ROOM TRANSFER FUNCTION MEASUREMENT FROM A DIRECTIONAL LOUDSPEAKER

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ABSTRACT

Room transfer function (RTF) is the room response observed at a particular listening point due to an impulse generated from an omnidirectional point source. Typically, measured RTFs in practice are often erroneous due to the directivity of the measurement loudspeaker. This paper formulates a spherical harmonic based parameterization of the room response for a directional loudspeaker, and provides a direct approach to derive the point to point RTF using measurements from a directional loudspeaker. Simulation results are presented for 2 directional loudspeakers with an active frequency bandwidth of $200 - 4000$ Hz.

***Index Terms*—** Room Transfer Function, Room Impulse Response, Directional Loudspeaker, Higher-order Loudspeaker

1. INTRODUCTION

The room transfer function (RTF) is an important measure that characterizes the room response between an omnidirectional loudspeaker and an omnidirectional microphone. Accurate measurement of the RTF is useful in many applications such as sound reproduction, soundfield equalization, echo cancellation, speech dereverberation etc. These applications use appropriate RTF deconvolution methods to cancel the effects of room reflections (reverberation), and therefore, are highly dependent on the accuracy of the RTF measurement.

In practice, any given loudspeaker radiates a unique frequency-dependent radiation pattern referred to as the *directivity* caused by its physical structure and manufacturing material. Therefore, the RTF measured by any loudspeaker will be contaminated by introducing undesirable uncertainties [1]. The directivity of most single driver loudspeaker largely differ from the omnidirectional pattern, hence the ISO 3382 standard [2] for RTF measurement specifies the measurement source to be composed of multiple oscillatory membranes mounted on the surface of a dodecahedron cabinet. As shown in [3, 4], even these multi-driver speaker arrangements produce omnidirectional responses only in the low frequencies, and with increasing frequency, they tend to become directional. In order to improve the accuracy of the RTF measurement, Klein *et. al* recently proposed an optimized measurement source composed of multiple transducers placed over a spherical surface. A practical experiment based on the above work was carried out in [5] where a spherical measurement source with 28 surface mounted transducers was used. While the optimized measurement system yields improved results, it has strict design constraints.

In this paper, we propose a novel method to derive the RTF from directional measurement sources with known directivity. To assist the formulation of this method, we first derive a spherical harmonic based parameterization of the room response for any arbitrary directional source, and later propose a practical approach to calculate the coefficients of the above parameterization, which finally leads to the estimated RTF. The derivation of the aforementioned parameterization was influenced by the work in [6, 7], where the authors introduced an efficient parameterization for the RTF between two arbitrary points from a pre-defined source region and a receiver region. Based on simulation results, the proposed method delivers significantly improved RTF estimates for variable directivity measurement sources.

2. PARAMETERIZATION OF THE ROOM RESPONSE FOR A DIRECTIONAL LOUDSPEAKER

In this section, we formulate the room response observed at the receiver location $\boldsymbol{O}_r$ due to a directional loudspeaker at $\boldsymbol{O}_s$ in terms of a spherical harmonic based parameterization. A directional loudspeaker can be generalized by a higher-order loudspeaker [8], which radiates multiple modes of sound at a given frequency. Based on the spherical harmonic decomposition of the wave equation [9], the soundfield due to an N^{th} order loudspeaker at $\boldsymbol{O}_s$ can be represented in terms of

$$S_{\text{out}}(\boldsymbol{x^{(s)}}, k) = \sum_{n=0}^{N} \sum_{m=-n}^{n} \beta_{nm}^{(s)}(k) h_n(kx^{(s)}) Y_{nm}(\theta^{(s)}, \phi^{(s)}) \quad (1)$$

where $\boldsymbol{x^{(s)}} = (x^{(s)}, \theta^{(s)}, \phi^{(s)})$ is an arbitrary observation location with respect to the source location $\boldsymbol{O}_s$, $k = 2\pi f/c$ is the wave number, f is the frequency, c is the speed of sound propagation, $\beta_{nm}^{(s)}(k)$ denotes the coefficients of the outgoing soundfield caused by the directional source, $Y_{nm}(\theta, \phi)$ denotes the spherical harmonic of order n and degree m, and $h_n(\cdot)$ represents the n^{th} order spherical Hankel function of the first kind. Note that the loudspeaker order N and the radiation pattern $\beta_{nm}^{(s)}(k)$ may be variable depending on the physical characteristics of the speaker. Typical loudspeakers have variable directivity patterns as shown in Fig. 1. and they can be measured inside an anechoic chamber [10]. For commercial loudspeakers, the directional response over the active bandwidth is often provided by the manufacturer.

As mentioned earlier, in [6], the authors presented a parameterization of the RTF between any 2 arbitrary points from a pre-defined source region and a receiver region. The source and receiver regions were two spatial regions within which the source and receiver may be arbitrarily positioned. Following a similar derivation, we aim to parameterize the room response at $\boldsymbol{O}_r$ due to an N^{th} order loudspeaker at $\boldsymbol{O}_s$. The main concept behind this derivation is as follows. If the

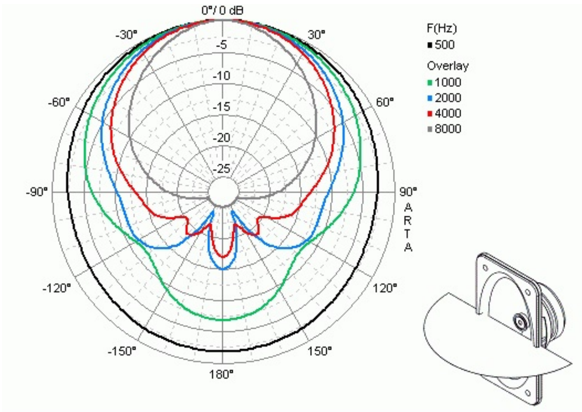

Fig. 1. Horizontal directivity pattern of a commercial loudspeaker (Visaton WB 10) [11].

room response at $\boldsymbol{O}_r$ due to each unit-amplitude outgoing spherical harmonic mode from the source location $\boldsymbol{O}_s$ can be formalized, then the room response at $\boldsymbol{O}_r$ due to any arbitrary directional/HO speaker can be represented in terms of the above formalization. To demonstrate the above statement, let us consider a unit amplitude outgoing wave of order n' and mode m'

$$\beta_{nm}^{(s)}(k) = \begin{cases} 1, & n = n' \text{ and } m = m' \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

producing

$$S_{\text{out}}(\boldsymbol{z}^{(s)}, k) = h_{n'}(kz^{(s)}) Y_{n'm'}(\theta_z^{(s)}, \phi_z^{(s)}). \tag{3}$$

For this particular outgoing source field, there will be a resulting room response present at the receiver location $\boldsymbol{O}_r$. Utilizing the spherical harmonic decomposition of a homogeneous incident soundfield [9], the room response observed at the omnidirectional receiver location $\boldsymbol{O}_r$ with respect to its local origin can represented in terms of

$$R_{n'm'}(\boldsymbol{x}, k) = \alpha_{00}^{n'm'}(k) j_0(k0) Y_{00}(0,0) \tag{4}$$

where $\alpha_{00}^{n'm'}(k)$ denotes the zero$^{\text{th}}$ order, zero$^{\text{th}}$ mode soundfield coefficient of the room response incident at $\boldsymbol{O}_r$ caused by an unit amplitude n'^{th} order and m'^{th} mode outgoing soundfield at $\boldsymbol{O}_s$ and $j_0(k0) = 1$ represents the spherical Bessel function of order 0. Note that (4) only considers the $00-$coefficient because the receiver signal is only observed over an omnidirectional receiver rather than a directional/HO receiver. If the room response was observed through a directional microphone over a spatial region, the incident soundfield will require a multiple modes of the form $\alpha_{v\mu}^{n'm'}(k)$ where v and μ denote the order and mode of the incident spatial soundfield centered at $\boldsymbol{O}_r$.

From (4), it is evident that, if $\alpha_{00}^{n'm'}(k)$ of (4) can be recorded for each unit amplitude outgoing mode from $\boldsymbol{O}_s$, the room response at $\boldsymbol{O}_r$ due to any arbitrary directional loudspeaker at $\boldsymbol{O}_s$ can be derived using (1), (3) and (4) as

$$P(\boldsymbol{O}_r, k) = \sum_{n=0}^{N_s} \sum_{m=-n}^{n} \beta_{nm}^{(s)}(k) \alpha_{00}^{nm}(k) Y_{00}(0,0). \tag{5}$$

The above result gives a spherical harmonic based parameterization for the room response at $\boldsymbol{O}_r$ due to a directional/HO loudspeaker at $\boldsymbol{O}_s$ with a directional response of the form (1). The terms $\alpha_{00}^{nm}(k)$ for $n = \{0, \cdots, N\}$ fully characterize the room response between the source location $\boldsymbol{O}_s$ and receiver location $\boldsymbol{O}_r$ up to a source order of N. Therefore, if $\alpha_{00}^{nm}(k)$ are known for a particular pair of $\boldsymbol{O}_s$ and $\boldsymbol{O}_r$, the room response for any N^{th} order source with arbitrary $\beta_{nm}^{(s)}(k)$, can be derived using (5).

3. DERIVATION OF THE RTF USING MEASUREMENTS FROM A DIRECTIONAL LOUDSPEAKER

In this section, we propose a practical approach to derive the ideal point-point RTF between $\boldsymbol{O}_s$ and $\boldsymbol{O}_r$ based on room measurements from a directional/non-ideal omnidirectional loudspeaker. The ideal RTF is defined as the room response at $\boldsymbol{O}_r$ due to an omnidirectional speaker at $\boldsymbol{O}_s$. Since the directionality of a unit amplitude omnidirectional loudspeaker is given by

$$\beta_{nm}^{(s)}(k) = \begin{cases} 1, & n = 0 \text{ and } m = 0 \\ 0, & \text{otherwise,} \end{cases} \tag{6}$$

the ideal RTF can be parameterized by simplifying (5) into

$$H(\boldsymbol{O}_s, \boldsymbol{O}_r, k) = \alpha_{00}^{00}(k) Y_{00}(0,0). \tag{7}$$

In practice, the challenge is to derive the ideal RTF $H(\boldsymbol{O}_s, \boldsymbol{O}_r, k)$ of (7) from a directional loudspeaker measurement $P(\boldsymbol{O}_r, k)$ of (5). This requires an accurate measurement of $\alpha_{00}^{00}(k)$ throughout the frequency bandwidth of interest.

The proposed approach is to take multiple room response measurements at $\boldsymbol{O}_r$ while rotating the look-direction of the loudspeaker. This would produce a set of linear equations of the form

$$\underbrace{\begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_Q \end{bmatrix}}_{\boldsymbol{P}} = Y_{00}(0,0) \underbrace{\begin{bmatrix} \beta_{00}^{(1)} & \beta_{1-1}^{(1)} & \cdots & \beta_{NN}^{(1)} \\ \beta_{00}^{(2)} & \beta_{1-1}^{(2)} & \cdots & \beta_{NN}^{(2)} \\ \vdots & \vdots & \vdots & \vdots \\ \beta_{00}^{(Q)} & \beta_{1-1}^{(Q)} & \cdots & \beta_{NN}^{(Q)} \end{bmatrix}}_{\boldsymbol{\beta}} \times \underbrace{\begin{bmatrix} \alpha_{00}^{00} \\ \alpha_{00}^{1-1} \\ \alpha_{00}^{10} \\ \vdots \\ \alpha_{00}^{NN} \end{bmatrix}}_{\boldsymbol{\alpha}} \tag{8}$$

where P_q for $\{q = 1, 2, \cdots, Q\}$ are the room response measurement for the q^{th} loudspeaker rotation and $\beta_{nm}^{(q)}$ is the loudspeaker response at the q^{th} rotation. Given the loudspeaker directionality β_{nm} is known, the rotated responses $\beta_{nm}^{(q)}$ can be derived using the following theorem [12, 13].

Theorem 1 *Let χ_{nm} denote the spherical harmonic coefficients in a coordinate system $\boldsymbol{E}$ and let ρ_{nm} denote the spherical harmonic coefficients in a new coordinate system $\boldsymbol{F}$ which is a rotated version of $\boldsymbol{E}$ with the same origin. Assume $(\vartheta, \psi, \gamma)$ are the standard Euler angles [13] that define the rotation from $\boldsymbol{E}$ to $\boldsymbol{F}$ using the $z-y-z$ convention in a right-handed frame. That is, the rotation is first done by an angle ϑ about the z-axis, then by an angle ψ about the new y-axis, and finally by an angle γ about the new z-axis. Then, the relationship between ρ_{nm} and χ_{nm} is given by*

$$\rho_{nm}(k) = \sum_{n=0}^{N} e^{im\gamma} d_n^{m'm}(\psi) e^{im\vartheta} \chi_{nm}(k) \tag{9}$$

with

$$d_n^{m'm}(\psi) = [(n+m')!(n-m')!(n+m)!(n-m)!]^{1/2}\cdots$$
$$(-1)^{m'-m} r \cdots$$
$$\sum_s \frac{(-1)^s (cos(\psi/2))^{2(n-s)+m-m'}}{(n+m-s)!s(m'-m+s)!(n-m'-s)!}$$

where r is the radius determining the order of the spherical harmonic decomposition and the range of s is determined by the condition that all factorials are non-negative.

In the current application, for each rotational instance q, $\chi_{nm}(k)$ is the known loudspeaker directionality $\beta_{nm}(k)$ with respect to the rotated Cartesian coordinate system, and ρ_{nm} is the new loudspeaker directionality $\beta_{nm}^{(q)}(k)$ with respect to the conventional Cartesian coordinate system with axes $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$.

Once $\boldsymbol{P}$ and $\boldsymbol{\beta}$ of (8) are obtained, the matrix equation can be solved to calculate the desired room response coefficients $\boldsymbol{\alpha}$. If (8) is overdetermined, that is $Q \geq (N+1)^2$, and $\boldsymbol{\beta}$ is well conditioned, the system of equations can be solved using the method of least squares. Once $\boldsymbol{\alpha}$ is calculated, the room response coefficient α_{00}^{00} can be extracted to formulate the desired point-point RTF (7).

4. SIMULATIONS

To demonstrate the applicability of the theory developed, simulation results were generated utilizing MATLAB. The objective was to utilize room response recordings from a directional loudspeaker to derive the ideal point-point RTF. A $6 \times 5 \times 2.5$ m rectangular room was considered as the reverberant environment with its center defined as the origin $\boldsymbol{O}$. The RTF was measured between the receiver location $\boldsymbol{O}_r = \boldsymbol{O}$, and the source location $\boldsymbol{O}_s = [1, 1, 0.5]$ m in Cartesian coordinates. Simulation examples are given for two directional loudspeakers, both operating over the frequency bandwidth $200 - 4000$ Hz.

The simulation of room reflections for rectangular rooms is well defined by the image-source method [14], which calculates the ideal point-point RTF based on the principal of image sources caused by each wall. This method was utilized to simulate desired ideal RTF as well as the room response due to a directional loudspeaker. In this paper, a directional loudspeaker was modeled as an array of point sources. Note that when a directional loudspeaker is modeled in terms of an array of L point sources, its directional response is given by [9]

$$\beta_{nm}^{(s)}(k) = \sum_{\ell=1}^{L} w_\ell(k) ikj_n(kr_\ell^{(s)}) Y^*(n, m, \theta_\ell^{(s)}, \phi_\ell^{(s)}) \qquad (10)$$

where $(\ell = 1, \cdots, L)$ denotes the ℓ^{th} point source at $(r_\ell^{(s)}, \theta_\ell^{(s)}, \phi_\ell^{(s)})$ with respect to $\boldsymbol{O}_s$ and $w_\ell(k)$ represents the driving signal at the ℓ^{th} point source. The order N of the loudspeaker will vary with frequency, and this variation is based on the relationship $N = \lceil ker_{\max}^{(s)}/2 \rceil$ [15], where $r_{\max}^{(s)}$ is the largest $r_\ell^{(s)}$[1].

In the first example, a directional loudspeaker was modeled in terms of 3 unit-amplitude point sources distributed over a spherical surface of radius 0.03 m with $(r_1^{(s)} = 0.03, \theta_1^{(s)} = \pi/4, \phi_1^{(s)} = \pi/3)$, $(r_2^{(s)} = 0.03, \theta_2^{(s)} = 2\pi/3, \phi_2^{(s)} = -\pi/3)$ and $(r_3^{(s)} = 0.03, \theta_3^{(s)} = \pi, \phi_3^{(s)} = 0)$. The order of this loudspeaker progressively varied from $N = 1-3$ with increasing frequency. The directional response of this loudspeaker was derived using (10) and the resulting room response at $\boldsymbol{O}_r$ was simulated using the image source method. In order to derive the ideal RTF using the above loudspeaker, multiple room response measurements $\boldsymbol{P}$ were simulated by considering a total of $(N+1)^2$ rotations. The resulting measurements were substituted in (8) along with the rotated loudspeaker responses $\boldsymbol{\beta}$, to derive the room response coefficients $\boldsymbol{\alpha}$, and finally the desired RTF (7) between $\boldsymbol{O}_r$ and $\boldsymbol{O}_s$. Figure 2 shows a comparison between the initial room response recording from the directional loudspeaker, the calculated point-point RTF and the ideal RTF based on the image-source method over the entire frequency bandwidth of interest. It is observed that the calculated RTF closely follows the desired ideal RTF.

For a better representation of the proposed method's broadband performance, we show the absolute squared error between the measured room response (for a directional loudspeaker) and the ideal RTF, and the absolute squared error between the calculated RTF and the ideal RTF. As shown in Fig. 3, the error from the measured room response is significantly larger compared to that from the calculated RTF. This result clearly represents the disadvantage of measuring the RTF from non-ideal point sources/directional sources, and such effects will largely impact the performance of applications involving room equalization, active noise cancellation, speech dereverberation etc.

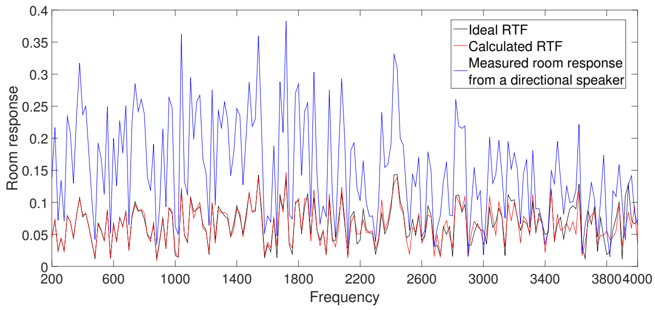

Fig. 2. Directional source 1: A comparison between the measured room response, calculated RTF and the idea RTF between the source and receiver locations $\boldsymbol{O}_s$ and $\boldsymbol{O}_r$

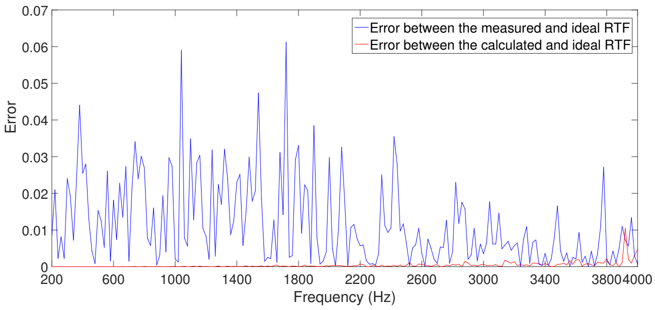

Fig. 3. Directional source 1: Absolute squared error between (i) the measured room response and the ideal RTF, (ii) the calculated RTF and the ideal RTF over the frequency bandwidth of interest.

[1]This truncation is based on the inherent properties of Bessel functions which tend to be negligible at orders after $N = \lceil ker_{\max}^{(s)}/2 \rceil$.

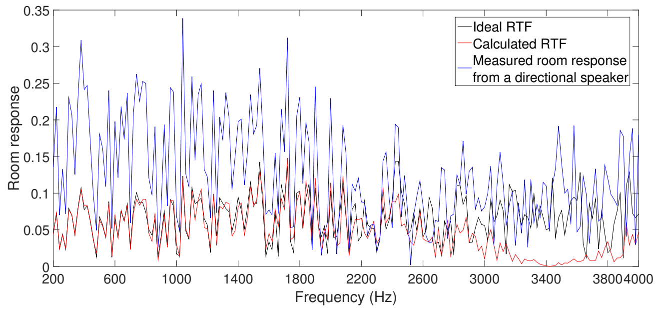

Fig. 4. Directional source 2: A comparison between the measured room response, calculated RTF and the idea RTF between the source and receiver locations $\boldsymbol{O}_s$ and $\boldsymbol{O}_r$

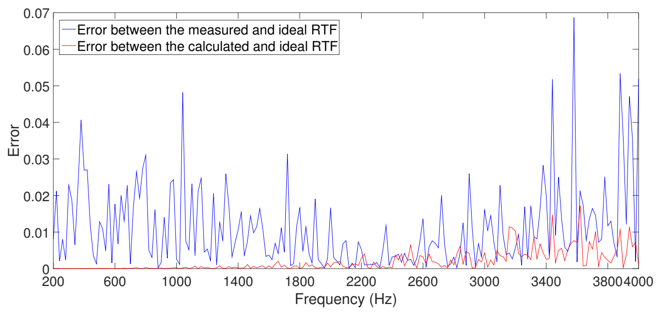

Fig. 5. Directional source 3: Absolute squared error between (i) the measured room response and the ideal RTF, (ii) the calculated RTF and the idea RTF over the frequency bandwidth of interest.

In the second example, a directional loudspeaker was modeled in terms of 3 unit-amplitude point sources distributed over a spherical surface of radius 0.05 m with $(r_1^{(s)} = 0.05, \theta_1^{(s)} = \pi/3, \phi_1^{(s)} = \pi/3)$, $(r_2^{(s)} = 0.05, \theta_2^{(s)} = \pi/2, \phi_2^{(s)} = -\pi/3)$ and $(r_3^{(s)} = 0.05, \theta_3^{(s)} = \pi, \phi_3^{(s)} = 0)$. The order of this loudspeaker was higher than the previous example with a variation of $N = 1 - 5$ over the frequency bandwidth $200 - 4000$ Hz. Figure 4 shows the measured, calculated and ideal RTF for this loudspeaker, and Fig. 5 shows the corresponding error. As observed in both of the above figures, the error with respect to the ideal RTF is significantly high for the measured RTF, compared to that of the calculated RTF. However, compared to the previous example, there appears a slight increase of error for the calculated RTF at high frequencies. This is mainly due to the increased order of the loudspeaker and the reasoning is as follows. Calculation of the desired RTF requires the zero$^{\text{th}}$ order room response mode $\alpha_{00}^{00}(k)$, which is due to the omnidirectional component of the directional speaker. The extraction of the effects due to this component from a directional speaker become difficult at higher frequencies because, with increasing order, the shape of the directional response become narrower. Therefore, the proposed method of RTF extraction based on room response measurements from rotated speaker positions is best effective for lower order loudspeakers. Since the order of practical non-ideal speakers is typically low, the above effect can be considered to be minimal.

In addition to the desired room response coefficients $\alpha_{00}^{00}(k)$, the solution to (8) also gives higher order coefficients $\alpha_{00}^{nm}(k)$ up to order N. The accuracy of these modes are however, largely affected by the aforementioned limitation with increasing order.

5. CONCLUSION

In this paper, we proposed a practical approach to measure the RTF from directional loudspeakers. Based on simulation examples, the proposed method yields largely improved estimates of the RTF compared to that obtained by the raw directional source. The main advantage of this approach is its applicability to any arbitrary measurement source. Practical implementation and performance analysis of the proposed method are topics of future research.

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