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Spatial quantum effects with continuous-wave laser beams

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The application of quantum optics to spatial measurements with continuous-wave laser beams is discussed. In particular, we summarize the work on measuring small displacements and tilt. We present the required theoretical multi-mode model and discuss how it applies to split detectors and spatial homodyne detectors. We show how spatial measurements can be improved using squeezed light. Finally, we present a demonstration of improved spatial homodyne measurements using squeezed light in a higher order spatial mode.

1. Introduction

Optics has many applications ranging from imaging to optical communication. At the same time, we are aware of quantum optics—the effects, limitations and opportunities created by the quantum nature of light. This applies not only to single photons, but also to continuous-wave optical beams, which have strong quantum properties, as demonstrated by the work on squeezed light and continuous-variable quadrature entanglement. This field of single spatial mode quantum optics has blossomed over the last twenty years [1]. Now we are exploring the possibility of using the quantum properties of light to improve spatial measurements and imaging.

This paper is a summary of the work that has been achieved in quantum imaging in more recent times, focussing on the author's work in the continuous-variable regime. We first present the quantum aspects of optical images and address the possibilities of improving the signal-to-noise ratio (SNR) of modulation measurements. Following this, the spatial multi-modal treatment is presented. We then discuss in detail the measurements of beam displacement using split detectors as well as ways to achieve sensitivities beyond the quantum noise limit (QNL) [2–4]. Finally, we describe a new optimal scheme to measure beam displacement [5]. This scheme involves homodyne detection with an appropriate mode-shape for the

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local oscillator beam. The preliminary results of an experimental implementation of this scheme are also presented.

1.1 Linking imaging with quantum optics

Images which typically have a two-dimensional intensity distribution are the most frequent form of information sent by light. Indeed, one of our own senses, sight, is based on detecting and interpreting images. There is no limitation to the structure of the intensity distribution of an image. We thus require a large set of functions to describe a complicated image, for example the harmonic functions, commonly used in Fourier optics. Recording, digitizing, storing and compressing the information of an image is a major activity in image processing and transfer.

There are two limitations in imaging—the ultimate resolution (which corresponds to the number and size of pixels used in detection) and the intensity noise of the optical beam. Only the latter is related to the quantum nature of light as the resolution is a consequence of the wave nature of light (i.e. diffraction effects). Improvements beyond the standard diffraction limit based on classical techniques, such as off-axis polarization effects [6] or near-field optics, have been demonstrated.

1.2 Measurements of modulation

Information in an optical beam is usually communicated in the form of temporal modulations of the intensity, phase, frequency or polarization of a single-mode light. The most convenient sets of modes are those which propagate without any change in shape (i.e. do not diffract). The most commonly used modal basis is the TEM_{pq} basis. The zeroth-order mode of this basis, the TEM_{00} mode (see figure 1), is most common as it is readily generated by optical resonators. In addition, in most cases, we ignore the spatial aspects of beams by using detectors which are much larger than the beam diameter. We now discuss other more complex modes where we have perfect knowledge of the spatial mode-shape.

Experimentally, the limitations to information transfer are the bandwidth and SNR of the modulation. In many modern instruments, the quantum noise dominates and thus our measurements are termed ‘shot noise limited’ or ‘at the QNL’. The quantum noise is directly related to the photon statistics which result in intensity fluctuations of the optical beam.

Figure 1. The TEM_{00} mode. Beam intensity $I(x, y)$ and beam size $W(z)$.

Measurements of an optical property ultimately involve intensity measurements. For example, the modulation in the phase (FM) is converted into AM using interferometers. Similarly, a modulation of the polarization is converted into AM using a polarimeter, which comprises a polarizing beam-splitter and difference detection. Figure 2 shows the schematic of these detection schemes. In all of these measurement schemes, beam-splitters are used. Therefore, there always exist open ports through which vacuum fluctuations are coupled into the measurement. These vacuum modes give rise to the QNL. In the following sections, we will show that it is possible to reduce the noise of these measurements by injecting squeezed modes with the appropriate mode-shape into these open ports.

The QNL is a consequence of the Heisenberg uncertainty principle. For two conjugate observables $\hat{X}$ and $\hat{P}$, the commutation relation is $[\hat{X}, \hat{P}] = i$. Rewriting the Heisenberg inequality using this commutation relation gives $V(\hat{X})V(\hat{P}) \geq 1$. This inequality states that there exists an uncertainty in the measurement of the $\hat{X}$ and $\hat{P}$ observables. For a coherent state which satisfies the Heisenberg uncertainty relation in the equality, the noise fluctuations of $\hat{X}$ and $\hat{P}$ are Gaussian, with equal power. When AM and FM are considered, the corresponding observables are the quadrature amplitude $\hat{X}_1$ and phase $\hat{X}_2$. For polarization, the observables are the Stokes parameters $\hat{S}_1, \hat{S}_2, \hat{S}_3$, which follow a cyclic commutation rule, $[\hat{S}_i, \hat{S}_j] = 2i\hat{S}_k$. The corresponding variances obey the inequality given by $V(\hat{S}_i)V(\hat{S}_j) \geq |\langle \hat{S}_k \rangle|^2$.

These temporal modulation measurements have benefited from the advances in quantum optics. Using the quantum correlations created in squeezed and entangled light beams to replace vacuum beams, improvements in SNR measurements have been demonstrated [1].

We now consider the spatial basis for an optical beam. We define the mode-shape and allow the beam to be displaced by d . The position of the laser beam, defined as the weighted centre of the beam, is given by

$$d = \langle x \rangle = \frac{\int_{-\infty}^{\infty} x I(x) dx}{\int_{-\infty}^{\infty} I(x) dx}, \quad (1)$$

Figure 2. Measurement schemes for (a) intensity, (b) polarization and (c) displacement modulation.

and this value can be measured with high precision, as small as a few nanometers, which is much less than the wavelength, and is ultimately limited by the quantum noise in the optical beam.

We will now discuss the wave-front measurements of an optical beam. Consider a spatial mask, such as the pits in a CD or DVD, that modifies the wave-front of the beam. By scanning this phase change we transform spatial information into a temporal modulation of the laser beam. This is done extensively for very simple masks, e.g. one bit per beam, and is the basis of what is currently used for optical storage technology. However, we could consider more complicated shapes affecting the beam, thus resulting in more complex forms of data storage with higher storage density [7, 8]. Here, the focussing of the beam is not improved, but the SNR of the information in the diffraction limited spot size is increased.

In order to apply quantum optics to this type of spatial measurements the following questions need attention. What is the most appropriate device for the detection of displacement and tilt? What are the corresponding spatial modes that should be used? How can the QNL be overcome by the use of spatial squeezed light or spatial entanglement?

2. Higher order spatial modes of the laser

There are many examples of laser beams whose transverse mode-shape remains unchanged as it propagates. These are known as the *propagating eigenmodes* of the electromagnetic field. The TEM_{00} mode, or Gaussian beam, is one of the most common examples of such modes.

Any beam of light can be decomposed into a sum of eigenmodes in a particular modal basis. Depending on symmetry, one would normally choose the suitable basis to describe a particular beam-shape. For instance, for a circularly symmetric beam, the Laguerre–Gauss LG_{pq} basis (see figure 3(b)) provides a convenient description. In contrast, for a beam with symmetry along one transverse axis, a more convenient basis would be the Hermite–Gauss TEM_{pq} basis (see figure 3(a)).

For Hermite–Gauss modes the transverse shape is given by a Gaussian function multiplied by a Hermite polynomial. The zeroth ordered TEM_{00} mode has, for instance, a Hermite polynomial of unity. We will find that any displacement, tilt or reshaping of a centred TEM_{00} beam will lead to a multi-modal decomposition of higher order TEM_{pq} centred modes. For small displacement and tilt compared with

Figure 3. (a) Hermite–Gauss and (b) Laguerre–Gauss modes (courtesy of A. White).

the waist size, along the x direction, the TEM_{10} mode contributes to the displacement and tilt signal with an appropriate phase factor (equations (3) and (4)) [5]. The contribution for a displacement or tilt along the y axis is similarly given by a TEM_{01} mode component. Only a larger displacement or tilt will produce even higher order modes.

3. Displacement measurements with a split detector

The simplest method of measuring beam displacement is to use a split detector, which consists of a large photo-detection region divided into left (L) and right (R) halves. The intensities of these two halves are given by the photo-currents i_L and i_R , respectively. Any small displacement of the beam will translate into a difference of the photo-currents given by $i_{\text{diff}} = i_L - i_R$, which is proportional to the displacement d_x , for $d_x \ll W$. However, the split detector is unable to detect a beam tilt which has a pivot point at the detector location. Tilt is converted into displacement with beam propagation. Thus, to perform a tilt measurement using split detection, one needs to relocate the detector to the appropriate plane, and infer the tilt from the displacement measurement performed in this plane.

This split detection measurement of beam displacement can be extended to both transverse axes using a quadrant detector that has four segments, A, B, C and D. Two displacement signals, d_x and d_y , for the x and y axes can be obtained with appropriate combinations of subtraction and addition of the individual photo-currents from the detector quadrants (see figure 4(a)).

The displacement signal is due to the shift of intensity, or photon distribution from one detector quadrant to the other. In figure 4(b) one can see the reduction of intensity on one side, and the corresponding increase on the other side, due to a displacement signal d_x . Without quantum effects we could imagine a perfectly still beam, $d_{x=0}$ with $i_{\text{diff}} = 0$ at all times. However, the photo-currents also contain noise which limits the accuracy of the measurement. It is most practical to analyse the noise not in the time domain but in terms of Fourier frequencies. All frequencies together produce the total noise in the measurement, as shown in figure 4(b) [9].

Figure 4. (a) Laser beam pointing on a quadrant detector. (b) The difference photo-current as a function of small displacement.

While quantum noise is present at all frequencies, the spectrum at very low frequencies is dominated by technical noise, intensity noise of the laser, vibrations, etc. At higher frequencies we can detect the quantum noise, which sets the limit to the accuracy of the measurement of d_x . This defines the smallest d_{QNL} , as shown in figure 4(b).

We can now define the smallest detectable spatial modulation d_{QNL} as the signal which is equal in size to the noise, that means it has a SNR of 1. This value will depend on the size W of the beam on the detector, the total optical power P , and the detection interval Δt . The result for a shot noise limited TEM₀₀ beam is [2, 10]

$$d_{\text{QNL}} = \frac{\sqrt{8}}{\pi} \frac{Whv}{\sqrt{P\Delta t}} = \frac{\sqrt{8}}{\pi} \frac{W}{\sqrt{N}}, \quad (2)$$

where N is the number of photons detected in the time Δt . An example is a typical laser pointer beam with $\lambda = 633 \text{ nm}$, $P = 1 \text{ mW}$, $W = 200 \mu\text{m}$ and $\Delta t = 1 \text{ ms}$, resulting in a quantum noise limited displacement measurement corresponding to $d_{\text{QNL}} = 0.1 \text{ nm}$. Such a very small value is readily achievable.

The quantum limit can be improved by using a higher optical power or a longer integration time. However, there are applications where radiation pressure plays a role, or where the objects measured are fragile to high intensities, which limits the power P . For stability reasons, measurements usually have to be fast, limiting Δt . The value d_{QNL} sets an absolute limit for the sensitivity of the measurement with coherent laser beams. This limit arises from the random distribution of the photons in the transverse plane of the light beam. An equivalent interpretation is that, in all laser beams, higher order spatial vacuum modes are present and contribute to the noise of the measurement.

4. A spatial multi-mode quantum model

We can improve beam displacement measurements with a split detector by using squeezed light in appropriate spatial modes. A simple explanation comes from the comparison of split detection with homodyne detection, as shown in figure 2(a). Using a 50:50 beam-splitter and two detectors, balanced in intensity, we can measure the modulation and the noise of the signal beam, for the amplitude quadrature $\hat{X}_1$ at the QNL. The advantage of homodyne detection is that we can substitute the vacuum input with squeezed vacuum beams and thereby improve the SNR at the modulation frequency. Here the squeezed fluctuations produce correlated photocurrent fluctuations between the two detectors which can cancel each other, and thus reduce the overall quantum noise.

We can now make an analogy with a one-dimensional split detector. Indeed, when the beam is centred, as in the homodyne detection configuration, the split detector measures two equal parts of the beam—it acts as a spatial 50:50 beam-splitter. In this case, the beam incident on the split detector is also playing the role of the local oscillator, and a noise analysis similar to the balanced configuration is possible.

Figure 5. Modes for a quantum laser pointer: a TEM_{00} beam, a flipped TEM_{f00} mode about the x axis and a flipped TEM_{0f0} about the y axis.

For instance, let us consider a TEM_{00} beam incident on a split detector. Moving the beam will result in transferring power into higher order mode components. A movement in the x and y direction will couple power into the TEM_{f0} and TEM_{0f} modes, respectively. We have to consider that all these other transverse modes contribute to the measurement noise. However, the analogy with homodyne detection implies that, in this specific case, there remains only one single vacuum port, and thus all these higher order modes can be written as one single spatial mode. This mode has been proven [4] to be the *flipped mode*, which is a Gaussian beam with a π phase flip at the centre of the detector. Such a mode, presented in figure 5, is denoted by TEM_{f00} for a flip about the x axis, and by TEM_{0f0} for a flip about the y axis.

If we create a beam which has squeezed light injected into the flipped mode, we are able to improve the SNR of the measurement with a split detector. This was first proposed by Fabre *et al.* [4]. A careful analysis shows that this can be done in two dimensions by considering a two-dimensional flipped mode [4], and that there is a complete set of two-dimensional flipped modes TEM_{lpfq} which can be used to describe the origin of quantum noise for two-dimensional split detection. Recent work [12] has shown that this set of modes is the optimum choice when we consider split or quadrant detectors. The conclusion is that the choice of detector determines the best set of modes.

5. Creating and combining spatial squeezed beams

With this theoretical analysis we set out to create spatial squeezed beams. The flipped mode was experimentally generated by using a phase plate which produced a π phase shift between the two halves, or even π phase shifts between the four quadrants. This custom-made phase plate was then inserted into the beam path. Flipped modes are not eigenmodes of the propagation, and thus are not immune to diffraction. The phase plate was therefore imaged onto the plane where the displacement occurred, and then again onto the detector. The requirement is to combine a coherent beam, which contains the optical power, with the squeezed flipped modes with

minimum losses. This new synthesized beam was then used to probe the displacement. The intense TEM_{00} and the squeezed flipped modes had to be carefully mode-matched and were detected by the quadrant detector [3].

The simplest solution to make the combination with low losses was to use a high reflectivity beam-splitter, which combines the majority (e.g. 95%) of the phase flipped squeezed beam with a small fraction (e.g. 5%) of the coherent beam [13]. Such an unequal mixing was necessary since a squeezed beam quickly loses its quantum properties with losses or attenuation, whereas a coherent beam remains coherent after any amount of loss. This produces good improvement in the SNR at the cost of wasting a large part of the laser power. However, there are disadvantages to this beam combination technique. One could argue that a coherent beam with large power could provide an equivalent or even better SNR. Furthermore, this technique does not work in the two-dimensional case where we want to combine, without losses, two squeezed beams with TEM_{f00} and TEM_{0f0} flipped modes with a coherent beam. No simple beam-splitter arrangement can achieve this.

The alternative is to use the spatial mode selection properties of optical resonators. A resonator with moderate finesse (about $F = 100$) can be designed to be impedance matched. That means the total internal losses are balanced by the transmission of the input mirror [14]. Such a resonator will transmit its eigenmode with high efficiency and simultaneously reflect any higher order modes with similar efficiency. As shown in figure 6, we combine a TEM_{00} with a TEM_{f0f0} squeezed beam to create a two-dimensional squeezed field. This beam is then combined with an intense coherent TEM_{f00} mode to form the two-dimensional spatially squeezed beam. Good mode matching and reliable locking of the resonators is required.

Let us now examine the technical details of the experimental demonstration of spatial squeezing [2, 3]. In order to generate conventional amplitude squeezed light we used a very stable Nd:YAG laser which produced ~ 500 mW of light at 1064 nm.

Figure 6. Schematic layout of the experiment for two-dimensional spatial squeezing. Two optical parametric amplifiers (OPAs) generated the squeezed beams where one was converted into the TEM_{f0f0} mode. The combined squeezed beams were superimposed with an intense beam in the TEM_{f00} mode [3].

This was first frequency doubled to 532 nm with one Li:NbO₃ crystal. Following this, the green light was converted back to 1064 nm in two separate OPAs using similar crystals. The OPAs were seeded by light at 1064 nm. The apparatus provided three beams, all at the same frequency, phase locked and mode-matched to each other. One was an intense coherent beam with a power of ~ 20 mW, while the other two were squeezed beams with a noise suppression of 3 dB below the QNL at detection frequencies of 3–6 MHz.

The TEM_{f00} mode was produced with an optical wave-plate, made from birefringent half-wave-plates that were cut and re-assembled with one half rotated by 90° with respect to the other half [9]. These plates have a very small gap, < 50 μm , and introduce minimal losses (see figure 6). To obtain the TEM_{f0f0} flipped mode, a four-quadrant wave-plate with alternate π phase shifts on the quadrants was constructed.

The mixing was achieved with the use of an optical ring cavity, impedance matched and locked to the TEM₀₀ mode. The finesse of the cavity was ~ 35 . The TEM₀₀ squeezed beam was transmitted with 95% efficiency through the cavity. The TEM_{f0f0} squeezed mode was reflected from the cavity with an efficiency of 94%. In our experiment, the limit of efficiency came essentially from the imperfections of the mode-shape of the squeezed beams. Finally, the combined squeezed beams were added on a 95% beam-splitter to a mode-matched coherent beam which had a TEM_{f00} mode-shape. This was the output beam of our *quantum laser pointer* [3].

We then displaced this beam in two dimensions at a frequency of 4.3 MHz, using a mirror mounted on a piezo-electric actuator. The very weak amplitude of the spatial oscillation in x and y directions was smoothly controlled using two separate signal generators. The beam was centred on a quadrant photo-detector and the wave-plates were carefully imaged onto the surface of the detector. The edges of the wave-plate were imaged to fit the gap between the quadrants of the detector.

The four quadrants of the detectors were electronically balanced and the combinations of currents for the x and y axis displacements were analysed using an electronic spectrum analyser. The modulation amplitude d_{mod} was increased in time. This forms the horizontal axis of figure 7. The vertical axis shows the power for the horizontal displacement signal for two separate experiments with (i) coherent and (ii) spatially squeezed light. At low modulation strengths, the measured power for the spatial squeezed beam was 3.3 dB below the QNL. Normalizing this to the quantum noise limit gives the signal-to-noise ratio (SNR) for the measurement. Thus curve (ii) has a larger SNR than curve (i).

This experiment demonstrates that the sensitivity of the laser pointer has improved by approximately a factor of 2, simultaneously in both transverse directions. The QNL can be surpassed for split detection when squeezed light in the appropriate spatial mode is used.

6. How to improve the split detector

Although split detection is the conventional technique for measuring the displacement of a laser beam, a pertinent question is whether this technique is optimal. Let us

Figure 7. Measurement of the horizontal displacement signal increased in time with (i) coherent and (ii) spatially squeezed light. The traces were averaged over 20 runs with $\text{RBW} = \text{VBW} = 100 \text{ kHz}$ and integration time $\Delta t = 10 \text{ ms}$ (from [4]).

now consider a detector with a large array of pixels, for example a CCD detector. If we assume the ideal situation where each pixel is arbitrarily small and there is an arbitrarily large number of pixels, this device is then able to accurately and fully measure the transverse intensity profile of a light beam [5]. By analysing the intensity results of these pixels, one is then able to determine the displacement of the beam, assuming *a priori* knowledge of the beam-shape.

Let us consider again the situation of figure 8(a), where the comparison between a centred and displaced beam is shown. In multi-pixel detection, points on the two curves represent the intensity detected by the pixels, before and after displacement. The horizontal axis represents the pixel number. The difference between the two curves gives a measure of the displacement signal (see curve (i) in figure 8(b)). The smallest displacement signal detectable by the array detector is equal to the power of the quantum noise. This quantum noise is proportional to the square root of the light intensity incident on the pixels (see curve (ii) in figure 8(b)).

An analysis of these two curves provides insight into the complexity of optimizing a beam displacement measurement. We see that the pixels at the centre of the detector see very small amounts of power change whilst registering a relatively large amount of quantum noise. These pixels should therefore be neglected. On the other hand, the pixels near the half intensity points of the beam see larger power differences. These pixels, therefore, carry most of the beam displacement information and should be preferentially selected.

We find that, in a multi-pixel detector, not all pixels carry the same amount of displacement information and contribute equally to the registered quantum noise. We therefore optimize the displacement measurement by increasing the electronic gain for pixels with good SNR. We also reduce the electronic gains of pixels with low SNR. In comparison with the split detector, where the entire half of a light beam is measured with equal weighting, we can now see that multi-pixel detection can be further optimized when the correct choice of pixel gains is made [12].

Figure 8. (a) A centred TEM_{00} beam is displaced by d_x . The shaded region indicates the power difference between the displaced and centred beam. (b) For small displacements, the displacement signal for a TEM_{00} beam is contained in the amplitude of the TEM_{10} mode (curve (i)). The noise is proportional to the square root of the light intensity incident on the pixel, which is a TEM_{00} beam (curve (ii)).

7. Spatial homodyne detection of squeezed light

Another interpretation of figure 8 uses the transverse modes which were used in the spatial description of the light. Indeed, the signal curve in figure 8, which corresponds to the difference between the centred beam and the displaced beam, also corresponds to the transverse mode which contains the displacement. In mathematical terms, this curve is the derivative of the Gaussian beam profile relative to the transverse variable [5, 15]. For $d \ll W_0$, the Taylor formula gives us the decomposition of a displaced TEM_{00} beam

$$\begin{aligned} \text{TEM}_{00}(d) &= \text{TEM}_{00} + d \frac{\partial}{\partial x} \text{TEM}_{00} \\ &= \text{TEM}_{00} + \frac{d}{W_0} \text{TEM}_{10}. \end{aligned} \quad (3)$$

This equation shows that displacing the TEM_{00} beam couples power into the TEM_{10} mode. The amplitude of the TEM_{10} mode is directly proportional to the displacement. A similar calculation for small tilt, θ , gives [15]

$$\text{TEM}_{00}(\theta) = \text{TEM}_{00} + i \frac{\pi W_0 \theta}{\lambda} \text{TEM}_{10}. \quad (4)$$

To measure the displacement and tilt optimally, we measure the signal in the TEM_{10} mode using a homodyne detector with a TEM_{10} local oscillator [5] (figure 9). This is the direct extension of the homodyne scheme we described above to higher order transverse modes. This scheme has been proven to be more efficient than the split detector scheme which is $\sim 80\%$ efficient, even at the classical level, as it fully detects the displacement/tilt signal contained in the TEM_{10} mode [5].

The challenge is now to generate a squeezed TEM_{10} mode. In principle, this should be possible by operating the OPA in the required higher order mode or by high efficiency mode conversion. Alternatively, one can start with the TEM_{00} mode

Figure 9. Spatial homodyne detector with a TEM₁₀ local oscillator beam [5].

Figure 10. Spatial homodyne measurement (i) with and (ii) without squeezing. The local oscillator has a TEM₁₀ mode. The phase of the local oscillator is scanned, the measurement cycles from displacement (close to minimum power) to tilt measurement (close to maximum power). For a coherent beam, the displacement and tilt signals are d_{CL} and θ_{CL} , respectively. For a squeezed beam, the displacement and tilt signals are d_{SQZ} and θ_{SQZ} , respectively. Trace (iii) is the quantum noise limit and trace (iv) is the locked signal for the displacement measurement. RBW = 100 kHz, VBW = 100 Hz.

and only measure the TEM₁₀ mode component, which represents $\sim 80\%$ of the power in the TEM₀₀ mode.

First preliminary experiments [16] have shown that it is indeed possible to operate a spatial homodyne detector for the measurement of displacement and that the SNR can be improved with spatially squeezed light. Figure 10 shows the measurement of a spatial modulation detected with and without squeezed light. Squeezed light was generated at 1064 nm and transformed into the TEM₀₀ mode (in the homodyne detection, we detect the squeezed TEM₁₀ component of this mode). It was combined with an intense TEM₀₀ beam and a constant relative phase between these two modes was maintained. A mirror mounted on a piezo-electric

actuator was used to displace and tilt the beam at a frequency of 4 MHz, corresponding to a mechanical resonance of the piezo-crystal. In this case, the tilt signal was much larger than the displacement signal as the PZT was modulated above its normal operating frequency range. The split phase-plate which produced the TEM_{10} mode was imaged onto the displacing mirror.

The homodyne detector has a TEM_{00} local oscillator beam with a power of 5 mW. The phase of the local oscillator beam was scanned at ~ 6 Hz, which corresponded to cycling between the measurement of the displacement (real part of TEM_{10} as shown in equation (4)) and the tilt (imaginary part of TEM_{10} as shown in equation (5)).

The results, with and without squeezing, are shown in figure 10. Traces (iii) and (iv) show the quantum noise and the reduction due to squeezing by 2 dB, respectively. Signals for pure displacement and tilt measurements are indicated by arrows. The displacement d_{CL} cannot be detected because the signal strength is buried within the quantum noise. In contrast, the tilt signal θ_{CL} is sufficiently large to be clearly resolved. With the squeezed light, a value d_{SQZ} beyond the QNL is obtained. At the same time, the noise in the complementary tilt measurement is increased. From equations (3) and (4) these are the conjugate quantities, in analogy to the amplitude and phase quadrature, and, consequently, tilt should see anti-squeezing. The increase shown in figure 10 of the value for θ_{SQZ} compared with θ_{CL} is consistent with the degree of squeezing used. This is a clear demonstration of the improvement in the SNR below the QNL due to the TEM_{10} component of the squeezed light.

8. Conclusions

We have first shown that spatial measurements are analogous to intensity and polarization measurements. With squeezed light we then proved that measurements beyond the QNL can be achieved. The crucial steps in the experiments are to find the correct mode for the squeezed light, generate this mode with minimum losses and combine it with a coherent TEM_{00} beam.

Each detection technique has an optimum eigenmode basis for the squeezed light, as illustrated in figure 11. We have shown that, for the simplest detection technique, namely the split detection, the eigenmodes are the flipped modes, and that a combination of these modes (figure 11(a)) is required to improve the measurement beyond the QNL. The technical challenge is to generate and combine the squeezed beams without losses and we have demonstrated that we can use these squeezed modes effectively. Next, we have shown that the spatial homodyne detector is optimal for small beam displacement measurements. Here the eigenmodes are the conventional TEM_{ij} and, for a small displacement, all the information can be measured using a TEM_{10} local oscillator beam. In order to improve this detection system beyond the QNL, squeezed light in the TEM_{10} mode (figure 11(b)) is required. Finally, we have presented preliminary results [16] showing that this new technique is indeed capable of displacement measurements beyond the QNL.

We have demonstrated how to handle spatial information with four pixels, by introducing quantum correlations between these pixels. We could also extend this

Figure 11. Illustration of the relevant eigenmodes for different spatial detectors.

to more than four pixels, and eventually would approach actual quantum images. This could lead to optical imaging systems with improved performance or even novel effects of image transformations, as described, for example, by Kolobov [10] and Lugiato *et al.* [17].

However, in our approach of mode synthesis, that is building up the non-classical multi-mode field from single mode squeezed fields, we will require one additional squeezer for each new spatial degree of freedom. This requires considerable resources. Alternatively, one could start with a spatially degenerate optical parametric oscillator and create the quantum correlations in one step. Some early results suggest that this might be feasible; Fabre *et al.* [18, 19] have already demonstrated signals below the QNL.

Finally, it has already been shown that spatial quantum correlations provide an alternative means of encoding, sending and manipulating quantum information using single photons. While we discuss here the use of CW laser beams, we find that the same concepts also apply to photon pairs created in optical downconverters, which contain correlations in the arrival time and direction (momentum). Already, a set of higher order LG_{pq} modes has been used to encode single photon quantum states with the aim of performing quantum logic experiments [20, 21].

All quantum protocols, such as quantum cryptography, teleportation or entanglement, will have an equivalent in the spatial domain, both with single photons and CW beams. In all these experiments we are using one of the biggest advantages of optics over electronics, namely direct parallel processing, and we conclude that spatial quantum optics is only at its beginning.

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