FOCUS ISSUE: Nonlinear Localized Modes: Physics and Applications

Introduction: Nonlinear localized modes

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The last ten years have seen remarkable developments in the research of nonlinear localized modes in different physical systems. This activity has blazed new trails in the nonlinear physics of spatially localized structures, after many years of active study of exactly integrable nonlinear equations and their localized solutions, *solitons*. One of the most exciting results of the past decade has been the discovery of stable highly localized nonlinear modes of discrete lattices, *discrete breathers* or *intrinsic localized modes*.[1–3]

The discreteness of space—i.e., the existence of an underlying spatial lattice—is crucial to the structural stability of these spatially localized nonlinear excitations. Spatial discreteness is obviously the standard situation for applications in solid state physics, but recent studies, both theoretical and experimental, have shown that the effects of spatial discreteness can be important in many other systems, including photonic crystals, coupled optical wave guides, and coupled Josephson junctions.

Discreteness is essential for avoiding resonances with plane wave spectra, which are bounded due to the discreteness, as opposed to the typical unbounded spectra of continuum field equations. Consider the case of the dynamics of nonlinear lattices, where the intrinsic localized modes/discrete breathers have been shown to be exact spatially localized, time-periodic excitations. Their structural stability and generic existence follow from the fact that all multiples of their fundamental frequencies are out of resonance with plane waves. Thus localization is obtained in a system without additional inhomogeneities. Notably, these excitations exist independent of the lattice dimension, number of degrees of freedom per lattice site, and other details of the system under consideration.

During the early years, studies of intrinsic localized modes were mostly of a mathematical nature, but the ideas of localized modes soon spread to theoretical models of many different physical systems, and the discrete breather concept has been recently applied to experiments in several different physics subdisciplines.

One of the first experiments came from the field of guided-wave optics. Light injected into a narrow waveguide which is weakly coupled to parallel waveguides (characteristic diameters and distances are of order of micrometers in a nonlinear optical medium based on GaAs materials) disperses to the neighboring channels for low field intensities, experiencing the so-called ''discrete diffraction.'' However, for large field intensities, the light remains localized in the initially injected waveguide.[4,5]

Spatially localized voltage drops in Nb-based Josephson junction ladders have been observed and characterized[6] (a typical size of junction is a few micrometers). These states correspond to generalizations of discrete breathers to dissipative systems.

Localized modes in antiferromagnetic quasi-one-dimensional crystals have been excited in Ref. 7, and more recently localized vibrational modes were observed in micromechanical oscillators arrays.[8]

In the quantum realm, bound phonon states (up to seven participating phonons) have been observed by overtone resonance Raman spectroscopy in PtCl mixed valence metal compounds.[9] These bound states are quantum versions of classical discrete breather solutions.

In the present volume, the reader will find a detailed analysis of the general properties of discrete breathers and examples of the above-mentioned and other physical models and materials in which intrinsic localized modes may play important roles.

Unsurprisingly, in a field that is moving as rapidly as this one, there have been several further developments that occurred after the composition of this focus issue. First, a novel design of waveguide arrays allowed the study of nonlinear localized modes in spectral bands as self-trapped states of Bloch waves.[10] Second, many properties of discrete modes are revealed in optically induced lattices, in which the so-called ''discrete solitons'' have been observed experimentally in both one and two dimensions.[11,12] These results show the way for the realization of a variety of nonlinear localization phenomena in photonic structures.

All these activities demonstrate that the concept of intrinsic localized modes and discrete breathers, as predicted by key researchers more than ten years ago, has broad generalizations to and applications in various areas of science. One particularly exciting prospect comes from the dramatic increase in research on artificial or man-made devices on the micrometer and nanometer scales (of both optical and solid state nature), together with a huge interest growing in the area of quantum information processing. We may expect interesting new developments in these areas, which will be connected in various ways to the understanding of the concept of nonlinear localized modes. Recent examples are the connection of one- and two-dimensional discrete breathers and the physics of Bose–Einstein condensates in periodic optical traps[13] as well as wave localization in photonic crystals.[14] Thus it is timely to give a survey of current research in these directions.

This Focus Issue includes both review articles and original results. The contributions can roughly be classified in *four broad categories*. *The first group of papers* is devoted to the analysis of general properties of intrinsic localized modes. A number of techniques have been developed in the last few years to study discrete systems with strong nonlinearities; some of the articles of this group contain an overview of these techniques and their applications. *The second and third groups of papers* span several areas of applications of the intrinsic localized modes. They cover several physical systems where a number of important experimental results have recently been demonstrated, including Josephson arrays and localized vibrational modes in micro-mechanical oscillator arrays. The concluding, *fourth group of papers* provides an important link to other fields where the ideas of intrinsic localized modes play an important role. This includes waveguide arrays in optics, localized modes in photonic crystals, and matter-wave breathers in Bose–Einstein condensates loaded onto optical lattices.

Importantly, many recent developments in different areas of physics are happening simultaneously and sometimes without close links between each other. For example, the first observation of self-trapping and localization in optical waveguide arrays came as a natural development of the physics of spatial optical solitons,[15] not from the theory of discrete breathers. Therefore, bringing researchers from different fields who are using different approaches to study similar physical objects, may greatly enhance progress in this field.

In preparing this Focus Issue, we have sought to capture the interdisciplinary relevance of nonlinear localized modes by choosing authors from a range of disciplines and by seeking to present several different new approaches to the problem of nonlinear localized modes and their properties. We hope this approach will make clear the many common properties shared by localized modes in different physical systems, ranging from highly discrete to almost continuous, and from conservative to dissipative.

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We believe that this issue will provide both a useful reference tool and an accessible introduction for scientists seeking to learn about this topic. We hope that readers will read the articles with interest and that the issue itself will help in developing this exciting field of physics.

[1] A. J. Sievers and J. B. Page, in *Dynamical Properties of Solids. VII Phonon Physics The Cutting Edge*, edited G. K. Horton and A. A. Maradudin (Elsevier, Amsterdam, 1995).
[2] S. Aubry, Physica D **103**, 201 (1997).
[3] S. Flach and C. R. Willis, Phys. Rep. **295**, 181 (1998).
[4] H. S. Eisenberg, Y. Silberberg, R. Morandotti, A. R. Boyd, and J. S. Aitchison, Phys. Rev. Lett. **81**, 3383 (1998).
[5] For a recent review of this field, see A. A. Sukhorukov, Yu. S. Kivshar, H. S. Eisenberg, and Y. Silberberg, IEEE J. Quantum Electron. **39**, 31 (2003).
[6] E. Trias, J. J. Mazo, and T. P. Orlando, Phys. Rev. Lett. **84**, 741 (2000); P. Binder, D. Abraimov, A. V. Ustinov, S. Flach, and Y. Zolotaryuk, *ibid.* **84**, 745 (2000).
[7] U. T. Schwarz, L. Q. English, and A. J. Sievers, Phys. Rev. Lett. **83**, 223 (1999).
[8] M. Sato, B. E. Hubbard, A. J. Sievers, B. Ilic, D. A. Czaplewski, and H. G. Craighead, Phys. Rev. Lett. **90**, 044102 (2003).
[9] B. I. Swanson, J. A. Brozik, S. P. Love, G. F. Strouse, A. P. Shreve, A. R. Bishop, W. Z. Wang, and M. I. Salkola, Phys. Rev. Lett. **82**, 3288 (1999).
[10] D. Mandelik, H. S. Eisenberg, Y. Silberberg, R. Morandotti, and J. S. Aitchison, Phys. Rev. Lett. **90**, 053902 (2003).
[11] J. W. Fleischer, T. Carmon, M. Segev, N. K. Efremidis, and D. N. Christodoulides, Phys. Rev. Lett. **90**, 023902 (2003); D. Neshev, E. A. Ostrovskaya, Yu. S. Kivshar, and W. Krolikowski, Opt. Lett. **28**, 710 (2003).
[12] J. W. Fleischer, M. Segev, N. K. Efremidis, and D. N. Christodoulides, Nature (London) **422**, 147 (2003).
[13] A. Trombettoni and A. Smerzi, Phys. Rev. Lett. **86**, 2353 (2001); E. A. Ostrovskaya and Yu. S. Kivshar, *ibid.* **90**, 160407 (2003).
[14] S. V. Mingaleev and Yu. S. Kivshar, Phys. Rev. Lett. **86**, 5474 (2001).
[15] Yu. S. Kivshar and G. P. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals* (Academic, San Diego, 2003), 560 pp.