RBCs AND DSGEs: THE COMPUTATIONAL APPROACH TO BUSINESS CYCLE THEORY AND EVIDENCE

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Abstract. Real business cycle (RBC) and dynamic stochastic general equilibrium (DSGE) methods have become essential components of the macroeconomist's toolkit. This literature review stresses recently developed techniques for computation and inference, providing a supplement to the Romer textbook, which stresses theoretical issues. Many computational aspects are illustrated with reference to the simple divisible labor RBC model. Code and US data to replicate the computations are provided on the internet, together with a number of appendices providing background details.

Keywords. Bayesian analysis; Computation; DSGE; RBC; Simulation

1. Introduction

Real business analysis now occupies a major position in the core curriculum of nearly every graduate program. At a recent National Bureau of Economic Research conference, a prominent Cambridge economist of the New Keynesian school described the RBC approach as the new orthodoxy of macroeconomics, without raising a challenge from the audience. (King and Rebelo, 1999, p. 930)

This review examines the real business cycle (RBC) and dynamic stochastic general equilibrium (DSGE) methods for analyzing business cycle behavior that have become necessary tools for graduate-trained macroeconomists. The default Master's level text, Romer (2006), provides an insightful characterization of the RBC literature, based heavily on Campbell (1994). However, the Romer text says

Table 1. Sources of Useful Code.

BACC	http://www2.cirano.qc.ca/~bacc/
DYNARE	http://www.ceprenmap.cnrs.fr/dynare/
IRIS	http://www.iris-toolbox.com/
QM RBC	http://dge.repec.org/
R software	http://www.r-project.org/
Anderson and Moore	http://www.federalreserve.gov/pubs/oss/oss4/about.html
Christiano	http://faculty.wcas.northwestern.edu/~lchrist/
Fernández-Villaverde	http://www.econ.upenn.edu/~jesusfv/companion.htm
LeSage	http://www.spatial-econometrics.com/
McGrattan	http://ideas.repec.org/e/pmc46.html
Rubio-Ramírez	http://www.econ.duke.edu/~jfr23/
Schorfheide	http://www.econ.upenn.edu/~schorf/research.htm
Sims	http://www.princeton.edu/~sims/
Smets	http://www.ecb.int/home/html/researcher_swm.en.html
Söderlind	http://home.tiscalinet.ch/paulsoderlind
Uhlig	http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm

little about the computational side of the RBC literature, to the frustration of researchers interested in using the models for business cycle analysis.

This paper describes the computational steps commonly used by RBC practitioners. In particular, it stresses recently developed (often Bayesian) techniques for computation and inference. A number of computational issues are discussed in the context of the simple divisible labor RBC model, which is often used for pedagogical purposes. Code and US data to replicate the computations for this paper are provided on the Reserve Bank of New Zealand's website at http://www.rbnz.govt.nz/research/discusspapers/supportinginfo/dp2007_15/3162093.html. Our aim is to describe the empirical tools required by the reader of Romer (2006) to build DSGE models. The paper also provides numerous internet links to helpful code for business cycle computation (in Table 1).

Section 2 provides a brief overview of RBC origins, including the contribution of Kydland and Prescott (1982). Section 3 presents a simple RBC model and considers solution methods. Section 4 examines how the model can be used to analyze the data by a variety of methods, including Bayesian estimation. Section 5 outlines several recent research agendas. Appendices to the discussion paper version of this paper provide details of background steps for our RBC example (see Table 2).

2. An Overview of RBC Models

The RBC approach is a flexible framework for quantitative business cycle analysis, which owes much to the pioneering work by Kydland and Prescott (1982). Their research agenda proposed novel techniques to examine both the theory and empirics of fluctuations. Prescott (1986) provides a summary. Rebelo (2005) surveys the

Table 2. Replication Files and Appendices.

Appendix	Program zip file
Solving for dynamics	(No applicable software files)
Bayesian estimation of model	Replication.zip
Markov chain diagnostics	McmcConvergence.zip
Assessing model fit	Replication.zip (simulate.m)

Files to replicate results are available at http://www.rbnz.govt.nz/research/discusspapers/supportinginfo/dp2007_15/3162093.html. The appendices are contained in the discussion paper version of this paper www.rbnz.govt.nz/research/discusspapers/dp07_15.pdf.

literature from a modern perspective, but does not focus on the computational issues that are the subject of this review.

Kydland and Prescott's theoretical framework was based on the idea that the neoclassical growth model could be used to study business cycles, following Brock (1974). Kydland and Prescott's use of stochastic technology and rational expectations produced a model that adhered to the Lucas micro-foundations research agenda. Like Lucas (1972, 1973), agent behavior in the RBC model was governed by the optimization under uncertainty framework straight from the micro-economist's toolbox.¹ Elements of this approach were foreshadowed by Robertson (1915), who noted that technology disturbances 'inventions' contributed to business cycles, and Frisch (1933), who studied business cycles within an optimizing framework. Unfortunately, optimization in the neoclassical growth model yields non-linear behavior, ruling out analytical solutions in general cases. The common approach is to linearize the model about the steady state of the system and consider an approximate solution. Researchers use computer programs such as GAUSS or MATLAB to solve and analyze these linearized systems.

The Kydland and Prescott (1982) approach to business cycle empirics became known as 'calibration'. This involves choosing parameters on the basis of long-run data properties and judgment (sometimes guided by microeconomic evidence). Judicious parameter selection is computationally convenient given numerical solution methods.

More recently, many RBC practitioners have turned to Bayesian methods to allow for a range of parameter values and to check the empirical validity of models. The Bayesian distributions are often estimated by Monte Carlo Markov chain (MCMC) simulation techniques: by choosing suitable transition functions for Markov chains it can be shown that the posterior distribution of a model's parameters coincides with the stationary distribution of the chain. Consequently, the posterior can be approximated by sampling from a suitably long realization of the chain (see Gelman *et al.* (2004) and Koop (2003) for a discussion of MCMC methods for Bayesian analysis). However, these techniques are very computationally intensive: the chains may need to be iterated for a very long time to approximate the stationary distribution. Other methods for RBC inference, such as limited information methods

and classical maximum-likelihood estimation with full information, are also often burdensome because they require the researcher to solve the forward-looking systems and may require numerical optimization techniques. In contrast to the impression left by Romer's textbook, analysis with modern macroeconomic models requires computational techniques.

The Kydland–Prescott model has just one source of uncertainty (a technology disturbance). Many practitioners inferred that the early vintage RBC models were inconsistent with the sample data, and subsequent models increased considerably in complexity to cope with the criticisms that arose. By the late 1990s, the RBC literature produced models with, for example, multiple shocks, price rigidities and monetary and fiscal policies. Since the term RBC was associated with real disturbances, the DSGE label became popular for these larger, computationally demanding models with multiple disturbances.

It is worth mentioning that some models drawing on RBC techniques abstract from the micro-foundations initially at the heart of the RBC research agenda. RBC may indeed be 'the new orthodoxy of macroeconomics' as King and Rebelo (1999) claim, but some DSGE models are more RBC than others. In particular, many of the models now incorporate the types of rigidities emphasized by the New Keynesian literature. With this point made, hereafter we simply use the term 'DSGE' to refer to an RBC model with multiple shocks.

3. A Simple RBC Model

The computational steps for theory and inference with RBC models are discussed below in relation to an explicit simple divisible labor example, encountered by many graduate students in studying (among others) King *et al.* (1988), Christiano and Eichenbaum (1992), Campbell (1994) and Romer (2006).² Given the functional forms that are employed in our simple model, analytical solutions are achievable for the log-linear approximation. However, with less restrictive assumptions on functional forms, computational techniques will typically be required. Since the theoretical specification – covered in detail by Campbell (1994) and Romer (2006) – will be familiar to most readers, a concise treatment will suffice.

We model a simple Walrasian economy subject to technological disturbances in discrete time. Following the literature, we make specific assumptions about the functional forms followed by preferences and constraints of the (here, but not necessarily) representative agent. These assumptions sometimes trouble students new to the RBC approach; the functional forms are unlikely to be exact. The 'keep-it-simple' approach owes something to the need for tractability. It also reflects the desire by RBC researchers to analyze the sample data by using incremental deviations from the well-understood neoclassical growth model.

The perfectly competitive economy contains a representative household that maximizes utility given an initial stock of capital. The household simultaneously participates in the goods, capital and labor markets. The economy also contains a representative firm, which sells output produced from capital, labor and technology.

The representative firm has a constant returns-to-scale Cobb–Douglas production function

$$Y_t = (A_t N_t)^\alpha K_t^{1-\alpha} \quad (1)$$

where Y_t is output in time t , A_t is technology, N_t is the number of labor hours worked, K_t is the capital stock and $0 < \alpha < 1$. Capital accumulates according to

$$K_{t+1} = (1 - \delta)K_t + Y_t - C_t \quad (2)$$

where δ is the depreciation rate of the stock of capital, and C_t is consumption.

The household has log utility in consumption and power utility in leisure:

$$\sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i}) + \theta \frac{(1 - N_{t+i})^{1-\gamma}}{1 - \gamma} \right] \quad (3)$$

where the elasticity of inter-temporal substitution of leisure is $\sigma \equiv 1/\gamma$ and β is the discount factor. The mechanism of inter-temporal labor supply is similar to that of Lucas and Rapping (1969) and labor is divisible. (Romer (2006, p. 210) discusses the indivisible labor case analyzed by Hansen (1985) and Rogerson (1988).)

We define the gross rate of return on investment capital, R_{t+1} , to be the marginal product of capital plus undepreciated capital:

$$R_{t+1} \equiv (1 - \alpha) \left(\frac{A_{t+1} N_{t+1}}{K_{t+1}} \right)^\alpha + (1 - \delta) \quad (4)$$

We consider a social planner maximizing the expected utility of the representative individual by picking a path for consumption and leisure subject to the two constraints (1) and (2). Stokey *et al.* (1989) and Ljungqvist and Sargent (2004) discuss a dynamic programming approach and Bellman's equation. A simple introduction to inter-temporal optimization is provided in supplement A of Obstfeld and Rogoff (1997). Note that the unique solution to the social planner's problem is the competitive equilibrium from the welfare theorems. Given the functional forms that have been assumed for preferences, the necessary first-order conditions for the problem can be written as³

$$C_t^{-1} = \beta E_t [C_{t+1}^{-1} R_{t+1}] \quad (5)$$

and

$$\theta(1 - N_t)^{-\gamma} = \frac{W_t}{C_t} = \alpha \frac{A_t^\alpha}{C_t} \left(\frac{K_t}{N_t} \right)^{1-\alpha} \quad (6)$$

where the marginal utility of leisure is set equal to the real wage W_t times the marginal utility of consumption. Given a competitive labor market, the real wage also equals the marginal product of labor. Note that equation (5) reflects the between-periods aspect of the problem, so that labor supply is dictated by inter-temporal substitution.

Following Campbell (1994) and Romer (2006), we assume that in the (unique) steady state, technology, capital, output and consumption all grow at the common constant rate, $G \equiv A_{t+1} A_t^{-1}$. The gross rate of return on capital is also constant in

the steady state, denoted by R . (We will consider the importance of this definition of the steady state below.)

The first-order condition (5) becomes

$$G = \beta R \quad (7)$$

In logs (denoted by lower case letters)

$$g = \log(\beta) + r \quad (8)$$

The definition of the return to capital (4) and the first-order condition (7) imply that the steady-state technology–capital ratio is constant

$$\frac{A}{K} = \left(\frac{G/\beta - (1 - \delta)}{1 - \alpha} \right)^{1/\alpha} \frac{1}{N} \approx \left(\frac{r + \delta}{1 - \alpha} \right)^{1/\alpha} \frac{1}{N} \quad (9)$$

with $R \approx 1 + r$. The production function and the technology–capital ratio also imply a constant steady-state output–capital ratio:

$$\frac{Y}{K} = \left(\frac{AN}{K} \right)^\alpha \approx \left(\frac{r + \delta}{1 - \alpha} \right) \quad (10)$$

The steady-state consumption–output ratio is

$$\frac{C}{Y} = \frac{C/K}{Y/K} \approx 1 - \frac{(1 - \alpha)(g + \delta)}{r + \delta} \quad (11)$$

Departures from the steady state can be modeled as a system of non-linear equations in the logs of technology, capital, output, labor and consumption. In some special cases, the model becomes linear (see, for example, McCallum, 1989). Non-linear variants can be log-linearized using first-order Taylor series expansions around the steady state. The non-steady-state behavior of the resulting linear system approximates the original non-linear specification.

The first-order log-linearization approximation can be inaccurate, especially when the economy is some distance from the steady state; see, for example, Den Haan and Marcet (1994). George and Oxley (1985, 1994) also discuss linearization difficulties in macro models. A number of researchers have considered higher order Taylor series approximations, including Collard and Juillard (2001) and Schmitt-Grohé and Uribe (2004a, b). Aruoba *et al.* (2006) describe less restrictive but typically more computationally burdensome approaches and horse-race a variety of methods. Fernández-Villaverde and Rubio-Ramírez (2006) provide a nice outline of their research agenda on estimating and analyzing non-linear and non-normal DSGE models. (Table 1 provides a link to Rubio-Ramírez's webpage.) A particularly appealing feature of their particle filter approach is that general equilibrium models can be estimated allowing for parameter change (see the discussion in Section 5). Nevertheless, for consistency with the Romer (2006) representation of RBC modeling, in this paper we use the standard linear approximation procedure.

Appendix A (available on the internet; see Table 2) provides details of the somewhat time-consuming log-linearization of equations (1), (2), (4)–(6) using first-order Taylor expansions around the steady state. The handy rules in Uhlig

(1999) make the process less burdensome than working through Campbell (1994) suggests. This approach yields the following linear (in logs) system:

$$y_t = \alpha(a_t + n_t) + (1 - \alpha)k_t \quad (12)$$

$$k_{t+1} = \lambda_1 k_t + \lambda_2(a_t + n_t) + (1 - \lambda_1 - \lambda_2)c_t \quad (13)$$

$$r_{t+1} = \lambda_3(a_{t+1} + n_{t+1} - k_{t+1}) \quad (14)$$

$$E_t \Delta c_{t+1} = \sigma \lambda_3 E_t [a_{t+1} + n_{t+1} - k_{t+1}] \quad (15)$$

$$n_t = v[(1 - \alpha)(k_t - n_t) + \alpha a_t - c_t] \quad (16)$$

where

$$\lambda_1 \equiv \frac{1+r}{1+g} \quad \lambda_2 \equiv \frac{\alpha(r+\delta)}{(1-\alpha)(1+g)} \quad \lambda_3 \equiv \frac{\alpha(r+\delta)}{r+1} \quad v \equiv \frac{1-N}{N}\sigma$$

For simplicity, we omit all constants; the variables can be thought of as zero-mean deviations from the steady-state growth path.

The log technology process follows

$$a_t = \phi a_{t-1} + \varepsilon_t \quad (17)$$

where ϕ measures the persistence of technology shocks, $-1 < \phi < 1$ and ε_t is an idiosyncratic disturbance. For simplicity, the technology process specified in equation (17) rules out unit root behavior that would make the model non-stationary.

The ‘technology shocks’ label is controversial. In this highly abstract model, any disturbances that affect the supply side – other than changes in capital and labor – will be lumped together under the label ‘technology’. See Summers (1986) for a critique of the contribution of technology in early RBC models. King and Rebelo (1999) argue that the RBC research agenda requires resuscitation because one-shock models require implausibly large and frequent technology shocks.⁴

For consistency with the Romer (2006) treatment of RBCs, we detrend by defining a deterministic steady state. Many RBC researchers prefer not to linearize about a deterministic steady state; see, for example, Cogley and Nason (1995a, b). Fukač and Pagan (2009) review methods for detrending models using a recent DSGE example.

Our system of linear difference equations (12)–(17) can be solved using the method of undetermined coefficients as described by Campbell (1994) and others. The researcher guesses the functional form of the solution and then verifies. The approach was developed in the rational expectations literature; see Blanchard and Fischer (1989, pp. 261–266) for a textbook treatment. Binder and Pesaran (1995), Uhlig (1999) and Sims (2002) discuss more general vector-based approaches for solving systems of linear difference equations; Blanchard and Kahn (1980) and Anderson and Moore (1985) provide the building blocks.⁵ Stokey *et al.* (1989),

Cooley (1995), Ljungqvist and Sargent (2004) and Canova (2007) contain useful discussions. Anderson (2006) compares the Anderson–Moore approach with a number of common alternatives. He finds the Anderson–Moore approach to be superior in numerical accuracy and computational efficiency.

Sims (2002) discusses conditions required to solve expectational difference equations, relating to the eigenvalues of the model. Note that there may be many solutions in complex DSGE models. In our simple example, we discard the explosive eigenvalue case since the competitive economy should have a unique solution. More complex models could give either only explosive solutions (usually interpreted as a misspecified model), or many stable solutions (which might make sense with multiple equilibria, or sunspot equilibria).

For our simple model, the researcher can, like Campbell (1994), derive expressions for the dynamics of consumption, output, capital and labor conditional on the ‘deep’ parameters. In a multi-shock model, these derivations by hand would be extremely time consuming, and unnecessary given that the model properties could simply be simulated.

4. Data Analysis

To solve the model by the methods described above, one requires values for the deep parameters (and hence the reduced-form parameters, the λ s) that specify the log-linear system. The early RBC literature focused on ‘calibration’, rather than estimation, of these key parameters.

This controversial step is driven by three main concerns. First, identification of deep parameters in estimated large dynamic systems can be troublesome. Second, there is often a conceptual mismatch between the theoretical variables and their sample counterparts. (For example, what is the sample equivalent of the ‘rate of return on one period real government debt’ if bonds are denominated nominally in the sample data?) And third, in practice the classical estimation of macro models can be plagued by badly behaved likelihoods – the researcher typically has very few business cycle fluctuations with which to estimate the highly abstract model.

Recent literature has focused more on the estimation of the key parameters, often by Bayesian methods. The use of ‘off-model’ or ‘prior’ information allows the researcher to mitigate, but not eliminate, the first and third of these issues (we return to these matters in Section 5). Not much can be done about conceptual mismatch in highly abstract aggregate models; the researcher simply hopes that the selected values of the parameters move the model closer to the data.

We start our review of empirical methods by describing a Bayesian full-information approach, keeping our review closer to more recent literature, and move on to discuss other estimation-based procedures. Then, we describe and contrast the calibration approach and what we term ‘Bayesian calibration’. The last of these deals with model uncertainty in a more formal way than calibration, but the full model is not estimated.

4.1 Bayesian Estimation

Returning to our simple RBC model, there are a number of unknown parameters in equations (12)–(16), namely $(\alpha, \delta, \sigma, r, N, g)$, and the parameters of the technology process, ϕ and the variance of ε , denoted by $\text{var}(\varepsilon) = \sigma_\varepsilon^2$. Since nearly all capital stock data are constructed by making assumptions about the rate of depreciation, most researchers fix δ (at a quarterly rate of around 2%). The remaining parameters can be estimated.

Koop (2003) provides an excellent general introduction to Bayesian estimation, highlighting the differences from the classical approach; see also Gelman *et al.* (2004). Since neither of these Bayesian textbooks consider explicitly DSGE estimation, the descriptions given by Lubik and Schorfheide (2005), An and Schorfheide (2007) and Canova (2007) are particularly welcome. Key influential papers include DeJong *et al.* (2000), Schorfheide (2000), Otrok (2001) and Rabanal and Rubio-Ramírez (2005).

To fix ideas without being too specific, consider a researcher wishing to estimate deep parameters of interest using additional ‘off-model’ information. The researcher’s priors encompass all the information that is not from the sample data. To implement the Bayesian approach, the researcher re-weights the likelihood using the ‘off-model’ priors and maximizes the re-weighted likelihood.

Since $P(Data)$ is simply a constant, Bayes’s rule implies that the posterior distribution over the parameters of model i , θ_i , given the *Data*, is proportional to the marginal likelihood, $p(Data|\theta_i)$, multiplied by the prior, $p(\theta_i)$,

$$p(\theta_i|Data) \propto p(Data|\theta_i)p(\theta_i) \quad (18)$$

Maximizing this re-weighted likelihood thus delivers the most likely values of the parameters given the data and the prior. In practice, the posterior mode for the parameters is often computed using the methods described by Schorfheide (2000):

1. solve the linear rational expectations system;
2. use the Kalman filter to find the numerical values that maximize the sum of the likelihood and the prior.

Given that the shape of the posterior density is determined by the prior and the likelihood, the posterior density for each parameter can be constructed by using an MCMC, typically the Metropolis–Hastings (MH) algorithm.

Juillard’s (1996) DYNARE package allows the Bayesian estimation along these lines using MATLAB. The researcher can input the model in linearized form or in levels, with some higher-order approximation methods as options. Appendix B (see Table 2) describes the results from the estimation of our simple RBC model using US data from 1959Q1 to 2006Q1. The DYNARE code and US data to replicate our results are provided on the website that accompanies this paper.

The main advantage of the Bayesian estimation approach is that the researcher can make (post-data) probabilistic statements about model parameters and related events of interest. To illustrate this, Figure 1 displays the prior (lighter line) and posterior densities for the parameters ϕ and σ analyzed in our simple RBC example.

Figure 1. Example RBC Prior and Posterior Densities.

In each case, the posterior mode value is shown by the lighter colored line. In the Romer (2006) textbook, the student is invited to explore the implications of different parameterizations of the model. But the student has no formal means to assess what values are reasonable, given the data. However, armed with the predictive densities for the parameters, the researcher can simulate the model at the parameter values that are supported by the data.

It is important to note that the posterior densities shown in Figure 1 are conditional on the priors for all the parameters, not just those for ϕ and σ . (Appendix B to this paper contains a complete list of the priors used for our analysis.) Some users find it difficult to specify priors since their specification requires a great deal of information for each parameter: the type of distribution, the support for the distribution, the mean of the distribution and perhaps higher moments too. In practice, prior elicitation can be troublesome. In response, DSGE practitioners sometimes resort to justifying their priors for a particular parameter with reference to other studies, even though the earlier research may have utilized a different model. Parameter priors are also typically assumed to be independent, even though the parameters may not necessarily be so.

In practice, the researcher also has to grapple with two other contentious issues: uninformativeness and Markov chain convergence problems. The former often causes the results to exhibit prior sensitivity. Consider the right-hand panel of Figure 1. The posterior density is pretty close to the prior density, often taken as a ‘warning indicator’ that the data are relatively uninformative about this parameter. In principle, prior sensitivity can be assessed by re-estimating the model using ‘reasonable’ (but not first choice) priors. Of course, there are many feasible prior densities. And the prior sensitivity for a specific parameter depends on the other priors in the system. So a thorough assessment carries a considerable computational burden.

A further worry is that a close match between the prior and the posterior of a particular parameter is neither necessary nor sufficient for prior sensitivity to be a problem. Canova and Sala (2009) discuss these and identification-related

issues for DSGE modeling; see also An and Schorfheide (2007) with a discussion, rejoinder, plus GAUSS and MATLAB replication programs available from Schorfheide's webpage (see Table 1 for a link). These uninformative problems occur with other types of modeling of course. Poirier (1998) provides the Bayesian perspective with examples based on different models. No doubt the DSGE literature will investigate more formal approaches to dealing with these issues in the future.

The second contentious issue is that chain convergence problems sometimes pollute reported posterior densities. Can the researcher be sure that Figure 1 is based on appropriate draws from the Markov chain? Ideally, the researcher should be convinced that the Markov chain has iterated long enough for the sample of parameters at the end of the simulated chain to approximate the stationary (posterior) distribution with sufficient accuracy. There are, however, two problems. First, has the chain run for long enough so that the effects of initial conditions have dissipated (an issue of bias)? And second, is the sample of simulated parameters from the Markov chain large enough to ensure that the properties of the stationary distribution are adequately captured (an issue of variance)?

Cowles and Carlin (1996), Guihenneuc-Jouyaux *et al.* (unpublished data) and Brooks and Roberts (1999) survey convergence diagnostics in general. A common approach is to employ a variety of diagnostics, including graphical representations of the Markov chains. Appendix C (see the associated website) shows how to implement a selection of diagnostics for our simple RBC model. (The DYNARE programs from the website accompanying this paper also supply some diagnostics.)

To summarize, although the Bayesian estimation provides the researcher with formal tools to make probabilistic statements about parameter values (and other features of interest), there are drawbacks. Would non-informative priors – classical estimation – be a better option?

Arguably, non-informative priors misrepresent the researcher's knowledge. For example, most researchers have some view of what constitutes a likely number for the elasticity of inter-temporal labor substitution. Nevertheless, there is a substantial literature using classical maximum-likelihood methods which pre-dates the Bayesian approach. Kim and Pagan (1995) review the early literature. Hansen and Sargent (1980), Sargent (1989), Altug (1989), McGrattan (1994) and Ireland (2001) are key papers in the development of the approach. McGrattan's webpage contains many useful programs (see Table 1 for a link). Convergence problems are also sometimes a practical difficulty with the classical estimation, even for relatively simple DSGE models.

An alternative strand of the DSGE estimation literature has focused on limited information methods. These include minimum distance estimation based on the discrepancy between vector autoregression (VAR) and DSGE model impulse response functions, used, for example, by Rotemberg and Woodford (1997) and Christiano *et al.* (2005). Christiano's webpage provides the code necessary for replications (see Table 1 for an internet link). Hall *et al.* (2007) propose an information criterion approach to facilitate matching. Ruge-Murcia (2007) argues that limited information methods are more reliable than either the Bayesian or

classical approaches to estimation of DSGE models in the presence of stochastic singularity. Measurement errors are typically added to the models to facilitate the implementation of the Bayesian and classical full-information methods; see also Bierens (2007) on singularity.

4.2 Calibration

Until the late 1990s, calibration was the most popular method for empirical analysis with DSGE models. Romer (2006, pp. 208–209) provides an introduction to calibration. In a nutshell, the researcher picks plausible values for the parameters by looking at the data or referring to other empirical studies. Ideally, this process should use features of the data other than those to be subsequently studied. The researcher simulates the model and examines additional properties of the model to assess its merits. For example, Kydland and Prescott (1982) employed the neoclassical growth model calibrated on ‘great ratios’ etc. and used the model to explore business cycle phenomena. Typically, the researcher compares the variance (or covariance) of the simulated data with equivalent sample statistics.

In the early literature, calibration was sometimes argued to be concerned with ‘measurement’ rather than ‘inference’. The exchange between Kydland and Prescott (1995), Hansen and Heckman (1996) and Sims (1996) captures nicely the 1990s debate about empirical methods. King and Rebelo (1999) also note the differences between calibration and traditional econometric methods. Cooley (1997) argues that estimation is actually complementary, since the former can help guide calibration choices. Cooley also emphasizes the bidirectional interplay between measurement and theory.

From the Bayesian perspective, picking plausible model parameters raises many of the issues encountered in prior mean elicitation. But where Bayesian would pick the combination of parameter values that reflects off-model beliefs, a calibrator would select a set of parameters that matches the sample data (albeit not the same features intended to be studied with the calibrated model). In this sense, the Bayesian counterpart to calibrated values is a set of parameter values comprising (some measure of) the center of the posterior density for each parameter. Papers in the calibration tradition typically justify the selected parameters with reference to the model and the sample data.

Notice that the pedagogical paper by Campbell (1994) understates considerably the attention given to parameter selection in the literature. Cooley and Prescott (1995) provide a more typical treatment. Many parameters are selected by matching the balanced growth path of the model to the long-run sample features (e.g. sample averages). Some calibrators refer to earlier papers; others use microeconomic evidence in calibration. In contrast, technology parameters are matched to the Solow residual (which captures total factor productivity). This step requires the researcher to carry out a growth accounting exercise; see Romer (2006, pp. 29–31 and 298).

Returning to our simple RBC example, the posterior means shown in Figure 1 (others are reported in Appendix B) are very close to the calibrated values selected

by Campbell (1994), which we used to determine the prior means. (Campbell calibrates a subset of the parameters considered in our Bayesian estimation example; we used off-model judgment to complete the prior specification.) There is little evidence to suggest that Campbell's preferred parameters are implausible, given the data and our prior specification.

Once the parameters have been chosen, the model can be solved and simulated. For researchers interested in experimenting with various calibrations in well-known models, Uhlig's DYNAMO package, see Table 1 for the link, provides menu-driven MS-WINDOWS-compatible software.

A controversial issue in the calibration literature is how to evaluate the resulting model. The early RBC literature judged model performance from the ability to match particular 'stylized facts' of the sample data. In particular, Prescott (1986) stressed that the one-shock RBC model produced a variance for real output not much smaller than in the detrended sample data. Early practitioners used the Hodrick and Prescott (HP) (1997) filter to extract a smooth non-linear trend from the sample data. Prescott (1986) argued that the HP filter resembles an approximate high-pass filter designed to eliminate stochastic components with periodicities greater than 32 quarters. Harvey and Jaeger (1993) and Cogley and Nason (1995a) note the sensitivity of the sample data characteristics to the filtering technique. Canova (1998a) and Burnside (1998) provide differing perspectives on the controversy surrounding HP filtering in the 1990s. The academic tension regarding filtering was sufficient for Canova to be labeled 'a skunk in a rose garden'; see Canova (1998b, p. 534). Fukač and Pagan (2009) discuss the implications of filtering with reference to more recent DSGE models.

The covariance statistics discussed in early vintage RBC models are just one measure of model fit. An influential paper by Cogley and Nason (1995b) shows that several well-known models from the early 1990s fail to reproduce the persistence and depth of business cycle fluctuations. A common interpretation is that the early models required more disturbances and frictions to match the data. (We return to this issue in the subsequent section.)

Another criticism of calibration is that although the parameters are selected with model uncertainty in mind, very little sensitivity analysis appeared in the published papers. Kim and Pagan (1995) discuss this in detail. A number of researchers responded by formalizing the contribution of model uncertainty by using Bayesian Monte Carlo techniques for calibration: the Bayesian calibration. Notice that this is distinct from the Bayesian estimation approach described above – there is no posterior simulation. So issues about the informativeness of the data, which often blight the Bayesian estimation of DSGE models, do not arise. Instead, the researcher generates (a large number of) draws from the priors – prior simulation – and solves the model for each draw. A good early reference is DeJong *et al.* (1996); a more recent example is Nason and Rogers (2006).

Bayesian calibration practitioners often assess model performance by estimating less-restrictive models, such as Bayesian vector autoregressions (BVARs), on the data simulated from the model and the sample. Geweke (2009) discusses the relationship with posterior odds comparisons for DSGE models relative

to the reference model. The BACC package developed by Geweke and co-authors can be used with MATLAB and GAUSS to conduct prior simulation (see Table 1) and to carry out the Bayesian estimation in a wide range of reduced-form specifications.

5. Some DSGE Research Agendas

In the final section of this paper, we give a sense of the direction of DSGE modeling by describing some (very broadly defined) research agendas, stressing computational issues.

5.1 *More Elaborate Models*

The early RBC models, with just one source of disturbance, were very parsimonious. A common perception was that the early models were too abstract for analysis of many economic issues. Subsequent models have been used to conduct experiments with multiple shocks, heterogeneous agents, wage and price rigidities and monetary and fiscal policies. The literature review in Cooley (1995) describes in detail many of the different aspects of this research initiative during the 1990s.

Rather than attempt a comprehensive review of the diverse strands of this research agenda, we limit our discussion to DSGE models for monetary policy analysis. Chari and Kehoe (1999) provide an overview of the late 1990s' optimal monetary (and fiscal) policy literature. Smets and Wouters (2007) summarize the more recent monetary-policy-related literature that, following Taylor (1993), has emphasized monetary-policy rules that 'often' perform 'well', rather than optimally. Kremer *et al.* (2006) provide a more detailed review of DSGE models as policy tools.

Smets and Wouters' own research design is summarized on the webpage listed in Table 1; see also Sims (2007b) for a critique. Smets and Wouters' ambitious agenda involves the construction of considerably larger models than those in Schorfheide (2000), but they utilize a similar Bayesian estimation strategy. There are similar initiatives to build 'medium-sized' models for a monetary policy analysis at the International Monetary Fund (see Bayoumi *et al.*, 2004), the Bank of Canada (Murchison and Rennison, 2006), Sveriges Riksbank (Adolfson *et al.*, 2007), the Norges Bank (Brubakk *et al.*, 2006), the Reserve Bank of New Zealand and other central banks. The rubric New Open Economy Models (NOEM) is sometimes used to describe open economy DSGE models, typically with New Keynesian elements. Examples include Lubik and Schorfheide (2005, 2007) and Justiniano and Preston (2009). Other interesting avenues include the introduction of fiscal policy, for example Schmitt-Grohé and Uribe (2004a, b), and the consideration of profound parameter uncertainty within the DSGE framework by Levin *et al.* (2005) and Edge *et al.* (2009).

A brief description of the Smets and Wouters (2003) model gives a feel for the scale, and illustrates some of the common features, of Bayesian DSGE models used for policy purposes. Their model has three agents: households, firms and the central bank. The policymaker sets the short-term interest rate in response to deviations

of output from its flexible price level (potential output) and inflation from its target rate. There are both nominal and real frictions: monopolistic competition, sticky wages and prices, partial indexation of prices and wages, cost adjustment in capital accumulation, habit formation and variable capital utilization. The eight disturbances are to general technology, labor supply, preferences, investment-specific technology, government consumption, price mark-ups, wage mark-ups, the equity premium, interest rates and the inflation target.

The search for richer dynamics has also focused on the labor market (e.g. Lubik and Krause, 2006, 2007) and the microfoundations of money (e.g. Lagos and Wright, 2005). In both cases, the new generation aims to recast some of the *ad hoc* elements in current DSGE models within an optimizing framework. These features seem likely to be incorporated in large-scale macro models in the future.

5.2 Fit Assessment with DSGE Models

The fit of DSGE models has played an important part in the development of the literature. The early papers by Kydland and Prescott focus on matching the variances and covariances of key macroeconomic aggregates. Cogley and Nason (1995b) drew attention to the limited internal propagation mechanisms within 1990s' vintage models by comparing the impulse response functions from a structural VAR and autocorrelation functions estimated on model-generated and the US sample data. (Appendix D to this paper on the internet discusses the fit of our simple example RBC model.) Watson (1993) provides a tool for assessing the fit of DSGE models related to the familiar R^2 statistic. More recently, Fukač and Pagan (2009) propose limited information techniques as supplements to measures of system fit.

A popular new tool for DSGE fit assessment comes from Del Negro and Schorfheide (2003, 2004). They estimate a structural VAR using priors based on a simple three-equation New Keynesian DSGE model. (GAUSS code for replication can be obtained from Schorfheide's webpage; see Table 1.) The intuition behind their approach is that the theoretical model can generate synthetic data. And then a VAR can be estimated on sample and synthetic data, controlling the ratio of simulated and sample data with a 'tightness' hyperparameter; see also Ingram and Whiteman (1994).

In practice, the hybrid DGSE-VAR is estimated by Bayesian methods. Del Negro and Schorfheide note that for their US data the resulting hybrid model betters the out-of-sample forecasting performance of their simple VAR. Lees *et al.* (2007) consider a NOEM variant on New Zealand data. Subsequent work by Del Negro *et al.* (2007) reinterprets the tightness hyperparameter as a measure of in-sample fit of the DSGE model, which could be used to guide DSGE model builders. Alternatively, Bayesian posterior odds rank candidate models; see Rabanal and Rubio-Ramírez (2005) and the small open economy application by Matheson (2006). Both Sims (2007a) and Del Negro and Schorfheide (2006) argue that posterior odds can overstate the difference in fit between candidate models within a sparse model space.

A common feature of many assessments of fit is that a constant parameter VAR is used as a benchmark. This choice can create difficulties, however. For example, under certain conditions a DSGE model does not have a reduced-form VAR representation; see Fernández-Villaverde *et al.* (2007), Chari *et al.* (2007b) and Christiano and Davis (2006). Another problem is that constant parameters may not be a very good restriction (a point considered further below).

Forecasting performance is often used to assess model fit, but by this metric it is not obvious that an unrestricted VAR should be used as a benchmark against which to assess DSGE models. Kilian (2007) notes the superior predictive properties of Bayesian VARs and factor models (among other techniques); see also the reply by Del Negro *et al.* (2007). Since factor models have also been shown to have relatively good forecasting performance, Giannone *et al.* (2006) argue that they provide a natural benchmark for DSGE validation. Boivin and Giannoni (2006) suggest that DSGE models should use factor model representations to aid measurement of the imperfectly observed theoretical variables.

5.3 *Consideration of Structural Breaks*

Since a number of papers in the Bayesian VAR literature argue that the Great Inflation and Great Moderation are distinct US regimes (see, for example, Cogley and Sargent, 2005; Sims and Zha, 2006), it is troubling that most DSGE models maintain the assumption of no structural change. Most existing DSGE models cannot handle expectations of parameter change. Reflecting the desire of researchers to replicate the business cycle characteristics of the recent US data, this area is already receiving a great deal of attention.

One strand of the research examines the impact of switches in policymaking regimes. For example, Lubik and Schorfheide (2004) and Lubik and Surico (2009) split the sample under consideration into sub-samples and consider DSGE parameters for each sub-sample. The forward-looking agents form expectations as if they did not expect the parameters to change. An alternative approach, influenced by the work of Svensson and Williams (unpublished data), uses minimal state variable solutions. Farmer *et al.* (2008) use this method to represent structural models with regime-switching VARs. Fernández-Villaverde and Rubio-Ramírez (2008) apply a particle filter to estimate a growth model with stochastic volatility; see also Justiniano and Primiceri (2008). A handy technical appendix provided by Fernández-Villaverde and Rubio-Ramírez (2008) reviews methods for estimating non-linear and non-normal general equilibrium models.

5.4 *Analysis of Historical Episodes*

Prescott (2002) makes the case for using the neoclassical growth model as a lens to analyze historical episodes. Although many recent papers examining modern business cycle behavior have utilized the Bayesian methods, most of the existing papers in this field do not. Given that analyses of historical episodes are often

based on a very small number of observations, it would seem that the Bayesian methods are particularly well suited to the task.

Some notable analyses of historical episodes utilize a deterministic growth model. Prominent examples include Cooley and Ohanian (1997), who analyze post World War II economic growth in the UK and the legacy of Keynes, the analysis of the Korean war by Ohanian (1997), and the accounts of the Great Depression from Cole and Ohanian (2002, 2004). Pensieroso (2007) surveys the causes of the Great Depression in the RBC literature. The survey encompasses deterministic models and discusses the macroeconomic implications in detail. Here, we focus more on distinguishing between the empirical methodologies.

Adopting a stochastic framework, Christiano *et al.* (2005) use the limited information estimation approach to match the impulse responses of a structural VAR to their growth model of the US Great Depression. Harrison and Weder (2006) and Weder (2006) provide further analysis of the same episode using calibrated DSGE models with sunspot equilibria.

Chari *et al.* (2002, 2007a) take a classical maximum-likelihood approach to estimating a DSGE model of the US Depression. Chari *et al.* (2007a) argue that their prototype DSGE model with time-varying efficiency, labor and investment wedges should be used as a complement to growth accounting to provide a starting point for analysis of historical episodes, i.e. as a precursor to a more complete DSGE analysis. Chari *et al.* (2007b) argue that structural VARs are ill-suited to this task; Christiano and Davis (2006) take the opposite view, drawing attention to model uncertainty issues in the classical estimation approach adopted by Chari *et al.* (2007a). Subsequent work in this area will doubtless extend to more formal treatments of model uncertainty in the analysis of historical episodes.

6. Conclusions

This review has examined the tools required by the reader of Romer (2006) to build modern DSGE models. Whereas the textbook treatment in Romer (2006) characterizes the RBC literature, this supplementary review has examined the computational issues commonly encountered by model builders, as well as providing insight into the future of DSGE research. The simple divisible labor RBC model has been used to illustrate a number of computational issues; code and US data have been provided to aid replication and further study. A number of popular research agendas have been described which, in addition to illustrating the computational subtleties of modern DSGE research, highlight the diverse methods for inference used by DSGE practitioners.

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Notes

1. The highly influential Lucas (1976) critique suggested that the empirical failures of modified IS-LM models, such as Mundell (1968) and Fleming (1962), stemmed from the absence of micro-foundations. Sims (1980) showed that the *ad hoc* nature of identification in their empirical counterparts made the parameter estimates unreliable.
2. The Christiano and Eichenbaum (1992) model has technology and government expenditure shocks; we abstract from the latter.
3. We are presuming that the shock process for technology takes the form given in equation (17).
4. King and Rebelo (1999) point to procyclical capacity utilization as a way of reconciling the one-shock model with the data. Long and Plosser (1983) offer an alternative way to build multi-shock models, exploiting disaggregate shocks. For simplicity, we do not follow this approach in our example. As noted earlier, most DSGE researchers prefer a multi-shock specification.
5. Juillard (1996) provides the MATLAB- and GAUSS-based DYNARE package, which automates first- and second-order log-linearization if required; see Table 1 for the DYNARE webpage address.

References

- Adolfson, M., Laséen, S., Lindé, J. and Villani, M. (2007) Evaluating an estimated new Keynesian small open economy model. *Journal of Economic Dynamics and Control* 32(8): 2690–2721.
- Altug, S. (1989) Time-to-build and aggregate fluctuations: some new evidence. *International Economic Review* 30(4): 889–920.
- An, S. and Schorfheide, F. (2007) Bayesian analysis of DSGE models. *Econometric Reviews* 26(2–4): 113–172.
- Anderson, G.S. (2006) Solving linear rational expectations models: a horse race. Working Paper 2006-26, Federal Reserve Bank Board of Governors.
- Anderson, G.S. and Moore, G. (1985) A linear algebraic procedure for solving linear perfect foresight models. *Economics Letters* 17: 247–252.
- Aruoba, S.B., Fernández-Villaverde, J. and Rubio-Ramírez, J.F. (2006) Comparing solution methods for dynamic equilibrium economies. *Journal of Economic Dynamics and Control* 30(12): 2477–2508.
- Bayoumi, T., with assistance from Laxton, D., Faruquee, H., Hunt, B., Karam, P., Lee, J., Rebucci, A. and Tchakarov, I. (2004) GEM: a new international macroeconomic model. Occasional Papers 239, International Monetary Fund.
- Bierens, H.J. (2007) Econometric analysis of linearized singular dynamic stochastic general equilibrium models. *Journal of Econometrics* 136(2): 595–627.
- Binder, M. and Pesaran, H.M. (1995) Multivariate rational expectations models and macroeconomic modeling: a review and some new results. In H.M. Pesaran and M. Wickens (eds), *Handbook of Applied Econometrics: Macroeconomics* (pp. 139–187). Oxford: Blackwell.
- Blanchard, O.J. and Fischer, S. (1989) *Lectures on Macroeconomics*. Cambridge, MA: MIT Press.
- Blanchard, O.J. and Kahn, C.M. (1980) The solution of linear difference models under rational expectations. *Econometrica* 48: 1305–1311.
- Boivin, J. and Giannoni, M. (2006) DSGE models in a data-rich environment. NBER Working Paper 12772.
- Brock, W.A. (1974) Money and growth: the case of long run perfect foresight. *International Economic Review* 15: 750–777.

- Brooks, S.P. and Roberts, G. (1999) Assessing convergence of Markov chain Monte Carlo algorithms. *Statistics and Computing* 8(4): 319–335.
- Brubakk, L., Husebø, T.A., Maih, J., Olsen, K. and Østnor, M. (2006) Finding NEMO: documentation of the Norwegian economy model. Norges Bank Staff Memo 2006/6.
- Burnside, C. (1998) Detrending and business cycle facts: a comment. *Journal of Monetary Economics* 41(3): 513–532.
- Campbell, J.Y. (1994) Inspecting the mechanism: an analytical approach to the stochastic growth model. *Journal of Monetary Economics* 33: 463–506.
- Canova, F. (1998a) Detrending and business cycle facts. *Journal of Monetary Economics* 41(3): 475–512.
- Canova, F. (1998b) Detrending and business cycle facts: a user's guide. *Journal of Monetary Economics* 41(3): 533–540.
- Canova, F. (2007) *Methods for Applied Macroeconomic Research*. Princeton, NJ: Princeton University Press.
- Canova, F. and Sala, L. (2009) Back to square one: identification issues in DSGE models. *Journal of Monetary Economics* 56(4): 431–449.
- Chari, V.V. and Kehoe, P.J. (1999) Optimal fiscal policy and monetary policy. In J.B. Taylor and M. Woodford (eds), *Handbook of Macroeconomics* (pp. 1671–1745). Amsterdam: Elsevier North Holland.
- Chari, V.V., Kehoe, P. and McGrattan, E.R. (2002) Accounting for the Great Depression. *American Economic Review* 92(2): 22–27.
- Chari, V.V., Kehoe, P. and McGrattan, E.R. (2007a) Business cycle accounting. *Econometrica* 75(3): 781–836.
- Chari, V.V., Kehoe, P. and McGrattan, E.R. (2007b) Are structural VARs with long-run restrictions useful in developing business cycle theory? Staff Report 364, Federal Reserve Bank of Minneapolis.
- Christiano, L.J. and Davis, J.M. (2006) Two flaws in business cycle accounting. NBER Working Paper 12647.
- Christiano, L.J. and Eichenbaum, M. (1992) Current real business cycle theories and aggregate labor market fluctuations. *American Economic Review* 82: 430–450.
- Christiano, L.J., Eichenbaum, M. and Evans, C.L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113(1): 1–45.
- Cogley, T. and Nason, J.M. (1995a) Effects of the Hodrick–Prescott filter on trend and difference stationary time series: implications for business cycle research. *Journal of Economic Dynamics and Control* 19: 253–278.
- Cogley, T. and Nason, J.M. (1995b) Output dynamics in real-business cycle models. *American Economic Review* 85(2): 492–511.
- Cogley, T. and Sargent, T.J. (2005) The conquest of US inflation: learning and robustness to model uncertainty. *Review of Economic Dynamics* 8(2): 528–563.
- Cole, H.L. and Ohanian, L.E. (2002) The U.S. and U.K. Great Depressions through the lens of neoclassical growth theory. *American Economic Review* 92(2): 28–32.
- Cole, H.L. and Ohanian, L.E. (2004) New deal policies and the persistence of the Great Depression: a general equilibrium analysis. *Journal of Political Economy* 112(4): 779–816.
- Collard, F. and Juillard, M. (2001) Accuracy of stochastic perturbation methods in the case of asset pricing models. *Journal of Economic Dynamics and Control* 25: 979–999.
- Cooley, T.F. (1995) *Frontiers of Business Cycle Research*. Princeton, NJ: Princeton University Press.
- Cooley, T.F. (1997) Calibrated models. *Oxford Review of Economic Policy* 13(3): 55–69.

- Cooley, T.F. and Ohanian, L.E. (1997) Post-war British economic growth and the legacy of Keynes. *Journal of Political Economy* 105: 439–472.
- Cooley, T.F. and Prescott, E.C. (1995) Economic growth and business cycles. In T.F. Cooley (ed.), *Frontiers of Business Cycle Research*. Princeton, NJ: Princeton University Press.
- Cowles, M.K. and Carlin, B.R. (1996) Markov chain Monte Carlo convergence diagnostics: a comparative review. *Journal of the American Statistical Association* 91(434): 883–904.
- DeJong, D.N., Ingram, B.F. and Whiteman, C.H. (1996) A Bayesian approach to calibration. *Journal of Business and Economic Statistics* 14: 1–9.
- DeJong, D.N., Ingram, B.F. and Whiteman, C.H. (2000) A Bayesian approach to dynamic macroeconomics. *Journal of Econometrics* 98: 203–223.
- Del Negro, M. and Schorfheide, F. (2003) Take your model bowling: forecasting with general equilibrium models. *Federal Reserve Bank of Atlanta Economic Review* 88(Fourth Quarter).
- Del Negro, M. and Schorfheide, F. (2004) Priors from general equilibrium models for VARs. *International Economic Review* 45(2): 643–673.
- Del Negro, M. and Schorfheide, F. (2006) How good is what you've got? DSGE-VAR as a toolkit for evaluating DSGE models. *Federal Reserve Bank of Atlanta Economic Review* 91(Second Quarter).
- Del Negro, M., Schorfheide, F., Smets, F. and Wouters, R. (2007) On the fit of new Keynesian models. *Journal of Business and Economic Statistics* 25(2): 143–162.
- Den Haan, W.J. and Marcet, A. (1994) Accuracy in simulations. *Review of Economic Studies* 61(1): 3–18.
- Edge, R., Laubach, T. and Williams, J.C. (2009) Welfare-maximizing monetary policy under parameter uncertainty. *Journal of Applied Econometrics* 24(7).
- Farmer, R.E.A., Waggoner, D.F. and Zha, T. (2008) Minimal state variable solution to Markov-switching rational expectations. Federal Reserve Bank of Atlanta Working Paper 2008-23.
- Fernández-Villaverde, J. and Rubio-Ramírez, J.F. (2006) The research agenda: Jesús Fernández-Villaverde and Juan F. Rubio-Ramírez on estimation of DSGE models. *Economic Dynamics Newsletter* 8(1), November.
- Fernández-Villaverde, J. and Rubio-Ramírez, J.F. (2008) Estimating macroeconomic models: a likelihood approach. *Review of Economic Studies* 74: 1059–1087.
- Fernández-Villaverde, J., Rubio-Ramírez, J.F., Sargent, T.J. and Watson, M.W. (2007) ABCs (and Ds) of understanding VARs. *American Economic Review* 97(3): 1021–1026.
- Fleming, M.J. (1962) Domestic financial policies under fixed and under floating exchange rates. *International Monetary Fund Staff Papers* 9: 369–379.
- Frisch, R. (1933) Propagation problems and impulse problems in dynamic economics. In *Economic Essays in Honor of Gustav Cassel*, 2nd edn (pp. 171–206). London: Allen and Unwin.
- Fukač, M. and Pagan, A. (2009) Limited information estimation and evaluation of DSGE models. *Journal of Applied Econometrics* 24(7).
- Gelman, A., Carlin, J.B., Stern, H.S. and Rubin, D.B. (2004) *Bayesian Data Analysis*, 2nd edn. Boca Raton, FL: Chapman and Hall/CRC.
- George, D.A.R. and Oxley, L. (1985). Structural stability and model design. *Economic Modelling* 2: 307–316.
- George, D.A.R. and Oxley, L. (1994). Linear saddle point dynamics 'on their head'. *European Journal of Political Economy* 10: 389–400.
- Geweke, J. (2009) *Complete and Incomplete Econometric Models* (pp. 66–72). Princeton, NJ: Princeton University Press.

- Giannone, D., Reichlin, L. and Sala, L. (2006) VARs, factor models and the empirical validation of equilibrium business cycle models. *Journal of Econometrics* 132(1): 257–279.
- Hall, A., Inoue, A., Nason, J.M. and Rossi, B. (2007) Information criteria for impulse response function matching estimation of DSGE models. Working Paper 2007-04, Duke University.
- Hansen, G.D., (1985) Indivisible labor and the business cycle. *Journal of Monetary Economics* 16: 309–327.
- Hansen, L.P. and Heckman J.J. (1996) The empirical foundations of calibration. *Journal of Economic Perspectives* 10: 87–104.
- Hansen, L.P. and Sargent, T.J. (1980) Formulating and estimating dynamic linear rational expectations models. *Journal of Economic Dynamics and Control* 2(1): 7–46.
- Harrison, S.G. and Weder, M. (2006) Did sunspot forces cause the Great Depression? *Journal of Monetary Economics* 53: 1327–1339.
- Harvey, A.C. and Jaeger, A. (1993) Detrending, stylized facts and the business cycle. *Journal of Applied Econometrics* 8: 231–247.
- Hodrick, R.J. and Prescott, E.C. (1997) Post-war U.S. business cycles: an empirical investigation. *Journal of Money, Credit and Banking* 29(1): 1–16.
- Ingram, B.F. and Whiteman, C.H. (1994). Supplanting the ‘Minnesota’ prior: forecasting macroeconomic time series using real business cycle model priors. *Journal of Monetary Economics* 34(3): 497–510.
- Ireland, P. (2001) Sticky-price models of the business cycle: specification and stability. *Journal of Monetary Economics* 45: 3–18.
- Juillard, M. (1996) DYNARE: a program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm. Couverture Orange 9602, CEPREMAP.
- Justiniano, A. and Preston, B. (2009) Monetary policy and uncertainty in an empirical small open economy model. *Journal of Applied Econometrics* 24(7).
- Justiniano, A. and Primiceri, G.E. (2008) The time varying volatility of macroeconomic fluctuations. *American Economic Review* 98(3): 604–641.
- Kilian, L. (2007) Comment on Del Negro, Schorfheide, Smets and Wouters. *Journal of Business and Economic Statistics* 25(2): 156–159.
- Kim, K. and Pagan, A. (1995) The econometric analysis of calibrated macroeconomic models. In M.H. Pesaran and M. Wickens (eds), *The Handbook of Applied Econometrics: Macroeconomics* (pp. 356–390). Oxford: Blackwell Publishers.
- King, R.G. and Rebelo, S.T. (1999) Resuscitating real business cycles. In J.B. Taylor and M. Woodford (eds), *Handbook of Macroeconomics* (Vol. 1B, pp. 927–1007). Amsterdam: Elsevier Science North-Holland.
- King, R.G., Plosser, C.I. and Rebelo, S.T. (1988) Production, growth and business cycles: I. The basic neoclassical model. *Journal of Monetary Economics* 21(2–3): 195–232.
- Koop, G. (2003) *Bayesian Econometrics*. Chichester: Wiley.
- Kremer, J., Giovanni, L., von Thadden, L. and Werner, T. (2006) Dynamic stochastic general equilibrium models as a tool for policy analysis. *CESifo Economic Studies* 52(4): 640–665.
- Kydland, F.E. and Prescott, E.C. (1982) Time to build and aggregate fluctuations. *Econometrica* 50: 1345–1370.
- Kydland, F.E. and Prescott, E.C. (1995) The econometrics of the general equilibrium approach to business cycles. In K.D. Hoover (ed.), *Macroeconometrics, Developments, Tensions and Prospects*. No. 46 in Recent Economic Thought. Boston, MA: Kluwer Academic.
- Lagos, R. and Wright, R. (2005) A unified framework for monetary theory and policy analysis. *Journal of Political Economy* 113(3): 463–484.

- Lees, K., Matheson, T. and Smith, C. (2007) Open economy DSGE-VAR forecasting and policy analysis – head to head with the RBNZ published forecasts. Reserve Bank of New Zealand Discussion Paper DP2007/01.
- Levin, A.T., Onatski, A., Williams, J.C. and Williams, N. (2005) Monetary policy under uncertainty in micro-founded macroeconomic models. In M. Gertler and K. Rogoff (eds), *National Bureau of Economic Research Macroeconomics Annual* (pp. 229–287). Cambridge, MA: MIT Press.
- Ljungqvist, L. and Sargent, T.J. (2004) *Recursive Macroeconomic Theory*, 2nd edn. Cambridge, MA: MIT Press.
- Long, J.B. and Plosser, C.I. (1983) Real business cycles. *Journal of Political Economy* 91(1): 39–69.
- Lubik, T.A. and Krause, M. (2006) The cyclical upgrading of labor and on-the-job search. *Labour Economics* 13(4): 459–477.
- Lubik, T.A. and Krause, M. (2007) The (ir)relevance of real wage rigidity in the new Keynesian model with search frictions. *Journal of Monetary Economics* 54(3): 706–727.
- Lubik, T.A. and Schorfheide, F. (2004) Testing for indeterminacy: an application to US monetary policy. *American Economic Review* 94(1): 190–217.
- Lubik, T.A. and Schorfheide, F. (2005) A Bayesian look at new open economy macroeconomics. In M. Gertler and K. Rogoff (eds), *National Bureau of Economic Research Macroeconomics Annual* (pp. 313–366). Cambridge, MA: MIT Press.
- Lubik, T.A. and Schorfheide, F. (2007) Do central banks respond to real exchange rate fluctuations? A structural investigation. *Journal of Monetary Economics* 54(4): 1069–1087.
- Lubik, T.A. and Surico, P. (2009) The Lucas critique and the stability of empirical models. *Journal of Applied Econometrics* 24(7).
- Lucas, R.E., Jr. (1972) Expectations and the neutrality of money. *Journal of Economic Theory* 4: 103–124.
- Lucas, R.E., Jr. (1973) Some international evidence on output–inflation tradeoffs. *American Economic Review* 63: 326–334.
- Lucas, R.E., Jr. (1976) Econometric policy evaluation: a critique. *Carnegie Rochester Conference Series on Public Policy* 1: 19–46.
- Lucas, R.E., Jr. and Rapping, L. (1969) Real wages, employment and inflation. *Journal of Political Economy* 77: 721–754.
- Matheson, T. (2006) Assessing the fit of small open economy DSGEs. Discussion Paper DP2006/11, Reserve Bank of New Zealand.
- McCallum, B.T. (1989) *Monetary Economics, Theory and Policy*. New York: Macmillan.
- McGrattan, E.R. (1994). The macroeconomic effects of distortionary taxation. *Journal of Monetary Economics* 33(3): 573–601.
- Mundell, R. (1968) *International Economics*. New York: Macmillan.
- Murchison, S. and Rennison, A. (2006) ToTEM: the Bank of Canada's new quarterly projection model. Technical Report 97, Bank of Canada.
- Nason, J.M. and Rogers, J.H. (2006) The present-value model of the current account has been rejected: round up the usual suspects. *Journal of International Economics* 68(1): 159–187.
- Obstfeld, M. and Rogoff, K. (1997) *Foundations of International Economics*. Cambridge, MA: MIT Press.
- Ohanian, L.E. (1997) The macroeconomic effects of war finance in the United States: World War II and the Korean war. *American Economic Review* 87(1): 23–40.
- Otrok, C. (2001) On measuring the welfare cost of business cycles. *Journal of Monetary Economics* 47(1): 61–92.
- Pensieroso, L. (2007) Real business cycle models of the Great Depression: a critical survey. *Journal of Economic Surveys* 21(1): 110–142.

- Poirier, D. (1998) Revising beliefs in nonidentified models. *Econometric Theory* 14(4): 483–509.
- Prescott, E.C. (1986) Theory ahead of business-cycle measurement. *Carnegie Rochester Conference Series on Public Policy* 25: 11–44.
- Prescott, E.C. (2002) Prosperity and depression. *American Economic Review* 92: 1–15.
- Rabanal, P. and Rubio-Ramírez, J.F. (2005) Comparing New Keynesian models of the business cycle: a Bayesian approach. *Journal of Monetary Economics* 52(6): 1152–1166.
- Rebelo, S. (2005) Real business cycle models: past, present and future. *Scandinavian Journal of Economics* 107: 217–238.
- Robertson, D.H. (1915) *A Study of Industrial Fluctuation*. London: King and Son; Reprinted 1948.
- Rogerson, R. (1988) Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics* 21: 3–16.
- Romer, D. (2006) *Advanced Macroeconomics*. New York: McGraw-Hill.
- Rotemberg, J.J. and Woodford, M. (1997) An optimization-based econometric framework for the evaluation of monetary policy. In B.S. Bernanke and J.J. Rotemberg (eds), *National Bureau of Economic Research Macroeconomics Annual* (pp. 297–346). Cambridge, MA: MIT Press.
- Ruge-Murcia, F.J. (2007) Methods to estimate dynamic stochastic general equilibrium models. *Journal of Economic Dynamics and Control* 31(8): 2599–2636.
- Sargent, T.J. (1989) Two models of measurements and the investment accelerator. *Journal of Political Economy* 97(2): 251–287.
- Schmitt-Grohé, S. and Uribe, M. (2004a) Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory* 114: 198–230.
- Schmitt-Grohé, S. and Uribe, M. (2004b) Optimal fiscal and monetary policy under imperfect competition. *Journal of Macroeconomics* 26: 183–209.
- Schorfheide, F. (2000) Loss function-based evaluation of DSGE models. *Journal of Applied Econometrics* 15: 645–670.
- Sims, C. (1980) Macroeconomics and reality. *Econometrica* 48: 1–48.
- Sims, C. (1996) Macroeconomics and methodology. *Journal of Economic Perspectives* 10: 105–120.
- Sims, C. (2002) Solving linear rational expectations models. *Computational Economics* 20: 1–20.
- Sims, C. (2007a) Comment on Del Negro, Schorfheide, Smets and Wouters. *Journal of Business and Economic Statistics* 25(2): 152–154.
- Sims, C. (2007b) Improving monetary policy models. *Journal of Economic Dynamics and Control* 32(8): 2460–2475.
- Sims, C. and Zha, T. (2006) Were there regime switches in U.S. monetary policy? *American Economic Review* 96(1): 54–81.
- Smets, F. and Wouters, R. (2003) An estimated dynamic stochastic general equilibrium model of the Euro Area. *Journal of the European Economic Association* 1: 1123–1175.
- Smets, F. and Wouters, R. (2007) Shocks and frictions in U.S. business cycles: a Bayesian DSGE approach. *American Economic Review* 97(3): 586–606.
- Stokey, N. and Lucas, R.E., Jr, with Prescott, E.C. (1989) *Recursive Methods in Economic Dynamics*. Cambridge, MA: Harvard University Press.
- Summers, L. (1986) Some sceptical observations on real business cycle theory. *Federal Reserve Bank of Minneapolis Quarterly Review* 10(4): 23–27.
- Taylor, J.B. (1993) Discretion versus policy rules in practice. *Carnegie-Rochester Series on Public Policy* 39: 195–214.

- Uhlig, H. (1999) A toolkit for analyzing non-linear dynamic stochastic models easily. In R. Marimón and A. Scott (eds), *Computational Methods for the Study of Dynamic Economies* (pp. 30–61). Oxford: Oxford University Press.
- Watson, M.W. (1993) Measures of fit for calibrated models. *Journal of Political Economy* 101(6): 1011–1041.
- Weder, M. (2006) The role of preference shocks and capital utilization in the Great Depression. *International Economic Review* 47: 1247–1268.