NON-RESONANT RADIATION
FROM \(^{14}\text{N}(p,\gamma)^{15}\).
PREFACE

The work described in this thesis is concerned with an investigation of non-resonant radiation from the $^N^{14}(p, \gamma)^{15}O$ reaction. These results, together with a theoretical model, have been used to extrapolate the reaction cross-section to stellar energies.

The investigation was undertaken jointly by the author and Dr. D.F. Hebbard who was responsible for suggesting the project. The work has been carried out entirely in the Department of Nuclear Physics at the Australian National University and covers the period September 1960 to May 1963. The author claims to have taken an active part in all aspects of the investigation reported.

No part of this work has been submitted for a degree at any other University.

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INTRODUCTION

It is of considerable importance in astrophysics to obtain reliable information on proton cross sections in light nuclei to enable worthwhile calculations to be made concerning stellar evolution processes.

The CNO cycle was first described by Bethe (Be39) and later by Burbidge et al. (Bu 57). This cycle in which the nuclei of carbon, nitrogen and oxygen act as agents in the conversion of hydrogen into helium, has been suggested as a possible energy source in the hotter stars, and has been the subject of a number of investigations.

The cross sections for the nuclear reactions which constitute the chain have been measured down to proton energies of about 100 keV. Below this energy the cross sections are so low, due to the exponential decrease in the Gamow penetration factor with energy, that reliable data has not been obtained experimentally. Information in the region of interest around 25 keV has been obtained by an extrapolation of the higher energy data.

These extrapolations show that at 25 keV $^{14}\text{N} (p,\gamma)^{15}\text{O}$ has the lowest cross section for any of the reactions involved in the CNO cycle. This reaction then largely controls the rate of energy generation as described by Fowler (Fo 54), and Caughlan and Fowler (Ca 62). For this reason it is of some importance
to obtain an accurate estimate of this cross section in the stellar energy region.

Cross section measurements for the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction by Lamb and Hester (La 57) and theoretical astrophysical calculations by Morton (Mo 59) indicate that this reaction is governed by a non-resonant process at low energies.

Recently Christy and Duck (Ch 61) have proposed a model which allows reasonably accurate direct-capture $(p, \gamma)$ cross sections to be calculated.

Hebbard and Povh (He 59) found that the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction showed evidence for a direct-capture process, having properties satisfying the requirements of Christy and Duck's theory. Their calculation of the theoretical cross section showed good agreement with experiment at 1 MeV, and on this basis it appeared worthwhile to study the reaction in more detail.

The experimental investigation which will be described in the chapters to follow can be divided into two distinct parts.

The first part was aimed at testing more conclusively for experimental evidence of a direct-capture process in $^{14}\text{N}(p, \gamma)^{15}\text{O}$. The second part, having shown the reaction to have direct-capture characteristics, was aimed at identifying those gamma rays which were contributing to the non-resonant process.
With the help of the model an attempt has been made to estimate their effect on the integrated \((p, \gamma)\) cross section as the proton energy was decreased. In this way it is hoped to obtain a more reliable extrapolation of the cross section down to stellar energies.
CHAPTER I

THE DIRECT-CAPTURE \((p, \gamma)\) PROCESS IN LIGHT NUCLEI

1.1 General Features

When certain light nuclei are bombarded with low energy protons up to a few MeV, the gamma ray yield as a function of energy often shows a slowly rising non-resonant background underlying the well defined resonance structure. The mechanism for the production of this background radiation is not well understood but it appears fairly well established that it often results from a direct-capture process involving an interaction between the incoming proton and the core of the target nucleus. It is this non-resonant process which is of particular interest here and the work to be described is aimed at trying to understand the process in more detail.

One of the first cases in which the non-resonant gamma radiation was observed was the \(D(p, \gamma)He^3\) reaction (Fowler, Lauritsen, and Tollestrup (Fo 49)). This is a particularly useful reaction to study since there are no known \((p, \gamma)\) resonances associated with the formation of \(He^3\). One might therefore reasonably hope to observe any gamma radiation which is the result of a direct-capture process.

Fowler et al. bombarded a heavy ice target with protons of energy up to 1.4 MeV and observed a weak gamma radiation of
energy about 6 MeV, identified as a radiative transition to the $^1$\textsuperscript{+} ground state of He$^3$. When allowance was made for the relatively poor angular resolution of their gamma ray detector, the radiation was observed to follow closely a $\sin^2 \Theta$ angular distribution, $\Theta$ being the angle of emission of the radiation with respect to the incoming proton direction. The integrated cross section for the $(p,\gamma)$ process was found to increase slowly with increasing proton energy being about 7 $\mu$-barns at 1 MeV. These results were later confirmed with more precise measurements by Griffiths and Warren (Gr 53), who obtained an angular distribution of the form $0.05 + \sin^2 \Theta$ at 1 MeV.

The interpretation of this reaction by Fowler et al. (Fo 49), Griffiths and Warren (Gr 53), and Wilkinson (Wi 52), was that of a direct interaction process involving the radiative capture of the incoming proton by the deuteron core. It was proposed that this occurred without appreciable interaction with the nuclear forces of the core, so that no change in the spin directions of the core or proton resulted, and no compound nuclear states were formed in He$^3$.

If one assumes the capture of an incident p-wave proton into an s-wave orbit around the $^1$\textsuperscript{+} deuteron core, electric dipole radiation will be emitted having a pure $\sin^2 \Theta$ distribution. This is only true provided that the $z$-component, $S_z$ of the channel spin $S$, remains fixed before and after the radiative
capture, i.e. a $\Delta S_z = 0$ transition.

When nuclear forces are present, as in the formation of a compound nucleus, $S_z$ is no longer fixed. Spin-orbit coupling forces may cause spin flips, and transitions for any $\Delta S_z$ can take place from zero up to some maximum value determined by the selection rules. These more complex transitions lead to less pronounced angular distributions. For example if the $D(p, \gamma)He^3$ reaction had proceeded by compound nucleus formation one would expect a distribution made up of contributions from $\sin^2 \Theta$ ($\Delta S_z = 0$), and $1 + \cos^2 \Theta$ ($\Delta S_z = \pm 1$) components. The purity of the $\sin^2 \Theta$ distribution for this reaction indicates that compound nucleus and other mixing effects must be small.

It is worth noting that such pure angular distributions are found when $\Delta S_z = 0$ is the only allowed transition. This occurs in direct-capture processes in which the final state is an s-state. The p to s transition then constitutes a rather special case in which the angular distribution of the gamma radiation is one of the most anisotropic that can be observed. A number of other light nuclei have been observed to show direct-capture characteristics when bombarded with low energy protons.

The $T(p, \gamma)He^4$ reaction has been the subject of considerable conjecture about the possible existence of a broad excited state in $He^4$. Argo et al. (Ar 50), first observed this reaction from $0.7 - 2.5$ MeV and attributed a rising intensity and a broad
maximum appearing in the \((p, y)\) excitation function as evidence for a level in \(He^4\) around 20 MeV excitation. The results of Perry and Bame (Pe 55) and Vlasov et al. (VL 55) on the other hand, favour a direct-capture process since they find no evidence for any excited states in \(He^4\). Furthermore, the ground state radiation for this reaction is observed to have a predominant \(\sin^2 \Theta\) angular distribution characteristic of a \(p\) to \(s\) transition. The values obtained for the \((p, y)\) cross sections are in good agreement with earlier theoretical calculations for the reaction based on the inverse \(He^4(y, p)T\) reaction, by Flowers and Mandl (FL 51). These latter workers favour a direct-capture process for the \(T(p, y)He^3\) reaction. Further evidence for the absence of excited states in \(He^4\) from photo disintegration and proton elastic scattering experiments with \(He^4\), by Fuller (Fu 54) and Benveniste and Cork (Be 53), appears to support the idea of a direct-capture process.

Warren et al. (Wa 54) using scintillation counters have observed non-resonant radiation following proton capture in \(O^{16}\). They found that about 90\% of the capture radiation is to the \(1^+\) first excited state in \(F^{17}\), the remainder being a weak transition to the \(5^+\) ground state. The low cross section of 6 \(\mu\) barns measured at 1.35 MeV and the \(\sin^2 \Theta\) distribution for the main capture radiation led Warren et al. to the conclusion that the process was one of direct-capture. They
interpreted the reaction as proceeding by the capture of p-wave protons to form the $2S_1$ first excited state in $F^{17}$.

Recently Tanner (Ta 59) has measured the cross section for proton capture in $O^{16}$ at a number of energies from 274 keV to 616 keV by detecting the positron activity of the $F^{17}$, and finds a smooth variation in the cross section with energy. This smooth cross section variation has been observed up to a proton energy of 3 MeV by Laubenstein and Laubenstein (La 51).

The cross section measurements for the $O^{16}(p,\gamma)F^{17}$ reaction are in good agreement with the theoretical calculations of Christy and Duck (Ch 61) using their direct-capture model. This model will be considered in more detail later in this chapter. Tanner (Ta 59) has also studied the $Ne^{20}(p,\gamma)Na^{21}$ reaction by activation measurements. On theoretical grounds this reaction is expected to show considerable direct-capture to the $\frac{1}{2}^+$ level in $Na^{21}$ at 2.42 MeV excitation, which lies only 26 keV below the proton threshold, and has a large proton reduced width.

Cross sections were measured at three proton energies from 1042 keV - 1136 keV giving cross sections of about 1 $\mu$ barn. The smaller cross section and the presence of positron activity from the $O^{16}(p,\gamma)F^{17}$ reaction which has a similar half life to the $Na^{21}$, prevented lower energy measurements being carried out. Pixley (Pi 57a) has reported a cross section measurement for the $Ne^{20}(p,\gamma)Na^{21}$ reaction at 400 keV as $(2.7 \pm 1.0) \times 10^{-2}$ $\mu$ barns.
These results are all in reasonably good agreement with Christy and Duck's calculations for this reaction, indicating support for a direct-capture process.

No angular distribution measurements have been reported for the capture radiation to the $1^+$ level in Na$^{21}$. This would be expected once again to show a $\sin^2 \Theta$ dependence, corresponding to the capture of incident p-wave protons into a $2S_\frac{1}{2}$ orbit around the Ne$^{20}$ core.

Kavanagh (Ka 60) has measured the Be$^7(p,\gamma)$B$^8$ cross section at 1 MeV. This reaction is interpreted as direct-capture of s-wave protons into a $P_{3/2}$ orbit of B$^8$. The direct-capture theory agrees within a factor of two with Kavanagh's results. The angular distribution was not measured but this would be predicted to be isotropic from the s-wave capture.

We can conclude this section by summarizing the essential features which characterize a non-resonant $(p,\gamma)$ process at low energies.

i) The process of non-resonant (direct) capture takes place at distances greater than the range of nuclear forces so that no appreciable spin-orbit coupling occurs, and all spin directions remain unchanged.

ii) The nuclear interior plays no essential role in the process.
iii) The total \((p, \gamma)\) cross section is small being typically a few \(\mu\) barns at 1 MeV, and increasing relatively slowly with proton energy. The energy dependence can be approximated over a limited energy range by a Gamow expression of the form
\[
\sigma = SE^{-1} \exp (-2\pi\eta),
\]
where \(S\) is a constant, \(E\) is the centre of mass proton energy, \(\eta = z_1 z_2 e^2/\hbar v\) with \(z_1, z_2\) the atomic numbers of the interacting nuclei, and \(v\) the speed of relative motion.

iv) A large proton reduced width, and a small proton binding energy for the capturing state is important in enhancing the direct-capture process.

v) The most favoured capturing processes correspond to \(s \to p\), and \(p \to s\) transitions with the emission of electric dipole radiation having isotropic, and \(\sin^2 \Theta\) angular distributions respectively.

1.2 The Direct-Capture Model of Christy and Duck.

In this section we describe the direct-capture model proposed by Christy and Duck (Ch 61) to account for the production of non-resonant radiation in a \((p, \gamma)\) reaction. We make no attempt to present the theoretical arguments in detail. These can be found by reference to the original paper. We describe features of the model, and in particular examine the
final expressions which are of importance in the calculation of the cross section.

The direct-capture model makes the basic assumption that the proton capture takes place at distances greater than the nuclear radius of the target, so that nuclear forces do not enter into the problem explicitly.

The reaction is assumed to proceed along the lines described in the previous section, that is to say, capture of an incoming proton in an orbit around an inert target core, the channel spin being fixed, but a change in momentum occurs corresponding to the emission of a gamma ray quantum.

For a proton of mass $M_1$, charge $z_1$ captured by a target of mass $M_2$ and charge $z_2$, the capture cross section for emission of electric dipole radiation of energy $E_y$ and polarization $m$ is,

$$
\sigma_{1m} = \frac{16\pi}{9} \left( \frac{E_y}{\hbar c} \right)^3 \frac{1}{\hbar v} |Q_{1m}|^2
$$

(1.1)

where $v$ is the speed of relative motion and the integrated cross section

$$
\sigma = \sum_m \sigma_{1m}.
$$

If $(r, \Theta, \phi)$ represents the relative co-ordinates of the proton and target nucleus then the electric dipole matrix element between the initial and final states takes the form

$$
Q_{1m} = \frac{\hbar}{M_1 M_2} \left( \frac{z_1}{M_1} - \frac{z_2}{M_2} \right) \int r Y_{1m}^*(\Theta, \phi) I_{1m}^*(r, \Theta, \phi) I_1(r, \Theta, \phi) d\Omega
$$

(1.2)
The cross section is determined by evaluating the matrix element over the extranuclear region $dT$. In this expression $Y_{1m}$ represents the electric dipole operator for the emission of EI radiation of polarization $m$, and $I_i$, $I_f$ are wave functions for the initial and final states of proton plus target core expressed in terms of channel spin states.

The angular integral may be separated out from the matrix element and evaluated with the aid of Racah algebra. The radial integral which remains can be written in the form,

$$R_{1 f i} = \int r^2 dr \ g_i^*(r) \ g_i(r)/k$$

$k$ being the wave number of the incident proton.

For the initial radial wave function $g_i(r)$ corresponding to an unbound proton state, Christy and Duck use Coulomb Functions with the inclusion of a nuclear phase shift $\delta_{J_1 i}$. Here $J_1$ refers to the spin of the initial state formed by vector coupling the incoming proton orbital angular momentum $\ell_1$ with the channel spin:

$$g_i(r) = \mathcal{R}_\ell_1(\eta, \varphi) + \left[ G_\ell_1(\eta, \varphi) + i F_\ell_1(\eta, \varphi) \right] e^{i \delta_{J_1 i}} \sin S_{J_1}. \quad (1.4)$$

with parameters, $\eta = z_1 z_2 e^2/\hbar v$, and $\varphi = kr$.

The final radial wave function $g_f(r)$, corresponding to a bound state for the proton having angular momentum $\ell_2$, is
written in terms of the Whittaker function $W_{\alpha \ell_2}(Kr)$.

$$g_f(r) = W_{\alpha \ell_2}(Kr)/r$$

The parameters are defined by the equations,

$$\alpha = z_1 z_2 e^{2\mu/k^2} K,$$

$$K = (2\mu E_B/h^2)^{\frac{1}{2}}$$

where $E_B$ is the binding energy of the proton in the final state and $\mu$ the reduced mass of the system.

The final expression for the integrated direct-capture cross section in terms of the radial integral becomes (equation 25 of Christy and Duck).

$$\sigma_1 = \frac{32\pi}{3} \left( \frac{E_v}{\hbar c} \right)^3 \frac{a^2}{\hbar c} \cdot \frac{c}{v} \cdot \mu^2 \left( \frac{z_1}{M_1} - \frac{z_2}{M_2} \right)^2 <\ell_1, \ell_2> \Theta^2 \frac{|R_{1f1}|^2}{W^2(Kr_n).r_n} \quad (1.5)$$

where $<\ell_1, \ell_2>$ is taken to mean the larger of $\ell_1$ and $\ell_2$, and $\Theta^2$ is the reduced proton width for the final state.

It should be noted that this expression is only valid for low energy capture-gamma rays such that $\lambda/2\pi \gg \Delta r$, where $\lambda$ is the wave length of the radiation and $\Delta r$ the extent of the radial integration. This condition will ensure that the vector potential of the radiation is essentially constant over the sphere of integration. Failure to satisfy this condition
implies that one must take into account other multipolarities in the radiation.

The cross section as a function of proton bombarding energy is largely contained in the term $\frac{1}{v}$, and the terms $\frac{1}{k}$ and the regular Coulomb function $F_{\nu}^1(\eta, \varphi)$ appearing in the expression for $R^{IF}$. $F_{\nu}^1(\eta, \varphi)$ which varies approximately as $\varphi \exp (-\pi \eta)$ for $\eta > \varphi$ is the most important term, and accounts for the rapid fall in cross section with decreasing proton energy. To enable reliable cross section extrapolations to be made to stellar energies, it is convenient to remove the known energy dependent factors and work in terms of the cross section factor $S(E)$. This changes only slowly with the centre of mass proton energy $E$ and is defined as,

$$S(E) = E \exp (2\pi \eta) \sigma_1$$ (1.6)

The usual units for this quantity are keV.barns. It is this term rather than the cross section which we will be concerned with here.

Christy and Duck have developed analytic expression for approximating the radial integral in (1.5) for a number of cases of interest. In the appendix we describe a computer programme designed to calculate these integrals more accurately. These results show that at distances greater than the nuclear radius the Coulomb function rises initially faster than the corresponding
fall in the Whittaker function. As a result the radial integrand rises fairly rapidly to a maximum falling more slowly to zero and thereafter undergoing small long period oscillations of decreasing magnitude about the zero position.

In Figure 1.1 we illustrate the typical form of the and Whittaker/Regular Coulomb functions which give rise to the radial integrand. As an example we have considered these functions for the direct-capture process to the 6.79 MeV level in $^{15}$O for a proton energy of 715 keV.

The main peak in the radial integrand corresponds to the wave mechanical prediction for the most probable region for capture of the incoming proton, and the area under the peak then gives a measure of the direct-capture contribution to the $(p,\gamma)$ process. It is when this contribution is large compared to the contribution from the nuclear interior that we speak of the process as being one of direct-capture.

For extrapolation purposes the variation of the cross section factor with proton energy is of interest and this depends on the relative changes in the $E^3\gamma$ and $|R_{1f1}|^2$ terms in (1.5). Calculations show that for a small proton binding energy $E_B$ in the final state, as well as the probability of direct-capture being large, there is often a considerable increase in $S(E)$ at lower energies. This appears to be due to the peak of the radial integrand moving out further and giving
Fig. 1. Representative Whittaker & Regular Coulomb functions leading to the radial integrand for the $^1_{14}(py)^{15}$ direct-capture to the 6.79 MeV level in $^{15}_0$. 

$N^{14}_p$, $E_p(C.M.) = 715$ KeV.

$\eta = 1.259$, $\lambda = 1.493$, $\kappa = 0.1506$,

$\ell_1 = 1$, $\ell_2 = 2$.
rise to a larger direct-capture contribution, which over compensates the fall in $E_\gamma^3$.

This is the case for $^{20}\text{Ne} \, ^{(p,\gamma)} \, ^{21}\text{Na}$ in which $E_B = 26$ keV, calculation showing an increase by a factor of four in $S(E)$ in going from 1 MeV to 25 keV.

The importance of a small proton binding energy also shows up in the $^{16}\text{O} \, ^{(p,\gamma)} \, ^{17}\text{F}$ reaction, the main direct-capture transition in theory and experiment being to the more weakly bound first excited state in $^{17}\text{F}$ than to the ground state.

As we have pointed out already the theory has been quite successful in predicting cross section factors for the $^{7}\text{Be} \, ^{(p,\gamma)} \, ^{8}\text{B}$, $^{16}\text{O} \, ^{(p,\gamma)} \, ^{17}\text{F}$, and $^{20}\text{Ne} \, ^{(p,\gamma)} \, ^{21}\text{Na}$. Satisfactory agreement with experiment has also been obtained for the alpha particle reactions, $^3\text{He} \, ^{(\alpha,\gamma)} \, ^{7}\text{Be}$, and $^3\text{H} \, ^{(\alpha,\gamma)} \, ^{7}\text{Li}$.

Christy and Duck point out that the largest uncertainty in their calculations are the values of the reduced widths, $\Theta^2$. To check the accuracy of their approximations used to evaluate the radial integral, the three $(p,\gamma)$ reactions above have been recalculated more accurately. These results will be found in the appendix and agree substantially with those of Christy and Duck showing that their results are quite reliable.
CHAPTER II

THE $^{14}\text{N}(p,\gamma)^{15}\text{O}$ REACTION AND EVIDENCE FOR A DIRECT-CAPTURE PROCESS

We consider previous work carried out on the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction at low proton bombarding energies. These include cross section measurements as well as investigations of the gamma radiation from both resonant and non-resonant processes.

2.1 Cross Section Measurements

The cross sections have been measured by counting the 120 sec $^{15}\text{O}$ positron activity.

Woodbury, Hall and Fowler (Wo 49) first measured the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ cross section using a thick nitrogen target bombarded with 128 keV protons. They reported a cross section of $7 \times 10^{-10}$ barns at 128 keV, this being considered a lower limit because of possible loss of $^{15}\text{O}$ from the target.

Duncan and Perry (Du 51) carried out systematic measurements of the cross section in the energy range $0.25 - 2.6$ MeV. They reported a number of resonances in this region superimposed on a rising background which was in evidence from 500 keV upwards. The background was attributed to the capture of s-wave protons forming a level resonant at about 2.6 MeV bombarding energy. In order to explain the contribution down in the region of 500 keV it was necessary to predict a large width of 1.2 MeV.
for this level. A second broad level of width 100 keV was also reported in the region of 700 keV, but this may have been due to radiation from the sodium content in the NaNO$_2$ target, used for these particular runs. The two lowest sharp resonances were reported at 280 keV and 1060 keV bombarding energy, and the measured cross section at 1 MeV was $4 \times 10^{-6}$ barns.

Lamb and Hester (La 57) using a 25 m.a. proton beam from the Livermore high current ion injector and thick targets of tantalum nitride, were able to measure cross sections in the energy range 100 - 135 keV. These varied from $(8.5 \pm 3.7) \times 10^{-12}$ barns at 100 keV to $(1.4 \pm 0.3) \times 10^{-10}$ barns at 135 keV, and gave an approximately constant cross section factor over this limited energy range. These results gave for the cross section factor at 25 keV, $S_0 = 2.7 \pm 0.2$ keV barns.

Pixley (Pi 57) has made similar cross section measurements in the ranges 220 to 285 keV and 400 to 650 keV in an attempt to link the data of Duncan and Perry with that taken at the lower energies. He finds a narrow resonance at 278 keV of width 1.7 keV superimposed on a non-resonant background which rises slowly with energy. The cross section was measured as $1.44 \times 10^{-7}$ barns at 450 keV rising to $4.74 \times 10^{-7}$ barns at 650 keV. These results are in satisfactory agreement with those of Duncan and Perry. When extrapolated they show good agreement with the results of Lamb and Hester, yielding a value
So = 2.8 keV barns. The cross section measured by Woodbury et al. however appeared inconsistent with these results being about a factor of ten higher.

Attempts have been made to explain this high value in terms of a resonance lying below the proton threshold (Fo 54). However, the later work of Lamb and Hester and Pixley showed that there is no evidence for such a resonant contribution. The results of Lamb and Hester and Pixley were therefore accepted in adopting a value of So = 3.0 ± 0.6 keV barns for the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction (Bu 57).

2.2 Gamma Radiation from $^{14}\text{N}(p,\gamma)^{15}\text{O}$

We refer to Lauritsen and Ajzenberg-Selove (La 62) for an energy level diagram of $^{15}\text{O}$. Part of the diagram relevant to $^{14}\text{N}(p,\gamma)^{15}\text{O}$ at low proton energies is reproduced in Figure 2.1. Gamma ray transitions reported from the $^{14}\text{N} + p$ resonances at 280, 1060 and 1550 keV are also included.

Johnson, Robinson and Moak (Jo 52) first investigated the gamma radiation from $^{14}\text{N}(p,\gamma)^{15}\text{O}$ using a scintillation detector. Working at the 280 keV resonance (7.55 MeV excitation in $^{15}\text{O}$) they reported radiation corresponding to transitions between the 7.55 MeV level and levels at 5.2, 6.16 and 6.79 MeV in $^{15}\text{O}$, without any noticeable ground state radiation.

Bashkin, Carlson and Nelson (Ba 55) and Tabata and Okano (Ta 60) have repeated these measurements, observing these same
Fig. 2.1 Energy levels of $O^{15}$.
transitions and giving relative intensities. On the basis of the isotropic nature of the gamma rays from the 7.55 MeV level in $^{15}O$ (Po 59b, Ta 60), and the proton elastic scattering experiments of Overley, Pixley and Whaling (Ov 56) and Pixley (Pi 57) an assignment of $\frac{1}{2}^+$ was made for this level.

Povh and Hebbard (Po 59b) have carried out angular correlation measurements on the gamma rays from the 280 keV resonance. Transitions are observed to known levels at 6.79 and 6.16 MeV and to a level at 5.18 MeV which is a member of the 5.2 MeV doublet in $^{15}O$ discovered by Povh (Po 59a) from the $^{16}\text{He}^3,\alpha)^{15}O$ reaction. With the aid of results from the $^{14}\text{N}(d,n)^{15}O$ stripping reaction (Ev 53) Povh and Hebbard were able to give firm assignments of $J^\pi = \frac{3}{2}^+$ and $\frac{3}{2}^-$ for the 6.79 and 6.16 MeV levels respectively. The 5.18 MeV level was restricted to be either $\frac{1}{2}^+$ or $\frac{3}{2}^+$, the $\frac{1}{2}^+$ assignment being preferred (La 62).

Li (Li 53) has measured the gamma-ray excitation function for the $^{14}\text{N}(p,\gamma)^{15}O$ reaction over the energy range 0.6 - 2.5 MeV, the results showing general agreement with the excitation function obtained by Duncan and Perry (Du 51). Li also investigated the gamma radiation at the 1060 keV resonance (8.28 MeV excitation in $^{15}O$). He observed a strong ground state gamma-ray of energy 8.3 MeV, a strong transition through a level at 5.2 MeV and a weak transition through a level at 6.8 MeV.

Hagedorn et al. (Ha 57a) assigned a value $J^\pi = \frac{3}{2}^+$ for the
8.28 MeV level in $^{15}_0$ on the basis of their elastic scattering experiments. Gorodetzky et al. (Go 57, Go 58) have carried out angular correlation measurements on the gamma rays from this level. They do not consider the possibility of two levels at 5.2 MeV, and their results analysed in terms of only one level contributing to the gamma-ray correlation are consistent with the $^2_2^+$ assignment for the 8.28 MeV level.

Hebbard and Povh (He 59) have made careful measurements of the gamma radiation following the proton bombardment of $^{14}_N$ at the 1060 keV resonance, and the 1550 keV resonance (8.74 MeV excitation in $^{15}_0$), to look for a "missing" level in $^{15}_0$. From comparison with the mirror nucleus $^{15}_N$, this was predicted to be somewhere in the region of 7 MeV excitation in $^{15}_0$. The position of this level was of astrophysical interest, for if it fell within the region lying 50 keV or so above the $^{14}_N + p$ threshold at 7.30 MeV, then the control rate of the CNO cycle might shift to the $^{12}_C(p, \gamma)^{13}_N$ reaction.

The missing level was found at 7.16 MeV which is too low in energy to be of astrophysical importance. The radiation to this level was detected by a coincidence experiment and found to be weakly resonant at both the 1060 and 1550 keV resonances. The mode of de-excitation of this level was not able to be determined because of its weak nature. Hebbard and Povh also observed strong transitions at these resonances to the ground
state and to the levels at 6.16 and 5.2 MeV. At the 1550 keV resonance there was evidence for a weaker transition to the 6.79 MeV level.

2.3 Evidence for Direct-Capture in $^{14}\text{N}(p,\gamma)^{15}\text{O}$

The experiments of Hagedorn et al. (Ha 57a) on the elastic scattering of protons by $^{14}\text{N}$ confirmed most of the resonances found by Duncan and Perry (Du 51). They failed to find any evidence for the broad level around 700 keV although the sensitivity of the equipment was adequate enough to detect a considerably smaller anomaly. In addition, the scattering experiments revealed a rising underlying background which was larger than that expected from pure Rutherford scattering. This could not be accounted for in a satisfactory way by s-wave capture alone as assumed by Duncan and Perry. It has been inferred that a direct-capture process could be partly responsible for this background, and the background observed in the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ excitation function (Aj 59).

Hebbard and Povh (He 59) looked at the non-resonant radiation from $^{14}\text{N} + p$ in the region between the 1060 and 1550 keV resonances and observed that this was largely the result of a transition to the 6.79 MeV level in $^{15}\text{O}$. In addition to this strong transition, weak transitions involving the 6.16, 5.2 and ground states in $^{15}\text{O}$ and having about one quarter the strength were also observed. These transitions are illustrated in
Hebbard and Povh have pointed out that the non-resonant radiation is probably the result of a direct-capture process because the main capturing level at 6.79 MeV is thought to have a large proton reduced width. On the basis of the shell model calculations of Halbert and French (Ha 57b) the 6.79 MeV level is expected to have a large reduced width for a 2s proton exterior to the core of the $^14_N$, with a smaller reduced width for a 1d proton. A similar deduction has been made by Hagedorn (Ha 57a) from the results of Inglis (In 53). Further evidence for a large proton reduced width for this level comes from the prominent threshold observed in the $^{14}_N(d,n)^{15}$ reaction (Ma 55).

The energy of the E1 non-resonant radiation to the 6.79 MeV level was sufficiently low to satisfy the requirements of Christy and Duck's theory and enabled a calculation of the cross section to be made by Hebbard and Povh. This was done for a proton energy of 1 MeV. Assuming a p to s transition and a maximum reduced width this gave a result of 6 $\mu$ barns as an approximate upper limit to the cross section, which compared favourably with Duncan and Perry's experimental value of 4 $\mu$ barns at this energy (Du 51).

From this result it appears likely that the non-resonant radiation from $^{14}_N + p$, is in fact the result of a direct-capture process and bears further investigation.
Fig. 2.2 The non resonant radiation from $^{14}\text{N} + \text{p}$ observed by Hebbard & Povh (He.59).
We will be mainly concerned with the direct-capture transition to the 6.79 MeV level and leave discussion of the other radiations until later.

Previous attempts to measure the non-resonant radiation from a singles spectrum have proved unsuccessful due to the presence of the intense 4.43 MeV gamma radiation from the competing $^{15}\text{N}(p,\alpha,\gamma)^{12}\text{C}$ reaction.

Bashkin, Povh and Hebbard (He 60) at California Institute of Technology made an unsuccessful attempt to observe the radiation using a target depleted in its natural $^{15}\text{N}$ content by a factor of thirty, whilst Povh at Freiburg, using a natural nitrogen target and a large beam current tried looking for the radiation in a region calculated to give the minimum 4.43 MeV background (He 60). It was apparent from these unsuccessful attempts that it would be necessary to do a coincidence experiment involving the non-resonance radiation and the 6.79 MeV ground state radiation in order to eliminate this background problem. The experimental aspects of this experiment are discussed in the chapters to follow.
CHAPTER III

THEORETICAL ANGULAR DISTRIBUTION OF THE
DIRECT-CAPTURE RADIATION TO THE 6.79 MeV STATE IN $^{15}\text{O}$.

Before describing the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ experiment in detail we first calculate the form of the angular distribution expected for the direct-capture radiation to the 6.79 MeV state in $^{15}\text{O}$.

To do this we set up the problem quite generally allowing for the possibility of contributions from different proton angular momenta in the initial and final states, and also the possibility of the reaction proceeding by different channel spin states. This has not been done with Christy and Duck's theory. As it stands this applies only to simple capture processes and does not contain explicit information about the properties of the emitted radiation. Furthermore because no well defined gamma emitting initial state is formed the usual angular distribution formulae such as those of Biedenharn and Rose (Bi 53) and Sharp et al. (Sh 53) are no longer directly applicable.

We have chosen to use a more elementary approach in which we consider individual transition paths between initial and final magnetic substates of the $^{14}\text{N} + p$ system expressed in channel spin notation. This is a convenient representation since we are not detecting the polarizations of the incident
proton or final radiation and therefore know in advance that different channel spin contributions to the angular distribution will add incoherently. Inclusion of the appropriate vector spherical harmonics describing the pattern of the emitted radiation and averaging over the initial and summation over the final magnetic substates will then reveal the required angular distribution function. We now describe the analysis in more detail.

3.1 Reaction Mechanism in Channel Spin Notation.

We define quite generally,

- $J_i$, spin of the target nucleus,
- $J_f$, spin of the final compound nucleus state,
- $S_i$, $S_f$, initial and final channel spins
- $\ell_i$, initial orbital angular momentum of the proton
- $\ell_f$, final orbital angular momentum of the proton after capture by the target core to form the compound state
- $s$, the intrinsic spin of the proton
- $L$, the angular momentum carried away by the direct-capture radiation.

The following vector equations must be satisfied to conserve angular momentum,

Channel spin equation, \[ \bar{S}_i = \bar{J}_i + \bar{s} \] (3.1)
Total spin equation, \( \bar{S}_i + \bar{\ell}_i = \bar{S}_f + \bar{\ell}_f + L \) \hfill (3.2)

Final state equation, \( J_f = S_f + \ell_f \) \hfill (3.3)

For the \( N^{14}(p, \gamma)O^{15} \) reaction we assume the direct-capture radiation to be \( E1 \) in accordance with Hebbard and Povh (He 59), and furthermore as discussed in section 1.1, we assume there is no appreciable spin-orbit coupling, so that the channel spin remains fixed before and after the interaction; that is to say \( S_i = S_f \).

Taking the spin of the \( 6.79 \text{ MeV} \) level of \( O^{15} \) as \( \frac{3^+}{2} \) and the ground state spin of \( N^{14} \) as \( 1^+ \), inserting these values in the equations (3.1) to (3.3) and applying the parity conservation law we arrive at the following conclusions:

(i) possible channel spins, \( S_1 = \frac{1}{2}, \ S_2 = \frac{3}{2} \)

(ii) \( |\ell_i - \ell_f| = 1 \)

(iii) \( \ell_f \) must be even.

We are now in a position to write down the angular momentum parameters in the two channels:

\[
\begin{array}{cccc}
\text{Channel spin } S & \frac{1}{2} & \frac{3}{2} & \frac{1}{2} & \frac{3}{2} \\
\ell_i & 1 & 1 & 3 & 3 \\
\ell_f & 2 & 0,2 & 2 & 2 \\
\end{array}
\]
Now, we consider only p-wave incident protons as being important in view of the higher centrifugal barrier for f-wave protons. This is indicated by an estimate of the ratio of p to f-wave barrier penetrability made from the curves of Sharp, Gove and Paul (Sh 61); this ratio being about $10^{-3}$. From these deductions it appears that we are concerned with p to s and p to d transitions forming the 6.79 MeV state.

3.2 Initial and Final State Wave Functions.

We define the co-ordinate system so that the target nucleus is centred at the origin. An incident proton moving along the positive z-axis, chosen also as the quantisation axis can then be represented by the plane wave expansion,

$$\psi_{\text{inc}}(x) = \frac{1}{2\pi} \sum_{\ell=0}^{\infty} i^{\ell} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_\ell(kr) \delta_\ell^{(3)}(\Theta, \Phi) \, dr \, d\phi$$

For the present calculation we consider only p-wave incident protons and factor out the radial term which contributes only a constant factor to the angular distribution. We then take $\phi_0(\Theta, \Phi)$ as the incident proton orbital wave function, $\Theta$ and $\Phi$ being the relative angular co-ordinates of the proton.

In what follows we neglect nuclear and hard sphere phase shift terms in writing down the various wave functions since they are not angle dependent. Using the symbols listed we
find expressions for the wave functions describing the initial
and final states, and the dipole operator connecting these states.

(a) Initial state wave function.

Call, $N^{14}$ ground state wave function, $\psi_{J_i m_i}$

and, Spin wave function of the proton, $\chi_{s m_s}$

The system of $N^{14} +$ incoming proton in channel spin
notation will then be described by vector coupling these
two wave functions. This can be written as

$$\psi_{SM} = \sum_{m_i} c(J_i s S; m_i, m_s; M) \psi_{J_i m_i} \cdot \chi_{s m_s}$$

where we have used the notation of Rose (Ro 58) for the
Clebsch-Gordan coefficient.

Finally we obtain the initial state wave function by
including the incoming orbital wave function of the
proton. Since this is not coupled to the channel spin
we write this in the form,

$$\psi (S, M) = \psi_{SM} \cdot Y_{10} (\Theta, \phi)$$

(3.4)

There will be six substates $\psi_{(SM)}$ corresponding to the
two channel spin states; $\psi (\frac{3}{2}, \pm \frac{3}{2})$, $\psi (\frac{3}{2} \pm \frac{1}{2})$, and
$\psi (\frac{1}{2}, \pm \frac{1}{2})$. 
(b) Electric dipole operator.

This we write as \( Y_{1m}(\Theta, \phi) \), where \( m = \pm 1, 0 \) defines the polarization of the emitted radiation.

(c) Final state wave function.

This is described by vector coupling the channel spin wave function \( \psi_{SM} \) with the angular wave function \( Y_{\ell_f} m_{\ell_f} \) describing the captured proton, to form the final state wave function \( \psi(J_f, m_f) \) of the compound nucleus.

\[
\psi(J_f, m_f) = \sum_{M} C(S, J_f; M, m_f - M, m_f) \psi_{SM} Y_{\ell_f} m_f - M \\
(3.5)
\]

In this expression we have set \( m_{\ell_f} = m_f - M \) from the property of the C-coefficient. There will be four substates \( \psi(J_f, m_f) \) corresponding to \( \psi(\frac{3}{2}, \pm \frac{3}{2}), \psi(\frac{1}{2}, \pm \frac{3}{2}) \).

To calculate the angular distribution we first compute the probability of the reaction proceeding from any one of the six initial substates \( \psi(S, M) \), to any one of the four final substates \( \psi(J_f, m_f) \) by emission of E1 radiation of polarization \( m \). The required probability will be,

\[
P(S, M, m_f) = \left| \int k \int \psi^*(S, M) Y_{1m} \psi(J_f, m_f) \, d\Omega \right|^2 \quad (3.6)
\]

where \( d\Omega = \sin \theta \, d\theta \, d\phi \), and the integration is performed over the proton co-ordinates, and \( k \) is a constant which is proportional to the radial integral \( R_{1ff1} \), defined in Chapter II.
3.3 Angular Distribution Function.

The emission of E1 radiation of polarization m will correspond to a radiation pattern at the angular co-ordinates \( \Theta_1, \phi_1 \) given by the square of the modulus of the vector spherical harmonic \( \bar{X}_{1m}(\Theta_1, \phi_1) \) defined by Blatt and Weisskopf (Bl 52). We write \( Z_{1m} = \left| \bar{X}_{1m} \right|^2 \), so that the product \( P(S, M, m_f)Z_{1m} \) will then correspond to a certain fraction \( P(S, M, m_f) \) of this radiation pattern being produced at the detector. It should perhaps be pointed out that since only one route is possible between pairs of initial and final states, interference terms which involve \( |\bar{X}_{1m}| |\bar{X}_{1m}'| \) do not occur. This allows us to work in terms of radiation intensities rather than amplitudes.

The required angular distribution of the radiation as a whole will be simply the sums of products of all the possible E1 transitions between the initial and final states, that is

\[
W(\Theta_1) = \sum_{SMm_f} P(S, M, m_f)Z_{1m} \tag{3.7}
\]

Figure 3.1 illustrates the various E1 transitions between the initial channel spin substates and the final compound nucleus substates which have to be considered. From this diagram it is apparent that we must make allowance for possible channel spin mixing, and angular momentum mixing for the
Fig. 3-1. Diagram illustrating the 16 dipole transitions leading to the final $3/2^+$ state in $^{15}$O.
captured proton, in forming the final state.

3.4 Allowance for Channel Spin Mixing and Capture of Different Proton Waves in the Final State.

We do this by modifying the final state wave function. Since there is no a priori reason to suppose the channel spin mixing will be the same for protons of different $\ell_f$, we must differentiate four possible channel spin amplitudes in the final state wave function.

We define, $a_{11}$, $a_{21}$ as the $S_1$, and $S_2$ channel spin probability amplitudes respectively for $\ell_f = \ell_1 = 0$, protons where $a_{11}^2 + a_{21}^2 = 1$, and $a_{12}$, $a_{22}$ as the $S_1$, and $S_2$ channel spin probability amplitudes respectively for $\ell_f = \ell_2 = 2$, protons where $a_{12}^2 + a_{22}^2 = 1$.

In addition we define the quantities $b_1$ and $b_2$ as the proton probability amplitudes for the final state to contain $\ell_1$ and $\ell_2$ protons respectively.

The final state wave function can then be written as,

$$\psi(J_f, m_f) = \sum_{i,j = 1, 2} a_{ij} b_j C(S_1, \ell_f; M_i, m_f; M_i, m_f) \psi S_i M_i, Y_{ij} M_i M_i - M_f (3.8)$$

The $C$ coefficient associated with $a_{11}$ $b_1$ vanishes, so that $a_{21}^2 = 1$. This corresponds to the fact that the final $J_f = \frac{3}{2}$ state cannot be formed by coupling $\ell_f = 0$ protons with channel spin $S_1 = \frac{1}{2}$. The $\psi(J_f, m_f)$ then reduces to the sum
of three terms corresponding to contributions from

\( (S_1 = \frac{3}{2}, \ell_f = 2), (S_2 = \frac{3}{2}, \ell_f = 0), \) and \( (S_2 = \frac{3}{2}, \ell_f = 2). \)

We are now in a position to evaluate \( P(S, M, m_f) \) by inserting (3.8) and the conjugate complex of (3.4) into (3.6).

We also apply the orthogonality property of \( \psi_{SM} \):

\[
\int \psi_{SM}^* \psi_{S_M M} \, d\Omega = \int \delta_{SS} \delta_{MM} \, d\Omega
\]

This eliminates contributions from differing initial and final channel spins, in agreement with our initial assumption in section 3.1 about the constancy of the channel spin. We then find,

\[
P(S, M, m_f) = \left| \int \psi_{SM}^* Y_{10}^* Y_{1m} \sum_{i,j} k_{ij} a_i b_j \langle S_i \ell_j f_f; M_1, M_r - M_1, m_f \rangle \right|^2
\]

Here \( k_1, k_2 \) represent the constant factors which are proportional to the radial integrals evaluated for \( \ell_f = 0 \), and \( \ell_f = 2 \) protons respectively.

Expanding (3.9) we find \( P(S, M, m_f) \) consists of the sum of four terms, which we write in the abbreviated form,

\[
P(S, M, m_f) = \left[ \int Y_a \sum_{M_1} C_1 \psi_{S_1 M_1} Y_{\ell_2 f_f} m_f - M_1 \int \delta_{SS} \delta_{MM} \, d\Omega \right]^2
\]

\[+ \left[ \int Y_b \sum_{M_2} C_2 \psi_{S_2 M_2} Y_{\ell_2 f_f} m_f - M_2 \int \delta_{SS} \delta_{MM} \, d\Omega \right]^2
\]

(both terms corresponding to pure p to d transitions)
\[ + \left[ \int \psi_{SM} Y_{\ell} Y_{m} \frac{S_{2} M_{2} Y_{\ell_{1}} m_{\ell} - M_{2} S S_{2} M M_{2}}{M_{2}} d\Omega \right]^2 \]

(corresponding to a pure p to s transition)

\[ + \left[ \int Y_{2bc} \cos S_{\ell} \frac{S_{2} M_{2} Y_{\ell_{2}} m_{\ell} - M_{2} S S_{2} M M_{2}}{M_{2}} \right. \]

\[ \times \left. \sum_{M_{2}} \frac{C_{3} S_{2} M_{2} Y_{\ell_{1}} m_{\ell} - M_{2} S S_{2} M M_{2}}{M_{2}} d\Omega \right] \]

(corresponding to interference between final s and d proton waves with unknown phase difference \( S_{\ell} \)) (3.10)

In this expression we have used the following abbreviations

\[ Y = \psi_{SM} Y_{10} Y_{1m}, \quad C_{1} = C(S_{1} \ell_{2} J_{f}; M_{1}, m_{\ell} - M_{1}, m_{\ell}) \]

\[ a = k_{2} a_{12} b_{2}, \quad C_{2} = C(S_{2} \ell_{2} J_{f}; M_{2}, m_{\ell} - M_{2}, m_{\ell}) \]

\[ b = k_{2} a_{22} b_{2}, \quad C_{3} = C(S_{2} \ell_{1} J_{f}; M_{2}, m_{\ell} - M_{2}, m_{\ell}) \]

\[ c = k_{1} a_{21} b_{1} \]

Equation (3.10) is a general expansion which can be used to calculate any of the individual transitions \( S M - J_{f} m_{\ell} \).

3.5 Calculation of Final Angular Distribution.

We illustrate the method by calculating \( P(S, M, m_{\ell}) \) for the transitions \( S_{1} M_{1} - J_{f} m_{\ell} \). In this case only the first term in (3.10) is non-zero, and we have
\[ P(S_1, M_1, m_f) = a^2 \left| \int \psi_{S_1 M_1}^* Y_{10}^* Y_{1m} \cdot C_1 \cdot \psi_{S_1 M_1} Y_{e_2 m_f - M_1} \, d\Omega \right|^2 \]

\[ P(\frac{1}{2}, M_1, m_f) = a^2 \cdot C(\frac{1}{2} 2 \frac{3}{2} M_1, m_f - M_1, m_f)^2 \left| \int Y_{10}^* Y_{1m} Y_{2 m_f - M_1} \, d\Omega \right|^2 \]

Now we use the identity

\[ \int Y_{e_3}^* Y_{e_2} Y_{e_1} \, d\Omega = \frac{(2\ell_1 + 1)(2\ell_2 + 1)}{4\pi(2\ell_3 + 1)} \cdot C(\ell_1 \ell_2 \ell_3; m_1, m_2, m_3) \]

\[ \cdot C(\ell_1 \ell_2 \ell_3; 000) \]

Hence \[ \int Y_{10}^* Y_{1m} Y_{2 m_f - M_1} \, d\Omega = \frac{5}{4\pi} \cdot C(211; m_f - M_1, m_1, 0) \cdot C(211; 000) \]

The first C-coefficient vanishes unless \( m_f - M_1 = -m \). The expression for \( P(\frac{1}{2}, M_1, m_f) \) then reduces to the product of terms involving three C-coefficients

\[ P(2, M_1, m_f) = \frac{5}{4\pi} \cdot a^2 \left| C(\frac{1}{2} \frac{3}{2} M_1, -m, m_f) \cdot C(211; -m, m_1, 0) \cdot C(211; 000) \right|^2 \]

It is now a straight forward calculation using tables of C-coefficients (Co 35) to arrive at the values of \( P(\frac{1}{2}, M_1, m_f) \) for the six transitions in this channel spin link. In the same way we can calculate the values of \( P(\frac{3}{2}, M_2, m_f) \) for the ten transitions involved in the second channel spin link, this involves the use of the last three terms in (3.10).

We list in table 3.1 the values of \( P(S, M, m_f) \) for all the sixteen transitions which emerge from these calculations, together with the associated quantity \( Z_{im} \).
### Table 3.1. Probabilities calculated for the sixteen allowed direct-capture transitions forming the 6.79 MeV state in $^0_{15}$, allowing in the final state for possible channel spin mixing and interference between s and d-wave protons.

<table>
<thead>
<tr>
<th>Transition $M_1 - \frac{3}{2} M_f$</th>
<th>Transition Probability $P(\frac{3}{2}, M_1, m_f)$</th>
<th>$Z_{1m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2} \quad \frac{3}{2}$</td>
<td>$1 \quad \frac{3}{4} a^2$</td>
<td>$Z_{1-1} = \frac{3}{16\pi} (1 + \cos^2 \Theta_1)$</td>
</tr>
<tr>
<td>$-\frac{1}{2} \quad \frac{3}{2}$</td>
<td>$1 \quad \frac{3}{4} a^2$</td>
<td>$Z_{10}$</td>
</tr>
<tr>
<td>$\frac{1}{2} \quad \frac{1}{2}$</td>
<td>$0 \quad 2 a^2$</td>
<td>$Z_{11} = \frac{3}{16\pi} (1 + \cos^2 \Theta_1)$</td>
</tr>
<tr>
<td>$-\frac{1}{2} \quad \frac{1}{2}$</td>
<td>$0 \quad 2 a^2$</td>
<td>$Z_{11}$</td>
</tr>
<tr>
<td>$\frac{1}{2} \quad -\frac{1}{2}$</td>
<td>$-1 \quad \frac{3}{4} a^2$</td>
<td>$Z_{11}$</td>
</tr>
<tr>
<td>$-\frac{1}{2} \quad -\frac{3}{2}$</td>
<td>$-1 \quad \frac{3}{4} a^2$</td>
<td>$Z_{11}$</td>
</tr>
<tr>
<td>$\frac{3}{2} \quad \frac{3}{2} M_1 - \frac{3}{2} M_f$</td>
<td>$P(\frac{3}{2}, M_1, m_f)$</td>
<td>$Z_{1m}$</td>
</tr>
<tr>
<td>$\frac{3}{2} \quad -\frac{1}{2}$</td>
<td>$1 \quad \frac{3}{2} b^2$</td>
<td>$Z_{1-1} = \frac{3}{16\pi} (1 + \cos^2 \Theta_1)$</td>
</tr>
<tr>
<td>$-\frac{1}{2} \quad -\frac{1}{2}$</td>
<td>$1 \quad 0$</td>
<td>$Z_{1-1}$</td>
</tr>
<tr>
<td>$\frac{1}{2} \quad -\frac{3}{2}$</td>
<td>$1 \quad \frac{3}{2} b^2$</td>
<td>$Z_{1-1}$</td>
</tr>
<tr>
<td>$\frac{3}{2} \quad \frac{3}{2}$</td>
<td>$0 \quad \frac{25}{4} b^2 + c^2 + 10bc \cos \theta_i$</td>
<td>$Z_{10} = \frac{3}{16\pi} (1 - \cos^2 \Theta_1)$</td>
</tr>
<tr>
<td>$\frac{1}{2} \quad \frac{3}{2}$</td>
<td>$0 \quad &quot; - 10bc \cos \theta_i$</td>
<td>$Z_{10}$</td>
</tr>
<tr>
<td>$-\frac{1}{2} \quad -\frac{3}{2}$</td>
<td>$0 \quad &quot; + 10bc \cos \theta_i$</td>
<td>$Z_{10}$</td>
</tr>
<tr>
<td>$\frac{3}{2} \quad \frac{3}{2}$</td>
<td>$0 \quad &quot; + 10bc \cos \theta_i$</td>
<td>$Z_{10}$</td>
</tr>
<tr>
<td>$\frac{3}{2} \quad \frac{3}{2}$</td>
<td>$-1 \quad \frac{3}{2} b^2$</td>
<td>$Z_{11} = \frac{3}{16\pi} (1 + \cos^2 \Theta_1)$</td>
</tr>
<tr>
<td>$\frac{1}{2} \quad -\frac{3}{2}$</td>
<td>$-1 \quad 0$</td>
<td>$Z_{11}$</td>
</tr>
<tr>
<td>$-\frac{1}{2} \quad -\frac{3}{2}$</td>
<td>$-1 \quad \frac{3}{2} b^2$</td>
<td>$Z_{11}$</td>
</tr>
</tbody>
</table>
We now use equation (3.7) with this table to arrive at the
equation for the angular distribution of the direct-capture
radiation. Performing the summation we observe that the
interference contribution vanishes. The angular distribution
then has the form,

\[ W(\Theta_1) = 25(a_2b_1k_1)^2 \sin^2 \Theta_1 + b_2^2 k_2^2 \left[ (a_{12})^2 + (a_{22})^2 \right] (7 - \cos^2 \Theta_1) \]

Noting that, \( a_{21}^2 = 1, (a_{12})^2 + (a_{22})^2 = 1 \) from the previous
section we arrive at the final result,

\[ W(\Theta_1) = \begin{align*}
25 \Theta \sin^2 \Theta_1 + (7 - \cos^2 \Theta_1) \\
\text{pure p to s direct-capture} & \quad \text{pure p to d direct-capture}
\end{align*} \quad (3.11) \]

where we have written \( \Theta = \frac{b_1^2 k_1^2}{b_2^2 k_2^2} \).

Examination of the above expression indicates that if the
gamma radiation to the 6.79 MeV level in \(^{15}\text{O}\) is largely the
result of a p to s direct-capture transition (\( \Theta \gg 1 \)), then it
should have a characteristic \( \sin^2 \Theta_1 \) angular distribution.

An experimental verification of this form of angular distribution
would then offer a good test for the direct-capture hypothesis.
Furthermore comparison of the theoretical angular distribution
(3.11) with experiment should give some indication of the s to
d-wave proton contributions forming the 6.79 MeV state, provided
the ratio $k_1^2/k_2^2$ can be determined from the radial integrals.
4.1 Triple Correlation Experiment.

The use of a coincidence technique means that one must perform a triple angular correlation experiment involving the proton and two coincident gamma rays. For the $^{14}\!\!N(p,\gamma)^{15}\!\!O$ reaction the two gamma rays of interest correspond to the direct-capture radiation to the 6.79 MeV level in $^0_{15}$ and the 6.79 MeV ground state radiation. From this correlation we extract the angular distribution of the direct-capture radiation by a suitable averaging process which eliminates the effect of the unwanted 6.79 MeV radiation.

To investigate the angular functions appearing in the triple correlation, we consider the general case of a $(p \gamma_1 \gamma_2)$ correlation and define the incident proton direction as the quantisation axis. Further, we call $\theta_1$ and $\theta_2$ the polar angles defining the first ($\gamma_1$), and second ($\gamma_2$), gamma rays respectively and $\phi$ the dihedral angle between the planes containing the proton and $\gamma_1$ and the proton and $\gamma_2$ respectively. This is illustrated in Figure 4.1.

The triple angular correlation function for the $(p \gamma_1 \gamma_2)$ process can be written following Biedenharn (Bi 60) as,
Fig. 4.1 Geometry for $(p_\Sigma, \Sigma_2)$ triple correlation.
The three angles \( \theta_1, \theta_2, \phi \) describing the directions of the two gamma rays are contained only in the last term. We can therefore examine this without being concerned with the preceding terms which describe the formation of the various wave mechanical states.

Neglecting phase terms and factors which do not depend on the three angles the expression for \( P_{KLM} (\theta_1, \theta_2, \phi) \) reduces to a form depending on a triple product of angle functions,

\[
P_K |\theta_1| P_M |\theta_2| \cos \theta \phi . \text{ with } |\theta| \leq \text{minimum } K, M.
\]

The acceptable range of values for the integers \( K, M \) in this product depend on angular momentum properties of the interacting nuclei and the multipolarity of the radiation. The limitation on the range of values for the integer follows from the property of the C-coefficient contained in the full expression for \( P_{KLM} (\theta_1, \theta_2, \phi) \).

For our purposes to be quite general we assume the possibility of all coincidence radiation mixtures up to and including a quadrupole-quadrupole transition, which will involve product terms up to \( P_4^4(\theta_1) P_4^4(\theta_2) \cos 4\phi \). These are listed
below in table 4.2 in which we have also included odd Legendre polynomials up to $P_3$ to allow for possible interference mixtures in both radiations. The correlation is made up of a sum of contributions from triple products, formed by any combination of single elements in the same row; this sum being extended over the five rows.

<table>
<thead>
<tr>
<th>$\Theta_2$ Dependence</th>
<th>$\Theta_1$ Dependence</th>
<th>$\phi$ Dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_0^0, P_1^0, P_2^0, P_3^0, P_4^0$</td>
<td>$P_0^0, P_1^0, P_2^0, P_3^0, P_4^0$</td>
<td>1</td>
</tr>
<tr>
<td>$P_1^1, P_2^1, P_3^1, P_4^1$</td>
<td>$P_1^1, P_2^1, P_3^1, P_4^1$</td>
<td>$\cos \phi$</td>
</tr>
<tr>
<td>$P_2^2, P_3^2, P_4^2$</td>
<td>$P_2^2, P_3^2, P_4^2$</td>
<td>$\cos 2\phi$</td>
</tr>
<tr>
<td>$P_3^3, P_4^3$</td>
<td>$P_3^3, P_4^3$</td>
<td>$\cos 3\phi$</td>
</tr>
<tr>
<td>$P_4^4$</td>
<td>$P_4^4$</td>
<td>$\cos 4\phi$</td>
</tr>
</tbody>
</table>

Table 4.2. Angular functions in $(p, \Theta_1, \Theta_2)$ correlation experiment.

Various methods of arriving at the angular distribution of the first radiation by choosing different angle combinations of the two detectors were considered. The procedure to be described proved to be the most satisfactory, requiring the least number of measurements for elimination of the major $\Theta_2$ and $\phi$ dependent terms in the correlation.

We consider a fixed value of $\Theta_1$, and first remove the $\phi$ dependence. If angles corresponding to $\phi = 45^0$ and $135^0$ are
chosen, terms involving \( \cos 3 \phi \) automatically vanish. In addition since the odd cosine functions change sign between these positions, arithmetic addition of the two coincidence counts of \( \gamma_1 \) will eliminate terms in \( \cos \phi \), and \( \cos 3 \phi \). With \( \cos 4 \phi = 1 \), we are left with terms in \( \Theta_1 \) and \( \Theta_2 \) in the first and last rows of the table. Next we attempt to remove the \( \Theta_2 \) dependence.

With \( \Theta_1 \) still fixed, we choose a pair of angles for \( \Theta_2 \) such that \( P_4^0(\Theta_2) = 0 \). It is convenient to choose the pair \( \Theta_2 = 109^{\circ}54', 149^{\circ}24' \) for geometrical reasons, since the \( \gamma_1 \) detector will cover the forward quadrant of angles. For these angles of \( \Theta_2 \), the \( P_2^0(\Theta_2) \) coefficients do not vanish but have opposite signs, and can be eliminated from the correlation by suitably weighting the coincidence counts of \( \gamma_1 \) at these two angles. If the \( \Theta_2 = 109^{\circ}54' \) \( [149^{\circ}24'] \) counts are multiplied by the coefficients of \( P_2^0(149^{\circ}24') \), \( [P_2^0(109^{\circ}54')] \) and then added, the \( P_2^0(\Theta_2) \) dependence is removed. Adopting this procedure we are then left with product terms of the form,

\[
P_1^0(\Theta_2) \cdot \left[ P_0^0, P_1^0, P_2^0, P_3^0, P_4^0(\Theta_1) \right]
\]
\[
P_3^0(\Theta_2) \cdot \left[ P_0^0, P_1^0, P_2^0, P_3^0, P_4^0(\Theta_1) \right]
\]

and the single term,

\[
P_4^4(\Theta_2) \cdot P_4^4(\Theta_1)
\]
This last term can be neglected for the \( N^{14}(p, \gamma)O^{15} \) reaction on the grounds that it would correspond to capture of d-wave protons to the final state followed by the emission of successive quadrupole radiations. If the odd Legendre Polynomials in both \( \Theta_1 \) and \( \Theta_2 \) can be shown to be absent by proving experimentally that the coincidence count of \( \gamma_1 \) is symmetrical about each of the directions \( \Theta_1 = 90^\circ \), and \( \Theta_2 = 90^\circ \), then the remaining terms will correspond to the angular distribution of the \( \gamma_1 \) radiation and have the form,

\[
W(\Theta_1) = A_0 P_0(\Theta_1) + A_2 P_2(\Theta_1) + A_4 P_4(\Theta_1)
\]

Summarizing, the angular distribution of the direct-capture radiation for the \( N^{14}(p, \gamma)O^{15} \) reaction can be obtained from experimental measurements of the coincidence counting rate for this radiation over a range of angles \( \Theta_1 \), for each of four positions \((\Theta_2, \phi)\) which define the direction of the 6.79 MeV gamma ray. For later use we refer to these angle pairs \((149^\circ 24', 135^\circ), (109^\circ 54', 135^\circ), (109^\circ 54', 45^\circ)\) and \((149^\circ 24', 45^\circ)\) as Positions I to IV respectively. Additional experiments are also required to test for the presence of odd Legendre polynomials occurring in \( \Theta_1 \) and \( \Theta_2 \).

If one limits \( \Theta_1 \) to five angles as in the present experiment, the triple correlation then requires a total of 20 coincidence measurements plus a minimum of two additional tests for the odd
Legendre functions.

It should be pointed out that if we had chosen to set \( \psi = 30^\circ, 90^\circ \) and \( 150^\circ \) and used six angle pairs for Detector II the \( \cos 4\psi \) term could have been completely eliminated. However this adds 50\% to the number of required measurements and since it became necessary to do an eight hour run at each of the 22 positions, this improvement was not considered worthwhile.

4.2 Target and Detector System.

Targets of tantalum nitride with thicknesses of 40 to 60 keV for 800 keV protons were used in the experiments. These were prepared by heating a freshly polished sheet of .005" tantalum to red heat (about 1100°C) in an atmosphere of a few cm of dry natural ammonia gas, using a conventional vacuum evaporator system with transformer heating. Preliminary vacuum outgassing of the tantalum and freezing of the ammonia before regeneration using a liquid air jacket considerably reduced the oxide deposit on the target. A satisfactorily nitrided target showed a brownish tinge on the grey tantalum surface, whilst excessive oxidation gave a blue surface layer.

Targets prepared in this way proved to be exceptionally stable running at a dull red heat for some 60 hours (typically 3 coulomb bombardment) before being replaced. Target replacement occurred when the yield of \( \gamma \)-rays from the
N\textsuperscript{14}(p,\gamma)O\textsuperscript{15} reaction was approximately 60% that of a fresh target. Running the targets at these temperatures considerably reduced carbon build up on the surface. Operation at higher temperatures in an effort to obtain increased yield was unsatisfactory leading to a rapid loss in nitrogen. The nitrogen profile as a function of depth for one of these targets is shown in Figure 4.2. This was determined from the 4.43 MeV gamma ray yield in the region of the 429 keV N\textsuperscript{15}(p,\alpha,\gamma)C\textsuperscript{12} resonance. The pronounced tail extending into deeper layers is characteristic of this type of target.

Targets in the form of \frac{\frac{1}{2}}{2}" diameter discs were clamped in a small \frac{\frac{1}{2}}{4}" copper holder mounted at the end of a \frac{\frac{1}{2}}{8}" steel rod which was free to move and rotate along the axis of a cylindrical pyrex glass target chamber. The target chamber having a wall thickness of 2 mm. and a diameter of 2" was smoothly capped with a hemispherical glass top centred on the target. The proton beam entered along a horizontal glass inlet tube of \frac{\frac{1}{2}}{3}" diameter which was coupled to the main 1" diameter brass beam tube by means of a small bellows unit. The inlet tube constituted the only asymmetrical object between the target and the two detectors. Collimation of the beam by \frac{\frac{1}{2}}{4}" tantalum stops located at the ends of the brass tube prevented protons from hitting the glass.

A second \frac{\frac{1}{2}}{6}" steel rod carrying a quartz disc could be
NH₃ PRESSURE = 2.5 cm.
0.005 TANTALUM SHEET
HEATED 3 MIN 1100 °C

Fig 4.2

RELATIVE COUNTS

ΔE = 50 KeV
E_p mean = 450 KeV
raised behind the target to facilitate beam alignment. The alignment could be carried out to better than $1/16''$ by visually observing the position of the target shadow with the target end on to the beam. Alternatively a clean tantalum target blank could be substituted for the quartz to enable background runs to be taken.

The lower end of the target chamber was vacuum sealed into a cylindrical aluminium plug made to be a close fit into the centre of a large angular distribution table. Figure 4.3 illustrates the target and detector system.

The two detectors, constrained to rotate about the target as centre consisted of 5" diameter x 4" sodium iodide crystals coupled to 3" Dumont type 6363 photomultipliers. They were lightly shielded with lead as illustrated in Figure 4.3 to reduce low energy background and eliminate true coincidences caused by energetic $\pi^-$-particles passing from one crystal to the other.

One crystal referred to as Detector I, mounted on the angular distribution table, was free to rotate in a horizontal plane. The height of the centre of the crystal and target centre was adjusted to be the same to within $\pm \frac{1}{64}''$. The design of the table ensured a negligible radial error on rotation, once the beam had been adjusted to hit the target centrally.
A, 1" DIA. BRASS BEAM INLET TUBE.
B, 1" COPPER BELLOWS.
C, 2" DIA. PYREX GLASS TARGET CHAMBER.
D, ANGULAR DISTRIBUTION TABLE.
R, TARGET CARRYING ROD.
S, 1/4" DIA. TANTALUM STOP.
T, TANTALUM NITRIDE TARGET.
X, 1" THICK CYLINDRICAL LEAD SHIELD.

Fig. 4-3.
The second crystal - Detector II, was mounted on an arc which could be rotated about a vertical axis, Plate 4.5. The arc was carefully aligned so that when it was centrally loaded with the crystal, its axis of rotation coincided with that of the table to within $^\pm 0.005"$. However when the crystal and its counterweight were moved along the arc surface to set the two dihedral angle positions for the experiment, it was found that the radial distances between the target and crystal face differed by about $0.020"$. This appeared to be due to flexing of the arc and its supports, and had the effect of changing the count rate in these two positions of Detector II by about $\frac{1}{2}\%$. This error amounted to about one quarter the statistical error in the number of counts in a typical run, and was automatically taken into account by inter-relating the Detector II count rates in the four positions.

Because of the acute background problems caused by the low $\gamma$-ray yield and the relatively large distance of the detectors from the target a third NaI(Tl) crystal and photomultiplier of the type already described was set up as a background monitor.

The detector was fixed in position some 10 ft. from the target and placed under the angular distribution table so that it was effectively shielded from the target radiation by several inches of steel. Pulses from the monitor were amplified and counted with a single channel analyser set to record pulses in
the energy range from 5.4 to 8 MeV. This counted pulses included in the energy region for the $^{14}\text{N}(p,\gamma)\text{O}^{15}$ monitor counter as well as fast neutrons and the cosmic ray flux.

Severe background problems from fast neutrons produced by the nearby tandem accelerator were encountered during preliminary runs. The fast neutron spectrum extended to about 6.5 MeV and increased considerably in intensity as the tandem proton energy increased, in most cases completely masking the $^{14}\text{N}(p,\gamma)\text{O}^{15}$ gamma radiation. Figure 4.4 illustrates typical backgrounds measured for a 2 hour period at different proton energies of the tandem. This can be compared with a representative $^{14}\text{N}(p,\gamma)\text{O}^{15}$ spectrum recorded under low background conditions with Detector II at a distance of 6" from the target. Tests showed that without construction of a large concrete shielding wall no significant improvement in the neutron background would be obtained. This did not prove to be a practicable measure, so the experimental programme was run during tandem shut down periods.

4.3 Design and Construction of the Angular Distribution Table and Counter Arc.

We describe the important features in the construction of the angular distribution table and the counter arc with the help of Plate 4.5.

The table is of a rigid construction designed to take up to
2,000 lb. of shielding material. When in use the table rests on steel pads, being supported by three 1" steel legs which screw into a lower box-like frame to give individual height adjustments. For mobility the table can be let down on to four heavy duty castors.

The upper table section consists of three 38" diameter steel discs of varying design, the top two being moveable but the lower disc being made part of the main supporting structure.

The lower disc of thickness 2" is supported near its circumference by four thick walled 2" steel tubes welded to the lower frame, whilst a series of smaller tube cross members provide support near the centre. To ensure rigidity the structure is held together by a series of 3/8" tapered dowel pins driven into the lower surface of the disc. On the upper surface thin circular brass strips are fastened near the centre and outside. This enables the upper pair of discs to slide horizontally together over a limited range to facilitate target alignment.

The central disc consists of a solid cartwheel-like structure. Inner and outer steel rings approximately 1" square in section, are welded on to a 1/4" thick steel disc. The outermost ring with a U groove cut in its top surface carries 3/8" steel balls which support the top plate at its circumference and allow it to rotate. The inner ring supports a tapered hollow steel cylinder
carrying a 5" diameter central bearing. This in turn supports the top disc of thickness 1\(\frac{7}{8}\)" allowing it to rotate above the central disc. The height of the main bearing was carefully adjusted by shimming so that the balls were in contact with the top plate and hence distributing the load evenly over the surface. Under these conditions height and radial variations of the table on rotation and lateral movement of the load were considerably better than .005".

The edge of the upper plate is graduated in degree intervals and estimates to a 1/5th of a degree can be made quite easily. Brass strips are fixed along one diameter of the table surface to act as guides for a sliding detector carrier. This can be moved either by hand or through small distances up to 2" by means of a relocatable screw adjustment. Screw adjustments and guides are also provided for the lateral movement (up to \(\frac{3}{8}\)" maximum) of the table, which is arranged to be perpendicular to the incident beam direction. A small screw clamp is also provided to prevent the table surface rotating.

The overall weight of the table with carrier is about 1,400 lbs and when in use stands 46" above floor level.

The counter arc is constructed out of 1" thick aluminium plate in the shape of a semicircular annulus of inside radius 22". One surface of the plate is machined flat and acts as a clamping face for the crystal support. The support is free
to slide over the surface being guided by a small locating
groove near the inside edge. A series of 7/16" diameter
holes are accurately positioned on a 23\(\frac{1}{8}\)" radius from the
arc centre. A matching set of holes in the support enable
it to be positioned and clamped in any position covering the
range 0° to 171° in 1° intervals, these being graduated on
the face of the arc. With this support radial distances in
the range \(\frac{1}{2}\)" to 6" from the arc centre can be covered with a
5" diameter x 4" crystal.

The arc is supported in a vertical plane symmetrically
about the 90° position by clamping inside a rectangular steel
slot. The remainder of the arc is completely unsupported, and
to reduce unnecessary weight its width is tapered progressively
from 90° to the horizontal positions.

The steel slot forms the lower section of an accurately
machined solid steel cylinder carried by two precision thrust
races. The whole of the assembly is capable of being rotated
inside a second large hollow cylinder fastened rigidly to the
lower of three moveable slides. These slides (similar in
construction to lathe slides) give the arc two horizontal and
one vertical degree of freedom and can be adjusted by hand with
the help of a knurled steel knob graduated in intervals of \(.005"\).
Once positioned these slides are locked firmly to take up any
unnecessary play which might affect the positional accuracy of
the detector. A fixed circular protector scale graduated in
degrees and a fine reference line attached to part of the arc
support enables the plane of the arc to be set to 1/5° with
care. With the 1° angle adjustment on the arc surface this
enables the detector to be set to any angle in the upper
hemisphere to the nearest 1/2°.

The uppermost slide is fastened beneath a small steel
trolley which moves along a length of steel girder fastened
rigidly to the ceiling so that it closely follows the projected
proton beam line direction.

Tolerances in design and manufacture of the arc and
supports were aimed at keeping the perpendicular distance
between the extended crystal axis and the arc centre less
than .015". Under these conditions the maximum change in
count rate is about 0.4% at a detector distance of 6".

With the crystal and support alone moving on the arc
errors of this order of magnitude were observed due to flexing
of the arc and its supports. The use of a suitable lead
counter weight situated on the opposite side of the arc was
found to give a considerable improvement.

4.4 Electronic Equipment.

The detecting system is shown in block diagram form in
Figure 4.6, together with the background monitor (Crystal III)
system.
Pulses from the two detectors I and II are first amplified in charge sensitive pre-amplifiers feeding double delay line amplifiers of Chalk River design (Go 60a). These then feed into a cross-over coincidence circuit also of Chalk River design (Go 60b), which forms the fast half of a fast-slow coincidence circuit. Pulse height selection in the two channels (SCA I) and (SCA II) forms the slow half. The resolving time of the cross-over coincidence circuit was set at 140 μsec. Shorter resolving times down to 40 μsec. could be obtained but the circuit as constructed became rather sensitive to the time of cross over and counting losses have been experienced as this drifted over long running periods.

Coincidence pulses from the triple coincidence circuit and all singles pulses in the two channels are fed into a quadrant selector designed by Hebbard (He 62). This unit serves the purpose of examining the singles and coincidence pulses of both detectors and routing them into one of four quadrants in the kicksorter memory. In the present experiment an R.I.D.L. 400 channel analyser was used with 100 channel quadrants recording separately each of the singles and coincidence spectra of the two detectors. By continuously recording this data it was possible to make accurate corrections for gain shifts in the system over the prolonged running times necessary in the experiment.
The four spectra which were recorded are as follows:

(i) Singles I, (Channels 0 - 99). This was used as an energy calibration and check on the gain of the Detector I system. For the triple correlation experiment Detector I covered the energy range 0 - 3.7 MeV.

(ii) Singles II, (Channels 100 - 199). This was used as an energy calibration and check on the gain of the Detector II system. In addition this acted as the monitor for the \( ^{14}\text{N}(p,\gamma)^{15}\text{O} \) reaction during the experiment. For the triple correlation experiment Detector II covered the energy range 0 - 8.4 MeV.

(iii) Coincidences I (Channels 200 - 299). This contained information on the direct-capture gamma-ray.

(iv) Coincidences II (Channels 300 - 399). This was used to display the position of the single channel analyser window in the Detector II channel. With the aid of the Singles II spectrum this enabled corrections to be made to the number of Coincidences I pulses caused by small drifts in this window position.

Spectra were recorded for a fixed line time (usually 120 minutes), on the kicksorter. A master switch on the Quadrant Selector controlled all gates to the kicksorter and also an input gate to the scaler recording pulses from the background monitor, crystal III. For this low yield reaction, with the
detectors 6" from the target kicksorter dead-times of 1 - 2% were typical. No corrections for dead-time counting losses were necessary since the reaction was monitored continuously by Detector II.

4.5 Experimental Measurements in General.

The single channel analyser in the high energy channel (SCA II in Figure 4.6) was set so that only low energy gamma rays from Detector I coincident with high energy gamma rays from Detector II in the range 5.0 - 8.0 MeV were recorded as low energy coincident events. A single channel analyser was later incorporated in the low energy channel (SCA I in Figure 4.6) and set to cover the range 0.6 - 3.7 MeV. This had the effect of eliminating the 0.51 MeV annihilation peak appearing in the low energy coincidence spectrum and therefore considerably improved the high energy coincidence spectrum.

The working distance of each of the crystals, measured from the target centre to the crystal face, was 6". This was the closest distance at which the required combination of angles could be obtained without the crystals interfering with one another.

For each of the four positions of Detector II measurements were taken for six angles $\Theta_1$ of Detector I corresponding to $0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$, $90^\circ$ and $135^\circ$ or $225^\circ$. Either of the last pair of angles, which test for odd Legendre polynomials in $\Theta_1$, by
comparison with the 45° measurement, could be used to suit the position of Detector II. The angles were covered in a random order to reduce any build up of systematic errors.

For angles in the forward direction and at 225° the target normal was set at 215°, and at 145° for the backward angle of 135°. Under these conditions the largest correction for target absorption in the .005" tantalum was $1 \frac{3}{2}$ (this corresponds to the emission of the low energy radiation at 90°). The symmetry of the glass target chamber ensured that it would attenuate equally in all directions, and so no correction was made for this absorption.

A proton energy of 800 keV was chosen for the measurements, this lying in the region between the $N^{14}(p, \gamma)O^{15}$ resonances at 280 and 1060 keV, and below the sharp $N^{15}(p, \alpha_{1} \gamma)C^{12}$ resonance at 898 keV. The proton beam was obtained from the Australian National University's 1.2 MV Cockcroft-Walton set. Stabilization of the 90° analysing magnet with only infrequent adjustments enabled the proton energy to be kept constant to within 1 keV, during an 8 hour run. Currents of 12 to 15 microamperes were used to give the required yield. Eight hour runs were carried out at each angle of Detector I, data being accumulated over this period. Print outs of the 400 channels were made at 2 hours, 4 hours, 6 hours and 8 hours, so that the individual sets of 2 hour data could be checked for consistency. The
data was collected over a continuous period of some 30 nights during shut-down of the tandem accelerator so that the neutron background problem discussed in section 4.2 was reduced.

With a proton energy of 800 keV, and a target thickness of 60 keV the mean energy of the direct-capture radiation to the 6.79 MeV state in $^{15}$O was 1.23 MeV. Using a target of this thickness did not appear to smear out the peak of the 1.23 MeV gamma ray appreciably in the low energy coincidence spectrum so that it was still well defined.

Figures 4.7, 4.8, 4.9 and 4.10 illustrate typical spectra obtained for Singles I, Singles II, Coincidences I, and Coincidences II respectively in an 8 hour run with Detector I at 90°. Also Figures 4.11 and 4.12 illustrate typical Coincidences I and II respectively in an 8 hour run at 0°.

4.6 Reaction Monitor.

Because of the gradual loss of nitrogen from the target, beam current integration was not a satisfactory monitor for the reaction. This however did prove useful in the estimation of the beam dependent background. The high energy gamma radiation from the $^{14}$N(p, $\gamma$)$^{15}$O reaction, observed in the Singles II spectrum was used as the monitor instead. A region in this spectrum corresponding to an energy range 5.0 - 7.0 MeV was found to give the most satisfactory $^{14}$N(p, $\gamma$) yield compared to background. This region was chosen to be clear of
Fig. 4.7 Typical 8HR Singles I spectrum with crystal 6" from target.

- $K^0_{40} 1.46$ MeV
- $\text{THC} 2.62$ MeV
- $N(p,x) 1.5 \times 10^5$ MeV

COUNTS IN 8 HRS
KICKSORTER CHANNEL
Fig 4.8 Typical 8Hr Singles II spectrum with crystal 6" from target.
Shaded area shows the summation region for the monitor.
Fig 4-9. 90° Coincidences I spectrum for a typical 8 Hr. run with the crystal 6" from the target. The shaded area represents the summation region for the 1.23 MeV. radiation. The 1.89 MeV gamma ray corresponds to a transition to the 6.16 MeV level in O\(^{15}\).

Fig 4-10. 90° Coincidences II spectrum for typical 8 Hr. run with the crystal 6" from the target & showing the position of the high energy window. The origin of the unlabelled gamma radiation is unknown.
Fig. 4-11 0° Coincidences I spectrum for a typical 8 Hr. run with the crystal 6" from the target. The intensity of the 123 MeV gamma ray can be seen to be much smaller than at 90°.

Fig. 4-12 0° Coincidences II spectrum for a typical 8 Hr. run with the crystal 6" from the target. The 6.2 & 7.2 MeV gamma rays are in coincidence with the 143 & 0.74 MeV gamma rays respectively in the Coincidences I spectrum & of unknown origin.
contributions from the 4.43 MeV radiation, and it contained about 80% of the pulses from the \( \text{N}^{14}(p,\gamma) \) reaction above 5 MeV.

To obtain the final value for the monitor count in a particular run it was necessary to make corrections for possible gain shifts and background contributions from radiation sources other than the \( \text{N}^{14}(p,\gamma) \) reaction. The following corrections were applied to the raw data taken over the summation region:

i) allowance for gain shifts in the detector

ii) subtraction of a time dependent background from cosmic rays and induced crystal activity

iii) subtraction of a beam dependent background from machine produced neutrons and gamma radiation from target contaminants.

Gain shifts over the 8 hour running period were checked by observing the positions of the prominent 2.62, 4.43 and 6.79 MeV gamma ray peaks in the Singles II spectrum (Figure 4.8). Shifts of the order of half a channel, corresponding to about a \( \frac{1}{2} \% \) change in the total count were typical. Day to day gain shifts during the course of the experiment tended to be slightly larger, the maximum shift for the entire running period requiring a correction of about \( 1\frac{1}{2} \% \).

A constant time dependent background was used in nearly all cases, the crystal activation being indistinguishable from the cosmic ray background. This accounted for about 25% of the
total counts recorded in the summation region of the singles spectrum and because of its large contribution this was rechecked at intervals throughout the experiment by recording pure background spectra. In three of the runs, immediately following above-average daytime neutron fluxes from operation of the tandem accelerator at high energies, higher backgrounds had to be subtracted to allow for the increased crystal activation.

The beam dependent background was estimated at the end of each 8 hour run from the spectrum obtained by bombarding a freshly polished tantalum disc. This was cut from the same sheet as that used for the target preparation. The background to be subtracted from the $^1_4N(p,\gamma)$ runs was found by normalizing to a constant current integrator reading in each case. This subtraction accounts for a further 20% of the monitor count.

It is difficult to estimate how reliable this last subtraction is, because different running conditions can conceivably affect the amount of radiation produced. If one assumes that the neutron contribution remains essentially constant and that the gamma radiation is derived from similar contaminants in the target and the tantalum blank, then this procedure should be fairly reliable; at the worst one can either over or under correct all the data by the same amount. If the neutron background is not constant then this procedure
is less reliable.

There is good evidence that the neutron background is in fact reasonably constant, from the similar readings obtained on the neutron monitor from day to day. In addition, before commencing the experiment precautions were taken to clean up the accelerating system to minimize neutron production. A total running time of 65 hours was spent bombarding various parts of the 90° magnet box and analysing slits in an effort to remove occluded deuterium. A noticeable initial drop in the machine background was observed which eventually levelled off after some 20 hours, giving an indication of a constant background. Constant background conditions during the experiment are also in evidence from a calculation of the angular distributions using the raw data uncorrected for background. The angular distribution calculated in this way turned out to be consistent with the background-corrected result to within statistics (Chapter V).

When Detector II was moved from one position to the next the target absorption and radiation emission changed, and it was therefore necessary to relate the count rates of the monitor in these two positions. This was done by keeping Detector I fixed and reducing its gain slightly so that it acted as temporary monitor of the high energy gamma radiation whilst Detector II was moved.
A similar procedure was carried out when the target orientation was changed, Detector I being fixed at 0° in this case so that the radiation it received was attenuated by the same target thickness in the two positions.

4.7 Low Energy Coincidences.

The gain of the low energy system was monitored by observing the positions of the 1.46 and 2.62 MeV background peaks in the Singles I spectrum (Figure 4.7). Using this information the counts of the 1.23 MeV gamma ray in summation region of the Coincidences I spectrum (Figure 4.9) were able to be corrected for small gain changes. These corrections were small in comparison with the relatively large statistical errors in the coincidence counts. Typical counts in this region were 75 per hour at 90° and 15 per hour at 0°.

The number of low energy coincidences recorded is sensitive to the position of the single channel analyser window associated with Detector II. Changes in the upper level did not contribute, since this was set considerably higher than the full energy of the 6.79 MeV gamma ray. Changes however in the lower level lead to corresponding changes in the number of events accepted in the high energy window and this in turn to changes in the number of low energy coincidences recorded.

At the start of each 8 hour run the window positions were checked with a pulser. Later drifts during the experiment
could be observed from the position of the window appearing in the Coincidences II spectrum (Figure 4.10). One of the earlier runs was chosen as standard, the position of the window in the Coincidences II spectrum noted, and the total number of counts in the corresponding region of the Singles II spectrum recorded. By doing this for slightly different positions of the window (assuming a constant width) a correction factor was obtained for the Coincidences I counts as a function of the window position. This was used in standardizing the results in later runs. The largest correction applied, amounted to a 2.6% change in the coincidence count.

Other corrections applied to the low energy coincidence count involved subtractions of a background and of contributions from the presence of other low energy gamma rays which appeared in the spectrum.

A constant background for the 1.23 MeV summation region was subtracted first for each individual 2 hour Coincidences I spectrum. The amount was determined by examination of the upper energy region which is clear of pronounced peaks. This was typically about 9 counts per hour and showed spherical symmetry.

Inspection of Figure 4.9 makes it apparent that three gamma rays of energy 0.51, 0.74 and 1.43 MeV are also present in the coincidence spectrum in addition to the 1.23 MeV gamma ray.
By limiting the summation region of the 1.23 MeV gamma ray to an energy range 0.97 - 1.34 MeV, contributions to the count in this region from both the 0.51 and 0.74 MeV gamma rays could be neglected. An estimate for a similar spectrum at 90° shows that about 0.4% of the counts in this region are due to the 0.51 and 0.74 MeV gamma rays. From an assumed line shape of a 1.43 MeV gamma ray, an estimate was made of the number of 1.43 MeV events contributing to the 1.23 MeV count. The 1.43 MeV gamma ray appeared to have an isotropic angular distribution, similar corrections being applied to the coincidence counts for all positions of Detector I.

A typical correction involved a subtraction of about 10% of the background-corrected counts at 90°, whilst at 0°, where the 1.23 MeV yield is considerably reduced, this subtraction amounts to about 60% of the counts.

The origin of the 0.74 and 1.43 MeV gamma rays were not identified, beyond showing that they were not associated with the \( N^{14}(p,\gamma) \) reaction since their energies did not change as the proton energy changed, and the energies of the coincidence pairs would not fit in with the known level scheme of \( 0^{15} \). By setting the low energy window over the 0.74 and 1.43 MeV peaks in turn it was established from the Coincidences II spectrum that the 1.43 MeV gamma ray was in coincidence with a gamma ray of 6.2 ± 0.1 MeV, and that the 0.74 gamma ray was in coincidence.

* See footnote on page 66.
with a gamma ray about 7.2 MeV. The very small number of coincidence counts recorded in the latter case making it difficult to define the energy with any certainty. Both these gamma rays are plainly visible in the $\Theta_1 = 0^\circ$ Coincidence II spectrum of Figure 4.12. No corrections for accidental coincidences were found to be necessary. The result of a test carried out over several hours using a Na$^{22}$ source to increase the accidental coincidence rate indicated that for the counting rates occurring in the $N^{14}(p,\gamma)$ reaction only 0.2 accidental coincidences per hour over the whole 100 channels were to be expected. This is in good agreement with a similar estimate from the resolving time of the coincidence circuit.

4.8 Angular Distribution Measurements on the 6.79 MeV and other Ground State Radiations.

To investigate the non-resonant radiation from the $N^{14}(p,\gamma)$ reaction more completely, angular distributions were measured at 800 keV for the 6.79, 8.05, 6.16 and 5.2 MeV radiations.

The four angular distributions were determined simultaneously from the singles spectrum of Detector I, the gain of which was adjusted to cover the energy range 0 - 9 MeV. The singles spectrum of Detector II was used as the monitor in exactly the same way as for the coincidence measurements. Both detectors were situated 6" from the target, Detector II being fixed on the arc in the position $\Theta_2 = 270^\circ$, $\phi = 135^\circ$ clear of the first two
quadrants over which the first detector moved. The angular distributions were measured at angles $\theta_1$ of $0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$, $90^\circ$, $120^\circ$, $135^\circ$, and $148^\circ$, the maximum backward angle which could be reached. The angles in the backward quadrant served to test for possible asymmetries about $\theta_1 = 90^\circ$.

A 60 keV thick target, also used in later coincidence runs, was used for these measurements. Similar running conditions to those described before were used, the target normal being set at $215^\circ$ for Detector I in the forward quadrant of angles, and at $145^\circ$ for the three backward angles. The Singles I and II spectra were recorded in separate groups of 100 channels in the kicksorter memory; a 1 hour run being made at each angle $\theta_1$. Background runs of $\frac{1}{2}$ hour duration were taken immediately after each target run by bombarding a clean tantalum disc. Beam current integration was used as a guide in estimating the beam dependent background as before.

Some early difficulties were experienced with extraneous radiation from the tantalum stop, angular distributions showing a marked rise at backward angles near the stop. A 1" thick lead shield around a freshly cleaned stop almost eliminated this problem (Figure 4.3), however some radiation (attributed to fluorine impurity in the tantalum) was later found to be still present. This required slight additional corrections to the counts of Detector I in the backward direction.
These angular distributions were taken before the start of the coincidence experiment. An angular distribution for the 6.79 MeV radiation symmetrical about $\theta = 90^\circ$ would then indicate the absence of geometrical asymmetries in the equipment.

Footnote.

It now appears that part of the unidentified coincident radiation can be associated with the 278 keV resonance in $^{14}\text{N}(p, \alpha)^{15}\text{O}$ from the semi-thick target. Transitions between levels in $^{15}\text{O}$ corresponding to, 7.55 - 6.79 - 0.0 MeV, 7.55 - 6.16 - 0.00 MeV lead to coincident pairs of gamma rays having energies, 0.76 and 6.79 MeV, 1.39 and 6.16 MeV respectively. These correspond closely to the energies of observed radiations.

The remaining 7.2 MeV gamma ray can probably be identified with the 710 keV $^{15}\text{N}(p, \gamma)^{16}\text{O}$ resonance, which is known to feed a level at 7.12 MeV in $^{16}\text{O}$. 
CHAPTER V

RESULTS OF ANGULAR DISTRIBUTION MEASUREMENTS ON THE DIRECT-CAPTURE AND GROUND STATE RADIATIONS FROM $^{14}\text{N}(p,\gamma)^{15}\text{O}$ AT 800 keV.

5.1 Angular Distribution of the 1.23 MeV Direct-Capture Radiation

Angular distributions were obtained from data collected in 8 hour runs by analysis of the $\gamma$-ray spectra taken at 2 hour intervals. All 20 positions of the two detectors defined by Positions I to IV of Detector II and the five angles $\theta_1 = 0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$ and $90^\circ$ of Detector I were used. In addition runs not directly included in the angular distribution analysis were made for $\theta_1 = 225^\circ$ in Positions III and IV to test for the presence of odd Legendre Polynomials $P_k(\cos \theta_1)$. Similarly additional runs were made at $\theta_1 = 0^\circ$ and $90^\circ$ with $\theta_2 = 70^\circ 6'$ instead of $109^\circ 54'$ ($\phi$ being unchanged), to test for odd Legendre Polynomials $P_k(\cos \theta_2)$. Two angles $\theta_1$ were used in each of these tests to reduce the possibility of any chance cancellation of terms. No evidence for the presence of odd Legendre Polynomials was found in any of these runs.

For each individual run the 1.23 MeV coincidence count and the monitor count of the high energy $^{14}\text{N}(p,\gamma)$ radiation were corrected as discussed in the previous chapter. A normalized set of 8 hour coincidence data was then obtained. Relations
connecting the counts of the monitor in the four positions then enabled all coincidence counts to be normalized to a chosen standard.

Angular distributions were calculated from least squares fits to the coincidence data for the five angles $\Theta_1$. This was done separately for each of the four positions of Detector II as well as for the weighted sum of the four positions which removes the $\Theta_2$ and $\phi$ dependence. The separate angular distributions were found to be all of the same form within statistics (Tables 5.1, 5.2), indicating that there was no significant dependence on the position of Detector II. For this reason the angular distributions for the equally weighted sum of the four positions, which achieves slightly better accuracy, was also calculated.

To test the effect of a variable beam dependent background on the angular distributions, similar calculations were carried out using a $N^{14}(p,\gamma)$ monitor count in which only the time dependent background had been subtracted. These angular distributions once again agreed within statistics with those calculated earlier (see Tables 5.1 to 5.3) indicating that errors made in correcting for the beam dependent background would not significantly alter the angular distributions.

A total of twelve angular distributions were obtained in this way. These were analysed by the method of Rose (Ro 53), and in all cases the statistics due to the counts were in
agreement with the errors deduced from the fit to the data points, the larger of these being quoted in the results. These results still contained the smoothing effect of Detector I. Least squares fits involving \( P_0, P_2 \) and \( P_4 \) terms in \( \Theta_1 \) were first carried out for the twelve data sets. These results gave \( P_4 \) coefficients in all cases which were the same order of magnitude as the statistical uncertainties in these terms, (Table 5.4). Least squares fits involving \( P_0 \) and \( P_2 \) terms in \( \Theta_1 \) were then tried. These indicated that there was no significant difference between corresponding values of \( P_0 \) and \( P_2 \) coefficients in each case, so that analysis in terms of \( P_0 \) and \( P_2 \) coefficients alone was sufficient. Tables 5.1 to 5.3 give the twelve least squares fits analysed in terms of \( P_0 \) and \( P_2 \) coefficients. Table 5.4 included for comparison with Table 5.1 involves \( P_0, P_2 \) and \( P_4 \) terms in the fit.
<table>
<thead>
<tr>
<th>Identification</th>
<th>Least Squares Fit</th>
<th>Error in Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$P_0 - 0.882 P_2$</td>
<td></td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>$P_0 - 0.875 P_2$</td>
<td>$\text{All} \pm 0.045 P_2$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$P_0 - 0.882 P_2$</td>
<td></td>
</tr>
<tr>
<td>$\beta^*$</td>
<td>$P_0 - 0.870 P_2$</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1. Least squares fits involving $P_0$ and $P_2$ terms in $\Theta_1$ for the weighted ($\alpha$) and equally weighted ($\beta$) 32 hour coincidence sums for the four positions of Detector II. These results correspond to normalization with a monitor count based on subtractions of a beam and time dependent background (no asterisk) and on subtraction of only a time dependent background (asterisk).

<table>
<thead>
<tr>
<th>Position</th>
<th>Least Squares Fit</th>
<th>Error in Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$P_0 - 0.865 P_2$</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>$P_0 - 0.864 P_2$</td>
<td>$\text{All} \pm 0.09 P_2$</td>
</tr>
<tr>
<td>III</td>
<td>$P_0 - 0.935 P_2$</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>$P_0 - 0.840 P_2$</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2. Least squares fits involving $P_0$ and $P_2$ terms in $\Theta_1$ for the 8 hour coincidence data from the individual positions I to IV of Detector II. Here the results are based on a beam and time dependent background correction for the monitor.
Position | Least Squares Fit | Error in Coefficient |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$P_0 - 0.867 P_2$</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>$P_0 - 0.826 P_2$</td>
<td>$\pm 0.09 P_2$</td>
</tr>
<tr>
<td>III</td>
<td>$P_0 - 0.931 P_2$</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>$P_0 - 0.830 P_2$</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3. As for Table 5.2, except that the results are based on a time dependent background correction only for the monitor.

<table>
<thead>
<tr>
<th>Identification</th>
<th>Least Squares Fit</th>
<th>Error in Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$P_0 - 0.907 P_2 + 0.041 P_4$</td>
<td></td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>$P_0 - 0.843 P_2 + 0.029 P_4$</td>
<td>$\pm 0.045 P_2$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$P_0 - 0.896 P_2 + 0.034 P_4$</td>
<td>$\pm 0.045 P_4$</td>
</tr>
<tr>
<td>$\beta^*$</td>
<td>$P_0 - 0.847 P_2 + 0.027 P_4$</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.4. Least squares fits involving $P_0$, $P_2$ and $P_4$ terms in $\theta_1$ for the weighted and equally weighted 32 hour coincidence sums for the four positions of Detector II.

The experimental results obtained for the coincidences over 32 hour and 8 hour periods are presented with their least squares fits in Figures 5.1 and 5.2. Here we have selected
Fig. 5-1 Least squares fits for 1.23 MeV coincidence data. The 225° points indicate the absence of appreciable odd Legendre polynomials in $\theta_1$. 

EQUALLY WEIGHTED SUM 
LEAST SQUARES FIT

$1036 - 914 P_2 = 1 - 0.882(\pm 0.045) P_2$

WEIGHTED SUM 
LEAST SQUARES FIT

$1343 - 1185 P_2 = 1 - 0.882(\pm 0.045) P_2$
COINCIDENCE COUNTS IN 8 HRS.

POSITION I
LEAST SQUARES FIT
274 - 237P₂

POSITION II
LEAST SQUARES FIT
309 - 267P₂

POSITION III
LEAST SQUARES FIT
372 - 348P₂

POSITION IV
LEAST SQUARES FIT
382 - 321P₂

θ, DEGREES.

Fig. 5-2
the data corresponding to normalizations with the monitor corrected for both beam and time dependent background.

As the result we present the least squares fit for the equally weighted 32 hour coincidence data (\( \alpha \) of Table 5.1) which involves beam and time dependent background corrections, i.e.

\[
1 - (0.882 \pm 0.045)P_2(\cos \theta_1)
\]

The required angular distribution of the 1.23 MeV radiation is obtained from this after allowance has been made for the smoothing effect of Crystal I. Using the curves given by Marion (Ma 60a) we find that with the experimental geometry used, detection of the 1.23 MeV radiation leads to a smoothing factor of 0.900 for the \( P_2 \) coefficient. We then obtain the required angular distribution as,

\[
W(\theta_1) = 1 - (0.974 \pm 0.050)P_2(\cos \theta_1)
\]

Because the form of the angular distribution does not depend on the orientation of Detector II, further runs were made with Detector II in a fixed position as close to the target as possible. Detector I was also moved in as close as possible. The limitation on distance is governed by the necessity to avoid detecting, in one crystal, Compton-scattered gamma-rays from the other crystal. The positions chosen for the crystals were Detector I at 4\( ^\circ \), Detector II at 1\( \frac{3}{2} \)\( ^\circ \) and
inclined at $\Theta_2 = 110^0$, $\varphi = 135^0$. We refer to this chosen position for Detector II as Position V. In Position V, calculations indicated that slightly less than 1% of the coincidences recorded in the 1.23 MeV summation region would be due to Compton events. This was considerably smaller than the best statistics on the counts, (about 4% at $90^0$). A factor of 7 increase in the counting rate was achieved in this position so that with 4 hour runs undertaken at each of the five angles $\Theta_1$, similar counting statistics were obtained as for the previous 32 hour total. Figures 5.3 and 5.4 illustrate the Singles II and Coincidences I spectra taken for the 4 hour run at $\Theta_1 = 90^0$.

The experimental data was analysed as before, this time only monitor counts corrected for beam and time dependent backgrounds were used. The ratio of background to true counts was smaller than before by a factor of four. Least squares analyses with terms involving $P_0$, $P_2$ and $P_4$ were carried out, but once again terms involving only $P_0$ and $P_2$ appeared sufficient. Figure 5.5 represents the least squares fit involving $P_0$ and $P_2$ coefficients for the 4 hour coincidence data taken in the Position V runs. The statistics of the counts were again found to be in agreement with the errors deduced from the fit to the data points. A 2 hour run at $\Theta_1 = 135^0$ indicated as before the absence of appreciable odd order $P_k(\cos \Theta_1)$ terms.
Fig. 5.3

Fig. 5.4. 90° Position Y coincidence spectra for 4-Hr run. Inset Coincidences II spectrum.
Fig 5-5.
The least squares fit to the Position V data was,

\[ 1 - (0.812 \pm 0.042)P_2(\cos \theta) \]

The smoothing factor for this geometry was 0.840 for the \( P_2 \) coefficient which gives for the angular distribution,

\[ W(\theta) = 1 - (0.967 \pm 0.050)P_2(\cos \theta) \]

The mean value for the Positions I to IV and the Position V data is then,

\[ W(\theta) = 1 - (0.97 \pm 0.03)P_2(\cos \theta) \quad (5.1) \]

This is the required angular distribution of the 1.23 MeV radiation to the 6.79 MeV level in \( ^{15}O \), at a proton bombarding energy of 800 keV.

5.2 Angular Distributions of Ground State Radiations.

Angular distributions of the main 6.79 MeV radiation and the weaker 8.05, 6.16 and 5.2 MeV radiations were determined by analyses of selected energy regions in the Singles I spectra. These spectra were obtained experimentally at \( E_p = 800 \) keV for the angles \( \theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 135^\circ \) and \( 148^\circ \) as described in section 4.8.

Figures 5.6 and 5.7 illustrate the Singles I and Singles II spectra obtained in a typical 1 hour run, together with their corresponding backgrounds taken by bombarding a tantalum blank.
Fig. 5-6 Typical Singles I spectrum showing the summation regions for the four gamma rays.

Fig. 5-7 Typical Singles II spectrum showing the monitor summation region.
The energy regions were chosen so that, as far as possible, the main contribution in each corresponded to one of the four γ-rays. These regions illustrated in Figure 5.6 were as follows,

- $8.05 \text{ MeV} = 7.0 - 8.2 \text{ MeV}$
- $6.79 \text{ MeV} = 6.3 - 7.0 \text{ MeV}$
- $6.16 \text{ MeV} = 5.3 - 6.3 \text{ MeV}$
- $5.2 \text{ MeV} = 4.85 - 5.5 \text{ MeV}$

Summation over the $8.05 \text{ MeV}$ region contains counts due only to the pure γ-ray (plus background), but the other regions contain counts from higher energy gamma-rays in varying amounts, which eventually have to be subtracted out. Because of the relatively poor statistics in the Singles I spectra the gamma-ray subtractions were left until after least squares fitting had been carried out on the data. Analysis with the gamma-ray subtractions done first leads to less reliable angular distributions because of the large fluctuations from the small number of counts which remain.

The monitor count from the Singles II spectrum was taken over the energy range $5.4 - 8.0 \text{ MeV}$ as illustrated in Figure 5.7. Corrections for the time dependent backgrounds were made from comparison with a standard background spectrum. Beam dependent backgrounds were obtained directly from beam current normalizations of the corresponding tantalum background runs.

The background corrections for the Singles I spectra were
determined separately for each of the summation regions of the four gamma-rays. The normalized tantalum backgrounds were fitted by least squares from $0^\circ - 90^\circ$ and joined by a smooth curve drawn through the rising background (caused by the stop radiation) at the three backward angles. The $0^\circ - 90^\circ$ background in all regions appeared to be isotropic. The anisotropic background for a given run was determined as an excess above this constant background after normalizing to a standard current integrator reading. Corrections where necessary were also made for small gain shifts and for target absorption in a similar way to the coincidence runs.

In Table 5.5 are listed the results obtained for the least squares fits involving $P_0$ and $P_2$ terms in $\Theta_1$, over the range $0^\circ - 90^\circ$ in the four gamma-ray regions. These are also illustrated together with the experimental results in Figure 5.8
Fig 5-8
Table 5.5. Least squares fits from the 1 hour Singles I spectra over the range 0° - 90° for the N°*(p, γ)°* ground state radiations included in the four summation regions. The results quoted still contain contributions from the isotropic beam dependent background.

The errors shown in the table were deduced from the statistics of the counts and are consistent with the errors deduced from goodness of fit tests over the full range of angles 0° - 148° with a single exception. For the angular distribution obtained for the 5.2 MeV region, the goodness of fit disclosed errors significantly larger than expected, probably due to the inclusion of a variable part of the 4.43 MeV gamma-ray spectrum. An error of 0.1 channel in the boundaries of the energy range begins to be significant here. The results as they stand point to isotropic distributions for the four gamma-rays. However, we still calculate the angular distributions for the pure
radiations so that their relative intensities may be estimated.

Subtractions are first made to remove the contribution of the 8.05 MeV radiation in each of the lower energy summation regions. This is done by subtracting a suitably weighted least squares fit for the 8.05 MeV radiation from each of the lower energy fits in turn. The separate weighting factors for the three energy regions involved, are obtained with the aid of the line shape of an 8.05 MeV gamma-ray. The relative fraction of counts appearing in the 8.05 MeV summation region and each of the other regions then give the required weighting factors. The $^{13}\text{C} (p, \gamma) ^{14}\text{N}$ resonance at 560 keV provides a convenient 8.05 MeV gamma-ray of sufficiently high intensity, this being measured under the same geometry conditions. Figure 5.9 illustrates the Singles I spectrum obtained for a 10 keV thick $^{13}\text{C}$ target at this resonance, and the estimate for a pure 8.05 MeV line shape.

These subtractions then leave angular distributions for a pure 6.79 MeV gamma-ray and for mixtures of $(6.16 + 6.79)$ MeV and $(5.2 + 6.16 + 6.79)$ MeV gamma-rays in the two remaining energy regions. A similar procedure was carried out to subtract the contributions from the 6.79 and 6.16 MeV radiations in turn. An additional line shape to aid these subtractions was taken from the 598 keV $^{19}\text{F} (p, \alpha \gamma) ^{16}\text{O}$ resonance, which provides a clean 6.13 MeV gamma-ray, illustrated in Figure 5.10. Interpolation between the 6.13 and 8.05 MeV gamma-ray spectra gave
Fig. 5-9
$\text{C}^{13}(p,\gamma)\text{N}^{14}$ 560 KeV RESONANCE.

COUNTS

KICKSORTER CHANNEL

ESTIMATED LINE SHAPES OF A PURE 0.05 MeV GAMMA RAY.

Fig. 5-10
$\text{F}^{19}(p,\gamma)\text{O}^{16}$ 598 KeV RESONANCE

COUNTS

KICKSORTER CHANNEL
the required 6.79 MeV line shape.

The final angular distributions which result for the pure gamma-rays are listed in Table 5.6. These have been corrected for the smoothing effect of Crystal I.

<table>
<thead>
<tr>
<th>Gamma-Ray Energy</th>
<th>Least Squares Fit</th>
<th>Error in Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.05 MeV</td>
<td>$1137 - 13P_2 = 1 - .01P_2(\Theta_1)$</td>
<td>$\pm .07 P_2$</td>
</tr>
<tr>
<td>6.79 MeV</td>
<td>$3220 + 10P_2 = 1 + .00P_2(\Theta_1)$</td>
<td>$\pm .03 P_2$</td>
</tr>
<tr>
<td>6.16 MeV</td>
<td>$853 + 20P_2 = 1 - .02P_2(\Theta_1)$</td>
<td>$\pm .15 P_2$</td>
</tr>
<tr>
<td>5.2 MeV</td>
<td>$670 + 17P_2 = 1 + .02P_2(\Theta_1)$</td>
<td>$\pm .20 P_2$</td>
</tr>
</tbody>
</table>

Table 5.6. Corrected angular distributions for the four ground state gamma-rays from $^{14}\text{N}(p, \gamma)^{15}\text{O}$ at 800 keV bombarding energy. The errors quoted are derived from the statistics of the least squares fits to the four gamma-rays in Table 5.5.

These results are all consistent with isotropic angular distributions as deduced before.

5.3 Relative Intensities of Ground State Radiations at 800 keV.

The relative intensities of the four gamma-rays can be obtained directly from the $P_0$ coefficient of the least squares fits in Table 5.6, allowance is made for the fractional intensity of the gamma rays appearing in the respective summation regions.

The fractional intensity of the 8.05, 6.79 and 6.16 MeV
gamma-rays were determined with the help of the previous line shapes. For a chosen gamma ray this required determining (from the line shape of the pure radiation), what fraction of the total number of counts in the spectrum appeared in the corresponding summation region. In the case of the 5.2 MeV radiation to obtain a satisfactory line shape it was necessary to interpolate between the 4.43 and 6.13 MeV spectra.

In Table 5.7 we list the relative intensities of the four gamma rays calculated from the least squares fits and the estimated fractional intensities. An estimate for a 6.79 + 1.23 MeV sum-peak contribution was subtracted from the 8.05 MeV radiation before arriving at these results.

The relative intensities for these gamma rays were also calculated from a separate singles spectrum obtained from a 2 hour run with Detector II set close in at Position V.

After subtraction of a constant background, starting from the highest energy radiation, successive gamma-ray subtractions were made from the spectrum with the aid of the line shapes of the 8.05 and 6.13 MeV gamma-rays, and the estimated line shape for the 5.2 MeV gamma-ray. The subtractions were estimated from the size of the peaks and carefully adjusted so that any counts remaining in the region of the full energy peaks could be accounted for by statistical fluctuations about the zero. Figure 5.11 illustrates the composite Singles II spectrum, and
Fig. 5.11

Counts

Kicksorter Channel
the relative intensities of the individual gamma-ray components used in the subtractions.

The relative intensities of these gamma-rays can then be determined from this break-up. A small correction for a sum-peak was again made to the 8.05 MeV spectrum before arriving at the final results. These are listed in Table 5.7 together with the averaged relative intensities from the two determinations.

<table>
<thead>
<tr>
<th>Gamma Ray Energy</th>
<th>Relative Intensity of Gamma Rays</th>
<th>Averaged Relative Intensities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>From Single Spectrum Decomposition</td>
<td>From Angular Distributions</td>
</tr>
<tr>
<td>8.05 MeV</td>
<td>15.5%</td>
<td>16.4%</td>
</tr>
<tr>
<td>6.79 MeV</td>
<td>52.7%</td>
<td>56.9%</td>
</tr>
<tr>
<td>6.16 MeV</td>
<td>17.5%</td>
<td>11.9%</td>
</tr>
<tr>
<td>5.2 MeV</td>
<td>14.3%</td>
<td>14.8%</td>
</tr>
</tbody>
</table>

Table 5.7 Relative intensities of the four ground state radiations from the $^{14}_N(p,\gamma)^{15}_O$ reaction at $E_p = 800$ keV.

The errors quoted for the averaged intensities are subjective estimates only and based on the method of carrying out the gamma-ray subtractions.

5.4 Discussion of Results.

The angular distribution of the 1.23 MeV radiation measured
as \( 1 - .97 P_2 \) is very nearly the \( \sin^2 \Theta \left[ \frac{2}{3} (1 - P_2) \right] \) distribution predicted by theory in Chapter III. This suggests that at 800 keV bombarding energy the \( N^{14}(p, \gamma)O^{15} \) non-resonant radiation to the 6.79 MeV state in \( O^{15} \) is almost entirely the result of a direct-capture process, involving a p to s transition.

Since the target core is not appreciably perturbed in a direct-capture process, it is reasonable to expect that the capturing state will possess a core largely composed of the \( N^{14} \) ground state wave function. The experimental results then suggest that, in that part of the 6.79 MeV wave function associated with the \( N^{14} \) ground state core, the additional proton is largely in an s-state. The result is not inconsistent with Halbert and French's (Ha 57b) structure of the lowest \( \frac{3}{2}^+ \) level in the \( A = 15 \) group.

In principle an estimate of the d-wave intensity in the wave function can be made by comparing the experimental angular distribution with the theoretical expression (3.11). However this is hardly worthwhile since there is no certainty that the non-\( \sin^2 \Theta \) part of the angular distribution is due to p to d capture as (3.11) would suggest. Furthermore a detailed calculation would be required to determine the ratio \( k_1^2/k_2^2 \) in (3.11) before the d-wave intensity could be extracted.

The isotropic angular distributions measured for the four ground state gamma-rays can be explained satisfactorily on the
The 6.79 MeV radiation.

Direct-capture to the 6.79 MeV level has been established. This predominantly involves capture of s-wave protons which dictates an isotropic angular distribution for the 6.79 MeV radiation.

The 8.05 MeV radiation.

On the shell model the $\frac{1}{2}^-$ ground state of $^{15}O$ would be expected to be formed by addition of a p-wave proton to the ground state of $^{14}N$, forming the configuration $(1s_\frac{1}{2}^2)^4 (1p_{3/2})^8 (1p_{\frac{1}{2}})^3$ which corresponds to a $(1p_{\frac{1}{2}})^{-1}$ hole in an otherwise complete $^{16}O$ ground state core. The 8.05 MeV radiation would then correspond to E1 emission to the ground state, following an s to p transition. This would result in an isotropic angular distribution for the radiation.

The 5.2 MeV radiation.

If one assumes that the $\frac{1}{2}^+$ 5.18 MeV level in $^{15}O$ is the one involved in the capture process then this would be expected to be a mixture involving the ground state of $^{14}N$ and an additional $(2s_\frac{1}{2})$ or $(1d_{3/2})$ proton, having configurations $(1s_\frac{1}{2})^4 (1p_{3/2})^8 (1p_{\frac{1}{2}})^2 (2s_{\frac{1}{2}})^1$ and $(1s_{\frac{1}{2}})^4 (1p_{3/2})^8 (1p_{\frac{1}{2}})^2 (1d_{3/2})^1$ respectively. The main direct-capture to this level might then be predicted to
follow a p to s transition as for the 6.79 MeV level, and give a $\sin^2 \Theta$ angular distribution. The 5.2 MeV ground state radiation would then be isotropic from both the $\frac{3}{2}^+$ spin assignment and the s-wave formation of the 5.18 MeV level. Alternatively one can assume that the 5.24 MeV level is the one involved, this being presumably the $\frac{5}{2}^+$ partner to the 5.28 MeV level in N$^{15}$. The part of the wave function appropriate to our direct-capture model would be expected to be composed of a N$^{14}$ ground state core with a mixture of $(1d_{5/2})$ and $(1d_{3/2})$ orbits for the additional proton. A p to d direct-capture process to form the 5.24 MeV level would then be allowed. In this case the ground state radiation would probably not be isotropic. The large errors involved in the present measurements do not exclude this possibility. More careful measurements of the gamma-ray energy would be required to establish the identity of the 5.2 MeV radiation.

The 6.16 MeV radiation.

The initial transition to the 6.16 MeV $\frac{3}{2}^-$ level in O$^{15}$ is more difficult to explain. However as one knows that the N$^{14}$ ground state as well as being composed of mainly $(p_{\frac{3}{2}})^{-2}$ holes in the O$^{16}$ ground state core, also consists of a small proportion of $(p_{\frac{3}{2}})^{-1} (p_{3/2})^{-1}$ and $(p_{3/2})^{-2}$ holes, then p-wave proton capture leaving a $(p_{3/2})^{-1}$ hole can account for the formation of this level. The form of the N$^{14}$ ground state wave function has been
studied by a number of workers in attempts to explain the long $\beta^-$-decay half life of $^{14}$C. Inglis (In 53), Jancovici and Talmi (Ja 54), Elliot (El 56) and Visscher and Ferrell (Vi 57) have presented theoretical wave functions for $^{14}$N whilst Sherr et al. (Sh 55) and Swann et al (Sw 61) have placed experimental limits on the wave function components. Some disagreement exists about the actual ground state composition. This has been discussed by MacFarlane and French (Ma 60b). For our present purposes we have considered the results of Visscher and Ferrell as being representative.

For the $^{14}$N ground state they give the wave function expressed in $L S$ coupling in the form,

$$\Psi(\text{LS}) = 0.920 \Psi(3D_1) + 0.355 \Psi(1P_1) + 0.173 \Psi(3S_1)$$

This may be transformed into an equivalent $jj$ coupled representation using a transformation formula such as that given by Rose (Ro 58b). These calculations show that the relative intensities of $(p_2^2)^{-2}$, $(p_2^2)^{-1}(p_{3/2})^{-1}$ and $(p_{3/2})^{-2}$ configurations in the $^{14}$N ground state are 85%, 13% and 1.4% respectively. These results serve to illustrate that formation of a $(p_{3/2})^{-1}$ hole by p-wave capture to the 6.16 MeV state is a possibility. On this model the direct-capture radiation to the 6.16 MeV state would correspond to an $s$ to $p$ transition, resulting in isotropic angular distributions for both initial and ground state radiations.
The direct-capture model can account for the 6.16, 5.2 and 8.05 MeV radiations but without further analysis of the experimental results there is no evidence to favour this mechanism over compound nucleus formation. Cascade radiations through the 6.79 MeV state may be ruled out. Previous $^{14}\text{N}(p,\gamma)$ work shows this level radiates entirely to the ground state. However possible contributions from the tails of prominent $^{14}\text{N}(p,\gamma)$ resonances (in particular the 280 keV resonance) need further investigation before any conclusions can be drawn.
CHAPTER VI

DETERMINATION OF CROSS SECTIONS FOR THE 6.79, 6.16, 5.2 MeV AND GROUND STATE GAMMA RAYS FROM N\(^{14}\)(p,\(\gamma\)) FOR PROTON ENERGIES IN THE RANGE 1070 – 300 KEV.

6.1 Experimental Measurements

Theoretical estimates of the cross sections for the 6.16, 5.2 and ground state radiations from N\(^{14}\)(p,\(\gamma\)) have been made assuming these are due to contributions from tails of nearby resonances. When compared with the experimental cross sections at 800 keV it is found that they are all too small by a factor of about ten. This large difference seemed to indicate that direct-capture involving the 6.16, 5.2 and ground state levels as well as the 6.79 MeV level was of importance in contributing to the N\(^{14}\)(p,\(\gamma\)) cross section. To obtain a reliable extrapolation of the \(\gamma\)-ray cross section it therefore became necessary to determine the cross section variation for each of the separate \(\gamma\)-rays. This was done by measuring the yield of the four \(\gamma\)-rays for a range of proton energies.

The gamma-ray yields were determined from analyses of spectra recorded in a random order for proton energies corresponding to 1070, 1000, 890, 800, 700, 600, 500, 400, 320, and 300 keV in the laboratory system. The \(\gamma\)-ray spectra were recorded by a single 5" x 4" crystal-Detector I, chosen for its superior
resolution. The crystal was set 2" from the target and at a fixed angle $\Theta_1 = 55^\circ$ to the proton beam, this being a zero of $P_2(\cos \Theta_1)$. The gain of the system was adjusted so that 200 channels in the kicksorter memory covered the energy range 0 - 9 MeV. Provision was made to record the 200-channel spectrum on punched tape for feeding directly into a computer, as well as recording it as a normal print-out.

A single tantalum nitride target of thickness 65 keV for 450 keV protons was used for these runs. The heat dissipation was kept at about 10 Watts whenever possible by varying the beam current with energy. This enabled the stability of the target to be readily monitored by repeat runs at a standard energy. This target had been bombarded previously for about 60 hours under similar conditions but still gave a satisfactory yield. No obvious loss in nitrogen content of the target was observed during the entire set of runs. Beam current integration was used to monitor the reaction, runs being usually two or three hours duration depending on the yield.

A prolonged run was attempted at 210 keV, using a fresh target of thickness 90 keV and accumulating the spectrum for 20 hours. However the beam focusing properties of the accelerator at this energy proved to be inadequate. The maximum beam current of 12 $\mu$amp available at the target gave a yield only slightly above room background. Estimates showed
that in the 20 hours only 2,500 additional counts were recorded above background in the energy range 5 - 8 MeV, and no obvious conclusions could be drawn from this.

Using the same detector geometry a series of standard spectra were also recorded for later use in the spectrum analysis. These consisted of the $^{13}\text{C}_1(p,\gamma)^{14}\text{N}$, $^{19}\text{F}_1(p,\alpha\gamma)^{16}\text{O}$, and $^{15}\text{N}_1(p,\alpha\gamma)^{12}\text{C}$ high energy $\gamma$-ray standards obtained as before and the $^{12}\text{C}_1(p,\gamma)^{13}\text{N}$, $^{22}\text{Na}$, and $^{137}\text{Cs}$ low energy $\gamma$-ray standards. In addition standard room background and prompt neutron background spectra were also recorded. The latter spectrum (with the room background contribution removed) was used to allow for the detection of machine produced neutrons.

6.2 Analysis of Complex Gamma-Ray Spectra

The spectrum analysis was programmed for a 1620 computer. Basically this involved breaking up a complex spectrum into a series of standard components having the required relative intensities by using a least squares method. The method is in principle the same as that described by Rose (Ro 53) for the analysis of angular distribution data in terms of Legendre functions. Here, the standard components take the place of the Legendre functions and the least squares fitting is done to minimize the residual counts in a selected region of the spectrum.
The complex spectrum of 200 channel extent is fed directly into the computer as recorded and stored as the 200-row data matrix \( D \), having elements \( D_i \) \((i = 0 - 199)\). Associated with this data but fed in separately are suitable channel weights \( W_i \).

The \( n \) separate component spectra having gains and zeros adjusted to match the complex spectrum being fitted are accepted one at a time, being stored as rows of an \( n \times 200 \) component matrix \( A \), having elements \( A_{in} \). The intensities of the components \( I_n \) which are the most probable values for the fit to the complex spectrum over the range \( i = i_1 \) to \( i_2 \) are determined from the least squares condition,

\[
\sum_{i=i_1}^{i_2} W_i(D_i - \sum_n I_n A_{in})^2 = \text{minimum}
\]

In addition to calculating the intensities \( I_n \), the mean square deviations in \( I_n \) and the individual fitted channel residuals are also calculated. This latter calculation enables the fitted spectrum to be plotted for comparison with the original complex spectrum.

Before this analysis could be performed it was necessary to generate the various \( \gamma \)-ray components having the required energies, gains and zeros. This was done with two additional programmes. The first of these generated pure \( \gamma \)-ray components from zero up to the required maximum energy by interpolating between standard spectra of energies 8.060, 6.135, 4.433, 2.365,
1.277, and 0.662 MeV \((\text{C}^{13}(p,\gamma)), \text{F}^{19}(p,\alpha\gamma), \text{N}^{15}(p,\alpha\gamma)\), \(\text{C}^{12}(p,\gamma), \text{Na}^{22}, \) and Cs\(^{137}\) spectra respectively). The second programme was designed to produce a folded spectrum of two \(\gamma\)-rays to allow for the simultaneous detection of two coincident \(\gamma\)-rays in the crystal. For each bombarding energy the \(\text{N}^{14}(p,\gamma)\) radiations of energy 6.79, 6.16 and 5.2 MeV were individually folded with their corresponding coincident \(\gamma\)-rays. These folded spectra were then used in the complex spectrum analysis. For the geometry used the efficiency of detecting the low energy \(\gamma\)-ray simultaneously with the high energy \(\gamma\)-ray (and hence the fraction of this low energy component used in the folding) was about 10\%. Equal weights were assigned to similar regions in the spectrum where the number of counts did not change appreciably. The reciprocal of the average number of counts in a given region was then used as the representative weight.

We illustrate typical results of the complex spectrum analysis with reference to the 3 Hr. runs at 800 and 600 keV. Figures 6.1 and 6.2 show the least squares fits to the experimental points and the position of the full energy peaks of the \(\gamma\)-ray components used in the analyses. These fits represents the efforts of a number of trials in which each of the prominent \(\gamma\)-ray peaks were individually fitted as closely as possible (within a fraction of a channel) by slight changes in the zeros of the component \(\gamma\)-rays.
Fig. 6.1 Least squares fit of γ-ray components to complex spectrum.
Fig. 6.2 Least squares fit to complex spectrum.

$E_p = 600 \text{ KeV}$

FIT CHANNELS 85 - 190.
being fed in. The range of fit to these and the other spectra were chosen to include as far as possible only the main contributions from the N\textsuperscript{14}(p,\gamma) gamma rays. Since it became desirable to determine the yield of 4.43 MeV radiation for a number of the runs a convenient lower bound for the fit was chosen below the full energy peak of this \gamma-ray.

Before a complex spectrum was analysed a smoothed time dependent room background was first subtracted from the raw data. Component \gamma-rays used in the fitting programme were as follows,

(i) neutron background spectrum
(ii) high energy \gamma-ray tail, consisting of a flat spectrum which allowed for possible \gamma-rays from the 1028 keV. N\textsuperscript{15}(p,\gamma\textsubscript{0}) resonance and from B\textsuperscript{11}(p,\gamma) radiation from the glass target chamber
(iii) 4.433 MeV, N\textsuperscript{15}(p,\alpha\gamma\textsubscript{1}) \gamma-ray with suitable gain and zero
(iv) 5.200 MeV
(v) 6.160 MeV suitably folded N\textsuperscript{14}(p,\gamma) components
(vi) 6.790 MeV
(vii) ground state N\textsuperscript{14}(p,\gamma\textsubscript{0}) \gamma-ray with suitable gain and zero.

Initially a \gamma-ray component was included in all fits to test for the presence of radiation from the 560 keV. C\textsuperscript{13}(p,\gamma\textsubscript{0}) resonance, due to possible carbon build up on the target. Only
for runs in the vicinity of the resonance energy was it necessary to retain this component in attempting later improved fits.

Inclusion of two additional $\gamma$-ray components in analysing the 700 and 800 keV data to allow for the detection of $N^{15}(p,\gamma\gamma)$ radiation from the 710 keV resonance gave considerably improved fits. This did not appreciably alter the intensities of the $N^{14}(p,\gamma)$ components or their statistical errors. This is illustrated in Fig. 6.1 for the 800 keV run. It is notable that the programme is sensitive to feeding in components not actually present in the complex spectrum. In this case the statistical errors associated with the intensities of the known gamma rays are made worse.

In table 6.1 we list the "best values" from these analyses of the $N^{14}(p,\gamma)$ yield, normalized to a proton charge of 60,000 $\mu$C, and including a correction for the kicksorter dead time.

For convenience we also include the resulting non-resonant cross-section factors in keV.barn which emerge from the target analysis discussed in the following sections.
<table>
<thead>
<tr>
<th>Ep(Lab.) keV</th>
<th>Ground State Yield per 60,000 $\mu$C and Cross-Section Factor keV.b.</th>
<th>5.2 MeV State</th>
<th>6.16 MeV State</th>
<th>6.79 MeV State</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1070</td>
<td>1,090,000 $\pm$ 13,200</td>
<td>862,000 $\pm$ 9,500</td>
<td>95,100 $\pm$ 8,500</td>
<td>81,900 $\pm$ 8,600</td>
<td>2,130,000</td>
</tr>
<tr>
<td>1000</td>
<td>38,400 $\pm$ 1,000</td>
<td>17,900 $\pm$ 1,500</td>
<td>22,000 $\pm$ 1,300</td>
<td>69,500 $\pm$ 1,500</td>
<td>147,700</td>
</tr>
<tr>
<td></td>
<td>0.451 $\pm$ 0.012 keV.b</td>
<td>0.210 $\pm$ 0.016 keV.b</td>
<td>0.257 $\pm$ 0.016 keV.b</td>
<td>0.817 $\pm$ 0.018 keV.b</td>
<td>1.73 $\pm$ 0.03 keV.b</td>
</tr>
<tr>
<td>890</td>
<td>20,000 $\pm$ 1,000</td>
<td>9,400 $\pm$ 1,600</td>
<td>21,400 $\pm$ 1,500</td>
<td>59,500 $\pm$ 1,700</td>
<td>109,100</td>
</tr>
<tr>
<td></td>
<td>0.316 $\pm$ 0.016 keV.b</td>
<td>0.149 $\pm$ 0.025 keV.b</td>
<td>0.340 $\pm$ 0.024 keV.b</td>
<td>0.928 $\pm$ 0.026 keV.b</td>
<td>1.73 $\pm$ 0.05 keV.b</td>
</tr>
<tr>
<td>600</td>
<td>14,500 $\pm$ 650</td>
<td>7,100 $\pm$ 1,200</td>
<td>15,200 $\pm$ 1,400</td>
<td>43,400 $\pm$ 1,200</td>
<td>80,000</td>
</tr>
<tr>
<td></td>
<td>0.315 $\pm$ 0.014 keV.b</td>
<td>0.157 $\pm$ 0.026 keV.b</td>
<td>0.336 $\pm$ 0.031 keV.b</td>
<td>0.959 $\pm$ 0.026 keV.b</td>
<td>1.77 $\pm$ 0.05 keV.b</td>
</tr>
<tr>
<td>700</td>
<td>11,500 $\pm$ 600</td>
<td>7,100 $\pm$ 1,200</td>
<td>20,500 $\pm$ 1,100</td>
<td>30,200 $\pm$ 950</td>
<td>69,400</td>
</tr>
<tr>
<td></td>
<td>0.390 $\pm$ 0.027 keV.b</td>
<td>0.240 $\pm$ 0.037 keV.b</td>
<td>0.694 $\pm$ 0.037 keV.b</td>
<td>1.023 $\pm$ 0.032 keV.b</td>
<td>2.35 $\pm$ 0.07 keV.b</td>
</tr>
<tr>
<td>600</td>
<td>3,160 $\pm$ 550</td>
<td>5,100 $\pm$ 600</td>
<td>14,500 $\pm$ 550</td>
<td>18,100 $\pm$ 550</td>
<td>41,200</td>
</tr>
<tr>
<td></td>
<td>0.228 $\pm$ 0.033 keV.b</td>
<td>0.310 $\pm$ 0.037 keV.b</td>
<td>0.870 $\pm$ 0.033 keV.b</td>
<td>1.100 $\pm$ 0.033 keV.b</td>
<td>2.50 $\pm$ 0.07 keV.b</td>
</tr>
<tr>
<td>500</td>
<td>1,370 $\pm$ 560</td>
<td>3,640 $\pm$ 970</td>
<td>12,000 $\pm$ 1,100</td>
<td>10,200 $\pm$ 900</td>
<td>27,400</td>
</tr>
<tr>
<td></td>
<td>0.175 $\pm$ 0.072 keV.b</td>
<td>(0.461 $\pm$ 0.173 keV.b)</td>
<td>(1.525 $\pm$ 0.141 keV.b)</td>
<td>(1.296 $\pm$ 0.115 keV.b)</td>
<td>(3.48 $\pm$ 0.23 keV.b)</td>
</tr>
<tr>
<td>400</td>
<td>1,720 $\pm$ 140</td>
<td>5,360 $\pm$ 250</td>
<td>17,500 $\pm$ 230</td>
<td>8,760 $\pm$ 190</td>
<td>33,300</td>
</tr>
<tr>
<td></td>
<td>0.664 $\pm$ 0.054 keV.b</td>
<td>(2.064 $\pm$ 0.96 keV.b)</td>
<td>(6.750 $\pm$ 0.90 keV.b)</td>
<td>(3.365 $\pm$ 0.72 keV.b)</td>
<td>(12.80 $\pm$ 1.6 keV.b)</td>
</tr>
<tr>
<td>320</td>
<td>8,870 $\pm$ 800</td>
<td>29,900 $\pm$ 1,000</td>
<td>108,000 $\pm$ 1,300</td>
<td>42,500 $\pm$ 1,200</td>
<td>189,300</td>
</tr>
<tr>
<td>300</td>
<td>8,170 $\pm$ 960</td>
<td>32,600 $\pm$ 1,100</td>
<td>120,000 $\pm$ 1,300</td>
<td>49,900 $\pm$ 1,200</td>
<td>211,000</td>
</tr>
</tbody>
</table>

Table 6.1. Yield of 5.2, 6.16, 6.79 MeV and ground state $\gamma$ -rays and the corresponding cross section factors for $^{11}B(p,\gamma)$. The errors quoted include only those from the least squares spectrum analysis. The results at 500 and 400 keV must be reduced to allow for contributions from the 276 keV resonance in the semi-thick target.
The relative $\gamma$-ray yields at 1070 and 300 keV from this semi-thick target are largely due to $N^{14}(p, \gamma)$ radiation from the 1058 and 278 keV resonances respectively. Following subtractions of estimates for non-resonant radiation these can be compared with the results of other workers at these resonances. We use the results of Hebbard and Povh (He 59) at the 1058 keV resonance and of Tabata and Okano (Ta 60) at the 278 keV resonance for the comparisons. These are listed in the table below which also includes a comparison with the earlier manual analysis at 800 keV.

<table>
<thead>
<tr>
<th>Workers</th>
<th>Energy</th>
<th>Relative Yields of Gamma Rays</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>keV</td>
<td>Ground State</td>
</tr>
<tr>
<td>Hebbard &amp; Povh</td>
<td>1058</td>
<td>57%</td>
</tr>
<tr>
<td>Present work</td>
<td>1070</td>
<td>53.2 ± .8%</td>
</tr>
<tr>
<td>Tabata &amp; Okano</td>
<td>278</td>
<td>3.0 ± .2%</td>
</tr>
<tr>
<td>Present work</td>
<td>300</td>
<td>3.7 ± .4%</td>
</tr>
<tr>
<td>Manual analysis</td>
<td>800</td>
<td>16 ± 5%</td>
</tr>
<tr>
<td>Present results</td>
<td>800</td>
<td>17.9 ± .8%</td>
</tr>
</tbody>
</table>

These results show quite good agreement. In particular we observe the reproduction of the small 3% ground state intensity of the 278 keV resonance, although approximately 80% of the pulses in the full energy peak are caused by coincident detection of two $\gamma$-rays in cascade.
The series of spectrum fits have been used as energy calibrations to see if there was any evidence favouring population of either the 5.24 or 5.18 MeV levels at different bombarding energies. The energy difference between $\gamma$-rays from these two levels would correspond to a peak shift of a little over a channel and this could easily be observed provided the fit to the 5.2 MeV peak was good. In table 6.2 we present a summary of this investigation.

<table>
<thead>
<tr>
<th>Ep(lab.) keV</th>
<th>&quot;5.2&quot; $\gamma$-ray MeV</th>
<th>Comments on Energy Extracted from Spectrum fit.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1070</td>
<td>5.24</td>
<td>Reliable fits. 8.28 MeV level in 0$^{15}$ (1058 keV resonance) feeding 5.24 MeV level.</td>
</tr>
<tr>
<td>1000</td>
<td>5.24</td>
<td></td>
</tr>
<tr>
<td>890</td>
<td>5.20-5.21</td>
<td>Reliable energy determination, fit good.</td>
</tr>
<tr>
<td>800</td>
<td>(5.25)</td>
<td>Fits unreliable as an energy calibration due to presence of 5 MeV N$^{15}$($p,\gamma\gamma\gamma$) components.</td>
</tr>
<tr>
<td>700</td>
<td>(5.22)</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>5.20</td>
<td>Reliable energy determinations, fits good.</td>
</tr>
<tr>
<td>500</td>
<td>5.21</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>5.20</td>
<td></td>
</tr>
<tr>
<td>320</td>
<td>-</td>
<td>No reliable energy estimate due to absence of 4.43 MeV peak.</td>
</tr>
<tr>
<td>300</td>
<td>-</td>
<td>Expect 7.56 MeV level (278 keV resonance) to feed 5.18 MeV level.</td>
</tr>
</tbody>
</table>

Table 6.2. Investigation of N$^{14}$($p,\gamma$) radiation from the 5.2 MeV doublet in 0$^{15}$. 
It would appear from these results that the production of 5.2 MeV non-resonant radiation is due to populating the 5.18 MeV $^{1/2+}$ state in $^0\text{^15}$ more strongly than the 5.24 MeV $(5/2^+)$ state. In addition we note that at the 1058 keV resonance the 5.24 MeV level is populated. This is in accordance with the results of Gorodetzky et al. (Go 57) who measured a non-isotropic angular correlation for the cascade radiation through the 5.2 MeV doublet.

6.3 Determination of $N^{14}(p,\gamma)$ Cross Sections

Originally it was aimed to extrapolate the $N^{14}(p,\gamma)$ cross-section factor by normalizing each of the theoretical 6.79, 6.16, 5.2 MeV, and ground state cross-section factor curves to selected values at 800 keV. Difficulties arose however in selecting suitable values for the cross-section factors at this energy. Use of values derived from Duncan and Perry's (Du 51) results gave a theoretical cross-section factor for $N^{14}(p,\gamma)$ which was well above the experimental values measured by Pixley (Pi 57) in the region 500-600 keV. On the other hand use of values based on a smooth curve extrapolated through Pixley's results produced erratic behaviour in the individual cross-section factor components with energy which was contrary to their expected smooth variations. This casts doubts on the results of both Pixley and of Duncan and Perry. For this reason it was decided to calculate the cross-section factors independently and use this as a basis for later extrapolations. This was done by first determining
the total nitrogen content in the target from analysis of the target excitation function based on the yield of 4.43 MeV radiation from the 429 keV $^15\text{N}(p,\alpha\gamma)^{12}\text{C}$ resonance. This information and relative $\gamma$-ray yields then enabled the individual absolute cross-section factors to be determined.

6.4 Measurement of Target Composition

The broken curve of Fig. 6.3 represents the measured excitation function of the target. This has been extended to higher energies to allow a reliable subtraction to be made for $\gamma$-ray contributions from sources other than the 429 keV resonance. The magnitude of this subtraction could be judged quite accurately from the requirement that the cross-section factors for $^14\text{N}(p,\gamma)$ at 400 and 500 keV should both correct to a smooth curve after removal of contributions from the 278 keV resonance. These resonant contributions are sensitive to the shape of the lower part of the target excitation function. The full curve of Fig. 6.3 then represents the estimate for the $\gamma$-ray yield from the 429 keV resonance alone, corrected to zero resonance width. The yield is expressed in $\gamma$-rays per $\mu$-coulomb. This is based on the results of the $\gamma$-ray analysis of the 500 keV run, which gives the total number of 4.43 MeV $\gamma$-rays detected for a given proton charge. The excitation function was measured at the end of the $^14\text{N}(p,\gamma)$ intensity runs and since repeated checks showed no noticeable loss in nitrogen content of the target this
should be a reliable representation during the entire set of runs.

To determine the target composition at various depths the yield curve was divided into a series of continuous layers as follows. Bombardment at 439 keV and a 10 keV energy loss in the target to reach the 429 keV resonance energy defined the extent of layer 1 and the start of layer 2. Similarly bombardment at 449 keV with an energy loss of 20 keV (part in each layer) to reach 429 keV energy defined the end of layer 2 and the start of layer 3 and so on. We illustrate the method of calculation in brief by evaluating the composition of layer 1.

For the narrow, isolated 429 keV resonance the yield of 4.43 MeV \( \gamma \) -radiation for a layer entrance energy \( E_{IN} \) can be written,

\[
Y(E_{IN}) = \left[ \frac{2\pi \lambda^2 \omega \int \gamma}{\Gamma} \right] \cdot \frac{1}{\varepsilon_{N15}} \cdot \frac{E \cdot W \cdot Q}{e} \text{ gammas/proton} \quad \text{(6.1)}
\]

where \( \varepsilon_{N15} \) = average stopping cross section per \( N^{15} \) atom in the layer

\[
\frac{2\pi \lambda^2 \omega \int \gamma}{\Gamma \varepsilon_{N15}} = \text{thick target yield in gammas per proton from the 429 keV resonance}
\]

\( E \) = efficiency of the \( \gamma \) -counter for the geometry used.

\( W \) = correction factor to allow for the angular distribution of the 4.43 MeV radiation.

\( Q \) = collected charge on the target in coulombs.

\( e \) = electronic charge in coulombs.
For the factor in brackets in equation (6.1) we use a value $2.59 \times 10^{-22} \text{ ev-cm}^2$ obtained from Hebbard's (He 60b) corrected value for the thick target yield of Scharf et al. (Sc 52) at the 429 keV resonance. For the 5" x 4" crystal detecting 4.43 MeV γ-rays at a distance of 5.55 cm we find an efficiency $E = 0.075$. The correction factor $W$ is evaluated at $\Theta_1 = 55^\circ$. We used the angular distribution obtained by Kraus et al. (Kr 53) at the 429 keV resonance, and modify this for the smoothing effect of the crystal, giving $W(\Theta_i) = P_0 + .211 P_2(\cos \Theta_i) + .170 P_4(\cos \Theta_i)$. The correction factor to be applied is then $W = \frac{W(55^\circ)}{P_0} = 0.934$.

Inserting these factors into equation (6.1) we find

$$Y(E_{\text{in}}) = \frac{1.13 \times 10^{-10}}{\varepsilon_{N^{15}}} \text{ ev-cm}^2 \cdot \text{ gammas/p.coulombs} \tag{6.2}$$

Taking a value of $Y(E_{\text{in}}) = 15.5 \text{ gammas/p.coulomb}$ from the target yield curve of Fig. 6.3 evaluated at the mean energy of the layer, 434 keV we find $\varepsilon_{N^{15}} = 7.29 \times 10^{-12} \text{ ev-cm}^2$. Multiplying this by $1/274$, the isotopic proportion of $N^{15}$ in natural nitrogen we obtain the average stopping cross-section for the target per nitrogen atom, $\varepsilon_{N_T} = 2.66 \times 10^{-14} \text{ ev-cm}^2$. Hence for the first layer corresponding to a 10 keV energy loss the total number of nitrogen atoms present $N = 10,000/\varepsilon_{N_T} = 3.76 \times 10^{17} \text{ atom.cm}^{-2}$. Using the tabulated stopping cross-sections for the pure elements $N$ and Ta evaluated at the mean layer energy 434 keV we obtain the
number of tantalum atoms in layer 1, $X_{Ta} = 2.54 \times 10^{17}$ atom cm$^{-2}$.

We observe that these proportions at the front of the target correspond to the formula $Ta_{0.62}N$ which is closely that of the stable nitride of tantalum $Ta_{3}N_{5}$. Pixley (Pi 57) found that fresh targets of tantalum nitride corresponded to the other stable nitride $TaN$. It would appear from this that heating under prolonged bombardment as suffered by the present target, leads to the compound $Ta_{3}N_{5}$, this being chemically more stable.

The calculation for the second layer follows along similar lines, the value of $E_{IN}$ entering layer 2 being determined from the pure element stopping cross-sections and the number of atoms of $N$ and $Ta$ just found in layer 1. By a reiterative calculation for succeeding layers the full target composition can be determined. This problem was programmed for the 1620 computer, a total of 27 layers being required to account for all the nitrogen in the target. Provision was made in the programme to feed in a selected proton bombarding energy and extract out the layer (if one existed) at which any desired energy was reached. This was used to calculate the $N^{14}(p, \gamma)$ thick target yield from the 278 keV resonance at 300 and 320 keV for comparison with the results of Duncan and Perry, Pixley, and Bashkin et al. These are listed in table 6.3 below.
<table>
<thead>
<tr>
<th>Worker</th>
<th>Radiation Detected</th>
<th>Target Used</th>
<th>Thick target yield for Pure N Target Reactions per proton</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duncan &amp; Perry (Du 51)</td>
<td>positrons</td>
<td>Be+N</td>
<td>$3.5 \times 10^{-11}$</td>
</tr>
<tr>
<td>Pixley (Pi 57)</td>
<td>prompt gammas</td>
<td>Ta+N</td>
<td>$1.8 \pm 0.4 \times 10^{-11}$</td>
</tr>
<tr>
<td>&quot;</td>
<td>annihilation quanta</td>
<td>Ta+N</td>
<td>$2.18 \pm 0.36 \times 10^{-11}$</td>
</tr>
<tr>
<td>&quot;</td>
<td></td>
<td>Be+N</td>
<td>$1.62 \pm 0.23 \times 10^{-11}$</td>
</tr>
<tr>
<td>Bashkin et al. (Ba 55)</td>
<td>prompt gammas</td>
<td>Ti+N</td>
<td>$2.7 \pm 0.6 \times 10^{-11}$</td>
</tr>
<tr>
<td>&quot;</td>
<td>positrons</td>
<td>Ti+N</td>
<td>$2.2 \pm 0.04 \times 10^{-11}$</td>
</tr>
<tr>
<td>Present results at 300 keV</td>
<td>prompt gammas</td>
<td>Ta+N</td>
<td>$2.12 \pm 0.25 \times 10^{-11}$</td>
</tr>
<tr>
<td>Present results at 320 keV</td>
<td>&quot;</td>
<td>Ta+N</td>
<td>$2.10 \pm 0.25 \times 10^{-11}$</td>
</tr>
</tbody>
</table>

Table 6.3. Comparison of thick target yields from the $^{14}N(p,\gamma)$ resonance at 278 keV.

The present results are seen to be some 10% higher than Pixley's mean value. The error quoted is almost entirely due to the estimated uncertainty in the thick target yield extracted from Schardt's data, on which the stopping cross-sections are based.

Once the total number of nitrogen atoms in the target $X_{NT}$ have been determined from the target analysis, the $^{14}N(p,\gamma)$ yields listed in Table 6.1 can be converted into absolute cross-sections, using the relation,
\[ Y = \sigma N T \frac{E W}{e} \] \hspace{1cm} (6.3)

where

\[ Y = \text{\(\gamma\) -ray yield per \(\mu\).coulomb} \]
\[ \sigma = \text{cross-section in cm}^2 \]

and \(E, W, e, \) and \(N T\) have been defined previously. In the present case the 6.79, 6.16, 5.2 MeV and ground state \(\gamma\) -rays are known to have isotropic angular distributions, so we set \(W = 1.0\) in the calculation of all four cross-sections. Finally we arrive at the individual cross-section factors \(S(E)\) listed in Table 6.1 using the relation defined in equation (1.6),

\[ S(E) = E \cdot \exp(2\pi \eta) \sigma. \]
CHAPTER VII

THEORETICAL $^{14}_N(p,\gamma)$ EXTRAPOLATION BASED ON THE EXPERIMENTAL CROSS-SECTION FACTORS FOR THE 6.79, 6.16, 5.2 MeV AND GROUND STATE GAMMA RAYS.

7.1 Direct-Capture Calculations

The theoretical variation of the cross-section factor $S$ with energy has been calculated for $(p,\gamma)$ direct-capture to each of the 6.79, 6.16, 5.2 MeV and ground states in $^0$. The method of calculation is discussed in the Appendix, where it is shown that the energy dependence of $S$ is proportional to an expression of the form,

$$ S(E) = |(I + I_N)|^2 $$

with $I = I_1 \cos \delta + I_2 \sin \delta$. Here $I_1$ and $I_2$ are energy dependent exterior radial integrals, $\delta$ the resultant phase shift between the incident and scattered proton waves and $I_N$ an estimate for the interior contribution of the nucleus assuming a simple square well shape.

Calculations of $S(E)$ were done for a series of proton energies covering the range 1000 - 25 keV in the laboratory system. Figure 1.1 of Chapter I is typical of the shape of the radial integrands which emerge. Examination of these radial integrands for the four direct-capture transitions indicates that the
condition $\kappa \gg \Delta r$ of Chapter I, is satisfied for the 6.79, 6.16 and 5.2 MeV levels, so that these calculations should be reliable. For the ground state in which $E_\gamma > 7$ MeV, $\kappa \sim \Delta r$ implying that these calculations may be less reliable. Examination of the results shows this to be the case. The peak of the radial integrand for the ground-state calculation turns out to be very close to the nuclear radius. This gives a large contribution from the nuclear interior which is sensitive to the choice of phase. In this case it is apparent that the assumptions of the direct-capture model (based on an extranuclear process) are not fulfilled. We have concluded that the ground state direct-capture calculations cannot be used with justification.

To calculate the resultant $^\alpha N + p$ phase shift $\delta$ the hard-sphere phase shift was combined with contributions from both bound and unbound levels in $^\alpha$. For the hard sphere phase shift $\delta_{HS}$ we have evaluated $\tan \delta_{HS} = -(F_e/G_e) r_N$, at the nuclear radius $r_N = 4.808$ fermi. For the bound and unbound levels the appropriate expression is

$$\tan \delta_o = -\left[ \frac{C \gamma_p^2}{A_e^2 (E - E_o - \Delta_e)} \right] r_N$$

where $\varphi = kr_N$, $k$ being the wave number of the incoming proton with relative angular momentum $e$, $\gamma_p^2$ the reduced proton width of the level in keV, $A_e^2 = \left[ F_e^2 + G_e^2 \right] r_N$. 
\[ \Delta \epsilon = - \left[ \phi \gamma_p^2 \left( E_\epsilon E_\gamma^1 + G_\epsilon G_\gamma^1 \right)/A_\epsilon^2 \right] rN \] the level shift in keV, \( E \) the proton bombarding energy in keV, and \( E_0 \) is defined in terms of the experimental resonance energy \( E_R \) (negative for bound states) by the condition \( E_R - E_0 - \Delta \epsilon (E_R) = 0 \).

The only levels in \( ^{15}O \) found to be of importance in contributing to the p and s wave phase shifts in the energy range 1000 - 25 keV are as follows:

- s-waves, channel spin \( \frac{1}{2} \), 7.55 MeV \( \frac{1}{2}^+ \).
- s-waves, channel spin \( \frac{3}{2} \), 6.79 MeV \( \frac{3}{2}^+ \), and 8.28 MeV \( \frac{3}{2}^+ \).
- p-waves, both channel spins 0.00 MeV \( \frac{1}{2}^- \), and 6.16 MeV \( \frac{3}{2}^- \).

The broad level at about 2300 keV in \( ^{15}O \) (Aj 60) may also be important. However there is no information on gamma ray transitions from this level to enable any calculations to be made.

For the unbound levels (7.55 and 8.28 MeV) \( \gamma_p^2 \) was calculated from the total width \( \Gamma \) using values 1.7 keV (Pi 57) and 3 keV (Ha 57a) respectively. For the bound states estimates for \( \gamma_p^2 \) were made from stripping reduced widths in the following way. Macfarlane and French (Ma 60b) define a dimensionless stripping reduced width \( \Theta^2 \) in the form,

\[ \Theta^2 = s' \Theta_o^2 \]

where \( s' \) is a multiplicative factor, usually less than one and
\( \Theta_0^2 \) is the single particle dimensionless stripping reduced width for the final state. Similarly we can define a dimensionless resonance reduced width \( \Theta_R^2 \),

\[
\Theta_R^2 = S' \Theta_{oR}^2
\]

where \( \Theta_{oR}^2 \) is the dimensionless single particle resonance reduced width. In terms of the definition of Lane (La 60) we can write

\[
\gamma_p^2 = \Theta_R^2 \left( \frac{\hbar^2}{\mu a^2} \right) \approx 2 \Theta_R^2 \text{ MeV}.
\]

For single particle 2s or 1p states, \( \Theta_{oR}^2 \approx 0.5 \) (Ba 63) so

\[
\gamma_p^2 \approx S' \text{ MeV} = \Theta^2 / \Theta_{oR}^2 \text{ MeV}.
\]

In this expression \( \Theta^2 \) represents the experimental stripping reduced width, whilst \( \Theta_{oR}^2 \) can be obtained from curves (Figures 55 and 56) given by Macfarlane and French. There are no absolute stripping reduced widths leading to final states in \( O^{15} \). We have therefore estimated \( \gamma_p^2 \) from the \( N^{14}(d, p) \) reaction (Ma 60b, Wa 57, Gr 56) using the absolute stripping reduced widths leading to the corresponding mirror levels in \( N^{15} \). For the 6.79 MeV level in \( O^{15} \) the mirror level is at 7.31 MeV in \( N^{15} \) (Pi 57, He 59). The \( N^{14}(d, p) \) reaction to this level gives \( \Theta^2 = 0.10 \), whilst the curves give \( \Theta_{oR}^2 (2s) = 0.16 \), hence we estimate

\[
\gamma_p^2 = 0.6 \text{ MeV}.
\]

In a similar way estimates for \( \gamma_p^2 \) of 0.2
and 1.0 MeV are obtained for the 6.16 MeV (1p) and ground state (2s) levels respectively. The values of $\gamma_p^2$ adopted for the 6.79 and 6.16 MeV states are thought to be reliable estimates, since the corresponding values of $\Theta_R^2 (= \gamma_p^2 / 2)$ used in conjunction with the results from the radial integrals yield (from equation A1) direct-capture cross-section factors which are within 10% of the experimental values.

Calculations of $| (I + I_N) |^2$ were performed for all possible channel spins which could contribute to the direct-capture with the emission of E1 radiation. The possible transitions are listed below, and in Figure 7.1 we illustrate the results of these calculations.

<table>
<thead>
<tr>
<th>Direct-Capture Level in $0^{15}$</th>
<th>$J^\pi$</th>
<th>Possible E1 Direct-Capture Transitions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Channel Spin $\frac{1}{2}$</td>
</tr>
<tr>
<td>6.79 MeV</td>
<td>$\frac{3^+}{2}$</td>
<td>Forbidden</td>
</tr>
<tr>
<td>6.16 MeV</td>
<td>$\frac{3^-}{2}$</td>
<td>2s - 1p</td>
</tr>
<tr>
<td>5.24 MeV (5.2 MeV)</td>
<td>$\frac{5^+}{2}$</td>
<td>1p - 1d</td>
</tr>
<tr>
<td>5.18 MeV</td>
<td>$\frac{1^+}{2}$</td>
<td>1p - 2s</td>
</tr>
<tr>
<td>0.0 MeV</td>
<td>$\frac{1^-}{2}$</td>
<td>2s - 1p</td>
</tr>
</tbody>
</table>

The ground state calculations were rejected for the reasons already given. Direct-capture to the 5.24 MeV level was included to obtain an estimate of its strength compared to direct-
Fig. 7.1 Theoretical direct-capture calculations for $N^7(p,\gamma)O^{15}$. 

CURVES NORMALISED AT 775 KeV PROPORTIONAL TO EXPT. INTENSITIES.
capture to the 5.18 MeV level. Comparison of the two radial integrals at 775 keV (after appropriate normalization to the same value of the Whittaker function at the nuclear radius) gives an intensity ratio of 15 to 1 in favour of direct-capture to the 5.18 MeV level. In view of this factor and the experimental evidence favouring population of the 5.18 MeV level below 1000 keV we have only considered this state as contributing to the $N^{14}(p, \gamma)$ cross-section.

The rapid change of the 6.16 MeV (channel spin $\frac{3}{2}$) direct-capture curve of Figure 7.1 is due entirely to the dominance of the 278 keV (7.55 MeV level) resonant phase shift. The contribution from this particular transition to the overall $N^{14}(p, \gamma)$ cross-section factor must be small, since the large increase predicted in $S(E)$ from 800 keV to lower energies is not observed experimentally. Inclusion of the nuclear phase shifts for the direct-capture transitions involving the 6.79, 5.2 and 6.16 MeV (channel spin $\frac{3}{2}$) levels has very little effect on the shape of the curves in Figure 7.1, a slightly increased slope in the 6.79 MeV curve being the only noticeable difference compared to the hard-sphere calculations.

7.2 Resonant Contributions to the $N^{14}(p, \gamma)$ Cross-Section Factor.

Contributions to the $N^{14}(p, \gamma)$ cross-section factor in the energy range of interest were mainly from the narrow resonances at 278 and 1058 keV which feed the 6.79, 6.16, 5.2 MeV and ground
state levels in $^0_{15}$. The effect of more distant $(p, \gamma)$ resonances is not well known but can be expected to lead to a fairly constant tail with energy. Such contributions are taken into account later. Curves were plotted (covering the energy range 0 - 1 MeV) showing the individual cross-section factors for radiation from the 278 and 1058 keV resonances feeding the 6.79, 6.16, 5.2 MeV and ground states in $^0_{15}$. The parameters used for these calculations were based partly on the present experimental results, and are listed in Table 7.1. The 278 keV resonance parameters are derived directly from the analysis of the 300 and 320 keV runs mentioned in Chapter VI. For the 1058 keV resonance, Duncan and Perry's value for the total gamma ray radiative width has been reduced by a factor 0.625 so as to make their cross-sections consistent with ours. We then use this revised figure with the relative intensities for the 1070 keV run to arrive at the radiative widths given in Table 7.1.

<table>
<thead>
<tr>
<th>Resonance Energy $E_R$ (keV)</th>
<th>Total Width $\Gamma$ (keV)</th>
<th>Partial Width $\omega \Gamma_\gamma(E_R)$ to $^0_{15}$ levels at $6.79$ MeV</th>
<th>$6.16$ MeV</th>
<th>$5.2$ MeV</th>
<th>$0.0$ MeV</th>
<th>Total Gamma Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>278</td>
<td>1.7</td>
<td>.0034 ev</td>
<td>.0084 ev</td>
<td>.0023 ev</td>
<td>.0004 ev</td>
<td>.0145 ev</td>
</tr>
<tr>
<td>1058</td>
<td>3</td>
<td>non-Resonant</td>
<td>.015 ev</td>
<td>.168 ev</td>
<td>.210 ev</td>
<td>.394 ev</td>
</tr>
</tbody>
</table>

Table 7.1. Parameters for the 278 and 1058 keV $^{14}_{N}(p, \gamma)$ resonances.
For a proton energy $E$, the $\left(p, \gamma\right)$ resonant cross-section $\sigma(E)$ to a given level was calculated using the single-level Breit-Wigner formula with energy dependent terms,

$$\sigma(E) = \frac{\pi \chi^2(E) \Gamma_p(E) \omega \gamma(E)}{\left[E - E_0 - \Delta\epsilon(E)\right]^2 + \frac{1}{4} \left[\Gamma_p(E) + \Gamma_{\gamma}(E)\right]^2}$$

Here $\chi^2(E) = \text{constant}/E$

$$\Gamma_p(E) = \frac{\Gamma_p(E_R)}{A\epsilon}$$

$$\Gamma_{\gamma}(E) = \left[\frac{E_{\gamma}(E)/E_{\gamma}(E_R)}{\gamma_0}\right]^{1/3} \Gamma_{\gamma}(E_R)$$

where $E_{\gamma}(E)$ and $E_{\gamma}(E_R)$ are the $\gamma$-ray energies to the level for proton energies $E$ and $E_R$ respectively.

$$\Delta\epsilon(E) = \left[2 \gamma^2_p \left(F \gamma^1_p + G \gamma^1_e\right)/A\epsilon^2\right]_{\text{rN}}$$

as defined previously.

The cross-section factor follows from the expression

$$S(E) = \sigma(E) E_{\text{CM}} \exp\left(2\pi\eta\right).$$

These calculations can then be used to determine the resonant contributions to the experimental cross-section factors since the $6.79$, $6.16$ and $5.2$ MeV levels decay directly to the ground state.

7.3 **Theoretical Extrapolation of $^{14}\left(p, \gamma\right)$ Cross-Section Factor.**

To obtain the required extrapolation, theoretical fits were made to the experimental cross-section factors for each of the $6.79$, $6.16$, $5.2$ MeV and ground state gamma rays. These fits
were then used to predict the cross-section factors at lower energies and the results combined to give an overall $N^{14}(p,\gamma)$ extrapolation to stellar energies.

To fit the experimental data, pure and interference contributions from (a) direct-capture, (b) the 278 and 1058 keV resonances and (c) constant tails from distant resonances were considered for the two possible channel spins of $\frac{1}{2}$ and $\frac{3}{2}$. The data was fitted by a least squares method using an expression of the form,

$$\left( |X_1 e^{i\Theta} \sqrt{S_{278}} + Y_1 + Z_1 e^{i\phi_1} \sqrt{DC_{1/2}} |^2 \right) +$$

channel spin $\frac{1}{2}$ contributions

$$\left( |X_2 e^{i\Theta} \sqrt{S_{1058}} + Y_2 + Z_2 e^{i\phi_2} \sqrt{DC_{3/2}} |^2 \right)$$

channel spin $\frac{3}{2}$ contributions.

The terms in the first bracket may be understood more clearly by expanding. The three squares taken in order correspond to pure contributions to the cross-section factors from, the 278 keV resonance, the constant tail, and direct-capture, all formed with channel spin $\frac{1}{2}$. The three cross terms correspond to interference contributions as follow

$$2 X_1 Y_1 \cos \Theta_1 \sqrt{S_{278}}$$

- interference arising from incoming $s$-waves forming the 278 keV resonance and

$$2 X_2 Y_2 \cos \Theta_2 \sqrt{S_{1058}}$$

- interference arising from incoming $s$-waves forming the constant tail with phase difference $\Theta_2$ of 0 or $\pi$. 


interference arising from incoming s-waves forming the constant tail \[ \text{forming the 278 keV resonance} \] and either incoming s or p-waves leading to direct-capture, with phase difference \( S_1 [S_1 + \Theta_1] \).

The terms in the second bracket have a similar meaning. Here channel spin \( \frac{3}{2} \) contributions are considered from the S-wave 1058 keV resonance, an S-wave constant tail and direct-capture.

For the analysis of a particular transition absolute values of \( \sqrt{S_{278}} \) and \( \sqrt{S_{1058}} \) were determined as discussed in the previous section, at energies corresponding to the experimental points. This required fixing \( X_1, X_2 = \pm 1 \), the sign being determined by trial and corresponding to \( \Theta_1, \Theta_2 = 0 \) or \( \pi \).

The quantities \( \sqrt{DC_{1/2}}, \sqrt{DC_{3/2}} \) were determined from curves similar to Figure 7.1, after normalizing to the experimental direct-capture cross-section factors at 775 keV. The phase shifts \( S_1, S_2 \) corresponding to interference arising from incoming S-waves forming either the 278 or 1058 keV resonances and incoming S-wave resulting in direct-capture, must be 0 or \( \pi \), when considering interference with incoming p-wave direct-capture, the phase shifts are given by

\[
\tan S_{1,2} = \tan \delta_{\text{coul.}} + \tan S
\]

where \( \tan \delta_{\text{coul.}} = \eta \) represents the s-p coulomb phase.
shift and \( \tan S \) includes nuclear and hard sphere phase shifts as discussed in Section 7.1.

We now consider the analysis of the individual sets of data in turn, summarizing first the mechanisms expected to contribute to the measured cross-section factors.

**6.79 MeV Transition**

Direct-capture \( p - s \), channel spin \( \frac{3}{2} \) only.

Resonant contributions.

- 278 keV resonance, s-wave, channel spin \( \frac{1}{2} \), adds incoherently to direct-capture
- 1058 keV resonance, ignored, no evidence for population of 6.79 MeV level.

This particular transition proved to be the simplest to analyse. An incoherent sum of contributions from the 278 keV resonance and direct-capture gave a good fit to the experimental points. This is illustrated in Figure 7.2. Here, and for succeeding transitions the experimental data has been plotted at the centre of target energies. The errors are shown corresponding to those quoted in Table 6.1. Because of the reliable fit this transition was made the basis for correction of the experimental points in the region of 400 and 500 keV. These points (ignored in the fit) were corrected on to the theoretical curve to allow for contributions from the 278 keV resonance caused by the semi-thick target. With a knowledge of these corrections and the relative intensities of the gamma-
6.79-MeV TRANSITION

- Experimental points
- Theoretical fit

- Points corrected for 278 KeV resonance contribution

Fig. 7-2. Energy dependence of cross-section factor for 6.79-MeV transition showing theoretical extrapolation.
rays at the 278 keV resonance, appropriate corrections were made to the corresponding pairs of points for the other transitions. In Figure 7.2 we have illustrated the theoretical extrapolation for the 6.79 MeV radiation, and find a value for the cross-section factor at $E_p = 25$ keV of $S_0 = 1.40$ keV barn. The direct-capture contribution for this transition amounts to 98% at 800 keV. Using this result the expected direct-capture angular distribution is calculated as $1 - 0.98 P_2$. This agrees well with the measured distribution of $1 - (0.97 \pm 0.03)P_2$ from Chapter V.

6.16 MeV Transition.

Direct-capture $s - p$, Both channel spins $\frac{1}{2}$ and $\frac{3}{2}$ allowed.

Resonant contributions

278 keV resonance, s-wave, channel spin $\frac{1}{2}$, would interfere with direct-capture, phase difference $0$ or $\pi$.

1058 keV resonance, s-wave, channel spin $\frac{3}{2}$, would interfere with direct-capture, phase difference $0$ or $\pi$.

This analysis proved to be the least satisfactory due to the large scatter of experimental points (Figure 7.3). The points around 600 and 700 keV appear to be high. This could be due to 6.13 MeV gamma radiation from fluorine contamination in the tantalum target material. Two alternative best fits are illustrated in Figure 7.3. One of these corresponds to the
Fig. 7.3 Energy dependence of cross section factor for 6.16 MeV transition showing theoretical extrapolations.
278 keV and a small 1058 keV resonance contribution interfering with direct-capture which is largely from channel spin \( \frac{3}{2} \). The other fit is obtained by replacing the direct-capture with contributions from constant s-wave tails in both channel spins. Both fits lead to quite different extrapolations, however there is evidence favouring the larger value of \( S_0 = 0.73 \) keV barns.

Careful re-examination of the 210 keV data gives indications of weak radiations corresponding to 6.79 and 6.16 MeV gamma rays which are statistically significant (at the 5% confidence level) above the background. There is no evidence for 5.2 MeV or ground state radiations. We may argue that since both 6.79 and 6.16 MeV transitions are observed they should have comparable cross-section factors of 210 keV. Examination of the extrapolation curves then favours the adoption of the larger value of \( S_0 \).

5.2 MeV Transition.

Direct-capture p - s, channel spin \( \frac{1}{2} \) only.

Resonant contributions

278 keV resonance s-wave, channel spin \( \frac{1}{2} \), interference with direct-capture for transitions with \( \Delta_m = 0 \), other transitions \( \Delta_m = \pm 1 \) add incoherently.

1058 keV resonance s-wave, channel spin \( \frac{3}{2} \), adds incoherently to direct-capture
In this case since $p - s$ direct-capture corresponds to a $\Delta_m = 0$ transition, this will be incoherent with 278 keV resonance radiation produced by $\Delta_m = \pm 1$ transitions and the intensities will simply add. The $\Delta_m = 0$ transitions however will produce coherent radiation with the direct-capture radiation and result in interference. The probability of the $\Delta_m = 0$ transition occurring is $\frac{1}{2}$ for initial and final states both of spin $\frac{1}{2}$.

In Figure 7.4 we again illustrate two fits. The partial interference fit, and a full interference fit between mainly the 278 keV resonance radiation and a constant tail. From these fits there is a slight preference for choosing a value $S_0 = 0.05$ keV barns as the extrapolated value.

**Ground State Transition.**

Direct-capture not calculable on the model

Resonance contributions. The 278 and 1058 keV contributions will interfere with the constant tails for each channel spin with phase difference 0 or $\pi$.

The two fits illustrated in Figure 7.5 correspond to the cases of constructive and destructive interference between contributions mainly from the 278 keV resonance and the channel spin $\frac{1}{2}$ constant tail. The 1058 keV resonance contribution is quite small but the channel spin $\frac{3}{2}$ constant tail appears
5.2-MeV TRANSITION

- Experimental points.
- Theoretical fit.
- Points corrected for 278 KeV resonance contribution.

---

Full interference - 278 KeV resonance & constant tail.
Partial interference - 278 KeV resonance & direct-capture.

---

Fig. 7.4 Energy dependence of cross-section factor for 5.2 MeV transition showing theoretical extrapolations.

$S_o = 0.12$ KeV·b
$S_o = 0.05$ KeV·b
Fig. 7-5. Energy dependence of cross-section factor for ground-state transition showing theoretical extrapolations.
significant at lower energies. Since there was no evidence for ground state radiation from the 210 keV data the lower value for the extrapolation, $S_0 = 0.21$ keV barns is preferred.

7.4 **Conclusions.**

In Figure 7.6 we have presented the overall $N^{14}(p,\gamma)$ cross-section factor extrapolation by combining the four preferred component curves. The resulting cross-section factor at 25 keV is $S_0 = 2.4 \pm 0.4$ keV barns. The error quoted is largely due to the uncertainty in the correction applied to Schardt et al's data (Sc 52) for the thick target gamma ray yield at the 429 keV $N^{15}(p,\alpha,\gamma)$ resonance on which the experimental results were based. Allowance has also been made for the uncertainty in the 6.16 MeV extrapolation due to the doubtful fit to the experimental data. The extrapolation curve can be seen to be in satisfactory agreement with the results of Lamb and Hester (La 57) but lies mid-way between the results of Pixley (Pi 57) and Duncan and Perry (Du 51). The value for $S_0$ from the present experiment appears lower but is not inconsistent with a value of $2.7 \pm 0.2$ keV barns obtained by Lamb and Hester and the value of 2.8 keV barns found by Pixley.

A careful measurement of the absolute thick target yield at the 429 keV resonance would remove a large amount of the uncertainty in the present result. Since the 278 and 1058 keV...
Fig 7.6 Energy variation of cross-section factor for $^{14}\text{N}(p,\gamma)^{15}\text{O}$ showing the theoretical extrapolation giving a value $S_0 = 2.4 \pm 0.4$ KeV.b
resonance yields have been corrected to be consistent with the present results, a revised 429 keV thick target yield could be used directly to fix a more accurate cross-section factor scale.
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</table>
Calculation of Cross-Section Factors Using the Direct-Capture Model.

On the direct-capture model of Christy and Duck (Ch 61), the cross-section \( \sigma_1 \) can be expressed in the form of equation (1.5),

\[
\sigma_1 = \frac{32 \pi}{3} \left( \frac{E \gamma}{\hbar c} \right)^3 \frac{e^2}{\hbar c} \cdot \frac{e}{v} \cdot \mu^2 \left( \frac{z_1}{M_1} - \frac{z_2}{M_2} \right)^2 \langle \ell_1, \ell_2 \rangle \Theta^2 \frac{|R_{1fi}|^2}{W^2(Kr_n) \cdot r_n}
\]

with the cross-section factor \( S \) being defined by

\[
S = E_{CM} \sigma_1 \cdot e^{2\pi \eta}
\]

The radial integral, \( R_{1fi} \) has been defined in Chapter I,

\[
R_{1fi} = \int r^2 \, dr \, g_f^*(r) \, g_i(r)/k \quad (1.3)
\]

where

\[
g_i(r) = F_{\ell_1}(\eta, \varphi) + G_{\ell_1}(\eta, \varphi) + i F_{\ell_1}(\eta, \varphi) e^{i \varphi} \sin \theta_i \quad (1.4)
\]

and \( g_f(r) = W_{\ell_2}(Kr)/r \).

It is convenient to write (1.4) in the form

\[
g_i(r) = e^{i \varphi} \left[ F_{\ell_1} \cos \varphi + G_{\ell_1} \sin \varphi \right]
\]

with \( \varphi \) representing the phase shift of the incoming proton.
A2.

wave which includes contributions from nuclear and hard sphere phase shifts. We can then write for (1.3)

$$|R_{1fi}|^2 = \left| \cos \int_{r_N}^{r_{max}} k^{-1} r F_{e_1} W \, d\ell_2 \, dr + \sin \int_{r_N}^{r_{max}} k^{-1} r G_{e_1} W \, d\ell_2 \, dr \right|^2$$

where the radial integration is taken from the nuclear radius \(r_N\) to some value \(r_{max}\) beyond which contributions to the integrals are sufficiently small to be neglected.

We next define the modified radial integrals,

$$I_1 = \frac{E}{c_0(\eta)} \int_{r_N}^{r_{max}} k^{-1} r F_{e_1} W \, d\ell_2 \, dr,$$

$$I_2 = \frac{E}{c_0(\eta)} \int_{r_N}^{r_{max}} k^{-1} r G_{e_1} W \, d\ell_2 \, dr,$$

where \(c_0(\eta) = \left[ \frac{2\pi \eta}{\exp (2\pi \eta) - 1} \right]^{\frac{3}{2}}\).

Referring back to equation (1.5), evaluating the physical constants and doing some rearranging, the cross-section factor can be expressed in units of keV.barn in the form

$$S = 6.789 \times 10^{-6} \sqrt[3]{z_1 z_2 M_1 \left( \frac{z_1}{M_1} - \frac{z_2}{M_2} \right)^2 \frac{\langle \ell_1 \ell_2 \Theta \rangle}{W^2} \frac{\alpha}{d\ell_2} \left( \frac{K \ell_1}{r_N} \right) \frac{r_N}{\ell_2}}$$ keV.barn (A1)

Here \(I^2 = \left| I_1 \cos \theta + I_2 \sin \theta \right|^2\) contains the energy
dependence of the cross-section factor so that for a particular reaction the energy dependence of \( S \) takes the form

\[
S(E) = \text{const.} \quad |I|^2
\]  
(A2)

Evaluation of the Radial Integrals.

The modified radial integrals \( I_1 \) and \( I_2 \) were calculated at selected energies with the aid of an I.B.M. 1620 computer in the following way. Input data corresponded to the quantities \( r_N, r_{\text{max}}, \gamma, \alpha, K, k, E, \ell_1, \ell_2, G_0(r_{\text{max}}), G_0^1(r_{\text{max}}) \) and \( h \) the interval of integration. Values of \( W \) at these intervals were computed by numerical integration of the differential equation,

\[
\frac{d^2 W}{dx^2} - \left[ 1 + \frac{2\alpha}{x} + \frac{\ell_2(\ell_2 + 1)}{x^2} \right] W = 0,
\]

where \( x = Kr \).

Starting values were computed at \( x_1 = Kr_{\text{max}} \), and \( x_2 = K(r_{\text{max}} - h) \) using the asymptotic expansion formula given in equation (17) of Christy and Duck's paper, and inward integration carried out in steps \( h \) to reach the nuclear radius at \( x = Kr_N \). Similarly \( F_0, G_0 \), and also \( F_0^1 \) and \( G_0^1 \) were calculated by numerical integration of the differential equations,

\[
\frac{d^2 \{ F \}}{d \zeta^2} + \left[ 1 - \frac{2\alpha}{\zeta} - \frac{\ell_1(\ell_1 + 1)}{\zeta^2} \right] \{ F \} = 0,
\]

where \( \zeta = kr \).
G₀ and G₀⁻¹ like W, being large and rapidly changing for small radial distances were calculated by inward integration from the starting values G₀(r_max) and G₀⁻¹(r_max) obtained from tables (Tu 58). F₀ and F₀⁻¹ were calculated by outward integration from the nuclear radius using a power series to obtain starting values at \( Q₁ = krₙ \) and \( Q₂ = k(rₙ + h) \). Higher orders Fₑ₁⁻¹, Gₑ₁⁻¹, Gₑ₁, were calculated from the computed F₀'s and G₀'s using a recurrence relationship.

The radial integration was carried out in 198 steps using Weddle’s Rule. The values of the step by step integrands could be printed out if desired, to enable the peak of the integrand to be found. Finally the values of I₁ and I₂ were printed separately. For convenience the hard sphere phase shift \( S_{HS} \) defined by \( \tan S_{HS} = - \left( \frac{Fₑ₁}{Gₑ₁} \right) rₙ \) was used in calculating the radial integrals. It was then a simple matter to modify the results later to include nuclear phase shifts.

Values of W, W⁻¹, Fₑ₁, Fₑ₁⁻¹, Gₑ₁, Gₑ₁⁻¹ were printed out at the nuclear radius so that by smoothly joining on a pair of sine functions to \( g₁(r) \) and \( g_f(r) \) an estimate could be made for a nuclear interior contribution. Single particle 2s or 2p initial or final states were represented by sine functions having two turning points, 1p and 1d states having only a single turning point. Evaluation of an expression of the form
\[ I_N = \frac{E^{3/2}}{\xi \frac{1}{2}} \int_0^N k^{-1} r A \sin ar B \sin br dr \]

gave the interior contribution. This could be included in the calculation of \( S \) by replacing \( |I|^2 \) by \( |I + I_N|^2 \) in equations (A1) and (A2).

**Theoretical Cross-Section Factors for the Reactions Be\(^7(p,\gamma)B\(^8\), O\(^{16}(p,\gamma)F\(^{17}\), and Ne\(^{20}(p,\gamma)Na\(^{21}\).**

Christy and Duck (Ch 61) have calculated cross-section factors for the reactions Be\(^7(p,\gamma)B\(^8\), O\(^{16}(p,\gamma)F\(^{17}\), and Ne\(^{20}(p,\gamma)Na\(^{21}\) using approximate methods to evaluate the radial integrals. To check their results and in particular their extrapolations of \( S \) to stellar energies we have computed these calculations using as far as possible the same parameters. The results of these calculations are reported in brief here. In each case \( S \) has been calculated from equation (A1) assuming hard sphere scattering at six approximately equally spaced energies from 1 MeV down to 25 keV. Because of the large extra nuclear distance to the peak in the radial integrand, contributions from the nuclear interior are expected to be small and have not been included. The possible exception is Be\(^7(p,\gamma)\) at the higher energies. The values of \( E_B \) and \( \Theta^2 \) in each case are those used by Christy and Duck and their value \( r_N = 1.41 (A_1^{1/3} + A_2^{1/3}) \) fermi, has also been used for
the nuclear radius. The resulting cross-section factors are given in the tables which follow. The input parameters have been listed together with values which emerge for \( r_p \), the peak position of the radial integrand, and \( W(Kr_N) \) the Whittaker bound state wave function at the nuclear radius.

\[ \text{Be}_7(p,\gamma)\text{B}_8 \]

Transition, S to p. Final state, \( 2^+ \) g.s. of \( \text{B}_8 \).

\( r_N = 4.107 \text{ fm} \). \( E_B = 136 \text{ keV} \), \( \alpha = 1.598 \)

\( r_{\text{max}} \) in range 140-185 fm. \( W(Kr_N) = 5.507 \times 10^{-1} \), \( K = 7.548 \times 10^{-2} \text{ fm}^{-1} \).

<table>
<thead>
<tr>
<th>( E_p \text{ keV C.M.} )</th>
<th>( r_p \text{ fm.} )</th>
<th>( S \left( \Theta^2 = 25/432 \right) \text{ keV b.} )</th>
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<tr>
<td>963</td>
<td>11.5</td>
<td>9.94 \times 10^{-3}</td>
</tr>
<tr>
<td>708</td>
<td>13.8</td>
<td>10.5</td>
</tr>
<tr>
<td>542</td>
<td>14.3</td>
<td>11.4</td>
</tr>
<tr>
<td>347</td>
<td>16.8</td>
<td>13.2</td>
</tr>
<tr>
<td>135</td>
<td>23.6</td>
<td>16.5</td>
</tr>
<tr>
<td>26.8</td>
<td>25.2</td>
<td>19.7</td>
</tr>
</tbody>
</table>
**O$_{16}$(p, X)$F_{17}^{17\ast}$**

Transition, p to s. Final state, $2^+$, 0.5 MeV level $F_{17}$.

$r_N = 4.963$ fm. 

$E_B = 98$ keV, \[ \alpha = 3.906 \]

$r_{\text{max}}$, in range 216-293 fm. $W(Kr_N) = 4.56 \times 10^{-3}$, $K = 6.645 \times 10^{-2}$ fm$^{-1}$.

<table>
<thead>
<tr>
<th>$E_p$ (keV C.M.)</th>
<th>$r_p$ (fm)</th>
<th>$S(\Theta^2 = \frac{1}{3})$ keV b.</th>
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<tr>
<td>761</td>
<td>18.0</td>
<td>2.57</td>
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<tr>
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<td>2.82</td>
</tr>
<tr>
<td>413</td>
<td>24</td>
<td>3.20</td>
</tr>
<tr>
<td>220</td>
<td>33</td>
<td>3.90</td>
</tr>
<tr>
<td>103</td>
<td>43</td>
<td>4.96</td>
</tr>
<tr>
<td>24.5</td>
<td>60</td>
<td>6.50</td>
</tr>
</tbody>
</table>

**Ne$_{20}$(p, X)$Na_{21}$**

Transition, p to s. Final state, $2^+$ 2.42 MeV level in Na$_{21}$.

$r_N = 5.237$ fm. 

$E_B = 26$ keV \[ \alpha = 9.538 \]

$r_{\text{max}} = 897$ fm. \[ W(Kr_N) = 5.583 \times 10^{-8}, K = 3.442 \times 10^{-2} \text{ fm}^{-1} \]

<table>
<thead>
<tr>
<th>$E_p$ (keV C.M.)</th>
<th>$r_p$ (fm)</th>
<th>$S(\Theta^2 = \frac{1}{3})$ keV b.</th>
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<td>981</td>
<td>16.5</td>
<td>7.4</td>
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<tr>
<td>728</td>
<td>20</td>
<td>8.3</td>
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<tr>
<td>487</td>
<td>25</td>
<td>9.8</td>
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<td>300</td>
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</tr>
<tr>
<td>122</td>
<td>55</td>
<td>17.6</td>
</tr>
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<td>24.5</td>
<td>110</td>
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</table>
Comparing with Christy and Duck's results, the Be\textsuperscript{7}(p,\gamma) S-values show good agreement except that below a few hundred keV Christy and Duck predict S to be constant. The ratio S(25 keV)/S(1 MeV) used as a basis for their extrapolation is 1.5 compared to the present value of 2. The S-values for O\textsuperscript{16}(p,\gamma) and Ne\textsuperscript{20}(p,\gamma) both appear about 50\% smaller than Christy and Duck's, however our values for the ratio S(25 keV)/S(1 MeV) of 2.7 and 4.1 respectively are in good agreement with their values of 2.5 and 4. It is likely that the discrepancies in the S-values are due to the different methods of calculating the Whittaker functions. Christy and Duck have used a W.K.B. approximation in both cases whereas we have used a series expansion. We may conclude that Christy and Duck's extrapolations for these reactions are not inconsistent with the present calculations.