SOME ASPECTS OF THE PURE THEORY OF INTERNATIONAL TRADE

Submitted by

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PREFACE

This study was carried out under the supervision of Professor I.F. Pearce to whom I am so indebted for his advice, encouragement and assistance that any general acknowledgement is inadequate. His influence upon this thesis must be evident to those acquainted with his work and ideas.

A further general acknowledgement is due to Professor T.W. Swan whose ingenuity is responsible partly for the solutions by determinants which replace the more cumbrous algebraic proofs.

Naturally enough, I am indebted also to other theorists in the field of international trade and value theory. The method of analysis adopted depends considerably upon the established work in value theory of Professor J.R. Hicks and upon its application generally to problems in international trade by Professors J.L. Mosak and J.E. Meade.

In addition to these general influences upon my work, however, it should be noted that some of the results of chapter six were obtained in collaboration with Professor Pearce and Mr S.F. Harris. The particular results concerned, upon which a forthcoming article [51] is based, are acknowledged in a footnote to the chapter.

The only previously published work to appear in the thesis is the material of chapter four which draws upon two articles of my own that have appeared respectively in the Economic Record [100] and the Journal of Political Economy [101].

Finally, in connection with acknowledgements, I should like to point out that a recent article by Professor H.G. Johnston and Dr J. Bhagwati [99] published in October 1961, anticipates to some degree the conclusions reached in chapters seven and eight. Nevertheless, these results were obtained independently; they were presented as they now stand in a seminar given to the Department of Economics at the Australian National University in October, 1960.

References are given in square brackets within the text of the thesis. In each case the first number given refers to the position of the reference in the bibliography. Different references are separated by semi-colon, while pagination is separated from the reference by a comma. Thus [10,99; 12] refers to reference ten, page ninety-nine and to reference 12.

In conclusion, I should like to express my gratitude to Mrs. M. Stern whose task of typing the thesis has been a difficult one.

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TABLE OF CONTENTS

1. INTRODUCTION

A traditional dichotomy exists in the theory of international trade between, on the one hand, the monetary theory and its associated problems which arise because different money circulates in different countries, because each country has its own central bank and controls its own monetary policy, and because money is treated as a commodity with a direct utility of its own and, on the other hand, the pure theory which is concerned with the real factors underlying the monetary problems. This thesis is concerned solely with the latter. More precisely, our interest centres upon the application of the neo-Walrasian analysis of value and welfare to some of the traditional questions posed in trade theory.

The pure theory of international trade itself can be subdivided broadly between 'positive' contributions, intended for purposes of explanation and prediction, and normative contributions relating international trade to economic welfare. In the positive field, and it is with problems arising here that we are primarily concerned, there has been a recent trend towards the empirical verification of different hypotheses. However, though this trend is quite marked in the theory of comparative costs [107] and in connection with the Heckscher-Ohlin theorem that a country's exports use intensively the country's abundant factor of production [108; 109], it is scarcely noticeable among the great bulk of pure theory writings which are concerned with the formulation of 'logically true' propositions which, in the nature of things, cannot be refuted empirically. The majority of these have been developed within the context of the familiar two-good, two-factor model with its additional assumptions of linear and homogeneous production functions with diminishing returns along isoquants, full employment, profit maximisation and perfect competition.

It is the purpose of this thesis to rework several of these propositions within the framework of a more general model, so as to provide theorems which not only give a more realistic interpretation of the problems, but which also are verifiable, if only under ideal conditions. The two chief innovations in our model are the introduction of commodities which, by reason of transport costs, tariffs or tastes, are excluded from international trade and the formulation of our results in terms of parameters which are readily recognisable, independent of the problem studied, and at least measurable conceivably.

The reader will soon note the similarity between our own model and that developed by Professor James Meade in the mathematical appendices of his celebrated treatise upon international trade problems [87; 62]. Commenting upon his decision to reduce his four-commodity model to a traditional two-commodity one, however, Meade states:

> If we are to use our model to illustrate any propositions in the theoretical analysis of international economic relationships we must further simplify the model, unless we are prepared to be content with the contemplation of a string of clumsy determinants without any very obvious conclusions to be drawn from them[87, 46].[1]

Contrary to Meade's dictum underlined above we shall demonstrate that when full use is made of the basic results of micro economic theory, general equilibrium theory in international trade is not too complex to handle. Much more important, it will become clear that modifications to basic formulae are essential if numerical estimates of elasticities are to be used to support qualitative results.

Broadly speaking, and apart from the inclusion of two additional commodities, our analysis differs from that of Meade in two important respects.

1. The underlining is my own.

First, no attempt is made to suggest different policy combinations which might achieve a desired result. Rather, we shall confine ourselves to providing (within the framework of our assumptions) unique solutions in terms of relative prices concerning the effect of a shift in one of the underlying equilibrium conditions of the model upon some dependent variable. From our model, which is of a comparative static type, we are ill-equipped to specify the best policies by which a necessary or desired adjustment is to be obtained. For instance, if a given transfer of purchasing power is to be effected between one country and another, our subsequent analysis (again, within the context of our assumptions) may allow us to say that expenditure must alter by a certain amount in each country and that certain adjustments to the relative prices both in the paying and in the receiving country are necessary. It will not enable us to specify precisely how these changes might best be effected.

Secondly, and quite unlike the comprehensive coverage given by Meade to international trade problems, our model concentrates on a mere handful of current (and not so current)issues. This restricted selection of topics, which is necessitated from both the point of view of space and the complexity of the analysis, nevertheless does provide an adequate insight into the additional problems arising from the inclusion of non-traded commodities.

There are three broad subdivisions of the logically true proposition. First, there is the static proposition which defines the properties of equilibrium in a given situation. Examples from this category are the classic formulation of the necessary conditions for equilibrium in a multi-commodity, international market by Pareto [20], and Samuelson's elegant theorem stating the sufficient conditions for the equality of international factor prices under free trade [110]. Secondly, there is the dynamic proposition where time enters explicitly into the model. Instances of such propositions are rare in the pure theory of international trade.[2] Finally, we have those hypotheses which are concerned with the differences between two equilibrium situations. Theorems in this section, which are by far the most numerous, can be subdivided further into those which concern the effects of autonomous changes in production functions, factor endowments, tastes or the distribution of factor ownership upon equilibrium prices and quantities and those which, taking the above parameters as given, study the effects of a change in the trade situation induced by a tariff, subsidy, export, consumption or production tax. Problems selected for study below are from both groups.

The next two chapters are introductory. Chapter Two provides a cursory and critical survey of different methods of analysis in the pure theory of international trade, emphasis being accorded generally to the work of those theorists using general equilibrium techniques. This leads, in the next chapter, to the development of a four-commodity, international, free-trade system which is obtained by coupling together two closed, three-commodity models. The assumptions of the model are carefully stated and their more important implications discussed. Additionally, the static (Hicksian) and dynamic stability of the system is examined. Finally, the chapter introduces a new parameter, analogous to an elasticity, which is called a coefficient of sensitivity. It is suggested that this parameter could have a wide and important range of application in international trade theory.

In Chapter Four the model is applied to that once controversial issue, the transfer problem. Apart from shedding new light upon the argument concerning the direction of the shift in the terms of trade, this application of the model serves also to

2. For exceptions see [106,72-73, footnote 5; 25,266-68].

emphasise the important role of non-traded commodities in the process of adjustment to equilibrium.

The next four chapters are concerned with the theory of tariffs. Chapter Five re-examines Metzler's contention that it is quite conceivable that a tariff would fail to protect the import-competing industries of a tariff-imposing country [16]. His argument is examined both for the two- and four-commodity cases and is rejected on *a priori* and empirical grounds.

Chapters Six, Seven and Eight deal with the effect of a tariff on the terms of trade when the assumptions of the classical, international trade theory model are relaxed. The analysis of Chapter Six, which examines the effect of a tariff on the terms of trade when non-traded commodities are explicitly allowed for, concludes in favour of the orthodox hypothesis that the terms of trade of the taxing country must improve; a finding which is contrary to the assertion of Graaf that:

> In a multi-commodity world ... it does not seem possible to generalise about the direction of the movement in the terms of trade [61,54].

Next, in Chapter Seven, the traditional assumption that the government and private sectors of the taxing economy have identical tastes or that the government redistributes its tariff revenue as subsidies to the private sector in a manner which is random with respect to tastes, is relaxed. Two cases are examined: first, where the demand of the private sector is independent of the amount and type of government expenditure; and secondly, where private consumption is influenced by these factors. Finally, in Chapter Eight, the assumption of a single representative consumer and producer (or identical consumers and producers) is relaxed and the effect of a tariff on the terms of trade is analysed when there exists a disaggregated private sector.

Our incursion into the theory of tariffs concludes with an analysis of the problem of an optimum tariff. In particular, we concern ourselves with the question of its probable size. New two- and four-commodity optimum tariff criteria are offered in place of the traditional Bickerdike-Edgeworth-Kahn-Graaf result which, it is suggested, is of extremely limited value when its parameters are interpreted meaningfully. Our conclusion is that Kahn [11] overstated considerably the case for a large optimum tariff.

Our attention turns, in Chapter Ten, to the question of the effect of consumption and production taxes upon the terms of trade, a subject which, as far as I am aware, has not been treated thoroughly in the literature. Criteria are established both for the two- and four-commodity cases: first, for the effect of a consumption tax on importables upon both the terms of trade and the domestic traded goods price ratio of the taxing country; and secondly, for the effect of a production tax on importables upon the terms of trade and the traded goods factor price ratio of the taxing country. These results are then compared with the corresponding effects of a tariff.

Finally, in Chapter Eleven, we study the effect of an increase in output upon the terms of trade and upon the relative prices of both the expanding country and the rest of the world (the foreign country). A considerable literature exists concerning this problem which is founded upon two- and quasi, four-commodity models. It is suggested that our own model is sufficiently general to include most of these, while at the same time, additional and important qualifications to the existing analysis are made.

We turn now to a brief review of analytic methods in the pure theory of international trade.

2. ANALYTIC METHODS IN THE PURE THEORY OF INTERNATIONAL TRADE

A. INTRODUCTION

Theoreticians in international trade are faced with problems which, if not peculiar to their own section of the discipline of economics, are perhaps more formidable than those encountered elsewhere. On the one hand, they may choose a model in which the number of variables is reduced sufficiently to allow for a simple exposition but in which, because of its simplicity, the complex reality of the world is ignored. This problem is aggravated particularly by the large number of important variables that must be considered in international trade and, further, because the nature of the international economy does not lend itself readily to an aggregative, macro-economic approach. On the other hand, attempts to construct a more general theory in terms of individual economic units, have resulted usually in answers of great complexity from which qualitative conclusions are extracted only with difficulty. Faced with this dilemma, the only justification for a new model or variation of an old one, is that it should enable us either to derive accepted theorems more readily, or to qualify or refute them. It appears that the best policy is to adopt a 'middle of the road' attitude. In due course, therefore, a model is developed in which the abstractions from reality are neither so severe that all connection between the real world and the model is sacrificed, nor so few, as to preclude the derivation of useful qualitative conclusions or to prohibit the possibility of empirical verification.

In this chapter we shall state criteria with which to assess different analytic methods in the pure theory of international trade and briefly review some of these to conclude that a general equilibrium type of analysis offers the greatest potential for the derivation of useful results.

What follows does not purport to be a general criticism of international trade methodology. As pointed out in the preceding chapter the scope of the present investigation is strictly limited and it may be that an analytic method which is inappropriate for the study of one problem could suffice for the study of another.

B. GENERAL CRITERIA FOR THE EVALUATION OF A HYPOTHESIS

There follows, a brief analysis of criteria that might be used to assess different hypotheses. In a well-known work [9,97-105] are listed several basic conditions which a good hypothesis should fulfil.

Whether a hypothesis be trivial or true, insignificant or significant, is relatively unimportant compared with the necessary condition that it be verifiable, even if only under ideal conditions. The question of verification introduces several subordinate considerations.

A theorem need not in itself be directly verifiable and, indeed, many of the most valuable hypotheses are not. However, it should be clearly stated so that its implications may be deduced and subjected to empirical confirmation. Thus, in economics, the hypothesis which states that a consumer will maximise his satisfactions from a given income is not directly verifiable, though many of its consequences are.

If a hypothesis is to be verifiable, it must be stated in terms of determinate operations, i.e. it must be expressed in terms of parameters which are not only unambiguously defined, but also readily recognisable in practice, conceivably measurable and independent of the problem studied.

A hypothesis should provide an answer to the problem under review. It would be a serious error, however, to contend that false hypotheses (the logical consequences of which have been refuted) are of no use. A false theorem may direct attention towards formerly unsuspected explanatory relations. Moreover, any newcomer to a field of research is, at least, aware of those theorems that do not explain the facts. It should be borne in mind, that no hypothesis exists by itself alone but is associated with or founded upon other theorems. Therefore, when a hypothesis is tested, it is not only the fate of a single hypothesis that is involved but also the relevant body of knowledge to which it is attached. It is always possible that the subordinate hypothesis is a good one and that its failure to meet the facts is due to unsound hypotheses on which it is based.

A hypothesis still has merit even where the existing state of knowledge is inadequate to ensure empirical testing. In the absence of the possibility of immediate testing, however, it would be of immense value if, as a result of qualitative restrictions imposed by assumption on our basic relationships, we could derive qualitative conclusions concerning the response of our system to changes in the underlying data. It is an unfortunate fact that many of the general equilibrium results in the theory of international trade fail to yield such qualitative conclusions. The reader is referred, in particular, to the work of Yntema and Mosak [23;24]. At the same time, given the existing state of econometrics, their general results cannot be tested. The remedy, at the expense of a loss in generality, is the imposition of increasingly severe restrictions upon each model.

Finally, it is important that a hypothesis should be as simple as possible. Simplicity in this sense does not mean the more familiar theorem, nor does it mean the selection of the one which contains the least number of variables. In deciding which is the more simple of two hypotheses one should choose whichever is related the more systematically to other theories not only in international trade but, if possible, throughout and beyond the discipline of economics. In this way many apparently unrelated incidents may find their explanation within the context of one general theory.

Let us summarize the criteria outlined above.

1. A hypothesis, or the consequences deduced from it, must be verifiable, even if only under ideal conditions. This implies that the parameters in which it is expressed must be readily identifiable, conceivably measurable, unambiguously defined and independent of the problem under review.

2. The hypothesis should explain the problem which prompted the enquiry.

3. Where the practicability of immediate testing is in doubt, the theorem will be of greater value if qualitative conclusions are possible.

4. The greater the simplicity of the hypothesis, the greater will be its generality, and the greater the extent of its integration within a given body of knowledge.

C. PSEUDO-GENERAL AND PARTIAL EQUILIBRIUM ANALYSIS

This section is concerned with a brief description and criticism of a number of approaches used to avoid more complex, full, general equilibrium analysis. The theorems in which we are interested study the rate of change in certain variables induced by some explicit change in the given data. Solutions are obtained normally from a set of assumed basic relationships which hold only within a given environment. To detail this environment completely would involve the specification of the entire universe. Economists have included as data, therefore, only those items which they consider to lie within the confines of their discipline. Furthermore, depending on the problem, certain economic relationships are excluded from consideration by the use of ceteris paribus assumptions which thus restrict the generality of the model. Logically, there is no difference between the method of general and partial equilibrium analysis, save that the former tends to exhaust usually the full content of the discipline of economics whereas the latter, by a more extensive use of the *ceteris paribus* clause, takes as given many of the variables of the general approach.

Any economic system contains a very large number of variables and, if worthwhile analysis is to take place, some reduction in the number of these by the introduction of restrictive assumptions is essential. At the same time as these restrictive assumptions make the analysis more manageable, however, they limit the area of applicability of the theory. It is important, therefore, that the full implications of the assumptions be recalled when interpreting the results.

There are a number of devices an economist may use to reduce to manageable proportions the number of variables involved in a problem. Broadly speaking, these may be classified into three groups: first, where even within a specified environment certain economic variables are treated as constants, we have the method of partial equilibrium analysis; secondly, where all specified variables are included 'totally' in the aggregate concept by the use of certain aggregated magnitudes such as Alfred Marshall's 'bales of output', we have a pseudo-general approach, typified in the pure theory of international trade by the use of offer curves, the equivalent algebraic models or the typical text-book arithmetic examples of comparative costs; finally, there is the traditional two-country, two-commodity approach to international trade problems within which environment all reactions among the variables are allowed for explicitly. Frequently, the difference between the first and last categories is one of degree only; the latter sometimes shading into the former in the case of complete specialisation in production in each of the two countries.

Consider first, the method of partial equilibrium analysis. Except where a single market is under consideration the partial approach is also partly aggregative for it deals in terms of aggregated exports, imports, consumption, etc. It remains distinct from the offer curve type of analysis, however, because it ignores the mutual interdependence of these aggregated variables. Normally, one variable is related functionally to one other and the only justification for neglecting this fact is that the unexplained residual shift in the dependent variable is small. For example, the demand for imports is defined often as a function of the price of imports and nothing else. This implies that any shift in the demand for imports not explained by a change in their price is negligible. Unfortunately, this is not so, as a country's demand for imports is influenced significantly by changes in the price of exportable or non-traded goods or by changes in income, etc. Where such important components of a country's national income as imports and exports are concerned, partial equilibrium techniques can be employed only at the risk of serious distortion. While of considerable use for studying the direct impact of commercial policy, on international trade, this type of analysis does not provide a full result. Valuable as a first step, it has failed to facilitate the development of hypotheses which can provide an adequate answer to the problem studied and, thereby, to form a basis for useful prediction. A particular example of this technique that is examined thoroughly in a later chapter is the well-known optimum tariff theorem enunciated by R.F. Kahn [11]. It is contended that this result is inapplicable in the real world, and that accordingly any policy recommendations that might be made on the basis of it need be scrutinised carefully in the light of the assumptions made.

The second and more ingenious method of reducing the number of variables to manageable proportions involves a combination of the use of the concept of a representative firm or individual, which acts similarly to all other units in its group, with aggregate composite magnitudes such as the Marshallian bale of output, which represents aggregate uniform investments of a country's labour (of various qualities) and her capital

[12,157]. Such assumptions have given us the familiar offer curve technique introduced by Marshall [12] and later, ingeniously combined with indifference curves by Edgeworth. In recent years, this method of analysis has been subjected to increasingly heavy criticism [13,369-388; 14,150-159]. It is not our purpose to review such criticism in detail. We shall confine ourselves, apart from a few remarks of a general nature, to whether or not hypotheses derived from such analysis fulfil the conditions established above.

It might be claimed validly that the offer curve technique is a form of general equilibrium analysis, as the shape and position of the curves is determined by the mutual interaction of all specified variables. Thus, a shift in one of the curves, let us say as the result of a tariff, would reflect all of the changes occurring in the system. Unfortunately, the apparent simplicity of these curves is deceptive. The shift in one of them has been likened, in a well-known passage, to the way in which the hands of a clock shift in response to the movements of unseen machinery [15,32]. In other words, the shape of such curves and their position is determined by unseen variables and without a knowledge of these components the shape and the position of the curves must remain unknown. Haberler put it succinctly:

> Guessing the shape of these curves and then reading off the result means simply jumping to the outcome of a complicated process without analysing it. By assuming Marshall's curves as given we really assume the result [14,159].

Some economists, with a form of algebraic analysis derived directly from offer curves,have established precise formulae for the effects of different disturbances in equilibrium conditions upon international and domestic price ratios. Two of these theorems are examined in detail later [16,1-29; 17,56-61]. The basic objection to both is that they cannot be tested because their consequences are not expressed in terms of parameters that can be measured independently of the problem studied. In both cases, the resulting formulae are given in terms of 'total' elasticities which embody all possible influences upon the demand for and supply of a commodity as the result of a change in the given data. Such elasticities are vulnerable to the same criticism as the offer curves from which they derive: that without a knowledge of their component parts we can, from complete ignorance, only guess at their magnitude. It is true that we might compute under ideal conditions a total elasticity of demand, for say imports, which would measure the proportionate change in the volume of imports due to a proportionate change in the terms of trade induced by, say, a tariff. This estimate, however, would be of historical value only, as it would not be independent of the problem studied, i.e. the effect of a tariff on the terms of trade. A similar proportionate change in the terms of trade caused this time by a devaluation might have an entirely different effect upon the demand for imports. To compare the two elasticities would involve the comparison of incomparables. Results expressed in terms of these concepts never could be falsified. Furthermore, prediction on the basis of such theorems is impossible. For instance, we may know the total elasticity applicable to a past devaluation. Should we intend now to devalue by the same amount we cannot ascertain the value of the present elasticity until the devaluation has been put into operation. The values of the component elasticities could have altered, perhaps, as a result of the structural changes that would have accompanied the past devaluation.[1]

1. For an interesting attempt to derive the shape of a country's foreign offer curve and to formulate policy predictions on the basis of it, the reader is referred to the work of R.L. Marris [102] and to the subsequent controversy which it aroused [103;104;105]. Marris calculates the simple regression of United Kingdom exports on United Kingdom terms of trade for the period 1920-38. These results are transformed into a foreign offer curve whose predictive value is examined in the light of post-war data. The inescapable conclusion which emerges from this ingenious attempt of Marris is that his foreign offer curve cannot be regarded as a satisfactory explanation of the effect of relative price

Finally, it remains to consider those models which allow explicitly for full general reactions among the variables within a specified environment comprising normally two countries and two commodities with a representative individual living in each country. We shall have occasion to examine a variety of these models, ranging from the simple Ricardian type, characterised by constant costs and the specialisation of each country in the production of its exportable good, to the more complex Heckscher-Ohlin model in which production of each commodity is variable, though the factors of production remain in inelastic supply.

An outstanding example of the application of such a model to problems of economic policy is contained in the celebrated two-volume study of Meade [86;54]. In the text of his first volume Meade confines himself, for the most part, to a two-country world in which each country produces only one good, thereby earning Johnson's stricture that he had simplified "... his problems to the point at which his results cannot be applied at all easily to practical problems." [88,827]. Though Johnson's criticism was directed specifically at the Ricardian form of Meade's analysis, which was not continued in his mathematical appendix [87], it is contended here that this criticism is mitigated only slightly in the case of the more complex Heckscher-Ohlin type model with its emphasis on increasing costs and some domestic production of the importable commodity for, while it is true that no country in the real world specialises completely in the production of one commodity, it is equally true that no single country in the real world is

changes upon the volume of United Kingdom exports and imports. Instead, it compounds the influence of other autonomous explanatory variables such as the fluctuations in business activity which occurred in the inter-war period, shifts in technology, tastes, etc., as well as price reactions. In other words, the value of such an estimate is historical rather than predictive and, even so, it is difficult to see how one might explain even the historical pattern of events unless some further knowledge is available concerning the subsidiary influences.

concerned solely with the production of importable and exportable commodities. In practice there will always be a composite class of goods which do not enter into international trade because of transport costs or artificial impediments.

Further, Johnson pointed out that

> ... the problems in which most theorists are interested require the specification of the direction of the net outcome of influences operating in opposite directions, and this in turn requires a specification of the magnitudes as well as the signs of the influences ... To determine the outcome of any particular case ... it is necessary to measure the quantity ... it follows immediately that the role of economic theory in the solution of practical problems is extremely limited: the important (and more difficult) part of the task becomes the problem of measurement [88,827].

This quotation raises two interesting questions. Supposedly, it is the role of the theoretician to provide answers which, even if they do not permit the drawing of unambiguous qualitative conclusions, are expressed at least in terms of parameters that are conceivably measurable and independent of the problem studied, thus permitting their theory to be tested empirically. If this is so, it may be asked what statistical meaning can be attached to a marginal propensity to consume <u>importables</u> or <u>exportables</u> in a world which includes non-traded commodities. As far as the author is aware, no such statistical estimate has ever been computed. At what stage, for instance, does an exportable become an importable or vicé versa? To compare such concepts with marginal propensities to consume <u>imports</u> or <u>exports</u> is to compare incomparables - a charge to which Johnson himself must plead guilty [89,215, footnote 8]. Similar difficulties would be encountered when computing elasticities of demand for, or supply of, importables and exportables, for again the almost insuperable problem would arise of interpreting the parameters of a two-commodity model in a world which includes, in each country, an aggregate class of non-traded goods. The remedy would appear to be a model which includes non-traded commodities and takes account of their relationships with the internationally traded goods sectors. We shall return to this suggestion after a brief examination of those attempts which have been made to construct a full general equilibrium model in international trade theory.

One point, however, should be made concerning the pseudo-general equilibrium model of the Heckscher-Ohlin variety. An apparent fault with several applications of this type of model is that they do not express their results in a form which is conducive to empirical testing. All too frequently, supply, demand and income reactions are aggregated into a form of total elasticity which obscures important relationships such as the relative size of the different markets. Moreover, nothing is known concerning the size of these total parameters and their magnitude must remain unknown until an assessment is made of the size of their components.

D. GENERAL EQUILIBRIUM MODELS

The most notable attempts to increase the rigour and generality of international trade theory are those of Pareto, Pietri-Tonelli, Yntema, Ohlin and, particularly, Mosak. Far ahead of its time, Pareto's pretentious mathematical analysis of the pure theory of international trade was to remain practically unknown to the English speaking world until the early nineteen thirties. Even today, his most comprehensive treatment of the subject is available only in Italian [19,142-73; 20,476-98]. As his theory of international trade follows naturally upon his description of equilibrium conditions in a single country, it anticipates, to a considerable extent, that of Mosak. The domestic economy is described by a Walrasian exchange equilibrium in which all markets are cleared and, since he assumes perfect competition, commodity prices for all individuals are proportional to marginal utilities. It follows, from the perfectly competitive assumption, that on the production side marginal productivities are proportional to factor prices and that all factor markets must be cleared. International trade is introduced to two similar closed economies by the addition of n plus 1 unknowns; the exchange rate between the currencies of the two countries and the quantities of the n commodities traded. n of the equations required for a solution derive from the fact that the price of a commodity must be the same in a now single international market. The final equation is obtained by assuming that the value of imports must equal the value of exports in either country when expressed in terms of international prices, which is similar to assuming that the excess demand for a commodity in one country is equal to the excess supply of it in another. The apparent simplicity of the method is dependent upon the fact that gold is included among the commodities traded. In equilibrium, the demand for and supply of gold are equal for the two countries combined. The equilibrating process that follows a disturbance in the equilibrium conditions consists of a shift in both domestic and international price levels due to a change in the utility and cost of production of gold and to a further shift in the prices of traded goods as a result of a change in the quantities bought and sold [23, 111-113].

The work of Pietri-Tonelli [21,522-29, 522-73] and of Ohlin [22,553-70] is similar to that of Pareto. In fact, the former's contribution may be dealt with summarily as it is no more than a clear restatement of Pareto's analysis. Ohlin, on the other hand, in the first appendix of his well-known text sets forth his model in the form of Casselian general equilibrium theory, thus avoiding the use of the concept of marginal utility. However, the manner in which the price systems of the two isolated markets are combined is remarkably similar to the method used by Pareto. Indeed, Ohlin was later to begrudge the unnecessary labour that he incurred through not discovering Pareto's work prior to commencing his own [22,566-67].

Though Pareto's contribution was to lie dormant for a number of years, it is to him that the honour must go for first having presented a concise general equilibrium theory of international trade in terms of a mutually interdependent, multiple-market theory of pricing. To the Italian pioneer must also go the credit for having stated the theory of comparative cost in terms of relative prices, thus becoming one of the first to break with the classical theory of real costs in international trade. Bearing in mind the problems with which we shall later be concerned, Ohlin's contribution is a limited one, though in the field of factor allocation and the distribution of income, his work opened up new vistas. It is noteworthy, however, that in this context Ohlin tended to abandon the full generality of his mathematical model and to revert to an argument in terms of a more simple verbal model which, to a large extent, depended upon the earlier work of Heckscher and Wicksell.

Unfortunately, despite their generality, the mathematical formulations of Pareto, Pietri-Tonelli, and Ohlin do little more than to state that there exists a final functional relationship between all variables and parameters. Merely to state that under certain conditions there are sufficient equations to determine the values of the unknowns is not, in itself, sufficient. This does not provide a verifiable hypothesis nor, in the cases of Pareto, Pietri-Tonelli or Ohlin, were their consequences verifiable. The problem of developing general equilibrium theory so that it could indicate the manner in which variables might alter, either qualitatively or quantitatively with respect to changes in the data, was to remain until restrictions were imposed upon empirically observable data by which hypotheses could be refuted conceivably. Thus, while a final assessment of the work of Pareto, Pietri-Tonelli and Ohlin (his mathematical model) must acknowledge the fact that they were among the first to apply general equilibrium analysis to the theory of international trade and thereby to describe the involved system of price interrelationships and the necessary conditions that must be fulfilled in equilibrium, it should point out also that they were unable to formulate testable hypotheses about the effects upon the equilibrium situation of a change in one of the initial conditions. Hence, problems such as the effect of tariffs, transfer payments, etc. on international prices could not be studied in terms of these models. It should be remembered, however, that their inability to do this was due, not to their failure to apply existing knowledge to their problems but, to the then inadequate development of the laws of change of equilibrium systems.

Theodore Yntema was the first to move in this direction [23]. His work, cast in the classical tradition rather than in the general equilibrium mould of Walras, describes, with the aid of special assumptions, the series of monetary adjustments that follow a change in the equilibrium conditions of an international system. Internal equilibrium is given by relating functionally the supply of and demand for each commodity to its price which is specified for a particular set of monetary conditions. All supply and demand schedules derived from such functions are assumed to move proportionately with respect to a change in the given monetary conditions. This assumption, together with the fact that only one price can exist for each commodity in an international market, is sufficient to determine the exchange rate, which will move proportionately to the common shift in all supply and demand schedules. Enough equations to depict international equilibrium are given by the requirements that for each commodity the sum of net exports for all countries combined must be zero and that for each country the value of imports must equal the value of exports when expressed in a common currency. Thus, in an m country, n commodity world there are (m plus n) equations to solve for (m plus n minus 1) unknowns - the n prices and (m minus 1) exchange rates. As one of these equations is not independent, the system is not overdetermined.

Yntema's model is superior to that of Pareto in three respects. First, as Pareto proceeds from a description of the individual economic unit to full general equilibrium analysis with no attempt to aggregate commodities, his system contains not only a greater, but an unmanageable number of equations for empirical purposes. This criticism is mitigated by his aggregation of countries as opposed to the m country model of Yntema. Secondly, Pareto's equations are not related clearly to the monetary system nor to changes in price levels and so, thirdly, the effect of a disturbance upon the initial equilibrium conditions cannot readily be connected with particular elasticities of supply and demand.

It is unfortunate, however, that despite the improved insight into the theory of international trade made possible by Yntema - particularly into the stability of international markets - he failed, as did the others, to derive testable hypotheses. His model is still too complex, the (m plus n) equations specifying too many relationships for practical purposes. In this respect, the same criticism can be sustained against Yntema that was made above concerning Pareto, there being a difference of degree only. Yntema did not venture far enough along the road from micro to macro economics. While he grouped individuals according to the countries in which they lived, he baulked at the creation of either country or commodity groups. Though there are strong grounds for such a decision, it will be argued later that some further degree of aggregation is necessary if worthwhile hypotheses are to be formulated.

Unfortunately, serious problems did arise where Yntema attempted to simplify his analysis by relating functionally the supply of and the demand for each commodity to its own price and nothing else. For the reasons set out above this is a questionable assumption in international trade theory.

As in the cases of the other authors discussed, Yntema did not have a knowledge of the laws of change of equilibrium systems. His model, however, was in a form that was to facilitate greatly the application of imminent and important discoveries in this field to problems of international economics.

We come now to the most elegant, rigorous and indeed inspiring formulation in mathematical terms of international trade theory [24]. Whereas Pareto superimposed international trade upon two Walrasian exchange economies and Yntema developed a classical model, Mosak's work is sufficiently general to include both, although it is cast primarily in the classical tradition. Mosak, like Pareto, proceeded from the equilibrium conditions for the individual economic unit to the conditions for equilibrium in a closed economy. Unlike Pareto, he aggregated individual supply and demand functions before coupling his system in trade with another closed economy. Essentially, however, his model is an elaboration of Yntema's and its advantages and improvements can be seen most readily when the two models are compared.

One significant improvement in a multi-country model is the exclusion of the balance of payments equations. While it is true that such a condition is necessary if equilibrium is to exist between two or more trading countries, Mosak dispenses with the need for an explicit formulation of it, by including equations for non-traded commodities. This, in conjunction with the requirement that money income must equal expenditure in each country, means that the balance of payment equations are contained implicitly in the equations of supply and demand.

It has been noted already, when comparing the work of Yntema and Pareto, that the latter derived his aggregate supply and demand functions from the individual utility and transformation functions whereas the former adopted a more simple, though less general, approach by commencing his analysis with given aggregate supply and demand functions. Mosak follows Pareto in this respect.

He records a further major improvement when he avoids the ambiguity of employing partial equilibrium analysis in a general model. The demand for and supply of any commodity is related functionally to <u>all prices and income</u>. Thus, cross elasticities of demand are not assumed to vanish identically nor are income effects, i.e. the rate of change in the quantity demanded of a commodity as the result of an uncompensated change in income, neglected.

Mosak's most important contribution is that his work actually studies the effects of a disturbance in the initial equilibrium conditions in terms of the recently discovered laws of change of equilibrium systems [25,90-124; 26,301-328; 27,577-616]. He extends the theory of value for a closed economy to an international one. His use of the Hicksian concept of income and substitution effects enables him to deal generally with problems of complementarity and substitutability in both production and consumption. For example, an increase in the price of a commodity will cause consumers to substitute out of it, to decrease their demand for complementary products and to increase their demand for close substitutes. Moreover, these substitution and income effects can be turned into elasticities that measure the change in the quantity demanded or supplied of a commodity in response to a change in any price <u>when all other prices and money income are held constant</u>. Hence, these partial elasticities are parameters that are independent of the problem under review, are readily recognisable, unambiguously defined and, at least, in principle, measurable.[2]

2. Cf. the work of R.J. Stone and others in the Department of Applied Economics, Cambridge [28].

Mosak's analysis clearly offers the greatest opportunity among the analytic methods examined for the formulation of hypotheses which might fulfil the conditions stated earlier. In the first place, the parameters in which his results are expressed are measurable under ideal conditions. It is conceivable that any one of the infinite number of parameters involved could be computed. Secondly, the theory is simple in the sense that the method employed has been used for the solution of economic problems outside the field of international trade, i.e. in the theory of value. Unfortunately, two problems remain. In the first place, the very large number of variables involved precludes the practicability of immediate testing. Answers to specific problems are not forthcoming. In the second place, in the absence of full quantitative information, he is unable to draw unambiguous qualitative conclusions about the behaviour of the solution values of the variables in response to changes in the given data because insufficient restrictions are imposed upon the original relationships to indicate definite limitations concerning the algebraic sign of these rates of change. It would appear that the remedy is to impose more severe initial assumptions.

E. CONCLUSION

In section two of this chapter, conditions were stated for the purpose of evaluating different hypotheses. Then, within specified limits, we examined different analytic methods in the pure theory of international trade, concluding:

1. that apparently simple methods of analysis such as the use of offer curves, partial equilibrium or two-country, two-commodity models either have failed to produce testable hypotheses or they have so simplified the analysis with restrictive assumptions that the resulting theorems are inapplicable in the real world;
2. that among general equilibrium theorists,

i. the works of Pareto, Pietri-Tonelli, and Ohlin fail to yield testable hypotheses, even though they were the first to indicate the mutual interdependence of all variables in an international price system;

ii. the respective models of Yntema and Mosak fail, in the absence of existing complete quantitative information, to produce results from which qualitative conclusions can be derived. Moreover, the work of the former is open to the objection that he omitted, by assumption, certain important variables.

We return now to the suggestion made in Section C of this chapter that a more realistic model should include a class of non-traded commodities and take account explicitly of the relationships existing between them and the internationally traded goods. The advantages of such a procedure are twofold. First, the presence of an aggregate, non-traded commodity is a small but nevertheless significant step toward reality: in succeeding chapters it will be shown that a failure to consider the influence of the non-traded sector could distort seriously both the direction and magnitude of the shift in certain dependent variables. Secondly, the inclusion of the non-traded sector enhances considerably the prospect of the empirical testing of the results of different applications of the Heckscher-Ohlin model, as the ambiguity involved in defining exportables and importables is removed.

At the same time, the reader should note that our excursion into a multi-commodity world does not subject the model to the criticism made earlier concerning Mosak's attempt to construct a more general model: namely, that in the absence of full quantitative information no qualitative conclusions can be obtained. For the most part, qualitative results follow as readily from the multi-commodity as from the two-commodity model. It is important to recall that this statement contradicts the conclusion of

Meade concerning the development of his own multi-commodity model:

> If we are to use our model to illustrate any propositions in the theoretical analysis of international economic relationships we must further simplify the model, unless we are prepared to be content with the contemplation of a string of clumsy determinants without any very obvious conclusions to be drawn from them [87,46].

Indeed, the fact that qualitative conclusions can be derived from our multi-commodity results that are subject, for the most part, to no greater restrictions than are the corresponding two-commodity answers, provides the principal justification for the model which is developed in succeeding chapters.

3. A FOUR-COMMODITY INTERNATIONAL MODEL

A. INTRODUCTION

In this chapter is set forth a model in which the reactions among the variables included are accounted for as in general equilibrium analysis but in which the number of such variables has been reduced severely by aggregation. Not only are individuals aggregated into community groups, as in more general theories, but also countries and commodities are treated similarly. The value of the conclusions reached must be measured against the loss of generality that is involved. The degree of aggregation, however, is less severe than that currently employed in the majority of trade models. The most significant alteration is the introduction of two non-traded commodities. We shall see that the presence of these emphasises the complexity of some apparently simple results at the same time as it introduces important qualifications to existing methods of analysis.

The next section gives the part of the notation of the model that occurs most frequently.

As the development of the model is dependent particularly upon recent improvements in the theory of value, one might have given, on the one hand, a brief resume of the theory of value for a two-country, four-commodity world which would serve to emphasise the assumptions underlying the different applications of it. On the other hand, such a procedure would be tedious and far from original. Though the different theorems of value theory are not commonplace, and though some of the proofs of them are not easily understood by the non-mathematical economist, such proofs are set out clearly in a number of standard texts. We shall confine ourselves, therefore, in Section C, to a statement of the theorems that we intend to use, referring the reader in each instance to a standard source for the proof. Moreover, the basic assumptions will be given but not discussed unless, as in some instances, the application of them is contentious.

In Section D we pause briefly to set out the equilibrium equations for a closed three-commodity system and, as the reader will see, there are good reasons for this apparent digression. First, it enables us to show how international trade is introduced by the linking together of two such closed economies. Secondly, and more important, conditions are derived for the stability of a closed, three-commodity economy which when contrasted, in Section E, with similar stability conditions obtained for the international model, shed new light upon the possible cause of instability in the international market.

Finally, the equilibrium equations of our basic free trade model are given and described in Section E, and the limited role of money in the model is analysed. Conditions are stated for Hicksian perfect and imperfect stability and the probability of these conditions being fulfilled in practice is discussed. In the course of this discussion a new parameter is introduced that would appear to have a wide range of applicability in the theory of international trade. The section concludes with a criticism of the Hicksian concepts of perfect and imperfect stability and an attempt is made to state necessary and sufficient conditions for a form of dynamic stability.

The reader should note that the careful development of the model from first assumptions, however, accomplishes nothing if it merely demonstrates the mutual interdependence of all included variables. An attempt must be made to determine how initial equilibrium positions might change as a result of changes in the given data and to express these rates of change in terms of conceivably quantifiable parameters upon which qualitative restrictions can be imposed. This means that restrictive assumptions must be introduced and is the reason for the increased degree of aggregation in our model when compared with a fully general one like Mosak's.

B. NOTATION

That part of the notation of the model which appears frequently is set out below though most terms are redefined on first appearance to save the reader the necessity of constant reference. Primes refer throughout to the foreign country.

X_1, X_2, X_3 - the respective total quantities consumed in Country One of exportables, importables, and non-traded goods.

X_1', X_2', X_3' - similarly for Country Two. It should be noted that X_1' being Country One's exportables, is Country Two's importables, etc.

x_1, x_2 - Country One's exports and imports respectively.

O_1, O_2, O_3 - the respective total quantities of good one (exportables), two (importables) and three (non-traded commodities) produced in the domestic country (Country One).

O_1', O_2', O_3' - similarly, for the foreign country (Country Two). It follows that

$$O_1 = X_1 + x_1$$
$$O_1' = X_1' - x_1, \text{ etc.}$$

Y - the value of production in the domestic country.

Y' - the value of production in the foreign country.

M - money expenditure in the home country, i.e. $\Sigma p_i X_i$.

M' - money expenditure in the foreign country, i.e. $\Sigma p_i X_i'$.

p_1, p_2 - the absolute prices of goods one and two in both countries (there being a common international currency and no impediments to trade).

p_3, p_3' - the prices of non-traded commodities in the domestic and foreign country respectively. These prices are different as by definition separate markets exist for each non-traded commodity.

T_{ij} - the total elasticity of demand for the i^{th} good with respect to the j^{th} price, which includes all supply, demand, income and other subsidiary reactions, i.e.

$$\left(\sum \frac{\partial X_i}{\partial p_k}\frac{dp_k}{dp_j} + \frac{\partial X_i}{\partial M}\frac{dM}{dp_j}\right)\frac{p_j}{X_i} + \beta$$

where β represents other subsidiary reactions.

ε_{ij} - the elasticity of demand of the i^{th} good with respect to the j^{th} price, all other prices and money income being held constant, i.e.

$$\frac{\partial X_i}{\partial p_j}\frac{p_j}{X_i}.$$

S_{ij} - the elasticity of supply of the i^{th} good with respect to the j^{th} price, all other prices being held constant, i.e.

$$\frac{\partial O_i}{\partial p_j}\frac{p_j}{O_i}.$$

σ_{ij} - the demand elasticity of substitution of the i^{th} good with respect to the j^{th} price, i.e. the proportionate response of the i^{th} good to a small, real income compensated change in the price of the j^{th} good.

X_{ij} - the rate of change in the demand for the i^{th} good due to a change in the price of the j^{th} good, all other prices and money income being held constant, i.e.

$$\frac{\partial X_i}{\partial p_j}.$$

O_{ij} - the rate of change in the supply of the i^{th} good due to a change in the price of the j^{th} good all other prices being held constant, i.e.

$$\frac{\partial O_i}{\partial p_j} .$$

a_{ij} - the effect on the demand for the i^{th} good of a real income compensated change in the price of the j^{th} commodity.[1]

M_i - the rate of change in demand for the i^{th} good due to a change in money income, i.e.

$$\frac{\partial X_i}{\partial M} .$$

C_i - the marginal propensity to spend on the i^{th} good, i.e. $p_i \frac{\partial X_i}{\partial M}$. Thus in the domestic country, C_1, C_2 and C_3 represent the marginal propensities to spend on exportables, importables and non-traded goods respectively. In the foreign country C_2', C_1' and C_3' are defined similarly.

K_{ij} - the aggregate demand-supply substitution effect; i.e. $a_{ij}-O_{ij}$.

E_{ij} - the quantity weighted elasticity form of K_{ij}. Thus,

$$E_{11} = \frac{X_1}{x_1} \sigma_{11} - \frac{O_1}{x_1} S_{11}$$

$$E_{22} = \frac{X_2}{x_2} \sigma_{22} - \frac{O_2}{x_2} S_{22}$$

and

$$E_{33} = X_3 \sigma_{33} - O_3 S_{33}.$$

Similarly for the foreign country.

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1. The reader should note that X_{ij}, O_{ij} and a_{ij} are respectively the non elasticity form of ε_{ij}, S_{ij} and σ_{ij}.

C. ASSUMPTIONS

The basic assumptions of the model are listed below, though many others are implied by our use of some of the fundamental theorems of value theory. The implications of the more contentious assumptions are noted, particularly those relating to the theory of international trade.

1. It is assumed that there are two countries: Country One (the domestic country) and Country Two (the rest of the world). This aggregative procedure has both advantages and disadvantages. An alternative approach would be to include a large number of countries at the expense of reducing the economic description of them to the barest of details. J.J. Polak [2] and F.D. Graham [3] adopted this method which is vulnerable to the criticism that it ignores the indirect influences of changes among the variables, some of which may be important.

The method used, in this thesis, of aggregating all countries apart from the domestic one into a foreign country, while it leaves greater scope for a more adequate description of reactions among the variables both at home and abroad, is not without its critics as anyone familiar with the work of Graham is no doubt aware. He has demonstrated on *a priori* grounds that under certain conditions a multi-country approach could lead to greater stability in the international terms of trade than is indicated by two-country models. Though existing empirical evidence of severe terms of trade fluctuations in agricultural economies scarcely sustains the generality of his thesis, it is conceded by most theorists that Graham's multi-country models have qualified significantly in some instances the traditional two-country results [4,27; 5].

Nevertheless, there is much to be said for the 'one thing at a time' approach. As our purpose is to rework several international trade theorems when the assumption of only two commodities is relaxed, there is some justification for adopting the conventional two-country procedure. This fact should be borne clearly in mind, however, when interpreting our results.

2. It is assumed that in the world as a whole there are four commodities, two of which are traded internationally. This means that there is a class of goods in each country which because of transport costs, lack of mobility, or peculiar tastes does not enter into international trade. These non-traded or domestic commodities may be defined as those for which the supply and demand is equal in each country. Therefore, in addition to its own exportable good, each country is assumed to produce some of the commodity which it imports (increasing costs exist throughout each economy) as well as a class of domestic goods. It is evident that the domestic commodity of each country will not enter into the utility functions of the other nor will its price be a datum for the producers of the other country.

This aggregation of commodities into four broad categories, though it represents a drastic departure from the n-commodity procedure of Mosak, adds considerably to the realism of the model when it is compared with conventional two-commodity formulations. Moreover, the reader should note that the severity of the aggregation is not due entirely to our search for unfettered qualitative conclusions. We have shown in the Appendix that the qualitative nature of our results is unaffected if the model is generalised to include n non-traded commodities. It is only for the purpose of a simplified exposition that aggregation occurs in the non-traded sector.

In practice, two diametrically opposed methods by which goods may be aggregated suggest themselves. First, all those commodities which exert an identical influence upon consumption preference fields and production functions may be grouped together. Such goods are perfect substitutes for one another. Normally, perfect substitutability of this type is unlikely to exist as it would prove impossible to identify the two commodities as separate products. Any grouping of commodities along these lines, therefore, ideally should concern those goods which are close substitutes for one another. Secondly, commodities might be combined because of a high degree of complementarity. Thus, bacon and eggs or knives and forks could be grouped as single commodities.

While the type of problem that we wish to examine necessitates the grouping of commodities into exportables, importables, and non-traded goods, a procedure which would appear to exclude an approximation towards either a high degree of substitutability or complementarity, it must be remembered that in international trade we are concerned frequently with situations in which all prices within the exportable and importable sectors do tend to move proportionately on impact as the result of a uniform tariff or exchange rate variation. Thus, in relation to the problems to be studied shortly, the assumption of aggregate exportable and importable commodities appears to be a reasonable one.

3. It is assumed that perfect competition exists in both countries and that full employment and balance of payments equilibrium are maintained by a flexible price mechanism.

4. All consumers and producers are assumed to have given respective utility and production functions, each producer maximising his profits and each consumer his level of satisfaction.

5. Factors of production are assumed to be in fixed supply.

6. By a suitable choice of quantity units it is assumed, without any loss of generality, that all prices in the free trade model are equal initially to unity. Thus $p_1 = p_2 = p_3 = p_3' = 1$.

7. It is assumed that there exists a 'neutral' fifth commodity called money, common to both countries which does not enter into the utility functions of any individual, i.e. a doubling of the commodity 'money' would have no effect upon the equilibrium quantities of the goods bought and sold. It is stressed that this assumption is in no way vital to

our analysis which is conducted throughout in terms of relative prices. These could be as readily defined in terms of physical units of a commodity arbitrarily selected as a numeraire. The introduction of money is a concession to the reader who would prefer to think in terms of money prices. It follows that in our real world, in which there exists a neutral fifth commodity called money which is common to both countries, there is no need for an exchange rate.

8. The following properties of individual demand substitution effects are assumed to hold for aggregate demand substitution effects.

i. a_{ii} is negative, where a_{ii} measures the response in demand for the i^{th} good due to a real income compensated change in the price of the i^{th} commodity. Thus a compensated increase in the price of a commodity must always lead to a decrease in the demand for it. As long as the real income effect is not stronger than, and does not operate in a direction contrary to, the substitution effect, X_{ii} (the partial derivative of the i^{th} good with respect to the i^{th} price) will always be negative. If X_{ii} is positive, the commodity is defined as an inferior good.

ii. $a_{ij}=a_{ji}$. It should be noted, however, that X_{ij} does not equal X_{ji} unless the two income elasticities of demand $M_i\frac{M}{X_i}$ and $M_j\frac{M}{X_j}$ are also equal.

iii. $\sum_{j=1}^{j=3} p_j a_{ij} = 0$.

iv. $\Sigma p_j a_{ij}$ (for all values of j except i) = $-p_i a_{ii}$, which is necessarily positive.

As the final results of the different applications of our model are expressed in terms of elasticities rather than partial derivatives, the above properties of the demand substitution term are restated below in elasticity notation. The elasticity of demand for the i^{th} good with respect to the j^{th} price is defined as the rate of change in the quantity of the i^{th} good demanded, divided by the rate of change in the price of the j^{th} commodity:

$$\varepsilon_{ij} = X_{ij} \frac{p_j}{X_i}$$

From the theory of demand we know that ε_{ij} comprises an income and a substitution effect:

$$\varepsilon_{ij} = -M_i \frac{p_j}{X_i} X_j + \sigma_{ij}$$

where

$$M_i = \frac{\partial X_i}{\partial M}$$

and

$$\sigma_{ij} = \frac{p_j}{X_i} a_{ij} .$$

Or,

$$\varepsilon_{ij} = - \frac{p_j X_j}{p_i X_i} C_i + \sigma_{ij} \qquad \ldots \qquad (1.3)$$

where $C_i = p_i M_i$ is the marginal propensity to spend on the i^{th} commodity.

If $i = j$ then (1.3) becomes

$$\varepsilon_{ii} = -C_i + \sigma_{ii}$$

The four properties of the substitution term in elasticity notation are

i. $\sigma_{ii} < 0$ $(i = 1,2,3)$

ii. $\sigma_{ij} = \sigma_{ji} \frac{p_j X_j}{p_i X_i}$ $(i \text{ and } j = 1,2,3)$

iii. $\sum_{j=1}^{j=3} \sigma_{ij} = 0$ $(i = 1,2,3)$ from which it follows that

iv. $\Sigma \sigma_{ij}$ (for all values of j except i) $= -\sigma_{ii}$ which is positive

9. Similarly, it is assumed that the following properties of individual supply substitution effects also hold in the aggregate.

i. O_{ii} is positive. Thus, the higher the price, the greater the supply. It should be noted that O_{ii} is all substitution effect there being no income effect on the supply side.

ii. $O_{ij} = O_{ji}$.

iii. $\sum_{j=1}^{j=3} p_j O_{ij} = 0$.

iv. $\Sigma\, p_j O_{ij}$ (for all values of j except i) = $-p_i O_{ii}$, which is negative.[1]

As in the case of demand, the properties of the substitution term are restated in elasticity form, where S_{ij} is defined as the elasticity of supply of the i^{th} good with respect to the j^{th} price.

i. $S_{ii} > 0$.

ii. $S_{ij} = S_{ji}$.

iii. $\sum_{j=1}^{j=3} S_{ij} = 0$ (i = 1,2,3) .

iv. $\Sigma\, S_{ij}$ (for all values of j except i) = $-S_{ii}$ which is negative.

10. It is assumed that the determinant

$$\begin{vmatrix} K_{ii} & K_{ij} \\ K_{ji} & K_{jj} \end{vmatrix}$$

is positive where $K_{ii} = a_{ii} - O_{ii}$. An outline of the proof follows. Consider the determinant

$$\begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix} \qquad (1)$$

This is a determinant of income compensated partial derivatives of demand functions. From the theory of demand we know that because indifference surfaces are convex to the origin this determinant is both symmetric and positive. It is, in fact, the discriminant of a negative definite quadratic form. Similarly, from the fact that production possibility

1. For the formal proof of these properties of demand and supply substitution effects see [26,310-11,321].

surfaces defining supply conditions are concave to the origin, we know that the determinant

$$\begin{vmatrix} -O_{ii} & -O_{ij} \\ -O_{ji} & -O_{jj} \end{vmatrix} \qquad (2)$$

is also the discriminant of a negative definite quadratic form. In writing (2) we have multiplied all coefficients by minus one, thereby turning a positive definite into a negative definite quadratic form.

As the sum of two negative definite quadratic forms must itself be negative definite, it follows that

$$\begin{vmatrix} K_{ii} & K_{ij} \\ K_{ji} & K_{jj} \end{vmatrix} \qquad (3)$$

is the discriminant of a negative definite quadratic form and is accordingly positive in sign.

Dividing (3) by K_{jj} which is necessarily negative, we obtain an expression:

$$K_{ii} - K_{ij}\frac{K_{ji}}{K_{jj}} = \phi_{ij} < 0 \qquad (4)$$

As all prices are unity initially, (3) can be rewritten in elasticity form:

$$\begin{vmatrix} X_i\sigma_{ii} - O_iS_{ii} & X_i\sigma_{ij} - O_iS_{ij} \\ X_j\sigma_{ji} - O_jS_{ji} & X_j\sigma_{jj} - O_jS_{jj} \end{vmatrix} \qquad (5)$$

with the property of sign unchanged. Dividing (5) by $x_i(X_j\sigma_{jj} - O_jS_{jj})$ which is necessarily negative, we obtain the expression:

$$\left(\frac{X_i}{x_i}\sigma_{ii} - \frac{O_i}{x_i}S_{ii}\right) - \left(\frac{X_i}{x_i}\sigma_{ij} - \frac{O_i}{x_i}S_{ij}\right)\left(\frac{\sigma_{ji} - O_{ji}}{\sigma_{jj} - O_{jj}}\right) = \psi_{ij} < 0 \qquad (6)$$

The expression ψ_{ij} appears in all applications of our model.

11. The assumption that community demand elasticities have the same properties as the individual concepts is, perhaps, our most controversial one. Two points, in particular, can be made. First, the reader should note that the use of aggregate elasticities is not without precedent; most models implicitly or explicitly adopt an aggregative procedure. There is, for instance, the authoritative assurance of Baumol that the use of community indifference curves in the theory of international trade is a legitimate procedure [87,19]. Secondly, the reader is referred to a forthcoming work [6, Ch.3] in which the author concludes that "Aggregate elasticities are likely to possess precisely the same properties of sign and symmetry as individual elasticities." This conclusion is subject to the assumption that any money income redistribution is random with respect to tastes. In a model such as our own, in which commodities are aggregated broadly, this does not seem to be an unreasonable economic assumption.

It is noted that problems of aggregation do not arise on the supply side.

12. It is assumed that the demand for each commodity is a function of all commodity prices and total expenditure (which need not be equal to the value of production in an open economy). As part of expenditure comprises investment it is assumed that marginal producers act as a class of 'consumers' whose 'indifference curves' are defined by production functions. In this way, any redistribution of investment expenditure due to relative price changes, is analogous to the redistribution of consumption expenditure referred to in the preceding assumption.

13. Finally, supply is assumed to be a function of all commodity prices. It is assumed that supply functions are homogeneous of order zero in prices so that a doubling of all prices would leave supply unchanged. Furthermore, the transformation surface defining optimum production possibilities is assumed to be concave to the origin.

D. MODEL OF A CLOSED, THREE-COMMODITY ECONOMY

Consider an economy in which there are three commodities of which the quantities O_1, O_2, O_3 are produced and the quantities X_1, X_2, X_3 are demanded for consumption. Perfect competition and profit maximization are assumed and there is full employment without inflation. The community's preference and supply functions are given.

1. Equilibrium equations

The following equations describe the equilibrium situation in our closed economy.

$$\left.\begin{aligned} X_1 - O_1 &= 0 \\ X_2 - O_2 &= 0 \\ X_3 - O_3 &= 0 \end{aligned}\right) \quad \ldots \quad (2.3)$$

$$M - p_1O_1 - p_2O_2 - p_3O_3 = 0 \quad \ldots \quad (3.3)$$

where $X_i = X_i(p_1,p_2,p_3,M)$ and $O_i = O_i(p_1,p_2,p_3)$.

Equations (2.3) are the equilibrium conditions which state that every market must be cleared, i.e. that in equilibrium there shall be zero excess demand in the system. Equation (3.3) is an identity, satisfied for any set of prices, which states that expenditure (M) is equal to $\sum_{j=1}^{j=3} p_jX_j$ which, in a closed economy, must be equal to the value of production $\sum_{j=1}^{j=3} p_jO_j$.

Treating all prices as variables, consider the simultaneous equilibrium of our system. We have four unknowns, p_1, p_2, p_3 and M and four equations. One of the supply-demand equations, however, is not independent. As the equilibrium conditions require that every excess demand be zero it is clear that if two of our markets are in equilibrium so, too, must be the third. We must, therefore, drop one of the supply-demand equations - it does not matter which. This, however, means that we cannot solve for the three absolute prices. Since the equations (2.3) are homogeneous of degree zero in prices, the number of variables can be reduced by one by using as numeraire the price of any arbitrarily selected commodity. Then, we may solve for two independent exchange ratios and for expenditure in terms of the numeraire. It can be shown that these two exchange ratios are sufficient to determine the barter terms of trade between every possible pair of commodities.

2. Stability conditions

The set of equations (2.3) and (3.3) are sufficient to determine two independent relative prices and expenditure (M) under a given set of conditions which includes the preference and supply functions of the community. A change in this given data will mean a new set of equilibrium price ratios and it is the nature and direction of the shift in these which we wish to describe. In order to do so, it is necessary first to specify the properties of the initial equilibrium position subject to the given conditions. These properties of the equilibrium position are known as stability conditions.

Hicks was the first economist to attempt a precise statement of stability conditions for a multi-market economy in which income effects are accounted for explicitly [26, 315]. First, he defined the excess demand for a commodity as the difference between the total quantity demanded and the total quantity supplied. Thus, in equilibrium, there would be zero excess demand for each commodity. In a single-commodity market the condition for stability is that a reduction in the price of a commodity should tend to increase the excess demand for it, this increase tending to restore the price to its equilibrium level. Thus a positive excess demand means that consumers desire to buy more than is being supplied at the current price. Competition among consumers will then lead to an increase in price which, if the market is stable, will increase the quantity supplied (a negative excess demand). Correspondingly, a situation characterized by negative excess demand means that suppliers are offering more than consumers wish to purchase at the current price. Competition among sellers will force down the price and cause, if the market is stable, a positive excess demand. In both cases price will adjust to its equilibrium level.

Stability conditions for a market in which there is more than one commodity, however, necessitate a consideration of the effect which a fall in the price of one commodity will have upon the excess demand for, and prices of, the other commodities in the system. Hicks, therefore, in an endeavour to surmount this difficulty, gave two definitions of stability, the one differing from the other according to the behaviour of the other prices in the system. First, he defined a market as imperfectly stable if a fall in the price of a good results in an excess demand for it, after all prices have shifted in such a way as to equate the quantities demanded and supplied in all other markets save the one under consideration. Secondly, Hicks defined a market as perfectly stable if a fall in the price of a good resulted in an excess demand for it after any given subset of prices in other markets is adjusted so that supply again equals demand in those markets, with all remaining prices held constant.

Consider the three-commodity system given by equations (2.3) and (3.3). If we choose to drop the supply-demand equations involving commodity one and select p_1 as numeraire, the effects of price changes upon the excess demands (D_i) are computed by total differentiation:

$$dD_2 = (X_{22}-O_{22})dp_2 + (X_{23}-O_{23})dp_3 + M_2 dM$$

$$dD_3 = (X_{32}-O_{32})dp_2 + (X_{33}-O_{33})dp_3 + M_3 dM$$

$$dM = O_2 dp_2 + O_3 dp_3$$

where

$$X_{ij} = \frac{\partial X_i}{\partial p_j}, \quad O_{ij} = \frac{\partial O_i}{\partial p_j}, \quad \text{and } M_i = \frac{\partial X_i}{\partial M}.$$

The reader should note that in differentiating the expenditure equation, use has been made of the relationship that $\sum_{i=1}^{i=3} p_i dO_i = 0$, which is equivalent to assuming that the ratio of changes in production must be equal to minus the ratio of marginal costs which, in turn, is equal to minus the ratio of prices.

Substituting for dM we have:

$$dD_2 = (K_{22}-X_2M_2)dp_2 + (K_{23}-X_3M_2)dp_3 + M_2(O_2dp_2+O_3dp_3)$$

$$dD_3 = (K_{32}-X_3M_3)dp_2 + (K_{33}-X_3M_3)dp_3 + M_3(O_2dp_2+O_3dp_3)$$

where $X_{ij} = - X_jM_i + a_{ij}$

and $K_{ij} = a_{ij} - O_{ij}$.

Making use of the fact that in a closed economy the demand for a commodity must equal the supply of it, we have:

$$\begin{aligned} dD_2 &= K_{22}dp_2 + K_{23}dp_3 \\ dD_3 &= K_{32}dp_2 + K_{33}dp_3 \end{aligned} \quad \ldots \quad (4.3)$$

Since K_{ij} may be assumed constant in a small neighbourhood around the equilibrium point, (4.3) forms a system of simultaneous linear equations in the variables dp_2 and dp_3. The coefficients of (4.3) form the Jacobian of (D_2,D_3) with respect to (p_2,p_3).

If equilibrium is displaced in the market for commodity two and all other prices are rigid (dp_3=0) then the first condition for the perfect stability of the market for good two is:

$$\frac{dD_2}{dp_2} = K_{22} < 0 \ .$$

This does not mean that the displacement of equilibrium in the market for good two has failed to react upon the equilibrium of the market for good three. It does mean that since the price of commodity three is rigid, the excess demand for good three will fail to react upon D_2.

The other condition for the perfect stability of the market for good two is when p_3 adjusts in response to the initial displacement. Solving, we have:

$$\frac{dD_2}{dp_2} = \frac{\begin{vmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{vmatrix}}{K_{33}} < 0$$

Obviously, a similar set of conditions can be obtained for $\frac{dD_3}{dp_3}$. Generalising, therefore, perfect stability for the closed, three-commodity system requires that the Jacobian determinants

$$K_{ii}, \quad \begin{vmatrix} K_{ii} & K_{ij} \\ K_{ji} & K_{jj} \end{vmatrix} \quad \ldots \quad (5.3)$$

be negative and positive respectively for all values of i and j.

The conditions for perfect stability in the Hicksian sense are more stringent than necessary for the consideration of many multimarket systems. If the system contains no rigid prices the necessary condition is that the i^{th} market is stable if

$$\frac{dD_i}{dp_i} = \frac{|A|}{A_{ii}} < 0 \quad \ldots \quad (6.3)$$

where $|A|$ is the Jacobian determinant of the complete system given by (4.3) and A_{ii} is cofactor of K_{ii} in $|A|$.

Can anything be said concerning the sign of the determinants:

$$K_{jj} \; (= A_{ii}) \text{ and } \begin{vmatrix} K_{ii} & K_{ij} \\ K_{ji} & K_{jj} \end{vmatrix} \; (=|A|)$$

From the theory of value we know that a_{jj} and -0_{jj} are negative (see assumptions 8i and 9i). Likewise, it has been shown that $|A|$ is positive (see assumption 10). We may conclude, therefore, that our closed, three-good system is both perfectly and imperfectly stable in the Hicksian sense.

For two reasons considerable care has been taken in the setting out of this closed economy model and in deriving the Hicksian stability conditions. First, we can, as a result, proceed more rapidly with the development of the international model. Secondly, the development of similar stability conditions for the international model emphasises clearly the source of possible instability in the international market.

At this juncture one of our assumptions should be recalled. The fact that any redistribution of money income should be random with respect to tastes is, when coupled with the identity of expenditure and the value of output in a closed economy, the reason for the non-appearance of possibly destabilizing income effects. We shall have cause to remember this when investigating the stability of our international system.

E. A FOUR-COMMODITY, INTERNATIONAL FREE TRADE MODEL

1. Equilibrium equations

Consider two closed economies of the type described in the preceding section of this chapter. Under pure competition the price ratios between internationally traded commodities in the absence of transportation costs must be the same for both countries. Now, it is a basic assumption of the work which follows that at all times expenditure in the domestic and the foreign country together must equal the value of production in the domestic and foreign country together. Consequently, at all times, the equilibrium of the system requires that the price of each commodity be such that all markets are cleared. The following six equations describe the equilibrium of our international system where in equilibrium all $D_i = 0$.

$$D_1 = X_1' - O_1' + X_1 - O_1 \quad \ldots \quad (7.3)$$

$$D_2 = X_2 - O_2 + X_2' - O_2' \quad \ldots \quad (8.3)$$

$$D_3 = X_3 - O_3 \quad \ldots \quad (9.3)$$

$$D_3' = X_3' - O_3' \quad \ldots \quad (10.3)$$

$$M = p_1O_1 + p_2O_2 + p_3O_3 \quad \ldots \quad (11.3)$$

$$M' = p_1O_1' + p_2O_2' + p_3'O_3' \quad \ldots \quad (12.3)$$

Two points should be noted. First, the fact that all markets must be cleared means that one supply and demand equation is not independent. Thus, we might choose to drop any one of the equations (7.3)to (10.3). Secondly, the balance of payments equation

$$B = p_1x_1 - p_2x_2 \quad \ldots \quad (13.3)$$

where B = 0 in this free trade case, is implied by the income-expenditure identities and the three independent supply-demand equations.

Our system does not determine absolute prices in money terms nor is it important for our purposes that it should do so because, as pointed out above, money is used in our model in a purely neutral or accounting sense. For what it is worth, the absolute level of money prices could be determined by the addition of what is known in monetary theory as an equation of exchange:

$$N = K(Y_w) \quad \ldots \quad (14.3)$$

in which the world volume of money (N) (there being a currency common to both countries) and the proportion of money (K) to world income (Y_w) are given. Equation (14.3) could be rewritten:

$$N = K\,\Sigma p_i X_i \qquad (i=1,2,3,3')$$

or

$$N = p_1 K \Sigma \frac{p_i}{p_1} X_i$$

from which it follows that

$$p_1 = \frac{N}{K\,\Sigma \frac{p_i}{p_1} X_i}$$

The above relationship states that, in terms of our token money, the level of absolute prices will be, given K, directly proportional to the volume of money. This is a rigorous formulation of the quantity theory. It is emphasised, however, that as long as we assume that money does not enter into the utility functions of individuals it can have no influence on the amounts of commodities supplied or demanded in equilibrium. These are dependent upon the independent price ratios and money expenditure. A doubling of the token money would leave these ratios unaffected. Only where money is considered in the Walrasian sense as having a direct utility of its own, can the relative price ratios alter with a change in the supply of money.

2. Hicksian stability of the international model

The Hicksian conditions for the equilibrium of the international economy are strictly analogous to those given for the closed economy. If we again choose to drop the demand-supply equation involving commodity one and to retain the price of domestic exportables ($p_1 = 1$) as numeraire, a total differentiation of equations (8.3)to (12.3) gives:

$$dD_2 = (X_{22}-O_{22}+X'_{22}-O'_{22})dp_2 + (X_{23}-O_{23})dp_3 + (X'_{23}-O'_{23})dp'_3 + M'_2dM' + M_2dM$$

$$dD_3 = (X_{32}-O_{32})dp_2 + (X_{33}-O_{33})dp_3 + M_3dM$$

$$dD'_3 = (X'_{32}-O'_{32})dp_2 + (X'_{33}-O'_{33})dp'_3 + M'_3dM'$$

$$dM = O_2dp_2 + O_3dp_3$$

$$dM' = O'_2dp_2 + O'_3dp'_3$$

Substituting in the first three equations for dM and dM' and remembering that

i. $(K_{ij}-X_jM_i) = (X_{ij}-O_{ij})$;

ii. X_i-O_i is equal to the relevant x_1, x_2, or zero as indicated by the equilibrium equations;

we have:

$$\left.\begin{aligned} dD_2 &= (K_{22}+K'_{22}+x_2M'_2-x_2M_2)dp_2 + K_{23}dp_3 + K'_{23}dp'_3 \\ dD_3 &= (K_{32}-x_2M_3)dp_2 + K_{33}dp_3 \\ dD'_3 &= (K'_{32}+x_2M'_3)dp_2 + K'_{33}dp'_3 \end{aligned}\right) \quad \ldots \quad (15.3)$$

The coefficients of (15.3) form the Jacobian $|A|$ of (D_2, D_3, D'_3) with respect to (p_2, p_3, p'_3). If the international market is to be perfectly stable in the Hicksian manner, the world excess demand for the i^{th} commodity must exist irrespective of whether or not the other price ratios remain constant or are adjusted so as to equate supply and demand in one or more of the other markets. For each additional price that is held constant there is one less unknown and one less equation to set equal to zero in (15.3). Solving for every such system the necessary conditions for Hicksian perfect stability are that the determinants

$$b_{ii}, \quad \begin{vmatrix} b_{ii} & b_{ij} \\ b_{ji} & b_{jj} \end{vmatrix}, \text{ and } |A| \quad \ldots \quad (16.3)$$

should be negative and positive alternately where b_{ij} is the element in the i^{th} row and j^{th} column of $|A|$ ($|A|$ being the Jacobian determinant of the full system given by (15.3)).

Imperfect stability requires that there shall be a world excess demand (negative or positive for a commodity following a change in its price, after all other price ratios are adjusted so as to equate the quantities demanded and supplied in the other markets. Thus, if equilibrium is displaced in the market for the i^{th} commodity, the system is imperfectly stable if

$$\frac{dD_i}{dp_i} = \frac{|A|}{A_{ii}} < 0 \quad \ldots \quad (17.3)$$

where $|A|$ is defined as in (16.3) and A_{ii} is the cofactor of b_{ii} in $|A|$.

In our closed economy we saw that the conditions necessary for both perfect and imperfect stability would be fulfilled. Can the same thing be said for our international system? Consider first, the requirements for perfect stability as given by (16.3). Written in full, these require that the determinants

$$K_{22} + K'_{22} + M'_2 - M_2 \,, \quad K_{33} \,, \quad K'_{33}$$

should be negative; that the determinants

$$\begin{vmatrix} K_{22}+K'_{22}+(M'_2-M_2)x_1 & K_{23} \\ K_{32}-M_3x_1 & K_{33} \end{vmatrix} , \quad \begin{vmatrix} K_{22}+K'_{22}+(M'_2-M_2)x_1 & K'_{23} \\ K'_{32}+M'_3x_1 & K'_{33} \end{vmatrix} , \quad \begin{vmatrix} K_{33} & 0 \\ 0 & K'_{33} \end{vmatrix}$$

should be positive; and that the determinant

$$\begin{vmatrix} K_{22}+K'_{22}+(M'_2-M_2)x_1 & K_{23} & K'_{23} \\ K_{32}-M_3x_1 & K_{33} & 0 \\ K'_{32}+M'_3x_1 & 0 & K'_{33} \end{vmatrix} \qquad (= |A|)$$

should be negative.

It is evident that these conditions differ significantly from those established in the closed economy. The reason for this difference (apart from the inclusion of an extra commodity) is the appearance of asymmetric income effects between the two countries. In the closed economy model all income effects were considered to be negligible or self-cancelling because of the assumption that any redistribution of income would be random with respect to tastes. In an international economy, however, where in any one country the demand for and supply of any internationally traded good is not equal, this assumption neither means that income effects will be negligible nor that they will cancel out. For instance, a rise in the price of domestic importables must result ~~(save where inferior commodities exist)~~ in negative income effects in the domestic country where the demand for exceeds the supply of importables, and in corresponding positive effects in the foreign country where the supply of the good necessarily exceeds the demand for it. In our model, where p_1 is selected as numeraire, this fact is reflected in $|A|$ by the appearance of the income effects in column one associated with the change in the price of good two (domestic importables). Only in the non-traded good sectors, where by definition the supply of equals the demand for the commodity, do the income effects cancel out. To remove these asymmetric income effects one would need to extend the assumption that any redistribution of income will be random with respect to tastes, so that it refers to the world as a whole. There appears to be little or no economic justification for this, particularly as the two non-traded goods are separate commodities.

If, therefore, our system is to be perfectly stable in the Hicksian sense, it is necessary to resort to probability type arguments. There are many economists who will not accept such arguments, holding that from ignorance only ignorance can result. Our position is not so grim, however. For instance, we do know the minimum and maximum limits of our variables, i.e. that K_{ii} can range from zero to minus infinity, while M_i must be positive and less than unity. We shall proceed, therefore, to consider the probable sign of the seven determinants set out above.

Consider first, the sign of the full determinant, $|A|$, which appears as the denominator in every application of our model. One of the conditions for perfect stability is that

$$|A| \quad = \quad \begin{vmatrix} K_{22}+K'_{22}+(M'_2-M_2)x_2 & K_{23} & K'_{23} \\ K_{32}-M_3x_2 & K_{33} & 0 \\ K'_{32}+M'_3x_2 & 0 & K'_{33} \end{vmatrix} \quad < \quad 0$$

An expansion of $|A|$ by the first column yields:

$$|A| = [K_{22}+K'_{22}+x_2(M'_2-M_2)]K_{33}K'_{33} - K_{32}(K_{23}K'_{33}) + M_3x_2(K_{23}K'_{33}) - K'_{32}(K'_{23}K_{33}) - M'_3x_2(K'_{23}K_{33})$$

which, when divided and multiplied throughout by $K_{33}K'_{33}$ gives:

$$|A| = \left[\frac{\begin{vmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{vmatrix}}{K_{33}} + \frac{\begin{vmatrix} K'_{22} & K'_{23} \\ K'_{32} & K'_{33} \end{vmatrix}}{K'_{33}} - \frac{x_2 \begin{vmatrix} M_2 & K_{23} \\ M_3 & K_{33} \end{vmatrix}}{K_{33}} + \frac{x_2 \begin{vmatrix} M'_2 & K'_{23} \\ M'_3 & K'_{33} \end{vmatrix}}{K'_{33}} \right] K_{33}.K'_{33}$$

$$= (\phi_{23} + \phi'_{23} + x_2\varphi_{23} - x_2\varphi'_{23})K_{33}.K'_{33}$$

where

$$\phi_{ij} = K_{ii} - K_{ij}\frac{K_{ji}}{K_{jj}} \; ; \quad \text{and} \quad \varphi_{ij} = -M_i + M_j \frac{K_{ij}}{K_{jj}} \; .$$

For purposes of discussion, and because most of our subsequent analysis appears in this form, $|A|$ is best considered in terms of elasticities. Since our choice of quantity units (which has been made with no loss of generality) implies that

i. M_i is both a marginal propensity to spend as well as an income effect;

ii. $K_{ij}=X_i\sigma_{ij}-O_iS_{ij}$, where σ_{ij} and S_{ij} are substitution elasticities of demand and supply with respect to p_j, and where X_i and O_i are quantities measured as multiples of the quantity of home exports x_1, and imports x_2;

it follows that $|A|$ can be expressed in terms of quantity weighted elasticities and marginal propensities to spend:

$$|A| = x_2(\Psi_{23} + \Psi'_{23} + \psi_{23} - \psi'_{23})K_{33}.K'_{33} \quad \ldots \quad (18.3)$$

where

$$\Psi_{ij} = (\frac{X_i}{x_i}\sigma_{ii} - \frac{O_i}{x_i}S_{ii}) - (\frac{X_i}{x_i}\sigma_{ij} - \frac{O_i}{x_i}S_{ij})(\frac{\sigma_{ji} - S_{ji}}{\sigma_{jj} - S_{jj}})$$

$$\psi_{ij} = -C_i + C_j(\frac{\sigma_{ji} - S_{ji}}{\sigma_{jj} - S_{jj}})$$

$$x_i\Psi_{ij} = \phi_{ij} \quad \text{(see assumption 10 above)}$$

and $$\psi_{ij} = \varphi_{ij} \; .$$

The reader should note that in converting ϕ_{ij} into ψ_{ij} the term K_{ji} was substituted for K_{ij} which, of course, we are entitled to do when dealing with partial derivatives.

In order to determine the sign of $|A|$ it is important to determine the sign and magnitude of its components. In assumption 10, it was shown that $\phi_{ij} = \psi_{ij}x_i$ must be negative. In addition to the property of sign, it is also possible to infer something about the magnitude of ψ_{ij} if we have some qualitative knowledge of the kind of commodities involved. The first expression in ~~square~~ brackets is negative (see assumption 8i). Likewise, the second term is negative, save where both complementarity in demand and/or joint supply exist. Thus, as ψ_{ij} must be negative, it follows thatit must be either less in absolute value than $(X_i\sigma_{ii} - O_iS_{ii})$ or equal to it.

Conditions for a large ψ_{ij} which will approximate in magnitude to the quantity weighted sum of the own substitution elasticities for the i^{th} good are:

i. that all own elasticities, both of supply and demand, should be high;

ii. that the amounts consumed and produced of the i^{th} commodity should be large relative to the amount traded, i.e. X_i and O_i which are multiples of x_i should be large;

iii. that there should be low cross elasticities, both of supply and demand, between non-traded and importable commodities, relative to the own elasticities of non-traded and importable commodities, i.e. σ_{ij} and S_{ij} low relative to σ_{ii} and S_{ii} respectively and σ_{ji} and S_{ji} low relative to σ_{jj} and S_{jj} respectively;

iv. from (iii) it follows that the cross elasticities of demand and supply between non-traded and exportable commodities should be high relative to the own elasticities of non-traded and exportable goods.

In summary, the expression ψ_{ij} which is called, henceforth, a coefficient of sensitivity, is an aggregation of substitution elasticities in the form of a determinant.

It reflects the sensitivity of the goods concerned to price changes in the system by considering not only the impact effect of a price change but also indirect effects which are reflected in the relative degree of competitiveness, both in production and consumption, between importable and non-traded goods on the one hand, and exportable and non-traded goods on the other. It has generally, the usual properties of an elasticity, being larger when substitution possibilities are greater, and smaller when there are both complementarity in consumption and/or joint supply.

Consider next the sign of

$$\psi_{ij} = -C_i + C_j\left(\frac{\sigma_{ji}-S_{ji}}{\sigma_{jj}-S_{jj}}\right) .$$

We know from the theory of demand that in a three-commodity world, $\sigma_{31}+\sigma_{32}+\sigma_{33}=0$ (see assumption 8iii), σ_{33} is inherently negative, and that σ_{31} and σ_{32} will be positive, save in the exceptional case where only three goods are consumed, of complementarity in consumption, in which case one of them could conceivably be negative. As long as they are positive, σ_{31} and σ_{32} each must be numerically less than σ_{33}. A parallel relationship holds for supply elasticities though they are of opposite sign. Thus, S_{31} and S_{32} each will be less in absolute value than S_{33}. Again, the case of joint supply, analogous to complementarity in demand, is excepted. Hence, the expression

$$\left(\frac{\sigma_{ji} - S_{ji}}{\sigma_{jj} - S_{jj}}\right)$$

normally will be less than unity and, since the numerator and denominator are of opposite sign, negative. As the multiplying term, C_j, must also be less than unity, this would reduce further the magnitude of the bracketed expression. The final value of ψ_{ij} is obtained by adding the negative value of the relevant marginal propensity to spend on importables. We may conclude, therefore, that the term ψ_{ij} will be negative and less than unity though it will exceed in magnitude the marginal propensity to spend on importables, the extent of this excess depending upon the relative degree of substitutability between importables and non-traded goods on the one hand, and exportables and non-traded goods on the other.

Thus, ψ_{23} and ψ'_{23} normally will be larger than C_2 and C'_2 respectively, but less than unity.

We are now in a position to examine the probable sign of $|A|$. The necessary conditions for a positive $|A|$ and, consequently, for possible instability are that

$$\Psi_{23} + \Psi'_{23} + \psi_{23} - \psi'_{23} > 0$$

which requires:

i. that the foreign country's marginal propensities to spend on foreign exportables and non-traded goods respectively should exceed considerably the domestic country's marginal propensities to spend on the same commodities;

ii. that in each country the quantity weighted own elasticities of demand for and supply of domestic importables be very small as this would ensure small coefficients of sensitivity.

There are strong a priori reasons for considering the fulfilment of these conditions to be abnormal. First, for $|A|$ to be positive both coefficients of sensitivity must be small, which implies complementarity in demand and joint production: if σ'_{22} and S'_{22} are very small it means that commodity two cannot be substituted for either goods one or three so that commodities one, two, and three all must be consumed and supplied in fixed proportions; similarly, in the domestic country. Now, it is improbable where such strong complementarity exists in each country that tastes and production conditions would not be similar also in the two trading areas. If this is so, it is easy to demonstrate that ψ_{23} and ψ'_{23} would tend to cancel each other out.

Secondly, by considering a sufficiently small volume of trade relative to the scale of the world economy, the negativeness of $|A|$ can always be secured, for every X_i and O_i in the model is measured relative to $x_1=x_2$ (providing the trade balance is small in relation to the volume of trade). Indeed, a sufficient condition is that one country produces the majority of its consumption of importables or that its consumption of exportables is large relative to its exports. As the foreign country in our model represents the rest of the world, these conditions are likely to be satisfied.

Finally, recourse is had to the probabilistic argument that the sum of two terms $(\Psi_{23} + \Psi'_{23})$, the value of each of which can range from zero to infinity, is almost certain to be greater than the difference between two terms $(\psi_{23} - \psi'_{23})$, the value of each of which cannot exceed unity. This line of reasoning, of course, is vulnerable to the sceptic's objection that from ignorance only ignorance can result. Nevertheless, we are not entirely ignorant - at least the range of values of the parameters is known.

Consider next the three second-order principal minors

$$\begin{vmatrix} K_{22}+K'_{22}+(M'_2-M_2)x_2 & K_{23} \\ K_{32}-M_3x_2 & K_{33} \end{vmatrix} (=|B|), \quad \begin{vmatrix} K_{22}+K'_{22}+(M'_2-M_2)x_2 & K'_{23} \\ K'_{32}+M'_3x_2 & K'_{33} \end{vmatrix} (=|C|), \quad \begin{vmatrix} K_{33} & 0 \\ 0 & K'_{33} \end{vmatrix} (=|D|),$$

whose signs must all be positive if the conditions for perfect stability are to be satisfied. $|D|$ is obviously positive by inspection. An expansion of $|B|$ yields

$$|B| = [K_{22}+K'_{22}+x_2(M'_2-M_2) - K_{23}\frac{K_{32}}{K_{33}} + x_2M_3\frac{K_{23}}{K_{33}}]K_{33}$$

which, when converted to elasticities, gives

$$|B| = x_2[\Psi_{23}+\psi_{23}+E'_{22}+C'_2]E_{33} \qquad \ldots \qquad (19.3)$$

A positive $|B|$ requires

$$\Psi_{23} + \psi_{23} + E'_{22} > -C'_2$$

For reasons advanced above it is scarcely conceivable that this condition would not be met. A similar expansion of $|C|$ gives

$$|C| = x_2[\psi'_{23}-\psi'_{23}+E_{22}-C_2]E'_{33} \quad \ldots \quad (20.3)$$

which, if $|C|$ is to be positive, requires

$$\psi'_{23} + E_{22} - C_2 > \psi'_{23}$$

From inspection it can be seen that $|B|$ is more likely to be positive, on probability grounds, than is $|C|$. Nevertheless, $|C|$ is almost certainly positive, the same arguments applying here as were used to justify the negativeness of the full determinant.

Finally, we must consider the three, first-order, principal minors

$$[K_{22}+K'_{22}+(M'_2-M_2)] = |E|, \qquad K_{33}, \qquad K'_{33},$$

of which the last two necessarily are negative. If $|E|$ is less than zero, all the conditions necessary for the perfect stability of our model are satisfied. Converting $|E|$ to elasticities we have

$$|E| = \frac{X_2}{x_2}\sigma_{22} + \frac{X'_2}{x_2}\sigma'_{22} - \frac{O_2}{x_2}S_{22} - \frac{O'_2}{x_2}S'_{22} + C'_2 - C_2 \quad \ldots \quad (21.3)$$

which, if $|E|$ is to be negative, requires

$$\frac{X_2}{x_2}\sigma_{22} + \frac{X'_2}{x_2}\sigma'_{22} - \frac{O_2}{x_2}S_{22} - \frac{O'_2}{x_2}S'_{22} > C_2 - C'_2$$

Once again, the arguments used above apply with undiminished force. All of our X_i are multiples of x_2, so that the magnitude of σ_{22} and σ'_{22} would be enlarged accordingly. At the same time, we have the sum of four parameters each of which can range in value from zero to minus infinity, offset only by the difference between two propensities which individually cannot exceed unity. Finally, it should be noted that there are grounds for considering the difference between the two income effects to be negligible. It was pointed out earlier that international trade introduces asymmetric income effects, a rise in the price of domestic importables necessarily reducing income in the home and increasing income in the foreign country. Where a common good is involved it seems plausible to argue that the income effects incurred by the surplus of consumption over production in the one country should cancel the income effects arising from the surplus of production over consumption in the other country. Necessary and important qualifications are, of course, that tastes are more likely to differ between trading areas than within them and that income effects involving the non-traded commodity of each country do not refer to a common good.

The arguments set out above hold a fortiori in the case of imperfect stability (see equation (17.3)) where it is only necessary to show that $|A|$ is negative and that all second order principal minors are positive. Hence, it is contended that on reasonable economic and probabilistic grounds our international system is both perfectly and imperfectly stable in the manner defined by Hicks.

3. Dynamic stability of the model

We must concern ourselves with a fundamental objection, first made by Samuelson [93;94], to the Hicksian concepts of stability.

Hicks pointed out in his text that the method of comparative statics has no meaning unless the economic system is dynamically stable, for only stable systems tend to approach equilibrium when disturbed. He concluded correctly, therefore, that the conditions for true dynamic stability would provide important information about the properties of static equilibrium:

> The laws of change of the price-system, like the laws of change of individual demand, have to be derived from stability conditions. We examine first what changes are necessary in order that a given equilibrium system should be stable; then we make an assumption of regularity, that positions in the neighbourhood of the equilibrium position will be stable also; and hence we deduce rules about the way in which the price-system will react to changes in tastes and resources [26,62].

Unfortunately, the Hicksian conditions of stability are not true dynamic stability conditions. Samuelson has shown that it is inadmissible to assume, as does Hicks, that when the price of one commodity is out of equilibrium the prices of all other commodities are either unchanged or are adjusted instantly to their new equilibrium [93,111-112]. When a dynamic system is involved true dynamic stability depends not only upon the slopes of the excess demand functions, D_i, but also upon the relative speeds of adjustment in the different markets. For instance, Samuelson gave examples showing that neither perfect nor imperfect stability of the Hicksian kind is sufficient to ensure true dynamic stability under all circumstances. Indeed, a system could be dynamically stable even though it were neither perfectly nor imperfectly stable in the Hicksian sense.

Since Hicks does not refer explicitly to speeds of adjustment in the different markets, he would appear to have developed stability conditions which are independent of reaction speeds. For instance, in assessing the effect of a change in price of the i^{th} commodity Hicks assumes that all other prices adjust to their new equilibria while no further change occurs in the price of the i^{th} commodity. Thus the reaction speed in the i^{th} market is assumed small, relative to other reaction speeds. But when considering the j^{th} market he must likewise assume that the speed of adjustment in this market also is small relative to other reaction speeds. Yet this is inconsistent - the reaction speed in the j^{th} market cannot be small relative to the reaction speed in the i^{th} market at the same time as the reaction speed in the i^{th} market is small relative to that in the j^{th} market. In fact, Hicks is postulating a different dynamic system for each market and it follows that the stability conditions he derives cannot be consistent with multi-market equilibrium unless stability is independent of the different reaction speeds.

If time is introduced to our model, an explicit statement must be made concerning the laws of price change before the time paths of the prices, following a disturbance, are investigated. Many different types of adjustment processes may be introduced [25,263-69]. Generally, a multi-market equilibrium is dynamically stable if every price approaches its equilibrium level over time following a slight displacement from equilibrium, i.e. if

$$\lim_{t\to\infty} p_{jt} = p_j^{\circ}$$

where p_{jt} is the price of X_j at time t and p_j° is the equilibrium price of X_j.

Consider again our international system, this time assuming that if the price of a good falls when its supply exceeds its demand, the system is dynamically stable. It is assumed that stability is dependent not only upon the slopes of the excess demand functions, but also upon the relative speeds of adjustment in each market. Assume further that the rate of change in price in each market is proportional to the amount of excess demand (negative or positive) in the market. Then, in such a system,

$$\frac{dp_2}{dt} = \alpha_2 D_2$$

$$\frac{dp_3}{dt} = \alpha_3 D_3$$

$$\frac{dp_3'}{dt} = \alpha_3' D_3'$$

where D_i is the (negative) excess demand function and α_i is the proportionality constant relating (in this case of excess supply) the rate of decrease in the i^{th} price to the amount of negative excess demand for the i^{th} commodity.

In our three-equation system, therefore, we have for small deviations from equilibrium

$$\left.\begin{aligned}
\frac{dp_2}{dt} &= \alpha_2 b_{22}(p_2-p_2^{\circ})+\alpha_2 b_{23}(p_3-p_3^{\circ})+\alpha_2 b_{23}'(p_3'-p_3^{\circ\prime}) \\
\frac{dp_3}{dt} &= \alpha_3 b_{32}(p_2-p_2^{\circ})+\alpha_3 b_{33}(p_3-p_3^{\circ}) \\
\frac{dp_3'}{dt} &= \alpha_3' b_{32}'(p_2-p_2^{\circ})+\alpha_3' b_{33}'(p_3'-p_3^{\circ\prime})
\end{aligned}\right\} \quad \ldots \quad (22.3)$$

where b_{ij} is defined as in (16.3) and the superscript ° refers to the equilibrium price.

By standard mathematical procedure the characteristic equation of the differential equations (22.3) is:

$$\begin{vmatrix} \alpha_2 b_{22}-\lambda & \alpha_2 b_{23} & \alpha_2 b_{23}' \\ \alpha_3 b_{32} & \alpha_3 b_{33}-\lambda & 0 \\ \alpha_3' b_{32}' & 0 & \alpha_3' b_{33}'-\lambda \end{vmatrix} \quad \ldots \quad (23.3)$$

which, if non-zero solutions are to be obtained, must be identically equal to zero.

Now, it is a well-known mathematical fact that if a differential equation system with constant coefficients such as (15.3) is to have stable solutions, the real parts of the roots of (23.3) (including the real parts of any complex roots), must be negative. Otherwise, the necessary dampening factors would not be present in the exponential solutions.

There exist necessary and sufficient conditions on the coefficients of a characteristic equation such as (23.3), known as the Routh-Hurwitz conditions [25,429-39], to guarantee stability. These are that in (23.3)

i. $-\Sigma\alpha_i b_{ii} > 0$

ii. $$-\alpha_2\alpha_3\alpha_3' \begin{vmatrix} b_{22} & b_{23} & b_{23}' \\ b_{32} & b_{33} & 0 \\ b_{32}' & 0 & b_{33}' \end{vmatrix} = -\alpha_2\alpha_3\alpha_3' \, |A| > 0$$

iii. $$(-\Sigma\alpha_i b_{ii})\left(\Sigma\alpha_i\alpha_j \begin{vmatrix} b_{ii} & b_{ij} \\ b_{ji} & b_{jj} \end{vmatrix}\right) + \alpha_2\alpha_3\alpha_3' \, |A| > 0 \qquad (i,j=2,3,3')$$

We proceed with the laborious task of expanding these different conditions.

(1) $-\Sigma\alpha_i b_{ii} > 0$.

If we revert to the elements of (16.3) this becomes:

$$-[\alpha_2(K_{22}+K'_{22}+x_2M'_2-x_2M_2) + \alpha_3K_{33} + \alpha'_3K'_{33}] > 0$$

or, in terms of quantity-weighted elasticities and marginal propensities to consume:

$$-[\alpha_2x_2(E_{22}+E'_{22}+C'_2-C_2) + \alpha_3E_{33} + \alpha'_3E'_{33}] > 0$$

where E_{ii} is the aggregate quantity-weighted sum of the demand and supply substitution elasticities for the i^{th} commodity with respect to the i^{th} price. It is most improbable that this condition would not be fulfilled. At a glance, it can be seen that the condition is stronger than the first order conditions needed to ensure Hicksian perfect stability for, even if $C'_2 > -E_{22}-E'_{22}+C_2$, the excess times α_2 would need to exceed $\alpha_3E_{33}+\alpha'_3E'_{33}$ - a most improbable situation.

(2) $$-\alpha_2\alpha_3\alpha'_3 \begin{vmatrix} b_{22} & b_{23} & b'_{23} \\ b_{32} & b_{33} & 0 \\ b'_{32} & 0 & b'_{33} \end{vmatrix} = -\alpha_2\alpha_3\alpha'_3\ |A| > 0.$$

Once again, replacing the b_{ij}'s by their respective elements in (16.3), we require:

$$-\alpha_2\alpha_3\alpha'_3 \begin{vmatrix} K_{22}+K'_{22}+x_2(M'_2-M_2) & K_{23} & K'_{23} \\ K_{32}-M_3x_2 & K_{33} & 0 \\ K'_{32}+M'_3x_2 & 0 & K'_{33} \end{vmatrix} > 0$$

An expansion of this determinant by the first column, when multiplied throughout and divided by $\alpha_3K_{33}.\alpha'_3K'_{33}$, and converted into elasticity form, yields the condition:

$$-x_2[\alpha_2(\phi_{23}+\psi_{23}+\phi'_{23}-\psi'_{23})]\alpha_3\alpha'_3(E_{33}.E'_{33}) > 0.$$

We have seen that the expression in curved brackets must be negative if the third order condition for Hicksian perfect stability is to be fulfilled (equation (18.3)). It follows that the similar condition for dynamic stability is neither more nor less restrictive. As in the static case, the expression can be of perverse sign if, and only if, ψ'_{23} exceeds $\phi_{23}+\phi'_{23}+\psi_{23}$ in absolute magnitude. Arguments advanced earlier suggest that this would not be the case in practice.

(3) Finally, we require:

$$(-\Sigma\alpha_i b_{ii})(\Sigma\alpha_i\alpha_j \begin{vmatrix} b_{ii} & b_{ij} \\ b_{ji} & b_{jj} \end{vmatrix}) + \alpha_2\alpha_3\alpha_3'|A| > 0.$$

A laborious expansion of this expression and a cancellation of like terms gives us, upon conversion to elasticities, the condition:

$$-\{[\alpha_2^2\alpha_3x_2^2\beta+\alpha_3^2\alpha_2x_2E_{33}](\gamma)E_{33}+[\alpha_2^2\alpha_3'x_2^2\beta+\alpha_3^{2\prime}\alpha_2x_2E_{33}'](\mu)E_{33}'+\alpha_3^2\alpha_3'(E_{33}^2E_{33}')+\alpha_3'^2\alpha_3(E_{33}'^2E_{33})$$
$$+2(E_{33}E_{33}')[\alpha_2\alpha_3\alpha_3'x_2\beta]\} > 0$$

where E_{ii} is the quantity-weighted elasticity form of K_{ii},

$$\beta = E_{22}+E_{22}'+C_2'-C_2 ,$$

$$\gamma = \Psi_{23}+\psi_{23}+E_{22}'+C_2',$$

and $$\mu = \Psi_{23}'-\psi_{23}'+E_{22}-C_2 .$$

It can be seen that our last condition could be negative if, and only if, β or γ or μ are negative. We have already had occasion to examine the probable signs of these expressions, concluding in each case that they would be negative. β is one of the first order conditions for Hicksian perfect stability and γ and μ are two of the second order conditions. It is noteworthy that while the non-fulfilment of any one of these conditions could destroy perfect stability in the Hicksian sense, this would not necessarily be so where dynamic stability is concerned. The final result would depend upon the different reaction speeds.

In the special case where all reaction speeds are the same, the unit of time could be selected in such a way that they all equal unity. Then the conditions for the dynamic stability of our system become:

(1) $-\{x_2\beta + E_{33} + E_{33}'\} > 0$.

(2) $-\{x_2[\Psi_{23}+\psi_{23}+\Psi_{23}'-\psi_{23}']E_{33}E_{33}'\} > 0$.

(3) $-\{E_{33}(x_2^2\beta+x_2E_{33})\gamma+E_{33}'(x_2^2\beta+x_2E_{33}')\mu+E_{33}E_{33}'(E_{33}+E_{33}')+2x_2(E_{33}E_{33}')\beta\} > 0.$

At this juncture, the following tentative conclusions are offered.

1. Subject to the nature of our dynamic assumptions, there is a strong case for expecting our international system to be dynamically stable.

2. These arguments would appear to be more conclusive than those offered to establish the perfect stability of our model in the sense used by Hicks. In other words, our conditions are less restrictive.

3. Obviously, different dynamic assumptions would yield different stability conditions. For instance, we might have assumed:

i. a cobweb relationship whereby supply adjusts to price after a given time lag;

ii. a situation in which price falls, not when supply exceeds demand, but when accumulated stocks exceed a normal value;

iii. a situation in which the rate of adjustment in one market depends upon the excess demand not only in that market but also in other markets; etc.

4. Finally, even within the limits of our dynamic assumptions, the stability of our model holds only in the 'small'. Where other than small deviations from equilibrium are contemplated our method of analysis is inadequate. However, it is worth mentioning in defence of linear approximations first, that several empirical investigations support this assumed relationship and, secondly, that a proof has been given that the stability of linear approximations is itself a necessary, if not sufficient, condition for the stability of more complex dynamic systems [94,256-257].

F. CONCLUSION

In concluding a long chapter the opportunity is taken to emphasise three points which have emerged in our analysis.

First, this chapter has served to introduce two expressions which appear consistently in later applications of the model - the coefficient of sensitivity ψ_{ij} and the propensity term ψ_{ij}. That qualitative conclusions can be drawn from a four-commodity model is due to the fact that we are able to establish certain qualitative and quantitative properties concerning these parameters.

Secondly, as pointed out in the last section, our model can be assumed to be dynamically stable subject to the nature of our dynamic assumptions. Moreover, in slightly varying form, the expression $\Psi_{23}+\Psi'_{23}+\psi_{23}-\psi'_{23}$ appears as the denominator in all developments of our model. In every case we assume it to be negative which is similar to assuming that our model is also imperfectly stable in the Hicksian sense (see equation (17.3)). One might argue that too much stress has been placed on establishing the stability of our system - that as we do in fact observe a stable pattern of behaviour in the real world it may be better to assume stability from the outset. While there are many precedents for this approach the primary objection to it is that instability has been acknowledged in the literature as a distinct possibility and that such an approach obscures its real cause.

This leads us to our third point which is that low elasticities are not in themselves, as is popularly believed, sufficient to ensure the instability of the international system. Low elasticities are necessary, but the other necessary condition is the presence of asymmetric income effects which arise, as long as we assume that within a country money income is redistributed at random with respect to tastes, because of the disparity between the supply of and the demand for internationally traded goods in each country. This point is shown clearly in our testing of the stability conditions where, in every case, one income effect must exceed another as a necessary condition (though not sufficient) for instability. Thus, in the cases cited, instability could only occur if

$$C'_2 > C_2, \quad C'_2 > \Psi_{23}, \quad \Psi'_{23} > C_2,$$

or

$$\psi'_{23} > \psi_{23}.$$

4. THE TRANSFER PROBLEM

A. INTRODUCTION

One hesitates before embarking upon an analysis of the transfer problem because that once controversial topic appeared to have been settled finally by two masterly articles of Samuelson [35;36] and a subsequent note by Johnson [37]. However, though they exhausted thoroughly the logical implications of a two-good, two-country model, these comprehensive surveys of the topic did not deal in a rigorous manner with a world in which non-traded commodities were included specifically. Among other qualifications of his analysis mentioned by Samuelson was the comment that

> ... we are still a long way from the conditions of the real world, involving many goods ... in the case where domestic goods are created by transport costs ... it is not clear that the terms of trade ... can be presumed to change in any one direction [36,288].

In this chapter the four-commodity model just developed is applied to the transfer problem, the analysis dividing readily into two parts. Section B is concerned with a rigorous examination of the effect of a transfer upon relative prices and real income in a world characterised both by the presence of non-traded commodities and the absence of all impediments to trade, whether of an artificial variety such as tariffs or of a natural variety such as transport costs. In Section C the model is expanded to include the effect upon the general solution of tariffs and transport costs. In the final section our results are summarized.

Two points should be noted. First, at no stage is an attempt made to review the extensive literature which centres upon this once controversial issue as this would involve a mere restatement of doctrinal history already covered comprehensively in several standard works [52,290-387; 14,63-83; 35; 36]. Secondly, the problem is approached from the classical viewpoint, it being assumed that the value of world production and world expenditure are equal and that all resources are fully employed. These assumptions contrast with those used in Keynesian-type models with constant price levels and national products determined by aggregate demand. In other words, the transfer problem which we shall investigate is a real as opposed to a monetary phenomenon.

B. THE ZERO-IMPEDIMENTS CASE

1. Development of the Model

It is our intention in this section to obtain a general four-commodity criterion for the effect of a given transfer (B) upon the terms of trade when there are no impediments to trade such as tariffs or transport costs. (B) appears as a trade deficit for the foreign (receiving) country and as a surplus for the domestic (paying) country. In fact, it could be considered either as a balance to be manipulated by relative price changes in which case it would concern the problem of the balance of payments, or as a given transfer requiring such relative price changes. We adopt the latter approach and consider B as an independent variable.

The set of equations (1.4)to (6.4) provides the equilibrium equations of the model:

$$x_1 = O_1 - X_1 = X_1' - O_1' \qquad \ldots \qquad (1.4)$$

$$x_2 = X_2 - O_2 = O_2' - X_2' \qquad \ldots \qquad (2.4)$$

$$0 = X_3 - O_3 \qquad \ldots \qquad (3.4)$$

$$0' = X_3' - O_3' \qquad \ldots \qquad (4.4)$$

$$M = O_1p_1 + O_2p_2 + O_3p_3 - B \qquad \ldots \qquad (5.4)$$

$$M' = O_1'p_1 + O_2'p_2 + O_3'p_3' + B \qquad \ldots \qquad (6.4)$$

The equations (1.4) define the paying country's exports, x_1, as equal on the one hand to the domestic supply of exportables, O_1, less the domestic consumption of exportables, X_1, and on the other hand, to the difference between foreign demand for home exportables, X_1', and the foreign supply of them, O_1'. Similarly, the domestic (paying) country's demand for imports, x_2, is defined as equal either to the home consumption of importables, X_2, less the domestic supply of them, O_2, or to the foreign supply of home importables, O_2', less the foreign consumption of them, X_2'. These equations reflect the fact that in equilibrium the world supply of and demand for each internationally traded good must be equal. The third and fourth equations are given by the requirement that the supply of and the demand for non-traded goods must be equal in each country. The only modification made to the free trade set of equations is because of the transfer. Expenditure is no longer equal to the value of production in each country, being less in the paying country and greater in the receiving country by the amount of the transfer itself. It is still true, however, that total world expenditure is equal to the total value of world production.

This time, in our development of the model, we select p_2, the home price of importables, as the numeraire and drop the supply-demand equation(1.4). Furthermore, by an appropriate choice of quantity units p_2 is set equal to unity. We know that in each country the demand for a commodity is a function of the two independent relative prices and of money expenditure, while the supply of a commodity is related functionally to the two relative prices. Thus, in the domestic country,

$$X_i = X_i(p_1, p_3, M)$$

and

$$O_i = O_i(p_1, p_3)$$

while in the foreign country

$$X_i' = X_i'(p_1, p_3', M')$$

and

$$O_i' = O_i'(p_1, p_3')$$

Differentiating totally the set of equations (2.4) to (6.4) we obtain, where

$X_{ij} = \dfrac{\partial X_i}{\partial p_j}$, $O_{ij} = \dfrac{\partial O_i}{\partial p_j}$, and $M = \dfrac{\partial X_i}{\partial M}$:

$$dx_2 = (X_{21}-O_{21})dp_1 + (X_{23}-O_{23})dp_3 + M_2dM \quad \ldots \quad (7.4)$$

$$dx_2 = -(X'_{21}-O'_{21})dp_1 - (X'_{23}-O'_{23})dp'_3 - M'_2dM' \quad \ldots \quad (8.4)$$

$$0 = (X_{31}-O_{31})dp_1 + (X_{33}-O_{33})dp_3 + M_3dM \quad \ldots \quad (9.4)$$

$$0 = (X'_{31}-O'_{31})dp_1 + (X'_{33}-O'_{33})dp'_3 + M'_3dM' \quad \ldots \quad (10.4)$$

$$dM = O_1dp_1 + O_3dp_3 - dB \quad \ldots \quad (11.4)$$

$$dM' = O'_1dp_1 + O'_3dp'_3 + dB \quad \ldots \quad (12.4)$$

In the last two equations use has been made of the condition that $\Sigma p_i dO_i = 0$. More simply, this means that in a world in which full employment without inflation is assumed, the ratio of changes in production must be equal to minus the ratio of marginal costs which, given perfect competition, is equal to minus the ratio of prices. This accounts for the omission of all terms in the last two equations involving a price times quantity change.

We now proceed to:

i. substitute equation (11.4) into equations (7.4) and (9.4);

ii. substitute equation (12.4) into equations (8.4) and(10.4);

iii. substitute into the resulting equations $(K_{ij}-X_jM_i)$ for $(X_{ij}-O_{ij})$ where K_{ij} is the aggregate demand-supply substitution effect; and for the terms (X_i-O_i) which arise after these substitutions, the relevant x_1, x_2 or zero as indicated by the equilibrium equations;

iv. cancel and collect terms wherever possible.

This yields the following rearranged set of equations.

$$(K_{21}+x_1M_2)dp_1 + K_{23}dp_3 - dx_2 = M_2dB \quad \ldots \quad (13.4)$$

$$(K_{31}+x_1M_3)dp_1 + K_{33}dp_3 = M_3dB \quad \ldots \quad (14.4)$$

$$(K'_{21}-x_1M'_2)dp_1 + K'_{23}dp'_3 + dx_2 = -M'_2dB \quad \ldots \quad (15.4)$$

$$(K'_{31}-x_1M'_3)dp_1 + K'_{33}dp'_3 = -M'_3dB \quad \ldots \quad (16.4)$$

The change in the price ratio p_1 is our measure of the change in the terms of trade due to a transfer from the domestic (paying) country to the foreign (receiving) country. A positive dp_1 means that the terms of trade of the paying country have improved, p_1 rising relative to p_2. Conversely, a negative dp_1 would mean a fall in p_1 (the price of domestic exportables) relative to p_2 (the price of domestic importables) and an adverse movement in the terms of trade of the paying country. It follows that an adverse movement in the terms of trade of the paying country must represent a favourable movement in the terms of trade of the receiving country and vice versa.

The solution for the effect of a transfer on the terms of trade obtained from the set of equations (13.4) to (16.4) is:

$$\frac{dp_1}{dB} = \frac{\begin{vmatrix} M_2 & K_{23} & 0 & -1 \\ M_3 & K_{33} & 0 & 0 \\ -M_2' & 0 & K_{23}' & 1 \\ -M_3' & 0 & K_{33}' & 0 \end{vmatrix}}{\begin{vmatrix} K_{21}+x_1M_2 & K_{23} & 0 & -1 \\ K_{31}+x_1M_3 & K_{33} & 0 & 0 \\ K_{21}'-x_1M_2' & 0 & K_{23}' & 1 \\ K_{31}'-x_1M_3' & 0 & K_{33}' & 0 \end{vmatrix}}$$

In the denominator of this result we make use of the homogeneity condition that $\Sigma_j p_j K_{ij}=0$ to add column two and three to column one, thereby obtaining aggregate demand-supply substitution effects with respect to the price change in commodity two in both the foreign and domestic countries. At the same time, providing the transfer is small relative to the total value of trade and given that prices are unity initially, x_1 must be approximately equal to x_2.

Our solution may now be written:

$$\frac{dp_1}{dB} = \frac{\begin{vmatrix} M_2 & K_{23} & 0 & -1 \\ M_3 & K_{33} & 0 & 0 \\ -M_2' & 0 & K_{23}' & 1 \\ -M_3' & 0 & K_{33}' & 0 \end{vmatrix}}{-\begin{vmatrix} K_{22}-x_2M_2 & K_{23} & 0 & -1 \\ K_{32}-x_2M_3 & K_{33} & 0 & 0 \\ K_{22}'+x_2M_2' & 0 & K_{23}' & 1 \\ K_{32}'+x_2M_3' & 0 & K_{33}' & 0 \end{vmatrix}} \qquad \begin{matrix} (= |A|) \\ \\ (= |B|) \end{matrix}$$

A Laplacian expansion from the first two rows of $|A|$ and $|B|$ gives:

$$\frac{dp_1}{dB} = \frac{-\begin{vmatrix} M_2 & K_{23} \\ M_3 & K_{33} \end{vmatrix} K_{33}' + \begin{vmatrix} M_2' & K_{23}' \\ M_3' & K_{33}' \end{vmatrix} K_{33}}{\begin{vmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{vmatrix} K_{33}' + \begin{vmatrix} K_{22}' & K_{23}' \\ K_{32}' & K_{33}' \end{vmatrix} K_{33} - x_2 \begin{vmatrix} M_2 & K_{23} \\ M_3 & K_{33} \end{vmatrix} K_{33}' + x_2 \begin{vmatrix} M_2' & K_{23}' \\ M_3' & K_{33}' \end{vmatrix} K_{33}}$$

Dividing top and bottom by $(K_{33}' \cdot K_{33})$,

$$\frac{dp_1}{dB} = \frac{-\dfrac{\begin{vmatrix} M_2 & K_{23} \\ M_3 & K_{33} \end{vmatrix}}{K_{33}} + \dfrac{\begin{vmatrix} M_2' & K_{23}' \\ M_3' & K_{33}' \end{vmatrix}}{K_{33}'}}{\dfrac{\begin{vmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{vmatrix}}{K_{33}} + \dfrac{\begin{vmatrix} K_{22}' & K_{23}' \\ K_{32}' & K_{33}' \end{vmatrix}}{K_{33}'} - \dfrac{x_2\begin{vmatrix} M_2 & K_{23} \\ M_3 & K_{33} \end{vmatrix}}{K_{33}} + \dfrac{x_2\begin{vmatrix} M_2' & K_{23}' \\ M_3' & K_{33}' \end{vmatrix}}{K_{33}'}}$$

Expanding the determinants of our answer, we have:

$$\frac{dp_1}{dB} = \frac{\phi_{23} - \phi_{23}'}{\phi_{23} + \phi_{23}' + x_2\phi_{23} - x_2\phi_{23}'} \quad \ldots \qquad (17.4)$$

where

$$\phi_{ij} = -M_1 + M_j \frac{K_{ij}}{K_{jj}}$$

and

$$\Phi_{ij} = K_{ii} - K_{ij} \frac{K_{ji}}{K_{jj}}$$

Since our choice of quantity units (which has been made with no loss of generality) implies that i. M_i is both a marginal propensity to spend as well as an income effect;

ii. $K_{ij}=X_i\sigma_{ij}-O_iS_{ij}$, where σ_{ij} and S_{ij} are substitution elasticities of demand and supply with respect to p_j, and where X_i and O_i are quantities measured as multiples of the quantity of home exports x_1 (=x_2 approximately, as long as prices are unity and B is small relative to the volume of trade);

it follows that $\frac{dp_1}{dB}$ can be expressed in terms of quantity weighted elasticities and marginal propensities to spend:

$$H = \frac{\psi_{23} - \psi'_{23}}{\Psi_{23} + \Psi'_{23} + \psi_{23} - \psi'_{23}} \quad \ldots \quad (18.4)$$

where H ($= \frac{dp_1}{p_1}\frac{x_1}{dB}$) is the rate of change in the terms of trade due to a transfer expressed as a proportion of the value of trade,

$$\psi_{ij} = -C_i + C_j\left(\frac{\sigma_{ji} - S_{ji}}{\sigma_{jj} - S_{jj}}\right)$$

$$\Psi_{ij} = \left(\frac{X_i}{x_i}\sigma_{ii} - \frac{O_i}{x_i}S_{ii}\right) - \left(\frac{X_i}{x_i}\sigma_{ij} - \frac{O_i}{x_i}S_{ij}\right)\left(\frac{\sigma_{ji} - S_{ji}}{\sigma_{jj} - S_{jj}}\right)$$

$$\psi_{ij} = \phi_{ij} \quad \text{and}$$

$$x_i\Psi_{ij} = \Phi_{ij}$$

The reader should note that in converting ϕ_{ij} into ψ_{ij}, the term K_{ji} is substituted for K_{ij} which, of course, we are entitled to do when dealing with partial derivatives.

2. The Direction of the Shift in the Terms of Trade - the Two-Commodity Case

Before the more complex multi-commodity criterion (equation 18.4) for the effect of a transfer on the terms of trade is analysed, it may be of assistance to the reader if the simple two-commodity answer is derived from it by setting to zero all terms which involve non-traded commodities:

$$H = \frac{C_2' - C_2}{(\frac{X_2}{x_2}\sigma_{22} - \frac{O_2}{x_2}S_{22}) + (\frac{X_2'}{x_2}\sigma_{22}' - \frac{O_2'}{x_2}S_{22}') + (C_2' - C_2)} \quad \ldots \qquad (19.4)$$

Given stability in the international system, the direction of the shift in the terms of trade is dependent upon the numerator of H. Consider for a moment the reason for a shift in the terms of trade. Obviously, such a movement will occur only in response to an excess demand, generated as a result of the transfer, for one of the traded commodities. As long as the receiving country increases its consumption of the two goods by the same amounts as the paying country reduces its consumption of them, there will be no excess demand, no induced price movement, and no shift in the terms of trade. This fact reflects itself in the criterion: as long as the marginal propensity to spend on importables in the paying (expenditure-reducing) country is the same as the marginal propensity to spend on the same good in the receiving (expenditure-increasing) country, there will be no excess demand for either commodity and no change in the terms of trade, money and real income declining by exactly the amount of the transfer in the paying country and increasing by exactly the amount of the transfer in the receiving country.

What if the paying country's marginal propensity to spend on importables exceeds that of the receiving country? In this event, the reduction in the amount of importables consumed in the paying country would exceed the increase in consumption of domestic importables in the receiving country, thereby generating an excess supply of importables upon the world market, or, necessarily, an excess demand for the paying country's exportables since income is assumed to be equal to expenditure, i.e. $C_1+C_2=C_1'+C_2'=1$. Given stability, this excess demand for domestic exportables can be eliminated only by a rise in their price relative to that of importables, the terms of trade necessarily improving for the paying country. Conversely, if C_2' should exceed C_2, the terms of trade movement would favour the receiving country.

Samuelson has pointed out that in the zero-impediments, two-commodity case there can be no a priori grounds for supposing that one marginal propensity should exceed the other. As long as we are ignorant concerning the quantitative magnitudes of the parameters, there can be no presumption in favour of the orthodox conclusion that the terms of trade should move adversely for the paying country [35,299]. Arguing on equiprobability lines, as does Samuelson, that the marginal propensities are identical in each country, means that there would be a zero terms of trade effect. We turn now to examine the effect upon the criterion of the introduction of non-traded commodities in each country.

3. The Direction of Shift in the Terms of Trade - the Four-Commodity Case

Remembering that the international system is assumed to be stable, the relevant criterion is now given by the numerator of equation (18.4):

$$\psi_{23} \gtreqless \psi_{23}' \quad \ldots \quad (20.4)$$

where $\psi_{ij} = -C_i + C_j \dfrac{\sigma_{ji} - S_{ij}}{\sigma_{jj} - S_{jj}}$.

This states that the terms of trade will improve (deteriorate) if the value of the paying country's propensity term exceeds (is less than) that of the receiving country. The influence of the non-traded commodity is best considered in two stages: the effect of the marginal propensities to spend on non-traded commodities; and the effect of the substitution possibilities in consumption and production upon the magnitude of the marginal propensities.

i. Superficially, from an examination of the propensity terms, it would appear that the larger the magnitude of the marginal propensity to spend on non-traded goods in the paying country and the smaller the non-traded propensity in the receiving country, the greater is the possibility of the terms of trade movement being favourable to the paying country. In fact, the larger the non-traded propensities, the greater will be the influence of the non-traded commodity upon the direction of the shift in the terms of trade but it is impossible to specify in which direction this influence will tend unless something is known of the substitution possibilities in the system.

ii. The direct effect of the transfer is to increase expenditure on commodities <u>at constant prices</u> in the receiving country and, likewise, to reduce expenditure upon commodities <u>at constant prices</u> in the paying country. Excepting the unlikely case in which one of the non-traded commodities is an inferior good, or where either the own elasticity of supply of, or demand for, the non-traded commodity is infinite, this increase in expenditure in the receiving country must increase the price of non-traded commodities and the decrease in expenditure in the paying country must reduce their price. As a result of these changes in the prices of the non-traded commodities, the consumption of traded commodities would increase in the receiving and decrease in the paying country; similarly, the supply of traded commodities would decrease in the receiving and increase in the paying country. The net effect upon the terms of trade of these shifts in consumption and production (which would tend to depress the price of traded goods in the paying and to raise the price of traded goods in the receiving country) would depend upon the substitution possibilities in consumption and production. Generally, given the magnitude of the non-traded propensities, the greater the substitution possibilities in consumption and production in each country between non-traded commodities and exportables relative to the substitution possibilities between non-traded commodities and importables, the more probable is the orthodox presumption that the terms of trade would turn against the paying country.

Let us now consider whether a presumption can be established that the terms of trade will move in a direction either adverse or favourable to the paying country. In our analysis of the two-commodity case it was pointed out that on equiprobability grounds there exists no reason for assuming a terms of trade movement in favour of either country when there are no impediments to trade. This conclusion was reached by Samuelson who rigorously terminated what had seemed to be an interminable squabble between the adherents of the orthodox (terms of trade movement adverse to the paying country) and modern (terms of trade movement favourable to the paying country) exponents.

Several economists, however, have endeavoured to create a presumption one way or the other by the addition of non-traded commodities to their models. Of these attempts only that of Viner [52,348-49] is logically successful in providing a defence for the orthodox position. His argument, which depends entirely upon the assumption that there exists in each country a class of non-traded goods which is infinitely substitutable in production with exportables, is seen as a special case of our own general, multi-commodity criterion. This assumption of Viner's would have the effect of reducing to zero the substitution elasticity weight in the paying country's marginal propensity term (ψ_{23}) while, at the same time, the corresponding weight in the receiving country's propensity term would be increased to minus unity. The criterion would then become:

$$C_2 \gtreqless C_2' + C_3' \quad .$$

Arguing along equiprobability lines that $C_2 = C_2'$, it follows that the terms of trade of the paying country would deteriorate, thus vindicating the orthodox point of view. Viner's approach, however, depends not only upon equiprobability arguments but also upon his restrictive assumption concerning production substitution possibilities, which is the same as assuming non-traded and exportable commodities to be one and the same good as far as producers are concerned. There would appear to be no economic justification for this. Moreover, even if a bias did exist so that, in production, non-traded commodities were more competitive with exportables than with importables, the effect of this could be more than offset by an opposite bias on the consumption side.

Before this restricted but nonetheless logically successful attempt of Viner's to support the orthodox viewpoint, Elliott [85] and Samuelson [35,302-3] with even more restrictive assumptions failed to achieve the same result. Again, their attempts appear as a special case within the context of our general model.

Samuelson introduced the non-traded commodity by assuming, first that supply elasticities were zero, and secondly, that the effect of any change in non-traded prices upon the marginal propensities to spend would be neutral, i.e. $\sigma'_{31}=\sigma'_{32}$ and $\sigma_{32}=\sigma_{31}$. In this case our formula reduces to:

$$-C_2 + C_3\left(-\tfrac{1}{2}\right) \gtreqless -C'_2 + C'_3\left(-\tfrac{1}{2}\right)$$

where again, there can be no presumption either way if we argue along equiprobability lines. This approach, of course, neglects entirely the influence of the substitution possibilities upon the direction of the shift in the international price ratio.

Are we then to conclude, when there exist no impediments to trade, that the introduction of non-traded commodities fails by equiprobability argument to indicate grounds for a presumption as to the direction of the movement in the terms of trade following a transfer? In fact, though we shall conclude that insufficient grounds exist for such a presumption, the writer would like to record first, a special case which does provide a slight bias in favour of the orthodox case and, secondly, another similar case which provides no presumption either way. Both examples depend entirely upon equiprobability type arguments, the first containing more restrictive assumptions than the second.

Consider the terms ϕ_{23} and ϕ'_{23} as they appear in equation (17.4). Forming elasticities our criterion becomes when written in full:

$$-C_2 + C_2' + C_3 \frac{X_2}{X_3} \frac{\sigma_{23}-O_2S_{23}}{\sigma_{33}-O_3S_{33}} - C_3' \frac{X_2'}{X_3'} \frac{\sigma_{23}'-O_2'S_{23}'}{\sigma_{33}'-O_3'S_{33}'} \gtreqless 0 .$$

Now, extend the equiprobability argument so that not only are marginal propensities to spend equal in each country but also average propensities to spend and demand and supply elasticities. In this case, the above criterion leaves a remainder (R):

$$R = \frac{C_3 x_2 S_{23}}{\sigma_{33}-S_{33}} \left(\frac{1}{X_3} + \frac{1}{X_3'} \right) > 0$$

which is due essentially to the fact that, given our assumptions, the sum of the supply and demand for domestic importables in the foreign (receiving) country necessarily is greater relative to the volume of non-traded goods than is the same sum in the paying country. Thus,

$$\frac{X_2}{X_3} = \frac{X_2'}{X_3'} \quad \text{(average propensities to spend equal)}$$

but $\qquad X_2 > O_2$

whereas $\qquad X_2' < O_2'$

hence,

$$\frac{O_2}{X_3} = \frac{O_2}{O_3} \neq \frac{O_2'}{X_3'} = \frac{O_2'}{O_3'}$$

The remainder would tend to disappear, given our assumptions, only if S_{23} were to exceed sufficiently S_{23}' and there does not appear to be any a priori reason why this should be the case. However, the magnitude of the remainder normally would be small unless the volume of trade approximates in size to the consumption of non-traded commodities in each country. Nevertheless, that is nevertheless for those who would accept this equiprobability type of argument, a slight presumption has been created in favour of the orthodox viewpoint that the terms of trade of the paying country would deteriorate.

At this juncture it may be thought that a further logical extension of the equiprobability argument, so that average propensities to produce were also equal, would re-establish Samuelson's two-commodity conclusion that no presumption can be made in either direction. In fact, if we assume the latter and equal average propensities to spend, it follows that $X_2=O_2$, etc., and that no trade would occur. Symmetry assumptions in terms of average propensities to spend and to produce are inconsistent with trade.

Next, we consider a less restrictive case which favours the 'no presumption' hypothesis. This time, in equation (17.4), before converting ϕ_{23} and ϕ'_{23} to elasticities ~~to elasticities~~ we substitute K_{32} and K'_{32} for K_{23} and K'_{23} respectively and our criterion in elasticity notation becomes:

$$-C_2 + C'_2 + C_3 \frac{\sigma_{32}-S_{32}}{\sigma_{33}-S_{33}} - C'_3 \frac{\sigma'_{32}-S'_{32}}{\sigma'_{33}-S'_{33}} \gtreqless 0$$

The reader will note that the quantity weights associated with the elasticities have been cancelled out. In this case, the assumption of equal average propensities to spend is no longer relevant, but the assumption of equal marginal propensities to spend and of equal supply and demand elasticities in each country leads to the conclusion that there will be no remainder, and hence, no presumption as to a terms of trade movement in either direction.

There is a tendency, because of the initial substitution, to identify the two cases set out above. In fact, $S_{23}=S'_{23}$ implies that $S_{32}\neq S'_{32}$ unless average propensities to produce are also identical, and this we have seen to be inconsistent with trade. The two cases involve, therefore, quite different assumptions.

This last conclusion should not be taken to mean, however, that because by equi-probability argument there can be established no or only a slight presumption as to the direction of the terms of trade change, the influence of the non-traded commodity upon the outcome is negligible, for as was shown earlier, the additional relationships introduced by the presence of non-traded goods could affect vitally both the magnitude and the direction of the shift in the terms of trade. To ignore these relationships when making a quantitative assessment would be to risk a serious error.

Finally, we note the 'few tentative speculations' of Samuelson upon the effect of non-traded commodities upon the transfer problem [36,289]. He was of the opinion that the determining factor would be the relative degree of substitutability, on the side of production only, between non-traded and exportable goods on the one hand, and non-traded and importable commodities on the other. Generally, he felt that the production relations between exportable and non-traded goods would outweigh the others, thereby supporting the orthodox conclusion. It has been shown clearly, however, that substitution effects on the side of consumption are equally important. One may presume that Samuelson's failure to perceive this was due to the special case of Viner in which the demand substitution possibilities do not affect the issue.

4. The Effect of a Transfer upon the Price of Non-Traded Goods

Conventional two-commodity or partial equilibrium analysis frequently has led economists to consider the terms of trade as the dependent variable by which international adjustments to equilibrium are secured. In recent balance of payments models this preoccupation with the internationally traded good sectors in each country emphasises the role of a relative price change among the traded commodities in relieving an excess supply of, or demand for these goods, whereas it may well be that shifts in the price level of non-traded commodities relative to the prices of the traded goods are the more important.

It is possible that recent writers, when considering the mechanism of adjustment to international equilibrium, have excluded non-traded commodities from their models because of the added complexity of the analysis generally and the apparent lack of precision in the results.[1] Nevertheless, the problem is an important one.

It is evident, of course, that an increase in expenditure in the receiving country and a decrease in expenditure in the paying country would tend to increase the price of non-traded commodities in the former and decrease it in the latter. But what, one may ask, is the extent of this shift in relation to the prices of the internationally traded commodities?

It should be noted that several classical [79,315] and neo-classical [52,324; 81,106] writers were aware of the important influence of non-traded commodities upon the adjustment process. Viner reflected the awareness of these earlier economists when he stated that there would be:

> ... for the borrowing country, a rise of export prices relative to import prices and of domestic commodity prices relative to both export and import prices [52,324].

Later, Wilson, in an empirical study, endorsed this conclusion noting that:

> ... some verification is found in Australian experience for the proposition that imports of capital tend to be positively correlated with increases in the ratio of the "domestic" price level to the price level of "international" commodities [81,106].

Should Viner's intuitive hypothesis be correct, it follows that the terms of trade are not the only dependent variable by which an adjustment to equilibrium is secured and, indeed, it may well be that the influence of the terms of trade is slight when compared with the effect upon the adjustment mechanism of a shift in the price of non-traded

[1] A notable exception to this generalisation is I.F. Pearce's paper on the problem of the balance of payments [82]. Pearce emphasises the importance of the role of the non-traded commodity in the equilibrating process, concluding that 'It may well be that the success of exchange depreciation as a policy rests more upon its power to reduce the price of non-traded goods relative to those traded than upon its power to affect the real terms of trade' [82,28]. As pointed out below, this conclusion, which is in the tradition of the Viner-Wilson hypothesis, fails to consider fully the indirect effects of the shift in the terms of trade and needs, therefore, to be qualified accordingly.

goods relative to the prices of the traded commodities. Viner's statement, however, is not without ambiguity. For instance, if the terms of trade should move favourably for the expenditure-increasing (receiving) country, one would expect the price of importables to fall relative to the price of non-traded goods, in the manner predicted by Viner. But what of the price of exportables? One must consider carefully not only the impact effects of the transfer but also the other reactions among the variables including, in particular, the shift in the terms of trade.

The remainder of this section is concerned with a rigorous examination of the Viner-Wilson hypothesis.

In solving for the change in price of the paying country's non-traded good, p_3, relative to the price of domestic importables, p_2, the solution for $\frac{dp_1}{dB}\frac{x_1}{p_1}$ in equation (18.4) can be substituted into equation (14.4) so that, when rearranged,

$$\frac{dp_3}{dB}\frac{x_1}{p_3} = \frac{x_1 M_3(H-1) + HK_{31}}{-K_{33}}$$

When converted to elasticities this yields:

$$\frac{dp_3}{dB}\frac{x_1}{p_3} = \frac{\frac{p_1 x_1}{p_3 X_3} C_3(H-1) + (\sigma_{31}-S_{31})H}{-\sigma_{33} + S_{33}} \quad \ldots \qquad (21.4)$$

Given stability in the international system and the absence of inferior goods or complementarity in demand or joint supply, an examination of H (equation(18.4))reveals that if the numerator is negative, H cannot exceed plus unity; and if the numerator is positive, H can range in value from minus infinity to zero. Thus, when the terms of trade of the paying country deteriorate, H conceivably could tend to minus infinity. Nevertheless, in all but exceptional cases, H would be less than unity, the numerator necessarily being less than unity and not very different from $C_2'-C_2$, while the denominator is almost certain to exceed unity. In what follows, therefore, H is assumed to range in value from minus to plus unity.

Consider first, in equation (21.4), the orthodox case in which the terms of trade of the paying country deterioriate (H<0). As both terms in the numerator are negative, the price of non-traded commodities will fall relative to the price of importables in the paying (expenditure-reducing) country. In this instance, the Viner-Wilson hypothesis is unambiguously correct.

Our formula reflects clearly the different influences which induce p_3 to fall relative to p_2 when the terms of trade move adversely for the paying country. The first term in the numerator shows the influence of two income effects, both of which reduce the demand for commodity three. There is the income effect associated with the fall in the price of domestic exportables and there is the other negative income effect associated with the reduction in expenditure due to the transfer. The magnitude of these effects will be augmented, the larger the marginal propensity to consume the non-traded commodity and the greater the importance of trade to the domestic country as measured by the ratio of the value of exports and non-traded commodity consumption. The second term in the numerator is a measure of the substitution effect of the fall in the price of domestic exportables relative to the price of importables (the numeraire) which causes a shift of consumption from, and of production towards the non-traded sector. Both influences would tend to depress the price of non-traded goods. Finally, in the denominator, are the substitution effects which measure the effect upon the consumption and production of the non-traded commodity of the fall in its price. Obviously, the larger is either own non-traded substitution elasticity, the less will be the effect of the other influences detailed above upon the price of the non-traded good.

What if there were no movement in the terms of trade (H=0)? In this case, both the income and the substitution effects of a change in the terms of trade are excluded from our formula. The direction of the shift in the non-traded/importable price ratio is determined unambiguously by the reduction in the demand for commodity three associated with the transfer, while the magnitude of the result is decided by the importance of trade to the country, by the size of the marginal propensity to consume non-traded goods and by the magnitude of the own non-traded substitution elasticities for good three.

Consider next the shift in the price of domestic non-traded goods relative to the price of domestic importables when the terms of trade improve for the paying country ($H>0$). This case is important, for it emphasises the fact that Viner and Wilson do not consider the indirect influence of the terms of trade. Our formula shows that the same influences are at work but that the direction of these effects is in conflict. In the first term of the numerator the income effects pull in contrary directions. While the impact effect of the transfer upon expenditure reduces demand for good three, the price induced income effect of the terms of trade change increases demand for good three. As long as the international market is stable, however, the net influence of this term must be to reduce the demand for commodity three. The magnitude of this net effect would depend upon the same factors as were detailed above. Unfortunately, the effect of the second term in the numerator, which reflects the substitution effect of the change in the terms of trade, is to increase the price of good three - the rise in the price of exportables increasing the demand for non-traded goods and reducing the supply of them. We see, therefore, that there are two forces - one an income, the other a substitution effect - which together might prevent a realization of the Viner-Wilson hypothesis. One point should be noted, however. When examining the determinants of the direction of the shift in the terms of trade, we saw that the smaller were σ_{31} and S_{31}, the greater was this shift likely to be. Thus for H to be large in (21.4), σ_{31} and S_{31} should be small. But, if p_3 is to rise relative to p_2 when the terms of trade are favourable to the paying country, we require σ_{31} and S_{31} large. It is therefore improbable that both H <u>and</u>

$(\sigma_{31}-S_{31})$ together can be large.

What of the relationship between the exportable and the non-traded goods price? We know

$$\frac{p_3}{p_1} = \frac{p_3/p_2}{p_1/p_2} .$$

Differentiating totally (logarithmically) with respect to a transfer expressed as a proportion of the value of trade,

$$\frac{d(p_3/p_1)}{p_3/p_1} \frac{x_1}{p_3/p_1} = \frac{dp_3}{dB}\frac{x_1}{p_3} - \frac{dp_1}{dB}\frac{x_1}{p_1} .$$

Substituting from (21.4) and from (18.4) for dp_1 and dp_3 respectively, and making use of the relationship that $\sum_j p_j K_{ij}=0$, we obtain:

$$\frac{d(p_3/p_1)}{p_3/p_1}\frac{x_1}{dB} = \frac{\frac{p_1x_1}{p_3X_3} C_3(H-1) - (\sigma_{32}-S_{32})H}{-\sigma_{33} + S_{33}} \quad \ldots \qquad (22.4)$$

This time, when the terms of trade change is favourable to the paying country $(H>0)$, the Viner-Wilson hypothesis needs no qualification. As H cannot exceed plus unity the numerator of (22.4) is negative, which means that the price of exportables must rise relative to the price of non-traded goods. The case in which H=0 again emphasises the fact that the Viner-Wilson hypothesis holds unambiguously only when the terms of trade effect is ignored. Trouble occurs when the terms of trade movement is adverse $(H<0)$. The shift in the terms of trade raises p_2 relative to p_1 and gives rise first to an income effect which tends to reduce the demand for and price of good three, and secondly to substitution effects which, by increasing the consumption and reducing the production of non-traded commodities, tend ~~also~~ to increase the price of good three. The other income effect, associated with the transfer, still reduces the demand for good three.

Similarly, results can be obtained for dp_3' (in terms of the price of domestic importables) and for $dp_3'-dp_1$ (which is a change relative to the price of home exportables). Table One summarizes these results:

Table One. Summary of shifts in traded/non-traded commodity price ratios

Relative price	Criteria	Effect on relative price $0<H<1$	$H = 0$	$-1<H<0$
$\frac{dp_3}{dB}\frac{x_1}{p_3} =$	$\dfrac{\frac{p_1x_1}{p_3X_3}C_3(H-1) + (\sigma_{31}-S_{31})H}{-\sigma_{33}+S_{33}}$	?	-	-
$\frac{d(p_3/p_1)}{dB}\frac{x_1}{p_3/p_1} =$	$\dfrac{\frac{p_1x_1}{p_3X_3}C_3(H-1) - (\sigma_{32}-S_{32})H}{-\sigma_{33}+S_{33}}$	-	-	?
$\frac{dp_3'}{dB}\frac{x_1}{p_3'} =$	$\dfrac{\frac{p_1x_1}{p_3'X_3'}C_3'(1-H) + (\sigma_{31}'-S_{31}')H}{-\sigma_{33}'+S_{33}'}$	+	+	?
$\frac{d(p_3'/p_1)}{dB}\frac{x_1}{p_3'/p_1} =$	$\dfrac{\frac{p_1x_1}{p_3'X_3'}C_3'(1-H) - (\sigma_{32}'-S_{32}')H}{-\sigma_{33}'+S_{33}'}$	?	+	+
$\frac{dp_3-dp_3'}{dB}x_1 =$	$\left(\dfrac{C_3\frac{p_1x_1}{p_3X_3}}{-\sigma_{33}+S_{33}} + \dfrac{C_3'\frac{p_1x_1}{p_3'X_3'}}{-\sigma_{33}'+S_{33}'}\right)(H-1) + \left(\dfrac{\sigma_{31}-S_{31}}{-\sigma_{33}+S_{33}} - \dfrac{\sigma_{31}'-S_{31}'}{-\sigma_{33}'+S_{33}'}\right)H$	?	?	-

These results are symmetrical. Should the terms of trade not change ($H=0$), the price of the receiving country's non-traded goods must rise relative to the price of its traded goods, and vice-versa for the paying country. This, however, is the only case in which the Viner-Wilson hypothesis is unambiguously correct. If the terms of trade change, the price of the receiving country's non-traded good will rise relative to whichever traded commodity has fallen in price relative to the other traded commodity as a result of the terms of trade change. Whether or not the rise in the price of the non-traded good will exceed the rise in the price of the traded good whose price has improved relative to the other traded good will be determined by the magnitude of the terms of of trade effect, by the importance of trade in the country concerned, by the magnitude of each country's marginal propensity to spend on non-traded goods, and by the relative degree of competitiveness between non-traded goods and exportables on the one hand, and non-traded goods and importables on the other. The reader can see from Table One that a corresponding degree of ambiguity exists when the price of the paying country's non-traded commodities is considered.

In the final row of Table One results are given for dp_3-dp_3'. When there is no terms of trade movement the price of non-traded goods in the receiving country rises relative to the corresponding price in the other country. This is still true when the terms of trade change, providing H is sufficiently small, and/or if the difference between substitution possibilities for non-traded goods in the two countries is sufficiently small, or if $H>0$ and the latter difference is negative or if $H<0$ and the latter difference is positive.

Finally, having pointed out the ambiguity implicit in Viner's original statement, we make an attempt to restate it. Knowingly or otherwise, Wilson commented accurately that there is some evidence of a positive correlation between the price of non-traded goods and <u>the</u> price level of traded goods. The difficulty, of course, is the concept of a single price for traded goods when the prices of imports and exports move in opposite directions as a result of the terms of trade change. In fact, it can be shown that there is a weighted index of the prices of traded goods taken together, in relation to which the price of non-traded goods must rise in the receiving country.[2] If such an index is to have positive weights which sum to unity for all possible magnitudes of income and substitution effects (subject to the assumption that all K_{ij}'s >0, that $i\neq j$, and that

[2] The author is indebted to Professor T.W. Swan for assistance with this point.

$H < 1$) there is only one such index, namely a Divisia index, defined by

$$dP = q dp_1 + (1 - q)\, dp_2$$

where

$$q = \frac{\sigma_{31} - S_{31}}{-\sigma_{33} + S_{33}}$$

and

$$1 - q = \frac{\sigma_{32} - S_{32}}{-\sigma_{33} + S_{33}} .$$

It is easily verified that for this index

$$\frac{dp_3 - dP}{dB} x_1 = \frac{\frac{p_1 x_1}{p_3 X_3} C_3 (H-1)}{-\sigma_{33} + S_{33}} < 0 .$$

Of course, the uncertainty of the sign of $dp_3 - dp_3'$, when $H \neq 0$, still persists, because $P \neq P'$ unless the ratios of the substitution possibilities are equal or the terms of trade movement is zero.

Three general conclusions emerge from our analysis:

1. that the Viner-Wilson hypothesis fails to consider the indirect effects of the terms of trade upon non-traded/traded relative prices and so at any one time (except where the terms of trade do not change) their hypothesis could be only partially fulfilled, unless
2. the hypothesis is reformulated to read: that there exists some index of the prices of traded goods taken together, in relation to which non-traded prices in the receiving country must rise. A similar index can be defined for the paying country, in relation to which the non-traded commodity price must fall;
3. that those factors which might cause a revision of the Viner-Wilson hypothesis, and about which some quantitative knowledge is required are:
    - i. the possibility of high own non-traded substitution possibilities in each country;

ii. the possibility of a small shift in the terms of trade, i.e. low coefficients of sensitivity;

iii. the relative degree of substitutability between non-traded commodities, and importables and exportables respectively; and

iv. the relative importance of ~~the~~ trade to the country concerned.

5. <u>The Effect of a Transfer Upon the Real Income of the Paying Country</u>

Real income in the paying country will decrease by an amount equivalent to the transfer itself, plus or minus an amount determined by the direction and magnitude of the shift in the terms of trade.[3] The latter is equal approximately to the increase or decrease in the cost of obtaining the initial volume of imports as a result of the shift in the terms of trade. It follows that the total change in the real income of the paying country, U, as the result of a transfer is given by:

$$dU = dp_1 x_2 - dB$$

so that

$$\frac{dU}{dB} = \frac{dp_1}{dB}\frac{x_1}{p_1} - 1$$

($x_1 = x_2$, since prices are unity initially and the change in the trade balance is assumed small relative to total trade.).

Substituting for $\frac{dp_1}{dB}\frac{x_1}{p_1}$ from equation (18.4) we obtain:

$$\frac{dU}{dB} = \frac{\psi_{23} - \psi'_{23}}{\Psi_{23} + \Psi'_{23} + \psi_{23} - \psi'_{23}} - 1$$

which, when rearranged, gives:

$$\frac{dU}{dB} = \frac{-\Psi_{23} - \Psi'_{23}}{\Psi_{23} + \Psi'_{23} + \psi_{23} - \psi'_{23}} \quad \ldots \quad (23.4)$$

As the coefficients of sensitivity must be negative, this equation states that the real

[3] By assumption we ignore the possible real income effects of a redistribution of income between the different individuals of each country.

income of the transferring country cannot increase as a result of the transfer unless the foreign market is unstable in the sense defined above, i.e. the denominator on the right-hand side is positive. This multi-commodity result confirms a similar conclusion of Samuelson in the two-commodity case [25,29].

A more important conclusion concerns the magnitude of this real income effect. Obviously if the terms of trade move adversely for the paying country ($\psi_{23}<\psi'_{23}$) real income will increase by more than the amount of the transfer; if the terms of trade move favourably, by less than the amount of the transfer. Given the difference in the propensities, the determinants of the amount by which real income falls short of or exceeds the transfer are the coefficients of sensitivity; the magnitude of this amount being related inversely to the size of them, being large when they are small and small when they are large. Conditions making for large or small coefficients of sensitivity have been established and discussed when these terms were defined. Generally, we know that they will be smaller than the sum of their respective own substitution elasticities of demand for, and supply of importables. This fact, given the sign and magnitude of the difference in the propensities, makes for a larger shift in the terms of trade and a larger real income effect. We are unable, however, to conclude that this will be the net effect of the introduction of non-traded commodities for we have seen that they could decrease or increase the movement in the terms of trade by their effects upon the propensity terms. Our single conclusion must be that to neglect the effect of non-traded goods both upon the magnitude and direction of the shift in the terms of trade is to risk a seriously distorted result.

C. THE TRANSFER PROBLEM: TARIFFS AND TRANSPORT COSTS

1. Development of the Model

It is our intention in this section to obtain a general four-commodity criterion for the effect of a given transfer upon the rate of change in the terms of trade of the paying (domestic) country when both transport costs and tariffs are allowed for explicitly. The assumptions and notation used in earlier developments of the model are retained with some exceptions. For instance, we assume, as does Johnson [37], that the transport of a commodity will absorb normally some of each country's internationally traded goods. For analytical reasons the transportation of commodities is assumed to involve two distinct processes, best thought of as sea transport provided by the domestic (paying) country from its own exportable good, and land transport provided likewise by the foreign (receiving) country from its own exportable commodity. Thus the act of transportation involves in each instance up to the trans-shipment point the use of the exporting country's exportable good and, after the trans-shipment point, the use of the importing country's exportable commodity.

Additionally, it is assumed that each country levies a constant ad valorem tariff. It follows, therefore, that each commodity has four distinct prices, each of which is applicable at some stage of the marketing process. The price of the paying country's exportables is given in the paying country by p_1, at the trans-shipment point by $\bar{p}_1$, at the border of the receiving country exclusive of tariffs by $\bar{p}_1'$, and in the market of the receiving country tariff inclusive by p_1'. Similarly, the price of the receiving country's exportables in the receiving country is p_2', at the trans-shipment point $\bar{p}_2'$, at the port of entry to the paying country $\bar{p}_2$, and in the market of the paying country p_2. Hence $\frac{\bar{p}_1}{\bar{p}_1'}$, $\frac{\bar{p}_2'}{\bar{p}_2}$ are the ratios of the values per unit of commodities one and two at the trans-shipment point to their respective transport-inclusive values at the port of destination. Similarly, $\frac{\bar{p}_1'}{p_1'}$ and $\frac{\bar{p}_2}{p_2}$ are the ratios of the transport-inclusive prices of good one and two respectively to their transport plus tariff-inclusive market prices. As the denominator in each of the above ratios exceeds the numerator, each is less than unity in magnitude.

In an analogous fashion, $\bar{x}_1$ and $\bar{x}_2$ respectively represent the quantities of exports of goods one and two which have not been used up in transport at the trans-shipment point. Note, however, that that part of the cost of transport of domestic imports which is incurred in the exportable good of the domestic country after the trans-shipment point is assumed to be provided for by the sale of a quantity of imports equal to $(\bar{p}_2-\bar{p}_2')\bar{x}_2$. Similarly, in the foreign country, a portion of imports equal to $(\bar{p}_1'-\bar{p}_1)\bar{x}_1$ is sold to defray that part of the cost of transporting foreign imports which is incurred in the foreign exportable good.

The equilibrium equations are:

$$\bar{x}_1 = X_1' - O_1' \quad \ldots \quad (24.4)$$

$$x_1 = O_1 - X_1 \quad \ldots \quad (25.4)$$

$$\bar{x}_2 = X_2 - O_2 \quad \ldots \quad (26.4)$$

$$x_2 = O_2' - X_2' \quad \ldots \quad (27.4)$$

$$0 = X_3 - O_3 \quad \ldots \quad (28.4)$$

$$0 = X_3' - O_3' \quad \ldots \quad (29.4)$$

$$M = p_1\bar{X}_1+(\bar{p}_2-\bar{p}_2')\bar{x}_2+\bar{p}_1\bar{x}_1+p_2O_2+p_3O_3+(p_2-\bar{p}_2)\bar{x}_2-B \quad \ldots \quad (30.4)$$

$$M' = p_1'O_1'+p_2'\bar{X}_2'+(\bar{p}_1'-\bar{p}_1)\bar{x}_1+\bar{p}_2'\bar{x}_2+p_3'O_3'+(p_1'-\bar{p}_1')\bar{x}_1+B \quad \ldots \quad (31.4)$$

$$p_2 = p_2'\left[1 + \left(\frac{p_2}{p_2'} - 1\right)\right] \quad \ldots \quad (32.4)$$

$$\bar{p}_2 = p_2'\left[1 + \left(\frac{\bar{p}_2}{p_2'} - 1\right)\right] \quad \ldots \quad (33.4)$$

$$\bar{p}_2' = p_2'\left[1 + \left(\frac{\bar{p}_2'}{p_2'} - 1\right)\right] \quad \ldots \quad (34.4)$$

$$p_1x_1 = \bar{p}_1\bar{x}_1 \quad \ldots \quad (35.4)$$

$$p_2'x_2 = \bar{p}_2'\bar{x}_2 \quad \ldots \quad (36.4)$$

$$B = \bar{p}_1\bar{x}_1 - \bar{p}_2'\bar{x}_2 \quad \ldots \quad (37.4)$$

Of these equations the first six are the basic supply-demand equations of the model. The next two are the expenditure equations suitably modified to include the presence of a tariff revenue term in each country. Thus in the domestic country, expenditure is equal to the value of output of good one (which comprises domestic consumption, $p_1\bar{X}_1$, plus exports, $p_1x_1=\bar{p}_1\bar{x}_1$, plus the value of exportables, $(\bar{p}_2-\bar{p}_2')\bar{x}_2$, used up in the transportation of imports from the trans-shipment point), good two, and good three, plus tariff revenue, $(p_2-\bar{p}_2)\bar{x}_2$, less the amount of the transfer. The foreign expenditure equation is defined similarly. Next come three price equations for p_2, $\bar{p}_2$ and $\bar{p}_2'$ respectively. Similar equations could be given for any two of p_1', $\bar{p}_1'$ and $\bar{p}_1$, but they are omitted because, by the assumption of constant tariff and transport cost loadings, any change in these price ratios is zero. Equations (35.4) and (36.4) state that the value of exports in each country before any transport costs are incurred must equal the value of exports at the trans-shipment point, the increased cost per unit of exports being inversely proportional to the reduction in the volume of exports. Finally, equation (37.4) states that the transfer, B, is equal to the value received by the exporters of the paying country for their exports, less the amount of money paid to the foreign (receiving) country's exporters for imports.

Not all of these equations are independent. We choose to omit numbers (25.4), (27.4), (35.4) and (36.4). This time, without loss of generality, the domestic price of exportables, p_1, is selected as numeraire and quantity units are chosen so that $p_1=p_2'=p_3'=p_3=1$. Differentiating totally the above equations we obtain:

$$d\bar{x}_1 = (X_{12}'-O_{12}')dp_2' + (X_{13}'-O_{13}')dp_3' + M_1'dM' \quad \ldots \quad (38.4)$$

$$d\bar{x}_2 = (X_{22}-O_{22})dp_2 + (X_{23}-O_{23})dp_3 + M_2dM \quad \ldots \quad (39.4)$$

$$0 = (X_{32}-O_{32})dp_2 + (X_{33}-O_{33})dp_3 + M_3dM \quad \ldots \quad (40.4)$$

$$0 = (X_{32}'-O_{32}')dp_2' + (X_{33}'-O_{33}')dp_3' + M_3'dM' \quad \ldots \quad (41.4)$$

$$dM = O_2dp_2+O_3dp_3+d(\bar{p}_2-\bar{p}_2')\bar{x}_2+d(p_2-\bar{p}_2)\bar{x}_2+(p_2-\bar{p}_2)d\bar{x}_2-dB \quad (42.4)$$

$$dM' = O_2'dp_2' + O_3'dp_3' + p_1'(1 - \frac{\bar{p}_1'}{p_1'})\, d\bar{x}_1 + dB \quad \ldots \quad (43.4)$$

$$dp_2 = p_2 dp_2' \quad \ldots \quad (44.4)$$

$$d\bar{p}_2 = \bar{p}_2 dp_2' \quad \ldots \quad (45.4)$$

$$d\bar{p}_2' = \bar{p}_2' dp_2' \quad \ldots \quad (46.4)$$

$$dB = \frac{\bar{p}_1}{p_1} d\bar{x}_1 + \bar{p}_2' d\bar{x}_2 - \bar{x}_2 d\bar{p}_2' \quad \ldots \quad (47.4)$$

where $X_{ij} = \frac{\partial X_i}{\partial p_j}$, $O_{ij} = \frac{\partial O_i}{\partial p_j}$ and $M_i = \frac{\partial X_i}{\partial M}$.

The reader should note first, that in equations (42.4) and (43.4) use is made of the homogeneity condition that $\sum_i p_i dO_i = 0$ where care should be taken to distinguish between the change in the production of that part of exportables measured in terms of imports which is included in the $\sum_i p_i dO_i$ and the change in tariff revenue due to the change in imports which is not part of the $\sum_i p_i dO_i$; and secondly, that in the equations (44.4) to (46.4) all changes in the relative prices, $\frac{p_2}{p_2'}$, $\frac{\bar{p}_2}{p_2'}$ and $\frac{\bar{p}_2'}{p_2'}$, are zero by the constant tariff and transport cost loading assumption.

We now proceed to

i. substitute in (42.4) for $d\bar{p}_2$ from (45.4) and into (42.4) and (47.4) for $d\bar{p}_2'$ from (46.4);

ii. substitute equation (42.4) into equations (39.4) and (40.4);

iii. substitute equation (43.4) into equations (38.4) and (41.4);

iv. substitute into the resulting equations for dp_2 from (44.4);

$(K_{ij} - X_j M_i)$ for $(X_{ij} - O_{ij})$ where K_{ij} is the net demand substitution effect; and for the terms $(X_i - O_i)$ which arise after these substitutions, the relevant x_1, x_2, $\bar{x}_1$, $\bar{x}_2$ or zero as indicated by the equilibrium equations;

v. cancel and collect terms wherever possible.

This yields the following rearranged set of equations:

$$(p_2K_{22}-x_2M_2)dp_2' + K_{23}dp_3 + M_2(p_2-\bar{p}_2)d\bar{x}_2 - d\bar{x}_2 = M_2dB \quad , \ . \ . \qquad (48.4)$$

$$(p_2K_{32}-x_2M_3)dp_2' + K_{33}dp_3 + M_3(p_2-\bar{p}_2)d\bar{x}_2 = M_3dB \quad . \ . \ . \qquad (49.4)$$

$$(K_{12}'+x_2M_1')dp_2' + K_{13}'dp_3' + M_1'(p_1'-\bar{p}_1')d\bar{x}_1 - d\bar{x}_1 = -M_1'dB \quad . \ . \ . \qquad (50.4)$$

$$(K_{32}'+x_2M_3')dp_2' + K_{33}'dp_3' + M_3'(p_1'-\bar{p}_1')d\bar{x}_1 = -M_3'dB \quad . \ . \ . \qquad (51.4)$$

$$\bar{p}_1d\bar{x}_1 + \bar{p}_2'd\bar{x}_2 - x_2dp_2' = dB \quad . \ . . . \qquad (52.4)$$

The rate of change in p_2' is our measure of the rate of change in the real terms of trade. As the $\sum_j p_jK_{ij}=0$, the solution for the rate of change in the terms of trade due to a transfer is:

$$\frac{dp_2'}{dB} = \frac{\begin{vmatrix} M_2 & K_{23} & 0 & 0 & M_2(p_2-\bar{p}_2)-1 \\ M_3 & K_{33} & 0 & 0 & M_3(p_2-\bar{p}_2) \\ -M_1' & 0 & K_{13}' & M_1'(p_1'-\bar{p}_1')-1 & 0 \\ -M_3' & 0 & K_{33}' & M_3'(p_1'-\bar{p}_1') & 0 \\ 1 & 0 & 0 & \bar{p}_1 & -\bar{p}_2' \end{vmatrix} \ (= |A|)}{\begin{vmatrix} (p_2K_{22}-x_2M_2) & K_{23} & 0 & 0 & M_2(p_2-\bar{p}_2)-1 \\ (p_2K_{32}-x_2M_3) & K_{33} & 0 & 0 & M_3(p_2-\bar{p}_2) \\ (-p_1'K_{11}'+x_2M_1') & 0 & K_{13}' & M_1'(p_1'-\bar{p}_1')-1 & 0 \\ (-p_1'K_{31}'+x_2M_3') & 0 & K_{33}' & M_3'(p_1'-\bar{p}_1') & 0 \\ -x_2 & 0 & 0 & \bar{p}_1 & -\bar{p}_2' \end{vmatrix} \ (= |B|)}$$

A Laplacian expansion of $|A|$ by the first two rows gives, when the result is multiplied by -1:

$$|A| = \begin{vmatrix} M_2 & K_{23} \\ M_3 & K_{33} \end{vmatrix} \begin{vmatrix} K_{13}' & M_1'(p_1'-\bar{p}_1')-1 \\ K_{33}' & M_3'(p_1'-\bar{p}_1') \end{vmatrix} (\bar{p}_2')$$

$$- \begin{vmatrix} K_{23} & M_2(p_2-\bar{p}_2)-1 \\ K_{33} & M_3(p_2-\bar{p}_2) \end{vmatrix} \left[\begin{vmatrix} K_{13}' & M_1'(p_1'-\bar{p}_1')-1 \\ K_{33}' & M_3'(p_1'-\bar{p}_1') \end{vmatrix} + \begin{vmatrix} -M_1' & K_{13}' \\ -M_3' & K_{33}' \end{vmatrix} (\bar{p}_1) \right]$$

Similarly,

$$|B| = \begin{vmatrix} (p_2K_{22}-x_2M_2) & K_{23} \\ (p_2K_{32}-x_2M_3) & K_{33} \end{vmatrix} \begin{vmatrix} K'_{13} & M'_1(p'_1-\bar{p}'_1)-1 \\ K'_{33} & M'_3(p'_1-\bar{p}'_1) \end{vmatrix} (\bar{p}'_2)$$

$$+ \begin{vmatrix} K_{23} & M_2(p_2-\bar{p}_2)-1 \\ K_{33} & M_3(p_2-\bar{p}_2) \end{vmatrix} \left[x_2 \begin{vmatrix} K'_{13} & M'_1(p'_1-\bar{p}'_1)-1 \\ K'_{33} & M'_3(p'_1-\bar{p}'_1) \end{vmatrix} + \begin{vmatrix} p'_1K'_{11}-x_2M'_1 & K'_{13} \\ p'_1K'_{31}-x_2M'_3 & K'_{33} \end{vmatrix} (\bar{p}_1) \right]$$

Dividing $|A|$ and $|B|$ by

$$\begin{vmatrix} K'_{13} & M'_1(p'_1-\bar{p}'_1)-1 \\ K'_{33} & M'_3(p'_1-\bar{p}'_1) \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} K_{23} & M_2(p_2-\bar{p}_2)-1 \\ K_{33} & M_3(p_2-\bar{p}_2) \end{vmatrix}$$

we obtain

$$\frac{dp'_2}{dB} = \frac{-1 + (\bar{p}'_2) \begin{vmatrix} M_2 & K_{23} \\ M_3 & K_{33} \end{vmatrix} \dfrac{1}{\begin{vmatrix} K_{23} & M_2(p_2-\bar{p}_2)-1 \\ K_{33} & M_3(p_2-\bar{p}_2) \end{vmatrix}} + (\bar{p}_1) \begin{vmatrix} M'_1 & K'_{13} \\ M'_3 & K'_{33} \end{vmatrix} \dfrac{1}{\begin{vmatrix} K'_{13} & M'_1(p'_1-\bar{p}'_1)-1 \\ K'_{33} & M'_3(p'_1-\bar{p}'_1) \end{vmatrix}}}{x_2 + (\bar{p}'_2)\left[\begin{vmatrix} p_2K_{22} & K_{23} \\ p_2K_{32} & K_{33} \end{vmatrix} - \begin{vmatrix} x_2M_2 & K_{23} \\ x_2M_3 & K_{33} \end{vmatrix} \right] \dfrac{1}{\begin{vmatrix} K_{23} & M_2(p_2-\bar{p}_2)-1 \\ K_{33} & M_3(p_2-\bar{p}_2) \end{vmatrix}} + (\bar{p}_1)\left[\begin{vmatrix} p'_1K'_{11} & K'_{13} \\ p'_1K'_{31} & K'_{33} \end{vmatrix} - \begin{vmatrix} x_2M'_1 & K'_{13} \\ x_2M'_3 & K'_{33} \end{vmatrix} \right] \dfrac{1}{\begin{vmatrix} K'_{13} & M'_1(p'_1-\bar{p}'_1)-1 \\ K'_{33} & M'_3(p'_1-\bar{p}'_1) \end{vmatrix}}} \quad \ldots \quad (53.4)$$

By expanding this result, forming elasticities (remembering that $p_2, p'_1 \neq 1$), and multiplying both sides by $x_2(=x'_1)$ we obtain:

$$H' = \frac{-1 - \dfrac{\bar{p}'_2}{\bar{p}_2}\dfrac{\bar{p}_2}{p_2}\psi_{23}\dfrac{1}{\lambda} - \dfrac{\bar{p}_1}{\bar{p}'_1}\dfrac{\bar{p}'_1}{p'_1}\psi'_{13}\dfrac{1}{\lambda'}}{1 + \bar{p}'_2\Psi_{23}\dfrac{1}{\lambda} + \bar{p}_1\Psi'_{13}\dfrac{1}{\lambda'} + \dfrac{\bar{p}'_2}{\bar{p}_2}\dfrac{\bar{p}_2}{p_2}\psi_{23}\dfrac{1}{\lambda} + \dfrac{\bar{p}_1}{\bar{p}'_1}\dfrac{\bar{p}'_1}{p'_1}\psi'_{13}\dfrac{1}{\lambda'}} \quad \ldots \quad (54.4)$$

where $H'\left(= \frac{dp'_2}{p'_2}\frac{x_2}{dB}\right)$ is the change in the terms of trade due to a transfer expressed as a proportion of the value of trade; where Ψ_{ij} and ψ_{ij} are defined as above,

$$\lambda = 1 + \psi_{23}\left(1 - \frac{\bar{p}_2}{p_2}\right) \quad \text{and} \quad \lambda' = 1 + \psi'_{13}\left(1 - \frac{\bar{p}'_1}{p'_1}\right).$$

The formula given in equation (54.4), though superficially quite different, in fact is identical to that obtained in the zero-impediments case (18.4) if all terms introduced by the presence of tariffs and transport costs are set to zero.[4]

Shortly, we shall see that the effect of the introduction of transport costs and tariffs upon the magnitude of the coefficients of sensitivity and upon the propensity terms is small. It follows that these assumptions do not normally affect significantly the stability of the model and so it is still assumed that our international system is stable, i.e. that the sign of our denominator is negative. In this event, the direction of the shift in the terms of trade is determined by the sign of the numerator, being favourable or adverse to the paying country according to whether

$$- \psi_{23} \frac{\bar{p}_2'}{\bar{p}_2} \frac{\bar{p}_2}{p_2} \frac{1}{\lambda} - \psi_{13}' \frac{\bar{p}_1}{\bar{p}_1'} \frac{\bar{p}_1'}{p_1'} \frac{1}{\lambda'} \gtreqless 1 \qquad \ldots \qquad (55.4)$$

As in the zero-impediments case use is made of the time-honoured method of gradually diminishing abstraction to examine the properties of (55.4). Accordingly, criteria are established for the multi-commodity case in which tariffs exist but in which there are no transport costs, and for the corresponding transport cost and/or tariff inclusive two-commodity cases.

Where transport costs are assumed to be negligible or zero, $p_1=\bar{p}_1=\bar{p}_1'$ and $p_2'=\bar{p}_2'=\bar{p}_2$, so that our criterion reduces to

$$- \psi_{23} \frac{p_2'}{p_2} \frac{1}{\lambda} - \psi_{13}' \frac{p_1}{p_1'} \frac{1}{\lambda'} \gtreqless 1 \qquad \ldots \qquad (56.4)$$

Similarly, in the case of zero tariffs, $p_1=p_1'$ and $p_2=p_2'$, the λ terms reducing to unity, and the criterion becoming:

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[4] Making use of the fact that $\sum_j p_j K_{ij}=0$, it can be shown that $\psi_{13}' \equiv \psi_{23}'$ and that $1+\psi_{13}' \equiv -\psi_{23}'$ (see page 115 below).

$$-\psi_{13}' - \psi_{23} \gtreqless 1 \qquad \ldots \qquad (20.4)$$

which is the zero-impediments result established in the previous section ($1+\psi_{13}'=-\psi_{23}'$).

If, in the three criteria (55.4), (56.4) and (20.4) given above, all terms involving the non-traded commodities are set to zero, analogous two-commodity criteria are obtained:

i. where both tariffs and transport costs exist:

$$C_2 \frac{\bar{p}_2'}{\bar{p}_2} \frac{\bar{p}_2}{p_2} \frac{1}{\lambda} + C_1 \frac{\bar{p}_1}{\bar{p}_1'} \frac{\bar{p}_1'}{p_1'} \frac{1}{\lambda'} \gtreqless 1 \qquad \ldots \qquad (57.4)$$

where $\lambda=1-C_2(1-\frac{\bar{p}_2}{p_2})$ and $\lambda'=1-C_1'(1-\frac{\bar{p}_1'}{p_1'})$;

ii. where only tariffs exist, transport costs being negligible or zero:

$$C_2 \frac{p_2'}{p_2} \frac{1}{\lambda} + C_1' \frac{p_1}{p_1'} \frac{1}{\lambda'} \gtreqless 1 \qquad \ldots \qquad (58.4)$$

iii. where both transport costs and tariffs are non-existent:

$$C_2 + C_1' \gtreqless 1 \qquad \ldots \qquad (19.4)$$

Consider the effect of tariffs upon the direction of shift in the terms of trade following a transfer, as reflected in the two-commodity criterion (58.4).

First, we examine the properties and influence of the λ terms which arise because of the existence of a prior tariff. From inspection, it can be seen that as long as each country's marginal propensity to spend on importables is less than unity~~, and the tariff does not exceed one hundred per cent,~~ the λ terms will be less than unity in magnitude, but positive. Normally, they would approximate to unity. Their influence is best explained thus: the effect of the transfer at constant prices is to decrease (increase) the tariff revenue receipts of the paying (receiving) country. This means that in the paying country the reduction in imports will be determined not only by the size of the transfer and the marginal propensity to spend on importables, C_2, but also by the reduction in tariff revenue. Similarly, in the foreign country the increase in the amount of imports demanded at constant prices is determined by the size of the transfer, by the foreign marginal propensity to spend on importables, and by the gain in tariff revenue upon each additional unit of imports. Thus the effect of a loss in tariff revenue in the transferring and a gain in tariff revenue in the receiving country is to increase the magnitude of C_2 and C_1' (the respective domestic and foreign marginal propensities to spend on importables). Arguing along equiprobability lines that these two propensities initially are equal to one half, it follows that there is a presumption in favour of the modern opinion that the terms of trade will move adversely for the receiving country. As one would expect, our λ terms become unity if there is no initial tariff.

Secondly, and offsetting this influence of the prior tariff, is the presence of the other weighting terms, the price ratios p_1/p_1' and p_2'/p_2 which both reflect the fact that more costly imports (due to the tariff)shift each country's marginal <u>physical</u> consumption propensities, M_2, M_1', in favour of their own exportable good. Thus if we argue on equiprobability grounds that $C_2=C_1'=\frac{1}{2}$, it follows that M_2 and M_1' are each less than one half and that, accordingly, the amount of the reduction in the demand for imports in the paying country and the amount of the increase in the demand for imports in the receiving country will be reduced.

Johnson [37,116] is of the opinion that nothing conclusive can be said about the net influence of these two effects upon the criterion, arguing that while dearer imports would tend to restore the orthodox conclusion, the effect of tariffs upon the marginal propensities to spend might offset it. In fact, it can be shown that the net influence would strengthen the orthodox presumption. The foreign propensity term from (58.4) can be rewritten:

$$C_1' \frac{1}{\lambda'} \frac{p_1}{p_1'} = C_1' \frac{1}{\frac{p_1'}{p_1} - C_1'(\frac{p_1'}{p_1} - 1)}$$

As long as C_1' is less than unity, it follows that $\frac{1}{\lambda'}\frac{p_1}{p_1'}$ must likewise be less than unity and positive. The net effect of the tariff-weighting term, therefore, is to reduce the magnitude of the foreign marginal propensity to spend on exportables while leaving its sign unchanged. Similarly, $\frac{1}{\lambda}\frac{p_2'}{p_2}$ reduces the magnitude of C_2, so that in each country the net effect of the tariff is to increase the probability of an adverse movement in the terms of trade of the paying country.

The effect of the tariff upon the direction of the shift in the terms of trade is the same in the multi-commodity case. This time the criterion is:

$$- \psi_{13}' \frac{p_1}{p_1'} \frac{1}{\lambda'} - \psi_{23} \frac{p_2'}{p_2} \frac{1}{\lambda} \gtreqless 1 \quad \ldots \quad (56.4)$$

where $\lambda = 1+\psi_{23}(1-\frac{p_2'}{p_2})$ and $\lambda' = 1+\psi_{13}'(1-\frac{p_1}{p_1'})$

Rewriting $$\frac{p_1}{p_1'}\frac{1}{\lambda'} = \frac{1}{\frac{p_1'}{p_1} + \psi_{13}'(\frac{p_1'}{p_1} - 1)}$$

it again follows that because ψ_{13}' is less than unity and negative, $\frac{1}{\lambda'}\frac{p_1}{p_1'}$ must be less than unity and positive. Similarly, $\frac{1}{\lambda}\frac{p_2'}{p_2}$ is less than unity and positive.

For the moment, let us continue to ignore the complex effects of transport costs in order that we may consider an attempt of Samuelson [36,280] to support the orthodox contention that the terms of trade of the paying country would deteriorate in a two-commodity world. It is suggested that by a logical extension of the restrictive assumptions used by Samuelson, the multi-commodity case also can be brought within the orthodox fold.

In view of our ignorance concerning the magnitude of the parameters appearing in the two-commodity criterion Samuelson suggested assuming that identical tastes exist in each country and that marginal and average propensities to consume are equal. In this manner, he was able to prove that the orthodox presumption in favour of an improvement in the terms of trade of the receiving country depended upon the presence of tariffs and that in their absence no presumption either way was possible.

Unfortunately, in a three-good world, these assumptions are not sufficiently restrictive. However, it appears logical to conclude that if we are ignorant concerning the different taste patterns in each country, we could be ignorant equally of the production conditions. If then we assume that demand and supply elasticities as well as marginal propensities to spend are equal in each country, it is possible to conclude that in the absence of tariffs there can be no presumption either way concerning the direction of the shift in the terms of trade, for where

$$\sigma_{ij} = \sigma'_{ij}$$

$$S_{ij} = S'_{ij}$$

$$\text{and} \quad C_i = C'_i$$

our numerator $-\psi'_{13}-\psi_{23}-1=0$. Only the introduction of tariffs can turn the presumption in favour of the orthodox viewpoint.

Finally, our attention centres upon the general, multi-commodity case in which both tariffs and transport costs are introduced explicitly to the model. The criterion is:

$$-\psi'_{13} \frac{\bar{p}_1}{\bar{p}'_1} \frac{\bar{p}'_1}{\bar{p}'_1} \frac{1}{\lambda'} - \psi_{23} \frac{\bar{p}'_2}{\bar{p}_2} \frac{\bar{p}_2}{p_2} \frac{1}{\lambda} \gtreqless 1 \quad \ldots \quad (55.4)$$

The influence of transport costs upon the criterion differs from that of tariffs because in the former case physical resources are actually consumed in the process of transportation. It should be recalled that we assumed that the transport of a commodity from one country to another would in the first instance absorb some of the exporting country's own exports and in the second instance absorb some of the exportable products of the importing country. From our criterion it is evident that the greater the proportion of transport cost provided from the exportables of the importing country, i.e. the smaller $\frac{\bar{p}_1'}{\bar{p}_1}$ and $\frac{\bar{p}_2'}{\bar{p}_2}$, the greater is the probability of an adverse terms of trade movement for the paying country. For instance, in the paying country the reduction in expenditure upon imports would reduce the demand for that part of the paying country's exportables which were used to provide transport for imports. In the receiving country, an increased expenditure upon imports would increase simultaneously the demand for exportables. As long as some of the cost of transport was incurred in the exportable good of the importing country, transport costs would increase the probability of an adverse terms of trade movement for the paying country. This case contrasts with that postulated by Samuelson, in which each country provides all the transport for its own exports from its exportable commodity. In this event $\frac{\bar{p}_1'}{\bar{p}_1}$ and $\frac{\bar{p}_2'}{\bar{p}_2}$ would go to unity and transport costs would have no effect upon the criterion expressed in terms of marginal propensities to spend; income in each country would be the same as for the case of zero transport costs since the cost of transport would be merely an export for the exporting country. Apart from this case, if use is made of the restrictive assumptions made above, namely equal marginal propensities to spend and the same supply and demand elasticities, a presumption can be established in favour of the orthodox case.

This completes our analysis of the effects of a transfer upon the direction of shift in the terms of trade. Table Two, below, summarizes our results for the multi-commodity case and compares them with the tentative conclusions established by Samuelson in the second of his celebrated articles upon the transfer problem.

D. CONCLUSIONS

We shall conclude this chapter with a few comments upon the relative value of the conclusions established above.

1. It is contended that the use of either pseudo general equilibrium two-commodity analysis or partial equilibrium analysis is inadequate for a study of the effect of a transfer upon the direction of shift in the terms of trade. On the one hand, if there is a postulated world in which only two-commodities exist the problem of qualitatively, to say nothing of quantitatively, assessing the magnitude of parameters such as marginal propensities to spend on importables and exportables is encountered. Without a knowledge of the complex substitution relationships introduced by the presence of a non-traded commodity, it is difficult to see how these concepts could be ever identifiable or analytically useful. Thus while the correctness of the two-commodity result is not disputed, its usefulness for purposes of prediction is. On the other hand, the partial equilibrium approach, in which all relationships between traded and non-traded commodities are ignored, is vulnerable to the grave objection that these non-traded/traded relationships could affect decisively, as we have seen, the direction of shift in the terms of trade.

Table Two. Summary of Transfer Effects upon the Terms of Trade in a multi-commodity world

Zero Impediments	Tariffs	Transport Costs	Marginal propensities to spend equal; demand and supply elasticities the same
Orthodox view favoured provided: i. C_1', C_3', and C_2, C_3 are small relative to C_2' and C_1 respectively. ii. Domestic goods are more competitive on the production and consumption side with exportables than with importables.	As for zero-impediments case except that the probability of the orthodox effect appearing is increased.	As for the tariff case, as long as transport costs are not incurred entirely in the good of the exporting country.	i. In the absence of trade impediments there exists a presumption that the terms of trade movement would be favourable to the receiving country. This presumption disappears when average propensities to consume differ between the two trading areas. ii. If either tariffs and/or transport costs exist the terms of trade will change in favour of the receiving country.
Samuelson's result Orthodox view favoured, provided domestic goods are 'more competitive on the production side' with exports than with import goods.			This case not considered.

2. Further, it is emphasised that Samuelson's tentative conclusion that the problem of the terms of trade shift in a multi-commodity world would be resolved by the production substitution possibilities in the system is only a part of the answer; substitution possibilities on the side of demand could be of equal or greater importance.

3. For those who will accept equiprobability type arguments, it has been shown that in the three-commodity, as opposed to the two-commodity zero-impediments case, a presumption can be established in favour of the orthodox conclusion that the terms of trade would improve for the receiving country. This presumption cannot be sustained, however, unless average propensities to spend (among other things) are equal in the two countries.

4. Additionally, it is argued that not only the direction but also the magnitude of the shift in the terms of trade and, consequently, the real income effect of a transfer, would be affected markedly by non-traded/traded commodity relationships. Any attempt to assess the real income effect of a transfer must have, therefore, some knowledge, either qualitative or quantitative, concerning these parameters.

5. Finally, and perhaps most important, a proof is given that the price level of non-traded commodities in the paying country would fall normally relative to an index of the prices of the internationally traded commodities. A corresponding rise in the price of non-traded commodities relative to an index of the price of the international goods would occur in the receiving country. This conclusion qualifies the Viner-Wilson hypothesis, which tends to neglect the indirect effect of the terms of trade shift in stating that the prices of the traded commodities would fall in the paying country relative to the non-traded price level and rise in the receiving country relative to the non-traded price level. The significance of this conclusion concerns the mechanism of adjustment in the international system, for it may well be that the adjustment to equilibrium secured

by means of a terms of trade shift is small compared with the adjustment secured by the shift in the non-traded/traded commodity price ratios.

5. TARIFFS AND THE DOMESTIC PRICE RATIO OF TRADED GOODS[1]

A. INTRODUCTION

In this chapter the effect of a tariff upon the domestic prices of importables and exportables in the tariff-imposing country is examined. First, the relevant literature is reviewed briefly and the more important assumptions and practical implications of established theorems are discussed. Secondly, a general four-commodity model is developed in which are included two classes of non-traded commodities and a criterion is derived for the effect of a tariff upon the domestic traded goods price ratio of the tariff-imposing country. Thirdly, the problem is analysed within the context of a classical, two-commodity, two-country model, which not only provides us with a sound foundation for the analysis of the more complex multi-commodity case, but also enables us to contrast the traditional result expressed in terms of 'total' elasticities with a two-good answer in which all of the relevant parameters are detailed explicitly. Finally, the multi-commodity criterion is analysed and compared with the two-good result.

B. THE ARGUMENT

Traditionally, the analysis of the effect of a tariff on the terms of trade and on the distribution of income within the taxing country were contained in separate economic boxes. Recent analysis,however, has demonstrated that both the magnitude and the direction of the latter are intimately associated with the size of the terms of trade movement, the critical factor being the direction of the change in the domestic traded goods price ratio.

The fact that a tariff might improve the terms of trade was recognised gene-

[1] The contents of this chapter form the basis of two articles written by myself and published recently [100;102].

rally by the classical economists; it is logically implicit in John Stuart Mill's theory of reciprocal demand and even such exponents of orthodoxy as Alfred Marshall and Frank W. Taussig acknowledge begrudgingly its theoretical possibility. At the same time, the classicists and neo-classicists ignored and, indeed, even rejected any idea that tariffs might influence the distribution of income within the taxing country. Taussig, for instance, was of the opinion that the factors determining income distribution "lie quite outside the tariff controversy ... " [29,54]. Their inability to deal with the income distribution question follows from the assumption of a single factor of production, labour, and from the then undeveloped theory of distribution. They assumed that the reward of each and every factor depended on the productivity of the entire economy.

Pigou first suggested that an increase in the output of one industry at the expense of another could raise the absolute share in the national income of the factor employed relatively more intensively in the expanding industry [30,85]. It was Heckscher [31] and Ohlin [22], however, who prepared the ground for a rigorous treatment by Stolper and Samuelson [32] of the effect of a tariff upon the distribution of the national income. As our concern is only the antecedents of the theorem which describes the effect of a tariff upon the domestic traded goods price ratio, no explicit account is given of the Heckscher-Ohlin model nor of the logical consequences derived from it by Stolper and Samuelson. It suffices to say that the latter demonstrated rigorously (subject to limiting conditions) that a tariff would increase the absolute as well as the relative share in the national income of the factor of production used relatively more intensively in the import-competing industry. Whereas the classicists, however, had ignored the income redistribution effects of a tariff and concentrated instead upon the terms of trade effect, Stolper and Samuelson reversed this procedure by assuming explicitly that the terms of trade did not change.

Metzler [16], in a recent well-known article, has integrated these different theorems to demonstrate that a terms of trade effect could cancel, or even reverse, the income redistribution effect postulated by Stolper and Samuelson. Our interest centres not in this qualification of the Stolper-Samuelson theorem,[2] however, but in a surprising consequence of it. The criterion Metzler developed showed that the Stolper-Samuelson conclusion would be reversed if the domestic price of importables fell relative to the domestic price of exportables. This, however, is precisely the condition for a tariff to fail to protect! More specifically, his formula purports to show that:

> when the foreign demand for a country's exports is inelastic ... a tariff far from protecting industries competing with exports at the expense of the export trades, may actually benefit the latter at the expense of the former [16,19].

He points out that the traditional analysis ignores the indirect effects that would follow the change in the terms of trade as the result of a tariff. An artificial restriction of demand for imports in the home country would cause an excess supply of the rest of the world's exports (i.e. imports) and a fall in their price. This would tend to raise the price of the home country's exports relative to that of its imports. If the latter terms of trade effect were sufficiently strong, the initial movement in the domestic price ratio (due to the tariff) could be not only arrested, but reversed, and a reduction in the domestic price of importables relative to the domestic price of exportables would occur. In this case, resources would flow from the 'protected' import-competing to the exportable goods industries. <u>The policy of protection would be not only ineffective but also actually harmful to those industries it sought to protect</u>.

[2] This theorem, with its severe limiting assumptions, already has been criticized extensively and any proof of the invalidity of Metzler's qualification of it would be pointless. The reader is referred to [57, 68-76] where most of the literature on the subject is listed.

Metzler continued to criticize the conclusions of the well-known Australian tariff enquiry that was published in 1929 [33]. As the findings of this report have continued to provide the theoretical justification of Australian tariff policy they are enumerated briefly below. The authors of the report were of the opinion that a tariff reduction would have:

1. affected unfavourably the Australian terms of trade;
2. reduced real wages and increased rents by lessening the relative scarcity of labour;
3. reduced the diversity of occupations and opportunities and decreased the stability of the national income by making it more dependent upon the seasons and upon the vagaries of the overseas markets;
4. reduced the standard of living of the existing population;
5. tended to eliminate certain strategic industries;
6. prevented the development of Australian industry and the attendant economies of scale.

Metzler argued that the first two conclusions were inconsistent. He maintained that the adverse terms of trade movement following a reduction in the tariff might protect industry and, if this were so, that the remaining conclusions also are inconsistent.

Though most of the above arguments are founded on particular Australian conditions, the infant industry one is not. Economists, normally, have taken for granted that

> An import duty ... will increase the marginal efficiency of investment in industries producing goods identical with or similar to those hit by the duty; it will diminish the marginal efficiency of investment in industries that make use of such goods and that produce goods complementary to them [34,107].

As the Metzler argument has been commented upon uncritically in recent tariff literature [91,16; 99,250; 64,76-78; 110,74-75], though more frequently in connection with income distribution than with trade protection, it seems a worthwhile task to attempt a reconciliation of these two divergent viewpoints. Our conclusion, reached below, is that Metzler failed to carry his analysis sufficiently far and that the likelihood of the appearance of his perverse case is remote in practice.

C. DEVELOPMENT OF THE MODEL

Basically, our assumptions are the same as those set out in the free trade model of chapter three, except that they are modified to account for the presence of an existing tariff levied upon the domestic country's imports (commodity two). The more important of these assumptions are: perfect competition; two countries; four commodities, of which two are non-traded goods excluded from international trade by transport costs, etc.; increasing costs; and a balanced balance of trade. Except for the presence of a prior tariff, and non-traded commodities in each country, these assumptions are identical to those used by Metzler. When comparing our criterion with his, however, it is possible to abstract from the complications introduced by the last two assumptions.

The equilibrium equations of the model are:

$$x_1 = O_1 - X_1 = X_1' - O_1' \quad \ldots \quad (1.5)$$

$$x_2 = X_2 - O_2 = O_2' - X_2' \quad \ldots \quad (2.5)$$

$$0 = X_3 - O_3 \quad \ldots \quad (3.5)$$

$$0 = X_3' - O_3' \quad \ldots \quad (4.5)$$

$$M = p_1 O_1 + p_2 O_2 + p_3 O_3 + (p_2 - p_2')x_2 \quad \ldots \quad (5.5)$$

$$M' = p_1 O_1' + p_2' O_2' + p_3' O_3' \quad \ldots \quad (6.5)$$

where $X_j = X_j(p_1, p_2, p_3, M)$

and $O_j = O_j(p_1, p_2, p_3)$.

The presence of the tariff is reflected by the addition first, of an extra price, p_2', which is less than the domestic price of good two by the amount of the tariff and, secondly, by an additional term in the domestic country's income equation equal to the tariff revenue, $(p_2-p_2')x_2$. Since $M \equiv \sum_j p_j X_j$, one of the supply-demand equations is not independent, and a balanced balance of trade is implied:

$$p_1 x_1 = p_2' x_2 \quad \ldots \quad (7.5)$$

We choose to drop the supply-demand equations involving the foreign country's importables and to use as numeraire the foreign price of the domestic country's importables, p_2'. Once again, we adopt the convention, without loss of generality, of choosing quantity units such that $p_1 = p_2' = p_3 = p_3' = 1$.

Before proceeding with the development of the model it may be of assistance to the reader, for comparative purposes, if we obtain Metzler's two-commodity criterion.

Take the balance of payments equation (7.5) and rearrange it so that

$$\frac{p_1}{p_2'} = \frac{x_2}{x_1}$$

Differentiating totally (logarithmically) with respect to a change in the tariff, p_2, we have, where $p_1 = p_2' = 1$,

$$\frac{dp_1}{dp_2} p_2 = \frac{dx_2}{dp_2}\frac{p_2}{x_2} - \frac{dx_1}{dp_2}\frac{p_2}{x_1} \quad \ldots \quad (8.5)$$

Now, where $p_2' = 1 = p_1$,

$$\frac{d(p_1/p_2)}{p_1/p_2} = dp_1 - \frac{dp_2}{p_2}$$

so that

$$p_2 \frac{dp_1}{dp_2} = \frac{d(p_1/p_2)}{p_1/p_2}\frac{p_2}{dp_2} + 1 \quad \ldots \quad (9.5)$$

Substituting in (8.5) for $\frac{dp_1}{dp_2} p_2$ we have

$$\frac{d(p_1/p_2)}{p_1/p_2}\frac{p_2}{dp_2} = \frac{dx_2}{dp_2}\frac{p_2}{x_2} - \frac{dx_1}{dp_2}\frac{p_2}{x_1} - 1 \quad \ldots \quad (10.5)$$

The left-hand side of equation (10.5) is our measure of the rate of change in the domestic price ratio of the internationally traded goods. If the domestic price ratio is to remain unaltered following a change in the tariff $\frac{d(p_1/p_2)}{p_1/p_2}\left(= -\frac{d(p_2/p_1)}{p_2/p_1}\right)$ must equal zero and the right-hand side of equation (10.5) likewise must be zero.

It is possible to derive Metzler's criterion directly from this equation. His formula refers to the intermediate case in which the protective and terms of trade effects of the tariff just cancel out - equality of the foreign elasticity of demand for exports with unity minus the domestic marginal propensity to spend on importables; if the elasticity is less than this critical value, the perverse case can result.[3]

As $p_2/p_2' = p_2/p_1 \Big/ p_2'/p_1$, it follows that at the critical point where $\frac{d(p_2/p_1)}{p_2/p_1}=0$, the proportionate change in p_2/p_2' must equal the reciprocal of the proportionate change in p_2'/p_1. Thus, in equation (10.5), the second term on the right-hand side can be interpreted as the foreign elasticity of demand for imports and the first term as the home country's elasticity of demand for imports. If the domestic price ratio does not change, the domestic elasticity of demand is all income effect and reduces to the marginal propensity to spend on importables, C_2. Hence, equation (10.5) may be rewritten as

$$E_{11}' = 1 - C_2 \qquad \ldots \qquad (11.5)$$

where E_{11}' is the foreign elasticity of demand for imports, $\frac{dx_1}{x_1}\frac{p_2}{dp_2}$, to which, in the conventional Marshallian manner, a positive sign has been given. This is the Metzler criterion.

One might interpret the elasticity of the above criterion in two ways: first, as Metzler himself chose to do [16,18], as a partial elasticity measuring the proportionate change in the quantity of imports demanded as the result of a proportionate

[3] Metzler assumes implicitly a stable international market.

change in their price, all other prices and money income being held constant; secondly, as the elasticity of the foreign reciprocal demand curve from which Metzler derived it originally [16,8]. The first interpretation is valid in a multi-commodity model only if the good is independent of other commodities in consumption and production. Such an assumption, however, is difficult to justify, for it is extremely unlikely that all other sectional price levels would be unaffected by a shift in the import price level. Thus, even if an accurate estimate of the foreign elasticity and of the domestic marginal propensity were available, any prediction formed on the basis of them could be seriously misleading. Furthermore, even in the two-good case postulated by Metzler, a partial interpretation of E'_{11} as the foreign elasticity of demand for imports must assume also that all supply reactions are negligible - an assumption which it is difficult to sustain having regard to the problem studied. Nor can the total interpretation (which measures the change in quantity due to the change in price when all of the repercussions of general equilibrium adjustment have worked themselves out) be considered as satisfactory, for this concept is not independent of the problem under review. A total elasticity of foreign demand for imports assessed after a devaluation is an entirely different concept from a total elasticity of foreign demand for imports measured ex post facto in the event of a tariff increase. Both measure the ultimate response of import demand to changing and different sets of circumstances. To use one for the prediction of the other normally would be quite wrong. Such elasticities are of historical value only. Nor, if one desired to predict the effect of a tariff on the domestic price ratio of the tariff-imposing country, could we use a total elasticity describing the previous response of the commodity concerned to a tariff change, for it is only reasonable to assume that autonomous influences, if not the previous tariff itself, would have affected the positions of supply and demand schedules and the level of income throughout the economy.

Only a prior knowledge of the partial elasticity components of the total elasticity affords a sufficient basis for prediction, these components being independent of the particular problem studied - measuring the rate of change in a variable in response to a price change, all other prices and income constant. This definition remains the same whatever the problem. We turn, therefore, to an analysis of the components of the terms of equation (10.5).

First, let us differentiate the set of equilibrium equations (2.5) to (6.5) making use of the fact that in a perfectly competitive world $\Sigma p_i dO_i = 0$. Writing $X_{ij} = \frac{\partial X_i}{\partial p_j}$ and $M_i = \frac{\partial X_i}{\partial M}$, we have

$$dx_2 = (X_{21}-O_{21})dp_1 + (X_{22}-O_{22})dp_2 + (X_{23}-O_{23})dp_3 + M_2 dM \quad \ldots \quad (12.5)$$

$$dx_2 = -(X'_{21}-O'_{21})dp_1 - (X'_{23}-O'_{23})dp'_3 - M'_2 dM' \quad \ldots \quad (13.5)$$

$$o = (X_{31}-O_{31})dp_1 + (X_{32}-O_{32})dp_2 + (X_{33}-O_{33})dp_3 + M_3 dM \quad \ldots \quad (14.5)$$

$$o = (X'_{31}-O'_{31})dp_1 + (X'_{33}-O'_{33})dp'_3 + M'_3 dM' \quad \ldots \quad (15.5)$$

$$dM = O_1 dp_1 + O_2 dp_2 + O_3 dp_3 + (p_2-1)dx_2 + x_2 dp_2 \quad \ldots \quad (16.5)$$

$$dM' = O'_1 dp_1 + O'_3 dp'_3 \quad \ldots \quad (17.5)$$

Next,

i. substitute (16.5) and(17.5) into the remaining four equations;

ii. substitute into the results for $X_{ij}-O_{ij}=K_{ij}-X_j M_i$, where K_{ij} is the aggregate demand-supply substitution effect;

iii. for the terms (X_i-O_i) which occur after these substitutions, substitute the appropriate x_1, x_2 or zero as indicated by the equilibrium equations; and

iv. cancel and collect terms wherever possible.

Then,

$$(K_{21}+x_1 M_2)dp_1 + K_{23}dp_3 + [M_2(p_2-1)-1]dx_2 = -K_{22}dp_2 \quad \ldots \quad (18.5)$$

$$(K'_{21}-x_1 M'_2)dp_1 + K'_{23}dp'_3 + dx_2 = 0 \quad \ldots \quad (19.5)$$

$$(K_{31}+x_1M_3)dp_1 + K_{33}dp_3 + M_3(p_2-1)dx_2 = -K_{32}dp_2 \quad \ldots \quad (20.5)$$

$$(K'_{31}-x_1M'_3)dp_1 + K'_{33}dp'_3 = 0 \quad \ldots \quad (21.5)$$

Our measure for the effect of a tariff upon the tariff-imposing country's domestic traded goods price ratio, $\frac{p_1}{p_2}$, is given in equation (9.5):

$$\frac{d(p_1/p_2)}{p_1/p_2}\frac{p_2}{dp_2} = p_2\frac{dp_1}{dp_2} - 1$$

where dp_2/p_2 is the proportionate change in the tariff.

Or,

$$\frac{d(p_2/p_1)}{p_2/p_1}\frac{p_2}{dp_2} = 1 - p_2\frac{dp_1}{dp_2} \quad \ldots \quad (22.5\ )$$

Solving for $\frac{dp_1}{dp_2}$ we obtain, after rearranging the order of the equations,

$$\frac{dp_1}{dp_2} = \frac{-\begin{vmatrix} K_{22} & K_{23} & 0 & M_2(p_2-1)-1 \\ K_{32} & K_{33} & 0 & M_3(p_2-1) \\ 0 & 0 & K'_{23} & 1 \\ 0 & 0 & K'_{33} & 0 \end{vmatrix} \; (= |A|)}{\begin{vmatrix} K_{21}+x_1M_2 & K_{23} & 0 & M_2(p_2-1)-1 \\ K_{31}+x_1M_3 & K_{33} & 0 & M_3(p_2-1) \\ K'_{21}-x_1M'_2 & 0 & K'_{23} & 1 \\ K'_{31}-x_1M'_3 & 0 & K'_{33} & 0 \end{vmatrix} \; (= |B|)} \quad \ldots \quad (23.5a)$$

A Laplacian expansion of $|A|$ from the first two columns gives

$$|A| = K'_{33}\begin{vmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{vmatrix}$$

A similar expansion from the first two rows of $|B|$, after columns two and three have been added to column one and use has been made of the fact that the $\Sigma p_j K_{ij}=0$, gives

$$|B| = -K'_{33}\begin{vmatrix} -p_2K_{22}+x_1M_2 & K_{23} \\ -p_2K_{32}+x_1M_3 & K_{33} \end{vmatrix} - \begin{vmatrix} K_{23} & M_2(p_2-1)-1 \\ K_{33} & M_3(p_2-1) \end{vmatrix}\begin{vmatrix} -K'_{22}-x_1M'_2 & K'_{23} \\ -K'_{32}-x_1M'_3 & K'_{33} \end{vmatrix}$$

Dividing top and bottom by $K_{33}.K'_{33}$

$$\frac{dp_1}{dp_2} = \frac{\dfrac{\begin{vmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{vmatrix}}{K_{33}}}{p_2\dfrac{\begin{vmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{vmatrix}}{K_{33}} - x_1\dfrac{\begin{vmatrix} M_2 & K_{23} \\ M_3 & K_{33} \end{vmatrix}}{K_{33}} + \dfrac{\begin{vmatrix} K_{23} & M_2(p_2-1)-1 \\ K_{33} & M_3(p_2-1) \end{vmatrix}}{K_{33}}\left[\dfrac{\begin{vmatrix} K'_{22} & K'_{23} \\ K'_{32} & K'_{33} \end{vmatrix}}{K'_{33}} + x_1\dfrac{\begin{vmatrix} M'_2 & K'_{23} \\ M'_3 & K'_{33} \end{vmatrix}}{K'_{33}}\right]} \quad \ldots \quad (23.5)$$

Let us pause for a moment to establish two propositions which will enable us to convert (23.5) into a more convenient form.

From the theory of demand we know, in a four-commodity world in which each country consumes and produces three commodities, that

$$p_iM_i + p_kM_k + p_jM_j = 1;$$

that

$$p_iK_{ii} + p_kK_{ik} + p_jK_{ij} = 0 \ ,$$

$$p_iK_{ki} + p_kK_{kk} + p_jK_{kj} = 0 \ ,$$

$$p_iK_{ji} + p_kK_{jk} + p_jK_{jj} = 0 \ ,$$

and that

$$K_{ij} = K_{ji} \qquad (i,j,k=1,2,3).$$

As long as prices are unity initially it can be shown easily by using these relationships that

$$\text{i.} \quad \frac{\begin{vmatrix} M_i & K_{ij} \\ M_j & K_{jj} \end{vmatrix}}{K_{jj}} = -\frac{\begin{vmatrix} M_k-1 & K_{kj} \\ M_j & K_{jj} \end{vmatrix}}{K_{jj}}$$

and

$$\text{ii.} \quad \frac{\begin{vmatrix} K_{ii} & K_{ij} \\ K_{ji} & K_{jj} \end{vmatrix}}{K_{jj}} = \frac{\begin{vmatrix} K_{kk} & K_{kj} \\ K_{jk} & K_{jj} \end{vmatrix}}{K_{jj}}$$

Making use of these results in (23.5)

$$\frac{dp_1}{dp_2} = \frac{\dfrac{\begin{vmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{vmatrix}}{K_{33}}}{p_2\dfrac{\begin{vmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{vmatrix}}{K_{33}} - \dfrac{x_1\begin{vmatrix} M_2 & K_{23} \\ M_3 & K_{33} \end{vmatrix}}{K_{33}} + \dfrac{\begin{vmatrix} K_{23} & M_2(p_2-1)-1 \\ K_{33} & M_3(p_2-1) \end{vmatrix}}{K_{33}}\left[\dfrac{\begin{vmatrix} K'_{11} & K'_{13} \\ K'_{31} & K'_{33} \end{vmatrix}}{K'_{33}} - \dfrac{x_1\begin{vmatrix} M'_1-1 & K'_{31} \\ M'_3 & K'_{33} \end{vmatrix}}{K'_{33}}\right]}$$

$$= \frac{\Phi_{23}}{p_2\Phi_{23} + x_1\phi_{23} + \gamma(\Phi'_{13} + x_1\phi'_{13} + x_1)} \qquad \ldots \qquad (24.5)$$

where

$$\Phi_{ij} = K_{ii} - K_{ij}\frac{K_{ji}}{K_{jj}}$$

$$\phi_{ij} = -M_i + M_j\frac{K_{ji}}{K_{jj}}$$

and

$$\gamma = 1 - (M_2 - M_3\frac{K_{23}}{K_{33}})(p_2-1) \ .$$

On conversion to elasticities and marginal propensities to spend (remembering that $p_2 \neq 1$) equation (24.5) reads:

$$\frac{dp_1}{dp_2} = \frac{1}{p_2}\ \frac{\Psi_{23}}{1 + \Psi_{23} + \psi_{23} + \lambda(\Psi'_{13} + \psi'_{13})} \qquad \ldots \qquad (25.5)$$

where

$$\Psi_{ij} = \frac{X_i}{x_i}\sigma_{ii} - \frac{O_i}{x_i}S_{ii} + \left(\frac{X_i}{x_i}\sigma_{ij} - \frac{O_i}{x_i}S_{ij}\right)\left(\frac{\sigma_{ji} - S_{ji}}{\sigma_{jj} - S_{jj}}\right)$$

$$\psi_{ij} = -C_i + C_j\left(\frac{\sigma_{ji} - S_{ji}}{\sigma_{jj} - S_{jj}}\right)$$

and

$$\lambda = 1 + \psi_{23}\left(1 - \frac{p'_2}{p_2}\right)$$

Substituting into (22.5) for this result we obtain

$$\frac{d(p_2/p_1)}{p_2/p_1}\frac{p_2}{dp_2} = 1 - \frac{\psi_{23}}{1 + \psi_{23} + \psi_{23} + \lambda(\psi'_{13}+\psi'_{13})}$$

$$= \frac{1 + \psi_{23} + \lambda(\psi'_{13}+\psi'_{13})}{1 + \psi_{23} + \psi_{23} + \lambda(\psi'_{13}+\psi'_{13})} \quad . . . \qquad (26.5)$$

which is an expression for the rate of change in the domestic price ratio p_2/p_1 due to a tariff.

By setting all terms which involve the non-traded commodity to zero in (26.5), an analogous two-commodity criterion is obtained:

$$\frac{d(p_2/p_1)}{p_2/p_1}\frac{p_2}{dp_2} = \frac{1 + \lambda(\frac{X'_1}{x_1}\sigma'_{11} - C'_1 - \frac{O'_1}{x_1}S'_{11}) - C_2}{1 + \lambda(\frac{X'_1}{x_1}\sigma'_{11} - C'_1 - \frac{O'_1}{x_1}S'_{11}) + (\frac{X_2}{x_2}\sigma_{22} - C_2 - \frac{O_2}{x_2}S_{22})} \quad . . . \qquad (27.5)$$

where $\lambda = 1 - C_2 + C_2\frac{p'_2}{p_2}$.

D. TWO-COMMODITY ANALYSIS

In this section the arguments for and against the appearance of Metzler's perverse case are examined with respect to a two-commodity world. First, his argument is examined and rejected on both *a priori* and probability grounds. Next, for the benefit of those readers who will not accept probability type arguments, an attempt is made to test empirically the conclusion reached by Metzler concerning the Australian case. Among other things, this emphasizes the practical difficulty of testing empirically theorems derived from two-commodity studies and leads to the analysis, in the next section, of the more complex, four-commodity result in which non-traded goods are included explicitly.

Consider the equation (27.5). As the proportionate change in p_2/p'_2 is positive for a tariff increase, a positive movement in p_2/p_1, p_2 rising relative to p_1, means that both sides of equation (27.5) must be positive. The absolute change in the domestic price ratio can be obtained by multiplying through by the proportionate change in p_2/p'_2.

As in other applications of the model, our international system is assumed to be stable. The only complication introduced by the tariff is the term λ whose influence upon the sign of the denominator can be neglected because, except for a very large tariff, it approximates to unity in magnitude. Since this term arises because of the assumption of a prior tariff, and since Metzler assumed free trade initially, its effect is ignored in the following comparison of the two formulae.

Given that the stability requirement is fulfilled, the direction of the movement in the domestic price ratio depends upon the sign of the numerator. An adverse protective effect, p_1 rising relative to p_2, requires that

$$- \frac{X_1'}{x_1} \sigma_{ii}' + \frac{O_1'}{x_1} S_{11}' + C_1' < 1 - C_2 .$$

It can be seen that the left-hand side of this condition is the breakdown of the total interpretation of Metzler's elasticity of demand for imports and that it comprises, in a two-good model, the sum of two substitution elasticities and a marginal propensity to spend on importables. Using the fact that the sum of the home country's marginal propensities to spend must equal unity, the criterion for the case in which the tariff fails to protect is

$$- \frac{X_i'}{x_1} \sigma_{11}' + \frac{O_1'}{x_1} S_{11}' < C_1 - C_1' \quad . \; . \; . \qquad (27.5a)$$

For purposes of discussion, this requirement is considered most conveniently in two stages: the requirement that the domestic marginal propensity to spend on exportables exceeds the foreign, and the requirement that the excess be larger than the sum of the foreign substitution terms.

The first requirement, which has been investigated exhaustively in the recent literature on the transfer problem [35;36;37], is the traditional one for a transfer (e.g. a reparations payment) to turn the terms of trade against the paying country. The outcome of that investigation has been the conclusion that in a two-good model in which tariffs and transport costs are absent initially, there is no presumption that one marginal propensity will exceed the other; in conjunction with the fact that the foreign substitution term must be positive, this would argue on a priori probability against the appearance of the perverse case.

The second requirement seems unlikely to be met for three reasons:

1. the difference between the marginal propensities must be a fraction, whereas the substitution elasticities can range from zero to infinity;
2. the weights attached to the substitution terms must exceed unity in the case of the demand elasticity and may do so in the case of the supply elasticity (particularly as the foreign country represents the rest of the world);
3. if the foreign substitution terms are small, high complementarity and joint supply are indicated and it is improbable, in a two-commodity world, that these conditions would not extend to the other country. In turn, this implies a similarity of tastes in each country which reduces the difference between the marginal propensities.

The above analysis would seem to tell heavily against the likelihood of the perverse case appearing, yet this is not entirely satisfactory because we are arguing in ignorance of the real magnitudes involved. Though the odds against the Metzler effect appearing in practice are exceedingly high, the facts in a particular case could be as required: low substitutability, and a preference for its own exportables by the domestic country. We shall digress, therefore, to examine the available facts in the Australian case.

Extensive use is made of the survey of international trade propensities and elasticities compiled recently by Hong Seng Cheng [38]. As this survey does not pretend to be exhaustive, it is possible that some relevant estimates are excluded.

For the perverse effect to operate, the rest of the world's substitution elasticities of supply of and demand for Australian exportables together must be less than the Australian marginal propensity to spend on exportables minus the foreign marginal propensity to spend on the same good. Table Three sets out the available data concerning two of these parameters, no estimates having been found for the rest of the world's substitution elasticity of supply of Australian exportables nor for its marginal propensity to spend on these commodities.

TABLE THREE

Estimates	Time Period	Australian Marginal Propensity to Import	World Substitution Elasticity of Demand for Australian Exports
Polak [2]	1920's	0.49	-0.48
	1930's	0.23	-
Tse Chun Chang [39]	1924-38	0.35	-0.66
Clark and Crawford [44]	1920's	0.21	-
	1930's	0.25	-
Tinbergen[40]	1924-37	-	-1.06*
Horner [41]	1936-38	-	-2.2 (wool) 64% -5.9 (wheat) of Australian -3.2 (butter) Exports
Brown [43]	1954-58	-	-1.69*

* As explained in the text, these estimates are derived from the work of Tinbergen and Brown who have no responsibility for this use, or misuse, of their original coefficients.

In our assessment of the probable sizes of the relevant marginal propensities we must confront the formidable fact that marginal propensities to spend on Australian exportables, not exports, are required. Any attempt to compute the parameters of the criterion, which holds strictly for a two-commodity world, depends among other things on the extent to which non-traded commodities are classified as exportables or importables. On a priori grounds, one might argue that the Australian marginal propensity to spend on its own export good is low because the majority of its exports consists of the four commodities: wool, wheat, butter and meat. Yet it was to avoid such an argument from ignorance that we turned to the available facts in the Australian case.

Fortunately, data is available concerning the Australian marginal propensity to spend on imports and, as we are arguing against the appearance of the Metzler effect in practice, it appears permissible to accept the most reliable estimate of the marginal propensity to spend on imports as the lower limit to the marginal propensity to spend on importables.

Of the estimates given, the first figure of Polak is suspect. Had he excluded the 1930/31 figures from his calculations, the value of the propensity would have fallen to approximately 0.29 [45,14]. We select the medial value of 0.25 as the lower limit to the Australian marginal propensity to spend on importables, which yields an upper limit of 0.75 for the Australian marginal propensity to spend on exportables. Polak [2,156, table facing] has estimated for the inter-war period a rest-of-the-world marginal propensity to allocate export expenditure on Australian products of 0.015. It follows from the composition of the Australian 'export basket' that the rest-of-the-world's marginal propensity to spend its income on the type of goods which Australia exports must be negligible. For practical purposes it is assumed zero.

The difference between the marginal propensities, therefore, has an upper limit of 0.75, though one would expect its actual value to be considerably below this figure.

Because of the paucity of data concerning the marginal propensities, an argument against the practical appearance of the Metzler effect must depend upon reliable estimates of the aggregate substitution term. Unfortunately, no data are available concerning that part of it which comprises the rest-of-the-world's supply elasticity and there is no alternative, in our ignorance, but to allocate it an assumed lower limit of zero.

Of the estimated substitution elasticities of demand given in Table Three, those of Polak and Chang have attracted the most criticism. A review of the 'elasticity pessimism' of the thirties, however, would serve no useful purpose. Instead, the reader is referred to Orcutt's masterly article in which he provides conclusive evidence that early empirical studies often severely biased their estimates toward zero [46,118]. Typical of the general awareness of this bias, is a recent comment by Warren L. Smith: "It has become increasingly apparent that studies of the international adjustment mechanism utilizing time series data for the 1930's...resulted in too pessimistic a view of the price sensitivity of both imports and exports" [47,S 127]. Polak himself placed little confidence in either his own or Chang's estimates. Of his own he wrote, "The weakness of many of the price coefficients found must be acknowledged... Conclusions on the effects of changes in relative prices...cannot safely be drawn from the data presented." [2,65]. His comment on the findings of Chang was that "recent studies...make it probable that Chang's findings cannot be taken as highly reliable estimates...and may be biased in the direction of zero." [48,20].

The estimate appearing opposite Tinbergen's name was originally a substitution elasticity, defined as the change in the ratio of the volume of Australian to total exports caused by a change in the export/import price ratio. It has been turned into an export elasticity of demand by the application of an ingenious method first used by Harberger [42,513-16]. Though Polak severely criticized Tinbergen's results by showing that income changes could account for a large part of what was apparently pure substitution effect, he also admitted that due allowance for income effects in countries exporting primary produce could increase the estimate because 1) the income elasticity of demand for food and raw materials is less than for manufactured commodities, and 2) the elasticity of supply of foodstuffs and raw materials is smaller than for manufactured goods [48,19].

Nevertheless, with no evidence to support it, Tinbergen's estimate provides a precarious basis for prediction.

The work of Horner represents a more realistic attempt to come to grips with the problems involved. Harberger was of the opinion that Horner's technique deserved a much wider recognition and, since then, V.W. Maluch has applied a similar technique to obtain estimates of Canadian price elasticities of export demand [50].[3] Working from the fact that demand for Australian exports is the difference between the rest of the world's demand for and supply of exportables, Horner estimated the separate supply and demand elasticities for the rest of the world. From these the price elasticity of demand for Australian exports was obtained. The reason for his method yielding individual commodity estimates so much higher than the aggregative estimates of Chang and Polak is due primarily to his explicit acknowledgement of, and provision for, the fact that where

> the exporting country supplies a more than negligible part of its export market, it can be readily seen that the price elasticity of demand for its own product will be greater than the price elasticity of demand on the export market for the commodity in general by an amount dependent on the proportion of that market it supplies. [41,326].

This fact is also reflected in the quantity weights which appear before the elasticities in our formula.

Evidence in support of one of Horner's estimates is provided by Stone [28] who

[3] It is noteworthy that the estimates he has derived from post-war data suggest far higher elasticities for Canadian exports (though admittedly only for a 35% sample) than indicated by the work of Chang.

obtained a price elasticity of demand of -0.41 for butter in the United Kingdom in the inter-war period. When this is weighted by the quantity consumed in the export market divided by the quantity exported by Australia for the three years ending 1938, an estimate similar to Horner's is obtained.

Finally, Table Four summarizes a crude attempt to compute a lower limit to the post-war aggregate export elasticity of demand for Australian exports. Extensive use is made of estimates given in a recent article by J.A.C. Brown [43] and of the technique established by Horner.

TABLE FOUR. LOWER LIMIT FOR A DERIVED AUSTRALIAN ELASTICITY OF EXPORT DEMAND

1	2	3	4	5	6	7	8	
Commodity	Time Period	U.K. Substitution Elasticity of Demand	Ratio of U.K. Consumption to Australian Exports	Australian Export Demand Elasticity	% of Australian Exports sold in UK	% Share of total Australian Exports	Weighted Elasticities of Australian Export Demand[4]	
Beef and Veal	1954-58	-1.4	8.07	11.29	81	3.6	-.41	(-.65)
Lamb and Mutton	"	-1.7	10.40	17.68	80	1.2	-.22	(-.33)
Butter	"	-0.59	5.51	3.25	85	2.9	-.09	(-.14)
Wheat	"		2.52[1]	2.87[2]		7.7	-.22	(-.35)
Wool	"		3.39[1]	1.59[2]		47.3	-.75	(-1.19)
Total other	"	0[3]		0[3]		37.3	0[3]	
Total						100.0	-1.69[5]	(-2.66)

[1] Ratio of rest-of-the-world's (i.e. the free world market) consumption to Australian exports for the period 1936-38.

[2] Price elasticity of demand computed by Horner for the period 1936-38.

[3] Assumed.

[4] Column (8) is derived by multiplying column (5) by column (7) and dividing by 100.

[5] The lower limit to the world export elasticity of demand for Australian exports.

All figures relate to 1954-58 data except the price elasticities of export demand for wheat and wool which are quoted directly from Horner's results. It is probable that these have altered, though in the case of wheat there is no apparent reason why such a shift would be significant in either direction. The wool estimate, on the other hand, if it were to be brought up to date, would reflect almost certainly the appearance of synthetic substitutes in the textile field in the post-war period. It would appear to be a safe lower limit.

For the first three products the world market is assumed to be the United Kingdom. Hence, for these commodities, the estimates in column five were derived by multiplying column three by column four for reasons indicated above. Columns six and seven are self explanatory but the last column requires some comment. To obtain a single lower limit for an export elasticity of demand some method of aggregation was necessary. This was done by

1. Assuming that the remaining 37.3% of exports had a zero demand elasticity
2. Multiplying column five by column seven and dividing by one hundred.[4]

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[4] This method of obtaining an aggregate elasticity depends on the assumption that all export prices move proportionately to the selected export price index.

Apart from the proportionality assumption, however, the method is open to the more serious objection that no account is taken of cross elasticities. Obviously, where two commodities are close substitutes and have high elasticities of demand the two facts are not unrelated. For instance, if the elasticities of demand for both lamb and beef were known to be large, it would be tempting to conclude that the aggregate elasticity of demand for meat in general was also large. In fact, the high elasticity of demand for beef could arise from its high substitutability for lamb, i.e. the cross elasticity of demand for beef with respect to the price of lamb could be high and vice versa. In such a case the combined elasticity would be less than either of the individual estimates. The true elasticity would be given by a determinant comprised of the different elasticities involved.

Despite the above objections it should be noted that the only close substitutes among the export commodities selected are lamb and mutton, and beef for which the cross elasticity of demand given by Brown [43] is only of the order of 0.3. This would not seriously affect our estimates.

The aggregated estimate of -1.69 can be taken as a lower limit to the elasticity of world demand for Australian exports. One might contend that this elasticity would be considerably larger because

1. it is certain that there would be some elasticity of supply, especially where long term elasticities were involved. In the case of wool and wheat Horner assumed these to be 0.56 and -2.33 respectively. However, as we are concerned with only a lower limit, they are best set at zero;
2. as stated above, it is probable that Horner's wool estimate would be larger if more recent data were available;
3. in Table Four the United Kingdom has been considered as the world market for the first three commodities. If this simplifying assumption is relaxed, the size of the free world market for these commodities, especially the meat products, would be increased considerably. This would increase the size of the weights and, accordingly, the elasticities;
4. finally, it is apparent that the world's demand for the remaining 37% of Australian exports would yield some elasticity coefficient. As a portion of them are manufactures, it would be surprising if their average elasticity were not greater than that of the agricultural group. If the weighted elasticities are derived for only the five products listed, i.e. if in Table Four column five is multiplied by column seven and divided by 63, the elasticity estimates appearing in the brackets are obtained and these sum to an elasticity of export demand of -2.66 for these products.

Table Five below summarizes our results. In each case, if the sum of the substitution elasticity terms appearing in column one is greater than the difference between the Australian and the rest-of-the-world's marginal propensities to consume exportables (column two) the perverse effect could not operate (column three).

Table Five

1 Estimated World Substitution Elasticities for Australian Exportables: Demand			Supply		2 Marginal Propensities to Consume Australian Exportables: Aust.		World's	3 Estimates Favour
-0.48	(Polak)	+	0	is less than	0.75	-	0	Metzler
-0.66	(Chang)	+	0	is less than	0.75	-	0	Metzler
-1.06	(Tinbergen)	+	0	is greater than	0.75	-	0	Orthodox
-2.20	(Horner)*	+	0	is greater than	0.75	-	0	Orthodox
-1.69	(Brown)	+	0	is greater than	0.75	-	0	Orthodox

* Horner's lowest individual commodity estimate.

Only the estimates of Polak and Chang indicate that a tariff might fail to protect the import-competing industries of Australia. Yet these estimates are the most dubious, the fact that they contain a bias toward zero being no longer disputed. Moreover, 0.75 is an extreme, even ridiculously high upper limit for the Australian marginal propensity to spend on exportables, which is certain to be reduced considerably in practice. Further, one must remember that the rest of the world's elasticity of supply of exportables and marginal propensity to consume these goods, in the absence of quantitative knowledge, have been set at zero. These facts, in conjunction with the evidence of our other estimates and the earlier *a priori* argument, suggest that the possibility of the appearance in Australia of the Metzler effect is remote.

E. FOUR-COMMODITY ANALYSIS

Not only does our empirical analysis support the *a priori* conclusion that the Metzler effect is unlikely to appear in practice, but also it serves to emphasize the inappropriateness of two-commodity analysis in a multi-commodity world. Apart from the fact that possibly vital relationships between traded and non-traded commodities are obscured, it is difficult to see what meaning can be given to the important concept of the marginal propensity to spend on importables or exportables. The purpose of this section is to derive a result for the effect of a tariff on the domestic traded goods price ratio when non-traded goods are included in the model. It is emphasized, however, that this is merely a step towards reality; the problems that remain of aggregation and of economic dynamics are considerable.

Let us now examine the multi-commodity criterion given in equation (26.5):

$$\frac{d(p_2/p_1)}{p_2/p_1}\frac{p_2}{dp_2} = \frac{1 + \psi_{23} + \lambda(\Psi'_{13}+\psi'_{13})}{1 + \Psi_{23} + \psi_{23} + \lambda(\Psi'_{13}+\psi'_{13})} \quad .$$

The reader will recall that the left-hand side of (26.5) can be interpreted as the rate of change in the domestic price ratio p_2/p_1 due to the tariff and that, as dp_2/p_2 is positive for a tariff increase, a positive movement in $d(p_2/p_1)/(p_2/p_1)$ requires both sides of (26.5) to be positive. Or, if the protective effect of a tariff is to be cancelled out exactly by a favourable terms of trade effect, both sides must be equal to zero.

As in the two-good case it is assumed that the international system is stable. This means that the denominator is negative. The direction of the change in the domestic traded goods price ratio p_2/p_1 is dependent, therefore, upon the sign of the numerator. Ignore, for the moment, the effect of λ which occurs because we have assumed a prior tariff to exist and which, in any case, would approximate to unity. If the perverse case is to occur the numerator must be positive, which requires

$$-(\Psi'_{13} + \psi'_{13}) < 1 + \psi_{23}$$

and, since $\psi'_{13}+\psi'_{23} = -1$, this also requires

$$-\Psi'_{13} < \psi'_{13} - \psi_{13} \quad \ldots \quad (27.5b)$$

For purposes of discussion, this requirement is considered most conveniently in two stages: the requirement that the weighted sum of the domestic marginal propensities to spend on exportables and non-traded goods exceed the corresponding weighted sum of the foreign propensities; and the requirement that the excess be greater than the sum of the quantity weighted foreign substitution terms.

The first requirement is identical to that needed, in the case of a transfer, to turn the terms of trade against the paying country when non-traded goods are allowed for explicitly (see page 71 above). Generally, ψ_{13} is the more likely to exceed ψ'_{13} the greater in the domestic and the smaller in the foreign country are the respective marginal propensities to consume commodity one (domestic exportables) and commodity three (non-traded goods). Ignore, for the moment, all price substitution effects of the tariff and concentrate upon the induced shift in expenditure from the foreign to the domestic country. To the extent that C_1 plus C_3 exceeds C'_1 plus C'_3, C_2 will be less than C'_2. This means that the expenditure-reducing country (the foreign country) reduces its consumption of commodity two by a greater amount than the expenditure-increasing country (the domestic country) increases its consumption of the same commodity. This will tend to aggravate further the excess supply of foreign exportables (commodity two) due to the tariff and, consequently, to induce a further improvement in the domestic country's terms of trade.

Unfortunately, one cannot neglect the effects of the substitution terms. Though it is conceivable that the increase in expenditure in the domestic country might scarcely raise the prices of commodity one (exportables) and two (importables) the excess demand for them at constant prices being eliminated, in the first case, by a diversion of exports to the home market, and in the second case, by an increased flow of imports - it is impossible that the increased expenditure on commodity three could fail to increase its price unless complete satiety existed or there were infinite substitution possibilities between the non-traded and one of the other commodities (in which case they would cease to be separate commodities). Similarly, in the foreign country, the reduction in expenditure must reduce the price of non-traded commodities.

The reader will recall that our marginal propensities to spend measure a country's rate of change in expenditure upon a commodity due to a change in income, <u>all prices being held constant</u>. We have shown, however, that the price of non-traded goods must change relative to those of the traded goods, thus inducing a change in the magnitudes of the marginal propensities to spend. This fact explains the presence of the substitution elasticity weights that appear before the marginal propensity terms and leads us to the further conclusion that ψ_{13} will be the more likely to exceed ψ'_{13} the greater in both countries are the substitution possibilities between each country's exportables and non-traded goods relative to those existing between each country's importables and non-traded commodities. Assume, for example, that these conditions do exist. In the domestic country consumers will substitute out of non-traded and into exportable rather than importable goods, thus increasing the marginal propensity to spend on exportables rather than the marginal propensity to spend on importables. At the same time, resources will flow to the non-traded goods industry from the exportable rather than from the import-competing sector, thereby increasing the price of exportables. Conversely, in the foreign country, where the price of non-traded commodities has fallen, the marginal propensity to spend on foreign exportables would tend to decrease more than the marginal propensity to spend on importables and resources would flow to the exportable rather than to the import-competing sector. The net effect would be an aggravated world excess supply of foreign exportables and an increase in the price adjustment necessary to restore equilibrium, i.e. an augmented favourable movement in the terms of trade of the taxing country.

The second requirement, that the excess of the domestic marginal propensity term over the foreign marginal propensity term should exceed the foreign coefficient of sensitivity (ψ'_{13}) is best considered in two stages. First, conditions for a low foreign coefficient of sensitivity would be ensured if:

1. the foreign own substitution elasticities of demand for and supply of importables were low. In this case, the magnitude of the other elasticities could not affect materially the size of ψ'_{13}. Thus the rise in the world price of foreign importables relative to foreign exportables as a result of the tariff would induce only a small transference of demand out of importables, and of resources into the import-competing industry. Both of these effects would increase the size of the movement in the terms of trade needed to restore equilibrium. At the same time, it is necessary that
2. the amounts of importables consumed and produced in the foreign country approximate in size to the volume of imports.

A priori considerations suggest that it is unlikely that the above two necessary conditions will be fulfilled in practice. When σ'_{11} and S'_{11} are small, it can be shown that all three classes of goods must be complementary to one another and subject to joint supply, a most unlikely event in a three-commodity system. For instance, if σ'_{11} is very small it means that commodity one cannot be substituted for either goods two or three, i.e. commodities one, two, and three are all consumed in fixed proportions. (An exception to this occurs where the demand for commodity one is completely satiated; an improbable situation in a three-commodity system.) Similarly, if S'_{11} is small, commodities one, two and three will be supplied jointly. Now if such a strong pattern of complementarity and joint supply exists in the one country in a four-commodity world, it is exceedingly improbable that these conditions would not be found also in the other country. Yet, if this were the case, it is just the situation in which $1+\psi_{23}+\psi'_{13}$ would tend to unity.[5] Thus the smaller the foreign coefficient of sensitivity, the smaller is the difference between the domestic and the foreign marginal propensity terms. Moreover, given that substitution possibilities are small, there is still the effect of the quantity weights X'_1 and O'_1, which are multiples of x_1, to be considered and, as the foreign country represents the rest of the world, it is improbable that these would be other than large, save where the domestic country's production of exportables constitutes an exceedingly high proportion of total world output of the commodity.

Nevertheless, though the practical possibility of the appearance of the perverse effect seems remote, it could be less improbable than in the two-good case because, at the critical point, the size of the terms of trade effect needed to restore equilibrium is increased by the addition of non-traded commodities in the foreign country,[6] the extent

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[5] This may be explained by looking at the two-good analogue of the situation, i.e. where $1+\psi'_{13}+\psi_{23}$ is equivalent to $1-C'_1-C_2$, which, since $C_1+C_2=1$ in each country in a two-good model, equals C'_2-C_2. With identical tastes, $C'_2=C_2$; and so $C'_2-C_2=0$, and, similarly, $1-C_1-C_2=0$. In the four-good case $C_1+C_2+C_3=1$ and so $-\psi_{13}-\psi_{23}=1$; with demand and production conditions identical $-\psi'_{13}=-\psi_{13}$ and, therefore, $1+\psi'_{13}+\psi_{23}=0$.

[6] The reader should note that this does not mean that the introduction of non-traded commodities will result in a net increase in the terms of trade movement. It will be shown in the next chapter that in the case of the domestic country, the introduction of non-traded goods would tend to reduce the magnitude of the terms of trade movement. The net effect would depend on many factors. At the critical position, however, where the effect of the tariff is just cancelled out, domestic substitution possibilities do not influence matters. This is borne out in equation (9.5) where, at the critical point, the rate of change in the terms of trade, dp_1/p_1, is equivalent to the rate of change in the tariff dp_2/p_2, the magnitude of the domestic coefficient of sensitivity having no effect whatsoever, though the domestic substitution possibilities do affect the domestic marginal propensity term (see p74 above). Generally, it can be said that the more $\psi'_{13}+\psi'_{13}+\psi_{23}$ tends to minus unity (and this is the more likely the smaller ψ'_{13}) the less is the effect of the domestic coefficient of sensitivity on the terms of trade. This point is discussed in the next chapter.

of this reduction depending upon the degree of substitutability between the traded commodities themselves and between each traded good and the non-traded commodity. In the foreign country, when there are only two goods, the rise in the price of importables relative to the price of exportables, diverts demand towards the exportable goods industry and diverts resources towards the import-competing industry, both of which effects reduce the excess supply of foreign exportables and the size of the shift in the terms of trade required to establish equilibrium. When non-traded goods are introduced, however, some of the transference of demand from the importable industry is absorbed in the non-traded goods sector, thus reducing the extent of the recovery in price of foreign exportables. Similarly, some of the flow of resources to the foreign import-competing industry, would be diverted to the non-traded goods sector, thereby enhancing the rise in price of foreign importables. Hence, the increase in the demand for and the decrease in the supply of foreign exportables would not be as noticeable as in the two-good case, with the result that the movement in the terms of trade would increase accordingly, increasing in turn the probability of the Metzler effect.

This leads us, secondly, to an interesting case in which the Metzler effect might operate despite high values for the foreign own substitution elasticities of demand for and supply of importables. The effect of these could be cancelled, or markedly reduced, by the presence of high foreign substitution possibilities in consumption and production between non-traded and importable goods relative to those between importable and exportable goods on the one hand, and to those between exportable and non-traded goods on the other. A rise in the price of foreign importables due to the excess demand for them resulting from the tariff, could see a transference of demand to the non-traded rather than to the exportable sector, while resources might flow to the import-competing industry from the non-traded rather than from the exportable sector. If these effects were sufficiently strong (ψ'_{13} being small despite high values for $(X'_1/x_1)\sigma'_{11}$ and $(O'_1/x_1)\ S'_{11}$) the adverse shift in the terms of trade of the foreign country would be augmented accordingly.

It should be noted, however, that one of the necessary conditions for a low foreign coefficient of sensitivity when the foreign own elasticities of demand for and supply of importables are large, is that non-traded goods should be relatively more substitutable in consumption and production with importables than with exportables, i.e. σ'_{31} and S'_{31} should be large relative to σ'_{32} and S'_{32} respectively. This condition, however, would increase the size of ψ'_{13} (see page 74) and, therefore, reduce the difference between the marginal propensity terms. Thus, even in this case, it appears extremely improbable that all of the necessary conditions for the appearance of the perverse case would be fulfilled simultaneously in practice.

6. TARIFFS AND THE TERMS OF TRADE WHEN NON-TRADED COMMODITIES ARE PRESENT

A. INTRODUCTION

Since the time of John Stuart Mill it has been generally accepted by economists that a tariff would improve the international terms of trade of the tariff-imposing country. This opinion is subject to the assumptions of the traditional neo-classical model from which the result derives, and when they are relaxed considerable disagreement exists in the literature. In the next three chapters, it is our intention to examine the effects of a tariff on the terms of trade when

1. non-traded commodities are introduced into the classical model;
2. the government is assumed to spend the tariff revenue, and private consumption is dependent upon the amount of this expenditure, government and private tastes differing;
3. there exist two representative individuals whose tastes and incomes differ, i.e. a disaggregated private sector.

In the last two cases the traditional conclusion regarding the direction of the terms of trade movement is modified, though on a priori grounds one would assume it to be favourable normally to the taxing country. Case (1) developed in this chapter, endorses the traditional result. It should be noted that until the question of an optimum tariff is considered the problem of equity, which involves the effects of redistributed income on welfare, is ignored. Our concern at present is with the direction of the shift in the terms of trade and not with its welfare implications.

B. THE ARGUMENT

The idea that a tariff might improve the taxing country's terms of trade was known to Ricardo [52,555] but was first formulated explicitly by John Stuart Mill [53,27].

Though Mill discounted its practical significance, this was because of real income losses due to decreased specialization and not, as in the case of Marshall later, due to the supposition that foreign elasticities would be so high as to make the improvement negligible [12,~~348~~]. The classicists argued that the tariff would increase the world supply of foreign exportables, thereby depressing the price and improving the foreign country's terms of trade. Subject to their assumptions, namely two countries and two commodities and identical government and private tastes, this conclusion has remained unchanged.

In a recent article, however, J. de V. Graaf claimed that the traditional conclusion is ambiguous when additional commodities are added to the model for

> In a multi-commodity world ... it does not seem possible to generalise about the direction of the movement in the terms of trade ... The crucial factors turn out to be the relations of complementarity and substitution existing between traded goods. They can turn the terms of trade in either direction [61,54].

In support of this contention, Graaf cites Mosak [24,65-7] whose conclusion, however, because it depends upon the manner in which the government spends the tariff revenue, does not prove that the classical proposition is invalid in a multi-commodity world. We turn, therefore, to an anlysis of this problem, proceeding from the traditional two-commodity case to a four-commodity model in which non-traded goods appear in each country.

C. TWO-COMMODITY ANALYSIS

The assumptions on which the classical model is based are identical with those given in chapter three, if due modifications are made for the presence of the non-traded commodities. The most important are: perfect competition and profit maximisation; two countries and two commodities; full employment without inflation; the balance of trade maintained in equilibrium by the price mechanism; zero transport costs; an initial tariff is levied by country one, but there are no tariffs in the foreign country; either the tariff revenue is redistributed by the government in the form of a subsidy to a single representative consumer, or government and private tastes are identical.

First, in this chapter, the traditional two-commodity result is examined. Equation (27.5) provides us with a ready-made criterion for the rate of change in the terms of trade of the tariff-imposing country (country 1) if we substitute for $\frac{d(p_2/p_1)}{p_2/p_1}$ from equation (22.5):

$$\frac{dp_1}{dp_2}\frac{p_1}{p_2} = \frac{\frac{X_2}{x_2}\sigma_{22} - \frac{O_2}{x_2}S_{22}}{1 + \lambda(\frac{X_1'}{x_1}\sigma_{11}' - C_1' - \frac{O_1'}{x_1}S_{11}') + \frac{X_2}{x_2}\sigma_{22} - C_2 - \frac{O_2}{x_2}S_{22}} \quad \ldots \quad (1.6)$$

where $\lambda = 1 + C_2(p_2'/p_2) - C_2$ and p_1 is the international price ratio (with zero transport costs and no tariffs in the foreign country and $p_2' = 1$). A favourable movement in the terms of trade means that p_1 (the foreign price of exportables) must rise relative to p_2' (the foreign price of importables). As the proportionate change in p_2'/p_2 is positive for a tariff increase, the left-hand side of equation (1.6) must be positive for a favourable movement in the terms of trade. In turn, this means that the numerator and denominator on the right-hand side must be of identical sign. Now, the numerator is negative unambiguously. Given that the international market is stable (that the denominator is negative) this means, therefore, that the terms of trade movement must be zero or favourable for the taxing country. It follows that the magnitude of the movement in the terms of trade would be the greater:

1. the larger are the domestic substitution possibilities both in consumption and production. Thus in the domestic country, a rise in the price of commodity two will see a large shift in demand to the exportable commodity and a heavy flow of resources from the exportable to the import-competing industry, both of which movements would increase the excess supply of commodity two and the favourable shift

in the terms of trade;

2. the larger are the marginal propensities to consume exportables in either country. In the foreign country, the decrease in expenditure would fall most heavily upon foreign exportables, thereby failing to relieve the excess supply of commodity two; in the domestic country, the increase in expenditure would be spent proportionately more upon good one, increasing both its price and the shift in the terms of trade;
3. the smaller are the foreign elasticities of demand and supply. This would reduce the transfer of demand from the import-competing industry whose product price has risen relatively due to the excess supply of foreign exportables, and decrease the flow of resources from the exportable to the import-competing industry. The adverse movement in the foreign country's terms of trade would be aggravated accordingly;
4. finally, the greater in the domestic country are the ratios of consumption and production of importables to imports and the smaller are the same ratios in the foreign country. In each case, this reflects the importance of the relative size of the market in determining the value of the elasticities.

Consider the effect of λ, the weight due to the assumption of initial tariffs. When a prior tariff exists, this term reflects the influence of the reduction in domestic imports upon tariff revenue and, consequently, on the magnitude of the shift in the terms of trade. To the extent that the increased domestic price of importables due to the tariff reduces the demand for them, tariff revenue levied at the initial rate will tend to fall. In turn, since we assume the redistribution of this tariff revenue to the private sector, this reduction in revenue receipts will reduce further the demand for importables, increasing the excess supply of them, and the subsequent shift in the terms of trade necessary to restore equilibrium to our international system. With no initial tariffs, λ becomes unity.

D. FOUR-COMMODITY ANALYSIS

Consider next the addition of non-traded commodities to the model. A slight rearrangement of equation (25.5) gives us the necessary criterion:

$$\frac{dp_1}{dp_2}\frac{p_2}{p_1} = \frac{\Psi_{23}}{1 + \lambda(\Psi'_{13}+\psi'_{13}) + (\Psi_{23}+\psi_{23})} \quad . . . \qquad (2.6)$$

in which λ, Ψ_{23}, Ψ'_{13}, ψ_{23} and ψ'_{13} are all defined as in equation (25.5) and in which the left-hand side is our expression for the rate of change in the terms of trade due to a tariff. Once more, assuming stability in the international market, it is evident that the movement in the terms of trade must be favourable to the taxing country, as the domestic coefficient of sensitivity (Ψ_{23}) is negative. ~~by the stability conditions.~~ This result is quite unambiguous, it supports the traditional, two-commodity conclusion while contradicting Graaf's statement that the result would be indeterminate because of complex substitution possibilities. The magnitude of the result, however, does depend upon the latter.

Of the conditions set out below for a large movement in the terms of trade of the tariff-imposing country, 1., 2., and 4. or 5. are both necessary and sufficient.

1. That the domestic own elasticities of demand for and supply of importables should be high. Reasons for this condition were outlined above in connection with the two-commodity result.
2. Additionally, in the taxing country, there must exist low substitution possibilities between non-traded commodities and importables relative to those existing between non-traded commodities and exportables, and high substitutability between the traded commodities. These conditions would ensure that the transfer of demand away from importables would be to the exportable good industry and not to the non-traded sector where its favourable influence on the terms of trade would be less noticeable. Furthermore, resources would be attracted to the import-competing

industry from the exportable rather than from the non-traded goods sector which would further increase the price of exportables, and at the same time add to the excess supply of domestic importables in the foreign market.

3. A further important, but not essential requirement for a large terms of trade effect is that the domestic ratios of consumption and production of importables to imports be high. This condition, which would be fulfilled if the domestic country consumed a large part of its production of importables, is a reflection of the influence of the relative size of the market upon the relevant elasticities.
4. The other necessary and essential condition for a large terms of trade effect is that the foreign own substitution elasticities should be very small. Reasons for this are given above in connection with the two-good model.
5. Should these own elasticities be large, however, their effects would be modified if the relative degree of competitiveness between non-traded and importable commodities exceeded considerably that existing between non-traded and exportable commodities. In this event, resources leaving the foreign country's exportable good industry would flow to the non-traded sector rather than to the import-competing sector, and similarly, purchasing power would transfer to exportables from the non-traded commodity rather than from the import-competing product. The net effect would be reflected in a small foreign coefficient of sensitivity.
6. As in the case of the domestic country, the quantity weights could play an important though not vital role in determining the size of the terms of trade movement. Generally, the smaller the ratio of foreign production of importables to imports, the greater would be the movement in the terms of trade.
7. Finally, the reader should note the effect of the marginal propensity terms, ψ_{23} and ψ'_{13} and of the tariff weight, λ. Their combined effect would not be important

in the case of a large terms of trade movement as their individual magnitudes cannot exceed unity unless exceptional demand and supply conditions exist. Ignoring λ, the movement in the terms of trade will be greater the smaller is the sum of ψ'_{13} and ψ_{23} which means

i. the greater is the foreign marginal propensity to consume exportables, C'_2, relative to the other foreign propensities, C'_1 and C'_3; and the greater is the domestic marginal propensity to consume exportables, C_1, relative to the other propensities, C_2 and C_3;

ii. the greater are the substitution possibilities between exportables and non-traded goods relative to those existing between importables and non-traded commodities.

The conditions upon the marginal propensities are explained: in the expenditure-reducing country (the foreign country) the reduction in purchasing power, given i. above, will fall most upon the exportable good industry while in the expenditure-increasing country (the domestic country) the increase in expenditure is concentrated similarly upon the exportable good industry. Both effects tend to increase the disparity in the international price ratio. The influence of the substitution weights upon the marginal propensity terms reflects the effect of a change in expenditure upon non-traded commodity prices and, via the latter, upon the size of the marginal propensities (see page 74 for a full discussion of this point).

8. Finally, it can be shown that the effect of the substitution weights in the λ term reflects similarly the influence of a change in expenditure upon non-traded commodity prices and, as a result, upon the propensities themselves. Apart from this, the effect of λ is analogous to its influence in the two-good case examined earlier.

D. CONCLUSIONS[1]

1. Our analysis, contrary to Graaf's, confirms the traditional conclusion that the terms of trade of the tariff-imposing country will improve when non-traded commodities are allowed for explicitly in the classical model.
2. At the same time, any assessment of the possible magnitude of the movement could not afford to ignore the vital role played by these non-traded commodities as they will always reduce the value of the coefficient of sensitivity to less than the value of the weighted own substitution elasticities of demand for and supply of the traded commodity concerned and in certain specifiable conditions they could reduce this value drastically.
3. Even so, the explicit recognition of the effects of the quantity weights and the breaking up of the elasticities suggests that the magnitudes of the coefficients of sensitivity need not be small.
4. The effect of the prior tariff is to increase, if only slightly, the magnitude of the favourable movement in the terms of trade.
5. Finally, the presence of non-traded commodities would affect the size of the marginal propensities depending upon the degree of competitiveness existing between importables and exportable commodities.

[1] Of the conclusions listed, 1. and 3. are contained in a joint paper submitted for publication recently [51].

7. GOVERNMENT EXPENDITURE, THE TERMS OF TRADE AND THE DOMESTIC PRICE RATIO

A. INTRODUCTION

This chapter examines the effect of a tariff upon the terms of trade and upon the domestic price ratio of the tariff-imposing country when the government spends the tariff revenue instead of either returning it as a subsidy to the private sector or remitting an equivalent amount of some other taxation. Because of the large number of variables involved a two-commodity model is developed. The sole justification for this is that it enables us to extend, if only slightly, our knowledge concerning the above problems. Obviously, the next desirable step would be to include a class of non-traded commodities.

Two cases are examined: first, where the government spends the tariff revenue and neither the amount nor the form of this expenditure is a datum in private utility functions; secondly, where the amount of government expenditure does affect individual utility functions and, therefore, private demand for imports.

Where private demand is independent of the amount of government expenditure, the traditional conclusion is that the tariff-imposing country's terms of trade will deteriorate if the government's marginal propensity to consume importables exceeds that of the private sector by more than the amount of the aggregate private supply-demand substitution elasticity for importables (considered as a total) [98, ; 64,67-8; 99,231].

Furthermore, should the government marginal propensity to consume exportables exceed the foreign aggregate supply-demand substitution elasticity for exportables the tariff could reduce the domestic price of importables relative to the domestic price of exportables [98, ; 99,247]. It is my intention to question the probability of the occurrence of these reversals in the normal terms of trade effect.

Throughout this chapter it is assumed that only two commodities are produced and consumed, though where private demand is influenced by government expenditure, this expenditure enters private utility functions as a third commodity.

Notation is the same as in previous applications of the model except that wherever confusion could arise, the public and private sectors are distinguished by the subscripts G and T respectively.

B. INDEPENDENCE OF PRIVATE DEMAND

1. Where the Government Purchases its Importables in the Foreign Market

The equilibrium equations of our model are:

$$x_1 = O_1 - X_{G1} - X_{T1} = X_1' - O_1' \quad \ldots \quad (1.7)$$

$$x_2 = O_2' - X_2' = x_{T2} + x_{G2} \quad \ldots \quad (2.7)$$

$$x_{T2} = X_{T2} - O_2 \quad \ldots \quad (3.7)$$

$$x_{G2} = X_{G2} \quad \ldots \quad (4.7)$$

$$M_T = O_1 p_1 + O_2 p_2 \quad \ldots \quad (5.7)$$

$$M_G = x_{T2} t \quad \ldots \quad (6.7)$$

$$M' = O_1' p_1 + O_2' p_2' \quad \ldots \quad (7.7)$$

$$t = p_2 - p_2' \quad \ldots \quad (8.7)$$

The first four of the above equilibrium equations are our basic supply-demand equations which state that in equilibrium the world supply and demand for each commodity must be equal. (1.7) states that the exports of the taxing country are equal either to the local supply of them less government and private consumption, or to the foreign consumption of domestic exportables less the foreign supply of them. Similarly, (2.7) defines domestic imports as equal to the foreign supply of the good less foreign consumption, or as equal to private plus government imports, while (3.7) states that private domestic imports are equal to the private consumption of importables less the domestic supply of them, it being assumed that there is no government supply of importables nor any government consumption of domestically produced importables. Obviously, as shown in (4.7), the government's consumption of importables is equal to government imports. Next, there are the three expenditure-income equations (5.7) to (7.7) which define respectively private income in the domestic country as equal to the value of domestic production; government income in the domestic country as equal to tariff revenue, there being no tariff levied on government imports; and income in the foreign country as equal to the value of foreign production. It should be noted that there is no government section in the foreign country. Finally, equation (8.7) defines the tariff, t, as equal to the difference between the domestic and foreign price of importables.

As $M = \sum_j p_j X_j$, one of the supply-demand equations is not independent. We choose to drop the x_1 equation and to use the price of domestic exportables, p_1, as numeraire. By a suitable choice of quantity units all prices except p_2 (the tariff inclusive price) are set, initially, to unity. Thus $p_1 = p_2' = 1$.

We now proceed to differentiate totally the equations (2.7) to (8.7) remembering that $X_{Ti} = X_{Ti}(p_2, M_T)$, that $O_i = O_i(p_2)$ and, more particularly, that $X_{Gi} = X_{Gi}(p_2', M_G)$, it being assumed that the government's demand for importables is a function of the price of exportables (our numeraire), of the foreign price of importables and of government income. This means that the government acts in a 'rational' manner and purchases its importables in the cheaper foreign market. We have:

$$dx_2 = dx_{T2} + dx_{G2} \quad \ldots \quad (9.7)$$

$$dx_2 = -(X_{22}' - O_{22}')dp_2' - M_2' dM' \quad \ldots \quad (10.7)$$

$$dx_{T2} = (X_{T22} - O_{22})dp_2 + M_{T2} dM_T \quad \ldots \quad (11.7)$$

$$dx_{G2} = (X_{G22})dp_2' + M_{G2} dM_G \quad \ldots \quad (12.7)$$

$$dM_T = O_2dp_2 \quad \ldots \quad (13.7)$$

$$dM_G = tdx_{T2} + dt(x_{T2}) \quad \ldots \quad (14.7)$$

$$dM' = O_2'dp_2' \quad \ldots \quad (15.7)$$

$$dt = dp_2 - dp_2' \quad \ldots \quad (16.7)$$

The reader should note that to simplify the three income-expenditure equations use has been made of the fact that the $\sum_j p_j dO_j = 0$. Next,

1. substitute in equations (10.7) to (12.7) for the expenditure equations (13.7) to (15.7) and into the resulting three equations

2. substitute $(X_{ij}-O_{ij}) = K_{ij}-X_jM_i$ (remembering that K_{Gij} will have no supply term), and for the terms (X_i-O_i) which arise after these substitutions, substitute the appropriate x_i or zero, as indicated by the equilibrium equations;

3. substitute into (10.7) for dx_2 from (9.7) and into the result substitute for dx_{G2} from (12.7) and for dx_{T2} from (11.7); finally,

4. cancel and collect terms wherever possible.

After rearranging,

$$(K_{G22}-x_{G2}M_{G2})dp_2'+(K_{22}'+x_2M_2')dp_2'+(1+M_{G2}t)(K_{T22}-x_{T2}M_{T2})dp_2 = -M_{G2}x_{T2}dt \quad \ldots \quad (17.7)$$

$$dp_2 - dp_2' = dt \quad \ldots \quad (18.7)$$

Our solution for the change in the terms of trade, dp_2', due to a tariff is:

$$\frac{dp_2'}{dt} = \frac{\begin{vmatrix} -M_{G2}x_{T2} & (1+M_{G2}t)(K_{T22}-x_{T2}M_{T2}) \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} (K_{G22}-x_{G2}M_{G2})+(K_{22}'+x_2M_2') & (1+M_{G2}t)(K_{T22}-x_{T2}M_{T2}) \\ -1 & 1 \end{vmatrix}} \qquad (=|A|)$$

$$= \frac{-(1+M_{G2}t)(K_{T22}-x_{T2}M_{T2}) - M_{G2}x_{T2}}{(K_{22}'+x_2M_2') + (K_{G22}-x_{G2}M_{G2}) + (1+M_{G2}t)(K_{T22}-x_{T2}M_{T2})}$$

which, when converted into elasticities, yields

$$\frac{dp_2'}{dt} = \frac{-\left(\frac{X_{T2}}{x_2}\sigma_{T22} - \frac{O_2}{x_2}S_{22} - C_{T2}\frac{x_{T2}}{x_2}\right) - \frac{1}{\lambda}\frac{p_2}{p_2'}\frac{x_{T2}}{x_2}C_{G2}}{\frac{1}{\lambda}\frac{p_2}{p_2'}\left(\frac{X_2'}{x_2}\sigma_{22}' - \frac{O_2'}{x_2}S_{22}' + C_2'\right) + \frac{1}{\lambda}\frac{p_2}{p_2'}\left(\frac{X_{G2}}{x_2}\sigma_{G22} - \frac{x_{G2}}{x_2}C_{G2}\right) + \left(\frac{X_{T2}}{x_2}\sigma_{T22} - \frac{O_2}{x_2}S_{22} - \frac{x_{T2}}{x_2}C_{T2}\right)} \quad \ldots (19.7)$$

where $\lambda = 1 + C_{G2}(p_2 - p_2') > 1$.

As dt is positive when the tariff increases, it is evident that the numerator and denominator of the right-hand side must be opposite in sign if the terms of trade are to move favourably for the tariff-imposing country (p_1 rising relative to p_2'). As in previous applications of the model, the denominator is assumed to be negative, i.e. stability exists in the international market. The movement in the terms of trade, therefore, depends upon the sign of the numerator and, from inspection, our analysis appears to support the traditional conclusion that the terms of trade movement could be adverse for the protecting country only if the government's marginal propensity to spend on importables exceeds that of the private sector by more than the aggregate private supply-demand substitution elasticity. The traditional analysis, however, obscures important weighting effects. Furthermore, the use of aggregate elasticities fails to stress the fact that two quantity weighted elasticities are involved. Moreover, the weight $\frac{x_{T2}}{x_2}$, which must be less than unity, would reduce the difference between the propensities.

The effect of the prior tariff is to increase the possibility of an adverse terms of trade movement because λ is less than p_2/p_2' as long as C_{G2} is less than unity.

If there were zero substitution possibilities in the domestic country, the conditions for an adverse terms of trade movement would be:

$$\lambda C_{T2} < \frac{p_2}{p_2'} C_{G2} \text{ .}$$

As λ is less than p_2/p_2', it follows that the presence of a [illegible] tariff means that the traditional requirement that the government's marginal propensity to spend on importables exceed that of the private sector, is no longer a necessary condition for an adverse terms of trade movement. Nevertheless, it is still essential that the government's marginal propensity to consume importables should exceed the private sector's marginal propensity to consume the same good, i.e. $M_{G2} > M_{T2}$. Only if the tariff rate were high could an adverse terms of trade movement occur when the sum of the weighted substitution elasticities was greater than unity. We conclude, therefore, that the possibility of an adverse terms of trade movement is negligible when the government purchases its importables in the foreign market.

2. Where the Government Purchases its Importables in the Domestic Market

Governments do not, however, always act upon rational economic motives. For instance, if the tariff is intended to protect local industry, the government may give a lead to its own policy by purchasing its importable goods requirements from the local manufacturers at the tariff-inclusive domestic price.

In this case the equilibrium equations are (if we omit the x_1 equations):

$$x_2 = O_2' - X_2' = X_{T2} + X_{G2} - O_2$$

$$M_T = O_1 p_1 + O_2 p_2$$

$$M_G = x_2 t$$

$$M' = O_1' p_1 + O_2' p_2'$$

$$t = p_2 - p_2'$$

The reader should note that private and total imports are now identical. Furthermore, in the domestic country, imports are defined as the difference between private plus government consumption of importables less the domestic supply of them.

Differentiating these equations totally and proceeding as above, a formula for the effect of a tariff upon the terms of trade is obtained:

$$\frac{dp_2'}{dt} = \frac{-\left(\frac{X_{T2}}{x_2}\sigma_{T22} - \frac{O_2}{x_2}S_{22} + \frac{X_{G2}-x_2}{x_2}C_{T2}\right)\frac{1}{\lambda} - \frac{X_{G2}}{x_2}\sigma_{G22} + \frac{X_{G2}}{x_2}C_{G2} - C_{G2}}{K} \quad \ldots \quad (20.7)$$

where λ (which reflects the effect of the prior tariff) $= 1 - M_{G2}t < 1$ and, apart from minor considerations, K is defined as the denominator of (19.7).

The principal difference between (19.7) and (20.7) is the appearance, in the numerator, of the quantity weighted government demand substitution elasticity and an additional income effect. These reflect the fact that the government's purchase of importables is now reduced as a consequence of their tariff-induced increase in price.

At first sight it may appear that, if the domestic supply of exceeds the private demand for importables (i.e. $X_{G2} > x_2$), the net result of the private income effect may be to increase private demand for importables. A moment's reflection, however, indicates that this is impossible as long as the income of the government is derived solely from tariff receipts. We may conclude, therefore, that an adverse movement in the terms of trade of the tariff-imposing country is most unlikely when the government purchases its importable requirements in its own domestic market.

Irrespective of where the government spends its tariff revenue, the terms of trade, if they are to deteriorate, must do so as the result of an excess world demand for importables. In the first case examined, the excess demand for (or supply of) importables is the net result of two effects: first, the effect of the change in tariff revenue (the net effect of a higher rate and a reduced volume) on government demand for importables; secondly, the reduction in imports demanded by the private sector due to the increase in their domestic price. In the second case, where the government purchases its importables in the domestic market, the effect of an increase in the price of importables upon the government's purchase of them reduces further any tendency towards an excess world demand for importables.

3. Effects on the Domestic Price Ratio

A criterion for the effect of a tariff upon the domestic price ratio, p_2, of the tariff-imposing country, can be obtained also from the equations (17.7) and (18.7):

$$\frac{dp_2}{dt} = \frac{\begin{vmatrix} (K_{G22} - x_{G2}M_{G2}) + (K_{22}' + x_2 M_2') & -M_{G2}x_{T2} \\ -1 & 1 \end{vmatrix}}{|A|}$$

On expanding this solution and converting it to elasticities we have:

$$\frac{dp_2}{dt} = \frac{\frac{p_2}{p_2'}\left(\frac{X_{G2}}{x_2}\sigma_{G22} - C_{G2}\right) + \frac{p_2}{p_2'}\left(\frac{X_2'}{x_2}\sigma_{22}' - \frac{O_2'}{x_2}S_{22}' + C_2'\right)}{p_2|A|} \quad \ldots \quad (21.7)$$

$|A|$ we know to be negative and so, if the domestic price ratio is to move in a perverse manner, the sum of the weighted foreign substitution elasticities and the government's weighted demand substitution elasticity must be less than the difference between the government's marginal propensity to spend on importables and the foreign marginal propensity to spend upon the same good.

The impact effect of a tariff is to create a world excess supply of the domestic country's importables. This is caused, in the case under consideration, by the reduction in domestic demands as consumers substitute out of importables, by the increase in domestic production, and by the reduced demand for importables arising from the reduction in real income due to their increase in price. This excess supply of importables can be eliminated only by a decrease in their price, or the same thing, by an improvement in the terms of trade of the taxing country. Now, a perverse movement in the domestic price ratio requires a terms of trade effect which is sufficiently strong, i.e. a fall in p_2 relative to p_1 which is sufficiently strong, to eliminate the initial effect of the tariff upon the domestic price ratio. The determinants of the strength of this terms of trade movement appear in our denominator. First, there is the foreign demand and supply substitution elasticities which reflect the extent to which a small rise in the price of domestic exportables (or fall in the price of importables) will induce a substitution by foreign consumers from domestic exportables to importables and by foreign producers from the exportable to the import-competing sector. If these effects are strong the terms of trade shift will be small, i.e. a small shift in price will eliminate the initial excess supply of importables, and the likelihood of a perverse price effect will be small. In a similar fashion, though there are no supply reactions, the government's demand substitution elasticity will help determine the extent of the necessary terms of trade movement. Obviously, its presence will reduce the size of the shift in the international price ratio compared with the conventional case in which the effect of the government sector is ignored. Secondly, there are income effects to be considered. These are three in number. There is the foreign income effect which arises because of the fall in price of good two: foreign producer's real income falls and less of good two is purchased thus tending to add to the excess supply of the commodity and to increase the magnitude of the necessary shift in the terms of trade. Next, there is an income effect whereby the government's real income rises as the result of the fall in the price of good two. Finally, there is a further government income effect arising from the additional revenue derived from the tariff increase. This will increase further the government's consumption of commodity two. Both of these last effects tend to reduce the necessary terms of trade movement.

To summarize: this case, in which the role of the government is treated explicitly, differs from the traditional one because the government's spending of the tariff revenue in the world market tends to decrease the terms of trade movement and, correspondingly, the possibility of the perverse case.

Where the government operates entirely in the domestic market the effect of a tariff upon the domestic price ratio is given by:

$$\frac{dp_2}{dt} = \frac{\frac{1}{\lambda}\frac{p_2}{p_2'}\left(\frac{X_2'}{x_2}\sigma_{22}' - \frac{O_2'}{x_2}S_{22}' + C_2'\right) - \frac{x_{T2}}{x_2}C_{G2}}{p_2|A|} \quad \ldots \quad (22.7)$$

where $\lambda = 1 - C_{G2}(1 - \frac{p_2'}{p_2}) < 1$.

As one would expect, both the government's demand substitution elasticity and its income effect which arises from the fall in price of commodity two in the foreign market, disappear from our numerator. Instead, the government is affected by the domestic tariff induced rise in the price of importables, switching part of its expenditure to exportables, and thereby increasing the shift in the terms of trade.

C. DEPENDENCE OF PRIVATE DEMAND

We consider next a case in which the amount of government expenditure is a datum in private utility functions. Normally, one would expect the private demand for commodities to be influenced by not only the amount but also by the type of government expenditure. For instance, the demand for a commodity would be affected differently depending on whether the government's service was complementary to or substitutable for it. We shall assume that the composition of the government's "consumption basket" is determined, like that of the private consumer, by relative prices. It follows that both the amount and the type of government consumption (or service to the community) are determined simultaneously.

Our equilibrium equations are identical to those set out for the traditional case (equations (1.7) to (8.7)). Once again we select p_1 as our numeraire and omit the supply-demand equation involving the exportable good of the taxing country.

This time, when differentiating (totally) our initial set of equations, we obtain a set of equations which is identical to those given in the set (9.7) to (16.7) except that (11.7) now reads:

$$dx_{T2} = (X_{T22}-O_{22})dp_2 + M_{T2}dM_T + T_2dM_G \qquad (11.7)$$

where $T_2 = (\partial X_{T2}/\partial M_G)$ is the rate of change in the private consumption of importables associated with a change in the amount of government expenditure.

We now proceed to

1. substitute for (13.7), (14.7), and (15.7) in (12.7), (10.7) and (11.7);
2. substitute (9.7) into the resulting equation for (10.7) and, into this, for (12.7);
3. substitute in the remaining equations for $(X_{ij}-O_{ij}) = (K_{ij}-X_jM_i)$ and, for the terms (X_i-O_i) which arise after these substitutions, substitute the appropriate x_i as indicated by the equilibrium equations;
4. rearrange, and cancel and collect terms wherever possible.

Then,

$$\left.\begin{aligned} dx_{T2}[1+M_{G2}(p_2-p_2')]+(K_{G22}-x_{G2}M_{G2})dp_2'+(K_{22}'+x_2M_2')dp_2' &= -x_{T2}M_{G2}dt \\ dx_{T2}[1-T_2(p_2-p_2')]-(K_{T22}-x_{T2}M_{T2})dp_2 &= x_{T2}T_2dt \\ dp_2 - dp_2' &= dt \end{aligned}\right\} \ldots \qquad (23.7)$$

Our solution for the rate of change in the terms of trade due to a tariff is

$$\frac{dp_2'}{dt} = \frac{\begin{vmatrix} 1+M_{G2}(p_2-p_2') & 0 & -x_{T2}M_{G2} \\ 1-T_2(p_2-p_2') & -(K_{T22}-x_{T2}M_{T2}) & x_{T2}T_2 \\ 0 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1+M_{G2}(p_2-p_2') & 0 & (K_{G22}-x_{G2}M_{G2})+(K_{22}'+x_2M_2') \\ 1-T_2(p_2-p_2') & -(K_{T22}-x_{T2}M_{T2}) & 0 \\ 0 & 1 & -1 \end{vmatrix}} \quad (=|A|)$$

On expanding these determinants from the first row, dividing top and bottom by $1-T_2(p_2-p_2')$, and converting to elasticities, we obtain:

$$\frac{dp_2'}{dt} = \frac{-\lambda\left(\frac{X_{T2}}{x_2}\sigma_{T22}-\frac{O_2}{x_2}S_{22}-\frac{x_{T2}}{x_2}(C_{T2}-D_2)\right)-\frac{x_{T2}}{x_2}C_{G2}\frac{p_2}{p_2'}}{\frac{p_2}{p_2'}\left(\frac{x_{G2}}{x_2}\sigma_{G22}-\frac{x_{G2}}{x_2}C_{G2}\right)+\frac{p_2}{p_2'}\left(\frac{X_2'}{x_2}\sigma_{22}'-\frac{O_2'}{x_2}S_{22}'+C_2'\right)+\lambda\left(\frac{X_{T2}}{x_2}\sigma_{T22}-\frac{O_2}{x_2}S_{22}-\frac{x_{T2}}{x_2}C_{T2}\right)} \quad \ldots(24.7)$$

where $D_2=T_2p_2=(\partial X_{T2}/\partial M_G)p_2$ and $\lambda=[1+C_{G2}(p_2-p_2')]/[1-D_2(1-\frac{p_2'}{p_2})]$ which is >1 as long as D_2 is positive, i.e. as long as government services are not substitutable in consumption for good two.

Where initial free trade exists, the criterion for the terms of trade movement, assuming a stable international market, is

$$\frac{X_{T2}}{x_2}\sigma_{T22} - \frac{O_2}{x_2}S_{22} - C_{T2}\frac{x_{T2}}{x_2} \gtreqless (D_2 + C_{G2})\frac{x_{T2}}{x_2}$$

The criterion differs from that developed in the case of independence (19.7) because an adverse movement in the terms of trade no longer requires the sum of the two weighted substitution elasticities plus the private marginal propensity to spend on importables to exceed unity: an adverse terms of trade movement could follow if government services were complementary with importables in private consumption. However, substitution possibilities would need to be abnormally small. It is no longer possible to generalise about the effect of λ for if government services were substitutable in consumption for commodity two, λ could be less than unity. In magnitude, however, it would still approximate to unity.

A similar criterion, in the dependence case, for the effect of a tariff upon the taxing country's domestic price ratio can be obtained from the set of equations (23.7):

$$\frac{dp_2}{dt} = \frac{\begin{vmatrix} 1+M_{G2}(p_2-p_2') & -x_{T2}M_{G2} & (K_{G22}-x_{G2}M_{G2})+(K_{22}'+x_2M_2') \\ 1-T_2(p_2-p_2') & x_{T2}T_2 & 0 \\ 0 & 1 & -1 \end{vmatrix}}{|A|}$$

where $|A|$ is defined as in the terms of trade solution (24.7). An expansion of these determinants from the first row gives, upon conversion to elasticities,

$$\frac{dp_2}{dt} = \frac{\frac{p_2}{p_2'}\left(\frac{X_{G2}}{x_2}\sigma_{G22} - C_{G2}\right) + \frac{p_2}{p_2'}\left(\frac{X_2'}{x_2}\sigma_{22}' - \frac{O_2'}{x_2}S_{22}' + C_2'\right) + \frac{x_{T2}}{x_2} D_2 \frac{p_2'}{p_2}\lambda}{\frac{p_2}{p_2'}|A|} \quad \ldots \quad (25.7)$$

where λ is defined as in (24.7). In this case, given initial free trade (λ=1), if importables are substitutable for government services in private consumption ($D<0$), a tariff increase could result in a perverse movement in the domestic price ratio, even when the difference between the foreign and the government's marginal propensities to spend on exportables is less than the sum of the foreign substitution elasticities and the government's demand substitution elasticity. If, however, an increase in government expenditure indirectly increases the private demand for importables ($D>0$), the case against the perverse effect holds a fortiori: the increased private demand for importables accompanying government expenditure mitigating those forces which make for a strong favourable movement in the terms of trade.

In conclusion, a comment is offered upon a recent article by R. Baldwin [64,69-71] in which he concludes that even where the private demand for commodities is partly dependent upon government expenditure, the criterion for the effect of a tariff on the terms of trade is the same as in the traditional, independent case. In fact, the case analysed by Baldwin assumes a different type of dependence from our own: he assumes that the government redistributes the tariff proceeds in kind. Given that this redistribution is random with respect to tastes the effect of the 'dependence' disappears. Our dependence, on the other hand, stems from the fact that consumers are influenced by the type and amount of government consumption itself. The two concepts are thus quite different.

8. DISAGGREGATION OF PRIVATE DEMAND AND SUPPLY, THE TERMS OF TRADE AND THE DOMESTIC PRICE RATIO

A. INTRODUCTION

In this chapter an attempt is made to examine the effects of a tariff on the terms of trade and on the domestic price ratio of the taxing country when the private sector of the economy no longer can be regarded as a homogeneous unit. As in the previous chapter, a two-commodity world is postulated but there are two different representative individuals. It is assumed that individual one specialises in the production of exportables; and that his consumption of importables comprises his own production, the total amount of the country's private imports, and a quantity of importables purchased from individual two. It follows that individual two supplies all of his own importable requirements. Throughout, one bar denotes individual one; two bars individual two. Thus,

$$\bar{O}_2 = \bar{X}_{T2} - \bar{x}_{T2}$$

states that individual one's supply of importables, $\bar{O}_2$, is equal to his consumption, $\bar{X}_{T2}$, minus his imports, $\bar{x}_{T2}$. It should be noted that individual one's imports, $\bar{x}_{T2}$, can be subdivided into his imports from abroad, x_{T2}, (which are equal to the country's total private imports) and into the amount of importables purchased from his fellow citizen, $\bar{\bar{x}}_{T2}$. Similarly,

$$\bar{\bar{O}}_2 = \bar{\bar{X}}_2 + \bar{\bar{x}}_{T2}$$

states that individual two's supply of importables is equal to his consumption of them plus the quantity supplied to individual one. In other words, the import-competing sector of the economy does not contribute to the country's demand for imports.

B. DEPENDENCE OF PRIVATE DEMAND

The more complex case in which private demands are related functionally to the amount of government expenditure is considered first.

The equilibrium set of equations are:

$$x_2 = O_2' - X_2' = x_{T2} + x_{G2} \quad \ldots \quad (1.8)$$

$$\bar{x}_{T2} = \bar{X}_{T2} - \bar{O}_2 \quad \ldots \quad (2.8)$$

$$\bar{\bar{x}}_{T2} = \bar{\bar{O}}_2 - \bar{\bar{X}}_{T2} \quad \ldots \quad (3.8)$$

$$x_{T2} = \bar{x}_{T2} - \bar{\bar{x}}_{T2} \quad \ldots \quad (4.8)$$

$$x_{G2} = X_{G2} \quad \ldots \quad (5.8)$$

$$x_1 = \bar{x}_1 - \bar{\bar{x}}_1 - X_{G1} \quad \ldots \quad (6.8)$$

$$\bar{x}_1 = \bar{O}_1 - \bar{X}_{T1} \quad \ldots \quad (7.8)$$

$$\bar{\bar{x}}_1 = \bar{\bar{X}}_{T1} - \bar{\bar{O}}_{T1} \quad \ldots \quad (8.8)$$

$$x_1 = X_1' - O_1' \quad \ldots \quad (9.8)$$

$$t = p_2 - p_2' \quad \ldots \quad (10.8)$$

$$\bar{M}_T = \bar{O}_1 p_1 + \bar{O}_2 p_2 \quad \ldots \quad (11.8)$$

$$\bar{\bar{M}}_T = \bar{\bar{O}}_1 p_1 + \bar{\bar{O}}_2 p_2 \quad \ldots \quad (12.8)$$

$$M_G = t x_{T2} \quad \ldots \quad (13.8)$$

$$M' = O_1' p_1 + O_2' p_2' \quad \ldots \quad (14.8)$$

These are identical to the set of equations (1.7) to (8.7) if due allowance is made for the fact that there are now two representative individuals. This time, the price of the taxing country's exportable commodity, p_1, is selected arbitrarily as numeraire and we choose to drop the supply-demand equations involving good one. If we remember that our demand functions for the taxing country are of the form $\bar{X}_i = \bar{X}_i(p_2, \bar{M}_T, M_G)$, a total differentiation of the equations (1.8) to (5.8) and (10.8) to (14.8) gives

$$dx_2 = dx_{T2} + dx_{G2} \quad \ldots \quad (15.8)$$

$$dx_2 = -(X'_{22}-O'_{22})dp'_2 - M'_2dM' \quad \ldots \quad (16.8)$$

$$d\bar{x}_{T2} = (\bar{X}_{T22}-\bar{O}_{22})dp_2 + \bar{M}_{T2}d\bar{M}_T + \bar{T}_2dM_G \quad \ldots \quad (17.8)$$

$$d\bar{\bar{x}}_{T2} = -(\bar{\bar{X}}_{T22}-\bar{\bar{O}}_{22})dp_2 - \bar{\bar{M}}_{T2}d\bar{\bar{M}}_T - \bar{\bar{T}}_2dM_G \quad \ldots \quad (18.8)$$

$$dx_{T2} = d\bar{x}_{T2} - d\bar{\bar{x}}_{T2} \quad \ldots \quad (19.8)$$

$$dx_{G2} = X_{G22}dp'_2 + M_{G2}dM_G \quad \ldots \quad (20.8)$$

$$dt = dp_2 - dp'_2 \quad \ldots \quad (21.8)$$

$$d\bar{M}_T = \bar{O}_2dp_2 \quad \ldots \quad (22.8)$$

$$d\bar{\bar{M}}_T = \bar{\bar{O}}_2dp_2 \quad \ldots \quad (23.8)$$

$$dM_G = tdx_{T2} + x_{T2}dt \quad \ldots \quad (24,8)$$

$$dM' = O'_2dp'_2 \quad \ldots \quad (25.8)$$

where $\bar{T}$ and $\bar{\bar{T}}$ are defined as in the last chapter.

We now proceed to:

1. Substitute in (16.8) for (15.8) and into the result for dx_{G2} from(20.8);
2. Subtract equation (18.8) from (17.8) and to substitute in the result for $(d\bar{x}_{T2}-d\bar{\bar{x}}_{T2})$ from (19.8).

This gives:

$$dx_{T2}+X_{G22}dp'_2+M_{G2}dM_G = -(X'_{22}-O'_{22})dp'_2-M'_2dM' \quad \ldots \quad (26.8)$$

$$dx_{T2} = (\bar{X}_{T22}-\bar{O}_{22})dp_2+\bar{M}_{T2}d\bar{M}_T+\bar{T}_2dM_G+ (\bar{\bar{X}}_{T22}-\bar{\bar{O}}_{22})dp_2+\bar{\bar{M}}_{T2}d\bar{\bar{M}}_T+\bar{\bar{T}}_2dM_G \quad \ldots \quad (27.8)$$

$$dt = dp_2 - dp'_2 \quad \ldots \quad (28.8)$$

$$d\bar{M}_T = \bar{O}_2dp_2 \quad \ldots \quad (29.8)$$

$$d\bar{\bar{M}}_T = \bar{\bar{O}}_2dp_2 \quad \ldots \quad (30.8)$$

$$dM_G = tdx_{T2} + x_{T2}dt \quad \ldots \quad (31.8)$$

$$dM' = O'_2dp'_2 \quad \ldots \quad (32.8)$$

Finally, substitute for (29.8) to (32.8) in (26.8) and (27.8) and, into the resulting equations, for$(X_{ij}-O_{ij}) = (K_{ij}-X_jM_i)$ and for the terms (X_i-O_i) which arise after these substitutions, the appropriate x_i or zero as indicated by the equilibrium equations. Rearranging, and cancelling or collecting terms wherever possible, we have

$$\left.\begin{aligned} dx_{T2} + (K_{G22}-x_{G2}M_{G2})dp_2' + M_{G2}t\,dx_{T2} + (K_{22}'+x_2M_2')dp_2' &= -x_{T2}M_{G2}dt \\ dx_{T2} - (\bar{K}_{T22}+\bar{\bar{K}}_{T22}-\bar{x}_{T2}\bar{M}_{T2}+\bar{\bar{x}}_{T2}\bar{\bar{M}}_{T2})dp_2 - t(\bar{T}_2+\bar{\bar{T}}_2)dx_{T2} &= (\bar{T}_2+\bar{\bar{T}}_2)x_{T2}dt \\ dp_2 - dp_2' &= dt \end{aligned}\right\} \quad \ldots \qquad (33.8)$$

Our solution for the change in the terms of trade, p_2', is

$$\frac{dp_2'}{dt} = \frac{\begin{vmatrix} 1+M_{G2}(p_2-p_2') & 0 & -x_{T2}M_{G2} \\ 1-(\bar{T}_2+\bar{\bar{T}}_2)(p_2-p_2') & -(\bar{K}_{T22}+\bar{\bar{K}}_{T22}-\bar{x}_{T2}\bar{M}_{T2}+\bar{\bar{x}}_{T2}\bar{\bar{M}}_{T2}) & (\bar{T}_2+\bar{\bar{T}}_2)x_{T2} \\ 0 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1+M_{G2}(p_2-p_2') & 0 & (K_{G22}-x_{G2}M_{G2})+(K_{22}'+x_2M_2') \\ 1-(\bar{T}_2+\bar{\bar{T}}_2)(p_2-p_2') & -(\bar{K}_{T22}+\bar{\bar{K}}_{T22}-\bar{x}_{T2}\bar{M}_{T2}+\bar{\bar{x}}_{T2}\bar{\bar{M}}_{T2}) & 0 \\ 0 & 1 & -1 \end{vmatrix} \ (=|A|)} \quad \ldots \qquad (34.8)$$

An expansion of these determinants from the first row yields, after division of the expansion by $1-(\bar{T}_2+\bar{\bar{T}}_2)(p_2-p_2')$:

$$\frac{dp_2'}{dt} = \frac{-\dfrac{1+M_{G2}(p_2-p_2')}{1-(\bar{T}_2+\bar{\bar{T}}_2)(p_2-p_2')}\,[\bar{K}_{T22}+\bar{\bar{K}}_{T22}-\bar{x}_{T2}\bar{M}_{T2}+\bar{\bar{x}}_{T2}\bar{\bar{M}}_{T2}+(\bar{T}_2+\bar{\bar{T}}_2)x_{T2}]-x_{T2}M_{G2}}{\dfrac{1+M_{G2}(p_2-p_2')}{1-(\bar{T}_2+\bar{\bar{T}}_2)(p_2-p_2')}\,[\bar{K}_{T22}+\bar{\bar{K}}_{T22}-\bar{x}_{T2}\bar{M}_{T2}+\bar{\bar{x}}_{T2}\bar{\bar{M}}_{T2}]+(K_{G22}-x_{G2}M_{G2})+(K_{22}'+x_2M_2')}\ .$$

On conversion to elasticities and marginal propensities to spend we obtain:

$$\frac{dp_2'}{dt} = \frac{-\lambda\left(\dfrac{\bar{X}_{T2}}{x_2}\bar{\sigma}_{T22}+\dfrac{\bar{\bar{X}}_{T2}}{x_2}\bar{\bar{\sigma}}_{T22}-\dfrac{\bar{O}_2}{x_2}\bar{S}_{22}-\dfrac{\bar{\bar{O}}_2}{x_2}\bar{\bar{S}}_{22}-\dfrac{\bar{x}_{T2}}{x_2}\bar{C}_{T2}+\dfrac{\bar{\bar{x}}_{T2}}{x_2}\bar{\bar{C}}_{T2}+(\bar{D}_2+\bar{\bar{D}}_2)\dfrac{x_{T2}}{x_2}\right)-\dfrac{x_{T2}}{x_2}C_{G2}\dfrac{p_2}{p_2'}}{\lambda\left(\dfrac{\bar{X}_{T2}}{x_2}\bar{\sigma}_{T22}+\dfrac{\bar{\bar{X}}_{T2}}{x_2}\bar{\bar{\sigma}}_{T22}-\dfrac{\bar{O}_2}{x_2}\bar{S}_{22}-\dfrac{\bar{\bar{O}}_2}{x_2}\bar{\bar{S}}_{22}-\dfrac{\bar{x}_{T2}}{x_2}\bar{C}_{T2}+\dfrac{\bar{\bar{x}}_{T2}}{x_2}\bar{\bar{C}}_{T2}\right)+\dfrac{x_{G2}}{x_2}(\sigma_{G22}-M_{G2})\dfrac{p_2}{p_2'}+\left(\dfrac{X_2'}{x_2}\sigma_{22}'-\dfrac{O_2'}{x_2}S_{22}'+C_2'\right)\dfrac{p_2}{p_2'}} \quad \ldots (35.8)$$

where $\lambda = \dfrac{1+C_{G2}(p_2-p_2')}{1-(\bar{D}_2+\bar{\bar{D}}_2)(1-\frac{p_2'}{p_2})}$ is positive in sign and either slightly in excess of or less than unity (depending upon the sign of $\bar{D}_2$ and $\bar{\bar{D}}_2$).

Disaggregation of the private sector in the tariff-imposing country introduces new complications. If as a result of income or taste factors the marginal propensities to consume importables of our two individuals differ, an alteration in the domestic price of importables will affect the real income distribution between them. Furthermore, each of our representative consumers is also a representative producer who owns a collection of the factors of production. Unless each producer holds these factors in the same proportions, the change in relative factor prices that follows the product price change must affect the income earned by each individual.

It is assumed that individual two (double-barred) owns a greater proportion of the factors engaged intensively in the import-competing industry and that his imports are zero. It follows that the net effect on his real income must be positive, the losses in consumption being more than compensated by the gains in production. If it should happen that he has a larger marginal propensity to spend on importables than does individual one, the total income effect may be positive; the necessary condition for this being that individual two's consumption of importables should exceed that of individual one by more than the amount of the country's private imports.

If the total income effect were positive it could exceed the negative sum of the elasticities, thereby causing an adverse movement in the terms of trade. Our formula confirms this line of reasoning. Ignore for the moment all the D terms, which reflect the government's influence on the private demand for importables, and our λ term which reflects part of the influence of the prior tariffs. It is no longer necessary that the government's marginal propensity to consume importables should exceed that of the private sector for an adverse movement in the terms of trade to occur; this could happen even where both of the private propensities exceed the public one as long as the individual with the highest marginal propensity to consume importables is also a net supplier of these commodities to the other individual. Once again, there is ambiguity concerning the effect of the prior tariff. If government services were substitutable for importables in consumption, λ could be less than unity.

When allowance is made for the partial dependence of private demand on public expenditure the D terms also must be considered. Their appearance could reduce the improbability of an adverse terms of trade movement (except in the case of strong complementarity between private consumption of importables and government services) but otherwise no modifications of our analysis are necessary.

A solution for the effect of a tariff upon the domestic price ratio of the taxing country, p_2, can be obtained also from the set of equations (33.8):

$$\frac{dp_2}{dt} = \frac{\begin{vmatrix} 1+M_G(p_2-p_2') & -x_{T2}M_{G2} & (K_{G22}-x_{G2}M_{G2})+(K_{22}'+x_2M_2') \\ 1-(\bar{T}_2+\bar{\bar{T}}_2)(p_2-p_2') & (\bar{T}_2+\bar{\bar{T}}_2)x_{T2} & 0 \\ 0 & 1 & -1 \end{vmatrix}}{|A|} \quad \ldots \quad (36.8)$$

where $|A|$ is defined as in (34.8). On expansion and conversion to elasticities, this yields, when the presence of the λ term is ignored,

$$\frac{dp_2}{dt} = \frac{\frac{p_2}{p_2'}\left(\frac{X_2'}{x_2}\sigma_{22}' - \frac{O_2'}{x_2}S_{22}' + C_2'\right) - (\bar{D}_2+\bar{\bar{D}}_2)\frac{x_{T2}}{x_2} + \frac{p_2}{p_2'}\left(\frac{x_{G2}}{x_2}\sigma_{G22} - C_{G2}\right)}{K} \quad \ldots \quad (37.8)$$

where K is the denominator in (35.8) which is negative. The shift in the domestic price ratio is governed, assuming stability, by the sign of the numerator. Imagine, for the moment, that the demand of the private sector is uninfluenced by government expenditure, the D terms equalling zero. In this case the criterion is identical to that given for the aggregated case by the numerator of equation (21.7), the reason for this being that the domestic distribution of income would be unaltered as long as the domestic price of importables were unchanged. Some modifications appear when allowance is made for the fact that government expenditure can influence private demands. For instance, if government services are substitutable for importables in the consumption of both individuals, the case for a perverse effect would be strengthened, the increased demand for exportables, as a consequence of government expenditure, increasing the favourable movement in the terms of trade of the taxing country.

C. INDEPENDENCE OF PRIVATE DEMAND

Let us consider next the effects of disaggregation when the government redistributes the tariff revenue as a subsidy, or where an equivalent amount of taxation is remitted elsewhere in the economy and the government's substitution demand elasticity and marginal propensity to spend on importables are identical to a weighted average of those of the private sectors.

Once more, individual two is a net supplier of importables which means that individual one is responsible for <u>all</u> of the country's imports. Thus we examine the case in which the import-competing sector of the economy is more than self-sufficient with respect to the goods it produces.

The equilibrium of our system is described by the following equations:

$$x_1 = \bar{x}_1 - \bar{\bar{x}}_1 = O_1' - X_1' \qquad \ldots \qquad (39.8)$$

$$\bar{x}_1 = \bar{O}_1 - \bar{X}_1 \qquad \ldots \qquad (40.8)$$

$$\bar{\bar{x}}_1 = \bar{\bar{X}}_1 - \bar{\bar{O}}_1 \qquad \ldots \qquad (41.8)$$

$$x_2 = O_2' - X_2' = \bar{x}_2 - \bar{\bar{x}}_2 \qquad \ldots \qquad (42.8)$$

$$\bar{x}_2 = \bar{X}_2 - \bar{O}_2 \qquad \ldots \qquad (43.8)$$

$$\bar{\bar{x}}_2 = \bar{\bar{O}}_2 - \bar{\bar{X}}_2 \qquad \ldots \qquad (44.8)$$

$$\bar{M} = \bar{O}_1 p_1 + \bar{O}_2 p_2 + \bar{K} t x_2 \qquad \ldots \qquad (45.8)$$

$$\bar{\bar{M}} = \bar{\bar{O}}_1 p_1 + \bar{\bar{O}}_2 p_2 + \bar{\bar{K}} t x_2 \quad \ldots \quad (46.8)$$

$$M' = O_1' p_1 + O_2' p_2' \quad \ldots \quad (47.8)$$

$$t = p_2 - p_2' \quad \ldots \quad (47.8a)$$

The reader should note first, that the constants $\bar{K}$ and $\bar{\bar{K}}$ refer to the proportions of tariff revenue redistributed to individuals one and two respectively and, secondly, that the subscript T, which was used to denote the private sector, is no longer necessary as the government sector is not included explicitly.

As one of the two supply-demand equations is implied by the fact that $\sum_i p_i X_i = M$, we choose to omit the equations involving the taxing country's exportable commodity. p_1 is selected arbitrarily as numeraire. Differentiating the remaining independent equations totally we obtain:

$$dx_2 = -(X_{22}' - O_{22}') dp_2' - M_2' dM' \quad \ldots \quad (48.8)$$

$$dx_2 = d\bar{x}_2 - d\bar{\bar{x}}_2 \quad \ldots \quad (49.8)$$

$$d\bar{x}_2 = (\bar{X}_{22} - \bar{O}_{22}) dp_2 + \bar{M}_2 d\bar{M} \quad \ldots \quad (50.8)$$

$$d\bar{\bar{x}}_2 = -(\bar{\bar{X}}_{22} - \bar{\bar{O}}_{22}) dp_2 - \bar{\bar{M}}_2 d\bar{\bar{M}} \quad \ldots \quad (51.8)$$

$$d\bar{M} = \bar{O}_2 dp_2 + \bar{K} x_2 dt + \bar{K} t dx_2 \quad \ldots \quad (52.8)$$

$$d\bar{\bar{M}} = \bar{\bar{O}}_2 dp_2 + \bar{\bar{K}} x_2 dt + \bar{\bar{K}} t dx_2 \quad \ldots \quad (53.8)$$

$$dM' = O_2' dp_2' \quad \ldots \quad (54.8)$$

$$dt = dp_2 - dp_2' \quad \ldots \quad (55.8)$$

Next,

1. substitute for (52.8) to (54.8) in (48.8), (50.8) and (51.8);
2. substitute for $d\bar{x}_2$ and $d\bar{\bar{x}}_2$ in (49.8);
3. substitute $(K_{ij} - X_j M_i) = (X_{ij} - O_{ij})$;
4. substitute for the terms $(X_i - O_i)$ the appropriate x_i or zero;
5. cancel and collect terms wherever possible.

Then, we have:

$$\left.\begin{aligned} & dx_2+(K'_{22}+x_2M'_2)dp'_2 = 0 \\ & dx_2-(\bar{K}_{22}+\bar{\bar{K}}_{22}-\bar{x}_2\bar{M}_2+\bar{\bar{x}}_2\bar{\bar{M}}_2)dp_2-(\bar{K}\bar{M}_2+\bar{\bar{K}}\bar{\bar{M}}_2)(tdx_2) = (\bar{K}\bar{M}_2+\bar{\bar{K}}\bar{\bar{M}}_2)(x_2dt) \\ & dp_2 - dp'_2 = dt \end{aligned}\right\} \quad \ldots \qquad (56.8)$$

The solution for the change in the terms of trade, p'_2, is:

$$\frac{dp'_2}{dt} = \frac{\begin{vmatrix} 1 & 0 & 0 \\ 1-(p_2-p'_2)(\bar{K}\bar{M}_2+\bar{\bar{K}}\bar{\bar{M}}_2) & -(\bar{K}_{22}+\bar{\bar{K}}_{22}-\bar{x}_2\bar{M}_2+\bar{\bar{x}}_2\bar{\bar{M}}_2) & x_2(\bar{K}\bar{M}_2+\bar{\bar{K}}\bar{\bar{M}}_2) \\ 0 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & K'_{22}+x_2M'_2 \\ 1-(p_2-p'_2)(\bar{K}\bar{M}_2+\bar{\bar{K}}\bar{\bar{M}}_2) & -(\bar{K}_{22}+\bar{\bar{K}}_{22}-\bar{x}_2\bar{M}_2+\bar{\bar{x}}_2\bar{\bar{M}}_2) & 0 \\ 0 & 1 & -1 \end{vmatrix}} \quad \ldots \qquad (57.8)$$

$$(=|A|)$$

On expanding these determinants and converting the result to elasticities and marginal propensities to spend we have:

$$\frac{dp'_2}{dt} = \frac{-\left(\frac{\bar{X}_2}{x_2}\bar{\sigma}_{22}+\frac{\bar{\bar{X}}_2}{x_2}\bar{\bar{\sigma}}_{22}-\frac{\bar{O}_2}{x_2}\bar{S}_{22}-\frac{\bar{\bar{O}}_2}{x_2}\bar{\bar{S}}_{22}-\frac{\bar{x}_2}{x_2}\bar{C}_2+\frac{\bar{\bar{x}}_2}{x_2}\bar{\bar{C}}_2\right)-(\bar{K}\bar{C}_2+\bar{\bar{K}}\bar{\bar{C}}_2)}{\left(\frac{\bar{X}_2}{x_2}\bar{\sigma}_{22}+\frac{\bar{\bar{X}}_2}{x_2}\bar{\bar{\sigma}}_{22}-\frac{\bar{O}_2}{x_2}\bar{S}_{22}-\frac{\bar{\bar{O}}_2}{x_2}\bar{\bar{S}}_{22}-\frac{\bar{x}_2}{x_2}\bar{C}_2+\frac{\bar{\bar{x}}_2}{x_2}\bar{\bar{C}}_2\right)+\lambda\left(\frac{X'_2}{x_2}\sigma'_{22}-\frac{O'_2}{x_2}S'_{22}+C'_2\right)\frac{p_2}{p'_2}} \quad \ldots \qquad (58.8)$$

where $\lambda=1-(\bar{K}\bar{C}_2+\bar{\bar{K}}\bar{\bar{C}}_2)(1-\frac{p'_2}{p_2})$ is less than unity.

As one would expect, the manner in which the tariff revenue is redistributed influences the demand for importables when the private marginal propensities to consume importables differ. If, in the special case considered, individual two, who is a net supplier of importables, has the larger propensity to consume and receives the larger share of redistributed tariff proceeds, the traditional conclusion that the terms of trade must move in favour of the tariff-imposing country is valid no longer, i.e.

$$\frac{\bar{x}_2}{x_2}\bar{C}_2 - \frac{\bar{\bar{x}}_2}{x_2}\bar{\bar{C}}_2 - \bar{K}\bar{C}_2 - \bar{\bar{K}}\bar{\bar{C}}_2 \text{ could be } \gtreqless 0 .$$

Furthermore, if individual two is not a net supplier of importables, both individuals consuming imports, the traditional conclusion still could be reversed if the individual with the higher marginal propensity to spend on importables finds that his share of the tariff proceeds is larger than the proportion of the initial quantity of imports that he consumes.

Finally, the solution for the effect of a tariff upon the domestic price ratio of the tariff-imposing country is obtained from the set of equations (56.8):

$$\frac{dp_2}{dt} = \frac{\begin{vmatrix} 1 & 0 & K'_{22}+x_2M'_2 \\ 1-(p_2-p'_2)(\bar{K}\bar{M}_2+\bar{\bar{K}}\bar{\bar{M}}_2) & x_2(\bar{K}\bar{M}_2+\bar{\bar{K}}\bar{\bar{M}}_2) & 0 \\ 0 & 1 & -1 \end{vmatrix}}{|A|}$$

where $|A|$ is defined as in (57.8). Expanding these determinants and converting the solution into elasticity form (but ignoring the effect of the prior tariff) we have:

$$\frac{dp_2}{dt} = \frac{\frac{p_2}{p'_2}\left(\frac{X'_2}{x_2}\sigma'_{22} - \frac{O'_2}{x_2}S'_{22} + C'_2\right) - (\bar{K}\bar{C}_2+\bar{\bar{K}}\bar{\bar{C}}_2)}{K} \quad \ldots \quad (59.8)$$

where K is the same as the denominator in equation (58.8). In this case, the possibility of a perverse movement in the domestic price ratio would be increased if the individual with the smaller marginal propensity to spend on importables received the larger portion of the redistributed tariff proceeds as this would increase the demand for exportables and augment the terms of trade movement.

9. THE PROBLEM OF AN OPTIMUM TARIFF

A. INTRODUCTION

The notion of an optimum welfare tariff has received considerable attention in literature dealing with the pure theory of international trade. It is possible, though in a somewhat arbitrary manner, to isolate three separate strands of thought. First, there is the problem of defining an optimum 'welfare' tariff with its attendant problems of equity and of impaired efficiency in resource allocation. Next, if one accepts the notion of an optimum tariff the problem arises of possible retaliation by the other country. Finally, there is the much debated speculative question concerning its probable size.

Inevitably, these issues overlap but for purposes of discussion an attempt is made to review them briefly and separately in Section B. Priority is given to the last aspect because our subsequent analysis provides a priori evidence which modifies Kahn's recent conclusion "that the optimum tariff will often be far from being 'small'" [11, 17-18]. It should be noted that our argument against his belief in a large optimum tariff is a theoretical one which contrasts with the more practical objections of Little [55] and Graaf [61] who, for the most part, did not seek to refute Kahn's statement but rather to show the practical problems that an optimum tariff policy would encounter. Section C develops, for the two and four-commodity cases, alternative optimum tariff criteria in which the explicit appearance and definition of all variables is seen as a considerable improvement upon other formulations. In Section D the analysis extends to the more complex four-commodity world which includes a class of goods in each country that does not enter into international trade. Finally, in Section E, the model is compared with the traditional Kahn-type formula.

B. A REVIEW OF SOME ASPECTS OF OPTIMUM TARIFF THEORY

1. The Notion of an Optimum Tariff

So far no attempt has been made to discuss the possible welfare effects of a shift in the terms of trade as the result of a tariff, the analysis in chapter six establishing merely the direction of this shift. Two separate income distribution effects are discernible. First, an import tax will affect favourably the terms of trade of the tariff-imposing country thereby effecting a distribution of real income from the foreign to the domestic country. Secondly, by its effect upon the demand conditions for each country's produce, a tariff may alter the distribution of real income as between one group of citizens and another within the taxing country; similarly, in the foreign country. Generally, the notion of an optimum 'welfare' tariff has excluded this second category of real income effects from consideration by assuming that each country follows a definite social welfare policy, i.e. maximizes a given social welfare function. Thus its preparedness to trade under different conditions is represented by a pattern of community indifference curves (or surfaces) which is assumed to have the properties of convexity and non-intersection.

We consider first the redistribution of income between countries.

a. Redistribution of World Income

Ignoring the redistribution of income as between different economic groups in the taxing country, one can discern two effects, one favourable to, the other unfavourable to the welfare of the country. On the one hand, the volume of trade is reduced normally, and gains from the international division of labour are sacrificed. These losses appear both in consumption and production where misallocation effects arise from the disparity in the domestic and international prices of the imported commodity. On the other hand, the country benefits from the improved ratio of exchange. When the gain derived from a small increase in the tariff is offset by the losses incurred, the tariff is at an optimum. In terms of the familiar offer curve analysis this is the point where the foreign offer curve touches the highest indifference curve of the tariff-imposing country.

More precisely, in a two-commodity world, this is where the marginal rate of transformation of one commodity into the other, through international trade, is equivalent to the marginal rate of substitution between the two commodities in the domestic market. Since the latter is equal also to the domestic exchange ratio, it follows that the optimum tariff will be where the marginal rate of transformation of one commodity into another, through foreign trade, is equal to the domestic exchange ratio [54,273-78].

b. Redistribution of National Income

Consideration of the effects of income redistribution within the tariff-imposing country introduces the thorny problem of equity and the concept of an optimum tariff loses its precision and ceases to be an operational concept. The alteration in domestic relative prices and in the distribution of the tariff revenue causes different individuals, as producers or consumers, to incur a loss or a gain in real income and the problem arises of comparing one person's loss with another's gain.

Kaldor was the first to acknowledge this difficulty [56]. He admitted that positions of constant real income for the community as a whole need not imply that the real income of each individual be unchanged. Nevertheless, he argued that one might consider the real income of the community constant if those who benefited from the change could compensate exactly those who lost, leaving aggregate real income unchanged. In the event of a surplus gain, the tariff could be regarded as efficient from the national viewpoint.

Compensation being unpaid, however, it is conceivable that the community might suffer a real income loss in the post-tariff situation [54,71-79]. To meet this criticism of the Kaldor criterion, Scitovsky added the additional requirement that it should not be possible for the potential losers to bribe the potential gainers into remaining in the free trade position, without thereby losing more than they would in the post-tariff situation [34]. Little [55] objected strongly to this amended version of the Kaldor criterion, emphasising that potential gains and losses were not proper measures of the adequacy of a tariff. One should compare the actual post-tariff distribution of income with that of the pre-tariff situation and select the more 'beneficial' of the two. In fact, this is what a government attempts to do. At this juncture, however, the economist can proceed no further and must await the decision of the politician on the aspect of equity. Meade [54,77-79] points out a further difficulty. Given that the post-tariff situation with compensation to be paid appears as the most desirable both on equity and efficiency grounds, can we be certain that the method of achieving the compensation will not itself lead to inefficiencies which would make the pre-tariff position the preferable one? Unfortunately, there is no clear-cut answer to this question - it would depend upon the particular circumstances of each situation.

2. Retaliation

As no contribution is made in this chapter to the theory of tariff retaliation, only a brief account of the principal conclusions is given. Kaldor [56,377-80] first acknowledged the possibility of a country gaining from the imposition of a tariff even when retaliation occurred. Despite this acknowledgment, however, economists continued to assume that all countries would lose unambiguously in the event of a tariff war [34; 58,272-3; 59,195]. Recently, this argument was challenged first, by Marsh [60,320] and later, more elaborately, by Johnson [17,31-61] who demonstrated that under certain conditions a country could gain from the imposition of a tariff even in the event of retaliation.

This conclusion has important practical implications. Scitovsky, in his celebrated article [34] contended, on the basis of the optimum tariff argument, that world free trade would not follow automatically as the result of economic self-interest but that it would need to be enforced by international agreement. In an enlightened world, therefore, in which every country suffered from trade restriction, one might expect international accord to be forthcoming readily. But, given that certain countries might gain real income despite retaliation, some incentive would need to be offered them in the way of a real income transfer before they would consent to a re-adoption of the free trade situation.

3. The Probable Size of the Optimum Tariff

That a tariff might improve the terms of trade of the tariff-imposing country was conceded even by such an arch-priest of orthodoxy as Ricardo [52,556]. The practical importance of this possibility, however, was discounted severely. As price effects were considered to be large, it was believed generally that losses due to decreased specialisation would all but cancel any limited gains arising from the slightly improved ratio of exchange. First Mill and later Marshall were to add their support to this opinion of Ricardo [53,27; 12,348] thereby indicating that the optimum tariff would be a small one. Of the neo-classicists only Edgeworth and Bickerdike sounded a warning note. Though both believed that the optimum tariff would normally be small, each hinted that it could be large. Edgeworth was inclined to treat this as a theoretical rather than a practical possibility [15,343] but Bickerdike was once of the opinion that "rather strong assumptions have to be made as to the elasticity of foreign supply and demand if the rate of tax affording maximum advantage is to come below ten per cent" [57,101].

A recent article by Kahn seeks to revive this conclusion of Bickerdike. Into the criterion which he develops, he substitutes assumed numerical estimates of the parameters, concluding 'that the optimum tariff will often be far from being "small", and in the case of some countries (large countries and countries which are idiosyncratic in the nature of their exports or imports) the optimum tariff will be surprisingly great' [11,17-18]. The remainder of this chapter is concerned with this statement and with the methodological implications of the analysis from which it derives.

It should be noted that whereas we dispute the fact that the optimum tariff need normally be large, others have indicated the practical difficulties of applying a large tariff. They do not, however, query the *a priori* reasoning which led Kahn to his conclusion. Little [55] points out that the elasticity coefficients of the formula are themselves related functionally to the tariff. Thus, though a large tariff might be indicated, this element of uncertainty could lead to the imposition of too large a tariff and to a decrease in real income. Hence, Little counselled a small rather than a large tariff because of the uncertainty of the result. That the potential gain could be negligible despite the size of the tariff emphasises his advice.

Graaf [61], on the other hand, argued that a correlation might be expected between the incidence of the tariff and the existence of monopolistic elements in the economy, in which case the mal-effects of impaired resource allocation would need to be deducted from the potential gain.

We turn now to the development of alternative optimum tariff criteria.

C. ALTERNATIVE OPTIMUM TARIFF CRITERIA

In this section alternative optimum tariff criteria are developed for two and four-commodity cases in which the usual assumptions apply: full employment, perfect competition, a balanced balance of trade, each country maximizing a given social welfare function, the government redistributing the tariff revenue to the private sector so that the distribution is random with respect to tastes, etc.

We shall develop first the four-commodity criterion. Consider the set of equations (1.5) to (6.5) which are the basic equilibrium equations of the model. Since $M \equiv \Sigma\, p_j X_j$ a balance of payments equation is implied:

$$p_1 x_1 = p_2' x_2 \qquad \ldots \qquad (1.9)$$

As in other applications of the model quantity units are selected so that $p_1 = p_2' = p_3 = p_3' = 1$ and p_2' is chosen arbitrarily as the numeraire. Bearing this in mind, let us differentiate (totally) equation (1.9) with respect to a change in the tariff, t:

$$x_1 \frac{dp_1}{dt} + \frac{dx_1}{dt} = \frac{dx_2}{dt} \qquad \ldots \qquad (2.9)$$

From our assumptions, it follows that this equation must be satisfied at all times.

Now, a necessary condition for the optimum tariff is that welfare (given a definite social welfare policy which assumes the maximisation of a social welfare function) should be maximised. This will occur when the marginal rate of substitution between goods in domestic consumption is equivalent to their marginal rate of transformation into one another as the result of foreign trade. As these two points must lie concurrently on the foreign offer curve (or surface) it follows that both will be equal to the slope of the tangent to the curve at that point. Moreover, as the domestic exchange ratio between the commodities is also equivalent to the marginal rate of substitution between them, it must also equal their marginal rate of substitution into one another through foreign trade. Thus, at the optimum point, the sum of the domestic prices times the change in quantities of imports and exports respectively must cancel out:

$$\frac{dx_1}{dt} = p_2 \frac{dx_2}{dt} = (1+t)\frac{dx_2}{dt} \qquad (p_1 = 1) \qquad \ldots \qquad (3.9)$$

where $t = (p_2 - p_2') = (p_2 - 1)$ is the tariff.

Substituting for (3.9) in (2.9) we obtain:

$$x_1 \frac{dp_1}{dt} + (1+t)\frac{dx_2}{dt} = \frac{dx_2}{dt}$$

from which it follows that:

$$t = -x_1 \frac{dp_1/dt}{dx_2/dt} \qquad \ldots \qquad (4.9)$$

If the tariff is to be at an optimum this condition must be satisfied. From the development of the model in chapter five, in which p_2' was used as numeraire, it is possible to substitute for the expressions dp_1/dt and dx_2/dt, solutions in terms of elasticities and marginal propensities to spend. $\frac{dp_1}{dt}$ $(= \frac{dp_1}{dp_2'})$ is given in equation (25.5) but it is necessary to obtain the solution for $\frac{dx_2}{dt}$ $(= \frac{dx_2}{dp_2'})$ from the set of equations (18.5) to (21.5):

$$\frac{dx_2}{dt} = \frac{\begin{vmatrix} K_{21}+x_1M_2 & K_{23} & 0 & -K_{22} \\ K_{31}+x_1M_3 & K_{33} & 0 & -K_{32} \\ K_{21}'-x_1M_2' & 0 & K_{23}' & 0 \\ K_{31}'-x_1M_3' & 0 & K_{33}' & 0 \end{vmatrix}}{|B|} \qquad (=|A|)$$

where $|B|$ is defined as in (23.5a). We now proceed to:

1. add column two and column three to column one in $|A|$ and $|B|$ respectively and to make use of the fact that $\sum_j p_j K_{ij} = 0$ to simplify the resulting expressions;
2. interchange columns two and four in $|A|$;
3. expand $|A|$ and $|B|$ from the first two rows according to Laplace;
4. divide top and bottom by $K_{33}K_{33}'$ and convert the resulting expressions into quantity-weighted elasticities and marginal propensities to spend.

Then,

$$\frac{dx_2}{dt} = \frac{x_2}{p_2} \frac{\Psi_{23}(\Psi'_{23}-\psi'_{23})}{\Psi_{23}+\frac{P'_2}{p_2}\psi_{23}+\lambda(\Psi'_{23}-\psi'_{23})} \quad \ldots \quad (5.9)$$

where $\lambda= 1+\psi_{23}(1- \frac{p'_2}{p_2})$.

Making use of the fact that $\Psi'_{23}=\Psi'_{13}$ and that $\psi'_{23}=-1-\psi'_{13}$ we can rewrite (5.9):

$$\frac{dx_2}{dt} = \frac{x_2}{p_2} \frac{-\Psi_{23}(\Psi'_{13}+1+\psi'_{13})}{1+\Psi_{23}+\psi_{23}+\lambda(\Psi'_{13}+\psi'_{13})} \quad \ldots \quad (6.9)$$

Substituting for (6.9) and (25.5) in (4.9) provides us with a four-commodity criterion for the optimum tariff, expressed in terms of quantity weighted elasticities and marginal propensities to spend:

$$t = \frac{1}{-\Psi'_{13} - \psi'_{13} - 1} \quad \ldots \quad (7.9)$$

If all terms involving non-traded commodities are set to zero an equivalent two-commodity criterion is obtained:

$$t = \frac{1}{-\frac{X'_1}{x_1}\sigma'_{11}+ \frac{O'_1}{x_1}S'_{11}+ C'_1 - 1} \quad \ldots \quad (8.9)$$

For the remainder of this section we shall analyse this more simple criterion.

The optimum tariff is indicated by the quantity weighted magnitude of the elasticities and the marginal propensity to spend. For example, at the optimum point if the value of these were two, p_2 must be twice the size of p'_2 and the tariff would be one hundred per cent. Table Six relates different hypothetical values of the weighted partial elasticities and the marginal propensity to spend to corresponding optimum *ad valorem* tariffs. These results may be checked by substituting into the formula.

TABLE SIX

TARIFF RATES AND THE CORRESPONDING ELASTICITY VALUES REQUIRED

Aggregate value of the weighted partial elasticities and propensity to consume	% Tariff
1.50	200
2.00	100
3.00	50
5.00	25
7.66	15
11.00	10
21.00	5

If, on the one hand, the value of the existing tariff should be less than the value of the right-hand side of the formula, an increase in the tariff would be the correct policy. As the elasticities and the amounts traded would not necessarily remain constant following the change in the tariff, a recalculation of the elasticities and an adjustment in the tariff would be necessary.[1] The larger the initial values of the elasticities, the smaller the optimum tariff and the smaller would be the probable change in the elasticities and the adjustment required. On the other hand, where the left-hand side of our formula sums to more than the value of the reciprocal of the elasticities and the marginal propensity to consume minus one, a decrease in the tariff would be the appropriate policy.

1. This point is relevant to the criticism of Little and Graaf [55,70;61,56-7] that as the elasticities of the optimum tariff formula are related functionally to the height of the tariff only the roughest of approximations would be yielded by such a formula. Hence, they argued that as too large a tariff could decrease welfare, a small tariff would be preferable. Assuming, however, that demand functions were reasonably continuous, a good approximation would be given - even in the case of a large tariff. In fact, there seems to be no reason why a few minor adjustments would not give a close approximation to the optimum position, more especially because each successive adjustment would mean smaller and smaller second order differences. It is conceded, however, that one might generally expect demand elasticity to vary inversely with price thus causing a series of oscillations from high to low tariffs dampening to the optimum tariff.

It should be noted that Johnson [17,58] has derived ingeniously from the foreign offer curve an optimum tariff formula that is similar to our own: namely that the tariff will be at an optimum when the tariff rate (p_2-1) is equivalent to the reciprocal of the foreign total elasticity of demand for imports expressed as a function of the barter terms of trade, minus one. This total foreign elasticity of demand is derived from the elasticity of the foreign country's offer curve.[2] Our result may be derived roughly from this formula for, in a two-commodity world, Johnson's total foreign elasticity of demand for exports comprises substitution elasticities of demand and supply and a propensity to consume.

There are several reasons why we have not derived our criterion by an extension of this apparently more simple technique. First, the answer is not intuitively obvious nor does it follow logically from geometric exposition. In particular, the effect of the quantity weights is obscured unless the more tedious algebraic method is used. Secondly, and most important, is the fact that even the most careful dissection of an offer curve (or surface) would fail to yield the complex four-commodity result presented in the next section. Finally, in its total form, the Johnson formula is vulnerable to the criticism that it is expressed in terms of parameters which are not independent of the problem studied and about which there is no knowledge either qualitative or quantitative.

D. FOUR-COMMODITY ANALYSIS

Consider the criterion:

$$t = \frac{1}{-\bar{\psi}'_{13} - \psi'_{13} - 1} \quad \cdots \qquad (7.9)$$

From the formula it can be seen that if the foreign coefficient of sensitivity and ψ'_{13} are small, the optimum tariff will be large:

[2] This relationship states that the elasticity of a reciprocal demand curve is equal to to the total foreign elasticity of demand divided by itself minus one.

$$\frac{2}{1} - 1 = \frac{1}{21 - 1}$$

for a 100% tariff; when they are large, the optimum tariff will be small:

$$\frac{1.05}{1.00} - 1 = \frac{1}{21 - 1}$$

for a 5% tariff.

The close analogy between the two and four-commodity results is apparent; they have the same properties of sign and an identical number of terms, the foreign coefficient of sensitivity being the aggregate quantity weighted substitution term which compares with the sum of the quantity weighted independent supply and demand elasticities of the two-commodity result.

It is possible to specify the precise conditions under which the optimum tariff would be small; namely, when the foreign coefficient of sensitivity (ψ'_{13}) and the foreign marginal propensity term (ψ'_{13}) are large. Conditions for a large ψ'_{13} are:

1. that the foreign own elasticities of supply of and demand for importables should be high - the higher the better. Thus, the rise in the price of importables would see a large transfer of demand away from imports to the other sectors in the economy, and at the same time, a large shift in resources from the rest of the economy to the import-competing sector, both of which effects would reduce the size of the necessary adjustment in the terms of trade. This requirement is the same as for the two good case, but while it is still a necessary condition it is not sufficient to ensure a large foreign coefficient of sensitivity, the other vital determinant being
2. that the degree of competitiveness between non-traded and exportable goods, both in supply and demand, should be considerably greater than the degree of competition between non-traded and importable goods. ~~Given these conditions in the foreign country, it is certain that the price of non-traded commodities would fall relative~~

For example, with low substitution possibilities between good one and the non-traded commodity but high values for the own elasticities of demand for and supply of importables, there would be a large transfer of resources from the exportable to the import-competing sector, rather than from the non-traded to the import-competing sector, and a large shift in demand away from importables to the exportable goods industry rather than to the non-traded sector.

All of these price effects would reduce the size of the necessary shift in the terms of trade. It can be seen how important are these complex relationships which exist between the traded and non-traded commodities. Should the above conditions concerning the cross-elasticities not be fulfilled, the foreign coefficient of sensitivity could be small despite high values for the foreign own elasticities of supply of and demand for importables. This would augment the adverse movement in the terms of trade and increase the size of the optimum tariff.

Finally, it is important but not essential if ϕ'_{13} is to be large; 3. that the foreign ratios of consumption and production of importables to imports be high. This would be ensured if the foreign country produced a large share of its consumption of importables. In this case, the effects of the domestic country's trade policies would have a relatively small effect upon the foreign (or international) prices of the commodities traded.

Let us examine the other component of our formula, ψ_{13}', the foreign marginal propensity term. Generally, ψ_{13}' will be large, and the optimum tariff small:

1. the greater are the foreign marginal propensities to consume non-traded and importable commodities relative to the marginal propensity to consume exportables. A small C_2' in the expenditure - reducing (foreign) country will mean a relatively smaller reduction in expenditure upon exportables at constant prices and, consequently, a smaller adverse movement in the foreign country's terms of trade;
2. the greater the degree of substitutability in production and consumption between non-traded and importable commodities relative to that existing between non-traded and exportable commodities. An explanation of this condition, which conflicts with 2. above, already has been given in detail (see pages 129-131). As its non-fulfilment would have only a minor effect upon the magnitude of ψ_{13}', reducing it in the extreme case approximately to the size of the foreign marginal propensity to consume exportables, C_1', its effect can be ignored.

We turn now to a comparison of our own and the traditional formula.

E. A COMPARISON WITH THE TRADITIONAL FORMULA

In this section we seek to prove that the value of the traditional formula is exceedingly dubious when a meaningful interpretation is given to the elasticity concepts in terms of which it is expressed. Further, it is contended that these improperly defined elasticity coefficients obscure vital relationships.

For purposes of comparison Kahn's derivation of the traditional formula is set out below. Our equations (2.9) and (3.9) are formally identical to those appearing in his paper [11,15-16, equations (1) and (2)] except that he uses a different numeraire, thus giving an additional term in p_2' when (2.9) is differentiated totally.

Rearranging equations (2.9) and (3.9) and including all prices even though they they are unity, we obtain:

$$p_1 dx_1\left(1+\frac{x_1 dp_1}{p_1 dx_1}\right) = p_2' dx_2\left(1+\frac{x_2 dp_2'}{p_2' dx_2}\right) \quad \ldots \quad (9.9)$$

and

$$p_2' dx_2 = \frac{p_1 dx_1}{(1+t)} \quad \ldots \quad (10.9)$$

where $t = (p_2/p_2'-1)$.

Denoting a 'foreign elasticity of demand for exports' by

$$\epsilon = -\frac{dx_1}{dp_1}\frac{p_1}{x_1}$$

and a 'foreign elasticity of supply of imports' by

$$\eta = \frac{dx_2}{dp_2'}\frac{p_2'}{x_2}$$

it follows from equations (9.9) and (10.9) that the optimum tariff requires that:

$$t = \frac{\frac{1}{7} + \frac{1}{\epsilon}}{1 - \frac{1}{\epsilon}}$$

which is the Kahn formula that has appeared so frequently in optimum tariff literature [17,60; 61; 15; 55; 62; 63; 100].

Trouble has been taken to derive the Kahn criterion in this manner because it enables us to link it the more readily with our own result and, at the same time, it facilitates an assessment of the elasticity coefficients.

It is difficult to be sure of Kahn's intention in his working of the problem as at no stage does he define clearly what is meant by his concept of elasticity. Johnson has commented:

> This formula must be interpreted with care, since the elasticities are defined in terms of the partial differentials of quantities with respect to prices, not in terms of partial derivatives as

> the conventional price elasticities are. The two definitions of elasticity are only identical when the good is independent of other goods in both consumption and production, otherwise, the elasticities of the formula must be interpreted as measures of the response of quantities to prices when all the repercussions of general equilibrium adjustment have been worked out [17,61].

In other words, Johnson would interpret the elasticities of the formula either as independent partials or as total elasticities which embody "all repercussions of general equilibrium adjustment".

Let us consider the total interpretation. Of the two elasticities enumerated above, one disappears completely from the final formula when care is taken to develop it further, i.e. if the terms dx_1/x_1 and dx_2/x_2 are broken into their component substitution elasticities and marginal propensities. The supply elasticity of the traditional formula (derived from dx_2/x_2) is in fact a redundant term. This fact is borne out by Table Seven below in which the numerical examples given are the same as those used in the text of Kahn's original paper and upon which he based his claim that the optimum tariff might normally be large. The reader should note that only where the supply term goes to the reciprocal of infinity and vanishes do the two formulae coincide.

TABLE SEVEN

COMPARISON OF THE TWO FORMULAE WHEN THE TRADITIONAL ELASTICITIES ARE CONSIDERED AS TOTAL ELASTICITIES

Values of the elasticities Demand	Supply	% Tariff Kahn Formula	% Tariff Our Formula
5 (∞) *	∞	25	25 (0)
20 (∞) *	∞	5	5 (approx),(0)
5	5	50	25 "
20	5	25	5 "

* As explained in the text these estimates are my own.

In this table it is assumed that the first terms in the denominator of the right-hand side of equation (7.9) are equivalent to Kahn's 'foreign elasticity of demand for exports' (derived from dx_1/x_1) when this elasticity is considered as a total. As the total foreign elasticity of supply of exports (derived from dx_2/x_2) is eliminated from our formula, the size of the elasticities in column two does not affect our optimum tariff.

It should be noted that if either total elasticity has a value of infinity, the other elasticity likewise must be infinite. In such a case, the optimum tariff would be zero (as shown by the bracketed figures). This, of course, is the classical case in which a tariff would fail to benefit the tariff-imposing country.

Generally, economists have interpreted the elasticities of the conventional formula as independent of other goods both in consumption and in production, i.e. as partial elasticities measuring the response of a commodity to a change in its own price, <u>all other prices and money spending being held constant</u>. More will be said concerning the potential dangers of such an approach. The point we are concerned with here is that the traditional formula is seriously misleading even when this interpretation is adopted. Graaf [61] was of the opinion that the traditional formula was correct if one considered all cross elasticities (and all terms involving the non-traded commodities) to vanish identically. If in our multi-commodity formula, however, all such terms are cancelled out, we are left with the two-commodity result (equation (8.9)). The reader is invited to try this experiment.

Table Eight presents the results of a comparison between the two formulae when the traditional elasticities are treated as independent partials. In our own formula (8.9) $\frac{X_1'}{x_1}\sigma_{11}'+C_1'$ is considered to be the partial elasticity of demand and $\frac{O_1'}{x_1}S_{11}'$ is the partial elasticity supply. (Though $\frac{O_1'}{x_1}S_{11}'$ refers to importables and the traditional elasticity refers to exportables, the two are not unrelated, for in a four-good world in which all elasticities involving the non-traded good vanish identically, $\frac{O_1'}{x_1}S_{11}' = \frac{O_2'}{x_2}S_{22}'$ (see pages 115-116)).

TABLE EIGHT

COMPARISON OF THE TWO FORMULAE WHEN THE TRADITIONAL ELASTICITIES ARE CONSIDERED AS MARSHALLIAN PARTIALS

Values of the elasticities Demand	Supply	% Tariff Kahn Formula	% Tariff Our Formula
5	∞	25	0
20	∞	5	0
5	5	50	11 (approx.)
20	5	25	4

Once again we have substituted Kahn's numerical examples into our own formula and it is evident that his result is generally misleading. Our formula shows that a zero tariff would be the optimum tariff when one of the elasticities is infinite, and confirms the classical conclusion that a tariff would fail to improve the terms of trade of the tariff-imposing country when the world's offer curve was infinitely elastic (as it must be when any of the partial elasticities are infinite).

It is emphasised that the formal validity of the traditional criterion is not disputed. Rather, an attempt has been made above to show that it is incorrect when the terms in which it is expressed are meaningfully interpreted. Its usefulness must be highly suspect as it provides us with little more than a tautology. Moreover, the elasticities which appear in it cannot be interpreted as parameters that are independent of the problem studied nor is anything known concerning them either of a qualitative or a quantitative nature. Finally, even if the assumption of vanishing cross elasticities were correct, to ignore them in the context of the present problem is unjustifiable. This leads us to another point concerning the probable size of the optimum tariff.

It is evident that only a rash prediction could be made on the basis of the two-commodity result, even when this is expressed in terms of conceivably measurable concepts. The complex relationships between traded and non-traded commodities could and probably would play a vital role in any necessary adjustments to equilibrium. It has been shown in an earlier chapter that the magnitude of the foreign coefficient of sensitivity will be less than the quantity weighted sum of the own elasticities of demand for and supply of importables in the foreign country. This does not mean, however, that it must be small. It could as conceivably be large as small and, at present, one can say no more.

In addition to the above reasons for not accepting the case for a large optimum tariff, one may query Kahn's conclusion that we would be concerned with short-term elasticities only. He felt that where tariff policy was concerned a country would tend to concentrate on the more immediate future the more serious were its economic difficulties. Today, however, tariffs normally are used to effect structural changes in an economy and not for the solution of short-term balance of payments problems. Hence, it is probable that the elasticities of our formula would be long-term elasticities and that their value, therefore, would be correspondingly higher.

Finally, it is claimed that our multi-commodity result provides an insight into the question of negative tariffs (subsidies). Graaf's conclusion that the terms of trade could move in either direction [61,54] led him to the further conclusion that the optimum tariff could be a subsidy. Discussing the four or n-commodity case, he expressed dissatisfaction with the traditional formula because:

> "... it obscures the fact that the optimum tariff is sometimes a subsidy. This fact is interesting because it throws light on the usual discussion of tariffs and the terms of trade, in which the possibility of a subsidy on imports being required to turn the terms of trade in one's favour is seldom considered." [61,54]

For a tariff to improve 'welfare', the terms of trade must move in favour of the tariff-imposing country. The optimum tariff will be a subsidy, i.e. a negative tariff, only if a tariff shifted the terms of trade in an adverse direction. In chapter six we saw that this would be the case in a four-commodity world only in exceptional if not impossible circumstances. It follows that it is highly improbable that the optimum tariff would ever be a subsidy.

The close analogy between the two- and four-commodity formulae has been remarked upon already. They have the same properties of sign and an identical number of terms, ψ'_{13} being the aggregate, quantity-weighted substitution term which compares with the sum of the quantity-weighted supply and demand elasticities of the two-commodity result.

Two further important implications are apparent from our reformulated criterion which do not follow from the traditional result because of its ill-defined elasticities. First, when the sum of the elasticities is less than unity, it is not possible to bring the two sides of the formula to equality, i.e. a result is not possible. This, of course, is what we should expect. Since the point of departure for the derivation of the formula is the usual one of assuming the tariff-imposing country's tariff to be at an optimum, it is not applicable to a situation where an optimum tariff cannot be defined. It is well known that when the foreign country's elasticities are less than unity, a tariff will always improve welfare [34]. In this situation the optimum tariff will appear to be infinitely large - until the elasticities change or until imports are nil. This may be explained more simply in terms of two-dimensional offer curve analysis. It can be shown [12,336-7] that the offer curve will bend backwards when the elasticity of it is less than unity. In this situation it is not possible for the foreign offer curve to be tangential at any point along the inelastic part of the curve to the tariff-imposing country's indifference curves; their general direction of slope is diagonally opposed and they will, in fact, intersect. Thus, no point along this range of the offer curve would represent a true equilibrium position.

In an analogous fashion, if ψ'_{13} and ψ'_{13} sum to less than unity, it is not possible to equate the two sides of the formula. If this situation prevailed the foreign offer surface would not be tangential to the tariff-imposing country's indifference surfaces; the two would always intersect.

In a particular situation of this type it is most probable that successive tariff increases will reveal the position where the foreign elasticities change so that they cease to be less than unity. An optimum position then becomes definable.

Secondly, the role of the domestic elasticities in the determination of the optimum tariff is indicated clearly by the presence of the quantity weights in our formula. These reflect the ratios of foreign consumption and production of exportables to the quantity of them imported, and as these amounts are dependent upon domestic as well as foreign elasticities, it follows that the optimum tariff is functionally related to the elasticities of both countries.

F. CONCLUSIONS

Briefly, the principal conclusions of this chapter are:

1. that the traditional optimum tariff formula, because of its improperly defined elasticity concepts, is definitely misleading and that it fails completely to provide a meaningful theorem concerning the optimum tariff;
2. that there are possibly fewer grounds for considering the optimum tariff to be large than there are for considering it to be small;
3. that any formula dealing with the problem of an optimum tariff cannot afford to ignore the relationships between traded and non-traded commodities;
4. that the optimum tariff will never be a subsidy.

10. CONSUMPTION AND PRODUCTION TAXES

A. INTRODUCTION

That part of the theory of international trade which has dealt with the effect of taxes upon the terms of trade has concentrated, as far as I am aware, almost entirely upon the effect of a tax or subsidy on imports or exports. The effect of consumption and production taxes upon different economic magnitudes appears to have been neglected. It is our purpose, therefore, in this chapter, to establish multi-commodity criteria for:

1. the effect upon (a) the international terms of trade and (b) the domestic price ratio of traded goods of a consumption tax levied on importables; and
2. the effect upon (a) the taxing country's terms of trade and (b) the taxing country's traded goods factor price ratio of a production tax levied on importables.

These criteria enable qualitative answers to be given to such questions as whether the terms of trade necessarily need improve as the result of consumption tax and, if so, whether the extent of the improvement could reverse the initial shift in the taxing country's traded goods price ratio, etc. We consider now a general consumption tax upon importables.

B. CONSUMPTION TAXES

1. Consumption Taxes and the Terms of Trade

The introduction of a general consumption tax upon importables necessitates a distinction between the price relevant to consumers and that relevant to producers. It means that p_2, the price of importables to consumers, exceeds p_2', the international price which enters into the supply functions of the taxing country, by the amount of the tax per unit. The reader will note that again we adopt the convention of assuming the domestic country to be the one which levies the tax.

The equilibrium equations of the model are:

$$x_1 = O_1 - X_1 = X_1' - O_1' \quad \ldots \quad (1.10)$$

$$x_2 = X_2 - O_2 = O_2' - X_2' \quad \ldots \quad (2.10)$$

$$0 = X_3 - O_3 \quad \ldots \quad (3.10)$$

$$0 = X_3' - O_3' \quad \ldots \quad (4.10)$$

$$M = O_1p_1+O_2p_2'+O_3p_3+(p_2-p_2')X_2 = O_1p_1+O_2p_2+O_3p_3+(p_2-p_2')x_2 \quad \ldots \quad (5.10)$$

$$M' = O_1'p_1+O_2'p_2'+O_3'p_3' \quad \ldots \quad (6.10)$$

These equations formally are the same as those for the model developed to study the effect of a tariff upon the terms of trade (see equations (1.5) to (6.5)). However, the difference in the supply functions in the two applications of the model is stressed. As before, $X_i=X_i(p_1,p_2,p_3,M)$ where p_2', the foreign price of importables, is selected as numeraire, and quantity units are chosen so that $p_1=p_2'=p_3=p_3'=1$; but $O_i=O_i(p_1,p_3)$, the foreign price of importables (which is a datum in the domestic supply functions) disappearing because of its role as numeraire.

Bearing this in mind, we now proceed to differentiate the equations (2.10) to (6.10) - the equation (1.10) being dropped because it is implied by the fact that $M\equiv\sum_i p_iX_i$. We have, after substituting for the differentiated income equations,

$$dx_2=(X_{21}-O_{21})dp_1+X_{22}dp_2+(X_{23}-O_{23})dp_3+M_2[O_1dp_1+O_2dp_2+O_3dp_3+(p_2-1)dx_2+x_2dp_2] \quad \ldots \quad (7.10)$$

$$dx_2=-(X_{21}'-O_{21}')dp_1-(X_{23}'-O_{23}')dp_3'-M_2'(O_1'dp_1+O_3'dp_3') \quad \ldots \quad (8.10)$$

$$0 =(X_{31}-O_{31})dp_1+X_{32}dp_2+(X_{33}-O_{33})dp_3+M_3[O_1dp_1+O_2dp_2+O_3dp_3+(p_2-1)dx_2+x_2dp_2] \quad \ldots \quad (9.10)$$

$$0 =(X_{31}'-O_{31}')dp_1+(X_{33}'-O_{33}')dp_3'+M_3'(O_1'dp_1+O_3'dp_3') \quad \ldots \quad (10.10)$$

Next,

1. substitute in the remaining equations for $X_{ij}-O_{ij}=K_{ij}-X_jM_i$, and for X_i-O_i the appropriate x_i or zero;

2. cancel and collect terms wherever possible; and

3. rearrange the equations so that

$$\left.\begin{aligned}
&(K_{21}+x_1M_2)dp_1+K_{23}dp_3-[1-(p_2-1)M_2]dx_2 = -a_{22}dp_2\\
&(K_{31}+x_1M_3)dp_1+K_{33}dp_3+M_3(p_2-1)dx_2 = -a_{32}dp_2\\
&(K'_{21}-x_1M'_2)dp_1+K'_{23}dp'_3+dx_2 = 0\\
&(K'_{31}-x_1M'_3)dp_1+K'_{33}dp'_3 = 0
\end{aligned}\right] \quad \ldots \qquad (11.10)$$

Our measure of the change in the terms of trade due to a consumption tax is: dp_1/dp_2.

Solving for this we obtain:

$$\frac{dp_1}{dp_2} = \frac{\begin{vmatrix} -a_{22} & K_{23} & 0 & -[1-M_2(p_2-1)] \\ -a_{32} & K_{33} & 0 & M_3(p_2-1) \\ 0 & 0 & K'_{23} & 1 \\ 0 & 0 & K'_{33} & 0 \end{vmatrix}}{\begin{vmatrix} K_{21}+x_1M_2 & K_{23} & 0 & -[1-M_2(p_2-1)] \\ K_{31}+x_1M_3 & K_{33} & 0 & M_3(p_2-1) \\ K'_{21}-x_1M'_2 & 0 & K'_{23} & 1 \\ K'_{31}-x_1M'_3 & 0 & K'_{33} & 0 \end{vmatrix}} \qquad (=|A|)$$

Making use of the fact that $\sum_j p_jK_{ij}=0$, but noting that $p_1O_{21}+p_3O_{23}=-p'_2O_{22}$, we can rewrite the above solution:

$$\frac{dp_1}{dp_2} = \frac{|A|}{\begin{vmatrix} -p_2a_{22}+p'_2O_{22}+x_1M_2 & K_{23} & 0 & -[1-M_2(p_2-1)] \\ -p_2a_{32}+p'_2O_{32}+x_1M_3 & K_{33} & 0 & M_3(p_2-1) \\ -K'_{22}-x_1M'_2 & 0 & K'_{23} & 1 \\ -K'_{32}-x_1M'_3 & 0 & K'_{33} & 0 \end{vmatrix}} \quad \ldots \qquad (12.10)$$

$$(|B|)$$

A Laplacian expansion from the first two rows gives:

$$\frac{dp_1}{dp_2} = \frac{\begin{vmatrix} a_{22} & K_{23} \\ a_{32} & K_{33} \end{vmatrix} (K_{33}')}{\begin{vmatrix} p_2a_{22}-p_2'O_{22}-x_1M_2 & K_{23} \\ P_2a_{32}-p_2'O_{32}-x_1M_3 & K_{33} \end{vmatrix} (K_{33}') + \begin{vmatrix} K_{22}'+x_1M_2' & K_{23}' \\ K_{32}'+x_1M_3' & K_{33}' \end{vmatrix} \begin{vmatrix} K_{23} & M_2(p_2-1)-1 \\ K_{33} & M_3(p_2-1) \end{vmatrix}}$$

Finally,

1. divide top and bottom by $x_2(K_{33}.K_{33}')$;

2. expand the resulting determinants; and

3. convert the result into quantity weighted substitution elasticities and marginal propensities to spend (remembering first, that because free trade prices are unity, $x_1=x_2$; and secondly, that $-\psi_{23}'=1+\psi_{13}'$, and $\Psi_{23}'=\Psi_{13}'$).

Then,

$$\frac{dp_1}{dp_2}\frac{p_2}{p_1} = \frac{\bar{\Psi}_{23}}{1+\lambda^*(\Psi_{13}'+\psi_{13}')+\Psi_{23}+\psi_{23}^*} \qquad \ldots \qquad (13.10)$$

where $\lambda^*=1+\psi_{23}^*(1-\frac{p_2'}{p_2})$ is normally positive and less than unity because

$$\psi_{23}^* = -C_2 + C_{32}\left(\frac{X_2\sigma_{23}-O_2S_{23}}{X_3\sigma_{33}-O_3S_{33}}\right)$$

is negative and less than unity (except for the abnormal case where $K_{23}<0$). Additionally,

$$\bar{\Psi}_{23} = \frac{X_2}{x_2}\sigma_{22}-\left(\frac{X_2}{x_2}\sigma_{23}-\frac{O_2}{x_2}S_{23}\right)\left(\frac{\sigma_{32}}{\sigma_{33}-S_{33}}\right)$$

is the same as the coefficient of sensitivity, Ψ_{23}, from which all supply reactions involving the change in price of the traded commodity are excluded. It follows that $\bar{\Psi}_{23}$ has the same sign property as its corresponding coefficient of sensitivity although its absolute value is less.

Equation (13.10) is our formula for the rate of change in the terms of trade, p_1, due to a consumption tax levied upon importables. By setting to zero all terms which involve the non-traded commodity, we obtain a two-commodity result:

$$\frac{dp_1}{dp_2}\frac{p_2}{p_1} = \frac{\frac{X_2}{x_2}\sigma_{22}}{1+\lambda\left(\frac{X_1'}{x_1}\sigma_{11}' - \frac{O_1'}{x_1}S_{11}' - C_1'\right) + \frac{X_2}{x_2}\sigma_{22} - \frac{O_2}{x_2}S_{22} - C_2} \quad \ldots \quad (14.10)$$

where $\lambda = 1 - C_2(1 - \frac{p_2'}{p_2})$.

Let us first analyse the more simple two-commodity result. As σ_{22} is negative, the sign of the formula is determinate, given that stability exists in the international market, i.e. that our denominator is negative. This means that p_1 will rise relative to p_2' when a general tax is put on the consumption of importables, the terms of trade improving unambiguously for the taxing country.

This result confirms one's intuitive expectation that a tax upon the consumption of importables would shift demand in the domestic country into exportables, thereby creating a world excess supply of the commodity and a subsequent improvement in the terms of trade.

The non-appearance of supply reactions in the numerator is explained by the fact that our numerator registers only impact effects. The change in the tax alters the relative price structure facing consumers, but leaves unaffected the relative prices which concern producers. The latter will alter only in response to a change in the terms of trade, the magnitude of the movement of which they partly determine. Hence their appearance in the denominator. A tariff, on the other hand, affects both consumers and producers, its direct effect being to reduce the demand for importables and to increase the supply of them. It follows that the excess demand generated by a consumption tax, which is determined by consumer reactions only, must be less than the excess demand generated by an equivalent tariff that affects producers as well as consumers. A consumption tax, therefore, creates a smaller excess world supply of importables and, as a result, needs a smaller shift in the terms of trade to restore equilibrium.

While one might derive intuitively the two-commodity result, it is difficult to see how the four-commodity case could be handled without recourse to the more complex analysis that we have used. There are several significant points both of similarity and of contrast between the four and the two-commodity formulae. In the first place, and most important, the result is qualitatively the same, $\bar{\psi}_{23}$ being negative by the stability conditions. This result is analogous to the conclusion that a tariff would improve the terms of trade of the taxing country in a four-commodity world (see equation (2.6)). Secondly, the more complex result differs from the two-commodity conclusion in that supply elasticities appear in the numerator as determinants of the direction of the shift in the terms of trade. Again, this is analogous to the four-commodity terms of trade result, except that in the case of the consumption tax there are no supply reactions with respect to a change in the domestic price of good two, p_2. This reflects the fact that production of commodity two is responsive to the tax-exclusive and not the tax-inclusive price of the good, and that the former can shift only if the terms of trade are affected. Finally, as in the two-good case, the fact that $\bar{\psi}_{23}$ is less in absolute value than ψ_{23} indicates that a consumption tax would generate a smaller excess supply of importables than would an equivalent tariff and, consequently, that it would require a relatively smaller corrective movement in the terms of trade. This conclusion would be mitigated if substitution possibilities are low generally in production relative to demand.

It is difficult, in the case of a consumption tax upon importables, to generalise about the effect of non-traded commodities upon the magnitude of the shift in the terms of trade for, as we have seen in our study of the tariff question, while the magnitude of each coefficient of sensitivity would tend to be less than the magnitude of the two-commodity term it replaces, there is no *a priori* reason for expecting the difference between the propensity terms in the formula to exceed or be less than their counterparts in a two-good result. This conclusion is subject to the qualification that where the coefficients of sensitivity are high, one might expect them to increase the magnitude of the terms of trade movement indicated by the two-commodity result because, in this case, the effect upon the final result of the propensity terms, which individually cannot exceed unity, is small.

2. Consumption Taxes and the Domestic Price Ratio of Traded Goods

Next, let us examine the effect of a general consumption tax on importables upon the market price ratio of the internationally traded goods in the domestic country. We ask whether a consumption tax upon importables could actually cause the market price of importables to fall relative to that of exportables. The problem is analogous to the Metzler argument considered earlier, which concluded that a tariff conceivably might result in a fall in the taxing country's market price of importables relative to its market price for exportables.

Our measure of the rate of change in the market price ratio of the internationally traded commodities in the taxing country is $d(p_2/p_1)/(p_2/p_1)$. As $p_1=p_2'=1$,

$$\frac{d(p_2/p_1)}{p_2/p_1} = \frac{dp_2}{p_2} - \frac{dp_1}{p_1}$$

or

$$\frac{d(p_2/p_1)}{p_2/p_1}\frac{p_2}{dp_2} = 1 - \frac{dp_1}{dp_2}\frac{p_2}{p_1} \quad .$$

Substituting for $\frac{dp_1}{dp_2}\frac{p_2}{p_1}$ from (13.10) and ignoring the effect of λ^*, which arises because of a prior tariff, we have:

$$\frac{d(p_2/p_1)}{p_2/p_1}\;\frac{p_2}{dp_2} = \frac{1+\Psi_{13}'+\bar{\bar{\Psi}}_{23}+\psi_{13}'+\psi_{23}^*}{1+\Psi_{13}'+\psi_{13}'+\Psi_{23}+\psi_{23}^*} \qquad \ldots \qquad (15.10)$$

where

$$\bar{\bar{\Psi}}_{23} = -\frac{O_2}{x_2}S_{22} - \left(\frac{X_2}{x_2}\sigma_{23} - \frac{O_2}{x_2}S_{23}\right)\left(\frac{-S_{32}}{\sigma_{33}-S_{33}}\right)$$

is the same as the coefficient of sensitivity, ψ_{23}, from which are excluded all demand reactions involving the change in the price of the traded good. Like $\bar{\psi}_{23}$, $\bar{\bar{\psi}}_{23}$ has the same properties of sign as its corresponding coefficient of sensitivity but is less in absolute magnitude. It follows that $\bar{\psi}_{23}+\bar{\bar{\psi}}_{23}=\psi_{23}$, and that $\bar{\psi}_{23}<\psi_{23}>\bar{\bar{\psi}}_{23}$.

A two-commodity result can be obtained by setting to zero all the terms in (15.10) which involve the non-traded commodities:

$$\frac{d(p_2/p_1)}{p_2/p_1}\frac{p_2}{dp_2} = \frac{1+\frac{X_1'}{x_1}\sigma_{11}' - \frac{O_1'}{x_1}S_{11}' - \frac{O_2}{x_2}S_{22} - C_1' - C_2}{1+\frac{X_1'}{x_1}\sigma_{11}' - \frac{O_1'}{x_1}S_{11}' + \frac{X_2}{x_2}\sigma_{22} - \frac{O_2}{x_2}S_{22} - C_1' - C_2} \quad \ldots \quad (16.10)$$

The condition under which a consumption tax on importables will increase the domestic market price of exportables relative to the price of importables is, given stability in the international market, that:

$$-\bar{\psi}_{13}' - \bar{\bar{\psi}}_{23} < \psi_{23}' + \psi_{23}^*(\psi_{23}' = -1-\psi_{13}') \quad \ldots \quad (17.10)$$

Similarly, in the two-commodity case, the criterion is:

$$-\frac{X_1'}{x_1}\sigma_{11}' + \frac{O_1'}{x_1}S_{11}' + \frac{O_2}{x_2}S_{22} < C_2' - C_2(C_2'=1-C_1') \quad \ldots \quad (18.10)$$

A comparison of these criteria with the corresponding ones derived for an import tax (equation (27.5a and b)) establishes the fact that a perverse movement in the domestic country's market price ratio is even less likely in the case of a consumption tax. The reason is obvious. In the tariff case, a perverse result depends upon a terms of trade effect sufficiently strong to reverse the initial tendency for the tariff to increase the domestic price of importables relative to the price of exportables in the taxing country. For reasons pointed out above, the magnitude of the terms of trade effect will be reduced where an equivalent consumption tax is used instead of a tariff. The argument against a consumption tax on importables having a perverse price effect holds, therefore, a fortiori. Thus, while it is true that if a tariff will cause the domestic price of importables to rise so too will a consumption tax, it is not true that in the unlikely event of a tariff increasing the price of exportables relative to the price of importables, a consumption tax upon importables necessarily will do likewise.

C. PRODUCTION TAXES

1. Production Taxes and the Terms of Trade

This time we seek criteria in the two and the multi-commodity cases for the effect upon the terms of trade of the taxing country of a production tax on importables. Once again, a distinction is necessary between the price ratios which confront consumers and producers; the price of importables, p_2, which enters as a datum into the taxing country's supply functions, being less than the market price, p_2', by the amount of the tax.

The equations which describe the equilibrium of our system are:

$$x_1 = O_1 - X_1 = X_1' - O_1' \quad \ldots \quad (19.10)$$

$$x_2 = X_2 - O_2 = O_2' - X_2' \quad \ldots \quad (20.10)$$

$$0 = X_3 - O_3 \quad \ldots \quad (21.10)$$

$$0 = X_3' - O_3' \quad \ldots \quad (22.10)$$

$$M = O_1p_1+O_2p_2+O_3p_3+(p_2'-p_2)O_2 = O_1p_1+O_2p_2'+O_3p_3 \quad \ldots \quad (23.10)$$

$$M' = O_1'p_1 + O_2'p_2' + O_3'p_3' \quad \ldots \quad (24.10)$$

These equations are the same as the set (1.10) to (6.10) except that the domestic expenditure equation now states that expenditure is equal to earned income plus the taxation receipts which we assume to be redistributed among consumers.

We select as numeraire p_2', the market price of importables, and choose to drop the supply-demand equation involving the taxing country's exportable commodity. Differentiating totally, making use of the fact that $\sum_i p_i dO_i = 0$, and substituting for dM and dM′ in the first four equations, we obtain:

$$dx_2 = (X_{21}-O_{21})dp_1 - O_{22}dp_2 + (X_{23}-O_{23})dp_3 + M_2(O_1dp_1+O_3dp_3) \quad \ldots \quad (25.10)$$

$$dx_2 = -(X'_{21}-O'_{21})dp_1 - (X'_{23}-O'_{23})dp'_3 - M'_2(O'_1dp_1+O'_3dp'_3) \quad \ldots \quad (26.10)$$

$$0 = (X_{31}-O_{31})dp_1 - O_{32}dp_2 + (X_{33}-O_{33})dp_3 + M_3(O_1dp_1+O_3dp_3) \quad \ldots \quad (27.10)$$

$$0 = (X'_{31}-O'_{31})dp_1 + (X'_{33}-O'_{33})dp'_3 + M'_3(O'_1dp'_1+O'_3dp'_3) \quad \ldots \quad (28.10)$$

Next, we proceed to

1. substitute in the remaining equations for $(X_{ij}-O_{ij})=(K_{ij}-X_jM_i)$, and for X_i-O_i the appropriate x_i or zero;
2. cancel and collect terms wherever possible; and
3. rearrange the equations so that

$$(K_{21}+x_1M_2)dp_1 + K_{23}dp_3 - dx_2 = O_{22}dp_2$$

$$(K_{31}+x_1M_3)dp_1 + K_{33}dp_3 = O_{32}dp_2$$

$$(K'_{21}-x_1M'_2)dp_1 + K'_{23}dp'_3 + dx_2 = 0$$

$$(K'_{31}-x_1M'_3)dp_1 + K'_{33}dp'_3 = 0$$

Since $p'_2=1$ is our numeraire, dp_2 is our measure of the production tax. Solving for the rate of change in the terms of trade, dp_1, which occurs as the result of a production tax, we have:

$$\frac{dp_1}{dp_2} = \frac{\begin{vmatrix} O_{22} & K_{23} & 0 & -1 \\ O_{32} & K_{33} & 0 & 0 \\ 0 & 0 & K'_{23} & 1 \\ 0 & 0 & K'_{33} & 0 \end{vmatrix}}{\begin{vmatrix} K_{21}+x_1M_2 & K_{23} & 0 & -1 \\ K_{31}+x_1M_3 & K_{33} & 0 & 0 \\ K'_{21}-x_1M'_2 & 0 & K'_{23} & 1 \\ K'_{31}-x_1M'_3 & 0 & K'_{33} & 0 \end{vmatrix}} \quad (=|A|)$$

which, if use is made of the relationship that $\sum_j p_j K_{ij}=0$, gives us:

$$\frac{dp_1}{dp_2} = \frac{|A|}{\begin{vmatrix} -a_{22}+p_2O_{22}+x_1M_2 & K_{23} & 0 & -1 \\ -a_{32}+p_2O_{32}+x_1M_3 & K_{33} & 0 & 0 \\ -K'_{22}-x_1M'_2 & 0 & K'_{23} & 1 \\ -K'_{32}-x_1M'_3 & 0 & K'_{33} & 0 \end{vmatrix}} \qquad (=|B|)$$

A Laplacian expansion of $|A|$ and $|B|$ from the first two rows yields:

$$\frac{dp_1}{dp_2} = \frac{\begin{vmatrix} -O_{22} & K_{23} \\ -O_{32} & K_{33} \end{vmatrix} K'_{33}}{\begin{vmatrix} a_{22}-p_2O_{22}-x_1M_2 & K_{23} \\ a_{32}-p_2O_{32}-x_1M_3 & K_{33} \end{vmatrix} K'_{33}+\begin{vmatrix} K'_{22}+x_1M'_2 & K'_{23} \\ K'_{32}+x_1M'_3 & K'_{33} \end{vmatrix} K_{33}}$$

Finally,

1. divide top and bottom by $K_{33}.K'_{33}$;
2. expand the resulting determinants; and
3. convert the result into quantity weighted substitution elasticities and marginal propensities to spend (remembering first, that because free trade prices are unity, $x_1=x_2$; and secondly, that $-\psi'_{23}=1+\psi'_{13}$ and $\Psi'_{23}=\Psi'_{13}$).

Then,

$$\frac{dp_1}{dp_2}\frac{p_2}{p_1} = \frac{\bar{\bar{\Psi}}_{23}}{1+\Psi'_{13}+\psi'_{13}+\Psi_{23}+\psi_{23}^{*}} \qquad \ldots \qquad (29.10)$$

where $\bar{\bar{\Psi}}_{23}$ is defined as in equation (15.10) and $\psi_{23}^{*}=-C_2+C_3\left(\frac{X_2\sigma_{23}-O_2S_{23}}{X_3\sigma_{33}-O_3S_{33}}\right)$ which is negative and less than unity unless $K_{23} < 0$.

As in the case of the consumption tax, the two-commodity result can be obtained by setting to zero all terms which involve a non-traded commodity. Thus,

$$\frac{dp_1}{dp_2}\frac{p_2}{p_1} = \frac{-\frac{O_2}{x_2}S_{22}}{1+\frac{X_1'}{x_1}\sigma_{11}' - \frac{O_1'}{x_1}S_{11}' + \frac{X_2}{x_2}\sigma_{22} - \frac{O_2}{x_2}S_{22} - C_1' - C_2} \quad \ldots \quad (30.10)$$

As dp_2 is negative for an increase in the production tax, p_2 falling relative to p_2', it follows from the negative numerator in both criteria that, if stability is assumed in the international market, the terms of trade of the taxing country must deteriorate, p_2' rising relative to p_1. This is the result that one would expect from a two-commodity analysis of the problem, the direct effect of the tax being to diminish the supply of importables (the consumption of them remaining constant), thereby creating (at constant cost) a world excess demand for them and a consequent deterioration in the terms of trade.

This unambiguous conclusion is not, however, evident intuitively when non-traded commodities are present, for in the multi-commodity case the result does not depend entirely upon the supply reactions of producers, consumers also being affected by the shift in relative prices which involve the non-traded commodity. Nevertheless, should all supply reactions be zero, there would be no change in the terms of trade and even if the taxing country's own importable supply substitution elasticity, S_{22}, is high, a negligible terms of trade movement could occur if substitution possibilities between importables and non-traded commodities are high compared with those existing between exportables and non-traded commodities and with those existing between the traded commodities.

In passing, it should be noted that the above criteria for the effect of a production tax upon the terms of trade are converted readily into criteria for the effect of a subsidy upon the terms of trade, by remembering that, for a subsidy, dp_2 will be positive. In this case, the terms of trade of the subsidising country would improve unambiguously.

2. Production Taxes and Relative Prices at Factor Cost

Finally, let us consider the effect of a production tax upon the factor cost price ratio of traded goods in the taxing country. Our measure of the rate of change in this price ratio is $d(p_2/p_1)/(p_2/p_1)$. As $p_1=p_2'=1$, we know that:

$$\frac{d(p_2/p_1)}{p_2/p_1} = \frac{dp_2}{p_2} - \frac{dp_1}{p_1}$$

or

$$\frac{d(p_2/p_1)}{p_2/p_1}\,\frac{p_2}{dp_2} = 1 - \frac{dp_1}{dp_2}\,\frac{p_2}{p_1}$$

Substituting for $\frac{dp_1}{dp_2}\,\frac{p_2}{p_1}$ from (29.10) we obtain:

$$\frac{d(p_2/p_1)}{p_2/p_1}\,\frac{p_2}{dp_2} = \frac{1+\Psi_{13}'+\psi_{13}'+\bar{\Psi}_{23}+\psi_{23}^*}{1+\Psi_{13}'+\psi_{13}'+\bar{\Psi}_{23}+\psi_{23}^*}$$

where $\bar{\Psi}_{23}$ is defined as in equation (13.10) and ψ_{23}^* as in equation (29.10).

Given stability in the international market, and making use of the fact that $1+\psi_{13}'=-\psi_{23}'$, we have as a criterion for the rate of change in the domestic factor price ratio of the taxing country:

$$-\Psi_{13}' - \bar{\Psi}_{23} \gtreqless -\psi_{23}' + \psi_{23}^* \qquad \ldots \qquad (31.10)$$

Similarly, making use of the relationship that $1-C_2'=C_1'$, we have an analogous two-commodity criterion:

$$-\frac{X_1'}{x_1}\sigma_{11}' + \frac{O_1'}{x_1}S_{11}' - \frac{X_2}{x_2}\sigma_{22} \gtreqless C_2' - C_2 \qquad \ldots \qquad (32.10)$$

In each case, if the difference between the propensity terms exceeds the ~~absolute~~ value of the sum of the quantity weighted substitution elasticities, the domestic factor price ratio of the taxing country would shift so that the price of exportables would fall relative to the domestic factor price of importables. This, of course, would need a strong adverse movement in the terms of trade and for reasons advanced earlier in the cases of a tariff and of a consumption tax on importables, it is extremely unlikely that the necessary conditions would be fulfilled.

Finally, the analogy between equations (31.10) and (32.10), and (17.10) and

(18.10) is pointed out; the only difference is that in the case of a consumption tax it is the additional supply reactions which decrease the probability of the perverse case, while in the case of a production tax, it is the addition of demand reactions which decreases such a probability.

11. ECONOMIC GROWTH AND THE TERMS OF TRADE

A. INTRODUCTION

The inaugural lecture of J.R. Hicks [67] was based on a model which would appear to have outlived the problem it purported to explain - namely, the post-war dollar gap. His contention that the dollar shortage may have arisen because of the tendency, reflected in the shift in the terms of trade, for technological improvement in the United States to be concentrated in the import-competing sector, has been subjected to a considerable amount of criticism, in the course of which a valuable collection of literature has been added to international trade theory [72; 73; 74; 71,512-19; 66; 70; 68; 69; 77; 78].

The effect of expanding productivity upon the international terms of trade had implications which extended beyond the immediate problem of the dollar gap. In particular, the answer to the question was of vital importance to the well-being of the emerging, underdeveloped areas of the world and it was with this point especially in mind that different theoreticians commenced to refine and extend Hicks' original stimulating contribution.

Today, a different aspect of the same problem concerns those primary producing nations whose economic interests are jeopardised by the impending entry of the United Kingdom to the European Common Market. Since the end of the war, no single group of countries has experienced increases in output comparable with the Common Market. Inevitably, Britain's entry to this expanding area would entail a closer economic association between it and the primary producing nations of the British Commonwealth. What, one might ask, would be the effect of continued economic expansion in Europe upon the terms of trade of these primary producers?

Having posed the problem let us be equally brief in stating that no solution to it will be offered here. Such a solution would involve, among other things, a detailed quantitative knowledge about existing tariff levels, demand and supply functions, etc., in each of these trading blocks. Moreover, not only do we lack the necessary quantitative information but, additionally, any conclusion would need to be mere conjecture until negotiations have been finalised.

Given our abysmal ignorance concerning the magnitude of the relevant parameters, however, a further problem still exists. Is our theoretical analysis sufficiently advanced to warrant its useful application to the problem? In particular, are the conclusions derived from the different models expressed in terms of parameters which are conceivably measurable in practice? Should this not be the case, the theorems involved will have a limited degree of practical application; their primary function being to indicate what might happen if certain unknown parameters were of such and such a magnitude.

It is the purpose of this chapter first, to review briefly some of the existing literature concerning the effect of economic expansion on a country's terms of trade and, secondly, to develop a model with the modest intent of extending slightly the degree of generality possessed by earlier formulations. At the same time results are expressed in terms of parameters which are independent of the problem under review and, at least, measurable conceivably. In this manner, a logical framework is provided within which a more comprehensive quantitative study of the problem might one day ensue.

Section B of this chapter reviews briefly some of the existing literature. This condensation is not intended to be comprehensive nor is it intended entirely as a review but rather to point out the reasons for the apparently contradictory results which derive from the two major approaches to the subject.

In Section C a four-commodity model is developed which provides criteria for the effect of an increase in productivity upon the expanding country's terms of trade and upon its real income. Finally, in Section D, these results are analysed and compared with other two and quasi-three-commodity ones, our conclusion being that even where in other models a non-traded commodity has been postulated, its important influence upon both the magnitude and the direction of the shift in the terms of trade has not been emphasised sufficiently.

B. THE LITERATURE

In reviewing the original conclusions of Hicks and the developments which ensued, it is the purpose of this section to emphasise that the answers to the problems which he raised depend on the type of model used. Our interest centres upon the original problem: namely, the effect of economic expansion on the terms of trade; other effects of economic growth upon international trade are ignored, including a country's relative dependence upon trade.

The formal problem which concerned Hicks was the effect of increasing productivity on a country's terms of trade when productivity in other countries remained static. His model was a two-country one in which the balance of payments was always in equilibrium. Additionally, the increase in output was assumed due to improved productivity - the supplies of the factors of production remaining constant. These assumptions are retained in this section.

His two principal conclusions were:

1. Should the productivity increase be uniformly distributed over all of the expanding country's industries, its terms of trade would most likely deteriorate.
2. Should the increase in output be concentrated entirely in the import-competing sector, the expanding country's terms of trade would improve. Moreover, this conclusion would still hold in the less extreme case in which growth occurred the more rapidly in the import-competing industry.

Subsequent analysis concentrated for the most part on four main types of model though, of course, an infinite variety of them is conceivable. That of Hicks envisaged three commodity groups comprising exportables, non-traded goods, and imports; the degree of substitutability between the latter two classes being less than infinite both in consumption and production. Throughout, Hicks assumed the domestic industries (exportable and non-traded) to be operating subject to constant costs. For this reason he described his model as Ricardian and this nomenclature is retained in this paper.

A further type of model, used by Mishan [72], Corden [73] and Johnson [66], assumes two countries and two commodities and may be described as of a 'Heckscher-Ohlin' variety, there being some production and consumption of both commodities in the expanding country, i.e. incomplete specialisation. It differs in two ways from the Ricardian model. In the first place, there are two instead of three commodities,[1] there being perfect substitutability between imports and import-competing goods. In the second place, there are increasing and not constant costs, which means that if factors move from the importable to the exportable sector, exportable production costs must rise while those of the importable sector must fall. This type of model lends itself readily to both algebraic and geometric exposition, and developments of it along these lines have extended considerably the theoretical literature on this topic.

The simplest model used might be described as ultra-Ricardian for it postulates the complete specialisation of each country in the production of its own export good. In relation to the Heckscher-Ohlin model this is the only modification. Again, this model has been most usefully developed by Johnson [66].

[1] One writer has interpreted the Ricardian model used by Hicks as a four-good one which implies, of course, that similar conditions exist in the foreign country. We shall see later that the presence of an extra commodity in the other country, while it affects the magnitude of the terms of trade movement, does not affect the direction of the shift in the terms of trade.

A fourth variety of model has been used by Black [75] and is called Marshallian because of its use of independent elasticities of supply and demand for imports and exports. Though it includes conceptually a class of non-traded commodities, it is subject to such rigorous and implausible assumptions that any conclusions derived from it must be evaluated with the utmost caution. Among the problems which it assumes away are all interrelations between traded and non-traded markets, and real income effects upon the terms of trade. A good deal has been written already in this thesis about the merits and demerits of the partial equilibrium approach. In this chapter we shall not engage in further controversy, concentrating instead upon comparing the other models described above with the four-commodity one developed in the next section. We now proceed to analyse the different models commencing with the simplest of the three.

1. The Ultra-Ricardian Model

In this case the expanding country produces a single commodity or group of commodities. A productivity increase means that the domestic (expanding) country can produce more of its exportable commodity with its given supply of factors than it could before. Consequently, the real cost of exportables declines. Imagine that factor incomes are expanded so as to maintain a constant export price level. Provided only that some of the increase in factor incomes is spent on imports, it follows that the demand for exportables will have expanded less than, and the demand for imports more than, the supply of them. Given stability in the foreign exchange market, equilibrium can be restored only by a deterioration in the terms of trade of the expanding country. The ultra-Ricardian model, therefore, supports the first contention of Hicks that a general productivity increase would affect unfavourably the terms of trade of the growing country.

2. The Ricardian Model

In the Ricardian model there are three commodities - exportables, non-traded goods, and imports, the latter being distinct from non-traded goods; and constant costs operate in all industries. Thus, a movement of factors from one industry to another will not affect their relative prices. Indeed, the assumption of constant costs infers that the two commodities produced in the expanding country must be perfectly substitutable for one another on the production side; the reason that non-traded goods are not exported being the existence of transport costs.

Consider first the case of a uniform increase in productivity at constant prices in the non-traded and exportable good industries. Once again factor incomes are assumed to have expanded accordingly. As long as a portion of the increased expenditure is allocated to <u>imports</u>, the terms of trade must deteriorate. This would be so even where an initial excess demand for exportables was created by the pattern of allocation of the new expenditure. With constant costs the excess demand would be eliminated by a transfer of resources from the non-traded goods industry, a net excess demand for imports always remaining. To rectify this, the terms of trade would need to deteriorate.

Next, consider the case of an increase in productivity within the non-traded good industry. Because of the presence of constant costs one can assume no longer that factor incomes would expand in such a way that the price of the non-traded good would remain constant. Instead, initially, the price of the non-traded good must fall. Hicks, considering only the substitution effects that would follow, concluded that the terms of trade of the expanding country would improve. It was left to A. Asimakopulos to point out that the real income effect of the price change could invalidate this conclusion [74, 228-33]. In other words, the final effect upon the terms of trade would depend upon the income and substitution effects of the change in the price of the non-traded commodity.

3. The Heckscher-Ohlin Model

Answers to the problems posed by Hicks have been obtained also within the context of a model whose antecedents are those of the Heckscher-Ohlin type: two commodities, two countries, some production of importables within each country, given supplies of the factors of production, increasing costs, etc. In each case, assuming stability in the international market and that productivity increases are not factor-biased,[2] the conclusions reached contradict those established by the Hicksian model:

1. where the increase in output is uniform in all industries, the direction of the shift in the terms of trade will depend on the proportions in which the increased expenditure is allocated to the consumption of either good; and
2. that when the increase in output occurs entirely in the importable good industry the terms of trade must improve (excepting the case where exportables are considered as inferior goods).

Consider the first conclusion. When productivity increases uniformly the proportionate rise in output will be the same in both industries. If factor incomes are raised in proportion to the productivity increase the relative prices of the commodities will not change. In other words, there will be an unbiased expansion of output at constant commodity prices.

What of the effect upon demand? Obviously the real income of the factors of production must have increased because of the rise in factor money earnings, the prices of the commodities being held constant. It is evident that if the increased supply of commodities is removed from the market at constant prices there can be no excess supply of or demand for either product and, consequently, no tendency for the terms of trade to change.

[2] See p.210 where it is noted that a factor-biased productivity increase could alter the results.

If the expansion of demand as the result of an increased expenditure is to be unbiased, the income elasticities of demand for both commodities must be unity - demand rising proportionately for each commodity. In such a case, the terms of trade must deteriorate unambiguously for, while the proportionate increases in the demand for and the supply of importables are equal, the absolute increase in demand must exceed the absolute increase in supply as long as domestic production is less than domestic consumption. This conclusion is subject, of course, to the requirement that neither the foreign elasticity of demand for nor supply of importables should be infinite.

It was Mishan [72] who pointed out first that a demand expansion biased sufficiently towards exportables would require an improvement in the terms of trade, thus qualifying the original proposition of Hicks that an unbiased productivity increase need cause the terms of trade to deterioriate necessarily.

Where the productivity increase occurs solely in the importable good industry, the conclusion derived from the analysis of the Heckscher-Ohlin type model is that the terms of trade will generally improve. The argument, in brief, is as follows. On the production side output in the importable sector rises while, initially, the level of industrial activity in the exportable good industry remains constant. Furthermore, the price of the importable commodity does not change, factor incomes expanding to take up the increase in productivity. This cannot be an equilibrium position, however, as perfect competition requires the same rate of return to factors in all industries. Factors would flow, therefore, from the exportable to the importable sector until increasing costs in the importable good industry and decreasing costs in the exportable good industry equated factor returns in the two sectors. This means that the output of importables would expand by more than the initial increase. It follows that if the marginal propensity to spend output on importables is less than unity in the domestic country, an excess supply of importables must occur, with a subsequent improvement in the expanding country's terms of trade.

A relaxation of the assumption that productivity increases are factor-neutral introduces an element of ambiguity into the above conclusions [77]. In fact, a productivity increase could result in a factor bias as readily as in a productivity bias. If, given the prices of capital and labour, the productivity increase raises the optimum ratio of capital to labour in the importable industry, the productivity increase may be termed capital-using in the importable sector. This means that labour becomes plentiful in relation to capital which causes the price of capital to rise and the output of the capital-intensive industry to contract relative to the labour-intensive one. To determine the total bias on the production side at constant prices, both product and factor biases need to be evaluated. The final assessment of the effect upon the terms of trade would need to take account also of the degree of bias on the consumption side.

4. Summary

There is much to be said for dealing with one problem at a time. Accordingly, we assume that a productivity increase will be factor-neutral. Subject to this assumption, Table Nine below summarizes the characteristics and results of each model.

TABLE NINE

A SUMMARY OF THE EFFECTS OF ECONOMIC EXPANSION UPON THE TERMS OF TRADE

Model	Characteristics of the Model		Uniform Productivity Increase. Must terms of trade deteriorate?	Productivity increase entirely in importable(non-traded)sector.Must terms of trade improve?
	Cost conditions	Number of Products		
Ultra-Ricardian	Constant	Two	Yes	N.A.
Heckscher-Ohlin	Increasing(at least in one industry)	Two	No	Yes
Ricardian	Constant	Three	Yes	No

One need not search far to account for the conflicting results presented in the Table. Obviously, those derived from the ultra-Ricardian model can be attributed to the assumption of complete specialisation. No one would contend that this is a realistic assumption and we proceed without comment to the other more complex models. On the one hand, in the event of a uniform productivity increase, the conclusions differ not because of the extra commodity in the Ricardian model, but because of the differing cost assumptions. In the Ricardian model the relief of any excess demand in the domestic market by the price mechanism is unnecessary, factors merely transferring under constant cost conditions to eliminate such an excess. On the other hand, where productivity increases entirely in the importable sector, it would appear that the difference in the results can be attributed to the number of commodities postulated in each model. This difference, however, is superficial rather than real, for the model used by Hicks is only two-commodity on the production side. The important point is that he assumes there is no domestic production of importables and that the adherents of the Heckscher-Ohlin type approach do. Obviously, an increase in productivity which is concentrated entirely in the import-competing sector will have vastly different results to one which is concentrated entirely in the non-traded good sector of an economy. The two problems are quite distinct and any comparison of the two models should bear this in mind.

An amalgam of the two approaches would appear to be a worthwhile task. Accordingly, in the next section, a general equilibrium model is developed in which there are three commodities consumed and produced in each country (there being a class of non-traded goods in each economy). The restrictive Hicksian assumption of constant costs is not retained but the specific inclusion of an import-competing as well as a non-traded industry enables the vital role of substitution and income effects to be emphasised fully. We turn now to the development of this model.

C. DEVELOPMENT OF THE MODEL

Throughout this section the convention of a foreign and a domestic country is adopted and no assumptions are made about the type of goods produced. The basic assumptions of the model, with one or two additions, are as set out in chapter three: two countries and four commodities, each country producing an importable, an exportable and a non-traded commodity; perfect competition exists both in the factor and product markets of each country; full employment of resources and a balanced balance of trade are ensured at all times by the free working of the price mechanism. Additionally, it is assumed that capital, technology and labour are immobile as between countries - only goods moving internationally. The increase in output is assumed due to improved productivity. Finally, it should be noted that we assume any increase in output to occur at constant relative commodity prices - factor incomes being adjusted accordingly. For the moment, our interest centres upon the establishment of a general criterion for the effect of an equal absolute increase in output in each country upon the domestic country's international terms of trade.

The equilibrium equations of our model are:

$$x_1 = O_1 - X_1 = X_1' - O_1' \quad \ldots \quad (1.11)$$

$$x_2 = X_2 - O_2 = O_2' - X_2' \quad \ldots \quad (2.11)$$

$$0 = X_3 - O_3 \quad \ldots \quad (3.11)$$

$$0 = X_3' - O_3' \quad \ldots \quad (4.11)$$

$$M = O_1 p_1 + O_2 p_2 + O_3 p_3 \quad \ldots \quad (5.11)$$

$$M' = O_1' p_1 + O_2' p_2 + O_3' p_3' \quad \ldots \quad (6.11)$$

These equations are the same as the set of equations (7.3) to (12.3) which describe the equilibrium of our basic free trade model, there being no tariffs, transport costs, or other impediments to trade.

Since $M \equiv \sum_j p_j X_j$, one of the supply-demand equations is not independent and a balanced balance of trade ($p_1 x_1 = p_2 x_2$) is implied. In our working of the problem the x_1 equations are discarded and p_2, the price of domestic importables, is used as numeraire. By a suitable choice of quantity units all prices are set equal to unity initially.

This time, we must allow for the effect of a given equal absolute increase in output in terms of the numeraire in each country upon the form of the supply and demand functions. Thus $X_j = X_j(p_1, p_3, M, G)$ and $O_j = (p_1, p_3, G)$, where G is the given increase in output in the domestic country, which is zero initially. Similarly, in the foreign country, $X'_j = (p_1, p'_3, M', G')$ and $O'_j = (p_1, p'_3, G')$.

Differentiating the equations (2.11) to (6.11) totally we have, after using the fact that the $\sum_i p_i dO_i = 0$ to reduce (5.11) and (6.11),

$$dx_2 = (X_{21} - O_{21})dp_1 + (X_{23} - O_{23})dp_3 + (m_2 - n_2)dG + M_2 dM \quad \ldots \quad (7.11)$$

$$dx_2 = -(X'_{21} - O'_{21})dp_1 - (X'_{23} - O'_{23})dp'_3 - (m'_2 - n'_2)dG' - M'_2 dM' \quad \ldots \quad (8.11)$$

$$o = (X_{31} - O_{31})dp_1 + (X_{33} - O_{33})dp_3 + (m_3 - n_3)dG + M_3 dM \quad \ldots \quad (9.11)$$

$$o = (X'_{31} - O'_{31})dp_1 + (X'_{33} - O'_{33})dp'_3 + (m'_3 - n'_3)dG' + M'_3 dM' \quad \ldots \quad (10.11)$$

$$dM = O_1 dp_1 + O_3 dp_3 \quad \ldots \quad (11.11)$$

$$dM' = O'_1 dp_1 + O'_3 dp'_3 \quad \ldots \quad (12.11)$$

where $n_i = \partial O_i / \partial G$ and $m_i = \partial X_i / \partial G$.

Next,

1. substitute in (7.11) to (10.11) for dM and dM′;
2. in the resulting equations substitute for $X_{ij} - O_{ij} = K_{ij} - X_j M_i$; and
3. for the terms $(X_i - O_i)$ which arise after these substitutions, substitute the appropriate x_1, x_2 or zero as indicated by the equilibrium equations;
4. cancel and collect terms wherever possible; and
5. rearrange the equations, so that

$$\left.\begin{aligned}
(K_{21}+x_1M_2)dp_1 + K_{23}dp_3 - dx_2 &= -(m_2-n_2)dG \\
(K_{31}+x_1M_3)dp_1 + K_{33}dp_3 &= -(m_3-n_3)dG \\
(K'_{21}-x_1M'_2)dp_1 + K'_{23}dp'_3 + dx_2 &= -(m'_2-n'_2)dG' \\
(K'_{31}-x_1M'_3)dp_1 + K'_{33}dp'_3 &= -(m'_3-n'_3)dG'
\end{aligned}\right\} \quad \ldots \qquad (13.11)$$

Now, by assumption, $dG=dG'$ so that it does not matter whether the change in the terms of trade, dp_1, is expressed with respect to dG or dG'. Solving, we have:

$$\frac{dp_1}{dG} = \frac{\begin{vmatrix} -(m_2-n_2) & K_{23} & 0 & -1 \\ -(m_3-n_3) & K_{33} & 0 & 0 \\ -(m'_2-n'_2) & 0 & K'_{23} & 1 \\ -(m'_3-n'_3) & 0 & K'_{33} & 0 \end{vmatrix}}{\begin{vmatrix} K_{21}+x_1M_2 & K_{23} & 0 & -1 \\ K_{31}+x_1M_3 & K_{33} & 0 & 0 \\ K'_{21}-x_1M'_2 & 0 & K'_{23} & 1 \\ K'_{31}-x_1M'_3 & 0 & K'_{33} & 0 \end{vmatrix}} \qquad (= |A|)$$

which, if we make use of the fact that $\sum_j p_j K_{ij}=0$, can be rewritten:

$$\frac{dp_1}{dG} = \frac{|A|}{\begin{vmatrix} -K_{22}+x_1M_2 & K_{23} & 0 & -1 \\ -K_{32}+x_1M_3 & K_{33} & 0 & 0 \\ -K'_{22}-x_1M'_2 & 0 & K'_{23} & 1 \\ -K'_{32}-x_1M'_3 & 0 & K'_{33} & 0 \end{vmatrix}} \qquad (= |B|)$$

Finally,

1. expand $|A|$ and $|B|$ from the first two rows according to Laplace;
2. divide top and bottom by $K_{33}.K'_{33}$; and
3. convert the resulting expressions into quantity weighted substitution elasticities and marginal propensities (remembering that all prices are unity, and because of this $x_1 = x_2$).

Then,

$$\frac{dp_1 x_1}{dGp_1} = \frac{-\nu_{23} - \nu'_{23}}{\Psi_{23} + \Psi'_{23} + \psi_{23} - \psi'_{23}} \qquad \ldots \qquad (14.11)$$

where $\nu_{ij} = -(c_i - s_i) + (c_j - s_j)\left(\frac{\sigma_{ji} - S_{ji}}{\sigma_{jj} - S_{jj}}\right)$,

$c_i = p_i m_i$ is the marginal propensity to spend an increase in output on the i^{th} good, and

$s_i = p_i n_i$ is the marginal propensity to produce the i^{th} good out of a given increase in output at constant prices.

If the increase in output should occur only in the domestic country ν'_{23} would be zero so that

$$\frac{dp_1 x_1}{dGp_1} = \frac{-\nu_{23}}{\Psi_{23} + \Psi'_{23} + \psi_{23} - \psi'_{23}} = H \qquad \ldots \qquad (15.11)$$

is our criterion for the effect of an increase in output, dG, expressed as a proportion of the value of trade, upon the terms of trade of the domestic (expanding) country. The corresponding formula for the foreign country is

$$\frac{dp_1 x_1}{dG' p_1} = \frac{-\nu'_{23}}{\Psi_{23} + \Psi'_{23} + \psi_{23} - \psi'_{23}} = H' \qquad \ldots \qquad (16.11)$$

From (15.11) and (16.11) it follows that if the change in output in each country expressed as a proportion of the value of trade should not be equal, the rate of change in the terms of trade is:

$$\frac{dp_1}{p_1} = \frac{-\nu_{23}(dG/x_1) - \nu'_{23}(dG'/x_1)}{\Psi_{23} + \Psi'_{23} + \psi_{23} - \psi'_{23}} = H\frac{dG}{x_1} + H'\frac{dG'}{x_1} \qquad \ldots \qquad (17.11)$$

For all these formulae, corresponding two-commodity results can be obtained by setting to zero all terms which involve the non-traded commodities. Thus, where the rate of growth of output as a proportion of the value of trade is the same in each country, (14.11), we have,

$$\frac{dp_1}{dG}\frac{x_1}{p_1} = \frac{(c_2-s_2) + (c_2'-s_2')}{E} \quad \ldots \quad (18.11)$$

where

$$E = \frac{X_2}{x_2}\sigma_{22} - \frac{O_2}{x_2}S_{22} + \frac{X_2'}{x_2}\sigma_{22}' - \frac{O_2'}{x_2}S_{22}' - C_2 + C_2' .$$

If the increase in output is in the domestic country only (15.11),

$$\frac{dp_1}{dG}\frac{x_1}{p_1} = \frac{c_2 - s_2}{E} \quad \ldots \quad (19.11)$$

if in the foreign country only (16.11),

$$\frac{dp_1}{dG'}\frac{x_1}{p_1} = \frac{c_2' - s_2'}{E} \quad \ldots \quad (20.11)$$

Finally, for the general case where output expands at a different rate in each country (17.11),

$$\frac{dp_1}{dG}\frac{x_1}{p_1} = \frac{(c_2-s_2)\frac{dG}{x_1} + (c_2'-s_2')\frac{dG'}{x_1}}{E} \quad \ldots \quad (21.11)$$

Assuming that our system is stable, it follows that in any of the criteria (14.11) to (21.11) the direction of shift in the terms of trade depends upon the sign of the numerator, being favourable to the domestic country when it is negative and adverse for the domestic country when it is positive.

D. ANALYSIS

1. The Terms of Trade, Two-Commodity Analysis

For purposes of discussion, let us concentrate upon the case in which the increase in output occurs entirely in the domestic country. We commence with the two-commodity result (19.11). From inspection, it can be seen that the direction of the shift in the terms of trade is dependent, assuming stability in the international market, upon the sign of the numerator: being favourable (adverse) to the expanding country if c_2 is less (more) than s_2.

If, in either a two or four-commodity world, the terms of trade are to be affected by an expansion in the output of one of the countries at constant relative commodity prices, an excess in the demand for or in the supply of one of the internationally traded commodities must be generated within the expanding country. Now, when a country's production is not completely specialised its demand for imports is equal to the difference between its total consumption and supply of the importable commodity. Hence, the net effect upon the demand for imports which follows an increase in output is determined first, by the amount of the increase in output which occurs in the import-competing industry and, secondly, upon the extent to which the increased income generated is spent, at constant prices, upon importables. Should the absolute increase in the demand for, exceed (be less than) the absolute increase in the supply of the commodity, an excess demand for (supply of) importables will arise in the expanding country. Given that equilibrium in the balance of payments is maintained by the price mechanism, it follows that this excess demand (supply) can be eliminated only by a deterioration (improvement) in the terms of trade of the domestic country. More specifically, in the two-commodity case, this would be when its marginal propensity to spend output on the consumption of importables, c_2, exceeds (is less than) its marginal propensity to produce importables, s_2, when output expands at constant prices.

2. <u>The Terms of Trade, Four-Commodity Analysis</u>

Even when the presence of non-traded goods is allowed for explicitly (equation (15.11)), the terms of trade can only change if, as a consequence of growth, there is an excess demand for or supply of one of the internationally traded commodities. This time, however, internal price effects operate so as to alter the magnitude of the critical propensity terms. Imagine a situation in which the values of the two marginal propensities were equal, the increase in expenditure due to the increase in output being allocated in such a way as to cancel exactly the increase in output in the import-competing industry. In a two-commodity world there would be no price change in either industry and no shift in the terms of trade. If non-traded commodities exist, however, it is reasonable to assume that some of the increase in expenditure would be allocated to them. Unless the marginal propensity to spend output on the consumption of non-traded goods, c_3, is equal to the marginal propensity to produce these goods, s_3, the price of non-traded commodities must be affected: rising (falling) if c_3 (s_3) is greater than s_3 (c_3). The only exception, as long as c_3 is not equal to s_3, to the necessity of this shift in the non-traded good price, is where either the own elasticity of demand for, σ_{33}, or supply of, S_{33}, the commodity is infinite (e.g. the special case of Hicks discussed above).

In the event of the price of the non-traded good rising (falling) demand would transfer away from (into) the non-traded good sector and resources would flow towards (away from) it, the direction of the shift in demand and supply depending upon the degree of substitutability existing, on the one hand, between non-traded and exportable commodities and, on the other hand, between non-traded and exportable goods. Inevitably, this shift in the consumption pattern and in the flow of resources would affect the size of the marginal propensity terms themselves, so as to reduce, cancel, or even outweigh the original difference between c_2 and s_2.

Generally, four cases can be distinguished.

1. Where c_3 exceeds s_3 and non-traded commodities are more substitutable (both in consumption and production) for exportables than for importables. In this case the rise in the price of the non-traded commodity due to the excess demand for it must result (excepting in all cases where infinite elasticities exist) in a proportionate flow of resources toward the non-traded sector that would come primarily from the exportable good

industry. On the demand side, purchasing power would transfer, for the most part, from the non-traded to the exportable sector. The net effect of these two reactions to the increase in the price of non-traded commodities would be, therefore, to increase proportionately the demand for exportables more than the demand for importables, at the same time as the proportionate reduction in the supply of exportables exceeded that of importables. It follows that if c_2 exceeded s_2 initially, the difference could be reduced, cancelled or even outweighed and that the possibility of a favourable movement in the terms of trade of the expanding country would be improved.

2. Where c_3 exceeds s_3 but non-traded commodities are more substitutable for importables than for exportables. In this case the flow of resources towards, and of demand from, the non-traded sector would concern the importable more than the exportable good industry; the probability of a favourable movement in the terms of trade of the expanding country being reduced accordingly.

3. Where c_3 is less than s_3 and non-traded commodities are more substitutable for exportables than for importables. This time the price of the non-traded commodity must fall as a result of the excess supply of it. As a consequence of the substitution bias, the flow of resources from and the transfer of demand to the non-traded sector would concern the exportable more than the importable sector. The possibility of a favourable terms of trade movement for the expanding country would be decreased.

4. Finally, where c_3 is less than s_3 but non-traded commodities are more substitutable for importables than for exportables. In this case, the fall in the price of the non-traded good would induce a proportionately heavier flow of resources from the non-traded to the import-competing than to the exportable sector, and a proportionately greater shift of demand from importables to non-traded goods than from exportables to non-traded goods. The possibility of a favourable movement in the terms of trade of the expanding country would be increased.

Table Ten summarizes these results:

TABLE TEN

INFLUENCE OF NON-TRADED COMMODITY ON THE MARGINAL PROPENSITIES

c_3 greater than or less than s_3	Non-traded goods relatively more substitutable with Exportables	Importables
c_3 greater than s_3	Probability of expanding country's terms of trade improving increased	Probability of expanding country's terms of trade improving decreased
c_3 less than s_3	Probability of expanding country's terms of trade improving decreased.	Probability of expanding country's terms of trade improving increased.

Though the above Table classifies in probability terms the different influences of the non-traded commodity upon the terms of trade, it does not enable us to determine whether or not its influence would be strong enough to reverse the effect of an initial difference in size between c_2 and s_2. What, for instance, can be said of a situation in which c_2 initially exceeds s_2 but c_3 is less than s_3? We have seen that the result will depend upon the substitution possibilities in the system, but to what extent is this so? Table Eleven classifies the different cases which might exist in a four-commodity world; in some of these cases, the magnitude of the substitution possibilities could be decisive.

TABLE ELEVEN

SUMMARY OF CRITERIA FOR EXPANDING COUNTRY'S TERMS OF TRADE CHANGE

Case	Criteria	Terms of trade effect on expanding country
1.	$c_2 > s_2$ $c_3 = s_3$ $c_2 + c_3 > s_2 + s_3$	Adverse
2.	$c_2 > s_2$ $c_3 > s_3$ $c_2 + c_3 > s_2 + s_3$	Adverse

TABLE ELEVEN (CONTINUED)

Case	Criteria	Terms of trade effect on expanding country
3.	$c_2 > s_2$ $c_3 < s_3$ $c_2 + c_3 > s_2 + s_3$	Adverse
4.	$c_2 > s_2$ $c_3 < s_3$ $c_2 + c_3 < s_2 + s_3$	Indeterminate. The result would depend on the substitution possibilities.
5.	$c_2 = s_2$ $c_3 = s_3$ $c_2 + c_3 = s_2 + s_3$	Zero terms of trade effect.
6.	$c_2 = s_2$ $c_3 > s_3$ $c_2 + c_3 > s_2 + s_3$	Adverse
7.	$c_2 = s_2$ $c_3 < s_3$ $c_2 + c_3 < s_2 + s_3$	Favourable
8.	$c_2 < s_2$ $c_3 = s_3$ $c_2 + c_3 < s_2 + s_3$	Favourable
9.	$c_2 < s_2$ $c_3 < s_3$ $c_2 + c_3 < s_2 + s_3$	Favourable
10.	$c_2 < s_2$ $c_3 > s_3$ $c_2 + c_3 > s_2 + s_3$	Indeterminate. The result would depend upon the substitution possibilities.
11.	$c_2 < s_2$ $c_3 > s_3$ $c_2 + c_3 < s_2 + s_3$	Indeterminate. The result would depend upon the substitution possibilities.

When comparing the two and four-commodity results it is important to be sure of an author's intentions when developing a two-commodity model. On the one hand, his result may refer to a world in which non-traded commodities are assumed absent. This is similar to the partial equilibrium approach which assumes that non-traded goods exist although their effects are negligible, i.e. that all substitution elasticities which involve the non-traded commodity are zero. On the other hand, it is conceivable that non-traded commodities might be classified either as importables or exportables depending on their relative degree of substitutability. It has been shown that this latter interpretation obscures vital relationships, so that no idea can be obtained about the manner in which the non-traded good affects the terms of trade reaction to economic expansion. Moreover, our results are incomparable - the parameters being defined differently in each case. The first interpretation, therefore, is favoured.

Of the eleven possible cases listed in the Table, numbers four, six, seven, ten and eleven all refer to situations in which the two-commodity criterion, namely, $c_2 \gtreqless s_2$, is invalidated by the addition of a non-traded commodity to the model. Five of the eleven cases, however, are special ones which involve equalities between the propensities and they are ignored throughout the remainder of the chapter. Of the remaining six possibilities there are three in which the two-good criterion is no longer applicable; in which a conclusion founded upon the two-commodity criterion could be invalidated. In case four, for instance, c_2 exceeds s_2 initially and one would expect an excess demand for importables with a subsequent deterioration in the terms of trade to follow. It is conceivable, however, that a high degree of substitutability between importables and non-traded goods relative to that existing between exportables and non-traded goods could result in a transfer of resources to the importable good industry, and a flow of purchasing power away from importables, sufficiently large to cancel and even outweigh the original difference between c_2 and s_2. The net effect of the non-traded commodity, therefore, would be to change an adverse into a favourable terms of trade movement.

The converse argument holds a fortiori in cases ten and eleven.

What are the implications of our analysis concerning the two problems to which Hicks addressed himself? Consider first, the case of a uniform increase in productivity in all industries. Our conclusion agrees with the one derived from the Heckscher-Ohlin type rather than the Ricardian model: that the terms of trade need not necessarily deteriorate. In addition to pointing out the importance of the manner in which demand expands, however, it is also necessary to stress the role played by substitution possibilities between non-traded and importable goods on the one hand, and between non-traded and exportable goods on the other. Their presence could enhance, diminish or even invalidate the qualification made to the original Hicksian proposition by Mishan and others.

When productivity increases solely in the import-competing industry our analysis supports, with qualifications, the conclusion of the Heckscher-Ohlin type model that the terms of trade of the expanding country improve. Once again the expansion of output at constant prices requires that resources flow from the exportable and the non-traded commodity sectors to the import-competing sector. Production of importables, therefore, increases by more than the initial expansion in output. As long as the marginal propensity to spend output on importables is less than unity it seems probable that the terms of trade would improve. To some extent, however, if non-traded goods were more substitutable with importables than with exportables, this terms of trade effect could be mitigated - the rise in the price of non-traded goods increasing the demand for importables and reducing the excess supply of them.

Finally, consider the case represented by equation (17.11) where both countries are expanding but the rate of growth in output is different in each country. The direction of the shift in the terms of trade would be determined by the following three factors.

1. The difference in either country between, on the one hand, the marginal propensity to spend output on importables and the marginal propensity to produce importables; and

on the other hand, between the marginal propensity to spend output on non-traded commodities and the marginal propensity to produce non-traded goods.

2. The degree of substitutability in each country, both in production and in consumption, between non-traded goods and exportables on the one hand and non-traded goods and importables on the other.

3. The annual rate of growth of output in each country expressed as a proportion of the initial value of trade.

3. Real Income Effect

We now leave the question of the direction of the shift in the terms of trade and concentrate instead upon the magnitude of such a movement, which is a determinant of the real income effect of an expansion in output. Real income in the domestic country when it alone is expanding, will depend both on the initial expansion in output and upon the direction and magnitude of the terms of trade movement. The loss or gain of real income in the expanding country due to the latter may be approximated on the compensation principle [54,71-2], as being equivalent to the increase or decrease in the cost of obtaining the initial volume of imports due to the shift in the terms of trade. Thus, the total real income change in the domestic country is approximated:

$$dU = dp_1 x_2 + dG$$

or

$$\frac{dU}{dG} = \frac{dp_1}{dG}\frac{x_1}{p_1} + 1 \qquad \ldots \qquad (22.11)$$

where $x_1 = x_2$; and dU = the change in real income in the domestic country, i.e. the change in the cost of securing the initial volume of imports plus the increase in output valued at constant prices.

Substituting in equation (22.11) for $\frac{dp_1}{dG}\frac{x_1}{p_1}$ from equation (15.11) we obtain:

$$\frac{dU}{dG} = \frac{-\,\nu_{23}}{\Psi_{23} + \Psi'_{23} + \psi_{23} - \psi'_{23}} + 1$$

which, when rearranged, yields:

$$\frac{dU}{dG} = \frac{\Psi'_{13} + \psi'_{13} + \Psi_{23} + \psi_{23} - \nu_{23} + 1}{1 + \Psi'_{13} + \psi'_{13} + \Psi_{23} + \psi_{23}} \quad \ldots \quad (23.11)$$

$$(\Psi'_{13} = \Psi'_{23} \text{ and } -\psi'_{23} = 1 + \psi'_{13}).$$

This is the criterion for the change in the real income of the domestic (expanding) country.

Let us consider the intriguing possibility that the effect of an expansion in output could reduce the expanding country's real income. From the assumption of stability in the international market it follows that the denominator on the right-hand side must be negative. For real income to decrease, therefore, the numerator must be positive, which requires:

$$\nu_{23} > 1 + \Psi'_{13} + \psi'_{13} + \Psi_{23} + \psi_{23}$$

in absolute magnitude or, expanding ν_{23} and rearranging:

$$(1+\Psi'_{13}+\psi'_{13}) - [s_2 - s_3(\frac{\sigma_{32}-S_{32}}{\sigma_{33}-S_{33}})] + [(c_2-C_2)-(c_3-C_3)(\frac{\sigma_{32}-S_{32}}{\sigma_{33}-S_{33}})] > -\Psi_{23} \quad \ldots \quad (24.11)$$

From the stability conditions we know that the foreign coefficient of sensitivity is inherently negative and, therefore, that the right-hand side of (24.11) must be positive. This means that if real income is to diminish as a result of growth at least one of the bracketed terms on the left-hand side must be positive. The reader can see for himself how unlikely it is that this condition would be fulfilled.

If in the above criterion, all terms which involve the non-traded commodity are set to zero, a two-commodity criterion is obtained which is identical to one derived by Johnson [17,76] except that our result expands his aggregate elasticity concepts:

$$\left(1+\frac{X_1'}{x_1}\sigma_{11}' - \frac{O_1'}{x_1}S_{11}' - C_1'\right) - s_2 + (c_2 - C_2) > -\frac{X_2}{x_2}\sigma_{22} + \frac{O_2}{x_2}S_{22} \qquad \ldots \qquad (25.11)$$

The importance of the inclusion of the non-traded commodity needs no emphasis, though it is difficult to generalize about its effect.

4. Solutions for Other Relative Price Changes

Instead of solving directly for the relative price changes dp_3, dp_3', etc., the solution for $x_1\frac{dp_1}{dG} = H$ is substituted into the appropriate equations in the set (13.11). Only the case in which the increase in output is concentrated entirely in the domestic country (dG' equalling zero) is considered. In a similar fashion, solutions are obtained for (dp_3-dp_1) and $(dp_3'-dp_1)$ which are changes in non-traded prices relative to the price of the domestic country's exportables. In these latter cases use is made of the fact that $\sum_j p_j K_{ij} = 0$ in order to simplify the results. Finally, a criterion is given for dp_3-dp_3'.

The relevant criteria are:

$$\frac{dp_3}{dG}\frac{x_1}{p_3} = \frac{(\sigma_{31}-S_{31})H + [C_3H+(c_3-s_3)]\frac{p_1x_1}{p_3X_3}}{-\sigma_{33}+S_{33}} \qquad \ldots \qquad (26.11)$$

$$\frac{dp_3-dp_1}{dG}x_1 = \frac{-(\sigma_{32}-S_{32})H + [C_3H+(c_3-s_3)]\frac{p_1x_1}{p_3X_3}}{-\sigma_{33}+S_{33}} \qquad \ldots \qquad (27.11)$$

$$\frac{dp_3'}{dG}\frac{x_1}{p_3'} = \frac{(\sigma_{31}'-S_{31}')H - C_3'H\frac{p_1x_1}{p_3X_3}}{-\sigma_{33}'+S_{33}'} \qquad \ldots \qquad (28.11)$$

$$\frac{dp_3'-dp_1}{dG}x_1 = \frac{-(\sigma_{32}'-S_{32}')H - C_3'H\frac{p_1x_1}{p_3X_3}}{-\sigma_{33}'+S_{33}'} \qquad \ldots \qquad (29.11)$$

$$\frac{dp_3-dp_3'}{dG}x_1 = \left[\left(\frac{\sigma_{31}-S_{31}}{-\sigma_{33}+S_{33}}\right)-\left(\frac{\sigma_{31}'-S_{31}'}{-\sigma_{33}'+S_{33}'}\right)\right]H+\left[\left(\frac{C_3}{-\sigma_{33}+S_{33}}+\frac{C_3'}{-\sigma_{33}'+S_{33}'}\right)H+\left(\frac{c_3-s_3}{-\sigma_{33}+S_{33}}\right)\right]\frac{p_1x_1}{p_3X_3} \qquad \ldots \qquad (30.11)$$

Few general conclusions emerge from these criteria. In the expanding country (26.11) indicates that if the terms of trade shift is favourable, the price of non-traded commodities will rise relative to the price of importables unless the increased supply of non-traded goods (due to the increase in output less the production substitution effect) exceeds considerably the increased consumption of them (due to both the rise in income as a result of the output increase, the improvement in the terms of trade, and to the price substitution effects in consumption). A fall in the terms of trade, on the other hand, would see the price substitution effects and the income effect of the terms of trade change reversed and c_3 would need to exceed s_3 considerably to prevent p_3 falling relative to p_2. At the same time, it should be noted that to the extent that either of the own non-traded substitution elasticities were high or infinite, the indicated shift in the traded/non-traded price ratio would be small or zero. Likewise, if H = 0, the result would be determined entirely (except where non-traded elasticities were infinite) by whether $c_3 \gtreqless s_3$.

In a similar though suitably modified fashion, we can analyse the effect of an increase in output upon the price ratio involving the non-traded and the exportable commodity in the domestic country. Even in the case of the foreign (and non-expanding) country it is not possible always to reach unambiguous conclusions. If the foreign country's terms of trade should improve, p_1 falling relative to p_2, where p_2 is the numeraire, price substitution effects would tend to depress the price of the non-traded commodity; to offset this there is the income effect of the favourable terms of trade movement which would increase the consumption of the non-traded good and tend to increase its price. These considerations apply in reverse if the terms of trade movement is adverse for the foreign country. Only in the case of the foreign non-traded/importable price ratio is the result definite, the income and substitution effects reinforcing one another both with respect to an improvement and a deterioration in the terms of trade, i.e. p_1 falls relative to p_3' when the foreign terms of trade improve and vice versa.

Finally, we consider the ratio of the prices of the non-traded commodities themselves and our conclusion is that there can be no general presumption as to the movement in this ratio. If, in ignorance, it is assumed that the quantity weighted substitution elasticities are the same in each country, the first term in our criterion disappears and we are left with the condition that p_3 will rise relative to p_3' if and only if

$$(C_3 + C_3')H + c_3 > s_3$$

The first expression contains income effects whose impact is determined by the terms of trade change: favouring a rise in p_3 relative to p_3' if the terms of trade change is favourable to the expanding country or a fall in p_3 relative to p_3' if the terms of trade movement is adverse. The other terms are our marginal propensities to consume and produce the non-traded good out of a given increase in output in the expanding country; p_3 will tend to rise (fall) the more c_3 exceeds (is less than) s_3.

The reader should note that in the analysis of the above criteria the possibility of complementarity in demand or of joint supply is excluded by assumption. In an earlier chapter it was argued that such conditions are considered abnormal in a four-commodity world.

Other criteria could be obtained by substitutions into the appropriate equations of (13.11) both for the case of an equal absolute increase in output in each country ($dG = dG'$) and for the case of divergent rates of growth. These additional criteria, however, would be of limited value as apart from the introduction of symmetrical 'effects' for the foreign country, the only new determinant would be, in the last case, a different growth rate in the foreign country. Both the basic analysis of the problem and the ambiguity of our conclusions would be the same.

E. CONCLUSIONS

The main conclusions of the chapter are summarized below.

1. It is contended that the contradictory results of some earlier models can be attributed, in the case of a uniform growth in the productivity of all industries, to different cost assumptions and, in the case of an increase of productivity which occurs entirely within the import-competing sector, to a failure to distinguish clearly between domestic production of importables and the domestic production of non-traded goods.
2. It is claimed that the presence of commodities which do not enter into international trade necessitates a careful restatement of established results. In particular, it is important within the context of the model, to consider all of the relevant parameters, the answer in many cases being affected vitally by the substitution possibilities which occur between the traded and non-traded commodities.
3. In some cases, a consideration of the traded/non-traded relationships may affect the direction of the shift in the terms of trade; in all cases the magnitude of the terms of trade movement will be affected, though it is not possible to generalize about this.
4. The addition of non-traded commodities enables us to consider the shifts in the non-traded/traded price ratios as well as in the terms of trade. The importance of this is that it focuses our attention on the role of the non-traded commodity in the equilibrating process.
5. Finally, it is argued that the expression of the results in terms of conceivably measurable parameters represents a step, if only a small one, toward the possibility of quantitative analysis.

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A.E.R.,	American Economic Review
E.,	Econometrica
E.I.,	Economia Internazionale
Ec.,	Economica
E.J.,	Economic Journal
E.R.,	Economic Record
G.D.E.,	Giornale Degli Economisti
I.E.R.,	International Economic Review
I.M.F.S.P.,	International Monetary Fund Staff Papers
J.P.E.,	Journal of Political Economy
M.S.E.S.S.,	Manchester School of Economic and Social Studies
O.E.P.,	Oxford Economic Papers
R.E.S.,	Review of Economic Studies
R.E.,	Review of Economics and Statistics
Q.J.E.,	Quarterly Journal of Economics

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APPENDIX

In chapter three it was suggested that our model is sufficiently general to include any number of non-traded commodities without affecting the qualitative nature of our conclusions.

For example, consider the problem of a tariff and the induced shift in the terms of trade of the taxing country. Rewriting the set of equations (18.5) to (21.5) so as to include n non-traded commodities we have:

$$
\begin{bmatrix}
K_{21}+x_1M_2 & K_{23}\ldots K_{2n} & 0 \ldots 0 & M_2(p_2-1)-1 \\
K_{31}+x_1M_3 & K_{33}\ldots K_{3n} & 0 \ldots 0 & M_3(p_2-1) \\
\vdots & \vdots & \vdots & \vdots \\
K_{n1}+x_1M_n & K_{n3}\ldots K_{nn} & 0 \ldots 0 & M_n(p_2-1) \\
K'_{21}-x_1M'_2 & 0 \ldots 0 & K'_{23}\ldots K'_{2n} & 1 \\
K'_{31}-x_1M'_3 & 0 \ldots 0 & K'_{33}\ldots K'_{3n} & 0 \\
\vdots & \vdots & \vdots & \vdots \\
K'_{n1}-x_1M'_n & 0 \ldots 0 & K'_{n3}\ldots K'_{nn} & 0
\end{bmatrix}
\begin{bmatrix}
dp_1 \\ dp_3 \\ \vdots \\ dp_n \\ dp'_3 \\ \vdots \\ dp'_n \\ dx_2
\end{bmatrix}
=
\begin{bmatrix}
-K_{22}dp_2 \\ -K_{32}dp_2 \\ \vdots \\ -K_{n2}dp_2 \\ \vdots \\ 0
\end{bmatrix}
$$

Let A_{rr} be the diminished determinant of substitution effects

$$
\begin{vmatrix}
K_{22} & \cdots & K_{2n} \\
\vdots & & \vdots \\
K_{n2} & \cdots & K_{nn}
\end{vmatrix}
$$

where in this case r=1; and let $A_{rr,ij}$ be the co-factor in A_{rr} of the element K_{ij}.

Then, proceeding exactly as in the four-good case, the solution for the effect of a tariff upon the terms of trade is:

$$\frac{dp_1}{dp_2} = \frac{\dfrac{A_{rr}}{A_{rr,ss}}}{\dfrac{p_s A_{rr}}{A_{rr,ss}} - x_1\left(M_s + \sum_3^n M_i \dfrac{A_{rr,is}}{A_{rr,ss}}\right) + \left[1-(p_2-1)\left(M_s+\sum_3^n M_i \dfrac{A_{rr,is}}{A_{rr,ss}}\right)\right]\left[\dfrac{A'_{rr}}{A'_{rr,ss}} + x_1\left(M'_s + \sum_3^n M'_i \dfrac{A'_{rr,is}}{A'_{rr,ss}}\right)\right]}$$

where, in this case, r=1, s=2, and i=3 .. to n.

This result is obtained by a Laplacian development from (n-1) rows or columns, after using the relationship $\sum_{j=1}^{n} p_j K_{ij} = 0$ and the convention that all prices except p_2 are unity initially by an appropriate choice of quantity units. The proof that the numerator and the denominator comprise only the terms shown is tedious but easy.

Since A_{rr} is the discriminant of a negative definite form, the ratio $A_{rr}/A_{rr,ss}$ must be negative. Now, it is easy to verify that for n=5, the terms $A_{rr,is}/A_{rr,ss} > 0$ if all $K_{ij} > 0$, i.e. all goods are substitutes. However, it is desirable to obtain a general proof concerning the positiveness of $A_{rr,is}/A_{rr,ss}$. Furthermore, if our conclusion about the terms of trade movement is to hold for the n-commodity case, we must show also that $A_{rr,is}/A_{rr,ss} < 1$.

Our proof that $0 < A_{rr,is}/A_{rr,ss} < 1$ depends upon two theorems concerning determinants. The first, established by Mosak [24, 49-51], is stated but the reader is referred to the reference cited for the proof. An outline is given, however, of the proof of the second theorem which is due, so far as I am aware, to the ingenuity of T.W. Swan.

Theorem 1

If in a full determinant of substitution effects (A) the $(n-1)^{th}$ determinant (A_{rr}) is negative definite and if all off-diagonal elements are non-negative (all $K_{ij} > 0$), then every non-principal minor has the same sign as every principal minor of the same order.

Theorem 2

If in a full determinant of substitution effects (A) the columns and rows sum to zero (prices being unity initially), then $A_{rs} = A_{ij}$ and $A_{rr,ss} = A_{ss,rr} = A_{rr,is} + A_{ss,ir}$ where A_{rs} is the co-factor of K_{rs} in A, and $A_{rr,is}$ is the co-factor of K_{is} in A_{rr}.

Proof. If A is the full determinant of substitution effects:

$$\begin{vmatrix} K_{11} & \cdots & K_{1n} \\ \vdots & & \vdots \\ K_{n1} & \cdots & K_{nn} \end{vmatrix}$$

and A_{nn} is the co-factor of K_{nn} in A, then

$$A_{n1} = (-)^n \begin{vmatrix} K_{12} & \cdots & K_{1n} \\ \vdots & & \vdots \\ K_{n-1,2} & \cdots & K_{n-1,n} \end{vmatrix}$$

From the theory of demand we know $\sum_{i=1}^{n} K_{ri} = 0$ (all prices unity),

$$\therefore \quad K_{rn} = -(K_{r1} + K_{r2} + \ldots K_{r,n-1})$$

$$\therefore \quad A_{n1} = (-)^n \begin{vmatrix} K_{12} & \cdots & -(K_{11} + K_{12} + \ldots + K_{1,n-1}) \\ \vdots & & \vdots \\ K_{n-1,2} & \cdots & -(K_{n-1,1} + K_{n-1,2} + \ldots + K_{n-1,n-1}) \end{vmatrix}$$

Adding columns to the last column

$$A_{n1} = (-)^n \begin{vmatrix} K_{12} & \cdots & -K_{11} \\ \vdots & & \vdots \\ K_{n-1,2} & \cdots & -K_{n-1,1} \end{vmatrix}$$

$$= \begin{vmatrix} K_{11} & \cdots & K_{1,n-1} \\ \vdots & & \vdots \\ K_{n-1,1} & & K_{n-1,n-1} \end{vmatrix} = A_{nn}$$

By permuting columns $A_{nr} = A_{nn}$. Similarly, by permuting rows and columns $A_{rs} = A_{nn}$.

Next, we see that for A, the determinants $A_{11,22} = A_{22,11}$ are

$$\begin{vmatrix} K_{33} & \cdots & K_{3n} \\ \vdots & & \vdots \\ K_{n3} & \cdots & K_{nn} \end{vmatrix}$$

and that the sum of the determinants

$$A_{11,32}+A_{22,31} = -\begin{vmatrix} K_{23}K_{24} & \cdots K_{2n} \\ K_{43}K_{44} & \cdots K_{4n} \\ \vdots \quad \vdots & \vdots \\ K_{n3} & \cdots K_{nn} \end{vmatrix} - \begin{vmatrix} K_{13}K_{14} & \cdots & K_{1n} \\ K_{43}K_{44} & \cdots & K_{4n} \\ \vdots \quad \vdots & & \vdots \\ K_{n3}K_{n4} & \cdots & K_{nn} \end{vmatrix}$$

$$= -\begin{vmatrix} (K_{13}+K_{23}) & (K_{14}+K_{24}) & \cdots & (K_{1n}+K_{2n}) \\ K_{43} & K_{44} & \cdots & K_{4n} \\ \vdots & \vdots & & \vdots \\ K_{n3} & K_{n4} & \cdots & K_{nn} \end{vmatrix}$$

As $\sum_i K_{ir} = 0$,

$$K_{1r} + K_{2r} = -(K_{3r}+ K_{4r}+\ldots+ K_{nr})$$

so that

$$A_{11,32}+ A_{22,31} = -\begin{vmatrix} -(K_{33}+ K_{43}+\ldots+K_{n3}) & \cdots & -(K_{3n}+ K_{4n}+\ldots+K_{nn}) \\ K_{43} & \cdots & K_{4n} \\ \vdots & \vdots & \vdots \\ K_{n3} & \cdots & K_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} K_{33} & K_{34} & \cdots & K_{3n} \\ \vdots & \vdots & & \vdots \\ K_{n3} & K_{n4} & \cdots & K_{nn} \end{vmatrix} = A_{11,22} = A_{22,11}$$

By permuting elements,

$$A_{rr,ss} = A_{ss,rr} = A_{rr,is}+ A_{ss,ir} .$$

Now, it follows from theorems 1 and 2 that

$$0 < \frac{A_{rr,is}}{A_{rr,ss}} < 1$$

The terms of trade, therefore, will move favourably for the taxing country even in the n-commodity case providing there is no complementarity and as long as the tariff (p_2-1) is not large.

The application of this general proof to other developments of the model is obvious.

One reservation should be noted, however. While the assumption that all $K_{ij} > 0$ is a reasonable one in a model characterised by a large degree of aggregation, this becomes less realistic an assumption as disaggregation becomes more general.