Squeezed light sources for current and future interferometric gravitational-wave detectors

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Declaration

This thesis is an account of research undertaken between February 2013 and December 2017 at the Centre for Gravitational Physics, Department of Quantum Science, in the Research School of Physics and Engineering at the Australian National University, Canberra, Australia.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not be submitted in whole or part for a degree in any university.

Georgia Leigh Mansell
January 2018
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Abstract

The era of gravitational-wave astronomy has begun, with the detection of 5 confirmed binary black holes and a binary neutron star coalescence by the Advanced Laser Interferometer Gravitational-wave Observatory (aLIGO), and later with the advanced Virgo detector. These detections have already revealed a wealth of discoveries across the fields of nuclear physics, general relativity and astrophysics. The work presented in this thesis is part of the ongoing effort to improve the sensitivity of ground-based interferometric gravitational-wave detectors.

The sensitivity of aLIGO, and other interferometric detectors, is broadly limited by quantum noise. Improving on the quantum noise will increase the astrophysical range of the detectors, and improve the source parameter estimation. One way to reduce quantum noise is to inject audio-band squeezed vacuum states into the detection port. This technique has been demonstrated on the initial LIGO and GEO600 detectors. A squeezed light source for aLIGO must meet stringent requirements in terms of optical loss, phase noise, and scattered light. The squeezer must produce high levels of audio-band squeezing and operate under vacuum, to take advantage of the excellent existing isolation systems and to minimise optical loss. At design sensitivity, squeezed states whose quantum noise depends on frequency will be required.

We have demonstrated an ultra-stable glass-based squeezed light source, the first experiment of this kind to operate under vacuum. The squeezer cavity is constructed quasi-monolithically, with optics and nonlinear crystal oven optically contacted to a breadboard base. The cavity is designed to have extremely low length noise, and to produce high levels of audio-band squeezing. We have measured $8.6 \pm 0.9 \text{ dB}$ of squeezing and infer the generation of $14.2 \pm 1.0 \text{ dB}$ after accounting for all known losses. The squeezer has demonstrated record phase noise performance of $1.3 \text{ mrad}_{\text{RMS}}$, dominated by sources other than cavity length noise. This exceeds the phase-noise requirement for a squeezer for aLIGO. A copy of this squeezer is currently being installed in a squeezing-ellipse rotation experiment to demonstrate frequency-dependent squeezing for aLIGO. Lessons learnt during the construction and operation of the in-vacuum squeezer have helped inform the design of a frequency-independent squeezed light source currently being installed at the LIGO sites.

Future gravitational-wave detectors will continue to use interferometric techniques, and will be limited by quantum noise for the foreseeable future. To improve on thermal noise limits and interferometer power handling, future detectors look to cryogenic silicon as a test mass material. To take advantage of the desirable properties of silicon, including low scatter and absorption, a longer operating wavelength is required. The proposed LIGO Voyager upgrade has an operating wavelength in the $2 \mu\text{m}$ region, with the specific wavelength to be determined. LIGO Voyager will require a squeezed light source in the $2 \mu\text{m}$ region to reach its design sensitivity.

We present the design, characterisation, and results of the first squeezed light source in the $2 \mu\text{m}$ region. Laser and detector technologies at $2\mu\text{m}$ are less developed than their 1064 nm counterparts, causing significant technical challenges. We have measured $4.0 \pm 0.2 \text{ dB}$ of squeezing at 1984 nm, limited by loss due to detector quantum efficiency. Accounting for known losses in the system, we infer the generation of 10 dB of squeezing. This is an important demonstration of quantum noise reduction for future detectors, and a pathfinder technology for the design choices of LIGO Voyager. So far we have found no reason why a $2 \mu\text{m}$ interferometer should not be feasible.
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Gravitational-wave astronomy is an emerging field, with a handful of detections of compact object coalescences revealing a wealth of new discoveries about black holes, neutron stars, and gravity. The work presented in this thesis is part of the ongoing effort to improve the sensitivity of current and future ground-based interferometric gravitational-wave detectors.

Gravitational waves were first postulated by Einstein in 1916, in his theory of general relativity [62], however it was not until 2015 that the first confirmed detections were made by the Advanced Laser Interferometer Gravitational-wave Observatory (aLIGO), and later in conjunction with the advanced Virgo detector. The first detected gravitational waves were generated by binary black holes [43, 42, 155, 156, 151], and later a binary neutron star system (BNS) [158]. These detections were the first “direct” detections of black holes, and allowed tests of general relativity in the strong field regime. The BNS detection, dubbed GW170817, put new limits on the equation of state of the neutron star. Its electromagnetic counterpart confirmed that gravitational waves travel at the speed of light, allowed a new independent measurement of the Hubble constant, and confirmed BNSs as the origin of kilonovae.

Future detections will reveal new physics on the formation mechanism behind compact binary systems, and the structure of neutron stars. Other new sources, which could include a stochastic background of gravitational waves, supernovae, or some unexpected source of gravitational waves, promise to enrich our understanding of astrophysics, nuclear physics, and gravitation.

Increasing the sensitivity of detectors will facilitate more detections and increase their astrophysical range. Laser interferometers, which form the basis of almost all current gravitational-wave detectors, are broadly limited by quantum noise. Quantum noise consists of shot noise at high frequencies and radiation pressure noise at low frequencies, and can be improved by injecting squeezed states of light into the detector, as realised by Caves et al. in 1981 [31].

The first experimental demonstration of squeezed light was achieved in 1985 [141], and the first squeezing from a below-threshold optical parametric amplifier was demonstrated by Wu et al. in 1986 [180]. Over the following decade increasing levels of squeezing in the MHz frequency band were achieved using optical parametric oscillation. The first squeezing measured at the Australian National University was reported by Lam et al. in 1999 [94]. The current record squeezing level is 15 dB measured by Vahlbruch et al. in 2016 [165].

It wasn’t until 2004 that squeezing was achieved in the audio frequency band, the sensitive band of ground-based gravitational-wave detectors. McKenzie et al. realised that channels which coupled technical noises at audio band frequencies into a squeezed state could be mitigated by removing the seed field, creating vacuum squeezed states [107]. Progress was made in controlling squeezed states of light in the audio band by Vahlbruch et al. in 2006, with the realisation of the coherent control scheme [164]. In 2011 Chua et al. demonstrated a backscatter-tolerant squeezed light source [41]; the current record squeezing level in the audio band, 11.6 dB at 10 Hz, was measured from this same squeezer system by Stefiszky et al. [145]. Each of these developments was a crucial step on the path to squeezing for gravitational-wave detectors.
Following these innovations, squeezed state injection has been demonstrated to improve the shot-noise limited sensitivity on two gravitational-wave detectors: GEO600 [148] and initial LIGO [149]. In GEO600 up to 4.4 dB of squeezing improvement beyond the shot noise limit has been achieved, and in LIGO 2.15 dB of squeezing improvement was achieved at the time of writing. The sensitivity improvement of both these experiments was limited by optical losses. The sources of loss include mode mismatch between the squeezed and interferometer beams, loss of injection optics, alignment fluctuations, imperfect cavities, and imperfect photodiodes.

In the era of advanced gravitational wave detectors higher levels of squeezing improvement are required, placing stringent requirements on the source of squeezed light being injected into the interferometer. Improved backscattered light mitigation, reduced phase noise, and frequency-dependent squeezed states will be required for aLIGO. Adaptive mode matching, low-loss Faraday isolators, and alignment sensing and control are active areas of research, in the future losses are projected to be sufficiently low as to allow for 10 dB of squeezing improvement. At high levels of squeezing, phase noise becomes a limiting factor [121]. To minimise some of the sources of loss and phase noise, the aLIGO squeezer will be operated under vacuum. We have demonstrated a stable, glass-based squeezer, the first of its kind to be operated under vacuum [170, 171, 123]. This proof-of-principle in-vacuum squeezer optical parametric oscillator (VOPO) experiment is an important step in informing the design of the aLIGO squeezer.

Frequency-dependent squeezing will be required to achieve broadband sensitivity improvement once aLIGO is limited by radiation pressure noise [150]. Increasing the level of frequency-independent squeezing (only improving high-frequency sensitivity, eventually at the expense of low-frequency sensitivity) will improve sky localisation, and some parameter estimation. Introducing frequency-dependent squeezed states additionally improves the broadband sensitivity and range of the detector [100]. Frequency-dependent squeezing can be achieved by combining the usual squeezer system with a filter cavity with a pole at roughly 50 Hz, to acquire the desired quadrature rotation. This requires a filter cavity with a long baseline and very high finesse; the first proof-of-principle audio-band squeezing ellipse rotation was achieved by Oelker et al. in 2015 [122]. The VOPO is currently being implemented on a longer-baseline filter cavity experiment, which aims to demonstrate the required squeezing levels and rotation frequency for aLIGO. New techniques to achieve frequency-dependent squeezing are being investigated, for example using the interferometer arms rather than an external filter cavity [101], optomechanically induced transparency [127], and electromagnetically induced transparency [81] and four wave mixing [46] in atomic vapour cells.

Future gravitational-wave detectors will continue to use laser interferometry, and will be limited by quantum noise for the foreseeable future. Proposed future detectors will utilise cryogenic silicon test masses to improve on noise sources across the spectrum, exploiting the low absorption and scatter of crystalline silicon [1, 152]. Silicon optics require longer operating laser wavelengths, as silicon has significant two-photon absorption below 1550 nm [25], excluding the current aLIGO wavelength of 1064 nm. While 1550 nm may be used, the wide-angle loss due to scattered light in silicon decreases for longer wavelengths, thus an operating wavelength in the 2 µm region is the design operation wavelength for the LIGO Voyager upgrade.

Design studies of LIGO Voyager assume the implementation of squeezing, and assume 8-10 dB of broadband squeezing improvement. The work presented in this thesis is the first step towards a squeezed light source for a 2 µm silicon interferometer. We show the design, implementation and first results from a squeezed light source in the 2 µm region, demonstrating 4.0 ± 0.2 dB of squeezing below shot noise, currently limited by photodetector quantum efficiency.
1.1 Thesis structure

This thesis consists of three background chapters, followed by three chapters summarising the design, construction and results from the first in-vacuum glass-based squeezed light source, and the first squeezed light source operating in the 2\(\mu\)m region.

Chapter 2 provides a summary of gravitational waves, methods to detect them, and noise sources in current and future interferometric gravitational-wave detectors. This chapter provides context and motivation for the work presented in later chapters.

Chapter 3 is an introduction to quantum states of light and quantum noise. First the electromagnetic field is quantised and squeezed states of light are introduced from a theoretical standpoint. The concepts of loss, phase noise, photodetection and balanced homodyne detection are introduced, and a model for the quantum noise of a Michelson interferometer is derived using the formalism introduced earlier in the chapter. The quantum noise of an arm-cavity dual-recycled Michelson, such as those used in advanced gravitational wave detectors, is obtained. Finally, methods for improving on quantum noise, with emphasis on squeezed state injection, are discussed.

Chapter 4 introduces the concepts of optical cavities, and optical cavities containing a nonlinear element. The production of squeezed states of light in an optical parametric oscillator (OPO) is derived, this is the method by which all squeezed states are produced in this thesis. Finally, control systems for locking optical cavities and controlling squeezed states of light are introduced, thus concluding the background chapters.

Chapter 5 summarises design considerations for squeezer OPOs, which is important to both the in-vacuum and 2\(\mu\)m OPOs presented in later chapters. First the nonlinear crystal is considered, and materials, phase matching, in-vacuum operation and optical coatings are discussed. The geometry of the OPO cavity is explored, with particular emphasis on the bowtie cavity geometry. Key geometric design factors including beam waist size, cavity stability, and higher order mode spacing are discussed, as well as optical parameters such as cavity escape efficiency and threshold power. Finally, tolerances on the cavity parameters are explored.

Chapter 6 discusses the first in-vacuum OPO, including motivation, construction, characterisation, operation, and results. The glass OPO was designed to meet the squeezed light source requirements of Advanced LIGO in terms of stability and vacuum compatibility. It was built using a novel glass-based architecture, and demonstrated high levels of squeezing under vacuum, and low squeezing ellipse phase noise.

Chapter 7 summarises the 2\(\mu\)m squeezer experiment, which is motivated by the requirements of future gravitational-wave detectors. Design choices and technical challenges are addressed, including choice of laser wavelength, crystal nonlinearity measurements, detector considerations and characterisation, laser frequency drift suppression, phase noise performance, and OPO characterisation. This chapter concludes with results showing the first demonstration of squeezed light in the 2\(\mu\)m region.

Chapter 8 concludes the work presented in this thesis, and presents some ideas of the further work to be completed in the field.

1.2 Publications

The results presented in this thesis are summarised in the following publications


- E Oelker, G L Mansell, M Tse, J Miller, F Matichard, L Barsotti, P K Fritschel, D E McClelland, M J Evans, and N Mavalvala. Ultra-low phase noise squeezed vacuum source for


Gravitational waves and their detection

The work presented in this thesis is motivated by the ongoing effort to improve the sensitivity of ground-based interferometric gravitational-wave detectors. This chapter summarises the physics of gravitational waves, including their formulation in Einstein’s theory of general relativity, their generation by astrophysical sources, and detection by ground- or space-based detectors. Finally, this chapter focuses on ground-based interferometric gravitational wave detectors, present and future, and the limiting noise sources of these devices.

Gravitational waves have recently been observed by the Advanced Laser Interferometer Gravitational-wave Observatory (aLIGO) and advanced Virgo detectors, opening a new window to the universe and launching the field of gravitational-wave astronomy. Gravitational waves from a binary black hole coalescence were detected on the 14th of September 2015 by the Advanced LIGO interferometers [43], shortly after the beginning of the first data-taking run. This event, dubbed GW150914, was the first direct observation of gravitational waves, the first direct observation of a binary black hole, the first observation of black holes in the 10s of solar masses regime, and the first test of general relativity in the strong dynamical field regime.

This first detection was followed by successive binary black hole coalescences: GW151226 [42], GW170104 [155], GW170608 [156], GW170814 [151], the last of which was observed jointly by the LIGO and Virgo interferometers.

The most recent event, GW170817, originated from a binary neutron star inspiral, and was observed across the electromagnetic spectrum, marking the beginning a new era of multi-messenger astronomy [158, 159]. The field of gravitational wave astronomy is new, and promises rich and engaging science for decades to come.

2.1 Gravitational waves

This section provides a brief introduction to the origin and nature of gravitational waves. Textbooks such as Maggiore [102], Misner, Thorne and Wheeler [113] and Weinberg [175] provide comprehensive guides to the subjects of general relativity, gravitation and cosmology in greater detail.

In 1916 Einstein published his revolutionary theory of general relativity, describing gravity in terms of the geometry of spacetime, and generalising his theory of special relativity from flat to curved spacetime [62]. Until then the gravitational force had been treated using Newton’s law of gravitation. General relativity accurately describes phenomena beyond classical physics, including gravitational lensing and gravitational time dilation. So far general relativity has passed all tests, correctly describing physical phenomena due to gravity.

The key results of general relativity are summarised by the Einstein field equations. These equations relate the curvature of spacetime to the distribution of mass-energy. A simplified form
of the Einstein field equations is given by

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

(2.1)

where \(G_{\mu\nu}\) is the Einstein tensor, which depends on the second order derivative of the spacetime metric, \(G\) is the Newtonian gravitational constant, \(c\) is the speed of light, and \(T_{\mu\nu}\) is the stress-energy tensor, which encapsulates the density of matter and energy in spacetime. In the words of John Archibald Wheeler, *spacetime tells matter how to move, and matter tells spacetime how to curve.*

One key result of general relativity is the prediction of gravitational waves. Fluctuations in the distribution of matter will cause fluctuations in the curvature of spacetime, leading to waves which propagate, analogous to waves in electromagnetism. These waves were postulated by Einstein, but due to the incredible stiffness of spacetime as a medium they were far from measurable with technology available at the time.

Gravitational waves are quadrupole in nature, as the lower order monopole radiation is forbidden by mass-energy conservation, and dipole radiation is forbidden by conservation of momentum [113]. As gravitational waves propagate through spacetime they perturb the proper length between two freely falling test masses. Gravitational wave detectors are designed to measure this perturbation.

Gravitational waves can be decomposed into their polarisation components, analogous to horizontal and vertical polarisations of plane waves in electromagnetism. The polarisation states of gravitational waves, however, are written in terms of the “+” and “x” polarisations. That is, any polarisation, including left- and right-handed circular polarisation, can be written as a combination of “+” and “x” components.

The effect of a passing gravitational wave on two test masses is to stretch and compress spacetime between them, depending on direction of their separation relative to the polarisation of the wave. It is convenient to visualise the effect of a gravitational wave on a ring of test masses. In the case of a “+” polarisation wave, the ring is deformed into an ellipse which pulses along the x and y axes as a function of time, as shown in the figure below.

The “x” polarised wave has a similar effect, along axes at 45° to x- and y-axis shown in the figure above.

In the case of a left-handed circular polarisation, the effect of a passing gravitational wave on a ring of test masses is to deform it into an ellipse which rotates clockwise as a function of time.

Most gravitational-wave detectors measure the separation of free test masses, thus the measured effect of the gravitational wave is usually expressed as a strain, the change in separation \(\Delta L\) divided by the total separation \(L\),

$$h = \frac{\Delta L}{L}.$$  

(2.2)
While huge amounts of energy can be emitted in gravitational waves, they only produce extremely small strains. The energy emitted in gravitational waves of the first detection, GW150914, was roughly equivalent to 3 times the rest mass energy of the sun, while the peak strain was $h \sim 10^{-21}$. This tiny strain produced a signal in both LIGO detectors with a combined signal-to-noise ratio of 24.

The first indirect evidence for gravitational wave emission was observed by Hulse and Taylor, who observed a pulsar in a binary system with a neutron star, labelled PSR B1913+16 [83]. The orbit was found to be shrinking at a rate which corresponded to the expected energy loss due to gravitational waves, as predicted by general relativity [147].

### 2.2 Sources of gravitational waves

Gravitational waves are produced by the acceleration of a mass with a non-constant quadrupole moment. The extreme stiffness of spacetime causes the strain induced by a passing gravitational wave to be minuscule, thus detectable sources are astrophysical in nature, and typically involve extreme dynamical conditions. So far observed gravitational waves originate from compact binary coalescences, where neutron stars or black holes of several solar masses move at an appreciable fraction of the speed of light.

In this section we will briefly discuss some known sources of gravitational waves, potential and observed, and describe qualitatively the interesting physics that can be learnt from their gravitational wave signals. This topic is reviewed in greater detail by Cutler and Thorne [50], and Saulson [132]. Below are sources at frequencies in the ground-based detector sensitivity band, which we will focus on.

**Compact binary coalescences (CBC)**

CBCs are the coalescence of two compact objects, consisting of a combination of black holes or neutron stars (or possibly a dense exotic form of matter). The gravitational-wave signal emitted from such sources is well known, producing a distinctive chirp waveform as a function of frequency and time. As the two progenitors coalesce their geodesics spiral in and they merge to form a single perturbed astrophysical object, which then rings down to equilibrium. The chirp of a CBC is extracted from the data using template matching with a filter bank of chirps with varying parameters. From the gravitational-wave signal the parameters of the astrophysical system can be inferred, including component masses, luminosity distance, and sky position. There is much that has been, and will be, learnt from CBC detections, including information about the formation mechanism of binary black holes, the equation of state of neutron stars, as well as tests of general relativity.

**Bursts**

Bursts of gravitational radiation are expected to originate from astrophysical sources such as spherically asymmetric core-collapse supernovae. The waveforms of burst signals are generally unknown and hence searches for such sources seek unmodeled excess signal in the detectors. Detecting the gravitational-wave signal from a supernova would help to understand the physical processes involved in their core collapse. The gravitational-wave signal reveals dynamics beyond what is visible electromagnetically, as electromagnetic signals originate from charged particles, while gravitational wave signals originate from the distribution of mass [126].

**Pulsars**

Spinning neutron stars, unless perfectly spherically symmetric, can produce periodic gravitational waves at an approximately persistent frequency. The frequency of the Crab and Vela pulsars are known (59.3 Hz and 22.4 Hz respectively), and with sufficient data might be observed with aLIGO [154].
2.3 Detecting gravitational waves

Stochastic background A background of gravitational wave signals due to a multitude of unresolvable sources will eventually become detectable in ground-based detectors. The stochastic background is measured as a correlated signal in the detectors, and is measured by cross-correlating the data from two or more interferometers [153]. The latest detections of binary black holes and the BNS GW170817 have refined the predictions on the stochastic background due to these sources, suggesting they may be detected in the next few years [157].

Other sources of gravitational waves, at frequencies below the ground-based detector band include supermassive black hole binary coalescences, white dwarf binaries, and primordial inflationary gravitational waves, which imprint a polarisation signature on the cosmic microwave background.

2.3 Detecting gravitational waves

The quest to detect gravitational waves has spanned several decades, with detectors proposed and built to target all of the sources discussed in the previous section. Most detectors are based on the principle of converting the strain induced by a passing gravitational wave to some measurable quantity, such as a voltage. While interferometric gravitational-wave detectors made the first confirmed detections, the first sensors were built by Weber and utilised resonant bar technologies [174].

From Weber’s initial bars, a network of resonant bar detectors was built and operated through the 1990s [35, 10, 17, 70, 53]. The resonant bars were constructed of metal, seismically isolated and cryogenically cooled. They had extremely high Q-factors, and were designed such that their resonant frequencies could be excited by a passing gravitational wave. The length change in the bar was then read out by sensitive piezo-electric transducers or superconducting transducers. The strain sensitivity of bar detectors was limited to gravitational wave sources within the galaxy, and their bandwidth limited by the quality factor of the bar [18]. Laser interferometers eventually proved the more robust detection method, with their broadband sensitivity and improved scalability.

Lower frequency sources are sought by space-based interferometric detectors such as the LISA mission [51], which targets supermassive black hole mergers, extreme mass ratio binary coalescences, and the inspirals of heavy solar mass black holes which will later merge in the LIGO band. LISA has the highest sensitivity in the mHz frequency band, where LIGO is unable to achieve high sensitivity due to seismic noise. At even lower frequencies, in the nHz band, pulsar timing array search for gravitational waves due to supermassive black hole binary coalescences. Pulsar timing looks for the effect of gravitational waves on the time of arrival of radio pulses from well-known millisecond pulsars [99]. Finally cosmic microwave background (CMB) telescopes look for the signature of primordial gravitational waves on the polarisation of CMB [136].

The remainder of this section presents an overview of laser interferometers for gravitational wave detection - their design, noise budgets and outlook.

2.3.1 Laser interferometers - past, present, and future

Interferometric gravitational-wave detectors measure the separation of free-falling masses, and search for the signal of a passing gravitational wave as a strain signal in this position measurement. Typical interferometers probe test mass position using a laser beam, imprinting the position information on the phase of the laser. In order for a test mass to be “freely falling” it may be
placed in space, isolated from external gravitational attractions, or be well isolated from the outside world on the ground. Ground-based interferometers operate with test masses and laser beams under vacuum. Multi-layer pendular suspensions, with active and passive seismic isolation, are used to isolate the test masses from the seismic environment, and to simulate free-fall at frequencies above the resonances of the pendulums.

The interferometers most relevant to this thesis read out test mass positions in a Michelson interferometer configuration, such as is pictured in the inset of figure 2.2. A laser beam is incident on a beamsplitter and sent along perpendicular arms. At the end of the arms are test masses, which the laser is reflected off and recombined at the beamsplitter. The power at the detection port is dependent on the interference condition between the two beams incident on the beamsplitter which is determined by the relative phase between them.

The effect of a passing gravitational wave in the “+” polarisation stretches and compresses spacetime along the x and y arms. As the speed of light is constant, the laser light reflecting off the test masses accrues a phase shift due to the gravitational wave. The Michelson topology minimises the noise common to both interferometer arms, such as laser intensity and frequency noise. The optical configuration of current gravitational-wave detectors will be further discussed in the following section.

The idea of using laser interferometers to detect gravitational waves was first formulated in the 1960s and 1970s [75] [115]. In 1972 Rainer Weiss performed the first analysis of the noise sources and performance of such interferometers [176], following an unconfirmed claim of detection of gravitational waves by Joseph Weber [174] and the invention of the laser [103]. This investigation prompted the first proposals for long baseline interferometric gravitational wave detectors. In the 1990s, construction of the network of gravitational-wave detectors began. The current network of gravitational-wave detectors will be discussed in section 2.3.2.

The “first generation” of interferometric gravitational-wave detectors operated through the 2000s. These detectors put new upper limits on the gravitational-wave strain produced by known sources, and spurred technological advances in many fields, including high-power single-frequency lasers, optical coatings, control systems, suspensions and seismic isolation.

The current generation of detectors (“second generation” or “advanced” detectors) are significant upgrades on their first generation counterparts, including new laser sources and components, improved seismic isolation and test mass suspension. At design sensitivity the Advanced LIGO (aLIGO) interferometers will have a strain sensitivity improvement of a factor 10 over initial LIGO in its most sensitive band [150]. The aLIGO detectors at design sensitivity will be able to observe binary neutron star coalescences up to a distance of 300 Mpc, and binary black holes out to 20 GPe\(^1\) [105].

There are several planned upgrades to the aLIGO detector for the mid- and long-term. The A+ upgrade, after the Advanced LIGO observing runs, involves the implementation of frequency-dependent squeezing and new optical coating technologies. LIGO Voyager - a mid-term upgrade relevant to the final part of this thesis - is further discussed in section 2.3.5.

Third generation detectors will require completely new sites and technologies, and are currently in early planning stages. The LIGO Cosmic Explorer is in very early planning stages; it is a 40 km detector which aims to be sensitive to all binary black hole mergers (within the frequency band of the detector) in the universe. The European-led Einstein telescope (ET) [1] is more developed, and is currently seeking a site. The ET uses a triangular xylophone detector topology, with high- and low-frequency interferometers housed in the same facility. The spaced-based LISA mission is another future detector, due for launch around 2030.

\(^1\)depending on the component masses of the black holes.
2.3 Detecting gravitational waves

Figure 2.1: Map of the network of current (red) and under-construction (yellow) gravitational-wave detectors and their approximate orientations. The LIGO and Virgo detectors have made the detections of gravitational waves so far, and are currently undergoing upgrades to reach their design sensitivity. KAGRA is currently under construction and LIGO-India is planned and soon-to-be under construction. Note the orientation of LIGO-India is unknown at the time of writing. Map from [87].

2.3.2 The current network of ground based interferometers

Figure 2.1 is a map of the network of current gravitational-wave detectors, operational, under-construction, and planned. A network of detectors is crucial to improve the sky localisation of gravitational-wave signals, and to detect gravitational waves of all polarisations. The value of multiple detectors in improvement of source sky localisation is demonstrated in the cases of GW170814 and GW170817, where the inclusion of Virgo’s data narrowed the source sky localisation significantly [151, 158].

LIGO

The twin LIGO interferometers are located on opposite sides of the USA, at the Hanford Site, Washington, and Livingston Parish, Louisiana, as shown in figure 2.1 [150]. The 4 km arms of the two LIGO interferometers are similarly oriented, so they are maximally sensitive to the same gravitational wave polarisation [167]. Conversely the two detectors are also minimally sensitive to the same gravitational wave polarisation (orthogonal to the optimum sensitivity), thus a network beyond the two detectors is required to be sensitive across the whole sky, and to all polarisations. The Advanced LIGO detectors are currently the most sensitive ground based interferometers, and have made the first detections of gravitational waves. The Advanced LIGO upgrade was installed in 2015, and the interferometers are currently alternating commission and observing runs as they seek to reach design sensitivity. The Advanced LIGO detectors are administered by Caltech and MIT, with collaborators in the broader LIGO Scientific Collaboration. Currently in the planning stages is an identical third Advanced LIGO detector, which will be built in the Hingoli district of India. An identical third detector will provide improved sky localisation and polarisation data.

Virgo

The advanced Virgo detector [160] in Cascina, Italy, is a 3 km laser interferometer op-
erated by the Virgo Scientific Collaboration and lead by groups in France, Italy, and the Netherlands. Advanced Virgo will have a comparable strain sensitivity to Advanced LIGO, and detections at all 3 sites provide valuable polarisation and sky localisation information about the source of gravitational waves. As such, the LIGO and Virgo Scientific collaborations co-ordinate observing runs for maximum joint observing time.

**GEO600** The GEO600 interferometer [178] is a 600 m detector located in Hannover, Germany and run by a German-British collaboration. Novel technologies now employed on the larger detectors, such as signal recycling and squeezed state injection, were first demonstrated on the GEO detector. While the other detectors are in commissioning phases GEO continues to take data.

**KAGRA** The KAGRA interferometer is located in the Kamioka mine in Gifu Prefecture, Japan, and is led by the Institute for Cosmic Ray Research and the University of Tokyo [9]. The interferometer is currently under construction and has many novel features. KAGRA is located underground to minimise seismic and Newtonian noise coupling. The test masses will be cryogenically cooled, and the test mass substrates are sapphire.

### 2.3.3 Advanced LIGO configuration

Figure 2.2 shows a simplified schematic the optical configuration of Advanced LIGO. Advanced LIGO’s optical configuration differs from that of a simple Michelson interferometer with the inclusion of Fabry-Pérot arm cavities, as well as partially transmissive mirrors on the symmetric (laser input) and anti-symmetric (detection) ports which form power- and signal-recycling cavities. An analysis of the electromagnetic fields propagating in Michelson interferometers is provided in chapter 3.

The arm cavities enhance the optical power incident on the test mass and increase the storage time of the gravitational wave signal in the interferometer arm. The power-recycling cavity enhances the optical power incident on the beamsplitter, and hence the power entering the arm cavities. Since the interferometer is operated close to a dark fringe, with almost complete destructive interference at the output port, most of the input power is directed back towards the laser, the power-recycling cavity sends much of this light back into the interferometer.

The signal recycling cavity partially reflects the signal at the anti-symmetric port back into the interferometer. This has the effect of resonantly enhancing the gravitational wave signal, while modifying the effective arm cavity finesse.

Optical and physical parameters of aLIGO are listed in table 2.1.

### 2.3.4 Noise sources and technical challenges

A noise budget for the strain sensitivity of aLIGO at design sensitivity is displayed in figure 2.3. In this section the individual noise sources, their coupling to the interferometer, and methods of minimising them will be discussed. The aLIGO detector is used for reference, but the same noise sources exist in advanced Virgo and the other ground-based laser interferometers, which have similar optical configurations.

**Quantum noise**

The interferometer quantum noise originates from the quantum nature of the light used to interrogate the test masses. At high frequencies, above roughly 80 Hz the quantum noise limit is due to shot noise, set by the Poissonian nature of photon counting statistics. Below 80 Hz the quantum
2.3 Detecting gravitational waves

![Figure 2.2: Schematic of Advanced LIGO optical configuration, adapted from [150]. PRM is the power recycling mirror, ITMX and ITMY are the input test masses on the x and y arms, ETMX and ETMY are the end test masses on the x and y arms, SRM is the signal recycling mirror, and BS is the beamsplitter.]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Advanced LIGO</th>
<th>LIGO Voyager</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm length [m]</td>
<td>3995</td>
<td>3995</td>
</tr>
<tr>
<td>Test mass temperature [K]</td>
<td>290</td>
<td>123</td>
</tr>
<tr>
<td>Laser wavelength [nm]</td>
<td>1064</td>
<td>2000 ± 200</td>
</tr>
<tr>
<td>Input laser power [W]</td>
<td>125</td>
<td>140</td>
</tr>
<tr>
<td>Arm cavity circulating power [kW]</td>
<td>710</td>
<td>3000</td>
</tr>
<tr>
<td>Test mass substrate</td>
<td>Fused silica</td>
<td>mCz Silicon</td>
</tr>
<tr>
<td>Test mass mass [kg]</td>
<td>40</td>
<td>200</td>
</tr>
<tr>
<td>Test mass radius [cm]</td>
<td>17</td>
<td>22.5 *</td>
</tr>
<tr>
<td>Test mass thickness [cm]</td>
<td>20</td>
<td>43 *</td>
</tr>
<tr>
<td>Test mass suspension material</td>
<td>Fused silica fibre</td>
<td>Silicon ribbon</td>
</tr>
<tr>
<td>Residual gas pressure [Pa]</td>
<td>$4 \times 10^{-7}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1: Summary optical and physical parameters of the Advanced LIGO interferometers and potential LIGO Voyager upgrade. aLIGO parameters from [150, 68], LIGO Voyager from [152], and starred parameters from [3]. Note that the specific wavelength for LIGO Voyager is yet to be selected, and mCz refers to the magnetic field induced Czochralski technique for growing single crystal silicon.
Figure 2.3: Noise sources in Advanced LIGO, noise budget created using Gravitational Wave Interferometer Noise Calculator (GWINC) software (version 3.1) [68]. Parameters are listed in table 2.1.
noise is due to radiation pressure noise, caused by the uncertainty in the momentum imparted to
the test masses by the photons. The frequency of the intersection of shot noise and radiation pres-
sure noise is set by the physical and optical parameters of the interferometer, such as the mass of
the test masses and the power circulating in the arm cavities.

Shot noise is generally constant as a function of frequency, however the shot noise spectrum
of aLIGO shown in figure 2.3 has a tail at high frequencies following the response of the Fabry-
Perot arm cavities. The shot noise signal-to-noise ratio scales as $1/\sqrt{P_{\text{circ}}}$, where $P_{\text{circ}}$ is the power
in the arm cavities. The shot noise level of the interferometer may be reduced by increasing
the circulating power, or injecting phase-squeezed states of light, a concept that will be explored
in chapter 3. Increasing circulating power in the interferometer is technically challenging and
eventually has adverse consequences associated by the extra heat deposited in the test masses.
High power problems include parametric instabilities, which have already observed in the aLIGO
detectors [66], as well as the increase of quantum radiation pressure noise.

The radiation pressure noise amplitude spectral density has a $1/f^2$ frequency dependence,
from the test mass mechanical response. The signal-to-noise ratio scales as $\sqrt{P_{\text{circ}}}$. Radiation
pressure noise is the limiting noise source between approximately 10 Hz and 40 Hz in aLIGO
at design sensitivity. To reduce the radiation pressure noise level one can increase the mass of
the test mass, decrease the power circulating in the interferometer, or inject amplitude-squeezed
states. Thus the compromise between shot noise and radiation pressure noise is considered when
choosing the power circulating in the interferometer.

The quantum noise floor is derived in section 3.4.3 and the proceeding sections.

**Test mass thermal noise**

Random Brownian fluctuations within an optical component set a limit on the precision with which
a measurement of its position can be made. This is known as thermal noise. Thermal noise
contributions from the optical coatings, suspensions, and the bulk test masses themselves set limits
on the sensitivity of aLIGO. These three contributions are shown in red (and teal), blue, and orange
in figure 2.3.

The coating thermal noise is the dominant source and is related to the mechanical loss of
the coatings, as summarised by the fluctuation-dissipation theorem [132]. Coating noise will be
the limiting noise source in the 50-80 Hz band when Advanced LIGO reaches design sensitivity.
Optical coatings are made of layers of high- and low-refractive index materials. The power spectral
density of the coating thermal noise is given by Levin’s formula [96], an embodiment of the
fluctuation-dissipation theorem,

$$S_x(f) = \frac{4k_B T \Phi d}{f Y w^2 \pi^2},$$

(2.3)

where $k_B$ is Boltzmann’s constant, $T$ is temperature in Kelvin, $f$ is frequency, $w$ is the beam dia-
meter on the optic, and $d$ is the coating thickness, $Y$ is the Young’s modulus, and $\Phi$ is mechanical
loss of the coating material.

To reduce the thermal noise displacement spectral density away from resonance, the mechanical
dissipation (or mechanical loss) must be as low as possible, while the beam size on the optic is
kept as large as possible. While decreasing temperature seems an easy option, the mechanical loss
of silica increases with decreasing temperature. Low mechanical loss optical coatings are an active
area of research, with promising approaches such as annealing coatings, different concentrations
of dopants, and crystalline coating technologies [128]. Coating thermal noise can be calculated
following the method of Harry et al. [80].

Below coating thermal noise is coating thermo-optic noise, which refers to the thermal effects
in the test mass coating caused by heat deposited by the laser power incident on the test mass.
Chapter 2 Gravitational waves and their detection

The suspension thermal noise is originates from the mechanical loss of the silica fibres used to suspend the test mass, and is also calculated from the fluctuation-dissipation theorem [48]. The final stage of the aLIGO suspension is monolithic fused silica which has a low mechanical loss. The shape of the suspension thermal noise, shown in blue in figure 2.3, is caused by the pendulum modes and violin modes of the suspension.

The test mass substrate thermal noise is caused by mechanical loss in the bulk fused silica. As fused silica is a low-loss material this level is much lower than the other sources of thermal noise.

Seismic noise

Seismic noise is caused by the vibrations of the earth. The effect of seismic noise is minimised by decoupling the test mass from its surroundings. The site of any detector will have a local characteristic seismic noise spectrum, caused by seismic activity, human activity, and weather. The microseismic peak is a site-specific peak in the seismic noise spectrum, typically around 0.1 Hz, caused by ocean waves impacting the coastline and characterised by the environment around the site. The seismic noise of the two LIGO sites has been extensively characterised [52]. To isolate the test masses from this motion, they are suspended from a quadrupole pendulum system, which is in turn attached to an active seismic isolation system [52, 150]. Each pendulum suppresses the amplitude of the seismic noise experienced by the test mass above its resonant frequency by \( \frac{1}{f^2} \), where \( f \) is frequency of the measurement. These pendular suspensions cause the steep roll-off of seismic noise seen by the interferometer shown in figure 2.3.

At very low frequencies, and around the microseismic peak, seismic noise is a limiting factor. Thus gravitational waves at frequencies below the LIGO band, are targeted by space-based detectors which are not limited by seismic noise.

Newtonian noise

Newtonian noise is the gravitational attraction between a test mass and other masses around it, it can be anthropogenic, seismic, or atmospheric in origin. While spectra of the Newtonian noise at the interferometer sites have not been directly measured, estimates of the Newtonian noise level can be made using sensitive microphones and seismometer arrays. Recent estimates suggest that Newtonian noise will be a limiting noise factor for aLIGO at design sensitivity near 10 Hz [58].

Residual gas noise

Residual gas in the vacuum system will cause phase noise in the beam path. Residual gas is also a problem between the test masses and their reaction masses, however this particular effect is not included in the model of residual gas noise in figure 2.3 [150]. The model used assumes a residual gas pressure of \( 4 \times 10^{-7} \) Pa, and only considers molecular hydrogen.

Other technical noises

There are other interesting and important noise sources below the noise curves shown in figure 2.3, including laser noise, noise due to the various controls, and the arm length stabilisation system. The aLIGO pre-stabilised laser (PSL) has requirements on the frequency noise, relative intensity noise (RIN), and beam pointing stability [93]. Asymmetries in the interferometer arms couple frequency noise to the gravitational wave signal, while the RIN couples to the gravitational wave channel via power mismatch in the arm cavities. The design and noise couplings in the interferometer arm-length stabilisation system are summarised in [143].
Another technical challenge faced by ground-based interferometers is scattered light. Light can be decoupled from the main interferometer beam by any source of loss. Spurious beams can acquire phase modulation when reflected or scattered from moving components, this effect is prominent in the audio frequency band where many mechanical components have resonances. This scattered light can be re-coupled to the main interferometer beam, introducing noise in the detection band. Scatter is a technical noise source and can be mitigated with careful alignment and removal of stray beams. Scattered light is important in the squeezer subsystem, as the squeezer itself can be a source of scattered light.

2.3.5 LIGO Voyager

The LIGO Voyager proposal is a potential future upgrade to the current aLIGO facilities, with major upgrades to the test masses, suspensions and laser systems. The aim of Voyager is to increase the binary neutron star inspiral range to 1.1 Gpc, and improve the low frequency cut off of the interferometer to 10 Hz [152]. A potential timescale for Voyager could involve installation as early as 2025. Research and development has begun with the aim of informing the design of future detectors. The Voyager design addresses and improves on almost all of the aLIGO noise sources listed in the previous section. Some of the optical and physical parameters of the LIGO Voyager design are listed in table 2.1, and a potential noise budget is shown in figure 2.4.

The Voyager design is based on a test mass material shift to cryogenic silicon. Silicon has several advantages over fused silica as a test mass substrate. At 123 K and 18 K the thermal expansion co-efficient of silicon is zero, and the mechanical dissipation due to thermoelastic damping is negligible [161, 131]. The thermal conductivity of silicon improves at low temperatures to $\sim$700 W/m/K [76], which is significantly higher than fused silica. There are several optical absorption processes in bulk silicon [134], which are being investigated in the context of cryogenic interferometers [54]. The zero thermal expansion coefficient coupled with low optical absorption and improved thermal conductivity allows for the use of higher laser powers with reduced severity of thermal effects such as thermal lensing. Large slugs of high purity silicon are readily available [65], driven by the semiconductor industry, allowing for larger, heavier test masses which reduce the radiation pressure noise coupling. While substrates which meet the LIGO-Voyager test mass requirements are still under development, their production is promising.

Coating thermal noise will be mitigated in LIGO Voyager by increasing the beam spot size on the test masses by 25% compared to Advanced LIGO, and through employment of new coating technologies. The baseline coating for the interferometer is amorphous silicon, which is an active area of research. Current results from experiments testing the loss of amorphous silicon coatings at 120 K suggest a 30% improvement on coating thermal noise compared to the Advanced LIGO coatings, with promise for improvements with techniques such as heat treating [116].

To facilitate the cryogenic operation of LIGO Voyager, and to support the heavier test masses, the final stage of the test mass suspensions are planned to be constructed of silicon ribbons. Heat is extracted from the test mass using radiative cooling. The heat dissipation and tensile stress of silicon ribbons is sufficient to meet the Voyager requirements, with an optimistic breaking stress of 450 MPa [49]. The suspension and seismic isolation systems will require redesign from the Advanced LIGO baseline, to support the much heavier test mass.

Silicon absorbs at the aLIGO wavelength of 1064 nm, hence a new main laser wavelength must be chosen. The operating wavelength for LIGO Voyager is to be determined, with wavelengths in the 1800 - 2100 nm region being considered. The specific wavelength choice is dependent on the availability of reliable high power single frequency lasers, the material properties of silicon, and the quantum efficiency of photodiodes. Further investigations into lasers in the 2 $\mu$m region are presented in section 7.2.1. High power lasers in the 2 $\mu$m region have been demonstrated with
Figure 2.4: Noise sources in LIGO Voyager, with aLIGO and A+ design sensitivities shown for comparison. Input laser power is 145 W at 2000 nm, with 10 dB of frequency dependent squeezing injected through a low loss (0.001% round trip loss) filter cavity. Newtonian noise subtraction has been implemented [58]. Suspension thermal noise calculated assuming silicon ribbons (compared with fibres in aLIGO) and blades held at 123 K. The test masses are 200 kg silicon at 123 K, with low loss amorphous silicon coatings. Noise budget created using Gravitational Wave Interferometer Noise Calculator (GWINC) software (version 3.1) [68].

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2.4 Conclusion

This chapter has summarised the formulation of gravitational waves through general relativity, the generation of gravitational waves by astrophysical sources, and the detection of gravitational waves, with emphasis on laser interferometers. While the first sources, compact binary coalescences, have recently been detected by the Advanced LIGO, the quest to improve the sensitivity of gravitational wave detectors is ongoing, and crucial to the achievement of further science goals. Limiting noise sources for the detectors have been explored, in particular for Advanced LIGO and the potential LIGO Voyager upgrade. Quantum noise is, for the foreseeable future, the most broadly limiting noise source of these interferometers. The following chapters will focus on the origin of quantum noise and methods of reducing it.
Chapter 3

Quantum states of light and quantum noise

3.1 Introduction

This chapter summarises key quantum optics and quantum measurement concepts relevant to squeezed light and gravitational-wave detection. Section 3.2 introduces the non-classical description of light by quantisation of the electromagnetic field, and introduces the formalism of quadrature operators and states of light. Section 3.3 discusses methods with which light can be detected, and the impact of loss and phase noise on squeezed states. Section 3.4 then explores the quantum noise of a Michelson interferometer and methods of manipulating it, with emphasis on the injection of squeezed states of light. Finally section 3.5 discusses the quantum noise of more complex Michelson interferometers used in gravitational wave detection, and finally squeezing injection into such complex Michelson interferometers is explored.

Many of the concepts presented here can be found in graduate-level textbooks and review papers on the topics of quantum optics [172] [69], quantum noise in interferometers [24] [27], and nonlinear optics [22].

3.2 Quantisation of the electromagnetic field

To establish a formalism that describes quantum states of light and the coupling of light to optical cavities we must first quantise the electromagnetic field. Electromagnetic fields may be described in terms of discrete spatial, frequency, and polarisation modes, which is convenient when discussing beams in terms of optical cavities or resonators. The quantisation of the electromagnetic field then follows from the equivalence of the Hamiltonian of an electromagnetic field, and that of a quantum harmonic oscillator. It can be shown [172] that the electric field may be written as

$$ E(r,t) = i \sum_k \sqrt{\frac{\hbar \omega_k}{2 \varepsilon_0}} \left( a_k u_k(r) e^{-i \omega_k t} - a_k^* u_k^*(r) e^{i \omega_k t} \right), \quad (3.1) $$

where $\hbar$ is the reduced Planck constant, $\omega_k$ is the angular frequency of the $k^{th}$ mode, $\varepsilon_0$ is the permittivity of free space, $a_k$ is the dimensionless amplitude of the $k^{th}$ mode, and $u_k$ is orthonormal mode function set. A similar function for the magnetic component of the field may be derived.

The Hamiltonian of the electromagnetic field, representing the total energy, is given by
3.2 Quantisation of the electromagnetic field

\[ H = \frac{1}{2} \int (\varepsilon_0 \mathbf{E}^2 + \mu_0 \mathbf{H}^2) d\mathbf{r} \]  
\[ = \sum_k \hbar \omega_k \left( \hat{a}_k^\dagger \hat{a}_k - \frac{1}{2} \right), \]  

(3.2)

where \( \mu_0 \) is the permeability of free space, and \( \mathbf{H} \) is the magnetic field strength. This Hamiltonian is analogous to that of a quantum harmonic oscillator, where the eigenstates and eigenvalues are found using ladder operators. The amplitudes \( a_k \) and \( a_k^\dagger \) are chosen to be boson operators, satisfying the following boson commutation relations

\[ [\hat{a}_k, \hat{a}_k^\dagger] = \delta_{kk'}, \quad \left[ \hat{a}_k, \hat{a}_k^\dagger \right] = 0. \]  

(3.3)

\( a_k \) and \( a_k^\dagger \) are non-commuting operators, which represent the annihilation and creation operators respectively. The eigenstates of the quantised Hamiltonian contain exactly \( N \) photons, and are known as Fock (or number) states,

\[ \hat{H}_k = \hbar \omega_k (N_k + 1/2). \]  

(3.4)

The Fock states form a complete orthonormal basis. Fock states, \( |n_k\rangle \), are eigenstates of the number operator \( N_k = \hat{a}_k^\dagger \hat{a}_k \). Note that the first term equation 3.4 gives the energy of the photons in the field. The second term implies that in the absence of photons (a vacuum state) there is still energy – the zero point energy.

### 3.2.1 Quadratures and quadrature operators

The annihilation and creation operators for the electromagnetic field, \( \hat{a} \) and \( \hat{a}^\dagger \), are analogous to the raising and lowering ladder operators for the harmonic oscillator. They satisfy the boson commutation relations defined in equation 3.3. These operators are not Hermitian (\( \hat{a} \neq \hat{a}^\dagger \)), and so they do not represent real observables. The quantised electric field of a single spatial optical mode may be written in terms of \( \hat{a} \) and \( \hat{a}^\dagger \),

\[ \hat{E}(t) = \sqrt{\frac{\hbar \omega}{2\varepsilon_0}} \left( \hat{a}(t)e^{-i\omega t} + \hat{a}^\dagger(t)e^{i\omega t} \right). \]  

(3.5)

Rewriting the electric field as the sum of two quadrature terms gives

\[ \hat{E}(t) = \sqrt{\frac{\hbar \omega}{2\varepsilon_0}} \left( \hat{X}_1(t)\cos(\omega t) + \hat{X}_2(t)\sin(\omega t) \right), \]  

(3.6)

where \( \hat{X}_1 \) and \( \hat{X}_2 \) are non-commuting Hermitian operators and hence represent observables. These quadrature operators are given by [11]

\[ \hat{X}_1 = \hat{a} + \hat{a}^\dagger, \]  

(3.7)

\[ \hat{X}_2 = i(\hat{a}^\dagger - \hat{a}). \]  

(3.8)

The amplitude quadrature information of the electric field of a single mode is contained in \( \hat{X}_1 \), while \( \hat{X}_2 \) contains the phase quadrature information. For example take a coherent field driven wholly in the cosine quadrature, for small angle fluctuations \( \hat{X}_2 \) will represent the phase of the field.

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An arbitrary quadrature may be defined using a linear combination of the basis formed by $\hat{X}_1$ and $\hat{X}_2$,

$$\hat{X}^\theta = \hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta}$$

$$= \hat{X}_1 \cos \theta + \hat{X}_2 \sin \theta.$$

### 3.2.2 Commutation relations and the uncertainty principle

One of the most profound results in physics is the Heisenberg Uncertainty Principle (HUP) which, in the most familiar application, relates the uncertainties in the position and momentum of a particle. The inequality is given by

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}.$$  \hspace{1cm} (3.10)

The HUP defines the fundamental limit to the precision with which one can simultaneously measure these two observables, and states that a quantum object cannot simultaneously have its position and momentum measured to arbitrarily high precision. If an experiment measures the position of a particle to arbitrary precision, the act of the measurement will perturb the momentum of the particle. As the measurement becomes more precise the back action effect on the momentum becomes more acute.

Any two non-commuting observables will have a corresponding uncertainty relation. Consider two operators $\hat{A}$ and $\hat{B}$, with commutation relation

$$[\hat{A}, \hat{B}] = C,$$  \hspace{1cm} (3.11)

the uncertainties in these two operators are then related by an uncertainty principle,

$$\Delta \hat{A}\Delta \hat{B} \geq \frac{|\langle C \rangle|}{2}.$$  \hspace{1cm} (3.12)

The relationship between uncertainty, variance and expectation value of an operator $\hat{X}$ is given by

$$V(\hat{X}) = \Delta \hat{X}^2 = \langle \hat{X}^2 \rangle - \langle \hat{X} \rangle^2,$$  \hspace{1cm} (3.13)

where $\Delta \hat{X}$ is the uncertainty in $\hat{X}$, and $\langle \hat{X} \rangle$ is the expectation value of $\hat{X}$. This is an important definition as in theory operators and their uncertainties are often discussed, while experimentally expectation values and variances are actually measured.

The amplitude and phase quadratures of light are non-commuting observables, with operators defined in equations 3.7 and 3.8. These two operators have the following commutation relation

$$[\hat{X}_1, \hat{X}_2] = 2i,$$  \hspace{1cm} (3.14)

and hence their uncertainties are related by the following uncertainty relation:

$$\Delta \hat{X}_1 \Delta \hat{X}_2 \geq 1.$$  \hspace{1cm} (3.15)

This uncertainty provides a fundamental quantum limit to the precision with which the amplitude and phase of an electromagnetic field can be simultaneously measured. If one wishes to measure both quadratures of a system there is an optimal accuracy with which each quadrature should be measured, this is known as the standard quantum limit (SQL). Measurement schemes
3.2 Quantisation of the electromagnetic field

designed to exceed the SQL are called quantum nondemolition experiments. More information about the SQL and nondemolition experiments can be found in Braginsky and Khalili [24].

Interferometric meters, such as gravitational-wave detectors, imprint small distance changes onto the phase of laser light. Hence the quantum limit of these detectors is the quantum noise limit set by the uncertainty principle associated with the phase and amplitude quadratures of the light. Squeezed light involves manipulating this uncertainty principle to reduce the noise in the quadrature of interest, and will be expanded on in the following sections.

3.2.3 States of light

The zero point energy in the quantised electromagnetic field, encapsulated in the Hamiltonian given in equation 3.4, was mentioned at the end of section 3.2. The Fock state with zero photons is found to have nonzero energy, hence the vacuum state has some energy associated, equivalent to half the energy of a single photon. In this section we will further explore some of the possible states of light relevant to this thesis, including the vacuum state.

For the purpose of this background section we only consider minimum uncertainty states of light where the uncertainty principle is treated as an equality

\[ \Delta \hat{X}_1 \Delta \hat{X}_2 = \frac{1}{\hbar}. \] (3.16)

A more complete mathematical treatment of minimum uncertainty states may be found in Walls and Milburn [172]. Examples of states with uncertainty greater than the limit of the uncertainty principle include thermal states or states with additional classical noise above the quantum noise limit.

Representing states of light

We will use two pictures describing these states of light: the ball-and-stick picture and the sideband picture. The ball-and-stick picture is conceptually intuitive, light is represented as a phasor, and uncertainty in phase and amplitude as a fuzzy ball at the end of the phasor. The sideband picture represents the real and imaginary parts of the carrier field amplitude from a reference frame co-rotating at the carrier frequency.

Modulation of the carrier field – that is phase, frequency or amplitude modulation – at Fourier frequencies of ±Δ may be represented simply in the sideband picture. The amplitude of the sidebands indicates the modulation depth, and the phase of the sidebands indicates the type of modulation. To introduce the sideband picture, amplitude and phase modulation are illustrated in figure 3.1a and 3.1b respectively. In the case of amplitude modulation the rotation of the sideband phasors causes the amplitude of the carrier to be modulated at the sideband frequency. The phase modulation sidebands result in the phase of the carrier being modulated at the sideband frequency. This is shown as a function of time by Chua [39].

Quantum noise may also be represented in the sideband picture as a spectrum of sidebands in phase and amplitude. For the minimum uncertainty state with equal noise in phase and amplitude, these sidebands are randomly oriented, and for squeezed states these sidebands are correlated, increasing the noise in one quadrature and reducing it in the other. The quantum noise sidebands in the sideband picture are shown in figure 3.2. These states of lights will be explored more completely in the following sections.
Chapter 3 Quantum states of light and quantum noise

Amplitude modulation

Phase modulation

Figure 3.1: Amplitude (left) and phase (right) modulation at frequency $\Delta$ in the sideband picture. As the phasors evolve in time the sidebands add in and out of phase. In the case of amplitude modulation this causes the amplitude of the carrier to be modulated at frequency $\Delta$, while phase modulation modulates the phase of the carrier.

The vacuum state

The ground state of the harmonic oscillator corresponds to the vacuum state of the field, defined as

$$\hat{a}_k |0\rangle = 0,$$  \hspace{1cm} (3.17)

where $\hat{a}_k$ is the dimensionless annihilation operator, and $|0\rangle$ is the vacuum state. While there are no photons in the vacuum state, there is energy. From the Hamiltonian of the quantised electromagnetic field (equation 3.2) the energy of the vacuum state is given by

$$\langle 0 | H | 0 \rangle = 1/2 \sum_k \hbar \omega_k,$$  \hspace{1cm} (3.18)

where $\omega_k$ is the angular frequency of the $k^{th}$ mode.

The variance (defined in equation 3.13) of the vacuum state in each quadrature may be calculated using the quadrature operators defined in equations 3.7 and 3.8. This calculation is shown completely in Wade [169], and the variances are found to be unity,

$$V(\hat{X}_1 |0\rangle) = 1 \quad V(\hat{X}_2 |0\rangle) = 1.$$  \hspace{1cm} (3.19)

Hence the vacuum state is a minimum uncertainty state with equal variance in each quadrature, and no coherent amplitude. The vacuum state is represented in the ball-and-stick picture in figure 3.3c.

The coherent state

A field from a well-stabilised laser can be represented by the coherent state, $|\alpha\rangle$. Coherent states are eigenstates of the annihilation operator,

$$\hat{a}|\alpha\rangle = \alpha |\alpha\rangle.$$  \hspace{1cm} (3.20)

The coherent state is generated mathematically from the vacuum state using the displacement operator,
3.2 Quantisation of the electromagnetic field

(a) A coherent state

(b) A coherent squeezed state

Figure 3.2: Coherent and squeezed states in the sideband picture. The carrier is shown in red, along the real axis, the quantum noise sidebands are uncorrelated in the case of the coherent state, and correlated when the state is squeezed.

\[ \hat{D}(\alpha) = e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}} \]

\[ = e^{-|\alpha|^2/2} e^{\alpha \hat{a}^\dagger} e^{-\alpha^* \hat{a}}, \]

where \( \alpha \) is the complex amplitude of the field. The coherent state is generated by applying the displacement operator to the vacuum state,

\[ |\alpha\rangle = \hat{D}(\alpha)|0\rangle. \]

Expanding the coherent state in the number state basis reveals that the state has a Poisson distribution of photons, with the mean number of photons given by \(|\alpha|^2\) [172].

The coherent state is a minimum uncertainty state of light with coherent amplitude and equal variance in the phase and amplitude quadratures,

\[ V(\hat{X}_1|\alpha\rangle) = V(\hat{X}_2|\alpha\rangle) = 1. \]

The variance of a coherent state hence is unity. This state may be represented by an uncertainty ball in the ball-and-stick phasor type picture as shown in figure 3.3a, or as uncorrelated sidebands about a carrier field represented in figure 3.2a.

The squeezed state

The squeezed state is a minimum uncertainty state with unequal variance in the two quadratures. One quadrature has less noise than a coherent state and, to satisfy the uncertainty principle, the noise in the other quadrature must be greater than a coherent state.

The squeezed state is generated mathematically with the unitary squeeze operator, defined as

\[ \hat{S}(\varepsilon) = e^{1/2r^2 \varepsilon^2} e^{-1/2r^2 \varepsilon^2}, \]

where \( \varepsilon = re^{2i\phi} \) quantifies the level of squeezing, \( r = |\varepsilon| \) is the “squeeze factor”, and \( \phi \) is the phase angle relative to the quadrature axis. The general squeezed state is generated by applying the squeeze operator to the vacuum state, and then applying the displacement operator [31]

\[ |\alpha, \varepsilon\rangle = \hat{D}(\alpha)\hat{S}(\varepsilon)|0\rangle. \]

The displacement operator does not modify the quadrature variances (which is why the coherent state has the same variance as the vacuum state), the squeeze operator does modify the
The coherent state

The squeezed coherent state

The vacuum state

The squeezed vacuum state

Figure 3.3: Ball-and-stick picture of four minimum uncertainty states, where $\alpha$ is the amplitude of the state, $\phi$ is the phase measured with respect to the basis defined by the orthogonal $\hat{X}^\theta$ and $\hat{X}^{\theta+\pi/2}$ quadratures. $\Delta \hat{X}^+$ is the uncertainty in the amplitude quadrature, and $\Delta \hat{X}^-$ is the uncertainty in the phase quadrature.

Applying the squeeze operator transformation to the quadrature operators yields

$$\hat{S}^\dagger(\epsilon)\hat{X}_{1,2}\hat{S}(\epsilon) = \hat{X}_{1,2} (\cosh(r) \mp e^{-2i\phi} \sinh(r)),$$

where the minus sign corresponds to the $\hat{X}_1$ quadrature. From these quadratures the quadrature variances can be calculated

$$V(\hat{X}_1|\alpha, \epsilon) = e^{-2r}, \quad V(\hat{X}_2|\alpha, \epsilon) = e^{2r}. \quad (3.28)$$

Hence the variance in one quadrature is attenuated by the squeeze factor, while the variance in the opposite quadrature is amplified by the same factor. The squeezed state is a minimum uncertainty state as the product of the two quadrature variances is unity.

Also note that the mean number of photons in the squeezed state, given by

$$\langle N \rangle = |\alpha|^2 + \sinh^2(r) \quad (3.29)$$
which is non-zero, even in the case of squeezing without a coherent amplitude, where $|\alpha| = 0$. Note that a squeezed state with a coherent amplitude ($|\alpha| \neq 0$) is referred to as a bright squeezed state, while a squeezed state with no coherent amplitude is referred to as a vacuum squeezed state.

The ball-and-stick representations of bright and vacuum squeezed states are shown in figure 3.3b and 3.3d, respectively. In the sideband picture there are correlations between the upper and lower sidebands, shown in figure 3.2b.

### 3.3 Measuring states of light

To calculate the optical power of a state of light from the field representation, the number operator is used. The number operator is the expectation value of the Hermitian conjugate of the creation operator multiplied by the annihilation operator, $N = \langle \hat{a}^\dagger \hat{a} \rangle$. The power in a field can then be calculated by multiplying the number operator by the energy of a single photon at frequency $\omega$,

$$P = \hbar \omega \langle \hat{a}^\dagger \hat{a} \rangle,$$

where $\hbar$ has units of $\sqrt{\text{photons/second}}$. This section discusses methods of photodetection to determine the amplitude and phase of an electromagnetic field.

#### 3.3.1 Photodetection

Photodetection makes use of the photoelectric effect to convert photons to an electrical signal. A photon incident on a photoelectric surface, such as a semiconductor, transfers energy to an electron, generating a current which can then be measured. Typically a photodiode is used as the sensor, and the current is read out using a transimpedance amplifier to convert the current to voltage. The photocurrent generated by a detector upon incidence of a beam of optical power $P$ is given by

$$i = \frac{e \eta_{PD} P}{\hbar \omega} = \frac{e \eta_{PD} \langle \hat{a}^\dagger \hat{a} \rangle}{\omega},$$

where $e$ is the charge of an electron, $\eta_{PD}$ is the quantum efficiency of the detectors, $\hbar \omega$ is the energy of a single photon at angular frequency $\omega$.

Two modes of operation explored in this thesis are photoconductive or photovoltaic detection. In a photoconductive detector the detector is operated with a reverse bias voltage across the diode and current from the diode is measured directly. Photovoltaic detectors use a transimpedance amplifier to convert the current to voltage, giving better dark current performance.

#### 3.3.2 Balanced homodyne detection

Balanced homodyne detection is used to measure the variances of vacuum squeezed states. Direct photodetection cannot be used in the squeezed vacuum case, even though there are photons present, they are too few to generate a measurable photocurrent. The balanced homodyne technique uses a bright local oscillator which is combined optically with a signal beam on a beamsplitter, both output ports are measured on two photodiodes as illustrated in figure 3.4. Subtracting the two currents allows a measurement of the quantum noise of one input beam to be amplified by the amplitude of the other input beam. The measurement quadrature is determined by the phase difference between the two input beams, $\phi$. This is shown mathematically below, following the treatment in [144].
Chapter 3 Quantum states of light and quantum noise

Figure 3.4: General homodyne detection schematic. \( \hat{A} \) and \( \hat{B} \) are the input fields, \( \hat{C} \) and \( \hat{D} \) are the output fields, \( \eta_{BS} \) is the reflectivity of the beamsplitter, \( i_1 \) and \( i_2 \) are the measured photocurrents from each detector.

The fields at the output ports of the beamsplitter are given by

\[
\hat{C} = \sqrt{\eta_{BS}} \hat{A} + \sqrt{1 - \eta_{BS}} \hat{B} e^{i\phi}, \quad \hat{D} = \sqrt{1 - \eta_{BS}} \hat{A} - \sqrt{\eta_{BS}} \hat{B} e^{i\phi}. \tag{3.32}
\]

where \( \eta_{BS} \) is the reflectivity of the beamsplitter, and \( \hat{A} \) and \( \hat{B} \) are the input fields. The number operators of the two output fields are given by

\[
\hat{C}^\dagger \hat{C} = \eta_{BS} \hat{A}^\dagger \hat{A} + \sqrt{\eta_{BS}(1 - \eta_{BS})} (\hat{A}^\dagger \hat{B} e^{-i\phi} + \hat{B}^\dagger \hat{A} e^{i\phi}) + (1 - \eta_{BS}) \hat{B}^\dagger \hat{B}, \tag{3.33}
\]

\[
\hat{D}^\dagger \hat{D} = (1 - \eta_{BS}) \hat{A}^\dagger \hat{A} - \sqrt{\eta_{BS}(1 - \eta_{BS})} (\hat{A}^\dagger \hat{B} e^{i\phi} + \hat{B}^\dagger \hat{A} e^{-i\phi}) + \eta_{BS} \hat{B}^\dagger \hat{B}. \tag{3.34}
\]

The photocurrents generated by each diode are proportional to the number operator of the field incident on the diode, that is \( i_1 \propto \hat{C}^\dagger \hat{C} \) and \( i_2 \propto \hat{D}^\dagger \hat{D} \). The photocurrents are subtracted electronically. Experimentally either the subtracted current is read out directly, or the two currents are measured separately and then subtracted. The subtracted current is given by

\[
i_1 - i_2 = (2\eta_{BS} - 1) \hat{A}^\dagger \hat{A} + (1 - 2\eta_{BS}) \hat{B}^\dagger \hat{B} + 2\sqrt{\eta_{BS}(1 - \eta_{BS})} (\hat{A}^\dagger \hat{B} e^{i\phi} + \hat{B}^\dagger \hat{A} e^{-i\phi}). \tag{3.35}
\]

The fields may be linearised by separating them into their steady state and time-varying components. Assuming that the time varying component of the field is much smaller than the steady state field, second order fluctuating terms can be neglected. The linearised input fields are given by \( \hat{A} = \bar{a} + \delta a \) and \( \hat{B} = \bar{b} + \delta b \), thus

\[
\hat{A}^\dagger \hat{A} = (\bar{a}^\dagger + \delta a^\dagger) (\bar{a} + \delta a) \tag{3.36}
\]

\[
= |\bar{a}|^2 + \bar{a}^\dagger \delta a + \bar{a} \delta a^\dagger
\]

\[
= |\bar{a}|^2 + a \delta \hat{X}_A^+, \]

recalling \( \delta \hat{X}_A^+ = (\delta \hat{A} + \delta \hat{A}^\dagger) \). The \( \hat{B} \) field can be treated similarly, hence the subtracted photocurrent may be rewritten in terms of quadrature operators,

\[
i_1 - i_2 = (2\eta_{BS} - 1) (|\bar{a}|^2 + a \delta \hat{X}_A^+) + (1 - 2\eta_{BS}) (|\bar{b}|^2 + b \delta \hat{X}_B^+)
\]

\[
+ 2\sqrt{\eta_{BS}(1 - \eta_{BS})} (2ab \cos \phi + \bar{a} (\delta \hat{X}_B^+ \cos \phi - \delta \hat{X}_B^- \sin \phi)
\]

\[
+ \bar{b} (\delta \hat{X}_A^+ \cos \phi - \delta \hat{X}_A^- \sin \phi)). \tag{3.37}
\]
Assuming a perfect beamsplitter, with $\eta_{BS} = 0.5$, eliminates the first two terms of equation 3.37. The remaining terms represent the DC field proportional to the phase difference between the two beams. The cross terms imply that the noise in $\hat{A}$ is amplified by the magnitude $\tilde{b}$ and the noise in $\hat{B}$ is amplified by the magnitude $\tilde{a}$. In the case of homodyne detection of vacuum squeezed states, such as in this thesis, we can also assume that $\tilde{b} \gg \tilde{a}$, and hence the subtracted current is given by

$$i_1 - i_2 \simeq 2\tilde{a}\tilde{b}\cos\phi + \tilde{b}(\delta\hat{X}_A^+ \cos\phi - \delta\hat{X}_A^- \sin\phi).$$

(3.38)

Note that the phase difference between the two input fields determines which quadrature of $\hat{A}$ is being observed.

### 3.3.3 Power spectral density

To extract frequency domain information of a signal the power spectral density of the noise is calculated. Experimentally this is done either in post-processing of a time-domain signal or using a spectrum analyser. The single-sided noise spectral density is defined as the Fourier transform of the time-domain auto-correlation function, and gives a measure of the noise of an operator as a function of Fourier frequency. The continuous Fourier transform of a time-domain operator $\hat{a}(t)$ is given by

$$\hat{a}(\omega) = \int_{-\infty}^{\infty} \hat{a}(t)e^{i\omega t}dt,$$

(3.39)

where $\omega$ is the Fourier frequency, relative to the carrier frequency. The power spectral density, $S(\omega)$, is then

$$S(\omega) = \frac{1}{2\pi} \int_{\tau} G(\tau)e^{i\omega\tau}d\tau,$$

(3.40)

where $G(\tau) = \langle \delta a(t)\delta a(t+\tau) \rangle$ is the autocorrelation between the time-varying component of $\hat{a}(t)$ over the measurement time, $\tau$. Note that $\delta a(t)$ is only the time-varying component of $\hat{a}(t)$, and has an average value of zero.

Measurements of the power spectral density are generally taken over some finite resolution bandwidth $B$ about the carrier frequency $\omega$. $B$ is chosen so that the power spectral density is roughly constant over this frequency band. It can be shown [26, 169] that the power spectral density over this bandwidth is given by

$$S_B(\omega) = S(\omega) \times B.$$

(3.41)

Thus such measurements must be normalised to the measurement bandwidth, the normalised power spectrum is given by

$$V(\omega) = \frac{S_B(\omega)}{B} = \langle |\delta\hat{X}_i(\omega)|^2 \rangle,$$

(3.42)

where $\delta\hat{X}_i(\omega)$ is the fluctuating component of the frequency-domain quadrature operator $\hat{X}$ of the $i^{th}$ quadrature. The power spectral density $V(\omega)$ is equivalent to the variance of a time-domain signal, thus for vacuum states $V(\omega) = 1$. 

30
3.3.4 Loss

Loss occurs when any light is coupled out of the main mode of a beam, by absorption, scattering, or imperfect reflection and transmission of optical components. Sources of loss can be modelled as a beamsplitter with non-unity reflectivity, and one input port consisting of vacuum, as in figure 3.4, letting $\hat{B}$ field be a vacuum state with fluctuating component $\delta \nu$. Assuming the beamsplitter is highly reflective, only the $\hat{C}$ field leaving the beamsplitter need be considered.

$$\hat{C} = \sqrt{\eta_{\text{loss}}} (\hat{a} + \delta a) + \sqrt{1 - \eta_{\text{loss}}} \delta \nu e^{i\phi}. \quad (3.43)$$

When a squeezed state encounters a source of loss, vacuum is coupled to the field, and the state is no longer minimum uncertainty. In terms of variances of a field, the effect of loss is given by

$$V_{\text{tot}} = \eta_{\text{loss}} V_{\text{in}} + (1 - \eta_{\text{loss}}) V_{\text{vac}}. \quad (3.44)$$

Where $V_{\text{tot}}$ is the output variance after accounting for the loss, $V_{\text{in}}$ is the initial variance without loss, $\eta_{\text{loss}}$ is the detection efficiency, $V_{\text{vac}}$ is variance of the vacuum, which is unity.

3.3.5 Phase noise

Phase noise, also known as squeezed-quadrature fluctuations or squeezing angle jitter, refers to fluctuations in the angle of the squeezing ellipse. When quadrature fluctuations occur faster than the measurement time, the anti-squeezed quadrature couples to the squeezed quadrature, and the measured level of squeezing is reduced, as shown in figure 3.5.

In terms of quadrature variances (defined in equation 3.13 with respect to the uncertainty in an arbitrary quadrature) the effect of a level of RMS phase noise $\tilde{\theta}_{sqz}$ on an arbitrary quadrature $V(\theta_{sqz})$ is given by

$$V_{\text{tot}}(\theta_{sqz}, \tilde{\theta}_{sqz}) = V(\theta_{sqz}) \cos^2 \tilde{\theta}_{sqz} + V(\theta_{sqz} + \pi/2) \sin^2 \tilde{\theta}_{sqz}, \quad (3.45)$$

where $V(\theta_{sqz})$ and $V(\theta_{sqz} + \pi/2)$ are the variances in the cosine and sine quadratures in the absence of phase noise.
The effect of phase noise is dependent on the level of squeezing, and is more important at higher levels of squeezing.

3.4 Quantum noise in a simple Michelson interferometer

Interferometric measurements, such as those in ground-based gravitational-wave detectors, are limited by quantum noise. In this section, the quantum noise in a Michelson interferometer is derived, and squeezed state injection to reduce the quantum noise is discussed. In section 3.5 this result will be extended to a compound Michelson interferometer with Fabry-Perot arms, power- and signal-recycling cavities, which better approximates the architecture of an advanced gravitational-wave detector. Methods for improving on the quantum noise limit in gravitational-wave detectors – specifically the injection of squeezed light – will then be discussed in section 3.5.2.

3.4.1 Back action noise and the standard quantum limit

The Heisenberg uncertainty principle states that if the relative position of a test-mass is measured to sufficient precision, the momentum will be perturbed by the measurement itself. This back-action defines the standard quantum limit (SQL) for the measurement of the position of a mass. The concept of a SQL was briefly introduced in section 3.2.1. It is the quantum noise when the measurement quadrature at each frequency is optimised. This was first derived for interferometric measurement by Braginsky [23] [24]. The single-sided noise spectral density\(^1\) of the SQL for a laser interferometer is simply

\[
S_{h}^{\text{SQL}} = \frac{8h}{M\Omega^{2}L^{2}},
\]

where \(M\) is the mass of the test mass, \(\Omega\) is the measurement frequency, and hence the frequency of the gravitational wave, and \(L\) is the arm length of the interferometer. The closer a measurement of a macroscopic system gets to this quantum limit, the more “quantum mechanical” the nature of the system becomes.

In 1981 Caves proposed a formalism for the quantum noise couplings into a Michelson interferometer [32]. Intuitively one might think that the quantum noise is inherent to the laser, however this cannot be the case. Laser noise is common to both arms of the Michelson, entering the bright port, and drives the symmetric mode of the arms. On a dark fringe, the interferometer is sensitive to asymmetric motion of the arms, which is driven by the dark port. Hence this is the port through which the quantum noise enters the interferometer.

In the case of gravitational-wave detectors, only a small amount of light exits the detection port. It is through the dark port that vacuum fluctuations are coupled into the experiment - similar to the irreversible coupling of vacuum to a squeezed state through a lossy beamsplitter, discussed in section 3.3.4.

3.4.2 The two-photon formalism

The following derivations of quantum noise follow the methods in Buonanno and Chen [27], using the Caves-Schumaker two-photon formalism [33, 135], as presented in Kimble [90]. The two-photon formalism utilises creation and annihilation operators to describe optical fields in terms of

\(^{1}\)The noise spectral density of an operator is the Fourier transform of the time-domain auto-correlation function of the operator, as discussed in section 3.3.3.
discrete polarisation, spatial or frequency modes. This was first introduced in section 3.2. Here
the operators represent Fourier sidebands around a carrier field.

In the Heisenberg picture the field is written as

\[ \hat{E}(t) = \sqrt{\frac{2\pi}{\hbar}} \left\langle \hat{a}_\omega e^{-i\omega t} + \hat{a}_\omega^\dagger e^{i\omega t} \right\rangle \frac{d\omega}{2\pi}, \] (3.47)

where \( A \) is the cross sectional area of the beam, and \( \hat{a}_\omega \) and \( \hat{a}_\omega^\dagger \) are the annihilation and creation
operators, which follow the usual boson commutation relations,

\[ [\hat{a}_\omega, \hat{a}_\omega^\dagger] = 0, \quad [\hat{a}_\omega^\dagger, \hat{a}_\omega^\dagger] = 2\pi \delta(\omega - \omega'). \] (3.48)

These creation and annihilation operators are fixed in time, while a gravitational wave signal
occurs in an interferometer as phase modulation sidebands at frequencies \( \pm \Omega \). We may write the
electromagnetic field in terms of upper and lower sidebands about a carrier frequency \( \omega_0 \), defining

\[ \hat{a}_1 = \hat{a}_{\omega+\Omega}, \quad \hat{a}_2 = \hat{a}_{\omega-\Omega}. \] (3.49)

From these operators we define the quadrature operators for the production of an upper and
lower sideband,

\[ \hat{a}_1 = \frac{\hat{a}_+ + \hat{a}_+^\dagger}{\sqrt{2}}, \quad \hat{a}_2 = \frac{\hat{a}_+ - \hat{a}_+^\dagger}{\sqrt{2t}}. \] (3.50)

Note that, as in the previous electromagnetic field quantisation, \( a_1 \) is the field amplitude for
photons in the cosine quadrature, chosen to be the amplitude quadrature, and \( a_2 \) is the field amplitude for photons in the sine quadrature, chosen to be the phase quadrature. The electromagnetic
field can be constructed from these quadrature operators,

\[ \hat{E}_j(a_j, t) = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}} \left\langle \hat{a}_j e^{-i\omega_j t} + \hat{a}_j^\dagger e^{i\omega_j t} \right\rangle \frac{d\Omega}{2\pi}, \] (3.51)

where \( j = 1, 2 \). The commutation relation for these fields reveal that there is an uncertainty trade-
off in the measurement of orthogonal quadratures.

### 3.4.3 Quantum noise in a Michelson interferometer

A diagram of a simple Michelson interferometer and the fields propagating therein is shown in
figure 3.6. The symmetric port of the interferometer refers to the laser input port, while the anti-
symmetric port refers to the output or detection port.

The field entering the symmetric port is linearised to a steady-state classical term and a fluctuat-
ing term,

\[ E_S(t) = E_0 \cos(\omega_0 t) + \delta E_S(t), \] (3.52)

where \( \omega_0 \) is the main laser carrier frequency, \( E_0 = \sqrt{\frac{8\pi P}{Ac}} \) is the electric field strength, \( P \) is the
input laser optical power, \( A \) is the mode area of the beam, and \( c \) is the speed of light.

Assuming the interferometer is held on a dark fringe, the field entering the interferometer at the anti-symmetric port consists only of vacuum fluctuations. The fields within the arms can be derived, assuming a perfect 50% beamsplitter, and the field exiting the interferometer can be
determined from this. The full derivation is shown in appendix C, here only the final result is shown.
3.4 Quantum noise in a simple Michelson interferometer

![Simple Michelson interferometer architecture](image)

Figure 3.6: Simple Michelson interferometer architecture, showing fields and directions, with Fabry-Perot cavities, power- and signal-recycling cavities shown transparently. The fields entering the symmetric and anti-symmetric ports are given by $E_S$ and $E_{AS}$ respectively, and the fields in the y and x arms are given by $E_1$ and $E_2$ respectively. The subscript $R$ indicates a reflected field.

The two quadratures of the field leaving the interferometer are given by

$$b_1(\Omega) = a_1(\Omega)e^{2i\beta}, \quad (3.53)$$
$$b_2(\Omega) = \left(a_2(\Omega) - \kappa(\Omega)a_1(\Omega) - \sqrt{2\kappa(\Omega)}\frac{h(t)}{h_{SQL}}\right)e^{2i\beta}, \quad (3.54)$$

where $a_1$ and $a_2$ are the quadratures of the field entering the interferometer, $\beta = \arctan(2\Omega L/c)$ is the arm-cavity single pass phase shift acquired at the sideband frequency $\Omega$, $L$ is the interferometer arm length, $h(t)$ is the strain sensitivity, $h_{SQL}$ is the strain equivalent standard quantum limit, and $\kappa(\Omega)$ is the radiation pressure coupling coefficient, dependent on the incident power and the mass of the test mass,

$$\kappa(\Omega) = \frac{4P\omega_0}{c^2M\Omega^2}. \quad (3.55)$$

Where $P$ is the input optical power, $\omega_0$ is the carrier frequency, $M$ is the mass of the test mass. The strain equivalent standard quantum limit is given by [90]

$$h_{SQL} = \sqrt{\frac{8h}{M\Omega^2 L^2}}. \quad (3.56)$$

The gravitational wave signal is present in the phase quadrature of the field leaving the anti-symmetric port ($b_2(\Omega)$). This quadrature has contributions from both the amplitude and phase quadrature of the field entering the anti-symmetric port which correspond to the shot noise and radiation pressure noise of the interferometer.

Measured in an arbitrary quadrature, $\zeta$, the output field can be expressed in the usual way,

$$b_{\zeta} = b_1 \cos \zeta + b_2 \sin \zeta. \quad (3.57)$$
The single sided power spectral density (PSD) of the interferometer output, $\sqrt{S_h}$, is the Fourier transform of the auto-correlation of the output fields in the time domain. Given the anti-symmetric port inputs $a_1$ and $a_2$ are vacuum, and the fields in equation C.10 and C.11, the PSD is calculated

$$S_h = \frac{\hbar^2 \text{SQL}^2}{2} \left( \frac{1}{\kappa} + \kappa \right).$$  

(3.58)

This is plotted with the strain-equivalent standard quantum limit, for an simple interferometer with unrealistically large input power, in figure 3.7.

If homodyne detection is used on the output, the output readout quadrature can be varied as a function of measurement frequency. The single sided power spectral density of an arbitrary readout quadrature, $\zeta$, is

$$S_{\zeta h} = \frac{\hbar^2 \text{SQL}^2}{2\kappa} \left( 1 + \left( \frac{\text{cot}\zeta - \kappa}{2} \right)^2 \right).$$  

(3.59)

Thus the standard quantum limit can also be beaten with an optimally chosen readout angle. This is called variational readout [168] and will not be further discussed here.

### 3.4.4 Squeezing in the interferometer arms

The quadrature components of the output field in equation 3.54, are modified from the input quadrature components by a factor which depends on the optomechanical response of the test mass. It can be shown that the optomechanical response causes ponderomotive squeezing of the field exiting the interferometer. The squeeze operator (from equation 3.24) can be written in terms of the upper and lower sidebands[34],

$$\hat{S}(\varepsilon) = e^{\varepsilon_1 \hat{a}_1 \hat{a}^\dagger} e^{-\varepsilon_1 \hat{a}^\dagger_1 \hat{a}},$$  

(3.60)

where $\varepsilon = re^{i\phi}$, $r$ is the squeeze factor and $\phi$ is phase. The squeeze operator transforms the two-photon quadrature fields as

$$\hat{S}^\dagger \hat{a}_1 \hat{S} = \hat{a}_1 (\cosh r + \sinh r \cos 2\phi) + \hat{a}_2 \sinh r \sin 2\phi,$$  

(3.61)

$$\hat{S}^\dagger \hat{a}_2 \hat{S} = \hat{a}_2 (\cosh r + \sinh r \cos 2\phi) + \hat{a}_1 \sinh r \sin 2\phi.$$  

(3.62)

Thus the quadrature fields leaving the interferometers can be written as a rotation of the field entering the interferometer, followed by a squeeze transformation,

$$\delta b_j = \hat{S}^\dagger (r, \phi) \hat{R}^\dagger (\theta) \hat{a}^\dagger_j e^{i\beta} \hat{R}(-\theta) \hat{S}(r, \phi),$$  

(3.63)

where $\theta = \arctan(\kappa/2)$, $\phi = 1/2\arccot(\kappa/2)$, and $r = \arcsinh(\kappa/2)$. Note that the angle of the ponderomotive squeezing does not correspond to the quadrature in which the gravitational wave signal resides, and choosing to read out the squeezed quadrature would sacrifice sensitivity to gravitational-wave strain.

### 3.4.5 Injection of squeezed states for quantum noise improvement

The quantum noise limit of a simple Michelson interferometer has contributions from the amplitude and phase quadratures of the field entering the anti-symmetric port, corresponding to the two noise sources: radiation pressure noise and shot noise. One method to modify the quantum noise of the interferometer is to inject squeezed states of light. In 1981 Caves proposed a method
3.4 Quantum noise in a simple Michelson interferometer

Figure 3.7: Model of the strain equivalent standard quantum limit with and without squeezing, and with frequency-dependent squeezing. The strain-equivalent SQL is shown in blue, the quantum noise PSD strain without squeezing in red, and with phase and amplitude squeezing in dashed purple and green respectively. The strain with frequency-dependent squeezing is shown in yellow. In this model the input power is 10 MW, test-mass mass is 40 kg, operating wavelength is 1064 nm, and interferometer arm length is 4 km.

of improving on the shot noise limit by injecting phase squeezed states of light in place of the vacuum entering the interferometer dark port [32]. This method was further expanded by others [163, 86].

The level of squeezing improvement in encapsulated in the squeeze factor, $r$, defined in section 3.2.3. The single-sided PSD of the noise of a squeezed-state enhanced interferometer is given by

$$S_h = \frac{h_{SQL}^2}{2} \left( \frac{1}{\kappa} + \kappa \right) \left( \cosh(2r) - \cos(2(\theta + \phi)) \sinh(2r) \right),$$

where $\phi = \arccot(\kappa)$ and $\theta$ is the angle of the squeezing. In the case of phase quadrature squeezing $\theta = 0$ while in the case of amplitude quadrature squeezing $\theta = \pi/2$, as illustrated by the dashed curves in figure 3.7.

For a fixed squeezing angle $\theta$, improvement in either the phase- or amplitude-quadrature, or a linear combination may be achieved. However the quantum noise of the interferometer is set by the amplitude quadrature at low frequencies and the phase quadrature at high frequencies. Thus there is a trade-off between shot noise and radiation pressure noise improvement, with frequency-independent squeezing sacrificing the low frequency noise floor for improvement at high frequencies or vice versa.

To achieve broadband squeezing improvement, the angle of squeezing must be frequency dependent, such that there is amplitude squeezing at low frequencies and phase squeezing at high frequencies (with the corner frequency set by the parameters of the interferometer encapsulated in $\kappa$). The optimum squeezing angle is $\theta_{opt}(\Omega) = -\arccot(\kappa)$, yielding broadband sensitivity improvement, shown in yellow in figure 3.7. The single sided PSD upon frequency-dependent
squeezed state injection is given by

\[ S_h = \frac{\hbar^2_{\text{SQL}}}{2} \left( \frac{1}{\kappa} + \kappa \right) e^{-2r}. \]  (3.65)

In a simple Michelson interferometer the optical power required to reach the standard quantum limit is extremely high. Instead the noise floor is typically limited by shot noise which is inversely proportional to the power in the interferometer. Once arm cavities and power recycling are added this limit is quickly approached in kilometre scale detectors.

3.5 Quantum noise in advanced gravitational wave detectors

There are three key differences between a simple Michelson and the optical layout of the second generation ground based detectors: Fabry-Perot arm cavities, power-recycling and signal-recycling. Each interferometer in the network has its own design features. The Advanced LIGO design is summarised in reference [150], advanced Virgo in reference [160], and KAGRA in reference [9].

Fabry-Perot arm cavities resonantly enhance the carrier fields circulating in the arms by a factor of \( 1/\sqrt{T} \), where \( T \) is the transmission of the arm cavity input test mass (ITM). This enhances the gravitational-wave signal, modifies the radiation pressure noise and standard quantum limit, and sets the bandwidth of the interferometer.

Power recycling is achieved with a partially reflective mirror on the symmetric port, resonantly enhancing the power entering the beamsplitter from the symmetric port. While the power recycling does increase the power circulating in the interferometer, it has no profound impact on the asymmetric mode of the interferometer or the standard quantum limit.

Signal recycling is achieved with a partially reflective mirror at the anti-symmetric port, first proposed by Meers [108]. The signal recycling cavity has a more profound impact on the quantum noise of the interferometer. At the signal recycling mirror fields exiting the dark port are reintroduced to the interferometer arm cavities, opening a channel through which the shot noise and radiation pressure noise can become correlated. The signal recycling cavity can be detuned to allow for signal extraction from the arm-cavities known as resonant sideband extraction, first proposed by Mizuno et al. [114].

3.5.1 Quantum noise in a dual-recycled Michelson

When including signal recycling, an additional signal recycling mirror (SRM) is added on the output port, creating an additional nested cavity in the interferometer. The signal recycling cavity reduces the effective finesse of the arm cavities, broadening the bandwidth of the detector. By tuning the length of the signal recycling cavity and the reflectivity of the signal recycling mirror the bandwidth of the detector can be tuned.

The SQL of a dual recycled, Fabry-Perot Michelson was derived following the methods of Kimble [90], and Buonanno and Chen [27]. The full derivation will not be repeated here, only the final results. The radiation pressure coupling constant and strain-equivalent SQL are of the same form for that of a simple Fabry-Perot Michelson, derived in appendix C and given in equations C.22 and C.23 respectively, with additional scaling due to the linewidth of the Fabry-Perot arm cavities,

\[ \kappa_{\text{DR}}(\Omega) = \frac{8\omega_0 P_{\text{circ}}}{mLc} \frac{2\gamma}{\Omega^2(\Omega^2 + \gamma^2)}, \]  (3.66)

\[ h_{\text{SQL,DR}} = \sqrt{\frac{8\hbar}{M\Omega^2 L^2}}. \]  (3.67)
3.5 Quantum noise in advanced gravitational wave detectors

The single sided PSD of the quantum noise of a dual-recycled lossless interferometer with arm cavities is given by [27]

\[ S_\eta^\zeta h = Q \left[ (C_{11} \sin \zeta + C_{21} \cos \zeta)^2 + (C_{21} \sin \zeta + C_{22} \cos \zeta)^2 \right], \tag{3.68} \]

where

\[ Q = \left( \frac{h_{\text{SQL,DR}}}{2 \kappa_{\text{DR}}(\Omega)} \right)^2 \alpha^2 |D_1 \sin \zeta + D_2 \cos \zeta|^2, \tag{3.69} \]

and where \( \zeta \) is the angle of the detection quadrature, \( \tau \) is the amplitude transmissivity of the signal recycling mirror (SRM), and

\[
C_{11} = (1 + \rho^2) \left[ \cos(2\phi) + \frac{\kappa_{\text{DR}}}{2} \sin(2\phi) \right] - 2\rho \cos(2\beta + 2\theta) \tag{3.70} \\
C_{12} = -\tau^2 \left[ \sin(2\phi) + \kappa_{\text{DR}} \sin^2(\phi) \right] \tag{3.71} \\
C_{21} = \tau^2 \sin(2\phi) - \kappa_{\text{DR}} \cos^2(\phi) \tag{3.72} \\
C_{22} = C_{11} \tag{3.73} \\
D_1 = -(1 + \rho e^{2i(\beta + \theta)}) \sin \phi \tag{3.74} \\
D_2 = (1 - \rho e^{2i(\beta + \theta)}) \cos \phi \tag{3.75}
\]

where \( \rho \) is the amplitude reflectivity of the SRM, \( \beta \) is the accumulated phase of the sideband frequency \( \Omega \) on a single round trip through the arm cavity, \( \theta \) is the single pass phase shift of the SRC, and \( \phi \) is the detuning of the signal recycling cavity.

Figure 3.8 shows the strain-equivalent noise for a signal-recycled Fabry-Perot arm cavity Michelson interferometer. Though power-recycling has not been included in the model its effect is included in the extra power at beamsplitter.

It is interesting to note that the inclusion of the signal recycling cavity opens an additional channel through which the radiation pressure noise and photon shot noise become correlated. The interferometer itself becomes a ponderomotive squeezer from the back action force on the test masses [28]. The standard quantum limit can also be shaped by modifying the properties of the signal recycling cavity. In particular the length of the SRC can be tuned to increase sensitivity at a specific frequency at the expense of broadband sensitivity.

3.5.2 Reduction of quantum noise in gravitational wave detectors

As shown through the previous sections there are several physical parameters of the interferometer that affect the quantum noise limit. There are both classical and quantum approaches to optimise the sensitivity.

Classically, one can increase the circulating power to reduce shot noise, which scales as \( 1/P_{\text{circ}} \), however increasing the power increases radiation pressure coupling 3.66. Increasing power also increases the thermal load on the test mass and compounds the problem of parametric instabilities – acoustic test-mass modes excited by the circulating laser power.

To reduce the effect of radiation pressure noise the mirror mass is maximised, limited by the availability of test masses of sufficient purity, coating technology and suspension technology.

In Advanced LIGO both of these parameters are optimised such that the corner frequency of quantum noise, where shot noise and radiation pressure noise meet, is around 100 Hz, as shown in figure 3.8. The full power circulating in the arm cavities in Advanced LIGO is 800 kW, set by the input laser power (125W), the arm cavity finesse, and the power recycling mirror reflectivity.
The injection of squeezed states, in place of vacuum, to improve the quantum noise limit of a lossless dual-recycled gravitational-wave detector is derived in [29, 79, 38]. The single-sided PSD of the quantum noise is given by

\[ S_{h, sqz} = \mathcal{Q} \left[ e^{-2r}(C_{11}^\alpha \sin \alpha + C_{21}^\alpha \cos \alpha)^2 + e^{2r}(C_{21}^\alpha \sin \alpha C_{22}^\alpha + \cos \zeta)^2 \right], \quad (3.76) \]

where \( r \) is the squeeze factor, the C coefficients are now weighted by the squeezing angle:

\[ C_{11}^\alpha = C_{11} \cos \alpha + C_{12} \sin \alpha \quad (3.77) \]
\[ C_{12}^\alpha = C_{12} \cos \alpha - C_{11} \sin \alpha \quad (3.78) \]
\[ C_{21}^\alpha = C_{21} \cos \alpha + C_{22} \sin \alpha \quad (3.79) \]
\[ C_{22}^\alpha = C_{22} \cos \alpha - C_{21} \sin \alpha \quad (3.80) \]

The quantum noise of a squeezing-enhanced dual-recycled arm-cavity Michelson is shown in figure 3.9. To cancel the effect of the anti-squeezed quadrature, an optimal frequency-dependent squeezed quadrature (\( \alpha \)), or readout quadrature (\( \zeta \)) can be used.

The quantum noise of a Michelson including losses is not modelled here, but can be found in appendix A of Chua [39].

### 3.6 Conclusion

In this section we have introduced the formalism for quantising the electromagnetic field, and modelled states of light using quantisation of the electromagnetic field. Squeezed states of light, their propagation, and their detection were introduced. The quantum noise of a Michelson interferometer was derived and the concept of a standard quantum limit introduced. These were then...
Figure 3.9: Model of the strain equivalent quantum noise in a signal recycled arm-cavity Michelson, with \((r = 1)\) and without \((r = 0)\) squeezing, measured at various squeezing injection angles, black dashed line shows quantum noise with signal-recycling cavity detuning but no squeezing injection, demonstrating sub-SQL performance. \(\alpha\) is the squeezing angle, and \(\phi\) is the SRC detuning. Model parameters include 40 kg test masses, 10 kW at the beamsplitter, 4km arm length, 55m signal recycling cavity length, 1.4% input test mass transmission, 35% signal recycling mirror transmission.
extended to a Fabry-Perot Michelson and then a dual recycled Fabry-Perot Michelson - the basic optical layout for a gravitational-wave detector.
Chapter 4

Generating and controlling squeezed states

4.1 Introduction

In this chapter we will derive how squeezed states of light are produced in nonlinear optical experiments. In section 4.2 the basics of optical cavities are discussed – their parameters, equation of motion and semi-classical behaviour. In section 4.3 second order nonlinear processes are introduced, with emphasis on second harmonic generation and optical parametric oscillation, and the equation of motion of an optical cavity containing a nonlinear element is derived. Finally section 4.4 introduces feedback control systems.

4.2 Optical cavities

Optical cavities, or resonators, are useful experimental tools for metrology, laser frequency stabilisation, selecting spatial or frequency modes, and for resonantly enhancing optical fields. This section presents the properties of optical cavities, and derives the response of an optical cavity from its equation of motion.

4.2.1 Free spectral range, finesse, linewidth

An optical cavity consists of partially reflective mirrors arranged such that the light in the cavity traces back over itself, making a self-repeating eigenmode. A diagram of a three mirror cavity is shown in figure 4.1, with one mirror representing the input coupler through which the beam enters the cavity, another mirror representing any additional loss, and the final mirror as the output coupler through which the beam exits the cavity.

When the length of the cavity is set such that it is an integer number of wavelengths of the field in the cavity, then the cavity is on resonance and the input field matches the cavity eigenmode. The free spectral range (FSR) of a cavity is the frequency (or length) spacing of successive resonances, and is defined as

$$v_{FSR} = \frac{c}{L},$$  \hspace{1cm} (4.1)

where $L$ is the round trip optical path length of the cavity. Note that for a linear cavity, $L$ is twice the physical length of the cavity.
4.2 Optical cavities

Figure 4.1: Model three-mirror optical cavity. $\kappa_{in}$ and $\kappa_{out}$ are the decay rates for in- and out-coupling fields, and $\kappa_l$ is the decay rate of loss in the cavity. $\hat{A}_{in}$, $\delta \hat{A}_{out}$, and $\delta \hat{A}_l$ are the fields entering the cavity via the input, output and loss ports, while $\hat{A}_{refl}$, $\hat{A}_{trans}$, and $\hat{A}_{loss}$ are the fields exiting the cavity from these ports. $\hat{a}$ is the intracavity field and $\tau$ is the cavity round trip time.

The finesse of an optical cavity is a measure of its optical quality factor, and is dependent on the reflectivity of the cavity mirrors. The finesse may be written as

$$F = \pi \sqrt{r}$$

(4.2)

where $r$ is the square root of the product of the cavity mirror power reflectivities,

$$r = \sqrt{R_{in} \times R_{out} \times R_l}.$$  

(4.3)

The bandwidth or linewidth of the cavity is usually defined in terms of its the full width at half maximum, which is given by the ratio of the FSR to the finesse,

$$\nu_{FWHM} = \frac{\nu_{FSR}}{F}.$$  

(4.4)

A cavity acts as an optical low pass filter for frequencies within the linewidth, as will be derived in section 4.2.3.

4.2.2 Optical cavity equation of motion

A quantum optical treatment of cavities is done rigorously in textbooks such as Walls and Milburn [172], and Bachor and Ralph [11]. The key results are summarised here. Optical cavities can be modelled in the same fashion as the quantum harmonic oscillator. Fields inside the cavity are represented as operators, and are calculated from the fields external to the cavity and the coupling rates of the cavity mirrors. Using this foundation, one can calculate Hamiltonians for the interactions in the system, and the interactions between the closed system and the outside world. From the Hamiltonians, the equation of motion of the field inside the cavity is calculated. This method will be extended to cavities containing nonlinear optical elements in section 4.3.

Consider a cavity such as in figure 4.1, with three coupling ports to the outside world – the input, output, and loss ports. $\kappa_{in}$, $\kappa_{out}$, and $\kappa_l$ are the coupling rates of their respective ports, $\hat{A}_{out}$, and $\hat{A}_l$ are the fields entering the cavity via each of these ports. $\hat{A}_{refl}$ is the field promptly reflected
from the input coupler, $\hat{A}_{\text{trans}}$ is the field transmitted through the output coupler, and $\hat{A}_{\text{loss}}$ is the field escaping the cavity through the loss port. $\hat{a}$ is the field circulating in the cavity, and $\tau = c/L$ is the round trip time of the cavity. The capitalised fields have units of $\sqrt{\text{photons/second}}$, while the circulating field has units of $\sqrt{\text{photons}}$.

Assuming the cavity is only driven by a coherent field entering from the input port, the fields entering the other two ports are assumed to be vacuum, and are represented in figure 4.1 by their fluctuating components only.

The decay rates of the mirrors, $\kappa_i$, have units of $s^{-1}$ and are defined as

$$\kappa_i = -\frac{1}{2\tau} \ln(R_i), \quad (4.5)$$

where $R_i$ is the power reflectivity of the $i^{th}$ mirror. The total decay rate of the cavity, $\kappa$, is the sum of the decay rates of the various mirrors

$$\kappa = \sum_i \kappa_i \quad (4.6) = \kappa_{\text{in}} + \kappa_{\text{out}} + \kappa_i,$$

The quantum Langevin equation for this cavity, written in a frame rotating with the frequency of the input field, as derived in [11]:

$$\dot{\hat{a}} = i\Delta \hat{a} - \kappa \hat{a} + \sqrt{2\kappa_{\text{in}}} \hat{A}_{\text{in}} + \sqrt{2\kappa_{\text{loss}}} \delta \hat{A}_l + \sqrt{2\kappa_{\text{out}}} \delta \hat{A}_{\text{out}},$$

(4.7)

where $\Delta$ is the detuning of the input field from resonance. The first term describes the dynamics of the closed system. The second term includes the out-coupling of the circulating field through all of the ports. And the final three terms include the external fields coupling to the cavity. This is the starting point from which the transfer function and quantum behaviour of the cavity may be derived.

If the cavity is held on resonance ($\Delta = 0$) then the frequency response of the cavity is found by taking a Fourier transform of equation of motion. Recall that $\hat{\mathcal{F}}(\hat{x}(t)) = i\omega \hat{x}(\omega)$, where $\hat{x}$ is the frequency domain operator, and $\hat{x}$ is the time derivative of the time domain operator $\hat{x}$. Thus the cavity equation of motion can be rewritten as

$$i\omega \ddot{\hat{a}} = -\kappa \dot{\hat{a}} + \sqrt{2\kappa_{\text{in}}} \hat{A}_{\text{in}} + \sqrt{2\kappa_{\text{loss}}} \delta \hat{A}_l + \sqrt{2\kappa_{\text{out}}} \delta \hat{A}_{\text{out}},$$

(4.8)

which can be rearranged and solved for the intra-cavity field

$$\ddot{\hat{a}} = \frac{\sqrt{2\kappa_{\text{in}}} \hat{A}_{\text{in}} + \sqrt{2\kappa_{\text{loss}}} \delta \hat{A}_l + \sqrt{2\kappa_{\text{out}}} \delta \hat{A}_{\text{out}}}{\kappa + i\omega}. \quad (4.9)$$

To calculate the response of the cavity, we use the boundary conditions at each port to relate the external fields to the intracavity field. These boundary conditions are defined in Walls and Milburn [172],

$$\hat{A}_{\text{in}} + \hat{A}_{\text{refl}} = \sqrt{2\kappa_{\text{in}}} \hat{a}, \quad (4.10a)$$

$$\hat{A}_{\text{trans}} + \delta \hat{A}_{\text{out}} = \sqrt{2\kappa_{\text{out}}} \hat{a}, \quad (4.10b)$$

$$\hat{A}_{\text{loss}} + \hat{A}_l = \sqrt{2\kappa_i} \hat{a}. \quad (4.10c)$$
4.2 Optical cavities

Figure 4.2: Cavity steady state response as a function of measurement frequency \((\omega/2\pi)\). The amplitude and phase response of the transmitted field, \(\mathcal{T}\), is shown on the left, and the amplitude and phase responses of the reflected field, \(\mathcal{R}\), is shown on the right. In this example the model cavity has alinewidth of 3.2 MHz (FWHM), a finesse of 313, and a FSR of 1GHz.

Combining these boundary conditions with 4.9, we may express the reflected and transmitted fields, \(\tilde{A}_{\text{refl}}\) and \(\tilde{A}_{\text{trans}}\) respectively, in terms of \(\tilde{A}_{\text{in}}\),

\[
\tilde{A}_{\text{trans}} = \frac{2\sqrt{\kappa_{\text{in}\kappa_{\text{out}}}}\tilde{A}_{\text{in}} + (2\kappa_{\text{out}} - \kappa - i\omega)\delta\tilde{A}_{\text{out}} + 2\sqrt{\kappa_{\text{in}}\kappa_{\text{out}}}\delta\tilde{A}_{\text{f}}}{\kappa + i\omega},
\]

\[
\tilde{A}_{\text{refl}} = \frac{(2\kappa_{\text{in}} - \kappa - i\omega)\tilde{A}_{\text{in}} + 2\sqrt{\kappa_{\text{out}}\kappa_{\text{in}}}\delta\tilde{A}_{\text{out}} + 2\sqrt{\kappa_{\text{in}}\kappa_{\text{out}}}\delta\tilde{A}_{\text{f}}}{\kappa + i\omega}.
\]

From these equations the transfer function of the optical cavity on transmission and reflection can be calculated.

4.2.3 Cavity steady state, semi-classical and quantum behaviour

First we consider the steady state response of the cavity, where the fluctuating terms are zero. The output steady state field divided by the input field gives the transfer function of the cavity. This can be evaluated for both the transmitted and reflected fields

\[
\frac{\tilde{A}_{\text{trans}}}{\tilde{A}_{\text{in}}} = \frac{2\sqrt{\kappa_{\text{in}}\kappa_{\text{out}}}}{\kappa + i\omega},
\]

\[
\frac{\tilde{A}_{\text{refl}}}{\tilde{A}_{\text{in}}} = \frac{2\kappa_{\text{in}} - \kappa - i\omega}{\kappa + i\omega}.
\]

The amplitude and phase response of the cavity, on transmission and reflection, around the resonant frequency is shown in figure 4.2.

To formulate the time-varying semi-classical noise properties of the cavity, the fields are linearly separated into steady state and fluctuating components

\[
\tilde{A}_{\text{i}} = \langle\tilde{A}_{\text{i}}\rangle + \delta\tilde{A}_{\text{i}}.
\]
Figure 4.3: Noise variances of an optical cavity on reflection and transmission as a function of detuning. A variance of 1 indicates a vacuum state, the input variance, $V_{Ain}$ is 10. The same model cavity as figure 4.2 is used, with a linewidth of 3.2 MHz (FWHM), a finesse of 313 and a FSR of 1GHz.

Considering only the fluctuating components of the reflected and transmitted fields, equations 4.12 and 4.11, gives

$$\delta \tilde{\Lambda}_{\text{refl}} = \frac{(2\kappa_{in} - \kappa - i\omega) \delta \tilde{A}_{in} + 2\sqrt{\kappa_{in}\kappa_{out}} \delta \tilde{A}_{out} + 2\sqrt{\kappa_{in}} \delta \tilde{\Lambda}_l}{\kappa + i\omega},$$  \hspace{1cm} (4.16)

$$\delta \tilde{\Lambda}_{\text{trans}} = \frac{2\sqrt{\kappa_{in}\kappa_{out}} \delta \tilde{A}_{in} + (2\kappa_{out} - \kappa - i\omega) \delta \tilde{A}_{out} + 2\sqrt{\kappa_{in}} \delta \tilde{\Lambda}_l}{\kappa + i\omega}. \hspace{1cm} (4.17)$$

Converting these to quadrature operators, using 3.7 and 3.8, gives

$$\delta \tilde{X}_{1,2}^{\text{refl}} = \frac{(2\kappa_{in} - \kappa - i\omega) \delta \tilde{X}^{in}_{1,2} + 2\sqrt{\kappa_{in}\kappa_{out}} \delta \tilde{X}^{out}_{1,2} + 2\sqrt{\kappa_{in}} \delta \tilde{X}^{l}_{1,2}}{\kappa + i\omega}, \hspace{1cm} (4.18)$$

$$\delta \tilde{X}_{1,2}^{\text{trans}} = \frac{2\sqrt{\kappa_{in}\kappa_{out}} \delta \tilde{X}^{in}_{1,2} + (2\kappa_{out} - \kappa - i\omega) \delta \tilde{X}^{out}_{1,2} + 2\sqrt{\kappa_{in}} \delta \tilde{X}^{l}_{1,2}}{\kappa + i\omega}. \hspace{1cm} (4.19)$$

The variances, $V^{(k)} = |\delta X^{(k)}|^2$, of the two quadratures of the reflected and transmitted fields are given by

$$V_{1,2}^{\text{refl}} = 1 + \frac{((2\kappa_{in} - \kappa)^2 + \omega^2)(V_{Ain}^{Aout} - 1)}{\kappa^2 + \omega^2}, \hspace{1cm} (4.20)$$

$$V_{1,2}^{\text{trans}} = 1 + \frac{4\kappa_{out}\kappa_{in}(V_{Ain}^{Aout} - 1)}{\kappa^2 + \omega^2}. \hspace{1cm} (4.21)$$

Plotting these variances as a function of input noise power demonstrates the role of the optical cavity as a low pass filter, as shown in figure 4.3. This semi-classical treatment will be applied to a cavity containing a nonlinear medium to derive the generation of light with noise properties below shot noise.
4.3 Nonlinear optics and generating squeezed states

Nonlinear optics is a field covering various nonlinear behaviours of materials, typically with the application of a large electric field, such as a high intensity optical field. Sum and difference frequency generation, including second harmonic generation and parametric amplification, are caused by the second order nonlinearity of a material. These allow for efficient generation of laser light at wavelengths not typically achievable by other means. The third order nonlinearity gives rise to the Kerr effect – where the refractive index has a dependence on intensity, this leads to other interesting nonlinear effects such as self- and cross-phase modulation, self-focusing and four-wave mixing. While this section will review the relevant sections for this thesis, text books such as Boyd [22] cover the subject in greater detail.

4.3.1 Second order nonlinear processes

We will focus on nonlinearities due to dependence of the induced polarisation of a material system \( \vec{P}(t) \) on applied optical electric field strength \( \vec{E}(t) \)

\[
\vec{P}(t) = \varepsilon_0 (\chi^{(1)} \vec{E}(t) + \chi^{(2)} \vec{E}(t)^2 + \chi^{(3)} \vec{E}(t)^3 ...),
\]

where \( \chi^{(n)} \) is the nth order of nonlinearity tensor, a property of the medium. The second order nonlinearity, \( \chi^{(2)} \) is typically the nonlinear process used in squeezed state generation.

The wave equation is used to describe the dynamics of the interaction between the induced polarisation and applied electric field in a nonlinear medium,

\[
\nabla^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial^2 \vec{P}}{\partial t^2},
\]

where \( n \) is the refractive index of the medium and \( c \) is the speed of light. This form of the wave equation – derived in [21] – reveals the relationship between the applied electric field, and an accelerating charge, given by \( \frac{\partial^2 \vec{P}}{\partial t^2} \). An accelerating charge generates an electromagnetic field, this is the origin of the newly generated field at a different wavelength. Given a general form of the generated electric field and the polarisation source term the wave equation can be solved for various conditions such as sum and difference frequency generation, resulting in coupled amplitude equations which can be used to calculate the intensity and frequency of the generated field.

As an applied electromagnetic field propagates through a \( \chi^{(2)} \) medium, energy is transferred between high energy photons and pairs of low energy photons, generating fields at new frequencies which also propagate through the medium. The transfer of energy between these fields is described by the conservation of energy and momentum.

Energy is conserved when the sum of the frequencies of the low energy photons is equal to the frequency of the high energy field:

\[
\sum_{\text{in}} \hbar \omega_i = \sum_{\text{out}} \hbar \omega_j,
\]

where \( \omega_i \) is the angular frequency of the \( i^{th} \) field, for \( i = 3, 2, 1 \) indicating the pump, signal, and idler photons. In the degenerate case, \( \omega_1 = \omega_2 \), and the produced field is at the fundamental wavelength.

Momentum conservation is encapsulated in the relative phase between the fields,

\[
\sum_{\text{in}} \hbar \mathbf{k}_i = \sum_{\text{out}} \hbar \mathbf{k}_j,
\]

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Figure 4.4: Parametric up- and down-conversion processes in a $\chi^{(2)}$ nonlinear medium. The left figure shows sum frequency generation of a high energy photon $\omega_3$ from two low energy photons $\omega_1$ and $\omega_2$. If $\omega_1 = \omega_2$ this degenerate process is second harmonic generation. The right figure shows one high energy photon $\omega_3$ is converted to two low energy photons $\omega_1$ and $\omega_2$, this down-conversion process is known as optical parametric oscillation.

where $k_i = n_i \omega_i / c$ is the wavevector of the $i^{th}$ field and $n_i = \sqrt{\varepsilon^{(1)}(\omega_i)}$ is the refractive index of the medium for the $i^{th}$ field. Matching the sum of the relative phases of input and output fields is important in optimising the efficiency of a preferred nonlinear process, this will be addressed in section 4.3.2.

For the purposes of this thesis we focus on two nonlinear processes: degenerate down-conversion and up-conversion processes respectively known as second harmonic generation and optical parametric oscillation. These two processes are shown diagrammatically in figure 4.4.

In 1968 Boyd and Kleinmann ([20]) derived equations for the interaction of beams in a nonlinear medium. Some of the key equations from this, and derived works, will be presented here.

First we consider sum-frequency generation, the general case of second harmonic generation. Two input waves, $\omega_1$ and $\omega_2$, are incident on a $\chi^{(2)}$ medium, of length $L$, and producing a third wave at frequency $\omega_3$. A solution to equation 4.23 for the generated electric field $\vec{E}_3(z,t)$ is

$$\vec{E}_3(z,t) = A_3 e^{i(k_3 z - \omega_3 t)}, \quad (4.26)$$

where $z$ is the propagation direction. Adding a source term for the pump field, substituting into the wave equation, and finding a solution yields a differential equation for the amplitude of the generated field $A_3$,

$$\frac{dA_3}{dz} = \frac{4 \pi i \chi^{(2)}_3 \omega_3^2 A_1 A_2 e^{i\Delta k z}}{k_3 c^2}, \quad (4.27)$$

where $\Delta k = k_1 + k_2 + k_3$ is the wavevector mismatch between the three fields. Similar equations can be found for the change in $A_1$ and $A_2$ along the length of the medium.

The efficiency of the nonlinear process is determined by the intensity of the generated field after the length of the crystal:

$$I_3 = I_{3,\text{max}} \sin^2(\Delta k L / 2) / (\Delta k L / 2)^2. \quad (4.28)$$

The dependence of the generated field on the phase, or wavevector, mismatch results in a characteristic shape of generated intensity as a function of crystal temperature, known as the phase matching curve.

4.3.2 Phase matching

Matching the relative phases between the three fields is key to obtaining nonlinear conversion as shown in equation 4.28. If there is no phase mismatch between the three fields ($\Delta k = 0$) the
amplitude of the generated field increases linearly with \( z \). Hence the intensity, \( |A_3|^2 \), increases quadratically along the length of the crystal.

Perfect phase matching is often impossible to achieve due to the normal dispersion in a nonlinear medium, where the refractive index (normally) increases with increasing optical frequency. For beams propagating co-linearly through the nonlinear medium, the phase matching condition may be written in terms of the pump, signal and idler refractive indices,

\[
 n_1 \omega_1 + n_2 \omega_2 = n_3 \omega_3. \quad (4.29)
\]

In a crystal with nonzero phase mismatch, the phases of the pump and fundamental fields diverge as they propagate through the crystal. When the relative phases reach a mismatch of \( \pi / 2 \) the nonlinear conversion reverses and energy is transferred to the opposite nonlinear process. In the case of the sum-frequency generation, light is converted from the desired pump field back into the signal/idler. The field exiting the crystal with a nonzero phase mismatch can be found by integrating equation 4.27 over the length of the nonlinear medium [22]:

\[
 A_3 = \frac{4\pi i \chi^{(2)} \omega_3^2}{k_3 c^2} A_1 A_2 \int_0^L e^{i \Delta k z} d z = \frac{4\pi i \chi^{(2)} \omega_3^2}{k_3 c^2} A_1 A_2 \left( e^{i \Delta k L} - \frac{1}{i \Delta k} \right). \quad (4.30)
\]

The intensity of the \( i^{th} \) field is calculated from the time-averaged Poynting vector,

\[
 I_i = \frac{n_i c}{2 \pi} |A_i|^2. \quad (4.31)
\]

For the generated field at the pump wavelength this is given by

\[
 I_3 = \frac{16 \pi (\chi^{(2)})^2 \omega_3^4 |A_1|^2 |A_2|^2 n_3}{k_3^2 c^3} \left| e^{i \Delta k L} - \frac{1}{i \Delta k} \right|^2. \quad (4.32)
\]

The modulus-squared term in this equation can be re-written as a sinc-squared function, and wavenumbers and angular frequencies rewritten in terms of wavelength, giving

\[
 I_3 = \frac{256 \pi^5 (\chi^{(2)})^2 I_1 I_2 n_1 n_2 n_3 \lambda^3 c}{n_1 n_2 n_3 A_3^2 L^2 \lambda^2 \text{sinc}^2 \left( \frac{\Delta k L}{2} \right)}. \quad (4.33)
\]

This is the full equation for the generated field intensity, and will be used in later chapters to calculate the nonlinearity of a crystal from measured powers.

There are several methods of achieving zero phase mismatch, or close to zero phase mismatch, in a nonlinear crystal. Two types of phase matching, and quasi-phase matching are summarised below.

**Type I and type II (birefringent) phase matching**

Type I and type II phase matching make use of the birefringence in an anisotropic crystal, that is a crystal with different refractive indices along the different crystal lattice directions. For a medium with normal dispersion \( \omega_3 \) is polarised in a direction with lower refractive index than \( \omega_1 \) and \( \omega_2 \).

Type I phase matching implies that \( \omega_1 \) and \( \omega_2 \) are polarised along the same crystal axis, while in type II phase matching they are polarised in different directions.

There are two methods to control the three refractive indices – angle tuning and temperature tuning. Tuning the angle of the crystal changes the refractive index seen by the input beam to
an angle-dependent combination of the refractive indices of the crystal axes. In this thesis we focus on temperature tuning, which exploits the temperature dependence of refractive index of the crystal.

Phase matching can be achieved by adjusting the temperature of the crystal such that the crystal refractive index for the pump and fundamental fields causes a zero relative phase shift. Experimentally accessible temperatures are limited, resulting in some crystal dimensions being inaccessible using this technique.

**Quasi phase matching**

Instead of modifying the crystal refractive indices, quasi phase matching (QPM) [7] flips the sign of the nonlinearity before the phase mismatch between the interacting fields becomes $\pi/2$ and the desired nonlinear process reverses. This technique allows access to crystal axes with high nonlinearity rendered unreachable using birefringent phase matching due to the extreme temperature requirements, since QPM does not tune the crystal index.

QPM is achieved using the periodic poling technique, where crystals are processed such that the direction of the effective nonlinearity is periodically flipped over the length of the crystal. The period of the periodic poling is set by rate of divergence of the phases of the interacting fields, which can be represented by the coherence length of the fields, given by [117]

$$L_c = \frac{2\pi}{|\Delta k|}. \quad (4.34)$$

$m^{th}$ order periodic poling flips the crystal domains every $m$ coherence lengths. For a degenerate process ($\omega_1 = \omega_2$), the length of poling regions is approximated by

$$\Gamma_0 = \frac{m\lambda_1}{2\times \Delta n}, \quad (4.35)$$

where $\Delta n = |n_{1,2} - n_3|$ is the refractive index difference between the pump and fundamental.

In the case of quasi-phase matched periodically poled materials, the effective nonlinearity ($d_{eff} = \chi^{(2)}/2$) is affected by deviation from perfect phase matching [84]. For $m^{th}$ order phase matching, the modified effective nonlinearity is given by

$$d_{eff} = \frac{2}{m\pi} d_{ij}, \quad (4.36)$$

where $d_{ij}$ is the unpoled nonlinearity along the $i, j^{th}$ crystal axes.

The poling length is optimised for a particular difference in refractive index between pump and fundamental, which depends on temperature. The sinc-squared phase matching curve has been recorded for all crystals used in this thesis to establish the optimal operating temperature. To measure the phase matching curve the crystal is operated in a single-pass SHG experiment, and the conversion efficiency measured as a function of crystal temperature.

### 4.3.3 Cavity equations of motion with a nonlinear element

An optical cavity is commonly used to increase the effective interaction length in a nonlinear medium. To analyse this type of system a quantum Langevin equation approach can be used, similar to that of 4.2.2, while including a nonlinear source term inside the cavity. Using this approach the production of squeezed light can be shown as a reduction in the variance of fields at the fundamental wavelength, using a degenerate sub-threshold optical parametric oscillator.
4.3 Nonlinear optics and generating squeezed states

Figure 4.5: Model three-mirror optical cavity containing a nonlinear medium. $\hat{a}$ and $\hat{b}$ are the co-propagating intracavity fields at the fundamental and second harmonic wavelengths. $\kappa^{a,b}_i$ and $\kappa^{a,b}_o$ are the decay rates for in- and out-coupling fields for the fundamental and pump fields, and $\kappa^{a,b}_l$ is the decay rate of loss in the cavity. $\hat{A}_{in}$ and $\hat{B}_{in}$, $\delta \hat{A}_{out}$ and $\delta \hat{B}_{out}$, $\delta \hat{A}_{l}$ and $\delta \hat{B}_{l}$ are the fundamental and second harmonic fields entering the cavity at the fundamental via the input, output and loss ports respectively. $\hat{A}_{refl}$ and $\hat{B}_{refl}$, $\hat{A}_{trans}$ and $\hat{B}_{trans}$, $\hat{A}_{loss}$ and $\hat{A}_{loss}$ are the fundamental and second harmonic fields exiting the cavity from the input, output and loss ports. $\tau$ is the cavity round trip time.

The Hamiltonian representing the energy of the overall system is the sum of the Hamiltonian of the closed system and the Hamiltonian of the interaction between the system and the outside world,

$$\hat{H} = \hat{H}_{cav} + \hat{H}_{ext}. \quad (4.37)$$

$\hat{H}_{cav}$ is the Hamiltonian of the internal modes of the closed lossless cavity, given by

$$\hat{H}_{cav} = \hbar \omega \hat{a}\dagger \hat{a} + \hbar (2\omega) \hat{b} \dagger \hat{b} + \frac{i\hbar}{2} (\varepsilon \hat{a} \dagger \hat{b} - \varepsilon^* \hat{a} \dagger \hat{b}^\dagger). \quad (4.38)$$

In this equation $\varepsilon$ is the nonlinear coupling rate, which depends on the second order nonlinearity of the medium ($\chi^{(2)}$), the phase mismatch ($\Delta k$) and the interaction length. The first two terms describe the number of photons in the fundamental and pump modes. The final terms describes the transfer of energy between the two wavelengths via the nonlinear process.

The second Hamiltonian in equation 4.37 is the coupling between the cavity and the outside world, also known as the heatbath. The heatbath is represented as a continuous spectrum of modes at each cavity mirror, coupling to the cavity via the decay rates $\kappa_i$. The Hamiltonian is

$$\hat{H}_{ext} = i\hbar \sum_i \int_{-\infty}^{\infty} \left[ \sqrt{2\kappa_i^a} (\hat{A}_i^\dagger \hat{a} - \hat{a}^\dagger \hat{A}_i) + \sqrt{2\kappa_i^b} (\hat{B}_i^\dagger \hat{b} - \hat{b}^\dagger \hat{B}_i) \right] d\omega \quad (4.39)$$

where $\hat{A}_i$ and $\hat{A}_i$ are the fields coupling in through the $\kappa^{a,b}_i$ mirrors, and $i = \text{in}, \text{out}, \text{l}$.

The cavity equation of motion can be found from these Hamiltonians using the Langevin approach, as shown in [73] and [44]. The derived equations of motion for the pump and fundamental
are given, assuming the only coherent field entering the cavity is through the input port.

\[
\dot{a} = \frac{1}{i\hbar} [a, \hat{H}_{cav}] - \kappa^a \dot{a} + \sqrt{2 \kappa_{in}^a} \hat{A}_{in} e^{i \omega_{in}^a t} + \sqrt{2 \kappa_{out}^a} \delta \hat{A}_{out} + \sqrt{2 \kappa_{i}^{\kappa}} \delta \hat{A}_i, \tag{4.40}
\]

\[
\dot{b} = \frac{1}{i\hbar} [b, \hat{H}_{cav}] - \kappa^b \dot{b} + \sqrt{2 \kappa_{in}^b} \hat{B}_{in} e^{i \omega_{in}^b t} + \sqrt{2 \kappa_{out}^b} \delta \hat{B}_{out} + \sqrt{2 \kappa_{i}^{\kappa}} \delta \hat{B}_i. \tag{4.41}
\]

Where \( \omega_{in}^{a,b} \) is the angular frequency of the driving fields at the fundamental and pump. Substituting the cavity Hamiltonian from equation 4.38, these equations can be simplified to

\[
\dot{a} = \epsilon \dot{\hat{a}} \dot{b} - (\kappa^a - i \omega_{cav}^a) \dot{a} + \sqrt{2 \kappa_{in}^a} \hat{A}_{in} e^{i \omega_{in}^a t} + \sqrt{2 \kappa_{out}^a} \delta \hat{A}_{out} + \sqrt{2 \kappa_{i}^{\kappa}} \delta \hat{A}_i, \tag{4.42}
\]

\[
\dot{b} = -\frac{\epsilon}{2} \dot{a}^2 - (\kappa^b - i \omega_{cav}^b) \dot{b} + \sqrt{2 \kappa_{in}^b} \hat{B}_{in} e^{i \omega_{in}^b t} + \sqrt{2 \kappa_{out}^b} \delta \hat{B}_{out} + \sqrt{2 \kappa_{i}^{\kappa}} \delta \hat{B}_i, \tag{4.43}
\]

where \( \omega_{cav}^{a,b} \) are the resonance frequencies of the cavity at the fundamental and second harmonic. These equations are for a general degenerate second order nonlinear process, will be used in the next subsection to show how squeezing is generated.

### 4.3.4 Optical parametric oscillation

In this section we will apply the nonlinear cavity equations of motion to the specific case of a below-threshold optical parametric oscillator (OPO), and show a reduction in the variance of one quadrature. First consider the steady state of the input field from resonance, analogous to the empty cavity in equation 4.7,

\[
\dot{a} = \epsilon \dot{\hat{a}} \dot{b} - (\kappa^a - i \Delta^a) \dot{a} + \sqrt{2 \kappa_{in}^a} \hat{A}_{in} + \sqrt{2 \kappa_{out}^a} \delta \hat{A}_{out} + \sqrt{2 \kappa_{i}^{\kappa}} \delta \hat{A}_i, \tag{4.44}
\]

\[
\dot{b} = -\frac{\epsilon}{2} \dot{a}^2 - (\kappa^b - i \Delta^b) \dot{b} + \sqrt{2 \kappa_{in}^b} \hat{B}_{in} + \sqrt{2 \kappa_{out}^b} \delta \hat{B}_{out} + \sqrt{2 \kappa_{i}^{\kappa}} \delta \hat{B}_i. \tag{4.45}
\]

We can make some simplifying assumptions. First assume the cavity is operated on resonance (\( \Delta^{a,b} = 0 \)), below threshold, with no pump depletion (\( \dot{b} = 0 \)), and with few photons at the fundamental frequency (\( \dot{a}^2 = 0 \)). The equations of motion simplify to

\[
\dot{a} = |g| e^{i \phi} \dot{\hat{a}}^{\dagger} - \kappa^a \dot{a} + \sqrt{2 \kappa_{in}^a} \hat{A}_{in} + \sqrt{2 \kappa_{out}^a} \delta \hat{A}_{out} + \sqrt{2 \kappa_{i}^{\kappa}} \delta \hat{A}_i, \tag{4.46}
\]

\[
\kappa_i \dot{b} = \sqrt{2 \kappa_{in}^b} \hat{B}_{in} + \sqrt{2 \kappa_{out}^b} \delta \hat{B}_{out} + \sqrt{2 \kappa_{i}^{\kappa}} \delta \hat{B}_i, \tag{4.47}
\]

where \( g = |g| e^{i \phi} = \epsilon b \) is the nonlinear gain factor written in terms of amplitude and phase components. The phase \( \phi \) is the phase difference between the pump and fundamental fields.

Below threshold we can treat \( a \) and \( b \) separately, and \( b \) classically. First consider the steady state of the pump field, by dropping fluctuating terms,

\[
b = \frac{\sqrt{2 \kappa_{in}^b}}{\kappa^b} B_{in}. \tag{4.48}
\]

The semi-classical equation of motion of the fundamental field is found by dropping the fluctuating terms from equation 4.46, and assuming the input fields are real.
\[ \dot{a} = |g|e^{i\phi} \dot{a}^\dagger - \kappa^a \hat{a} + \sqrt{2\kappa_{in}^a} \hat{A}_{in}, \]
\[ \dot{a}^\dagger = |g|e^{-i\phi} \dot{a} - \kappa^a \hat{a} + \sqrt{2\kappa_{in}^a} \hat{A}_{in}. \]

Let \( P_{out}(g = 0) \) be the power in a seed field in the cavity before experiencing the nonlinear gain, and \( P_{out}(g) \) be the seed field after the nonlinear element. The nonlinear gain is then
\[ g' = \frac{P_{out}(g)}{P_{out}(g = 0)} = 1 + \frac{(|g|/\kappa^a)^2 + 2|g|\kappa^2 \cos \phi}{(1 - |g|^2/(\kappa^a)^2)^2}. \]

When the nonlinear gain factor approaches the cavity decay rate (\( g \to \kappa^a \)) the nonlinear gain experienced by a seed field (\( g' \)) asymptotes infinity. This defines the threshold power of the OPO.

A convenient way of writing the threshold power (\( P_{crit}^b \)) in terms of the cavity coupling rates is
\[ P_{crit}^b = \left( \frac{\kappa^a \kappa^b}{E \sqrt{2 \kappa_{in/out}^b}} \right)^2 \frac{hc}{\lambda^b}. \]

Treating the OPO semi-classically reveals the noise couplings from each port, from which the noise of the output fields can be calculated. The equation of motion of the fluctuating component of \( a \) and it’s complex conjugate \( a^\dagger \) can be calculated from equation 4.46,
\[ \delta \dot{a} = -\kappa_a \delta a + g \delta a^\dagger + \sqrt{2\kappa_{in}^a} \delta A_{in} + \sqrt{2\kappa_{out}^a} \delta A_{out} + \sqrt{2\kappa_{in}^a} \delta A_l, \]
\[ \delta \dot{a}^\dagger = -\kappa_a \delta a^\dagger + g \delta a + \sqrt{2\kappa_{in}^a} \delta A_{in}^\dagger + \sqrt{2\kappa_{out}^a} \delta A_{out}^\dagger + \sqrt{2\kappa_{in}^a} \delta A_l^\dagger. \]

We can calculate the quadrature fluctuations from these fields, recalling the definitions from section 3.2.1, \( \hat{X}_1 = \hat{a} + \hat{a}^\dagger \) and \( \hat{X}_2 = i(\hat{a}^\dagger - \hat{a}) \),
\[ \delta \hat{X}_1^a = (-\kappa^a + g) \delta \hat{X}_1^a + \sqrt{2\kappa_{in}^a} \delta \hat{X}_{in}^a + \sqrt{2\kappa_{out}^a} \delta \hat{X}_{out}^a + \sqrt{2\kappa_{in}^a} \delta \hat{X}_l^a, \]
\[ \delta \hat{X}_2^a = (-\kappa^a + g) \delta \hat{X}_2^a + \sqrt{2\kappa_{in}^a} \delta \hat{X}_{in}^a + \sqrt{2\kappa_{out}^a} \delta \hat{X}_{out}^a + \sqrt{2\kappa_{in}^a} \delta \hat{X}_l^a. \]

To calculate the spectra of the fluctuating components of the quadratures we take the Fourier transform, using the tilde to denote operators in the Fourier domain,
\[ \delta \hat{X}_{1,2}^\omega = \sqrt{2\kappa_{in}^a} \delta \hat{X}_{1,2}^{A_{in}} + \sqrt{2\kappa_{out}^a} \delta \hat{X}_{1,2}^{A_{out}} + \sqrt{2\kappa_{in}^a} \delta \hat{X}_{1,2}^{A_l}. \]

The boundary conditions for the reflection and transmission ports (equation 4.10c) are used with equation 4.57 to evaluate the fluctuating components of the quadratures upon transmission and reflection from the cavity in terms of the other input fields, eliminating the intracavity field,
\[ \delta \hat{X}_{trans}^{1,2} = \frac{2\sqrt{\kappa_{in}^a \kappa_{out}^a}}{\kappa^a + g + i\omega} \delta \hat{X}_{1,2}^{A_{in}} + \frac{2\sqrt{\kappa_{in}^a \kappa_{out}^a}}{\kappa^a + g + i\omega} \delta \hat{X}_{1,2}^{A_{out}} + \frac{2\sqrt{\kappa_{in}^a \kappa_{out}^a}}{\kappa^a + g + i\omega} \delta \hat{X}_{1,2}^{A_l}. \]
\[ \delta \hat{X}_{refl}^{1,2} = \frac{(2\kappa_{in}^a - \kappa^a - i\omega)}{\kappa^a + g + i\omega} \delta \hat{X}_{1,2}^{A_{in}} + \frac{2\sqrt{\kappa_{in}^a \kappa_{out}^a}}{\kappa^a + g + i\omega} \delta \hat{X}_{1,2}^{A_{out}} + \frac{2\sqrt{\kappa_{in}^a \kappa_{out}^a}}{\kappa^a + g + i\omega} \delta \hat{X}_{1,2}^{A_l}. \]
We consider the specific case of the reflected field, with no coherent amplitude at the fundamental wavelength – that is an unseeded optical parametric oscillator. Taking the variances \( V(1) = \langle |\delta X(1) |^2 \rangle \) for each quadrature on reflection:

\[
V_{1}^{\text{out}} = 1 + \frac{\kappa_{0}^{\text{out}}}{\kappa^{a}} \frac{4\left(g / \kappa^{a}\right)}{\left(\omega / \kappa^{a}\right)^{2} + \left(1 - g / \kappa^{a}\right)^{2}},
\]

(4.60)

\[
V_{2}^{\text{out}} = 1 - \frac{\kappa_{0}^{\text{out}}}{\kappa^{a}} \frac{4\left(g / \kappa^{a}\right)}{\left(\omega / \kappa^{a}\right)^{2} + \left(1 + g / \kappa^{a}\right)^{2}},
\]

(4.61)

Recall that the variance of the vacuum state is unity. The second term in equation 4.60 shows that the variance of that quadrature has been increased by a factor depending on the cavity decay rate and the nonlinear gain. Equation 4.61 show that the variance in the opposite quadrature has been decreased by that same factor.

Simplifying the expressions for variance in terms of the normalised pump parameter and the cavity escape efficiency \( \eta_{\text{esc}} \) gives

\[
V_{1}^{\text{out}} = 1 + \eta_{\text{esc}} \frac{4\chi}{\left(\omega / \kappa^{a}\right)^{2} + \left(1 - \chi\right)^{2}},
\]

(4.62)

\[
V_{2}^{\text{out}} = 1 - \eta_{\text{esc}} \frac{4\chi}{\left(\omega / \kappa^{a}\right)^{2} + \left(1 + \chi\right)^{2}},
\]

(4.63)

where the escape efficiency is the quantum efficiency with which light is coupled out of the cavity. These are the variances in the case of an unseeded optical parametric oscillator, used in all the experiments presented in this thesis. Figure 4.6 shows the variances of the two quadratures on reflection, as a function of cavity detuning, pump power, and cavity escape efficiencies.

### 4.4 Control systems

In the previous section the quadrature variances were calculated assuming no cavity detuning from resonance \( \Delta^{a} = \Delta^{b} = 0 \). Generally, to achieve this condition a feedback control system
4.4 Control systems

Figure 4.7: Block diagram showing the key components of a generic feedback loop in the Laplace domain. Comparing the reference \( r(s) \), and the state of the system generates an error signal \( e(s) \), this is fed electronically to the controller \( K(s) \), which makes actuates on the plant, \( P(s) \), to drive the error signal to zero.

is required, for example to force the laser frequency to track and follow any drifts in the cavity length.

Several feedback control systems are used in each experiment presented in this thesis – to maintain nonlinear crystals at their phase matching temperatures, to set the phase condition between the squeezed beam and local oscillator for balanced homodyne detection, to maintain cavities on resonance, and to stabilise lasers. Feedback works to maintain a system (or “plant”) at its operating point by comparing the current state to some reference desired state, and actuating on the system to push it to the desired state. A block diagram of a generic feedback control loop is shown in figure 4.7.

Through this thesis control loops are characterised through Bode plots of their transfer functions, that is the amplitude and phase responses as a function of frequency. From these transfer functions the unity gain frequency, a measure of the bandwidth of the loop, and the gain and phase margins can be determined.

In this section we introduce the concepts of Pound-Drever Hall-locking, dither locking and coherent locking, that are used in multiple experiments in this thesis. More specialised control systems will be discussed in the chapters of their respective experiments. Examples of more thorough introductory textbooks include Bechhoefer [14] and Abramovici and Chapsky [2].

4.4.1 Pound-Drever-Hall locking

Pound-Drever-Hall (PDH) locking is a technique which can be used to stabilise the frequency noise of a laser by locking it to an optical cavity reference [57]. Conversely, the technique can be used to stabilise the length of an optical cavity by locking it to the frequency of the laser. The scheme works by using radio frequency (RF) phase modulation sidebands, added to the field incident on the cavity, as a reference from which a phase-sensitive measurement of the field resonant in the cavity can be made.

The amplitude of the complex optical transfer function of a cavity on reflection and transmission is shown in figure 4.2. The reflected and transmitted signals are symmetric as a function of detuning, thus from this signal alone a controller cannot discern which side of resonance a detuned system is on, and hence which direction to actuate on the system. Instead the phase response of the cavity is used to generate an error signal asymmetric with detuning.

A PDH scheme where an optical cavity is locked to a laser is shown schematically in figure
4.8. An electro-optic modulator adds phase modulation sidebands to the field incident on the cavity. The phase modulation sidebands are typically set well outside the linewidth of the cavity, so they are not resonant in the cavity. The signal reflected by the cavity is then monitored with a photodiode.

On the photodiode the beat note between the reflected carrier and the sidebands is detected. When the cavity is on resonance the reflected carrier field acquires no additional phase shift, and at the photodetector there is only phase modulation. When the cavity is detuned from resonance the carrier field acquires a phase shift that depends on the direction of the detuning, this introduces amplitude modulation at the sideband frequency. Hence, the information to generate the error signal is encapsulated in the phase of the beat note between the carrier and the sidebands.

The voltage from the photodiode is then mixed down electronically at the modulation frequency, which generates a signal proportional to the sum and difference frequencies of the photodiode signal and the modulation signal. Applying a low-pass filter to this signal, we focus on the difference frequency which contains information of the relative phase difference between the photodiode signal and the modulation signal. This is derived mathematically in [16].

The voltage of the PDH error signal, as a function of angular frequency detuning $\omega$, can be approximated by

\[
V_{err} \simeq \Re \left[ R^\ast (\omega) \times R(\omega + \Omega_m) - R(\omega) \times R^\ast (\omega - \Omega_m) \right] \cos \phi + \Im \left[ R^\ast (\omega) \times R(\omega + \Omega_m) - R(\omega) \times R^\ast (\omega - \Omega_m) \right] \sin \phi,
\]

where $R$ is the complex cavity transfer function on reflection, as in equation 4.13, $\Omega_m$ is the phase modulation sideband frequency, and $\phi$ is the demodulation phase. This error signal is plotted in figure 4.9 with the cavity reflection signal. Components proportional to the imaginary part of the cavity response generate an error signal asymmetric with detuning. The demodulation phase is typically tuned so that the error signal has a steep slope.

The advantages of the PDH technique are its high bandwidth – set by the sideband modulation frequency. The technique can also be performed on transmission of the cavity, provided the RF sidebands are sufficiently within the cavity linewidth.

4.4.2 Dither locking

Dither locking is a simple method which has the same applications as Pound-Drever-Hall locking, that is, locking a the length of an optical cavity to the frequency of a laser, or the frequency of a laser to a reference optical cavity.
4.4 Control systems

Figure 4.9: Model of cavity transfer function on reflection, as a function of frequency detuning from resonance, and PDH error signal, showing fundamental resonance and sidebands. Cavity parameters, linewidth $\Delta \nu = 32\text{MHz}$, Finesse $\mathcal{F} = 312$, free spectral range $\nu_{\text{FSR}} = 1\text{GHz}$, and phase modulation sideband frequency $\Omega_m = 50\text{MHz}$.

To generate an error signal, the laser frequency (or the cavity length) is modulated at frequency $\Delta \omega$, much less than the cavity linewidth. The response of the cavity converts this dither to amplitude modulation of the transmitted field, shown in figure 4.10. The magnitude of this amplitude modulation is proportional to the derivative of the transmitted power, and when the cavity is on resonance the dither produces no amplitude modulation. Thus the transmitted signal, proportional to the derivative of the cavity response, can be used as an error signal to lock the cavity on resonance.

The AC component of the field transmitted by the cavity is mixed down with the original dither frequency, to produce an error signal, as with PDH locking. The bandwidth of a dither locked loop is limited by the modulation frequency, which is in turn limited by the bandwidth of the actuator being dithered. Dither locking is used in chapter 7 to lock the $2\ \mu\text{m}$ squeezer SHG to the laser frequency.

In the case of dither locking the frequency modulation sidebands must be within the linewidth of the cavity, this limits the bandwidth of the technique. Higher bandwidths can be achieved using the Pound-Drever-Hall technique where phase modulation sidebands can be well outside the cavity linewidth.

4.4.3 Squeezing angle control

When measuring squeezing, or implementing squeezed light in a gravitational-wave detector, the squeezed quadrature must be locked to the phase of the local oscillator, or the main interferometer beam. This is nontrivial in the case of vacuum squeezing where there is no coherent amplitude associated with the squeezed beam. An early method used to control the squeezing angle was quantum noise locking, as presented in [106] and [146], however the coherent locking technique developed by Vahlbruch et al. [164, 36] is a higher bandwidth system, with no need for a seed field. Coherent locking uses a frequency detuned coherent locking field (CLF) to interrogate the OPO cavity without interacting with the squeezing beam and causing seeding. The CLF may be generated by a second laser or by frequency shifting the main laser, and must be within the linewidth of the OPO cavity while being sufficiently detuned from the squeezing frequency so as
to not interact with and seed the squeezed vacuum. The CLF co-propagates with the squeezing and the beat note between the CLF and the local oscillator is detected at the homodyne detector. This beat note can be demodulated and fed back to the local oscillator phase or pump phase so that measurement and squeezing quadratures can be controlled.

The two coherent locking schemes used in experiments in this thesis are shown diagrammatically in figures 4.11 and 4.12, both are variations on the original coherent locking scheme.

Coherent locking

In the case of coherent locking, a single sideband which is frequency shifted from the main laser by $\Omega_{CLF}$, is injected into OPO through the CLF input coupler. The CLF input coupler has slightly less than high reflectivity, and is shown as the second flat mirror in the OPO in figure 4.11. As the CLF is detuned within the linewidth of the cavity this signal field is resonant and a second (idler) sideband is generated by the nonlinear process in the crystal, following a steady state condition of equation 4.46, the signal and idler fields inside the cavity are given by

$$\begin{bmatrix} \bar{a}_s \\ \bar{a}_i \end{bmatrix} = \frac{\sqrt{2} \kappa_{CLF} A_s}{(1 - x^2) \kappa_{tot}} \begin{bmatrix} e^{i\phi_s} \\ xe^{-i(\theta_p - \phi_s)} \end{bmatrix},$$

(4.65)

where $\bar{a}_s$ and $\bar{a}_i$ are the signal and idler fields in the cavity, $\kappa_{CLF}$ is the decay rate of the CLF input coupler, $A_s$ is the incoming signal field, $x = |g|/\kappa^d$, $\kappa^d$ is the total cavity decay rate, $\phi_s$ is the signal phase, $\theta_p$ is the pump phase, which gives the phase of the squeezed beam, and hence the squeezing angle.

The generated idler field has frequency detuning $-\Omega_{CLF}$ and phase $\theta_p - \phi_s$. Thus the phase of the generated idler field varies linearly with the pump phase, which in turn indicates the squeezing phase. Equation 4.65 neglects the frequency detuning of the CLF from the OPO, and assumes perfect phase matching and co-resonance with the pump field.
**Figure 4.11:** Schematic of the coherent locking scheme. 1064 nm beams are shown in red, the frequency-detuned CLF field shown in orange, and pump beam in green. Electrical connections shown in black. Two AOMs are used to generate the frequency-offset field, which is injected into the OPO, and locked on reflection. The local oscillator is locked to the CLF using an error signal generated from the homodyne signal. Note that this diagram shows a double AOM based scheme, while the same lock can be achieved using a second auxiliary laser, phase locked the main squeezer laser.

The reflected CLF is calculated from the cavity boundary conditions,

\[
\tilde{A}_{\text{ref}}^{\text{CLF}} = \sqrt{2\kappa_{CLF}} \tilde{a} - \tilde{A}_{\text{in,CLF}} \quad (4.66)
\]

\[
= \frac{\sqrt{2\kappa_{CLF}} A_x}{(1-x^2)\kappa_{tot}} \begin{bmatrix} e^{i \phi_s} \\ 0 \end{bmatrix} - \begin{bmatrix} A_x e^{i \phi_s} \\ 0 \end{bmatrix}.
\]

This field is measured on a photodetector (B in figure 4.11), and demodulated at \(2\Omega_{CLF}\) – the beat frequency between the generated and injected sidebands – to produce an error signal proportional to the pump phase. The pump and CLF phases are locked either by feeding back to a voltage controlled oscillator (VCO) controlling the phase of one of the two AOMs, as in figure 4.11, or by feeding back to the laser frequency, if a second auxiliary laser is used to generate the CLF field.

A portion of the CLF is transmitted through the cavity via the pump/squeezing input/output coupler and co-propagates, but does not interact, with the squeezed beam. The CLF on transmission is given by

\[
\tilde{A}_{\text{trans, CLF}} = \sqrt{2\kappa_{CLF}^{\text{in}}} \begin{bmatrix} a_x \\ a_x^* \end{bmatrix} \quad (4.67)
\]

\[
= \frac{\sqrt{2\kappa_{CLF}^{\text{in}}} A_x}{(1-x^2)\kappa_{tot}^{\text{in}}} \begin{bmatrix} e^{i \phi_s} \\ 0 \end{bmatrix} - \begin{bmatrix} A_x e^{i \phi_s} \\ 0 \end{bmatrix}.
\]

At the homodyne these fields are interfered with the local oscillator, generating a beat note at \(\Omega_{CLF}\), which is then demodulated to create the second coherent locking error signal proportional to the phase difference between the CLF and the local oscillator. This is fed back to a PZT on either the local oscillator or pump paths.
Modified coherent locking

Modified coherent locking, shown in figure 4.12 and described in [144], is a variation on the coherent locking technique whereby a portion of the coherent locking field is frequency doubled using single-pass second harmonic generator. The co-propagating CLF, frequency offset from the carrier field by $\Omega_{CLF}$, and second harmonic CLF, with frequency $2(\omega + \Omega_{CLF})$, are combined with the pump using a mostly-transmissive beamsplitter. One port of the beamsplitter goes to the OPO, while the unused port is used to detect the beat note between the pump and the frequency-doubled CLF, labelled as B in figure 4.12. This beat note is demodulated at $2(\omega + \Omega_{CLF})$ fed back to the auxiliary laser frequency to phase lock the CLF to the main laser. Since the path between the beamsplitter and OPO is common to both the CLF and the pump this establishes a phase lock between the two beams.

A small amount of CLF at the (detuned) fundamental enters the OPO cavity and undergoes amplification and de-amplification via nonlinear interaction, as in the original coherent locking scheme. Note that there are several dichroic mirrors between the CLF beamsplitter and the OPO, highly reflective at the pump wavelength and transmissive at the CLF frequency, to reduce the CLF power incident on the OPO to prevent seeding and pump depletion.

At the homodyne detector the beat note between the CLF and LO is detected. This field differs slightly from that of the unmodified coherent locking scheme due to the change in input coupler, this field is now given by

$$\tilde{A}_{\text{trans,mCLF}} = \sqrt{2\kappa^{in/out}} \left[ \frac{\tilde{a}_s}{\tilde{a}_i^2} \right]$$

$$= \frac{2\kappa^{in/out}}{(1 - x^2)\kappa^{tot}} \left[ e^{i\phi_s} \right].$$

This field is demodulated at $\Omega_{CLF}$. This error signal can also be fed back to either the LO or pump phases.

The CLF second harmonic is only used to establish a phase lock between the auxiliary and main lasers. Compared to an original coherent lock with an auxiliary laser, as opposed to an AOM-based scheme, this simplifies the required electronics. The modified coherent locking scheme also

Figure 4.12: Schematic of the modified coherent locking scheme. A second laser is used to generate the CLF, which is frequency doubled, and beat with the pump field to phase lock the CLF to the pump field. The LO is locked to the CLF using an error signal generated from the homodyne.
has no requirements on the reflectivity of the second flat mirror, as the pump/squeezing input coupler is also used as the CLF input coupler. This allows access to higher OPO escape efficiencies. The modified coherent locking scheme was used in one of the two in-vacuum OPO experiments presented in chapter 6.

4.5 Conclusion

This chapter has introduced several nonlinear optics concepts and briefly introduced control systems. A formalism for fields propagating in optical cavities was derived and extended to cavities with a nonlinear element. The process of optical parametric oscillation was introduced, and shown to produce a field with the noise variance in one quadrature reduced while the noise variance in the opposite quadrature is increased. This is how squeezed light is produced in all of the experiments presented in this thesis. Finally several types of control systems were introduced, and the key components of a squeezer combined to introduce the basic experimental configurations of the main experiments presented in later chapters of this thesis.
5.1 Overview

This chapter describes some of the optical and mechanical design choices made when building the squeezer optical parametric oscillators (OPOs) presented in this thesis. The OPO is the heart of the squeezer and consists of a nonlinear medium surrounded by an optical cavity to resonantly enhance the interacting fields. Section 5.2 discusses the nonlinear crystal parameters including material, phase matching method, dispersion compensation and coating loss. Section 5.3 discusses the design choices made with respect to the optical cavity geometry: the advantages of a bowtie configuration, and the choice of the dimensions of the bowtie, considering beam spot sizes, cavity stability, higher order mode content, and tolerances on mirror alignment. Section 5.4 discusses the choice of mirror reflectivities in the cavity, in particular the input coupler of the cavity, which sets the coupling, escape efficiency, and threshold. We also consider the choice of coherent locking scheme, however this, and the strengths and weaknesses of a glass-based mechanical construction compared to a metal-based construction, will be further discussed in the next chapter. This chapter will be written with the context of the 1064 nm in-vacuum OPO design choices; the operation and results from this squeezer, and the 1984 nm squeezer are discussed in chapters 6 and 7 respectively.

Part of the work presented in this chapter is summarised in:


5.2 Nonlinear crystal considerations

The nonlinear element is the key component of the OPO. The principles behind the use of optical parametric oscillators to generate squeezing are detailed in section 4.3, here the focus is on the engineering specifications of nonlinear crystals for our particular application. First the crystal material is considered, a decision based on the damage threshold, nonlinearity and prevalence of photothermal effects. Then the crystal dimensions are discussed, including the optimal length for the nonlinear interaction and the design of a wedged crystal for dispersion compensation.
5.2 Nonlinear crystal considerations

Table 5.1: Summary of periodically poled crystals. Crystal nonlinearities can be found in [139], other crystal parameters can be found in [56]. \( n \) is the refractive index, \( d_{\text{eff}} \) is the effective nonlinearity, \( \alpha \) is the absorption per unit length at 1064 nm.

<table>
<thead>
<tr>
<th>Material</th>
<th>( n_{1064\text{nm}}/n_{532\text{nm}} )</th>
<th>( d_{\text{eff}} ) [pm/V]</th>
<th>( \alpha ) [%/cm]</th>
<th>Dam. Thresh. [W/cm(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPMgO:LN</td>
<td>2.1494 / 2.2279</td>
<td>15.9</td>
<td>&lt; 0.1 [5]</td>
<td>&gt; 400 [117]</td>
</tr>
<tr>
<td>PPKTP</td>
<td>1.8296 / 1.8868</td>
<td>9.3</td>
<td>0.02 [173]</td>
<td>89 k [124]</td>
</tr>
<tr>
<td>PPLT</td>
<td>2.1399 / 2.2078</td>
<td>8.8</td>
<td>0.1 [4]</td>
<td></td>
</tr>
<tr>
<td>PPKN</td>
<td>2.1201 / 2.2041</td>
<td>12.5</td>
<td>&lt; 0.5 [30]</td>
<td></td>
</tr>
</tbody>
</table>

5.2.1 Choice of nonlinear material

The first experiment demonstrating squeezed states of light used four wave mixing in a sodium gas cell [141]. In 1986 magnesium oxide doped lithium niobate (MgO:LN) was used to produce 3dB of squeezing in the first nonlinear crystal based experiment [180]. Since then nonlinear crystals have prevailed as the dominant technique for squeezed state production due to their compactness and convenience. With the development of the periodic poling technique, quasi-phase matching was developed, allowing access to crystal axes with higher nonlinearity at lower phase matching temperatures [7]. A summary of quasi phase matching is presented in section 4.3.2. Periodically-poled potassium titanyl phosphate (PPKTP) and periodically-poled lithium niobate have emerged as standard crystals in degenerate sub-threshold OPOs used for production of vacuum squeezing.

Parameters for various periodically poled crystals are shown in table 5.1. Note that measurements of crystal absorption are difficult to obtain, especially at longer wavelengths, as the absorption is extremely low.

Despite the high damage threshold of KTP, it does exhibit a photothermal effect known as grey tracking when exposed to high intensity light, particularly at shorter wavelengths in the visible and UV. This effect has been documented in second harmonic generation from 1064 nm to 532 nm [15, 130, 162]. Heating of the crystal reverses the grey-tracking photorefractive damage.

Another property many nonlinear crystals can exhibit is green induced infra-red absorption, whereby the absorption of the crystal in the infrared is increased by up to a factor of 35 in the presence of a green field [71]. In lithium niobate the effect can be eliminated by the additional doping of magnesium oxide, while in PPKTP the effect can be reduced depending on the growth method of the crystal, and is only present at extremely high green intensities.

Ultimately PPKTP is chosen as the nonlinear crystal in all squeezer OPOs presented in this thesis for its high effective nonlinearity \( (d_{\text{eff}}) \), low phase matching temperature, low absorption, and high damage threshold. High \( d_{\text{eff}} \) allows high levels of squeezing to be achieved for lower pump powers. The periodic poling technique yields easily accessible phase matching temperatures, simplifying the electronics required to control crystal temperature. PPKTP has low absorption at the infrared wavelengths critical for these squeezing experiments (1064 nm and 1984 nm), minimising intra-cavity loss and increasing achievable squeezing levels.

5.2.2 Optimum crystal length for squeezing

In the case of an ideal lossless nonlinear interaction the optimal interaction length would be such that all the pump photons are converted to fundamental photons, intuitively this would best be achieved with a long nonlinear medium such as an optical fibre. Optical fibre has been doped and used as a nonlinear medium, however in practice fibre exhibits high optical loss and scattering compared to nonlinear crystals. Introducing a resonant cavity around a nonlinear crystal increases
Chapter 5 Design considerations for squeezer optical parametric oscillators

Figure 5.1: Waist size as a function of crystal length for 1064nm/532nm nonlinear processes in KTP. Calculated using equation 5.2 with refractive indices from table 5.1. Squeezing experiments at 1064 nm in this thesis used 10 mm long crystals.

The effective interaction length while introducing lower loss compared to fibre-based systems.

The conversion efficiency of a nonlinear medium is dependent on the parameters of the pump beam, including power density and waist position. If the pump power density is too high the crystal and coatings are at risk of photorefractive effects and damage, while if the power density is too low the nonlinear effect will be weaker. This balance is a primary consideration when setting crystal lengths and beam waist sizes when designing a nonlinear optics experiment. The focus of a Gaussian beam is often represented by the Rayleigh range \( z_R \), the distance from the beam waist at which the beam radius increases by a factor of \( \sqrt{2} \),

\[
z_R = \frac{\pi \omega_0^2}{\lambda}.
\] (5.1)

A beam of small waist diverges quickly, which limits the possible length of the crystal while a beam of large waist may not reach sufficient power density for nonlinear interaction.

The optimal focusing of the pump beam in the nonlinear crystal is discussed by Boyd and Kleinmann [20], the optimum ratio of crystal length to Rayleigh range is \( \xi = l/(2z_R) = 2.84 \), where \( \xi \) is known as the Boyd-Kleinmann focusing parameter. This can be rearranged in terms of more familiar waist sizes and wavelengths to give

\[
\omega_{0,b} = \sqrt{\frac{l_c \lambda_p}{2\pi n_b \zeta}},
\] (5.2)

where \( l_c \) is the crystal length, \( n_b \) is the crystal refractive index at the pump wavelength, and \( \zeta \) is 2.84 for optimum conversion. Figure 5.1 shows how the optimal waist size changes as a function of crystal length.

In the 1064 nm glass squeezer the crystal is 10mm long, and the optimal waist size of 18 \( \mu \)m. The 1984 nm squeezer the crystal is 12 mm, giving an optimal pump size of 26 \( \mu \)m.
5.2 Nonlinear crystal considerations

5.2.3 Periodic poling and phase matching

Phase matching and the periodic poling technique were discussed in section 4.3.2. The length of the poling period is determined by the coherence length of the pump and fundamental beams. The crystal refractive index changes as a function of wavelength, thus for crystals operating at different wavelengths the poling length will differ.

The poling length as a function of wavelength and refractive index is given by

$$\Gamma_0 = \frac{m \lambda_f}{2 \Delta n} = \frac{m 2 \pi}{\Delta \kappa},$$

(5.3)

where \(m\) is the order of the periodic poling (typically \(m = 1\)), \(\lambda_f\) is the fundamental wavelength, \(\Delta n\) is the difference in refractive index between the pump and fundamental, and \(\Delta \kappa\) is the phase mismatch between the pump and fundamental.

A crystal with given poling length will provide adequate phase matching for a specific wavelength. Beyond the design wavelength the poling length is no longer optimal and some phase mismatch will be introduced. Nonlinear conversion is proportional to \(\text{sinc}^2(\frac{\Delta \kappa L}{2})\), where \(L\) is the length of the crystal. The phase matching dependence on wavelength is analogous to that of temperature.

5.2.4 Crystal wedge for dispersion compensation

Like previous ANU OPO designs [41], a dual resonant cavity was employed for both the 1064 nm VOPO and the 1984 nm OPO, where both the pump and fundamental fields are resonantly enhanced. Making the cavity dually resonant lowers the requirements on the pump power. Operating the cavity at a dual resonance point adds an additional complication to the cavity design – the cavity needs to be held at the dual resonance point while the crystal is at the correct phase matching temperature. In the absence of all dispersion the cavity would be dual resonant without any compensation as the fundamental wavelength is exactly twice that of the pump. Realistically the nonlinear crystal, mirror coatings and air all have wavelength dependent refractive indices, which results a different resonance condition for the pump and fundamental.

To compensate for dispersion the nonlinear crystal used was wedged, as pictured in figure 5.2. A section of the crystal is left unpoled and cut at a wedge, so that the crystal is translated laterally across the beam, the path length through the crystal is changed, allowing for the dispersion of the cavity to be selected. Leaving the section unpoled ensures that the beam does not exit the crystal across multiple polling regions, or on the border between two poling regions. The phases of the pump and fundamental exiting the crystal change as a function of crystal length, and at certain lengths they are aligned. At these points through the crystal dual resonance occurs.

To ensure against the possibility of defects in the crystal or damage to the crystal coating, the wedge angle is set such that there are several paths through the crystal that correspond to dual resonance. The distance between consecutive dual resonances across the crystal is given by

$$d_{\text{trans}} = \frac{\lambda_f}{\Delta n \times \theta_{\text{wedge}}},$$

(5.4)

where \(\Delta n\) is the refractive index difference between pump and fundamental. In the 1064 nm OPO a crystal wedge of 1.15° was sufficient, however for the 1984 nm OPO the refractive index difference between pump and fundamental is roughly a factor of 4 smaller. Increasing the crystal width is expensive and would interrupt the beam at the centre of the bow tie while increasing the angle of the bowtie would introduce further astigmatism. Hence a steeper wedge is required to produce the same number of dual resonances across a crystal of the same width.
5.2.5 Operation under vacuum

For optimum conversion efficiency the crystal is positioned in the beam such that there is a dual resonance at the peak of the phase matching curve. The temperature is then tuned to find the dual resonance operating point. However as the atmospheric pressure changes, such as when pumping down to vacuum, the refractive index of air changes, shown in figure 5.3.

As the refractive index of air is dependent on pressure and wavelength, a slight round-trip path length change, different for the pump and fundamental, is introduced going between air and vacuum. Pumping down from atmospheric pressure (roughly 101 bar) to the high vacuum under which the VOPO was operated (1 × 10⁻⁶ mbar) corresponded to a refractive index change of \( \Delta n = 4.1517 \times 10^{-6} \). This shift is appreciable enough that this dual resonance temperature will no longer be at the optimum phase matching temperature, and hence lower nonlinear gains achievable.

By adjusting the crystal position, a refractive index difference \( \Delta n_{\text{vac}} \) can be corrected with lateral crystal translation \( \Delta x \). The translation required for one pump wavelength free spectral range is given by

\[
\Delta x_{\text{vac},\text{FSR}} = \frac{\Delta n_c}{L_c \tan \theta_{\text{wedge}}},
\]

where \( \Delta n_c \) is the refractive index difference between the pump and the fundamental of the nonlinear crystal. The cavity length change when pumping down to vacuum is given by

\[
\Delta x_{\text{cav}} = \frac{\Delta n_{\text{vac}}}{L_{\text{cav}}}. \tag{5.6}
\]

This length change, modulo the green FSR (given by \( \lambda_{2v} \)) is compensated for by adjusting the crystal position. Note the crystal need not be positioned back at the original dual resonance point, only to the nearest dual resonance point.

In a singly resonant cavity this effect would not be a problem as the temperature of the crystal is only set to maximise nonlinear gain. However in a dual resonant system there are only discrete temperatures for which dual resonance occurs which shift as the cavity is put under vacuum, while the optimal phase matching temperature for the crystal remains constant, shown graphically in figure 5.4.

In initial measurements taken under vacuum the crystal was adjusted manually to the approximate in-vacuum position before pumping down. Since then remote actuation has been achieved using pico-motor controlled screws in the oven vacuum mounts, allowing for the crystal position to be optimised while the experiment is under vacuum.
5.2 Nonlinear crystal considerations

**Figure 5.3:** Difference in refractive indices of air at 1064 nm and 532 nm as a function of pressure. The red dashed line indicates atmospheric pressure. At low pressure the refractive index difference asymptotes to zero.

**Figure 5.4:** The effect of evacuation on dual resonance temperature. The blue sinc function, and black markers show the phase matching curve of the nonlinear crystal; the red lines show the free spectral range of 1064 nm with the crystal positioned such that the dual resonance condition overlaps with the peak of the phase matching curve in air. When the system is pumped down to vacuum, the dispersion of the cavity changes and the dual resonance temperatures shift by 1.4 FSRs shown by the yellow line. The transition from air to vacuum is indicated with a black arrow.
5.2.6 Effect of crystal coating loss

There are two traditional options for optical coatings – electron beam evaporation (e-beam) and ion beam deposition (IBS) – the former is cheaper, but the optical losses in the coating greater. The reflectivity of e-beam coatings may also be dependent on the humidity of the air. For antireflective coatings e-beam loss can be up to 0.5% while IBS can be up to 0.1% (as quoted by the manufacturer Raicol). Intra-cavity loss is, unsurprisingly, critical for squeezing, as the squeezed beam encounters each source of loss multiple times, compounding the effect. Intra-cavity loss scales linearly with the escape efficiency of the cavity (as defined in section 5.4.1), but squeezing escaping the cavity scales exponentially with loss, as shown in figure 5.9 with reference to the choice of cavity mirror reflectivities.

5.3 Cavity geometry

This section summarises the design choices of the OPO cavity, including choice of geometry (travelling wave vs standing wave), cavity dimensions and mirror radii of curvature. These choices are made by taking into account backscattered light, cavity stability, higher order mode spacing and mirror alignment tolerances.

5.3.1 Bowtie cavity motivation and effect of backscattered light

A squeezed light source for the purpose of gravitational-wave detection must produce squeezing in the audio frequency band. This presents a considerable technical challenge as many sources of classical noise are present in the audio band, such as stray light at the main laser frequency scattered into the squeezed beam. Experiments thus are designed to first produce as little scattered light as possible, and to be immune to the presence of scattered light. The choice of cavity geometry is inherently linked to the production of scattered light. There are merits to both linear and travelling-wave geometries. In the case of the GEO600 squeezer a compact linear hemilithic cavity was chosen [89]. The linear cavity has fewer surfaces and hence is less susceptible to imperfections on the mirror surface, and can have lower intracavity loss, allowing for improved cavity escape efficiency.

The primary advantage of the bow tie cavity is its improved optical isolation from backscatter and back reflections. Preventing spurious beams incident on the OPO from re-entering the interferometer reduces parasitic interferences contaminating the gravitational wave signal. The backscatter noise requirement in advanced interferometers is strict: nominally a factor of 10 below the squeezed quantum noise [13]. The relative intensity noise (RIN) of the backscattered light, relative to quantum noise is given by [61]:

\[
\frac{RIN_{BS}}{RIN_{QN}}(f) = 4\pi \delta z_{BS}(f) \frac{\eta_{PD} P_{BS}}{\lambda h c},
\]

where \(\delta z_{BS}\) is the motion of the OPO relative to the interferometer, \(\eta_{PD}\) is the quantum efficiency of the detection photodiode, and \(P_{BS}\) is the power in the backscattered light that reaches the detection photodiode. The backscattered power is given by

\[
P_{BS} = P_{BS, inc} \times R_{OPO} \times \eta_{loss},
\]

where \(P_{BS, inc}\) is the power incident on the OPO from the detection port, \(R_{OPO}\) is the reflectivity of the OPO and \(1 - \eta_{loss}\) is the loss between the OPO and the read out photodetector.

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Equations 5.7 and 5.8 show that to minimise the impact of backscattered light we can either reduce the power incident on the OPO from the IFO, reduce the reflectivity of the OPO, or reduce the motion of the OPO relative to the IFO.

The effect of backscatter noise is reduced by both optical and seismic isolation, however the requirements for optical isolation are more stringent. In a travelling wave cavity, the physical separation of input and output beams mean that any spurious back reflections incident on the OPO from the interferometer path will not retroreflect back into the interferometer. The bow tie cavity provides at least 40 dB of optical isolation [41]. In the GEO detector two Faraday isolators are placed between the squeezer and the interferometer, each with intrinsic loss which reduces the amount of squeezing entering the interferometer.

The disadvantages of the bow tie geometry are an increased number of optical surfaces in the cavity, increased loss, and beam astigmatism. Highly reflective mirrors coated with the ion beam sputtering (IBS) technique are readily available, and typically have losses on the order of 0.01%. The astigmatism of the cavity mode is due to the non-normal incidence of the beam on the cavity mirrors, and is dependent on the angle of the bow tie. Keeping the half-angle of the bow tie down to 6 degrees allows for a very small ellipticity in the beam, as shown later in figure 5.7, which is easily matched to the interferometer beam.

5.3.2 Mirror spacings, stability and spot size

When designing the cavity geometry the spot size in the crystal, stability and footprint are considered. The parameters which can be set in a bowtie cavity are the mirror radius of curvature, flat mirror separation, and curved mirror separation, shown diagrammatically in figure 5.5. To minimise astigmatism cavity angle is set to be as acute as possible, while preventing the diagonal beams from being interrupted by the long edge of the nonlinear crystal, and to allow for beam clearance of the components of the cavity. This angle was chosen to be 6° which introduces sufficiently low beam astigmatism.

A cavity model using ray tracing ABCD matrices was made and verified using OptoCad [133]; methods detailed in appendix A. The waist size of a Gaussian beam scales linearly with the wavelength, hence when constructing a cavity which has the same waist size for a longer central wavelength, such as when going from 1064 nm to 1984 nm, larger focusing power and some modifications to cavity geometry are necessary.

![Figure 5.5: Bowtie cavity footprint, showing VOPO tombstone and mirror style, with labelled dimensions, mirror radii of curvature. Note that in the bowtie configuration there are two waists, \( \omega_c \) and \( \omega_{ext} \), or the primary are secondary waists, half way between the two curved mirrors and two flat mirrors respectively.](image)

The radii of curvature of the two curved mirrors in the cavity were chosen to be \(-50\) mm as this is a standard component, and focuses the beam to the optimal waist size with a reasonable separation of curved mirrors. Figure 5.6 shows contours of the waist size in the crystal as a
The range of possible curved and flat mirror separations is then limited by the footprint of the cavity. If the curved mirror separation is too short, the crystal will interrupt the cavity mode when it is fully inserted into the cavity, limiting the range of useful crystal positions in the beam. Based on these limitations the minimum curved mirror spacing is 57mm and the minimum flat mirror separation is 90mm.

Figure 5.6 shows how the waist size is strongly dependent on curved mirror separation, and only weakly dependent on flat mirror separation. As the cavity is operated further from optimal stability sensitivity to curved mirror separation increases. To achieve the desired waist size of 30µm in the VOPO a curved mirror separation (free space, not including the optical path through the crystal) of 60.42 mm and a flat mirror separation of 110 mm are chosen.

Waist size astigmatism in the secondary waist must also be considered when selecting mirror separation. Figure 5.7 shows the primary and secondary waist sizes as well as the stability parameter, \( m \), as a function of curved mirror separation, for a given flat mirror separation. There is an optimal curved mirror separation for astigmatism in the beam, where the waist sizes in each plane are closest.
Figure 5.7: Primary waist, secondary waist and cavity stability as a function of the curved mirror separation. Solid lines are sagittal plane waist sizes, dashed lines are tangential plane waist sizes. For the bottom plot, the line of stability for the fundamental sagittal case is behind the line of pump stability in the sagittal plane.
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Figure 5.8: Model of higher order mode spacing given cavity parameters shown in table, for the OPO cavity at 1984 nm (left) and 992 nm (right). Number indicates the order of the $m, n^{th}$ mode, power in each mode $1/n$ of the fundamental which is normalised, this assignment is arbitrary. The round trip accumulated Gouy phase of the cavity at 1984 nm is 112.93 degrees in the sagittal plane, and 117.93 degrees in the tangential plane, and at 992 nm the accumulated Gouy phase is 116.3 degrees in the sagittal plane and 111.75 degrees in the tangential plane. Note that as the mode order increases the accumulated phase for tangential and sagittal drift further apart.

5.3.3 Higher order mode spacing

Another factor considered when designing the cavities is the higher order mode spacing. In theory a cavity can be aligned such that only the TEM$_{00}$ lowest order Hermite-Gaussian mode is resonant. In practice any resonator can support a complete family of higher order Hermite-Gaussian modes. This can be problematic if a higher order mode frequency overlaps with the fundamental mode, in this case there is potential for some of the higher order mode to be transmitted by the cavity when the cavity is locked on the fundamental. Higher order mode content will be present in the pump transmission and the squeezing mode, causing problems when mode matching the local oscillator to the squeezed beam at the homodyne.

Power couples to higher order modes via two processes: mode mismatch between the cavity eigenmode and the incident beam, and misalignment of the cavity mirrors. The higher order mode resonance frequency can be written as a function of its accumulated Gouy phase:

$$\omega_{qnm} = \frac{2\pi c}{L_{cav}} \left( q + (m + n + 1) \times \frac{\Delta \phi}{\pi} \right), \quad (5.9)$$

where $m, n$ indicate the order of the Hermite-Gaussian mode, and $q$ is an integer, $L_{cav}$ is the cavity round trip length, $\phi$ is the Gouy phase of the higher order mode relative to the fundamental. The $n,m^{th}$ higher order mode accrues $(n + m) \times \Delta \phi$ degrees of phase shift.

To ensure that on resonance the mode of the squeezed beam is purely in the fundamental the cavity geometry must be chosen such that the Gouy phase of low order modes, which are more likely to have extra power, does not overlap them with the fundamental. Figure 5.8 shows the phase shift of the first 6 higher order modes with respect to one free spectral range, at the pump and fundamental of the VOPO. The $4^{th}$ order higher order mode overlaps with the fundamental, which is suboptimal. Mode overlap is hard to avoid with such a low-finesse cavity.

While setting the higher order mode spacing optimally in theory, the accumulated Gouy phase is strongly dependent on curved mirror separation, and so small deviations in the curved mirror
position can move the higher order mode spacing sub-optimally. This was shown in the second VOPO, where a 1mm shift in the curved mirror separation caused the third order higher order mode to overlap with the fundamental. For the 2 µm OPO the higher order mode spacings of pump and fundamental were considered when selecting cavity mirror spacings. This will be discussed in chapter 7.

5.4 Mirror reflectivities

Another set of parameters of the OPO cavity are the mirror reflectivities, in particular the input/output coupler whose reflectivity at the pump and fundamental set the escape efficiency and threshold power. Both the escape efficiency and threshold power of the cavity decrease with increasing input coupler reflectivity, creating a trade-off for optimum squeezing. Ideally a squeezer OPO has high escape efficiency and low threshold.

The second flat mirror of the cavity has, in some squeezer designs, been used as a second input coupler for the coherent locking field (CLF) and as such has slightly lower than standard high reflectivity [144]. In this section these design choices will be addressed.

5.4.1 Escape efficiency

Escape efficiency is a measure of the quantum efficiency of an optical cavity, and is defined as the ratio of the decay rate of the input coupler to the total decay rate of the cavity. The escape efficiency is given by

$$\eta_{esc} = \frac{\kappa_{in/out}}{\kappa_{total}} = \frac{\kappa_{in/out}}{\kappa_{loss} + \kappa_{in/out}},$$

(5.10)

where \(\kappa_{in/out}\) is the input/output coupler decay rate, and \(\kappa_{total}\) is the decay rate of all the loss in the cavity, including the input coupler. The mirror decay rate was defined in equation 4.5, but repeated here for reference:

$$\kappa_i = -\frac{1}{2\tau} \ln(R_i),$$

(5.11)

where \(\tau\) is the cavity round trip time, and \(R_i\) is the reflectivity of the \(i^{th}\) mirror where \(i\) can indicate the physical cavity mirrors or also the effective reflectivity of other sources of loss in the cavity such as the crystal (see the cavity model in figure 4.1). For moderate finesse cavities the decay rate can be approximated as

$$\kappa_i = \frac{1 - \sqrt{R_i}}{\tau}.$$

(5.12)

The importance of escape efficiency in squeezed state generation is shown in figure 5.9 which shows the squeezing and anti-squeezing levels as a function of cavity escape efficiency. At low escape efficiency squeezing (and anti-squeezing) are not coupled out of the cavity efficiently enough.

Measuring escape efficiency is useful, as calculation based on coating reflectivities and crystal loss has large uncertainty. The measurement of escape efficiency is described in section 6.3.2

5.4.2 Threshold

The threshold power of the optical parametric oscillator was defined in section 4.3.4. As the nonlinear gain factor approaches the cavity decay rate, the nonlinear gain asymptotes to infinity.
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Figure 5.9: Squeezing and anti-squeezing as a function of OPO escape efficiency, assuming no other intra-cavity loss. To produce as high levels of squeezing as possible escape efficiencies of close to 100% are required, for example the first VOPO escape efficiency was measured to be \(97.9 \pm 0.1\%\).

The input pump power which is required to meet this condition is known as the threshold power. The threshold power is defined in terms of the cavity coupling rates,

\[
P_{\text{crit}}^b = \left( \frac{\kappa^a \kappa^b}{\epsilon \sqrt{2 k_{\text{in/out}}^b}} \right)^2 \frac{hc}{\lambda^b}.
\]  

(5.13)

To produce squeezed vacuum states the OPO is operated in a sub-threshold regime. Above threshold the number of fundamental photons produced increases, creating state with a coherent amplitude. The closer the OPO is to threshold the higher nonlinear gains can be achieved, hence a greater number of fundamental photons are produced, and the greater squeezing levels can be achieved. When designing the OPO cavities presented in this thesis mirror reflectivities are chosen to be high enough for a low threshold power (on the order of 10s of mW), but low enough to have a high escape efficiency (>97%). Hence squeezing is typically generated inside a moderate-finesse cavity. The highest squeezing levels presented to date use a very low threshold cavity [165].

Figure 5.10 shows the trade-off between escape efficiency and threshold in the case of the VOPO. An input coupler reflectivity of 84% was chosen, which theoretically corresponds to a threshold of 160 mW and an escape efficiency of 97.7%.

5.5 Tolerances on mirror parameters

Tolerances on the alignment of the cavity mirrors are important when constructing any optical cavity. In this section we discuss the tolerances on the positions and angles of the four OPO mirrors. The glass OPO had additional requirements on the mirror position and angle, compared to the standard metal OPO, due to the fixed vertical alignment of the mirrors, as will be discussed.
5.5 Tolerances on mirror parameters

As shown in figure 5.6, the waist size in the cavity is strongly dependent on the curved mirror separation and weakly dependent on the flat mirror separation. This holds true for other cavity parameters including stability and transverse mode spectrum. For example, the transverse mode spectrum of the glass OPO can have a significant overlap between the fundamental and a third order mode with a shift in the curved mirror separation of merely 0.5 mm. Thus to be within a safety factor of 2, a 250 µm tolerance on the position of the curved mirrors was set. Conversely, flat mirror separation is barely critical to the waist size and transverse mode spectrum, and their separation only needs an accuracy of ±5 mm.

The OPO cavity is comparatively forgiving to angular misalignments of the mirrors. In the case of an angular misalignment of a single mirror, the other mirrors can be also slightly misaligned to compensate. While this kind of misalignment will still allow for a cavity eigenmode to form, this eigenmode will differ from the ideal mode of 6 degree bowtie angle, parallel to the plane of the board, with modelled waist sizes. There are hard limits on the path of the cavity eigenmode, set by the crystal aperture and the mirror aperture, particularly for beams transmitted through the mirrors and tombstones. The latter requirement is stricter for the glass OPO compared to previous OPO designs because the mirrors were glued to tombstone mounts, narrowing the aperture the beam must clear.

To model the angular alignment tolerances of the mirrors, each mirror angle was deviated from optimal, and the effect on the position and angle of the eigenmode beam on the subsequent mirrors modelled using ray tracing techniques, as done in [169]. The relation between angular deviation of a single mirror and beam angle and spot position at all mirrors is encapsulated in the matrix.
Chapter 5 Design considerations for squeezer optical parametric oscillators

derived in [169] and repeated here:

\[
\begin{bmatrix}
  x_1 \\
  \theta_1 \\
  x_2 \\
  \theta_2 \\
  x_3 \\
  \theta_3 \\
  x_4 \\
  \theta_4
\end{bmatrix} =
\begin{bmatrix}
  0.14 & -0.016 & -0.14 & 0.089 \\
  0.59 & 1.41 & 2.07 & -2.07 \\
 -0.016 & 0.14 & 0.089 & -0.14 \\
 -1.41 & 1.41 & 2.07 & -2.07 \\
 -0.13 & 0.089 & 0.27 & -0.32 \\
 -1.414 & -0.59 & 2.07 & -2.07 \\
 0.089 & -0.14 & -0.32 & 0.27 \\
 4.11 & -4.11 & -10.61 & 10.61
\end{bmatrix}
\begin{bmatrix}
  \alpha \\
  \beta \\
  \gamma \\
  \delta
\end{bmatrix},
\]

(5.14)

where \( \alpha, \beta, \gamma, \) and \( \delta \) are angular deviations of the first flat mirror, second flat mirror, first curved mirror, second curved mirror, labelled as mirror 1, 2, 3, and 4 respectively in figure 5.5. Equation 5.14 shows that angular misalignment of each mirror in turn affects the spot positions of the cavity eigenmode on each of the four mirrors consistently, with the curved mirror angles generally more critical than the flat mirrors. Similarly for three of the four mirrors the angle of the beam off the mirror is affected consistently by misalignments, with the curved mirror angles being slightly more important. The angle of the beam from mirror 4 is most strongly affected by the misalignment of the mirrors, and in particular by misalignment of the curved mirrors.

5.6 Choice of coherent locking scheme

To produce stable vacuum squeezed states in the audio band a control scheme is required to lock the squeezing angle, defined by the phase of the pump light entering the OPO, to the homodyne local oscillator phase. This is typically done using a coherent locking scheme, where a bright proxy beam is injected into the OPO and experiences the same phase jitter as the squeezing beam, and is detected on the homodyne. Section 4.4.3 summarises two variations on the coherent locking scheme.

In the context of cavity OPO design the choice of coherent locking scheme defines which cavity mirror acts as the input coupler for the coherent locking field. The original scheme uses a different input coupler from the pump and squeezing input coupler, requiring one of the cavity mirrors to be slightly reduced from maximal reflectivity to allow for the coherent locking error signal to be detectable on reflection. This has the disadvantage of lowering the cavity escape efficiency, however the required reduction in reflectivity is low.

The modified coherent locking scheme uses the pump and squeezing input coupler as the coherent locking field input coupler, allowing the other 3 mirrors to be of maximal reflectivity.

5.7 Chapter summary

This chapter has outlined the design considerations for squeezer optical parametric oscillators that have been taken into account when designing the experiments presented in this thesis. Nonlinear crystal considerations set physical characteristics, cavity geometry sets the cavity eigenmode, the cavity stability and higher order mode spacing, optical characteristics of the OPO are set by the mirror reflectivities. All of these parameters are optimised to generate squeezing in the OPO and efficiently couple it out of the cavity. The work presented in this thesis will be drawn on in the following chapters on the in-vacuum glass-based squeezer OPO, and the 2 \( \mu \)m OPO.
Development of a glass-based squeezed light source for Advanced LIGO

This chapter details the motivation, design, construction, and results from the in-vacuum glass-based OPO (VOPO) experiments. The VOPO was designed to meet Advanced LIGO’s strict requirements on a squeezed light source, in terms of length stability, phase noise performance, vacuum compatibility, and squeezing level. Section 6.1 provides motivation for the in-vacuum glass-based OPO. Section 6.2 summarises the mechanical construction and optical design. Section 6.3 summarises the characterisation of the length noise, escape efficiency, and threshold of the VOPO. Section 6.4 provides an overview of the experiment operation. Finally section 6.5 presents the squeezing results and phase noise performance of the VOPO.

Two VOPO cavities were constructed: a proof-of-principle experiment at the ANU with only the OPO under vacuum, henceforth known as VOPO1, and a second experiment at MIT for injection into a long baseline filter cavity squeezing ellipse rotation experiment, referred to as VOPO2. The VOPO2 additionally placed the homodyne detector under vacuum with the OPO.

The content of this chapter is summarised in the papers:


6.1 Motivation

To date, squeezed vacuum injection has been demonstrated in both the GEO600 [148] and initial LIGO Hanford detectors [149]. The LIGO H1 squeezing experiment achieved 2.15 dB of squeezing improvement below shot noise, which limited the experiment at Fourier frequencies above roughly 300 Hz. The GEO600 squeezing injection effort is ongoing, 3.5 dB of squeezing was reported in [148] and the highest squeezing improvement to date is 4.4 dB.

To increase the level of squeezing improvement and to develop a squeezed light source appropriate for second-generation gravitational-wave detectors, several factors need to be addressed:
6.1 Motivation

loss, phase noise (or quadrature fluctuations), and scatter noise. The vacuum compatible glass OPO was designed to address the requirements of such detectors, as summarised in [121].

The GEO and initial LIGO squeezing injection experiments had high levels of optical loss encountered by the squeezed beam – 38% and 56% respectively. The effect of loss on squeezed states is discussed in section 3.3.4. The sources of loss include mode mismatch between the squeezed beam and interferometer beam, loss of injection optics including Faraday isolators, alignment fluctuations which couple the squeezed beam out of the output mode cleaner (OMC) TEM00 mode, transmission of the OPO and OMC, and imperfect photodiodes. Adaptive mode matching and low loss Faraday isolators are active areas of research for this purpose.

Including the squeezed light source in the main interferometer vacuum envelope will reduce alignment fluctuations and remove some sources of loss originating from the injection optics. The total loss experienced by a squeezed state entering the aLIGO interferometer is expected to be reduced to 20-30% in the near future, which would allow for 6 dB of squeezing improvement [121]. Further reduction to 10% would allow 10 dB of squeezing improvement if achieved in parallel with improved squeezing ellipse phase noise and backscatter noise, shown in figure 6.1.

Sources of phase noise include OPO cavity length noise, sensor noise of various locking fields, alignment jitter between the squeezer and the interferometer, OPO detuning noise and coherent locking sideband noise. The glass OPO addresses the cavity length noise contribution to the overall phase noise by using a rigid, stable, quasi-monolithic cavity design.

Imperfect optics, in addition to introducing loss, are a source of backscatter noise. Spurious reflections and backscatter from scatter points in these optics can couple back into the main interferometer beam. When encountering optical surfaces with motion relative to the interferometer, scattered fields acquire additional phase modulation which then interferes with the interferometer carrier field, adding noise to the gravitational wave signal [40]. The bowtie design of the glass OPO adds inherent backscatter noise isolation, as discussed in section 5.3.1. Additionally, a vacuum-compatible squeezer can be integrated into the LIGO vacuum system, making use of excellent pre-existing seismic and acoustic isolation.

6.1.1 Squeezing, loss, and phase noise

The variances of the field exiting an optical parametric oscillator in the squeezed and anti-squeezed quadratures are defined in equations 4.62 and 4.63 respectively. Including terms for loss and quadrature fluctuations in these equations gives an insight into the importance of these two factors. Loss is added as vacuum noise, where the source of loss can be represented as an optical element with transmissivity $1 - \eta_{\text{loss}}$.

\[ V_{\text{out}} = \eta_{\text{loss}} V_{\text{in}} + (1 - \eta_{\text{loss}}) V_{\text{vac}}, \quad \text{(6.1)} \]

The variances in each quadrature, including loss, may be written as

\[ V_{\text{out,1}} = 1 + \eta \frac{4x}{(\omega/\kappa_{\text{tot}})^2 + (1-x)^2}, \quad \text{(6.2)} \]

\[ V_{\text{out,2}} = 1 - \eta \frac{4x}{(\omega/\kappa_{\text{tot}})^2 + (1+x)^2}, \quad \text{(6.3)} \]

where $\eta$ is the total detection efficiency including external losses and OPO escape efficiency, $x$ is the pump power as a fraction of threshold pump power (which may also be written as $1 - \frac{1}{\sqrt{g}}$, where $g$ is the nonlinear gain), $\omega$ is the carrier angular frequency, and $\kappa_{\text{tot}}$ is the cavity decay rate at the fundamental wavelength.
Figure 6.1: Squeezing level contours as a function of phase noise and loss, in this model the pump power fraction, \( x \), is optimised for each loss and phase noise value, the power reflectivity of the OPO input coupler at the fundamental, \( R_a = 0.85 \), is assumed to be the same as the glass OPO. Coloured boxes indicate loss levels for enhanced LIGO H1, GEO600, aLIGO expected and future aLIGO squeezing injection, from [149], [148], and [121] respectively.

Including squeezing ellipse phase noise in the expression for the squeezed quadrature variances in an arbitrary quadrature yields

\[
V_{\text{tot}}(\theta_{sqz}, \tilde{\theta}_{sqz}) = V(\theta_{sqz})\cos^2\tilde{\theta}_{sqz} + V(\theta_{sqz} + \pi/2)\sin^2\tilde{\theta}_{sqz} \\
= 1 + 4\eta_x \left( \frac{\tilde{\theta}^2}{(\omega/\kappa_c)^2 + (1-x)^2} - \frac{1 - \tilde{\theta}^2}{(\omega/\kappa_c)^2 + (1+x)^2} \right),
\]

where \( \theta_{sqz} \) is the measurement angle, and \( \tilde{\theta}_{sqz} \) is the RMS phase noise, both in radians. Equation 6.4 assumes small angle fluctuations in the squeezing ellipse phase noise. Figure 6.1 shows the level of squeezing as contours as a function of loss and phase noise, and illustrates the importance of phase noise in experiments with low loss. In the case of a squeezed light source for Advanced LIGO, the input loss levels are projected to approach 10%, hence the total squeezing ellipse phase noise must be kept below the few mrad level.

A realistic budget of squeezing ellipse phase noise sources in advanced detectors is presented in [121].

### 6.1.2 Cavity length fluctuations

The motivation for building the VOPO as a semi-monolithic glass resonator is the improved cavity stability compared to a cavity design utilising traditional mechanical mirror mounts. A fluctuation in the OPO cavity length causes a small detuning of the cavity, this translates to phase fluctuations on the generated squeezed field. For a dual-resonant OPO cavity locked using the Pound-Drever-
Hall technique on reflection of the pump field, the change in squeezed quadrature angle as a function of the fluctuation in cavity length is given by [60]

$$\frac{d\theta_{SQZ}}{d\delta L} = \frac{\omega}{\bar{L}} \left( \frac{1}{\gamma^b_{tot}} + \frac{1}{\gamma^a_{tot}(1 + x^2)} \right),$$  \hspace{1cm} (6.5)

where $\omega$ is the angular frequency of the carrier at the fundamental, $\bar{L}$ is the nominal cavity length, $\gamma^b_{tot}$ is the cavity decay rate at the pump wavelength, $\gamma^a_{tot}$ is the cavity decay rate at the fundamental wavelength, and $x$ is the nonlinear interaction strength normalised to the threshold power.

In the case of the H1 squeezing experiment the OPO was constructed from standard low drift optical mounts bolted to a heavy aluminium base plate. The phase noise contribution per nm of cavity length deviation of the H1 squeezer is 90 mrad/nm, and the OPO cavity was estimated to contribute $24.6 \pm 3$ mrad of squeezing ellipse phase noise below 100 kHz [60]. To achieve 6 dB of squeezing improvement in Advanced LIGO, a more stable OPO cavity is required.

The VOPO mechanical design was inspired by the aLIGO output mode cleaner (OMC). The OMC has a length noise of less than $1 \times 10^{-15} \text{m}/\sqrt{\text{Hz}}$ at 100 Hz when operated under vacuum and in a seismically and acoustically isolated environment [6]. If we assume the length noise of stable reference cavity approximately scales as $1/\sqrt{f}$ below 100 Hz, the upper bound on the absolute length noise contribution of the VOPO is estimated to be $10^{-12} \text{m}$. If the VOPO had an equal absolute length noise to the OMC this would translate to $< 0.1$ mrad of RMS phase noise on the generated squeezed field. To reach this goal the OPO was built with similar optical mount design, quasi-monolithically, and using the same low-noise PZT actuators as the OMC.

### 6.1.3 Scatter noise and isolation

When incorporating any subsystem into LIGO, the scattered light impact is evaluated. Sources of scatter on the squeezing injection path include the injection Faraday isolator and other optics between the OPO and the interferometer, as well as the squeezer itself. Scattered light may acquire phase modulation due to the relative motion of optics between the source and the interferometer, such as the motion of the platform which houses the OPO. When scattered or backreflected light is reflected off the OPO it can couple back into the main interferometer mode and add phase and amplitude noise at the frequency of the motion of encountered optics, creating parasitic signals in the audio band which mask the gravitational wave signal.

The importance of scattered light as the motivation for the choice of a bowtie cavity geometry was discussed in section 5.3.1. The angular separation of the input and output fields of a previous ANU-built travelling wave bowtie cavity was measured to provide 41 dB of isolation from backscatter incident on the OPO, and 47 dB of isolation from backscatter originating from inside the OPO [41]. The Advanced LIGO requirement on backscattered light power is expressed as a ratio of the relative intensity noise ($RIN_{BS}$) of backscattered light to the relative intensity noise of the quantum noise of the interferometer ($RIN_{QN}$). Nominally $RIN_{BS}$ must be a factor of 10 below $RIN_{QN}$. As higher levels of squeezing improvement are achieved and the quantum noise is reduced, the requirements on backscattered light become more stringent.

The mitigation of noise due to backscattered light requires both optical and motional isolation. By reducing the relative motion between the squeezer and the interferometer the relative intensity noise of the backscattered light is reduced as demonstrated in equation 5.7. Reduction in squeezer motion is achieved by improving the seismic isolation of the squeezer platform.
Figure 6.2: Solidworks model of the second VOPO. Half inch cavity optics are attached to rectangular tombstones, which have chamfered holes to allow the beam to pass through. Two of the cavity mirrors are PZT-mounted for cavity actuation. The breadboard base is a round polished optical flat while the VOPO1 was on a rectangular breadboard. The crystal oven (copper piece) is attached to the translation stage (Newport New Focus 9071) which is attached to optical flat feet. All components are optically contacted to the breadboard. The bolts attaching the translation stage to the oven legs are not pictured.

6.2 Glass OPO construction

This section focuses on the techniques used to construct the cavity, the mounting and alignment of the mirrors and the nonlinear crystal, the construction tolerances, and the efforts to use only vacuum compatible materials in VOPO. The optomechanical design of the VOPO is summarised in [171]. The design choices behind the optical parameters of a general squeezer OPO cavity are discussed in chapter 5.

The mechanical design of the VOPO followed other ultra-stable in-vacuum experiments including the aLIGO OMC [6] and LISA optical bench testbeds [138], where low resonance frequency tombstone optics are adhered to a stable glass breadboard base. While the LISA testbed tombstone optics have one face optically coated and act as the optical elements themselves, the VOPO cavity has the necessary complexity of curved mirrors and PZT actuators. Two of the VOPO cavity mirrors are curved to focus the beam through the crystal, and at least one mirror requires a piezo-electric transducer (PZT) to control the cavity length. While curved tombstones may be possible their development would be costly. For the initial VOPO build, half inch optics are adhered to the tombstones using degassed epoxy (MasterBond EP30-2). Finite element analysis completed by the LIGO OMC group of a tombstone with attached PZT and mirror gives a 10 kHz resonant frequency. A CAD rendering of the VOPO is shown in figure 6.2.

6.2.1 Optical contacting technique

Solution-assisted optical contacting is an adhesive-free bonding technique used to construct the VOPO. This technique was chosen due to the strength and reversibility of the bonds formed, and the ease of optical alignment while the bonds set. To bond two surfaces, they must have excellent surface flatness and be brought into sufficiently close contact that a covalent intermolecular bond forms between them [78]. Solution-assisted optical contacts make use of an intermediate solvent between the two surfaces. The Van der Waals force between the two surfaces is facilitated by the residual liquid between them. As the liquid evaporates and the surfaces are brought closer,
6.2 Glass OPO construction

<table>
<thead>
<tr>
<th></th>
<th>Side 1</th>
<th>Side 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS flatness</td>
<td>0.032 λ</td>
<td>0.01 λ</td>
</tr>
<tr>
<td>PV flatness</td>
<td>0.155 λ</td>
<td>0.047 λ</td>
</tr>
</tbody>
</table>

Table 6.1: Breadboard flatness measured with \( \lambda = 633 \) nm laser using a Zygo interferometer by Sydor optics.

increasing the surface tension between them. Eventually, as the two surfaces are “wrung” together – their separation is small enough that a covalent bond is formed.

Ideally the RMS surface roughness of the two bonding surfaces is \(<0.5\) nm, though contacts have been formed between surfaces of roughness \(2\) nm [78, 138], this surface roughness figure is typically achieved with high-precision laser grade optical surfaces. The base of the VOPO is a highly polished UV fused silica breadboard (Sydor Optics and Edmund Optics for the VOPO1 and VOPO2 respectively). The flatness of the breadboard measured over every 2” aperture by the manufacturer is detailed in table 6.1. The tombstones are highly polished on the bonding surface with flatness of \(\lambda/10\) and scratch/dig of 10/5. Note that the surface flatness is a larger scale figure of merit than the surface roughness discussed above.

When constructing the two VOPO cavities, 99.9% purity “spectroscopy grade” methanol was used as the intermediary solvent. Using a solvent allows the bond to remain adjustable until the solvent evaporates, “wringing” the two surfaces together [137]. Upon the initial contact between the two surfaces the interface was very slippery, as the liquid evaporates the bond becomes increasingly tacky and more conducive to fine alignment. The time from optic placement on the breadboard to the setting of the bond was several minutes, depending on the volume of methanol between the surfaces. A photo of a practice contact between a mirrorless tombstone and the breadboard is shown in figure 6.3, demonstrating methanol distribution via pipette. The contacts were left overnight, under compression to allow the bonds to fully set. Longer chain solvents can be used and would give a longer evaporation time, and hence more time to position the optic.

The quality of a contact is directly visible. Early in the bonding process, Newton’s rings are visible due to the small gap between the two surfaces. These rings, or fringes, dissipate as the bond forms. An example of the fringes is shown in figure 6.4. The shear strength of an optical contact between two wedged flats, polished to \(\lambda/10\) flatness, was measured to be 0.13 MPa in a rudimentary experiment.

The reversibility of the bonds depended on several factors. Theoretically after a contact is formed, it can be flooded with solvent and gently rocked loose. In practice we found that the ease of removal varied from contact to contact and depended on the lab conditions under which the contact was made and the time between contacting and decontacting. There were several instances in both VOPO constructions where a bond needed to be broken and reformed, for example to optimise the cavity lengths, and after repeated flooding with methanol the bond still held fast. In other cases bonds did not immediately form correctly and needed to be reset. It was found that after repeated de-contacting it became increasingly difficult to form good bonds, such as the bond on the right in figure 6.5. This is expected to be caused by micro scratches on the bonding surfaces which could form with excessive sliding between the surfaces.

A clear view of two contacts is shown in figure 6.5. In future, if optical contacting is to be used in cavity construction, a lever arm tool which sits over the tombstone, without damaging the tombstone and optic, could be useful to allow more efficient breaks of optical contacts. Application of heat may also assist in breaking optical contacts [55].

Alternative techniques for bonding the glass tombstones to the breadboard were considered. In the OMC and for the LISA testbed interferometers with similar construction the hydroxide
Figure 6.3: Photograph of practice optical contacting of a bare tombstone, showing methanol distribution via pipette, alignment mask details and micrometers which provide finely adjustable reference points for the tombstone to rest against. The micrometer configuration shown is for the input/output coupler (M1).

Figure 6.4: Photograph of in-progress optical contact between two 1” wedged optics. The fringes are Newton’s rings due a narrow methanol gap between the two surfaces, they indicate an area where the contact has not yet formed. The width of the fringes is a useful diagnostic to gauge the separation between the two surfaces.
Figure 6.5: Photograph of the footprints of two optical contacts, dark sections are contacted to the breadboard while the pale areas are uncontacted. The input coupler (left) has a small uncontacted defect near the front edge and a wide uncontacted border on three of the four edges of the tombstone footprint. The PZT mirror (right) is a cleaner contact with only uncontacted regions in a narrow border.
catalysis bonding technique (also known as silicate bonding) was used [63]. This forms a strong but irreversible bond between two similar surfaces and is procedurally more complicated than optical contacting. A second alternative adhesion technique considered was UV curable glue, which allows the optics to be positioned while the glue is still slippery. Once the cavity alignment is finalised the glue is set with a UV lamp. A disadvantage of UV-cured glue is that it is harder to reverse.

### 6.2.2 Crystal oven design

The function of the VOPO oven assembly is to maintain the temperature of the nonlinear crystal in air and under vacuum, and to translate the crystal under vacuum for dispersion compensation. Drawings of the initial version of the crystal oven are given in figure 6.6, the full oven construction is shown on the left, and an exploded view of the copper heat sink and crystal holding pieces is shown on the right.

A thermistor is used to read out the temperature, and a Peltier element (or thermoelectric cooler) is used to actuate on the temperature. In order to maintain the set point temperature, the Peltier element draws heat from the reservoir and pumps it to the top copper block.

The oven must be able to translate the wedged nonlinear crystal laterally across the beam to find the dual resonance point that corresponds to the crystal phase matching temperature. The crystal translation must also be able to operate remotely under vacuum, as the optimal path through the crystal is different between in vacuum and in air operation. This effect is due to the lack of dispersion under vacuum, as discussed in section 5.2.5. The initial VOPO oven design, pictured in figure 6.6 was not remotely actuatable and so the crystal was approximately positioned for in-vacuum operation before the chamber was pumped down to vacuum. Later iterations of the VOPO oven used picomotor actuators to accommodate in-vacuum actuation.

When the oven is operated in air, heat is dissipated to the surroundings via convection and radiation. Under vacuum the only heat dissipation mechanism is radiation. The radiative power dissipated by the oven is determined by the Stefan-Boltzmann law,

$$P_{net} = \epsilon \sigma (T^4 - T_{env}^4) \approx 3.1 \text{mW},$$  

(6.6)
6.2 Glass OPO construction

Figure 6.7: First test of the VOPO oven under vacuum. Poor thermal contact resulted in a stable, but poorly damped loop which could not be fixed by adjusting the PID controller parameters. Indium foil was placed between the copper components and the Peltier, thermistor, and crystal allowing for improved thermal contact. After the installation of the indium foil the oven settle time under vacuum was comparable to in air.

where $A = 725 \text{ mm}^2$ is the surface area of the object dissipating heat, $\varepsilon = 0.05$ is the emissivity of copper, $\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ is Boltzmann’s constant, $T$ is the temperature of the oven and $T_{env}$ is the environmental temperature, here taken as 305 K and 294 K respectively. The Peltier is designed to pump up to 2.25 W of heat, and so will easily overcome the 3.1mW of dissipation to maintain the temperature of the crystal.

Delays between the crystal temperature and the thermistor temperature are minimised by placing the thermistor as close to the crystal as possible. For faster reaction times, the size of the top block which houses and clamps the crystal was minimised. In early versions of the oven, the crystal, the Peltier and thermistor were all in direct contact with the copper surfaces of the oven. The surface roughness of the unpolished copper was high enough to limit the thermal conductivity between the crystal, copper, Peltier and thermistor. In air the oven performed well, however once pumped down to vacuum the oven control loop was would oscillate, with a settle time of several minutes, as shown in figure 6.7. The poor thermal contact introduced a delay between the thermistor readout and Peltier actuation. To mitigate this, clean indium foil was placed between the crystal and the copper top block, the Peltier and the copper blocks and the thermistor and the top block to maximise thermal conduction and hence oven response time. This may also help prevent thermal lensing, due to temperature gradients in the crystal.

Thermal expansion of the oven pieces was also considered. As the oven must operate over a large temperature range ($20 - 50^\circ\text{C}$) thermal expansion of the copper and steel components is non-negligible. To prevent these expansions from propagating through the oven legs and putting strain on the optical contact, vacuum compatible plastic washers were used between the translation stage and the legs.
6.2.3 Construction tolerances

The tolerances on alignment of the cavity mirrors for a generic bowtie cavity are discussed in section 5.5. There are additional requirements for the glass OPO, as the tombstone optics do not allow for vertical alignment of the cavity mirrors. Beyond a 1 mrad deviation from perpendicularity the vertical misalignment may prevent a cavity eigenmode from forming (this can be corrected by misalignment of the other cavity mirrors) [111]. This sets a requirement on the perpendicularity of the front face to the base of the tombstone, and the flatness of the glue bond between the tombstone face and the optic. The tombstones used in the glass OPO have 30 arcsecond tolerance on this perpendicularity, so the limiting factor is the glue bond between the optic and the tombstone.

6.2.4 Vacuum compatibility

A key design point of the VOPO is vacuum compatibility. As the second largest vacuum system in the world housing the highest quality precision optics, LIGO has extremely stringent vacuum standards. Allowed materials and quantities of restricted materials are closely monitored and a list of allowed materials can be found in [47].

To our knowledge, the VOPO is the first nonlinear optics experiment of such complexity to be run under vacuum. While the VOPO prototype was tested in a smaller vacuum chamber, effort was invested into maintaining the cleanliness of the VOPO to the LIGO vacuum standard and sourcing LIGO vacuum compatible materials for the VOPO construction. Table 6.2 is a summary of the components of the VOPO, their constituent materials and, where known, vacuum compatibility status.

The thermistor and Peltier used in the current VOPO are so far untested for hydrocarbon outgassing under vacuum. The Peltier consists of an array of bismuth telluride semiconductor blocks sandwiched between two ceramic plates. With the application of current, heat is pumped from one side of the Peltier to the other. The chosen Peltier for this experiment contained no hydrocarbon fillers or grease and is theoretically vacuum compatible. A similar Peltier (Marlow Industries) has been operated under vacuum in the Virgo system for many years as part of the OMC [19].

<table>
<thead>
<tr>
<th>Component</th>
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<th>Product</th>
<th>Material</th>
<th>LIGO Vac approved</th>
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<tr>
<td>Mirrors</td>
<td>Laseroptik</td>
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<tr>
<td>PZT</td>
<td>Noliac</td>
<td>NAC2124</td>
<td>NCE51F</td>
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</tr>
<tr>
<td>Epoxy</td>
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<td></td>
<td></td>
<td>F2A.0909</td>
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</tr>
<tr>
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<td>10kOhm</td>
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</tr>
<tr>
<td>Oven connector,</td>
<td>Accuglass</td>
<td>110000,</td>
<td>glass-filled</td>
<td>Yes</td>
</tr>
<tr>
<td>cable</td>
<td></td>
<td>123502</td>
<td>dyaithilate</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.2: Details of the components of the VOPO, including manufacturer, product number, material and LIGO vacuum compatibility status.

If the Peltier element is found to outgas unacceptable quantities of hydrocarbons, resistive
heating is being considered an alternative method to heat the crystal. The disadvantage of resistive heating is the resistor only has the ability to heat, not cool, and so under vacuum the system would rely on radiative cooling. Equations describing radiative cooling of the oven are given in section 6.2.2.

The oven electrical connectors and cables are made from low-outgassing materials designed for high vacuum operation. The screws and washers are made of PEEK, a plastic which is LIGO-vacuum approved. To facilitate thermal contact between the crystal surfaces, thermistor, Peltier and copper oven pieces indium foil is used rather than thermally conductive paste. Indium foil is allowed in the LIGO vacuum system in limited quantities.

6.2.5 Cavity alignment

An alignment mask was used to position the optics of the VOPO cavities while their optical contacts were forming. The mask, made of 1mm thick aluminium, sat over the glass breadboard resting on PEEK stoppers. Holes were cut at the position of each optic, and a larger hole to accommodate the oven, as shown in figure 6.8, and figure 6.3 with micrometers for fine alignment. The micrometers were attached to the mask, and used in conjunction with mask extrusion points to provide three points of contact for each optic. As demonstrated in chapter 5, the position of the two curved mirrors is critical to the cavity parameters including stability, higher order mode spacing and waist size. The alignment procedure was as follows:

1. The visible pump beam was mode matched to the secondary waist of the cavity, as calculated using an Optocad model.

2. A practice cavity was built, with lens tissue between the optics and the board, allowing for the position of the mask with respect to the board and reference beams to be finalised.

3. The mask position was set with fixing screws, such that the input beam was centred on the first two mirrors.
4. The breadboard and tombstones were cleaned with methanol and lens tissues immediately before placement – both bonding surfaces must be free of dust and other contaminants, otherwise a defect will form in the contact.

5. The mirror M2 was positioned first. 1-2 drops of solvent were placed on the board in the approximate bond location, then the tombstone was placed on the board. While the optic was still able to slide freely it was placed against the micrometers and mask alignment points such that the reflected beam made the correct cavity angle. As the solvent evaporated positioning the optic required increasing force, allowing for increasingly fine adjustments. Once the mirror was positioned a weight was placed on top of the tombstone, pressing the bonding surfaces together while the contact was allowed to set overnight.

6. The first curved mirror M3 was placed in a similar fashion to M2, with the beam aligned to be parallel to the input beam of the cavity.

7. The second curved mirror M4 was placed such that the reflected beam exited at the correct cavity angle and overlapped with the input beam roughly at the position of the input coupler.

8. The final mirror M1, the input coupler, was placed and finely adjusted until the transmitted beams from several cavity round trips were visible. Note that while the VOPO cavity is unstable without the crystal, several passes of the cavity can be seen and a rough alignment of the four mirrors achieved.

9. Finally the crystal oven was placed. The feet of the oven are placed with the oven loosely attached and contacted. As the oven can be translated after contacting, the positioning of the oven feet was less crucial than the optics.

After the cavity was aligned, following this procedure the crystal was positioned in the beam, and the cavity parameters were characterised.

6.2.6 Nonlinear crystal positioning

As discussed in section 5.2.5, to optimise squeezing production the crystal temperature for dual resonance must overlap with the peak phase matching temperature, shown in figure 5.4. To meet this condition, first the crystal phase matching temperature is determined by operating the cavity in a SHG configuration. A seed field at the fundamental wavelength is injected into the crystal, and the produced optical power at the pump wavelength is monitored. The SHG conversion efficiency is measured as a function of crystal temperature, with the input fundamental optical power low to prevent local heating and thermal lensing in the crystal.

The crystal temperature is set for optimal phase matching, and beams at the pump and fundamental wavelengths are injected into the cavity. The crystal is positioned laterally such that both the pump and fundamental beams are resonant. This measurement is also performed at low pump and fundamental optical powers to minimise local heating in the crystal.

This measurement is performed in air. After pumping down to vacuum the crystal must be re-positioned as the dual resonance temperature will be modified by the lack of dispersion of air under vacuum.

6.3 OPO characterisation

After construction, several parameters of the OPO were measured to fully diagnose and understand the cavity and the squeezing it can produce. These include length noise, phase noise contribution, cavity escape efficiency and threshold. This section summarises these measurements.
6.3 OPO characterisation

Figure 6.9: Experimental schematic of the aluminium OPO length noise measurement made with a pre-stabilised laser, delivered to the OPO via a fibre link, to which the OPO is locked and the error signal spectrum calibrated to give a length noise spectrum. This specific layout was used for the aluminium OPO length noise measurement.

6.3.1 Measurement of cavity length fluctuations

Measuring the length noise of a stable cavity is nontrivial as a more stable reference with which to compare with the cavity is required. Independent length noise characterisations of an aluminium OPO and a glass OPO were attempted. Both of these measurements were made by Pound-Drever-Hall locking the cavities to pre-stabilised lasers, and calculating the RMS fluctuations in the error signal. An experimental schematic of these tests is shown in figure 6.9.

The error signal spectrum was calibrated from volts to meters and plotted as a length noise spectrum. The noise of the error signal spectrum has contributions from the frequency noise of the laser, the length noise of the cavity, and any additional phase noise introduced along the fibre link. Additionally the length noise of the cavity will depend on its coupling to the seismic environment.

In the case of the VOPO length noise measurement, the laser travelled through tens of meters of optical fibre which may have been the limiting noise source. The VOPO cavity was locked to the laser using a SR560 pre-amplifier with a UGF of 200Hz. This measurement was not done under vacuum, but in air with the vacuum tank lid on. Results are shown in figure 6.10.

A similar measurement was taken for the LIGO H1 squeezer, however these two measurements were taken at different times, under different conditions and are not directly comparable. The phase noise intrinsic to the H1 squeezer was estimated at 19 mrad, where the cavity length noise is thought to have dominated this estimate [60]. This corresponds to 0.5 nm of RMS cavity length noise. The glass OPO measurement showed 5pm RMS of length noise. Note that both of these measurements are only upper limits to the length noise as contributions from the laser and fibre are unaccounted for.

A similar experiment was done at ANU using an aluminium OPO cavity, with the same design and mechanical construction as the H1 squeezer. The OPO cavity was locked with a 700 Hz UGF control loop to a pre-stabilised laser, brought to the OPO via an optical fibre. The length noise was found to be on the order of 0.01 nm RMS, significantly lower than the H1 OPO, but higher than glass OPO. The results of the aluminium cavity length noise measurement are shown in figure 6.11.

While the VOPO had a lower measured length noise than the aluminium OPO, an improved measurement without the fibre link between the pre-stabilised laser and OPO is ongoing, and is currently limited by environmental coupling.
Figure 6.10: Glass OPO length noise, as measured with the MIT VOPO. The blue trace shows the error signal amplitude spectral density after conversion from volts to meters. The red trace removes the noise suppression from the control loop. Green and purple are the RMS of the blue and red traces respectively.

Figure 6.11: Aluminium cavity length noise measurement. The blue trace shows the error signal amplitude spectral density after conversion from volts to meters. The red trace removes the noise suppression from the control loop. Green and purple are the RMS of the blue and red traces respectively.
6.3 OPO characterisation

6.3.2 Measurement of escape efficiency

Escape efficiency is a measure of the quantum efficiency with which light is coupled out of an optical cavity. While it can be calculated from the theoretical loss contributions of each of the components of the cavity, measuring escape efficiency is a helpful diagnostic tool for finding unexpected additional intracavity loss. Examples include dirty cavity mirror or paths through a nonlinear crystal with a high density of crystal defects.

To measure the escape efficiency of a cavity, the field reflected off the input coupler is measured on- and off-resonance. This is achieved most simply by scanning the cavity length or laser frequency, typically with a piezo-electric actuator, and looking at the minima in the reflected photodiode signal. The efficiency is calculated using the ratio of the off-resonance and on-resonance voltages. As shown in [144] the decay rates can be related to the ratio of on- and off-resonance powers,

\[ \frac{P_{\text{refl}}}{P_{\text{inc}}} = \left( \frac{\kappa_{\text{in/out}} - \kappa_{\text{loss}}}{\kappa_{\text{in/out}} + \kappa_{\text{loss}}} \right)^2. \]

(6.7)

where \( P_{\text{inc}} \) is the incident power and \( P_{\text{refl}} \) is the reflected power. Any mode mismatch or cavity misalignment takes power out of the fundamental and, if unaccounted for, will give an overestimate of the escape efficiency. To include the effect of higher order modes (HOMs) in the escape efficiency calculation the higher order mode power (relative to the off resonance offset) is subtracted from both reflected powers. We define this ratio, once corrected for higher order modes, as \( PD \):

\[ PD = \frac{P_{\text{refl}} - \sum \text{HOM}}{P_{\text{inc}} - \sum \text{HOM}}. \]

(6.8)

The decay rate loss may then be defined in terms of \( PD \) and \( \kappa_{\text{in}} \)

\[ \kappa_{\text{loss}} = \frac{\kappa_{\text{in}} - \sqrt{PD} \kappa_{\text{in}}}{1 + \sqrt{PD}}, \]

(6.9)

this can be rearranged and escape efficiency calculated in terms of \( PD \) only without dependence on the input coupler reflectivity required,

\[ \eta_{\text{esc}} = \frac{1}{2} (1 + \sqrt{PD}). \]

(6.10)

An example of an escape efficiency measurement is shown in figure 6.12, which shows data taken for the VOPO2. A separate 1064 nm beam path incident on the input coupler of the VOPO was prepared specifically for this measurement. In this case the mode matching was sufficient that no higher order mode subtraction was necessary. The measured escape efficiency was \((98.51 \pm 0.09)\%\) for the VOPO1, and \((99.1 \pm 0.5)\%\) for the VOPO2, after mirror cleaning.

6.3.3 Measurement of OPO threshold

The threshold of the OPO depends on the crystal nonlinearity and the cavity coupling rates, as shown in equation 5.13. To measure threshold, a small seed signal is injected and the nonlinear gain is measured as a function of input pump power. The threshold measurement of the VOPO1 is shown in figure 6.13, which corresponds to 157 mW threshold power. From the other cavity parameters – length and decay rate at the pump and fundamental – the crystal nonlinear co-efficient is inferred to be \(1030 \text{ s}^{-1}\) from this measurement.
Figure 6.12: Reflected signal at the fundamental while scanning the second VOPO, measured using a photodiode. An initial measurement (before mirror cleaning) gave a poor escape efficiency of 94.7%, shown in blue. After cleaning the cavity mirrors this was dramatically improved to 99.1%, shown in red.

Figure 6.13: Threshold power measurement of the VOPO1, nonlinear gain is measured as a function of increasing pump power. The threshold power is evaluated using it as a fitting parameter in a least squares fit. This data has been corrected for mode-mismatch between the pump beam and the OPO where not all the power incident on the cavity coupled to the TEM00 mode. This calibration was done by multiplying by the mode matching efficiency.
6.4 The in-vacuum glass OPO experiments

The two glass-based VOPO experiments were built and tested in separate configurations. The VOPO1 was an initial proof-of-principle experiment. It has been run as a vacuum squeezer designed to address the requirements of Advanced LIGO, as well as as a bright squeezer experiment testbed intended for radiation-pressure noise reduction in optomechanical cavity experiments, the results of which will not be discussed here.

The VOPO2 is being used in conjunction with a long baseline filter cavity to demonstrate low-frequency squeezed state rotation. The initial tests of the VOPO2 tested new technologies, important for the implementation of squeezing on the advanced gravitational-wave detectors: in-vacuum homodyne detection, fibre delivery of key beams to the OPO and homodyne, and a novel modification of the coherent locking scheme. This section describes the experimental components of the VOPO experiments.

Figure 6.14: Schematic of the VOPO1 experiment. The main laser is frequency doubled with an external SHG which is PDH locked to the laser (red section of diagram). The pump beam exiting the SHG is steered into the vacuum tank where the OPO is placed. The OPO is PDH locked to the pump beam on reflection from the bowtie cavity (green). The pump beam is power stabilised using a Mach-Zehnder interferometer, which is locked using DC voltage offset lock and the signal transmitted by the OPO cavity (dark blue). A mode cleaner cavity prepares the beam for the homodyne local oscillator. The mode cleaner cavity (yellow) is PDH locked to the main laser using the same modulation sidebands as the SHG lock. The coherent lock (see section 4.4.3 for theoretical details) is achieved in two parts – first the CLF is locked to the squeezing laser using the beat note between the pump field and the frequency doubled CLF (pale blue). The local oscillator is then locked to the squeezed beam, via the CLF, by detecting the beat note between the CLF and the LO at the homodyne detector. The beat note is mixed down with a signal at the offset frequency generated with a signal generator phase locked to the first CLF signal generator, and the error signal fed back to a phase actuator on either the pump or LO path (brown).
6.4.1 Operation of the first VOPO

A schematic of the VOPO1 experiment is shown in figure 6.14 with the key parts of the experiment highlighted in colours; each part is expanded on in this section.

The main laser is a nonplanar ring oscillator (NPRO), with a Nd:YAG laser crystal forming a stable monolithic cavity. The operating wavelength is 1064 nm. Part of the main laser was frequency doubled using an Innolight second harmonic generator (SHG) which produced roughly 160 mW of pump power. The SHG is PDH locked to the laser, using 12 MHz phase modulation sidebands on the main laser, detected on transmission through the cavity.

The pump beam is steered into the vacuum tank through low loss vacuum windows. Various mode matching and steering optics are in the vacuum tank with the VOPO cavity. The VOPO is dually resonant and PDH locked using the reflected pump beam, which is steered out of the vacuum tank and detected on a photodiode. The reflected beam is mixed down with the 70 MHz signal to generate an error signal which is fed back through a servo and amplifier and used to drive the two cavity PZTs.

At high pump powers the phase noise due to pump intensity fluctuations can limit the squeezing level. The pump power is stabilised using a Mach-Zehnder interferometer on the pump path. When the OPO cavity is locked, the field transmitted by the OPO fluctuates with the pump power fluctuations. This field is detected on a photodiode, this signal is then subtracted from a fixed DC voltage, to generate a rudimentary error signal. This is sent to a servo, amplified, and fed back to a PZT on one of the mirrors of the Mach-Zehnder interferometer. This simple lock is known as a DC voltage subtraction lock. In a later modification to the VOPO1 experiment the sensor for this loop was moved to the unused output port of the interferometer, to remove the dependence on the OPO control loop.

The homodyne local oscillator is provided by a tap off of the main beam. A mode cleaner cavity is used to limit the spatial mode of the local oscillator (LO) to TEM00 and mitigate beam pointing jitter (the homodyne is, in reality, further from the main laser than depicted in figure 6.14). The mode cleaner cavity is PDH locked using the 12 MHz phase modulation sidebands on the main laser, and feeding back to a PZT on the cavity. The cavity is constructed on a spacer.

The coherent locking scheme is described in section 4.4.3. The first VOPO used the modified coherent locking scheme, which consists of two control loops and requires an auxiliary laser and single-pass SHG [41].

The first loop phase locks the auxiliary beam and the pump beam, thus making the auxiliary laser a proxy for the squeezed beam. The beat note between the pump and the frequency doubled auxiliary laser is detected on a photodiode on the second output port of a beamsplitter where the CLF is combined with the pump beam. The beat note is mixed down at 59.6 MHz, twice the offset locking frequency at the fundamental, to generate an error signal. The actuator of this phase lock is the auxiliary laser PZT for fast control, and the laser crystal thermal tuning for slow control.

The second part of the coherent locking scheme locks the CLF phase, and hence the squeezing phase, to the LO phase, allowing the measurement quadrature to be chosen. The beat note between the CLF and LO is detected on the homodyne. The RF component of the subtracted signal is mixed down with the signal from a 29.8 MHz signal generator which is phase locked to the 59.6 MHz signal generator. The error signal is filtered and fed back to either the local oscillator phase or the pump phase via PZT-mounted steering mirrors on either path.

The squeezed light was coupled out of the vacuum tank using super-polished highly reflective dichroic beamsplitters to separate the squeezing from the pump light with as little loss and scatter as possible. The squeezed beam was combined with a bright local oscillator and detected using a balanced homodyne detector following the AEI design by Henning Vahlbruch [166]. Both the mode cleaner cavity and homodyne detector were housed in an heavy aluminium box, acoustically isolating the apparatus.
6.4 The in-vacuum glass OPO experiments

![Diagram](image-url)

**Figure 6.15:** The second VOPO experimental configuration. The main laser and pump are delivered by an Innolight Diabolo laser, shown in the red box. The OPO (green segment) is locked on reflection of the pump, the reflected beam is coupled out of the vacuum via a vacuum window. The coherent locking beam and diagnostic seed beam are delivered to the vacuum tank by optical fibre, and the beam passing through to the cavity is set by a MEMS fibre switch. The first stage of the coherent lock is detected on reflection, and fed back to the AOM frequency (blue). For the second stage of the coherent lock the beat between the coherent locking field and local oscillator is detected at the homodyne, and fed back to the local oscillator phase (brown). Key differences between this and the first VOPO are the AOM-based coherent locking scheme, fibre coupling to the OPO under vacuum, and in-vacuum homodyne operation.

High vacuum was achieved in the first OPO experiment with a combination of a turbo and a titanium ion pump. The turbo pump (HiCube 80 Eco), which had an internal backing roughing pump, reached $3 \times 10^{-7}$ mbar after 2 days of operation. Despite isolation and baffles the vibrations of the backing pump still coupled mechanically to the vacuum tank and made squeezing measurements with the turbo pump on impossible. An ion pump (Gamma vacuum) was therefore used to maintain the pressure below $1 \times 10^{-5}$ mbar while data was taken.

### 6.4.2 Features of the second VOPO

The VOPO2, shown in figure 6.15, was a similar experimental layout to the VOPO1, but tested several new technologies which are important for squeezing injection into aLIGO. The main beams were delivered to the OPO via optical fibre, minimising free-space coupling between the in-air and in-vacuum components of the experiment which can add significant squeezing ellipse phase noise. The VOPO2 also included an in-vacuum homodyne detector, which also minimises phase noise between the OPO and detector. The OPO was built on a smaller footprint glass breadboard, reducing the VOPO footprint left space for the in-vacuum homodyne. This section discusses the key novel features of the VOPO2, and highlights the lessons learnt from this experiment.
Chapter 6 Development of a glass-based squeezed light source for Advanced LIGO

Figure 6.16: Photo of the second VOPO in the vacuum chamber. The OPO and preparation optics were assembled on one breadboard while the homodyne and preparation optics were on another. Preparation optics and black glass dumps for beams rejected from polarisers after the delivery fibres can be clearly seen. Anodised aluminium mounts shown in this picture were replaced by un-anodised aluminium before pumping down.

VOPO mounting

The vacuum tank housing the VOPO2 was deeper, allowing for improved seismic isolation. The VOPO1 rested on PEEK feet directly on the base of the vacuum tank. The vacuum tank was clamped to the optical table, resting on brass wedges placed on the table, with multiple sorbothane pads between the table and the tank to damp the acoustic mode of the tank.

The VOPO2 vacuum chamber sat on a table separate from the preparation optics. Inside the tank three cones of clean viton supported an initial 100 kg mass, on top of which 3 more cones of viton supported a second 100 kg mass. Three cubes of clean viton sat between the top mass and the aluminium optical breadboard onto which the VOPO preparation optics were mounted.

The glass breadboard of the VOPO was kinematically mounted to a smaller aluminium breadboard. Three tungsten carbide pads – one plane, one v-groove, and one cone – were glued to the base of the VOPO using heat-cured epoxy (EPO-TEC 353ND), corresponding reference ball-bearings were press-fit into aluminium dog clamps that were bolted onto the breadboard. The homodyne detector, beamsplitter and LO delivery fibre and preparation optics were mounted on a separate smaller breadboard.

Improved seismic isolation of this experiment reduced the environmental coupling, improving the cavity length noise and phase noise performance, as measured and discussed in section 6.5.2.

Fibre delivery

Fibre coupling the pump, seed/CLF, and LO beams removes the need for external realignment through the vacuum windows when pumping the experiment down to vacuum or relocating the experiment. The second VOPO experiment showed that the pointing stability of optical fibres is sufficient to maintain alignment into the OPO over long periods, and revealed some issues with operating optical fibres at moderate powers at short wavelengths such as 532 nm.

The in-vacuum fibres were polyamide coated SMF28 and SM600 fibres, designed to be LIGO
6.4 The in-vacuum glass OPO experiments

vacuum compatible. SMF28 is a standard fibre designed for single mode operation at 1310 nm and 1550 nm and SM600 is single mode fibre at 633 nm. These fibres support several spatial modes at 1064 nm and 532 nm. Additional modes were stripped off by coiling the fibre prior to the output collimator. To ensure reliable single mode operation fibres that are single mode at 532 nm and 1064 nm will be used in future.

Fibres located outside the vacuum tank were isolated from air currents with soft acoustic damping tubes, additional acoustic damping has been shown to dramatically improve polarisation stability compared to undamped fibres [120]. Polarisation maintaining fibres were initially used in place of single mode fibres, but were rejected due to poor power stability. This choice was made before the implementation of the acoustic isolation, and so polarisation maintaining fibres may be a viable option.

An initial build of the VOPO experiment used commercially available weld-able fibre feedthroughs (Vacom). These have female fibre connectors on either side of the feedthrough and are similar to standard fibre mating sleeves with a short piece of fibre between them. They were easily contaminated with dust, difficult to clean, and had varying power transmission depending on the tightness of the fibre connector. Improved feedthroughs were later developed in-house. A piece of unjacketed fibre was passed through a small hole drilled into a blank feedthrough, the hole was then sealed with epoxy (353ND). The main issue with the house-made design is support of the delicate fibres on either side of the feedthrough.

Another challenge encountered working with the fibres was damage consistently caused to the cores of fibres used with the 532 nm pump light. Damage occurred spontaneously when operating at moderate powers on the order of 100 mW. Typically damage was present on the perimeter of the core of the output end of the fibre when at an interface with another fibre, sometimes damaging both fibre interfaces. It is suspected that dust on the end of the fibre ignited and caused this failure. Multiple high-power tests have tried to recreate this situation with limited success, and investigations are ongoing.

In-vacuum homodyne

In-vacuum homodyne detection minimises the phase noise due to the path length between the squeezer and the detector, and will be a useful diagnostic when injecting squeezed light into aLIGO. The homodyne detector used in the VOPO2 experiment was hermetically sealed in a nitrogen-filled box. The circuit needed careful testing to ensure electronics would not over heat. The homodyne detector schematic is shown in appendix B.

6.4.3 Coherent locking scheme

Two variations on the original coherent locking scheme [164] were used in the two VOPO experiments: the “modified coherent lock” used on the VOPO1 and pictured in figure 4.12, and the double acousto-optic modulator (AOM) coherent lock used on the VOPO2 and pictured in figure 4.11. The robustness of the coherent lock is important as any noise or lock-point errors, where the CLF phase acquires a phase shift independent of the squeezing phase, will cause squeezing ellipse phase noise.

Both VOPOs were originally designed for use with the modified coherent locking scheme where the CLF is injected into the OPO cavity via the pump and squeezing input output coupler. The second flat mirror of the VOPO (M2) had a reflectivity slightly below maximum achievable high reflectivity (HR). This reflectivity is higher than optimal for unmodified coherent locking where M2 is used as the input coupler for the CLF. The extra reflectivity results in the VOPO being an extremely under-coupled cavity for the CLF, so the reflected signal is dominated by the
promptly reflected beam and the cavity signal is weaker than optimal. Despite this, the signal was still high enough to establish the first stage of the coherent lock.

A new double-AOM scheme for coherent locking was first implemented on the VOPO2 [123]. The scheme is described theoretically in section 4.4.3. This scheme has the advantages that it does not require a second laser, reducing cost and control loop complexity. The second auxiliary laser is replaced with two AOMs, which up-shift and down-shift the frequency of a portion of the main laser beam. The difference between the two AOM frequencies is the CLF offset frequency—far enough from the squeezing as to not interact with it, but close enough to be within the cavity linewidth. This technique has been tried with a single AOM with poor results for squeezing experiments in the audio band due to zeroth order contamination entering the cavity and causing seeding at the fundamental squeezing wavelength. With two AOMs this becomes a second order problem, however care must be taken to implement proper electromagnetic shielding and grounding to prevent cross-talk between the two AOMs. Using two AOMs also allows for operation at higher shift frequency as the frequency offset between the carrier and the CLF is the difference between the two AOMs. Higher shift frequency gives a better spatial separation of zeroth (unshifted) and first order beams. Once properly shielded cables were installed between radio frequency signal generators and the AOMs, the scheme was found to be comparable to traditional coherent locking.

The double-AOM scheme was locked with a high bandwidth control loop, which allowed a reduction in the CLF contribution to squeezing ellipse phase noise. The scheme also utilised a smaller CLF frequency offset from the main squeezing laser which reduces the detuning noise contribution to phase noise, which was significant in the LIGO H1 squeezer [60]. The new coherent locking scheme contributes roughly 1 mrad of phase noise, due to OPO detuning noise (measured in a similar way to the OPO length noise spectrum) and coherent control sidebands (dominated by the shot noise on the in-loop sensor, and length noise due to the SHG) [123].

### 6.4.4 Scattered light mitigation

In order to obtain a measurement limited by quantum noise down to 1 Hz, much effort is put into finding and eliminating sources of scattered light. Backreflections from optics on the local oscillator or squeezing paths can scatter light back into the main beam and introduce parasitic beams.

There are several ways to prevent this, by directing and dumping stray beams. The light reflected promptly off the homodyne photodiode surface is dumped using thermal glass V’s, as pictured in figure 6.17b. Similar beam dumps were implemented on the VOPO2, black welding glass beam dumps are pictured above plate-polarisers on the three input paths in figure 6.16. While not pictured on the promptly reflected beam from the homodyne detectors, these were later implemented before pumping down to vacuum. The VOPO1 experiment additionally employed an optical isolator on the local oscillator path to mitigate reflections off other surfaces such as a half waveplate. One inch diameter beam tubes along the squeezing path and strategically placed black foil (for in-air experiments) helped prevent stray light from coupling back into the squeezing path.

### 6.5 Squeezing results

The key results of the two VOPO experiments are presented in this section, including the first data of squeezing generated under vacuum, and preliminary squeezing data generated and measured under vacuum with an in-vacuum homodyne. The phase noise measurement of the VOPO2 experiment is presented, and the future plans for in-vacuum squeezing in aLIGO discussed.
6.5 Squeezing results

(a) Layout of the homodyne tank  (b) Thermal glass beam dumps

Figure 6.17: Scattered light mitigation techniques in the first VOPO. Photographs of the homodyne tank showing the squeezing and local oscillator paths (left), and the thermal glass beam dumps for the beam promptly reflected from the diode (right).

6.5.1 Squeezing under vacuum

The following two sections describe the independent squeezing results from the two in-vacuum OPO experiments.

VOPO1

The results from the VOPO1 squeezer are shown in figure 6.18, and are published in [170] and [171]. The experimental configuration was as shown in figure 6.14. 8.6 ± 0.9 dB of squeezing was measured from 10 Hz to 100 kHz, with 15.9 ± 0.7 dB of corresponding anti-squeezing. These power spectral densities, normalised to shot noise, were measured using a Stanford Research Systems SR785 Network Analyser, with several measurement bandwidths stitched together.

The local oscillator power incident on the beamsplitter was 1.9 mW, corresponding to -120.6 dBm of shot noise power measured at the homodyne detector, and a homodyne dark noise clearance of 22 dB. The common mode rejection on the local oscillator path was >70 dB, and on the squeezing path was 50 dB, providing sufficient immunity to intensity noise and stray light from these paths. The common mode rejection is measured using an optical transfer function. The field incident on the homodyne, from either the LO or squeezing path, is amplitude modulated with a swept sine signal. The signal measured by the homodyne is measured with one diode blocked, and then with both diodes unblocked. The difference in these un-subtracted and subtracted signals gives a measure of the common mode rejection. To optimise the subtraction we make use of the homodyne beamsplitter's reflectivity, which is dependent on angle of incidence. The beamsplitter angle is adjusted for optimal subtraction. Fine adjustments to subtraction can be made by adjusting the position of the beam on the diode, exploiting slight defects in the uniformity of the diode.
The nonlinear gain for the measurement shown in figure 6.18 was 16.2. This was measured by injecting a small seed field at the fundamental wavelength into the OPO, and measuring the amplification upon the injection of pump light.

To measure shot noise of the local oscillator, the squeezing path is blocked and only the local oscillator is incident on the homodyne detector. To measure squeezing and anti-squeezing, the squeezing path is unblocked, allowing squeezed light and the co-propagating coherent locking field to be detected on the homodyne detector. The CLF is locked to the local oscillator, and the measurement quadrature determined by setting the local oscillator phase. The local oscillator phase which corresponds to the squeezing angle is determined by minimising the homodyne noise power measured, and the anti-squeezing quadrature is $90^\circ$ out of phase with the squeezing quadrature.

The total measurement efficiency was estimated to be $(91.7 \pm 3)\%$, consisting of the OPO escape efficiency $(98.51 \pm 0.09\%)$, fringe visibility between the local oscillator and squeezed beams $(97.4 \pm 0.5\%)$, efficiency of the path between the OPO and the homodyne $(97.5 \pm 0.5\%)$, and homodyne detector quantum efficiency $(98 \pm 1\%)$. Later, a dirty optic was found on the squeezing path which may have contributed up to 3% loss, which would explain the lower than expected squeezing level. Correcting for 3% loss gives 9.2 dB of squeezing. Correcting for the cumulative sum of all the losses, and 20 mrad of squeezing ellipse phase noise (the measurement of which is discussed in section 6.5.2), the inferred squeezing leaving the OPO cavity is 14.2 dB. If detection and propagation losses can be reduced, up to 12.65 dB of squeezing could be measured with this configuration.

**VOPO2**

Preliminary in-vacuum and in-air results from the VOPO2 experiment are shown in figure 6.19. The experiment was configured as in figure 6.15. In air the squeezing level was 6.3 dB and the anti-squeezing level was 11.5 dB. While pumping down to vacuum, drifts in component alignment caused a worsening of the homodyne subtraction, fringe visibility and pump alignment into the OPO, causing reduced measured squeezing levels and additional scatter noise. Under vacuum the squeezing level was 5.5 dB and the anti-squeezing was reduced to 10.3 dB. The total efficiency of the measurement shown in figure 6.19 was $(76 \pm 4)\%$, with contributions from fringe visibility $(92 \pm 2\%)$, photodiode quantum efficiency $(95 \pm 3\%)$, propagation loss $(99 \pm 0.2\%)$, and OPO escape efficiency $(98 \pm 1\%)$. Note the slightly lower homodyne detector photodiode quantum efficiency in this experiment was due to the use of ETX500 photodiodes compared to Laser Components IGHQEX1000 photodiodes as in the VOPO1. The nonlinear gain for this measurement was 7. Accounting for the known losses in this experiment the inferred squeezing level exiting the OPO is 11 dB.

The squeezing levels measured in both in-vacuum squeezing experiments was lower than the aLIGO requirement, however these modest levels are due to losses within the experiments rather than low generated squeezing.

### 6.5.2 Phase noise measurement

A direct measurement of cavity length noise was elusive due to the ultra-stable nature of the cavity. The length noise can also be inferred from the squeezing ellipse phase noise, if other sources of phase noise can be eliminated or independently measured. The effect of squeezing ellipse phase noise on the measured squeezing level is to modify the measured variance,

$$V_{tot}(\tilde{\theta}_{sqz}) = V_1 \cos^2 \tilde{\theta}_{sqz} + V_2 \sin^2 \tilde{\theta}_{sqz},$$

(6.11)
Figure 6.18: Results of first VOPO experiment, measured with the homodyne outside vacuum [170]. Traces show the local oscillator shot noise, squeezing, anti-squeezing and dark noise levels, all normalised to the shot noise level. To generate this figure 5 separate measurements of different frequency spans were stitched together. At each span the data has been averaged 100 times, except the data of the lowest span (400 Hz) which has only 25 averages.
Figure 6.19: In-air and in-vacuum shot noise, squeezing and anti-squeezing spectra from the second VOPO
Shot noise, squeezing and anti-squeezing measurements before and after vacuum tank pump
down. Additional scattered light is apparent in the shot noise and squeezing channels around
10 Hz.
6.5 Squeezing results

where $V_{1,2}$ are the variances of squeezed and anti-squeezed quadratures, and $\tilde{\theta}_{sq}$ is the RMS squeezing ellipse phase noise. Phase noise projects anti-squeezing into the squeezed quadrature, reducing the measurable squeezing. The effect of phase noise is more pronounced in the squeezed quadrature, rather than the anti-squeezed quadrature, and is significant at high nonlinear gains where $V_1$ is smallest and $V_2$ is largest. To acquire a precise measure of phase noise, squeezing and anti-squeezing data at nonlinear gains over 100 are required.

To measure the phase noise, squeezing and anti-squeezing levels are recorded as a function of nonlinear gain. The measured variances ($V_{tot}$) can be fit for the loss and phase noise experienced by the squeezed beam. The loss of the measurement can be calculated from the loss along the optical path between the squeezer and the homodyne, the fringe visibility between the squeezed and local oscillator beams, and the photodiode quantum efficiency. Loss dominates the variance of the anti-squeezed quadrature, and the squeezed quadrature at low nonlinear gains. The RMS phase noise is used as a fitting parameter to fit the squeezed quadrature data at high nonlinear gains.

The phase noise measurement is sensitive to other sources of noise beyond the cavity length fluctuations, including lock point errors due to the frequency offset of the CLF, fluctuations in the OPO cavity length due to the cavity control sidebands, path length fluctuations between the OPO and the homodyne, and fluctuations in the pump phase.

Figure 6.20 shows the phase noise measurement for the VOPO2. To take each data point in figure 6.20, a fixed pump power is injected into the OPO, and shot noise, squeezing, and anti-squeezing are measured. At high nonlinear gains the pump beam is intensity stabilised using a free-space Mach-Zehnder interferometer. Each data point represents an average of the squeezing and anti-squeezing levels over 2-10 kHz, where the spectrum is flat.

The phase noise is measured to be $1.3^{+0.7}_{-0.5}$ mrad, almost 2 orders of magnitude improvement over the measured phase noise for the previous LIGO squeezer [60], and nearly an order of mag-

![Figure 6.20: Phase noise measurement of the MIT VOPO [123]. Loss and phase noise are fitting parameters to fit the squeezing and anti-squeezing levels as a function of nonlinear gain. The squeezing and anti-squeezing levels at low nonlinear gains indicate the total detection loss (82.9%), while the tail in the squeezing curve at the highest gains indicates the phase noise.](image)
nitude better than any previously reported squeezing ellipse phase noise measurement [89]. This measurement is dominated by the SHG length noise, coherent locking field shot noise, OPO detuning, and OPO control sidebands. This low phase noise level meets the requirements of a squeezed light sources for aLIGO.

A similar measurement was made for the VOPO1. This measurement corresponded to $(21.8 \pm 0.9)$ mrad of phase noise and, like the VOPO2, is dominated by sources of phase noise beyond the VOPO length noise.

### 6.5.3 Informing the design of the aLIGO squeezer

The lessons learnt when building and testing the VOPO motivated the design of a second type of in-vacuum OPO to be installed for early injection of frequency independent squeezing at the LIGO sites [112]. The construction of the new VOPO, dubbed VOPO3, is an intermediate between the H1 squeezer and the VOPO design, as the optical contacting procedure was found to be too variable for the production of multiple identical cavities. By reducing the CLF detuning 29.5 MHz (H1 squeezer) to 3.1 MHz (aLIGO squeezer), the length noise coupling in the VOPO3 is less critical, thus the requirements on the cavity length noise are reduced [111]. The metal design length noise coupling is 7 mrad/nm. Given a requirement of <1 mrad of phase noise contribution from the OPO length noise, the length noise requirement is <0.14 nm. The crystal oven has a lower thermal mass than the VOPO, but still follows the Peltier/thermistor design of the VOPO. The oven is also designed to be remotely translatable under vacuum, using picomotor actuators.

The original VOPO was designed to be optimally stable with the nonlinear crystal in $(m \simeq 1$, see section 5.3.2), which resulted in an unstable cavity with the crystal removed. The VOPO3 cavity is designed to be stable with and without the nonlinear crystal, so the loss and phase noise contribution of the crystal can be determined independent of the cavity. The finesse of the VOPO3 at the pump has been doubled, compared to the H1 squeezer, to reduce the threshold of the cavity.

### 6.6 Conclusions and future work

In this chapter the motivation, construction, characterisation and results from the first in-vacuum squeezers have been explored. These experiments have successfully demonstrated high squeezing levels while operating under vacuum. While higher levels of squeezing have been measured in-air, the generated squeezing of the VOPO is comparable, accounting for known losses in the experiments. The VOPO has revealed the challenges associated with operating such a sensitive experiment under vacuum, including dual-resonance temperature shifts due to dispersion, oven instabilities, alignment drifts when pumping down, and operation of optical fibres at moderate powers. The VOPO1 experiment is currently being used to generate a noise budget for the cavity length noise measurements; the VOPO2 experiment is being implemented on a long-baseline filter cavity to demonstrate frequency-dependent squeezing for aLIGO. The design of these cavities has informed the design of the VOPO3 for early injection into aLIGO which is currently being installed at the two LIGO sites.
Chapter 7

Development of a squeezed light source at 2 µm

This chapter presents the characterisation of, and results from, the first squeezed light source in the 2 µm wavelength region. The motivation for the experiment is discussed, then the design choices including choice of laser, nonlinear crystal, optical parametric oscillator (OPO) parameters, and detector considerations are explored. The characterisation of the OPO threshold and escape efficiency, and crystal nonlinearity are presented. Finally the first results from the squeezer, and the phase noise performance, are shown.

There were several significant technical challenges overcome to operate this experiment, including reduced availability of optics, detectors, and modulators, increased laser frequency and intensity noise, reduced nonlinear crystal performance, and poor photodetector quantum efficiency, dark noise and power handling. Despite this, 4.0 ± 0.2 dB of squeezing was measured in the 700 Hz – 100 kHz frequency band, limited by detector quantum efficiency.

The contents of this chapter are presented in the paper:


7.1 Motivation

The 2 µm squeezer is a pathfinder technology for the proposed LIGO Voyager upgrade – a potential mid-term future detector upgrade introduced in section 2.3.5. The squeezer is a necessary component of LIGO Voyager to reach the design sensitivity in terms of quantum noise performance. Design and operation of the squeezer will help inform the design of LIGO Voyager, and potentially other silicon detectors, including choice of laser wavelength. It is the first quantum optics experiment of its kind performed at this wavelength. Some of our findings may be useful to fields beyond gravitational wave detection, including carbon dioxide spectroscopy [110]. This research is, to our knowledge, the first attempt to generate squeezed light in the 2 µm region.

The aim of LIGO Voyager is to achieve the best sensitivity using the current LIGO sites and vacuum infrastructure. The primary differences from the Advanced LIGO design are the test mass material, size and temperature, as well as the coating design, and laser wavelength. The LIGO Voyager proposal features 200 kg silicon test masses, operated at low temperature (123 K), with silicon ribbon suspensions. The main laser wavelength must be longer than aLIGO to make use of the transmission window of silicon – roughly 1.5 – 6 µm. Below 1550 nm, silicon has significant two-photon absorption [25], excluding the current aLIGO laser wavelength (1064 nm) as an option for LIGO Voyager.
7.2 Experiment design

This section explores the design choices of the 2 µm squeezer experiment, including the choice of laser wavelength, laser source, nonlinear crystal material and dimensions, optical parametric oscillator design, and detector considerations, including dark noise and quantum efficiency.

7.2.1 Choice of wavelength

The specific wavelength for the proposed LIGO Voyager upgrade, at the time of writing, has yet to be determined. The centre wavelength for this squeezer was therefore chosen based on our own experimental constraints. Key parameters in this decision are the commercial availability of lasers, the scalability of technologies to meet stringent LIGO requirements, and the wavelength dependence of absorption in air, shown in figure 7.1.

Given these constraints, three wavelength options were considered:

1. **1984 nm** is an accessible wavelength for comparatively stable, narrow linewidth thulium fibre lasers, and is in a transmission window of atmospheric absorption, as shown in the inset.
Chapter 7 Development of a squeezed light source at 2 µm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>1984 ± 1 nm</td>
</tr>
<tr>
<td>Spectral linewidth</td>
<td>&lt;50 kHz</td>
</tr>
<tr>
<td>Frequency drift</td>
<td>&lt;100 MHz/min</td>
</tr>
<tr>
<td>Power drift</td>
<td>±5%</td>
</tr>
<tr>
<td>Relative intensity noise</td>
<td>-80 dB/Hz @ 10 Hz</td>
</tr>
<tr>
<td>Polarisation extinction ratio</td>
<td>&gt; 15 dB</td>
</tr>
</tbody>
</table>

Table 7.1: Summary of quoted parameters of the AdValue Photonics 1984 nm laser used for the first 2 µm squeezing experiment.

of figure 7.1. Thulium lasers at this wavelength are cheaper than at longer wavelengths, as (depending on dopant concentrations, and the host material of the gain medium) the lasing ion has peak emission around 1984 nm [181].

2. **2097 nm** is an attractive wavelength as it corresponds to a cryogenic holmium solid state laser under development within the LIGO scientific collaboration [72]. 2097 nm is beyond the main CO$_2$ and water absorption peaks at 2 µm. The disadvantage of this wavelength is it is on the edge of the wavelength range of the thulium lasing ion. While commercially available, thulium lasers at this wavelength are more expensive and have poorer noise properties than shorter wavelength systems.

3. **2128 nm** is an alternative idea for a 2 µm squeezing experiment which could utilise a stable Nd:YAG NPRO 1064 nm pump source for the OPO. This would still require a 2128 nm light source to act as the homodyne local oscillator, diagnostic OPO seed, and coherent locking field, generated either from an external laser or an above-threshold OPO. While diode lasers with linewidths on the order of 1 MHz exist at this wavelength, they are limited to low powers, insufficient for this application. If a higher power diode could be found and appropriately locked to the NPRO, then this would be another option for this experiment.

Based on these considerations, a squeezed light experiment at 1984 nm laser was constructed.

7.2.2 The 2 µm laser

The 2 µm laser used in this experiment was a 1984 nm 2 W system from AdValue Photonics, with a fibre laser seed and fibre amplifier configuration. Phase and amplitude fluctuations of the pump field place limits on squeezing produced in the OPO, as discussed in [74, 179]. These in turn place limits on the main laser frequency noise and intensity noise. The linewidth is 50 kHz, which is well below other noise sources in squeezing production [59]. The full list of laser parameters is presented in table 7.1, and the laser and amplifier datasheets are given in appendix B.

The error on the centre wavelength, quoted in table 7.1, is the manufacturing tolerance. The second harmonic wavelength was measured with an optical spectrum analyser to be between 992.04 nm and 992.21 nm, depending on the laser thermal tuning.

Constraints on the laser frequency drift are set by the cavities in the experiment – the SHG and the OPO. The frequency drift must be sufficiently low such that the SHG and OPO can be locked and maintained on resonance, limited by experimenter reaction times and actuator range. Assuming a few seconds reaction time between finding resonance and employing feedback, the laser frequency drift needs to be stabilised to 25 MHz/min, determined by the linewidth of the SHG.
7.2 Experiment design

<table>
<thead>
<tr>
<th></th>
<th>$n_{\text{pump}}$</th>
<th>$n_{\text{fund}}$</th>
<th>$\Delta n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{\text{fund}} = 1984,\text{nm}$</td>
<td>1.8328</td>
<td>1.8061</td>
<td>0.0267</td>
</tr>
<tr>
<td>$\lambda_{\text{fund}} = 1064,\text{nm}$</td>
<td>1.8868</td>
<td>1.8296</td>
<td>0.0572</td>
</tr>
</tbody>
</table>

Table 7.2: Refractive indices of KTP at the pump and fundamental, at 1984 nm and 1064 nm for comparison. Values calculated from reference [88].

<table>
<thead>
<tr>
<th></th>
<th>$\frac{dn}{dT}_{\text{pump}}[1/\degree\text{C}]$</th>
<th>$\frac{dn}{dT}_{\text{fund}}[1/\degree\text{C}]$</th>
<th>$\Delta \frac{dn}{dT}[1/\degree\text{C}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{\text{fund}} = 1984,\text{nm}$</td>
<td>$1.5057 \times 10^{-5}$</td>
<td>$1.3209 \times 10^{-5}$</td>
<td>$1.848 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\lambda_{\text{fund}} = 1064,\text{nm}$</td>
<td>$2.4188 \times 10^{-5}$</td>
<td>$1.4774 \times 10^{-5}$</td>
<td>$9.414 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Table 7.3: Change in KTP refractive index with temperature at pump, fundamental, and the difference between pump and fundamental for 1984 nm and 992 nm, and 1064 nm and 532 nm for comparison. Values calculated from reference [88].

The free running laser frequency drift does not meet this requirement, hence a laser frequency stabilisation system is required as described in section 7.3.1.

7.2.3 Nonlinear crystal

Due to its wide transparency range (400 nm – 4 µm), [56] periodically-poled KTP was used as the nonlinear crystal for the 2 µm squeezer, in both the SHG and OPO. At 1984 nm the refractive index of KTP is lower than that at 1064 – a comparison is shown in table 7.2.

The refractive index difference between the pump and fundamental in PPKTP is also much less for 1984 nm than for 1064 nm. The phase mismatch ($\Delta k$) between the pump and fundamental at 1984 nm determines the optimal periodic poling length (defined in section 4.3.2). The poling length for PPKTP at 1984 nm is longer than at 1064 nm, resulting in a broadening of the phase matching temperature curve, as shown in figure 7.2. When pumping down to vacuum the broadening could be advantageous, as the softer maximum increases the tolerance in the mismatch between the dual resonance and phase matching temperatures. One disadvantage is the crystal temperature must be set comparatively high to operate in a region where there is no nonlinear gain, this is required to complete diagnostic measurements such as measuring fringe visibility between the local oscillator and squeezed beam.

The change in refractive index with temperature is another key parameter which has implications when using crystal temperature to acquire dual resonance. At 1984 nm the difference in $\frac{dn}{dT}$ between the pump and fundamental is roughly 20% of the $\frac{dn}{dT}$ difference between 1064 nm and 532 nm.

To acquire dual resonance in squeezing experiments, the nonlinear crystal temperature is adjusted. The temperature change causes a change in the cavity optical path length via the thermal expansion of the crystal, and the change in refractive index, which is different between the pump and the fundamental wavelengths due to dispersion. The lesser difference in $\frac{dn}{dT}$ results in a broader optimum dual resonance temperature (blue curve in figure 7.2), reducing the impact of temperature fluctuations on squeezing. The temperature difference between consecutive dual resonance points is also high (\sim 33\degree C), this is important when operating the crystal away from the optimum phase matching temperature for diagnostic measurements such as fringe visibility.
Chapter 7 Development of a squeezed light source at 2 μm

Figure 7.2: Phase matching models and data for 1984 nm single pass second harmonic generation as a function of crystal temperature. Phase matching temperature curve shown in red assuming 33 μm periodic poling frequency, and measured data in black. The FSRs of the OPO cavity are shown in blue. The width of the phase matching curve and the FSRs are approximately 4 times that of 1064 nm.

7.2.4 Optical parametric oscillator design

The OPO cavity was designed after consideration of parameters and characteristics discussed in chapter 5. The mechanical construction followed that of the previous ANU aluminium OPO [41].

A bowtie geometry was again chosen for backscatter tolerance, and ease of alignment. The length of the cavity is 459 mm, slightly longer than the in-vacuum OPO (presented in chapter 6) to achieve the correct waist size in the crystal while maintaining a stable cavity. The specifications of the cavity are summarised in table 7.5, and a photo of the cavity is shown in figure 7.8.

The Gouy phase shift (discussed in section 5.3.3) of the first 6 higher order modes is shown in figure 7.3. These plots demonstrate that with the design cavity geometry no low-order mode has the same resonant frequency as the fundamental, though the 3rd order mode partially overlaps with the TEM00 at the fundamental wavelength.

The concave mirrors have 50 mm radius of curvature, allowing for a waist of roughly 29 μm in the crystal. The two flat mirrors (input coupler and flat HR) are both PZT-mounted, since the 2 μm cavity requires twice the length actuation range compared to a cavity at 1 μm.

The input coupler reflectivity is 85% at both the pump and fundamental. The cavity has higher finesse at the pump wavelength than the VOPO, this increase in finesse reduces the threshold power, hence reducing the requirements on the incident pump power. The input coupler reflectivity at the fundamental was chosen to have a high escape efficiency (96.7%) while maintaining a low threshold power (designed to be 15.2 mW, assuming the same crystal nonlinearity as 1064 nm).
7.2 Experiment design

Figure 7.3: Model of higher order mode spacing for the OPO cavity at 1984 nm (left) and 992 nm (right). Number indicates the order of the $m^{th}$ mode. Power in each mode is $1/n$ of the fundamental, which is normalised, this assignment is arbitrary. The round trip accumulated Gouy phase of the cavity at 1984 nm is 112.93 degrees in the sagittal plane, and 117.93 degrees in the tangential plane. At 992 nm the accumulated Gouy phase is 116.3 degrees in the sagittal plane and 111.75 degrees in the tangential plane. As the mode order increases the accumulated phase for tangential and sagittal drift further apart.

7.2.5 Detector considerations

A significant technical challenge in designing a gravitational-wave detector in the $2\mu m$ wavelength region is the requirement for high quantum efficiency, low dark noise photodetectors. This problem was investigated from the perspective of the $2\mu m$ squeezer and remains an ongoing area of investigation.

Semiconductor materials

Optics experiments at more exotic wavelengths typically suffer from less developed laser and detector technology as well as fewer available optical components. In the case of photodetection at $2\mu m$, a variety of semiconductor-based detectors exist, including PbSe, PbS, InSb, InAs, HgCdTe (MCT), and extended InGaAs [45]. Some of these detectors can be cooled to improve their shunt impedance and dark current, however this complicates the detector electronics significantly. InSb sensor arrays have demonstrated broadband high quantum efficiency [129], with the added complication of cryogenic operation, requiring bulky liquid nitrogen cooling.

A promising semiconductor for high quantum efficiency operation is extended InGaAs, due to its low noise and comparatively high detectivity [45]. InGaAs is a common detector material used at 1-1.5 $\mu m$. By modifying the relative concentrations of indium and gallium, the responsivity of InGaAs can be extended out to 2.6 $\mu m$ [12, 97]. While currently available InGaAs sensors have much lower quantum efficiency and higher dark noise compared to their counterparts for shorter wavelengths, there is potential for the material to be further developed in the future, and no fundamental reason why high quantum efficiency photodiodes cannot be manufactured at $2\mu m$ [8].
Quantum efficiency

The quantum efficiency ($\eta_{PD}$) of a photodetector is the efficiency with which an incident photon (with energy $E = \hbar \omega$) is detected as an electron, as introduced in section 3.3.1. The current produced by a photodetector with incident optical power $P$ is then given by

$$i = \frac{e \eta_{PD} \hbar \omega P}{\hbar \omega},$$

(7.1)

where $e$ is the charge of an electron, $\omega$ is the angular frequency of the optical field, and $i$ is the generated photocurrent. In many optics applications, detector quantum efficiency is not critical. For squeezed state detection every source of loss, including reduced diode quantum efficiency, couples vacuum into the measurement, destroying squeezed states.

Defects in quantum efficiency can be intrinsic or extrinsic to the photoelectric material. Intrinsic quantum efficiency defects result when a photon is absorbed, but the corresponding current is not produced. Example processes include free carrier absorption, where no electron-hole pair is produced when the photon is absorbed, or electron-hole pair recombination, where the photon is absorbed, the electron-hole pair is created, but then recombines before the corresponding current can be measured. Extrinsic quantum efficiency defects occur when the incident photon is not absorbed: it is either reflected or scattered off the diode surface.

Experimental techniques can be employed to improve on extrinsic quantum efficiency defects, including removing windows from the diode casing, applying anti-reflective coatings to the diode surface [177], and recycling the light reflected from the surface of the diode using a highly focusing mirror [118].

With current technology, extended InGaAs has a low quantum efficiency at wavelengths longer than 1550 nm compared to 1550 nm and below. Despite this, extended InGaAs has comparable quantum efficiency to other 2 $\mu$m sensors. Based on suggestions from photodiode manufacturers, we expect that this defect in quantum efficiency in extended InGaAs is due to extrinsic defects, and with sufficient investment high quantum efficiency diodes can be developed.

Dark noise

Detector dark noise is caused by the dark current of the photodiode and the Johnson noise of the circuits used to convert the diode current into a voltage. Dark noise clearance is crucial in squeezing experiments: to measure signals below shot noise, the dark noise must be well below the expected squeezing level. A shot noise clearance above dark noise of 15-20 dB is typically sought.

Photodetectors are typically operated in one of two modes: photoconductive or photovoltaic. Photoconductive detectors use a reverse bias voltage applied across the diode. The photocurrent generated by light incident on the diode is then read out across a load resistance. Photovoltaic detectors have no reverse bias voltage across the diode and incident light generates a photocurrent which in turn builds up a voltage which is then directly measured. Photovoltaic detectors have improved dark noise performance at the expense of bandwidth and detector linearity [82]. To reduce the homodyne dark noise in the 2 $\mu$m squeezer experiment, the detector was operated in a photovoltaic mode.

The homodyne detector will be further discussed in section 7.3.4.

7.3 The 2 $\mu$m squeezer experiment

This section details the 2 $\mu$m squeezer experiment configuration, including the laser frequency stabilisation, nonlinear optical stages, electronic control loops, and homodyne detector stage. The
7.3 The 2 \( \mu \)m squeezer experiment

![Schematic of the 2 \( \mu \)m squeezer experiment. 1984 nm beams are shown in red, 992 nm beams in orange, electrical connections in black, optical fibre in blue. The main laser operates at 1984 nm and is stabilised to a passive fibre Mach-Zehnder interferometer, shown in yellow. The laser is frequency doubled at the SHG to produce the 992 nm pump beam for the OPO, the SHG is locked using a 9 kHz dither on the laser frequency (red). The pump beam acquires 21 MHz sidebands via an electro-optic phase modulator. These sidebands are used to lock the OPO via PDH locking on reflection; the OPO section of the experiment is shown in green. To generate squeezed vacuum the OPO is locked at a dual resonance point at the crystal phase matching temperature. The squeezed beam is reflected off multiple highly reflective dichroic beamsplitters, to remove residual pump light, and mixed with a fibre-delivered local oscillator. The shot noise, squeezing and anti squeezing are then detected using balanced homodyne detection. Complete (but simplified) experiment schematic is shown in figure 7.4.

7.3.1 Fibre Mach-Zehnder laser frequency stabilisation

To meet the requirements discussed in section 7.2.2, the main laser required frequency stabilisation. A passive fibre Mach-Zehnder interferometer (MZI), thermally and acoustically isolated, was used as a frequency discriminator and reference to which the laser is locked, similar to [95] and [37]. The MZI is a cheap, simple-to-assemble, wavelength independent alternative to a reference cavity. The arm length mismatch of the MZI sets the steepness of the error signal as a function of frequency detuning, analogous to the finesse of a reference cavity. The fringe spacing of the interferometer is given by

\[
\Delta \nu_{\text{fringe}} = \frac{c}{2\Delta L},
\]  

(7.2)

where \( \Delta L \) is mismatch between the two arms of the interferometer. In this experiment an arm length mismatch of 3 m was employed, resulting in a fringe spacing of 50 MHz. The short arm-length mismatch is required as the laser experiences large frequency excursions which would cause fringe-hopping for a longer arm-length mismatch. Provided the servo can maintain the lock to within half a fringe, this arm length mismatch is sufficient for this experiment.

The MZI is locked using a simple DC lock, shown schematically in figure 7.4. Photodiodes on both outputs of the MZI are sensitive to the laser frequency drift. The gain and offset of the signal from each photodiode is adjusted electronically to match the amplitudes of the two signals,
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Figure 7.5: Bode plot of the open loop transfer function of the frequency stabilising fibre Mach-Zehnder interferometer, with 100 Hz unity gain frequency and 90° of phase margin (note that the phase margin is the phase lag at the unity gain frequency, a phase margin of 180° causes instability). A resonance of the laser PZT is visible at 3 kHz.

compensating for inequalities in the 50/50 fibre couplers\(^1\). Subtracting these two signals generated an error signal, which was low-pass filtered, amplified, and fed back to the laser PZT. The open loop transfer function of the MZI is shown in figure 7.5.

When implementing the MZI feedback to the laser PZT, there was some coupling between the frequency actuation and the intensity of the laser. This was minimised by electronically balancing the signals on the photodiodes, and was only problematic when the laser entered regions of high intensity noise thought to be due to mode-hopping.

7.3.2 The second harmonic generator (SHG)

Generation of the 992 nm pump field was achieved by second harmonic generation (SHG) of the main laser, following the same principle as the glass OPO. The SHG utilised a PPKTP nonlinear medium (Raicol, \(1 \times 2 \times 10\) mm) in a near-concentric, singly resonant (at 1984 nm) cavity. The cavity parameters are listed in table 7.4.

The SHG was constructed with stable mounts attached to a base plate, which was then housed inside a dye-cast box. Some additional optics were also attached to the base plate – a collimating lens, dichroic mirror (highly reflective at 1984 nm, anti-reflective at 992 nm), focusing lens, and a photodiode on transmission for locking. The box was clamped to the optical table.

The cavity mounts were coarsely aligned at 1064 nm, using alternative mirrors and crystal, to generate 532 nm. This assisted with the alignment for 1984 nm, which is significantly harder to image. With the mounts aligned, the mirrors and crystal were switched out for the 1984 nm mirrors and crystal, and the final fine alignment was done using the 1984 nm beam.

The SHG cavity was locked using the dither locking technique, introduced in section 4.4.2.

\(^1\)Note that the couplers used were designed for an operating wavelength of 1550 nm, and were significantly unbalanced at 1984 nm, with splitting ratios up to 85/15.
7.3 The 2 \mu m squeezer experiment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity length</td>
<td>33 mm</td>
</tr>
<tr>
<td>Mirror ROC</td>
<td>-15 mm</td>
</tr>
<tr>
<td>\omega_0 in crystal (1984 nm)</td>
<td>26.6 \mu m</td>
</tr>
<tr>
<td>Finesse (1984 nm)</td>
<td>101.4</td>
</tr>
<tr>
<td>Linewidth (1984 nm)</td>
<td>30.0 MHz</td>
</tr>
<tr>
<td>Free spectral range (1984 nm)</td>
<td>3.7 GHz</td>
</tr>
</tbody>
</table>

**Table 7.4:** Modelled physical and optical parameters of the 2 \mu m second harmonic generator. The cavity is singly resonant at the fundamental, and utilises a smaller PPKTP nonlinear crystal in a moderate-finesse, nearly concentric cavity.

**Figure 7.6:** Photograph of the SHG cavity box showing cavity optics, collimating and focusing lenses, and dichroic mirror.
Dither locking was chosen over PDH due to the lack of availability of phase modulators in the 2 µm wavelength region. A schematic of the dither locking electronics is shown in the red section of figure 7.4. Originally the laser frequency was dithered at 84 kHz using the seed laser PZT. This exploited the laser PZT resonance, requiring only a small dither for a large frequency modulation. Later, after the laser PZT dither was found to be a significant contributor to squeezing ellipse phase noise, the cavity length was dithered instead. This dither was converted to amplitude modulation by the cavity and detected on transmission using a Thorlabs extended InGaAs photodiode (PDA10D). The AC component of the photodiode signal was electronically mixed with the original dither signal, producing an error signal which was fed back to the SHG PZT to lock the cavity length to the laser.

The conversion efficiency from 1984 nm to 992 nm was sufficient to produce up to 200 mW of pump power. The power was sufficient to reach moderate nonlinear gains in the OPO, enough gain to produce high levels of squeezing, but not enough to precisely characterise the squeezing ellipse phase noise.

The SHG was found to introduce additional intensity noise to the pump field, increasing the relative intensity noise (RIN) by roughly an order of magnitude over the field entering the SHG. Pump intensity noise can limit the squeezing produced in an OPO, due to thermal effects in the nonlinear crystal [77]. Intensity fluctuations in the pump field cause local heating in the nonlinear crystal, which couples intensity noise to the nonlinear gain. While the pump intensity noise in this experiment was relatively high, in our vacuum-seeded OPO the pump noise coupling problem becomes a second order effect [107].

### 7.3.3 The optical parametric oscillator (OPO)

The optical parametric oscillator (OPO) is pictured in figure 7.8. It consists of a bowtie cavity and nonlinear crystal held in a temperature controlled oven. Design of the OPO cavity was discussed in section 7.2.4, and characterisation of the OPO cavity will be discussed in section 7.4. The modelled parameters of the OPO cavity are summarised in table 7.5.

The crystal oven consists of several copper pieces which house the crystal, the copper heat...
7.3 The 2 μm squeezer experiment

**Figure 7.8:** The 2 μm OPO cavity, with crystal oven close up (inset), and cavity dimensions. Both flat mirrors of the cavity are PZT-mounted to give sufficient length actuation range. The crystal oven is the same design as previous ANU aluminium OPO cavities.
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<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
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<td>Cavity length</td>
<td>459 mm</td>
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<tr>
<td>Mirror ROC</td>
<td>-50 mm</td>
</tr>
<tr>
<td>$\omega_0$ in crystal (1984 nm)</td>
<td>28.5 (s) 30.2 (t) µm</td>
</tr>
<tr>
<td>$\omega_0$ external (1984 nm)</td>
<td>512.59 (s) 463.05 (t) µm</td>
</tr>
<tr>
<td>Finesse (1984 nm)</td>
<td>38.6</td>
</tr>
<tr>
<td>Linewidth (1984)</td>
<td>16.54 MHz</td>
</tr>
<tr>
<td>Free spectral range (1984)</td>
<td>639 MHz</td>
</tr>
<tr>
<td>Input coupler reflectivity (1984 nm)</td>
<td>85%</td>
</tr>
</tbody>
</table>

Table 7.5: 2 µm OPO cavity parameters. The design considerations for OPOs was discussed in chapter 5, the 2 µm OPO has a longer cavity length compared to the 1064 nm to maintain optimal waist size in the crystal and cavity stability.

reservoir, and a translation stage for crystal positioning. The crystal is mounted with indium foil between it and the copper surfaces to enhance thermal contact and maintain uniform temperature through the crystal. The temperature is read out by a thermistor, placed in the copper close to the crystal, and actuated on using a Peltier, sandwiched between the top block and the heat reservoir. A thin layer of thermally conductive paste is placed between the Peltier faces and the copper of the oven top block and heat reservoir, and between the thermistor and the copper of the top block. A Newport 3040 commercial feedback system is used to control the crystal temperature.

The OPO is Pound-Drever-Hall (PDH) locked to the pump beam using 21 MHz phase modulation sidebands on the pump beam, detected on reflection.

7.3.4 The homodyne detector

The homodyne detector used in this initial proof-of-principle experiment consisted of a circuit board designed for use with InGaAs diodes [85], modified to increase the transimpedance gain, and extended InGaAs sensors. The circuit diagram of the unmodified homodyne detector is shown in appendix B. Extended InGaAs has a larger capacitance, increased dark (leakage) current, and lower reverse bias voltage compared to InGaAs. This caused some technical problems.

The original detector design operated the diodes with a 5V bias voltage, which needed to be lowered to comply with the maximum ratings of the extended InGaAs diodes (1.8 V). With this bias voltage applied across the diode, the photodetector dark noise was higher than the shot noise of a 1 mW beam, as shown in figure 7.9. This was far too high to measure squeezing. The spectrum of the electronic noise with a bias voltage had a characteristic slope depending on the manufacturer or the size of the photodiode. This additional noise is thought to be due to noise in the diode material itself, analogous to Johnson noise in a resistor. The initial homodyne was hence operated with zero bias voltage, which reduces the leakage current [82] at the expense of photodiode bandwidth. The bandwidth of the detector was roughly 500 kHz when operated with zero bias.

wThe 1 mm diodes have a higher quoted responsivity. The dark noise of the 1 mm diodes was significantly higher than the 500 µm diodes, with and without a bias voltage, shown in figure 7.9. Both types of diodes come packaged in a can with a protective window. To improve the quantum efficiency the windows of both diodes were removed.

Additional noise at higher laser powers was observed with both sizes of diode, however the effect appeared at lower powers on the 1 mm photodiodes. Above incident powers of roughly 121
7.4 Characterising the OPO

Several characterisations of the OPO are made before measuring squeezing. Before aligning the cavity, the crystal nonlinearity and phase matching temperature were measured in a single-pass SHG measurement. Once the cavity was aligned, the threshold and the escape efficiency were measured. The threshold gives an indirect measure of the crystal nonlinearity, dependent on the other cavity parameters. The escape efficiency is a measure of how much of the squeezed field exits the cavity, a critical source of loss. This section summarises the results of these experiments.
Figure 7.10: Responsivity of one of the extended InGaAs photodiodes in the homodyne. Optical power is measured with a power meter (Thorlabs S148C), a calibrated neutral density (ND) filter is placed between this and the photodiode so as not to saturate the diode. The ND filter (power) transmission was measured at each power level, and was on average $T_{ND} = 0.180 \pm 0.005$. This plot has been calibrated to remove the effect of the filter.

7.4.1 Measurement of effective nonlinearity

A simple method of measuring a crystal’s nonlinearity is to operate it under single-pass SHG conditions, and consider the output power at the second harmonic as a function of the input power. The formula relating these two quantities is \[ P_{2\omega} = \frac{16\pi^2 d_{eff}^2 h}{n_{2\omega}n_{\omega} \varepsilon_0 c \lambda_{2\omega}} l e^{-\alpha l} P_{\omega}^2, \] (7.3)

where $P_{2\omega}$ is the power at the second harmonic, $P_{\omega}$ is the power at the fundamental, $d_{eff} = 1/2\chi^{(2)}$ is the effective nonlinearity, $h$ is the Boyd-Kleinmann confocal parameter, $n_{2\omega}$ and $n_{\omega}$ are the crystal refractive indices at each wavelength, $\varepsilon_0$ is the permittivity of free space, $l$ is the crystal length, and $\alpha$ is the crystal absorption. When performing this measurement, care must be taken in the calibration of the measured optical powers ($P_{2\omega}$ and $P_{\omega}$), and to factor in other losses along the beam path, including crystal anti-reflective coatings, and dichroic mirror reflectivities.

To set up the single pass measurement, a 1984 nm “seed” beam was injected through the flat HR mirror, and mode-matched to the OPO cavity. The two flat cavity mirrors were then removed, such that the seed beam was allowed to pass through the crystal, without the added complication of the cavity. The seed beam was well mode-matched to the cavity, which gives a good estimate of the beam waist size. The focusing parameter $\xi$, on which the confocal $h$ parameter depends, is 2.5843 for a waist size in the crystal of 30 \( \mu \text{m} \). This is found by rearranging equation 5.2.

Removing the flat HR mirror, changes the optical path length of the beam by 3mm. This will shift the beam waist off the centre of the crystal by 3mm, and is accounted for in the $h$ parameter.

The result of the 2 \( \mu \text{m} \) OPO crystal nonlinearity measurement is shown in figure 7.11. An

\[ ^2h \text{ is a function of the focusing in the crystal, the waist position, and the phase mismatch between the two beams. It is calculated using equation (2.23) in reference [20].} \]
Effective nonlinearity of 2.26 pm/V fits the data very well. To our knowledge, there is no data on the nonlinearity of PPKTP at longer wavelengths in the literature. In theory it is expected to be consistent with the nonlinearity at 1064 nm which is generally reported to be around 10 pm/V, more than a factor of 4 above the nonlinearity which we have measured here.

### 7.4.2 Measurement of escape efficiency

The escape efficiency of the OPO gives an indication of the intra-cavity loss. The method of measurement for the 2 micron OPO was the same as for the VOPO, described in section 6.3.2. The ratio of the on-resonance and off-resonance cavity reflection is related to the escape efficiency using equation 6.10.

The escape efficiency was measured at both the pump and fundamental wavelengths. To measure the escape efficiency at the fundamental a separate tap off from the seed beam was mode matched and injected into the cavity through the input coupler.

Figure 7.12 shows the reflected and transmitted fields measured while scanning the cavity length. The escape efficiency was calculated to be (96.7 ± 0.8)% which is consistent with the expected intra-cavity losses, ruling out intracavity loss as the cause of the low measured crystal nonlinearity.

The noise on the reflected field is indicative of the typical relative intensity noise on the pump field. To calculate the depth of the resonance dips a least-squares fit is used, as shown in the inset of figure 7.12.
Figure 7.12: Cavity scan which was used to calculate the escape efficiency. Transmitted field shown in yellow, reflected field in blue, and a moving average of the reflected field in red. Inset is a zoomed in plot of the first dip, with a fit to the data shown in purple. The average on- and off-resonance values are indicated with black dashed lines. The measurement was confirmed at several optical powers. The apparent difference in linewidth between successive resonances is due nonlinearity in the PZT scan.
7.5 Results from the 2 µm squeezer

Figure 7.13: Measurement of the OPO threshold. The nonlinear gain is measured as a function of input pump power. The amplification and de-amplification fits correspond to a threshold power of 315 mW.

7.4.3 Measurement of threshold

The threshold of the OPO is measured by fitting the nonlinear gain as a function of pump power. Measurement of threshold is a useful characterisation of the OPO cavity, and a higher than expected threshold may indicate unaccounted-for loss or low crystal nonlinearity. The classical nonlinear gain, ingoing pump power, and threshold power are related,

\[ G = \frac{P_{\text{out}}(g)}{P_{\text{out}}(g = 0)} = \frac{(1 \pm \sqrt{P/P_{\text{crit}}})^2}{(1 - P/P_{\text{crit}})^2}. \] (7.4)

A plot of the nonlinear gain and pump power is shown in figure 7.13, fitting this data gives a threshold of 315 mW. The design threshold was 15.2 mW, assuming a nonlinear strength of \( \varepsilon = 1090 \text{s}^{-1} \) and intracavity losses of 1.39% and 1.44% at the pump and fundamental respectively, based on the quoted loss numbers for the optical coatings on the mirrors and crystal.

The threshold may also be used to back-calculate the nonlinear coupling rate, \( \varepsilon \), via a rearrangement of equation 4.52, restated here,

\[ \varepsilon = \frac{\kappa_{\text{tot}}^a \kappa_{\text{tot}}^b}{\sqrt{2\kappa_{\text{in}}}} \sqrt{\frac{\hbar c}{\lambda}} P_{\text{crit}}. \] (7.5)

The threshold we have measured yields a coupling rate of 277 s\(^{-1}\), assuming the same theoretical amount of loss. This is also roughly a factor of 4 lower than expected, and combined with the single pass SHG measurement suggests that the crystal nonlinearity is lower than expected.

7.5 Results from the 2 µm squeezer

The experiment was configured according to figure 7.4. Squeezing data was taken using two spectrum analysers, producing zero-span and audio band spectral measurements of the noise power spectral density measured using the homodyne detector. Initial squeezing levels were very low, and
it was later realised that the squeezing had extremely high phase noise. This section discusses the initial squeezing measurements, characterisation and reduction of squeezing ellipse phase noise, and the final squeezing measurement of $4.0 \pm 0.2$ dB of squeezing and $10.5 \pm 0.5$ dB of anti-squeezing relative to the shot noise level.

### 7.5.1 Initial squeezing results

The first results from the $2 \mu m$ OPO displayed very low levels of squeezing, but expected levels of anti-squeezing. Several sources of additional loss were identified, including polarisation mismatch between the local oscillator beam and squeezed beam, and additional homodyne electronic noise (discussed in section 7.3.4).

The beamsplitter used for the homodyne detection was specified to have 50% reflectivity for unpolarised light, which resulted in an uneven splitting ratio for the s-polarised light exiting the OPO. This problem was solved by operating the beamsplitter at $15^\circ$ angle of incidence. After solving this problem, and lowering the local oscillator power to prevent additional homodyne detector noise, the squeezing level was still found to be low. Detuning the temperature of the crystal appeared to improve the squeezing level, which indicated that squeezing was improved at lower nonlinear gains, a symptom of high squeezing ellipse phase noise.

### 7.5.2 Squeezing ellipse phase noise

The squeezing ellipse phase noise was characterised first with an optical measurement where squeezing and anti-squeezing levels were measured as a function of nonlinear gain. Figure 7.14 shows this phase noise measurement. At high nonlinear gains the coupling between anti-squeezing and squeezing was so severe that only anti-squeezing was measured. The total phase noise was measured to be $(280 \pm 40)$ mrad$_{RMS}$, extremely high compared to 1064 nm squeezing experiments where phase noise levels of less than 10 mrad$_{RMS}$ are routinely measured. The error in this measurement could be reduced if higher nonlinear gains were accessible, however this was not possible with the crystal nonlinearity and laser power available.

The relative phase noise between the local oscillator and the squeezing path was independently measured using a diagnostic seed beam. The seed was injected into the OPO while locked on dual resonance away from the phase matching temperature ($\sim 65^\circ$C), to avoid the effect of nonlinear gain. As the seed field is resonant in the OPO, and the OPO is locked to the pump field, the seed field acquires the same phase jitter that would normally be present on the squeezed field. The beat between the LO and the seed field is measured on the homodyne detector. The subtraction of the two diodes provides an error signal, much like that of the fibre Mach-Zehnder used to frequency stabilise the laser. While the loop was not robust, it maintained lock for the necessary time to take a phase noise spectrum. This was then calibrated using the error signal peak-to-peak voltage. Figure 7.15 shows the phase noise spectrum.

Four dominant sources of phase noise were identified. The laser PZT resonance at roughly 85 kHz contributed significantly to the phase noise. This was eliminated by introducing a notch filter on the feedback signal to the laser. The OPO and SHG servos also introduced phase noise at 3 kHz and 7 kHz, which was minimised by adjusting the loop gain and reducing the dither amplitude respectively. Finally, a 300 kHz source of phase noise, also originating from the laser, contributes roughly 10 mrad$_{RMS}$, as shown in figure 7.15. The height of the 300 kHz phase noise peak, and the noise performance of the laser, varies over time. This appears to depend on thermal tuning and environmental conditions but is still being investigated.
7.5 Results from the 2 $\mu$m squeezer

**Figure 7.14:** Squeezing and anti-squeezing levels as a function of nonlinear gain showing excess phase noise. The nonlinear gain of each data point has been scaled to match the anti-squeezing to theory, as the anti-squeezing levels are only weakly dependent on phase noise. The theoretical curves are generated using equation 6.4. Note that the fringe visibility was much lower for this measurement compared to the final squeezing data, which causes the lower squeezing levels in the absence of phase noise.

**Figure 7.15:** Phase noise RMS spectrum and RMS, measured before (light colours) and after (dark colours) suppression of the 85 kHz laser PZT resonance. Additional contributions at 3 kHz and 7 kHz are due to the OPO control loop and SHG dither respectively. These are more severe in the dark trace due to the loop gains and SHG dither magnitude, and were later improved prior to taking squeezing data. The 300 kHz peak originates from the laser, the cause is unknown.
Chapter 7 Development of a squeezed light source at 2 µm

7.5.3 Squeezing results

The results from the 2 µm experiment presented here were taken without locking the squeezing angle relative to the local oscillator phase. Instead, squeezing phase angle was scanned relative to the local oscillator phase via a PZT-mounted mirror, mapping out squeezing and anti-squeezing in the time domain.

The first squeezing measurements show 4 dB of squeezing and 10.5 dB of anti-squeezing measured in the 10-80 kHz frequency band. The total efficiency of this initial measurement was approximately 68 ± 6%, dominated by the photodiode quantum efficiency. If the diode quantum efficiency could be improved to 99%, the expected measurable squeezing level would be 8.9 dB. Sources of loss are listed in table 7.6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homodyne fringe visibility</td>
<td>98% ± 1.3%</td>
</tr>
<tr>
<td>Photodiode quantum efficiency</td>
<td>73.5 ± 5%</td>
</tr>
<tr>
<td>OPO escape efficiency</td>
<td>96.7 ± 0.8%</td>
</tr>
<tr>
<td>Local oscillator (LO) power</td>
<td>200 µW</td>
</tr>
<tr>
<td>Homodyne dark noise clearance</td>
<td>13.1 dB</td>
</tr>
<tr>
<td>LO common mode rejection</td>
<td>60 dB</td>
</tr>
<tr>
<td>Crystal temperature</td>
<td>33.8°C</td>
</tr>
</tbody>
</table>

Table 7.6: List of measured and inferred homodyne and OPO parameters of the 2 µm squeezer experiment. These parameters correspond to the data shown in figure 7.16.

The local oscillator power was kept low to prevent excess photodiode noise, as mentioned in section 7.3.4. The 20k transimpedance gain of the photodiode gave a dark noise clearance of 13.1 dB for only ~ 100 µW of local oscillator power on each diode. The local oscillator common mode rejection was required to be greater than 50 dB to sufficiently suppress the LO intensity noise.

Figure 7.16 shows a zero-span measurement at 30 kHz where the squeezing angle is scanned as a function of time, and a spectrum of squeezing in the audio band. Squeezing was observed in the kHz band of spectral frequencies, below these frequencies scattered light peaks can occasionally be visible, and higher frequencies are beyond the bandwidth of the homodyne detector. Squeezing could not be measured at lower frequencies without a coherent locking system to stabilise the squeezing angle with respect to the local oscillator phase.

Shot noise data was taken with the squeezing path blocked and only the local oscillator incident on the homodyne. The zero-span scanning arches as a function of time were acquired using an Agilent E4402B spectrum analyser. The resolution bandwidth was 1 kHz and the video bandwidth was 30 Hz. The phase of the local oscillator was scanned at 500 mHz to map out the curves over 10 seconds. The spectrum shown in figure 7.16 was taken using an SR785 network analyser. The squeezing and anti-squeezing traces were acquired by slowly scanning the phase of the local oscillator and capturing the trace at its minimum and maximum.

Note that the traces are normalised to the average shot noise level, which is averaged over the flat part of the spectrum. At low frequencies the shot noise dips slightly, however there are fewer data points at this part of the spectrum.
7.5 Results from the 2 μm squeezer

Figure 7.16: Zero span squeezing arches (top) and squeezing spectrum (bottom), normalised to shot noise (blue trace). Dark noise is shown in purple. Both plots show 4 dB of squeezing and 10.5 dB of anti squeezing. The nonlinear gain for these measurements was 7. The zero span measurement was taken at 30 kHz with a 1kHz resolution bandwidth and 30 Hz video bandwidth.
7.6 Conclusions and future work

This chapter has presented results from the first 2 $\mu$m squeezed light source. Design choices and technical challenges associated with the laser, detectors and nonlinear crystals were discussed. There is further work required to fully understand and optimise the system, including:

- Understanding and characterising the crystal nonlinearity and loss at 1984 nm. The nonlinearity measured here was a factor of 4 below expected values, the physical reason for this is as yet unknown.

- Investigating further options for optimising the quantum efficiency of photodiodes at 2 $\mu$m, and understanding the current limits on diode quantum efficiency and dark noise.

- Implementing an intensity noise stabilisation system on the main laser. While vacuum squeezing is not limited by pump intensity fluctuations, reducing the RIN of the laser will improve the reliability of the experiment.

- Improving the laser frequency stabilisation, increasing the range and steepness of the Mach-Zehnder frequency stabilisation will improve the time over which the experiment can be run.

- Characterising and removing the 300 kHz source of squeezing ellipse phase noise present on the laser.

With improvements to quantum efficiency, 10 dB of squeezing should be achievable at 2 $\mu$m. Based on this early work there appears to be no fundamental reason why 2 $\mu$m should not be used as the base wavelength for future gravitational wave detectors, though further investigations into nonlinear crystals and KTP nonlinearity at this wavelength are required.
Conclusions and future work

In this thesis, new technologies for squeezed light sources for current and future gravitational-wave detectors have been demonstrated. The first in-vacuum squeezing generation and detection was achieved, and a low phase noise squeezed light source demonstrated. This work was a key step on the path to an optimal squeezer for Advanced LIGO, that will include the implementation of frequency dependent squeezing. Squeezing implementation on Advanced LIGO will improve the range of the interferometers, directly translating to more detections. The first squeezed light generation and detection in the 2 \textmu m region was demonstrated. This experiment revealed several of the challenges of moving to longer wavelengths, and is an important milestone on the design of future cryogenic silicon detectors, including the proposed LIGO Voyager upgrade.

This chapter discusses the results, technical challenges, and future work for the in-vacuum squeezer effort for Advanced LIGO and its upgrades, and for the 2 \textmu m squeezed light source for future gravitational-wave detectors.

8.1 In-vacuum squeezer

We have constructed and tested an ultra-stable in-vacuum squeezed light source, producing squeezing in the audio band and demonstrating low phase noise operation, which meets the Advanced LIGO squeezer requirements. We have measured 8.6 ± 0.9 dB of squeezing, and can infer the generation of 14.2 ± 1.0 dB of squeezing based on known losses in the experiment.

To generate and measure audio-band in-vacuum squeezing, several technical challenges were overcome in terms of cavity design, construction techniques, in-vacuum operation, and coherent control.

- The design of the optical parametric oscillator cavity geometry took into account beam waist size in the crystal, escape efficiency and threshold power. In the second VOPO problems were encountered with higher order mode spacing, this was then taken into account in the designs of subsequent OPO cavities.

- The construction of the cavity utilised the optical contacting technique to attach the cavity mirrors and crystal oven to a glass base plate. To our knowledge, this was the first nonlinear optics experiment to use optical contacting in this way, and the first nonlinear optics experiment to be built in this quasi-monolithic style. Challenges associated with removal and attachment of mirrors were overcome to produce two VOPO squeezers.

- Operation of a dual resonant cavity under vacuum revealed the need for a remotely translatable crystal oven, to allow for dispersion compensation for operation in-air and under vacuum. This was retro-fitted to the existing crystal oven, but reveals some of the subtleties associated with operating a dual resonant cavity in air and under vacuum.
Future work on the VOPO is ongoing. The VOPO2 is being installed in a long baseline filter cavity experiment to demonstrate audio-band squeezing ellipse rotation for aLIGO. A third VOPO, with a design based on the lessons learnt from the first two VOPO experiments, is currently being installed at the LIGO sites for early frequency-independent injection. Further work on these endeavours includes:

- A full length noise characterisation of the VOPO1. Current measurements are upper limits, limited by environmental coupling, measurement techniques, and the stability of the laser. Measurements of cavity length noise with and without the nonlinear crystal will be useful to discern length noise due to the cavity length and due to the nonlinear crystal temperature fluctuations.

- Experimental comparison of the new VOPO3 and VOPO designs in terms of squeezing level, cavity stability, and phase noise performance will be useful to decide which cavity construction should be used for future squeezing efforts.

- Demonstration of frequency-dependent squeezing with a long baseline filter cavity, with corner frequency as required for aLIGO will be an important milestone for future squeezing efforts. This effort is ongoing at the MIT LIGO lab.

- Reduction of optical losses for squeezing injection into aLIGO will be critical to reach 10 dB of squeezing improvement desired in future detectors. Adaptive mode matching, low-loss Faraday isolators, and improved control systems are currently being researched for this purpose.

### 8.2 2 μm squeezer

We have demonstrated the first squeezed light source operating in the 2 μm region, a pathfinder technology for quantum noise reduction techniques required for future generations of gravitational-wave detector, such as the proposed LIGO Voyager upgrade. We measured 4.0 dB of squeezing at 1984 nm, and 10 dB of squeezing production is inferred from known losses in the system.

As lasers and detectors are less developed in this wavelength region, the experiment is limited by loss due to low quantum efficiency detectors, and by phase noise due to the laser source. Many challenges associated with available technologies in the 2 μm region were overcome to produce this result.

- The laser source used in these experiments has a large free-running frequency drift. This would prevent the various optical cavities of the experiment from being locked, without additional stabilisation. The laser was stabilised to a passive fibre Mach-Zehnder interferometer, which proved difficult to lock due to the large frequency excursions of the laser. With a short arm-length mismatch fibre Mach-Zehnder the laser frequency was sufficiently stabilised to achieve squeezing.

- Low availability of electro-optic phase modulators with sufficient power handling capabilities complicated the control of the second harmonic generator cavity, ruling out Pound-Drever-Hall locking. The SHG was instead locked using dither locking. The initial implementations of the dither locking technique, where the laser frequency was the source of the dither, added significant squeezing ellipse phase noise. While dithering the cavity length reduced the phase noise contribution, the system would be improved with the implementation of a higher bandwidth control system.
Photodiodes at 2 µm have comparatively low quantum efficiency, and high dark noise. To overcome the dark noise problem the homodyne detector was modified to operate with a zero bias voltage across the photodiodes. Further improvement of the homodyne electronics and optical implementation is ongoing.

Future work on the 2 µm squeezer is required to both improve the experiment and to investigate potential issues for a 2 µm silicon-based gravitational-wave detector.

- The 2 µm laser used in this proof-of-principle experiment was sufficient to provide a source to measure squeezing. Further research into 2 µm lasers is required, to improve this experiment and for LIGO Voyager. Improved laser frequency and intensity stability is required to increase the reliability of the various control loops, and improve the squeezing ellipse phase noise performance. Improved stability and performance is a step on the path to a laser appropriate for LIGO Voyager, which has the additional requirement of high-power single-frequency operation. Thulium fibre lasers and amplifiers show promise for higher power scalability, but further research is required. The exact wavelength of LIGO Voyager is yet to be decided. This experiment has shown no fatal flaws with the 1984 nm wavelength in terms of laser technology.

- The nonlinearity of the periodically-poled KTP crystals used in the 2 µm squeezer was measured to be 25% of the expected value. The origin of this low nonlinearity needs to be understood; it is not yet clear whether this is caused by this particular batch of crystals or is a property of KTP generally. In the latter case, other nonlinear crystal options will need to be investigated, or higher pump powers will be required for squeezed light sources.

- Further research into photodetector materials will be required to produce sensors appropriate for future gravitational-wave detectors for LIGO Voyager. While high quantum efficiency sensors have been demonstrated [129], more development is needed to produce detectors of sufficient speed and noise performance for squeezing, and for gravitational-wave detection.
Appendices
ABCD Matrix Methods

Ray tracing, or ABCD, matrices are a common technique used to model Gaussian beams. Comprehensive summaries of ray tracing techniques may be found in most optics textbooks including [125], [140], and papers such as [91]. Using this technique, beams are defined as vectors with two parameters: the distance from the system’s optical axis, and the angle made with the optical axis. Optical components are then defined as matrices whose elements describe the effect of the component on these beam parameters. A ray tracing matrix model was used to complement a model made using OptoCad [133] when choosing the parameters for both the glass and 2 µm OPO cavities. Matrices relevant to our model are summarised below:

- Propagation over a distance $d$:
  $$M_p = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \quad (A.1)$$

- Refraction at a normal interface, from a medium of refractive index $n_1$ to a medium of refractive index $n_2$:
  $$M_{refr} = \begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix} \quad (A.2)$$

- Refraction at an angled interface, for angle of incidence $\theta$:
  $$M_{t,\theta} = \begin{pmatrix} 1 & 0 \\ 0 & \sin^{-1}\left(\frac{n_1}{n_2} \sin(\theta)\right) \end{pmatrix} \quad (A.3)$$

- Reflection off a curved mirror, where $R_{eff}$ is the effective radius of curvature of the mirror. In the tangential plane (the plane of the cavity) $R_{eff} = R \cos(\theta_{aoi})$ while in the sagittal plane (normal to the cavity) $R_{eff} = \frac{R}{\cos(\theta_{aoi})}$.
  $$M_{refl} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{2}{R_{eff}} \end{pmatrix} \quad (A.4)$$

These matrices can be combined to find a single matrix representing the operation of the total cavity. For example a bowtie cavity, traced out starting in the centre of the wedged crystal, can be represented by:

$$M_{cavity} = M_{p,Lc/2} \times M_{t,\text{crystal} \rightarrow \text{air}} \times M_{p,da} \times M_{refl,M4} \times M_{pb} \times M_{refl,M3} \times M_{p,da} \times M_{t,\text{air} \rightarrow \text{crystal}} \times M_{p,Lc/2}. \quad (A.5)$$
To form a cavity the beam must be self repeating. This is represented by the cavity eigenmode, which can be solved for in the usual Eigen equation way, using

\[ M_{\text{cavity}} \vec{r} = \lambda \vec{r} \]  
(A.6)

\[ \rightarrow (M_{\text{cavity}} - \lambda I) \vec{r} = 0 \]  
(A.7)

\[
\begin{pmatrix}
A - \lambda & B \\
C & D - \lambda
\end{pmatrix}
\begin{pmatrix}
r \\
r'
\end{pmatrix}
=
\begin{pmatrix}
0 \\
0
\end{pmatrix},
\]  
(A.8)

where \( M \) is the round trip matrix of the cavity, \( \vec{r} \) is the eigenray, and \( \lambda \) is the eigenvalue. Solutions to equation A.8 are only possible if the determinant is 1. The stability of a cavity can be characterised by the \( m \) parameter

\[ m = \frac{A + D}{2}. \]  
(A.9)

A cavity is stable if \( |m| < 1 \). The stability of a cavity may also be represented by the \( g \) factor of the cavity, as defined in [91].
Data-sheets and schematics
Figure B.1: VOPO2 homodyne schematic, designed by Tomoki Isogai
# Chapter B: Data-sheets and Schematics

**Figure B.2:** Seed laser datasheet from the manufacturer

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**Product Delivery Data Sheet**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Test Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OPTICAL CHARACTERISTIC</strong></td>
<td><strong>SPECIFICATION</strong></td>
</tr>
<tr>
<td>Output Power (mW)</td>
<td>~30</td>
</tr>
<tr>
<td>Center Wavelength (nm)</td>
<td>1984±1</td>
</tr>
<tr>
<td>PER (dB)</td>
<td>≥15</td>
</tr>
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</table>

**Delivery Contents**

- Fiber laser unit (1)
- Power cord (1)
- User Guide (1)
- Keys (2)

**Special Instruction**

1. Do not open the instrument box without permission from the manufacturer
2. Please read the User Guide before turn on the amplifier

---

**Single-frequency operation**

![Graph showing single-frequency operation](image-url)
Figure B.3: Seed laser characterisation from the manufacturer 1/3
Figure B.4: Seed laser characterisation from the manufacturer 2/3
Figure B.5: Seed laser characterisation from the manufacturer 3/3
# Chapter B Data-sheets and schematics

## Figure B.6: AdValue amplifier datasheet

![Product Delivery Data Sheet](image)

<table>
<thead>
<tr>
<th>Product Name</th>
<th>2 μm fiber amplifier</th>
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</thead>
<tbody>
<tr>
<td>Product Part Number</td>
<td>AP-AMPI-1984-02-LP</td>
</tr>
<tr>
<td>Time of Manufacture</td>
<td>August 2016</td>
</tr>
<tr>
<td>Test Date</td>
<td>6/25/2016</td>
</tr>
<tr>
<td>Test Personnel</td>
<td>Todd M.</td>
</tr>
</tbody>
</table>

### Description
2 μm fiber amplifier. AP-AMPI LP. Operating wavelength 1984±1 nm, output power ≈2 W, single longitudinal mode, linearly polarized, PM1550 fiber output, armored cable, 1 m cable length, collimator termination, beam diameter 4-5 mm. Power stability ±5% over minutes.

### Specs and Test Data

<table>
<thead>
<tr>
<th>OPTICAL CHARACTERISTIC</th>
<th>SPECIFICATION</th>
<th>TEST DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Testing Wavelength (nm)</td>
<td>1984±1</td>
<td>1948.1</td>
</tr>
<tr>
<td>Max Avg. Power (W) with 30mW seed</td>
<td>≥2</td>
<td>2.3</td>
</tr>
<tr>
<td>Polarization extinction ratio (dB)</td>
<td>≥15</td>
<td>15.4</td>
</tr>
</tbody>
</table>

### Delivery Contents

| 1 | Fiber Amplifier unit (1) |
| 2 | Keys (2) |
| 3 | Power cord (1) |
| 4 | User Guide (1) |
| 5 | Delivery Data Sheet (1) |
| 6 | FC/APC mating sleeve (1) |

### Special Instructions

1. Do not open the instrument box without permission from the manufacturer.
2. Always connect and turn on the input signal before turning on the amplifier! (Otherwise damage to amplifier may occur.)
3. Always turn off the amplifier before turning off the input signal! (Otherwise damage to amplifier may occur.)
4. Please make sure the loaded seed is over 15 mW (Otherwise damage to the amplifier may occur.)
5. Avoid connecting and disconnecting fiber patch cables frequently. Make sure the connector is clean before connecting.

### Other Notes

1. Please read the User Guide before turn on the amplifier.

### Spectrum

With seed at wavelength 1948.1nm, 10mW
Quantum noise in Interferometers

C.1 Quantum noise in a simple Michelson interferometer

The fields within a Michelson are shown in figure C.1. The fields entering the symmetric port is linearised to a steady state classical term and a fluctuating term,

\[ E_S(t) = E_0 \cos(\omega_0 t) + \delta E_S(t), \quad (C.1) \]

where \( \omega_0 \) is the main laser carrier frequency, \( E_0 = \sqrt{\frac{8\pi P}{Ac}} \) is the electric field strength, \( P \) is the input laser optical power, \( A \) is the mode area of the beam, and \( c \) is the speed of light.

Assuming the interferometer is held on a dark fringe, the field entering the interferometer through the dark port is defined in terms of quadrature operators (as in equation 3.51),

\[ E_{AS}(t) = \sqrt{\frac{\pi \hbar \omega_0}{Ac}} \left[ \cos(\omega_0 t) \int_0^\infty (a_1(\Omega)e^{i\Omega t} + a_1^*(\Omega)e^{-i\Omega t}) \frac{d\Omega}{2\pi} \sin(\omega_0 t) \right] \left[ \int_0^\infty (a_2(\Omega)e^{i\Omega t} + a_2^*(\Omega)e^{-i\Omega t}) \frac{d\Omega}{2\pi} \right] \]

where \( \omega_0 \) is the carrier frequency of the laser, \( \Omega \) is the Fourier frequency of the measurement (and hence the sideband frequency of the phase modulation generated by an incoming gravitational wave), \( a_i \) and \( a_i^* \) are the annihilation and creation operators for the \( i \)th quadrature of light.

Assuming a perfect 50% beamsplitter, the fields entering the interferometer arms are given by

\[ E_1(t) = \frac{1}{\sqrt{2}} (E_S(t) + E_{AS}(t)), \quad (C.3) \]
\[ E_2(t) = \frac{1}{\sqrt{2}} (E_S(t) - E_{AS}(t)). \quad (C.4) \]

Upon reflection from the end mirrors, the fields acquire a phase shift corresponding to any test mass displacement, given by \( x_1(t) \) and \( x_2(t) \). The reflected fields are

\[ E_{1,R}(t) = \frac{1}{\sqrt{2}} \left( E_S \left( t - \frac{2x_1(t)}{c} \right) + E_{AS} \left( t - \frac{2x_1(t)}{c} \right) \right), \quad (C.5) \]
\[ E_{2,R}(t) = \frac{1}{\sqrt{2}} \left( E_S \left( t - \frac{2x_2(t)}{c} \right) - E_{AS} \left( t - \frac{2x_2(t)}{c} \right) \right). \quad (C.6) \]

The outgoing field of the interferometer can then be calculated by rearranging and expanding equations C.5 and C.6, and substituting them into equation C.2,

\[ E_{AS,R}(t) = E_{AS}(t) + E_0 \frac{\omega_0(x_2(t) - x_1(t))}{c} \sin(\omega_0 t). \quad (C.7) \]
C.1 Quantum noise in a simple Michelson interferometer

**Figure C.1:** Simple Michelson interferometer architecture, showing fields and directions. The fields entering the symmetric and anti-symmetric ports are given by $E_S$ and $E_{AS}$ respectively, and the fields in the y and x arms are given by $E_1$ and $E_2$ respectively. The subscript $R$ indicates a reflected field.

The differential displacement of the test masses $\Delta x = x_2(t) - x_1(t)$ may originate from three sources – classical displacement, displacement due to quantum radiation pressure noise, and displacement due to the gravitational wave signal. To present this result in terms of familiar amplitude and phase quadratures, the displacement due to radiation pressure noise is given by,

$$\Delta x(t) = \chi(\Omega) \frac{2\Delta \hat{P}_1(\Omega)}{c} = \frac{2\sqrt{\bar{P}\omega_0}}{cM\Omega^2}, \quad (C.8)$$

where $\chi(\Omega)$ is the mechanical susceptibility of the test masses, $\Delta \hat{P}_1(\Omega)$ is time-varying component of the power incident on the test mass, and $M$ is the mass of the test mass. In terms of quadrature operators, the field leaving the anti-symmetric port has the form

$$E_{AS,R}(t) = \sqrt{\frac{4\pi h_0}{Ac}} \left[ \cos(\omega_0 t) \int_0^\infty (b_1(\Omega)e^{i\Omega t} + \bar{b}_1(\Omega)e^{i\Omega t}) \frac{d\Omega}{2\pi} + \sin(\omega_0 t) \int_0^\infty (b_2(\Omega)e^{i\Omega t} + \bar{b}_2(\Omega)e^{i\Omega t}) \frac{d\Omega}{2\pi} \right]. \quad (C.9)$$

Equating the quadrature terms between equation C.9 and equation C.7, with reference to equation C.2, the quadrature components of the field exiting the interferometer are given by

$$b_1(\Omega) = a_1(\Omega)e^{2i\beta}, \quad (C.10)$$

$$b_2(\Omega) = \left( a_2(\Omega) - \kappa(\Omega)a_1(\Omega) - \sqrt{2\kappa(\Omega)}\frac{h(t)}{h_{SQL}} \right)e^{2i\beta}. \quad (C.11)$$

$h(t)$ is the strain sensitivity, $h_{SQL}$ is the strain equivalent standard quantum limit, $\beta = \arctan(2\Omega L/c)$ is the single pass phase shift acquired at the sideband frequency, and $\kappa(\Omega)$ is the radiation pressure coupling coefficient, dependent on the incident power and the mass of the test mass,

$$\kappa(\Omega) = \frac{4P\omega_0}{c^2M\Omega^2}. \quad (C.12)$$
C.2 Quantum noise in a dual-recycled Michelson

The input-output relations of an arm-cavity Michelson interferometer without signal recycling are derived. The fields in one arm of such an interferometer are shown in figure C.2.

The fluctuating components of each field leaving the beamsplitter and entering the arm cavities is given by the contributions due to the incoming field from the dark port, assuming the interferometer is operated on a dark fringe,

\[ f_x^j = -\frac{a_j}{\sqrt{2}}; \quad f_y^j = \frac{a_j}{\sqrt{2}}, \]  \hfill (C.13)

where \( j = 1,2 \) indicates the quadrature, and the minus sign is due to the convention of the beamsplitter. The fields inside the arm cavities, and leaving the arm cavities are hence

\[ j_j = \sqrt{T} f_j + \sqrt{R} k_j, \quad g_j = -\sqrt{R} f_j + \sqrt{T} k_j, \]  \hfill (C.14)

We will neglect arm cavities losses besides the input coupling mirror. At the end mirror the fields accumulate a phase change due to propagation along the length of the arm, and a phase shift due to the movement of the mirror, similar to the simple Michelson case.

\[ k_j e^{-\frac{\omega \delta t}{T}} = j_j e^{\frac{\omega \delta t}{T}} + \delta k_j, \]  \hfill (C.15)

where the movement phase shift due to mirror displacement, \( \delta k \), couples into the quadratures in the same manner as for a simple Michelson,

\[ \delta k_1 = 0; \quad \delta k_2 = \frac{2}{\sqrt{T}} D \frac{2\omega \chi}{c}. \]  \hfill (C.16)

Substituting C.16 into C.14, rearranging to find the field circulating in the arm cavity, and approximating \( \sqrt{R} e^{\frac{\omega \delta t}{T}} \approx 1 \), it can be shown that

\[ j_j = \frac{\gamma}{\pi} \sqrt{T} f_j + \delta k_j. \]  \hfill (C.17)

\( \gamma = \frac{T_i}{T_e} \) is the arm cavity pole frequency, that is the corner frequency at which the cavity response rolls off with a \( 1/f \) slope in the frequency domain, \( \mathcal{F} \) is the finesse of the arm cavity, defined as the free spectral range divided by the cavity FWHM bandwidth (see section 4.2 for background on optical cavities).
From equation C.17 it is evident that the gravitational wave signal, imprinted on the \( \delta k_j \) term of the circulating field, is resonantly enhanced by the arm cavity finesse. The interferometer bandwidth is now set by the pole of the cavity – if \( \Omega \gg \gamma \) the sensitivity diminishes as \( 1/\Omega \).

To calculate the fields exiting the arm cavities, we insert C.17 into C.14 using similar approximations as previously,

\[
g_j = f_j e^{2i\beta} + \frac{T}{\pi} \sqrt{1 + (\Omega/\gamma)^2} e^{i\beta} \delta k_j, \tag{C.18}
\]

where \( \beta = \arctan(\Omega/\gamma) \) is the phase accumulated by the field at sideband frequency as it propagates through the arm cavity. From this the field exiting the interferometer can be calculated,

\[
b_j = \frac{1}{\sqrt{2}} (g_j^y - g_j^x) \tag{C.19}
\]

\[
= a_j e^{2i\beta} + \frac{T}{\pi} \sqrt{1 + \Omega/\gamma} e^{i\beta} \delta k_j - \delta k_j^x \sqrt{2}.
\]

Subtracting the fluctuating terms for each arm, as given by equation C.16, gives

\[
\frac{\delta k_j^x - \delta k_j^y}{\sqrt{2}} = 2 \sqrt{\frac{2I_0}{\hbar_0 T}} \omega_0 \delta x. \tag{C.20}
\]

Considering only quantum noise and gravitational waves as sources of mirror motion, \( \delta x \) contains two terms: the gravitational wave strain \( Lh(\Omega) \) and the radiation pressure noise \( \delta \hat{x}(\Omega) \). The radiation pressure noise in the Fabry-Perot Michelson interferometer is the same as for a simple Michelson, with scaling for mirror mass, incident power and quadrature fluctuations coupling in, that is

\[
\delta \hat{x}(\Omega) = \frac{4\sqrt{2}\hbar_0 a_1 \omega_m}{m\Omega^2 L(\gamma - i\Omega)} \tag{C.21}
\]

\[
= \sqrt{\frac{\kappa(\Omega)}{2}} Lh_{SQL} a_1 e^{i\beta},
\]

where the radiation pressure noise coupling constant and free mass strain-equivalent SQL are now given by

\[
\kappa(\Omega) = \frac{8\omega_m^2 P_{circ}}{m^2 L^2 (1 + (\Omega/\gamma)^2)} \tag{C.22}
\]

and

\[
h_{SQL} = \sqrt{\frac{8\hbar}{M\Omega^2 L^2}}. \tag{C.23}
\]

Figure C.3 shows the strain equivalent standard quantum limit and the power spectral density of the quantum noise for an arm-cavity Michelson, showing the effect of the arm cavity on the bandwidth of the interferometer.
The final form of the output quadratures is then (as derived in Kimble [90])

\[ b_1 = a_1 e^{2i\beta}, \]  
\[ b_2 = (a_2 - Ka_1) e^{2i\beta} + \sqrt{2K} \frac{h}{h_{\text{SQL}}} e^{i\beta}. \]  
\[ (C.24) \]
\[ (C.25) \]

Thus in a Fabry-Perot arm cavity interferometer, the SQL is given simply by \( h_{\text{SQL}} \), and the quadratures of the field exiting the interferometer anti-symmetric port by \( b_1 \) and \( b_2 \). This can be extended to include squeezed light injection in much the same way as section 3.4.5.
Bibliography


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