A CYCLOTRON INJECTOR FOR
A PROTON SYNCHROTRON.

Being a thesis submitted for the degree of
Doctor of Philosophy in the Australian
National University.

A.H. MORTON.
To Callinica.
The problem, set at the beginning of 1953, was to construct and bring into operation a cyclotron which would be suitable as an injector for the Canberra proton synchrotron. The main content of this thesis is concerned with work which has been done towards the solution of the problem.

At the time a choice of machine for injection was made the requirement that the injection energy should be in the region of 8 Mev ruled out Cockcroft-Walton and electrostatic generators. Thus the choice was between a linear accelerator and a cyclotron. As the resources of the laboratory were to be heavily committed in building the proton synchrotron itself it was felt that it would be better to avoid such an undertaking as the construction of a linear accelerator if a cyclotron could meet the situation. Having on hand a magnet which was used for the model experimental homopolar generator, and which was suitable for a 30 inch cyclotron, further supported the selection of this machine.

Generally the extraction of beam from a cyclotron is inefficient, and the external beam suffers from excessive horizontal divergence. However, it was considered that these disadvantages of the machine might be reduced, or removed, by proper understanding of the beam orbits, and their control, during the early part of the acceleration, at small particle radii, supported by care in the extraction process. It was decided to build what might be termed a simple machine, with nothing more complicated than would be necessary in order to achieve a beam. Then, when the machine was working, some modifications, properly based on an understanding of the machine's operation, might be introduced in order to improve the beam characteristics.

The cyclotron which was designed and constructed is described in Chapter V. Methods used for design calculations, and the methods of making R.F. measurements are given. The apparatuses used for measuring the magnetic field are described in Appendix I.

The discussion of the machine's operation, in Chapter VI, follows roughly the chronological order of progress. In this
chapter are discussed three new techniques which bring about considerable improvement in the cyclotron beam.

(a). It is shown how the beam characteristics were improved by replacing the more conventional type of "feeler" by a plate on the dee facing the ion source, the plate having a rectangular hole in it through which the ions pass into the field free region of the dee after travelling only about a cm. from the ion source. As well as increasing the electric field at the ion source this arrangement substitutes a useful spacial bunching of ions for the phase bunching which occurs in cases where the electric field is extensive.

(b). In order to define the beam orbits, as well as to cut down the amount of waste beam accelerated, defining slits were introduced on the early revolutions. The defining and restricting of the beam in this manner greatly facilitated the measurements of the positions of orbit centres, leading to greater extraction efficiency. A discussion of the measurements of orbit centres is given in Appendix II.

(c). The fraction of beam which survived to large radii was increased several-fold following the fitting of slits giving vertical focusing at the early dee gap crossings.

Without vertical focusing, which has only recently been introduced, extraction efficiencies have varied from about 50 percent with 250 micro-Amp. extracted to 100 percent when accelerating about 20 micro-Amp. of beam.

Of the earlier chapters of the thesis the first two are quite general. A sketch of the history of high energy accelerator development is given in Chapter I, while some theory of acceleration is presented in Chapter II.

Although the theme of Chapter III is "injection", some general remarks are offered concerning proton synchrotrons to illustrate the desirability of such an undertaking as the Canberra air-cored synchrotron. This machine is briefly described.

Chapter IV provides some discussion of ion motion in cyclotrons.

The quadrupole focusing lenses, discussed in Appendix III, have so far been used with only small external beams of tens of micro-Amp.

In Appendix IV the investigation of the \( \text{B}^{11}(p,\gamma)\text{C}^{12} \) reaction is described. One reason for doing the experiment was
to determine the value of the cyclotron in this type of work, when the machine was operated with a very restricted beam which was completely extracted and then refocused by quadrupole lenses.

The Canberra cyclotron is about to be moved to a site more adequately shielded against radiation where consistent operation with larger beams will be possible. When the machine has been reassembled after the move attention will be devoted to the examination of possible means of improving the characteristics of large external beams. In particular the optical problems of bringing particles through the cyclotron’s fringing magnetic field without loss of beam quality will be investigated.

CANDIDATE’S CONTRIBUTION TO PROJECT.

The design, construction and operation of the Canberra cyclotron has been essentially a joint project. The candidate has worked alongside his supervisor almost throughout, and, even when sections of work were undertaken separately, joint discussion was maintained. Many new ideas came from discussion and in such cases it is, of course, difficult to allocate precise responsibility for the ideas.

When the candidate arrived at the National University in April of 1953 the cyclotron had been designed only in broad outline. During the remainder of that year he worked on detailed design, made drawings, and spent some six hundred hours machining the pole tips and vacuum system components to offset limited workshop resources. Broadly speaking 1954 was spent designing, constructing, testing and measuring components and ancillary equipment. For three months of 1955 effort was given to calculating machine operation and table reading for computations associated with the design of the proton synchrotron orbital magnet. During this year the cyclotron was assembled and tested. Work after this period is described in the thesis, mainly in Chapter VI.

In so far as it can be set down the candidate’s responsibility for work referred to in the thesis is shown in the table which follows.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapters II, III, IV.</td>
<td>Where source not acknowledged by reference to literature the candidate is responsible for the presented development of theory.</td>
<td></td>
</tr>
</tbody>
</table>

Chapter V. Detailed design and drawing of pole tips, vacuum system, ion source, deflector, water cooling system.
Magnetic field measurements.

Chapter VI. Orbit centre measurements.
Development of and measurements with multi internal defining slits.
Most of extraction.
Development of and operation with focusing slits.
Measurements on external beam.

Appendix I. Field measuring equipment design. entirely responsible for flip coil method.

Appendix II. Discussion of orbit centre measurements. Candidate entirely responsible.

Appendix III. Quadrupolo focusing lenses. Candidate entirely responsible.

Appendix IV. $B^{11}(p, \gamma)C^{12}$
Preparation of machine for experiment, operation of machine, design of target chamber, making of Boron targets.
Conduct of experiment. Candidate entirely responsible.

Discovery and employment of $Al^{27}$ reaction for energy measurements. Joint project with Mr. D. Gemmell, a student in Nuclear Physics. Candidate entirely responsible.
Table continued:

| Treatment of results and other material presented | Candidate entirely responsible |

ACKNOWLEDGEMENTS.

The candidate wishes to express his indebtedness to the following members of the Research School of Physical Sciences. Professor M.L.Oliphant for his initiating the project, his interest in it and his helpful encouragement. Professor E.W.Titterton who suggested the $B^{11}(p,\gamma)C^{12}$ experiment and assisted with helpful discussion throughout its conduct. Dr. W.I.B.Smith who has been supervisor, mentor, critic and friend. Mr. R.W.Parks who joined the cyclotron crew as a technician in 1954 and has willingly and competently helped since then. Mr. D.Gemmell for a happy association during the running of the nuclear physics experiment. All others, particularly those of the workshops, who have helped in various ways.

The candidate also wishes to thank the National University for a scholarship which enabled him to come here in the first instance.

(A.H.Morton)

April 1958.
CONTENTS.

Chapter. Page.

I. Historical Summary of High Energy Accelerator Development. 1

II. Some General Considerations of High Energy Accelerators. 14

III. Canberra Proton Synchrotron and Injection Requirements. 37

IV. Particle Motion in the Cyclotron. 65

V. Description of the Canberra 30 inch Cyclotron. 86

VI. Operation of the Canberra Cyclotron. 113

Appendix.

I. Measurement of the Magnetic Field. 147

II. The Measurement of Orbit Centres. 152

III. Quadrupole Focusing Lenses. 163

IV. The Reaction $^{11}\text{Be}(p,\gamma)^{12}\text{Be}$. 170
LIST OF FIGURES.

The subject of figure shown in list is not necessarily the title of the figure itself. Each figure follows as closely as possible the page on which reference is first made to it. Hence the page numbers of the table refer to pages immediately preceding the figures. Figures of appendices are not listed.

<table>
<thead>
<tr>
<th>Figure No.</th>
<th>Subject of Figure.</th>
<th>Page.</th>
</tr>
</thead>
<tbody>
<tr>
<td>II-1</td>
<td>Stable R.F. phase for accelerators.</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>R.F. accelerating gap</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>Particle motion in A.G. synchrotron</td>
<td>36</td>
</tr>
<tr>
<td>III-1</td>
<td>Orbital magnet of Canberra proton synchrotron</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>Homopolar generator</td>
<td>41</td>
</tr>
<tr>
<td>3</td>
<td>Pulse from homopolar generator</td>
<td>41</td>
</tr>
<tr>
<td>4</td>
<td>Beam loss by scattering</td>
<td>53</td>
</tr>
<tr>
<td>5</td>
<td>Synchrotron oscillations</td>
<td>55</td>
</tr>
<tr>
<td>6</td>
<td>Horizontal betatron oscillations for injection parallel to the equilibrium orbit</td>
<td>57</td>
</tr>
<tr>
<td>7</td>
<td>Horizontal betatron oscillations for injection at an angle to equilibrium orbit</td>
<td>57</td>
</tr>
<tr>
<td>8</td>
<td>Vertical betatron oscillations</td>
<td>58</td>
</tr>
<tr>
<td>IV-1</td>
<td>Axes for particle motion in cyclotron</td>
<td>66</td>
</tr>
<tr>
<td>2</td>
<td>Phase of R.F. for ions second crossing of dee gap.</td>
<td>66</td>
</tr>
<tr>
<td>3</td>
<td>Particle phase vs R.F. phase</td>
<td>71</td>
</tr>
<tr>
<td>4</td>
<td>Ion orbit centre</td>
<td>71</td>
</tr>
<tr>
<td>5</td>
<td>Maximum distances of ion and orbit centre from ion source</td>
<td>74</td>
</tr>
<tr>
<td>6</td>
<td>Phases of ions entering dee</td>
<td>76</td>
</tr>
<tr>
<td>7</td>
<td>Precession of orbit centre</td>
<td>81</td>
</tr>
<tr>
<td>V-1</td>
<td>Steps on face of pole tips</td>
<td>88</td>
</tr>
<tr>
<td>2</td>
<td>Effect of steps on face of pole tips</td>
<td>89</td>
</tr>
<tr>
<td>3</td>
<td>Rose shim as linear dipole</td>
<td>89</td>
</tr>
<tr>
<td>4</td>
<td>Rose shim dimensions for uniform field</td>
<td>89</td>
</tr>
<tr>
<td>5</td>
<td>Design shape of pole tips</td>
<td>89</td>
</tr>
<tr>
<td>6</td>
<td>Circuit for control and stabilization of magnetic field</td>
<td>91</td>
</tr>
<tr>
<td>7</td>
<td>Field near edge of pole tips</td>
<td>91</td>
</tr>
<tr>
<td>8</td>
<td>Azimuthal variation of magnetic field</td>
<td>91</td>
</tr>
<tr>
<td>9</td>
<td>Radial variation of magnetic field</td>
<td>92</td>
</tr>
<tr>
<td>10</td>
<td>Field variation to large radii</td>
<td>92</td>
</tr>
</tbody>
</table>
List of figures continued:

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-11</td>
<td>Main components of vacuum system</td>
<td>92</td>
</tr>
<tr>
<td>12</td>
<td>Ion source</td>
<td>93</td>
</tr>
<tr>
<td>13</td>
<td>R.F. cavity</td>
<td>94</td>
</tr>
<tr>
<td>14</td>
<td>Circuits of cavity and loop</td>
<td>95</td>
</tr>
<tr>
<td>15</td>
<td>Transformation $z = (Z+1)/(Z - 1)$</td>
<td>96</td>
</tr>
<tr>
<td>16</td>
<td>Apparatus for R.F. measurements</td>
<td>98</td>
</tr>
<tr>
<td>17</td>
<td>Initial check of resonant frequency of cavity</td>
<td>98</td>
</tr>
<tr>
<td>18</td>
<td>Cavity impedance</td>
<td>99</td>
</tr>
<tr>
<td>19</td>
<td>Impedance for comparison with cavity impedance</td>
<td>100</td>
</tr>
<tr>
<td>20</td>
<td>Point on admittance plot where susceptance is equal to conductance</td>
<td>101</td>
</tr>
<tr>
<td>21</td>
<td>Plot to obtain Q of cavity</td>
<td>101</td>
</tr>
<tr>
<td>22</td>
<td>Equivalent circuit of oscillator</td>
<td>103</td>
</tr>
<tr>
<td>23</td>
<td>Conditions of oscillation</td>
<td>106</td>
</tr>
<tr>
<td>24</td>
<td>Oscillator plate circuit</td>
<td>106</td>
</tr>
<tr>
<td>25</td>
<td>Calculated power curves for 3Q/260-E triode</td>
<td>107</td>
</tr>
<tr>
<td>26</td>
<td>Grid circuit</td>
<td>107</td>
</tr>
<tr>
<td>27</td>
<td>Complete oscillator circuit</td>
<td>107</td>
</tr>
<tr>
<td>28</td>
<td>Values of grid circuit parameters for correct feedback ratio</td>
<td>107</td>
</tr>
<tr>
<td>29</td>
<td>Admittance of grid circuit</td>
<td>108</td>
</tr>
<tr>
<td>30</td>
<td>Admittance of plate circuit</td>
<td>108</td>
</tr>
<tr>
<td>31</td>
<td>Circuit for dee voltage regulator</td>
<td>110</td>
</tr>
<tr>
<td>32</td>
<td>R.F. voltmeter</td>
<td>110</td>
</tr>
<tr>
<td>33</td>
<td>Deflector and splitter</td>
<td>111</td>
</tr>
<tr>
<td>34</td>
<td>Radii of curvature of deflector and splitter</td>
<td>111</td>
</tr>
<tr>
<td>VI-1</td>
<td>Resonance plots for 36 and 46 K.V. dee potential</td>
<td>115</td>
</tr>
<tr>
<td>2</td>
<td>Resonance plot for 80 K.V. dee potential</td>
<td>115</td>
</tr>
<tr>
<td>3</td>
<td>Effect of shim on magnetic field</td>
<td>117</td>
</tr>
<tr>
<td>4</td>
<td>Effect of shim on orbit centre</td>
<td>117</td>
</tr>
<tr>
<td>5</td>
<td>Orbit centres for 36 and 46 K.V. dee potential</td>
<td>118</td>
</tr>
<tr>
<td>6</td>
<td>Orbit centres for 80 K.V. dee potential</td>
<td>119</td>
</tr>
<tr>
<td>7</td>
<td>Beam profile at 135°</td>
<td>121</td>
</tr>
<tr>
<td>8</td>
<td>Beam profile at 225°</td>
<td>121</td>
</tr>
<tr>
<td>9</td>
<td>Resonance plot with 1/4&quot; defining slit</td>
<td>122</td>
</tr>
<tr>
<td>10</td>
<td>Loss of beam with radius, with and without defining slit</td>
<td>122</td>
</tr>
<tr>
<td>11</td>
<td>Beam vs radius for various positions of defining slit</td>
<td>122</td>
</tr>
<tr>
<td>12</td>
<td>Beam cross sections with 1/4&quot; slit</td>
<td>123</td>
</tr>
<tr>
<td>13</td>
<td>Beam cross sections for various positions of defining slit</td>
<td>123</td>
</tr>
<tr>
<td>14</td>
<td>Orbit centres when using 1/4&quot; defining slit</td>
<td>123</td>
</tr>
</tbody>
</table>
List of figures continued:

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>VI-15</td>
<td>Initial orbits with two defining slits</td>
<td>124</td>
</tr>
<tr>
<td>16</td>
<td>Resonance plots with two defining slits</td>
<td>125</td>
</tr>
<tr>
<td>17</td>
<td>Beam vs radius with two defining slits</td>
<td>126</td>
</tr>
<tr>
<td>18</td>
<td>Motion of orbit centres with two slits</td>
<td>126</td>
</tr>
<tr>
<td>19</td>
<td>Effect of deflector on precession</td>
<td>129</td>
</tr>
<tr>
<td>20</td>
<td>Beam accelerated and extracted (2 slits)</td>
<td>129</td>
</tr>
<tr>
<td>21</td>
<td>Beam from ion source</td>
<td>132</td>
</tr>
<tr>
<td>22</td>
<td>Graphical solutions of equations showing</td>
<td>134</td>
</tr>
<tr>
<td>23</td>
<td>spacial bunching of ions</td>
<td>134</td>
</tr>
<tr>
<td>24</td>
<td>Relative positions of ion source and ion</td>
<td>136</td>
</tr>
<tr>
<td>25</td>
<td>extractor plate</td>
<td>136</td>
</tr>
<tr>
<td>26</td>
<td>Cross sections of beam intensity</td>
<td>136</td>
</tr>
<tr>
<td>27</td>
<td>Focusing slit system</td>
<td>138</td>
</tr>
<tr>
<td>28</td>
<td>Effect of focusing slits without defining slits</td>
<td>139</td>
</tr>
<tr>
<td>29</td>
<td>Effect of focusing slits with 1 m.m. defining</td>
<td>139</td>
</tr>
<tr>
<td>30</td>
<td>Beam cross sections at 090°</td>
<td>140</td>
</tr>
<tr>
<td>31</td>
<td>Beam at first external slit system</td>
<td>142</td>
</tr>
<tr>
<td>32</td>
<td>Beam at second external slit system</td>
<td>142</td>
</tr>
</tbody>
</table>
CHAPTER I.

HISTORICAL SUMMARY OF HIGH ENERGY ACCELERATOR DEVELOPMENT.

"It has long been my ambition to have available for study a copious supply of atoms and electrons which have an individual energy far transcending that of the Alpha and Beta particles from radioactive bodies. I am hopeful that I may yet have my wish fulfilled, but it is obvious that many experimental difficulties will have to be surmounted before this can be realized, even on a laboratory scale." — Rutherford

Within a very few years of Rutherford expressing his wish for artificially accelerated particles there emerged two classes of machines which would provide these. The direct accelerators, such as the voltage multiplier or Cockcroft-Walton generator, and the Van der Graaff electrostatic generator promised to meet the more modest requirements of the period but had obvious

limitations imposed on their attainable energies by insulation problems. The multiple (or cyclic, or resonance) accelerators, on the other hand, were not faced with this difficulty, and it appeared as though the upper limits of energies which might be achieved were almost unrestricted. It is this class of machines which we shall discuss as high energy accelerators.

The fundamental principles of a number of the high energy accelerators were first published in 1928, by Wideroe. The linear accelerator principle which involves the application of a radio frequency potential to a series of electrodes, such that the charged particles see an accelerating field as they cross each electrode gap, was soon adopted for the acceleration of heavy ions to low energies. However, although the linear accelerator is now producing both high energy electrons and protons, many years of effort and research were necessary to overcome the problems inherent in this type of machine. The induction accelerator or betatron, so enthusiastically considered at conception, was soon found to be anything but simple to bring into operation. It was the cyclotron, employing a

radio frequency electric field between two duants, or "dees",
to accelerate particles constrained to approximately spiral
orbits by a magnetic field, which was almost immediately
successful among the new high energy accelerators. For some
ten years this machine was the only cyclic, or resonant,
accelerator to be used as a tool by nuclear physicists.

Initial experiments to test the feasibility of the cyclotron
were carried out early in 1930, at Yale University, Newhaven,
under the direction of E.O. Lawrence. The projected construction
was reported in September of that year to the American National
Academy of Sciences. The following year Lawrence, now at the
University of California, Berkeley, had a cyclotron accelerating
molecular hydrogen ions to 80 KeV. Within a short time a
second experimental machine was giving 0.1 micro-Amp of 0.5 MeV
protons, which convinced Lawrence that it was only a matter of
bigger magnets and better R.F. oscillators before 10 MeV
particles were obtained.

After producing helium ions and protons with energies
greater than 1 MeV with their machine of 11 inch pole,
the Berkeley team embarked on a more ambitious programme of

constructing a machine with 45 inch pole tips in an endeavour to exceed 10 Mev in proton energy. The further developments at Berkeley\textsuperscript{14,15} and the building of a machine at Cornell\textsuperscript{16} might be regarded as concluding the initial development of the cyclotron. By the end of 1936 it was well established as a successful and promising source of energetic positive ions.

During the immediate prewar years cyclotrons were built at many centres\textsuperscript{X}, and a number of improvements in design introduced. Probably the most important of these was the replacement of glass insulators by inductances, within the vacuum system, for supporting the dees\textsuperscript{18}.

Theoretical contributions of this period by Rose\textsuperscript{19}, Wilson\textsuperscript{20}, Bethe\textsuperscript{21}, and Thomas\textsuperscript{22} led to a better understanding of the part played by the magnetic and electric fields in focusing the beam. Qualitative estimates of desired field variation with radius could be made, and shimming became a somewhat more orderly procedure. A greater area of the magnet gap became available

\begin{itemize}
\item \textsuperscript{14} E.O.Lawrence, M.S.Livingston, Phys. Rev. 45, 608 (1934).
\item \textsuperscript{15} E.O.Lawrence, D.Cooksey, Phys. Rev. 50, 1131 (1936).
\item \textsuperscript{16} M.S.Livingston, Rev. Sc. Instrum. 7, 55 (1936).
\item \textsuperscript{X} In April, 1939, there were 26 cyclotrons either in operation or under construction\textsuperscript{17}.
\item \textsuperscript{17} W.B.Mann, Nature 143, 583 (1939).
\item \textsuperscript{18} M.S.Livingston, J.H.Buck, R.D.Evans, Phys. Rev. 55, 1110 (1939).
\item \textsuperscript{19} M.E.Rose, Phys. Rev. 53, 392 (1938).
\item \textsuperscript{20} R.R.Wilson, Phys. Rev. 53, 408 (1938).
\item \textsuperscript{21} L.H.Thomas, Phys. Rev. 54, 580 and 588 (1938).
\item \textsuperscript{22} H.A.Bethe, M.E.Rose, Phys. Rev. 52, 1254 (1937).
\end{itemize}
with the introduction of edge (or ring or Rose) shims\textsuperscript{23,24}.

Towards the end of this term of activity the state of the cyclotron was reviewed by Mann\textsuperscript{25}, and Livingston\textsuperscript{26} showed the field at a slightly more advanced state. The culmination of the period's achievement was the reported successful operation of the Berkeley 60 inch machine\textsuperscript{27}. A 25 micro-A. beam of 8 Mev protons was claimed, as well as 16 micro-A of 16 Mev deuterons, extracted. Those responsible for the machine felt confident that 25 Mev deuterons and 50 Mev alpha particles were possible with this machine, and that much higher energies could be achieved simply by constructing larger ones. The optimism of scientists at this period is reflected in Lawrence's mention of a 120 inch cyclotron with dee voltages between one and two $10^6$ volts (Foreword of second reference 25). Before America was involved in hostilities a start had been made on a 184 inch cyclotron at Berkeley. The majority of the present day fixed frequency cyclotrons are, in essentials, the same as the type of machine that was developed during this period.

\begin{thebibliography}{9}
\bibitem{24} W.J.Henderson et alia, J. Frank. Inst. 228, 563 (1939).
\bibitem{26} M.S.Livingston, J. App. Phys., 15, 2 and 128 (1944).
\bibitem{27} E.O.Lawrence et alia, Phys. Rev. 56, 124 (1939).
\end{thebibliography}
Inspite of the great effort and money directed to science and technology during the years 1940-1945, there was not much stimulation in the field of particle acceleration. The paucity of unclassified literature of this period may tend to obscure some of the advances made, but there is no doubt that machine building was virtually halted till "after the war", with perhaps the betatron as the exception.

The betatron took its place as a practical machine in 1940, when 2.3 Mev electrons were produced at Illinois. The appealingly simple principle of induction acceleration required the addition of Kerst's careful design studies, based on a theoretical consideration of the electron's orbital behaviour, and the focusing characteristics of this type of machine, before success could be achieved. After the initial operation of the Illinois betatron other machines, on a grander scale, were soon built. Kerst constructed a 20 Mev accelerator for the General Electric Company at Schenectady, and one for 100 Mev electrons was completed at the same establishment in 1945. Although the limit of the betatron was not reached at this stage, a limit

32. In 1950 a 315 Mev machine came into operation at Illinois.
was forseen due to radiation energy loss\textsuperscript{33,34}.

The pattern for the post war development in particle acceleration was set by Veksler. In two papers in 1944\textsuperscript{35,36} he pointed out that the problems associated with acceleration to very high energies in the betatron and cyclotron were not necessarily limiting ones, and could, at least in principle, be overcome. A third paper appeared under his name in 1945\textsuperscript{37}, and was followed by one on similar lines by McMillan\textsuperscript{38} who was unaware that he had been preceded\textsuperscript{39,40}. The theme of Veksler and McMillan was phase stability, as an inherent property of the R.F. acceleration of particles in a magnetic field. They showed that if an accelerated particle gets out of phase with the radio frequency accelerating field it tends to adjust its orbit so that its phase deviation is reduced. This means that the magnetic field, or the frequency of the electric field, or both, might be varied, and the particle not necessarily be lost by acquiring wrong phase characteristics.

\textsuperscript{33} D.Iwanenko, I.Pomeranchuk, Phys. Rev. 65, 343 (1944).
\textsuperscript{34} J.P.Blewett, Phys. Rev. 69, 87 (1946).
\textsuperscript{36} do. 44, No. 9, 365 (1944).
\textsuperscript{39} V.I.Veksler, Phys. Rev. 69, 244 (1946).
\textsuperscript{40} E.Mc.Millan, Phys. Rev. 69, 534, (1946).
The principle of phase stability, under conditions of varying magnetic field, or electric field frequency, or both, is employed in three types of accelerators. The frequency modulated, or synchro-, cyclotron employs a magnetic field which is stationary in time, together with a varying frequency electric field, to accelerate protons, or heavier ions, from virtually zero energy well into the relativistic range. The electron and proton synchrotrons maintain the accelerated particles on approximately constant radius orbits by means of rising magnetic fields, the particles being "injected" into the synchrotron orbit after initial acceleration by other means. Once electrons have an energy of a few Mev their velocity is almost that of light, and further acceleration produces only a negligible change in the particles' resonant frequency. For this reason the electron synchrotron can operate with the frequency of the accelerating electric field constant. In the case of the proton synchrotron, however, since the heavier particles do not reach near light velocity till they have a few Bev of energy, both the accelerating field frequency and the magnetic guide field need to increase during the accelerating cycle.

X. Except in the case of strong focusing electron synchrotrons, and race-track type machines, the initial acceleration is achieved by means of a betatron incorporated in the synchrotron unit, and a proton synchrotron may accept particles from any suitable lower energy accelerator.
It is probable that the first electron synchrotron to operate, apart from Goward’s 41 8 Mev experimental model at Malvern (T.R.E.), was a 30 Mev machine at the Physical Institute of the Russian Academy of Sciences, the construction of which Veksler reported as nearing completion at the beginning of 1946. 39
By the end of 1951 there were more than twenty machines either completed, or under construction, to produce from 20 to 340 Mev electrons 42.

At the end of the war Berkeley was in an ideal position to enter the field of frequency modulated cyclotrons. The 184 inch cyclotron was at a stage of construction which allowed ready modification. Experiments were carried out using the 37 inch cyclotron, the pole tips of which were redesigned to give a radial field decrease of 13 percent, thus simulating the relativistic decrease in the resonant frequency which would be expected in the frequency modulated machine. 43 The initial performance of the 184 inch accelerator was reported from Berkeley 44 early in 1947. 195 Mev deuterons, and 390 Mev alpha particles were obtained. Subsequently, after modification, the machine produced 350 Mev protons. Livingston 45 lists nine

43. J.R.Richardson et alia, Phys. Rev. 73, 424 (1948).
synchrocyclotrons as operating with proton energies of 100 MeV or more, in 1954. The Berkeley 184 inch was being converted at that time to produce 730 MeV protons.

Although there was immediate enthusiasm for the electron synchrotron and the synchrocyclotron, laboratories were hesitant about embarking on construction of proton synchrotrons. These machines would be very expensive, and optimism concerning their practicability was not high. The plunge was taken at Birmingham, England, in 1946, when Oliphant made the decision to build a 1 BeV proton synchrotron. Following the theoretical and design studies for the British machine, decisions were taken in the United States to build two proton synchrotrons: the "Cosmotron" at Brookhaven, and the "Bevatron" at Berkeley. Successful acceleration of protons to 2 BeV was achieved at Brookhaven by mid 1952, and 3 BeV particles were obtained in 1954. An extensive review of the Cosmotron, edited by Blewett appeared in 1953. The Berkeley machine attained 4.9 BeV in 1954, and in 1955 more than 6 BeV. The Birmingham proton synchrotron was operating at 1 BeV in 1953.

With the discovery of the principle of strong focusing\(^{50}\) it was reasonable to expect that it would be possible to construct higher energy synchrotrons without unduly increasing their dimensions. The first strong focusing electron synchrotron was built at Cornell. Two 25 Bev strong focusing proton synchrotrons have now been under construction for some time: one at Brookhaven, the other at Geneva. However these do not yet belong to history.

Whether proton energies much in excess of 25 Bev will be obtained in the foreseeable future is very likely to depend on the success of the Canberra air-cored synchrotron. This machine, designed to produce 10 Bev protons, departs markedly from contemporary design. A brief description of it will be given in Chapter III.

The eventual success of the linear accelerator as a machine for producing high energy protons and electrons can be directly attributed to the ultra high frequency techniques developed during the war. The availability of ultra high frequencies, together with the discovery of the principle of phase focusing, have led to such machines as the 32 Mev proton accelerator at Berkeley\(^{51}\), and the 600 Mev electron accelerator at Stanford.\(^{50}\) E.C.Courant, M.S.Livingston, S. Snyder, Phys. Rev. 88, 1190 (1952).\(^{51}\) L.W. Alvarez, Phys. Rev. 70, 799 (1946).\(^{51}\)
As examples of post war fixed frequency cyclotrons which reflect some excellence of design and construction one might quote the low magnetic field (8709 gauss) 86 inch machine at Oak Ridge\textsuperscript{52} which gives an internal beam of more than 1 \text{m}\mathring{\text{A}}\textsuperscript{23} Mev protons, and the Livermore (California) machine\textsuperscript{53}, with its cam shaped pole tips. Both of these machines have their dees suspended vertically in a horizontal magnetic field.

Although the cyclotron has for many years been the only machine which is widely used in the energy range of 6 to 20 Mev protons, its value as a nuclear physics tool has been severely limited by the high level of background radiation which accompanies its operation. Many excitation curves for proton induced reactions, for example, are still not extended beyond 5 Mev, up to which energy the direct accelerators can be used. It is as a source of radio isotopes that the cyclotron has perhaps proved most valuable. Also as a neutron source it has been outstanding, being able to compete with the atomic pile.

To the very high energy accelerators the subject of particle physics owes its development. During the last few

\textsuperscript{52} R.S. Livingston, A.L. Boch, O.R.N.L.-1196 (June, 1952).
years investigations of mesons have been carried to a stage that would not have been reached in centuries had cosmic rays remained the only source of particles. To date the crowning achievement of the high energy accelerator is surely the production of the first observed antiproton at Berkeley towards the end of 1955.

Two types of machines which, if they ever become practicable, will have a great impact on the future of the accelerator field are the F.F.A.G. X cyclotron and the stabilized electron beam accelerator. F.F.A.G. machines were discussed at an early stage of consideration in 1955, and since then the Mid-Western Universities of America have made some progress with their development. The idea of using an intense electron beam to provide guiding fields for heavier particles has been discussed by Budker.

CHAPTER II.

SOME GENERAL CONSIDERATIONS OF HIGH ENERGY ACCELERATORS.

All the high energy accelerating machines, except the linear accelerator, employ a magnetic guide field; all, except the betatron, feed the accelerated particles with energy from a radio-frequency electric field. As is to be expected the machines generally have a number of features in common. Wherever a magnetic field is used to constrain particles to circular type orbits betatron oscillations are a feature of the particle motion; and, if the magnetic field varies with time, solonoidal acceleration will take place. Accompanying radio-frequency acceleration there is phase focusing and, as well, some degree of spacial focusing associated with the non-uniformity of the electric field in the accelerating gaps. The extent to which these features are important depends upon the behaviour of the
fields, magnetic and radio-frequency electric, and whether both fields are present, as well as upon the specific charge of the accelerated particles and the energy range through which they are taken in any particular machine.

In Table II-1 the main accelerator types are listed together with their characteristic field behaviour. The last column of the table indicates the approximate energy range through which the machines can be expected to accelerate their particles.

<table>
<thead>
<tr>
<th>ACCELERATOR</th>
<th>BEHAVIOUR OF FIELD</th>
<th>RANGE OF ACCELERATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Betatron</td>
<td>Magnetic: variable</td>
<td>zero to a few-100 Mev</td>
</tr>
<tr>
<td></td>
<td>Electric: none</td>
<td></td>
</tr>
<tr>
<td></td>
<td>R.F.: zero to a few-100 Mev</td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>Magnetic: none</td>
<td>Protons: zero to few-25 Mev</td>
</tr>
<tr>
<td></td>
<td>Electric: variable*</td>
<td>Electrons: &lt; 1 Mev to several-700 Mev</td>
</tr>
<tr>
<td></td>
<td>R.F.: variable</td>
<td>Protons: &lt; 1-few Mev to few-30 Mev</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Protons: zero to few-25 Mev</td>
</tr>
<tr>
<td>Cyclotron</td>
<td>Magnetic: fixed</td>
<td>Electrons: fixed**</td>
</tr>
<tr>
<td></td>
<td>Electric: variable</td>
<td>Protons: zero to few-25 Mev</td>
</tr>
<tr>
<td></td>
<td>R.F.: zero to few-25 Mev</td>
<td></td>
</tr>
<tr>
<td>Synchro-</td>
<td>Magnetic: fixed</td>
<td>Protons: zero to few-25 Mev</td>
</tr>
<tr>
<td>cyclotron</td>
<td>Electric: variable</td>
<td>Protons: zero to few-25 Mev</td>
</tr>
<tr>
<td>Electron</td>
<td>Magnetic: variable</td>
<td>Protons: zero to few-25 Mev</td>
</tr>
<tr>
<td>synchrotron</td>
<td>Electric: fixed***</td>
<td>Protons: zero to few-25 Mev</td>
</tr>
<tr>
<td>Proton</td>
<td>Magnetic: variable</td>
<td>Protons: zero to few-25 Mev</td>
</tr>
<tr>
<td>synchrotron</td>
<td>Electric: variable</td>
<td>Protons: zero to few-25 Mev</td>
</tr>
</tbody>
</table>

* R.F. is actually constant. However the electrode assembly makes it appear as variable to the accelerated particles.
** If betatron injection is not used some frequency variation is necessary during initial acceleration of electrons.

This chapter will be devoted to the development of some of the equations describing particle motion under various conditions of acceleration. Where applicable they will be used to illustrate
features of machine operation, some of which we have referred
to above. The symbols which will be used are more or less
conventional and are listed, with their meanings, in the next
section.

Symbols.

\( m_0 \) rest mass of particle, \( 1.6724 \times 10^{-24} \) gm for proton,
\( 9.1085 \times 10^{-28} \) gm for electron.

\( v \) speed of particle.

\( c \) speed of light.

\( m \equiv m_0/(1 - v^2/c^2)^{1/2} \).

\( V \) electric potential.

\( H \) magnetic field strength, gauss.

\( p \) momentum.

\( E \) energy of particle including rest energy.

\( E_0 \) rest energy = \( m_0 c^2 \), 938 Mev for proton, 0.51 Mev for electron.

\( T \) K.E. of motion = \( E - E_0 \).

A useful relation between momentum and energy for a
relativistic particle is obtained by writing

\[ m^2 = m_0^2/(1 - v^2/c^2), \] or

\[ m^2c^4 - m_0c^2v^2 = m_0^2c^4. \]

i.e. \( p^2c^2 = E^2 - E_0^2 \), or \( p^2 = T(T + 2E_0)/c^2 \).
e.....electronic charge, 4.8029x10^{-10} e.s.u.

r.....radius of particle orbit.

R.....radial distance from magnetic centre of machine, where
the magnetic centre is the centre of curvature of field
contours in the median plane*.

w.....angular frequency of particle, v/r.

**Equilibrium Orbits and Betatron Oscillations.**

For a particle moving with constant speed in a uniform
magnetic field the Lorentz and centrifugal forces balance.

This is expressed by

\[ \frac{Hev}{c} = \frac{mv^2}{r}, \]  

**II-1.**

r being perpendicular to H.

Consistent with this equation are the following useful relation-
ships:

(a) \( p = \frac{He\nu}{c} \)

(b) \( w = \frac{He\nu}{E} \)

(c) \( r = \left( E^2 - E_o^2 \right)^{1/2}/He \)

(d) \( r = c\left( E^2 - E_o^2 \right)^{1/2}/wE \)

* In alternating gradient synchrotrons and EFAG machines there
is no single magnetic centre. The magnetic centre of a
standard synchrotron with straight sections is simply the
centre of curvature of each curved sector. In the case of
the betatron, standard cyclotron, synchrocyclotron, and
standard circular synchrotron the magnetic centre coincides
with the geometric centre of the machine.
When the condition of magnetic field uniformity required by equation II-1 does not hold, the particle will experience an acceleration, in the plane of its orbit, at right angles to its path, equal to

\[ \frac{d^2r}{dt^2} = -\frac{Hev}{c} \cdot \frac{v^2}{r} \]  \[ \text{II-3.} \]

In accelerators where the field varies radially there is, for a given particle energy, one particular orbit for which the radial acceleration is zero. The centre of this orbit coincides with the magnetic centre of the machine. This is the so-called equilibrium orbit.

The radial variation of an accelerator's magnetic field is conventionally expressed in terms of an index, \( n \), evaluated at a radius \( R_0 \), where the field is \( H_0 \), by the relation

\[ H = H_0 (R_0/R)^n, \]

whence the "n value", in terms of the variation of field with radius, is expressed as

\[ n = -(R_0/H_0) \cdot \frac{\delta H}{\delta R} \]  \[ \text{II-4.} \]

If we consider orbits whose centres are very close to the magnetic centre, so that we may write, as a reasonable approximation, equation II-3 may be varied to yield

\[ \frac{d^2}{dt^2}(\delta r) = \left( -\frac{ev}{mc} \frac{\delta H}{\delta r} - \frac{v^2}{r^2} \right) \delta r \]

\[ = (n\frac{Hev}{mc} - \frac{v^2}{r^2}) \delta r \]
Thus for small radial displacements from the equilibrium orbit the particle will oscillate about it with a frequency of \((1 - n)^{1/2}w\).

Any radial field component, \(H_r\), will produce a force on a particle in the direction of \(z\), perpendicular to the median plane. This means an acceleration

\[
\frac{d^2z}{dt^2} = H_r \frac{ev}{mc},
\]

or, by variation with respect to \(z\),

\[
\frac{d^2}{dt^2} \delta z = (ev/mc)(\frac{\partial H_r}{\partial z}) \delta r.
\]

With irrotational fields, and in the absence of a circumferential field component, \(\frac{\partial H_r}{\partial z} = \frac{\partial H}{\partial r}\). Using this relation, and equation II-4, we obtain

\[
\frac{d^2}{dt^2} \delta z = (ev/mc)(-nH/r) \delta z
\]

\[
= -nw^2 \delta z
\]

This equation shows that particles off the median plane will oscillate about it with a frequency of \(n^{1/2}w\).

These two oscillations, of frequencies \((1 - n)^{1/2}w\), and \(n^{1/2}w\), are the "Betatron oscillations". In machines employing very small \(n\) values, e.g. cyclotrons, where \(n\) is of the order of 0.01, the radial betatron oscillation frequency will be
almost that of the resonant frequency of the machine, while
the vertical oscillation frequency will be only about one tenth
of this value. With an \( n \) value of about \( 1/2 \), which is in the
region of that found in standard proton synchrotrons, the two
betatron oscillation frequencies are nearly equal. High \( n \)
values, much greater than unity, are used in alternating
gradient machines to which reference will be made later.

Increasing the magnetic field during acceleration brings
about a damping of betatron oscillations. The manner in which
this occurs can be readily illustrated by applying the W.K.B.
approximation in solving equations II-5 and II-6.

Writing, in equation II-5, \( y \) for \( \delta r \), and expressing \( w \) as
\( f(t) \), since \( w \) will vary with the magnetic field, we have
\[
d^2y/dt^2 + f(t) = 0, \quad \text{...............II-7.}
\]
with \( f(t) \) positive.

Assuming \( y = Ae^{1B} \) as a solution equation II-7 becomes
\[
A'' + i(AB'' + 2A'B') + (f - B'^2)A = 0 \quad \text{............... II-8.}
\]
where primes denote differentiation of \( y \) with respect to \( t \).

Imposing one arbitrary condition \[ \text{it.} \quad B' \text{ may be related to} \]
\( f(t) \) by putting \( f - B'^2 = 0 \) \( \text{i.e.} \)
\[
B = \int_{t_0}^{t} f^{1/2} \cdot dt
\]
Since \( f(t) \), the resonant frequency, is slowly varying with
time, A also will be, and A" may be neglected in comparison with A'. Equation II-8 then becomes

\[ AB'' + 2A'B' = 0. \]

Integration gives \( A = K(B')^{-1/2} \), where \( K \) is a constant,

\[ = Kf^{-1/4}, \]

giving for an approximate solution of II-5

\[ y = Kf^{-1/4} \exp \left( \int_{t_0}^{t} f^{1/2} \, dt \right) \] ....................II-9.

This equations shows that the amplitude of oscillation, varying as \( f^{-1/4} = w^{-1/2} \), will decrease with rising magnetic field, as the inverse of the square root of the field value. A similar treatment for equation II-6 shows the same form of compression of the vertical betatron oscillations. The oscillation amplitude damping in betatrons and synchrotrons contributes considerably to the small beam cross sections achieved in these machines.

**Solenoidal Acceleration.**

Accompanying any variation of magnetic field with time there is a force along the orbit of particles moving in the field equal to \( (e/2\pi r) d\phi/dt \), where \( \phi \) is the magnetic flux threading the orbit. This force will retard or increase the orbital velocity depending upon the direction of the flux change and the sign of the charge of the accelerated particle. The
The betatron employs this type of acceleration, with the rate of change of particle momentum being given by

\[ \frac{dp}{dt} = \frac{e}{2\pi r} \frac{d\phi}{dt}, \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \ \text{II-10.} \]

if the energy loss by radiation from the accelerated electrons is neglected. Under such circumstances equation II-10 can be integrated, Her/c substituted for p, and the total flux change expressed as

\[ \phi = 2\pi r^2 H/c. \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \ \text{II-11.} \]

Here \( r = R \) and is approximately constant during acceleration. Towards the end of the acceleration cycle \( r \) is changed sufficiently, by changing the relationship between \( \phi \) and \( H \), to bring the electron beam onto a target. Equation II-11 shows that the change of flux through the orbit to any particular stage of acceleration must be equal to twice the flux that would be given by taking the guide field over the whole orbital area.

The changing flux is usually carried by "flux bars" through the orbit. As these are steel, and saturate at flux densities of about 20,000 gauss, a restriction exists on the total flux change possible. In some betatrons the central field is biased negatively at the start of each cycle in order to extend the possible flux change. The energy limit of the betatron is due to radiation loss of energy by the electrons becoming equal to
the rate at which energy can be provided by a physically achievable central flux change. Iwanenko and Pomeranchuk have estimated the limiting energy value of the betatron to be about $5 \times 10^8$ electron volts. The rate of energy radiation by an electron is proportional to $(e^2/r)(E/E_0)^4$. Schwinger gives a convenient expression for the energy radiated per revolution,

$$\delta E = 88.5E^4/R,$$

where $E$ is in Bev, $R$ in metres and $\delta E$ in Kev per electron revolution.

Most electron synchrotrons use betatron injection in order to avoid the necessity of varying the frequency of the electric accelerating field. Quite a small betatron acceleration is sufficient in such cases as electrons approach the velocity of light at energies of only a few Mev.

**Orbital Acceleration Equations.**

Particles accelerated by radio frequency electric fields receive energy over only discrete sections of their orbits, wherever accelerating gaps are situated. However it is useful to regard the addition of energy, say $W$ per revolution, as due

---

to an average force \( \frac{dW}{ds} = \frac{W}{2\pi r} \), where \( s \) is distance measured along the orbit. The rate of particle momentum increase can then be expressed in terms of three forces: the force \( \frac{dW}{ds} \), just mentioned, a force due to any time variation of the magnetic field, and the loss of energy by radiation, say \( U \) (energy) per revolution, considered as a retarding force \( \frac{dU}{ds} = \frac{U}{2\pi r} \). In terms of these we have
\[
\frac{dp}{dt} = \frac{W}{2\pi r} \pm \left( \frac{e}{2\pi r} \right) \frac{d\phi}{dt} - \frac{U}{2\pi r} \ldots \ldots II-12.
\]
If \( r \) is constant the rate of increase of angular momentum is given by
\[
\frac{d}{dt}(pr) = \frac{dW}{d\theta} \pm \left( \frac{e}{2\pi} \right) \frac{d\phi}{dt} - \frac{dU}{d\theta} \ldots \ldots II-13.
\]
or the rate of energy increase by
\[
\frac{dE}{dt} = \frac{wW}{2\pi} \pm \left( \frac{we}{2\pi} \right) \frac{d\phi}{dt} - \frac{dU}{dt} \ldots \ldots II-14.
\]
In the second and third equations \( U \) is the energy loss by radiation, but not per revolution.

Of the terms on the right hand side of the above equations the first is the most important for accelerators other than the betatron, where it does not have any place. For machines accelerating protons the second and third terms are of very minor importance, indeed negligible. The last term is of considerable importance once electrons reach high energies.
Phase focusing and phase oscillations have been discussed in detail by several authors, the first two contributing to the subject being Veksler\(^3\) and McMillan\(^4\). As focusing here implies an acceleration towards some equilibrium value it is convenient to discuss the matter in terms of the quantity \(\frac{d^2\phi}{dt^2}\), where \(\phi\) is the phase of the electric field as the particle crosses the region of acceleration. The expression for \(\frac{d^2\phi}{dt^2}\) can be obtained, in an approximate manner, with little algebra, as follows.

The rate of change of \(\phi\), \(\frac{d\phi}{dt}\), is equal to the difference between the existing angular frequency, \(w\), of the particle being considered and the angular frequency of the electric field. This latter is the angular frequency of a particle on the equilibrium orbit enjoying acceleration at the equilibrium phase, and will be denoted by \(w_0\).

Thus

\[
\frac{d\phi}{dt} = w_0 - w
\]

and

\[
\frac{d^2\phi}{dt^2} = \frac{dw_0}{dt} - \frac{dw}{dt}.
\]

In machines where phase focusing is important either the frequency \(w_0\), or the magnetic field, or both are varied in a manner consistent with acceleration at an equilibrium phase.

φ. We shall first of all determine $dw/dt$ for the general case where $w_0$, and $H$, the field at the equilibrium orbit, are both regarded as variables.

Assuming the radial acceleration accompanying phase oscillations to be negligible we may start from the relations of equation II-2. Using $w = Hec/E$ we obtain

$$\frac{dw}{dt} = (ec/E)\frac{dH}{dt} - (Hec/E^2)\frac{dE}{dt}$$

$$\frac{dw}{dt} = \frac{dc}{E} \left[ \frac{dH}{dt} \frac{dr}{dt} + \frac{r}{\rho} \frac{dH}{dt} \right] - \frac{Hec}{E^2} \frac{dE}{dt}$$

i.e.

$$\frac{dw}{dt} = (w/H)\frac{dH}{dt} + (w/H)\frac{dH}{dt} - (w^2/Hec)\frac{dE}{dt} \quad \text{II-15.}$$

From II-2(c) we have

$$\frac{dr}{dt} = \frac{E}{(E^2 - E_0^2)\sqrt{2}} \cdot H \cdot \frac{dE}{dt} - \frac{E^2 - E_0^2}{E_0^2} \cdot \frac{He}{E} \cdot \frac{dE}{dt} \quad \text{II-16.}$$

$$\frac{dr}{dt} = \frac{c}{wHe} \cdot \frac{dE}{dt} - r/H \cdot \frac{dH}{dt} - \frac{r}{H} \cdot \frac{dH}{dt}$$

i.e.

$$\frac{dr}{dt} = \frac{c}{(1-n)wHe} \cdot \frac{dE}{dt} - \frac{r}{(1-n)H} \cdot \frac{dH}{dt} \quad \text{II-16.}$$

Substitution of equation II-16 into equation II-15 yields
\[
\frac{dw}{dt} = \left(\frac{w}{H}\right)\left(-n\frac{H}{r}\right)\frac{\partial H}{\partial t} + \left(\frac{w}{H}\right)\frac{\partial x}{\partial t} \\
+ \left(\frac{w}{H}\right)\left(-n\frac{H}{r}\right)\left(c/(1-n)wH\right)\frac{dE}{dt} - \left(\frac{w^2}{He}\right)\frac{dE}{dt} \\
= \left(\frac{w}{H}\right)\left(1 + n/(1-n)\right)\frac{\partial H}{\partial t} - \left(\frac{1}{He}\right)\left(nc/(1-n)r^2 + w^2/c\right)\frac{dE}{dt}
\]

i.e.

\[
\frac{dw}{dt} = \frac{w}{H} \cdot \frac{1}{1-n} \frac{\partial H}{\partial t} - \frac{c}{He} \left[\frac{n}{1-n} + \frac{v^2}{c^2}\right] \frac{dE}{dt} \quad \text{.................................. II-17.}
\]

There will be an equation similar to II-17 for \(dw_0/dt\), with terms evaluated on the equilibrium orbit. The only significant difference between the values of the two expressions will be due to \(dE/dt\). Substituting from equation II-14, and taking \(W = eV_0 \sin \phi\), and \(W_0 = eV_0 \sin \phi_0\), where \(V_0\) is the equivalent R.F. accelerating voltage, we obtain the phase acceleration equation as

\[
\frac{d^2\phi}{dt^2} = \frac{woeV_0}{2H_0r_0^2} \left(\frac{n}{1-n} + \frac{v^2}{c^2}\right) (\sin \phi - \sin \phi_0) \quad \text{.......................... II-18.}
\]

It will be noticed that this expression implies focusing provided \(\sin \phi\) decreases as \(\phi\) increases. This requires that the value of \(\phi_0\) should lie between 90° and 180°.

The damping of phase oscillations with increasing magnetic

\* This is of the same form as the equation for a pendulum experiencing a constant torque.
field does occur, the phase amplitude decreasing approximately as $H^{-1/4}$. Twiss and Frank\textsuperscript{5} discuss fully the damping of both betatron oscillations and phase oscillations in their paper on the orbital stability of the proton synchrotron.

From Equation II-18 the frequency of phase oscillations, $w_{ph}$, is obtained approximately as

$$w_{ph} = \left( \frac{cV_0}{2r_0^2H_0w_0} \right)^{1/2}(\frac{n}{1-n} + \frac{v^2}{c^2})^{1/2}w \quad \text{II-19.}$$

For low energy particles, as for example in not very high energy synchrocyclotrons, $v^2/c^2$ can be neglected, while for relativistic energies, such as met in the electron synchrotron, $v^2/c^2$ can be taken as unity, and the second right hand term of II-19 becomes $1/(1-n)^{1/2}$. In general the frequency of phase oscillations, for existing machines, lies between $2\times10^{-2}$ and $2\times10^{-3}$ times the orbital frequency.

The amplitude of the radial synchrotron oscillations which accompany the phase oscillations can be expressed as a variation in orbit radius as

$$\frac{\Delta r}{r} = \frac{\Delta w}{w}.$$
\( \Delta w \) is readily obtained by integration of Equation II-18.

It is useful to rewrite II-18 as

\[
\frac{d^2 \phi}{dt^2} = \left( \frac{ec^2V_0}{\pi \rho c E} \right) \left( \frac{n}{1-n} + \frac{v^2}{c^2} \right) (\sin \phi - \sin \phi_0)
\]

\( E = He c/w \)

in order to involve the particle energy.

If this expression is multiplied by \( \frac{d \phi}{dt} \) it can be directly integrated to yield

\[
(1/2) \left( \frac{d \phi}{dt} \right)^2 = \left( \frac{ec^2V_0}{\pi \rho c E} \right) \left( \frac{n}{1-n} + \frac{v^2}{c^2} \right) (\cos \phi_0 - \cos \phi - (\phi - \phi_0) \sin \phi_0)
\]

or, since \( \frac{d \phi}{dt} = \omega_0 - w = \Delta w \), we have

\[
\Delta r = \frac{(eV_0)^{1/2} c}{w(\pi E)^{1/2}} \left[ \frac{n}{1-n} + \frac{v^2}{c^2} \right]^{1/2} (\cos \phi_0 - \cos \phi - (\phi - \phi_0) \sin \phi)^{1/2}
\]

........................II-19(a).

In the linear accelerator phase focusing is achieved by having the particles cross the accelerating gaps as the electric field is rising. This means that \( \phi_0 \) lies between 0° and 90°. Phase focusing here, as will be shown in the next section, precludes any possibility of spatial focusing by the normal
type of accelerating gap.

Figure II-1 shows the allowable radio frequency phases for acceleration in orbital machines and the linear accelerator. As a result of phase focusing there is, of course, a bunching of the accelerated particles.

Phase focusing which occurs in the cyclotron will be discussed in chapter IV.

**Focusing Effects of R.F. Accelerating Fields.**

We shall consider the electric accelerating field to be uniform across the accelerating gap, and, with this approximation, shall determine the instantaneous changes of momentum perpendicular to the median plane, or the axis of acceleration, as the particle enters and leaves the gap. The net change in this momentum as the particle passes through the gap will be equal to the sum of the two instantaneous changes. The two changes are of opposite sign and differ by an amount determined by changes in the values of significant variables during the crossing. Referring to Figure II-2, we have \( p_B = -(p_A + \Delta p_{AB}) \), and are interested in the value of \( \Delta p_{AB} \).

As the accelerating gaps used are symmetrical about the
(a) **ORBITAL ACCELERATOR.**

(b) **LINEAR ACCELERATOR.**

[arrows show direction of phase restoring "forces"]

**FIGURE II-1.**

**FIGURE II-2.**
median plane, or have cylindrical symmetry, we shall concern ourselves with two dimensions only, \(x\), in the direction of the axis of motion, and \(z\), perpendicular to it, taken as vertical.

The accelerating potential will be considered to change from \(1/2V_0\sin \varphi\) to \(-1/2V_0\sin \varphi\), where \(\varphi\) is the phase of the R.F. at the time of acceleration, and \(V_0\) is the maximum voltage amplitude.

For slit type electrodes, such as in the cyclotron, the Laplacian of the potential, in two dimensions, is

\[
\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial z^2} = 0.
\]

In the region of \(A\) the vertical field is

\[
F_z = -\frac{\partial V}{\partial z}
\]

\[
= -\int (\frac{\partial^2 V}{\partial z^2}) dz = \int (\frac{\partial^2 V}{\partial x^2}) dz = z \frac{\partial^2 V}{\partial x^2}.
\]

Thus

\[
P_A = ez \left[ (\frac{\partial^2 V}{\partial x^2}) \frac{dx}{v} \right]^{A+\varepsilon}_{A-\varepsilon} = (ez/v)(\frac{\partial V}{\partial x})^{A+\varepsilon}_{A-\varepsilon} = -(ez/v)V_0 \sin \varphi /d,
\]

where \(v\) is the particle velocity, and \(d\) the gap length.

When the electrodes are cylindrical, such as in the linear accelerator, the Laplacian, in two dimensions is

\[
(1/z)\frac{\partial}{\partial z}(z \cdot \frac{\partial}{\partial z})V + \frac{\partial^2 V}{\partial x^2} = 0
\]
where $z$ is the axis and $x$ the radius of the electrode system.

This is in conformity with Figure II-2.

From the Laplacian we obtain

$$\frac{\partial}{\partial z}(zF_x) = -z\frac{\partial F_x}{\partial x}$$

or

$$zF_x = -\int z\frac{\partial F_x}{\partial x}dz$$

$$= -x^2/2\frac{\partial F_x}{\partial x} + \int z^2/2\frac{\partial^2 F_x}{\partial x^2}dz$$

As $z$ is small higher than the first power terms in $z$ will be neglected, giving

$$F_x = -(x/2)\frac{\partial F_x}{\partial x}$$

Thus $p_A$ for cylindrical type electrodes becomes

$$p_A = -(ez/2\pi)(V_o \sin\phi)/d$$

i.e. there is a factor of two in the difference between $p_A$ for the two types of electrodes. We shall neglect this and continue to consider the case of the slit type lens.

Since $p_A$ is a function of $\phi$, $z$, and $v$, $\Delta p_{AB}$ is equal to

$$\frac{\partial p_A}{\partial \phi}\Delta \phi + \frac{\partial p_A}{\partial z}\Delta z + \frac{\partial p_A}{\partial v}\Delta v$$

Substituting for

$$\frac{\partial p_A}{\partial \phi} = -eV_o \cos\phi/vd, \quad \Delta \phi = wd/v,$$

$$\frac{\partial p_A}{\partial z} = -eV_o \sin\phi/vd, \quad \Delta z = p_A d/mv = -(eV_o \sin\phi)/mv,$$
\[ \frac{\partial P_A}{\partial v} = e z V_o \sin \theta / v^2 d, \quad \Delta v = v \Delta E / 2E = e V_0 \sin \theta / mv, \]

\[ \Delta P_{AB} \text{ is given by} \]

\[ \frac{-e z V_o \cos \theta \cdot w d}{v d} + \frac{e z V_o \sin \theta \cdot e V_0 \sin \theta}{mv \cdot v d} + \frac{e V_0 \sin \theta \cdot e z V_o \sin \theta}{mv \cdot v^2 d} \]

whence

\[ P_A + P_B = \frac{2e V_0}{v^2} \left[ w \cos \theta - \frac{2e V_0 \sin \theta}{mv d} \right] \]

In this expression negative terms are focusing ones. This means that the second term in the brackets is always focusing, for any practical cases. However it is so small, except for exceedingly low energies, that its effect is generally unimportant. The cosine term will be focusing or defocusing according to whether the phase of the R.F. field is greater or less than \( \pi / 2 \).

With the cyclotron, ions during the initial stages of acceleration cross the dee gap on both sides of the 090° phase angle. Later in the acceleration magnetic vertical focusing becomes of importance. In the case of the linear accelerator, however, with no magnetic field normally present, and with an equilibrium phase between 0° and 090°,(the cosine term of equation II-20 is of necessity defocusing) there has been considerable difficulty in achieving survival for more than a small percentage of the starting beam. When accelerating electrons the problems have
been less difficult to solve. This is presumably in part due to the very high velocities reached by electrons for considerably less acceleration than in the proton situation. Such circumstances would quickly reduce the value of the defocusing term in Equation II-20. In addition to this advantage held by electrons there are those naturally associated with a shorter flight path.

The early proton machines ran with an R.F. phase angle of nearly 090°, with a constant spilling of particles down the falling slope of the accelerating potential. Current practice is to operate with an equilibrium phase of about 030°. Two methods have been employed to help offset the defocusing problem. One which consists of incorporating foils or grids in the down beam electrode of accelerating gaps to produce a converging electric field, has brought about improvement in the focus of the surviving beam but many ions are lost by collision with the focusing agents. Much better results have been obtained by the second method. This is the use of quadrupole focusing lenses. The lenses, either magnetic or electrostatic, provide focusing without interfering with the beam aperture. Quadrupole lenses are also used in the early stages of electron accelerators with satisfactory results.
Alternating Gradient Magnetic Focusing.

The principle of alternating gradient focusing is discussed in conjunction with the design of the quadrupole focusing lenses for the A.N.U. cyclotron in Appendix III. The first article published on the subject in a recognised journal is by Courant, Livingston and Snyder.

In synchrotrons employing strong focusing the n value is much greater than unity and reverses sign from segment to segment of the guide field magnet. Although this condition brings about an increase in betatron oscillation frequency the accompanying reduction in amplitude is only achieved through great accuracy in magnet design and construction. Due to the increased possibility of resonances between the two betatron oscillations, and between them and the particle's orbital motion, stringent limits are imposed on the allowable relationships between the chosen n value and the number of magnet sectors used. Similar limits are imposed on the permissible errors in magnet alignment, and field irregularities due to other causes.

An interesting feature of the A.G. synchrotrons is the required behaviour of the equilibrium phase of the accelerating.

field. A qualitative explanation of this behaviour can be obtained with reference to Figure II-3. It will be seen that for a particle displaced outwards from the true equilibrium orbit, more time will be spent outside the displaced orbit than inside, and vice versa for a particle displaced inwards. If $s$ is the orbit length this means that $\Delta s/s$ is greater than for a standard machine. Now if we consider a change $\Delta t$ in the period of revolution, $t$, accompanying a change in momentum from the equilibrium value we have

$$\frac{\Delta t}{t} = \frac{\Delta s}{s} - \frac{\Delta v}{v}.$$ 

In a standard $\Delta t/t$ is always positive. For low energies in A.G. machines it is negative. Thus the equilibrium phase needs to lie between zero and $\pi/2$. With increasing energy $\Delta v/v$ decreases, and at the "transition energy" of the machine $\Delta t/t$ changes from negative to positive. At this stage of acceleration a suitable R.F. phase change to a value greater than $\pi/2$ is necessary to recover phase focusing, and hold the accelerated beam.
FIGUR II-3. PARTICLE MOTION IN A.G. SYNCHROTRON.
CHAPTER III.

CANBERRA PROTON SYNCHROTRON AND INJECTION REQUIREMENTS.

The description of the Canberra proton synchrotron will be restricted to a summary of its main features, with some brief remarks about its major components. At the present stage of the machine's construction it is necessary, when discussing injection, to generalize to some extent. Later, when the orbital magnet has been installed and tested, the additional information available will permit a re-examination of the injection problem in more detail. Before presenting the account of the machine, or considering injection, we shall make some comments on proton synchrotrons in general.

Features of Proton Synchrotrons.

As mentioned in Chapter II the orbit for particles in a synchrotron is of practically constant radius. This requires
that a correct relationship, consistent with the machine's equilibrium orbit and the particles' energy gain per revolution, is maintained between the magnetic guide field and the frequency of the electric accelerating field. The rate of rise of the magnetic field is practically determined by its very heavy power supply and the circuit containing the magnet windings, making fast correction impossible. Therefore it is to the electric field frequency that control and correction must be applied. This is done by varying the rate of change of frequency in accordance with the monitored rate of magnetic field change or the orbital position of the beam. Errors remaining in the rate of frequency rise, provided they are not excessive, are compensated for by the particles' phase oscillations.

Radial synchrotron oscillations, accompanying the phase oscillations, together with betatron motions, can make considerable demands on vacuum space, and the volume of magnetic field which must be provided. As well as being caused by R.F. errors, phase oscillations are introduced by phase spread at injection. There are three main sources of betatron oscillations. These are irregularities in the magnetic field, gas scattering, and conditions at injection. The last two mentioned will be
discussed later in the chapter.

Although the ratio of beam energy to magnet steel is better in the synchrotron than in the synchrocyclotron, which preceded it as the highest energy proton accelerator, the amount of steel required for very high energy, standard type synchrotrons is exceedingly great. A 36,000 ton magnet is used for the Moscow 10 Bev machine. The alternating gradient accelerators requiring smaller beam apertures, and so permitting a reduced magnet gap, use comparatively less steel. At Brookhaven a 3,000 ton magnet is provided for the 25 Bev proton machine. No matter whether standard or alternating gradient accelerators are built, so long as steel magnets are retained, limiting the field strength to something less than 20,000 gauss, the dimensions of the machines, and the cost, must keep increasing with the sought after energy.

The Canberra Proton Synchrotron.

Air cored guide field magnets offer a means of escape beyond the upper field limit of 20,000 gauss. This way out has been chosen for the Canberra proton synchrotron\(^1\),\(^2\),\(^3\).

---

Designed for a peak field of 80 K-gauss, the machine has an orbit radius, for 10 Bev protons, of 480 cm, compared with 2,800 cm in the standard type accelerator at Moscow.

The field magnet for the Canberra synchrotron consists of $1\frac{1}{6}$" square copper conductors. These are arranged to simulate, in section, two overlapping circular conductors, the common parts of which are free of copper to provide a region of nearly uniform magnetic field, in which the vacuum chamber is located. The arrangement of the conductors is sketched in Figure III-1. The magnet is separated into quadrants by straight sections, of length 250 cm. Initially rising at a rate of $2 \times 10^5$ gauss/second, the guide field reaches its peak value in 0.8 seconds, requiring a current of $1.6 \times 10^6$ amp. Security for the magnet conductors against the very large mechanical forces is provided by a bed of duralumum plates, stacked edgewise and boxed in stainless steel plates.

The power supply for the orbital magnet is a homopolar generator, now reaching the final stages of construction. The generator has four 139" mild steel discs, 10" thick, forming two rotors. These, when spinning at 900 r.p.m., in a 16,000

* The conductors actually have slightly elliptical sections to give the requisit n-value of 0.56.
Figure III-1. Orbital Magnet Conductors and General Arrangement.
FIGURE III-1. ORBITAL MAGNET CONDUCTORS AND GENERAL ARRANGEMENT.
gauss field, store some $5 \times 10^8$ joules of energy. Liquid sodium-potassium alloy is jetted onto the rotors to provide a brush system through which current is fed to the generator, while "running up", or drawn from it during the main magnet pulse. The arrangement of the rotors is shown in Figure III-2. Power for the run up, of about ten minutes duration, is provided by a battery of 480 K.\textsuperscript{wa}t rectifier sets. When the homopolar is running at full speed the rotors are shorted through the orbital magnet windings to give the voltage and current pulses shown in Figure III-3.

The vacuum chamber is of circular cross section, 22 cm diameter. It probably will be fabricated from stainless steel tubing. The limitation which the small lateral aperture imposes on the horizontal beam oscillations which can be endured is, of course, considerable. The ratio of horizontal aperture to orbit radius for the Canberra machine is about half that pertaining to the other weak focusing proton synchrotrons.

To accelerate the protons a ferrite loaded, tuned cavity provides a peak voltage of 8 K.V. Accelerating with a stable phase angle of 120\(^\circ\) this means an energy gain of 4 Kev per beam revolution. Of some 40 K.W. of radio frequency power
FIGURE III-2. Homopolar Generator - Part Section.

FIGURE III-3. Pulse from Homopolar Generator.
energy, determined by parameters of the synchrotron, and to deliver them into the parent machine so that they are acceptable for subsequent synchrotron acceleration. The successful operation of the synchrotron is very largely dependent on the manner in which injection is achieved.

Injection into the synchrotron takes place before acceleration by that machine begins, during the early part of the current pulse producing the magnetic guide field. As the magnetic field rises the equilibrium orbit for a particle of given energy contracts. Injection can usefully begin when the equilibrium orbit includes the exit of the inflector, which is situated as near as practicable to the outer limit of the acceleration space of the synchrotron vacuum box. Particles entering the machine at this time will remain on the shrinking equilibrium orbit as the field rises. Particles leaving the inflector after this time will be starting in the synchrotron at a radius greater than that of the equilibrium orbit and will oscillate about it, horizontally. By the time the equilibrium orbit lies about half way between the inflector and the inner wall of the vacuum chamber the amplitude of horizontal oscillations will be sufficient to carry particles now entering the machine to the inner wall where they will be
lost. As the magnetic field varies in space as well as time, depending on the n-value used, the orbit shrinking is not a linear function of the rate of rise of field and, consequently, the injection period is reduced as will be noticed in later discussion. As the useful injection period ends the synchrotron acceleration starts and the beam from the injection apparatus is cut off. The amount of injected beam that can be accepted, and accelerated by the synchrotron without loss due to striking the wall of the chamber, assuming that the correct relationship between particle energy and guide field is maintained, depends upon the amplitude of oscillations about the equilibrium orbit.

In connection with betatron oscillations we shall now give some consideration to the two causes which have a bearing on injection. These are (a) injection errors, and (b) gas scattering.

(a) Injection errors. For the time being we shall assume a circular orbit. Later in obtaining figures for the Canberra machine the effect of the straight sections will be included.

* The injection could take place during synchrotron acceleration if done with rising injector energy.
For particles injected off the equilibrium orbit, and tangential to it, the amplitude of oscillation will be just the distance they are initially from it. In the case where particles enter the synchrotron at an angle to the orbit tangent the oscillation amplitude depends on the particle velocity and the n-value pertaining in the guide field.

If the entering proton diverges from the tangent by \( \theta \), horizontally, the induced horizontal betatron oscillation amplitude is \( a = \frac{v\theta}{(1 - n)^{1/2}} \). For vertical divergence of \( \theta \), \( a = \frac{v\theta}{n^{1/2}} \). Here \( v \) is the particle velocity, and \( w \) the angular frequency. \( \theta \) is assumed small as in fact it must be. The beam divergence, and displacement, and angular variation which cannot be controlled at injection places a limit on the amount of intentional displacement or directional variation which can be used to make the beam clear the inflector during early revolutions in the synchrotron.

(b) Gas scattering. Scattering of protons by residual gas molecules in the vacuum chamber is almost all of the Rutherford type. Following considerations of this problem by Blachman and Courant\(^1,2\), and Greenberg and Berlin\(^3\), Courant\(^4\).

in connection with the Brookhaven project, produced a revised formula. The probability, $P$, of a proton surviving being multiply scattered from its permitted aperture is expressed in terms of a parameter $\eta$, given by

$$P(\eta) = 1 - (2\pi / \eta)^{1/2} \exp(-1/2\eta) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .$$

for $\eta$ less than $0.2$. $\eta$ is given by

$$\eta = \left\{ \left[ \pi N R^3 (1+\alpha)^2 e^4 \right] / \left[ 4nA^2 T^2 \right] \right\} \left\{ \log(e^2/\chi^2) - 1 \right\} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .$$

$$\theta = (An)^{1/2} / (1+\alpha)^{1/2} R$$

and $$X = 1.20(\chi / a_o Z)^{-1/2} (1+3.33\chi^2)^{1/2}$$

The meanings of the terms used are as follows:

- $N$ = number of scattering atoms per c.c.
- $R$ = radius of proton orbit.
- $\alpha$ = $4L/2\pi R$, $L$ = length of synchrotron straights.
- $2e$ = charge of scattering atoms.
- $n$ = n-value of magnetic field.
- $A$ = semi-aperture.
- $e\nu$ = energy gain of proton per revolution.
- $a_o = $ Bohr radius of Hydrogen atom, = $5.3X10^{-9}$ cm.
- $2\pi \chi$ = wavelength of proton.
- $\gamma = (Ze^2/\hbar \nu)$, where $\nu$ is the proton velocity, taken at twice injection energy.

It will be seen from equation III-2 that $\eta$ is proportional to the residual pressure and the cube of the orbit radius, and inversely proportional to injection energy, energy gain per revolution and the square of the allowed aperture. For
Preparatory to considering in some detail the problem of injection for the Canberra synchrotron we shall briefly look at four existing injector systems, those belonging to the machines listed in Table III-1.

(a) Birmingham. Here injection is from a 460 Kev Cockroft-Walton renator into a space horizontally (free of obstruction) of 30 cm and vertically 10 cm. The rising magnetic field shrinks the orbit by about 3 mm per revolution, during some 70 revolutions at injection. By increasing the energy of the enjector by some 30 Kev during the pulse, oscillations due to injection off the equilibrium orbit are avoided. The frequency at injection is 330 Kc/s. Owing mainly to the low injection energy, the theoretical value of pressure for ten percent loss of beam is lower than for the other existing machines, namely $2 \times 10^{-6}$ mm Hg. In practice, because of magnet misalignments, and small errors in the R.F. control system, the beam is lost at this pressure, and for useful operation the pressure must be better than $8 \times 10^{-7}$ mm. The incoming beam is maintained on a tangential path by a $\frac{1}{8}$" thick mumetal cone and a set of electrostatic guide plates.

(b) Brookhaven. The Cosmotron receives its beam from a horizontally mounted, pressurised Van de Graaff, at 3.5 MeV. The spread in energy is not more than ±3 Kev (0.1 percent), and the divergence is kept to about 0.001 radian, leading to betatron oscillation amplitudes of about 1 inch. The beam from the electrostatic accelerator is analysed magnetically, the proton part undergoing a 25° deflection: the mass 2 beam being used to control the accelerator energy. A drop of nearly 400 Kev occurs when 2 mA current is drawn for 100 microseconds. The inflection is by means of a 30° electrostatic deflector, delivering the beam some ten inches outside the final equilibrium orbit. The theoretical pressure for ten percent loss of beam due to gas scattering is $8 \times 10^{-6}$ mm Hg.

Soon after commencing operation this machine was accelerating about $10^{10}$ protons from an injection pulse of $3 \cdot 4 \times 10^{11}$.

(c) Berkeley. Two machines are involved in injection for the Bevatron. A 500 Kev Cockroft-Walton set delivers protons to a 9.9 Mev Linear accelerator. A 20° magnetic analyser separates the protons from other ions after the particles leave the Cockroft-Walton set. A 35° electrostatic inflector is used. The energy spread is better than 30 Kev.

and the divergence about 0.0015 radian. It is found that the use of quadrupole focusing magnets between the linear accelerator and the inflector increases the beam by a factor of two. This is taken to indicate that beam divergence is still an important factor. On the assumption that 7" of the 12" vertical aperture may be allowed for gas scattering the theoretical ten percent loss occurs for a pressure of 8.5 \times 10^{-6} \text{ mm Hg}. Performance figures published in 1955 show that for a peak beam of 430 micro-Amp from the linear accelerator 400 micro-Amp enter the inflector and 300 micro-Amp leave it. Typical figures are 120-260 micro-Amp from linear accelerator, 100-240 to the inflector and 60-165 from the inflector. It was expected that with a pressure of 2 \times 10^{-6} \text{ mm Hg} and injected 300 micro-Amp pulse would yield 0.5-1.5 \times 10^{10} accelerated protons. The injection pulse is for 100 beam revolutions, i.e., about 230 micro-seconds.

(d) Moscow. Protons with energy of less than 1 Mev are received from a Cockcroft-Walton set by a linear accelerator. This raises the particles to injection energy of 9 Mev. The beam is initially at 90° to the synchrotron orbit. 80° of

deflection is done magnetically, and the last 10° by a system of five electrostatic deflector pieces forming the inflector. The proton orbit shrinks by about 1 cm per beam revolution for the fifty revolutions covering the injection period.

The Brookhaven and CERN alternating gradient machines are designed for injection from 50 MeV linear accelerators. As a result of the slow rate of field rise anticipated in these machines, and the high injection energy, it is possible that injection will not continue for more than one particle revolution. That is "single shot" injection will be used.

Concerning the Canberra proton synchrotron there are three factors which might be considered as important in deciding the injection. These are the R.F. range necessary for acceleration, gas scattering, and the behaviour of the magnetic field at low values. The range through the R.F. must sweep while complying with the demands of the fast rising magnetic field, has an important bearing on the complexity and cost of the R.F. system as a whole. It was decided that the lower limit of the frequency should be about 1 Mc/s. This would require an injection energy of about 8 MeV. Gas scattering for this energy, at reasonably easily maintained pressures
will be shown to be relatively unimportant. There are considerable eddy currents associated with the Canberra orbital magnet at low fields. To avoid, completely, the field irregularities due to these, as an injection hazard, would necessitate an injection energy comparable with that for the 25 A.G. synchrotrons. However, they are considerably less for 8 Mev injection than for 1 Mev, and it was decided that with programmed trimming of the guide field at low values the figure of 8 Mev would be tolerable.

As a magnet, suitable for an 8 Mev cyclotron was available the emphasis of the injection problem was altered. Instead of a determination of what sort of pre-accelerator should be used, it became an investigation into the possibility of using a cyclotron for injection.

For convenience we shall relist the more important parameters of the Canberra synchrotron immediately below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of equilibrium orbit in quadrants</td>
<td>480 cm</td>
</tr>
<tr>
<td>Length of straight sections</td>
<td>250 cm</td>
</tr>
<tr>
<td>Half aperture, radius of vacuum chamber</td>
<td>11 cm</td>
</tr>
<tr>
<td>Field at injection (for 8 Mev protons)</td>
<td>850 gauss</td>
</tr>
<tr>
<td>Initial rate of rise of guide field</td>
<td>$2 \times 10^5$ gauss sec$^{-1}$</td>
</tr>
<tr>
<td>n-value</td>
<td>0.56</td>
</tr>
<tr>
<td>Energy gain per particle revolution</td>
<td>4 Kev</td>
</tr>
</tbody>
</table>

* The magnet was constructed for use with the experimental model of the homopolar generator.*
We shall regard the energy of 8 Mev as fixed in the calculations which follow, thus determining the synchrotron orbital frequency at slightly below 1 Mc/s.

The "Hr" value for an 8 Mev proton is $4.093 \times 10^5$ gauss-cm. Hence the magnetic field for an orbit radius of 480 cm is 852.7 gauss. Assuming an injection radius of 488 cm the field, at this radius, for injection is 838.7 gauss.

The angular frequency of a proton on the equilibrium orbit, neglecting straights, is

$$w = \frac{He(1 - \frac{v^2}{c^2})^{1/2}}{m_0 c}$$

$$= 8.1 \times 10^6 \ \text{rad. sec}^{-1},$$

whence $f_0 = 1.29 \times 10^6 \ \text{c/sec}.$

With $2\pi R = 3.019 \times 10^3 \ \text{cm}$, and straights of 4 x 250 cm, the actual orbital frequency is $f_T = 0.97 \ \text{Mc/s}$.\(^{\#}\)

In order to have some idea of the aperture into which injection may be undertaken it is well at this stage to

\(^{\#}\) If there are N straights of length L, and $f_0$ is the resonant frequency of a particle in the magnet sectors, the actual orbital frequency is $f_T = f_0 \cdot \frac{2\pi R}{(2\pi R + NL)}$. In the curved sectors the constants of the acceleration causing betatron oscillations are $nw^2$, and $(1-n)w^2$. If these are averaged over the whole orbit the vertical oscillation frequency becomes $n^{1/2} \cdot \frac{1 + NL/2\pi R}{1 + NL/2\pi R}^{1/2} \cdot f_T$, and the horizontal oscillation frequency becomes $(1-n)^{1/2} \cdot \frac{1 + NL/2\pi R}{1 + NL/2\pi R}^{1/2} \cdot f_T$.\]
consider the gas scattering expected in the synchrotron. Using Courant's formula, the value of \( \eta \) for the Canberra machine is \( 0.1 \times 10^5 \) p, where p is the vacuum chamber pressure in mm Hg., and the half aperture taken as 1 cm. With a vacuum chamber radius of 11 cm, amplitudes of betatron oscillations, due to gas scattering, of greater than about 1 cm are likely to be intolerable. In Figure III-4 the fraction of beam lost outside a 1 cm half aperture is plotted as a function of pressure. The curve shows that providing the tank pressure is maintained at \( 10^{-5} \) mm Hg., or better, the loss of particles by gas scattering can be expected to be negligible.

The period available for injection is readily obtained, to an approximation, by regarding \( H \) as constant, in time, in the expression \( \frac{\partial H}{\partial R} = -nH/R \), giving the variation of field with radius as nearly 1 gauss cm\(^{-1}\). This means that the field at the machine's equilibrium orbit, radius \( R_0 = 480 \) cm, is about 8 gauss higher than at the particle equilibrium orbit which includes the inflector exit, at \( R = 488 \) cm. Thus the field must rise only 6 gauss with time, before the equilibrium orbit, initially including the inflector exit, has shrunk to the machine's equilibrium orbit. With the rate of field rise equal to \( 2 \times 10^5 \) gauss sec\(^{-1}\), this would indicate an injection
Fraction of beam lost = \( \left( \frac{2 \pi}{\eta} \right)^{\frac{1}{2}} e^{-\frac{1}{2} \eta} \) (for \( \eta < 0.2 \)).

\[ \eta = 0.1 \times 10^{-5} \rho \text{ (mm Hg)} \]

**FIGURE III-4.** Beam Loss by Gas Scattering.
6(\cos \phi - \cos \phi - (\phi - \phi) \sin \phi)\cdot

In Figure III-5 the amplitude of synchrotron oscillations, which can be expected near injection energy, is plotted in terms of phase angle, for \( \phi_0 = 150^\circ \). If the amount of aperture allowed for these oscillations is taken as 1.5 cm, it will be seen from the figure that a phase spread of only 45° can be accepted satisfactorily for acceleration. This reduces the number of particles which might be accepted to about \( 2 \times 10^7 \) per micro-amp. of injected beam.

In order to accommodate the variously caused horizontal oscillations of the proton beam it might be necessary to reduce the radius for injection, so decreasing the injection period. A reduction in the initial radius might well be necessary to allow for betatron oscillations, intentionally introduced to make the protons miss the inflector during their early revolutions. Nevertheless, for the time being, there is no apparent reason for abandoning the injection period figure of 31.5 micro-sec.

The amplitude of betatron oscillations due to beam divergence, or to particle entry at an angle \( \theta \) to the equilibrium orbit, is \( v \theta / 1.16 n^{1/2} w \), for vertical oscillations,
FIGURE III-5. SYNCHROTRON OSCILLATIONS.
and \( v \theta / 1.16(1 - n)^{1/2} \) for horizontal oscillations. Here 
\( w = 2 \pi f_p \). Thus for oscillation amplitude equal to 1 cm, 
\( \theta_{\text{vertical}} = 0.00135 \) radian, and \( \theta_{\text{horizontal}} = 0.00123 \) radian. 
Alternatively a divergence of 0.1° will lead to amplitudes 
of 1.3 cm (vertical), and 1.4 cm (horizontal).

The frequency of horizontal betatron oscillations is 
0.76 Mc/s, of vertical 0.84 Mc/s, compared with \( f_T \) of 0.97 
Mc/s. Table III-2 shows the phases of betatron oscillations 
for particles passing the inflector during some early 
revolutions.

<table>
<thead>
<tr>
<th>Particle phase</th>
<th>2\pi</th>
<th>4\pi</th>
<th>6\pi</th>
<th>8\pi</th>
<th>10\pi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horiz. B-osc. phase</td>
<td>233°</td>
<td>-154°</td>
<td>129°</td>
<td>52°</td>
<td>-25°</td>
</tr>
<tr>
<td>= -77°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vert. B-osc. phase</td>
<td>312°</td>
<td>-96°</td>
<td>-144°</td>
<td>168°</td>
<td>120°</td>
</tr>
<tr>
<td>= -48°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To clear the inflector on subsequent revolutions the 
particles must pass some distance inside (or above, or below) 
their initial path as each revolution is completed. Physical 
dimensions of the inflector might require this distance to be
of the order of a centimeter. Figure III-6 shows the amplitudes of horizontal betatron oscillations associated with the injection of protons parallel to the equilibrium orbit, which is shrinking at the rate of 0.27 cm per beam cycle. The inflector is regarded as presenting an obstruction for a distance of 1 cm, inwards, from the ion path at the beginning of the synchrotron orbit. The figure shows that particles entering on the equilibrium orbit are unlikely to clear the inflector on their first, second, or third repassing of it, while particles entering at the beginning of the three subsequent cycles will tend to collide with the inflector at the end of one revolution, thereafter missing it. Particles entering at the end of four or more cycles will clear the inflector on all revolutions.

The effect of impressing a horizontal betatron oscillation of amplitude 0.5 cm is shown in Figure III-7. Curves D and E correspond to the case where the particles are injected so that their initial motion is with increasing radius; for F the initial radius is decreasing. It is seen that an outward deflection improves clearance of the inflector for the first and second passings, while decreasing chances of clearance on the third and fourth passings. Where the beam is initially
A Injected at end of first cycle.
B " " " " second cycle.
C " " " " third "

**FIGURE III-6** Horizontal Betatron Oscillations — Injection Parallel to Equilibrium Orbit.
Figure III-7: Horizontal Betatron Oscillations - Inversion at an Angle to Equilibrium Orbit.
directed inwards (F), collision is enhanced on the first and second passings, and thereafter clearance is assured.

Figure III-8 shows the vertical betatron oscillations resulting from injection on the median plane, but at an angle to it. If the inflector occupied 4 cm of the vertical aperture then a vertical oscillation of 3 cm in amplitude would clear the inflector on the first, second, and fifth passing, but would be within the vertical dimensions of it at the third and fourth passing.

The results of the betatron oscillations just discussed are summarized in Table III-3. The summary suggests the following possible means of achieving injection without undue loss of protons.

(a). Inject parallel to the equilibrium orbit in the horizontal plane, at an angle to the median plane, sacrificing possibly most of the first cycle of injection.

(b). Inject with radius decreasing, and at an angle to the median plane.

(c). Sacrifice the first four cycles after the equilibrium orbit coincides with the inflection path, and inject parallel to the median plane and the equilibrium orbit.
We shall now briefly enumerate the particle oscillations, and their amplitudes where possible, which have been mentioned in relation to the Canberra synchrotron.

(a) Radial synchrotron oscillations—amplitude 8 cm for acceptance of a 45° phase spread.

(b) Horizontal betatron oscillations due to injection off the equilibrium orbit—amplitudes up to 8 cm for injection period of 31.5 micro-sec.

(c) Perhaps 0.5 cm horizontal and vertical oscillation to enable beam to clear inflector.

The above mentioned claim practically the whole of the horizontal aperture available. In addition to these are further betatron oscillations of a more random character, due to

(d) Divergence of beam. A half angle divergence of not more than 0.1° might be regarded as a reasonable demand. It would lead to oscillation amplitudes of up to 1.5 cm.

(e) Energy spread in the beam from the injector. A spread of ± 2 percent would be equivalent to a position error of 0.5 cm.

(f) Errors in the magnetic field.

(g) Gas scattering. This is negligible.

In view of the demands made on the aperture by the first three mentioned oscillations, any oscillations arising from
the causes (d) to (g) are likely to bring about loss of beam. Admittedly some of the oscillations can be expected to cancel each other, and indeed, in some cases, to aid survival. However such effects are not estimable.

It is worthy of note that the permitted synchrotron oscillations, for beam accepted near the equilibrium orbit, can be larger than for beam accepted later. This could well mean that particles with a phase spread exceeding 45° could be accelerated from a considerable part of the injection pulse.

Assuming loss during acceleration to reduce the beam by a factor of between 10 and 100, the number of particles surviving could be expected to be in the region of $10^6$ per micro-ampere of injected beam.

If an unlimited supply of beam from an injector is on hand, two factors limit the number of ions that can be accelerated in a synchrotron. These are the R.F. power provided for acceleration, and the beam instability that might be brought about by space charge effects. The average power needed to accelerate a micro-coulomb of protons ($6.25 \times 10^{12}$) to 10 Bev, in 0.8 second is 12.5 K.W. Thus the Canberra machine has sufficient power (5 K.W.) to accelerate some
2.5 \times 10^{12} \text{ particles.} \text{ Space charge effects have been discussed by Moon}^{14}, \text{ who shows that the space charge brings about a virtual change in the } n \text{ value of a machine, which can lead to beam instability. The maximum number of particles which can be accelerated stably is given by}

\[ N = \frac{vHA}{4\pi Re}, \text{ (electrostatic units)} \]

where \( v \) is the particle velocity, \( A \) the cross sectional area of uniform beam, \( 1/2\pi R \) the fraction of orbit length occupied by particles. The space charge limit for the Canberra machine, during the early stages of acceleration, where effects would be expected to be most serious, taking \( v = 0.12c \) and \( 1/2\pi R = 1/4 \), is

\[ N = 8 \times 10^{12} \text{ particles.} \]

In view of the two limitations just mentioned, \( 10^{12} \) particles might be regarded as the maximum number which could be usefully injected. As this would require an injector beam of the order of the order of 6 m.A, the limitations are unlikely to require further consideration.

We must now answer the question, will the cyclotron be satisfactory as an injector? Firstly, in regard to the amount of beam which can be delivered, we are in a position

to make a reasonably substantiated claim (see Chapter VI) that the Canberra cyclotron is likely to offer greater beam intensities than the existing injection machines. Indeed it seems that the limit to the beam which may be had from the cyclotron is at present set by the power available, and this can be increased. There is no reason for thinking that a mA of beam cannot be offered to the synchrotron if desired.

The cyclotron certainly holds advantages in the absence of energy spread, and multiplicity of ion types, in the accelerated beam. One can be sure that the beam contains only protons, and, with the momentum analysis inherent in the machine, the spread in energy is likely to be less than 0.2 percent.

It is the external beam divergence characteristic of cyclotrons generally, that could be the greatest drawback in using such an instrument for injection. However, with the advent of quadrupole focusing lenses, this can be regarded as less serious. Divergence measurements, for small external beams, finely collimated during the early revolutions in the cyclotron, showed nothing greater than 0.25° half angle. With greater beams, and less "stopping down", the divergence is
likely to increase considerably. At the same time it is probably not unreasonable to suggest that, with focusing of the external beam, a final half angle figure of not more than $0.5^\circ$ should be achieved.

It does appear that the cyclotron is likely to be eminently suited for its role as an injector.
In the fixed frequency cyclotron, as in the synchrocyclotron, the ions are formed near the centre of the machine. From the ion source they are extracted by the radio-frequency electric accelerating field. It is convenient to consider the particle motion during three stages. The initial motion, during which electric focusing forces are important and the rate of increase in orbit radius is high, occupies only several cycles, then the major part of the acceleration takes place, i.e. there are many particle revolutions, in a region of slowly decreasing magnetic field, accompanied by considerable vertical focusing, and finally the ions move into a region of more rapidly falling magnetic field, which can induce large horizontal particle oscillations, before extraction of some
\[-m \ddot{x} = y e x H / c\]
\[\ddot{x} = -\omega y\]
\[y = \omega x\]

**FIGURE IV.1. AXES FOR CONSIDERATION OF ION MOTION.**

**FIGURE IV.2. PHASE OF R.F. FOR 2\textsuperscript{nd} GAP CROSSING VS. $m^* \%$.**
\[ \ddot{x} = (eF/m)\cos(\Theta t + \phi) - w\dot{y} \]
\[ \dot{y} = w\dot{x}, \]
where \( F \) is the electric field, \( w \) is the resonant angular frequency of the ion in the magnetic field, \( \Theta \) the angular frequency of the electric field, and \( \phi \) the value of \( \Theta \) when the time \( t \) is taken as zero. It will be noticed that the R.F. phase is taken as zero when the electric field is maximum instead of zero, as previously (Chapters II and III). This has been done to be consistent with what appears to be the traditional notation used in the literature.

Integration of \( \dot{y} = w\dot{x} \) gives
\[ \dot{y} = w\dot{x} + c_0, \]
where \( c_0 = \dot{y}_0 - wx_0 \). The subscript "o" is used with variables to indicate their values at \( t = 0 \).

The differential equation for \( x \) can now be written as
\[ \ddot{x} = K\cos(\Theta t + \phi) - w^2x - wc_0, \]
where \( K = eF/m \). This is a standard second order equation, having the complementary function \( c_1\cos wt + c_2\sin wt \), and a particular integral
\[ \frac{1}{(D^2 + w^2)}(K\cos(\Theta t + \phi) - wc_0) = \frac{K}{(w^2 - \Theta^2)}(\cos(\Theta t + \phi)) - c_0/w, \]
giving the solution
\[
x = \frac{K}{(w^2 - \phi^2)} \cos(\phi t + \phi) + c_1 \cos wt + c_2 \sin wt - c_0/w \quad \cdots \cdots \text{IV-1}
\]

Differentiating IV-1 we have

\[
\dot{x} = -\frac{K\phi}{(w^2 - \phi^2)} \sin(\phi t + \phi) - c_1 w \sin wt - c_2 w \cos wt \quad \cdots \cdots \text{IV-2}
\]

Evaluating the constants \(c_1\) and \(c_2\), at \(t = 0\),

\[
c_1 = -\frac{K}{(w^2 - \phi^2)} \cos \phi + \frac{\dot{y}_0}{w}, \text{ since } \dot{y}_0 = w x_0 + c_0,
\]

and \(c_2 = \frac{\phi}{w} \frac{K}{(w^2 - \phi^2)} \sin \phi + \frac{\dot{x}_0}{w}\)

whence

\[
x = \frac{K}{(w^2 - \phi^2)} \left[ \cos(\phi t + \phi) - \cos \phi \cos wt + (\phi/w) \sin \phi \sin wt \right]
+ x_0 + \left(\frac{\dot{x}_0}{w}\right) \sin wt - \left(\frac{\dot{y}_0}{w}\right) (1 - \cos wt) \quad \cdots \cdots \cdots \text{IV-3}.
\]

Substituting for \(x\) in \(\dot{y} = wx + \dot{y}_0 - wx_0\), we obtain

\[
y = \frac{wK}{(w^2 - \phi^2)} \left[ \cos(\phi t + \phi) - \cos \phi \cos wt + (\phi/w) \sin \phi \sin wt \right]
+ \dot{x}_0 \sin wt + \dot{y}_0 \cos wt,
\]

which on integration yields

\[
y = \frac{K}{(w^2 - \phi^2)} \left[ (\phi/w) \sin(\phi t + \phi) - \cos \phi \sin wt - (\phi/w) \sin 0 \cos wt \right]
- \dot{x}_0/w \cos wt + \dot{y}_0/w \sin wt + k \quad \cdots \cdots \cdots \cdots \cdots \text{IV-4}.
\]

Evaluated at \(t = 0\), \(k\) is given by

\[
k = y_0 + \dot{x}_0/w + \frac{K}{(w^2 - \phi^2)} \frac{\phi}{w} \sin \phi,
\]

and Equation IV-4 becomes
Equations IV-3 and IV-5 give the position of a particle, 
\((x, y)\), in terms of the resonant frequency \((w)\) of the particle, 
the electric field frequency \((\theta)\), and the initial conditions 
of R.F. phase \((\phi)\), the particle position \((x_0, y_0)\) and velocity 
\((\dot{x}_0, \dot{y}_0)\), existing at the instant from which the time \((t)\) is 
measured.

If we express \(\theta\) as \(w + q\) equation IV-3 can be rearranged 
as
\[
x = \frac{-K}{q(2w^2 - q)} \left[ \cos wt \cdot \cos q \phi (\cos qt - 1) + \sin wt \cdot \sin q \phi (1 - \cos qt) 
- \sin qt \cdot \sin (wt + \phi) + q/w \cdot \sin wt \cdot \sin \phi \right]
\]
plus terms not involving \(\phi\).

By expanding \(\cos qt\) and \(\sin qt\), in terms of \(q\), and then 
letting \(q\) go to zero we obtain
\[
x = \frac{(K/2w^2)}{w^2} \left[ \sin wt \cdot \cos \phi - \sin wt \cdot \sin \phi \right] + x_0 + (\dot{x}_0/w) \sin wt
- (\dot{y}_0/w)(1 - \cos wt) \quad \text{IV-6}
\]

Similarly treating Equation IV-5 we obtain
\[
y = \frac{(K/2w^2)}{w^2} \left[ \sin wt \cdot \cos \phi - wt \cos (wt + \phi) - 2 \sin \phi (1 \cos wt) \right]
+ y_0 + (\dot{y}_0/w) \sin wt + (\dot{x}_0/w)(1 - \cos wt) \quad \text{IV-7}.
\]
For \( x_o = y_o = \dot{x}_o = \dot{y}_o = 0 \), Equations IV-6 and IV-7 are the ones given by Livingston. If \( \theta \) is expressed as \( w(1 - \epsilon) \), Equations IV-6 and IV-7 lead to the approximate solutions of Cohen.

As the ion enters the field free region of the dee the above equations for \( x \) and \( y \), with \( K = 0 \), still apply, if \( t \) is now measured from the instant of entry.

When ions complete their first, or first and second, half revolutions within the region of the electric field (considered uniform) a considerable amount of phase bunching takes place. If we look at the phase of the electric field at the time the ions cross the centre line of the dee gap, an indication of the amount of bunching which can be expected to occur under these circumstances is obtained. Expressing \( w \) as \( w = \theta/m \), and taking the cases for \( \phi = -\pi/2 \), and \( \phi = 0 \), we have, by virtue of Equation IV-3, for ions completing their first revolution without leaving the uniform electric field region,

\[
x(\phi=0) = K/(w^2 - \theta^2)\cdot(\cos\theta t - \cos\theta t), \text{ and} \\
x(\phi=-\pi/2) = K/(\theta^2 - w^2)\cdot(\sin\theta t - \theta/w\sin\theta t).
\]

Thus \( x(\phi=0) \) is zero when \( \cos\theta t = \cos(\theta/m)t \), i.e. when

\[
\theta t = 2\pi n/(1 + 1/m), \quad n = 0, 1, 2, \ldots .
\]
$x(\phi=\pi/2)$ is zero when $\sin \phi = m \sin(\phi t/m)$.

In Figure IV-2 the phase of the electric field, as an ion makes its second gap crossing is plotted against $m$ for \( \phi = 0 \), and \( \phi = -\pi/2 \). It will be noticed that for ions starting 90° ahead of the peak of the electric field, the phase of the R.F., when they make their second crossing of the gap, differs from that for ions starting with zero phase by something less than 12°, when $m=0.9$. It is obvious from the figure that for values of $m$ likely to be met with in practice there is little effect on the initial ion revolutions due to the difference between $\phi$ and $w$.

When $w = 0$ we see, from Equation IV-6, that $x$ is zero when $\sin(wt+\phi) = (1/wt) \sin wt \cdot \sin \phi$. The R.H.S. of this expression cannot be greater than $1/wt$, or $1/n$. Thus ($wt+\phi$), the R.F. phase, will not be more than $18\frac{1}{2}°$, for the first crossing, and 9° for the second crossing, different from 180°, and 360° respectively, providing, of course, that the ions' entire motion has been in the region of uniform electric field. Figure IV-3 shows the phase of the R.F. as ions of various starting phases make first and second gap crossings, without having left the uniform field region.

When the particles pass beyond the extent of the uniform
FIGURE IV-3.

First Crossing

Second Crossing

FIGURE IV-4. ORBIT CENTRE.
electric field, on their first half revolution, soon after leaving the ion source, phase bunching occurs to a lesser extent, and may be practically absent. Before discussing this condition we shall derive expressions for the position of ion orbit centres, as such information is necessary in order to picture the characteristics of the motion of the ion as it passes into the field free region of the dee.

The position of the centre of the ion orbit can be determined by reference to Figure IV-4, where C(x_c, y_c) is the centre of revolution for an ion at (x, y), r is the radius of revolution, and B the angle r makes with the x-axis, which is perpendicular to the centre line of the dee gap.

We have

\[ x - x_c = r \cdot \sin B = \dot{y}/w, \]  
\[ y - y_c = r \cdot \cos B = -\dot{x}/w, \]

or \[ x_c = x - \dot{y}/w \]

and \[ y_c = y + \dot{x}/w. \]

Substituting for the variables, from Equations IV-3 and IV-5, we have

\[ x_c = x_0 - \dot{y}_0/w, \quad \text{and} \]
\[ y_c = y_0 + \dot{x}_0/w + (K/w) \left[ \sin(\theta t + \phi) - \sin \phi \right]. \]
Thus for an ion moving in a uniform electric field, with initial conditions \( x_0 = y_0 = \dot{x}_0 = \dot{y}_0 \), the centre of revolution remains on the y-axis and oscillates, with the same frequency as that of the electric field, about its mean position, determined by the start phase \( \varphi \). While an ion is moving in the field free region of the dee the orbit centre is stationary except for effects which will not be discussed till later, as they are unimportant at small radii.

It is usual to have attachments ("feelers") on the dee, facing the ion source, to increase the electric field experienced by particles as they are extracted, and accelerated, from the ion source. Among the effects of the feelers is a decrease in the extent of the field in the x-direction, making more valid the assumption of field uniformity. If the field, taken as uniform, extends a distance \( x_d \) from the ion source, the x-co-ordinate of a particle, between the ion source and feelers, may be given by the expression

\[
xx/k = wtsin(\omega t + \varphi) - sin\omega t . \sin\varphi
\]

obtained from Equation IV-3, with \( k = eV/2\omega^2m \), where

\[
V/x_d = F, \text{ the electric field. } x \text{ will be equal to } x_d \text{ at the }
\]

\[ ^\text{\# e.g. orbit precession and magnetic field irregularities.} \]
instant the particle leaves the electric field for the field free space of the dee.

In conformity with Equation IV-9, we rewrite Equation IV-8 as

\[ \frac{y_c x_d}{2k} = \sin(w t + \phi) - \sin \phi \] \hspace{1cm} \text{IV-10.}

Another quantity in which we shall be interested as the particles leave the influence of the electric field is \((y_c - y)\), which is given by

\[ \frac{x_d}{k_0} (y_c - y) = w t \cos (wt + \phi) + \sin w t \cos \phi \] \hspace{1cm} \text{IV-11.}

\[ (= \frac{\pi}{w}) \]

The quantities given by Equations IV-9, IV-10 and IV-11 are shown in Figure IV-5. It is to be noted that from \((y_c - y)\) the radius of the ion orbit, and so the particle energy, can be determined. Graphical solutions of the above three equations are used in Chapter VI (Figures VI-, VI-, and VI-), and show how, for given \(x_d\) values, the position of the orbit centre, \(y_c\), and the magnitude of \((y_c - y)\), both have stationary values (maxima) with respect to variation in the starting phase, \(\phi\). The value of \(\phi\) corresponding to the maxima of \(y_c\) and \((y_c - y)\) are easily found as follows.

Differentiating Equation IV-9 with respect to \(\phi\), and
$\phi$ vs. $xxd/k$ for maximum values of $y_c$ and $(y_c - y)$.

$x_d =$ Extent of uniform field from Ion Source, due to $D_e$

$V =$ Potential = $V$.

$k = eV/2\omega^2m$. 

**FIGURE IV-5**
taking $dx/d\phi$ as zero, we have $d(wt)/d\phi$ given by

$$
\frac{d(wt)}{d\phi} = \frac{\sin wt \cdot \cos \phi - wt \cos(wt + \phi)}{\sin wt \cdot \cos \phi + wt \cos(wt + \phi)} \quad \text{(}x \text{ const)} \tag{IV-12}
$$

This gives the rate of variation of $wt$ with respect to $\phi$ if $x$ is to be constant, for example when $x = x_d$.

Differentiating IV-10 and IV-11, and taking $dy_c/d\phi$ and $d(y_c-y)/d\phi$, respectively, as zero, we obtain

$$
\frac{d(wt)}{d\phi} = \frac{-\cos(wt + \phi) + \cos \phi}{\cos(wt + \phi)} \quad \text{(}y_c \text{ const)} \tag{IV-13}
$$

and

$$
\frac{d(wt)}{d\phi} = \frac{\sin wt \cdot \sin \phi + wt \sin(wt + \phi)}{2 \cos wt \cdot \cos \phi - \sin wt \cdot \sin \phi - wt \sin(wt + \phi)} \quad \text{(}y_c-y \text{ const)} \tag{IV-14}
$$

If now we equate the expressions for $d(wt)/d\phi$ of Equations IV-12 and IV-13, we obtain

$$
-\tan \phi = \cot wt - 1/(2\sin wt - wt) \quad \text{IV-15}
$$

while equating the expressions of IV-12 and IV-14 leads to

$$
-\tan \phi = \cot wt - wt/\sin^2 wt \quad \text{IV-16}
$$

Equations IV-15 and IV-16 give the relationship between $\phi$ and $wt$ for the turning points in the values of $y_c$ and $(y_c - y)$ respectively (with change of $\phi$) for a constant value
of $x$, the distance of the ions from the ion source. By using these equations to evaluate $\phi$ in terms of $w_t$, and then determining the corresponding values of $xx_d/k$, the curves of Figure IV-5 are obtained.

The curves show for what values of $\phi$ there is a bunching of ions, in terms of orbit centres, and orbit radii, corresponding to particular values of $xx_d/k$. If the extent of the electric field is less than that for which $xx_d/k$ is equal to about 0.3 it will be seen that both types of bunching occur for the same value of $\phi$. This corresponds to a case where ions enter the dee bunched radially about a common centre of revolution.

That the spacial bunching of the above type is achieved at the expense of phase bunching is clearly shown in Figure IV-6. Here $wt$, the change in R.F. phase between the time the ions leave the ion source and enter the dee at $x = x_d$, is plotted against the initial R.F. phase, $\phi$, for various values of $x_d^2/k$. It will be noticed that $w_t$ is practically constant, with constant $x_d^2/k$, for values of $\phi$ which are of interest. We have not shown, of course, that there will be an absence of phase bunching as the ions re-enter the electric field and make their
\( x_d = \text{Distance of Ion Source from Dee.} \)

**Figure IV-6.** Values of \( \omega t \) when Ion Enters Dee for Various \( \phi \) and \( x_d^2/k \).
first complete crossing of the dee gap. However the effect of feelers, particularly when not restricted to only one dee, will be to limit the extent of the electric field for this crossing also, and possibly the next. This will mean that the ions will be making considerable excursions into the dees before they experience an extensive electric field.

The type of phase focusing found in the synchrotrons and the synchrocyclotron does not occur in the fixed frequency cyclotron as the constancy of electric field frequency and magnetic field preclude the existence of a constant equilibrium R.F. phase. Generally the ions' resonant frequency is higher than the electric frequency for the early acceleration, tending to produce a forward phase shift, while, at greater radii, the decreasing magnetic field, and relativistic effects, produce a lag in the particle frequency.

(b). Vertical Motion. The vertical motion of ions in the cyclotron has been discussed by several authors


and, apart from some very broad generalities, each machine
has to be treated separately, with a knowledge of the expected vertical ion motion being gained only after a great deal of tedious arithmetic. (See for example references 4, 6).

As far as the magnetic field is concerned, in all standard cyclotrons it decreases with radius, and produces vertical focusing. There are various estimates of the best shape of field to use, with fairly general agreement, backed by experience, that the field should decrease some one to two percent, between the centre of the machine and the final particle orbit. This sort of decrease means a very small $n$ value, and will lead to negligible focusing forces at small radii.

Although the assumption of a uniform electric field is reasonable when considering horizontal motion, it is the non-uniformity which is important when considering the vertical motion of ions at small radii. The effect of the electric field, once the ions are spending a reasonable part of each revolution within the dees, can be determined with some degree of success by recourse to Equation II-20. Taking into account the different convention we have adopted here for the R.F. phase, the change in vertical momentum of an ion crossing the dee gap is expressed by

\[ \text{dee gap is expressed by} \]

\[ \Delta p_z = \frac{zeV_o}{v^2} \left[ -w\sin(\theta t + \phi) - 2eV_o \cos^2(\theta t + \phi) \right] \]

Re-writing this expression with \( v = wr \) we have

\[ \Delta p_z = \frac{zeV_o}{wr^2} \left[ -\sin(\theta t + \phi) - \frac{2eV_o \cos^2(\theta t + \phi)}{mdw^3r^3} \right] \]  

..........IV-17.

The second term on the R.H.S. of this equation is always focusing, however it very quickly diminishes with radius, and is almost ineffective by the time it becomes valid. The first term is focusing for ions crossing the dee gap after the R.F. peak, and defocusing for ions which are ahead of the R.F. in phase. Thus it will, in general, be a defocusing term in the early stages of acceleration, where the term is valid, because of the tendency for ions to cross the dee gap at negative R.F. phases. Equation IV-17 is, in essentials, the same as those of Rose and Wilson.

There is reason to believe that loss of ions by excessive vertical motions is initiated by conditions prevailing as the particles leave the ion source, and enhanced, rather than caused, by the vertical defocusing forces. The three dimensional solutions of the initial ion motion equations which were carried out at Hammersmith\(^7\), as well as experimental R.Gallop, (unpublished).
observations with our own machine, support this view. In Chapter VI we shall discuss this subject again, in reference to steps which can be taken to correct the defocusing forces.

Motion of Ions at Larger Radii.

Once the ions have orbit radii equal in magnitude to several dee gap widths their vertical motion is influenced by the magnetic field rather than by the electric. Vertical betatron oscillations occur with amplitudes decreasing with increasing radius, so that ions which survive to this stage are unlikely to be lost as a result of vertical displacement from the median plane.

The horizontal motion of particles is most easily visualized in terms of orbit centres. The motion of the centres is influenced by horizontal betatron oscillations, irregularities in the magnetic field, and acceleration by the electric field. Magnetic field irregularities are mentioned in relation to shimming in Chapter VI and will not be discussed here.

A precession of orbit centres can be regarded as arising from the difference between the frequency of horizontal betatron oscillations and the resonant particle frequency. If the period
of revolution of an ion is $t_0$, the period of horizontal betatron oscillations is $(1 - n)^{-1/2}t_0$, which for small $n$ is very nearly equal to $(1 + n/2)t_0$. Now when a particle is at the position corresponding to maximum amplitude of horizontal oscillation its orbit radius is colinear with the radius of the equilibrium orbit. (See figure IV-7). Thus during the course of one complete betatron oscillation, from a position of maximum amplitude, the radius vector from the machine centre to the ion orbit centre will describe an angle of $n\pi$, measured in the direction of the particle's motion.

If we look upon the electric acceleration as taking place at the instant the particle is mid-way between the dees, it will result in a displacement of the orbit centre, parallel to the dee edges, and equal to $r\Delta E/2E$, where $E$ is the non-relativistic energy of the particle. In cases where the dee connections to the R.F. lines are made nearer to one end of the dee edge than the other, there is likely to be a variation in potential along the dee edge. Under these conditions the ions pass through a stronger, and then a weaker field at successive gap crossings, and consequently there is a general migration of orbit centres away from the higher potential end of the dee gap.
Precessing Orbit of Particle:

Equilibrium Orbit:

AB is net movement of Orbit Centre during one Betatron oscillation (exaggerated value of π).

O is machine centre.

FIGURE IV-7. Precession of Orbit Centre.
The phase of the electric accelerating field, as the ions cross the dee gap, will change with each cycle, except for the region of acceleration where the magnetic field is such that the ions' resonant frequency is equal to the electric field frequency. We shall discuss the changing phase in this section, which deals with the region of acceleration in which particles spend the major part of their time, during acceleration. However, it is to be remembered that the phase change is not restricted to this region. Indeed, the actual turn to turn change of phase is greatest just before extraction, where the magnetic field gradient is steepest.

The rate of change of phase is given by \(-\frac{d\phi}{dt} = w - \theta\), or the change in phase per revolution by

\[-\frac{d\phi}{d(\text{rev})} = \frac{2\pi(w - \theta)}{w} = \frac{2\pi\Delta H}{w} = \frac{2\pi H}{H}\.\]

From this expression the variation in phase with radius can be obtained. We have

\[\frac{d\phi}{d(r^2)} = \frac{(d\phi/d(\text{rev}))(d(r^2)/d(\text{rev}))}{(d(r^2)/d(\text{rev}))}, \text{ and} \]

\[d(r^2)/d(\text{rev}) = (d(r^2)/dE)(dE/d(\text{rev})), \text{ where } E \text{ is the particle energy.}\]

Since \(H^2e^2r^2 = p^2c^2 = E^2 - E_0^2\), where \(p\) is the particle momentum, and \(E_0\) its rest energy
\[ \frac{d(r^2)}{dE} = \frac{2E}{H^2}e^2. \]

Thus, with \( \frac{dE}{d(\text{rev})} = 2eV_0 \cos\phi \), we have

\[ -\frac{d\phi}{d(r^2)} = \left( \frac{\pi eH^2}{2eV_0 \cos\phi} \right) (\Delta H/H), \]

or

\[ \sin\phi - \sin\phi_o = -\frac{\pi eH^2}{2eV_0} \int_{r_o}^{r^+} (\Delta H/HE) d(r^2) \]

For a given field shape, and dee potential, \( V_o \), this equation can be integrated numerically, to obtain the value of \( \phi \) as a function of \( R^2 = r^2 \). Once \( \phi \) lies outside the range \(-90^\circ \) to \( 90^\circ \) the particles will not be accelerated. Thus for a particular value of dee potential, \( V_o \), and resonant field, \( H_o \), there will be a limited range of values of \( H \) for which ions will be accelerated to a specified radius. If \( H \) is too low, particles will drop behind the R.F. phase (\( \phi \) positive), and if \( H \) is too high the particles will arrive at the dee gap too early (\( \phi \) negative). Where strong phase bunching exists, the cut off of beam with field variation will be sharp. For all conditions of bunching, the high field cut off, for radii beyond where \( H = H_o \), will not vary greatly with radius, since the number of particle revolutions before \( H = H_o \) will be fairly constant. The low field cut off value will, of course, increase considerably with radius. These effects are apparent in the "resonance plots" shown in Chapter VI.
Motion of Ions Near Maximum Radius.

When ions are accelerated into the region of rapidly decreasing magnetic field the betatron oscillation effects are considerably greater. The beam is further compressed vertically, and the rate of precession of orbit centres increases. If the centres of the ion orbits are not very close to the machine centre the particles will experience such large changes in field, as well as changes in n-value, during one revolution that no satisfactory quantitative estimate can be made of the beam behaviour. If correction of the beam orbits is necessary it is generally done by using shims to vary the magnetic field.

Extraction of the cyclotron beam usually starts with entry of particles into a channel, across which is applied an electrostatic field. If the orbits are centred precisely on the machine centre it is conceivable that particles just not entering the extractor channel on one revolution would do so during the next, due to their turn to turn increase in radius. However with machines extracting at radii where the field is falling rapidly, so that orbit centres need differ only slightly from the machine centre for considerable precession to occur, it is doubtful whether the orbit at which
extraction will start can be selected or predetermined.

While the beam is passing through the deflector channel particles on the outer side of it will, of course, experience a weaker magnetic field than those on the inside. This can lead to undesirable beam divergence. By making the extraction path as short as practicable, so that the beam direction differs no more than necessary from that of the field gradient, the amount of divergence will be kept to a minimum. Until the advent of quadrupole focusing lenses the divergence of external beams of cyclotrons was an obstacle of considerable proportions for much experimental work.
CHAPTER V.

DESCRIPTION OF THE CANBERRA 30 INCH CYCLOTRON.

While describing the Canberra cyclotron we shall not limit the subject matter for discussion to components as they exist at the time of writing. Some space will be devoted to justifying designs chosen, involving, at times, pre-design calculations. The behaviour of the components in operation, together with modifications made, and possible future changes, will be covered where it is considered desirable. However, any comments on the actual acceleration of protons, and the characteristics of the beam, will be reserved for the next chapter where such material properly belongs.

The Magnet.

The thirty ton magnet, designed by J.W. Blamey, was

X. Senior Fellow, Australian National University.
constructed at H.M.A. Naval Yard, Sydney in 1951. Some details follow:

Construction....... H-Type. Top of yoke and pole pieces demountable.
Pole pieces......... Root dia. 35", stepped down to 32" face.
Magnetizing coils... 1.25X0.75" aluminium bar, 0.4" bore for water cooling. 2 windings of 16 layers X 18 turns connected in series. Resist. 0.075 ohms.
Rated operation.... 843 Amp. (53 K.W.) for 20° C temp. rise in cooling water. 8.3 gal. per min. (43 p.s.i.)
Power supply....... Separately excited motor-generator set.

With 28 inch pole tips for the model homopolar generator the magnet poles approximated the shape given by Rose and Bethe for minimum saturation effects. With the larger diameter pole tips of the cyclotron this was not possible. However as the cyclotron operates with a fixed field this is not important in our case.

Contour of Pole Tips.

Although it is not possible to achieve magnetic focusing at small radii it was considered an advantage to have the central field rising as sharply as practicable, if only to make the magnetic forces predominate at a slightly earlier stage. In machining the pole tips shims were incorporated, as steps in the pole tip face. A reasonable calculation of the field shape due to a given pole tip shape, over areas inside the region where saturation occurs, can be made by regarding the innermost raised portion of the pole tips as a dipole sheet superimposed on the uncontoured metal, and the edges of subsequent steps in the profile as line currents. This permits
determination of the effect of the central shims, on the magnitude of the field, and the effects of the steps on the field gradient. Thus $\Delta H/H$ is found for the central region, and thereafter $d(\Delta H/H)/dR$.

The calculation for the central region is made while referring to Figure V-1(a). The moment (m) per unit area, due to a dipole sheet is $m = hH/4\pi$, where $h$ is half the dipole length. The potential on the axis of a single uniform circular dipole sheet is

$$\phi = 2\pi m(1-a)^{1/2}/(a^2 + z^2)^{1/2}.$$ 

Including both sheets, and their immediate images, we have

$$\Delta H = -\partial \phi/\partial z = 2hHa^2/(a^2 + z^2)^{3/2},$$

while, including other images, $\Delta H/H$ is given by

$$\Delta H/H_{axis} = 2f^2(h/z)[1/(f^2 + 1)^{3/2} + 1/(f^2 + 3^2)^{3/2} + 1/(f^2 + 5^2)^{3/2} + \ldots \ldots]$$

where $f = a/z$. This is the expression used for a practical case.

Referring to Figure V-1(b) for the case of a step in the profile we have

$$\Delta H/H = 2hx/\pi \left[1/(x^2 + z^2) + 1/(x^2 + 3^2z^2) + 1/(x^2 + 5^2z^2) + \ldots \ldots \right]$$

or, the expression required,
Potential on axis for uniform dipole sheet = m\omega, where m is pole per unit area, and \omega is solid angle subtended.

\[ \omega = \frac{1}{4\pi} \int_0^{2\pi} r^2 \cos \theta \, d\theta = 2\pi (1 - \cos \theta). \]

(a) Centre region of Pole-tips.

\[ \Delta H = \frac{2i}{r} \]

\[ i = m = \frac{HH}{4\pi} \]

(b) Step in Pole-tip

FIGURE V-1.
\[ \frac{d(\Delta H/H)}{dx} = 2h/\pi x^2 \left[ \frac{(1-f^2)/(1+f^2)^2 + (3^2-f^2)/(3^2+f^2)^2}{(5^2-f^2)/(5^2+f^2)^2} \right], \]

where \( f = x/z \).

Figure V-2 shows \( \Delta H/H \), and \( d(\Delta H/H)/dx \), as functions of \( a/z \) and \( x/z \) respectively.

Near the pole tip edge, when the pole tip is of square cross section, there is no simple way of calculating the field, because of the unknown effects of saturation of the steel. In the case of the Canberra cyclotron an estimate of the expected unshimmed field, near the pole tip edge, was made from data available from the Birmingham cyclotron.

A very rough calculation of the effect of Rose\(^2\) shims can be made by regarding the shim as a linear dipole (Figure V-3). For a linear dipole of moment \( M \) the field \( \Delta H \) is given by

\[ \Delta H = \frac{d\phi}{dz} = 2M(x^2 - z^2)/(x^2 + z^2), \]

where \( M = HV/4\pi \), and \( V \) is the linear volume.

Rose has calculated the size of shims to produce a uniform field. A plot of his values of shim dimensions is shown in Figure V-4. Rose's method, like the rough calculation above,

FIGURE V-2. EFFECT OF STEPS IN POLE PIECE.
M = HV/4π where V is linear Volume.

**FIGURE V-3.** Rose Shim as Linear Dipole.

**FIGURE V-4.** Rose Shim Dimensions for Uniform Field.
FIGURE V-5. Design Shape of Pole Tips.
assumes no saturation. In the belief that flatter shims would lead to less saturation, Rose's curve was extrapolated in choosing design values for edge shims in our case. The design pole shape for the Canberra cyclotron is shown in Figure V-5.

Control and Stabilization of Magnetic Field.

The current for exciting the cyclotron magnet generator is provided by five 12El's in parallel. These are grid controlled by the differential output of a 12-AT7 double triode. The grids of the 12-AT7 are connected to the photo-electrodes of a 5584 photo tube, illuminated from a Cambridge "D'Arsonnel" 463 ohm galvanometer. In series with the galvanometer are connected a potentiometer, for control, loops from the generator field and magnet pole, a shunt (186 mV/2000Amp) which is also in series with the magnet supply, as well as a portion of the differential voltage output of the amplifier, giving a fast feedback. One grid of a second 12-AT7 is connected to the cathodes of the main amplifier, its other grid being held at a constant reference voltage. If the light spot leaves the photo electrodes completely, the negative signal to the grids of the main amplifier is reflected, via the cathodes to the second difference amplifier. The difference output of
this tube then energizes a relay which circuits the galvanometer, for protection. The magnet control circuit is shown in Figure V-6.

The short term stability of the magnetic field is of the order of 1/10,000. Long term stability is somewhat less.

**Measurements of Magnetic Field.**

Absolute measurements of the field were made using nuclear (proton) resonance. For point to point field differences a double flip coil assembly was used. These apparatuses are described in Appendix I.

Figure V-7 shows the field near the edge of the pole tips for various Rose shims. A shim of 0.50 inch mild steel was added to the one machined on the pole tips for operation of the cyclotron, to date (Feb. 1958).

In Figure V-8 is shown the azimuthal variation of field, together with variation in gap dimension. Initially four spacers were used in the shimming gap and some bending of the pole tips took place. The bending was corrected by addition of more spacers, and shimming (mechanical) in the region of the original spacers.
FIGURE V-6. CIRCUIT FOR CONTROL AND STABILIZATION OF MAGNETIC FIELD.
FIGURE V-7. FIELD NEAR EDGE OF POLE TIPS FOR VARIOUS ROSE SHIMS.
FIGURE V-8. AZIMUTHAL VARIATION OF MAGNETIC FIELD AND GAP.
The radial variation of the field used for operation is shown in comparison with the original design field in Figure V-9, while Figure V-10 shows the field variation over larger radii, extending beyond the edges of the pole tips.

The above measurements (Figures V-7 to V-10) were made on the median plane for fields \( H_0 \), at 20 cm radius, of 12,600 gauss. Measurements made a few cm above and below the median plane indicated that it was midway between and parallel to the pole tip faces.

The Vacuum System.

The brass vacuum chamber is of conventional design. The total system has a volume of some 400 litres. It is pumped by a 9 inch oil diffusion pump followed by an Edwards "Speedivac", Type 1545-A. Pumping speed at the entrance to the manifold is 1,000 litres per second. Ultimate non-operating pressure is about 3 \( \times 10^{-6} \) mm Hg. With hydrogen entering the system at the rate of 1 c.c. per hour (atmospheric pressure) the system pressure rises to 2-4 \( \times 10^{-5} \) m.m. Hg. A sketch of the vacuum system is shown in Figure V-11.

The Ion Source.

This is of the conventional hooded arc type. A sketch

Initially a Molybdenum hood of circular cross section was used. The metal was not available in sizes suitable for the design shown in Figure V-12.
FIGURE V-10. FIELD VARIATION TO LARGE RADIUS.
II. Main Components of Vacuum System.

FIGURE V-11. MAIN COMPONENTS OF VACUUM SYSTEM.
of the ion source is given in Figure V-13. The supply for the 0.1 inch tungsten filament is a six volt plating generator, excited by a transformer-rectifier set with a variac in the primary supply for control. A 300 Volt, D.C. generator, excited from a voltage stabilized power supply, supplies the arc potential. The arc is operated at about 140 Volts, for arc currents from 0.5 to 1.5 Amp., depending on requirements. Hydrogen is supplied from a simple generator via a needle valve. The oxygen formed during electrolysis of a sulphuric acid solution escapes to the atmosphere. "Tripping" electrodes are used, in conjunction with relays, to switch the electrolysis current supply as the electrolyte level varies. The hydrogen is washed by concentrated sulphuric acid which it enters through scinted glass. There appears to be very little water, if any, carried over to the washing agent. A sweep electrode, evolving hydrogen to the oxygen side of the generator was dispensed with as the amount of oxygen entering the ion source, without it, had no noticeable damaging effect on the filament.


The single dee of the Canberra cyclotron is operated at
a nominal frequency of 19.4 Mc/s, with a peak voltage of 80 K.V. The throat of the dee is screwed to a five inch diameter copper tube which, together with a twenty inch diameter copper tube, forms a coaxial line. Support for the five inch line is obtained by the "shorting plate" and the "back plate", shown in Figure V-13. Transfer of power from the oscillator to the cavity is by means of a loop, inductively coupled to the line of the cavity, and condenser connected to the plate of the oscillator valve.

Regarding the cavity as forming a series resonant circuit, the capacity, C, the inductance, L, and the resistance, r, were calculated from the physical dimensions, chosen to be consistent with required operating characteristics. Calculated values were:

\[ C = 260 \text{ pf}, \text{ mainly associated with the dee.} \]

\[ L = 2.77 \times 10^{-3} \text{ micro-Henri per cm of line, together with some 0.06 micro-H. due to the region of dee-line connection.} \]

\[ r = \text{about} \ 5 \times 10^{-3} \text{ ohms.} \]

As the dee voltage is specified, it is convenient to

X. The "Cavity" refers to the R.F. combination of dee, coaxial line, together with capacity, inductance and resistance of associated components which are included in the resonant circuit.
(a) Vertical Section sketch of Dee and Line.

(b) Cavity Circuit.

(c) Cavity Circuit — Shunt Resistance.

FIGURE V-13. R.F. CAVITY.
replace the series circuit of the cavity by a parallel circuit, i.e. \( r \) is replaced by a shunt resistance \( R_s \), as in Figure V-13 (b), and (c).

If \( V \) is the dee potential, the power consumed in the circuit of Figure V-13(b) is \( \frac{i_2^2 r}{2} \), where \( i_2 = jwCV \). In (c) the power loss is \( \frac{V^2}{2R_s} \). Equating the two expressions for power, we have \( \frac{1}{R_s} = \frac{w^2 C^2 r}{2} = \frac{r}{w^2 L^2} \), at resonance.

For \( C = 260 \text{ pf}, \) and \( r = 5 \times 10^{-3} \text{ ohms}, \) \( R_s \approx \text{about 150,000 ohms} \), when the frequency is \( 19.4 \text{ Mc/s} \). Thus for 80 K.V. peak dee potential, and no beam load, the expected power loss would be some 21 Kilo-watt.

Defining the "Q" of the circuit as the ratio of the energy stored to the energy dissipated per radian we have

\[
Q = \frac{(1/2.CV^2)}{(1/2.V^2/R_w)} = \frac{CwR_s}{R_w} = 5,000. 
\]

As electrical measurements made on the cavity will have to be considered with the coupling loop as an intermediary, it is necessary to introduce an equivalent circuit which includes the loop. This is done by considering the standard transformer circuit\(^3\), as follows. The actual circuit under consideration (Figure V-15(a)) gives the equations

3. e.g. G.P. Harnwell, "Principles of Electricity and Magnetism", Mc-Graw-Hill.
(a) Circuit of Cavity and Loop.

(b) Equivalent Circuit of Cavity and Loop.

FIGURE V-14. Circuits of Cavity and Loop.
\[ V_1 = z_{L1}i_{L1} - jwM_2 \quad \text{------------------------ V-1.} \]
\[ 0 = -jwM_2 i_{L1} + z_{2i2} \quad \text{------------------------ V-2.} \]

where \( z_{L1} = r_1 + jwL \), and \( z_2 = r + jwL + 1/jwC \).

Equations V-1 and V-2 lead to \( z'' = z_1 + \frac{w^2M^2}{z_2} \),

where \( z'' \) is the impedance looking into the loop from the

R.H.S. of \( C_1 \),

or \( \tilde{z}'' = z_1 + z_2^* \)

where \( \frac{1}{z_2^*} = \frac{r}{w^2M^2} + \frac{jwL}{w^2M^2} - (\frac{j}{w^2M^2})(\frac{1}{wC}) \)

\[ = \frac{1}{R} + \frac{1}{Z_1} + \frac{1}{Z_2}, \]

if \( R = M^2/L^2 \),

\[ z_1 = -jwM^2/L = -j/wC'' \quad (C'' = L/w^2M^2) \]

and \( z_2 = jwC\cdot w^2M^2 = jwL'' \quad (L'' = Cw^2M^2) \)

This means the circuit of Figure V-14(a) can be replaced by

the equivalent circuit of Figure V-14(b), where the original
cavity parameters appear separately, and the circuit lends
itself to the treatment of admittances.

The employment of graphical methods in handling R.F.
measurements is more convenient than using algebra, which is
inclined to become tedious. Impedances can be plotted on the
complex Z-plane (Figure V-15). However a simplification is

achieved by recourse to the use of a Smith\textsuperscript{4, 5} chart. This

4. P.H. Smith, Electronics, 12, 29 (1939).
5. P.H. Smith, Electronics, 19, 130 (1944).
FIGURE V-15. TRANSFORMATION FROM Z-PLANE TO z-PLANE. \( z = \frac{Z+1}{Z-1} \)
involves the Mobius Transformation, the most general of the transformations giving only one value of $z$ (on $z$-plane) corresponding to each value of $Z$, and conversely, namely $z = (aZ + b)/(cZ + d)$, specialized in this case by having $a = b = c = -d = 1$. This transformation converts the imaginary axis of the $Z$-plane to a unit circle on the $z$-plane, centred on the $z$-origin. Lines of constant $X = k$ (see Figure V-15) transform to circles of radius $1/(k+1)$, centred on the real axis, and tangent to the line $x = 1$ ($z = x - iy$). The line $Y = k'$ transforms to a circle of radius $1/k'$, with centre $(1,1/k')$, tangent to the $x$-axis. $Z = 0$ transforms to $z = -1$, $Z = \infty$ to $z = 1$, while $Z = 1$ becomes $z = 0$.

Comparison between the two planes, shown in Figure V-15, shows that circles to the left of the line $x = 1$ are lines of constant resistance or conductance, depending on which is plotted. Circles, again tangent at $x = 1$, above and below the $x$-axis are lines of constant reactance or susceptance, positive above the real axis, negative below. Furthermore, points of impedance will lie diametrically opposite their counterparts in admittance, with respect to the $z$-origin.

The above transformation is particularly useful for
interpreting measurements made when a coaxial line of negligible losses is used. If we express impedances in units of $Z_0$, the characteristic impedance of the line, and write $\theta = 2\pi l / \lambda$, we have $Z_s$, the "sending end" impedance

$$Z_s = (Z_\ell \cos \theta + jZ_\ell \sin \theta) / (\cos \theta + jZ_\ell \sin \theta),$$

which transforms, when $z = (Z + 1)/(Z - 1)$, to

$$z_s = z \cdot \exp(-2\theta) = z_\ell \cdot \exp(-4\pi l / \lambda),$$

where $\lambda$ is the wavelength corresponding to the R.F. being used, and $Z_\ell$ and $z_\ell$ are the "load end" impedances. The expression for $z_s$ is a vector, $z_\ell$, rotating in a clockwise direction about the origin, at the rate of one revolution per half wavelength. Thus the sending end impedance of a line is obtained on the Smith chart by rotating the load impedance, clockwise, an amount equivalent to the length of the line.

Electrical measurements on the cavity and loop were made with the apparatus as in Figure V-16. An initial check was made of the resonant frequency of the cavity, assuming the value of the loop inductance. See Figure V-17.

By shorting the dee the admittance measured at the H.F. bridge, with a half wave-length line, was that of the loop. By successive shortenings of the line, till it was resonant with
FIGURE V-16. APPARATUS FOR R.F. MEASUREMENTS ON CAVITY.
Impedance Plot.

\[ Z_L (\text{at loop}) = Z_0^2 \left( \frac{1}{2s} \right) \text{ for } \frac{\lambda}{4} \text{ Line.} \]

Bridge measures \( \frac{1}{2s} \)

**Figure V-17. Initial Check of Resonant Frequency of Cavity.**
the loop, at the resonant frequency of the cavity, the effects of the loop were cancelled. Under these circumstances the admittance measured was that of the cavity. If a quarter wave line was used, with length adjusted for resonance, the bridge measurement was then the impedance of the cavity. (See Figure V-18)

The shunt resistance of the cavity was measured by four methods which will be described briefly ((a)--(d)) below. The results of the measurements will be given at the end of the discussion of methods.

(a) Using a single resistor across dee. Regarding the line and loop as a passive four terminal network, we have

\[ Z_1 = \frac{(aZ_2 + b)}{(cZ_2 + d)} \]

together with the fact, when a balanced quarter wave line is used, that \( Z_1 = 0 \) when \( Z_2 = \infty \), and vice versa. Under these conditions \( Z_1 = K/Z_2 \).

If a resistance \( R_x \) is placed across the dee, in parallel with the shunt resistance, \( R_s \), we have

\[ Z_{EX}/Z_{BOO} R_s = 1/R_s + 1/R_x \]

or

\[ R_s = R_x (Z_{EX}/Z_{BOO} - 1) \]

where \( Z_{BOO} \) is the measurement on the bridge. Three measurements were made in this way with \( R_x = 4,350, 71,500 \) and \( 73,000 \) ohms.
Impedance of loop (Admittance Bridge) for unbalanced 1/4 line.

Loop Reactance balanced by shortening 1/4 line.

Resistance of 50 Q presented to loop by resonant cavity & 1/4 line (see fig V-18 B)

Impedance of cavity as presented to loop
(Measured as Admittance Bridge, using balanced 1/4 line)

FIGURE V-18. CAVITY IMPEDANCE. (using balanced 1/4 line).
Resistances of the order of magnitude of $R_x$ in the second and third readings, which are sufficiently large for satisfactory comparison with $R_s$, had to be measured on a Q-meter, as they were outside the range of the R.F. bridge.

(b). Using a composite resistor across $Q$. In order to achieve a resistance comparable with $R_s$, and one whose value could be checked independently of the Q-meter, a resistance of the order of 1,000 ohms was used with a small capacity ($C_1$), of 1.1 pf, in series. The resistance was mounted as shown in Figure V-19.

From Figure V-19(b) we have

$$\frac{i}{V} = \frac{R + jX_2}{jR(x_1 + x_2) - x_1 x_2},$$

$$i_1 = \sqrt{\frac{V x_2 ((x_1 + x_2)^2 R^2 + x_1^2 x_2)}{1^2}},$$

and

$$i_{1R/2}^2 = \left[ \frac{\sqrt{V x_2^2 (x_1 x_2^2 + R^2(x_1 + x_2)^2)}}{R} \right]^2 = \frac{V^2}{2R_x}.$$

This gives for the value of $R_x$

$$R_x = R(x_1/x_2 + 1)^2 + (x_1/R)^2 = R(C_2/C_1 + 1) + \frac{1}{w^2} C_1^2 R.$$ 

With $C_1 = 1.1$ pf, $C_2 = 6$ pf, and $R = 1,040$ ohms the value of $R_x$, as used for measurement, was 92,000 ohms.
**FIGURE V-19. Composite Impedance for Comparison with Cavity Impedance.**
(c) By determination of Q of cavity. Referring to the equivalent circuit of the cavity, Figure V-13(c), the admittance of the circuit is

\[ jwC + \frac{1}{jwL} + \frac{1}{R_s} = jwC \left( 1 + \frac{\omega_0^2}{\omega^2} \right) + \frac{1}{R_s}, \]

which can be written as \( \frac{1}{R_s} \left( 1 - \frac{Q^2 \Delta \omega}{\omega} \right) \), where \( \Delta \omega = \omega_0 - \omega \), near resonance, and \( Q = \frac{CV^2}{2} / \left( V^2 / 2R_s \omega \right) = \omega CR \). Thus, near resonance, \( Q = \omega_0 / 2\Delta \omega \) when the real and imaginary components of the admittance are equal.

The impedance of the cavity is plotted on the Smith chart of Figure V-20, and transposed to the admittance plane. In Figure V-21 susceptance is plotted as a function of frequency in order to obtain \( \omega_0 \) and \( \Delta \omega \). The value of Q obtained, with 31.5 ohms for \( \omega_0 / \omega_C \), gave the value of \( R_s \) shown in Table V-1.

(d) Using resistance at loop and mutual inductance. Taking a calculated value of \( 1.36 \times 10^{-8} \) Henri for the mutual inductance of the Loop-cavity, together with \( \omega L = 31.5 \) ohms, and the resistance presented by the cavity, at the loop (see Figures V-14(b) and V-17) equal to 525 ohms, a value of 173,000 ohms was obtained for the shunt resistance of the cavity.

The errors shown in the table below represent the maximum
Impedance of Cavity (Measured with $\frac{3\lambda}{4}$ Line).

**FIGURE V-20. ADMITTANCE PLOT TO OBTAIN POINT WHERE SUSC.$\cdot$COND.**
FIGURE V.21. Plot to Obtain Q of Cavity.

\[ Q = \frac{19.9969}{0.0039} = 5130 \]
errors which were calculated from estimated uncertainties in all components used and measurements made.

Table V-1. Shunt Resistance, Rg, of Cavity.

<table>
<thead>
<tr>
<th>Method of measurement</th>
<th>Value obtained for Rg (K-ohms)</th>
<th>Calculated error (k-ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>150</td>
<td>± 23</td>
</tr>
<tr>
<td>(i)</td>
<td>154</td>
<td>± 23</td>
</tr>
<tr>
<td>(ii)</td>
<td>166</td>
<td>± 20</td>
</tr>
<tr>
<td>Value accepted as reasonable</td>
<td>163 ± 15 K-ohms.</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>177</td>
<td>± 27</td>
</tr>
<tr>
<td>(c)</td>
<td>163</td>
<td>± 14</td>
</tr>
<tr>
<td>(d)</td>
<td>173</td>
<td>± 32</td>
</tr>
<tr>
<td>calculated</td>
<td>150</td>
<td>± 30, -10</td>
</tr>
</tbody>
</table>

The capacity and inductance of the loop were estimated from admittance measurements made at three different frequencies, using a quarter wave line to the loop with the dee shorted. By regarding the loop as a pure inductance, shunted by a capacity, the individual values of these were obtained. Results of measurements are shown immediately below:

<table>
<thead>
<tr>
<th>Angular frequency</th>
<th>Admittance at bridge m-mho</th>
<th>pf</th>
<th>(1/C^2L)</th>
<th>w^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.18X10^7</td>
<td>0.25</td>
<td>55</td>
<td>3.12X10^6</td>
<td>38.2X10^14</td>
</tr>
<tr>
<td>12.55X10^7</td>
<td>0.20</td>
<td>59</td>
<td>2.93X10^6</td>
<td>157.8X10^14</td>
</tr>
<tr>
<td>17.96X10^7</td>
<td>0.15</td>
<td>62</td>
<td>2.78X10^6</td>
<td>322.5X10^14</td>
</tr>
</tbody>
</table>

These values were used in the expressions 1/X_L = wC - 1/wL and 1/\omega^2_C = w^2C - 1/L, where \omega = \omega^2/L_g. X_L and X_g
are the load end, and the sending end reactances respectively, while \( Z_0 \), the characteristic impedance of the line, is 76 ohms. From the three equations obtained values of \( C \) and \( L \) were found. As the values of \( C \) are dependent on the small differences between values of \( 1/Z_0^2 C \), results were not accurate. Figures for \( C \) were 15±9 pf, 12±3 pf and 9±6 pf, with a value of about 0.32 micro-Henri for the inductance.

The capacity and inductance of the dee were obtained by measurement, using a quarter wave line coupled directly to the dee. A measurement of the admittance at a frequency of 9.812 Mc/s, together with the resonant frequency gave values of \( C = 250 \text{ pf} \), \( L = 0.250 \text{ micro-Henri} \). Thus \( 1/w_0 C = w_0 L = 31.5 \text{ ohms} \).

**Radio Frequency System. (2) The Oscillator.**

The oscillator is required to resonate with the cavity as a load. It is required to provide energy for the circulating proton beam as well as providing sufficient power to make good resistive losses in the circuit.

The oscillator may be represented as in Figure V-22(a). If the impedance presented to the plate by the external circuit
FIGURE V-22. EQUIVALENT CIRCUIT OF OSCILLATOR.
is $Z'$, and if $Z_e$ is the impedance of the oscillator valve as seen by the electron stream, the condition for oscillation is $Z' = -Z_e$. This gives an infinite impedance across the terminals AB of the equivalent circuit shown in Figure V-22(b). This impedance is $Z'Z_e/(Z' + Z_e)$, where

$$Z' = \frac{Z_1(Z_2 + Z_3)}{Z_1 + Z_2 + Z_3}.$$

$Z_2$ and $Z_3$ are chosen so that $Z_2/(Z_2 + Z_3)$ gives the required feedback ratio, $n = e_g/e_p$. At the same time $Z_2$ and $Z_3$ are kept purely reactive and their values kept within limits such that, while satisfying the feedback requirement, $Z_2/(Z_2 + Z_3)$ will not vary quickly with frequency near resonance. Thus $Z_e$ is practically independent of frequency. However $Z_e$ is a function of oscillation amplitude, being dependent on the mutual conductance of the tube, which changes considerably from when the valve operates class A, for small oscillation amplitudes, to class C at large amplitudes, with the grid swinging negative beyond cut-off. With the grid 180° different, in phase, from the plate, and class C operation, the magnitude of the negative resistance $Z_e(A)$ increases with amplitude. Hence $\frac{\partial Z_e}{\partial A}$ is positive, where $A$ is the oscillation amplitude.

$Z'$ is a function of frequency but not, in our case, a
The condition for oscillation may now be expressed as
\[ Z'(w) = X(w) + jY(w) = -Z_0(A) \] ........................V-3.

However this does not necessarily represent a condition for stable oscillations. The additional requirement for stability can be found by considering near resonant conditions in terms of a complex frequency, \( W = a - jB \). Then the oscillation is expressed by \( V = V_0 e^{jW} = V_0 e^{(ja + B)} \), which is of exponentially varying amplitude according to the value of \( B \).

If Equation V-3 is varied with respect to amplitude we have
\[
\delta_W = \frac{(-\partial Z_0/\partial A)(\partial X/\partial w - j\partial Y/\partial w)}{(\partial X/\partial w)^2 + (\partial Y/\partial w)^2} \cdot \delta_A \] ........................V-4.

Since \(-\partial Z_0/\partial A\) is positive, the amplitude of oscillation included in a complex \( W = a - jB \), will increase with \( A \) for \( \partial Y/\partial w \) positive, and decrease with \( A \) for \( \partial Y/\partial w \) negative. Thus the conditions for stable oscillation are
\[
X(w) = -Z_0(A),
\]
\[
jY(w) = 0,
\]
and \( \partial Y(w)/\partial w \) negative.

This means that the point of stable oscillation corresponds to
the situation where the reactance across the valve is changing from positive to negative, i.e. from inductive to capacitive. (See Figure V-23.)

The valve used for the self excited oscillator was the S.T.C. 3Q/260-E triode. The choice was mainly influenced by the fact that this valve was readily obtainable in Australia. From the valve's characteristic curves provided by the manufacturers the curves of Figures V-25, -26, -27, were plotted for the three values of feedback ratio (n) indicated in the figures. A value of $n = 0.125$ was selected as most suitable, considering permissible plate and grid dissipation figures. A grid resistor of 1,000 ohms was used initially.

For a cavity shunt resistance of about 160,000 ohms, and a circulating beam of 2 m.A. of 8 Mev protons, the valve would have to supply some 40 K-watts of R.F. power. Thus for a peak R.F. plate potential of 11.5 K.V., a resistance of about 1650 ohms had to be presented to the plate. The main item involved in the transformation of the cavity shunt resistance to the plate load was the coupling loop. Final adjustments to this were made, in conjunction with bridge measurements of the impedance at the plate of the valve, when the valve was in its
FIGURE V-23. CONDITIONS OF OSCILLATION.

FIGURE V-24 OSCILLATOR PLATE CIRCUIT.
operating position.

The loop reactance, together with the inter-electrode capacities, and the electrode inductances, determined the plate circuit shown in Figure V-24, except for the value of the blocking condenser, \( C_4 \). The cavity is not shown in the circuit because, except at resonance, it makes almost zero contribution to the plate impedance. For zero reactance to the plate \( C_4 \) should have been \(-61j\) ohms. However this value would have introduced a resonance into the plate circuit close to 20 Mc/s. Consequently, a value of \(-54j\) ohms was selected to push the resonance above 30 Mc/s. \( C_4 \) was made cylindrical, coaxial with the valve anode, with polythene for dielectric.

The valve grid circuit is shown in Figure V-28, with \( C_1 \), \( C_2 \), and \( L_1 \) initially undetermined. The two major requirements of the grid circuit were:

(a). At the resonant frequency the admittance had to be \(-9/266j\) mhos, to give the correct feedback ratio (0.125).

(b). The admittance had to become positive with increase in frequency before the plate circuit resonance was reached.

This was actually a loop resonance.

\( X. \ Z_2 \) (Figure V-22(a) was the plate grid capacity of 30 pf., giving, at 20 Mc/s, an admittance of \( 1/Z_2 = j/266 \). Thus \( 1/Z_3 = -9j/266 \).
FIGURE V-25. CALCULATED POWER CURVES FOR 3Q/260-E TRIODE
FIGURE V-26. CALCULATED POWER CURVES FOR 3Q/260-E TRIODE.
Figure V.27. Calculated Power Curves for 3Q760C tube.

Grid Dissipation, watt.  Plate Dissipation, Kilo-watt.

Plate Swing: A0. Loop Voltage: VQ = 2.7.

Grid Resistor values (in) shown on curves.

Calculation Power Curve:

Feed back Ratio: T = 0.125.
**FIGURE V-28. GRID CIRCUIT.**

Y is point of connection to bridge for measurements.

$L_1$, $C_1$, and $C_2$ are added reactances to give correct feedback ratio.

**FIGURE V-29. COMPLETE OSCILLATOR CIRCUIT.**
To satisfy condition (a) the admittance at Y in Figure V-28 had to be 0.05j Mho. Taking reasonable values for C₂, L₁ was plotted as a function of C₁, as in Figure V-30, so that condition was met. Thus values of the three undetermined parameters could be chosen to meet both requirements, as indicated in the figure.

The admittance of the grid circuit is shown, as a function of frequency, in Figure V-31. It will be noted that the curve takes on positive values of admittance at \( m = 1.4 \), where \( m \) is the ratio of frequency to resonant frequency \( (w/w₀) \), thus oscillations would not occur in the plate circuit at the frequency of the first loop resonance of something over 30 Mc/s.

In Figure V-32 the susceptance presented to the plate, with the grid circuit included, is plotted as a function of frequency. The condition for oscillation (cavity excluded) occurs at \( m = 1.6 \). However, as the grid circuit admittance is positive in this region, no "take-off" would be expected. Another resonance is shown for \( m \) equal to about six. Here the grid circuit admittance is negative (see Figure V-31), and conditions are favourable for oscillations. This danger was not expected to be serious as the valve would deliver
FIGURE V-30. VALUES OF $L_1$, $C_1$ AND $C_2$ FOR CORRECT FEEDBACK RATIO.
Figure V-31
little power at such a high frequency. In operation some annoyance was experienced due to very high frequency potentials occurring in the grid circuit, and causing flash-overs. These were stopped by rather critical trial-and-error adjustments to the grid circuit parameters.

The grid resistance, for which radiator elements were used was isolated from the radio frequency by an inductance. At a later stage the grid resistance was made adjustable, up to 3,000 ohms. Also the cathode was lifted above earth by an adjustable resistance.

The complete oscillator circuit as it went into operation is shown in Figure V-29. Bridge measurements were made of admittances at various points of the circuit during assembly, to ensure that design figures were maintained.

Although the oscillator did operate unaided, initially, it was obvious that an inordinately long time would be needed to get beyond the "clean-up" stage, and operation was not as spontaneous as was desired, due to multipacting and gas discharges. To improve this state of affairs an R.C.A. 350-w radio transmitter, operating at half the oscillator frequency, was inductively coupled to the grid circuit, to act as booster.
The transmitter is switched by a relay which operates on the difference between the oscillator grid and cathode current, so that it injects a signal into the grid circuit whenever the oscillator stops operating.

In using the machine with fine internal slit systems, and in order to maintain constant orbit centres, a constant dee voltage is of considerable importance. To achieve this a dee voltage control and regulator system has been added. It is a feedback circuit from the dee to the oscillator grid, as shown in Figure V-33.

The oscillator power supply is a 50 K.W. English-Electric mercury arc rectifier set with a maximum voltage output of 14.6 K.V.

The R.F. potentials of the dee, and the oscillator plate and grid, are sampled by capacity coupling between the components and three quarter wave lines, each of which is terminated by a crystal diode and resistance in parallel, as shown in Figure V-34. With $R \gg Z_0$, the input impedance at $C$ for a three quarter wave line is close to zero, and stray capacity to earth does not affect the calibration of the voltmeter. The crystal rectifier causes the line to be charged to a
FIGURE V-33. CIRCUIT FOR DEE VOLTAGE REGULATOR.
FIGURE V-34. R.F. VOLTOMETER.
D.C. level closely approximating the peak R.F. level at the end of the line. Thus the current through the meter, M, is equal to \( \frac{V_{Z_0}}{Z_C} \), where \( Z_C \) can be calculated from the geometry of the pickup condenser. The possibility of spurious readings, due to harmonic voltages, if present on the oscillator plate or grid, can be minimized by placing a condenser to earth at the input end of the cable.

**The Extractor System.**

The layout of the splitter and deflector is shown in Figure V-35. The expressions needed to find the path of an ideal beam through the extraction channel can be obtained with reference to Figure V-36. We have

\[
\delta \psi = \delta \theta - \delta \phi = \left(\frac{\delta R}{R \tan \psi}\right) \left(1 - \frac{R}{\rho \cos \psi}\right),
\]

since \( \delta \theta = \frac{\delta R}{R \tan \psi} \), and \( \delta \phi = \frac{R \delta \theta}{\rho \cos \psi} \).

Whence \( \frac{d \psi}{dR} = \frac{1}{R \tan \psi} - \frac{1}{\rho \sin \psi} \), and

\[
\delta (R \cos \psi) = \frac{R \delta R}{\rho}.
\]

Using these relationships, a step by step plot of the beam in the channel can be made, being determined by the electric and magnetic fields prevailing at each point.

The size of the magnetic deflector required was estimated
FIGURE V-36. Radii of Deflector and Splitter.
from magnetic field measurements made in the vicinity of various pieces of steel placed in the magnet gap. The final dimensions were corrected so that the beam was started on the desired external path.

The deflector potential supply is an oil immersed voltage doubler, with variac control in the primary side. Extraction requires from forty to sixty K.V., depending on the position of the deflector, and the direction of beam entry into the extraction channel. Cooling for the deflector is a closed oil circuit, including a heat exchanger.

Although a slot was cut in the tungsten septum when extraction was first undertaken, this was subsequently dispensed with. Some evidence was found to suggest that the deflector field perturbed the final orbit(s) of the beam, when the septum was slotted. If the beam intercepted the unslotted septum a ragged slot was soon formed by melting of the metal. However, with care in operation, i.e. with correct deflector volts, and with suitable beam direction at entry, no damage is done by beams of a few hundred micro-Amp. When larger beams are extracted it might be necessary to return to the use of a slotted septum.
CHAPTER VI.

OPERATION OF THE CANBERRA CYCLOTRON.

The cyclotron was operated, at first, without a dummy dee. In this condition a circulating beam was first detected during November of 1955. By the following April beam had been accelerated to full radius, and some extraction achieved. In June of 1956 a dummy dee was fitted. Following this, a couple of months were spent making measurements on the internal beam. During the remainder of the year the use of beam defining slits, at small radii, made it possible to study the behaviour of the beam in more detail, as well as to accelerate, and extract, more efficiently. The intense internal beams, of up to 3 mA, or even smaller ones, reduced to a few hundred micro-amps by the internal slits, accompanied by extracted beam of up to 250 micro-amps, produced a very high level of radiation in the space available around the initial site of the machine. Because
of this the decision was taken to run with only very restricted beams until after the cyclotron had been moved from its temporary site, to the position where it would be used as an injector for the proton synchrotron. Using a much reduced beam, extraction to a point some fifteen feet from the cyclotron was achieved in early 1957, and an investigating run made for the $^{11}\text{B}(p,\gamma)^{12}\text{C}$ experiment which is discussed in Appendix IV. Electrical breakdown over the deflector insulators held up operation for some time in the second half of 1957. This was caused by the build up of an oil film on the insulators. Since the regrinding of the porcelain insulator surfaces, and the fitting of a venetian blind type of baffle at the entrance to the pumping manifold, so that the R.F. on the dee cannot penetrate the manifold to the refrigerated baffles, there has been no indication of further trouble. Following the repairs, and modification, effort was directed towards achieving a clean external beam, with as low a level of background as possible, for the nuclear physics experiment. This work was not without a great deal of value in assisting the development of the machine as an injector, as it involved careful measurement, and manipulation of the beam, as will be required for injection. The experiment was carried out during late 1957.
and early 1958. During this period the quadrupole focusing magnets were fitted and their usefulness confirmed.

In the following pages of this chapter, techniques used in the machine operation, as well as behaviour of the machine, and its accelerated beam, will be discussed.

Once a circulating beam is detected in a cyclotron there is likely to be some anxiety as to how much of it can be accelerated to full radius. With a fixed frequency machine, having chosen an operating value of dee voltage, the only remaining variable affecting the radius to which acceleration can be achieved is the magnetic field. To determine an operating value of field, the beam can be measured, at a particular radius, as the magnetic field is varied. Results of this type of measurement are shown in the resonance plots of Figures VI-1 and VI-2. It will be noticed that they display the characteristics mentioned in Chapter IV following the derivation of Equation IV-18.

In Figure VI-1 resonance curves for 36 K.V. and 46 K.V. dee potentials are shown. In order to accelerate a beam to more than 12 1/2 inches it is obvious that 36 K.V. dee potential is inadequate, because the phase lag of particles
Figure VI-1

(a) 36 KV accelerating potential

(b) 46 KV accelerating potential
mV (indicating magnet current).

**Resonance Plot for Dee Potential of 80 KV.**

**Figure VI-2.**
making the great number of revolutions necessary, for this acceleration per revolution, would exceed 90°. With 46 K.V., although it would be expected that some beam would reach 12 1/2 inches, there would be considerable losses over the final part of the acceleration, due to ions losing excessive phase with respect to the R.F. Using the design potential of 80 K.V., the resonance curves of Figure VI-2 indicate negligible ion losses during the latter stages of acceleration, as well as not too critical a value of magnetic field. None of the curves shows measurements at a radius of greater than 11 inches, because, when the measurements were made, the position of the orbit centres was not known with any accuracy, and it was desired to keep the beam away from the extraction system.

The method employed for measuring the position of orbit centres is discussed in Appendix II. The earlier determinations of the positions of mean orbit centres were made with dee potentials of 36 K.V. and 46 K.V. Although these values were used because, at the time of measurement, the full dee volts had not been obtained, there is some advantage in using lower potentials as they present a somewhat magnified picture of the orbit precession to be expected with higher values.
Likewise the effect of shimming is magnified.

The magnetic shims which were used for control of the position of orbit centres consisted of two mild steel discs, twelve inches in diameter, and 0.200 inches thick. The type of field variation caused by these shims is shown in Figure VI-3. Before discussing the measured positions of orbit centres, we shall make some comments on the effects to be expected from shimming.

If we regard the change of magnetic field, due to the presence of shims, as a step equal to $\Delta H$, the orbit centre can be considered as moving a distance $\Delta r = (r/H)\Delta H$ towards the ion, in order to bring about the necessary change in orbit radius. As the ion passes out of the region of influence of the shim the orbit centre will move $\Delta r$ away from the ion. As shown in Figure VI-4, the net result is a movement of the orbit centre of $2\Delta r \sin(\theta/2)$, in a direction at 90° to the direction of the shim centre from the machine centre. Here $\theta$ is the angle subtended at the orbit centre by the ion path across the shim region. In view of the foregoing it is apparent that wedge-shaped shims would have certain advantages. However, the circular shims were easily made and adjustment
FIGURE VI.3  EFFECT OF SHIM ON FIELD.
FIGURE VI-4. EFFECT OF SHIM ON ORBIT CENTRE.
was sufficiently easy for them to be retained in our case. The adjustment of shims is, in practice, something of a "move-and-try" operation. At the same time adjustment is greatly facilitated if it is a "sensible-move-and-try" procedure.

In regard to the orbit centres, when shims are present, as shown in Figure VI-5, it is to be remembered that the unshimmmed precession is still present, with the effect of shims added. In curve B, the effect of the shim appears to be dominant, but in the case of C the unshimmmed precession seems to predominate. The apparent predominance of the unshimmmed precession for C might be explained on the grounds that many revolutions occur between $8\frac{7}{6}$ inches and $10\frac{7}{6}$ inches, due to loss of phase by the ions. Over these radii the extent of the shims' influence would be decreasing. The starting points of both curves B and C do show the effect of shimming, but to note this is perhaps pushing the accuracy of measurement too far. The starting point of curve D, for a radius of $7\frac{3}{2}$ inches, is inexplicable, and best written off as an error of measurement. The general trend of the curve is consistent with the shimmed precession effect. The translation of curve E from curve A can be satisfactorily regarded as due to the effect of shimming in a $337\frac{1}{2}^\circ$ position. All the curves of the
**Figure VI-5.** Motion of Orbit Centres.
figure indicate that the centre of the orbits is initially about half an inch towards $270^\circ$ from the machine centre, and somewhat towards $180^\circ$. This is to be expected, as the ion source was first installed on the North-South centre line of the machine, and somewhat South of the machine centre.

The precession of the beam centres, with 80 kV. dee potential, is shown in Figure VI-6. Comparison between curve A, in this figure, and curve A of Figure VI-5, illustrates the reduction in precession with increased acceleration voltage. Curve B, with shims 5 inches towards $315^\circ$, at first sight might appear irreconcilable. However, it will be noted that the shim extends one inch beyond the machine centre, away from $315^\circ$, and the effect on early orbits will be considerable, with $\theta$ approximately $180^\circ$. If it is assumed that this effect moves the orbit centres strongly towards $045^\circ$, possibly nearly to the machine centre, the path taken on curve B, when the shims are no longer effective, is reasonable, within the accuracy of measurement. Curve C, corresponding to measurements made after the ion source was moved $5/8$ inch towards $090^\circ$, shows the orbit centres moving about the machine centre on a much reduced radius. It indicates, however, that the ion source is still South of the machine centre. More will be
A Shims central.
B Shims 5” to 3/15°
C Shims central after Ion Source moved 5/8” to 090°

Dee Potential 80 KV in each case.

**FIGURE VI-6. MOTION OF ORBIT CENTRES.**
said concerning orbit centres later, when discussing the use of internal slits. Firstly some further remarks will be made on the subject of the unrestricted beam.

When the beam was accelerated without any restrictions, such as slits on the early orbits, the beam intensity near full radius could be varied from something under 1/2 mA to about 3 mA, by varying the operating conditions of the ion source. This involved varying the arc voltage and the hydrogen supply rate. Reduction to much below 1/2 mA led to unstable ion source operation. The greater part of the ion loss during acceleration occurred during the very early orbits, with radii less than three to four inches. Measured currents near the ion source were of the order of 10 to 20 mA. These figures were regarded as unreliable, and the usual practice was to measure the beam power, when working at small radii, in the belief that this would avoid measurement of low energy components of doubtful origin. Later experience showed that there was no reason to regard current measurements as suspect, except when made well away from the ion source, along the 225°-270° radius, where there did seem to be a component of low energy ions.
Figures VI-7 and VI-8 give some indication of the beam behaviour during the first three revolutions. The power to the two targets is indicated by m.V. readings from thermocouples across their cooling water. In the measurements plotted the thermocouples had different calibrations. Thus the two curves of each figure are in different units.

Referring to Figure VI-7, the curves represent the sum of plots of intercepts by targets $135^\circ$ and $225^\circ$ of the first three orbits. The procedure adopted was as follows. Target $225^\circ$ was set at a radius which did not interfere with the first orbit. Target $135^\circ$ was moved in quarter inch steps of increasing radius, the beam to both targets being measured for each position. Target $225^\circ$ was then set beyond the second orbit, and $135^\circ$ moved out till it received no beam again. This was then repeated for the third orbit. The first steep slopes on the curves of Figure VI-7 correspond to the first orbit, the second steep slopes to the second, and the third to the third. The hatched part at the bottom of the figure indicates the profile of the beam in the three orbits.

In Figure VI-8 target $135^\circ$ was set to intercept successively the first, second, and third orbits, while target
FIGURE VI-7. Beam Profile at 135°
Beam to Target 135°

Beam Profile at 225°

Indicated Profile at 225°
225° was moved outwards through the orbit in each case. The results of three pairs of curves so obtained are shown, summed, in this figure, as in the previous one. As the successive orbits overlap considerably at 225°, they are not clearly defined in the figure, and the profile curve is meaningless, apart from indicating the separation between the highest intensity portions of the beam on successive orbits.

The introduction of a slit, 1/4 inch wide by 1/2 inch high, defining the first orbit of the beam, at 225°, was intended to remove the very late phase ions from the beam, to give a greater separation between the edges of successive orbits, to reduce the total beam, and to assist in simplifying the problem of extraction.

Resonance curves are given in Figure VI-9, for a target setting of 9 inch radius, for comparison between the unrestricted beam and that passing the quarter inch slit. It will be seen that the slit reduced the beam by about a factor of two. In Figure VI-10 it is shown that the greater part of the beam which is normally lost during the early acceleration was stopped by the slit. The curves of Figure VI-11, showing the beam fall off with radius, for various slit positions, afford a
FIGURE VI-9. RESONANCE CURVES
FIGURE VI-10. LOSS OF BEAM WITH RADIUS.
FIGURE VI-11

BEAM VS RADIUS WITH 1/4" INTERNAL SLIT.
comparison between the amount of beam accelerated from different parts of the total beam cross section on the first revolution. The scale for \( mV/R^2 \) in this figure is not the same as in Figures VI-10 and VI-11.

Beam cross sections are shown for one position of the slit, with the slit centre about \( \frac{1}{2} \) inch radius, in Figure VI-12. Comparison with Figures VI-7 and VI-8 shows the orbit separation about the same as determined for the unrestricted beam, though with a much sharper definition of separate orbits.

In Figure VI-13 the beam cross sections, as determined at target \( 135^\circ \), are shown. The lack of any appreciable change of beam radius, with change of slit radius, points to this position as that of beam "cross-over" for the first orbit.

Orbit centres were measured at radii of about five and seven inches. These are shown in Figure VI-14. The positions of the curves, for two lines of centres, suggest that the magnetic centre of the machine lies somewhat to the East and North of its geometrical centre. This, of course, assumes the accuracy of measurement to be of the order of the line thickness used in the figure. Further work with the quarter inch slit was not done, as it was thought that a greater
FIGURE VI-12. Beam Cross-section (¼" slit).
FIGURE VI-13  BEAM CROSS-SECTION AT 135°
Unbracketed figures indicate Beam orbit radius.

Curve A: Slit moved 5/8" outward in selecting centres from Right to Left.

Curve B: Radius of centre of slit shown by bracketed figures.

**FIGURE VI-14. ORBIT CENTRES—¼" INTERNAL SLIT.**
reduction in beam, by finer slits, would lead to higher accuracy of measurement, as well as a greater reduction in the radiation level at large beam radii.

The system which replaced the quarter inch slit, at 225°, consisted of an eighth inch wide slit at 225°, followed by a sixteenth inch slit at 180°. Both slits were half an inch high. Eight positions were arbitrarily selected for the 180° slit, at intervals of 1/16 inch. The 225° slit was adjusted, for each setting of the one at 180°, to give maximum beam through the two slits. The slit positions so obtained were used during the measurements which will be discussed in the immediately following pages. The positions, i.e. the radii, of the slits corresponding to the various settings, are listed in Figure VI-15. Position (i) is not given, as so little beam accompanied this setting, and it was not used. A blank "stop" was used in conjunction with the 180° slit, which could be moved radially to give virtual extension to the 180° slit, piece, either inwards or outwards. For slit settings (ii) and (iii), the blank stop was set inside the 180° slit, and for other slit positions it was set to the outside of the slit, to stop early phase, larger radii ions, from passing outside the slit body on their first revolution.
FIGURE VI-15  INITIAL ORBITS WITH TWO SLITS.
Resonance plots, of beam against magnetic field, are shown in Figure VI-16, for three slit positions (iii, v, and vii). The beam was measured, as current, and power, to the 180° target, at a radius of eleven inches. With the numbering of the slit positions increasing as the slits are moved to smaller radii, position (iii) allows the acceleration of ions which leave the ion source at earlier R.F. phases than in the case of (v) and (vii). Thus, as shown in the figure, the beam through the slits in the (iii) position can be accelerated for a lower value of magnetic field without being lost by falling behind the phase of the R.F. As the field is increased, it will be noticed, there is a fall off in the beam of (iii), across the resonant field region. This is consistent with a much greater phase spread for these ions than in the case of the other two curves shown. It is possible that with the slits in positions (ii) and (iii) some ions, on their first orbit, passed by slit 180°, on the outside, as for these positions the blank stop was on the inside. Beam measurements at small radii, made when the positions of the 225° slit and the stop were being determined, did not support this possibility, though they were unable to rule it out entirely. As the phases of the ions from the ion source become more negative, from zero,
Target 180° at 11" radius.

Upper curve of each pair is mV(power) plot.

FIGURE VI-16 . RESONANCE PLOTS — TWO SLITS .
the energy gained on the first half revolution at first increases, but with very negative phases the energy gained, and so the radius, is likely to be less\textsuperscript{X}. Thus there is a probability that ions of considerably different phases passed through the slits in position (iii). The reduction in beam intensity, with radius, shown in Figure VI-17, for stop positions (ii) and (iii), also suggest a phase spread, with the early, or more negative, phased ions dropping from the accelerated beam at smaller radii, where the field is higher than the resonance value. The figure shows the beam intensity to be about the same for slit positions (iv), (v), and (vi), suggesting that the ions in the beam defined by these slit positions are of near zero phase at their start.

With the beam stopped down by the two slits, measurements of orbit centres became a much simpler operation, as well as a more accurate one. Results of some of the measurements made are shown in Figure VI-18. Most of the measurements taken at small radii were for only the (iii), (v), and (vii) slit positions. At larger radii the orbits were checked for all slit positions, but only three are shown in the figure, for the sake of clarity. It was assumed from the figure that \textsuperscript{X} This has been discussed in Chapter IV, and will be mentioned again, later in this chapter.
FIGURE VI-17. Beam vs Radius with Two Internal Slits.
FIGURE VI-18. Motion of Orbit Centres with Two Slits.
the starting line of orbit centres was displaced about 0.05 inch North of the machine centre. The low radius points below the East-West centre line are regarded as in error, the accuracy of measurement being slightly less at smaller radii. It was this starting line which was used, together with the East-West positions of the (iii), (v) and (vii) orbit centres, when estimating the orbits shown in Figure VI-15. The precession of the three orbit centres shown in Figure VI-18 suggests that the ions corresponding to slit position (v) have their orbits centred on the magnetic centre of the machine which is apparently displaced slightly to the North-East of the machine's geometrical centre. As mentioned earlier, this displacement of the magnetic centre is roughly supported by the measurements made with only one slit (see Figure VI-14). The position of the magnetic centre is, of course, influenced by the position of the shims used. They were central for the above measurements.

It is interesting to note the agreement between the position of centres of eleven inch orbits, as indicated in Figure VI-18, and calculated from Figure VI-16. The ratios \( \frac{mV}{mA} \) (Figure VI-16), for slit positions (iii), (v), and (vii), corresponding to a 1095 mV magnetic field value, are 6.15, 6.32, and 6.52 respectively. These values are proportional
to the beam energy, i.e. the square of the orbit radius.

Taking the orbit radius for slit position (v) as 11.1 inches, the other indicated radii are 10.97 inches for the orbit corresponding to slit position (iii), and 11.28 inches for the orbit corresponding to slit position (vii). These two values give a very reasonable confirmation of the centres shown in Figure VI-18.

Measurements of the distance of the leading edge of the splitter septum from the machine centre showed it to be at a radius which was certainly not greater than 12.8 inches. This would mean, even allowing for small errors in orbit centre measurements, that the larger radii orbits of Figure VI-18 were passing exceedingly close to, if not through, the septum slot. Extraction was carried out without further measurements of orbit centres.

When extracting without internal slits it had been necessary to stop most of the beam on one of the probe targets, allowing only a small amount to reach the splitter. Otherwise, part of the full beam which was not centred satisfactorily for entry into the extractor channel tended to strike the septum, and vaporize it. A septum without a pre-cut slot was
used for most extractions, as it appeared that the deflector field penetrated sufficiently far through the slot to perturb the ion orbits before the ions entered the channel. The only measurements made to detect this effect are shown in Figure VI-19. The unrestricted beam was stopped with target 135°, just inside the radius at which some extraction appeared. Target 225° was moved in till it intercepted about fifty percent of the beam, with the deflector volts set for extraction. The beam to target 225° was measured for various settings of the deflector voltage. It is possible, since the measurements were made with no beam restriction, that the beam which was perturbed was initially so centred that it passed through the entrance to the deflector channel, and the slot of the septum, and then back into the dee region inside the splitter. This explanation of the effect is not favoured. In any case it was considered that beam which could not be extracted with an unslotted septum was unlikely to be extracted very satisfactorily at all. Whether a return to some form of slotted splitter will be necessary for extraction of large beams is still a matter of conjecture.

The extraction achieved with the two slits is shown in Figure VI-20, for various slit settings. It will be seen that
FIGURE VI-19. EFFECT OF DEFLECTOR ON PRECESSION.
Figure VI-20. Beam Accelerated and Extracted.
the best ratio of extracted to accelerated beam occurs for the (v) slit position. Just how much this is due to using this slit position when adjusting the magnetic field for extraction is not known, but the orbit centre for the (v) slit position (Figure VI-18) appears to be the best position of centres for extraction. With the slits in positions (iii) and (iv) it was difficult to hold the deflector voltage, due to overloading. Persistent operation in one of these positions caused the leading edge of the septum to melt. If the extracted and accelerated beam values, for the points in Figure VI-20, are separately added it will be seen that for a total acceleration of about one and three quarter mA, some seven hundred micro-A of beam was extracted. Had a single slit, which covered the range of the five slit positions, been used it is unlikely that extraction would have been satisfactory, because of the behaviour of the beam passing through slits in the (ii) and (iii) positions. However, a single slit, covering slit positions (v) and (vi), with perhaps part of (iv), could well be expected to yield an extracted beam of some 500 micro-A. for a total accelerated beam of about 900 micro-A. Further increases in beam could be achieved by increasing the ion source output, or decreasing the ion loss at small radii. This
suggests that the production of an external beam of about a m.A., for injection, is within the capabilities of the machine. It is well to remember, however, that the opening of internal slits could be accompanied by an increase in the horizontal divergence of the external beam.

After the extraction discussed immediately above, attention was turned to the ion source region of the machine. The copper feelers on the dee were replaced by a carbon plate, with a slit in it, through which the ions pass after leaving the ion source. The electric field, between the parallel flat faces of the ion source and the carbon plate, is rendered relatively uniform. Thus the general vertical divergence of the field, from the ion source to the dee, is removed. However, the lens formed by the hole in the slit is a defocusing one, as is shown in elementary texts. This is probably an advantage, if the slit in the carbon extraction plate is correctly situated with respect to the ion source, as can be seen from a consideration of the behaviour of the arc plasma.

When the ion source is running, but no R.F. is applied, the plasma can be seen, bulging from the extraction slot of

the ion source. When the R.F. is applied, the beam emerging from the ion source shows the influence of strong focusing forces. As it leaves the ion source, the beam is convergent, as seen from the side (see Figure VI-21), and appears to have a vertical cross-over, within a cm. or so from the ion source. This suggests that the surface of the plasma, from which the ions are escaping, has the concave, approximately hemispherical shape discussed, for example, by Thonemann. However, with the arc source used in the cyclotron, the plasma is restricted to a narrow vertical pencil, except at the exit slot, and is not as extensive as in the types of sources where the plasma has been studied. The result of this, when ions are drawn, it is believed, is that the arc is practically extinguished above the exit slot, and by far the greater part of the beam leaves from the lower part of the slot. This view is based on visual observations, described below.

When the ion source cap is loose (see Figure VI-21) the brightness of the arc can be seen between the cap and the ion source body, while the R.F. is not on. Once the R.F. is on, however, the brightness almost entirely disappears. The ionization of residual gases, by the beam, which provides a

Brightness of arc is visible only when R.F. not applied.

Carbon Plate on Dee

Median Plane

Apparent Cut-off of Arc

Arc

Filament

A: Direction of Beam entering Dee when ion source slot is symmetrical about median plane (ie. opposite DE)

B: Direction of Beam entering Dee when Ion Source lowered with respect to Carbon Plate DE.

FIGURE VI-21. Beam from Ion Source.
means of observing it on its first half revolution, shows the lower part of the beam from the ion source to be much more intense than that from the upper part. With the slit used on the dee it was found that the ion source had to be lowered to the position shown in the figure (VI-21), in order to keep the most intense part of the beam approximately horizontal on its first half revolution. A slit of smaller vertical extent might avoid the necessity to do this.

If it becomes desirable to operate the ion source without the "pinch-off" of the plasma reducing the beam, and making it asymmetrical, a hood with a filament at each end of the arc channel could be used. It would not be difficult to fit such an ion source in the Canberra cyclotron.

The main purpose of the slit type feeler on the dee is, of course, to increase the electric field strength near the ion source in order to extract more particles from it. As well as doing this the reduction in the extent of the electric field has profound effects on the subsequent ion orbits, as mentioned in Chapter IV. Phase bunching almost entirely disappears, while a radial bunching of orbits for particles of different phases is brought about.
For the purpose of discussion here, we repeat the relevant equations from Chapter IV (IV-9, -10, -11) in a slightly different form. Expressing the R.F. phase as \( \psi = wt + \phi \), the equations become

\[
xx_d/k = (\psi - \phi) \sin \theta - \sin(\psi - \phi) \sin \phi \quad \text{VI-1.}
\]

\[
x_d y_c/k = 2(\sin \psi - \sin \phi) \quad \text{VI-2.}
\]

and

\[
-x_d(y - y_c) = (\psi - \phi) \cos \theta + \sin(\psi - \phi) \cos \phi \quad \text{VI-3.}
\]

where \( x_d \) is the extent of the uniform electric field, and \( k = eV_c/2\pi m \). For the first acceleration, from the ion source, \( x_d \) is very nearly the distance of the carbon extraction plate from the ion source.

Graphical solutions of the above equations, covering the useful values of the variables, are given in Figures VI-22, VI-23, and VI-24. The values of \( \psi \) and \( \phi \), corresponding to a selected value of \( x \), are obtained from the curves of Figure VI-22. The corresponding values \( y_c \) and \( y_c - y \) can then be read off Figures VI-23, and VI-24, in terms of the \( \psi \) and \( \phi \) values. The \( y_c \) values pertaining to \( x = x_d \) are the centres about which the ions rotate in the field free region, preceding their first full crossing of the dee gap. From the
Figure VI-22: Graphical Solution for $xx_d/k$

\[
\frac{xx_d}{k} = (\psi - \phi) \sin \psi - \sin(\psi - \phi) \sin \phi
\]
Figure VI-23:
Graphical Solution for $x dy_e / dx$

\[
\frac{x}{y} \frac{dy_e}{dx} = 2 (\sin \psi - \sin \phi)
\]

Lines of constant $x/d$ shown dashed.

With values in brackets: ( )
FIGURE W-24. Graphical Solution for $\frac{x}{y}(y^2-x^2)$.

Lines of constant $\frac{x}{y} x$ shown dashed with values in brackets. ( )
(y_c - y), and \( x = x_d \) values, the radius of the field free region orbit, for various initial phase angles, can be determined. This gives, as well, a measure of the particle energy at this stage.

Even when the extent of the R.F. field is considerable, for the ions' first full crossing of the dee gap, phase bunching is negligible, unlikely to amount to more than a few degrees. In view of this situation, and of the fact that the electric field shape, and extent, are unlikely to be known with any accuracy, it is reasonable to take the energy gain by the particle at this crossing, and subsequent ones, as \( eV_0 \cos \phi \), where \( \phi \) is the R.F. phase at the instant the ion crosses the centre line of the gap, assuming a constant particle angular frequency, after the point where \( x = x_d \). For practical values of \( x_d \) it is found that \( \omega t \) at \( x = x_d \) is approximately twice the angle that the orbit radius makes with the line through the ion source, parallel to the dee edge. This means that there is a backward phase shift (i.e. to later R.F. phase) of about \( \omega t/2 \), on the first half revolution, which tends to offset the general forward movement in phase due to the difference between the resonant frequency, \( \omega \), and the electric field frequency, \( \Theta \).
Figure VI-25 shows the arrangement of the ion source and the extraction plate and slit on the dee. The orbits plotted are calculated for a value of \( x_d = 1 \text{ cm} \), and a dee potential of 80 K.V.

The reversal of the distance of orbit centres, along the machine radius, with changing start phases, shows up very clearly when defining slits are used. The sharp cut off on the outside of the beam cross section, at radii 225° and 180° illustrates the reversal effect. This is shown in Figure VI-26, in contrast with the 225° radius cross section, without the carbon plate, as in Figure VI-8.

It will be noticed, from Figures VI-7, VI-10 and VI-11, that the greatest loss of beam occurs during the acceleration to a radius of a few inches. This is due to the absence of any magnetic focusing in the region where the accelerating gap is acting as a defocusing lens, as discussed in Chapter IV (see Equation IV-17). Cohen\(^3\) has illustrated a type of exponential increase in the vertical extent of the beam over this region.

To overcome some of the defocusing, and resulting loss

---

$\phi$ for orbits shown in order of increasing radius for solid curves, decreasing radius for dashed curves, at $270^\circ$:
- $68^\circ$, $52^\circ$, $40^\circ$, $28^\circ$
- $17\frac{1}{2}^\circ$, $7^\circ$, $2^\circ$, $11\frac{1}{2}^\circ$, $20^\circ$

(Some calculated first revolution orbits shown)
(for $x=x_d=1.05\text{cm}$)

**FIGURE VI-25. RELATIVE POSITIONS OF ION SOURCE AND ION EXTRACTION PLATE.**
FIGURE VI-26. CROSS SECTIONS OF BEAM INTENSITY.

(Ion Extraction Plate 1 cm from Ion Source)
of beam, in the Canberra cyclotron, a system of slits was fitted to the dummy dee, and dee, on the down beam sides of the gap. These slits remove the defocusing fields in the latter half of the gap crossing, much as focusing grids do in the linear accelerator. In the case of the cyclotron, however, they can be arranged so that they do not intercept beam.

With the well defined orbits on the early beam crossings of the gap from the dee to the dummy dee, it was possible to form the dummy dee slits by means of carbon rods, which were independently adjustable along the dummy dee edge. After the first crossing of the gap, into the dee, on the Eastern side of the ion source, the orbits tended to overlap. A carbon focusing plate was used this side. It was adjustable along the dee edge. The plate was positioned so that the beam entering the dee, at the end of its first revolution, passed between the inner edge of the focusing plate, and the Eastern edge of the carbon extractor plate facing the ion source. The position of the beam striking the focusing plate, on its second revolution, was noted, and carbon cut away to make a hole through which the beam would pass. Then the position of the beam striking the plate on the next revolution was noted, and
more carbon removed. By repeating this procedure it was arranged that the beam passed through the plate on the first five revolutions. The beam passed through slits on the Western side of the ion source on the first three revolutions. The position of the Western slits and the Eastern focusing plate are shown in Figure VI-27.

Observations of the beam cross sections were made during tests of the effectiveness of the focusing system, by running a carbon plate in, along the 090° radius, between the top and bottom of the dummy dee, and noting the pattern of the intercepted beam on it. This plate tended to perturb the ion orbits somewhat, and would have a defocusing effect on the beam of the orbit passing close to its inner edge. However, it indicated the general pattern of the beam cross sections.

Before discussing results it is worthwhile to refer to the expression for $p^A$, the change in vertical momentum of an ion as it crosses an accelerating gap (see Chapter II), namely $p^A = -(ez/v)V_o \sin \phi/d$. Remembering that we have taken zero phase for the cyclotron R.F. corresponding to the voltage peak, and writing $p^z$ for the vertical momentum component introduced into the motion of a particle entering
FIGURE VI-27. FOCUSING SLIT SYSTEM. (Looking into Dee from 180°.)
the accelerating gap, parallel to, and distance \( z \) from, the median plane, we have

\[
p_z = -(z e/\gamma) V_0 \cos \phi / d.
\]

This equation expresses the vertical momentum component due to an accelerating gap with focusing slits, since there is no vertical field component at the second electrode. If the focal length of the lens so formed is \( f \), then \( z/f = -p_z/p \), or, substituting \( p = H e r/c \), and using the above expression for \( p_z \),

\[
z/f = (e V_0 \cos \phi / w H d) (z/r^2).
\]

Thus \( f = w H r^2 d / e V_0 \cos \phi \) .................................................. VI-4.

Taking the dee gap, in our case, as 5 cm., we have the lens focal length approximately equal to 0.88r^2, or 0.28rD, where \( D \) is the distance between successive gap crossings, or the lens separation. This means, with \( r \) of the order of three to four cm. on the first revolution, that the first Eastern slit is almost at the focal point of the first Western one.

The results of measurements made with, and without, the focusing slits are shown in Figures VI-28, and VI-29, while
Curve Beam at 1.75" Focusing Slits

A 2.4 mA East and West.
B 3.0 mA West only East dee aperture as for A.
C 3.0 mA " E. aperture increased by ½" for 3½"
D 2.8 mA none.
E 3.1 mA none with feelers on Dummy Dee.

FIGURE VI-28. EFFECT OF FOCUSING SLITS WITHOUT DEFINING SLIT.
**FIGURE VI-29. EFFECT OF FOCUSING SLITS WITH DEFINING SLIT.**
sketches of the beam cross sections, at 090°, appear in Figure VI-30. It was unfortunate that the ion source filament had to be changed after the measurements made with the slits fitted, as this meant that the subsequent measurements, without slits, were not made under identical ion source conditions. However it was felt that they were sufficiently unvaried to introduce no discrepancies into the comparisons made.

Figure VI-28 shows plots of the beam against radius, when no defining slit was used. The loss of beam shown by curve B, in comparison with curve C, between five and six inch radii, was due to the interception of it by the mounting for the Eastern plate, which was left on the dee to preserve the same aperture as was used when all slits were in place. The beam melted the copper mounting on the lower dee, whereas, with the focusing plate in, it had given no indication of striking it at all. A comparison of the curves A and C, with D, shows a threefold improvement in the fraction of surviving beam, with one set of slits only (Western set), and a fivefold improvement with both sets, over the fraction which survives without focusing slits. The slits gave a ninefold
**FIGURE VI-30. BEAM CROSS SECTION AT 090°.**

A and B - with East and West focusing slits.

C - with West slits only.

D - with no slits.

E - with no slits (feeters on dummy dee)  

F - with no slits  

High Ion Source Arc.
improvement over the situation with feelers on the Western side dummy dee.

Figure VI-29 shows results of measurements made with a half m.m. wide defining slit, at 225°, on the first revolution. The focusing slits appear to give the same sort of improvement as without the defining slit. Curves F and G are equivalent to D and E except that the ion source output was increased considerably. The readings taken with feelers on the dummy dee would only include beam which had passed through the defining slit, whereas, without feelers, readings would include some beam which had passed above, and below the slit. This would tend to give a higher survival fraction in the former case.

In Figure VI-30 the pattern in A was sketched from the beam intercepts on the focusing plate while it was being observed prior to carbon removals. The other patterns were obtained by using the 090° carbon plate. It was necessary to increase the output of the ion source to obtain much beyond the second revolution pattern when operating without the focusing slits. No defining slit was used when observing the beam cross sections.
As internal defining slits were used to stop down the beam, the external measurements made on it would present only a restricted picture of what could be expected with a more intense beam. For the measurements two sets of external slits were used. The first set was just outside the vacuum box, and the second some five feet away, clear of the magnetic fringing field. Each set of slits consisted of two vertically, and two horizontally moving gates, which could be adjusted to provide horizontal and vertical slits of variable position, and opening.

The variation of beam intensity, perpendicular to its axis, as well as the extent of the beam, vertically and horizontally, was determined by measuring the beam to the first set of gates, as their position was varied. The results of these measurements are shown in Figure VI-31. These indicate a beam height, at this point, of a quarter of an inch, with a horizontal spread of three quarters of an inch. The beam intensity appears to be uniform over its vertical extent. The horizontal variation shows the peak intensity to be about one third of the beam width from its inner edge.

The divergence of the beam was found by setting the first
**FIGURE VI-31.** BEAM AT FIRST EXTERNAL SLIT SYSTEM.
First External Slit open 0.050".

The 0.050" translation of curves showing horizontal variation corresponds to change in position of first external slit.

FIGURE VI-32. Beam at Second External Slits.
external slits, and measuring the beam to the second slit
gates, as they were given various settings. The results
obtained, with a first slit aperture of 0.050 inch, are
shown in Figure VI-32. The translation of the second slits
is not shown. The beam appears to diverge, vertically, from
a point 7.5 inches on the machine side of the first slits,
with a total divergence of 0.4°, and horizontally from a
point 6 inches behind the first slits, with a divergence of
0.5°.

The horizontal variation of beam intensity at the second
external slit system (Figure VI-32) suggests the possibility
of a double beam. This could be due to the splitter peeling
off part of the beam on each of two revolutions, as the orbit
centres process. No attempt was made to correct this
situation, as with the narrow slits used for experimental
purposes (see Appendix IV) only one peak of the external
beam would be involved.

When being used as an injector the cyclotron will be
pulse operated. Some consideration will be given to this
matter when the machine has been resited. At present it is
thought that the ion source will be pulsed, with "on" periods of the order of a second or so, while the actual period of delivery of beam to the synchrotron will be controlled by a deflector system between the two machines.

As the presentation of the material in this chapter has followed the chronological order of development of the cyclotron's operation, it has not been possible to divide the subject matter into completely separate sections. This will now be done in a summarized form.

(a). Operation with Unrestricted Beam.

With a first revolution beam of the order of 20 m\(\cdot\)A., some 3 m\(\cdot\)A. of protons have been accelerated to full radius. Extraction with a large unrestricted beam is not satisfactory due to beam striking the splitter.

(b). Internal Beam Defining Slits.

When the beam is restricted on one, two, or three of its first revolutions, measurements of the orbit centres (see Appendix II) are simplified, and shimming for extraction made easier. Extraction is more efficient, varying from 50 to nearly 100 per cent (see Appendix IV) of the accelerated
beam, depending on the size and number of slits used. A further advantage of defining slits is that the R.F. is not unduly loaded by beam which is unlikely to be extracted.

(c). Ion Extraction Plate on Dee facing Ion Source.

When this is used the pattern of the first revolution orbit is completely changed. Phase bunching is replaced by a radial (and energy) bunching of beam. This makes it possible for more beam to pass through narrow defining slits on early revolutions than could otherwise be so.

(d). Beam Focusing Slits.

With the cyclotron it is possible to fit focusing slits, or plates, to the second electrode of the accelerating gap without intercepting beam. These increase the fraction of beam which survives to regions where magnetic focusing prevents loss of particles. Measurements which have been made show that the beam intensity at larger radii is considerably increased by this means.

(e). Conclusion.

With the combination of ion extraction plate, focusing slits, and beam defining slits, as well as careful measurements of orbit centres, and proper shimming for extraction
it is believed that a useful external beam of 1 m•A• can be obtained. When focused by quadrupole lenses it should be possible to deliver this beam to the synchrotron in a satisfactory manner.

By using very narrow defining slits, of a m.m., and less, the background radiation from the cyclotron can be reduced to a level where it becomes much less of a drawback to experimental work.
APPENDIX I.

MEASUREMENT OF THE MAGNETIC FIELD.

Using Nuclear Resonance.

The principle of nuclear magnetic induction has been treated by, for example, Bloch\(^1\). He shows that nuclei which are subjected to a strong magnetic field $H_0$, in the z-direction, can have a forced precession, about $H_0$, impressed on them by the application of a small oscillating field, in the x-direction, equal to $H'\cos wt$. This is equivalent to two fields, each of magnitude $H'/2$, rotating in opposite directions. The direction of precession will coincide with the direction of rotation of one of the fields. If the angular frequencies, $w$ and $w_L$, also coincide, where $w_L$ is the Larmor precession frequency for nuclei in the field $H_0$, resonance exists, and an energy transfer is possible between the radio-frequency field $H'$ and

and the nuclear spin system.

In measuring the magnetic field, $H_0$, of the cyclotron, this field was varied through the resonance value by the addition of a small 50 cycle "wobbling" field resulting from the application of an A.C. voltage to a coil, with its axis parallel to $H_0$, wound around the sample. The apparatus used is shown in Figure AI-1. The R.F. field $H'$ is provided by a R.F. voltage, from the tuned oscillator, applied across the coil, with axis in the $x$-direction, immediately about the sample. At resonance the energy fed into the nuclear spin system, and finally from this into the molecular thermal motion, produces an apparent change of impedance in the circuit used to excite $H'$. This means a change in the signal appearing at the plate of the EA50, for detection and subsequent amplification. For a relatively large signal it is essential to have sufficient coupling between the spins and the thermal motions of the liquid molecules. The addition of ferric chloride to the water sample provided this, by virtue of the high magnetic moment of the ferric ions. The slight broadening of the resonance peak was not of consequence in our case.

In determining the value of $H_0$, the frequency of the
FIGURE AI-1. CIRCUIT FOR MEASURING FIELD, USING NUCLEAR RESONANCE.
oscillator was varied till \( w = w_L \), at which stage the variation in signal was detected visually on the C.R.O. At resonance the value of the field is given in terms of the frequency and the gyromagnetic ratio of the proton as

\[
H_0 = \frac{w}{\gamma} \quad (\gamma \text{ was taken as } 2.6753 \times 10^4)^2.
\]

The accuracy of field measurements depended mainly on the accuracy to which the frequency could be measured, except where the magnetic field was non-uniform over the area of the sample, as near the edge of the pole tips. In this case the nuclear resonance signal became very broad and difficult to set on with any accuracy. The oscillator frequency was measured with a Bendix frequency meter.

**Using Flip Coils.**

Two identical coils were mounted with their axes parallel to the direction of the magnetic field, so that they could be rotated through 180° about an axis perpendicular to the field, and passing through each coil. The apparatus is shown diagrammatically in Figure AII-2. If the coils, 1 and 2, connected in series opposition, are in fields of strength \( H_1 \)

---

Figure AI-2. Diagram of Flip Coil Circuit.
and $H_2$ respectively, the charge which will flow through the circuit containing the galvonometer, resistance $R$, and the secondary of the mutual inductance $M$, as the coils are yurned through 180° is

$$\left(1/R\right)\int_{0}^{N} A(N - H_2) \cos \theta d\theta - \left(1/R\right)\int_{0}^{0} LdI = 2(N - H_2)AN/R,$$

where $A$ is the cross sectional area of each coil and $N$ is the number of turns on each coil.

The charge induced in the coil circuit by a reversal of current $i$ through the primary of the mutual inductance is

$$\left(1/R\right)\int_{-i}^{i} Mdi - \left(1/R\right)\int_{0}^{0} LdI = 2Mi/R.$$

By adjusting $i$ so that a null reading is obtained on the galvanometer the relation $(H_1 - H_2) = Mi/NA$ holds.\(^X\)

With the coils used, $N = 1,000$ turns, $A = 0.458$ cm\(^2\), $M = 10$ mH., $i$ was about 0.46 mA. per gauss difference between $H_1$ and $H_2$.

To ensure that the coils were nearly identical they were positioned in the field and the primary current $i$ noted for a null galvanometer reading when the coils were rotated. The positions of the coils were interchanged and a second reading \(^X\). This method has been reported\(^3\) as a means of measuring absolute field values, employing only one coil.

3. C.T. Lane, J. Sc. Instrum., 5, 214 (1928)
of \( i \) taken. If the two readings were not the same, turns of winding were appropriately added to or subtracted from one coil. This procedure was repeated until interchange of coil positions gave no change in \( i \) for a zero reading on the galvanometer. This still left the possibility that a difference in the number of turns on each coil was used to balance a lack of parallelism between the coil axis. However, numerical checks showed that for a difference in turn number of up to half a dozen turns, accompanied by an angle between coil axes of \( 0.2^\circ \) would lead to negligible errors.

Both of the field sampling systems were mounted so that their positions in the magnet gap could be varied radially and in azimuth. Whereas the nuclear resonance method was useful for obtaining absolute field values, the flip coils were more satisfactory for measuring the difference between field strengths of different points. Apart from the relative simplicity of the latter system, it had the advantage that any small drift in the absolute field strength did not affect the difference values obtained. Measurements within the gap, and inside the region of large rate of field decrease with radius, were considered to be accurate to about one in ten thousand.
APPENDIX II.

THE MEASUREMENT OF ORBIT CENTRES.

If extraction is to be efficient, and the extracted beam mono-energetic, and not made up of particles extracted over several cycles, it necessary to know, and control, the position and precession of the orbit centres. In this respect a single dee machine has an advantage in that there is ample space available for using probe targets to investigate the behaviour of the beam orbits.

Ideally four targets are used. One is left in a predetermined position, which need not be known with any great accuracy, while the others, one at a time, are moved in till half the total beam, initially measured on the fixed target, has been intercepted. In this way three points on the mean orbit are obtained, defining the orbit and its centre. If only three targets are used, the position of the fixed one, with a correction for beam width, is required as a defining point.
When the accelerated beam is confined to particles whose orbit centres cover a very small area, and orbit precession is negligible, the interpretation of measurements made using probe targets is straightforward and simple. However, when the orbit centres cover an area whose dimensions are considerably greater than the turn to turn increase in orbit radius, and precession moves the centres at a rate comparable with the increase in beam radius per revolution, the interpretation of measurements is less straightforward. The greater part of the discussion in this appendix is devoted to effects of spread of orbit centres and precession on measurements made with probe targets.

Initially we shall consider the case where precession is absent or negligible. It will also be assumed that the radii at which measurements are being made are much greater than the dimensions of the orbit centre area, and much greater than the distance of the orbit centres from the machine centre. In Figure AII-1 let $T_S$ be the fixed target, or "stop target", and $T_q$ a target which moves along a radius making an angle $\theta$ with the axis of $T_S$. The orbit centres are regarded as being distributed over the area extending from $X$ to $X'$ along the $T_S$ axis and from $Y$ to $Y'$ at right angles to
FIGURE AII-1. ARRANGEMENT OF TARGETS FOR MEASURING ORBIT CENTRES.
the $T_S$ axis. Concerning the interception of beam by target $T_S$ it is well to note the following. If $XT_S$ is equal to $R$, and if particles centred on $X$ are regarded, for simplicity, as just missing $T_S$ on one revolution, then on the next revolution, after an acceleration giving a radius increase of $\Delta R$, those particles centred on the strip $XX_1$, of width $\Delta R$, will be stopped by the target $T_S$. On the next revolution the particles centred on the strip $X_2X_2$ will strike $T_S$, and so on, till particles centred on the strip $X_n-1X_n$, which includes $X'$ are stopped.

Concerning the interception of beam by the target $T_O$, let us consider first of all those particles centred on the line $XX'$. As $T_O$ is moved in, the first particles intercepted will be those centred on the $X_{n-1}X_n$ section. These will have an orbit radius of $R+n\Delta R$. If $\Theta$ is less than $90^\circ$ the first of the particles centred on $X_{n-1}X_n$ to be intercepted will be those revolving about the centre $X_{n-1}$, whereas if $\Theta$ is greater than $90^\circ$ the first intercepted will be those centred on $X'$. After its first stopping of particles centred on the strip $X_{n-1}X_n$ the target $T_O$ will have to be moved inwards a distance equal to $\Delta R(1 - \cos \Theta)$ before it will stop any particles centred on $X_{n-2}X_{n-1}$. In order to intercept the total beam
centred on XX' it will be necessary to move $T_\theta$ a distance of
$$\Delta R \left[ (n-1)(1-\cos \theta) + |\cos \theta| \right].$$

Extension of the area of orbit centres perpendicular to XX' increases the distance $T_\theta$ has to be moved to cover the circulating beam. If, for example, strip $X_{n-1}X'$ is rectangular and measures $y_n y'_n$ at right angles to XX', target $T_\theta$ will move a distance equal to $y_n y'_n \sin \theta + |\Delta R| \cos \theta$, to cover all of the beam centred on this area. Of course, if $y_n y'_n$ is considerably larger than $\Delta R$, beam from other strips will be intercepted before all of strip $X_{n-1}X'$ has been stopped. It will be noted that, providing the shape of the area covered by the orbit centres is not greatly different from rectangular, of dimensions XX' by YY', the total distance it will be necessary to move $T_\theta$ is
$$(XX'-\Delta R)(1-\cos \theta) + |\Delta R\cos \theta| + YY'\sin \theta.$$  

Referring to a distribution of centres only on the line XX' again, it is well to note that although those particles centred on the interval $X_{n-1}X'$ are first intercepted by $T_\theta$ when they have a radius of $R+n\Delta R$, they will be intercepted at progressively earlier stages of their acceleration, as $T_\theta$ is moved inwards. Finally, when the whole beam has been just
stopped by this target, the radius of the $X_{n-1}X'$ particles will have been reduced by approximately the amount $T_\Theta$ has moved. For the more real case when the centres spread also in the $Y$ direction, it is unlikely that all of the orbits centred on one strip will have the same radius when intercepted by $T_\Theta$. For example, when the $X_{n-1}X'$ strip has a second dimension of $y_n y'_n$, and $T_\Theta$ is moved to stop all the beam centred on the strip, the radius of the first particles to be intercepted will have been reduced to approximately $R + \Delta R = y_n y'_n \sin \Theta$.

When the spread of orbit centres being measured is considerable there will be more than a negligible reduction in the energy delivered to the probe target by particles as they are intercepted at earlier stages of their acceleration. If, accompanying the acceleration, there is a loss of particles from the beam, the current to the probe target, as its radius is decreased, will increase at a greater rate than is warranted by the interception of beam to the stop target. By using measurements of beam power and current to the stop target, rather than the probe target, these effects are avoided. At the same time it should be remembered that, for a large area of orbit centres, particles reaching the stop target, $T_\Theta$, will
Figure AII-2. Interception of beam by probe. ($\theta$=45°, $\theta$=90°)
Figure AII-3. Interception of beam by probe 
\( (\theta = 135^\circ, \Theta = 180^\circ) \)
Thus for $\theta = 180^\circ$, the distance $T_0$ must move for total interception is equal to $(2n - 1)\Delta R$, i.e. twice the distance covered by the centres in the axial direction, less one turn to turn beam radius increase.

At large radii where precession is likely to be rapid, and $\Delta R$ small, measurements of orbit centres are apt to be very approximate, unless a small portion of the beam only has been selected, during the early stages of acceleration. Measurements under such conditions have been successfully made, as discussed in Chapter VI. Rather than attempt any sort of systematic treatment of the effects of precession on measurements, we shall make some general remarks on the phenomenon now, and refer to it again when presenting examples of actual measurements made.

If the precession is in the direction of the stop target its effect is to cause an apparent change in the value of $\Delta R$. Taking the linear movement of the centres towards $T_S$, per revolution, as $p_S$, we may replace $\Delta R$ by $\Delta \rho = \Delta R + p_S$. Should $p_S$ be negative and greater than $\Delta R$ the beam cannot be intercepted by $T_S$ until the value of $p_S$ changes. An example of these conditions prevailing will be given later.
Any component of precession perpendicular to the axis of $T_S$ might be regarded as modifying the value of $YY'$. However, it is well to note that the direction, and rate, of precession will be different for different parts of the orbit centre area. Thus it is only under conditions of restricted acceleration, i.e. with only part of the full beam being accelerated, that an intelligent guess is likely to be successful in interpreting the precession effect on measurement.

At the same time it is safe to assume that any otherwise unaccountable large movement of a probe target to remove completely the beam from the stop target is due to precession, with a large component in the direction of the probe axis.

From the foregoing it is obvious that the measurement of orbit centres is not a simple geometrical operation. In making such measurements one must be prepared to take into account the following:

(i). The extent of area covered by orbit centres, and possible variations in density of centres.

(ii). The turn to turn increase of orbit radius.

(iii). The precession of orbit centres per cycle.

(iv). The different acceleration ages of particles.
If all of the available beam is accelerated these factors can contribute to large errors in the determination of the position and motion of orbit centres. By restricting the beam, with defining slits at small radii, the spread of the orbit centre area is reduced, as is the variation in energy of particles which left the ion source during the same cycle. Under these circumstances the effects of the above factors can be allowed for with relative accuracy.

In making measurements, the results of some of which will now be discussed, the methods used for detecting the beam, as well as the positions of the targets, were not always ideal. Although there were ports capable of taking targets at $090^\circ$, $135^\circ$, $180^\circ$, $225^\circ$, and $257^\circ$, their availability was dependent on other current requirements. The $090^\circ$ target had to be displaced with respect to the actual $090^\circ$ radius inorder to give clearance to the dee.

In Figure AII-4(a) the ratio of the beam reaching target $090^\circ$, as stop, to the total beam, i.e. $(t-1)/t$, is plotted against the radius of target $257^\circ$, as a probe. Hence $\Theta = 167^\circ$. The plot looks, at first sight, as a near perfect example of a typical $T_\Theta = 180^\circ$ case. However the dee voltage at the X. Zero degrees is taken in the direction of the dee line axis.
Measurements of Full Beam and Restricted Beam.

**Figure AII-4.**
time the measurement was made, was only 36 kV., so that $\Delta R$ would not be more than 0.072 inch, at a radius of about 10 inches, and the 0.3 inch steps in the curve would require a precession component towards the stop target of about 1/4 inch per cycle. Calculation of the orbit centres showed a precession, which while being in the right direction, was almost an order too small. It is with some reluctance that one is forced to account for the attractive curve by assuming that the distinctive features are due to fluctuations in the total beam intensity.

The next example, which appears to be valid, is taken from measurements for which the beam was restricted by a 1/4 inch slit on the first revolution. It is shown in Figure AII-4(b). The steps in the curve of beam power to $T_{60}$ plotted against radius of $T_6$ appear consistent with a $\Delta R$ value of about 0.1 inch. Precession was towards the stop target.

Figure AII-5 shows plots which are typical of a series obtained, using unrestricted beam, with the 090° target as stop, and with probe targets at 180° and 257° (with respect to the machine). The interesting features are the relationships between the curves indicating power to the probes, and
Solid curves: Current to $T_o$.

Dashed curves: Power to $T_o$.

FIGURE AII-5. MEASUREMENTS OF BEAM CURRENT AND POWER.
those indicating current. For $T_\theta=90^\circ$ the ratio of power to current is greater for greater radii of $T_\theta$, as would be expected. However, in the case of $T_\theta=167^\circ$ the ratio decreases for larger radii. The reason for this was found to be due to the presence of low energy ions which had apparently migrated along the dee edge in the $270^\circ$ direction. At no time has there been evidence of electron emission from targets, which would of course affect results.

Figure AII-6 shows results of a measurement made at the $135^\circ$ target, using a block of copper, in the dee opposite the target, as a stop, giving $T_\theta=180^\circ$. Taking the extrapolated values of cut-off for beam to $T_\theta$, we have $2XX'-\Delta R = 0.5$ inch. With $\Delta R$ not greater than 0.2 inch this indicates the extent of the area of orbit centres in the $XX'$ direction to be not greater than 0.35 inch.

Finally, Figure AII-7 shows a beautiful example of a case where the precession component away from the stop target becomes greater than the turn to turn increase in orbit radius. The curves are for power measured to the stop target, as a function of the radius of $T_\theta$. Below the intercept curves is shown the path of the orbit centres, illustrating how the stop intercepted the beam at two different occasions: firstly as the centres move away from it, and later towards it.
FIGURE AII-6. MEASUREMENTS WITH PROBE AT 180° TO STOP.
FIGURE AII-7. MEASUREMENTS SHOWING EFFECT OF PRECESSION.
APPENDIX III.

QUADRUPOLE FOCUSING LENSES.

The principle of quadrupole focusing\(^1,2,3,4\) is much the same as that of the strong focusing employed in A.G. synchrotrons. However, as the beam passes through each sector of the lens system only once, in the case of quadrupole focusing, the theory is simpler in this case, and combinations of lenses are amenable to the type of treatment that is used in optics. For a single lens, if focusing is achieved in one plane, defocusing must be tolerated in a plane perpendicular to it. By rotating the second lens of a pair through ninety degrees about the lens axis, with respect to the first, overall focusing can be achieved in both planes, providing certain

---

geometrical requirements are met. In what follows we shall regard the focusing forces as varying linearly with the particles' distance from the central beam axis. This is convenient, and conforms with practice.

Considering the beam as travelling along the z-axis of a Cartesian co-ordinate system, with the requirements that there should be focusing in the z-x plane, and defocusing in the z-y plane, we have the focusing force F, to which a particle is subjected, given by \( F_x = kx \), and \( F_y = -ky \), where \( k \) is a constant depending on geometry, and field values used.

If an electric focusing field is used, we have
\[
\frac{e\partial V}{\partial x} = kx, \quad \text{and} \quad \frac{e\partial V}{\partial y} = -ky,
\]
where \( e \) is the particle charge, and \( V \) is the electric potential. For both field and potential to be zero on the z-axis we have \( V = (k/2e)(x^2 - y^2) \). Thus the equipotentials are hyperbolae, with axes coinciding with the \( x \) and \( y \) axes, and the required field shape is given by hyperbolic electrodes, as in Figure AIII-1.

In the case where a magnetic field is used, we have
\[
\frac{e\nu \times H_y}{c} = kx, \quad \text{and} \quad \frac{e\nu \times H_x}{c} = -ky \quad \text{AIII-1.}
\]
or
\[
\partial V/\partial x = (c/\nu)ky, \quad \text{and} \quad \partial V/\partial y = (c/\nu)kx.
\]
FIGURE AIII-1    ELECTRODES FOR ELECTROSTATIC LENS.

FIGURE AIII-2.    POLE TIPS FOR MAGNETIC LENS.
In the above relations $H$ is in gauss and $e$ is in $e\cdot s\cdot u$.
If $V = H = 0$ on the $z$-axis the relations are satisfied by
\[ V^2 = \left( \frac{ck}{ev} \right)^2 x^2 y^2. \]
Again the equipotentials are hyperbolae, but in this case they are asymptotic to the $x$ and $y$ axes, and the field requirements are met by pole tips as in Figure AIII-2.

For focusing the external beam of the Canberra cyclotron magnetic lenses were chosen. It was considered that they were preferable to the electric type of lens in that insulation problems would be negligible with them, and there would be no need to take precautions against stray fields from the cyclotron magnet, which, if more than about 100 gauss, might lead to P.I.G. type discharges with electric lenses.

Considering the motion of an ion in the $x$ direction, as it passes through a single lens we have the equation of motion
\[ \frac{md^2x}{dt^2} + kx = 0, \]
or
\[ \frac{d^2x}{dz^2} + K^2 x = 0 \]
..........................AIII-2.
where $K^2 = k/mv^2$, and the sign of $k$, and so $K^2$, will depend on the direction of the $y$ component of the field $H$.

Solutions of Equation AIII-2 are
\[ x = AsinKz + BcosKz, \]
when $K^2$ is positive (focusing).
We shall consider a focusing lens as in Figure AIII-3(a), where the focusing field extends for a distance $s$ along the $z$-axis, and $z$ is measured from the side of the lens facing the oncoming beam. Referring to the figure and evaluating the constants of the focusing equation above, and of its derivative $dx/dz = AK\cos Kz - BK\sin Kz$, we have,

$x = B$ and $dx/dz = 0$, when $z = 0$. Thus $A = 0$, and within the lens $x = B\cos Kz$, and $dx/dz = -BK\sin Kz$, where $B$ is the distance of the approaching particle from the $z$-axis. When $z = s$, we have $dx/dz = -BK\sin Ks = -B/(B - b)/X_p$, and $x = B\cos Ks = b$, where $f$, and $X_p$ are the focal length of the lens, and the distance of the principal plane from the surface $z = s$, respectively, as with the optical equivalent.

Thus $f = 1/(K\sin Ks)$,

\[\text{AIII-3.}\]

and $X_p = (1 - \cos Ks)/K\sin Ks = f(1 - \cos Ks)$

Treating the defocusing lens with reference to Figure AIII-3(b), we obtain

$f = 1/(K\sinh Ks)$

\[\text{AIII-4.}\]

and $X_p = (\cosh Ks - 1)/K\sinh Ks = f(\cosh Ks - 1)$

$K^2$ is evaluated as follows. The field in the direction
(a) Converging Lens.

(b) Diverging Lens.

**Figure AIII-3** Path of ion through magnetic lenses.
of the $r$-axis, Figure AIII-4, and on the axis $r$ is

$$H_r = \frac{(H_x + H_y)}{\sqrt{2}},$$

which, from Equations AIII-1, is equal to

$$(ck/\sqrt{2}\epsilon v)(x + y).$$

Thus $H_r = ckr/\epsilon v$.

Assuming there is no flux leakage the magnetomotive force
along $r$ is

$$4\pi n_i/10 = \int_0^r H_r dr = (ck/\epsilon v)\int_0^r rdr = (ck/\epsilon v)r^2/2.$$

Thus $k = 8\pi n_i\epsilon v/10cr^2$, and $K^2 = 8\pi n_i/10HR$,

where $HR = mv\epsilon/e$ gauss-cm.

For 8 Mev protons from the Canberra machine $HR = 4.1\times10^5$,
and $K^2 = 6.15(ni/r^2)\times10^{-5}$.

It will be noticed that $X_p$, of Equations AIII-3, and
AIII-4, has a limiting value, as $K$ tends to zero, of $s/2$.
With decreasing $K$, $X_p$ (focusing) decreases to $s/2$, while
$X_p$ (defocusing) increases to $s/2$. In either case, for
practical values of $K$ and $s$, no great error is introduced
by taking $X_p = s/2$, i.e. by regarding the focal planes as
being situated at the centre of a single lens element. In
what follows we shall assume that the focal planes of single
lens elements are at the centres of the elements for all values
of $ni/r^2$.

In Figure AIII-5 the focal length of single element lenses,
converging and diverging, is shown as a function of $ni/r^2$, i.e.
FIGURE AIII-4.  TYPICAL MAGNETIC LENS.
\( r \) is \( \frac{1}{2} \) aperture.

**Figure AIII-5.** Focal Length of Single Lens Element.
the ratio of the Ampere-turns associated with each pole to
the square of the half aperture of the lens.

Once the focal lengths and the positions of the focal
planes have been determined for the individual lens elements,
the treatment of combinations of elements is the same as in
optics. The notation used in treating combinations of elements
is shown in Figure AIII-6. In this notation \( f \) indicates
focal lengths of single element lenses, \( F_1 \) and \( F_2 \) refer
to two element lenses, while \( F \), without a subscript, is used
for the focal length of the four element lens system. \( p \) and
\( P \) are similarly used to indicate principal planes: unprimed
letters refer to object space, primed letters to image space.
\( g \) and \( G \) indicate focal planes, with priming and subscripts
similar to those used with \( p \) and \( P \). \( z_1 \) and \( z_2 \) mark the
physical limits of a two element system, \( Z_1 \) and \( Z_2 \) mark
the limits of a four element system. The elements of a system
are numbered consecutively from left to right, it being
assumed that the beam approaches from the left.

In deriving the curves showing focal lengths and the
positions of focal planes, the lens elements are taken as
10 cm. in length, and 10 cm apart, i.e. \( a = p_1p_2 = p_3p_4 = 20 \text{ cm.} \)
Principal Planes: \( p \) and \( P \).
Focal Planes: \( g \) and \( G \).
Focal Lengths: \( f = pg \)
\[ F_1 = P; G_1 \]
\[ F = PG \]

**FIGURE AIII-6.** Four Element Lens System Showing Notation.
This is the value of $a$ used in constructing the lenses for focusing the cyclotron beam. The value of $A = \Phi_1 \Phi_2$, which is used in calculations with a four element lens system, will vary with operating conditions.

Figures AIII-7 and AIII-8 show the variation of focal length with Ampere-turns for a two and a four element system respectively. Figure AIII-9 shows the position of focal planes of a two element system, while the four element case is illustrated in Figure AIII-10.

If $u$ and $v$ are the distances of the object and image respectively, from the focal planes, then $v$ can be determined, for a given $u$ from the relation $uv = F^2$. The final quantities in which we are interested are the distances of the object and image from the physical ends of the lens system, i.e. from $Z_1$ and $Z_2$, for a four element system. If we call the distance of the object from $Z_1$ and the distance of the image from $Z_2$, $m$ and $n$ respectively, we have $m = u - GZ_1$, and $n = v - Z_2G$.

The effect of the lenses on the external beam initially used for the $B^{11}(p, \gamma)\text{Cl}^2$ experiment is shown in Figure AIII-11. The prints were obtained by exposing dye-line paper to the beam immediately after it had passed through an 0.007" aluminium window into the air.
FIGURE AIII-7. Focal Length of Two Element Lens.
FIGURE AIII-8. FOCAL LENGTH OF FOUR ELEMENT LENS.

- Figures on curves give \( \frac{m \cdot e}{T} \) values for 1 and 4 (Amp-turn-cm\(^{-2}\) x 10\(^3\))
- Lens separation = 20 cm.

---

I and 4. focus
- elements 1 and 4 Focus
- elements 1 and 4 Defocus.
- Same focal lengths for 1 and 4 focusing and defocusing.

---
Figures on curves give \( \text{mI} / \text{r}^2 \) values in units of \( 10^3 \text{amp-turns} \cdot \text{cm}^{-2} \).

**FIGURE AIII-9.** Positions of Focal Planes of a Two Element Lens.
FIGURE AIII-10. Positions of Focal Planes for a 4-Element Lens.
In (a) the sides of the beam are cut off by the first external slits, 0.075" wide. Distance from slits to point of exposure = 13.4". Top and bottom of beam cut off by frame of aluminium window, where exposure made.

In (b) \( \frac{n \lambda}{r^2} \) for lens system = \( \frac{1800 \times 2.5 \text{ amp-turn-cm}^{-2}}{(3.8)^2} \) (elements 1 and 4)

and \( \frac{1800 \times 2.5 \text{ amp-turn-cm}^{-2}}{(3.8)^2} \) (elements 2 and 3)

Distance of 1st lens element from slits = 28.5".

FIGURE AIII-11. FOCUSING BY QUADRUPOLE LENS.
APPENDIX IV.

GAMMA RAYS FROM TRANSITIONS TO THE GROUND STATE OF \(^{12}\text{C}\)

IN THE REACTION \(^{11}\text{B}(p, \gamma)^{12}\text{C}\).

The yield of gamma rays from the reaction \(^{11}\text{B}(p, \gamma)^{12}\text{C}\) has been measured by Huus and Day\(^1\) for proton energies up to 2.8 MeV., and for proton energies from 2 MeV. to 5.4 MeV. by Blair, Kington and Willard\(^2\). The reverse reaction, \(^{12}\text{C}(\gamma', p)^{11}\text{B}\), examined by Cohen et alia\(^3\) suggested the shape of the "giant resonance" for photon absorption and implied the possibly of some form of fine structure. It is claimed that fine structure in the giant resonance is responsible for the "breaks" detected in the neutron yield curves following irradiation with betatron bremsstrahlung, for the

reaction $^{12}(\gamma,n)^{11}$, at Saskatchewan. It was considered that an examination of the gamma ray yield from the $^{11}(p,\gamma)^{12}$ reaction, as a function of proton energy up to 7.5 Mev would indicate more directly the shape of the giant resonance than the photon initiated reactions. Having available a large NaI crystal and a multi-channel pulse height analyser made possible resolution of the gamma ray spectrum so that only ground state transition photons would be included in the measured yield. In this appendix the experiment to this end is described.

**Apparatus.**

(a) **Proton source.**

The cyclotron was operated with a 1 m.m. by 1 cm. defining slit on the first beam revolution, followed by half m.m. wide slits to define the beam on its second and third revolutions. The first and third slits were adjusted for maximum full radius beam, then the second slit was introduced and adjusted in steps of 0.002 inch, moving towards larger

4. L. Katz et al., Phys. Rev., 95, 464 (1954), and


* After the last pull-down of the cyclotron the shorting plate had not been re-adjusted for full energy of 8 Mev, this being delayed till "after the move".
radii, from the position where it allowed maximum beam through, until the desired amount of beam was obtained. Under these conditions, and with relatively low power to the ion source arc, it was possible to reduce the stable beam to some 10 to 15 micro-Amps. There was no detectable loss of beam from a radius of about three inches to final radius, although no focusing slits were used on the dees. Except when stopped on an internal target the whole of the beam was extracted.

The first external slits were open 0.075 inch horizontally by 0.100 inch vertically, and stopped about 90 percent of the beam which was extracted. The gates of these slits were metered and in operation the deflector potential was adjusted to maintain equal beam to each of the side gates, which corresponded to maximum beam through the slits.

The quadrupole focusing magnets were adjusted to give the focus shown in Appendix III (Figure AIII-11), a few inches on the down beam side of the position of the boron target.

Beam energy was varied by passing the protons through various numbers of aluminium foils, 1.64 mg·cm\(^{-2}\) thick, just before the target. After passing through the target, kept at 80 to 100 volts positive to prevent the escape of
delta rays, the beam passed into the lead lined catcher which was metered to give readings of instantaneous beam current and integrated beam current. The general arrangement of the apparatus is shown in Figure AIV-1, while Figure AIV-2 shows the experimental region in more detail.

Initially the machine was operated without stopping any beam internally. The magnetic field was set to be consistent with a fixed dee potential and deflector potential. However, as the measured deflector potential was influenced slightly by deflector current, which, though exceedingly small, was more detectable just after start-up, and at times of tank discharge, it was decided to have a tighter method of control. Using the internal target to stop 50 percent of the total beam there was only one operating point consistent with this requirement, and equal beam to the horizontal gates of the first external slits. The effect of variation of the magnetic field when using the internal target is shown in Figure AIV-3.

(b) Gamma Ray Detection.

The gamma rays were detected by a cylindrical NaI crystal (Tl activated), 3.5 inches high by 5 inches in diameter. The crystal, mounted vertically in a steel case, 6 inches high
FIGURE AIV-1. SKETCH OF GENERAL ARRANGEMENT OF APPARATUS.
**PLAN VIEW**

- **NaI(Tl) Crystal**
- **Dumont 6364 Pmt.**
- **From Cathode**
- **Follows output of Pmt.**
- **Non Overload Amplifier**
- **Kicksorter**

**Perspex Windows**
- **Beam Catcher**
- **Target**
- **X** = Foil Rack containing 1, 2, 3, 4, 5, and 6 foils.
- **Y** = Foil Rack containing 7, 14, 21, 28, 35, 42 foils.

**VIEW AA' SIDE ELEVATION OF TARGET TUBE**

**FIGURE A IV-2. SKETCH SHOWING TARGET TUBE AND CRYSTAL.**
FIGURE AIV-3. MACHINE OPERATION FOR CONSTANT BEAM ENERGY.
by 9 inches square, was situated at the same height as the
target, in a direction at 90° to the beam direction, 17 inches
from the target. A five inch Dumont 6364 phototube was
mounted directly on top of the crystal. From the cathode
follower output of the phototube pulses were fed into an
amplifier and thence to the multi-channel pulse height
analyzer. Between the perspex window of the target tube (see
Figure AIV-2) and the steel casing of the crystal shielding
consisted of 8 inches of paraffin, an 0.010 inch sheet of
cadmium, and three inches of lead. The three inch layer of
lead surrounded the steel casing. Between the foil racks and
the crystal three inches of paraffin was replaced by three
inches of lead, immediately next to the target tube. A one
inch collimating hole through steel and lead permitted the
crystal to see the target.

(c) Targets.

Early runs were made using $\text{B}_2\text{O}_3$ targets evaporated
onto 0.02 mg. $\text{cm.}^{-2}$ gold foil. After some experimenting some
carbon crucibles were made which were satisfactory for evaporat-
ing pure boron. The greatest difficulty was in achieving a
proper balance between the electrical resistance of the
crucible and its heat conductivity to the water cooled copper
electrodes at its ends. The target used for most of the experimental runs was put down on the gold foil as a 1.25 inch diameter disc of thickness 0.8 mg. cm.$^{-2}$. Chemical tests on the material which was evaporated to a copper plate holding the foil indicated that only about 50 percent of the target was boron, the remainder being carbon. Comparison between yields from the boron and $\text{B}_2\text{O}_3$ targets confirmed this figure for the fraction of boron.

**Experimental Procedure.**

Runs of six hundred micro-coulombs integrated beam were made for the majority of exposures of the boron target to the proton beam. These usually took 10 to 12 minutes with beams of 0.8 to 1 micro-Amp. Larger beam currents were not used as any further increase in the gamma ray flux tended to produce gain shifts in the phototube.

The pulse height analyser, operated with sixty channels, was biased so that only the leading edge of the 18.86 Mev. (for 7.5 Mev. protons) gamma ray peak due to the $\text{Al}^{27}(p, \gamma)\text{Si}^{28}$ reaction, at the energy attenuating foils, was displayed. As the energy was reduced, with more foils, and the background
increased, there was considerable increase in the gain of the phototube resulting in the background display advancing towards higher channel numbers.

In counting the gamma ray yield all pulses (above the cosmic ray background) beyond the fifteenth channel back from half height of the leading edge of the ground state transition peak were included. This amounted to counting everything above the peak due to transitions via the 4.4 Mev. level of C\textsuperscript{12}. It was found that background runs were not necessary for proton energies greater than about 4.5 Mev. as the background, other than that due to cosmic rays, contributed practically nothing to the energy region included in the count. Typical spectra are shown in Figure AIV-4.

Results.

In Figure AIV-5 the yield of counted gamma rays is shown as a function of proton energy. The gamma ray energies shown are not corrected for target thickness. The open circles indicate the results of three runs using the internal target to maintain constant energy of protons from the cyclotron. The barred circles are results of three runs made without
600 micro-coulombs of Protons.

\[ 600 \text{ micro-coulombs of Protons.} \]

Background

\[ \text{Background} \]

50 Kicksorter channels.

\[ 50 \text{ Kicksorter channels.} \]

\[ E_p \approx 4.28 \text{ Mev spectra displaced to Right so that ground state transition peak under that for } E_p \approx 7.55 \text{ Mev.} \]

\[ E_p \approx 4.28 \text{ Mev, } E_p \approx 7.55 \text{ Mev, } E_p \approx 19.6 \text{ Mev, } E_p \approx 18.4 \text{ Mev, } E_p \approx 22.8 \text{ Mev} \]

\[ \text{Level of Cosmic Ray Background.} \]

\[ \text{Histograms of Typical } \gamma \text{-ray Spectra.} \]

\[ \text{FIGURE AIV-4. Histograms of Typical } \gamma \text{-ray Spectra.} \]
**FIGURE AIV-5. GAMMA RAY YIELD FROM $B^\alpha(p,\gamma)C^{12}$.**

- ○ $50\%$ Beam stopped internally.
- ○ All beam extracted.
- ● Result of single run only.
employing the internal target. The filled circles are the results of a single run with the internal target used. As there is a possibility of error in choosing the fifteenth channel back from the leading edge of the counted peak this has been allowed for in the error limits. Thus the limit signs represent $\frac{1}{2} \sqrt{(n + (n'/2)^2)}$, where n is the total number of pulses counted and n' is the number of pulses contributed by the lowest energy channel included in the count. It will be noticed that apart from where the points fall below a smooth curve at 21.4 and 22 Mev gamma ray energy the deviations in the yield are within the limits of error. At the same time the irregularities at about 20.1 and 20.7 Mev gamma ray energy tended to appear in most of the individual runs over the range. The irregularity at 22.5 Mev showed no consistency with a number of runs.

**Constancy of Cyclotron Beam Energy.**

A check of the constancy of the energy of the beam from the cyclotron was made using the $\text{Al}^{27}(p,\gamma)\text{Si}^{28}$ reaction. The tendency for the $\text{Si}^{28}$ nucleus, when excited to energies up to 15 Mev., to de-excite by cascade through 4 and 7 Mev. levels evidently persists for higher excitation energies. This makes

possible the observation of considerable variations in the gamma ray yield from direct transitions to the ground state, and cascade transitions directly through the 1.78 Mev. level. The largest contribution to the yield is from de-excitation through the 1.78 Mev. level, with the direct ground state transition gamma ray yield showing up as a shoulder to the larger peak. As the proton energy is reduced from 7.39 Mev. to 7.23 Mev. the yield from the observed transitions drops sharply to about half value. Foils were used to reduce the proton energy so that the yield was half way between the maximum and minimum, and its variation was measured for different operating conditions of the cyclotron. Results of these measurements, for an 80 K.V. target, are shown in Figure AIV-6.

Measurements of the variations in beam deflection by an analysing magnet confirmed the results of Figure AIV-6. In the magnetic measurements the position of the beam was detected by dye-line papers exposed to the beam at an aluminium window fitted with cross-wires. After passing through a 1.6 m.m. lead slit at the normal target position the beam was focused to a 2.6 m.m. wide line by the magnet system. This was

* The yield of gamma rays for the reaction has since been measured for proton energies down to 5.5 Mev.
FIGURE AIV-6. INDICATION OF BEAM ENERGY VARIATION USING REACTION $^{27}\text{Al}(p,\gamma)^{28}\text{Si}$. 

<table>
<thead>
<tr>
<th>Run No</th>
<th>% Beam to Internal Target</th>
<th>mag. field (m.V)</th>
<th>Dee Pot. (k.V)</th>
<th>Deflector Pot. (k.V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 4, 6</td>
<td>46</td>
<td>108.18 (av.)</td>
<td>81.0</td>
<td>55.5</td>
</tr>
<tr>
<td>2, 3, 5</td>
<td>15</td>
<td>108.11 (av.)</td>
<td>81.0</td>
<td>55.1</td>
</tr>
<tr>
<td>7</td>
<td>46</td>
<td>108.13</td>
<td>81.5</td>
<td>55.5</td>
</tr>
<tr>
<td>8</td>
<td>46</td>
<td>108.14</td>
<td>80.5</td>
<td>55.3</td>
</tr>
<tr>
<td>9</td>
<td>31</td>
<td>108.14</td>
<td>81.0</td>
<td>55.3</td>
</tr>
</tbody>
</table>
consistent with the geometry of the system. Foils were placed in the beam and the deflection, as shown on the dye-line papers, was measured as the energy was attenuated. Thus the solid curve of Figure AIV-7 was obtained. The measured deflection of the beam with 15 percent of the beam stopped internally, instead of 46 percent, was less than half of that caused by one foil. This placed an upper limit of something less than 40 K.V. change in energy. The measured spread of the image on the dye-line paper when all machine controls were varied to their limit indicated variation of energy of no more than 40 K.V.

The measurements by the above two methods, in conjunction with observed normal operation of the cyclotron, indicate an energy stability during a run within the limits + zero and - 30 K.V. For most runs it would be considerably less than this.

From measurements of increase in width of the analysed beam pattern for different foil thicknesses the points of the dashed curve of Figure AIV-7 were obtained. The conversion of linear measurements to K.V. was made using the solid curve. This energy spread is in reasonable agreement with the straggling curve of Figure AIV-8, which was calculated
Deflection of beam (m.m) from position at full energy.

Energy spread of beam due foils (K.V.)

Beam Deflection

Energy Spread

Deflection of beam (m.m) from position at full energy.

Proton Energy (Mev)

Number of foils.

6 7.1 7.2 7.3 7.4 7.5

0 1 2 3 4 5 6

50 40 30 20 10 0

FIGURE AIV-7. MAGNETIC ANALYSIS OF PROTON BEAM.
FIGURE IV-8. STRAGGLING CURVE.
according to the Bohr theory. The lower energy part of this curve is too high as it was plotted without allowance being made for the integrated reduction of energy.

Discussion of Results and Conclusions.

In considering the results obtained for the gamma ray yield in the region of the giant resonance(Figure AIV-5) it is interesting to compare them with the relevant experiments which have been conducted elsewhere.

In so far as the low energy end of the yield curve is concerned there is quite good agreement with the results of Bair, Kington and Willard.

Apart from the 400 K.V. wide shoulder below 22 Mev excitation of C$^{12}$ there is nothing to suggest structure of the type shown in reference (3). Even when allowance is made for target thickness, variation of cyclotron energy and the effects of straggling down to low energies the resolution is sufficient to detect structure of separation equal to 200 K.V.

The 1953 results of Katz et alia might be considered as showing some small agreement with our lower energy results.

However the \((\gamma,n)\) results do not show a break at 22 Mev where the \((p,\gamma)\) yield increases more sharply, and does show one at 22.4 Mev. where the gamma ray yield of the inverse type of reaction is decreasing at an increasing rate.

An estimate of the cross section for the reaction we have studied gives a value of 0.42 mb. This assumes that the crystal is black to 22 Mev. gamma rays, that the radiation is isotropic, that the contribution due to radiation penetrating the shielding is negligible compared with the amount incident on the crystal through the collimating hole, and that the pulses included in the selected count represent 0.3 of the total number of pulses to the phototube. This last assumption is based on previous work with the crystal. Using detailed balancing the cross section for the inverse reaction is about 40 mb., or twice that obtained by Cohen et alia.

The examination of the gamma ray yield from the \(^{11}_{\text{B}}(p,\gamma)^{12}_{\text{C}}\) reaction has established the general shape of the \(^{12}_{\text{C}}\) giant resonance up to nearly 23 Mev. The peak occurs at about 22.3 Mev. and the width at half height of the resonance, assuming symmetry about the peak, is some 3.3 Mev. Although no strong evidence has been found for fine structure the results do not rule it out entirely. If \(^{12}_{\text{C}}\) levels above 22 Mev, of separation
less than about 100 K.V., contribute to the examined reaction they might not be resolved. At lower energies the spacing would need to be greater for resolution. Between 21 and 27.5 Mev. two variations of level density are indicated, while there is some evidence for variation at 20.2 and 20.8 Mev.
Canberra Cyclotron. Region of Experimental Target.