. The presentation of big data, is mostly unstructured text (Asadabadi, Saberi, and Chang, 2017) and, so, MCDM methods should be modified to handle such data. The uncertainty of information in the comparison matrices can be addressed using Z-numbers. These can be performed by adding a reliability level to fuzzy linguistic values. We suggest future studies in the field of MCDM methods examine the application of interval-valued fuzzy-rough numbers on the fuzzy part of z-number and address this subjectivity.

2.3. The Relevant Concepts

In this section, the concepts which are utilized in this study, such the selected fuzzy membership function, The Graded Mean Integration Representation (GMIR), Transformation rules, and fuzzy comparisons are explained.

2.3.1. The Choice of the Fuzzy Membership Function

A fuzzy set 'A' is defined on a universe X as presented below.

$$A\ =\ \{(x, \mu_A(x)) |\ x\ \in\ X\} \quad (7)$$

Fuzzy numbers are defined based on sets of real numbers. We consider the triangular type of fuzzy numbers. The membership function of a triangular fuzzy number A, μ_A: R → [0,1] can be represented by the expression.

$$\mu_A(x)\ = \begin{cases} 0 & x < l \\ \dfrac{x-l}{m-l} & l \leq x < m \\ \dfrac{u-x}{u-m} & m \leq x \leq u \\ 0 & x \geq u \end{cases} \quad (8)$$

Where $l\ <\ m\ <\ u$

Let $A = (l, m, u)$ be the symbol representing this type of fuzzy number (an alternative notation for fuzzy set A is $A(x)$). Therefore, a triangular fuzzy number is fully characterized by a triple: (l, m, u). The parameter 'm' gives the grade of $\mu_A(x)$ where parameters 'l' and 'u' are the lower and upper bounds (Zadeh, 1965).

2.3.2. The Graded Mean Integration Representation (GMIR)

In GMIR, $R(\tilde{a})$ of a TFN $\tilde{a}$ represents the ranking of triangular fuzzy number (Zhao and Guo, 2014; Chen and Hsieh, 2000; Liao, Liang and Chen, 2013). Where $\tilde{a}_i = (l_i, m_i, u_i)$, the GMIR $R(\tilde{a}_i)$ of TFN $\tilde{a}_i$ can be calculated as follows.

$$R(\tilde{a}_i) = \frac{l_i + 4m_i + u_i}{6} \qquad (9)$$

2.3.3. Transformation Rules for Z-numbers Linguistic Variables

To illustrate further, we discuss how Z-numbers are converted to regular fuzzy numbers based on a previous study (Kang et al., 2012). Consider a Z-number $Z = (A, B)$ and let $\{\tilde{A} = (x, u_{\tilde{A}}) | x \in [0,1]\}$, $\{\tilde{B} = (x, u_{\tilde{B}}) | x \in [0,1]\}$ be a triangular membership function. The second component of the Z-numbers (reliability) is converted into a crisp number.

$$\alpha = \frac{\int x\mu_\beta \, dx}{\int \mu_\beta \, dx} \qquad (10)$$

Following that, the weights of the second part are added to the first part (restriction). Weighted Z-numbers are as follows.

$$\tilde{Z}^\alpha = \{(\mathrm{x}, \mu_{\tilde{A}^\alpha}) | \mu_{\tilde{A}^\alpha}(\mathrm{x}) = \alpha\mu_{\tilde{A}}(\mathrm{x}), \mathrm{x} \in [0,1]\} \qquad (11)$$

Using a combination of transformation rules (Table 1) and the reliabilities of the restrictions (Table 2), the transformation rules of linguistic variables of decision makers for Z-numbers are obtained. Then, they are transformed into the fuzzy ratings. For the first component (restriction), suppose there are 'n' criteria for a research object. Assume that the fuzzy pairwise comparison of these 'n' criteria are based on the linguistic variables of decision makers, such as 'Equally Important (EI)', 'Weakly Important (WI)', 'Fairly Important (FI)', 'Very Important (VI)', and 'Absolutely Important (AI)'. After this, the linguistic evaluations of decision makers need to be transformed to fuzzy ratings (represented by TFNs). The rules of transformation are listed in Table 1 (Lootsma, 1980; Van Laarhoven and Pedrycz, 1983; Guo and Zhao, 2017).

Table 1. Transformation rules of linguistic variables of constraints

Linguistic terms	Membership function
Equally Important (EI)	(1,1,1)
Weakly Important(WI)	(2/3,1,3/2)
Fairly Important (FI)	(3/2,2,5/2)
Very Important(VI)	(5/2,3,7/2)
Absolutely Important(AI)	(7/2,4,9/2)

The second component can be performed based on five linguistic variables. The rules of transformation are listed in Table 2 (Sahrom et al., 2015).

Table 2. Transformation rules of linguistic variables of reliabilities

Linguistic terms	Membership function
Very Low (VL)	(0,0,0.3)
Low (L)	(0.1,0.3,0.5)

Medium (M)	(0.3,0.5,0.7)
High (H)	(0.5,0.7,0.9)
Very High (VH)	(0.7,1.0,1.0)

Assume a Z-number, $Z = (\tilde{A}, \tilde{R})$ for the restriction. There is a linguistic term 'Very important (VI)' ($\tilde{A} = VI$) with a reliability 'High' ($\tilde{R} = H$). This is expressed below.

$$Z = [(5/2,3,7/2;1),(0.5,0.7,0.9)]$$

First, the reliability is converted to a crisp number according to Equation 10.

$$\alpha = \frac{\int x\mu_\beta \, dx}{\int \mu_\beta \, dx} = 0.7$$

Second, the weight of reliability is added to the constraint.

$$\tilde{Z}^\alpha = (5/2,3,7/2;0.7)$$

Then, the weighted Z-number is converted to the regular fuzzy number.

$$\tilde{Z}' = (\sqrt{0.7} * 5/2, \sqrt{0.7} * 3, \sqrt{0.7} * 7/2) = (2.09,2.51,2.93)$$

Repeating the same procedure, all the members of Table 1 and 2 are transformed as presented in Table 3.

Table 3. Transformation rules for Z-number linguistic variables to fuzzy numbers

Linguistic terms	Membership function
(EI,VL)	(1,1,1)
(EI,L)	(1,1,1)
(EI,M)	(1,1,1)
(EI,H)	(1,1,1)
(EI,VH)	(1,1,1)
(WI,VL)	(0.21,0.32,0.47)
(WI,L)	(0.37,0.55,0.82)
(WI,M)	(0.47,0.71,0.82)
(WI,H)	(0.56,0.84,1.26)
(WI,VH)	(0.63,0.95,1.43)
(FI,VL)	(0.47,0.63,0.79)
(FI,L)	(0.82,1.10,1.37)
(FI,M)	(1.07,1.42,1.78)
(FI,H)	(1.26,1.68,2.10)
(FI,VH)	(1.43,1.90,2.38)
(VI,VL)	(0.79,0.95,1.11)
(VI,L)	(1.37,1.64,1.92)
(VI,M)	(1.78,2.13,2.49)
(VI,H)	(2.10,2.52,2.94)
(VI,VH)	(2.38,2.85,3.33)
(AI,VL)	(1.11,1.26,1.42)
(AI,L)	(1.92,2.19,2.47)
(AI,M)	(2.49,2.84,3.20)
(AI,H)	(2.94,3.36,3.78)

(AI,VH)	(3.33,3.80,4.28)

2.3.4. Fuzzy Reference Comparison

The fuzzy comparison matrix is as follows.

$$\tilde{A} = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{a}_{nn} \end{bmatrix} \end{array} \quad (12)$$

$\tilde{a}_{ij}$ represents the relative fuzzy preference of criterion 'i' to criterion 'j', which is a triangular fuzzy number; $\tilde{a}_{ij} = (1, 1, 1)$ when $i = j$. Based on the basic principle of BWM as detailed by Rezaei (2015), we know that it is not necessary to execute 'n' fuzzy pairwise comparisons in order to obtain a completed matrix $\tilde{A}$. A pairwise comparison $\tilde{a}_{ij}$ is defined as a fuzzy reference comparison if 'i' is the best element and/or 'j' is the worst element. For $\tilde{A}$, there are a total of $2n - 3$ fuzzy reference comparisons ($n - 2$ Best-to-Others fuzzy comparisons $+ n - 2$ Others-to-Worst fuzzy comparisons $+1$ Best to Worst fuzzy comparison), which need to be performed for Fuzzy BWM.

With respect to different criteria, both the fuzzy weights of criteria and the fuzzy weights of alternatives can be determined using Fuzzy BWM. In order to determine the fuzzy weights of criteria, fuzzy comparisons on relative criteria should be executed. To find the fuzzy weights of alternatives, the related alternatives should be compared against each criterion. Finally, the fuzzy ranking scores of alternatives can be derived from the fuzzy weights of different criteria multiplied by the fuzzy weights of the corresponding criteria. Then, the crisp ranking scores of alternatives, where needed, can be calculated by employing the GMIR method for optimal alternative selection. Thus, we see that the logic and procedure of fuzzy comparisons for determining the weights of criteria and alternatives are similar.

3. Methodology

In this section, the concept of Z-numbers is combined with BWM. In the proposed approach, similar steps to the Fuzzy BWM (Guo and Zhao, 2017) are followed in order to make the Z-numbers linguistic variables transformation rules (Table 3).

3.1. ZBWM Steps

The method can be summarized as follows:

Step 1. Build the decision criteria

The decision criteria consist of a set of criteria. Assume that there are 'n' decision criteria: $\{c_1, c_2, \cdots, c_n\}$.

Step 2. Determine the best (most important) criterion and the worst (least important) criterion

Based on the decision criteria, the decision maker should identify the best and worst criterion from their own perspective. The best criterion represents the most desirable or the most important criterion (c_B), and the worst criterion presents the least desirable or the least important criterion for the decision (c_W).

Step 3. Execute Z-numbers reference comparison for the best criterion

As mentioned previously, reference comparisons include two parts: one part is the paired comparisons: $\tilde{a}_{ij}$ where 'i' is the best element, and here c_i is the best criterion (c_B); the other is the pairwise comparison $\tilde{a}_{ij}$ in the case that 'j' is the worst element, and here c_j is the worst criterion (c_W). In this step, the first part will be performed.

By using the linguistic terms of decision makers listed in Table 3, the Z-numbers' preferences of the best criterion over all the criteria can be determined. By using Formula 11, the obtained fuzzy Best-to-Others vector is presented below.

$$\widetilde{A}_B = (\tilde{\mathrm{a}}_{\mathrm{B1}}, \tilde{\mathrm{a}}_{\mathrm{B1}}, \cdots, \tilde{\mathrm{a}}_{\mathrm{Bn}}) \tag{13}$$

Where $\tilde{\mathrm{a}}_{\mathrm{Bj}}$ represents the fuzzy preference of the best criterion (c_B) over criterion j ($j = 1,2, \dots, n$). It can be known that $\tilde{\mathrm{a}}_{\mathrm{BB}} = (1,1,1)$.

Step 4. Execute Z-numbers reference comparison for the worst criterion

In this step, the other part of fuzzy reference comparison is performed. By using linguistic evaluations of decision makers listed in Table 3, fuzzy preferences of all criteria over the worst criterion can be determined. By using Formula 11, the fuzzy Other-to-Worst vector is as follows.

$$\widetilde{A}_W = (\tilde{\mathrm{a}}_{\mathrm{1W}}, \tilde{\mathrm{a}}_{\mathrm{2W}}, \cdots, \tilde{\mathrm{a}}_{\mathrm{nW}}) \tag{14}$$

Where $\tilde{\mathrm{a}}_{\mathrm{iW}}$ represents the fuzzy preference of criterion i ($i = 1,2, \dots, n$) over the worst criterion (c_W), it can be known that $\tilde{\mathrm{a}}_{\mathrm{WW}} = (1,1,1)$.

Step 5. Determine the optimal fuzzy weights ($\widetilde{\boldsymbol{W}}_1^*, \widetilde{\boldsymbol{W}}_2^*, \dots, \widetilde{\boldsymbol{W}}_n^*$)

The optimal fuzzy weight for each criterion is the one for which the following equations are true: $\widetilde{W}_B / \widetilde{W}_j = \tilde{\mathrm{a}}_{\mathrm{Bj}}$ and $\widetilde{W}_j / \widetilde{W}_W = \tilde{\mathrm{a}}_{\mathrm{jW}}$. To satisfy these conditions for all j, we determine a solution where the maximum absolute gaps $\left|\frac{\widetilde{W}_B}{\widetilde{W}_j} - \tilde{\mathrm{a}}_{\mathrm{Bj}}\right|$ and $\left|\frac{\widetilde{W}_j}{\widetilde{W}_W} - \tilde{\mathrm{a}}_{\mathrm{jW}}\right|$ for all j are minimized ($\widetilde{W}_B$, $\widetilde{W}_j$ and $\widetilde{W}_W$ are triangular fuzzy numbers). However, based on the linguistic variables of decision makers in some instances, we need crisp values after obtaining the fuzzy weight of criterion. That is to say, the fuzzy weight of criterion represented by $\widetilde{W}_j = (l_j^W, m_j^W, u_j^W)$ need to be transformed into a crisp value. In this paper, the transformed crisp value of fuzzy weight $\widetilde{W}$ of criterion is needed. This is because we need to build the constraint conditions for the solution as used by Rezaei et al. (2015). We use the graded mean integration representation (GMIR) (see Equation (9)) to transform the fuzzy weights of criterion to crisp weights.

Therefore, the constrained optimization problem for determining the optimal fuzzy weights ($\widetilde{\mathrm{W}}_1^*, \widetilde{\mathrm{W}}_2^*, \dots, \widetilde{\mathrm{W}}_n^*$) is as follows.

$$Min\ Max \left\{ \left|\frac{\widetilde{W}_B}{\widetilde{W}_j} - \tilde{\mathrm{a}}_{\mathrm{Bj}}\right|, \left|\frac{\widetilde{W}_j}{\widetilde{W}_W} - \tilde{\mathrm{a}}_{\mathrm{jW}}\right| \right\}$$

$$s.t. \begin{cases} \sum_{j=1}^{n} R(\widetilde{W}_j) = 1 \\ l_j^W \le m_j^W \le u_j^W \\ l_j^W \ge 0 \\ j = 1,2, \dots, n \end{cases} \tag{15}$$

Where $\widetilde{W}_B = (l_B^W, m_B^W, u_B^W)$, $\widetilde{W}_j = (l_j^W, m_j^W, u_j^W)$, $\widetilde{W}_W = (l_W^W, m_W^W, u_W^W)$, $\tilde{a}_{Bj} = (l_{Bj}, m_{Bj}, u_{Bj})$ and $\tilde{a}_{jW} = (l_{jW}, m_{jW}, u_{jW})$.

Equation (15) can be transferred to the following nonlinear constrained optimization problem.

$$Min\ \tilde{\xi}$$

$$s.t.\begin{cases} \left|\frac{\tilde{W}_B}{\tilde{W}_j} - \tilde{a}_{Bj}\right| \le \tilde{\xi} \\ \left|\frac{\tilde{W}_j}{\tilde{W}_W} - \tilde{a}_{jW}\right| \le \tilde{\xi} \\ \sum_{j=1}^{n} R(\tilde{W}_j) = 1 \\ l_j^W \le m_j^W \le u_j^W \\ l_j^W \ge 0 \\ j = 1,2,\dots,n \end{cases} \tag{16}$$

Where $\tilde{\xi} = (l^{\xi}, m^{\xi}, u^{\xi})$.

Considering $l^{\xi} \le m^{\xi} \le u^{\xi}$, we assume $\tilde{\xi}^* = (k^*, k^*, k^*), k^* \le l^{\xi}$, then Equation (16) can be transferred as follows.

$$Min\ \tilde{\xi}^*$$

$$\text{s.t.}\begin{cases} \left|\frac{(l_B^W, m_B^W, u_B^W)}{(l_j^W, m_j^W, u_j^W)} - (l_{Bj}, m_{Bj}, u_{Bj})\right| \le (k^*, k^*, k^*) \\ \left|\frac{(l_j^W, m_j^W, u_j^W)}{(l_W^W, m_W^W, u_W^W)} - (l_{jW}, m_{jW}, u_{jW})\right| \le (k^*, k^*, k^*) \\ \sum_{j=1}^{n} R(\tilde{W}_j) = 1 \\ l_j^W \le m_j^W \le u_j^W \\ l_j^W \ge 0 \\ j = 1,2,\dots,n \end{cases} \tag{17}$$

Solving Equation (17), the optimal fuzzy weights $(\tilde{W}_1^*, \tilde{W}_2^*, \dots, \tilde{W}_n^*)$ can be obtained.

3.2. ZBWM Strengths and Weaknesses

The main contribution that BWM has made to the literature is the reduction in number of comparisons that have consequently resulted in a lower inconsistency level. By integrating Z-numbers with BWM, we have added two more features to the merits of the method. Firstly, it is possible to address human linguistic uncertainty. Secondly, the component of reliability of Z-numbers has strengthened the new method to be capable of translating human knowledge and, therefore, more suitable for unstructured text. In addition to these, there is now room for further improvements due to subjectivity involved in the fuzzy part of Z-numbers in ZBWM. Recent advancements and findings with this regard can be employed to empower ZBWM improving computed ranking accuracy.

Table 4. ZBWM Strengths and Weaknesses

	Strengths	Weaknesses
ZBWM	• Less pairwise comparisons • Higher consistency • Ability to handle linguistic uncertainty • Ability to be applied in a big data environment	Fuzzy part has the subjectivity issue during the concept translation process

3.3. ZBWM Pseudo Code

To simplify the programming of the proposed approach, the steps of ZBWM are represented in the following pseudocodes. ZBWM pseudocode calls two sub-algorithms in lines 4 and 5. The five steps of ZBWM are associated with the relevant lines of the pseudocode.

Algorithm. 1 ZBWM

Input: linguistic variables of constraints and reliabilities: T,

Output: optimal weights:

1: $C \leftarrow assings\ a\ set\ of\ decision\ criteria$ **(Step 1)**

2: $B \leftarrow$ determine the best criterion **(Step 2)**

3: $W \leftarrow$ determine the worst criterion **(Step 2)**

4: $\tilde{A}_B \leftarrow$ preference_best_criterion(C, B, W, T) **(Step 3)**

5: $\tilde{A}_W \leftarrow$ preference_worst_criterion(C, B, W, T) **(Step 4)**

6: $n \leftarrow |C|$

7: $\boldsymbol{for}\ i = 1{:}n$

8: $\widetilde{\mathrm{W}}_{\mathrm{i}}^{*} \leftarrow Opt(\tilde{A}_B, \tilde{A}_W)$ **(Step 5)**

9: **end**

10: Return $\widetilde{W}^*$

The *Opt* function in line 8 finds the optimum weights following Equation 17. Algorithm 2 is invoked in line 4 of ZBWM pseudocode to identify the user's preference of best criterion over other criteria.

Algorithm 2. preference_best_criterion(C, B,W,T)

Input:
Weights: $\mathbf{wt_i}$
Output:

$n \leftarrow |C|$

$\boldsymbol{for}\ i = 1{:}n$

$println('What\ is\ the\ preference\ of\ the'\ \boldsymbol{B}\ 'over'\ \boldsymbol{C_i}\ '?\ ')$

$println$ ('$pick\ your\ preference\ from'\ T$)

$\mathrm{ZA}_{Bi} \leftarrow$ User's input

$\tilde{A}_{Bi} \leftarrow \mathrm{Ztransform}(\mathrm{ZA}_{Bi})$

end

Return $\tilde{A}_B$

Algorithm 3 is invoked in line 5 of ZBWM Algorithm pseudocode to identify the user's preference of other criteria over the worst criterion. Ztransform function converts a given Z-number, $\mathrm{ZA}_{\mathrm{Bi}}$, into a regular fuzzy number, $\tilde{A}_{Bi}$.

Algorithm 3. preference_worst_criterion (C, B,W,T)

Input:
Weights: $\mathbf{wt_i}$
Output:

$n \leftarrow |C|$

$\boldsymbol{for}\ i = 1{:}n$

$println('What\ is\ the\ preference\ of\ the'\ \boldsymbol{C_i}\ 'over'\ \boldsymbol{W}\ '?\ ')$

$println$ ('$pick\ your\ preference\ from'\ T$)

$\mathrm{ZA}_{Wi} \leftarrow$ User's input

$\tilde{A}_{Wi} \leftarrow \mathrm{Ztransform}(\mathrm{ZA}_{Wi})$

end

Return $\tilde{A}_W$

3.4. Consistency Ratio for ZBWM

Consistency Ratio (CR) is an important indicator to check the inconsistency level of pairwise comparisons. In this section, the computation of CR for ZBWM is explained. A comparison is fully consistent when $\tilde{a}_{Bj} \times \tilde{a}_{jW} =$

$\tilde{a}_{BW}$, where $\tilde{a}_{Bj}$, $\tilde{a}_{jW}$ and $\tilde{a}_{BW}$ present the preference of the best criterion over criterion j, the preference of the criterion j over the worst criterion, and the preference of the best criterion over the worst criterion, respectively.

According to Table 3, the maximum possible fuzzy value of $\tilde{a}_{BW}$ is $(3.33, 3.80, 4.28)$, which corresponds to the linguistic term 'Absolutely Important (AI)' with the reliability linguistic term 'Very High' given by decision makers. When $\tilde{a}_{Bj} \times \tilde{a}_{jW} \neq \tilde{a}_{BW}$, which means $\tilde{a}_{Bj} \times \tilde{a}_{jW}$ may be higher or lower than $\tilde{a}_{BW}$, the inconsistency of fuzzy pairwise comparisons will occur. When both $\tilde{a}_{Bj}$ and $\tilde{a}_{jW}$ are equal to $\tilde{a}_{BW}$, inequality will reach the greatest amount, which results in $\tilde{\xi}$. Considering the occurrence of the greatest inequality, according to the equality relation $(\widetilde{W}_B/\widetilde{W}_j) \times (\widetilde{W}_j/\widetilde{W}_W) = (\widetilde{W}_B/\widetilde{W}_W)$, the following equation can be obtained.

$$\left(\tilde{a}_{Bj} - \tilde{\xi}\right) \times \left(\tilde{a}_{jW} - \tilde{\xi}\right) = \left(\tilde{a}_{BW} + \tilde{\xi}\right) \qquad (18)$$

For the maximum fuzzy inconsistency $\tilde{a}_{Bj} = \tilde{a}_{jW} = \tilde{a}_{BW}$, Equation (18) can be written as follows.

$$\left(\tilde{a}_{BW} - \tilde{\xi}\right) \times \left(\tilde{a}_{BW} - \tilde{\xi}\right) = \left(\tilde{a}_{BW} + \tilde{\xi}\right) \qquad (19)$$

Derived from that, Equation (19) is formulated as follows.

$$\tilde{\xi}^2 - (1 + 2\tilde{a}_{BW})\tilde{\xi} + \left(\tilde{a}_{BW}{}^2 - \tilde{a}_{BW}\right) = 0 \qquad (20)$$

For $\tilde{a}_{BW} = \left(l_{BW}, m_{BW}, u_{BW}\right)$, the maximum possible fuzzy value is $(3.33, 3.80, 4.28)$, which indicates $l_{BW} = 3.33$, $m_{BW} = 3.80$, $u_{BW} = 4.28$. It shows that the maximum value of $l_{BW}, m_{BW}, and\ u_{BW}$ cannot exceed 4.28. In this case, if we use the upper boundary u_{BW}, we can find the maximum possible $\tilde{\xi}$, This is because u_{BW} is the largest in the interval $[l_{BW}, u_{BW}]$. While, $\tilde{\xi}$ can also be represented by a crisp value ξ. In other cases such as $\tilde{a}_{Bw} = (1,1,1)$, $\tilde{a}_{Bw} = (0.21, 0.32, 0.47)$, and $\tilde{a}_{Bw} = (0.37, 0.55, 0.82)$, we can perform a similar process. Therefore, Equation (20) can be transferred to the following equation.

$$\xi^2 - (1 + 2u_{BW})\xi + \left(u_{BW}{}^2 - u_{BW}\right) = 0 \qquad (21)$$

Where $u_{BW} = 1, 0.47, 0.82, 1.07$, and so forth.

By solving Equation (21) for different u_{BW}, the maximum possible ξ can be found and is used as the consistency index for ZBWM (Table 5). Then the consistency ratio can be calculated using $\tilde{\xi}^*$ as follows:

$$Consistency\ Ratio = \frac{\tilde{\xi}^*}{Consistency\ Index} \qquad (22)$$

Given that, the bigger the $\tilde{\xi}^*$ is, the higher the consistency ratio will be, and the less reliable the comparisons becomes (Rezaei, 2015).

Table 5. Consistency index (CI) for ZBWM

Linguistic terms	**(EI,VL)**	**(EI,L)**	**(EI,M)**	**(EI,H)**	**(EI,VH)**	**(WI,VL)**	**(WI,L)**	**(WI,M)**	**(WI,H)**	**(WI,VH)**	**(FI,VL)**	**(FI,L)**	**(FI,M)**
$\tilde{u}_{BW}$	**1**	**1**	**1**	**1**	**1**	**0.47**	**0.82**	**1.07**	**1.26**	**1.43**	**0.79**	**1.37**	**1.78**
CI	**3**	**3**	**3**	**3**	**3**	**2.07**	**2.7**	**3.11**	**3.42**	**3.68**	**2.64**	**3.6**	**4.22**
Linguistic terms	**(FI,H)**	**(FI,VH)**	**(VI,VL)**	**(VI,L)**	**(VI,M)**	**(VI,H)**	**(VI,VH)**	**(AI,VL)**	**(AI,L)**	**(AI,M)**	**(AI,H)**	**(AI,VH)**	
$\tilde{u}_{BW}$	**2.10**	**2.38**	**1.11**	**1.92**	**2.49**	**2.94**	**3.33**	**1.42**	**2.47**	**3.20**	**3.78**	**4.28**	
CI	**4.71**	**5.11**	**3.17**	**4.44**	**5.27**	**5.92**	**6.45**	**3.68**	**5.24**	**6.27**	**7.07**	**7.74**	

4. Case Study

In this section, we use the method to address the supplier development problem discussed by Rezaei et al. (2015). Doing so, we need to determine all the important criteria and their relationships with the decision variables. Supplier development is an important part of any supplier relationship and is a crucial component of supply chain management. In order to be a market leader, improving supplier capabilities and its willingness to collaborate are crucial (Rezaei et al. 2015). Eight criteria for capabilities dimension and four criteria for willingness dimension proposed by Rezaei (2015) are shown in Tables 6 and 7.

Table 6. Main criteria considered to evaluate capabilities

Main capability criteria
Technical capability (C_1^C)
Product quality capability (C_2^C)
Delivery capability (C_3^C)
Intangible capability (C_4^C)
Service capability (C_5^C)
Financial/cost capability (C_6^C)
Sustainable capability (C_7^C)
Organizational capability (C_8^C)

Table 7. Main criteria considered to evaluate willingness

Selected willingness criteria
Willingness to improve performance (C_1^W)
Willingness to share information (C_2^W)
Willingness to rely on each other (C_3^W)
Willingness to get involved in long term relationship (C_4^W)

'Product quality capability' (C_2^C) is selected as the best criterion and 'Organizational capability' (C_8^C) is regarded as the worst capabilities criterion. The linguistic terms of a decision maker for Z-number preferences of the best criterion over all other criteria are listed in Table 8.

Table 8. Best-to-Others vector (capabilities)

Criteria	C_1^c	C_2^c	C_3^c	C_4^c	C_5^c	C_6^c	C_7^c	C_8^c
Best criterion C_2^c	(VI,VH)	(EI,M)	(VI,VL)	(AI,VH)	(VI,H)	(FI,M)	(FI,VH)	(AI,VH)

According to Table 3, the fuzzy Best-to-Others vector for the capabilities criteria can be obtained as follows.

$$\tilde{A}_B^c = [(2.38,2.85,3.33),(1,1,1),(0.79,0.95,1.11),(3.33,3.80,4.28),(2.10,2.52,2.94),(1.07,1.42,1.78),(1.43,1.90,2.38),(3.33,3.80,4.28)]$$

Table 9. Others-to-Worst vector (capabilities)

Criteria	Worst criterion C_8^c
C_1^c	(WI,VH)
C_2^c	(AI,VH)
C_3^c	(AI,VH)
C_4^c	(VI,VL)
C_5^c	(FI,M)
C_6^c	(VI,H)
C_7^c	(FI,VH)
C_8^c	(EI,M)

The linguistic terms of a decision maker for Z-number preferences of all the criteria over the worst criterion are presented in Table 9. According to Table 3, the fuzzy Others-to-Worst vector for the capabilities criteria can be obtained as presented.

$$\tilde{A}_W^c = [(0.63,0.95,1.43),(3.33,3.80,4.28),(3.33,3.80,4.28),(0.79,0.95,1.11),(1.07,1.42,1.78),(2.10,2.52,2.94),(1.43,1.90,2.38),(1,1,1)]$$

$$min \quad k^*$$

$$\begin{cases} -k*u_1 \le i_2 - 2.38*u_1 \le k*u_1 \\ -k*m_1 \le m_2 - 2.85*m_1 \le k*m_1 \\ -k*i_1 \le u_2 - 3.33*i_1 \le k*i_1 \\ -k*u_3 \le i_2 - 0.79*u_3 \le k*u_3 \\ -k*m_3 \le m_2 - 0.95*m_3 \le k*m_3 \\ -k*i_3 \le u_2 - 1.11*i_3 \le k*i_3 \\ -k*u_4 \le i_2 - 3.33*u_4 \le k*u_4 \\ -k*m_4 \le m_2 - 3.80*m_4 \le k*m_4 \\ -k*i_4 \le u_2 - 4.78*i_4 \le k*i_4 \\ -k*u_5 \le i_2 - 2.10*u_5 \le k*u_5 \\ -k*m_5 \le m_2 - 2.52*m_5 \le k*m_5 \\ -k*i_5 \le u_2 - 2.94*i_5 \le k*i_5 \\ -k*u_6 \le i_2 - 1.07*u_6 \le k*u_6 \\ -k*m_6 \le m_2 - 1.42*m_6 \le k*m_6 \\ -k*i_6 \le u_2 - 1.78*i_6 \le k*i_6 \\ -k*u_7 \le i_2 - 1.43*u_7 \le k*u_7 \\ -k*m_7 \le m_2 - 1.90*m_7 \le k*m_7 \\ -k*i_7 \le u_2 - 2.38*i_7 \le k*i_7 \\ -k*u_8 \le i_2 - 3.33*u_8 \le k*u_8 \\ -k*m_8 \le m_2 - 3.80*m_8 \le k*m_8 \\ -k*i_8 \le u_2 - 4.78*i_8 \le k*i_8 \\ -k*u_8 \le i_1 - 0.79*u_8 \le k*u_8 \\ -k*m_8 \le m_1 - 0.95*m_8 \le k*m_8 \\ -k*i_8 \le u_1 - 1.11*i_8 \le k*i_8 \\ -k*u_8 \le i_3 - 3.33*u_8 \le k*u_8 \\ -k*m_8 \le m_3 - 3.80*m_8 \le k*m_8 \\ -k*i_8 \le u_3 - 4.78*i_8 \le k*i_8 \\ -k*u_8 \le i_4 - 0.63*u_8 \le k*u_8 \\ -k*m_8 \le m_4 - 0.95*m_8 \le k*m_8 \\ -k*i_8 \le u_4 - 1.43*i_8 \le k*i_8 \\ -k*u_8 \le i_5 - 1.07*u_8 \le k*u_8 \\ -k*m_8 \le m_5 - 1.42*m_8 \le k*m_8 \\ -k*i_8 \le u_5 - 1.78*i_8 \le k*i_8 \\ -k*u_8 \le i_6 - 2.10*u_8 \le k*u_8 \\ -k*m_8 \le m_6 - 2.52*m_8 \le k*m_8 \\ -k*i_8 \le u_6 - 2.94*i_8 \le k*i_8 \\ -k*u_8 \le i_7 - 1.43*u_8 \le k*u_8 \\ -k*m_8 \le m_7 - 1.90*m_8 \le k*m_8 \\ -k*i_8 \le u_7 - 2.38*i_8 \le k*i_8 \\ \frac{1}{6}*i_1 + \frac{4}{6}*m_1 + \frac{1}{6}*u_1 + \frac{1}{6}*i_2 + \frac{4}{6}*m_2 + \frac{1}{6}*u_2 + \\ \frac{1}{6}*i_3 + \frac{4}{6}*m_3 + \frac{1}{6}*u_3 + \frac{1}{6}*i_4 + \frac{4}{6}*m_4 + \frac{1}{6}*u_4 + \\ \frac{1}{6}*i_5 + \frac{4}{6}*m_5 + \frac{1}{6}*u_5 + \frac{1}{6}*i_6 + \frac{4}{6}*m_6 + \frac{1}{6}*u_6 + \\ \frac{1}{6}*i_7 + \frac{4}{6}*m_7 + \frac{1}{6}*u_7 + \frac{1}{6}*i_8 + \frac{4}{6}*m_8 + \frac{1}{6}*u_8 = 1 \\ i_1 \le m_1 \le u_1 \\ i_2 \le m_2 \le u_2 \\ i_3 \le m_3 \le u_3 \\ i_4 \le m_4 \le u_4 \\ i_5 \le m_5 \le u_5 \\ i_6 \le m_6 \le u_6 \\ i_7 \le m_7 \le u_7 \\ i_8 \le m_8 \le u_8 \\ i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8 > 0 \\ k \ge 0 \end{cases} \tag{23}$$

Now, solving the nonlinear constrained optimization problem (Equation 23), the optimal fuzzy weights of eight main capabilities criteria can be calculated as presented below.

$$w_1^{c*} = (0.063, 0.069, 0.085), w_2^{c*} = (0.218, 0.218, 0.229)$$

$$w_3^{c*} = (0.217, 0.217, 0.229), w_4^{c*} = (0.050, 0.061, 0.061)$$

$$w_5^{c*} = (0.085, 0.095, 0.095), w_6^{c*} = (0.151, 0.156, 0.162)$$

$$w_7^{c*} = (0.098, 0.057, 0.070), w_8^{c*} = (0.050, 0.057, 0.070)$$

$$\tilde{\xi}^{c*} = (0.267, 0.267, 0.267)$$

The crisp weights (GMIRs) for the main capabilities criteria can be calculated as presented below.

$$w_1^{c*} = 0.071, w_2^{c*} = 0.220, w_3^{c*} = 0.219, w_4^{c*} = 0.059, w_5^{c*} = 0.093, w_6^{c*} = 0.156, w_7^{c*} = 0.120, w_8^{c*} = 0.058$$

$$\tilde{\xi}^{c*} = 0.267$$

Therefore, the weights of eight criteria 'Technical capability' (C_1^C), 'Product quality capability' (C_2^C), 'Delivery capability' (C_3^C), 'Intangible capability' (C_4^C), 'Service capability' (C_5^C), 'Financial/cost capability' (C_6^C), 'Sustainable capability' (C_7^C), and 'Organizational capability' (C_8^c) are respectively 0.071, 0.220, 0.219, 0.059, 0.093, 0.156, 0.120, and 0.058. The preference orders of the eight criteria are the same between BWM and ZBWM.

Given $\tilde{a}_{BW}$ = (AI,VH), the consistency index for this case is 7.74 (Table 5). The consistency ratio is 0.267/7.74 = 0.034, which implies a very good consistency. The consistency ratio for this same case using BWM is 0.86/5.23 = 0.164 (Rezaei et al., 2015), which is larger than that of ZBWM. Therefore, the ZBWM shows more consistency in comparisons than the BWM because the ZBWM can take vagaries, uncertainty and the reliability of the decision makers into consideration.

Table 10. Result of BWM – weights of capabilities criteria

Main capabilities criteria	Criteria weights
Technical capability (C_1^C)	0.071
Product quality capability (C_2^C)	0.220
Delivery capability (C_3^C)	0.219
Intangible capability (C_4^C)	0.059
Service capability (C_5^C)	0.093
Financial/cost capability (C_6^C)	0.156
Sustainable capability (C_7^C)	0.120
Organizational capability (C_8^C)	0.058

'Willingness to improve performance' (C_1^W) is selected as the best criterion while 'willingness to share information' (C_2^W) is regarded as the worst criterion. The linguistic terms (variables) of the decision maker for Z-number preferences of the best criterion over all other criteria are listed in Table 11.

Table 11. Best-to-Others vector (willingness)

Criteria	C_1^W	C_2^W	C_3^W	C_4^W
Best criterion C_1^W	(EI,M)	(VI,VH)	(FI,M)	(WI,VH)

Then, according to Table 3, the fuzzy Best-to-Others vector can be obtained as follows.

$$\tilde{A}_B^w = [(1,1,1), (2.38,2.85,3.33), (1.07,1.42,1.78), (0.63,0.95,1.43)]$$

Table 12. Others-to-Worst vector (willingness)

Criteria	Worst criterion C_2^W
C_1^W	(VI,VH)
C_2^W	(EI,M)
C_3^W	(VI,H)
C_4^W	(VI,M)

The linguistic terms of a decision maker for Z-number preferences of all the criteria over the worst criterion are listed in Table 12. According to Table 3, the fuzzy Others-to-Worst vector can be obtained as is presented.

$$\tilde{A}_W^w = [(2.38,2.85,3.33), (1,1,1), (2.10,2.52,2.94), (1.78,2.13,2.49)]$$

$$\min \ k^*$$
$$\begin{cases} -k * u_2 \leq i_1 - 2.38 * u_2 \leq k * u_2 \\ -k * m_2 \leq m_1 - 2.85 * m_2 \leq k * m_2 \\ -k * i_2 \leq u_1 - 3.33 * i_2 \leq k * i_2 \\ -k * u_3 \leq i_1 - 1.07 * u_3 \leq k * u_3 \\ -k * m_3 \leq m_1 - 1.42 * m_3 \leq k * m_3 \\ -k * i_3 \leq u_1 - 1.78 * i_3 \leq k * i_3 \\ -k * u_4 \leq i_1 - 0.63 * u_4 \leq k * u_4 \\ -k * m_4 \leq m_1 - 0.95 * m_4 \leq k * m_4 \\ -k * i_4 \leq u_1 - 1.43 * i_4 \leq k * i_4 \\ -k * u_2 \leq i_3 - 2.1 * u_2 \leq k * u_2 \\ -k * m_2 \leq m_3 - 2.52 * m_2 \leq k * m_2 \\ -k * i_2 \leq u_3 - 2.94 * i_2 \leq k * i_2 \\ -k * u_2 \leq i_4 - 1.78 * u_2 \leq k * u_2 \\ -k * m_2 \leq m_4 - 2.13 * m_2 \leq k * m_2 \\ -k * i_2 \leq u_4 - 2.49 * i_2 \leq k * i_2 \\ \frac{1}{6} * i_1 + \frac{4}{6} * m_1 + \frac{1}{6} * u_1 + \frac{1}{6} * i_2 + \frac{4}{6} * m_2 + \\ \frac{1}{6} * u_2 + \frac{1}{6} * i_3 + \frac{4}{6} * m_3 + \frac{1}{6} * u_3 + \frac{1}{6} * i_4 + \\ \frac{4}{6} * m_4 + \frac{1}{6} * u_4 = 1 \\ i_1 \leq m_1 \leq u_1 \\ i_2 \leq m_2 \leq u_2 \\ i_3 \leq m_3 \leq u_3 \\ i_4 \leq m_4 \leq u_4 \\ i_1, i_2, i_3, i_4 > 0 \\ k \geq 0 \end{cases} \tag{24}$$

Then, by solving the nonlinear constrained optimization problem (Equation 24), the optimal fuzzy weights of four main willingness criteria can be calculated as presented below.

$$w_1^{w*} = (0.261,0.334,0.355), \ w_2^{w*} = (0.113,0.121,0.121)$$
$$w_3^{w*} = (0.228,0.278,0.308), \ w_4^{w*} = (0.227,0.285,0.307)$$

$$\tilde{\xi}^{w*} = (0.221, 0.221, 0.221)$$

The crisp weights (GMIRs) for the main willingness criteria are calculated as presented below.

$$w_1^{w*} = 0.325, w_2^{w*} = 0.120, w_3^{w*} = 0.275, w_4^{w*} = 0.279, \tilde{\xi}^{w*} = 0.221$$

Thus, the weights of four criteria 'Willingness to improve performance' (C_1^W), 'Willingness to share information' (C_2^W), 'Willingness to rely on each other' (C_3^W), and 'Willingness to get involved in long term relationship' (C_4^W) are respectively 0.325, 0.120, 0.275, and 0.279 in BWM method (Rezaei et al., 2015). It is observed that the preference order of four criteria is the same between BWM and ZBWM. Because $\tilde{a}_{BW}$ = (VI,VH), the consistency index for this case is 6.45 (Table 5). Given that, the inconsistency level using consistency ratio is 0.221/6.45 = 0.034. The consistency ratios for this same case using BWM and Fuzzy BWM are 1.146/3 = 0.382 and 0.2361/6.69 = 0.035 respectively (Rezaei et al., 2015; Guo and Zhao, 2017), which are larger than that of ZBWM. Therefore, it can be concluded that ZBWM presents greater consistency than the BWM and Fuzzy BWM.

Table 13. Result of BWM – weights of willingness criteria

Main willingness criteria	Criteria weights
Willingness to improve performance (C_1^W)	0.325
Willingness to share information (C_2^W)	0.120
Willingness to rely on each other (C_3^W)	0.275
Willingness to get involved in long term relationship (C_4^W)	0.279

As we summarized, Z-number is more capable of managing text data. The results show that it provides more consistency and greater reliability. Using the proposed method for the case study, consistency ratios are lower than those obtained using the BWM and Fuzzy BWM. It can affect the decision making process by providing more accurate data.

5. Discussion and Future Study

The vagaries of natural language is a pervading phenomenon in the real-world decision-making process (Asadabadi et al. 2017). Given that, fuzzy sets can be applied to address the problem. However, it often occurs that fuzzy numbers have different reliabilities. The reliability of information has been taken into consideration through applying Z-numbers (Azadeh et al., 2013). This paper presented a combined MCDM method, namely ZBWM. At first, we proposed transformation rules to enable conversion of the Z-number linguistic variables to fuzzy numbers. Then a five step procedure was used to derive the weights of the criteria. In order to analyze the reliability of the results, a consistency ratio was calculated by use of the proposed consistency index table (Table 5). A practical case was selected to show the applicability of this method.

BWM was employed by Rezaei (2015) to develop the suppliers in different segments. For the purpose of comparison, we adopt the example mentioned by Rezaei (2015). This case was also selected by Guo and Zhao (2017) for the application and verification of Fuzzy BWM. At first we compare the ranking of the criteria in all mentioned methods. The ranks obtained with all models are almost the same. For the willingness criteria, the willingness to improve performance and for the capabilities criteria, the product quality capability is best ranked according to three methods. The comparison of ranking the criteria is shown in Table 14 and Table 15. In the case study, we observe that the consistency indices for the capabilities and willingness criteria are 7.74 and 6.45 respectively and, therefore, the inconsistency levels using consistency ratios are 0.267/7.74 = 0.034 for capabilities and 0.221/6.45 = 0.034 for willingness criteria. In comparison with Rezaei et al. (2015) where the consistency ratios for capabilities and willingness are 0.86/5.23 = 0.164 and 1.146/3 = 0.382 respectively, ZBWM resulted in a lower ration and, hence, lower inconsistency. In comparison with Guo and Zhao (2017) where the consistency ratio for willingness is 0.2361/6.69 = 0.035, again ZBWM resulted in higher consistency. The comparison of consistency ratios is shown in Table 16. This means that the ZBWM presents more consistency than the BWM and Fuzzy BWM. The lower inconsistency is because the ZBWM can take vagueness, uncertainty and the reliability of the decision makers into consideration.

Table 14. Ranking of capabilities criteria in BWM and ZBWM

Main capabilities criteria	BWM rank	ZBWM rank

Technical capability (C_1^C)	6	6
Product quality capability (C_2^C)	1	1
Delivery capability (C_3^C)	2	2
Intangible capability (C_4^C)	7	7
Service capability (C_5^C)	5	5
Financial/cost capability (C_6^C)	3	3
Sustainable capability (C_7^C)	4	4
Organizational capability (C_8^C)	8	8

Table 15. Ranking of the willingness criteria in BWM, ZBWM and Fuzzy BWM

Main willingness criteria	BWM rank	ZBWM rank	Fuzzy BWM
Willingness to improve performance (C_1^W)	1	1	1
Willingness to share information (C_2^W)	4	4	3
Willingness to rely on each other (C_3^W)	3	3	2
Willingness to get involved in long term relationship (C_4^W)	2	2	2

Table 16. Consistency ratios for BWM, ZBWM and Fuzzy BWM

	BWM	ZBWM	Fuzzy BWM
Consistency ratio (willingness criteria)	0.382	0.034	0.035

5.1. Important Direction for Future Studies: Dealing with hidden information in unstructured data

Currently, about 80% of data is semi-structured or unstructured (Herschel and Jones, 2005). Such data contains a wealth of information that can boost the quality of decisions. Current MCDM techniques are not capable of using such information (Ho, Xu, and Dey, 2010). Application of techniques such as Natural Language Processing (NLP) for MCDM method can address this problem. For that purpose, techniques such as fuzzy information processing should be utilized. Examining the main steps of ZBWM techniques reveals that steps two, three, and four which were explained in the methodology, can be benefit unstructured information. Especially, when it is not possible to obtain the decision maker preferences (which are essential in completing steps 2-4), using techniques to extract these preferences from unstructured text is helpful. Such situations occur when the person with the essential information is not interested or not available to answer all the questions and, hence, some of the questions are partially unanswered. It should be noted that in some instances, we cannot find information to compare two options in the comparison matrix. An example of this is when the user with the required information does not specify their preferences with regard to some of the elements. This requires future research by using techniques for handling missing data. In this research, we extended BWM in a way that enables the method to accept extracted information from unstructured text using Z-numbers. However, to have a MCDM method capable of finding information from unstructured text, techniques in NLP, and missing data handling can be utilized. This interesting research line can connect two communities in MCDM and big data that seem to be considerably beneficial in the current era of big data. Zadeh, the originator of fuzzy logic, predicted the value of his proposal in the field of soft computing about 15 years ago "*In coming years, computing with words and perceptions is likely to emerge as an important direction in science and* *technology*" (Zadeh, 2001). In recent years, he proposed new lines of research such as fuzzy logic based formalism and Z-numbers to enable performing computation with natural language (CNL) (Zadeh, 2011).

6. Conclusion

The vagaries and ambiguity of real world decision making processes encourages the application of fuzzy sets in the area of decision-making. However, fuzzy sets do not consider the reliability of information in the decision making process. To deal with this, we presented an integrated approach combining Z-numbers, developed by Zadeh (2011), with the BWM. The application of Z-numbers removes the ambiguity and improves the objectivity of the decision-making process. This is due to the fact that the certainty and reliability of the available data to make a particular decision in a given environment is improved. The proposed method is then employed to address a supplier development problem that was previously discussed by Rezaei et al. (2015). This case was also being used by Guo and Zhao (2017) to verify the Fuzzy BWM. At first, the ranks obtained by the proposed method were compared with those of BWM and Fuzzy BWM. The ranking of the criteria are nearly the same in all three methods. There is a negligible difference in the ranks of the third and forth willingness criteria in Fuzzy BWM. Then the superiority of the method in achieving less inconsistency when compared with other methods is exposed. The consistency ratios for the willingness and capability comparisons by BWM were equal to 0.382 and 0.164 respectively and the willingness consistency ratio for the Fuzzy BWM was 0.035. By considering the uncertainty of the real world and also the reliability of the data we observed, the consistency ratios for the willingness and capability comparisons became 0.034. This shows that the comparisons were considerably consistent. Therefore, the ZBWM method has presented a more consistent approach when compared with BWM and Fuzzy BWM. The proposed ZBWM method can be considered the first step in bridging the MCDM and big data communities. For further research, we suggest that in order to validate the process, data be collected from different. Another possible direction for future work would be to apply Z-numbers in combination with other MCDM methods in order to efficiently calculate the criteria weights and reduce the ambiguities of the data.

We have used Z-numbers in combination with BWM when information is in the form of linguistics. Future research could be undertaken to address the subjectivity issue of the fuzzy part of Z-numbers in ZBWM by utilizing interval-valued fuzzy-rough. NLP techniques could be examined to see how NLP and ZBWM can be synchronized in order to automatically handle unstructured text. Further, the method could be combined with the QFD (Quality Function Deployment) method. Previously there has been considerable research on integrating different MCDM methods with QFD. However, we believe that the data coming from customers can be relatively subjective especially where customers are allowed to express opinions. ZBWM can be used to capture the involved subjectivity in customer requirements, and based on customer desires, provide a ranking of product requirements.

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