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Modulation stability analysis and solitary wave solutions of nonlinear higher-order Schrödinger dynamical equation with second-order spatiotemporal dispersion

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Abstract: In optical fibers, the higher-order nonlinear Schrödinger (NLS) dynamical equation which describes the beyond the classic slowly varying envelopes and spatiotemporal dispersion of pulses is investigated. By applying the proposed modified extended mapping method, the optical soliton solutions of higher-order NLS dynamical equation with the coefficients of group velocity dispersion, second-order spatiotemporal dispersion and cubic nonlinearity are deduced. The obtained solutions have important applications in applied sciences and engineering. The formation conditions are specified on parameters in which optical solitons can exist for this media. The moments of some constructed solutions are presented graphically which facilitate the researchers to comprehend the physical phenomena of this equation. The modulation instability analysis is utilized to discuss the model stability, which verifies that all obtained solutions are stable and exact. Other such forms of the system arising in sciences and engineering can also be solved by this steadfast, influential and effective method.

Keywords: Modified extended mapping method; Higher-order nonlinear Schrödinger equation; Solitons; Solitary wave solutions

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1. Introduction

The nonlinear Schrödinger equations (NLSEs) having the coefficients of group velocity dispersion and second-order spatiotemporal dispersion are important physical models and illustrate the dynamics of optical soliton promulgation in the optical fibers for trans-continental communication [1–5]. In optical fibers, most of these models are regularly expressed in the time domain, and when fields at different frequencies propagate through the fiber the common practice is also to write a distance equation for each field component. The nonlinear transformation of dielectric of the fiber termed as the Kerr effect is applied to neutralize the dispersion effect; in this state, the optical pulse might lean to form a steady nonlinear pulse known as an optical soliton. The bit rate of transmission is restricted by the dispersion of the fiber material. The fiber loss is the only factor that contributes to the drop in the pulse quality by expansion in the pulse width (for more details see references [6–8]).

In optical fibers, the dynamical models of soliton propagation are an area of enormous curiosity because of the broad applications in ultrafast signal routing systems, trans-continental and short-light-pulse telecommunication [9, 10]. These systems are mostly articulated in time domain, and when different frequency fields propagate throughout the fiber the ordinary practice is also to inscribe a distance equation for every field component. In dielectric fibers, the authors in [10] examined the optical solitons experimentally and theoretically. Solitons in homogeneous and Hamiltonian systems are localized solitary waves

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having special properties such as enormously robust to perturbations, without changing structure during propagation at a constant speed, stable with respect to collision with other solitons. If the length of dispersion is similar to nonlinear length [11] then a chirp-free pulse results. In last few years, the field of optical fiber communication systems attained much concentration of researchers for investigation.

In recent decades, several researchers have paid much attention to constructing the exact solutions to a number of nonlinear Schrödinger models. Various systematic and powerful techniques have been formulated to obtain exact explicit solutions in many forms such as the trial equation technique, mapping method, inverse scattering method, variational method, semi-inverse variational principle, Bäcklund transformations, Darboux transformation, Hirota's bilinear method, the extended tanh method, expansion methods, simple equation method, auxiliary equation method, direct algebraic method, mapping method, modified simple equation method, the rational expansion method [12–23]. Soliton solutions gained a lot of interest of researchers to study interactions, structures and more properties. Exact solitary wave solutions of few nonlinear equations via an expansion just about an integrable ODE are of current interest to the community of engineering and applied science, and the studies give improvements to the accessible literature on related topics [24–38].

The governing NLSE with group velocity dispersion coefficient and second-order spatiotemporal dispersion coefficient [39] is as

$$i(q_x + \beta_1 q_t) + \beta_2 q_{tt} + \beta_3 q_{xx} + |q|^2 q = 0, \tag{1}$$

where the dependent function is $q(x, t)$ that signifies the macroscopic complex-valued wave profile. Furthermore, the β_1 is proportional to the ratio of group speed and β_2 and β_3 are the coefficients of group velocity dispersion and spatial dispersion, respectively. For more details, see references [39–43].

In the current study, the soliton solutions of nonlinear higher-order NLS equation (1) with group velocity dispersion coefficient, second-order spatiotemporal dispersion, and cubic nonlinearity are constructed by the proposed modified extended mapping method.

This article is devised as follows: In Sect. 2, the key steps of the proposed method are given. The application of the proposed method to higher-order NLSE is presented in Sect. 3, and exact soliton and solitary wave solutions are constructed. In Sect. 4, the behavior of the solitons is discussed physically. Lastly, the conclusions are revealed.

2. Description of the proposed method

We give the algorithm of the proposed modified extended mapping method for constructing the soliton solutions of nonlinear partial differential equations (PDEs). Let us assume a nonlinear PDE in general form having two independent variables t and x as

$$G(q, q_t, q_x, q_{xx}, q_{xxx}, \ldots) = 0, \tag{2}$$

where the $q(x, t)$ is a function and G is a polynomial function. Consider that Eq. (2) has the form

$$q(x,t) = \psi(\xi) = \sum_{i=0}^{M} A_i \varphi^i(\xi) + \sum_{i=-1}^{-M} B_{-i} \varphi^i(\xi) + \sum_{i=-1}^{-M} C_{-i} \varphi^i(\xi)\varphi'(\xi), \tag{3}$$

where

$$\varphi'(\xi) = \sqrt{\alpha_0 + \alpha_1\varphi(\xi) + \alpha_2\varphi^2(\xi) + \alpha_3\varphi^3(\xi) + \alpha_4\varphi^4(\xi)}$$

and

$$\xi = k(x - \omega t), \tag{4}$$

where $A_i, B_i, C_i, \alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, k$ and ω are arbitrary constants. Utilizing the homogeneous balancing principle on Eq. (2), the series of coefficients $A_0, A_1, \ldots, A_M, B_1, \ldots, B_M, C_1, \ldots, C_M, k, \omega$ can be obtained.

3. Application of the proposed method to higher-order NLSE

As Eq. (1) is complex, assume the traveling wave solution of Eq. (1) in the form as

$$q(x,t) = \psi(\xi) e^{i\tau}, \quad \tau = \gamma x - vt + \vartheta, \tag{5}$$

where $\psi(\xi)$ is the amplitude components of the wave profiles, τ is the phase factor, and γ, v, ϑ indicate the frequency of solitons, the wave number and the phase constant, respectively. Substituting Eq. (5) into Eq. (1) and separating the real and imaginary parts yields

$$\left(\beta_2 k^2\omega^2 + \beta_3 k^2\right)\psi''(\xi) - \left(\beta_3\gamma^2 + \beta_2 v^2 - \beta_1 v + \gamma\right)\psi(\xi) + \psi^3(\xi) = 0, \quad \gamma = \frac{\beta_1\omega - 2\beta_2 v\omega - 1}{2\beta_3}. \tag{6}$$

Applying homogeneous balancing principle on Eq. (6), the solution of Eq. (6) is

$$\psi(\xi) = A_0 + A_1\varphi + \frac{B_1}{\varphi} + \frac{C_1\varphi'}{\varphi}. \tag{7}$$

Putting Eq. (7) into Eq. (6) and setting the coefficients of $\varphi^i \varphi^{(i)}$ to zero, we got a system of equations in parameters $A_0, A_1, B_1, C_1, \alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, k, \omega, \gamma, v, \beta_i (i = 1, 2, 3)$. Mathematica software is utilized to solve this system of equations. We obtained the following cases of solutions:

Case 1 $\alpha_0 = \alpha_1 = 0$,

Set 1

$$A_0 = 0, \quad A_1 = \frac{ik\sqrt{\alpha_4(\beta_2\omega^2 + \beta_3)}}{\sqrt{2}}, \quad B_1 = 0, \quad C_1 = \frac{ik\sqrt{\beta_2\omega^2 + \beta_3}}{\sqrt{2}}, \quad v = \frac{\beta_1(\beta_2\omega^2 + \beta_3) \pm \sqrt{-(\beta_2\omega^2 + \beta_3)(\beta_2(2\alpha_2\beta_3 k^2(\beta_2\omega^2 + \beta_3) - 1) - \beta_1^2\beta_3)}}{2\beta_2(\beta_2\omega^2 + \beta_3)}. \tag{8}$$

Set 2

$$A_0 = 0, \quad A_1 = -\frac{ik\sqrt{\alpha_4(\beta_2\omega^2 + \beta_3)}}{\sqrt{2}}, \quad B_1 = 0, \quad C_1 = \pm\frac{ik\sqrt{\beta_2\omega^2 + \beta_3}}{\sqrt{2}}, \quad v = \frac{\beta_1(\beta_2\omega^2 + \beta_3) - \sqrt{-(\beta_2\omega^2 + \beta_3)(\beta_2(2\alpha_2\beta_3 k^2(\beta_2\omega^2 + \beta_3) - 1) - \beta_1^2\beta_3)}}{2\beta_2(\beta_2\omega^2 + \beta_3)}. \tag{9}$$

The following soliton and solitary wave solutions of Eq. (1) are obtained from set 1:

$$q_{11}(x,t) = -\frac{ik\sqrt{\beta_2\omega^2 + \beta_3}\left(2\alpha_2\sqrt{\alpha_4} - \sqrt{\alpha_2(\alpha_3^2 - 4\alpha_2\alpha_4)}\sinh(\sqrt{\alpha_2}\xi)\right)}{\sqrt{2}\left(\alpha_4 - \sqrt{\alpha_3^2 - 4\alpha_2\alpha_4}\cosh(\sqrt{\alpha_2}\xi)\right)} e^{i(\gamma x - vt + \vartheta)}, \quad \alpha_2 > 0, \quad \alpha_3^2 > 4\alpha_2\alpha_4. \tag{10}$$

$$q_{12}(x,t) = -\frac{ik\sqrt{\beta_2\omega^2 + \beta_3}\left(\sqrt{\alpha_2(\alpha_3^2 - 4\alpha_2\alpha_4)}\sinh(\sqrt{\alpha_2}\xi) + 2\alpha_2\sqrt{\alpha_4}\right)}{\sqrt{2}\left(\sqrt{\alpha_3^2 - 4\alpha_2\alpha_4}\cosh(\sqrt{\alpha_2}\xi) + \alpha_4\right)} e^{i(\gamma x - vt + \vartheta)}, \quad \alpha_2 > 0, \quad \alpha_3^2 > 4\alpha_2\alpha_4. \tag{11}$$

$$q_{13}(x,t) = -\frac{ik\sqrt{\beta_2\omega^2 + \beta_3}\left(2\alpha_2\sqrt{\alpha_4} - \sqrt{\alpha_2(4\alpha_2\alpha_4 - \alpha_3^2)}\cosh(\sqrt{\alpha_2}\xi)\right)}{\sqrt{2}\left(\alpha_4 - \sqrt{4\alpha_2\alpha_4 - \alpha_3^2}\sinh(\sqrt{\alpha_2}\xi)\right)} e^{i(\gamma x - vt + \vartheta)}, \quad \alpha_2 > 0, \quad \alpha_3^2 < 4\alpha_2\alpha_4. \tag{12}$$

$$q_{14}(x,t) = -\frac{ik\sqrt{\beta_2\omega^2 + \beta_3}\left(\sqrt{\alpha_2(4\alpha_2\alpha_4 - \alpha_3^2)}\cosh(\sqrt{\alpha_2}\xi) + 2\alpha_2\sqrt{\alpha_4}\right)}{\sqrt{2}\left(\sqrt{4\alpha_2\alpha_4 - \alpha_3^2}\sinh(\sqrt{\alpha_2}\xi) + \alpha_4\right)} e^{i(\gamma x - vt + \vartheta)}, \quad \alpha_2 > 0, \quad \alpha_3^2 < 4\alpha_2\alpha_4. \tag{13}$$

$$q_{15}(x,t) = \frac{ik\sqrt{\beta_2\omega^2 + \beta_3}\left(\sqrt{\alpha_2\alpha_4} \pm \operatorname{sech}^2\left(\frac{\sqrt{\alpha_2}\xi}{2}\right) - 2\alpha_2\left(1 \pm \tanh\left(\frac{\sqrt{\alpha_2}\xi}{2}\right)\right)^2\right)}{2\sqrt{2\alpha_4}\left(1 \pm \tanh\left(\frac{\sqrt{\alpha_2}\xi}{2}\right)\right)} e^{i(\gamma x - vt + \vartheta)}, \quad \alpha_2 > 0, \quad \alpha_3^2 = 4\alpha_2\alpha_4. \tag{14}$$

$$q_{16}(x,t) = -\frac{ik\sqrt{\beta_2\omega^2 + \beta_3}\left(2\alpha_2\left(1 \pm \coth\left(\frac{\sqrt{\alpha_2}\xi}{2}\right)\right)^2 + \sqrt{\alpha_2\alpha_4} \pm \operatorname{csch}^2\left(\frac{\sqrt{\alpha_2}\xi}{2}\right)\right)}{2\sqrt{2\alpha_4}\left(1 \pm \coth\left(\frac{\sqrt{\alpha_2}\xi}{2}\right)\right)} e^{i(\gamma x - vt + \vartheta)}, \quad \alpha_2 > 0, \quad \alpha_3^2 = 4\alpha_2\alpha_4. \tag{15}$$

Similarly, one can achieve more new solutions of Eq. (1) from set 2.

Case 2 $a_0 = a_1 = a_3 = 0$,

Set 1

$$A_0 = B_1 = C_1 = 0, \quad k = \mp\frac{A_1}{\sqrt{2}\sqrt{-\alpha_4(\beta_2\omega^2 + \beta_3)}}, \quad v = \frac{\beta_1 \pm \frac{\sqrt{\alpha_4(\beta_2\omega^2+\beta_3)\left(\alpha_4\left(\beta_3\beta_1^2+\beta_2\right)-2\alpha_2 A_1^2\beta_2\beta_3\right)}}{\alpha_4(\beta_2\omega^2+\beta_3)}}{2\beta_2}. \tag{16}$$

Set 2

$$A_0 = A_1 = B_1 = 0, \quad C_1 = \pm\frac{\sqrt{\omega^2(\beta_1 - 2\beta_2 v)^2 + 4\beta_3 v(\beta_2 v - \beta_1) - 1}}{2\sqrt{\alpha_2\beta_3}}, \quad k = \mp\frac{\sqrt{-\omega^2(\beta_1 - 2\beta_2 v)^2 + 4\beta_3 v(\beta_1 - \beta_2 v) + 1}}{2\sqrt{2\alpha_2\beta_3(\beta_2\omega^2 + \beta_3)}}. \tag{17}$$

Set 3

$$A_0 = B_1 = 0, \quad C_1 = \pm\frac{A_1}{\sqrt{\alpha_4}}, \quad k = -\frac{\sqrt{2}A_1}{\sqrt{-\alpha_4(\beta_2\omega^2 + \beta_3)}}, \quad v = \frac{\beta_1 \mp \frac{\sqrt{\alpha_4(\beta_2\omega^2+\beta_3)\left(\alpha_4\left(\beta_3\beta_1^2+\beta_2\right)+4\alpha_2 A_1^2\beta_2\beta_3\right)}}{\alpha_4(\beta_2\omega^2+\beta_3)}}{2\beta_2}. \tag{18}$$

The following soliton and solitary wave solutions of Eq. (1) are obtained from set 2

$$q_{21}(x,t) = -\sqrt{-\frac{\alpha_2}{\alpha_4}}A_1 \csc(\sqrt{-\alpha_2}\xi)e^{i(\gamma x - vt + \vartheta)}, \quad \alpha_2 < 0. \tag{19}$$

$$q_{22}(x,t) = \sqrt{-\frac{\alpha_2}{\alpha_4}}A_1 \operatorname{csch}(\sqrt{-\alpha_2}\xi)e^{i(\gamma x - vt + \vartheta)}, \quad a_2 < 0. \tag{20}$$

$$q_{23}(x,t) = \sqrt{-\frac{\alpha_2}{\alpha_4}} A_1 \sec(\sqrt{-\alpha_2}\xi) e^{i(\gamma x - vt + \vartheta)}, \quad \alpha_2 < 0. \tag{21}$$

$$q_{24}(x,t) = \frac{2\alpha_2 A_1 e^{\sqrt{\alpha_2}\xi}}{1 - \alpha_2\alpha_4 e^{2\sqrt{\alpha_2}\xi}} e^{i(\gamma x - vt + \vartheta)}, \quad \alpha_2 > 0. \tag{22}$$

One can construct more soliton solutions of Eq. (1) from sets 2 and 3 in the same way.

Case 3 $\alpha_0 = \alpha_1 = \alpha_4 = 0,$

Set 1

$$A_0 = A_1 = B_1 = 0,$$
$$C_1 = \frac{\sqrt{\omega^2(\beta_1 - 2\beta_2 v)^2 + 4\beta_3 v(\beta_2 v - \beta_1) - 1}}{2\sqrt{\alpha_2}\sqrt{\beta_3}}, \tag{23}$$
$$k = \pm\frac{\sqrt{-\omega^2(\beta_1 - 2\beta_2 v)^2 + 4\beta_3 v(\beta_1 - \beta_2 v) + 1}}{\sqrt{2}\sqrt{\alpha_2\beta_3(\beta_2\omega^2 + \beta_3)}}.$$

Set 2

$$A_0 = A_1 = B_1 = 0,$$
$$C_1 = -\frac{\sqrt{\omega^2(\beta_1 - 2\beta_2 v)^2 + 4\beta_3 v(\beta_2 v - \beta_1) - 1}}{2\sqrt{\alpha_2}\sqrt{\beta_3}},$$
$$k = \mp\frac{\sqrt{-\omega^2(\beta_1 - 2\beta_2 v)^2 + 4\beta_3 v(\beta_1 - \beta_2 v) + 1}}{\sqrt{2}\sqrt{\alpha_2\beta_3(\beta_2\omega^2 + \beta_3)}}. \tag{24}$$

The following solitary wave solutions of Eq. (1) are constructed by substituting Eq. (23) into Eq. (7)

$$q_{31}(x,t) = \frac{\sqrt{-\alpha_2}\tan\left(\frac{\sqrt{-\alpha_2}}{2}\xi\right)\sqrt{\omega^2(\beta_1 - 2\beta_2 v)^2 + 4\beta_3 v(\beta_2 v - \beta_1) - 1}}{2\sqrt{\alpha_2}\sqrt{\beta_3}} e^{i(\gamma x - vt + \vartheta)}, \quad \alpha_2 < 0. \tag{25}$$

$$q_{32}(x,t) = -\frac{\tanh\left(\frac{\sqrt{\alpha_2}}{2}\xi\right)\sqrt{\omega^2(\beta_1 - 2\beta_2 v)^2 + 4\beta_3 v(\beta_2 v - \beta_1) - 1}}{2\sqrt{\beta_3}} e^{i(\gamma x - vt + \vartheta)}, \quad \alpha_2 > 0. \tag{26}$$

Similarly, one can construct more soliton solutions of Eq. (1) from set 2.

Case 4 $\alpha_3 = \alpha_4 = 0,$

Set 1

$$A_0 = A_1 = 0, \quad B_1 = -\frac{ik\sqrt{\alpha_0(\beta_2\omega^2 + \beta_3)}}{\sqrt{2}},$$
$$C_1 = \pm\frac{ik\sqrt{\beta_2\omega^2 + \beta_3}}{\sqrt{2}},$$
$$v = \frac{\beta_1(\beta_2\omega^2 + \beta_3) \mp \sqrt{-(\beta_2\omega^2 + \beta_3)\left(\beta_2(2\alpha_2\beta_3 k^2(\beta_2\omega^2 + \beta_3) - 1) - \beta_1^2\beta_3\right)}}{2\beta_2(\beta_2\omega^2 + \beta_3)}. \tag{27}$$

Set 2

$$A_0 = A_1 = 0, \quad B_1 = \mp\frac{ik\sqrt{\alpha_0(\beta_2\omega^2 + \beta_3)}}{\sqrt{2}}, \quad C_1 = \frac{ik\sqrt{\beta_2\omega^2 + \beta_3}}{\sqrt{2}},$$
$$v = \frac{\beta_1(\beta_2\omega^2 + \beta_3) \pm -\sqrt{-(\beta_2\omega^2 + \beta_3)\left(\beta_2(2\alpha_2\beta_3 k^2(\beta_2\omega^2 + \beta_3) - 1) - \beta_1^2\beta_3\right)}}{2\beta_2(\beta_2\omega^2 + \beta_3)}. \tag{28}$$

We achieved the following solitary wave solutions of Eq. (1) from set 1:

$$q_{41}(x,t) = \frac{(\alpha_1^2 - 4\alpha_0\alpha_2)\sec^2\left(\frac{1}{2}\sqrt{4\alpha_0\alpha_2 - \alpha_1^2}\xi\right) \pm \frac{ik\sqrt{\beta_2\omega^2+\beta_3}}{\sqrt{2}} + 2\sqrt{2}i\sqrt{\alpha_0}\alpha_2 k\sqrt{\beta_2\omega^2 + \beta_3}}{2\left(\alpha_1 - \sqrt{4\alpha_0\alpha_2 - \alpha_1^2}\tan\left(\frac{1}{2}\sqrt{4\alpha_0\alpha_2 - \alpha_1^2}\xi\right)\right)} e^{i(\gamma x - vt + \vartheta)}, \quad 4\alpha_0\alpha_2 > \alpha_1^2. \tag{29}$$

Similarly, one can achieve more new exact solution of Eq. (1) from set 2.

Case 5 $\alpha_0 = \frac{\alpha_2^2}{4\alpha_4}, \alpha_1 = \alpha_3 = 0,$

Set 1

$$A_0 = A_1 = C_1 = 0, \quad k = \pm\frac{\sqrt{2\alpha_4}B_1}{\sqrt{-\alpha_2^2(\beta_2\omega^2 + \beta_3)}},$$
$$v = \frac{\beta_1 \mp \frac{\sqrt{\alpha_2(\beta_2\omega^2+\beta_3)\left(\alpha_2\left(\beta_3\beta_1^2+\beta_2\right)-8\alpha_4\beta_2\beta_3 B_1^2\right)}}{\alpha_2(\beta_2\omega^2+\beta_3)}}{2\beta_2}. \tag{30}$$

Set 2

$$A_0 = 0, \quad A_1 = \pm\frac{i\sqrt{\alpha_4}k\sqrt{\beta_2\omega^2 + \beta_3}}{\sqrt{2}},$$
$$B_1 = \mp\frac{i\alpha_2 k\sqrt{\beta_2\omega^2 + \beta_3}}{2\sqrt{2}\sqrt{\alpha_4}}, \quad C_1 = -\frac{ik\sqrt{\beta_2\omega^2 + \beta_3}}{\sqrt{2}},$$
$$v = \frac{\beta_1(\beta_2\omega^2 + \beta_3) - \sqrt{(\beta_2\omega^2 + \beta_3)\left(\beta_3\beta_1^2 + \beta_2(4\alpha_2\beta_3 k^2(\beta_2\omega^2 + \beta_3) + 1)\right)}}{2\beta_2(\beta_2\omega^2 + \beta_3)} \tag{31}$$

Set 3

$$v = \frac{\beta_1(\beta_2\omega^2 + \beta_3) - \sqrt{-(\beta_2\omega^2 + \beta_3)\left(\beta_2(8\alpha_2\beta_3 k^2(\beta_2\omega^2 + \beta_3) - 1) - \beta_1^2\beta_3\right)}}{2\beta_2(\beta_2\omega^2 + \beta_3)},$$
$$A_0 = A_1 = B_1 = 0, \quad C_1 = \pm ik\sqrt{2(\beta_2\omega^2 + \beta_3)}. \tag{32}$$

Set 4

$$A_0 = B_1 = C_1 = 0, \quad k = \pm \frac{A_1}{\sqrt{2}\sqrt{-\alpha_4(\beta_2\omega^2+\beta_3)}},$$
$$v = \frac{\beta_1 \mp \frac{\sqrt{\alpha_4(\beta_2\omega^2+\beta_3)\left(\alpha_4\left(\beta_3\beta_1^2+\beta_2\right)-2\alpha_2 A_1^2\beta_2\beta_3\right)}}{\alpha_4(\beta_2\omega^2+\beta_3)}}{2\beta_2}. \quad (33)$$

We construct the following solitary wave solutions of Eq. (1) from set 1:

$$q_{51}(x,t) = \pm\sqrt{\frac{2\alpha_4}{\alpha_2}}B_1 \cot\left(\sqrt{\frac{\alpha_2}{2}}\xi\right)e^{i(\gamma x - vt+\vartheta)}, \quad \alpha_2 > 0, \quad \alpha_4 > 0. \quad (34)$$

$$q_{52}(x,t) = \pm\sqrt{-\frac{2\alpha_4}{\alpha_2}}B_1 \coth\left(\sqrt{-\frac{\alpha_2}{2}}\xi\right)e^{i(\gamma x - vt+\vartheta)}, \quad \alpha_2 < 0, \quad \alpha_4 > 0. \quad (35)$$

One can construct more new solutions of Eq. (1) from sets 2, 3 and 4 similarly.

Case 6 $\alpha_0 = \alpha_1 = \alpha_2 = 0$,

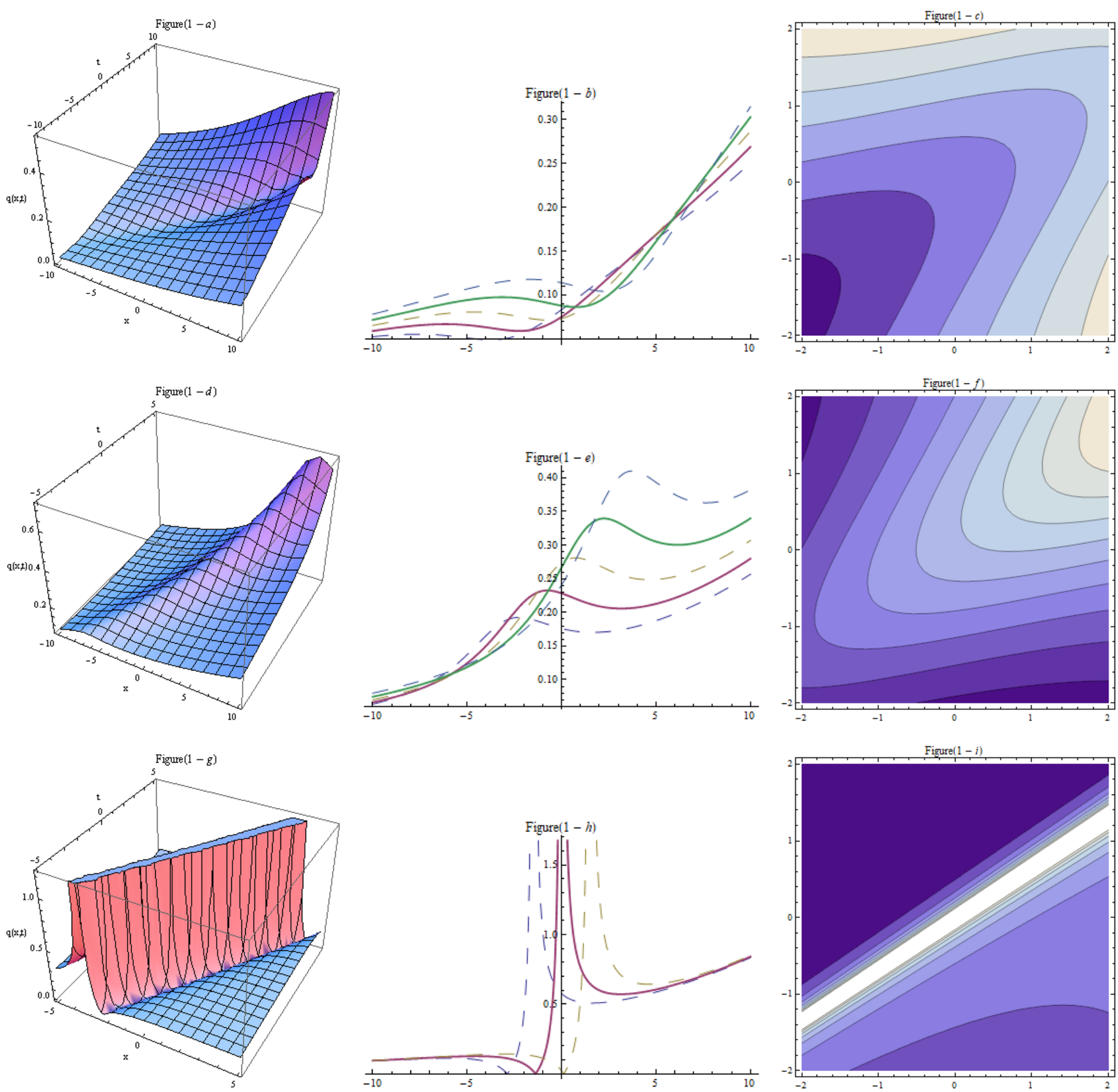

Fig. 1 Exact solitons in various shapes based on solutions (10), (11) and (15)

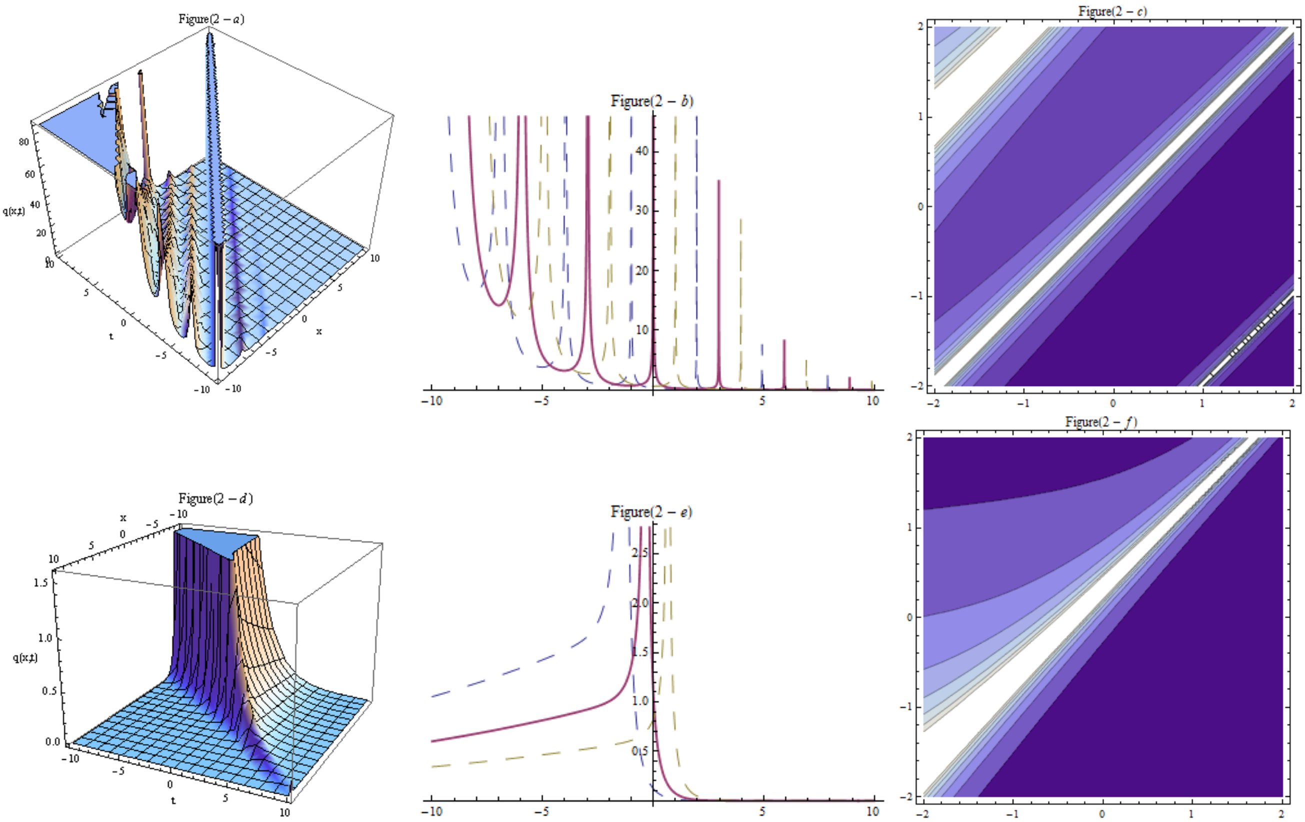

Fig. 2 Exact solitary wave in different shapes based on solutions (20) and (22)

Set 1

$$A_0 = B_1 = 0, \quad A_1 = \frac{i\sqrt{\alpha_4(\beta_3\beta_1^2+\beta_2)}k}{\sqrt{2}(\beta_1-2\beta_2 v)},$$
$$C_1 = \pm\frac{i\sqrt{\beta_3\beta_1^2+\beta_2}k}{\sqrt{2}(\beta_1-2\beta_2 v)}, \tag{36}$$
$$\omega = -\frac{\sqrt{4\beta_3 v(\beta_1-\beta_2 v)+1}}{\beta_1-2\beta_2 v}.$$

Set 2

$$A_0 = B_1 = 0, \quad A_1 = \pm\frac{i\sqrt{\alpha_4(\beta_3\beta_1^2+\beta_2)}k}{\sqrt{2}(\beta_1-2\beta_2 v)},$$
$$C_1 = \pm\frac{i\sqrt{\beta_3\beta_1^2+\beta_2}k}{\sqrt{2}(\beta_1-2\beta_2 v)}, \tag{37}$$
$$\omega = \mp\frac{\sqrt{4\beta_3 v(\beta_1-\beta_2 v)+1}}{\beta_1-2\beta_2 v}.$$

The following solitary wave solutions of Eq. (1) are constructed by substituting set 1 into Eq. (7):

$$q_{61}(x,t) = -\left(\frac{\sqrt{2(\beta_3\beta_1^2+\beta_2)}\,\mathrm{ik}(2\alpha_3\sqrt{\alpha_4}\mp\alpha_3^2\xi)}{(\alpha_3^2\xi^2-4\alpha_4)(\beta_1-2\beta_2 v)}\right) e^{i(\gamma x - vt+\vartheta)}, \quad a_4>0. \tag{38}$$

Similarly, one can achieve more solitary wave solution of Eq. (1) from set 2.

4. Modulation instability

Several higher-order nonlinear systems show an instability that leads to the investigation of the modulation of the steady state as a consequence of interaction among the dispersive and nonlinear effects. To obtain the modulation instability of higher-order NLSE (1) by utilizing the standard linear stability analysis [5, 42] to scrutinize how weak and time-dependent perturbations establish along the propagation distance. The steady-state solution of higher-order NLSE has the form

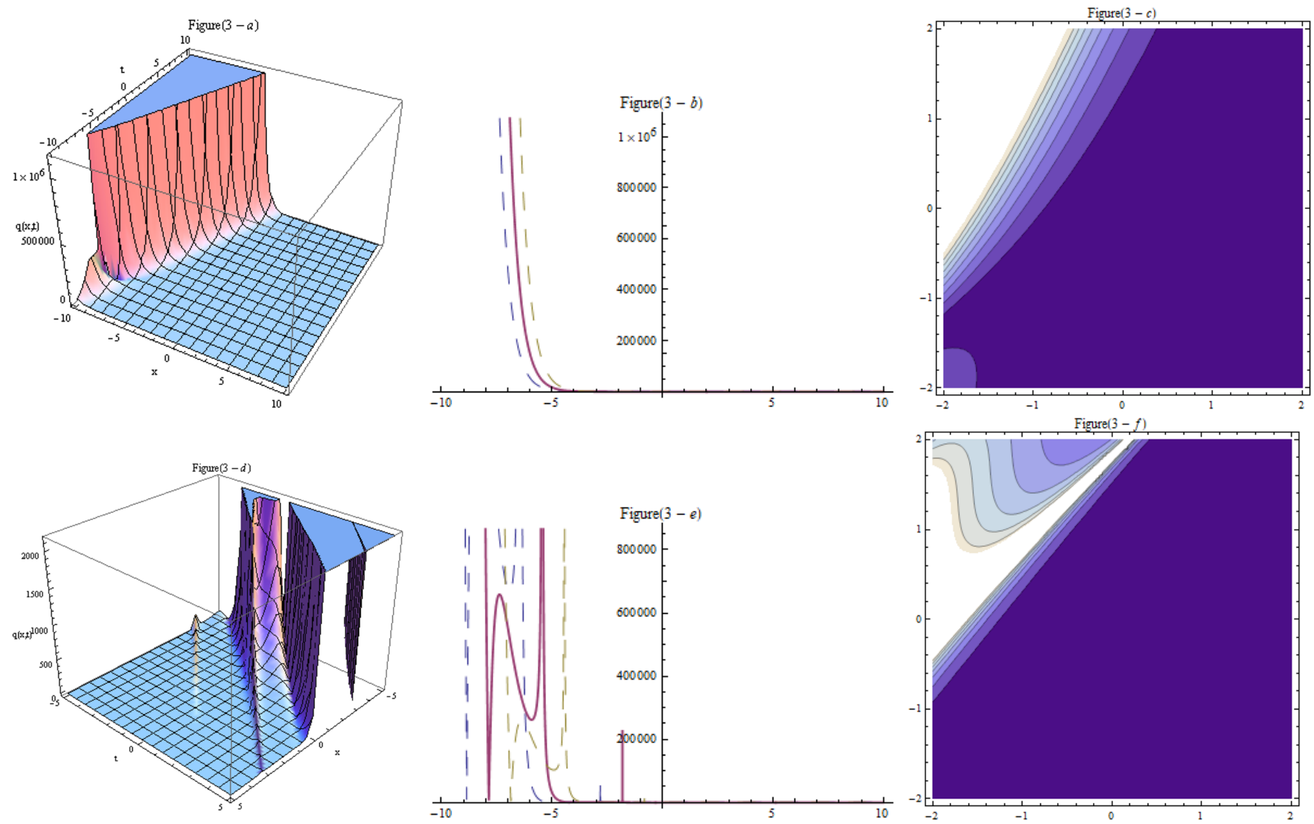

Fig. 3 Exact solitary wave in different shapes based on solutions (25) and (29)

$$q(x,t) = \left(\sqrt{P} + \psi(x,t)\right)e^{i\varphi(t)}, \quad \phi(z) = P\alpha t, \tag{39}$$

where the optical power P is normalized. The perturbation $\psi(x,t)$ is investigated by using linear stability analysis. Substituting Eq. (39) into Eq. (1) and linearizing, we obtain

$$i(\beta_1 + 2\alpha\beta_2 P)\frac{\partial\psi}{\partial t} + i\frac{\partial\psi}{\partial x} + \beta_2\frac{\partial^2\psi}{\partial t^2} + \beta_3\frac{\partial^2\psi}{\partial x^2} - P\alpha(\beta_1 + P\alpha\beta_2)\psi + P(2\psi + \psi^*) = 0, \tag{40}$$

where $*$ denotes complex conjugate. Assume that the solution of Eq. (40) has the form

$$\psi(x,t) = \delta_1 e^{ik(x-\omega t)} + \delta_2 e^{-ik(x-\omega t)}, \tag{41}$$

where k and ω are the normalized wave number and frequency of perturbation. The dispersion relation $\omega = \omega(k)$ of a constant coefficient linear evolution equation determines how time oscillations e^{ikx} are associated with spatial oscillations $e^{ik\omega t}$ of wave number k; putting Eq. (41) in Eq. (40), we achieved a dispersion relation. The graph of the achieved dispersion relation is shown in Fig. 5.

5. Results and discussion

The extracted solitary wave solutions via the modified extended mapping method are different from the achieved solutions of various researchers by other methods because the considered solution (7) of the current method is different from the existing methods. Equation (4) gives some special type of solutions such as rational functions and trigonometric and hyperbolic trigonometric functions via choosing different values of parameters. So, our achieved solutions are new and have not been formulated previously.

Figure 1 evaluates the solitary wave solutions in various shapes based on Case 1 solutions. Figure 1a–c denotes the dark solitary wave, bright solitary wave and solitary wave of solutions (10), (11) and (15) at $\alpha_2 = 0.5,\ \alpha_3 = 1, \alpha_4 = -1, k = 0.5,\ \beta_1 = 1, \beta_2 = -1.5, \beta_3 = 1, \omega = 1.5, \vartheta = -1$, $\alpha_2 = 0.5, \alpha_3 = 1, \alpha_4 = -1, k = 0.5,\quad \beta_1 = 1, \beta_2 = -1.5, \beta_3 = 1, \omega = 1.5, \vartheta = -1$ and $\alpha_2 = 1, \alpha_3 = 2, \alpha_4 = 1, k = 0.5,\quad \beta_1 = 1, \beta_2 = -1.5, \beta_3 = 1, \omega = 1.5, \vartheta = -1$, respectively. Figure 1b, e, h and c, f, i evaluates the solitons in one-dimensional and contour plots of the same solutions, respectively.

Figure 2 evaluates the exact solitary wave solutions in different forms based on Case 2 solutions. Figure 2a, d

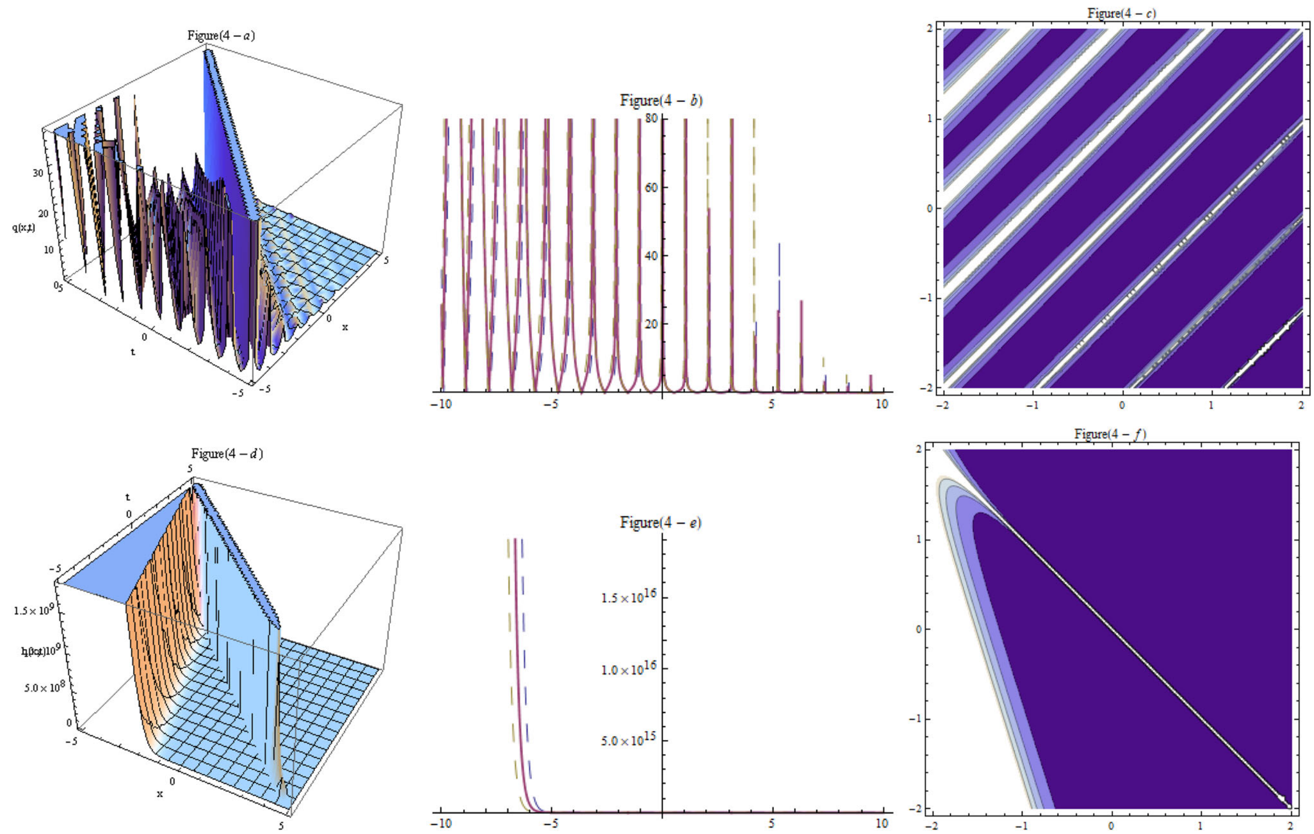

Fig. 4 Exact solitary wave in different shapes based on solutions (34) and (35)

denotes the periodic solitary wave and solitary wave of solutions (20) and (22) at $\alpha_2 = -0.5, \alpha_4 = -1, \beta_1 = 1, \beta_2 = -1.5$, $\beta_3 = 1, \omega = 1, \vartheta = -1, A_1 = 1.5$ and $\alpha_2 = 0.5, \alpha_4 = 1, \beta_1 = 1, \beta_2 = -1$, $\beta_3 = 0.5, \omega = 1, \vartheta = -1, A_1 = 1.5$, respectively. Figure 2b, e and c, f evaluates the solitary wave in one-dimensional and contour plots of the same solutions at same parameters, respectively.

In Fig. 3, the exact solitary wave solutions in different shapes are drawn based on Case 3 and Case 4 solutions. Figure 3a, d denotes the solitary wave and periodic solitary wave of solutions (25) and (29) at $\alpha_2 = -0.5, \alpha_3 = 1, \beta_1 = 0.5$, $\beta_2 = -2, \beta_3 = 1, \omega = 1, \vartheta = -1, A_1 = 1.5, v = -1$ and $\alpha_0 = \alpha_1 = \alpha_2 = 1, k = 1$, $\beta_1 = 1, \beta_2 = -1.5$, $\beta_3 = 0.75, \omega = 1, \vartheta = 1$, respectively. Figure 3b, e and c, f evaluates the solitary wave in one-dimensional and contour plots of the same solutions, respectively.

Figure 4 evaluates the exact solitary wave solutions in different forms based on Case 5 solutions. Figure 4a, d signifies the periodic solitary wave and solitary wave of solutions (34) and (35) at $\alpha_2 = 0.5, \alpha_4 = 1, \beta_1 = 1$, $\beta_2 = -1, \beta_3 = 0.5, \omega = 1, \vartheta = -1, B_1 = 1.5$ and $\alpha_2 = -1, \alpha_4 = 1, \beta_1 = 1$, $\beta_2 = -1, \beta_3 = 0.5, \omega = -1, \vartheta = 1, B_1 = 1.5$, respectively. Figure 4b, e and c, f evaluates the solitary wave in one-dimensional and contour plots of the same solutions at same parameters, respectively. The dispersion relation $\omega = \omega(k)$ between frequency ω and wave number k of perturbation is presented in Fig. 5.

6. Conclusion

In this article, we have effectively extracted the optical soliton and solitary wave solutions of higher-order NLSE with the coefficients of group velocity dispersion, second-order spatiotemporal dispersion and cubic nonlinearity via employing the effective and powerful method, namely modified extended mapping method. The constructed solitons and solitary wave solutions are of scrupulous interest experimentally and theoretically due to their potential applications in high-speed optical fiber transmission system, trans-oceanic distances, and so on. The moments of some achieved solutions are presented graphically and also specified the formation conditions for solitons which help the researchers to know the physical phenomena of this model. Numerous solutions are novel from achieved solutions. The modulation instability (MI) analysis is employed, and an analytic expression for the MI gain has been established which is sensitive to the septic nonlinearity. The constructed solutions and computational

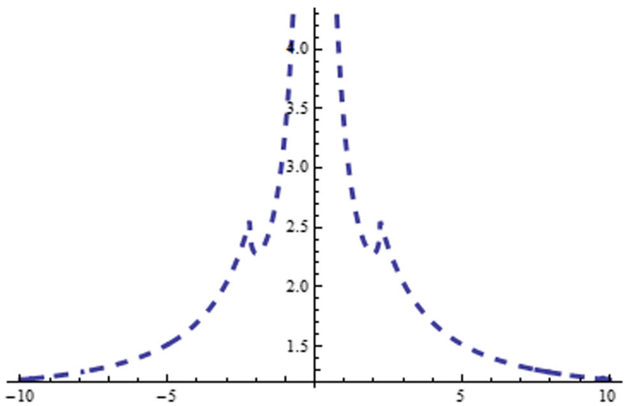

Fig. 5 Graph of dispersion relation $\omega = \omega(k)$

work authenticate the effectiveness, simplicity, and power of the proposed method. The method can also be functional to other sorts of higher-order nonlinear models in current areas of research.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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