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S. Hüller, P. E. Masson-Laborde, D. Pesme, M. Casanova, F. Detering, and A. Maximov

Citation: [Physics of Plasmas \(1994-present\)](#) **13**, 022703 (2006); doi: 10.1063/1.2168403

View online: <http://dx.doi.org/10.1063/1.2168403>

View Table of Contents: <http://scitation.aip.org/content/aip/journal/pop/13/2?ver=pdfcov>

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Harmonic decomposition to describe the nonlinear evolution of stimulated Brillouin scattering

S. Hüller

Centre de Physique Théorique, Ecole Polytechnique, CNRS UMR 7644, 91128 Palaiseau Cedex, France

P. E. Masson-Laborde

Commissariat à l'Energie Atomique, BP 12, 91680 Bruyères-le-Châtel, France

D. Pesme

Centre de Physique Théorique, Ecole Polytechnique, CNRS UMR 7644, 91128 Palaiseau Cedex, France

M. Casanova

Commissariat à l'Energie Atomique, BP 12, 91680 Bruyères-le-Châtel, France

F. Detering and A. Maximov

Centre de Physique Théorique, Ecole Polytechnique, CNRS UMR 7644, 91128 Palaiseau Cedex, France

(Received 27 September 2005; accepted 5 January 2006; published online 8 February 2006)

An efficient method to describe the nonlinear evolution of stimulated Brillouin scattering (SBS) in long scale-length plasmas is presented in the limit of a fluid description. The method is based on the decomposition of the various functions characterizing the plasma into their long- and short-wavelength components. It makes it possible to describe self-consistently the interplay between the plasma hydrodynamics, stimulated Brillouin scattering, and the generation of harmonics of the excited ion acoustic wave (IAW). This description is benchmarked numerically in one and two spatial dimensions [one dimensional (1D), two dimensional (2D)], by comparing the numerical results obtained along this method with those provided by a numerical code in which the decomposition into separate spatial scales is not made. The decomposition method proves to be very efficient in terms of computing time, especially in 2D, and very reliable, even in the extreme case of undamped ion acoustic waves. A novel picture of the SBS nonlinear behavior arises, in which the IAW harmonics generation gives rise to local defects appearing in the density and velocity hydrodynamics profiles. Consequently, SBS develops in various spatial domains which seem to be decorrelated one from each other, so that the backscattered Brillouin light is the sum of various backscattered waves generated in several independent spatial domains. It follows that the SBS reflectivity is chaotic in time and the resulting time-averaged value is significantly reduced as compared to the case when the IAW harmonics generation and flow modification are ignored. From the results of extensive numerical simulations carried out in 1D and 2D, we are able to infer the SBS reflectivity scaling law as a function of the plasma parameters and laser intensity, in the limit where the kinetic effects are negligible. It appears that this scaling law can be derived in the limit where the IAW harmonics generation is modeled simply by a nonlinear frequency shift. © 2006 American Institute of Physics. [DOI: [10.1063/1.2168403](https://doi.org/10.1063/1.2168403)]

I. INTRODUCTION

The parametric scattering instabilities excited by laser-plasma interaction can no longer be described by means of the simple coupled mode equations for three-wave interaction whenever the longitudinal plasma wave is driven to an amplitude so large that its nonlinearity cannot be ignored. The longitudinal wave nonlinearity may give rise to various effects that can be of fluid and/or of kinetic type. These effects are expected to reduce the growth of the parametric instability, either because they correspond to a nonlinear frequency shift (e.g., in the case of partial particle trapping), or because they give rise to the effective damping associated with the energy transfer from the excited longitudinal wave into further spectral longitudinal wave components. Such

spectral components originate, e.g., from the parametric decay of the excited longitudinal wave, and cannot interact resonantly with the incoming laser wave. In the present article we consider stimulated Brillouin scattering (SBS); that is, the process by which the incident laser wave couples to an ion acoustic wave (IAW) and a scattered transverse wave. We denote by ω_s and k_s the linear frequency and wave number of this IAW, respectively, henceforth called the “fundamental” ion acoustic wave. These two quantities, ω_s and k_s , are determined by the usual resonant three-wave coupling conditions $\omega_0 = \omega_{sc} + \omega_s$ and $k_0 = k_{sc} + k_s$, where ω_0 (ω_{sc}) and k_0 (k_{sc}) denote the laser (scattered transverse) frequency and wave number, respectively; ω_α and k_α satisfy the dispersion relation corresponding to the transverse waves for $\alpha=0$ and $\alpha=sc$, and corresponding to the ion acoustic wave for $\alpha=s$. The fact that the longitudinal wave nonlinearity reduces the parametric instability excitation is one of the reasons in-

^{a)}Permanent address: Laboratory for Laser Energetics, University of Rochester, 250 East River Road, Rochester, NY 14623.

voked to explain why the standard three-wave interaction modeling overestimates the scattering levels of SBS observed experimentally.^{1–5} The IAW fluid-type nonlinearity gives rise to the generation of harmonics of the fundamental IAW, behaving, therefore, as $\exp[-il(\omega_s t - k_s z)]$, with $|l| > 1$. The effect of harmonics generation on SBS has already been investigated in detail.^{6–11} In particular, harmonics generation has been shown to be able to reduce the SBS reflectivity significantly as compared with the results obtained with the fundamental IAW only. However, the previous studies^{6–10} are incomplete because they retained the harmonics $l \neq 0$ only, and consequently ignored the component $l=0$. This latter component describes the long scale-length perturbation, and therefore corresponds to the hydrodynamic flow modification^{12,13} caused by the momentum deposition from the incident transverse wave. All the mentioned models^{6–11} ignored the multidimensional effects. The IAW fluid-type nonlinearity may also give rise to subharmonics generation by means of parametric instabilities that can be of decay type (the so-called “two-ion decay”)¹⁴ or of modulational type.¹⁵ On the other hand, the IAW nonlinearity of kinetic type, corresponding to partial particle trapping, is known to give rise to a nonlinear IAW frequency shift¹⁶ (for Stimulated Raman Scattering (SRS) we refer to a recent work¹⁷), which is also able to deeply modify the SBS nonlinear behavior.^{18,19} In particular, the nonlinear frequency shift due to kinetic effects can strongly enhance the two ion wave decay and subharmonic generation.^{15,18,20,21}

In the present paper, we restrict ourselves to the proper modeling of the IAW fluid-type nonlinearity, so that the nonlinear processes of kinetic type are not taken into account. We therefore reconsider the effect of the IAW fluid-type nonlinearities on SBS by accounting properly for the flow modification caused by SBS, corresponding to the “harmonic” $l=0$. We first derive approximate equations describing simultaneously the plasma hydrodynamics (i.e., the long-wavelength density and flow profiles), SBS, and the harmonic generation of the excited IAW resulting from the fluid-type nonlinearity. Our method consists in decomposing the fluid variables into long and short-wavelength components, the latter corresponding to the SBS-generated IAW and its harmonics.²² We refer to this method as the “harmonic decomposition” method. Our modeling can be extended by introducing a mesh of subharmonics, so that subharmonic generation caused by the parametric instabilities of the primary IAW could be studied in a straightforward way. The kinetic effects could also be taken into account phenomenologically, in particular in the perturbative limit where they can be modeled by a nonlinear frequency shift.^{16,20} Our new code, called HARMONY1D/2D, based on this harmonic decomposition method, makes it possible to describe plasmas of spatial sizes of the order of realistic laser-produced plasmas (of millimeter size, typically), because it does not require to resolve the IAW micrometer scale. Based on a systematic numerical study in one (1D) and in two spatial dimensions (2D) for a single laser hot spot, we have derived the scaling law of the SBS reflectivity as a function of the plasma parameters and laser intensity in the limit where the kinetic effects can be ignored.

In the next section, we present the model equations. In Sec. III, we describe the numerical results obtained with the decomposition model. We benchmark our results in 1D and in 2D against a reference fluid-type code, in which no decomposition into separate spatial scales is made. In the concluding section (Sec. IV), we discuss the applicability and limits of our modeling.

II. HARMONIC DECOMPOSITION MODELING

Because the IAW frequency ω_s is very small compared to the laser wave frequency ω_0 , one has $k_{sc} \approx -k_0$ and $k_a \approx 2k_0$. It is therefore convenient to decompose the total transverse electric field as

$$E(\mathbf{x}, t) = e^{-i\omega_0 t} (E_+ e^{ik_0 z} + E_- e^{-ik_0 z}) + \text{c.c.}, \quad (1)$$

where $E_+(\mathbf{x}, t)$ and $E_-(\mathbf{x}, t)$ are the forward- and backward-propagating light field components, respectively, both enveloped in time and space with respect to the light frequency ω_0 and wave numbers k_0 and $-k_0$. The z coordinate corresponds to the direction of propagation of the incident laser light. Without any loss of generality, we may assume $k_0 > 0$. The difference ω_s between the actual frequency ω_{sc} of the scattered light and ω_0 is accounted for by the slow time variation of the envelope $E_-(\mathbf{x}, t)$; similarly, the difference $|k_{sc} + k_0| \approx 2(c_s/c_+) |k_0|$ between the actual wave number k_{sc} of the scattered light and $-k_0$ is accounted for by the slow spatial variation of $E_-(\mathbf{x}, t)$. Here, c_s denotes the ion sound velocity: $c_s \equiv [c_{se}^2 + c_{si}^2]^{1/2}$ with $c_{se}^2 = ZT_e/M_i$ and $c_{si}^2 = 3T_i/M_i$, Z and M_i being the ion charge and mass, respectively, $c_+ \equiv c^2 k_0 / \omega_0$ is the group velocity of the incoming laser light, c denoting the speed of light. The wave number k_0 is determined from a fixed reference plasma density, denoted as N_{eq} ; namely, $k_0^2 \equiv (1 - N_{eq}/n_c) \omega_0^2 / c^2$, n_c denoting the critical electron density corresponding to the incoming laser light. Concerning the plasma density $n(\mathbf{x}, t)$ and velocity $\mathbf{v}(\mathbf{x}, t)$, we decompose these two functions into their long-wavelength components, denoted as $N_0(\mathbf{x}, t)$ and $\mathbf{v}_0(\mathbf{x}, t)$, and their short-wavelength components associated to the IAW harmonics $n_l(\mathbf{x}, t)$ and $\mathbf{v}_l(\mathbf{x}, t)$, with $|l| = 1, 2, \dots$, as follows:

$$n = N_0 + (n_1 e^{ik_a z} + n_2 e^{2ik_a z} + \dots + \text{c.c.}),$$

$$\mathbf{v} = \mathbf{v}_0 + (\mathbf{v}_1 e^{ik_a z} + \mathbf{v}_2 e^{2ik_a z} + \dots + \text{c.c.}),$$

in which all the quantities N_0 , $\mathbf{v}_0$, n_l , and $\mathbf{v}_l$ are functions that are slowly varying in space, as compared with $\exp(ik_a z)$. The terms N_0 and $\mathbf{v}_0$ are real. They correspond to $l=0$ and describe the hydrodynamic evolution. The terms n_1 and $\mathbf{v}_1$ are complex and correspond to the spatial and temporal envelopes of the fundamental IAW component excited by SBS, respectively, whereas the terms n_l and $\mathbf{v}_l$ with $l > 1$ are the envelopes describing its harmonics. The harmonics components $n_l(\mathbf{x}, t)$ and $\mathbf{v}_l(\mathbf{x}, t)$, with $l = -1, -2, \dots$, are given by $n_l(\mathbf{x}, t) \equiv n_{-l}^*(\mathbf{x}, t)$ and $\mathbf{v}_l(\mathbf{x}, t) \equiv \mathbf{v}_{-l}^*(\mathbf{x}, t)$. The wave number k_a for the IAW fundamental component is defined as $k_a \equiv 2k_0$, because the fundamental IAW component is driven by the ponderomotive force, which behaves as $\propto E_+ E_-^* \exp(i2k_0 z)$ according to the decomposition (1). Again, the small difference $|k_s - k_a| \approx 2(c_s/c_+) |k_0|$ between the actual IAW wave

number k_s and k_a will be accounted for by the slow spatial variation of the envelopes n_l and $\mathbf{v}_l$.

We use the paraxial approximation to reduce the wave equation for the total electromagnetic field E to two paraxial equations for $E_+(\mathbf{x}, t)$ and $E_-(\mathbf{x}, t)$,

$$\mathcal{L}_{\text{par}}(E_+) = -i(\omega_0/2n_c)[n_1 E_- + (N_0 - N_{\text{eq}})E_+], \quad (2)$$

$$\mathcal{L}_{\text{par}}(E_-) = -i(\omega_0/2n_c)[n_1^* E_+ + (N_0 - N_{\text{eq}})E_-], \quad (3)$$

with the paraxial operator $\mathcal{L}_{\text{par}}(E_{\pm}) = [\partial_t + c_{\pm} \partial_z + \nu_t - i(c^2/2\omega_0)\nabla_{\perp}^2]E_{\pm}$, where c_+ and $c_- \equiv -c_+$ stand for the group velocity of the forward- and backward-propagating light, respectively; and ν_t denotes the damping of the transverse waves. The right-hand side (r.h.s.) source terms in Eqs. (2) and (3) account for (i) the resonant three-wave interaction corresponding to SBS in which the incident wave E_+ is coupled to the backscattered wave E_- and to the fundamental IAW component n_1 , and for (ii) the refraction of the incident wave on the long-wavelength density modifications, $N_0 - N_{\text{eq}}$, giving rise to self-focusing in particular. The decomposition of the transverse wave into the two components E_+ and E_- , together with the paraxial approximation for each of these components, allows a significantly coarser spatial resolution and consequently leads to much less numerical expense. These approximations are correct in the undercritical domain only, $N_0 \ll n_c$.

The equations describing the fundamental IAW component n_1 , and its harmonics, n_l with $|l| > 1$, can be derived in a straightforward way from the set of the isothermal fluid equations for the ions, coupled to the Boltzmann equation for the electron density, and to the Poisson equation. Assuming that each component behaves at lowest order as a low-frequency normal mode, and dealing with the fluid nonlinearity along the standard perturbative procedure, one obtains

$$\begin{aligned} &(\partial_t + i[l\{\omega_a + k_a v_{0z}(z)\} + \delta\omega_l] + \nu_{a,l}) \\ &+ (v_{0z} + v_{gl})\partial_z - i(c_s/2lk_a)\nabla_{\perp}^2 n_l \\ &= -i\frac{\omega_a}{2}N_0\left(\delta_{l,1}\frac{k_a^2 c_{se}^2}{\omega_a^2}\frac{\epsilon_0 E_+ E_-^*}{n_c T_e} + Q_l\right), \end{aligned} \quad (4)$$

where the quadratic coupling term Q_l is given by

$$Q_l = p_l \sum_{h \neq 0, h \neq l} n_h n_{l-h}/N_0^2,$$

where p_l denotes a coupling coefficient, the expression of which is given further on. In Eq. (4), v_{0z} denotes the z component of the flow $\mathbf{v}_0$; ω_a is the eigenfrequency corresponding to the fundamental component wave number k_a . Denoting by $\omega(k)$ the positive solution to the dispersion relation $D(\omega, k) = 0$, with

$$D(\omega, k) \equiv \omega^2 - k^2(c_s^2 + c_{si}^2 k^2 \lambda_D^2)/(1 + k^2 \lambda_D^2), \quad (5)$$

ω_a is given by $\omega_a \equiv \omega(k_a) = [(c_s^2 k_a^2 + c_{si}^2 k_a^4 \lambda_D^2)/(1 + k_a^2 \lambda_D^2)]^{1/2}$ [Note that ω_s , mentioned above, is not exactly ω_a since $|\omega_s - \omega_a| = |\omega(k_s) - \omega(k_a)| \approx 2(c_s/c_+)c_s|k_0|$.] In all these expressions, λ_D denotes the local electron Debye length $\lambda_D^2 \equiv \epsilon_0 T_e / N_0 e^2$, computed in terms of the local density $N_0(z)$. The term $\delta\omega_l$ appearing in the r.h.s. of Eq. (4) corresponds to

the fact that the l th IAW harmonic is driven slightly off resonance, for $|l| \neq 1, 0$, because the dispersive effects due to the $k_a^2 \lambda_D^2$ corrections appearing in the expression for ω_a lead to a mismatch $\omega(lk_a) - l\omega_a \neq 0$ for $|l| \neq 1, 0$. The expression of $\delta\omega_l \approx \omega(lk_a) - l\omega_a$ is

$$\begin{aligned} \delta\omega_l(z) &= - \left[\frac{D(l\omega_a, lk_a)}{(\partial D/\partial \omega)_{(\omega=l\omega_a, k=lk_a)}} \right] \\ &= - \frac{c_{se}^2 k_a^4 \lambda_D^2}{2\omega_a} \frac{l(l^2 - 1)}{(1 + k_a^2 \lambda_D^2)(1 + l^2 k_a^2 \lambda_D^2)}. \end{aligned} \quad (6)$$

This expression can be simplified into $\delta\omega_l(z) \approx -(c_s k_a^3 \lambda_D^2/2)l(l^2 - 1)(1 + k_a^2 \lambda_D^2)^{-1/2}(1 + l^2 k_a^2 \lambda_D^2)^{-1}$ in the limit $T_i \rightarrow 0$ because one has $c_s = c_{se}$ and $\omega_a \rightarrow c_{se} k_a/(1 + k_a^2 \lambda_D^2)^{1/2}$ in the case of cold ions. The term v_{gl} plays the role of the group velocity of the l th harmonic, and is given by

$$v_{gl} \equiv - \left[\frac{\partial D/\partial k}{\partial D/\partial \omega} \right]_{\omega=l\omega_a, k=lk_a} = \frac{k_a}{\omega_a} \left[\frac{c_{se}^2}{(1 + l^2 k_a^2 \lambda_D^2)^2} + c_{si}^2 \right], \quad (7)$$

which again can be simplified into $v_{gl} \approx c_s(1 + k_a^2 \lambda_D^2)^{1/2}(1 + l^2 k_a^2 \lambda_D^2)^{-2}$ in the limit $T_i \rightarrow 0$. The expression of the coupling coefficient p_l appearing in front of the quadratic coupling term Q_l is

$$\begin{aligned} \frac{p_l}{l} &= \left(\frac{3}{2} - \frac{1/2}{[1 + l^2 k_a^2 \lambda_D^2][1 + k_a^2 \lambda_D^2]} + \frac{c_{si}^2}{c_{se}^2} \right. \\ &\quad \times (1 + k_a^2 \lambda_D^2) \Bigg) / \{ [1 + k_a^2 \lambda_D^2]^{1/2} \\ &\quad \times [1 + (1 + k_a^2 \lambda_D^2)c_{si}^2/c_{se}^2][1 + c_{si}^2/c_{se}^2] \}, \end{aligned} \quad (8)$$

so that one has approximately $p_l \approx l$, with corrections in $k_a^2 \lambda_D^2$ increasing with increasing l . The equation of evolution (4) for the harmonic l th envelope is valid in the so-called weak coupling regime for which the SBS growth rate is smaller than the IAW frequency ω_a .

Concerning the long-wavelength hydrodynamic quantities, we simply describe them here by isothermal fluid equations. Using their conservative form for what concerns their left-hand side (l.h.s.), we write

$$\partial_t N_0 + \nabla N_0 \mathbf{v}_0 = (\partial_t n)_{\text{IAW}}, \quad (9)$$

$$\partial_t (N_0 \mathbf{v}_0) + \nabla (N_0 \mathbf{v}_0 \mathbf{v}_0) + c_s^2 \nabla N_0 = -N_0 c_s^2 \nabla U_0 + (\partial_t n \mathbf{v})_{\text{IAW}}, \quad (10)$$

where the r.h.s. source terms $(\partial_t n)_{\text{IAW}}$ and $(\partial_t n \mathbf{v})_{\text{IAW}}$, account for the existence of the IAW fundamental component excited by SBS and of its harmonics. These two source terms therefore describe the momentum transfer into the flow caused by the IAW excitation due to SBS. They are given by

$$(\partial_t n)_{\text{IAW}} \equiv -\nabla \cdot \sum_{l \geq 1} (n_l \mathbf{v}_{-l} + \text{c.c.}), \quad (11)$$

$$\begin{aligned} (\partial_t n \mathbf{v})_{\text{IAW}} &\equiv \sum_{l \geq 1} [\mathbf{v}_0 (\partial_t n)_{\text{IAW}} + 2\nu_{a,l} (n_l \mathbf{v}_{-l} + \text{c.c.}) \\ &\quad + N_0 (c_s^2 \nabla |n_l/N_0|^2 - \mathbf{v}_l \cdot \nabla \mathbf{v}_{-l} + \text{c.c.})], \end{aligned} \quad (12)$$

where $\nu_{a,l}$ denotes the IAW damping of the harmonic com-

ponent n_l . Using the approximation $\mathbf{v}_l \approx c_s(n_l/N_0)\mathbf{e}_z$ underlying the derivation of Eq. (4), one obtains $(\partial_t n)_{\text{IAW}} \approx -2c_s \partial_z \sum_{l \geq 1} (|n_l|^2/N_0)$, and $(\partial_t n \mathbf{v})_{\text{IAW}} \approx 2c_s \sum_{l \geq 1} (2\nu_{a,l} - \nu_{0z} \partial_z) |n_l|^2/N_0$. The ponderomotive force is given by $\nabla U_0 = \epsilon_0 \nabla (|E_+|^2 + |E_-|^2)/n_c T_e$.

Equations (2)–(4), (9), and (10) describe what we call the harmonic decomposition modeling. They form a closed system describing SBS in a temporally and spatially evolving plasma. They can be shown to conserve momentum¹³ at the lowest order in $1/(k_a \ell_{\parallel})$ and in $1/(k_a \ell_{\perp})^2$ the inhomogeneity lengths $\ell_{\parallel}$ and $\ell_{\perp}$ being defined as $\ell_{\parallel} = |\partial_z v_0/v_0|^{-1}$ and $\ell_{\perp} = |\nabla_{\perp} v_0/v_0|^{-1}$. Theoretically, the harmonic decomposition is valid only in the regime $|n_1/N_0| \ll \sqrt{6}(k_a \lambda_D)^2$, for which the ordering $n_1 \gg n_2 \gg n_3 \dots$ can be proved to hold. One has, therefore, to check *a posteriori* that this condition was satisfied during the simulation. Practically, one retains only a finite number of harmonics, up to the order l_{max} . Harmonics n_l with $|l| > l_{\text{max}}$ then are considered to have a vanishing amplitude; i.e., $n_{|l| > l_{\text{max}}} = 0$. We have investigated the convergence of the numerical solution as a function of l_{max} and found that the condition $l_{\text{max}} \geq 1/k_a \lambda_D$ is a sufficient condition to ensure a good accuracy of the solution in the laser intensity range $I_L \lambda^2 \sim [10^{14} - 1.5 \times 10^{15}] \text{ W cm}^{-2} \mu\text{m}^2$. Thus, for the range of plasma parameters corresponding to our simulations, namely, $N_0/n_c \sim [0.01 - 0.15]$, $T_e \sim 1 \text{ keV}$, corresponding to $k_a \lambda_D \geq 0.25$, we found by varying $\nu_{a,1}/\omega_a$ in the range $\nu_{a,1}/\omega_a \sim [0 - 0.1]$ that $l_{\text{max}} = 5$ always leads to a good accuracy of the solution, even in the extreme limit $\nu_{a,1}/\omega_a = 0$.

III. NUMERICAL RESULTS

Our simulations were aimed at investigating the relative importance (i) of the SBS-induced flow modification, originating from the two source terms $(\partial_t n)_{\text{IAW}}$ and $(\partial_t n \mathbf{v})_{\text{IAW}}$ appearing in the r.h.s. of Eqs. (9) and (10), and (ii) of the IAW harmonic generation. In order to accentuate the potential effect of each mechanism, we neglected the IAW damping—or set it to extremely small values in some simulations, $\nu_{a,1}/\omega_a = 0.001$ —although being aware that the IAW damping coefficient is usually of the order of a few percents of the IAW frequency. The reasons for taking this dampingless limit were twofold:

- (i) The SBS-induced flow modification due to momentum transfer, first pointed out by Rose in Ref. 12, cannot be ignored in the regime of absolute instability corresponding to weak IAW damping. It is indeed in this regime that the stationary 1D limit of Eqs. (9) and (10) exhibits the most pronounced flow modification. Namely, the generation of the backscattered light gives rise to a transfer of momentum to the bulk plasma in the spatial domain of SBS activity. This momentum transfer results in a decrease $\Delta v \equiv v_{0,\text{out}} - v_{0,\text{in}} < 0$ of the flow v_0 in the direction of propagation of the laser, the net flow decrease being given by

$$\Delta v/c_s \approx -2R_{\text{SBS}}(2\epsilon_0|E_+|^2/N_{\text{eq}}T_e)(1 - N_{\text{eq}}/2n_c).$$

Here, R_{SBS} denotes the SBS reflectivity corresponding to the considered SBS active region. As expected, the

flow modification increases with increasing reflectivity, so that minimizing the damping should maximize the role of flow modification.

- (ii) It is already known from previous studies that harmonic generation in the limit of a strongly damped IAW, while yielding a reduction of the SBS reflectivity as compared with the simple three-wave coupling,¹¹ does, however, not cause a strong nonstationary behavior of the backscattered light,^{9,10} and henceforth does not require a particularly robust numerical scheme.

Our simulations were carried out on the basis of the harmonic decomposition modeling equations (2)–(4), (9), and (10). In the simulations presented and discussed in the following, the IAW was expanded up to its fifth harmonic, resulting in a set of equations for n_1, n_2, n_3, n_4 , and n_5 . In this limit, the r.h.s. nonlinear coupling terms Q_l read $Q_1 = p_1(2n_2n_1^* + 2n_3n_2^* + 2n_4n_3^* + 2n_5n_4^*)$, $Q_2 = (p_2/2)(n_1^2 + 2n_3n_1^* + 2n_4n_2^* + 2n_5n_3^*)$, $Q_3 = (p_3/3)3(2n_2n_1 + 2n_4n_1^* + 2n_5n_2^*)$, $Q_4 = (p_4/4)4(n_2^2 + 2n_1n_3 + 2n_5n_1^*)$, and $Q_5 = (p_5/5)5(2n_1n_4 + 2n_2n_3)$. The coefficients p_l are computed from Eq. (8); namely, $p_1 \approx 1.022$, $p_2/2 \approx 1.078$, $p_3/3 \approx 1.141$, $p_4/4 \approx 1.197$, and $p_5/5 \approx 1.240$ (for the case $T_i/T_e = 0.05$ and $k_a \lambda_D = 0.25$). The frequency mismatch $\delta\omega_l$ and the group velocity v_{gl} are given by Eqs. (6) and (7), respectively.

A. Simulations in 1D: Benchmark

We began our study by restricting ourselves to one-dimensional simulations in order to benchmark our harmonic decomposition code HARMONY1D against a reference code. In the following, our reference code is referred to as the “complete” 1D code, in the sense that it does not make use of the harmonic decomposition presented in the previous section. Our reference code solves Helmholtz’s equation for the total transverse electric field $E(z, t)$ as well as the system of ion fluid equations for continuity and momentum, with the electrostatic field in the source term. This field is computed by solving Poisson’s equation in which the complete ponderomotive potential, $\nabla|E(z, t)|^2$, is accounted for in the electron density (in 1D, the operator ∇ reduces to the partial derivative $\mathbf{e}_z \partial_z$).

In order to ensure equivalent boundary and initial conditions, we considered a realistic case similar to an “exploding foil,” where an initially heated plasma expands in vacuum, starting from an almost box-like density profile, with smooth shoulders, in the interval $z_1 < z < z_2$ along the laser axis. The plasma profile, with the initial plateau width $L_{\text{ini}} \approx 160\lambda_0$, successively undergoes rarefaction from each side, so that the velocity profile eventually tends to a monotonous curve varying from negative to positive values with $v_0 = 0$ in the center. The simulation box is chosen in such a way that the rarefaction of the profile does not significantly change the boundary conditions for the light fields at the entrance ($z_{\text{ent}} = 0 < z_1$) and the rear side ($z_{\text{rear}} > z_2$). The total box size is $z_{\text{rear}} = 2000/k_0 \approx 320\lambda_0$, where $\lambda_0 \equiv 2\pi c/\omega_0$ denotes the laser wavelength in vacuum. The boundary condition for the incident light at $z = 0$ is constant with time, $E_+(0) = \text{const}$, whereas the backscattered light is seeded with a noise source

at the level $\langle |E_-(z=z_{\text{rear}})|^2 \rangle \sim 10^{-6} |E_+(z=0)|^2$ and with a spectral bandwidth sufficiently larger than the IAW frequency, in order to cover all possible SBS resonances in the profile. In the density profile, wings left and right of the central plateau (for times $t < L_{\text{ini}}/2c_s$), the plasma is strongly inhomogeneous in velocity and density, so that SBS is inhibited by the strong flow gradient.

We carried out our simulations in the absolute instability regime of SBS with almost undamped IAWs ($\nu_{a,1}/\omega_a \sim 10^{-3}$), both to examine the role of flow due to momentum transfer, and to benchmark the robustness of our decomposition code. Before examining the results of our numerical simulations, it should be recalled that in the case of completely undamped IAWs, and in the simple limit of three-wave coupling, i.e., in the absence of flow modification and of harmonics generation, the SBS saturation level is independent of the noise level $\langle |E_-(z=z_{\text{rear}})|^2 \rangle$ imposed at the r.h.s. boundary condition for the backscattered light.²³ More precisely, in the case of the electron density $N_{\text{eq}}/n_c = 0.1$ and of the plasma length chosen for our simulations, the standard three-wave interaction model for undamped IAWs predicts a steep increase of the SBS reflectivities R_{SBS} as a function of the laser intensity, varying from $R_{\text{SBS}} \ll 1$ for small laser intensities, to $R_{\text{SBS}} \approx 1$ for normalized laser intensities above $a_0^2 \equiv \epsilon_0 |E_0|^2 / n_c T_e \approx 0.003$, corresponding to $I_L \lambda^2 \approx 3 \times 10^{13} \text{ W cm}^{-2} \mu\text{m}^2$ for $T_e = 1 \text{ keV}$, E_0 denoting the laser light amplitude at the plasma entrance boundary $E_0 \equiv E_+(z=0)$.

Our simulations comparing the decomposition code HARMONY1D and the complete code show a very good agreement, even in the extreme case shown in Fig. 1, corresponding to an almost undamped IAW ($\nu_{a,1}/\omega_a \sim 10^{-3}$). In these simulations, the plasma parameters were $N_0/n_c = 0.12$, $T_e = 1 \text{ keV}$, yielding $k_a \lambda_D = 0.25$, $T_i = 0.05 \text{ keV}$, the collisional damping for the transverse waves was vanishing. The initial plasma length was the one mentioned above; namely, $L_{\text{ini}} \approx 160\lambda_0$. The laser intensity was $I_L = 2.5 \times 10^{14} \text{ W/cm}^2$ with $\lambda_0 = 1.064 \mu\text{m}$, leading to the normalized intensity $a_0^2 \equiv \epsilon_0 |E_0|^2 / n_c T_e \approx 0.025$. For these parameters, the reflectivity would be 99% in the absence of any IAW nonlinearity or flow modifications. By contrast, it can be seen in Fig. 1 that, past an initial transient period, the HARMONY1D results obtained with five harmonics (continuous thick curve) and those obtained with the complete code (continuous thin curve) both lead to a reflectivity fluctuating in time, the time averaged of which being significantly smaller; namely, $\langle R_{\text{SBS}} \rangle \sim 0.6$. We did not observe any significant changes whenever at least three harmonics were retained, while restricting to less than three harmonics led to important differences. In particular, we show, in Fig. 1, the results corresponding to the case where no harmonics are retained, although the flow modification due to SBS is taken into account (monotonic dotted curve). It can be seen that this latter curve leads to results close to the standard three-wave coupling description, $R_{\text{SBS}} \rightarrow \approx 100\%$. Finally, we checked that the results do not depend on the noise level $\langle |E_-(z=z_{\text{rear}})|^2 \rangle$. In the absolute instability regime considered here, the growth of the fundamental IAW does not depend on the noise level,

FIG. 1. SBS reflectivity versus time from 1D simulations for the case of an almost undamped IAW ($\nu_{a,1}/\omega_a \sim 10^{-3}$). The simulation parameters are: laser intensity $I_L = 2.5 \times 10^{14} \text{ W/cm}^2$ for the laser wavelength $\lambda_0 = 1.064 \mu\text{m}$ at $N_0/n_c = 0.12$, electron temperature $T_e = 1 \text{ keV}$, yielding $k_a \lambda_D = 0.25$, ion temperature $T_i = 0.05 \text{ keV}$, and $Z = 1$, no collisional damping for the transverse waves. The initial plasma length is $L_{\text{ini}} \approx 160\lambda_0$. The comparison shows the results from the complete code (continuous thin curve), the decomposition code HARMONY1D with five harmonics (continuous thick curve), and without any harmonics (monotonic dotted line). For comparison, also shown are the results from a hybrid PIC code (fluctuating dotted line). Subplot (a) instantaneous reflectivity and (b) time-averaged reflectivity.

so that the harmonics growth can be expected not to depend on it either, because the harmonics are nonlinearly driven by the fundamental component. On the other hand, the subharmonic generation due to the parametric instabilities of the primary IAW might depend on the noise. In order to investigate this possibility, we added a source of noise in volume in the equation of conservation of momentum in the complete code. We did not observe any modifications of the results, and the Fourier spectrum of the ion density in the case of the complete code does not exhibit any significant subharmonics generation. We may therefore conclude that the main mechanism responsible for the reduction of the SBS growth is the harmonics generation.

In Fig. 2, we present a direct comparison of the spatial density profiles computed with the complete code (continuous line), as well as the envelopes of the density profiles reconstructed from the simulations carried out with the harmonic decomposition code. To reconstruct this density profile from the decomposed quantities N_0 and n_1 , we plot the local maxima, $\{N_0(z) + \max[n_1(z)]\}/N_{0,\text{max}}$ (dotted line), and

FIG. 2. Spatial profiles of the electron density obtained from the complete code (continuous curve), showing the rapid oscillations with $\lambda_0/2$ periodicity. Spatial profiles of the electron density reconstructed from the decomposition code: the dotted curve represents $\{N_0(z) + \max[n_1(z)]\}/N_{0,\max}$, the dashed curve represents $\{N_0(z) + \min[n_1(z)]\}/N_{0,\max}$, the normalization density $N_{0,\max}$ being the initial maximum density. With “max” and “min” we indicate that the local maxima/minima within a wavelength and within an oscillation period of the density perturbation n_1 corresponding to the fundamental IAW component. Subplot (a) corresponds to the time instant $\omega_a t = 60$, subplot (b) to $\omega_a t = 150$; subplot (c) corresponds to $\omega_a t = 120$, with a zoom in the spatial domain $900 < z < 1600$.

the local minima, $\{N_0(z) + \min[n_1(z)]\}/N_{0,\max}$ (dashed line), of the density $[N_0(z) + n_1(z)]/N_{0,\max}$ normalized to the initial maximum density $N_{0,\max} \equiv \max[N_{\text{eq}}(t=0)]$. Here, the symbols max and min denote the maxima and minima, respectively, computed locally in space and time over one wavelength and one oscillation period. When doing this

reconstruction, we did not retain the harmonics contribution, $n_{l>1}$ —although they were taken into account in the simulations—in order to investigate the order of magnitude of the departure from linear behavior. It is particularly striking that this reconstruction (without the higher harmonics) does correctly reproduce the phase of the local wave trains, i.e., the knots and the extrema, which keep on forming as time goes on. In our article, we call “defects” the knots appearing in the density profiles.

In the case of lower intensity values, the agreement between the complete and the HARMONY1D simulations is even more striking. This excellent agreement between the two hydrodynamic codes gives us confidence in the robustness of the harmonic decomposition modeling.

We checked in the simulations presented in Figs. 1 and 2 as to whether the maximum amplitudes of the fundamental IAW satisfied the inequality²⁴ $|n_1/N_{\text{eq}}| < |n_1/N_{\text{eq}}|_{\text{harm}} \equiv 6^{1/2}(k_a \lambda_D)^2$, which is usually thought to be the validity condition for the perturbative harmonic expansion of an IAW. For the parameters corresponding to the simulation results displayed in Figs. 1 and 2, one has $|n_1/N_{\text{eq}}|_{\text{harm}} = 15\%$, while the maximum amplitude of $|n_1/N_{\text{eq}}|$ observed in Fig. 2 is $|n_1/N_{\text{eq}}| \approx 20\%$, so that this validity condition is not satisfied. However, this validity condition was derived^{24,25} by assuming *a priori* the ordering $|n_1| \gg |n_2| \gg |n_3| \dots$, by expanding the density up to the third harmonic, and then by computing *a posteriori* the condition such that the contribution of the third harmonic to be small as compared to the second harmonic contribution. On the other hand, we found, in our current simulations, that in fact n_1 , n_2 , and n_3 show locally comparable amplitudes, so that this validity condition should not be strictly applied here. Thus, the condition $|n_1/N_{\text{eq}}| < |n_1/N_{\text{eq}}|_{\text{harm}}$, requiring the convergence of the harmonic development from the second order, appears to be too severe, because it does not account for the class of solutions showing convergence of the subsequent harmonics n_l starting at an order $l \geq 3$. Our numerical results show that the class of solutions satisfying the ordering $|n_1| \approx |n_2| \approx |n_3| \gg |n_4| \gg |n_5| \dots$ describe correctly the IAW nonlinear behavior, because we did not observe any significant differences between our current simulations results, obtained with five harmonics, with those provided by simulations in which we retained nine harmonics, even in the extreme cases such as the one shown in Figs. 1 and 2. In conclusion, we believe that retaining $l_{\max}$ harmonics, with $l_{\max} \geq 1/k_a \lambda_D$, is a sufficient condition to ensure the validity of the harmonic decomposition method in the range of laser intensities considered in this article.

It can be observed, in the spatial profiles of the fundamental IAW amplitude $|n_1|$, that the IAW behavior, and hence the flow modifications, are both entirely connected with the existence of defects in their spatial profiles and with their nonmonotonic behavior in space. By examining the density profile as a function of time, one clearly observes that SBS develops in distinct spatial domains, interrupted by phase defects. The latter originate in the density profile shoulder at the laser entrance side, and then propagate into the profile plateau. Thus, the SBS activity is localized in several adjacent domains that appear as almost uncorrelated wave trains.

FIG. 3. Spectrum of the backscattered light determined from simulations carried out with the complete code and corresponding to Figs. 1 and 2. The frequency coordinate is shifted by the incident laser frequency ω_0 .

This feature is reflected in the structured nature of the temporal spectrum of the backscattered light, shown in Fig. 3, in which distinct peaks appear. Consequently, the reflectivity exhibits a temporally chaotic behavior. We found that all these features—the existence of defects, the temporally chaotic reflectivity, the structured SBS light spectrum—are controlled by the size of a parameter defined in Sec. III C and denoted as Γ [see Eq. (13)]; namely, they can be observed in the regime $\Gamma > 1$, and the higher this parameter Γ (in the domain $\Gamma > 1$), the more pronounced are these features.

In Ref. 9, the nonstationary behavior of the SBS reflectivity and the resulting reduced average backscatter level (as compared to the linear IAW case) were explained by the detuning originating from the coupling with the second harmonic of the IAW. In our modeling, we retained the IAW harmonics up to higher orders, and we solved, in contrast to Ref. 9, a set of differential equations for these harmonics. In the following, we show, however, that the scaling between the quantities governing the average SBS reflectivity involves two dimensionless parameters, introduced later in Sec. III C: the dimensionless nonlinear frequency shift Γ and the dimensionless length L/L_0 . Both parameters, in the asymptotic limit $\Gamma \gg 1$, combine to a single one, $\Gamma^{-1/3}L/L_0$, which is the same one as in the case of the simple frequency shift model of Ref. 9.

Finally, before presenting 2D results obtained with our decomposition code, it is interesting to compare our fluid-like results with those obtained from simulations carried out with the 1D version of our hybrid kinetic code KOLIPIC,¹⁹ in which the ions are dealt with along the particle-in-cell (PIC) method, and the electrons form a Boltzmann gas (as in the complete hydrocode). The ion temperature $T_i = 0.05$ keV is sufficiently low so that the ion kinetic effects can be thought to be negligible. It can be seen in Fig. 1 that the complete code and HARMONY1D code results are in very good agreement with those obtained from KOLIPIC (oscillating dotted line), in particular the time-averaged reflectivity, $\langle R(t) \rangle = \int_0^t R(t') dt'$, which converges for all three simulations to the same level. Therefore, this comparison shows that the ion

kinetic effects do not play any role in the time interval $\omega_a t \lesssim 300$, for these parameters (low ion temperature, in particular).

B. Two-dimensional simulations

We also carried out simulations in two spatial dimensions in order to benchmark the 2D version of our harmonic decomposition code, HARMONY2D, against a complete code. The latter reference code is based on the code KOLIBRI,^{26,27} using (i) a nonparaxial wave solver for the Helmholtz equation in 2D, and (ii) a 2D nonlinear hydrodynamic code solving the equations of conservation for ion density and momentum, with the assumption of isothermal electrons, for simplicity.²⁸ Thus, the electrons form a Boltzmann gas, and Poisson's equation is solved. We have chosen an exploding foil profile shorter than in the 1D case, $z_{\text{rear}} = 102\lambda_0$, with an initially homogeneous plasma in the interval $z_1 < z < z_2$, with $z_1 = 34\lambda_0$ and $z_2 = 67\lambda_0$. In both codes we have imposed a temporally incoherent seed in the r.h.s. boundary condition for the backscattered light component, with a temporal bandwidth of about $\sim 2\omega_a$. The incident laser light (coming from the left) is focused at the center of the plasma, and before any nonlinear effects take place, it forms a single Gaussian laser beam with the beam waist $w_0 = 5\lambda_0$ at the focal point, corresponding to the vacuum Rayleigh length $L_R = \pi w_0^2 / \lambda_0 \approx 80\lambda_0$. In our article, we define the beam waist $w(z)$, at a given longitudinal coordinate z , in such a way that the radial intensity shape is of the form $I_L(r) \propto \exp[-2r^2/w^2(z)]$. The longitudinal plasma size ($33\lambda_0$) is therefore significantly shorter than the longitudinal extension of the laser speckle, so that this physical configuration can be considered to be close to a 1D case, at least in the limit where self-focusing does not take place. The question as to what extent SBS can be described by a 1D modeling in a multidimensional plasma is discussed in detail at the end of this subsection.

Concerning the results of our 2D simulations, we find again a very good agreement between the HARMONY2D and the complete 2D code results, both for what concerns the temporal evolution of the SBS reflectivity, as can be seen in Figs. 4 and 5, and the spatial behavior of the waves, as shown in Figs. 6–9. The representative case illustrated in Figs. 4–9 corresponds to the laser intensity $I_{L,\text{in}} = 1.5 \times 10^{15}$ W/cm², $I_{L,\text{in}}$ denoting the laser intensity at the entrance boundary, the laser wavelength $\lambda_0 = 1 \mu\text{m}$, the electron temperature $T_e = 1$ keV, and the maximum electron density in the plateau $n_e = 0.1n_c$, yielding $k_a \lambda_D = 0.27$. For the simulations carried out with KOLIBRI, the spatial step size was such that the wavelength corresponding to the fifth harmonic—and therefore the Debye length as well—could be resolved. Consequently, as in the 1D simulations, we choose to consistently retain five harmonics in the simulations carried out with HARMONY2D. We checked that the harmonics amplitudes $|n_l|$ are rapidly decreasing as a function of their order l , for $l > 5$, because of the strong dispersion. In the simulations with HARMONY2D, we introduced a very small damping, $\nu_{a,1}/\omega_a \approx 0.008$, to the fundamental IAW in order to account for the fact that the numerical scheme of the full hydrocode does always cause a nonzero damping to short-wavelength

FIG. 4. SBS reflectivity in logarithmic scale, comparing the instability growth for the complete code KOLIBRI 2D results (dotted line), with those obtained with the decomposition code HARMONY2D with five harmonics (thin line), for initial times: $\omega_a t \leq 40$. The curve “no harmonics” (thick line) shows the case in which no harmonics are taken into account in the code HARMONY2D. The parameters are the following: laser intensity $I_L = 1.5 \times 10^{15}$ W/cm², laser wavelength $\lambda_0 = 1$ μ m, electron temperature $T_e = 1$ keV, maximum electron density $N_0 = 0.1n_c$, and IAW damping $\nu_{a,1}/\omega_a \approx 0.008$. The laser hot spot beam waist is $w_0 = 5\lambda_0$ for the amplitude.

density perturbations. In contrast to the physical damping of IAWs, which increases almost linearly with the wave number $\propto k$, the usual hydrodynamic schemes introduce a numerical dissipation that may increase nonlinearly with k , in most cases scaling as $\propto k^2$ due to viscosity. In the case in which one wants to retain the smallest possible damping, it is therefore important to match the damping of the fundamental IAW in both the decomposition and the full hydrocode, which we found to be $\nu_{a,1}/\omega_a \approx 0.008$ in our case. For the harmonics, however, this damping is of minor importance as long as dispersion effects due to not so small values of $k_a \lambda_D$ are dominating.

Concerning the SBS reflectivity, the agreement between the two codes is excellent for the initial temporal growth, as it can be best seen in the logarithmic scale in Fig. 4; the agreement between the saturation values, shown in Fig. 5, is remarkable as well. Figure 4 also shows the behavior of the

FIG. 5. SBS reflectivity, similar to Fig. 4, but in linear scale and for longer times: $\omega_a t \leq 110$. It compares the instability growth for the complete code KOLIBRI 2D results (dotted line), with those obtained with the decomposition code HARMONY2D with five harmonics (thin line). The curve “no harmonics” (thick line) shows the case in which no harmonics are taken into account in the code HARMONY2D. The parameters are identical to those of Fig. 4.

FIG. 6. Snapshot of the total transverse field $|E|$ (i.e., incident and scattered) in 2D space computed with the complete code KOLIBRI for the same parameters as in Figs. 4 and 5, at the time $\omega_a t = 40$. The z axis is the incident laser light direction of propagation, and the x axis is the transverse direction. The field values are normalized to the maximum intensity of the incident light (in vacuum). The longitudinal oscillations in the field correspond only to a stroboscopic image of the beating between the incident and backscattered light fields, the real periodicity of this beating being $\lambda_0/2$.

SBS reflectivity when no harmonics are taken into account. In this case, the reflectivity reaches much higher values than when the five first harmonics are retained. In the snapshots showing the spatial profiles of the light field (Figs. 6 and 7)

FIG. 7. Superposition of incident light field (gray scale) and of the backscattered light field (contour lines) computed with the decomposition code HARMONY2D, at the time $\omega_a t = 40$. The parameters are the same as in Figs. 4 and 5. This figure is the exact HARMONY2D counterpart to Fig. 6 (obtained with the complete code KOLIBRI). The field values are normalized to the maximum intensity of the incident light (in vacuum). The longitudinal oscillations in the field correspond only to a stroboscopic image of the beating between the incident and backscattered light waves, the real periodicity of this beating being $\lambda_0/2$.

FIG. 8. Snapshot of the electron density (normalized to $N_{0,\max}$ in the initial profile) in 2D space computed with the complete code KOLIBRI, at the time $\omega_a t = 40$. The parameters are the same as in Figs. 4 and 5. The oscillations in the density correspond only to a stroboscopic image of the real oscillations caused by SBS, the periodicity of the latter being $\lambda_0/2$.

and the density (Figs. 8 and 9), it can be seen how well the counterpropagating field components E_+ and E_- , as computed from the two codes (with five harmonics retained in HARMONY2D), match up when superimposed. The long-wavelength hydrodynamics and the short-wavelength SBS-driven IAWs also fit very well. Concerning the harmonic decomposition code results, instead of reconstructing completely the fields from their components $|E_+|$, $|E_-|$, N_0 , and n_1 , we preferred in Figs. 7 and 9 to draw the contours of $E_- \cos(k_a z)$ and $n_1 \cos(k_a z)$ upon the shadow graphs of $|E_+|$

FIG. 9. Superposition of the mean density, $N_0(z, x_{\perp})$, and of the first harmonic component of the IAW density perturbation $n_1(z, x_{\perp})$, both normalized to $N_{0,\max}$ in the initial profile. The parameters are the same as in Figs. 4 and 5. This figure is the exact HARMONY2D counterpart to the Fig. 8 (obtained with KOLIBRI). The oscillations in the density correspond only to a stroboscopic image of the real oscillations caused by SBS, the periodicity of the latter being $\lambda_0/2$.

and N_0 , respectively. In conclusion, the excellent agreement between the HARMONY2D results with those obtained with the reference code gives us confidence in the reliability of the decomposition method in 2D. In addition, the numerical algorithm used in HARMONY2D proves to be much more efficient than the complete code KOLIBRI, the computing time being divided by a factor of >200 . This factor is based on simulations carried out in single- and multi- (OpenMP) processor mode, on a IBM Regatta 4 parallel machine (at IDRIS, Orsay, France).

We now comment on the differences which may exist between the 1D and 2D or three-dimensional (3D) simulations due to the density profile modifications taking place in the transverse direction in 2D or 3D modeling. Such transverse density-profile modifications can be caused by the ponderomotive force associated to the incident beam, in which case they may lead to the self-focusing (SF) of this beam. The SF effects represent the main source of difference with respect to 1D simulations, for which the ponderomotively driven profile modifications in the direction perpendicular to the propagation axis cannot take place. In the real 3D space, self-focusing is the nonlinear process by which the incident laser light may form steady state filaments when propagating inside the plasma. In 3D geometry, self-focusing takes place when the power P of the incident laser exceeds the critical power P_c , given by^{30–33}

$$P_c = 4\pi N_c (c/\omega_0)^2 (n_c/N_0) n_c c_+ T_e (1 - N_0/2n_c),$$

with $N_c = 1.90$ in the case of an incident Gaussian laser beam. Characterizing the incident Gaussian beam by its intensity $I_{L,\text{in}}$ and its beam waist w_{in} at the plasma entrance point, P is given by $P = (\pi/2) I_{L,\text{in}} w_{\text{in}}^2$, so that it is convenient to introduce the threshold parameter for self-focusing in 3D, $p_{3D} \equiv P/P_c$, which reduces in the limit $N_0/n_c \ll 1$ to

$$p_{3D} \approx 0.5 \frac{I_{L,\text{in}} \lambda_0^2}{10^{15} \text{ W cm}^{-2} \mu\text{m}^2} \frac{N_0}{n_c} \frac{1 \text{ keV}}{T_e} \left(\frac{w_{\text{in}}}{\lambda_0} \right)^2$$

in practical units. The beam intensity $I_{L,\text{in}}$ and waist w_{in} at the plasma entrance point are related to $I_{L,0}$ and w_0 by flux conservation; namely, $I_{L,\text{in}} w_{\text{in}}^2 = I_{L,0} w_0^2$, $I_{L,0}$ and w_0 denoting what would be the laser beam intensity and beam waist at the focal point in the absence of nonlinear effects. Thus, the condition for self-focusing in 3D is simply $p_{3D} \geq 1$, the filament formation being then the result of the nonlinear saturation of the so-called collapse phenomenon. In the context of self-focusing, collapse means that the beam waist $w(z)$ would vanish to zero in a finite propagation distance within the plasma, and correspondingly the laser intensity would increase to infinity, in the absence of the nonlinear saturation caused by the low-frequency response of the plasma. Here $w(z)$ denotes the beam waist as a function of the longitudinal coordinate z . Collapse can take place only if the dimension $D_{\perp}$ of the transverse directions satisfies $D_{\perp} \geq 2$, 2 being the so-called critical dimension. Therefore, self-focusing and filament formation, which occur naturally in the real 3D space for $p_{3D} > 1$, may be correctly described in 3D modelings only. On the other hand, even though the ponderomotive force associated to the incident laser beam cannot lead, in a

2D modeling, to the same self-focusing as it would occur in 3D, the ponderomotive force is able to give rise, in 2D, to density profile modifications in the transverse direction, and consequently to a nonlinear modification of the laser beam propagation as compared with the linear case where the ponderomotive force is ignored. Therefore, we will keep referring to this nonlinear propagation effect as “self-focusing” in the context of 2D simulations as well, this effect being exactly of same nature as the one leading to self-focusing in real 3D space.

The question is now to quantify the importance of the self-focusing effects occurring in 2D simulations when comparing the 1D with the 2D results. By means of a standard variational method such as used in Refs. 33 and 34, one easily finds that the importance of the self-focusing effects in 2D modeling are controlled, in the absence of SBS, by the the parameter p_{2D} defined as $p_{2D} \equiv 2N_c p_{3D}$, which reduces in the limit $N_0/n_c \ll 1$ to

$$p_{2D} \approx 1.8 \frac{I_{L,in} \lambda_0^2}{10^{15} \text{ W cm}^{-2} \mu\text{m}^2} \frac{N_0}{n_c} \frac{1 \text{ keV}}{T_e} \left(\frac{w_{in}}{\lambda_0} \right)^2$$

in practical units. Namely, self-focusing effects are found to be negligible in the regime $p_{2D} \ll 1$ while in the opposite regime $p_{2D} > 1$, they remain negligible as long as the inequality $w_{in}/w_0 > p_{2D}$ is fulfilled. Thus, the linear laser beam focal point has to be far enough away from the plasma entrance so that the self-focusing effects remain negligible in the regime $p_{2D} > 1$. The self-focusing effects are not negligible in the regime $p_{2D} > 1$ and for $w_{in}/w_0 \leq p_{2D}$. In this limit, and in the case where the linear focusing point z_{foc} is inside the plasma, the minimum beam waist, denoted as w_{min} , is reduced as compared with what it would be in the absence of SF effects: w_0 . Correspondingly, the maximum laser intensity, denoted as $I_{L,max}$ is enhanced as compared with the linear result $I_{L,0}$, with $I_{L,max} = I_{L,0}/(w_{min}/w_0)$. In the regime $p_{2D} > 1$ and for $w_{in}/w_0 \leq p_{2D}$, one obtains $w_{min}/w_0 \approx (w_{in}/p_{2D}w_0)$. Finally, in the regime $p_{2D} > 1$ and in the stronger limit $w_{in}/w_0 < \sqrt{p_{2D}}$, self-trapping of the light occurs, under the form of a sequence of focusing/defocusing. For the parameters of the 2D simulations presented above, one obtains $p_{3D} = 1.9$, $p_{2D} = 6.7$, and $w_{in}/w_0 = 1.02$. Thus, our 2D simulations correspond to a regime where self-focusing, in the absence of SBS, would give rise to filament formation in 3D and to self-trapping of the light in 2D modeling.

At this point, it should be pointed out that the decomposition method of HARMONY2D makes it possible to study easily the interplay between self-focusing and stimulated Brillouin scattering: (i) SF effects can be taken into account or ignored by keeping or suppressing the corresponding r.h.s. source terms proportional to $|E_+|^2$, in Eqs. (9) and (10); and (ii) SBS can be suppressed by switching off the coupling with the IAW amplitude n_1 in Eqs. (2) and (3) for the fields E_+ and E_- . From Fig. 9 it can be observed, first of all, that the long-wavelength density component N_0 undergoes a ponderomotively driven profile modification in form of a density depression along the light propagation axis ($x_\perp = 0$). Correspondingly, this density depression can be seen in Figs. 6 and 7 to give rise to an enhancement of the incident wave inten-

FIG. 10. Average SBS reflectivity as a function of the laser intensity in the range $[10^{14} - 10^{16}] \text{ W/cm}^2$ comparing 1D (“+”) and 2D (“Δ”) simulations with HARMONY1D/2D; the plasma parameters are the same as in Fig. 4 and 5: laser wavelength $\lambda_0 = 1 \mu\text{m}$, electron temperature $T_e = 1 \text{ keV}$, maximum electron density $n_e = 0.1n_c$, and IAW damping $\nu_{a,1}/\omega_a \approx 0.008$.

sity maximum, $I_{L,max} \approx 1.4$, with respect to the corresponding value in the initially unperturbed plasma, $I_{L,0} \approx 1.1$ (the maximum value at focus *in vacuo* and the plasma entrance point being $I_{L,0} = 1$ and $I_{L,in} = 0.9$, respectively). We then carried out simulations without SBS, for the same parameters as in Figs. 6–9 and we observed (not shown here) that the modifications of the long-wavelength hydrodynamics and of the shape of the incident field E_+ associated to self-focusing are much more pronounced in the case in which SBS is suppressed than in the case in which it is retained: in the case in which SBS was suppressed, we obtained $I_{L,max}/I_{L,0} \approx 2.6$. Therefore, we may conclude that the depletion of the pump field amplitude $|E_+|$ caused by the SBS growth reduces dramatically the self-focusing effects, for the parameters considered here. This fact explains why the 2D simulations results do not differ significantly from the 1D simulations ones, as will be discussed in the following subsection.

C. Scaling laws for the reflectivity

We undertook a systematic study of the SBS reflectivity by varying the laser intensity, the plasma density, and the plasma box size in order to explore the scaling of the SBS reflectivity with these parameters. In all the simulations carried out with HARMONY2D, we retained five harmonics to describe the IAW nonlinearity. As already mentioned, we counterchecked that five harmonics were sufficient by retaining up to nine harmonics in the cases with the highest intensity, and we did not observe any significant differences as compared with five harmonics. Concerning the results, we found, first of all, that the differences between the 1D and 2D simulations are very small. Figure 10 shows the time-averaged²⁹ SBS reflectivity as a function of the laser intensity for the same parameters as mentioned above (T_e

$=1$ keV, $n_e=0.1n_c$ at $\lambda_0=1\text{ }\mu\text{m}$, $k_a\lambda_D=0.27$, $\nu_{a,1}/\omega_a \approx 0.008$, $L_{\text{ini}}=33\lambda_0$), comparing the 1D (“+”) and 2D (“ Δ ”) results. The time-averaged reflectivity values were determined within a time interval δt in such a way that the following counteracting constraints were satisfied: the time averaging of the reflectivity had to be made (1) after the saturation of the SBS growth had occurred, and (2), before the hydrodynamic expansion had deeply modified the density and velocity profiles. Typically, the averaging time $\delta t = 100\omega_a^{-1} - 200\omega_a^{-1}$ was found to satisfy these two constraints.

It can be seen in Fig. 10 that the 1D and 2D results are very close to each other, within a laser intensity interval extending over two orders of magnitude. On the first hand, this result may not seem to be really surprising, because the 1D and 2D parameters were chosen in such a way that the 1D case could be expected to be close to the 2D case; namely, (i) the density profile lengths were identical in both kinds of simulations, and (ii) the Rayleigh length of the 2D speckle was larger than the density profile length, so that in the absence of SBS and SF, the intensity of the incident 2D beam would vary only slightly along the propagation axis, similar to the 1D case in which the laser intensity does not vary spatially. On the other hand, we have seen in the previous subsection that the parameters p_{2D} and $w_{\text{in}}/w_0=1.02$, controlling the SF effects, are such that SF would be expected to lead to significant density depletion and intensity enhancement in the absence of SBS. Indeed, for the parameters $N_0/n_c=0.1$, $T_e=1$ keV, and $w_0/\lambda_0=5$, corresponding to Fig. 10, the condition $p_{2D} > 1$ reduces to $I_L\lambda_0^2/10^{15}\text{ W cm}^{-2}\text{ }\mu\text{m}^2 \geq 0.22$, so that the condition $p_{2D} > 1$ is satisfied for most of the 2D simulations, the results of which are displayed in Fig. 10. Therefore, we interpret the fact that the 1D and 2D simulation results do not significantly differ from each other as being due to the fact that SBS suppresses the self-focusing of the incident beam in the regime $p_{2D} > 1$.

We carried out an additional set of 2D simulations in which we varied the initial maximum electron density, in the cases corresponding to two laser intensities, $I_L=5 \times 10^{14}\text{ W/cm}^2$ and $I_L=1.5 \times 10^{15}\text{ W/cm}^2$, the other parameters being the same as in Fig. 10 ($T_e=1$ keV, $\lambda_0=1\text{ }\mu\text{m}$, $w_0/\lambda_0=5$, $\nu_{a,1}/\omega_a \approx 0.008$, $L_{\text{ini}}=33\lambda_0$). The corresponding results are displayed in Fig. 11, under the form of two curves, the lower (upper) one corresponding to the lower (higher) intensity. Because our 2D and 1D simulation results appeared to be very close to each other, we then proceeded as previously done in the case of 1D simulations⁹ in order to investigate the existence of a scaling law for the reflectivity. Namely, we displayed all our 2D results, thus corresponding to various key parameters such as the laser intensity and the plasma density, in the single Fig. 12, in which the reflectivity is plotted as a function of the dimensionless length L/L_0 , where L denotes the plasma profile length and L_0 the characteristic length for the spatial growth of the SBS instability, related by $L_0=(c_+c_s)^{1/2}/\gamma_0=(2/\pi)L_c$ to the critical length L_c for the existence of unstable normal modes in the limit of a plasma slab of length L .^{23,24,35,36} The length L_0 depends on the laser intensity and wavelength, on the plasma temperature, and density N_0 via the classical SBS growth rate γ_0 , and the group velocities c_+ and $v_{g1} \approx c_s$. Its expression is given

FIG. 11. Average SBS reflectivity as a function of n_e/n_c , for 2D simulations with intensities $I_L=5 \times 10^{14}\text{ W/cm}^2$ (lower branch) and $I_L=1.5 \times 10^{15}\text{ W/cm}^2$ (upper branch). The other parameters are the same parameters as in Fig. 10: laser wavelength $\lambda_0=1\text{ }\mu\text{m}$, electron temperature $T_e=1$ keV, and IAW damping $\nu_{a,1}/\omega_a \approx 0.008$.

further on in terms of physical units. In Fig. 12, we plotted the results corresponding to simulations in which (a) the intensity was varied, keeping fixed the density $N_0/n_c=0.1$ (simulation data: “ $\times$ ”), and (b) the density was varied, keeping fixed the laser intensity $I_L=5 \times 10^{14}\text{ W/cm}^2$ (simulation data: “ Δ ”); the other parameters are the same as in Fig. 10: $T_e=1$ keV at $\lambda_0=1\text{ }\mu\text{m}$, $k_a\lambda_D=0.27$, $\nu_{a,1}/\omega_a \approx 0.008$, $L_{\text{ini}}=33\lambda_0$. On the other hand, it has been shown in Refs. 9 and 24 that IAW harmonics generation is able to transform the coherent three wave coupling into an interaction of incoherent type, whenever the condition $\Gamma > 1$ is satisfied, with

FIG. 12. Average SBS reflectivity as a function of the parameter L/L_0 for two sets of 2D simulations: in the set (a) the intensity was varied, keeping fixed the electron $N_0/n_c=0.1$; in the set (b) the density was varied, keeping fixed the intensity $I_L=5 \times 10^{14}\text{ W/cm}^2$. This second set of simulations is identical to the set shown in the lower branch of Fig. 11. The other parameters are the same parameters as in Fig. 10. Laser wavelength $\lambda_0=1\text{ }\mu\text{m}$, electron temperature $T_e=1$ keV, and IAW damping $\nu_{a,1}/\omega_a \approx 0.008$. The simulation results are represented by “ $\times$ ” [set (a)] and “ Δ ” [set (b)]. The scaling law (16) is represented by the thin [set (a)] and the thick [set (b)] continuous curves. For the latter, the data pairs of R and Γ from the simulation were inserted in Eq. (16) to determine the corresponding L/L_0 value.

$$\Gamma \equiv [I_L \lambda_0^2 / (I_L \lambda_0^2)_{\text{inc}}]^{1/2}, \quad (13)$$

where $(I_L \lambda_0^2)_{\text{inc}}$ represents the intensity threshold above which harmonics generation is strong enough to cause chaotic coupling between the coherent incident laser wave with the backscattered wave and the IAWs. The expression of $(I_L \lambda_0^2)_{\text{inc}}$ reads

$$(I_L \lambda_0^2)_{\text{inc}} \equiv 2.9 \times 10^{12} \text{ W cm}^{-2} \mu\text{m}^2 (N_0/n_c) (1 - N_0/n_c)^{1/2} \\ [T_e(\text{keV})(1 + 3T_i/ZT_e)]^3 [1 + (3/2)T_i/ZT_e]^{-4}. \quad (14)$$

This result has been derived in Ref. 9 in the limit of weak harmonic generation, for which the effect of the second harmonics n_2 can be reduced perturbatively to a nonlinear frequency shift appearing in the equation for the fundamental IAW component n_1 . In this limit, the harmonics of order higher than 2 are neglected. In this perturbative regime, the following approximate scaling law for the reflectivity was derived:⁹

$$R \approx 1 - \Gamma^{1/3} \frac{\pi L_0}{2L}, \quad (15)$$

valid in the regime $R \geq 0.1$, for moderate values of Γ ; i.e., $\Gamma > 1$ with $\Gamma = O(1)$. For now, laser intensities, Γ can take values far beyond the regime $\Gamma = O(1)$, so that we were led to generalize the latter scaling law in the nonperturbative regime $\Gamma \gg 1$. We found that the following scaling law

$$L/L_0 - \Gamma^{1/3} \approx \frac{3\pi}{4} \frac{\Gamma^{1/3} R^{1/3}}{(1-R)^{1/3}}, \quad (16)$$

relating implicitly the reflectivity R to the two dimensionless parameters Γ and L/L_0 , reproduces very closely our numerical results in the whole space of parameters spanned in our simulations. In practical units, the scaling parameters L_0 and Γ read

$$L_0 = 0.74 \lambda_0 \left(\frac{T_e(\text{keV})(1 + T_i/ZT_e)}{I_{15} \lambda_{\mu\text{m}}^2 (1 - N_0/n_c)^{1/2} N_0/n_c} \right)^{1/2} \quad (17)$$

and

$$\Gamma = 18.6 \left(\frac{I_{15} \lambda_{\mu\text{m}}^2 [1 + (3/2)T_i/ZT_e]^4}{[T_e(\text{keV})(1 + T_i/ZT_e)]^3 (1 - N_0/n_c)^{1/2} N_0/n_c} \right)^{1/2},$$

in which we used the notations $I_{15} \lambda_{\mu\text{m}}^2 \equiv I_L \lambda_0^2 / 10^{15} \text{ W cm}^{-2} \mu\text{m}^2$ and $T_e(\text{keV}) \equiv T_e / 1 \text{ keV}$ for simplicity.

The scaling law (16) is displayed in Fig. 12 in the form of two curves: the thinner one corresponds to the simulations (a) in which the density was kept fixed ($N_0/n_c = 0.1$) and the intensity was varied, while the thicker one corresponds to the simulations (b) in which the intensity was kept fixed ($I_L = 5 \times 10^{14} \text{ W/cm}^2$). As said above, the other parameters are the same as in Fig. 10 ($T_e = 1 \text{ keV}$ at $\lambda_0 = 1 \mu\text{m}$, $k_a \lambda_D = 0.27$, $\nu_{a,1}/\omega_a \approx 0.008$, and $L_{\text{ini}} = 33\lambda_0$). Figure 12 reproduces also the simulation data, indicated with “ $\times$ ” and “ $\triangle$,” already shown in the previous figures. The coordinate of the horizontal axis in Fig. 12 is the rescaled length L/L_0 , L_0 being then computed, for each numerical simulation, from the physical parameters corresponding to the simulation, according to the

expression (17). Figure 12 demonstrates the excellent agreement between the scaling law (16) and the numerical results, whereas the domain of values of Γ spanned by these numerical simulations is very broad, extending over the interval $\Gamma = [20, 200]$.

The main difference between the perturbative result (15), valid in the perturbative regime $R \geq 0.1$, $\Gamma > 1$, with $\Gamma = O(1)$, and the scaling law (16) is that the latter predicts significantly smaller reflectivity values than the perturbative result in the domain $\Gamma \gg 1$ and $R \geq 0.5$. On the other hand, it should be pointed out that both scaling laws (16) and (15) lead to the same characteristic length for a non-negligible SBS reflectivity, in the regime $\Gamma > 1$ of incoherent interaction; namely,

$$L_{\text{inc}} \equiv \Gamma^{1/3} L_0 \approx \frac{2\lambda_0}{(I_{15} \lambda_{\mu\text{m}}^2)^{1/3} (N_0/n_c)^{2/3}}. \quad (18)$$

Recalling that L_0 is the characteristic length for SBS growth in the limit where the harmonics generation is negligible, one may summarize the effect of harmonics generations on SBS as follows: the harmonics generation is able to transform the coherent three-wave coupling into an interaction of incoherent type, whenever the condition $\Gamma > 1$ is satisfied. In the incoherent regime $\Gamma > 1$, the SBS growth is reduced, the characteristic plasma length for SBS growth being increased from L_0 to L_{inc} . Thus, the parameter controlling the SBS growth reduction due to the IAW harmonics generation is the dimensionless parameter Γ , and L_{inc} can be significantly longer than L_0 in the regime $\Gamma \gg 1$.

IV. DISCUSSION AND CONCLUSIONS

The harmonic decomposition modeling makes it possible to discriminate the relative importance of the various effects contributing to the nonstationary behavior in the SBS reflectivity, as it can be seen in Fig. 1. By suppressing parts of these effects in different runs, we found that the most important effect is the the IAW harmonics generation: namely, retaining the harmonics generation and neglecting the SBS-induced flow modification lead to results that remain in reasonably good agreement with the complete modeling, whereas, retaining the flow modification, but ignoring the harmonics leads to unphysically high levels of IAW amplitudes. From these observations, it follows that a realistic modeling of SBS requires the proper description of the IAW harmonics.

A. Numerical and computational aspects

Concerning the harmonics description, we found a great sensitivity of the results to the correct evaluation of the “group” velocities v_{gl} and of the frequency shift $\delta\omega_l$, and only to a lower degree to the coupling coefficients p_l of each harmonic. Thus, although early models (see, e.g., Refs. 6, 14, and 37) had clearly pointed out the potential importance of the IAW harmonics generation, they restricted the evaluation of v_{gl} and $\delta\omega_l$ to the limit of the Korteweg–de Vries equation, which leads to less satisfactory agreement than the results obtained with the expressions (6) and (7) when compared with the complete code results.

Let us comment now on the slight discrepancies between the harmonic decomposition results and those obtained with the complete code, as can be observed, e.g., in the SBS reflectivity in Fig. 1, for times beyond $2k_0c_s t \approx 150$ in 1D simulations. By means of extensive simulations, we discovered that these discrepancies are due mainly to the unavoidable dissipation of standard numerical schemes in the complete codes. In particular, most schemes designed to avoid shock formation introduce a numerical dissipation that is typically of viscous type, therefore increasing as k^2 with the wave number k . We therefore came to the conclusion that a complete code needs good precision and high spatial resolution in order (i) to avoid undesired numerical dissipation, and (ii) to resolve correctly the wavelength of the highest IAW harmonic. This is why a simulation carried out with a complete code leads to a numerical expense increasing heavily with the physical time described. By contrast, the harmonic decomposition codes have only to resolve long-wavelength hydrodynamics, so that requirements for precision and spatial resolution can be handled by conventional numerical schemes for hydrocodes.

B. Physical modeling and choice of the physical parameters

In the simulations presented in our article, we restricted ourselves to “exploding foil”-type velocity and density profiles. This choice was just a matter of convenience, making it possible to obtain comparable results without difficulties arising from boundary conditions. Concerning more general realistic situations, our 1D simulations can be thought of as describing the SBS development in a long laser hot spot: as explained in Sec. III B, the 2D simulations parameters have been chosen in such a way that the 2D simulations could be expected to be correctly mimicked by the 1D simulations. On the other hand, exploding foil profiles undergo rarefaction. The latter takes place at both density profile shoulders, progressing from both sides towards the plasma center with the ion acoustic speed c_s , which is close to the IAW group velocities. One could therefore speculate that the rarefaction taking place at the shoulder on the side of the laser light entrance interferes with the SBS-driven IAWs. In order to investigate the role of the rarefaction on the SBS evolution, we carried out additional simulations in which we artificially blocked the rarefaction,³⁸ and we still observed the defects in the spatial wave profiles and the nonstationary behavior of the SBS reflectivity. Thus, we may conclude that these characteristic features are not due to an interplay between the temporally evolving rarefaction and the SBS generated IAWs, but originate from the nonlinear dephasing caused by the coupling of the fundamental IAW component with its harmonics.

We have carried out simulations in the regime of almost undamped IAWs, what requires particularly good precision and robustness of the numerical codes—in contrast to the regimes of heavily damped IAWs or other regimes in which convective SBS amplification is established. In the regime of almost undamped IAWs, the effect of the fluid-type nonlinearity on the SBS growth is particularly important, and we

find that the harmonics generation becomes efficient to reduce the SBS reflectivity as compared with the standard three-wave description whenever the laser intensity $I_L \lambda_0^2$ exceeds the incoherent threshold $(I_L \lambda_0^2)_{\text{inc}}$. In the limit $\Gamma \equiv [I_L \lambda_0^2 / (I_L \lambda_0^2)_{\text{inc}}]^{1/2} \gg 1$, the characteristic plasma length needed for significant SBS growth is $L_{\text{inc}} \equiv \Gamma^{1/3} L_0$, which is longer than the characteristic length L_0 for significant SBS growth in the three wave description. Thus, for given plasma physical parameters and a given plasma length L , the SBS reflectivity can be significantly lower, in the regime $(I_L \lambda_0^2) \gg (I_L \lambda_0^2)_{\text{inc}}$, than in the three-wave description prediction. However, it should be remarked that for given physical parameters of the plasma and given laser intensity, our scaling law for the reflectivity predicts a 100% reflectivity in the limit $L \gg L_{\text{inc}} \approx 2\lambda_0 / [(I_{15} \lambda_{\mu\text{m}}^2) (N_0/n_e)^{2/3}]$. Thus, we find, as in all the previous studies on this subject,^{6,8–11,14} that the harmonics generation cannot saturate just by itself the SBS reflectivity below 100% in the limit of a long plasma.

Our scaling law for the reflectivity is valid in the following limit. (i) The IAW damping ν_a has to be small enough for the condition for absolute instability to be satisfied; namely, $\nu_a / \omega_a < 1 / (k_0 L_0) \approx \lambda_0 / (2\pi L_0)$, with $L_0 / \lambda_0 \approx 0.74 [(n_e / N_0) \times T_e (\text{keV}) (1 + T_i / ZT_e) / (I_{15} \lambda_{\mu\text{m}}^2)]^{1/2}$. (ii) The plasma inhomogeneity has to be weak enough for the amplitude gain factor to be large; namely, $G_{\text{inh}} \equiv \pi / \kappa' L_0^2 \gg 1$, with $\kappa' \equiv k_a L_v$, where L_v denotes the characteristic length of the expansion velocity inhomogeneity $L_v^{-1} \equiv \partial_z \ln(v_0 + c_s)$. Thus, one has $G_{\text{inh}} \approx (1/4) (L_v / \lambda_0) / (L_0 / \lambda_0)^2$, so that the inequality $L_v / \lambda_0 \gg 4 (L_0 / \lambda_0)^2$ has to be satisfied. (iii) The kinetic effects must be negligible.

The main objective of the present article was to check the validity of the harmonic decomposition method in the framework of a fluid description, so that it is beyond the scope of this paper to investigate the relative importance between the nonlinear kinetic effects and IAW harmonic generation. Although it is the subject of work in progress, we may already state: (i) the ion kinetic effects are expected to be ignorable in the limit $3T_i / ZT_e \rightarrow 0$; and (ii) in the limit where the kinetic effects are not ignorable and can be modeled by adding a nonlinear frequency shift of the form $-i\eta |n_1 / N_{\text{eq}}|^{1/2}$ (see Ref. 16) in the propagator appearing in the l.h.s. of Eq. (4) describing the evolution of the IAW components n_l with $l \geq 1$. It is, however, necessary to also take into account the subharmonic generation, because a frequency mismatch is able to enhance the parametric decay of the fundamental IAW component into its subharmonics, as shown in Refs. 15, 18, 20, and 21. (The expression $-i\eta |n_1 / N_{\text{eq}}|^{1/2}$ of the nonlinear frequency shift is valid only during the phase of monotonic growth of the fundamental component n_1 ; it has to be generalized in order to be able to describe the phase of temporally nonmonotonic behavior of n_1 , in a way such as proposed in Ref. 39.) The subharmonic generation is therefore an important issue in the context of kinetic effects. The subharmonics generation can be easily described within the harmonic decomposition method by introducing a finite number of subharmonics together with their own harmonics, denoted as $n_{l/N}$ and behaving as $\exp[i(l/N)k_a z]$, k_a / N being the minimum wave number retained in the harmonic/subharmonic decomposition.

C. Conclusions

In conclusion, we have shown that the SBS modeling presented here, based on the harmonic decomposition of the fluid variables, represents a promising way to describe laser plasma interaction in long scale-length plasmas. We benchmarked our code based on the harmonic decomposition method with a reference code, in the extreme limit of vanishing, or very small IAW damping. A novel picture of SBS arises in which the incoherent superposition of scattered waves generated in distinct spatial domains in the density/velocity profiles leads to a temporally chaotic SBS reflectivity and to a significant reduction in the time averaged reflectivity. We obtained a scaling law for the SBS reflectivity which is in very good agreement with our simulation data in 1D and 2D. The harmonic decomposition description appears to be sufficiently robust and versatile to allow further sophistication such as including (i) a nonlinear frequency shift in order to model kinetic effects and (ii) the subharmonic generation.

Recently, we benchmarked our code HARMONY2D against experimental results obtained at LULI/Ecole Polytechnique, in which SBS was studied from a random phase plates (RPP) smoothed laser beam in a well-characterized 2 mm scale plasma. We took into account all available information concerning the laser beam and the plasma parameters (electron and ion temperature, the plasma velocity and density profiles, in particular). Our results are reported in Ref. 40. We found a very good qualitative and quantitative agreement with the experimental results in the limit of the accuracy corresponding to the limited experimental resolution.

ACKNOWLEDGMENTS

The authors would like to acknowledge fruitful discussions with D. Teychenné, P. Loiseau, R. Sentis, O. Morice, L. Divol, J. Myatt, C. Riconda, H. A. Rose, and W. Rozmus. The numerical simulations were carried out thanks to the access to the IDRIS facilities at Orsay, France.

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- ⁴⁰P. E. Masson-Laborde, S. Hüller, D. Pesme, M. Casanova, P. Loiseau, O. Morice and Ch. Labaune, “Modeling parametric scattering instabilities in large-scale expanding plasmas” *J. Phys. IV* (to be published).