Three Essays on Firm Dynamics and Development

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I hereby declare that this thesis is my own original work.

Li Anpeng

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Abstract

This thesis is a collection of three essays on firm dynamics and development. These essays consider the interaction between the dynamics of allocative efficiency and economic growth from the three different perspectives.

The first essay quantitatively analyses the role of allocative efficiency in explaining growth miracles. It builds a heterogeneous firm model with entry and exit. The model economy converges to a more efficient steady state by selecting more productive firms and reallocating resources to them. Frictions obstruct firm selection and labour reallocation and delay the convergence for decades. Meanwhile, slow efficiency improvement continuously increases productivity and contributes to miraculous growth. In counterfactual experiments, higher-level frictions decrease both the aggregate productivity in the new steady state and the speed of convergence.

The second essay investigates how technological diffusion could shape the high productivity dispersion and exaggerate the so-called allocative inefficiency in emerging economies. It develops a growth model with heterogeneous firms and simulates the dynamics of its productivity distribution during a catch-up process. Firms in the model economy learn about new technology from the world frontier. Their learning speeds differ. When they start to catch up to the frontier, fast learners get close to the frontier in a short time while slow learners remain close to their original low productivity. Consequently, productivity dispersion increases. Furthermore, when adjustment costs exist, marginal productivity is correlated with productivity, so its dispersion increases as well. The economy appears more inefficient. After a long period of learning, slow learners ultimately narrow the gap to frontier. In the new steady state, the productivity dispersion is low again and the economy once again appears efficient even without the reductions to adjustment costs. In the simulation of China, the economy has already passed the bottom of U-shaped pattern. The
productivity dispersion and so-called allocative inefficiency keeps decreasing until convergence. The result suggests that the different productivity dispersions in emerging and developed economies can be a consequence of their different stages of development.

The third essay explores the role of recessions in resource reallocation. It focuses on capital reallocation from the low-productivity state sector to the high-productivity private sector. During recessions, state-owned enterprises liquidate capital to repay debt. Private firms take over the realised resources. This improves allocative efficiency. The timing of a recession is important. In the early stage of transformation, private firms are too small to take over all the liquidated capital. The impact of the insufficient resource reallocation is limited. However, the recession influences the economy for a longer period, so the cumulative welfare gain is large. By contrast, a late recession without fire sales generates a large temporary welfare gain, but a relatively small cumulative welfare gain.
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Empirical evidence suggests that the allocative efficiency of an economy is related to its stage of development. Cross-country studies (Bartelsman, Haltiwanger, & Scarpetta, 2013; Hsieh & Klenow, 2009) show that low productivity firms hold too many resources in emerging economies. This so-called misallocation drives large losses on productivity and output. Conversely, resource allocation in developed economies is closer to optimal distribution. The efficiency difference is able to explain a large proportion of cross-country income differences. Historical analyses reveal a similar relationship between allocative efficiency and the stages of development. For example, Ziebarth (2013) finds that allocative efficiency of the U.S. in the late 19th century is similar to that of China and India currently. The three economies are different in terms of in market structure and institution but similar in levels of development. Now the U.S. economy reaches a more advanced level and its efficiency is much higher.

This thesis studies the relationship between efficiency and development. It is a collection of three self-contained essays that explore the phenomenon in three directions. Chapter 2 analyses the contribution of efficiency improvement in economic growth, particularly in miraculous growth. Chapter 3 studies the reverse impact how a catch-up process influences the so-called allocative efficiency. Chapter 4 studies the mechanism of efficiency improvement and explores how recessions influence
reallocation.

The three chapters adopt the same methodology. First, I build models for each chapter. In Chapter 2, I extend the Hopenhayn model (Hopenhayn, 1992; Hopenhayn & Rogerson, 1993) by adding adjustment frictions and entry/exit barriers. In Chapter 3, I add both technological diffusion and adjustment costs to the Hopenhayn model. In Chapter 4, I model the transformation of low-productivity firms (Song, Storesletten, & Zilibotti, 2011) with an imperfect financial market (Kiyotaki & Moore, 1997). Then, I set the parameter values to match the data. In Chapter 2 and 3, the major parameters are estimated from the firm-level data by the Olley and Pakes’ (1996) method, the ordinary least squares regression, the maximum likelihood estimation (MLE), and the simulated method of moments (SMM). In Chapter 4, parameter values are chosen based on data moments or model assumptions. Finally, I simulate the model economy and study the dynamics. In Chapter 2 and 3, I simulate the transitional dynamics and study the co-movement of economic growth and so-called allocative efficiency. In Chapter 4, I simulate the transformation with different shocks and study the impact of recessions.

The remaining of this chapter provides separate summaries for each essay.

1.2 Allocative efficiency and growth miracles

Many economies have experienced a period of high-speed growth. The causes of such unexpected economic performance remains controversial. In Chapter 2, I post a new hypothesis that the improvement of allocative efficiency plays an important role in miraculous growth. I study firm dynamics in an environment that labour adjustment and firm entry and exit are all costly. The analytical framework is extended from the Hopenhayn model. I estimate the model parameters to match Chinese firm-level data. The firm level productivity is estimated by Olley and Pakes’ (1996) method. The dynamics of productivity is estimated by the ordinary least squares regression. The adjustment, entry, and exit costs are estimated by the SMM. Then, I
simulate the transition from the current stage to the new steady state and decompose the growth to ascertain the contribution of efficiency improvement.

Efficiency improvement is the major cause of the miraculous growth in the model economy. Allocative efficiency is improved when the market selects high-productivity firms and reallocates labour to them. Adjustment costs slow down the process for decades. Meanwhile, slow efficiency improvement continues to contribute to economic growth. The contribution is higher in the early stage of the transition. In the same stage, the economy experiences high-speed growth. Then, efficiency improvement slows down until it finally stops when the economy reaches a new stationary distribution. Miraculous growth disappears as well. In counterfactual experiments, higher-level adjustment costs delay the convergence and lower the aggregate productivity and output in the new steady state.

1.3 Technological diffusion, productivity dispersion, and so-called allocative inefficiency

In Chapter 3, I explore how economic growth shapes the dispersed productivity distribution and inefficient resource allocation in emerging economies. Particularly, the study highlights the role of technological diffusion. I extend the Hopenhayn model by including technological adoption and labour adjustment costs. The estimation strategy is similar to that applied in Chapter 2. I estimate the firm-level productivity by the Olley and Pakes’ (1996) method, the dynamics of productivity by the MLE, and the adjustment costs by the SMM.

I simulate the economic transition from a low-tech steady state to a high-tech steady state and analyse the dynamics of the dispersion of labour productivity. During the catch-up process, the dispersion of productivity increases at the beginning, and then, decreases to a low level again in the new steady state. The adjustment costs remain constant over time, so the dynamics of the dispersion is purely driven by the technological diffusion. In the economy, firms learn about new technology from the
world frontier. Their learning speeds differ. In the transition, fast learners catch up to the frontier very quickly while slow learners remain close to their original low technology. The dispersion of productivity increase. Then, slower learners also move closer the frontier after a long period of learning. The distribution of productivity slowly narrows until it finally reaches the stationary distribution. Furthermore, firms face adjustment costs and delay labour adjustment as a response, so the marginal productivity of labour comoves with the productivity dynamics. The pattern of the dispersion movement is U-shaped too. This indicates that the cross-country differences on productivity dispersion may be partially driven by the different stages of development.

In the simulation of China, the economy has already passed the bottom of the U-shaped curves. The productivity dispersion keeps decreasing until converge. The resource allocation also becomes more efficient.

1.4 Reallocation through recession

In Chapter 4, I study how recessions influence the transformation of state-owned enterprise (SOEs). SOEs are less productive than private firms in general, but they dominate the economies of many countries, such as pre-reform Russia or China (Djankov & Murrell, 2002; Megginson & Netter, 2001; Murrell, 1993; Rodrik, 2006). Hence, the issue of how to reallocate resources from SOEs to private firms is a key to improve the efficiency of these countries. A number of literature (see the reviews written by Hopenhayn, 2014a; Restuccia & Rogerson, 2017) consider how reducing distortions improves allocative efficiency. This chapter introduces a new channel whereby recessions can boost the capital allocation from SOEs to private firms.

I model the capital reallocation from SOEs to private firms (Song et al., 2011) with an imperfect financial market (Kiyotaki & Moore, 1997). The parameter values are chosen to match the transformation of SOEs in China. I simulate the transformation of the model economy with negative shocks, and then, analyse the impact of these
shocks on capital reallocation and social welfare. I also compare the influence of the recessions at different stages of the transition. Some model assumptions and estimations are simplified and will be adjusted in future works.

At the beginning of the transformation, SOEs hold most capital of the economy. However, private firms have higher productivity, so they accumulate capital faster and ultimately dominate the market. Recessions boost the transformation. When SOEs are affected by a negative shock, they liquidate capital to repay debt. Private firms are more productive but face the same collateral constraint, so they have more liquidity and are able to purchase the liquidated capital. Consequently, the liquidation market improves allocative efficiency and boosts long-run growth. While recessions cause a temporary welfare loss, the cumulative effect is positive.

The timing of a recession is important. In the early stage of transformation, private firms are too small to take over all the liquidated capital. In this scenario, a recession causes a large temporary welfare loss and a small efficiency gain. However, the recession boosts the transformation from an early period, so the cumulative welfare gain is relatively large. By contrast, a late-arriving recession can fully reallocate the liquidated capital to private firms, resulting in a small temporary welfare loss and a large efficiency gain. However, since the recession only influences the late segment of the transition, the cumulative welfare gain is relatively small. The opposite short-run and long-run effects reveal the importance of the time horizon in welfare and policy analyses.

1.5 Organisation

This thesis is organised as follows. Chapters 2 to 4 present the core research materials. Chapter 2 evaluates the role of allocative efficiency in relation to growth miracles. Chapter 3 studies the relationship between technological diffusion and productivity dispersion. Chapter 4 explores how recessions influence resource reallocation. Finally, chapter 5 sets out the conclusions that can be drawn from these essays.
The road to efficiency: allocative efficiency and growth miracles

2.1 Introduction

Many economies experience dramatic high-speed growth for decades, like the Chinese economic boom since 1978. The explanations of these growth miracles are still controversial. This paper posts a new hypothesis that growth miracle is also a process of slow efficiency convergence. Transitional economies increase efficiency by selecting high-productivity firms and reallocating resource to them. Costly labour adjustment and firm selection delay the efficiency improvement. During the slow convergence, efficiency improvement keeps boosting productivity growth. Eventually, the economic growth slows down while reaching steady state.

The study is motivated by two stylised facts. First, developed economies are more efficient than emerging countries in the sense of resource allocation across firms and selection of good firms. The phenomenon has been shown in many cross-country empirical studies (e.g. Akcigit, Alp, & Peters, 2016; Bartelsman, Haltiwanger, & Scarpetta, 2013; Hsieh & Klenow, 2009). The difference indicates that the process of catch-up is also a process of efficiency improvement. Second, efficiency improvement takes time. It is much slower than the prediction of neoclassical models (Buera & Shin, 2013). For example, China, the largest emerging economy, experienced miraculous growth for four decades, but its allocative efficiency is still much lower than in
the United States (Hsieh & Klenow, 2009). India, the second largest emerging economy, has enjoyed economic boom since 1991. Yet, its current allocative efficiency is also significantly lower than in the United States (Bils, Klenow, & Ruane, 2017). The slow efficiency improvement can contribute to growth for a long time.

In this paper, I build a model which endogenously generates temporary high speed growth. The economy comprises heterogenous firms with entry and exit, as set by Hopenhayn and Rogerson (1993). Firms face productivity shocks and incumbents adjust labour or exit the market as a response. The labour adjustment is costly, so firms cannot adjust labour immediately to the optimal level. The productivity shocks are persistent, so they will adjust labour to a more efficient level in the long run. When a shock is very negative, firms can exit the market as a response. It is also costly. In addition to the exit cost, firms have to pay adjustment costs to lay off all the employees. When the exit costs are higher than the deficit, a firm will stay in the market even if it is unprofitable. On the other hand, the economy allows entry of new firms. Potential entrants make the decision of entry based on the expected value of a new firms and entry costs. Entry costs comprise two elements. First, an entrant has to pay a fixed cost to release its productivity and the value of the firm. Second, it has to pay the adjustment costs for its initial recruitment. These costs block the entry of new firm, and reduce the size of the market. Because of the costly entry, exit, and labour adjustment, efficiency improvement takes decades in the simulations. Meanwhile, the increasing efficiency consistently contributes to economic growth.

The estimation of value of parameters targets Chinese manufacturing firms. The estimation is divided into three stages for the sake of reducing computational complexity and increasing robustness. First, discounting factor, labour supply, and initial distribution are predetermined as the real values in the data. Second, the firm-level productivity is estimated using the Olley and Pakes’ (1996) method. The stochastic process of productivity is estimated using a regression on the firm-level productivity
panel. Third, since the model does not have an analytical form solution, the parameters of adjustment costs, entry cost, and exit cost are estimated by simulated method of moment (SMM). The simulation with all the estimates shows a similar pattern of firm dynamics as in the real data.

I simulate the firm dynamics after 2007, the most recent year before the Global Financial Crisis. The simulated economy converges to a more efficient stationary distribution. The economy enjoys high-speed growth during the convergence process and stops growing up when reaching steady state.

I decompose the growth to see the contribution of efficiency improvement. In the model economy, growth is driven by five engines: (i) improvement of allocative efficiency, (ii) expansion of market size, (iii) exit of low-productivity firms, (iv) entry of high-productivity firms, and (v) productivity progress of survival firms. The growth decomposition shows that allocative efficiency plays the most important role in the periods of growth miracle.

I run counterfactual experiments with different levels of adjustment costs. In the experiments, adjustment costs influence both the convergence speed and the allocative efficiency in the new steady state. When labour adjustment and firms selection are cost-free, the economy directly reaches the optimal distribution. Otherwise, the transition takes time. Higher adjustment costs lead to slower convergence. In the new steady state, firms still face idiosyncratic shocks and have to adjust labour or exit the market. Higher costs reduce the efficiencies of resource reallocation and firm selection, so reduce the aggregate productivity in the steady state.

The literature suggests that growth miracles happens when an economy converges to its balanced growth path (e.g. Hausmann, Pritchett, & Rodrik, 2005). This study follows this line of thought and suggests a new channel, efficiency improvement during the convergence. It links the discussion of growth miracles to recent literature on efficiency and productivity (Restuccia & Rogerson, 2008; Hsieh & Klenow, 2009; Bartelsman et al., 2013).
Buera and Shin (2013), and Moll (2014) study reallocation dynamics and productivity growth and both focus on financial frictions. In both studies, frictions do not exercise much influence on the long-run performance, as firms could release financial constraint by self-saving (Kiyotaki & Moore, 1997). Conversely, in the model of this study labour adjustment costs still matter in steady state. The result seems more consistent with empirical evidence from labour market (Prescott, 2004).

The rest of the paper is organised as follows. In Section 2, I build a model of firm dynamics. In section 3, I introduce a productivity accounting approach for the model. In Section 4, I describe the estimation of the parameters of the model. In Section 5, I present the simulations of reallocation dynamics and growth of the model economy. In Section 6, I demonstrated the impact of frictions in counterfactual experiments. Finally, Section 7 concludes the paper.

2.2 Model

I construct a heterogeneous firm model with adjustment frictions and productivity shocks, which is built on the Hopenhayn model (Hopenhayn, 1992; Hopenhayn & Rogerson, 1993).

Households.– A measure 1 of households live in the economy. Every household provides a measure $L$ of labour. The households are consumers, labour providers, and owners of establishments in the economy. Since the study focuses on firm behaviour, I simplify the household behaviour by three assumptions. First, the economy only produces a nondurable good, so the households have to consume all the products at the end of each period. Intertemporal transfer does not exist in the economy. Second, leisure does not generate utility, so the labour supply is perfectly inelastic. Third, households own all the establishments and equally distribute profits, so the welfare maximisation problem is equivalent to the output maximisation problem.
Incumbent firms—A firm is characterised by its productivity $z_{i,t}$ and last-period labour $l_{i,t-1}$. It uses its output to pay the wage $w_t l_{i,t}$, and the adjustment costs $c(l_{i,t}, l_{i,t-1})$. When a firm with productivity $z_{i,t}$ and last-period labour $l_{i,t-1}$ employs $l_{i,t}$ labour at time $t$, it gets one-period profit:

$$\pi(z_{i,t}, l_{i,t}, l_{i,t-1}; w_t) = f(z_{i,t}, l_{i,t}) - w_t l_{i,t} - w_t c(l_{i,t}, l_{i,t-1}).$$

(2.1)

The production technology is Cobb-Douglas with labour as the only input:

$$f(z_{i,t}, l_{i,t}) = z_{i,t}^{\alpha} l_{i,t}^{\alpha} \quad \alpha \in (0, 1).$$

(2.2)

The analysis focuses on the efficiency of labour allocation, so I neglect other production factors. I assume a decreasing return to scale technology for the sake of keeping the existence of low-productivity firms. In addition, the assumption is consistent with empirical evidence. The estimate of the output elasticity of labor is smaller than one. Each firm faces an idiosyncratic productivity shock in each period. The shock follows an AR(1) process:

$$z_{i,t} = \bar{z}^{1-\rho} z_{i,t-1}^\rho \ln \epsilon_{i,t} \sim_{i.i.d.} N(0, \sigma^2), \quad \rho \in (0, 1).$$

(2.3)

The range of $\rho$ makes the process stable. Then, the joint distribution of productivity and labour gets convergence. The assumption also makes the process mean reverting. When the average productivity is smaller than $\bar{z}$ at the initial stage, it will keep increasing on the road of convergence. Furthermore, the assumption of the range is consistent with empirical evidence. The estimate of $\rho$ is in this range. Parameter $\rho$ also measures the persistency of idiosyncratic productivity. If $\rho$ is closer to one, the productivity shock is more persistent, and then, firms are more willing to adjust labour to fit the productivity shocks.

Labour adjustment is costly. When firms adjust labour, they have to pay adjustment costs. The costs are proportional to wage. When wage is higher, labour
adjustment cost is also higher. This study focuses on the consequence rather than cause of frictions, so I use broadly defined adjustment costs to represent many types of labour market frictions, such as hiring cost (Oi, 1962), firing cost (Hopenhayn & Rogerson, 1993), and search friction (Cooper, Haltiwanger, & Willis, 2007). The labour adjustment behaviour is diverse among Chinese firms. Cooper, Gong, and Yan (2015; 2017) find many firms adjust their labour smoothly and continuously. However, a large number of firms do not adjust labour. To fit the pattern, I use a rich setup of adjustment costs in this study, following Cooper and Haltiwanger (2006) and Bloom (2009). The costs contain three components. The first is the disposable fixed cost, \( c_0 \delta_{i,t} \neq \delta_{i,t-1} \). Firms have to pay the cost when they hire/fire employees regardless of the quantity of hiring/firing. When the fixed cost is high, the majority of firms do not adjust labour, while a fraction of firms make a huge labour adjustment at one period. The second component, \( c_1 |\Delta I_{i,t}| \), is proportional to the gross firing/hiring. For example, training costs are proportional to the number of new employees. Unemployment compensations are proportional to the number of unemployed workers. The proportional cost also can generate inaction like the fixed cost does. The third component is the quadratic cost \( c_2 \frac{(\Delta I_{i,t})^2}{I_{i,t} + I_{i,t-1}} \), which makes the sharp labour adjustment costlier. If the quadratic costs are high, most firm will smoothly adjust their labour.

The total adjustment costs are the summation of the three components:

\[
c_{ad}(I_{i,t}, I_{i,t-1}) = c_0 \delta_{i,t} \neq \delta_{i,t-1} + c_1 |\Delta I_{i,t}| + c_2 \frac{(\Delta I_{i,t})^2}{I_{i,t} + I_{i,t-1}}.
\]

(2.4) Firing and hiring costs are assumed to be symmetric for the sake of reducing computational complexity of the latter estimation.

Firms will exit the market when continued operation is unprofitable. They consider the following Bellman equation for the decision of exit and labour adjustment,
\[ V_t(z_{i,t}, l_{i,t-1}; \bar{w}^t) = \max_{l_{i,t} \geq 0} \{ \pi(z_{i,t}, l_{i,t}, l_{i,t-1}; w_t) \]
\[ + \beta \max \{ \mathbb{E} V_{t+1}(z_{i,t+1}, l_{i,t}; \bar{w}^{t+1}), -w_t c_{ad}(0, l_{i,t-1}) - w_t c_{ex} \} \}, \tag{2.5} \]

where \( \bar{w}^t \) is the wages from period \( t \) to infinity. Firms make the exit decision at the end of every period based on the comparison between the expected future value, \( \mathbb{E} V(z_{i,t+1}, l_{i,t}; \bar{w}^{t+1}) \), and the exit cost, \( -w_t c_{ex} \), plus the cost for laying off all employees, \( -w_t c_{ad}(0, l_{i,t-1}) \). If operation is costlier, a firm will fire all employees and exit the market.

The decision of exit is determined by two costs, adjustment and exit costs. Adjustment costs influence exit decision in two ways. First, if adjustment costs do not exist, firms with decreasing return to scale production technology will never exit the market. When they are hit by a negative shock, they can make positive profit by reducing labour. Adjustment frictions obstruct prompt labour adjustment, so generate the possibility of negative profit and exit. Second, adjustment costs also block firm exit since firms have to consider firing costs when they plan to exit. Exit cost, \( w_t c_{ex} \), measures the rest of the costs that firms have to pay when they exit the market (e.g., loss in fire sales). Positive exit cost makes more firms stay in the market. It is assumed to be proportional to wage. When wage is higher, exit cost is higher too.

**Entering firms.**– The economy also allows entry of new firms. New firms can be created by paying an entry cost, \( c_{en} \). It is also assumed to be proportional to current wage, \( w_t \). Potential entrants realise their productivity after paying the cost, so the decision of entry is based on the expected value of a new firm. The value is equal to the expected value of an incumbent with zero existing labour:

\[ V_{t}^e = \mathbb{E} V_t(z_{i,t}, 0; \bar{w}^t) - w_t c_{en}. \tag{2.6} \]
Firms can freely enter the market, so the value of entry is zero in equilibrium,

\[ V_t^e = 0. \]  \hspace{1cm} (2.7)

The free entry condition determines the measure of entering firms.

*Time line.* Figure 2.1 describes the time line of a firm. When the firm decides to enter the market, it pays the entry cost at the beginning of the period. Then, the firm realises its productivity, makes the employment decision, and produces goods. The exit decision is made at the end of each period. If it choose to stay, the process will restart from realise the productivity at the beginning of the next period.
Distribution dynamics.– The firm distribution of firms updates every period until reaching the new steady state. The law of motion is:

\[ \mu_t = T_t(\mu_{t-1}) + \nu_t, \]  

(2.8)

where \( \mu_t \) be the joint distribution of labour and productivity at period \( t \), \( \nu_t \) is the distribution of the new entrants at period \( t \), and \( T_t \) is the transitional function of incumbent firms which is solved from the Bellman equation (2.5).

Equilibrium.– This study analyses both the steady state and the transitional path. Definition 1 defines the stationary equilibrium in the steady state.

**Definition 1** A stationary equilibrium of the model is a wage system, which satisfies the following conditions: (i) household optimisation; (ii) firm optimisation; (iii) labour market clear; (iv) free entry \( V^e(\bar{w}_t) = 0 \); and (v) invariant distribution over time.

Definition 2 defines the recursive competitive equilibrium on the transitional path.

**Definition 2** A recursive competitive equilibrium of the model is a wage system, which satisfies the following conditions: (i) household optimization; (ii) firm optimization; (iii) labour market clear; (iv) free entry \( V^c(\bar{w}_t) = 0 \); and (v) law of motion \( \mu_t = T_t(\mu_{t-1}) + \nu_t \).

When the free entry condition holds and the entry cost keeps constant in the transitional path, wage is always equal to the steady-state wage. The computation is simplified because of the constant wage.

### 2.3 Growth decomposition

I decompose economic growth of the model economy to identify the quantitative importance of efficiency improvement. I calculate the contribution of improving allocative efficiency à la Hopenhayn (2014b). Furthermore, I decompose other growth channels partially following Melitz and Polanec (2015).
Five engines can drive economic growth in the model economy. Since the aggregate labour supply is fixed, all the growth engines work on increasing the aggregate productivity. The first is the productivity growth of the incumbent firms. This paper studies labour productivity, so the productivity growth contains both technology progress and capital accumulation. The second is the exit of the low-productivity firms. The selection of exit firms will boost average productivity growth if these firms are less productive than the survivors on average. The third component is the entry of the high-productivity firms. The entry of new firms raises the aggregate productivity if the entrants are, on average, more productive than incumbents. The forth is the improvement of allocative efficiency. The aggregate productivity will increase if resources are reallocated from the low-productivity firms to the high-productivity firms. The fifth growth engine is expansion of the market size. When more firms produce in the market, the average size of these firms will be smaller. Then, marginal productivity will be higher on average since the technology is decreasing return to scale.

To set an efficient economy as the benchmark, I assume a social planner who can reallocate resource without any costs. As the paper focuses on resource allocation rather than market selection, the social plan takes productivity distribution and the number of firms as given and only adjusts labour allocation. Given the productivity distribution, the distance from the aggregate productivity in reality to the aggregate productivity in the social planner economy measures the allocative inefficiency of the economy. The social planner solves the following problem:

$$\max_{l_i,t} \sum_i z_i l_i^a,$$

subject to

$$\sum_i l_i,t \leq L.$$
The social planner problem has an analytical solution. The maximised output is:

$$Y = \left( \bar{z}_{i,t} \cdot N_t \right)^{1-\alpha} L^\alpha,$$

in which

$$\bar{z}_{i,t} \equiv z_{i,t}^{\frac{1}{1-\alpha}},$$

and $N_t$ is the number of firms at period $t$. A competitive equilibrium without any frictions also can achieve this result. The allocative efficiency can be measured by a relative productivity:

$$A_{rel,t} \equiv \frac{A_{sim,t}}{A_{opt,t}},$$

where $A_{sim,t}$ is the actual aggregate productivity of the simulated economy at the same period. Then, the growth of aggregate productivity $\Delta \ln(A_{sim,t})$ can be decomposed by two components:

$$\Delta \ln(A_{sim,t}) = \Delta \ln(A_{rel,t}) + \Delta \ln(A_{opt,t}).$$

The first component measures the contribution of improving allocative efficiency.

According to equation (2.9), $A_{opt,t}$ can be further decomposed by two parts, $N_t$ and $\bar{z}_{i,t}$:

$$\Delta \ln(A_{sim,t}) = \Delta \ln(A_{rel,t}) + (1-\alpha)\Delta \ln(N_t) + (1-\alpha)\Delta \ln(\bar{z}_{i,t}).$$

The second term in equation (2.12) represents the size effect. Larger $N_t$ indicates smaller average size and higher marginal productivity on average. The last term in equation (2.12) measures the productivity effect. It could increase in three ways: productivity growth of the incumbent firms, exit of the low-productivity firms, and entry of the high-productivity firms. Equation (2.13) decomposes the three channels:
\[ \Delta \ln(A_{\text{sim},t}) = \underbrace{\Delta \ln(A_{\text{rel}})}_{\text{allocative efficiency}} + (1-\alpha) \Delta \ln(N_t) + (1-\alpha) \Delta \ln(\mathbb{E}_{z_i,t}) \underbrace{\ln(\mathbb{E}_{z_i,t}/\mathbb{E}_{z_i,t-1})}_{\text{technology progress}} + (1-\alpha) \ln(\mathbb{E}_{z_i,t-1}/\mathbb{E}_{z_i,t-1}) \underbrace{\ln(\mathbb{E}_{z_i,t-1}/\mathbb{E}_{z_i,t-1})}_{\text{firm entry}}. \] (2.13)

The last term does not include all impacts of firm entry. This channel boosts growth only if the productivity of the entering firms are higher than the productivity of incumbent firms. Firm entry also increase aggregate productivity by increasing the market size. This impact is in the second component. For the same reason, the forth term of equation (2.13) is also not the full impact of exit.

### 2.4 Estimation

The target of the estimation is fitting the dynamic pattern of Chinese manufacturing firms. I estimate the parameter values in three stages for the sake of robustness and computational simplicity. First, I predetermine some parameters directly from values in the data. Second, I estimate the firm-level productivity using Olley and Pakes’ (1996) method and the productivity dynamics using regression. Third, I estimate the adjustment costs by SMM.

#### 2.4.1 Data

The estimation targets the pattern of firm dynamics in reality. The data are from the Chinese Industrial Enterprises Database, an annual survey of Chinese industrial firms. It is an imbalanced panel data containing all public firms and private firms which annual sales larger than five million RMB. The data are frequently used in the analysis of productivity and resource allocation (e.g. Hsieh & Klenow, 2009; Song, Storesletten, & Zilibotti, 2011). I select only manufacturing firms to ensure homogeneity of firms. The model is not designed for business cycles, so I select the
Table 2.1: Data descriptives

<table>
<thead>
<tr>
<th>Year</th>
<th>Labour Mean</th>
<th>Std. dev.</th>
<th>Output (million RMB) Mean</th>
<th>Std. dev.</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>241.65</td>
<td>295.80</td>
<td>7.12</td>
<td>12.11</td>
<td>125894</td>
</tr>
<tr>
<td>2001</td>
<td>211.56</td>
<td>266.51</td>
<td>7.03</td>
<td>12.14</td>
<td>140125</td>
</tr>
<tr>
<td>2002</td>
<td>204.73</td>
<td>257.73</td>
<td>7.54</td>
<td>13.00</td>
<td>148694</td>
</tr>
<tr>
<td>2003</td>
<td>198.76</td>
<td>249.85</td>
<td>8.70</td>
<td>15.18</td>
<td>167238</td>
</tr>
<tr>
<td>2004</td>
<td>171.19</td>
<td>232.02</td>
<td>NA</td>
<td>NA</td>
<td>245765</td>
</tr>
<tr>
<td>2005</td>
<td>173.50</td>
<td>215.06</td>
<td>11.70</td>
<td>20.58</td>
<td>230898</td>
</tr>
<tr>
<td>2006</td>
<td>164.59</td>
<td>204.28</td>
<td>13.91</td>
<td>24.51</td>
<td>257143</td>
</tr>
<tr>
<td>2007</td>
<td>158.21</td>
<td>196.33</td>
<td>16.65</td>
<td>28.86</td>
<td>290160</td>
</tr>
</tbody>
</table>

period from 2000 to 2007 to avoid influences from the Asian Financial Crisis and Global Financial Crisis. The data in 2000 are the baseline of the estimation. The data processing mainly follows Brandt, Van Biesebroeck, and Zhang (2014).

Table 2.1 displays the data by years. The number of firms keeps increasing throughout the whole period. This increases faster than labour supply, so the average size of firms keeps decreasing. However, the lower average input (labour) does not block output growth, rather average output increases to more than double the original value during the data periods. This indicates massive productivity growth in the economy. The simulations in the next section will attempt to fit these patterns.

2.4.2 Predetermined parameters

Other parameters determined before estimation are shown in Table 2.2. All the values are straightforwardly from the real data. The discounting factor $\beta$ is chosen to match the average annual interest rate from 2000 to 2007. Other values are based on the data in 2007, the baseline year of the simulation (the most recent year before the Global Financial Crisis). The labour supply $L$ is the total employees of all firms. The initial distribution $\mu_0$ is the real distribution of the 2007 data.
The road to efficiency: allocative efficiency and growth miracles

Table 2.2: Value assignment of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discounting factor</td>
<td>0.96</td>
<td>Annual interest rate</td>
</tr>
<tr>
<td>$L$</td>
<td>Labour supply (million)</td>
<td>45.91</td>
<td>Total labour supply in 2007</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Initial distribution</td>
<td>NaN</td>
<td>Real distribution in 2007</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Output elasticity of labor</td>
<td>0.64</td>
<td>Chinese firm-level panel data, (\ln(\bar{z})) AR(1) constant from 2000 to 2007</td>
</tr>
<tr>
<td>$\rho$</td>
<td>AR(1) coefficient</td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation of AR(1) shock</td>
<td>2.03</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>Fixed adjustment costs</td>
<td>0.21</td>
<td>Moments of Chinese firm-level data</td>
</tr>
<tr>
<td>$c_1$</td>
<td>Proportional adjustment cost</td>
<td>0.29</td>
<td>See table 2.3</td>
</tr>
<tr>
<td>$c_2$</td>
<td>Quadratic adjustment costs</td>
<td>1.81</td>
<td></td>
</tr>
<tr>
<td>$c_{ex}$</td>
<td>Exit cost</td>
<td>163.58</td>
<td></td>
</tr>
<tr>
<td>$c_{en}$</td>
<td>Entry cost</td>
<td>385.45</td>
<td></td>
</tr>
</tbody>
</table>

2.4.3 Productivity: Olley and Pakes’ (1996) method

I estimate the firm-level productivity, $z_{i,t}$, using Olley and Pakes’ (1996) approach, and then, run a regression on the firm-level panel data to get the parameters of the stochastic process of technological progress. The results are shown in Table 2.2.

The literature highlights the endogeneity problem of productivity estimation, that input level is correlated with unobserved productivity shocks. Olley and Pakes (1996) introduced investment as a proxy of unobserved shocks and develop a consistent semiparametric estimator. This study uses this approach to estimate labour productivity. When adjustment costs exist, labour is no longer a freely variable input. Thus, labour is treated the same as capital in the original version. Based on the same argument Olley and Pakes (1996) made on investment, I use the change of employment as a proxy of unobserved productivity shocks. I control for other production factors in the productivity regression. Theoretically this allows a consistent estimator of the output elasticity of labour $\hat{\alpha}$. As shown in Table 2.2, the estimate is close to the labour income share and most estimates in the literature. The productivity is
calculated from output, labour, and estimated output elasticity of labour:

\[ \hat{A}_{i,t} = \left( y_{i,t} \cdot L^{-\hat{\alpha}}_{i,t} \right)^{\frac{1}{1-\hat{\alpha}}}. \]

The productivity shocks are independent and identically distributed as assumed in equation \ref{eq:2.3}, so the AR(1) process can be estimated simply by a regression on the lagged productivity. The results are shown in the Table \ref{tab:2.2}.

2.4.4 Frictions: Simulated method of moments

Since the model does not have an analytical form solution, I estimate adjustment costs, entry cost, and exit cost by SMM, a simulation-based estimation. The main idea is selecting parameters to minimise the weighted distance between the moments of simulated data and real data (McFadden 1989, Pakes & Pollard 1989). The estimator is solved from the following minimisation problem:

\[ \hat{\theta} = \arg \min_{\theta \in \Theta} [M_{real} - M_{sim}(\theta)]^\prime W [M_{real} - M_{sim}(\theta)]. \]

where \( \theta \) is the set of parameters, \( M_{sim}(\theta) \) is the moments of the simulation with parameter \( \theta \), \( M_{real} \) is the moments of the data, \( W \) is the optimal weight matrix Lee and Ingram (1991) provided. Under the null hypothesis, the real data and simulated data are independent and from the same data generation process. Based on this condition, Lee and Ingram (1991) proved that the efficient weight matrix is a function of the inverse of the variance-covariance matrix of \( [M_{real} - M_{sim}(\theta)] \). The variance-covariance matrix is estimated by bootstrap method.

As shown in Table \ref{tab:2.3}, seven target moments are chosen for the five parameters. All the moments describe labour adjustment and exit/entry of firms. They do not directly target the relationship between labour and output, that is, they do not directly target productivity. This is to avoid overfitting. The first moment is the proportion of firms that do not adjust labour. For the sake of consistency and for robustness, I
ignore the small labour adjustments within the grids set in the simulations. Hence, the data moment here is larger than the value of the real data. The second moment is the first-order autocorrelation of labour adjustment. The third is the same autocorrelation across the firms with non-zero labour adjustment. The two moments measure firms’ intertemporal decision on labour adjustment. The fourth moment is the average ratio of labour adjustment to existing labour. The fifth is the same ratio across the firms with non-zero labour adjustment. If the non-convex adjustment costs are high, the second and third moments would be low, while the fourth and fifth would be high. If the convex (quadratic) adjustment cost are high, the second and third would be high, while the fourth and fifth would be low. The last two moments are the average exit and entry rates across the simulation periods. The two describe firm entry and market selection.

The simulation process is as follows. The real distribution of 2000 data is taken as the initial distribution of labour and productivity, and then, firm dynamics are simulated based on the model and given parameters. The first eight periods of the simulations are taken to calculate the moments. In this way, the simulated moments can match the moments of real data from 2000 to 2007.

Table 2.3 reports the data moments and simulated moments. Among the seven moments, five fit data well that are the proportion of firms that do not adjust labour, the first-order autocorrelation of the ratio of employment to labour stock (all the firms and adjusting firms only), entry and exit rate. On the other hand, two moments do not fit data very well that are the average ratio of labor adjustment (all the firms and adjusting firms only). The two simulated values are nearly a half of the real values. As I use five variables to match seven moments, it is understandable to get five good matches only. The estimates are shown in Table 2.2. The parameter of the quadratic term is the largest among the three components of adjustment costs. The impact of this component is also large in the later quantitative analysis. The parameter of the fixed adjustment cost is the smallest. In other words, proportional component
dominates the non-convex adjustment costs. Entry and exit costs are much higher than adjustment costs. They are compared to the future value of a firm, so have to be large enough to generate impact.

### 2.5 Reallocation dynamics

I simulate the firm dynamics after 2007. The simulation does generate a temporary high-speed growth, mainly because of the efficiency improvement.

#### 2.5.1 Distorted labour reallocation

Figure 2.2 plots the labour adjustment decision of firms in the simulated economy. The horizontal axis is the productivity labour ratio $z_{i,t-1}$, which measures the imbalance between productivity and labour. Higher $z_{i,t-1}$ indicates higher demand of labour. The vertical axis is the labour adjustment ratio, $l_{i,t}/l_{i,t-1}$, $(l_{i,t} + l_{i,t-1})$ is used as the denominator rather than $l_{i,t-1}$ or $l_{i,t}$ as the latter two can be zero. Both ratios are rescaled by a concavification transformation, $\tilde{x} = \ln(x + 1)$.

The pattern of labour adjustment is clear, while not smooth enough because of the numerical error. First, the relationship between the two ratios is positive. A firm is more likely to hire more labourers when its productivity is higher among same size firms, and vice versa. Second, the relationship is nonlinear. Compared with a middle-scale imbalance between productivity and labour, large scale imbalance will not generate same scale labour adjustment. This is driven by the quadratic
adjustment cost, which punishes large scale hiring/firing. Third, the middle segment of the curve is flat. Firms adjust labour only if the productivity shock exceeds a threshold. Although the inaction region is not large, the impact can be big. In the benchmark economy, 25.39% of firms do not adjust their labour. This is because the idiosyncratic shock $\epsilon$ concentrates around one, so most shocks are small. The inaction region is driven by both proportional and fixed adjustment costs, but mainly from the proportional component. If the disposable fixed component dominates the decision, firms would hire/fire a lot once they exceed the threshold of action since they want to divide the fixed cost. However, the graph is continuous around the thresholds. It is consistent with the small fixed cost in the former estimation.

The decision rules indicate labour hoarding behaviour. If the negative shock is not large enough, firms tend to hoard labour to avoid adjustment costs. Even if the economy faces an aggregate negative shock, the unemployment rate might not
Figure 2.3: Dynamics of the number of firms.

increase sharply, such as what happened in Germany after the Global Financial Crisis (Burda & Hunt, 2011).

2.5.2 Reallocation dynamics

The simulation captures basic features of firm dynamics in the real data, although that is not the main target of the analysis. Figure 2.3 shows the dynamics of the number of firms. The market size keeps growing in the simulation at the early stage. There are large inflows and outflows in the market, but firm entry dominates the change of market size. The pattern is the same as in the real data shown in Table 2.1. Then, the increasing trend slows down. Finally, the market size starts to become stable.

Labour reallocation is summarised in Figure 2.4. At the beginning, a lot of firms enter the market. Most of these firms are small businesses since they cannot grow up soon with adjustment frictions. The large entry pulls down the size distribution, and makes the average firm size decreases as the pattern of the real data. However, after the market size becomes stable, labour slowly moves from large firms to small but more productive firms. Finally, the labour allocation converges to a stationary distribution after many years.
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The firm dynamics generates a short period high-speed growth, as shown in Figure 2.5. The sharply increasing productivity drives the same level output growth, since labour supply is constant in the economy. The increasing trend in the early periods is consistent with the real data. For the future, the model predicts the end of the growth miracle when the economy reaches the steady state.

2.5.3 Growth decomposition

The model generates a temporary high-speed growth. Now I decompose the miraculous growth and identify the contribution of efficiency improvement. Following equation (2.13), the productivity growth is decomposed by five components: (i) improving allocative efficiency, (ii) increasing market size, (iii) technological growth of incumbent firms, (iv) entry of high-productivity firms, and (v) exit of low-productivity firms. Figure 2.6 shows the result of the decomposition during the periods of miraculous growth. The allocative efficiency plays the most important role at the beginning of miraculous growth. Then, its absolute contribution decreases along with the decreasing growth rate of aggregate productivity, but its relative contribution remains high. This is supportive to the hypothesis of efficiency-driven miraculous growth and consistent with some previous studies (e.g. Song et al., 2011). The increasing market
size makes the second largest contribution of productivity growth. Furthermore, its relative contribution becomes more important in the later periods. However, its contribution is still lower than the contribution of efficiency improvement on average.

2.6 The role of frictions

Frictions are crucial in the model. Without any frictions, the economy will always stay with the first-best resource allocation. The transitional will be much faster than in the real data. In this section, I analyse the impact of adjustment frictions. I simulate economies with different levels of frictions from the same initial distribution, and compare the simulations. I consider four levels of adjustment costs: cost-free, half of the benchmark costs, the benchmark costs, and double of the benchmark costs. The analysis focuses on the impact on the new steady state and the convergence speed.

Table 2.4 summarises the simulations on the new steady state. In these experiments, higher frictions lower the aggregate productivity in the future steady state. The impact is from both allocative efficiency, $A_{sim}/A_{opt}$, and optimal productivity, $A_{opt}$. 

---

Figure 2.5: Dynamics of aggregate productivity.
The road to efficiency: allocative efficiency and growth miracles

Figure 2.6: Growth decomposition during the periods of miraculous growth.

Table 2.4: Counterfactual experiments on adjustment frictions

<table>
<thead>
<tr>
<th></th>
<th>Adjustment frictions</th>
<th>None</th>
<th>Half</th>
<th>Benchmark</th>
<th>Double</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate productivity (A_{\text{sim}} \times 10^3)</td>
<td></td>
<td>3.97</td>
<td>2.78</td>
<td>1.89</td>
<td>1.10</td>
</tr>
<tr>
<td>Optimal productivity (A_{\text{opt}} \times 10^5)</td>
<td></td>
<td>3.97</td>
<td>3.36</td>
<td>2.61</td>
<td>1.83</td>
</tr>
<tr>
<td>Average productivity (10^9)</td>
<td></td>
<td>3.01</td>
<td>1.94</td>
<td>1.94</td>
<td>1.99</td>
</tr>
<tr>
<td>Number of firms (million)</td>
<td></td>
<td>1.46</td>
<td>1.41</td>
<td>0.70</td>
<td>0.25</td>
</tr>
<tr>
<td>Allocative efficiency (A_{\text{sim}}/A_{\text{opt}} \times %)</td>
<td></td>
<td>99.99</td>
<td>82.60</td>
<td>72.20</td>
<td>60.03</td>
</tr>
<tr>
<td>Incumbent firms with hiring (%)</td>
<td></td>
<td>32.80</td>
<td>17.92</td>
<td>16.17</td>
<td>13.95</td>
</tr>
<tr>
<td>Incumbent firms with firing (%)</td>
<td></td>
<td>46.37</td>
<td>34.76</td>
<td>32.44</td>
<td>20.39</td>
</tr>
<tr>
<td>Incumbent firms with fixed labour (%)</td>
<td></td>
<td>12.13</td>
<td>47.18</td>
<td>50.89</td>
<td>64.59</td>
</tr>
</tbody>
</table>
Resource allocation is closer to the first-best distribution when adjustment costs are lower. In the scenario of cost-free (no adjustment costs), firms can adjust their labour to the optimal level directly. As a result, the simulated productivity should (theoretically) equal the optimal productivity. However, in Table 2.4, $A_{\text{sim}} / A_{\text{opt}}$ is 99.99% rather than 100%. This is because the labour choice is restricted on the grid points rather than the whole state space. When labour adjustment frictions increase, more and more firms choose to hoard their labour rather than fitting the shocks. The allocative efficiency decreases in the stickier labour market. The impact of allocative efficiency is bounded. It achieves its maximum in the cost-free economy, as shown in column 1 of Table 2.4. It reaches its minimum when the costs are too high to adjust any labour for all the firms. In this scenario, even higher adjustment costs cannot distort the market any more.

Adjustment costs also reduce the optimal productivity $A_{\text{opt}}$. It lowers the average productivity through two channels. First, it distorts entry and exit. This channel dominates the impact on $A_{\text{opt}}$ when the frictions are low. Second, adjustment frictions reduce the number of firms in the market. This channel dominates the impact on $A_{\text{opt}}$ when the adjustment frictions are high.

Adjustment frictions do not only lower the steady state productivity, but also the convergence speed. Figure 2.7 compares the dynamics of aggregate productivity in the markets with different levels of adjustment frictions. All the economies show the same pattern. The aggregate productivity grows up fast at beginning, and then, converges to a higher value. The different levels of frictions lead to the different levels of aggregate productivity in the steady states and the different speeds of convergence. Convergence speed falls when the frictions are higher.

The different dynamic patterns are from both allocative efficiency, $A_{\text{sim}} / A_{\text{opt}}$, and optimal productivity, $A_{\text{opt}}$. Figure 2.8 shows the impact on the dynamics of allocative efficiency. In the friction-free economy, firm distribution directly goes to the optimal level, $A_{\text{sim}} / A_{\text{opt}}$, is close to one from the first period of the simulation. Ad-
Justment costs delay the efficiency improvement. The convergence is slower when the adjustment costs are higher. In the scenario of double of the benchmark value, the convergence is much slower than in the other three simulations.

Optimal productivity is also influenced by the adjustment frictions, but the influence is not as sharp as in allocative efficiency. Figure 2.9 shows the impact. Optimal productivity grows more slowly when adjustment costs are higher. In the scenario of highest adjustment costs, the dynamics do not show a convergence pattern until the end of the same simulation periods.

2.7 Conclusion

The chapter builds a link between efficiency improvement and growth miracles. I create, present, and analyse a heterogenous-firm model with adjustment frictions and entry/exit barriers. The model can generate a short period of high-speed growth. Efficiency improvement is found to play the most important role in growth during the miracle period.

Figure 2.7: Counterfactuals on the dynamics of aggregate productivity with different levels of adjustment frictions.
§2.7 Conclusion

Figure 2.8: Counterfactuals on the dynamics of allocative efficiency with different levels of adjustment frictions.

Figure 2.9: Counterfactuals on the dynamics of optimal productivity with different levels of adjustment frictions.
Technological diffusion, productivity dispersion, and so-called allocative inefficiency

3.1 Introduction

Literature has found massive productivity dispersion across firms in emerging economies (Bartelsman, Haltiwanger, & Scarpetta, 2013; Hsieh & Klenow, 2009) but has not found primary cause yet (Restuccia & Rogerson, 2017). Some emphasise the role of frictions (Banerjee & Duflo, 2005; Jones, 2016), while others highlight the nature of the economies (Asker, Collard-Wexler, & De Loecker, 2014; David, Hopenhayn, & Venkateswaran, 2016). This paper links the two lines and suggests that technological catch-up would boost productivity dispersion and exaggerate the friction-driven misallocation of resource in emerging economies. The cross-country differences of dispersions may represent different stages of development.

I study an emerging economy with technological diffusion and constant adjustment costs. In the economy, firms learn about new technology from the world frontier. Their learning speeds differ. In the transition, fast learners catch up to the frontier very quickly while slow learners remain close to their original low technology. The dispersion of productivity increase. Then, slower learners also move closer
Technological diffusion, productivity dispersion, and so-called allocative inefficiency

the frontier after a long period of learning. The distribution of productivity slowly narrows until it finally reaches the stationary distribution. Furthermore, firms face adjustment costs and delay labour adjustment as a response. Hence, the marginal productivity of labour comoves with the productivity dynamics. The movement of its dispersion is U-shaped too. Since adjustment costs remain constant over time, the dispersion dynamics represent only the stages of development.

The U-shaped pattern relies on two key assumptions. First, firms improve productivity by learning from the frontier. Many studies found that this is the major engine of productivity progress in emerging economies (See the evidence in the next session). Existing models (e.g. Acemoglu, Aghion, & Zilibotti, 2006) typically assume a common probability of technological adoption. This paper, by contrast, assumes firm-specific learning abilities, which gives extra flexibility. This assumption is consistent with the estimate from firm-level data. Second, adjustment costs exist in factor markets. This is another stylised fact which has been supported by the empirical evidence from many markets. This paper focuses on Chinese labour market. In this context, Cooper, Gong, and Yan (2015; 2017) find significant adjustment costs as well.

To formalise the above mechanism, I extend Hopenhayn and Rogerson’s (1993) model and assume technical progress is the only source of growth. Then, the dynamics of resource allocation becomes a consequence of economic growth. The model economy is populated by heterogeneous entrepreneurs and homogenous labourers. Each entrepreneur runs an enterprise and hires labourers as the only production factor. Enterprises learn new technology from the frontier while how much they can learn is the different realisations of a random process. To fit new technology and productivity, they adjust labour and pay adjustment costs. For the sake of data fitness, the costs are assumed to be a combination of both convex and non-convex components. The firm-specific learning speeds generate more dispersed productivity distribution during catch-up process. The learning and adjustment frictions together make the distribution of marginal productivity broader as well. The two
distributions comove and the dynamics of dispersions both show U-shaped pattern. Furthermore, technological catch-up also introduces a persistent uncertainty shock on future productivity, so the dispersion of marginal productivity raises sharply in the beginning.

The estimation of parameter values targets Chinese manufacturing firms. For the sake of robustness and computational simplicity, the estimations are separated by four steps. First, some parameters are predetermined based on real data. Second, firm-level productivity is estimated by the Olley and Pakes’ (1996) approach. Third, given idiosyncratic productivity, the deeper parameters of learning process are estimated by maximizing likelihood. Fourth, since the model does not have analytical form solution, the parameters of adjustment costs are estimated by simulated method of moments.

I start the analysis with a thought experiment of firm dynamics and catch-up. I simulate an emerging economy that initially stays in a steady state with lower technology level, and then, starts to catch up the world frontier. In this simulation, the dynamics of productivity dispersion and so-called allocative inefficiency are both U-shaped. The new technology brings high dispersions and inefficiency at first, and then, slowly reduces dispersions and boosts efficiency. Since technological change is the only source of growth, the relationship suggests causality; that is, the U-shaped movements are a byproduct of technical progress. As discussed previously, the U-shapes are driven by both technological diffusion and adjustment frictions. In both the low-tech and high-tech steady states, firms’ productivity levels are close to each other so resource allocation is less important. However, during the transition, productivity highly differs across firms due to different learning speeds. The larger dispersion makes resource allocation more important in the middle part of the transition.

Then, I simulate the transitional dynamics of future China. The simulation suggests that the economy was already on the upwards part of the U-shaped pattern.
The economy will converge to a more efficient allocation even without reducing frictions. The magnitude of dispersion reduction and efficiency gain are substantially high when compared to the literature on misallocation and frictions (Restuccia & Rogerson, 2017). The result indicates that the differences of productivity dispersions between emerging and developed economies are not only due to the different levels of distortions, but also largely due to the different stages of development. Furthermore, economic development itself can be a solution of resource misallocation.

The level of frictions is still important in the simulation, but the importance depends on the stage of development. Frictions are more influential in the middle part of the transition. Whether the level of frictions are high or low, the low-tech and high-tech steady states are always efficient but the transitional paths are different. Higher frictions generate the higher dispersion of marginal productivity during transitional dynamics. By contrast, lower frictions lead to smoother transition.

I make three specific contributions in this paper: First, this study links allocative efficiency to the stage of development and suggests that resource allocation is naturally more inefficient in emerging economies. Quantitatively, this channel can explain a large proportion of cross-country efficiency differences. The analysis follows the efforts by Asker et al. (2014), Buera and Shin (2013) and Moll (2014), and highlights dynamic rather than static resource allocation. Second, this study suggests that economic growth itself can solve the problem of resource misallocation. When an economy converges to steady state, the allocation of production factors can return to a more efficient level even without the reduction of frictions. Third, this model can capture the firm dynamics in emerging economies, particularly consistent and large firm entry and exit. I model the economy by the transitional dynamics of a canonical model. The simulation shows the similar pattern as in the real data.

The remaining of the paper is organized as follows. In section 2, I illustrate the motivating facts. In section 3, I build a theoretical model for the later analysis. In section 4, I describe the estimation of parameters. In section 5, I simulate the
model economy and show the relationship between technical progress and allocative efficiency. Finally, I conclude in section 6.

3.2 Motivating facts

In this section, I demonstrate the motivating facts of the study. I firstly provide the evidence of the link between the stages of development and allocative efficiency, and then, discuss the literature on technology diffusion and resource allocation.

Economic development and productivity dispersion – To the author’s knowledge, due to data availability, the analysis of the firm dynamics during whole catch-up process is still missing. Instead, I will provide some suggestive evidences that can indicate the importance of the stage of development.

In general, productivity dispersions tend to be similar in economies that are in a similar stage of development. As an example, Ziebarth (2013) found that the level of productivity dispersion in the 19th century U.S. is very close to that in contemporary China and India. What is similar about the three economies is their stages of development not institutions or policies. Particularly, it is a stylised fact that productivity distribution is more dispersed in emerging economies than in developed economies (Bartelsman et al., 2013; Hsieh & Klenow, 2009). In addition, productivity dispersion also can be low in less-developed economies when they are in steady state. Brown, Dinlersoz, and Earle (2016) found that after East European upheaval and Soviet disorganization, the level of dispersion became higher in Georgia, Hungary, Lithuania, Romania, Russia, and Ukraine. This finding also suggests that the level of distortions is not the whole story of resource misallocation, as market distortions were diminished in these markets in that era. To sum up, empirical evidence suggests that productivity dispersion is higher in transitional economies than in steady-state economies.
Technology diffusion and resource allocation – In this model, I assume that economic growth is triggered by international technology adoption. This type of growth has been found in nearly all the economies (Comin & Hobija, 2010; Eaton & Kortum, 1999; Fagerberg, 1994; Keller, 2004). Particularly, it is more important in emerging economies, as they are far from the world technological frontier (Acemoglu et al., 2006; Howitt, 2000).

When the new technologies arrive an economy, firms never embrace them immediately. They keep investing in old technology for decades, and then slowly reallocate resources to new technology. At the industry level, this phenomenon happened when tractors went into American agriculture (Olmstead & Rhode, 2001) and when steam engines went into British industry (Crafts, 2004; Crafts & Mills, 2004). At the country level, it happened in the first industrial revolution in Britain (see the case study of the steam engine), and in the second (Atkeson & Kehoe, 2007) and third industrial revolutions (Greenwood & Yorukoglu, 1997; Gordon, 2012) in America. This study is a natural extension of this line of literature in the cross-country context.

3.3 Model

Based on the discussion above, I build a model of heterogeneous firms with international technology diffusion and adjustment friction. The analytical framework is extended from Hopenhayn and Rogerson’s (1993) model.

Agents – The economy is composed of two kinds of agents, a measure $M_t$ of entrepreneurs and a measure $L_t$ of labourers. Both grow at the same constant growth rate $g_t > 1$. ¹ I simplify their behaviour by the following three assumptions. First, labourers cannot get utility from leisure, thus, the labour supply is perfectly inelas-

¹Population growth does not influence any agent in the economy and will not boost the growth of per capita output. I make this assumption for the sake of data fitness. Because of baby boom and rural-urban migration, labour force increase dramatically in the data period. The number of firms also substantially increases. This change can be easily captured by this assumption. This is substantial to validate the latter quantitative results.
tic. Second, entrepreneurs are risk neutral, so their utility maximization problem is equivalent to the output maximization problem. Third, the economy only produces a nondurable good, so households have to consume all the products at the end of each period. In other words, no intertemporal transfer exists in the economy.

**Time line** – Figure 3.1 describes the life cycle of an entrepreneur. An entrepreneur can only manage one firm in each period. When he/she launches a firm, he/she draws productivity from the stationary distribution, and then decides whether to exit the market or not. If the entrepreneur decides to stay (with probability 1 in the first period), he/she adjusts employment, and then produces goods. In the next period, the entrepreneur draws a new productivity $z_{i,t}$ based on the existing productivity $z_{i,t-1}$, and repeats the cycle. The firm keeps working until the entrepreneur reaches a too low productivity $z_{i,t} < z^*_t(l_{i,t-1})$. Then, he/she closes the firm, and organizes a new one in the next period.

**Technology** – A firm is characterized by its productivity $z_{i,t}$ and last-period employees $l_{i,t-1}$. Using labour $l_{i,t}$ as the only input it produces final good, and using the output it pays wages $w_t l_{i,t}$, and adjustment costs $c(l_{i,t}, l_{i,t-1})$. I neglect other production factors, so the analysis can focus on resource allocation of one factor. When a firm with productivity $z_{i,t}$ and existing employees $l_{i,t-1}$ employs $l_{i,t}$ labour at time $t$, it gets one-period profit,

$$
\pi(z_{i,t}, l_{i,t}, l_{i,t-1}; w_t) = f(z_{i,t}, l_{i,t}) - w_t l_{i,t} - c(l_{i,t}, l_{i,t-1}).
$$

(3.1)

Firms’ production functions are in Cobb-Douglas form with the same labour supply elasticity $\alpha$,

$$
f(z_{i,t}, l_{i,t}) = z_{i,t} l_{i,t}^\alpha, \quad \alpha \in (0, 1).
$$

(3.2)

I assume that the production functions are decreasing returns to scale. Theoretically,
Technological diffusion, productivity dispersion, and so-called allocative inefficiency

Figure 3.1: Life cycle of an entrepreneur

the assumption keeps the existence of heterogeneity. Empirically, the assumption is consistent with real data in the later quantitative analysis.

The productivity follows a random coefficient autoregressive process,

\[
\ln z_{i,t} = \ln z_{i,t-1} + (\ln z_{f,t} - \ln z_{i,t-1})\epsilon_{i,t} - \eta_{i,t}. \tag{3.3}
\]

where

\[
z_{f,t} = z_{f,0}g_t^t, \quad z_{i,0} \in (0, z_{f,0}),
\]

\[
\epsilon_{i,t} \sim_{i.i.d.} B(e_1, e_2), \quad \eta_{i,t} \sim_{i.i.d.} \Gamma(e_3, e_4). \tag{3.4}
\]

A firm improves productivity by learning from the world frontier \(z_{f,t}\). I ignore research and development. The reason is that most technology progress is from learning (Comin & Hobija, 2010; Eaton & Kortum, 1999; Fagerberg, 1994; Keller, 2004). This is valid particularly in developing economies, since they are far from the fron-
§3.3 Model

tier. The frontier grows from the initial level $z_{f,0}$ with a constant rate $g_z \geq 1$. How much a firm can narrow the distance to the frontier ($\ln z_{f,t} - \ln z_{i,t-1}$) is a random variable $\epsilon_{i,t} \in [0, 1]$. An independent shock $\eta_{i,t}$ covers the rest of variation in technical progress. I choose beta and gamma distribution to reduce the restriction on the shape of the distributions. The progress is tight, $z_{i,t} \in (0, z_{f,t}]$. Define the operator $\tilde{\cdot}$ by

$$\tilde{x}_t \equiv g_z^{-t} x_t.$$  

Then, $z_{i,t}$ follows a stationary process,

$$\ln \tilde{z}_{i,t} = \ln \frac{z_{i,t-1}}{g_z} \cdot (1 - \epsilon_{i,t}) + \ln z_{f,0} \cdot \epsilon_{i,t} - \eta_{i,t}, \quad (3.5)$$

Adjustment frictions – Labour adjustment is costly. When a firm wants to adjust labour as a response to a productivity shock, it has to pay adjustment costs. The study focuses on the consequences rather than cause of friction, so I use broad adjustment costs rather than open the black box of deeper mechanisms. The costs can include hiring cost (Oi, 1962), firing cost (Hopenhayn & Rogerson, 1993), and search friction (Cooper, Haltiwanger, & Willis, 2007). Labour adjustment behaviour is diverse in reality. For example, in Chinese firm-level data, many firms adjust their labour smoothly and continuously, but also a significant amount of firms do not adjust labour in a particular year (Cooper et al., 2015; 2017). To fit the pattern, I use a rich setup of adjustment costs following Cooper and Haltiwanger (2006), Bloom (2009). Three kinds of adjustment costs are considered. The first component is the disposable fixed cost $c_0 \mathbb{1}_{l_{i,t} \neq l_{i,t-1}}$. Firms have to pay the cost when they hire/fire employees regardless of the quantity of adjustment. When fixed cost are high, the majority of firms do not adjust labour, and a small fraction of firms make a huge labour adjustment in one period. The second component $c_1 |\Delta l_{i,t}|$ is proportional to the gross firing/hiring. For example, training costs are proportional to the number of new employees, and unem-
Employment compensation is proportional to the number of unemployed workers. The proportional cost also can generate an inaction region like the fixed cost. The third component is the quadratic cost $c_2 \frac{(\Delta l_i)^2}{l_{it} + l_{i,t-1}}$, which makes sharp labour adjustments more costly. If the quadratic costs are high, most firms will smoothly change their labour. The total adjustment cost is the summation of the three costs,

$$
c(l_{it}, l_{i,t-1}) = c_0 1_{l_{it} \neq l_{i,t-1}} + c_1 |\Delta l_i| + c_2 \frac{(\Delta l_i)^2}{l_{it} + l_{i,t-1}}.
$$

(3.6)

I assume that firing and hiring costs are symmetric in the concern of computational complexity in the later estimation.

**Incumbent’s problem** – An incumbent entrepreneur will adjust labour for the maximal value. He/she will close a firm when keeping operating is not profitable. The incumbents consider the following Bellman equation for labour adjustment and exit decisions,

$$
V_t(z_{i,t}, l_{i,t-1}; \bar{w}^t) = \max_{1_{\text{exit}} \in \{0, 1\}} \{(1 - 1_{\text{exit}}) \cdot \max_{l_{i,t} \geq 0} \{\pi(z_{i,t}, l_{i,t}, l_{i,t-1}; w_t) + \beta \mathbb{E} V_{t+1}(z_{i,t+1}, l_{i,t}; \bar{w}^{t+1})\} + 1_{\text{exit}} \cdot [\beta \mathbb{E} V_{t+1}(z_{i,t+1}, 0; \bar{w}^{t+1}) - c(0, l_{i,t-1})]\}.
$$

(3.7)

where $\bar{w}^t$ is the wage vector from period $t$ to infinity.

A firm makes the exit decision once it realizes its productivity at the beginning of each period. The decision is based on the comparison between the expected future value, $\mathbb{E} V_{t+1}(z_{i,t+1}, l_{i,t}; \bar{w}^{t+1})$, and the exit cost, $-c(0, l_{i,t-1})$, which is the layoff cost for all employees when it exits. If operation is more expensive, they will clear all their labour ($l_{i,t}=0$) and exit the market. Adjustment costs influence exit decisions in two ways. On the one hand, if costs do not exist, a firm with a decreasing return to scale production function will never leave the market. Even if it is hit by a huge negative shock, it still can be profitable by reducing labour. Adjustment frictions obstruct
prompt labour adjustment, and then generate the possibility of negative profit and exit. On the other hand, adjustment costs are also barriers to exit decisions since exit firms have to pay adjustment costs for firing all workers.

Given the existing employees $l_{i,t-1}$, there exists a threshold of productivity $z^*_t(l_{i,t-1})$. A firm will exit the market if it draws productivity $z_{i,t} < z^*_t(l_{i,t-1})$. Section 4 will provide a more detailed discussion on the threshold.

Entrant’s problem – At the very beginning of each period, newborn entrepreneurs enter the market. The old entrepreneurs who exited in the last period also enter the market again. The two groups are identical. Each firm draws a detrended productivity from the stationary distribution of $\tilde{z}_{i,t}$ (See equation (3.5)). Then, it solves the Bellman equation (3.7) with zero existing labour ($l_{i,t-1} = 0$). No one will exit the market right after entry since $z^*_t(0) = 0$.

Law of motion – The distribution of firms updates every period until it reaches a new steady state. Define $\mu_t$ as the measure of the joint distribution of labour and productivity. Then, the distribution in period $t$ is

$$\mu_t = T_t(\mu_{t-1}) + \nu_t, \quad (3.8)$$

where $T_t(\cdot)$ is the transition function of the incumbent firms, which is determined by the Bellman equation (3.7). $\nu_t$ is the distribution of new entering firms at period $t$, which is determined by the initial draw and the Bellman equation.

Equilibrium – The study discusses both the steady state and the transitional path to the steady state, so I define two kinds of equilibria. Definition 3 is used to calculate the steady state.

Definition 3 A stationary equilibrium of the model is a wage system in steady state, which
satisfies the following conditions: (i) labourer optimization; (ii) entrepreneur optimization; (iii) labour market clear; (iv) invariant distribution over time.

Definition 4 describes the relative recursive competitive equilibrium used in the computation of the transitional dynamics.

**Definition 4** A recursive competitive equilibrium of the model is a wage system, which satisfies the following conditions: (i) labourer optimization; (ii) entrepreneur optimization; (iii) labour market clear; (iv) law of motion $\mu_t = T_t(\mu_{t-1}) + \nu_t$.

**Growth decomposition** – Productivity is the only source of growth in the model, since the total production input is fixed. Two channels contribute to productivity growth. The first channel is the technical progress of the incumbent firms. The second channel is the allocative efficiency. Reallocating more resources to high-productivity firms raises the aggregate productivity. I decompose the two channels à la Hopenhayn (2014b).

I assume a social planner who can reallocate resources without any costs. Given the productivity distribution, the distance between real productivity and social optimal productivity measures the allocative efficiency of the economy. The social optimal productivity is achieved in the competitive equilibrium without any friction. It also can be solved as a social planner problem (Hopenhayn, 2014b),

$$\max_{l_{it}} \sum_i z_i l_{it}^a,$$

subject to

$$\sum_i l_{it} \leq L_t.$$

The social planner problem has an analytical solution, that the optimal output is

$$Y = \left(\frac{1}{\Lambda_{opt}} \sum_i z_i l_{it}^a \cdot N_t\right)^{\frac{1}{1-a}} L_{t}^{\alpha},$$

(3.9)
where \( N_t \) is the number of firms at period \( t \). The allocative efficiency can be measured by the relative productivity,

\[
A_{rel,t} \equiv \frac{A_{sim,t}}{A_{opt,t}}
\]  

(3.10)

The measure includes not only reallocation within the surviving firms, but also the exiting and entering firms. Now the growth of simulated productivity \( \Delta \ln(A_{sim,t}) \) can be decomposed by relative two components,

\[
\Delta \ln(A_{sim,t}) = \Delta \ln(A_{rel,t}) + \Delta \ln(A_{opt,t}).
\]  

(3.11)

### 3.4 Estimation

This section explains the estimation of the parameters. The estimation targets Chinese manufacturing firms. I separately estimate three parts of the model for the purpose of robustness and computational complexity. I estimate labour productivity using the Olley and Pakes’ (1996) approach, estimate adjustment costs by simulated method of moments (henceforth SMM), and calibrate other parameters directly from data.

#### 3.4.1 Data

The estimation is based on the Chinese Industrial Enterprises Database, an annual survey of Chinese industrial firms. The data preprocessing mainly follows Brandt, Van Biesbroeck, and Zhang (2014). I include all manufacturing firms for the purpose of consistency. The data are frequently used in the analysis of productivity and resource allocation (e.g. Hsieh & Klenow, 2009; Song, Storesletten, & Zilibotti, 2011). The imbalanced panel data include all state-owned enterprises and most non-state firms with annual sales larger than five million RMB. I exclude observations with missing values and outliers in key variables. Since my model cannot handle aggregate shocks, I choose the period from 2000 to 2007 to avoid influences from the Asian Financial Crisis and the Global Financial Crisis. Data from 1999 provide a baseline.
for estimation.

Table 3.1 describes the data, grouped by years. As shown in the table, the distribution of China’s manufacturing firms is not stationary during this period. Both the first-order and first-order moments of the size distribution keeps changing. The average size of firms keeps decreasing, although both the number of firms and the total labour supply increase. The standard deviation of the size of firms keeps decreasing. On the other hand, the data also indicate massive productivity growth in the economy, since the average output goes up although average input (labour) goes down. The simulations in the next section will try to fit these patterns.

<table>
<thead>
<tr>
<th>Year</th>
<th>Labour Mean</th>
<th>Std. dev.</th>
<th>Output Mean</th>
<th>Std. dev.</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>241.65</td>
<td>295.80</td>
<td>7.12</td>
<td>12.11</td>
<td>125894</td>
</tr>
<tr>
<td>2001</td>
<td>211.56</td>
<td>266.51</td>
<td>7.03</td>
<td>12.14</td>
<td>140125</td>
</tr>
<tr>
<td>2002</td>
<td>204.73</td>
<td>257.73</td>
<td>7.54</td>
<td>13.00</td>
<td>148694</td>
</tr>
<tr>
<td>2003</td>
<td>198.76</td>
<td>249.85</td>
<td>8.70</td>
<td>15.18</td>
<td>167238</td>
</tr>
<tr>
<td>2004</td>
<td>171.19</td>
<td>232.02</td>
<td>NA</td>
<td>NA</td>
<td>245765</td>
</tr>
<tr>
<td>2005</td>
<td>173.50</td>
<td>215.06</td>
<td>11.70</td>
<td>20.58</td>
<td>230898</td>
</tr>
<tr>
<td>2006</td>
<td>164.59</td>
<td>204.28</td>
<td>13.91</td>
<td>24.51</td>
<td>257143</td>
</tr>
<tr>
<td>2007</td>
<td>158.21</td>
<td>196.33</td>
<td>16.65</td>
<td>28.86</td>
<td>290160</td>
</tr>
</tbody>
</table>

3.4.2 Predetermined parameters

The top panel of Table 3.2 shows the calibration of the baseline parameters. The initial distribution $\mu_0$ is taken from the real data in 2007, the last year before the Global Financial Crisis. The initial labour supply $L_0$ and initial number of entrepreneurs are also taken from the same year data. The population growth rate $g_l$ is calibrated by the average labour force growth rate in the data from 2000 to 2007. Lastly, the discount factor $\beta$ is calculated from the average annual interest rate from 2000 to 2007.
### Table 3.2: Value assignment of parameters

<table>
<thead>
<tr>
<th>Para.</th>
<th>Explanation</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>initial distribution</td>
<td>NaN</td>
<td>real distribution in 2007</td>
</tr>
<tr>
<td>$L_0$</td>
<td>initial labour supply (m.)</td>
<td>45.91</td>
<td>total labour supply in 2007</td>
</tr>
<tr>
<td>$M_0$</td>
<td>the ini. num. of entrepreneurs (m.)</td>
<td>0.29</td>
<td>num. of firms in 2007</td>
</tr>
<tr>
<td>$g_l$</td>
<td>the growth rate of population</td>
<td>1.06</td>
<td>avg. lab. force growth rate (00-07)</td>
</tr>
<tr>
<td>$g_z$</td>
<td>the growth rate of frontier</td>
<td>1.02</td>
<td>the prod. growth rate in the U. S.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.96</td>
<td>annual interest rate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>the output elasticity of labor</td>
<td>0.64</td>
<td>Chinese firm-level panel data</td>
</tr>
<tr>
<td>$\tilde{\xi}_{f,0}$</td>
<td>technology frontier</td>
<td>8.41</td>
<td>the highest grid point in 2007</td>
</tr>
<tr>
<td>$e_1$</td>
<td>the 1st shape parameter of $B$ dis.</td>
<td>1.17</td>
<td>the likelihood of firm panel</td>
</tr>
<tr>
<td>$e_2$</td>
<td>the 2nd shape parameter of $B$ dis.</td>
<td>10.08</td>
<td>the likelihood of firm panel</td>
</tr>
<tr>
<td>$e_3$</td>
<td>the shape parameter of $\Gamma$ dis.</td>
<td>0.93</td>
<td>the likelihood of firm panel</td>
</tr>
<tr>
<td>$e_4$</td>
<td>the inverse scale parameter of $\Gamma$ dis.</td>
<td>0.51</td>
<td>the likelihood of firm panel</td>
</tr>
<tr>
<td>$c_2$</td>
<td>quadratic adjustment cost</td>
<td>4.60</td>
<td>the moments of labour adjustment</td>
</tr>
<tr>
<td>$c_1$</td>
<td>proportional adjustment cost</td>
<td>2.20</td>
<td>(see table 3.3)</td>
</tr>
<tr>
<td>$c_0$</td>
<td>fixed adjustment cost</td>
<td>0.82</td>
<td></td>
</tr>
</tbody>
</table>

#### 3.4.3 Firm-level productivity: Olley and Pakes’ (1996) method

I estimate firm-level productivity $z_{i,t}$ à la Olley and Pakes (1996). The results are shown in the upper middle panel of Table 3.2.

I use a semiparametric method to estimate the production function parameters for consistency. The literature highlights the endogeneity problem of productivity estimation, that input levels are correlated with unobserved productivity shocks. Olley and Pakes (1996) introduced investment as a proxy of unobserved shocks, and then, develop a consistent estimator. This study uses the labour version of the Olley and Pakes’ (1996) approach. Labour is no longer a freely variable input when adjustment costs exist, so I can treat labour as the same as capital. Based on the same argument as Olley and Pakes (1996) made, I use employment as the proxy of unobserved productivity shocks. I also control for other production factors. Then, theoretically, I can get a consistent estimator of the output elasticity of labour $\hat{\alpha}$. As shown in Table 3.2
the estimate is near the labour income share and near most estimates in the literature. The estimator of productivity is

\[ \hat{z}_{i,t} = (y_{i,t} - \hat{\alpha}_{i,t})^{\frac{1}{1 - \hat{\alpha}}}. \]

### 3.4.4 Learning process: maximum likelihood estimation

I estimate the learning process \((3.3)\) \((3.4)\) by maximum likelihood estimation. The likelihood function is as follows,

\[
\ln L(e_1, e_2, e_3, e_4; \cdot) = \sum_i \sum_t \ln \left[ \int_0^1 f_B(x; e_1, e_2) f_T(\ln z_{i,t-1} - \ln z_{i,t}) + (\ln z_{f,t} - \ln z_{i,t-1}) x; e_3, e_4) dx \right].
\]

I pool all the data with previous year productivity. Firm entry and exit can drive data selection issues, but modelling this part takes much more computational power, so I ignore the issue to reduce computational complexity.

I assume some of Chinese manufacturing firms have already reached the world technology frontier. In the estimation, I use the 99th percentile productivity in that industry in that year as the frontier. I chose the 99th percentile rather than maximum point to rule out the impact of outliers. In the simulations, I calibrate the baseline frontier \(z_{f,0}\) by the highest grid point in the baseline year. In the estimation, all the frontiers are calibrated from the data, so the growth rate \(g_z\) is not necessary. In the simulations, I assume the frontier grows at the same rate as labour productivity growth in the United States.

The estimates of \(e_1, e_2, e_3,\) and \(e_4\) are shown in the lower middle panel of Table 3.2. The distributions of \(\epsilon_{i,t}\) and \(\eta_{i,t}\) are shown in Figure 3.2. Both variables are skewed and closed to zero, this indicates that productivity is persistent across periods. The value of the learning process \(\epsilon_{i,t}\) is small; most firms cannot learn much from the frontier, so the catch-up process is slow. The shape parameter of \(\eta_{i,t}\) is closed to 1, so the Gamma distribution here is closed to exponential distribution, but the peak is
slightly higher than zero. Section 4 provides a sensitivity test of the impact of these parameters.

![Graphs](a) The pdf of \( \epsilon_{i,t} \), \( e_1 = 0.13 \), \( e_2 = 10.10 \). (b) The pdf of \( \eta_{i,t} \), \( e_3 = 1.07 \), \( e_4 = 0.06 \).

Figure 3.2: The probability density functions of \( \epsilon_{i,t} \) and \( \eta_{i,t} \).

Notes: See Equation (3.3) and (3.4)

### 3.4.5 Adjustment frictions: simulated method of moments

I estimate adjustment costs by simulated method of moments, henceforth SMM, since the analytical form solution does not exist in the model. The main idea of SMM is selecting parameters to minimize the weighted distance between the moments of simulated data and real data (McFadden, 1989; Pakes & Pollard, 1989). The estimator is solved from the following minimization problem,

\[
\hat{\theta} = \arg\min_{\theta \in \Theta} |M_{real} - M_{sim}(\theta)|' WV[M_{real} - M_{sim}(\theta)].
\]

I use the optimal weight matrix which Lee and Ingram (1991) provide. Under the estimating null, the real data and simulated data are independent and from the same data generation process. Based on this condition, Lee and Ingram (1991) proved that the efficient weight matrix is a function of the inverse of the variance-covariance matrix of \([M_{real} - M_{sim}(\theta)]\). I estimate the variance-covariance matrix by the bootstrap method.

As shown in Table 3.3, five target moments are chosen for the five parameters. If the estimation targets the relationship between labour and output, I might target
productivity directly. To avoid the possibility, I use five moments on labour market and exit/entry only. The first is the proportion of firms which do not adjust the previous year’s labour. The number would be high if adjustment costs, in particular the fixed disposable component, are high. The second is the first order autocorrelation of labour adjustment, since adjustment costs mainly influence firms’ intertemporal labour decisions. The third is the same autocorrelation across firms with labour adjustments. The fourth is the average ratio of labour adjustment to last year’s labour. If the fixed adjustment cost is high, the third moment would be high. If the quadratic component is high, the forth moment would be low. The fifth is the same ratio across the firms with labour adjustment.

The simulation process for SMM is as follows. I take the 1999 real data as the initial distribution of labour and productivity, and then simulate the whole transitional path of the economy. I use the first 8 periods to calculate the simulated moments, so the periods are the same as in the data.

Table 3.3 reports the data moments and simulated moments. The simulated moments and data moments match well.

The bottom panel of Table 3.2 reports the estimates. The parameters values are not very close to the values in Chapter 2. The difference comes from the different model setups. The multiplier of quadratic adjustment cost is the largest among the three components, and its effects are even larger in the later quantitative analysis. Fixed adjustment costs are small which is related to the fact that only 27.88% of firms fix their labour in the real data. For the purpose of consistency and for robustness,
I ignore small labour adjustments within the grids in the former numerical work. Entry and exit costs are higher because they are compared to the future value of the firms rather than only one-period profit.

### §3.5 Results

In this section, I simulate the catch-up process based on estimates in the last section, and check the relationship between technical progress and allocative inefficiency. I will discuss some features of the model first, and then, simulate the convergence path of an emerging economy. Lastly, I will simulate the firm dynamics of China after 2007.

#### 3.5.1 Technical progress and distorted labour adjustment

This subsection provides some discussion about the basic features of this model. First, I discuss how the distributions of learning skills and the growth rate of frontier influence the catch-up processes. Then, I show the relationship between distance to frontier and volatility of productivity. Finally, I illustrate the distorted labour adjustment.

![Figure 3.3: The stationary distribution of logarithmic productivity with different learning skills $\epsilon_{i,t}(e_1, e_2)$.](image)

Notes: Slow learning: $e_1 = 1.06, e_2 = 11.09$; benchmark learning: $e_1 = 1.17, e_2 = 10.08$; fast learning: $e_1 = 1.29, e_2 = 9.07$. 

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[302x768]Results

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Technological diffusion, productivity dispersion, and so-called allocative inefficiency

Figure 3.4: The stationary distribution of logarithmic productivity with different growth rates of the frontier $g_z$.

Notes: Slow growth: $g_z = 0.92$; benchmark growth: $g_z = 1.02$; fast growth: $g_z = 1.12$.

Catch-up – Where the economy will catch up to is determined by three factors: the learning skills $\epsilon_{i,t}(e_1,e_2)$, the growth rate of the frontier $g_z$, and idiosyncratic shock $\eta_{i,t}(e_3,e_4)$. The first two relate to the learning process. Figure 3.3 shows the impacts of the learning skills. I simulate the stationary distributions of productivity with three learning distributions. When the learning process is faster (the learning distribution is more right skewed), the stationary distribution is closer to the frontier. Figure 3.4 shows the impacts of the growth rate of the frontier. When the frontier grows faster, the stationary distribution is further from the frontier.

Distance to frontier and uncertainty – The volatility of productivity is going down along with the catch-up process. This is a key assumption of this model. This subsection provides evidence to support the assumption. Equation (3.3) decomposes the volatility into two parts, $\epsilon_{i,t}$ and $\eta_{i,t}$. $\epsilon_{i,t}$ is related to distance to frontier. $\eta_{i,t}$ catches the rest. The variance of $\epsilon_{i,t}$ is $1.09e - 3$. The variance of $\eta_{i,t}$ is $3.98e - 3$. The contribution of $\epsilon_{i,t}$ depends on the distance to frontier. Figure 3.5 (a) shows the distribution of
distance to frontier in the real data. I rule out the top 1% and the bottom 1% for robustness. Figure 3.5 (b) shows the volatility decomposition in the same range. The distance to frontier plays a significant role in the whole range. Based on equation (3.3), the variance in productivity decreases when the technological gap is lower. It contributes 49.36% of the variance in productivity even in the highest productivity firms. The contribution is 70.58% in the lowest productivity firms. Uncertainty will go down significantly in all these firms if they can catch up to the frontier. This result is not sufficient to show the causality from technology to uncertainty, but at least the relationship is consistent with my assumptions, and the magnitude is large enough.

Figure 3.5: Distance to frontier and volatility of productivity.

Notes: The domain is from the 1st percentile -8.76 to the 99th percentile -3.56.

*Distorted labour adjustment* – Figure 3.6 plots the labour adjustment of a firm in the new steady state. The pattern is clear, while it is not smooth enough according to the computational accuracy. First, higher productivity indicates higher labour adjustment in the same size firms, matching the increasing trend. Second, the trend is not linear. Firms are more sensitive to smaller productivity shocks than larger shocks, since quadratic adjustment costs punish large scale hiring/firing. Third, the middle segment is flat. Firms adjust labour only if productivity shocks exceed a
Figure 3.6: Policy function for labour adjustment.

Notes: The horizontal axis is the productivity to labour ratio $z_i, t / l_{i, t-1}$. The vertical axis is the labour adjustment ratio $\Delta l_i, t / l_{i, t-1}$. Both the two ratios are rescaled by a concavification transformation $\tilde{x} = \ln(x + 1)$.

threshold.

3.5.2 From low-tech to high-tech

This subsection describe the firm dynamics during catch-up process by a thought experiment. I assume that an economy starts from a steady state in which the technology frontier is half of the world frontier. Then, the economy opens the gate to the world frontier, starting the catch-up process. The cross-country technology difference in the simulation is much smaller than in the real world, so it is a conservative estimate of the magnitude of the impact.

Theoretically, the simulation studies the impact of a permanent shock to the technology frontier. According to equations (3.3) and (3.5), the shock at the frontier also brings an uncertainty shock. Unlike the temporary shock discussed by Bloom (2009),
Section 3.5 Results

Figure 3.7: Proximity to frontier and allocative efficiency, from the steady state with a lower frontier.

the shock here is gradually decayed.

**U-shaped curve** – As shown in Figure 3.7, the movement of allocative efficiency is U-shaped. The start of the catch-up process generates a dramatic efficiency reduction at the beginning of the simulation. As discussed before, according to uncertainty and adjustment frictions, the economy cannot immediately move resources to firms with new technology. The uncertainty shock generates efficiency losses. Since technical progress is gradual, the loss cannot be huge at the beginning, but keeps accumulating. It takes several periods until reaching the bottom. Then, technical progress reduces uncertainty gradually, and pushes up the efficiency. Along with the catch-up process, allocative efficiency dramatically reduces at the beginning, and then gradually increases.

**Distribution dynamics** – Figure 3.8 shows the dynamics of productivity distribution when an economy grows from poor to rich. I increase the initial technological gap to show the pattern more clearly. The initial frontier here is half of the world frontier.
in the logarithmic space. The change happens in both the first order moment and the second order moment. On the one hand, the mean of the distribution keeps increasing during the catch-up process, so the uncertainty of the economy keeps decreasing because of equation (3.3). This drives the U-shape as discussed in the last subsection. On the other hand, the dynamics of variance follows an inverted U-shape. The distribution is relatively narrow in both the lo-tech and hi-tech steady state. This is because of different learning speeds across firms. Some lucky firms learn fast, so they can reach the frontier quickly. Some unlucky firms learn slowly, so they continue using old technology for a long time. Thus, the distribution spread when they catch up to the new frontier. Eventually, everybody reaches relatively high technology in the new steady state. This is another driving force of the U-shape. Resource allocation is less important when productivity distribution is concentrated, such as in two steady states in the economy. However, when some firms are much more productive than others, it is more important that they have enough production factors. This is what happened in the catch-up periods.
§3.5 Results

Figure 3.9: Proximity to frontier and allocative efficiency, from a steady state with a lower frontier, with half the overall estimated adjustment costs.

Notes: The initial allocation is theoretically optimal. The trend is slightly non-monotonic because of the numerical error from discretization.

Figure 3.10: Proximity to frontier and allocative efficiency, from a steady state with a lower frontier, with double the overall estimated adjustment costs.

Notes: The initial allocation is theoretically optimal. The trend is slightly non-monotonic because of the numerical error from discretization.
Technological diffusion, productivity dispersion, and so-called allocative inefficiency

Figure 3.11: Growth decomposition.

Notes: The decomposition follows Equation (3.11) in the Section 3.

The role of frictions – Although the paper highlights the importance of the stage of development, the level of frictions is still crucial. Figure 3.9 shows the transitional dynamics with half of the overall estimated adjustment costs. Figure 3.10 shows the transitional dynamics with double of the overall estimated adjustment costs. The pattern of the two transitions is the same as the former one, but the level of inefficiency is different. Lower friction helps the economy pass catch-up periods more smoothly. By contrast, higher friction generates higher inefficiency in the transition.

3.5.3 China in the future

In subsection, I simulate the world largest emerging economy, China, and show how technological diffusion will eventually reduce the dispersion of productivity. The simulation suggests that China has already passed the bottom of the U-shaped curve and will converge to a more efficient distribution even without the reduction of frictions.


**Growth decomposition.**— Aggregate productivity keeps growing in the simulation. The average growth rate is 3.32%. The growth is driven by both technology progress and efficiency improvement. Figure 3.11 decomposes the growth during the whole transitional dynamics (see section 3 for the derivation of the decomposition). At the beginning, the improvement in allocative efficiency contributes a large proportion because of the transition from initial inefficiency. When the economy is close to steady state, the impact from the efficiency channel becomes less significant. Technical progress makes the main contribution in later economic growth.

**Technological progress and allocative efficiency**— Figure 3.12 shows the relationship between technical progress and allocative efficiency. Technology level is measured by the average distance to the frontier, \( \text{mean}(\frac{z_{i,t}}{z_{f,t}}) \). Allocative efficiency is measured by the ratio of real productivity over optimal-allocation productivity (see equation (3.10)). There is a clear positive relationship between technical progress and allocative efficiency. Since learning from the frontier is the only source of growth in the model, the relationship can be explained as causal. When the emerging economy catches up to the frontier, efficiency improvement can be a byproduct of technical progress. The level of improvement is higher than in most literature (see the review written by Restuccia & Rogerson, 2017), and even beyond the estimate by Hsieh and Klenow (2009).

**Firm dynamics**— Figure 3.13 shows more details of the dynamics of the size distribution. The market reallocates resources from the largest firms to relatively smaller ones. It indicates that large firm occupied too many resources in the real data. The size distribution keeps concentrating until it reaches stationary distribution. Along with the increasing size of small firms, allocative efficiency also increases.

Figure 3.14 shows the dynamics of the Pearson correlation coefficient between productivity \( z_{i,t} \) and labour \( l_{i,t} \). The correlation keeps increasing along with the con-
Figure 3.12: Proximity to frontier and allocative efficiency.

Notes: The horizontal axis measures technology level of the economy (see Equation (3.9)). The vertical axis measures allocative efficiency (see Equation (3.10)). Section 3 shows the derivation of the measures. The initial distribution is from the real data in 2007.

Figure 3.13: The dynamics of size distribution
Figure 3.14: The dynamics of the Pearson correlation coefficient between productivity $z_{i,t}$ and labour $l_{i,t}$.

Notes: The initial distribution is from real data in 2007.

Convergence. The correlation is relatively weak in the real data in 2007, since the allocation is quite inefficient. It increases dramatically at the beginning of the simulation, and then, keeps increasing during the whole simulation. Finally, labour allocation is highly determined by the productivity distribution. High-productivity firms occupy more resources, low-productivity firms have less resources. The trend in the correlation explains the dynamics of allocative efficiency, which also increases dramatically at first, and then keeps increasing.

3.6 Conclusions

This study posts a new hypothesis on growth theory, that technical catch-up can cause productivity dispersion temporarily high in emerging economies, and that the dispersion can be low again in the long run. Simultaneously, new technology leads severer friction-driven misallocation in the beginning and finally brings efficiency a-
gain. The U-shaped patterns exist when technological diffusion is firm-specific and factor market is frictional. Numerical experiments suggest that different stages of development can largely explain the cross-country difference in productivity dispersion and so-called allocative efficiency.

There are two natural directions for further investigation. First, analysing the firm dynamics during whole catch-up process. This study suggests a pattern of the dynamics of productivity distribution, yet the direct empirical evidence is still missing due to data availability. Second, decomposing the impact of level of distortions and stage of development. This study suggests that both are important in explaining inefficiency in emerging economies, but the relative importance of each component is still an open empirical question.
Appendix. Numerical method

The appendix describes the numerical methods used to solve the steady state and transitional dynamics. I illustrate the structure of the algorithm first, and then, discuss some details.

Steady state – I make the economy stationary by removing growths of population and technological frontiers, and then searching for the equilibrium wage in steady state by Brent’s method. The labourers’ problem is trivial. The firms’ problem (3.7) is solved by value function iteration. Given a guess of steady state wage, the algorithm used to compute the steady state is as follows:

1. Guess the steady state wage \( w^* \).
2. Guess value function by on-grid iteration.
3. Off-grid value function iteration from the initial guess.
4. Use the decision rules computing the new state.
5. Iterating the firms dynamics until converging to the stationary distribution.
6. Calculate the extra labour supply in the steady state.

The algorithm also provides the stationary distribution and the value function in the steady state.

Transitional dynamics – I compute the transitional dynamics using the algorithm posted by Conesa and Krueger (1999). I assume that the transition takes 100 periods which is long enough to obtain convergence numerically. The algorithm includes the following steps:

1. Use the steady state wage as the initial guess of the wage path.
2. Given the wage path and the steady state value function, backwards compute the value function for each period.

3. Given the value function, forwards compute the path of equilibrium wages from the initial state.

4. Update the guess of the wage path until convergence.

I do one-step interpolation in the third step to improve the efficiency.

Discretization – I discretize the state space by a finite grid of labour $l_{i,t-1}$ and productivity $z_{i,t}$. However, the optimal choice is not restricted on the grid points. I approximate the off-grid points by interpolation of a cubic spline. When the optimal point is out of the grid, it will be assigned to the closest two grid points. Suppose the optimal labour is $l^*$ for a measure 1 of firms, the closed two points are $l_1$ and $l_2$, where $l_1 < l^* < l_2$. Then, $\frac{l_2-l^*}{l_2-l_1}$ of the firms will be assigned to $l_1$, $\frac{l^*-l_1}{l_2-l_1}$ of the firms will be assigned to $l_2$. To balance accuracy and computational speed, I use more grid points in firm dynamics than in the value function iteration.

Measure of convergence.– Given two matrixes, I calculate the distance of each pair of elements, and then, use the L2-norm of the distance matrix to measure the distance of the matrixes. If the sum of the absolute value of the two elements is larger than one, I use the sum as the denominator to normalize their distance. In this way, I can control the scale effect.

The distance of the two distributions is measured by the distance of their moments. Since the first order moments (total labour supply, average productivity) are fixed, I use the covariance matrix representing the distribution.

The baseline requirement of accuracy is $1e-6$. To reduce the computation time, I use $1e-3$ when searching the wage path.
Chapter 4

Reallocation through Recession

4.1 Introduction

In emerging economies, low-productivity firms hold too many production resources. Inefficient allocation generates significant loss on aggregate productivity (Bartelsman, Haltiwanger, & Scarpetta, 2013; Hsieh & Klenow, 2009). In many countries, the low-productivity firms are mainly state-owned enterprises (SOEs). As such, the transformation of the state sector becomes a major engine of resource reallocation and productivity growth. This transformation is likely to take place during an economic recession. For example, the market-owned reforms of the Soviet Union and Eastern Europe began during the economic recessions in the 1980s. China’s major SOE reform was implemented during the Asian Financial Crisis. This paper studies the relationship between recessions and reallocation and suggests that recessions not only inspire reforms, but also boost reallocation.

In this study, I build a growth model with two types of producers. The first, SOEs, are less productive but initially larger, and the second, private firms, are more productive but originally smaller. Along with the economic growth, private firms accumulate more capital and finally dominate the market. As a result, the economy achieves more efficient resource allocation and higher aggregate productivity.

Recessions boost reallocation. During a recession, SOEs are hit by a negative productivity shock. They have to liquidate capital to repay their debt. Private firms purchase capital from the liquidation market. As a result, capital is reallocated to
more productive producers and aggregate productivity increases.

Recessions also lead to negative outcomes. Asides from direct productivity loss, recessions can lead to fire sales and slow capital accumulation. If private firms do not have enough liquidity to purchase all the liquidated capital, households enter the liquidation market. They do not hold production technology on capital, so only treat capital as a form of saving. Their entry reduces production capital, drives the capital (collateral) price down, and cuts external finance. The risk of fire sales only exists in the early periods of the transformation. Once private firms become large enough to purchase all liquidated capital, the risk disappears.

Because recessions create both positive and negative outcomes, the aggregate effect becomes a quantitative problem. I set the values of parameters based on Chinese data, and then, quantitatively analyse the influence. The initial size difference is taken from national account data in 1978. The number and productivity of the two types of firms are taken from firm-level data from 2000 to 2007, the period between the two financial crises. The rest are normalised or arbitrarily chosen to match the model assumptions.

I simulate three transitional dynamics with different shocks: no shock, an early-period shock with fire sales, and a late-arriving shock without fire sales. I use the first scenario as the benchmark, and calculate the impact by comparing the latter two with the benchmark. On the productivity side, both shocks boost later productivity growth. The effect is smaller in the first scenario as capital cannot be fully reallocated to private firms during fire sales. However, the effect influences the economy from the beginning of the transition, so the cumulative effect can be significant. By contrast, in the second scenario, the temporary productivity gain is larger, but the cumulative effect can be small. On the capital side, only the early shock leads to capital loss. After fire sales, capital accumulation remains lower than the benchmark economy for a considerable amount of time but finally becomes higher due to the increased aggregate productivity. The late shock does not lead to fire sales, so does
not generate capital loss. The economy accumulates more capital immediately after the recession.

The welfare effect is mixed. Both recessions harm the economy in the short term but benefit the economy in the long term. The recession with fire sales generates larger temporary loss and smaller temporary gain afterwards but, overall, larger cumulative gain. The recession without fire sales generates smaller temporary loss, larger temporary gain afterwards but relatively small cumulative gain.

This paper is related to discussions on the transformation of the state sector (Djankov & Murrell, 2002; Megginson & Netter, 2001; Murrell, 1993; Rodrik, 2006). Many studies evaluate the transition based on short-term performance. This paper suggests that the long-term outcome can be different to the short-term effect. A recession that generates a short-term welfare loss may improve welfare in the long term.

This paper is also related to the literature on resource allocation. Most studies in this field suggest to improve efficiency by reducing distortions (Banerjee & Duflo, 2005; Hopenhayn, 2014a; Hsieh & Klenow, 2009; Restuccia & Rogerson, 2008; 2017). This paper provides a new perspective: recessions also can improve efficiency and boost long-term growth, despite causing temporary welfare loss.

Finally, this paper sits alongside the literature on the welfare and policy analysis of recessions, specifically in the context of fire sales (Bianchi, 2016; Dávila & Korinek, 2017; Farhi & Tirole, 2012; Jeanne & Korinek, 2013; Lorenzoni, 2008; Shleifer & Vishny, 2011). Compared to other discussions in the literature, this paper considers a new channel that recessions can boost resource reallocation and benefit the economy in the long term. The quantitative analysis suggests that even if a recession results in fire sales and short-term welfare loss, the cumulative welfare effect still can be positive.

I build a theoretical model in Section 2 and set parameter values in Section 3. In Section 4, I discuss the impact of recessions in a simulated economy. Finally, Section
5 concludes and discusses possible directions of future works.

4.2 Model

Agents – The model economy is populated by households with overlapping generations and two types of firms: private firms and SOEs. The measure of new households is \( m \). The measures of SOEs and private firms are \( m_l \) and \( m_h \). Every household lives in the economy for two periods. They are consumers and debt holders. Firms play the roles of producers and debt issuers. They hold the same production technology but with different levels of productivity. The initial sizes are also different. SOEs are less productive but larger in the beginning. By contrast, private firms are more productive but originally smaller.

Technology – The economy produces nondurable and durable goods, say consumption goods and capital goods (capital). Households can store capital as part of their savings, but do not hold production technology. By contrast, firms are able to use capital to produce consumption goods. They consume a portion of their production and use the rest for debt repayment and investment. For the sake of simplicity, self-consumption is assumed to be nontradeable, and fixed as a proportion \( \gamma \) of the total output (Kiyotaki & Moore, 1997). The technology is constant return to scale. The production function of tradeable consumption goods is given as:

\[
f(k_{i,t}) = z_{i,t}k_{i,t},
\]

where \( z_{i,t} \) equals \((1-\gamma)\) times the productivity of all consumption goods. This study focuses on the transformation of SOEs, so ignores the uncertainty of the productivity of private firms and assumes it is equal to a constant value \( z_h \). The productivity of SOEs is independent and identically distributed over time, as described in the
following stochastic process:

\[ z_{i,t} = z_l b^\eta_t, \quad b \in (0, 1), \quad \eta_t \sim_{\text{i.i.d.}} B(1, p). \]  (4.2)

\( z_l \) is the baseline productivity of SOEs that is strictly lower than \( z_h \). When a negative shock hits the economy, the productivity of SOEs becomes an even lower value, \( z_l b \).

The occurrence of recession follows Bernoulli distribution with the probability \( p \). The productivity process is common knowledge in the economy.

Firms accumulate capital through two channels. They can build capital from final goods with a constant return to scale technology. The productivity is normalised as one. In other words, firms evaluate capital as the final goods used in its construction. Alternatively, firms can purchase capital from the market when other firms liquidate assets. More details about the liquidation market will be discussed later.

*Financial markets* – Firms can issue debts in the debt market. Young households will purchase debt to smooth consumption across periods. Households live for two periods, so all debts must be repaid in the next period. Firms face two kinds of constraints in the financial market: liquidity and collateral constraint.

Final goods are the liquidity of the economy. Firms repay their debt with the final goods they produce. They are not allowed to reschedule or refinance their debt. If the liquidity is not enough as in the following equation:

\[ z_{i,t} k_{i,t} < R_t d_{i,t-1}, \]  (4.3)

in which \( R_t \) is the interest rate at period \( t \) and \( d_{i,t-1} \) is the level of debt at period \( t - 1 \), they must liquidate a part of their capital \( (R_t d_{i,t-1} - z_{i,t} k_{i,t}) / q_t \) to cover the rest of their debt, where \( q_t \) is the asset price at period \( t \). As previously assumed, the negative shock only applies to SOEs. When SOEs face a liquidity shortage, private firms and households have enough liquidity to purchase the liquidated assets from
SOEs. The asset price is determined by demand side. Private firms would like to pay one unit of final goods for one unit of liquidated assets, the same as how much they spend for building new capital. If private firms have enough liquidity to purchase all liquidated assets, the liquidation market will reallocate capital to more productive producers without generating any externality or social welfare loss, otherwise the market suffers from the loss of fire sales.

Young households enter the liquidation market when private firms cannot provide enough liquidity:

$$\sum_i z_i k_{i,t} < \sum_i R_i d_{i,t-1}. \quad (4.4)$$

They purchase the remaining liquidated assets as a part of saving. Households evaluate capital the same as debt, so they are willing to pay the price $E(q_{t+1}/R_{t+1})$. For the sake of simplicity, their predictions on capital price and interest rate are assumed to be adaptive. This assumption will be adjusted in the future works. They bid $q_t/R_t$ for one unit of capital. This is lower than the capital price in normal periods. Thus, the entry of households drives the dramatic devaluation of capital that leads to three outcomes. First, private firms can afford more capital than before and more resources are reallocated to more productive producers. Second, the devaluated collateral reduces external finance of all firms, damaging capital accumulation in the economy. Third, households hold capital but do not use that capital for production. This directly reduces total output, consumption, and social welfare. The last two negative effects leave a space of government intervention. The assumption that households do not hold production technology captures the same idea of assuming expertise on operating capital. This kind of assumption underpins most models on fire sales since Shleifer and Vishny (1992) and Kiyotaki and Moore (1997) made the original contributions.

The risk of fire sales only exists in the early periods of transition. In that time, the liquidated assets of SOEs are too large for private firms due to the imbalance of initial firm sizes. Private firms narrow the size gap by their higher productivity and
are eventually able to afford all liquidated assets. The absence of the risk of fire sales eliminates the necessity of government intervention.

Aside from liquidity constraint, firms also collateral constraint in the debt market. They use capital as collateral. When a firm issues a debt, the amount cannot exceed a proportion $\phi$ of the collateral value, as given by:

$$R_{t+1}d_{i,t} \leq \phi \cdot q_t k_{i,t}.$$  (4.5)

The further capital value $k_{i,t+1}q_{t+1}$ is uncertain because of the risk of liquidation. The market are assumed to use the current value $q_t k_{i,t}$ as the adaptive expectation of the future value. The pecuniary externality of liquidation generates a real effect on capital accumulation in the same period. When a fire sale occurs, $q_t$ reduces from the normal value of 1 to the fire-sale price $q_t / R_t$. The devaluation of collateral means that it is more difficult to get external finance. This increases the necessity of government intervention. The leverage ratio $\phi$ is assumed in the following range:

$$\phi \in (z_l b, z_l].$$  (4.6)

This range determines the liquidation risk of firms. Under this assumption, SOEs have abundant liquidity in normal periods but face liquidity shortage when they are hit by a negative productivity shock. They need to liquidate assets during recessions. Private firms always hold extra liquidity after debt repayment. Although they are buyers in the liquidation market, they may not possess enough funds to purchase all liquidated assets. In this case, a fire sale occurs.

Household’s problem – A representative household lives in the economy for two periods. When it is born at the period $t$, it receives an endowment of asset $a_t$ that increases along with economic growth. By this assumption, households also enjoy the benefit of economic growth. In the benchmark set-up, endowment increases at
the same rate as capital accumulation,

\[ a_t = z_t \sum_i k_{i,t}. \] (4.7)

Young households spend a portion of their endowment to purchase debt or capital for the sake of smoothing consumption.

During a normal period, firm debts are the sole financial tool. Households consider the following problem:

\[
\max_{c_t, c_{t+1}} u(c_t) + \frac{1}{R_{t+1}} u(c_{t+1}),
\]

subject to

\[ c_t + \frac{1}{R_{t+1}} c_{t+1} \leq a_t, \] (4.9)

where \( c_t \) represents consumption in the young period, \( c_{t+1} \) represents consumption in the old period. The utility of the representative household is a weighted summation of utilities in the two periods. The one-period utility function \( u(\cdot) \) is increasing, strictly concave, and satisfies the Inada condition. The logarithmic form is used as the baseline case:

\[ u(c) = \ln(c). \] (4.10)

The subjective discount rate is assumed to be the inverse of the current interest rate. As the result of the problem, households equally allocate consumptions over the two periods, as shown in Equation (4.11):

\[ c_t = c_{t+1} = \frac{R_{t+1}}{1 + R_{t+1}} a_t. \] (4.11)

The debt \( d_{hh,t} \) they hold is determined by the following equation:

\[ d_{hh,t} = \frac{1}{1 + R_{t+1}} a_t. \] (4.12)
As shown in Equation (4.12), the demand of intertemporal transfer decreases when the interest rate $R_{t+1}$ increases. This is due to wealth effect. When $R_{t+1}$ increases, the future value of the endowment $a_t$ also increases, resulting in households that prefer to consume more and to save less.

During fire sales, households achieve the goal of consumption smoothing in both liquidation and debt markets. They purchase capital in liquidation market first and purchase debt in debt market later. When households enter the debt market, they can adjust the expectation of the interest rate, but not the asset price. As such, the budget constraint becomes:

$$c_t + \frac{1}{R_{t+1}} c_{t+1} \leq a_t + q_t k_{hh,t} \Delta \frac{1}{R_{t+1}},$$

where $k_{hh,t}$ is the capital households purchase in the liquidation market. The last term represents the prediction error of the interest rate. As a result, the consumption of young households becomes:

$$c_t = \frac{R_{t+1}}{1 + R_{t+1}} (a_t + q_t k_{hh,t} \Delta \frac{1}{R_{t+1}}).$$

The debt they purchase becomes:

$$d_{hh,t} = a_t - q_t k_{hh,t} \frac{1 + R_t}{(1 + R_{t+1}) R_t}.$$ 

The consumption of old households becomes:

$$c'_{t+1} = R_{t+1} a_t + q_{t+1} k_{h,t} - q_t k_{h,t} \frac{R_{t+1}(1 + R_t)}{(1 + R_{t+1}) R_t}.$$ 

Compared with the economy without fire sales, the solutions of $c_t$ and $d_{hh,t}$ include prediction error in the interest rate. The solution of $c'_{t+1}$ includes prediction errors of both the interest rate and the asset price.
Firm’s problem – Figure 4.1 depicts the time line of a firm. At the beginning of each period, the firm realizes its productivity and produces consumption goods. Production depends on realised productivity and the capital accumulated in previous period. It consumes the nontradeable consumption goods, and repays their debt with what remains. If that is not enough (for a SOE), the firm liquidates capital. If it is too much (for a private firm), the firm will use the excess liquidity to purchase the liquidated capital. At the end of the period, it issues new debt based on the remaining capital (collateral) and invests in new capital for the next period.

Given the productivity $z_{i,t}$, capital $k_{i,t}$, and debt $d_{i,t-1}$ issued during the last period, a firm considers the following Bellman equation,

$$V(z_{i,t}, k_{i,t}, d_{i,t-1}) = \max_{k_{i,t+1}} u \left( \frac{\gamma z_{i,t} k_{i,t}}{1 - \gamma} \right) + \mathbb{E} \left[ \frac{1}{R_{t+1}} V(z_{i,t+1}, k_{i,t+1}, d_{i,t}) \right], \quad (4.17)$$
where $\frac{1}{1-\gamma} z_{i,t} k_{i,t}$ is the self consumption of the firm. The utility function $u(\cdot)$ is assumed to be the same as that of households. The budget constraint is as follows:

$$R_t d_{i,t-1} + l_t^I \cdot 1_{R_t > 0} + \Delta k_{i,t+1} \leq z_{i,t} k_{i,t} + d_{i,t}. \quad (4.18)$$

The firm obtains the output $z_{i,t} k_{i,t}$ and debt $d_{i,t}$. They use the income to repay the debt issued in the last period $R_t d_{i,t-1}$, and invest in new capital $\Delta k_{i,t+1}$. It also needs to pay the value loss $l_t^I$ in the case of liquidation, in which:

$$l_t^I = q_t \cdot \frac{R_t d_{i,t-1} - z_{i,t} k_{i,t}}{1 - q_t}. \quad (4.19)$$

The firm is also subject to the collateral constraint shown in Equation (4.5). Given these assumptions, collateral constraint is binding. As such, the firm’s problem is trivial. The optimal debt level is given by:

$$d_{i,t} = \phi q_t k_{i,t} \frac{R_{t+1}}{R_{t+1}}. \quad (4.20)$$

Equilibrium – Definition 5 describes the recursive competitive equilibrium of the model.

**Definition 5** A recursive competitive equilibrium of the model is an interest rate system that satisfies the conditions of (i) household optimisation; (ii) firm optimisation; (iii) liquidation market clear; (iv) debt market clear.

### 4.3 Parameter values

I set the parameter values to match the SOE transformation in China. The initial distribution is from the national account data provided by the Chinese National Bureau of Statistics (NBS). The definition of ownership here follows the NBS. The number and productivity of the firms are from NBS firm-level data. The data period is from
2000 to 2007, a stable period between the Asian Financial Crisis and the Global Financial Crisis. The data processing follows Brandt, Van Biesebroeck, and Zhang (2014). In the data, a firm is defined to be a SOE if the state owns no less than half of its shares or is the controlling share holder. The rest of parameter values are normalised or arbitrarily chosen to match the model assumptions.

Table 4.1 summarises the choices of the values. The measure of households \( m \) is normalised as 1, as is the measure of SOEs \( m_l \). The measure of private firms \( m_h \) is the ratio of the number of SOEs to the number of private firms in NBS firm-level data. The production function is assumed to be constant return to scale, so the productivity is the output capital ratio. \( z_{i,t} \) is the productivity of tradable consumption good, so equals to the original productivity times \( 1 - \gamma \). The productivity of private firms \( z_h \) is \( 1 - \gamma \) times the average output capital ratio of all private manufacturing firms in NBS firm-level data. The baseline productivity of SOEs \( z_l \) is calculated by the same way. The negative productivity shock \( b \) is arbitrary. I assume the shock halves productivity. The initial capital of private firms \( k_{h,0} \) is normalized as 1. Therefore, the initial capital of SOEs \( k_{l,0} \) is a ratio of the initial sizes of the two types of firms. The average initial size of SOEs is calculated from the total capital of SOEs and the number of SOEs in 1978. There were almost no private firms at that time. As such, I use the average disposable income per capita in 1978 as the initial investment of private firms. The leverage ratio \( \phi \) is arbitrary. This value satisfies condition \((4.6)\). Thus, only SOEs suffer the risk of liquidation.
Table 4.1: Value assignment of the parameters

<table>
<thead>
<tr>
<th>Para.</th>
<th>Explanation</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>Measure of households</td>
<td>1</td>
<td>Normalisation</td>
</tr>
<tr>
<td>$m_h$</td>
<td>Measure of private firms</td>
<td>4.42</td>
<td>Numbers of SOEs and private firms from 2000 to 2007</td>
</tr>
<tr>
<td>$k_{h,0}$</td>
<td>Initial capital of private firms</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Proportion of self consumption</td>
<td>0.50</td>
<td>Arbitrary</td>
</tr>
<tr>
<td>$z_h$</td>
<td>Productivity of private firms</td>
<td>0.70</td>
<td>$\gamma$ and average output capital ratio of private firms from 2000 to 2007</td>
</tr>
<tr>
<td>$m_l$</td>
<td>Measure of SOEs</td>
<td>1</td>
<td>Normalisation</td>
</tr>
<tr>
<td>$k_{l,0}$</td>
<td>Initial capital of SOEs</td>
<td>7743.56</td>
<td>Average capital of SOEs and average disposable income per capita at 1978</td>
</tr>
<tr>
<td>$z_l$</td>
<td>Baseline productivity of SOEs</td>
<td>0.46</td>
<td>$\gamma$ and average output capital ratio of SOEs from 2000 to 2007</td>
</tr>
<tr>
<td>$b$</td>
<td>Negative shock</td>
<td>.5</td>
<td>Arbitrary</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Leverage ratio</td>
<td>.3</td>
<td>Arbitrary, within the range $(z_lb, z_l)$</td>
</tr>
</tbody>
</table>
4.4 Simulations

In this section, I simulate the economy with different shocks and identify the impact by comparing transitional paths. The first scenario is the transition without any shocks. This is the benchmark of the analysis. In the second scenario, a negative productivity shock hits SOEs in the second period of the transition. Because of the imbalanced initial sizes, private firms are too small to purchase all liquidated assets. The shock leads to fire sales. In the third scenario, a shock hits the economy in the 34th period. This is the first period in which private firms are large enough to purchase all liquidated assets.

4.4.1 The transformation of the state sector

Figure 4.2, 4.3, and 4.4 demonstrate the transformation of the state sector with recessions occurring in different periods. The blue lines show the benchmark transformation without recessions, the yellow lines show the transformation with a recession and fire sales, and the red lines show the transformation with a recession but without fire sales.

Figure 4.2 shows the dynamics of the size difference between the two types of firms. The economy begins in an unbalanced state in which SOEs are much larger than private firms. Both SOEs and private firms continue to accumulate capital during the transition but the speeds of capital accumulation are different. The private sector accumulates capital faster due to higher productivity. Finally, the size difference reverses to the opposite direction and the curve passes the horizontal line of 0. The size of private firms becomes larger than the size of SOEs. Before reverting, private firms can already afford all liquidated assets, as they are more than SOEs. Capital accumulation will be discussed in further detail later.

Recessions boost resource reallocation but timing remains important. In the fire-sale scenario, private firms cannot afford all liquidated assets. As such, the impact on the size difference is relatively smaller. In the scenario without fire sales, private
§4.4 Simulations

Figure 4.2: The dynamics of size difference between SOEs and private firms.

Notes: The difference is measured by logarithmic capital ratio.

firms are large enough to take over all liquidated capital. The recession generates a relatively larger impact on resource reallocation, as shown in Figure 4.2.

Figure 4.3 demonstrates the dynamics of the proportion of capital held by SOEs. At the start, SOEs hold most capital of the economy. Later, the share decreases due to different productivity. Finally, private firms own most capital of the economy. Recessions boost this transition. The effect also depends on timing. The pattern is the same as that shown in Figure 4.2.

Figure 4.4 shows the dynamics of the market share of SOEs. Because the productivity difference between the two types of firms is fixed during the booming periods, the dynamics of market share represent capital dynamics. At the start of the transformation, SOEs occupy most of the market share. Later, private firms slowly take over the market and finally occupy almost the entire market. Recessions boost this process. A recession without fire sales generates a greater effect on the transformation, as it reallocates more capital to private firms.
Reallocating through Recession

Figure 4.3: The dynamics of the proportion of capital occupied by SOEs.

Figure 4.4: The dynamics of the market share of SOEs.
4.4.2 Productivity growth

Figure 4.5 demonstrates the dynamics of aggregate productivity that is a weighted combination of the productivity of SOEs and private firms. The weight is the capital share. Larger shares of private firms indicate higher aggregate productivity. Through this channel, the economic transformation boosts productivity growth. In the beginning, aggregate productivity represents the productivity of SOEs, as SOEs hold most of the capital. Next, it increases with the increasing share of high-productivity private firms. Finally, private firms dominate the market. Aggregate productivity represents their productivity.

Recessions reduce short-term productivity and boost the transition but they cannot influence long-term productivity. The impact is through two channels. First, recessions directly reduce the productivity of SOEs for one period. Second, recessions reallocate more capital to high-productivity producers. The two channels work in opposite directions. The first channel dominates in the short term. Therefore, productivity decreases during recessions before the productivity of SOEs recovers to the normal level. More efficient capital allocation boosts the transformation. Productivity growth is faster than in the benchmark economy. However, the recessions only
influence the speed rather than the target of the transition. In the long term, private firms dominate the market in all cases. Hence, long-term aggregate productivity is determined by the productivity of private firms rather than recessions.

As shown in Figure 4.5, the size of the impacts depend on timing. A recession with fire sales generates larger short-term productivity loss and smaller productivity gain afterwards. The large immediate loss is primarily from the productivity loss of SOEs. This channel is more influential in the early stage because of the large initial size of SOEs. By contrast, the impact of capital reallocation is limited. Liquidated capital cannot be fully reallocated to private firms during fire sales. Insufficient capital reallocation is powerless to substantially boost productivity growth. Thus, the aggregate effect on productivity is extremely negative during the recession. Afterwards, productivity recovers to a normal value and grows faster due to more efficient capital allocation. This acceleration is less than in the scenario without fire sales because of the insufficient reallocation. However, as the recession boosts transformation at the early stage, the cumulative effect can be considerate. By contrast, the later recession generates less short-term productivity loss and more productivity gain afterwards. Smaller loss is achieved with a smaller share of SOEs and sufficient capital reallocation. Later, more efficient capital allocation further boosts transformation and productivity growth. However, the greater acceleration only influences the later segment of the transition. As such the cumulative effect may not be significant.

### 4.4.3 Capital accumulation

Recessions affect capital accumulation but the impact is limited. Figure 4.6 shows three dynamics of capital. Capital continues to increase in all three economies and recessions do not considerably change this trend. In the scenario without fire sales, the recession boosts capital reallocation without any loss. In the scenario with fire sales, the recession generates a negative effect on capital accumulation. The effect is limited for two reasons. First, fire sales only apply to the extra asset which private
firms cannot afford, so the amount is limited. Second, the fire-sale assets will be
resold to firms again in the next period, rendering the negative effect temporary. As
a result, recessions mainly influence capital allocation rather than accumulation.

Figure 4.7 shows further details of the impact of recessions. For comparison and
detrend, I use the economy without any recessions as the benchmark and calculate
the relative capital accumulation. Subgraph 4.7(a) and 4.7(b) show the relative capi-
tal accumulation of SOEs and private firms. The capital of SOEs relatively declines
during recessions, and never recovers to the benchmark level. The impact is similar
in private firms, but in the opposite direction. The recessions influence capital accu-
cummulation in both the short and long term, while the short-term effect is larger. The
pattern matches the impact on transformation. The recessions reallocate capital to
high-productivity producers, a change that is permanent.

Subgraph 4.7(c) shows the dynamics of total capital. The peak of the impact is
within four percent of the benchmark total capital and long-term effects are within
two percent of the benchmark value. This matches the small movement shown in
Figure 4.6. In the scenario without fire sales, capital is directly reallocated to the pri-
ivate sector without any loss. Because high-productivity firms hold more production
resources, later capital accumulation is faster than in the benchmark economy. In
the scenario with fire sales, the recession leads to temporary loss of total capital. It comes close to returning to the benchmark level when households resell the capital in the following period. The more efficient capital allocation boosts capital accumulation in which the speed of accumulation is faster than in the benchmark economy. Finally, the total capital becomes higher than in the benchmark economy. However, due to fire-sale loss, the amount of capital remains lower than in the economy without fire sales. In the long term, all economies are dominated by private firms, so the allocation effect is weakened. The difference among the three economies reduces.
4.4.4 Output

The total output of the economy is determined by aggregate productivity and total capital. Continuous productivity growth and capital accumulation force output to continue to increase in all three scenarios. Figure 4.8 shows the output dynamics of three simulations. Output is measured by the logarithmic output divided by the benchmark value. The pattern of the dynamics is similar to the pattern of capital dynamics and the scale is larger due to the productivity channel.

![Figure 4.8: The dynamics of relative output.](image)

Notes: Output is measured by the ratio of logarithmic output to the benchmark value.

Subgraph 4.8(a) and 4.8(b) show the output dynamics of SOEs and private firms. The dynamics of private firms are exactly the same as the capital dynamics, as their
productivity is constant. The difference between the graphs is purely driven by the logarithmic transformation. For SOEs, output drops further than capital due to productivity decline during recessions.

Subgraphs 4.8(c) shows the aggregate effect. In the scenario with fire sales, logarithmic output drops to 92.07 percent of the benchmark output. This is a consequence of both direct productivity loss of SOEs and capital loss during fire sales. The output decline is greater when the recession hits the economy during the early stage of the transformation, as productivity shocks target SOEs and SOEs are relatively larger in this stage. Immediately after the recession, output recovers and almost returns to the benchmark value. The recovery is in both the productivity and capital sides: productivity returns to the normal value and the fire-sale capital is resold to to firms. Afterwards, output grows faster than in the benchmark economy and finally reaches a higher value in the long term.

In the scenario without fire sales, short-term output loss is purely driven by productivity loss rather than capital accumulation. As shown in Figure 4.5, productivity decreases slightly during the recession, so output also decreases slightly. Then, productivity growth and capital accumulation together boost output growth. As such output quickly recovers and becomes higher than both the benchmark value and the output of the fire-sale economy. However, the output boom only occurs later in the transformation, so the cumulative effect may not be significant.

4.4.5 Consumption

Figure 4.9 shows the consumption dynamics of the economy. Subgraphs 4.9(a) and 4.9(b) show the consumption dynamics of young and old households. Households distribute endowments to equalise their consumptions over two periods. As a result, the pattern of the two dynamics are similar but the dynamics of old households delay for one period. Because of prediction errors, the dynamics of the two generations are slightly different during recessions. As assumed in Equation (4.7), household
endowment represents capital accumulation in the economy. The future value also depends on interest rate. In booming periods, interest rate is constant constant over time, so consumption dynamics represent the capital dynamics. In a recession period, future value decreases due to lower interest rate, so households consume less and save more. In the scenario with fire sales, logarithmic total capital declines to 96.14 percent of the benchmark value. Household consumption decreases even further, equals to 95.13 percent of the benchmark value. In the scenario without fire sales, capital does not decline but household consumption decreases due to lower interest rate and lower future value of their endowments.

Subgraphs 4.9(c) and 4.9(d) show the consumption dynamics of SOEs and private firms. All firms consume a fixed portion of their production, so consumption dynamics directly represent output dynamics. The differences to output dynamics are purely driven by logarithmic transformation.

Subgraph 4.9(e) plots the dynamics of total consumption, that is a weighted summation of the above four dynamics and the weight is their population. The pattern is similar to the dynamics of total output.

### 4.4.6 Welfare

The dynamics of individual utilities are represented by their consumptions, which are discussed in previous subsection. As assumed in Equation (10), utilities are the logarithmic transformation of consumptions, so their dynamics are identical to the dynamics of logarithmic consumption.

Figure 4.10 demonstrates the dynamics of single-period social welfare. The social welfare function is utilitarian that is the weighted summation of individual utilities and the weight is the population of each group. In the scenario with fire sales, capital and output are lower in total than the benchmark level for a long time. However, social welfare declines for only two periods, and then, grows even higher than the benchmark value. The rapid recovery is driven by the considerable utility gain of
private firms. Although all others suffer utility loss for a long time, the utility gain of private firms is large enough to boost the catch-up of social welfare. In the scenario without fire sales, the pattern is similar to output dynamics: social welfare slightly declines, and then, grows higher than other two economies soon after the recession.

The cumulative effect on social welfare is different with the short-term effect. The recession with fire sales generates a greater welfare loss during the recession, a smaller short-term gain afterwards, but a larger cumulative gain. By contrast, the recession without fire sales generates a smaller welfare loss during the recession, a larger temporary gain afterwards, but a smaller cumulative gain. In the scenario with fire sales, welfare drops to 93.18 percent of the benchmark value and increases to up to 102.26 percent afterwards. The cumulative gain is 14.98 percent.\footnote{The value is the overall welfare gain over one-hundred simulation periods and discounted to the net value in the first period. When the simulation is expanded to longer horizon, the result nearly does not change. The reason is that the net values of long-run gains are tiny after discounted exponentially and the welfare effect is dominated by early-period gains.} In the scenario without fire sales, welfare declines to 98.79 percent of the benchmark value and soon grows to up to 102.54 percent. However, the cumulative gain is only 3.77 percent of the benchmark value. The different cumulative effect is due to timing. The first recession arrives at the beginning of the transition, influencing almost the entire transitional dynamics. Although the short-term effect is limited, the cumulative effect is greater. By contrast, the second recession hits the economy during the late segment of the transition, and while the short-term effect is greater, the cumulative effect is relatively small in limited time.

### 4.5 Conclusion

This chapter studies the role of recessions in the transformation of the state sector. In the model economy, recessions reallocate capital to the private sector and boost long-term growth. On the other hand, recessions cause short-term loss of output, consumption, and social welfare. The positive effect dominates in all simulations. Recessions generate gains on cumulative welfare. The timing of a recession is impor-
tant. In the early stage of the transformation, a recession leads to larger temporary loss, as private firms are too small to take over all the resources released by SOEs. However, it boosts transformation for a longer length of time, so generates more significant cumulative welfare gain than later recessions.

Model assumptions and estimations are simplified in the current version. These will be adjusted in future versions. For instance, the assumption of adaptive expectation will be modified to rational expectation. In addition, the current welfare effect depends on the parameter environment. A natural extension is the sensitivity test of important parameters, such as the productivity difference between SOEs and private firms and the size of negative shocks. This can provide more insight on the welfare and policy analysis.
Figure 4.9: The dynamics of relative consumption.

Notes: Consumption is measured by the ratio of logarithmic consumption to the benchmark value.
Figure 4.10: The dynamics of relative social welfare

Notes: The measures are the ratios to the benchmark values.
Conclusion

This thesis comprises three essays on firm dynamics and development. Each essay extends the literature on allocative efficiency and development in a different direction. In this concluding chapter, I summarise the key findings of each essay and suggest avenues for further research.

In Chapter 2, I find that the improvement of allocative efficiency is able to explain a large proportion of miraculous growth. I study the transitional dynamics of an economy that is populated by heterogenous firms. Allocative efficiency improves when only more productive firms survive and they are allocated more production resources. Adjustment frictions and entry/exit barriers delay the efficiency improvement for decades. Meanwhile, the slow efficiency improvement continues to contribute to growth. The growth rate is consequently high. Eventually, the growth miracle disappears once the allocative efficiency reaches the stationary level.

Chapter 2 reveals the importance of efficiency improvement in a simulated economy, yet the quantitative importance of this channel in the real world remains an empirical question. The answer of this question could be country-specific. Decomposing the growth of a miraculous economy will provide further insights into the contribution of efficiency improvement. Such analysis relies on rich and sufficiently long firm-level panel data.

In Chapter 3, I find that technological diffusion could cause higher productivity dispersion in emerging economies. In these economies, the productivity distribution may become narrow again even when distortions remain the same. Firms in the
model learn about new technology from the world frontier. Different learning speeds lead to a dispersed productivity distribution in the catch-up process. When firms face adjustment costs, the dispersion of marginal productivity also increases. After a long period of learning, slow learners also move closer to the frontier. Finally, the dispersions are low again. This result suggests that high productivity dispersion does not necessarily indicate high-level distortions, but may also represent a stage of development.

Chapter 3 provides a hypothesis on productivity dispersion in emerging economies. More empirical evidences are necessary to show the quantitative importance of the hypothesis. Future investigations should seek to decompose the dispersion differences between emerging and a developed economies, examine the contribution of the stages of development.

In Chapter 4, I suggest that recessions also can be a channel of improving efficiency. In the model economy, recessions boost the transformation of the state sector by reallocating more capital to private firms. During a recession, state-owned enterprises liquidate capital for debt repayments. Private firms take over the liquidated capital. The reallocation improves efficiency and boosts long-run growth. On the other hand, recessions lead to temporary loss of productivity, output, and welfare, but the cumulative welfare effect is nonetheless positive. The timing of a recession is important. An early recession with fire sales results in large temporary losses, slow recovery, but large cumulative welfare gain, as it boosts the transformation for a longer time. By contrast, a late-arriving recession results in smaller temporary losses, faster recovery, but relatively small cumulative welfare gain.

Some assumptions and estimations are simplified in Chapter 4, which will be adjusted in further works. For example, the assumption of adaptive expectation will be modified to rational expectation. Furthermore, it should be noted that the current results are calculated based on the assigned parameters. A sensitivity test of the parameter environment could better the understanding of welfare effect.
References


