Suppression of left-handed properties in disordered metamaterials

Alexander A. Zharov, Ilya V. Shadrivov, and Yuri S. Kivshar

Citation: Journal of Applied Physics **97**, 113906 (2005); doi: 10.1063/1.1923591
View online: http://dx.doi.org/10.1063/1.1923591
View Table of Contents: http://scitation.aip.org/content/aip/journal/jap/97/11?ver=pdfcov
Published by the AIP Publishing

Suppression of left-handed properties in disordered metamaterials

Alexander A. Zharov,[a] Ilya V. Shadrivov,[b] and Yuri S. Kivshar
Nonlinear Physics Centre, Research School of Physical Sciences and Engineering, Australian National University, Canberra ACT 0200, Australia

(Received 8 November 2004; accepted 7 April 2005; published online 1 June 2005)

We show that disorder can modify dramatically the effective magnetic response of a composite metamaterial including its specific property of negative refraction. We demonstrate that a relatively weak disorder in the parameters of the split-ring resonators can reduce and even completely eliminate the finite frequency range where the metamaterial possesses the left-handed properties. This effect is expected to be crucially important for a design of metamaterials operating in the terahertz or visible region of the frequency spectrum. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1923591]

I. INTRODUCTION

Recently fabricated microstructured materials open a unique possibility to study experimentally the so-called left-handed metamaterials.[1–3] The composite materials created by arrays of wires and split-ring resonators (SRRs) have been shown to possess a negative real part of the magnetic permeability and negative dielectric permittivity in the microwave range, and they demonstrate many unusual properties predicted theoretically a long time ago,[4] including negative refraction.

The microstructured design based on the lattices of the split-ring resonators and wires has already been demonstrated to give negative magnetic permeability in the terahertz region.[5,6] It is believed that this concept can be extended into the infrared, bringing us close to the realization of magnetism at *optical frequencies*. Indeed, there has already been a proposal that silver nanowires could be used to produce anomalous magnetic effects in the visible region.[7]

In the metamaterials, simultaneously negative dielectric permittivity and magnetic permeability occur in a finite frequency range. Within the effective-medium approximation, the effective dielectric permittivity becomes negative in a relatively large frequency range due to the linear response of wires, whereas the effective magnetic permeability can become negative in a relatively narrow frequency range near a resonance. It is this specific frequency range where the induced magnetic momentum of each SRR in the structure is directed opposite to the external magnetic field being strong enough to produce collectively negative values of the effective magnetic permeability.

In this paper, we study the effect of random variation of the structure parameters on the major properties of left-handed materials and the existence of the frequency domain where the composite material exhibits the specific left-handed properties such as negative refraction.[8] We show, that even the relatively small variation of the SRR parameters can result in a dramatic decrease of the size of the left-handed frequency domain and, above some threshold value, such a disorder can suppress and even eliminate completely the left-handed properties of the composite material.

[a]Permanent address: Institute for Physics of Microstructures, Russian Academy of Sciences, Nizhny Novgorod 603950, Russia.
[b]Electronic mail: ivs124@rsphysse.anu.edu.au

II. MODEL FOR METAMATERIALS

We study a composite structure made of metallic wires and single-ring microwave resonators as shown schematically in Fig. 1(a). This model is qualitatively similar to the model of double-ring resonators usually studied in the theory of left-handed composite media.[9] Within the effective-medium approximation, the dielectric permittivity of this structure can be calculated[10,11] and presented in the form

$$\epsilon_{\text{eff}} \approx 1 - \frac{\omega_p^2}{\omega(\omega - i\gamma_e)}, \quad (1)$$

where $\omega_p = 2\pi c/d \ln(d/r_\omega)$, d is the size of the unit cell, r_ω is the radius of a metallic grid, $\gamma_e = c^2/2\sigma S \ln(d/r_\omega)$, σ is conductivity of a wire, S is the effective wire cross section, ω is the angular frequency, and c is the free-space speed of light. Usually, the effective plasma frequency, $f_p = \omega_p/2\pi$, is between 10 and 15 GHz.

The most crucial properties of the left-handed composite metamaterials are due to a response of the SRR arrays which determine its effective magnetic permeability. Magnetization of the metamaterial with three SRRs per unit cell [see Fig. 1(a)] can be presented in the form,[12] $\mathbf{M} = \chi(\omega)\mathbf{H}'$, where

$$\chi(\omega) = \frac{\eta\omega^2}{\omega_0^2 - \omega^2 + i\gamma_m\omega}, \quad (2)$$

$$\eta = \frac{\pi}{8}\left(\frac{a}{d}\right)^3 [\ln(8a/r) - 7/4]^{-1/2}. \quad (3)$$

Here, $\mathbf{H}'$ is the acting (microscopic) magnetic field, $\omega_0 = \{d_g c^2/\pi\epsilon r^2 a[\ln(8a/r) - 7/4]\}^{1/2}$ is the SRR eigenfrequency, a is the SRR radius, $\gamma_m = c^2/2\sigma_r S_r[\ln(8a/r) - 7/4]$ is the damping coefficient, d_g is the size of SRR slot, and ϵ is permittivity of the dielectric infilling the SRR slot in the structure.

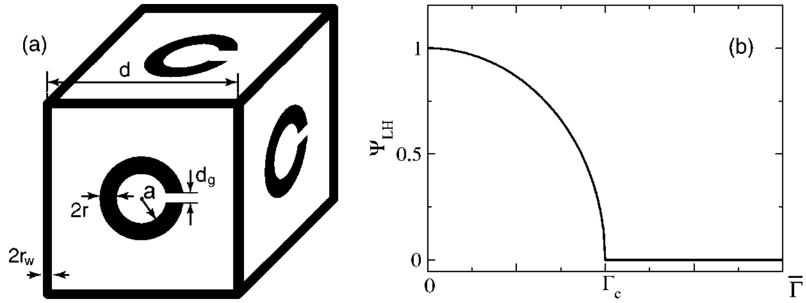

FIG. 1. (a) Schematic of the metamaterial structure. (b) Dependence of the real part of the order-parameter Ψ_{LH} on the effective losses Γ in the composite structure.

III. EFFECT OF DISORDER

Results (2) and (3) are obtained under the major assumption that all resonators in the structure are identical. Now we consider the case when the size of the slot d_g is a random function that is characterized by a statistical distribution function. Our studies are motivated by recent efforts to create metamaterials with the left-handed properties for shorter wavelengths when the key parameters are difficult to control in fabrication. In our structure, the parameter η does not depend on d_g [see Eq. (3)], and only SRR eigenfrequency ω_0 is affected by fluctuations of d_g. Then, the equation for the magnetic susceptibility (2) can be generalized for the case of randomly varying eigenfrequency:

$$\chi(\omega) = \eta\omega^2 \int_0^\infty \frac{F(X)dX}{X^2 - \omega^2 + i\gamma_m\omega}, \quad (4)$$

where $F(X)$ is the normalized distribution function of the SRR eigenfrequencies, i.e., $\int_0^\infty F(X)dX=1$. In the standard case when all SRRs are identical, the distribution function can be represented as $F(X)=\delta(X-\omega_0)$.

To describe the coupling between the acting magnetic field $\mathbf{H}'$ and the macroscopic magnetic field $\mathbf{H}$, we use the Lorentz–Lorenz relation,[13] $\mathbf{H}'=\mathbf{H}+(4\pi/3)\mathbf{M}$, and present the effective magnetic permeability in terms of magnetic susceptibility (4) as follows:

$$\mu_{\text{eff}}(\omega) = \frac{1 + (8\pi/3)\chi(\omega)}{1 - (4\pi/3)\chi(\omega)}. \quad (5)$$

For definiteness, we consider the Lorenz-type distribution of the SRR eigenfrequencies in the form $F(X)=(\Gamma/\pi)[(X-\omega_0)^2+\Gamma^2]^{-1}$, with a narrow width for $\Gamma \ll \omega_0$ such that the eigenfrequencies of all resonators are close to some mean value ω_0. As a result, a nonvanishing contribution to the integral are given by the values of X in the vicinity of $X=\omega_0$ and we introduce a variable Δ, $X=\omega_0+\Delta$, where $|\Delta| \ll \omega_0$. We are interested in the behavior of the magnetic susceptibility in the vicinity of $\omega=\omega_0$, and we introduce $\omega=\omega_0+\Omega$, where $|\Omega| \ll \omega_0$. In this approximation, Eq. (4) can be rewritten as

$$\chi(\omega) \approx \frac{\eta}{2\pi}\omega_0\Gamma \int_{-\infty}^{\infty} \frac{d\Delta}{(\Delta^2+\Gamma^2)(\Delta-\Omega+i\gamma_m/2)}. \quad (6)$$

Using the contour integration, the integral in Eq. (6) can be calculated explicitly, and the expression for the magnetic susceptibility can be obtained in the following form:

$$\chi(\Omega) = \frac{\omega_0\eta}{2}\frac{1}{(-\Omega+i\overline{\Gamma})}, \quad (7)$$

where $\overline{\Gamma}=\Gamma+\gamma_m/2$. The result (7) shows that the random variation of the SRR eigenfrequencies is *equivalent* to the introduction of additional losses in the structure. Even for an ideal case of lossless resonators, i.e., when $\gamma_m=0$, the composite structure with random variation of the SRR frequencies possesses effective losses. From the physical point of view, such effective losses resemble the collisionless Landau damping in plasma[14] caused by the presence of resonant particles and, simultaneously, it follows from the Kramers–Kronig relations.

From Eqs. (5) and (7), we find the frequency domain where the real part of magnetic permeability becomes negative, $\Omega_1<\Omega<\Omega_2$, where

$$\Omega_{1,2} = \frac{1}{3}\Gamma_c \mp \sqrt{\Gamma_c^2 - \overline{\Gamma}^2}, \quad (8)$$

where $\Gamma_c=\pi\eta\omega_0$. Thus, the width of the frequency domain where the effective magnetic permeability becomes negative is $\Delta\Omega_{\text{LH}}=2\text{Re}\sqrt{\Gamma_c^2-\overline{\Gamma}^2}$. The value Γ_c has the meaning of the critical losses in the composite structure, above which the domain with negative values of the magnetic permeability disappears, and the relative critical parameter Γ_c/ω_0 depends only on the structure of the composite. Taking the characteristic values $d=0.5$ cm, $a=0.2$ cm, and $r=0.05$ cm, we obtain $\Gamma_c/\omega_0=0.046$. This result indicates that even for the case of lossless resonators, i.e., $\gamma_m=0$, relative fluctuations of the SRR eigenfrequencies cannot exceed 4.6%. Accordingly, if the slot size is $d_g=0.01$ cm (then $\omega_0/2\pi=9.6$ GHz) the critical variation can be estimated as $\Delta d_g \sim 5$ μm. The existence of such a critical distribution width places strict requirements on the manufacturing of such materials. Although for microwaves such requirements can be easily met, it can result in substantial experimental difficulties for shorter wavelengths. In particular, the recent proposal to fabricate left-handed metamaterials in optics using nanowires and π-shaped particles,[7] would require an accuracy better than 5 nm, which is a real challenge for the current technology.

We note that premeditative introduction of disorder does not allow to increase the size of the left-handed domain for *any type* of the distribution function. Such a conclusion has a simple physical explanation. Indeed, disorder in the eigenfrequencies for any given SRR density results in a decrease of the effective number of resonators, which contribute into the negative magnetization. In the recent experiment,[15] combin-

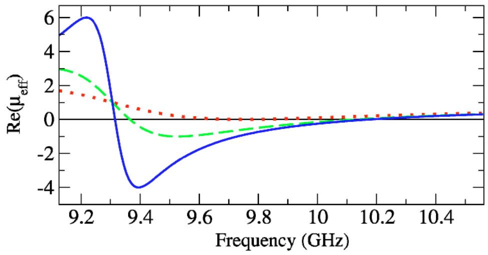

FIG. 2. Real part of the effective magnetic permeability as a function of the wave frequency for $\overline{\Gamma}/\Gamma_c=0.3$ (solid), $\overline{\Gamma}/\Gamma_c=0.5$ (dashed), and $\overline{\Gamma}/\Gamma_c=1$ (dotted).

ing *s*-shaped resonance particles, the authors fabricated the metamaterial with two domains of the negative refraction. Using the results obtained above, we may come to the conclusion that the total size of the frequency domain with the negative magnetic permeability in the fabricated structure[15] is less than it would be for the metamaterial where all resonators are identical. Moreover, using the resonators with more than two different eigenfrequencies can eliminate completely the left-handed properties.

IV. EFFECTIVE ORDER PARAMETER

Reduction and even complete suppression of the frequency domain with negative magnetic permeability with a growth of the value of $\overline{\Gamma}$ can also be explained in a different way. Indeed, the left- and right-handed properties of the metamaterial can be treated as two different "phase states" of the structure. Thus, a transition from one state to the other one can be interpreted as the phase transition of the second kind. The parameter $\overline{\Gamma}$ describes the disorder in the structure, and it can be treated as "effective temperature." Such comparison is adequate since the effective temperature is determined as the width of statistical frequency fluctuations including both homogeneous and inhomogeneous line broadening. To describe different phase states of the structure, we may introduce the effective order parameter, as is usually accepted in the theory of phase transitions. In the absence of homogeneous $(\gamma_m=0)$ and inhomogeneous $(\Gamma=0)$ SRR line broadening (i.e., in the absence of disorder), the metamaterial has the maximum width of the left-handed domain, which decreases and then disappears at the critical "effective temperature" Γ_c termed as the temperature of the phase transition. We use the ratio of the left-handed frequency range to the maximum left-handed frequency domain as the order parameter,

$$\Psi_{\rm LH}=\frac{\Delta\Omega_{\rm LH}}{\Delta\Omega_{\rm LH}^{(\max)}}=\mathrm{Re}\left[1-\left(\frac{\overline{\Gamma}}{\Gamma_c}\right)^2\right]^{1/2}. \tag{9}$$

so that the metamaterial displays the left-handed properties when $\Psi_{\rm LH}\neq 0$, i.e., below the effective critical temperature Γ_c. Dependence of the order parameter $\Psi_{\rm LH}$ on Γ is shown in Fig. 1(b). The real part of the magnetic permeability as a function of the wave frequency is shown in Fig. 2 for different values of the ratio $\overline{\Gamma}/\Gamma_c$. For large values of the ratio $\overline{\Gamma}/\Gamma_c$, the frequency domain where the material possesses negative magnetic permeability is eliminated due to disorder.

The result (9) suggests two ways for increasing the width of the effective frequency domain where the composite material possesses the left-handed properties, namely, (i) decreasing the effective temperature $\overline{\Gamma}$, e.g., by improving the manufacturing technology for shorter wavelengths, and (ii) increasing the effective critical temperature Γ_c, e.g., by a better design of the resonators.

V. CONCLUSIONS

We have analyzed the effect of disorder on the properties of the left-handed composite structures. We have demonstrated that even relatively weak disorder in the eigenfrequencies of the split-ring resonators can result in a dramatic reduction of the size of the left-handed frequency domain. More importantly, above a certain threshold value of the disorder strength, the left-handed frequency domain can disappear completely. We believe our results provide a useful guide for a design of metamaterials operating for shorter wavelengths where the effects of disorder are expected to be crucially important.

ACKNOWLEDGMENTS

One of the authors (A.A.Z.) acknowledges a partial support from the Russian Foundation for Basic Research (Grant No. 05-02-16357) and thanks Nonlinear Physics Centre at the Australian National University for hospitality and fellowship. This work has been supported by the Discovery grant of the Australian Research Council.

[1] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).
[2] M. Bayindir, K. Aydin, E. Ozbay, P. Markos, and C. M. Soukoulis, Appl. Phys. Lett. **81**, 120 (2002).
[3] C. G. Parazzoli, R. B. Greegor, K. Li, B. E. C. Koltenbah, and M. Tanielian, Phys. Rev. Lett. **90**, 107401 (2003).
[4] V. G. Veselago, Usp. Fiz. Nauk **92**, 517 (1967) [Sov. Phys. Usp. **10**, 509 (1968)].
[5] T. J. Yen, W. J. Padilla, N. Fang, D. C. Vier, D. R. Smith, J. B. Pendry, D. N. Basov, and X. Zhang, Science **303**, 1494 (2004).
[6] H. O. Moser, B. D. F. Casse, O. Wilhelmi, and B. T. Saw, Phys. Rev. Lett. **94**, 063901 (2005).
[7] V. A. Podolsky, A. K. Sarychev, and V. M. Shalaev, Opt. Express **11**, 735 (2003).
[8] After completing this work, we became aware of independent numerical studies of this effect by K. Aydin, K. Guven, N. Katsarakis, C. M. Soukoulis, and E. Ozbay, Opt. Express **12**, 5896 (2004).
[9] P. Markos and C. M. Soukoulis, Phys. Rev. E **65**, 036622 (2002).
[10] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999).
[11] A. A. Zharov, I. V. Shadrivov, and Yu. S. Kivshar, Phys. Rev. Lett. **91**, 037401 (2003).
[12] I. V. Shadrivov, N. A. Zharova, A. A. Zharov, and Yu. S. Kivshar, Phys. Rev. E **70**, 046615 (2004).
[13] M. Born and E. Wolf, *Principles of Optics*, 7th ed. (Cambridge University Press, Cambridge, 2002).
[14] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon, Oxford, 1981).
[15] H. Chen, L. Ran, J. Huangfu, X. Zhang, K. Chen, T. M. Grzegorczyk, and J. A. Kong, J. Appl. Phys. **96**, 5338 (2004).