1. Introduction

Low-energy heavy-ion reactions provide us a rich laboratory to study the equilibration dynamics of strongly interacting many-body systems. In addition, these reactions probe an intriguing interplay between the microscopic single-particle dynamics and collective motion at time scales too short for complete equilibration. In order to elucidate trends and systematics in these reactions both theoretical and experimental studies must be undertaken for an assortment of projectile and target combinations. Such studies is expected to be undertaken at the current and future radioactive ion-beam (RIB) facilities [1].

In this manuscript we will discuss the equilibration dynamics and time-scales for various quantities that are connected to the experimentally observable entities. These include the study of mass, isospin, and total kinetic energy (TKE) equilibration time-scales. In most of these studies one is essentially dealing with the transport phenomena of isospin asymmetric systems. Recently, charge equilibration, driven by the nuclear symmetry-energy, has been experimentally studied near the Fermi energy [2]. At these energies the sticking or contact times of the participating nuclei are sufficiently short to induce a partial charge equilibration [3]. Charge equilibration has also been studied with deep inelastic collisions at lower energies, but with large isospin asymmetry in the...
entrance channel [4-6]. In recent years a number of transport models have been employed to investigate the density dependence of the symmetry energy away from the saturation density [7-13]. While considerable success has been achieved in obtaining information about the EOS from these calculations more refinement of the models are needed to make a deeper connection to fundamental aspects of nuclear many-body physics.

For the low-energy heavy-ion collisions the relative motion of the centers of the two nuclei is characterized by a short wavelength and thus allows for a classical treatment, whereas the wavelength for the particle motion is not small compared to nuclear sizes and should be treated quantum mechanically. The mean-field approach such as the time-dependent Hartree-Fock (TDHF) theory [14,15] and its extensions provide a microscopic basis for describing the heavy-ion reaction mechanism at low bombarding energies. In this manuscript we provide studies of a variety of nuclear reactions to address some of the issues discussed above.

2. – Equilibration dynamics

In this section we discuss equilibration times for mass, isospin, and TKE in low-energy heavy-ion reactions. Figure 1 shows the general time-scales associated with various reaction types as a function of increasing inelasticity. It is important to state that somewhere in the quasifission time-scale range (slow quasifission) [16,17] we observe a total TKE loss of the reaction products, TKE following the Viola systematics. In this sense quasifission reactions are most suitable for the study of time-scales for mass equilibration, whereas time-scales for isospin and TKE equilibration can be investigated in deep-inelastic reactions.

2.1. Quasifission. – Quasifission, which occurs typically for systems with product of the charges \( Z_1 Z_2 > 1600 \), is characterized by two final state fragments that emerge after a long lived composite system (typically longer than 5 zs) and final fragment masses \( A_f = A_{CN}/2 \pm 20 \) or more, and thus occupy the regime between quasielastic and fusion/fission. In addition, final TKE’s distinguish quasifission from highly damped deep-inelastic collisions, which have a smaller mass and charge difference between initial and final fragments. In TDHF the mass and charge differences between the initial nuclei and the final fragments measure the number of nucleons transferred. In recent years a compelling number of TDHF calculations of quasifission have shown that TDHF is an excellent predictor for the experimentally measured quantities, such as the mass-angle distributions [18-28]. Due to the long contact times the quasifission process is suitable to study mass equilibration. In fig. 2 we plot the ratio of final and initial mass differences defined generally by

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\Delta A(t) = A_{TLF}(t) - A_{PLF}(t),
\]

Fig. 1. – Time-scales for various reaction types as a function of increasing inelasticity.
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Fig. 2. - The ratio of the final and initial fragment masses as a function of contact time for the $^{48}\text{Ca} + ^{249}\text{Bk}$ system at $E_{c.m.} = 234\text{ MeV}$ and for two orientations of the $^{249}\text{Bk}$ nucleus. The dashed line shows one possible fit.

as a function of contact time for the $^{48}\text{Ca} + ^{249}\text{Bk}$ system at $E_{c.m.} = 234\text{ MeV}$, and for two extreme orientations of the deformed $^{249}\text{Bk}$ nucleus indicated by the angle $\beta$ which is the angle between the symmetry axis of the nucleus and the collision axis. The points correspond to the impact parameters used, ranging from head-on collisions to more peripheral collisions. The horizontal bars on the right side of the figure indicate the number of particles transferred between the target and the projectile. We observe that more mass transfer happens at larger contact times as expected. The dashed line shows a typical fit of a function in the form of $c_0 + c_1 \exp(-\tau/\tau_0)$. Depending on the quality of the fit we obtain equilibration times in the range 5–10zs. In fig. 3 we plot the

Fig. 3. - The ratio of the final and initial fragment masses as a function of contact time for the $^{54}\text{Cr} + ^{186}\text{W}$ system at $E_{c.m.} = 218.6\text{ MeV}$ and for two orientations of the $^{186}\text{W}$ nucleus. Dashed lines show two possible fits.
same fragment mass difference ratio for the $^{54}$Cr + $^{186}$W system at $E_{cm} = 218.6$ MeV and for two orientations of the $^{186}$W nucleus. On the plot we show two possible fits to the points. The obtained equilibration times are in the range 7–12 zs. More calculations are underway to study the orientation angle dependence of the equilibration times in more detail. However, based on these and other results obtained from TDHF calculations we can safely conclude that mass equilibration times are long, and a typical time of around 10 zs could be argued. Mass equilibration times can also be influenced by shell effects. While the preference of Pb isotopes as quasifission product was theoretically observed in TDHF calculations of $^{48}$Ca + $^{238}$U system [20,29-32], it was recently confirmed experimentally by explicit charge measurement of the quasifission products [27] in the $^{48}$Ti + $^{238}$U system. Naturally, influence of shell effects will depend on the target projectile combinations but it is clear that shell effects do introduce a delay in the equilibration process.

2.2. *Deep-inelastic reactions.* – Study of strongly damped collisions of nuclei or so-called deep-inelastic collisions can play an important role in elucidating the dynamics of charge and mass exchange, dissipation of energy and angular momentum, degree of isospin equilibration, and the dependence of these quantities on the properties of the reactants such as the neutron-to-proton ratio ($N/Z$) [33-35]. Here, we focus on the study of equilibration times for isospin and TKE. Recently, an experimental study [2] in the Fermi energy range have obtained an isospin equilibration time of about 0.3 zs. We have performed studies of the $^{78}$Kr + $^{208}$Pb system at 8.5 MeV/nucleon [36]. In fig. 4 we plot the $(N - Z)/A$ value of the primary PLF (full circles) and TLF (open squares) formed in $^{78}$Kr + $^{208}$Pb at $E = 8.5$ MeV/nucleon as a function of the contact time between the collision partners. The solid lines show fits to the TDHF results. For this system the mean life time of the charge equilibration process, obtained from the final $(N - Z)/A$ value of the fragments is $\sim 0.5$ zs. This and other studies suggest that for low-energy heavy-ion collisions isospin equilibration occurs in the time-scale range 0.5–1.0 zs.

![Fig. 4.](image-url)

Fig. 4. – The $(N - Z)/A$ value of the primary PLF (full circles) and TLF (open squares) formed in $^{78}$Kr + $^{208}$Pb at $E = 8.5$ MeV/nucleon are plotted as a function of the contact time between the collision partners. The solid lines show fits to the TDHF results.
In order to study TKE equilibration times in fig. 5 we plot the exit channel TKE for the $^{78}$Kr + $^{208}$Pb and $^{92}$Kr + $^{208}$Pb reactions at $E = 8.5$ MeV/nucleon as a function of the contact time. The calculations are done for two extreme orientations of the deformed Kr nuclei. The points correspond to calculations for impact parameters in the range 0–10 fm. We observe that for both systems the TKE rapidly falls initially as a function of contact time but slowly stabilizes around and after the contact times 1.5–2 zs. From this we may conclude that the equilibration time for TKE is in the range of 1.5–2 zs. Similar results have been recently found for the Ni + Ni system [31].

3. – Summary

We have presented a discussion of equilibration times for mass, isospin, and TKE using the TDHF approach. We find that mass equilibration times are much longer than those for isospin and TKE. The fully microscopic TDHF theory has shown itself to be rich in nuclear phenomena and continues to stimulate our understanding of nuclear dynamics. The time-dependent mean-field studies seem to show that the dynamic evolution builds up correlations that are not present in the static theory. Although there is evidence that one-body dissipation can properly account for the transport phenomena seen in these reactions, further experiments are needed to test this conclusion. We plan to supplement the studies mentioned in this manuscript with more TDHF calculations and provide a more detailed analysis of equilibration times in low-energy heavy-ion reactions.

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