Robust generation of orbital-angular-momentum entangled biphotons in twisted nonlinear waveguide arrays

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We describe the generation of photon pairs entangled in optical angular momentum (OAM) through spontaneous four-wave mixing (SFWM) in circular arrays of nonlinear waveguides. We show that the incorporation of the array twist along the waveguides can enable robust generation even in presence of defects and inhomogeneities. We present analytical results and numerical simulations based on coupled-mode equations for the classical pump field and a discrete Schrödinger equation for the biphoton wave function.

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I. INTRODUCTION

Quantum biphoton states with orbital-angular-momentum (OAM) entanglement offer important benefits for a variety of quantum applications. These states can benefit the quantum cryptography applications [1–4] by extending the alphabet to increase the transmission rate [5–8], while providing absolutely secure communications. Whereas OAM states can be fragile under the presence of defects and inhomogeneities, a solution to this issue was recently suggested. It was shown that twisted multi-core photonic-crystal fibers provide robust transmission of OAM states [9]. This happens because the twist introduces a difference in propagation constants between the OAM with positive and negative charges, such that an undesirable scattering between these states gets suppressed.

Integration of generation and transmission of photon states can offer further benefits in terms of compactness and stability of quantum devices. It was shown that entangled biphoton states can be created in arrays of coupled waveguides with quadratic nonlinearity through spontaneous parametric down-conversion (SPDC) [10–15] or spontaneous four-wave mixing (SFWM) in media with cubic or Kerr-type nonlinearity [16–19]. However, generation of OAM-entangled biphotons in twisted waveguide arrays remained unexplored.

In this paper, we investigate theoretically the generation of photon pairs through SFWM and their propagation in twisted waveguide arrays with cubic nonlinearity, as shown schematically in Fig. 1. Such structures can be created through fs laser writing in glass [20] or by twisting multi-core fibers [9]. In Sec. II we formulate the model equations for the classical pump and quantum wavefunction of the biphotons. Then in Sec. III we present analytical solutions for homogeneous (defect-free) circular

waveguide arrays, and formulate conditions for generation of signal and idler photons with particular OAM. In the following Sec. IV we present numerical simulations demonstrating the effects of inhomogeneities in waveguide parameters and illustrate robust OAM generation in twisted arrays, in contrast to straight arrays where OAM states become strongly distorted due to scattering on defects. We present conclusions and outlook in Sec. V.

FIG. 1. Scheme of twisted circular array of coupled waveguides. Pairs of signal and idler photons are generated through spontaneous four-wave mixing from the pump.

II. MODEL EQUATIONS

We consider a linear polarization of light and small twist of waveguides, assuming that the structure of the pump and biphoton guided modes is essentially the same as in the straight array. We also assume that the pump amplitude is relatively weak such that only one pair of photons can be created through SFWM, and accordingly neglect pump depletion and nonlinear self- and cross-phase modulation effects. Then, the classical pump evolution can be modelled by linear coupled-mode equations [21], where the effect of twist is reflected through phase modulation of the coupling coefficients between the

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waveguides [22, 23]:

$$i \frac{dE_n}{dz} + C (e^{-i\phi_0} E_{n+1} + e^{i\phi_0} E_{n-1}) + \rho_n E_n = 0. \quad (1)$$

Here n is the waveguide number, N is the total number of waveguides ($0 \leq n \leq N-1$), E_n is the complex field amplitude in the n -th waveguide, $E_N \equiv E_0$ and $E_{-1} \equiv E_{N-1}$ due to closed-loop boundary conditions, and the real-valued coefficient C defines a coupling strength between the straight neighboring waveguides. We also take into account possible fabrication inhomogeneities defined through the mismatches ρ_n of the propagation constants between different waveguides. The phase factor of the coupling coefficients is defined as

$$\phi_0 = \frac{2\pi R^2 K}{Z_p} \sin\left(\frac{2\pi}{N}\right), \quad (2)$$

where R is the radius of the circular waveguide array, K is the propagation constant of an individual waveguide, and Z_p is the propagation distance where the array is rotated by 2π . Note that for a straight waveguide array $Z_p \rightarrow \infty$, and accordingly $\phi_0 = 0$. The derivation applies under the assumption of small twist rate, meaning that $|\phi_0| \ll \pi$.

We consider the generation of photon pairs through SFWM in the near-degenerate regime, when the signal and idler mode profiles of optical waveguides are essentially the same as at the pump frequency. Then, we use the approach developed for modelling of photon-pair generation and their quantum walks in waveguide arrays [10, 11, 16, 24–28] and incorporate the effect of twist through the phases of the coupling coefficients [22, 23] to obtain a Schrödinger-type equation for the evolution of a biphoton wave function along the waveguides,

$$\begin{aligned} i \frac{d\psi_{n_s, n_i}(z)}{dz} = & -C \left(e^{i\phi_0} \psi_{n_s-1, n_i} + e^{i\phi_0} \psi_{n_s, n_i-1} \right. \\ & \left. + e^{-i\phi_0} \psi_{n_s+1, n_i} + e^{-i\phi_0} \psi_{n_s, n_i+1} \right) \\ & - (\rho_{n_s}^{(s)} + \rho_{n_i}^{(i)}) \psi_{n_s, n_i}(z) \\ & + i\gamma e^{i\Delta\beta^{(0)}z} E_{n_s}^{(p)}(z) E_{n_i}^{(p)}(z) \delta_{n_s, n_i}, \end{aligned} \quad (3)$$

where n_s and n_i are the waveguide numbers describing the positions of the signal and idler photons ($0 \leq n_s \leq N-1$ and $0 \leq n_i \leq N-1$), $\Delta\beta^{(0)}$ is the four-wave mixing phase-mismatch in a single waveguide, γ is a normalized nonlinear coefficient, $\rho_{n_s}^{(s)}$ and $\rho_{n_i}^{(i)}$ are the propagation constant mismatches due to possible waveguides differences for the signal and idler photons, respectively. Similar to Eq. (1), we consider the closed-loop boundary conditions for the biphoton wavefunction as follows: $\psi_{-1, n_i} \equiv \psi_{N-1, n_i}$, $\psi_{N, n_i} \equiv \psi_{0, n_i}$, $\psi_{n_s, -1} \equiv \psi_{n_s, N-1}$, $\psi_{n_s, N} \equiv \psi_{n_s, 0}$.

III. HOMOGENEOUS TWISTED WAVEGUIDE ARRAYS

We first analyze the case of a homogeneous waveguide array composed of identical waveguides all having the same propagation constants, such that $\rho_{n_p} \equiv 0$, $\rho_{n_s}^{(s)} \equiv 0$, and $\rho_{n_i}^{(i)} \equiv 0$. Then, the eigenmode solutions of Eq. (1) can be found as

$$E_n^{(m_p)}(z) = \varepsilon_n^{(m_p)} \exp(i\beta_{m_p} z), \quad (4)$$

where the mode profiles are

$$\varepsilon_n^{(m)} = \exp\left(i \frac{2\pi m n}{N}\right), \quad (5)$$

the mode propagation constants are

$$\beta_{m_p} = 2C \cos\left(\frac{2\pi m_p}{N} - \phi_0\right), \quad (6)$$

and $m_p = 0, \dots, N-1$ are the mode numbers. The eigenmodes $E_n^{(m_p)}(z)$ represent discrete optical vortices [11, 29, 30] with a charge m_p , and carry a corresponding optical angular momentum (OAM).

The pump profile can be represented as a superposition of eigenmodes as

$$E_{n_p}^{(p)}(z) = \frac{1}{N} \sum_{m_p=0}^{N-1} A_{m_p} \varepsilon_{n_p}^{m_p} e^{i\beta_{m_p} z}. \quad (7)$$

Here A_{m_p} are the z -independent mode amplitudes, which values can be determined from the initial conditions by performing an inverse discrete Fourier transform:

$$A_{m_p} = \sum_{n=0}^{N-1} E_n(0) \left(\varepsilon_{n_p}^{m_p} \right)^*. \quad (8)$$

Next, we seek solution of Eq. (3) in the modal representation as

$$\psi_{n_s, n_i} = \frac{1}{N^2} \sum_{m_s=0}^{N-1} \sum_{m_i=0}^{N-1} \Phi_{m_s, m_i}(z) \varepsilon_{n_s}^{m_s} \varepsilon_{n_i}^{m_i}, \quad (9)$$

such that the biphoton mode amplitudes can be expressed through a discrete Fourier transform as follows,

$$\Phi_{m_s, m_i}(z) = \sum_{n_s=0}^{N-1} \sum_{n_i=0}^{N-1} \psi_{n_s, n_i}(z) \left(\varepsilon_{n_s}^{m_s} \varepsilon_{n_i}^{m_i} \right)^* \quad (10)$$

We now apply the discrete Fourier transform to Eq. (3), and obtain the evolution equations for the biphoton modal amplitudes:

$$\begin{aligned} \frac{d\Phi_{m_s, m_i}}{dz} = & i\beta_{m_s, m_i} \Phi_{m_s, m_i} \\ & + \gamma e^{i\Delta\beta^{(0)}z} \sum_{n_p=0}^{N-1} [E_{n_p}(z)]^2 \left(\varepsilon_{n_p}^{m_s} \varepsilon_{n_p}^{m_i} \right)^* \end{aligned} \quad (11)$$

where

$$\beta_{m_s, m_i} = 2C \left[\cos \left(\frac{2\pi m_s}{N} - \phi_0 \right) + \cos \left(\frac{2\pi m_i}{N} - \phi_0 \right) \right] \quad (12)$$

is the biphoton propagation constant.

Since the evolution of different modes is uncoupled in the framework of Eq. (11), it is straightforward to find its analytical solution as:

$$\begin{aligned} \Phi_{m_s, m_i}(z) &= \exp(i\beta_{m_s, m_i} z) \\ &\gamma \int_0^z \exp \left[i(\Delta\beta^{(0)} - \beta_{m_s, m_i}) z' \right] \\ &\sum_{n_p=0}^{N-1} [E_{n_p}(z')]^2 \left(\varepsilon_{n_p}^{m_s} \varepsilon_{n_p}^{m_i} \right)^*, \end{aligned} \quad (13)$$

where we assumed that there are no signal and idler photons at the input, $\Phi_{m_s, m_i}(z=0) \equiv 0$.

We now investigate the possibility to generate the biphotons in a certain OAM state, which is important to many applications. We consider a pump beam with a particular OAM, such that only one mode amplitude $A_{m_p} \neq 0$, and all others a vanishingly small. Then, we can calculate the pump term in Eq. (13) as follows,

$$\begin{aligned} &\sum_{n_p=0}^{N-1} [E_{n_p}(z')]^2 \left(\varepsilon_{n_p}^{m_s} \varepsilon_{n_p}^{m_i} \right)^* \\ &= \sum_{n_p=0}^{N-1} \left[\frac{1}{N} A_{m_p} \varepsilon_{n_p}^{m_p} e^{i\beta_{m_p} z'} \right]^2 \left(\varepsilon_{n_p}^{m_s} \varepsilon_{n_p}^{m_i} \right)^* \\ &= \frac{1}{N} A_{m_p}^2 e^{i2\beta_{m_p} z'} \delta_{2m_p - m_s - m_i, N}, \end{aligned} \quad (14)$$

where due to periodic boundary conditions we denote $A_{m_p} \equiv A_{m_p \pm N}$. Similarly, the Kronecker delta is defined as

$$\delta_{m, N} = \begin{cases} 1, & \text{if } \{m/N\} = 0, \\ 0, & \text{otherwise,} \end{cases} \quad (15)$$

where $\{ \}$ denotes the fractional part of a number.

Now we substitute Eq. (14) into Eq. (13) and obtain:

$$\begin{aligned} \Phi_{m_s, m_i}(z) &= z\gamma \frac{A_{m_p}^2}{N} \delta_{2m_p - m_s - m_i, N} \text{sinc} \left[\frac{z}{2} \Delta\beta_{m_s, m_i, m_p} \right] \\ &\exp \left[\frac{iz}{2} (\Delta\beta_{m_s, m_i, m_p} + 2\beta_{m_s, m_i}) \right]. \end{aligned} \quad (16)$$

Here, the phase mismatch is defined as

$$\Delta\beta_{m_s, m_i, m_p} = \Delta\beta^{(0)} - \beta_{m_s, m_i} + 2\beta_{m_p}. \quad (17)$$

According to Eq. (16), a pump beam with a particular OAM will generate only pairs of signal and idler

photons which satisfy the angular momentum conservation expressed through the Kronecker delta function. We illustrate this relation by summarizing in Table I the possible states of generated biphotons for different OAM in a three-waveguide structure. In Table I, the first column represents the OAM $|m_p\rangle$ of a pump beam coupled to the array, and the second column shows the corresponding pump propagation constants β_{m_p} . The third column lists the possible OAM states $|m_s, m_i\rangle$ of the generated biphotons, and the fourth column presents the corresponding biphoton propagation constants β_{m_s, m_i} . For clarity, we explicitly write down the equivalence of pump states $|1\rangle \equiv |-2\rangle$ and $|2\rangle \equiv |-1\rangle$ due to the discrete periodic symmetry. It then becomes easy to see how the selection rules work based on the OAM conservation for the generated biphotons: $m_s + m_i = 2m_p$ up to multiples of N . We see that two different momentum states of biphotons can be generated for each pump OAM. Further selectivity in excitation of particular photon states can be achieved by adjusting the phase mismatch, as we discuss below.

IV. ROBUST GENERATION IN PRESENCE OF INHOMOGENEITIES

In this section, we illustrate the generation of photon pairs with particular OAM through a choice of the pump OAM and the nonlinear phase-matching. We first consider the case of perfectly symmetric arrays using the solutions derived in Sec. III, and then analyze the effect of possible defects due to the fabrication imperfections.

FIG. 2. Evolution of OAM correlations along the waveguides for the biphotons generated through SFWM in arrays of straight ($\phi_0 = 0$, left column) and twisted ($\phi_0 = 0.1$, right column) three identical waveguides. The normalized phase mismatch $\Delta\beta^{(0)}$ is (a) 3, (b) 3.5037, (c,d) 0. The pump beam OAM is $|m_p\rangle = |2\rangle$, $C = 1$, and $\gamma = 0.01$.

The most efficient generation of photon pairs should occur in the regime of phase matching. This is indeed observed from the biphoton solution in a homogeneous array according to Eq. (16). Specifically, the best generation conditions are for $\Delta\beta_{m_s, m_i, m_p} = 0$, which is achieved

Pump state $ m_p\rangle$	Propagation constant β_{m_p}	Biphoton state $ m_s, m_i\rangle$	Propagation constant β_{m_s, m_i}
$ 0\rangle$	$2C \cos(\phi_0)$	$ 0, 0\rangle$	$4C \cos(\phi_0)$
		$ -1, 1\rangle + 1, -1\rangle$	$-2C \cos(\phi_0)$
$ -2\rangle \equiv 1\rangle$	$-C[\cos(\phi_0) - \sqrt{3} \sin(\phi_0)]$	$ 0, -1\rangle + -1, 0\rangle$	$C[\cos(\phi_0) - \sqrt{3} \sin(\phi_0)]$
		$ 1, 1\rangle$	$-2C[\cos(\phi_0) - \sqrt{3} \sin(\phi_0)]$
$ 2\rangle \equiv -1\rangle$	$-C[\cos(\phi_0) + \sqrt{3} \sin(\phi_0)]$	$ 0, 1\rangle + 1, 0\rangle$	$C[\cos(\phi_0) + \sqrt{3} \sin(\phi_0)]$
		$ -1, -1\rangle$	$-2C[\cos(\phi_0) + \sqrt{3} \sin(\phi_0)]$

TABLE I. Summary of biphoton states which can be generated by pump beams with different OAM in a three-waveguide structure.

FIG. 3. Effect of inhomogeneities on the biphoton generation in straight ($\phi_0 = 0$, left column) and twisted ($\phi_0 = 0.1$, right column) three coupled waveguides. (a-d) Evolution of the biphoton OAM correlations for the normalized phase mismatch $\Delta\beta^{(0)}$ equal to (a) 3, (b) 3.5037, and (c,d) 0. (e,f) Evolution of the pump spectrum in OAM space. The input pump beam OAM is $|m_p\rangle = |2\rangle$, $C = 1$, and $\gamma = 0.01$. The propagation constant mismatch in the second waveguide is $\rho_2 = \rho_2^{(s)} = \rho_2^{(i)} = 0.1$.

when

$$\Delta\beta^{(0)} = \beta_{m_s, m_i} - 2\beta_{m_p}. \quad (18)$$

By satisfying this condition for a required state according to Table I, its preferential generation is therefore expected.

We illustrate the biphoton state generation in Fig. 2, where the evolution of OAM biphoton correlations along the waveguides is demonstrated under different phase-matching conditions for straight (left column) and twisted (right column) structures. In all cases, the input pump OAM is chosen as $|m_p\rangle = |2\rangle$. For a homogeneous waveguide array, the pump OAM is preserved along the propagation direction as discussed in Sec. III. We select the SFWM phase mismatch in a single waveguide following Eq. (18), in order to provide the phase matching for either the biphoton state $|m_s, m_i\rangle = |0, 1\rangle + |1, 0\rangle$ (fifth

row in Table I) or the state $|m_s, m_i\rangle = |-1, -1\rangle$ (sixth row in Table I). The corresponding simulation results are presented in Figs. 2(a,b) and 2(c,d), respectively. We observe efficient generation of the phase-matched state satisfying OAM conservation rules, whereas the generation of all other states is suppressed. We note that in case of ideally homogeneous waveguides considered in Fig. 2, there are essentially no differences in the biphoton generation efficiencies, provided that Eq. (18) is satisfied while taking into account the effect of the twist on the propagation constants, as summarized in Table I.

Next, we consider the effect of inhomogeneities on the biphoton generation. As an example, we introduce a mismatch of the propagation constant in the second waveguide relative to the other two waveguides, which could appear due a variation in optical density or size (thickness) of the waveguide. We assume that this difference is the same for the pump, signal, and idler waves ($\rho_2 = \rho_2^{(s)} = \rho_2^{(i)}$). It should be noted that all states of photon pairs will remain pure (not mixed), because we consider static defects (not dynamical vibrations).

We present in Fig. 3 a comparison of the biphoton generation in straight (left column) and twisted (right column) structures. We observe that in straight waveguides, inhomogeneities lead to a generation of different OAM photon states compared to a single-state generation in a homogeneous structure, c.f. Figs. 3(a,c) and 2(a,c). It happens because the defect in the waveguide propagation constant results in scattering between the OAM of opposite signs (i.e. $|1\rangle$ and $|-1\rangle$). This is observed for the pump beam in Fig. 3(e), and is even more pronounced for the biphotons. In contrast, we find that the generation of states with pure OAM is achieved when twist above a certain threshold value is introduced, see Figs. 3(b,d). In twisted structures, the pump OAM is also preserved as shown in Fig. 3(f).

Such robustness originates in the fact that the structure twist introduces a mismatch of the propagation constants between states with positive and negative OAM ($\beta_{m_p} - \beta_{-m_p}$ and $\beta_{m_s, m_i} - \beta_{\pm m_s, \pm m_i}$) already in homogeneous structures according to Eqs. (6) and (12). Accordingly, if inhomogeneities introduce propagation constant differences less than the twist-induced mismatch, then the OAM states are approximately preserved. Indeed, we have verified that twisted structures provide

robust generation in the general case when the propagation constant mismatches are different at the pump, signal, and idler wavelengths.

FIG. 4. Fidelity for biphoton wavefunction at the distance $z = 20$ vs. the defect strength ($\rho = \rho_2 = \rho_2^{(s)} = \rho_2^{(i)}$) and twist angle (ϕ_0). Other simulation parameters correspond to Fig. 3(d).

We further quantify the fidelity of biphoton generation as

$$F = \frac{|\sum_{n_s, n_i} \psi_{n_s, n_i} (\psi_{n_s, n_i}^{(t)})^*|}{(|\psi| \cdot |\psi^{(t)}|)^{1/2}}, \quad (19)$$

where $\psi^{(t)}$ is the desired target state (which would be obtained in a homogeneous array of waveguides), and ψ is the state that is generated in the presence of inhomogeneities. The fidelity reaches the maximum value of $F = 1$ in case of perfect state generation, if $\psi = \psi^{(t)}$.

We present in Fig. 4 the fidelity dependence on the defect strength ($\rho = \rho_2 = \rho_2^{(s)} = \rho_2^{(i)}$) and twist an-

gle (ϕ_0) at a fixed propagation distance ($z = 20$). We observe that in a straight structure ($\phi_0 = 0$), fidelity decreases as the defect strength is increased. Importantly, high-fidelity state generation is recovered when the twist angle above a certain threshold is introduced. These results also indicate that the state generation in twisted waveguide arrays should be robust even if the values of inhomogeneities (ρ) vary along the waveguides, provided they remain below a particular maximum value.

V. CONCLUSIONS

In this paper, we predict that robust generation of photon pairs with desired OAM can be achieved through spontaneous four-wave mixing in circular twisted arrays of waveguides, in contrast to straight waveguide arrays suffering from scattering between opposite OAM states at inhomogeneities. Building on a recent experimental demonstration of classical OAM state transmission through twisted multi-core photonic-crystal fibers [9], our results suggest a potential for developing quantum communications for cryptography and other applications by combining robust photon-pair generation in twisted structures and their subsequent transmission through twisted optical fibers.

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- [1] A. K. Ekert, J. G. Rarity, P. R. Tapster, and G. M. Palma, *Phys. Rev. Lett.* **69**, 1293 (1992).
 - [2] M. Giustina, A. Mech, S. Ramelow, B. Wittmann, J. Kofler, J. Beyer, A. Lita, B. Calkins, T. Gerrits, S. Nam, R. Ursin, and A. Zeilinger, *Nature* **497**, 227 (2013).
 - [3] R. Fickler, M. Krenn, R. Lapkiewicz, S. Ramelow, and A. Zeilinger, *Sci. Rep.* **3**, 1914 (2013).
 - [4] A. V. Gleim, V. I. Egorov, Y. V. Nazarov, S. V. Smirnov, V. V. Chistyakov, O. I. Bannik, A. A. Anisimov, S. M. Kynev, A. E. Ivanova, R. J. Collins, S. A. Kozlov, and G. S. Buller, *Opt. Express* **24**, 2619 (2016).
 - [5] A. Vaziri, G. Weihs, and A. Zeilinger, *Phys. Rev. Lett.* **89**, 240401 (2002).
 - [6] J. Leach, B. Jack, J. Romero, A. K. Jha, A. M. Yao, S. Franke-Arnold, D. G. Ireland, R. W. Boyd, S. M. Barnett, and M. J. Padgett, *Science* **329**, 662 (2010).
 - [7] J. Leach, E. Bolduc, D. J. Gauthier, and R. W. Boyd, *Phys. Rev. A* **85**, 060304 (2012).
 - [8] A. E. Willner, H. Huang, Y. Yan, Y. Ren, N. Ahmed, G. Xie, C. Bao, L. Li, Y. Cao, Z. Zhao, J. Wang, M. P. J. Lavery, M. Tur, S. Ramachandran, A. F. Molisch, N. Ashrafi, and S. Ashrafi, *Adv. Opt. Photon.* **7**, 66 (2015).
 - [9] X. M. Xi, G. K. L. Wong, M. H. Frosz, F. Babic, G. Ahmed, X. Jiang, T. G. Euser, and P. S. J. Russell, *Optica* **1**, 165 (2014).
 - [10] A. S. Solntsev, F. Setzpfandt, A. S. Clark, C. W. Wu, M. J. Collins, C. L. Xiong, A. Schreiber, F. Katzschnmann, F. Eilenberger, R. Schiek, W. Sohler, A. Mitchell, C. Silberhorn, B. J. Eggleton, T. Pertsch, A. A. Sukhorukov, D. N. Neshev, and Y. S. Kivshar, *Phys. Rev. X* **4**, 031007 (2014).
 - [11] D. M. Markin, A. S. Solntsev, and A. A. Sukhorukov, *Phys. Rev. A* **87**, 063814 (2013).
 - [12] H. Jin, F. M. Liu, P. Xu, J. L. Xia, M. L. Zhong, Y. Yuan, J. W. Zhou, Y. X. Gong, W. Wang, and S. N. Zhu, *Phys. Rev. Lett.* **113**, 103601 (2014).
 - [13] R. Kruse, L. Sansoni, S. Brauner, R. Ricken, C. S. Hamilton, I. Jex, and C. Silberhorn, *Phys. Rev. A* **92**, 053841 (2015).

- [14] F. Setzpfandt, A. S. Solntsev, J. Titchener, C. W. Wu, C. L. Xiong, R. Schiek, T. Pertsch, D. N. Neshev, and A. A. Sukhorukov, *Laser Photon. Rev.* **10**, 131 (2016).
- [15] A. S. Solntsev and A. A. Sukhorukov, *Rev. Phys.* **2**, 19 (2017).
- [16] A. S. Solntsev, A. A. Sukhorukov, D. N. Neshev, and Y. S. Kivshar, *Opt. Express* **20**, 27441 (2012).
- [17] M. J. Collins, C. Xiong, I. H. Rey, T. D. Vo, J. He, S. Shahnia, C. Reardon, T. F. Krauss, M. J. Steel, A. S. Clark, and B. J. Eggleton, *Nat. Commun.* **4**, 2582 (2013).
- [18] J. W. Silverstone, D. Bonneau, K. Ohira, N. Suzuki, H. Yoshida, N. Iizuka, M. Ezaki, C. M. Natarajan, M. G. Tanner, R. H. Hadfield, V. Zwiller, G. D. Marshall, J. G. Rarity, J. L. O'Brien, and M. G. Thompson, *Nature Photonics* **8**, 104 (2014).
- [19] C. L. Xiong, B. Bell, and B. J. Eggleton, *Nanophotonics* **5**, 427 (2016).
- [20] T. Meany, M. Gräfe, R. Heilmann, A. Prez-Leija, S. Gross, M. J. Steel, M. J. Withford, and A. Szameit, *Laser Photon. Rev.* **9**, 363 (2015).
- [21] D. N. Christodoulides, F. Lederer, and Y. Silberberg, *Nature* **424**, 817 (2003).
- [22] S. Longhi, *J. Phys. B* **40**, 4477 (2007).
- [23] I. L. Garanovich, S. Longhi, A. A. Sukhorukov, and Y. S. Kivshar, *Phys. Rep.* **518**, 1 (2012).
- [24] S. Longhi, *Phys. Rev. Lett.* **101**, 193902 (2008).
- [25] Y. Bromberg, Y. Lahini, R. Morandotti, and Y. Silberberg, *Phys. Rev. Lett.* **102**, 253904 (2009).
- [26] A. Peruzzo, M. Lobino, J. C. F. Matthews, N. Matsuda, A. Politi, K. Poulios, X. Q. Zhou, Y. Lahini, N. Ismail, K. Worhoff, Y. Bromberg, Y. Silberberg, M. G. Thompson, and J. L. O'Brien, *Science* **329**, 1500 (2010).
- [27] J. O. Owens, M. A. Broome, D. N. Biggerstaff, M. E. Goggin, A. Fedrizzi, T. Linjordet, M. Ams, G. D. Marshall, J. Twamley, M. J. Withford, and A. G. White, *New J. Phys.* **13**, 075003 (2011).
- [28] M. Grafe, A. S. Solntsev, R. Keil, A. A. Sukhorukov, M. Heinrich, A. Tunnermann, S. Nolte, A. Szameit, and Y. S. Kivshar, *Sci. Rep.* **2**, 562 (2012).
- [29] F. Lederer, G. I. Stegeman, D. N. Christodoulides, G. Asanto, M. Segev, and Y. Silberberg, *Phys. Rep.* **463**, 1 (2008).
- [30] A. S. Desyatnikov, M. R. Dennis, and A. Ferrando, *Phys. Rev. A* **83**, 063822 (2011).