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Tip-tilt mirror suspension: Beam steering for advanced laser interferometer gravitational wave observatory sensing and control signals

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We describe the design of a small optic suspension system, referred to as the tip-tilt mirror suspension, used to isolate selected small optics for the interferometer sensing and control beams in the advanced LIGO gravitational wave detectors. The suspended optics are isolated in all 6 degrees of freedom, with eigenmode frequencies between 1.3 Hz and 10 Hz. The suspended optic has voice-coil actuators which provide an angular range of ± 4 mrad in the pitch and yaw degrees of freedom.

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I. INTRODUCTION

Second generation interferometric gravitational wave detectors, such as advanced LIGO,¹ are currently under construction. These detectors use optical metrology to achieve displacement sensitivities of $\sim 10^{-20}$ m/ $\sqrt{\text{Hz}}$ at 100 Hz, with a signal bandwidth from 10 Hz to 10 kHz. The detectors use large 34 cm diameter suspended optics to form a Michelson interferometer with 4 km long Fabry–Perot cavities in each arm. The gravitational wave signal is obtained via a “dc-readout” scheme.² This scheme registers small power fluctuations on dc coupled readout photodiodes at the output of the interferometer. In order to prevent non-TEM₀₀ mode light and light in the RF control sidebands from reaching the readout photodiodes, a short (~ 1 m) Fabry–Perot cavity is placed just in front of the readout photodiodes. This “output mode cleaner” is typically a monolithic cavity, mounted in a suspension for vibration isolation. Mode-matching and beam steering optics are required to direct the optical beam from the output of the large suspended interferometer into the suspended output mode-cleaner. Reflective optics are used to provide actuation for an active alignment control system. In addition to steering the beam into the output-mode-cleaner, suspended optics are required for beam steering of other interferometer sensing ports such as the “reflected port” and “pick-off port.” To reduce any coupling of vibrational motion into the gravitational wave readout, the optics must be isolated in all 6 degrees of freedom. In this paper, we discuss the design of such a suspended steering mirror, the tip-tilt mirror suspension (or tip-tilt for short), which will be used in the advanced LIGO detector.

The mechanical design and the horizontal and vertical isolation of the tip-tilt mirror suspension are described first. This is followed by a discussion of local sensing and actuation. We conclude by presenting the overall isolation and steering performance.

II. MECHANICAL DESIGN

The tip-tilt mirror suspension has been designed to provide isolation of a 2 in. diameter mirror in all 6 degrees of freedom, and control of 3 of them. All the eigenfrequencies of these 6 degrees of freedom are designed to be below 10 Hz, the lower end of the gravitational wave signal band. The quality factors of the eigenmodes of the tip-tilt mirror suspension are damped to values less than 100. This reduces the sensitivity (around the resonances) to external motion of the tip-tilt mirror suspension as a whole.

The tip-tilt mirror suspension is designed around an aluminium ring, which holds a standard 2 in. $\phi \times 3/8$ in. thick (50.8×9.52 mm) optic. Mechanical isolation of the suspended aluminium ring is provided by means of a single stage pendulum and cantilever blade springs. A schematic of the suspension is shown in Fig. 1. The hexagonal (blue) “ground” points indicate where the base of the blades are attached to the mounting structure. The suspension wires are attached to the tips of the suspension blades via a clamp, as indicated by the green circles. The other end of the suspension wire is clamped to the aluminium ring. The mechanical parameters are listed in Table I.

The tip-tilt mirror suspension is manufactured from standard ultrahigh vacuum compatible materials (aluminium and stainless steel). The overall structure is ~ 250 mm in height. Including a 2 in. mirror and four actuators, the total mass of the unit is ~ 2.3 kg, with a footprint of $124 \text{ mm} \times 95 \text{ mm}$. Figure 2 shows an engineering rendering of the tip-tilt mirror suspension.

The tip-tilt mirror suspension provides isolation and control of the mode-matching and steering mirrors in the interferometric sensing chain. The suspension can also be designed to perform two other tasks. With sufficiently strong actuators (i.e., with large enough magnets), the pitch and yaw angles of the suspended mirror can be modulated at high frequencies (~ 1 kHz) to generate alignment error signals. The tip-tilt can also act as a shutter (e.g., to protect sensitive photodetectors) by rapidly applying a large alignment offset. Shutter speeds

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FIG. 1. (Color online) Schematic overview of the suspended optic with the 6 degrees of freedom shown (yaw, roll, transverse, vertical, longitudinal, and pitch). Centre of mass (CM), the two hexagonals (blue) are local “ground,” and the circle (green) are wire clamps.

as low as ~ 2.5 ms have been observed in the laboratory depending on the optical configuration.

A. Horizontal isolation: Mirror suspension

Horizontal isolation is achieved by suspending the mirror by a simple pendulum with two wires, one on either side of the aluminium ring. The wire ends are clamped between two metal plates.

The optic is clamped inside the aluminium ring by a small cylinder of teflon (PFA-440 HP) whose position is locked by a set screw. The two suspension wires are clamped onto the sides of the aluminium ring. The pendulum has a length of 140 mm and provides a longitudinal resonant frequency of 1.3 Hz. The suspension wire is a $127\ \mu\text{m}$ diameter stainless steel piano wire. This diameter is small enough to reduce flex-

FIG. 2. (Color online) An engineering rendering of the tip-tilt mirror.

ing of the wire near the wire clamps, so that the violin mode frequencies are an order of magnitude higher than the vertical blade resonance, and so the vertical motion is dominated by the blade spring. The wires are thick enough to reduce the risk of the wire breaking during handling and assembling.

The resonant frequency of the pitch mode, which is coupled to the longitudinal mode, is predominantly determined by the height of the wire clamping above the centre of mass of the mirror assembly. This distance is denoted in Fig. 1 as d-pitch. In addition, the resonance frequency of the pitch mode is influenced by the flexing of the wire, just above the wire clamp. This distance can be approximated by^{3,4}

$$d_{flex} = \sqrt{\frac{EI}{T}}, \quad (1)$$

in which T is the wire tension, E is the Young's modulus of the wire material, and I is the cross section moment of inertia. For the tip-tilt mirror suspension this distance is 2.3 mm. The wire stiffness is strongly dependent on the Young's modulus of the suspension wires, and determines the distance where the wire will flex above the wire clamp. Measurements have shown that the Young's modulus of the suspension wires can vary by almost 10%, affecting the pitch resonance by up to 1.5%.

The resonant frequency of the yaw mode is controlled by the separation of the suspension points at the top (d-yaw) and the effective optic diameter. Table II lists the calculated fundamental eigenmodes.

TABLE I. Basic tip-tilt mirror suspension mechanical parameters.

Parameter	Value
Effective optic diameter (inc. ring) (wire separation at the optic)	76.2 mm (3 in.)
Effective optic mass (inc. ring)	~ 96 g
Total assembly mass	~ 2.3 kg
Optic size	50.8 mm $\times$ 9.5 mm (2 in. $\times$ 3/8 in.)
Transmission aperture (diameter, centred)	28 mm vertical ~ 35 mm horizontal
Beam height	101.6 mm (4 in.)
Pendulum length	140 mm
Suspension wire diameter	$127\ \mu\text{m}$
d-pitch	1.3 mm
d-yaw	30 mm
Moment of inertia	
$I_{\text{long, pitch}}$	3.1×10^{-6} kg m ²
I_{yaw}	3.8×10^{-6} kg m ²
$I_{\text{transverse, roll}}$	6.4×10^{-6} kg m ²

TABLE II. Basic tip-tilt mirror suspension modelled and measured eigenmodes.⁵

Degree-of-freedom	Modelled (Hz)	Measured (Hz)
Longitudinal	1.31	1.3
Pitch	1.65	1.63
Yaw	1.6	1.59
Transverse	1.34	...
Vertical	6.0	5.9
Roll	8.7	8.6

B. Vertical isolation: Suspension blade design

Vertical isolation for the suspended mirror is provided by two triangular shaped beryllium copper cantilever blade springs. These blades are of a similar design to those used in the VIRGO and advanced LIGO test mass suspensions.⁶ In these systems, the suspension blades are made from maraging steel, pre-cured and heat treated to reduce material creep. Material creep has not been addressed in this design due to the more relaxed requirements.

Modelling of the blades was used to determine the vertical resonance and required blade parameters.^{7,8} The blade vertical resonance frequency is given by

$$f = \sqrt{\frac{Eah^3}{16\pi^2 ml^3 \alpha}} \text{ [Hz]}, \quad (2)$$

where E is the Young's modulus, a is the width of the blade at the base, h is the blade thickness, m is the mass supported by the blade, l is the length of the blade, and α is the shape factor where a triangular blade has a shape factor of 1.5, while a rectangular blade has a shape factor of unity.⁷

By setting the blade thickness, the width of the base and length of the triangular suspension blade, the vertical resonance frequency was obtained. The key parameters are shown in Table III. The vertical resonance frequency is strongly dependent on the blade thickness and length. A practical balance was achieved between the length of the blades (to simplify manufacturing and to keep the unit small) and the requirement to keep the vertical resonance frequency under 10 Hz.

Each blade is mounted in an aluminium structure which allows the height of the wire suspension point (clamping of the wire at the tip of the blade) to be adjusted and locked, see Fig. 3. This also provides a means of adjusting the height of

TABLE III. The vertical blade parameters and modelling results.

Parameter	Set values
Base width	23 mm
Blade length	50 mm
Shape factor (triangular ⁶)	1.5
Blade thickness	254 μ m
Parameter	Modelled
Vertical resonance	5.9 Hz
Effective spring constant (single blade)	65.8 N/m
Deflection at tip (with 48 g load)	7 mm
Blade curvature radius (under load)	177.3 mm
Angle of the blade-tip (under load)	16.4°

FIG. 3. (Color online) Single blade units with height adjuster.

the optic with respect to the coil-magnet actuators. To reduce the bending of the wire at the suspension point, the blades are launched at an angle such that the wires leave the clamps at 90°, when they are loaded with the mirror assembly.

The vertical suspension mode has no active damping, however, passive damping is achieved through the use of a 12 mm diameter viton o-ring. A small ledge is mounted under the original blade, and separated by a spacer. This ledge is a shorter version of the original blade. The viton o-ring is placed between the ledge and the suspension blade. Due to the load on the blade the o-ring is slightly compressed. Vertical motion at the blade tip, will cause the o-ring to be twisted (shear) between the ledge and the blade, and provide damping that reduces the quality factor. Using this method the quality factor of the vertical mode is reduced from 180 down to 67, well below the required 100.

III. LOCAL SENSING AND ACTUATION

Control of the mirror is achieved by four voice-coil actuators, which provide control over longitudinal, pitch, and yaw degrees of freedom. Each voice-coil is paired with a magnet mounted to the suspended aluminium ring. A position sensor is also incorporated in the actuator assembly, in the form of an optical shadow sensor. A small flag is mounted to the aluminium ring. The flag is positioned such that it blocks half of the light falling on a photodiode from an opposing light-emitting diode (LED), acting as a shadow sensor, as shown in Fig. 4. Movement of the flag (and hence the aluminium ring) will be registered by the photodiode. The combination of the shadow sensor and the coil-magnet actuator provide local sensing and control of the mirror.

The shadow sensor-coil combination is built by the University of Birmingham and integrated into a single unit, referred to as the Birmingham Optical Sensor and Electro-Magnetic actuator (BOSEM).^{9,10} The four BOSEMs are mounted in a square with 48.22 mm sides, with the centre of the BOSEMs at a radius of 34.1 mm from the centre of the optic.

A. Eddy current damping of the eigenmodes

The longitudinal, pitch, and yaw motion of the suspended mirror can be actively controlled by the four BOSEMs.

FIG. 4. (Color online) A cross section of the aluminium ring with flags and the BOSEMs. Visible in the BOSEM are the photodiode, LED, and coil. The flag is attached to the aluminium ring with the magnet press fitted into it.

However, the transverse (sideways), vertical and roll modes of the suspended mirror cannot be controlled or damped by the BOSEMs. Additional magnets are installed to provide eddy current damping for these modes. These smaller additional magnets ($\varnothing 3 \text{ mm} \times 3 \text{ mm}$ long) are located on either side of the optic and mounted on the aluminium ring. The polarity of the magnets are arranged such that the total magnetic momentum of the mirror assembly is minimised. Eddy current damping is provided by placing small aluminium disks just in front of each of these magnets. Near-critical damping is achieved by optimising the distance between the magnets and the aluminium plates.

B. Actuation magnets

The standard magnets used in the BOSEMs are nickel plated SmCo ($\text{Sm}_2\text{Co}_{17}/\text{SmCo } 2:17$), and are $\varnothing 2 \text{ mm} \times 3 \text{ mm}$ long, and provide a force coefficient of 21 mN/A, in combination with the BOSEM coils. The small dimensions (e.g., magnetic strength) are chosen to mitigate the magnetic couplings with the environment. The magnets are arranged with alternating polarities, to reduce the magnetic moment of the mirror assembly.

To optimise the current-force coupling from the voice-coil to the magnet, the centre of the standard magnet needs to be $\sim 6.7 \text{ mm}$ in front of the centre of the BOSEM coil.¹¹ This is accomplished by press fitting the magnet into the back of the flags (Fig. 4).

C. Shadow sensor flag

The shadow sensor flags are made from a polymer thermoplastic (PEEK) and are screwed into the mirror holder, as seen in Fig. 4. The magnet is press fitted into the flag to locate the magnet with respect to the mirror holder. The position of the BOSEM with respect to the flag may be tuned, adjusting the BOSEM along with the length of the flag.

The operation point is such that the flag covers the shadow sensor at 50% of its full range with the mirror freely

FIG. 5. (Color online) Tip-tilt longitudinal transfer function, driving the four BOSEM coils, and reading out the shadow sensors. The coils were driven, and the sensors were readout, uniformly.

hanging. A deviation from a uniform LED intensity profile is addressed by screening the LEDs and using a lensing system in front the LED (see Ref. 9 for more details). In this position, the distance between the BOSEM and the mirror holder is $\sim 2 \text{ mm}$, which is sufficient to prevent it from interfering with the suspended mirror. The linear range of the BOSEM, around its operating point, is $0.7 \text{ mm}_{\text{pk-pk}}$, with a nominal displacement sensitivity of 23.4 V/mm .^{9,10}

IV. PERFORMANCE

The measured performance of a prototype tip-tilt mirror suspension was compared to numerical models as discussed below.

A. Transfer functions

An electrical current in each voice-coil of the BOSEMs, generates a magnetic field, which will act on the magnets in the aluminium ring. Applying the current through the BOSEMs in the appropriate polarity allows the aluminium ring (and optic) to be controlled in longitudinal, pitch, and yaw.

Figures 5–7 show the longitudinal, pitch, and yaw transfer functions of the tip-tilt mirror suspension measured in the laboratory at the Australian National University. The transfer functions were obtained by driving the coils and reading out the shadow sensors in the BOSEMs. The coils were diagonalised to account for the different dc and frequency dependent gains of the individual current drivers and fundamental mechanical resonances. Recorded data have been fitted with second order functions to obtain the frequency and quality factor of each resonance and plotted against the modelled response.

The fitting results are presented in the legends of the figures. The quality factors of all modes are below 100, meeting the system requirements.

FIG. 6. (Color online) Tip-tilt pitch transfer function, the top two coils are driven in a positive direction, while the lower two coils are driven in a negative direction. The shadow sensors are read out differentially between the top pair and the lower pair.

B. Pitch and yaw hysteresis

Figure 8 shows two graphs of the hysteresis exhibited by the tip-tilt mirror suspension. The figures show minimal hysteresis in both the pitch and yaw degrees. The hysteresis was measured by applying a series of dc pitch (or yaw) offsets via the four BOSEM coils and recording the pitch (yaw) values sensed by the shadow sensors. After each offset step, the tip-tilt mirror suspension was allowed to reach equilibrium, then the sensed offset was measured with a 5 s average time. As shown in Fig. 8, both degrees of freedom are linear and show negligible hysteresis over a range of 8 mrad_{pk-pk}.

C. Angular pointing per voice-coil current

Figure 9 shows the pitch and yaw angle of the tip-tilt mirror suspension per applied BOSEM coil current.

FIG. 7. (Color online) Tip-tilt yaw transfer function, the left two coils are driven in a positive direction, while the lower two coils are driven in a negative direction. The shadow sensors are read out differentially between the left pair and the right pair.

FIG. 8. (Color online) Hysteresis of the tip-tilt mirror suspension measured using the shadow sensors in the BOSEMs, (a) pitch hysteresis and (b) yaw hysteresis. The start and stop position is indicated by an arrow. Offsets were first increased to a positive maximum before cycling through a minimum and back to zero. The turnarounds of the measurements are due to the sensor limits, and do not represent the actual motion of the optic. The residual is shown between the forward and backward offset, when going through the cycle (scale on the right-hand side of each graph).

This was obtained by utilising the transfer functions from Figs. 6 and 7 and applying the BOSEM optimal coil-current to force coefficient (modelled) to generate the graphs.¹¹ From Fig. 9, the angle-per-current coefficient below resonance is 0.05 mrad_{pk}/mA. The maximum coil current for dc steering can be obtained by using the linear pointing range (from the hysteresis), which results in a maximum coil current of 80 mA_{pk}. To increase the angle-to-current coefficient, larger magnets can be used.

D. Vertical mode damping

Figure 10 shows the performance of the o-ring damper for the vertical isolation of a single blade. The measurement was obtained by creating a shadow sensor at the wire clamp

FIG. 9. (Color online) Tip-tilt mirror suspension pitch and yaw angle per applied coil current (modelled), using $\phi 2 \text{ mm} \times 3 \text{ mm}$ long magnets.

FIG. 10. (Color online) Measurement of the viton o-ring damped vertical blade resonance (bounce) as well as the roll mode resonance of a single suspension blade (while installed within the tip-tilt mirror suspension assembly). Included in the graph is the fitted model with the results listed in the legend. The o-ring damping has reduced the vertical mode resonance to 67 (down from 180).

at the tip of the suspension blades using a small laser and a photodiode. The wire clamp blocked the laser approximately at 50% in front of the photodiode. Changes in intensity on the photodiode, due to the motion of the wire clamp, were recorded. The mirror assembly was excited by ambient air turbulence and seismic noise in the laboratory.

The figure indicates that the quality factor of the damped vertical resonance is 67, down from 180 without the damper (data not shown). In addition to damping the vertical mode, the roll mode of the suspended mirror is also damped.

E. Eigenmodes of the tip-tilt structure

The tip-tilt structure is designed to be stiff so that its mechanical modes will have a minimal impact on the platform to which it is mounted; the design goal was a lowest structural mode eigenfrequency of 150 Hz.^{12,13}

The tip-tilt structure was mounted horizontally (without BOSEMs) on a vertically actuated vibration table (TIRA TV 55240/LS-340) using a rigid L-bracket. The tip-tilt structure was rotated such that the vertical motion of the structure was in the beam direction (backward and forward motion of the structure, see Fig. 2) and rotated by 90° for the transverse motion (sideways motion of the structure). Three accelerometers were used to record the vibrations: (1) one was mounted on the base of the vibration platform, (2) one on the base of the tip-tilt mounting bracket, and (3) one on top of the tip-tilt structure. The results, shown in Fig. 11, were obtained by taking the ratios ((3)/(1) : (2)/(1)) between the three accelerometer measurements.

The principal resonances were found to be at 306 Hz along with the beam direction and at 249 Hz in the transverse direction, well above the 150 Hz target. Both resonances have a quality factor of around 40, as can be seen in Fig. 11. The features around 50 Hz are instrumental and do not represent

FIG. 11. (Color online) Measured modal response of the tip-tilt structure.

any modes in the structure. Due to the limitations of the available apparatus, low frequency modes (below ~5 Hz) were not explored.

V. CONCLUSION

We present the design and the measured performance of the tip-tilt mirror suspension used in the advanced LIGO gravitational wave detectors. These suspended steering mirrors are used in the optical chain in the interferometric sensing and control system. The six eigenmodes of the suspended mirror are all below 10 Hz, and have quality factors well below 100. The suspended mirror is controlled in longitudinal, pitch, and yaw degrees of freedom via optical sensors and electromagnetic actuators. Due to the small and rigid design of the tip-tilt mirror suspension, they can be useful in other optics experiments.

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