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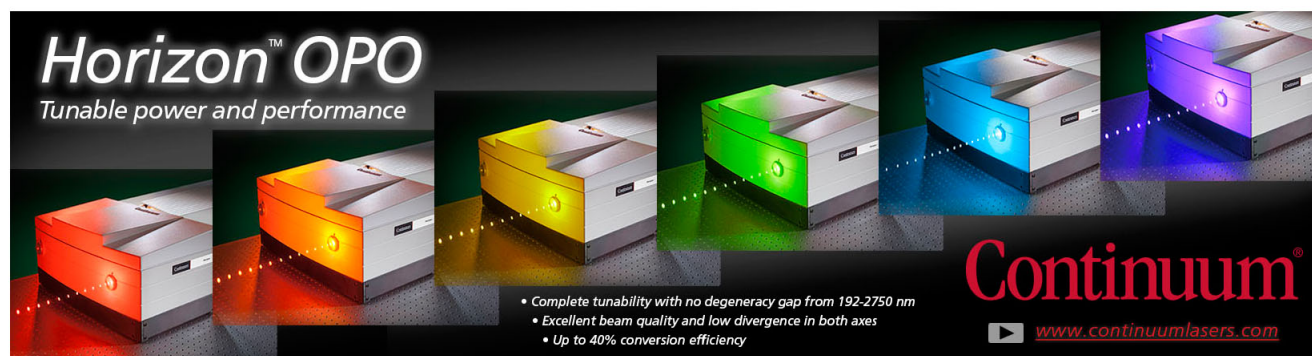
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Competing nonlinearities with metamaterials

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We suggest an approach for creating metamaterials with sign-varying nonlinear response. We demonstrate that microwave metamaterials with such competing nonlinearities can be created by loading split-ring resonators (“meta-atoms” of the structure) with pairs of varactor diodes and photodiodes exhibiting nonmonotonic resonance frequency shift with changing incident microwave power. Additionally, the nonlinear response of such metamaterials can be controlled by illuminating the meta-atoms by light. © 2012 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4768945>]

Nonlinear optics describes the propagation of light in nonlinear media which electric polarization responds nonlinearly to the electric field of light,¹ and this optical nonlinearity is hard to control or engineer. Natural materials usually have either focusing or defocusing type of Kerr-like nonlinear response, with theoreticians extensively exploring more exotic nonlinearities, which can rarely be found in natural dielectric materials.²

Recently emerged physics of metamaterials offers exceptional opportunities for creating artificial materials with a wide range of macroscopic parameters through appropriate arrangement of their subwavelength elements (also called “meta-atoms”).^{3–5} However, it has not been recognized yet that the concept of metamaterials allows engineering a nonlinear response as well. In this Letter, we employ a design of microwave metamaterials as a proof of principle to realize competing nonlinearities in metamaterials by employing two different mechanisms of nonlinear response. Our work opens a path for implementing complex nonlinear response functions for engineering nonlinear properties of metamaterial integrated circuits and metadevices.

The nonlinear response of artificial microwave metamaterials have been shown to depend on the circuitry employed in a design of each split-ring resonator (SRR).⁶ In brief, the resonant frequency of the SRR with a nonlinear varactor diode shifts to higher values for increasing intensity. At the same time, adding a simple inductor in parallel to the varactor changes the behaviour to the opposite, i.e., the resonant frequency shifts to the lower values with intensity. In terms of nonlinear optics, if for a given frequency, the former case corresponds to *defocusing nonlinearity*, then the latter case—to *focusing nonlinearity*. Recently, we have demonstrated that both linear and nonlinear responses of SRRs containing photodiodes can be tuned by changing the intensity of an external light source.^{7–9} In particular, it was shown experimentally that an increase in the light illumination intensity shifts the SRR’s resonance towards lower frequencies, while an increase in the applied microwave signal shifts the resonance response to higher frequencies.⁸ This effect opens a way towards a design of more complex nonlinear response.

In this Letter, we study the nonlinear behaviour of a split-ring resonator (metamaterial “meta-atom”) containing a varactor diode, which is *reverse-biased* by photodiodes. We demonstrate that the effective nonlinearity can change its sign as we increase the intensity of a microwave signal. As expected, the nonlinear response can be controlled additionally by illuminating photodiode with an external light source. We perform numerical simulations and demonstrate that such nonmonotonic nonlinear response corresponding to the competing nonlinearities is caused by effective switching between the two nonlinear regimes.⁶

Schematic of the SRR is shown in Fig. 1(a), with the photograph of the experimental setup presented in Fig. 1(b). The SRR is formed by two side-coupled broken rings made of copper, which are printed on a dielectric substrate (FR4 fiberglass, $\epsilon_r \approx 4.4$). To achieve tunability, we solder a varactor diode (D1) in an additional gap, which we made in the

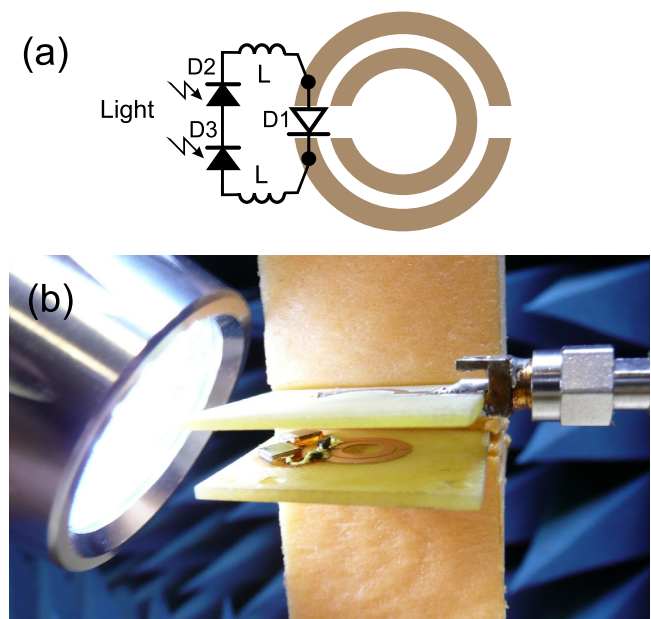


FIG. 1. (a) Schematic of the light-tunable SRR and attached electronic components. (b) Photograph of a SRR sample (bottom dielectric board) excited by a loop antenna (top dielectric board) and illuminated by a light source.

outer ring of the SRR. The bias voltage for the varactor diode is produced by two photodiodes (D2 and D3, see Fig. 1(a)), which operate in the photovoltaic mode. In order to enhance the effect of illumination, we use two photodiodes connected in series, which double the generated DC voltage. The polarity of the photodiodes is chosen, so that they provide the reverse voltage on the varactor. To prevent shunting the varactor by the large capacitance of the photodiodes, we use chip inductors (L) connected in series with the photodiodes.

We study SRR identical to that used in our earlier work:⁷ the radius of the inner ring is 2.56 mm and the width of the metal strip is 1.44 mm, the gap between the inner and outer rings is 0.32 mm. The upper split in the outer ring, and the symmetric split in the inner ring have the same width of 0.32 mm. The varactor diode D1 is a SkyworksTM SMV1233 varactor mounted in the additional gap in the outer ring. To provide the reverse voltage to the varactor, we use two BPW-34-S photodiodes (Opto SemiconductorsTM), D2 and D3 in Fig. 1(a). The inductors L are 22 nH chip inductors (LQG18HN22NJ00D from MurataTM), whose self-resonant frequency is around 2.3 GHz. These components are soldered on small metallic patches near the SRR.

In order to measure the magnetic response of the light-tunable SRR, we design a symmetric microstrip loop antenna. The antenna is fixed 5 mm above the plane of the SRR and is connected to an Agilent PNA E8362C vector network analyzer (VNA) through a coaxial cable, as shown in Fig. 1(b). To study experimentally the self-induced nonlinear effects in the light-tunable SRR, we sweep the applied power from the VNA and measure the reflection coefficient spectrum for different values of light intensity. To illuminate the sample in our measurements, we use a light-emitting diode light torch and control the light intensity near photodiodes using a lux meter.

The reflection coefficient S_{11} measured by the VNA when the loop antenna is coupled to the SRR is shown in Fig. 2 for different values of the microwave power and for different intensities of light illuminating photodiodes. The minimum of the reflection coefficient corresponds to the maximum of the power that couples to the resonator, which occurs at the resonance of the SRR. First, we measure the reflection coefficient of the SRR in the absence of light (0 lx). We increase the applied power of VNA from 5 dBm

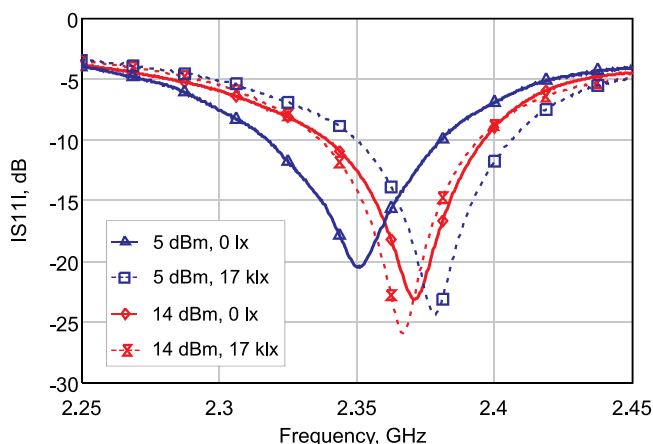


FIG. 2. Measured reflection coefficient for two different values of the input microwave signal power and for two different intensities of the external light source.

to 14 dBm and observe a shift of the resonant frequency to higher values (corresponding curves are shown by solid lines in Fig. 2). Next, we repeat the measurements under the light intensity of 17 klx, and notice that the resonant frequency shifts to lower frequencies when the incident power increases from 5 dBm to 14 dBm (dashed curves in Fig. 2). This result indicates that the type of the nonlinear response of the SRR can be changed by light illumination.

To explain the observed nonlinear behaviour, we perform numerical simulations. The nonlinear equivalent circuit of the SRR is shown in Fig. 3(a). The ring is represented by $R_{SRR}L_{SRR}C_{SRR}$ circuit. To simulate the excitation by a loop antenna, we use inductance $L1$ to model antenna response, and mutual inductance M , which describes antenna coupling to SRR. To model varactor $D1$, we use a SPICE model of a varactor provided by the manufacturer. To model the photodiodes, we employ the photodiode SPICE model and data from the photodiode data sheet using approach presented in Ref. 10. The ring inductance and capacitance can be analytically

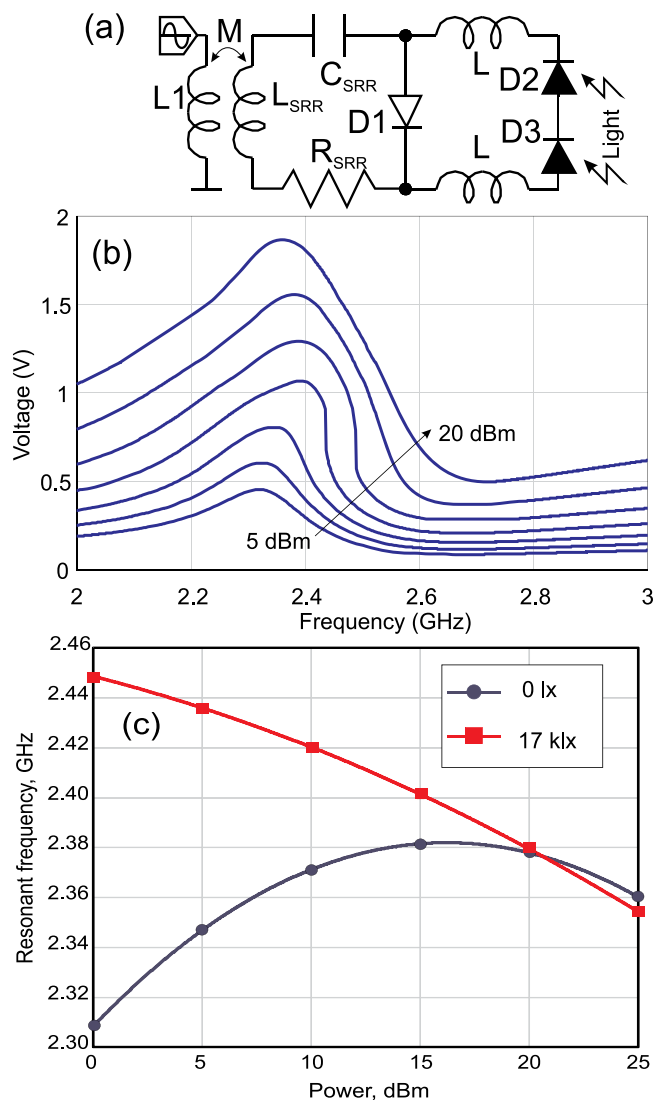


FIG. 3. (a) Nonlinear equivalent circuit of SRR excited by a loop antenna. (b) Voltage across the varactor vs. frequency simulated numerically for input powers from 5 dBm to 20 dBm with a step of 2.5 dBm and light intensity 0 lx. (c) Simulated shift of the resonant frequency of the nonlinear SRR vs. the input power, with and without external illumination.

estimated as $L_{SRR} = 9.6$ nH and $C_{SRR} = 0.5$ pF.¹¹ Next, a linear circuit simulation has been performed to find the values of the equivalent circuit elements. We simulate the reflection coefficient numerically and fine-tune the parameters of the effective circuit, so that the simulated reflection coefficient matches that measured in experiment in linear regime (at -15 dBm) and 0 lx illuminance. We find that the following parameters give the best agreement with the experiment: $LI = 0.44$ nH; $M = 0.84$ nH; $L_{SRR} = 7$ nH; $C_{SRR} = 0.62$ pF; and $R_{SRR} = 0.84$ Ohm.

In order to understand the nonlinear behaviour of the SRR, we perform harmonic balance simulations and find the voltage generated across the varactor for the input powers changing between 5 dBm and 20 dBm. The results obtained without light illumination (0 lx) are shown in Fig. 3(b). Remarkably, the voltage across the varactor can reach up to 2V when the input power is increased up to 20 dBm. This voltage is above the critical forward voltage of the two photodiodes, and this should lead to the DC current bypass via these photodiodes. Next, we use the harmonic balance simulations to obtain the large signal reflection coefficient of the nonlinear circuit. The simulated results without light illumination (0 lx) and for the light intensity of 17 klx are shown in Fig. 3(c). For dark state, we observe that the resonant frequency initially shifts up, when the input power changes from 0 dBm to 15 dBm. Further increase of the input power above 15 dBm leads to a shift of the resonant frequency down to lower values. At the same time, the resonance shift for the bright state (17 klx) is monotonous, as shown in Fig 3(c).

To test our predictions experimentally, we measure the current vs. voltage ($I-U$) characteristic of our photodiode (see Fig. 4). In the dark state (0 lx), the observed $I-U$ behaviour is similar to that of a conventional rectifier diode. However, when the photodiode is illuminated, the $I-U$ curve shifts. If the circuit is open (operation in a photovoltaic mode), the open circuit voltage $U_{oc} = 0.4$ V will be generated with positive polarity at the anode. On the other hand, if the circuit operates in a biased regime and the forward voltage is applied to the photodiode, the breakdown voltage can be estimated as $U_d = 0.44$ V.

Thus, a change in the sign of nonlinear frequency shift can be explained by the regimes of the photodiodes operation.

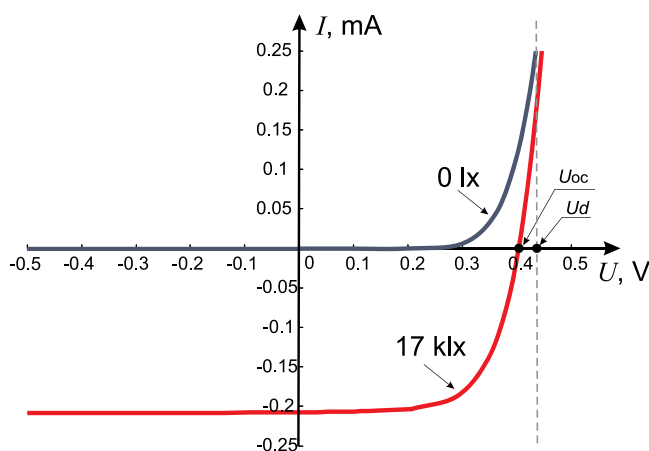


FIG. 4. Experimentally measured current vs. voltage ($I-U$) characteristics of a BPW-34-S photodiode for two values of light intensity.

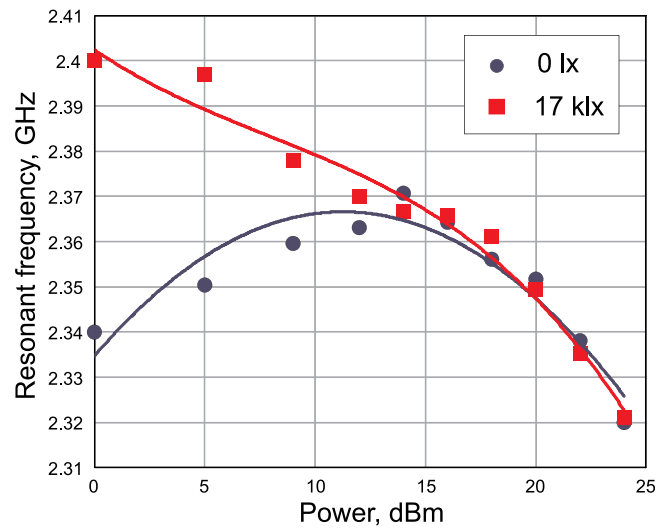


FIG. 5. Experimentally measured shift of the SRR resonant frequency as a function of input power, for two values of light source intensity. Measured data are shown by markers, whereas the lines are guide for eye.

In the dark state, when the input power is low, the photodiodes operate in the bias-free regime providing zero reverse voltage to the varactor. The constant voltage across the varactor appearing due to the signal rectification shifts the operating point of the varactor to the region of the $I-U$ curve with low effective conductance and lower capacitance, and the SRR resonant frequency grows. After the input power increases up to 15 dBm, the voltage rectified by the varactor is of the order of 1 V, which is above $2U_d$. This bias applied to photodiodes is sufficient to make them conducting. As soon as this happens, the characteristic impedance of photodiodes significantly decreases and DC current may flow via this network causing the varactor discharge. As a result, with further power increase, the resonance frequency decreases. In the light illuminated state, the varactor operates around the zero voltage point, this leads only to the negative sign of nonlinearity.

Finally, in order to verify this nonlinear behaviour, we studied experimentally the resonant shift of the nonlinear SRR. For the incident power changing from 0 dBm up to 24 dBm, we measure the reflection coefficients of the SRR in the dark (0 lx) and bright (17 klx) states. The resonant frequency is determined from the minimum of the reflection coefficient, and its shift is represented in Fig. 5. As was predicted numerically, in the dark state, the resonant frequency grows when the input power increases up to 14 dBm and decreases with further power increase. In the bright state, the resonant frequency decreases monotonically with the increase of the input power. While we observe an excellent qualitative agreement of the behaviour of our resonator with that predicted numerically, there is a small disagreement in absolute values of the resonant frequency. We suppose that this mismatch occurs due to the presence of parasitics in electronic components, which are not taken into account by our simplified model shown in Fig. 3(a).

In conclusion, we have demonstrated that the split-ring resonator incorporating electronic components may exhibit nonmonotonic dependence of its resonant frequency on the incident microwave power. In addition, by employing

photodiodes, it becomes possible to tune the overall nonlinear response by illuminating the structure with light. The proposed meta-atoms would allow creating metamaterials with competing nonlinearity, the type of material that was long considered by theoreticians, but which was never found in nature. Since such material was not found in nature, there are no applications of such type of the nonlinearity. We can suggest that our meta-atoms may potentially be used for creating new functionalities of nonlinear metamaterials, such as nonlinear mirrors, frequency selective surfaces, antenna arrays, cloaking devices, etc. As an example, if the nonlinear metamaterial mirror with just the varactor diodes exhibits saturation of the transmission, similar to that shown in Ref. 12, then by adding the photodiode and changing the illumination, one can control the saturation regime of such a mirror, which adds an extra degree of freedom for controlling the metamaterial response. We believe that our approach will be useful for other types of nonlinear metamaterials also allowing to control and engineer both wave mixing and second-harmonic generation.^{13–15}

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