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Green's functions of magnetoelectroelastic solids with a half-plane boundary or bimaterial interface

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Abstract

Green's functions for magnetoelectroelastic medium with an arbitrarily oriented half-plane or bimaterial interface are presented in this paper. The derivation is based on an extended Stroh's formalism and coordinate-transform technique. In particular, a new coordinate variable is introduced to handle vertical or other boundary problems. These Green's functions satisfy related boundary or interface conditions. The Green's functions obtained can be used to establish boundary-element formulation and to analyse fracture behaviour involving half-plane boundaries or bimaterial interfaces.

§ 1. Introduction

Green's functions play an important role in the solution of numerous problems in the mechanics and physics of solids. It is the heart of many analytical and numerical techniques such as singular-integral-equation methods, boundary-element methods, eigenstrain approaches, and dislocation methods (Mura 1987, Qin 2001, Qin and Mai 2002). Pan (2002) derived three-dimensional (3D) Green's functions for anisotropic magnetoelectroelastic materials with a horizontal boundary or bimaterial interface based on an extended Stroh's formalism and two-dimensional Fourier transforms. Soh *et al.* (2003) also presented 3D explicit Green's functions for an infinite 3D transversely isotropic magnetoelectroelastic solid based on the potential theory. Huang *et al.* (1998) obtained magnetoelectroelastic Eshelby tensors in an inclusion resulting from the constraint of the surrounding matrix of piezoelectric–piezomagnetic composites. Li and Dunn (1998) and Li (2000) studied coupled magnetoelectroelastic behaviour arising from an inclusion or an inhomogeneity using Eshelby's tensor approach. Based on Stroh's formalism, conformal mapping, and Laurent series expansion, Liu *et al.* (2001) obtained Green's functions for an infinite 2D anisotropic magnetoelectroelastic medium containing an elliptical cavity or a crack. The aforementioned studies concerned half-plane boundaries or bimaterial interfaces in the horizontal direction only. When the boundaries or interfaces are vertical (or even arbitrarily oriented), the approaches reported in the literature are not applicable. In the present paper, Green's functions for 2D anisotropic magnetoelectroelastic solids with a vertical (or arbitrarily oriented) half-plane boundary or bimaterial interface are presented, using the coordinate technique.

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The Green's functions presented here are suitable for implementing into standard boundary-element formulation and computer programming for numerical analysis.

§ 2. Basic formulations

The governing equations and general solutions of 2D magnetoelectroelastic solids where all fields are functions of x_1 and x_2 only, are summarized briefly here. Throughout this paper the shorthand notation introduced by Barnett and Lothe (1975) and the fixed Cartesian coordinate system (x_1, x_2, x_3) are adopted. Lower-case Latin subscripts always range from 1 to 3, upper-case Latin subscripts range from 1 to 5, and the summation convention is used for repeating subscripts unless otherwise indicated. In the stationary case when no free electric charge, electric current, and body force are assumed to exist, the complete set of governing equations for uncoupled electromagnetoelastic problems is (Chung and Ting 1995, Li 2000, Liu *et al.* 2001):

$$\Sigma_{iJ,i} = 0, \tag{1}$$

together with

$$\Sigma_{iJ} = E_{iJMn} U_{M,n}, \tag{2}$$

in which

$$\Sigma_{iJ} = \begin{cases} \sigma_{ij}, & J \le 3, \\ D_i, & J = 4, \\ B_i, & J = 5, \end{cases} \qquad U_M = \begin{cases} u_m, & M \le 3, \\ \phi, & M = 4, \\ \varpi, & M = 5, \end{cases} \tag{3}$$

$$E_{iJMn} = \begin{cases} C_{ijmn}, & J, M \le 3, \\ e_{nij}, & J \le 3, M = 4, \\ q_{nij}, & J \le 3, M = 5, \\ e_{imn}, & J = 4, M \le 3, \\ -\kappa_{in}, & J = 4, M = 4, \\ -a_{in}, & J = 4, M = 5, \\ q_{imn}, & J = 5, M \le 3, \\ -a_{in}, & J = 5, M = 4, \\ -\mu_{in} & J = 5, M = 5, \end{cases} \tag{4}$$

where σ_{ij}, D_i, and B_i are elastic stress tensors, electric displacement vectors, and magnetic induction vectors respectively; u_m, ϕ, and ϖ denote elastic displacement vector, electric potential, and magnetic potential; C_{ijmn} are elastic moduli, e_{nij} are piezoelectric coefficients, q_{nij} are piezomagnetic coefficients, a_{in} are magnetoelectric coefficients, κ_{in} are dielectric constants, and μ_{in} are magnetic permeabilities. A general solution to equation (1) can be expressed as (Liu *et al.* 2001):

$$\mathbf{U} = 2\mathrm{Re}[\mathbf{Af(z)q}], \tag{5}$$

with

$$\mathbf{A} = [\mathbf{A}_1\ \mathbf{A}_2\ \mathbf{A}_3\ \mathbf{A}_4\ \mathbf{A}_5]$$

$$\mathbf{f}(\mathbf{z}) = \langle f(z_\alpha) \rangle = \mathrm{diag}[f(z_1)\, f(z_2)\, f(z_3)\, f(z_4)\, f(z_5)]$$

$$\mathbf{q} = \{q_1\ q_2\ q_3\ q_4\ q_5\}^T$$

$$z_i = x_1 + p_i x_2,$$

in which "Re" stands for the real part of the complex number, the prime ($'$) denotes differentiation with respect to the argument, $\mathbf{q}$ denotes unknown constants to be found by boundary conditions, $\mathbf{f}$ is an arbitrary function to be determined, and p_i and $\mathbf{A}$ are constants determined by

$$\left[\mathbf{Q} + \left(\mathbf{R} + \mathbf{R}^T\right) p_i + \mathbf{T} p_i^2\right] \mathbf{A}_i = 0, \tag{6}$$

in which the superscript "T" denotes the transpose, and $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{T}$ are 5×5 matrices defined by

$$(\mathbf{Q})_{IK} = E_{1IK1}, \quad (\mathbf{R})_{IK} = E_{1IK2}, \quad (\mathbf{T})_{IK} = E_{2IK2}. \tag{7}$$

The stress–electric displacement–magnetic induction (SEDMI), Σ, obtained from equation (2) can be written as

$$\boldsymbol{\Sigma}_{1J} = -\boldsymbol{\varphi}_{J,2}, \quad \boldsymbol{\Sigma}_{2J} = \boldsymbol{\varphi}_{J,1}, \tag{8}$$

where $\boldsymbol{\varphi}$ is the SEDMI function given as

$$\boldsymbol{\varphi} = 2\ \mathrm{Re}\,[\mathbf{B}\mathbf{f}(\mathbf{z})\mathbf{q}], \tag{9}$$

with

$$\mathbf{B} = \mathbf{R}^T \mathbf{A} + \mathbf{T}\mathbf{A}\mathbf{P} = -(\mathbf{Q}\mathbf{A} + \mathbf{R}\mathbf{A}\mathbf{P})\mathbf{P}^{-1}$$

$$\mathbf{P} = \langle p_\alpha \rangle = \mathrm{diag}[p_1\ p_2\ p_3\ p_4\ p_5]. \tag{10}$$

§ 3. Green's function for half-plane and bimaterial problems

The half-plane or bimaterial interface considered in this section is different from those reported in the literature (Gao and Fan 1998, Qin and Mai 1998, Qin 1999, Qin 2001, Pan 2002). The half-plane boundary (or bimaterial) is in the vertical ($x_1 = 0$ on the boundary in our analysis) rather than the horizontal direction (see figure 2). It is obvious that $z_k = x_1 + p_k x_2$ becomes a real number on the horizontal boundary $x_2 = 0$. However, z_k is, in general, neither a real number nor a pure imaginary number on the vertical boundary $x_1 = 0$, which complicates the related mathematical derivation. To bypass this problem, a new coordinate variable is introduced:

$$z_k^* = \frac{z_k}{p_k}. \tag{11}$$

In this case z_k^* is a real number on the vertical boundary $x_1 = 0$. This coordinate transformation is used for both the half-plane and bimaterial problem below.

3.1. *Green's function for full space*

For an infinite magnetoelectroelastic solid subjected to a line force $\mathbf{q}_0$ and a line dislocation $\mathbf{b}$ both located at $z_0(x_{10}, x_{20})$ (see figure 1), the solution in the form of equations (5) and (9) is (Qin 1999, 2001):

$$\mathbf{U} = \frac{1}{\pi}\mathrm{Im}\left[\mathbf{A}\langle\ln(z_\alpha^* - z_{\alpha 0}^*)\rangle\mathbf{q}\right], \quad \boldsymbol{\varphi} = \frac{1}{\pi}\mathrm{Im}\left[\mathbf{B}\langle\ln(z_\alpha^* - z_{\alpha 0}^*)\rangle\mathbf{q}\right], \tag{12}$$

where $\mathbf{q}$ is a complex vector to be determined. Since $\ln(z_\alpha^* - z_{\alpha 0}^*)$ is a multivalued function we introduce a cut along the line defined by $x_2 = x_{20}$ and $x_1 \le x_{10}$. Using the polar coordinate system (r, θ) with its origin at $z_0(x_{10}, x_{20})$ and with $\theta = 0$ being parallel to the x_1-axis, the solution (12) applies to

$$-\pi < \theta < \pi, \quad r > 0. \tag{13}$$

Therefore

$$\ln(z_\alpha^* - z_{\alpha 0}^*) = \ln r \pm i\pi, \quad \text{at } \theta = \pm\pi \text{ for } \alpha = 1-5. \tag{14}$$

Owing to this relation, equation (12) must satisfy the conditions

$$\mathbf{U}(\pi) - \mathbf{U}(-\pi) = \mathbf{b}, \quad \boldsymbol{\varphi}(\pi) - \boldsymbol{\varphi}(-\pi) = \mathbf{q}_0, \tag{15}$$

which lead to

$$2\mathrm{Re}(\mathbf{Aq}) = \mathbf{b}, \quad 2\mathrm{Re}(\mathbf{Bq}) = \mathbf{q}_0. \tag{16}$$

This can be written as

$$\begin{bmatrix} \mathbf{A} & \bar{\mathbf{A}} \\ \mathbf{B} & \bar{\mathbf{B}} \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \bar{\mathbf{q}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{b} \\ \mathbf{q}_0 \end{Bmatrix}. \tag{17}$$

It follows from the relation

$$\begin{bmatrix} \mathbf{B}^T & \mathbf{A}^T \\ \bar{\mathbf{B}}^T & \bar{\mathbf{A}}^T \end{bmatrix} \begin{bmatrix} \mathbf{A} & \bar{\mathbf{A}} \\ \mathbf{B} & \bar{\mathbf{B}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{I} \end{bmatrix}, \tag{18}$$

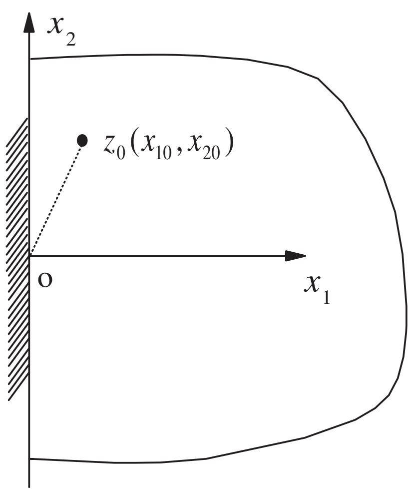

Figure 1. Magnetoelectroelastic half-plane.

that

$$\begin{Bmatrix} \mathbf{q} \\ \bar{\mathbf{q}} \end{Bmatrix} = \begin{bmatrix} \mathbf{B}^T & \mathbf{A}^T \\ \bar{\mathbf{B}}^T & \bar{\mathbf{A}}^T \end{bmatrix} \begin{Bmatrix} \mathbf{b} \\ \mathbf{q}_0 \end{Bmatrix}. \tag{19}$$

Hence

$$\mathbf{q} = \mathbf{A}^T \mathbf{q}_0 + \mathbf{B}^T \mathbf{b}. \tag{20}$$

3.2. *Green's function for half-space*

Let the material occupy the region $x_1 > 0$, and a line force-charge $\mathbf{q}_0$ and a line dislocation $\mathbf{b}$ apply at $z_0(x_{10}, x_{20})$. To satisfy the boundary conditions on the infinite straight boundary of the half-plane, the solution should be modified as follows

$$\mathbf{U} = \frac{1}{\pi}\mathrm{Im}\{\mathbf{A}\langle \ln(z_\alpha^* - z_{\alpha 0}^*)\rangle \mathbf{q}\} + \sum_{\beta=1}^{5} \frac{1}{\pi}\mathrm{Im}[\mathbf{A}\langle \ln(z_{\alpha 0}^* - \bar{z}_{\beta 0}^*)\rangle \mathbf{q}_\beta], \tag{21}$$

$$\boldsymbol{\varphi} = \frac{1}{\pi}\mathrm{Im}\{\mathbf{B}\langle \ln(z_\alpha^* - z_{\alpha 0}^*)\rangle \mathbf{q}\} + \sum_{\beta=1}^{5} \frac{1}{\pi}\mathrm{Im}\{\mathbf{B}\langle \ln(z_{\alpha 0}^* - \bar{z}_{\beta 0}^*)\rangle \mathbf{q}_\beta\}, \tag{22}$$

where $\mathbf{q}$ is given in equation (20) and $\mathbf{q}_\beta$ are unknown constants to be determined.

Consider first the case in which the surface $x_1 = 0$ is traction-free, so that (Qin 2001)

$$\boldsymbol{\varphi} = 0 \quad \text{at } x_1 = 0. \tag{23}$$

Substituting equation (22) into equation (23) yields

$$\boldsymbol{\varphi} = \frac{1}{\pi}\mathrm{Im}\{\mathbf{B}\langle \ln(x_2 - z_{\alpha 0}^*)\rangle \mathbf{q}\} + \sum_{\beta=1}^{5} \frac{1}{\pi}\mathrm{Im}\{\mathbf{B}\langle \ln(x_2 - \bar{z}_{\beta 0}^*)\rangle \mathbf{q}_\beta\} = 0. \tag{24}$$

Noting that $\mathrm{Im}(f) = -\mathrm{Im}(\bar{f})$, we have

$$\mathrm{Im}\{\mathbf{B}\langle \ln(x_2 - z_{\alpha 0}^*)\rangle \mathbf{q}\} = -\mathrm{Im}\{\bar{\mathbf{B}}\langle \ln(x_2 - \bar{z}_{\alpha 0}^*)\rangle \bar{\mathbf{q}}\}, \tag{25}$$

and

$$\langle \ln(x_2 - z_{\alpha 0}^*)\rangle = \sum_{\beta=1}^{5} \ln(x_2 - z_{\beta 0}^*)\mathbf{I}_\beta, \tag{26}$$

where

$$\mathbf{I}_\beta = \langle \delta_{\beta\alpha}\rangle = \mathrm{diag}[\delta_{\beta 1}, \delta_{\beta 2}, \delta_{\beta 3}, \delta_{\beta 4}, \delta_{\beta 5}]. \tag{27}$$

Equation (24) now yields

$$\mathbf{q}_\beta = \mathbf{B}^{-1}\bar{\mathbf{B}}\mathbf{I}_\beta \bar{\mathbf{q}} = \mathbf{B}^{-1}\bar{\mathbf{B}}\mathbf{I}_\beta(\bar{\mathbf{A}}^T \mathbf{q}_0 + \bar{\mathbf{B}}^T \mathbf{b}). \tag{28}$$

If the boundary $x_1 = 0$ is a rigid surface, then

$$\mathbf{U} = 0, \qquad \text{at } x_1 = 0. \tag{29}$$

The same procedure shows that the solution is given by equations (21) and (22) with

$$\mathbf{q}_\beta = \mathbf{A}^{-1}\bar{\mathbf{A}}\mathbf{I}_\beta(\bar{\mathbf{A}}^T \mathbf{q}_0 + \bar{\mathbf{B}}^T \mathbf{b}). \tag{30}$$

Therefore the final version of the Green's function can be written in terms of z_k as

$$\mathbf{U} = \frac{1}{\pi}\mathrm{Im}\{\mathbf{A}\langle \ln(z_\alpha - z_{\alpha 0})/p_\alpha\rangle \mathbf{q}\} + \sum_{\beta=1}^{5}\frac{1}{\pi}\mathrm{Im}\{\mathbf{A}\langle \ln(z_\alpha/p_\alpha - \bar{z}_{\beta 0}/\bar{p}_\beta)\rangle \mathbf{q}_\beta\} \tag{31}$$

$$\boldsymbol{\varphi} = \frac{1}{\pi}\mathrm{Im}\{\mathbf{B}\langle \ln(z_\alpha - z_{\alpha 0})/p_\alpha\rangle \mathbf{q}\} + \sum_{\beta=1}^{5}\frac{1}{\pi}\mathrm{Im}\{\mathbf{B}\langle \ln(z_\alpha/p_\alpha - \bar{z}_{\beta 0}/\bar{p}_\beta)\rangle \mathbf{q}_\beta\}. \tag{32}$$

3.3. *Green's function for a bimaterial problem*

We now consider a bimaterial solid whose interface is on x_2-axis ($x_1 = 0$). It is assumed that the left half-plane ($x_1 < 0$) is occupied by material 1, and the right half-plane ($x_1 > 0$) by material 2 (figure 2). They are rigidly bonded together so that

$$\mathbf{U}^{(1)} = \mathbf{U}^{(2)}, \quad \boldsymbol{\varphi}^{(1)} = \boldsymbol{\varphi}^{(2)}, \quad \text{at } x_1 = 0, \tag{33}$$

where the superscripts (1) and (2) label the quantities relating to materials 1 and 2 respectively. The equality of traction continuity comes from the relations $\mathbf{t} = \partial\boldsymbol{\varphi}/\partial s$. When points along the interface are considered, integration of $\mathbf{t}^{(1)} = \mathbf{t}^{(2)}$ provides equations $(33)_2$ since the integration constants corresponding to rigid motion can be neglected.

For a magnetoelectroelastic bimaterial plate subjected to a line force-charge $\mathbf{q}_0$ and a line dislocation $\mathbf{b}$ both located in the left half-plane at $z_0(x_{10}, x_{20})$ (figure 1), the solution may be assumed, in a similar treatment to that in the half-plane problem, in the form

$$\mathbf{U}^{(1)} = \frac{1}{\pi}\mathrm{Im}\left\{\mathbf{A}^{(1)}\left\langle \ln(z_\alpha^{*(1)} - z_{\alpha 0}^{*(1)})\right\rangle \mathbf{q}\right\} + \sum_{\beta=1}^{5}\frac{1}{\pi}\mathrm{Im}\left\{\mathbf{A}^{(1)}\left\langle \ln(z_\alpha^{*(1)} - \bar{z}_{\beta 0}^{*(1)})\right\rangle \mathbf{q}_\beta^{(1)}\right\}, \tag{34}$$

$$\boldsymbol{\varphi}^{(1)} = \frac{1}{\pi}\mathrm{Im}\left\{\mathbf{B}^{(1)}\left\langle \ln(z_\alpha^{*(1)} - z_{\alpha 0}^{*(1)})\right\rangle \mathbf{q}\right\} + \sum_{\beta=1}^{5}\frac{1}{\pi}\mathrm{Im}\left\{\mathbf{B}^{(1)}\left\langle \ln(z_\alpha^{*(1)} - \bar{z}_{\beta 0}^{*(1)})\right\rangle \mathbf{q}_\beta^{(1)}\right\}, \tag{35}$$

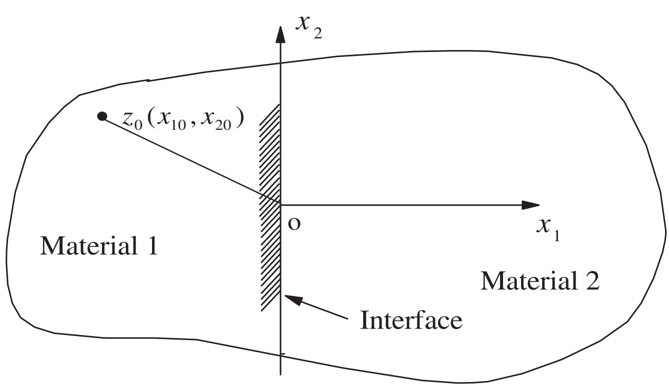

Figure 2. Magnetoelectroelastic bimaterial plate.

for material 1 in $x_1 < 0$ and

$$\mathbf{U}^{(2)} = \sum_{\beta=1}^{5} \frac{1}{\pi} \mathrm{Im}\left\{ \mathbf{A}^{(2)} \left\langle \ln\left(z_\alpha^{*(2)} - z_{\beta 0}^{*(1)} \right) \right\rangle \mathbf{q}_\beta^{(2)} \right\}, \tag{36}$$

$$\boldsymbol{\varphi}^{(2)} = \sum_{\beta=1}^{4} \frac{1}{\pi} \mathrm{Im}\left\{ \mathbf{B}^{(2)} \left\langle \ln\left(z_\alpha^{*(2)} - z_{\beta 0}^{*(1)} \right) \right\rangle \mathbf{q}_\beta^{(2)} \right\}, \tag{37}$$

for material 2 in $x_1 > 0$, where $z_{\beta 0}^{*(1)} = z_{\beta 0}^{(1)}/p_\beta^{(1)}$, $z_\alpha^{*(i)} = z_\alpha^{(i)}/p_\alpha^{(i)}$ $(i = 1, 2)$. The value of $\mathbf{q}$ is again given in equation (20) and $\mathbf{q}_\beta^{(1)}$, $\mathbf{q}_\beta^{(2)}$ are unknown constants which are determined by substituting equations (34)–(37) into equation (33). Following the derivation in section 3.2, we obtain

$$\mathbf{A}^{(1)}\mathbf{q}_\beta^{(1)} + \bar{\mathbf{A}}^{(2)}\bar{\mathbf{q}}_\beta^{(2)} = \bar{\mathbf{A}}^{(1)}\mathbf{I}_\beta\bar{\mathbf{q}}, \qquad \mathbf{B}^{(1)}\mathbf{q}_\beta^{(1)} + \bar{\mathbf{B}}^{(2)}\bar{\mathbf{q}}_\beta^{(2)} = \bar{\mathbf{B}}^{(1)}\mathbf{I}_\beta\bar{\mathbf{q}}. \tag{38}$$

Solving equation (38) yields

$$\mathbf{q}_\beta^{(1)} = \mathbf{B}^{(1)-1}\left[\mathbf{I} - 2\left(\mathbf{M}^{(1)-1} + \bar{\mathbf{M}}^{(2)-1} \right)^{-1} \mathbf{L}^{(1)-1} \right] \bar{\mathbf{B}}^{(1)} \mathbf{I}_\beta \bar{\mathbf{q}}, \tag{39}$$

$$\mathbf{q}_\beta^{(2)} = 2\mathbf{B}^{(2)-1}\left(\bar{\mathbf{M}}^{(1)-1} + \mathbf{M}^{(2)-1} \right)^{-1} \mathbf{L}^{(1)-1} \mathbf{B}^{(1)} \mathbf{I}_\beta \mathbf{q}, \tag{40}$$

where $\mathbf{M}^{(j)} = -i\mathbf{B}^{(j)}\mathbf{A}^{(j)-1}$ is the surface impedance matrix.

§ 4. Green's function for a solid with an arbitrarily oriented half-boundary or biomaterial interface

If the half-boundary is in an angle θ_0 $(\theta_0 \neq 0)$ with positive x-axis, the corresponding Green's function can be obtained by introducing a new mapping function

$$z = \zeta^{\theta_0/\pi} \quad (\theta_0 \neq 0), \tag{41}$$

which maps the boundary $\theta = \theta_0$ in the z-plane onto the real axis in the ζ-plane $(\xi + i\eta)$ (figure 3).

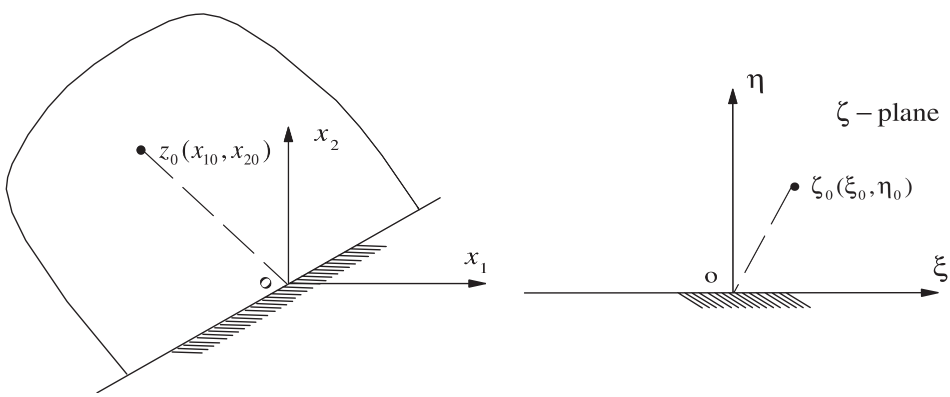

Figure 3. Magnetoelectroelastic solid with arbitrarily oriented half-plane.

Following the procedure in sections 3.2 and 3.3 it can be shown that the resulting Green's functions can be expressed as

$$\mathbf{U} = \frac{1}{\pi}\mathrm{Im}\left\{\mathbf{A}\left\langle \ln\left(z_\alpha^{\pi/\theta_0} - z_{\alpha 0}^{\pi/\theta_0}\right)\right\rangle \mathbf{q}\right\} + \sum_{\beta=1}^{5}\frac{1}{\pi}\mathrm{Im}\left\{\mathbf{A}\left\langle \ln\left(z_\alpha^{\pi/\theta_0} - \bar{z}_{\beta 0}^{\pi/\theta_0}\right)\right\rangle \mathbf{q}_\beta\right\}, \tag{42}$$

$$\boldsymbol{\varphi} = \frac{1}{\pi}\mathrm{Im}\left\{\mathbf{B}\left\langle \ln\left(z_\alpha^{\pi/\theta_0} - z_{\alpha 0}^{\pi/\theta_0}\right)\right\rangle \mathbf{q}\right\} + \sum_{\beta=1}^{5}\frac{1}{\pi}\mathrm{Im}\left\{\mathbf{B}\left\langle \ln\left(z_\alpha^{\pi/\theta_0} - \bar{z}_{\beta 0}^{\pi/\theta_0}\right)\right\rangle \mathbf{q}_\beta\right\}, \tag{43}$$

for the half-plane problem, and

$$\mathbf{U}^{(1)} = \frac{1}{\pi}\mathrm{Im}\left\{\mathbf{A}^{(1)}\left\langle \ln\left(z_\alpha^{(1)\pi/\theta_0} - z_{\alpha 0}^{(1)\pi/\theta_0}\right)\right\rangle \mathbf{q}\right\} + \sum_{\beta=1}^{5}\frac{1}{\pi}\mathrm{Im}\left\{\mathbf{A}^{(1)}\left\langle \ln\left(z_\alpha^{(1)\pi/\theta_0} - \bar{z}_{\beta 0}^{(1)\pi/\theta_0}\right)\right\rangle \mathbf{q}_\beta^{(1)}\right\}, \tag{44}$$

$$\boldsymbol{\varphi}^{(1)} = \frac{1}{\pi}\mathrm{Im}\left\{\mathbf{B}^{(1)}\left\langle \ln\left(z_\alpha^{(1)\pi/\theta_0} - z_{\alpha 0}^{(1)\pi/\theta_0}\right)\right\rangle \mathbf{q}\right\} + \sum_{\beta=1}^{5}\frac{1}{\pi}\mathrm{Im}\left\{\mathbf{B}^{(1)}\left\langle \ln\left(z_\alpha^{(1)\pi/\theta_0} - \bar{z}_{\beta 0}^{(1)\pi/\theta_0}\right)\right\rangle \mathbf{q}_\beta^{(1)}\right\}, \tag{45}$$

for material 1 in $x_1 < 0$ and

$$\mathbf{U}^{(2)} = \sum_{\beta=1}^{5}\frac{1}{\pi}\mathrm{Im}\left\{\mathbf{A}^{(2)}\left\langle \ln\left(z_\alpha^{(2)\pi/\theta_0} - z_{\beta 0}^{(1)\pi/\theta_0}\right)\right\rangle \mathbf{q}_\beta^{(2)}\right\}, \tag{46}$$

$$\boldsymbol{\varphi}^{(2)} = \sum_{\beta=1}^{5}\frac{1}{\pi}\mathrm{Im}\left\{\mathbf{B}^{(2)}\left\langle \ln\left(z_\alpha^{(2)\pi/\theta_0} - z_{\beta 0}^{(1)\pi/\theta_0}\right)\right\rangle \mathbf{q}_\beta^{(2)}\right\}, \tag{47}$$

for material 2 in a biomaterial problem, where $\mathbf{q}_\beta$, $\mathbf{q}_\beta^{(1)}$, and $\mathbf{q}_\beta^{(2)}$ have, respectively, the same forms as those given in equations (30), (39), and (40).

§ 5. Conclusions

Green's functions in closed form for magnetoelectroelastic solids with a vertical or arbitrarily oriented boundary or interface have been derived through the use of Stroh's formalism, conforming mapping and a coordinate-transform technique. These Green's functions satisfy the related boundary or interface conditions. Introduction of a new coordinate variable simplifies the mathematical calculations.

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