Spontaneous formation and synchronization of vortex modes in optically induced traps for exciton-polariton condensates

Alexey V. Yulin,¹ Anton S. Desyatnikov,² and Elena A. Ostrovskaya³

¹*ITMO University 197101, 49 Kronverksky avenue, St. Petersburg, Russian Federation*

²*Physics Department, Nazarbayev University, 53 Kabanbay Batyr Avenue, Astana 010000, Kazakhstan*

³*Nonlinear Physics Centre, Research School of Physics and Engineering, Australian National University, Canberra, Australian Capital Territory 2601, Australia*

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We study the dynamics of exciton-polariton condensates in reconfigurable potentials created by an incoherent optical pump. The mode selection in the effective optically induced trap is shown to be determined by the largest growth rates of the linear eigenmodes of the effective complex potential created by the pump. We demonstrate that selection of higher order nonlinear modes with nonzero orbital angular momenta in such traps leads to formation of vortices in a single-component condensate and spin textures in exciton-polariton condensates consisting of two polarization components. We also show that cross-polarization coupling can lead to the synchronization of the modes belonging to different polarizations. We highlight clear signatures of the various spontaneously formed trapped states in the cavity photoluminescence spectrum, which should aid in their experimental observations.

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I. INTRODUCTION

Optical trapping and manipulation of exciton-polariton condensates in semiconductor microcavities [1,2] has developed into an active area of research due to its enormous potential for engineering flexible and reconfigurable exciton-polariton confinement [3]. Optically induced potentials form because of the strong nonlinear interaction between the condensed polaritons and higher energy excitonic reservoir [4], which is created by an incoherent optical pump and replenishes the condensate through the stimulated scattering process. In the short time since discovery of this powerful trapping technique, it has been used for a range of fundamental and applied studies of microcavity polaritons, such as direct observation of quantum eigenstates of a confining potential [5], demonstration of phase transitions [6], study of non-Hermitian quantum physics and quantum chaos [7], direct measurement of polariton interaction strength [8], as well as construction of a prototype interferometer [9] and an all-optical transistor switch [10,11].

Understanding of conditions for formation and stability of trapped states in the optically defined potentials is critical for the progress towards controlled all-optical manipulation of polariton condensates [12]. A range of experiments with optical confinement targeted controlled excitation of ground and higher-order trapped states, as well as pattern formation in the annular geometry [13–15]. Moreover, controlled excitation of states with nonzero orbital angular momentum (polariton vortices) has been achieved in leaky traps with discrete boundaries and broken chiral symmetry [16]. It is acknowledged that, in general, the optically trapped condensate is formed in a multimode regime, and a single-mode condensation is rarely achieved [15]. Theoretical considerations [17] and experimental investigations of the annular [15,18] and large-area multimode traps [7,19,20] suggest that the process of selection of a dominant mode of the trapped condensate strongly relies on an overlap of the *linear* mode of the effective complex (non-Hermitian) potential with the gain region created by the pump. Depending on the shape of

the pump, condensation in a particular trapped mode can be favored. In addition, various energy relaxation [21] and parametric scattering processes [22] compete in the dynamical population of multiple trapped modes. The complexity of the mode selection process in pump-induced traps is highlighted by comparison with experimental and theoretical results on the selection of the higher-order modes in microstructured, pump-independent traps [23,24,25]. In this work, we elucidate the process of spontaneous mode selection in tight, radially symmetric, optically induced traps operating in a nearly single-mode regime. Conceptually, this setup is close to that considered in Ref. [16], where the trap area was comparable to polariton wavelength, and very few lowest-order modes were populated. We consider a single component, as well as a two-component (spinor) polariton condensate, the latter being created in a microcavity with a significant anisotropy-induced linear polarization splitting [26]. We would like to note here that the linear coupling breaks the symmetry in respect of the spin polarization of the modes. However, the chiral symmetry in respect to the sign of orbital angular momentum (vorticity) is preserved. We show that, in this system, higher-order modes with nonzero orbital angular momentum (vortex modes) can be selected spontaneously, and the condensate can be driven to predominantly occupy these modes in a steady state by controlling the *area* of the trapping potential *without* breaking the chiral symmetry.

In a two-component scenario, spontaneous vortex mode selection can lead to formation of nontrivial vortex textures previously considered in the context of untrapped polariton condensates in the presence of impurities [27]. Persistent dynamical (time-varying) states and their spectral signatures are also identified. We demonstrate that, depending on the coupling strength, the optically trapped spinor condensate can undergo the process of dynamical synchronization [28]. In a synchronized state, both components of the condensate have the same energy (frequency), which can be considered as a chemical potential of the condensate. In a desynchronized state, the spectrum contains a number of different frequencies, and in some cases can be treated as a weakly nonequilibrium

state consisting of two subsystems characterized by two different chemical potentials.

II. THE MODEL

We start with a general mean-field model describing the dynamics of two coupled polarizations of the polariton condensate. This model, based on a single-component open-dissipative Gross-Pitaevskii equation [29], was introduced in Ref. [30] to describe the dynamics of polariton condensates replenished by a spin-polarized (two-component) reservoir, and used, in particular, to describe the process of dynamical synchronization between (pseudo)spin states in a homogeneous condensate [28]. It is based on the assumption that the reservoir relaxation (combining radiative and nonradiative decay, as well as scattering into the condensate mode) is fast compared to the spin relaxation, so that the two condensate components of the orthogonal circular polarizations are created and maintained by the respectively polarized pumping. In the dimensionless form, the equations read

$$\begin{aligned} i\partial_t \psi_{\pm} = & \left[-\frac{\nabla^2}{2} + i(Rn_{\pm} - \gamma_c) + g_c|\psi_{\pm}|^2 + g_r n_{\pm} \right] \psi_{\pm} \\ & + (\alpha|\psi_{\mp}|^2 + \beta n_{\mp}) \psi_{\pm} + \sigma \psi_{\mp}, \\ \partial_t n_{\pm} = & p_{\pm} - [\gamma_r + R|\psi_{\pm}|^2] n_{\pm}, \end{aligned} \quad (1)$$

where ψ_{pm} are the normalized order parameters for the condensate in the two orthogonal circular polarization components (+) and (−), $n_{\pm}$ are the spin-polarized reservoir densities, and $p_{\pm}$ are the dimensionless pump rates. The condensate dynamics is determined by the coherent polaritons relaxation rate γ_c , incoherent reservoir relaxation rate γ_r , the rate of stimulated scattering from the reservoir into the condensate R , the strength of the effective nonlinearity (blue shift caused by the condensed polaritons) g_c , and blue shift due to the incoherent reservoir g_r . The coefficient of nonlinear interaction between the condensed polaritons of different polarizations, α , and the blue shift of coherent electrons caused by the incoherent reservoir components of different polarizations, β , are assumed to be small [31]. The linear coupling between the polarization components is controlled by the σ coefficient, while anisotropic spin coupling within each component is assumed to be negligible. The dimensionless variables and coefficients are obtained by using the time scale $\tau = 1/\Gamma_c$ and the length scale equal to the diffraction length of the polariton matter wave $L = \sqrt{\hbar/(m_p \Gamma_c)}$; Γ_c is the coherent polariton relaxation rate in physical units. In what follows, we use the dimensionless parameters $\gamma_c = 0.5$, $g_c = 1$, $\alpha = -0.1$, $g_r = 4$, $\beta = -0.4$, $\sigma = 0.15$, $\gamma_r = 3$, $R = 1.1$, corresponding to the physical parameters of the system defined in Ref. [32] and the chosen scaling. This model lacks the phenomenological energy relaxation, which is important for describing the dynamical transition between steady states in a multimode condensation regime (see, e.g., Ref. [33]); however, here our primary goal is to establish the *existence* of such steady or quasisteady states. Our results will hold validity for highly photonic condensates displaying inefficient phonon-assisted relaxation, as observed for highly photonic condensates obtained under conditions of large negative detuning between the cavity photon mode and the exciton resonance.

FIG. 1. Panel (a) shows the spatial distribution of the pump p , and the cross section of the pump is shown in panel (b). The dependencies of the real ω_r and imaginary parts ν of the eigenfrequencies of the linear modes are shown in panels (c) and (d). The numbers n, m marking the curves are the indices of the modes; i.e., the curves show the dependencies $\omega_{r, n, m}$ and $\nu_{r, n, m}$ for the modes with the angular index n and radial index m as defined in (2). The rings and squares in panel (c) mark the frequencies of the stationary nonlinear states obtained by direct numerical simulations. The amplitude of the pump is $P = 3.27$, the width of the pump ring is $w_p = 1$, and the pump displacement $\tilde{r}_0 = 0$.

Throughout this work, we will consider a ring-shaped pump shown in Figs. 1(a) and 1(b), which not only replenishes the condensate but also creates an effective potential confining the condensate due to the reservoir-induced blue shift [6,14]. The shape of the pump defines the pump rate (the number of reservoir excitons created per unit area per time unit)

$$p_{\pm} = P \left(e^{-\frac{(\tilde{r} \pm \tilde{r}_0/2) - r_p)^2}{w_p^2}} + e^{-\frac{(\tilde{r} \mp \tilde{r}_0/2) + r_p)^2}{w_p^2}} \right),$$

where P is the amplitude of the pump, the parameter r_p is the radius of the ring, w_p is the width of the ring, and $\tilde{r}_0$ is the relative shift of the pump beams of orthogonal polarizations.

III. MODE SELECTION IN A SPIN-INSENSITIVE TRAP

First of all, we will analyze the dynamics of the condensate in the single-component case, i.e., $\alpha = 0$, $\beta = 0$, and $\sigma = 0$. This corresponds to a common situation of a polarization-insensitive reservoir created, e.g., by a linearly polarized pump.

In this case, the model (1) reduces to a single equation for the condensate mode coupled to a reservoir density. Close to the condensation threshold, i.e., for a vanishingly small population of condensed polaritons, one can treat this equation as a linear eigenvalue problem with an *effective complex potential* and study how the frequency (real part of the eigenvalue) and linewidth (imaginary part of the eigenvalue) of the linear eigenmodes depend on the width of the ring. Similarly to the treatment in Ref. [17], to examine the behavior of the system in the linear regime, we linearize Eq. (1)

around the trivial stationary state with zero coherent polariton component $\psi = 0$ but nonzero density of the incoherent excitonic reservoir $n_0 = p/\Gamma$.

Because of the azimuthal symmetry of the equations it is convenient to look for the eigenmodes in the form $\psi_{n,m} = \rho_{n,m}(r) \exp(im\theta) \exp(i\omega_{n,m}t)$, where m is the orbital angular momentum (OAM) of the mode. The eigenvalue problem can then be written in the form

$$\omega_{n,m} \rho_{n,m} = \left[i(Rn_0 - \gamma_c) + g_r n_0 + iU_a(r) + \frac{m^2}{2r^2} - \frac{1}{2r} \partial_r - \frac{1}{2} \partial_r^2 \right] \rho_{n,m} \quad (2)$$

with the boundary conditions $\rho_{n,m} = 0$ at $r \rightarrow \infty$ and at $r = 0$ for $m > 0$, and for $m = 0$ the second boundary conditions changes to $\partial_r \rho = 0$. The linear modes found for each m can be characterized by the number n of radial nodes along r . Thus each of the modes is characterized by a radial index n (the first index) and by the OAM of the mode m (the second index).

Calculating the eigenmodes numerically, we have to work with a finite integration window and to emulate an infinite problem we introduce artificial losses U_a growing adiabatically at large r . This allows use of calculation windows of smaller size and facilitates the calculations.

The dependencies $\omega_{r,n,m}$ of the real part of the eigenfrequencies of the linear lower-order modes are presented in Fig. 1(c) for the chosen set of parameters. Figure 1(d) shows the imaginary parts $\nu_{n,m}$ of the eigenfrequencies and indicates that some of the eigenmodes are growing $\nu > 0$. In a spontaneous condensation process, the modes with the largest growth rate, ν , will dominate the dynamics. Importantly, both the imaginary and the real parts of the eigenfrequencies depend of the pump radius strongly and can be controlled by this parameter. Therefore, one can selectively excite modes with different OAM by controlling the shape of the optical excitation beam.

To confirm this mode selection mechanism, we performed direct numerical simulations of the nonlinear equations (1) with absorbing boundary conditions using weak random noise as the so-called initial condensation seed. We found that, depending of the pump radius, different nonlinear modes can be excited. In particular we observed the formation of the ground-state mode with the angular momentum $m = 0$ is favored when the radius of the pump ring is relatively small.

For larger radii of the pump ring the modes with higher angular momenta win the competition. Figures 2(a) and 2(b), for example, demonstrate a *stationary vortex* with the angular momentum $|m| = 1$ forming in the pump ring of the radius $r_p = 1.7$. We stress that the radial symmetry of the pump is intact, and the selection of the vortex mode in this case is completely spontaneous. The vortex states with $m = \pm 1$ are energy degenerate and, in an experiment, a superposition of these states (a dipole mode) is likely to be observed in the spatial distribution of the condensate [16].

Let us mention here that for the symmetric pump the symmetric solution with zero angular momentum always exists. However, this solution may not necessarily be stable, and the state growing from weak noise may not be symmetric. Apart from the symmetric solution there may exist stable states with a nonzero angular momentum. Of course, in a symmetric

FIG. 2. (a) The polariton density and (b) phase distribution corresponding to a stationary vortex mode with $m = 1$ supported by the pump ring with the width $r_p = 1.7$. Panels (c) and (e) are the k -space density distributions of the stationary states with $m = 0$ and $m = 1$, respectively. The corresponding spectra are shown in panels (d) and (f). The width of the pump ring supporting the ground state shown in panel (c) is $r_p = 1.15$.

system, the existence of a steady state with orbital momentum $+m$ means that there is also an energy-degenerate steady state with the orbital momentum $-m$. Each of these states is an attractor, and the system achieves either one or another state depending on the initial conditions. If there are multiple stable states, then the growth rate becomes important since it defines the volume of the attraction basin for each of these states. This is a classical example of a system with spontaneously broken symmetry. In the case considered here, it happens that the growth rate of the state in linear regime is a crucial parameter defining the angular momentum of the final state.

It is instructive to examine the spectral signatures of different nonlinear modes. In Figs. 2(c) and 2(e), and Figs. 2(d) and 2(f), the k -space density distributions and spectra of the modes with $m = 0$ and $|m| = 1$ are shown, respectively. These spectral signatures are typical of a trapped ground-state condensate and the vortex mode observed experimentally [16].

For the chosen set of the parameters, the frequency of the nonlinear stationary states happens to be close to the frequencies of the corresponding linear modes, which is expected close to the bifurcation point where the linear mode growth rate is small. It is seen that, for the lower pump intensities, the frequencies of the stationary states follow closely the dependency of the real part of the frequency of the mode with the appropriate symmetry. For large pumps the deviation between the frequencies grows, which can be explained by the fact that the mean field of the polariton component modifies the density of the effective pump induced by the incoherent excitons. Importantly, the frequency of the nonlinear modes can also be tuned by the width of the pump, and for an appropriate choice of the pump widths the modes with $m = 0$ and $|m| = 1$ can have the same frequency. This presents an opportunity to synchronize modes belonging to two different polarizations and having different angular momenta. Thus, as we show in the following section, one can obtain complex vortex structures in a two-component condensate.

Finally, we would like to note that in the traps of relatively large radii support a number of modes with similar growth

rates and their competition can result in the formation of complex states, including dynamical ones showing periodic, quasiperiodic, or even stochastic behavior. However, the detailed consideration of these possibilities is beyond scope of the present study.

IV. MODE SYNCHRONIZATION IN A TWO-COMPONENT CONDENSATE

Next, we analyze the full problem of two-condensate polarizations interacting both linearly and nonlinearly. We fix the radius of one of the spin-polarized pumps at a value supporting a vortex state $r_- = 1.7$. The radius of the pump in the orthogonal polarization, r_+ , is varied around the value $r = 1.2$, which ensures formation of the ψ_+ mode with zero OAM.

Let us first consider the case when the frequencies of the $m = 0$ and $|m| = 1$ states in both polarization are far detuned, so that their mutual synchronization is impossible. Then the linear coupling results in the *hybridization* of the modes with and without the OAM. The spectra of the two components of the stationary states are shown in Figs. 3(a) and 3(b). As can be seen from the figure, the spectra contain *incommensurable* frequencies.

The dependence of the spectrum on the wave vector k allows us to conclude that the ψ_+ component does not contain a vortex. Indeed, the k -space density distribution in the (+) polarization corresponding to Fig. 3(a) (not shown) is very similar to that of a ground state shown in Fig. 2(c).

The structure forming in the orthogonal polarization is an eccentric vortex with the spectral pattern having the signature of the vortex state; see Fig. 3(b). The corresponding intensity and phase distributions are shown in Figs. 3(d) and 3(e). In contrast to the monochromatic steady state shown in Fig. 2(a), in the case of two interacting polarizations the vortex is a *dynamical state* undergoing a steady rotation.

It worth noting here that each of the polarizations contains more than one frequency component, and that spatial field distributions corresponding to the different frequencies can have different angular momenta. The total field of the state represents a so-called dynamical spin vortex texture, whereby the vorticity of the *dominant* spectral components in the (+) and (−) spin states are different.

By characterizing each polarization component by the frequency (energy) of the most intense spectral line, we can trace how the detuning between the polarizations depends on the radius of the pump ring. This dependence, shown in Fig. 4, demonstrates that the polarizations can be *synchronized* by changing the shape of the pump. Indeed, as seen in Fig. 4, there is a range of the pump widths where the detuning of the polarizations is equal to zero. In the case of the synchronization, the spectrum contains only one frequency [see Figs. 5(a) and 5(b)]. The synchronous states are spontaneously formed *half-charge vortices* [34–36] also known as polarization dislocations [37]. Importantly, if the linear coupling between the polarization components is absent, then no synchronization occurs, as demonstrated by the blue squares in Fig. 4(a). This observation is in line with the previous studies of synchronization in spatially homogeneous two-component polariton condensates [28].

FIG. 3. Spectra of the nonsynchronized nonlinear states for $r_p = 1.08$ are shown in panels (a) and (b) for the first (+) and the second (−) polarizations correspondingly. The relative displacement of the pumps is $r_0 = 0.3$. Panels (c) and (d) show the distributions of the densities of the coherent polaritons in the (+) and (−) polarizations, and the phase distributions of the coherent polaritons are shown in panels (e) and (f). The displacement of the pumps is $r_0 = 0.3$.

FIG. 4. The dependence of the detuning of the (+) polarization frequency on the frequency of the (−) polarization is shown in panel (a) as a function of the radius of the (+) pump. The detuning is defined as the difference between the frequencies of most intense spectral lines of the first and the second polarizations. The black circles are the measured detunings for $\sigma = 0.15$. The detunings for the case $\sigma = 0$ are shown by the blue squares. The lines are guides for the eye. Panel (b) shows the detuning of the frequencies of different polarizations as a function of the shift of the pump centers. The relative displacement of the pumps is $r_0 = 0.3$.

FIG. 5. The spectra (a, b) and polariton density distributions (c, d) in the two polarization components under the same conditions as in Fig. 3, but for the radius of (+)-polarization pump $r_p = 1.17$, allowing for the formation of a synchronized state.

In addition, the two polarization components can also be moved in and out of synchronization by the relative displacement of the pump ring centers, $r_0 \equiv |\vec{r}_0|$. If the displacement is reduced below a certain threshold (i.e., the pumps of the orthogonal polarizations are coalined), then the synchronization disappears. This can be explained by the fact the states with $m = 0$ and $|m| = 1$ have different symmetries, and therefore the corresponding overlap integral defining their interaction strength is equal to zero. As mentioned above the synchronization disappears if the linear coupling between the polarizations is equal to zero but it is a difficult problem to estimate the synchronization range as a function of the linear coupling strength analytically. This is so because, apart from the coupling strength, the interaction depends on the overlap integral between the polarizations. The latter can be calculated either numerically or by involving perturbation theory; however, such calculation is beyond scope of the present paper.

Of course, the synchronization also disappears when the shift is too large, so that the polarizations do not talk to each other. Let us also stress that the small displacement allowing for the synchronization of the state does not break the symmetry in the sense that the nonlinear stationary states are invariant with respect of the signs of their orbital angular momenta (vorticity). The sign of the vorticity of the final state depends on the initial conditions.

In the intermediate range, for the nonzero and moderate values of the displacement, the effect of vorticity switching is observed. In Fig. 6(a), the dependence of the vorticity m is the (−) polarization is shown as a function of time. The vorticity is calculated as

$$m = \frac{1}{2\pi} \int_S (\partial_x J_y - \partial_y J_x) dx dy, \quad (3)$$

FIG. 6. (a) Vorticity of the (−) polarization component as a function of time. The shift between the pump centers is $r_0 = 0.7$. The radius of the spatial area used for vorticity calculation is $r = 3$. (b) The spectra of the two polarization components for the dynamical state with oscillating vorticity.

where the polariton flow is defined as $J = \text{Im}(\psi^* \nabla \psi)$, and S is the area enclosing the spatially localized condensate density distribution. The value of m is the topological charge of the state and is identical to the value of the OAM. The value of m changes when the amplitude singularity in the wave function ψ moves outside the integration area S . The deviation of m from the integer values in our simulations can be explained by small inaccuracies of the calculation of the integral. Despite the deviations, it is clearly seen that periodic switching between the vortex $m = +1$ and the antivortex $m = -1$ states takes place. This dynamics corresponds to a periodic switching and mutual transformations between different topological types of polarization dislocations [37].

As before, the dynamically evolving state has a distinct spectral signature $\omega(\vec{k})$, which is strikingly different from a steady-state mode characterized by a single spectral line. In particular, the charge flipping dynamics results in the appearance of the side bands in the spectrum, as shown in Figs. 6(b) and 6(c). The distance between the additional spectral lines is proportional to the frequency of the so-called vorticity flips. This observation supports the conclusion that the origin of the sidebands is not the beating between two polarization components but rather switching between the states with different vorticities.

The time-dependent dynamics of the polariton density distribution and phase shown in Fig. 7 reveals that the intermediate state with zero vorticity [Fig. 7(c)] corresponds to a *dipole azimuthon* [38] containing a vortex-antivortex pair. This state is a member of a general family of density-modulated vortex states well studied in the context of coherent optical waves [39], ultracold atomic matter waves [40], and exciton-polariton condensates [41]. The dynamical behavior demonstrated in Figs. 6 and 7 is reminiscent of the charge flipping phenomenon observed in optical systems [38].

FIG. 7. The phase (top row) and polariton density distributions (bottom row) in the (—) spin component for different moments of time corresponding to the points marked (b, c, d) in Fig. 6(a).

V. CONCLUSIONS

In conclusion, we have illustrated the mechanism for selection of a dominant mode populated by a polariton condensate in

a tight optically defined trap. The dominant mode is determined by the maximum growth rate of the linear eigenmodes in the effective complex potential, and therefore population of vortex modes with nontrivial orbital angular momenta can be controlled by the trapping geometry. Moreover, we show that two modes supported by spatially overlapping traps created, e.g., by spin-polarized excitonic reservoirs, can synchronize, which leads to controlled formation of half-vortices and other nontrivial steady-state spin textures *without* coherent imprinting of orbital angular momentum onto the polariton condensate [42] or defect-induced pinning [27]. Intermittent synchronization leads to formation of vortex-antivortex states with periodic charge flipping dynamics. Finally, we have identified clear spectral signatures of the different stationary and dynamical states, which should assist their observation in experiments.

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- [1] H. Deng, G. Weihs, Ch. Santori, J. Bloch, and Y. Yamamoto, Condensation of semiconductor microcavity exciton polaritons, *Science* **298**, 199 (2002).
 - [2] J. Kasprzak, M. Richard, S. Kundermann, A. Baas, P. Jeambrun, J. M. J. Keeling, F. M. Marchetti, M. H. Szymańska, R. André, J. L. Staehli, V. Savona, P. B. Littlewood, B. Deveaud, and L. S. Dang, Bose-Einstein condensation of exciton polaritons, *Nature (London)* **443**, 409 (2006).
 - [3] C. Schneider, K. Winkler, M. D. Fraser, M. Kamp, Y. Yamamoto, E. A. Ostrovskaya, and S. Holing, Exciton-polariton trapping and potential landscape engineering, *arXiv:1510.07540* (unpublished).
 - [4] E. Wertz, L. Ferrier, D. D. Solnyshkov, R. Johne, D. Sanvitto, A. Lemaître, I. Sagnes, R. Grousson, A. V. Kavokin, P. Senellart, G. Malpuech, and J. Bloch, Spontaneous formation and optical manipulation of extended polariton condensates, *Nat. Phys.* **6**, 860 (2010).
 - [5] G. Tosi, G. Christmann, N. G. Berloff, P. Tsotsis, T. Gao, Z. Hatzopoulos, P. G. Savvidis, and J. J. Baumberg, Sculpting oscillators with light within a nonlinear quantum fluid, *Nat. Phys.* **8**, 190 (2012).
 - [6] P. Cristofolini, A. Dreismann, G. Christmann, G. Franchetti, N. G. Berloff, P. Tsotsis, Z. Hatzopoulos, P. G. Savvidis, and J. J. Baumberg, Optical Superfluid Phase Transitions and Trapping of Polariton Condensates, *Phys. Rev. Lett.* **110**, 186403 (2013).
 - [7] T. Gao, E. Estrecho, K. Y. Bliokh, T. C. H. Liew, M. D. Fraser, S. Brodbeck, M. Kamp, C. Schneider, S. Höfling, Y. Yamamoto, F. Nori, Y. S. Kivshar, A. G. Truscott, R. G. Dall, and E. A. Ostrovskaya, Observation of non-Hermitian degeneracies in a chaotic exciton-polariton billiard, *Nature (London)* **526**, 554 (2015).
 - [8] Y. Sun, Y. Yoon, M. Steger, G. Liu, L. N. Pfeiffer, K. West, D. W. Snoke, and K. A. Nelson, Polaritons are not weakly interacting: Direct measurement of the polariton-polariton interaction strength, *arXiv:1508.06698* (unpublished).
 - [9] C. Sturm, D. Tanese, H. S. Nguyen, H. Flayac, E. Galopin, A. Lemaître, I. Sagnes, D. Solnyshkov, A. Amo, G. Malpuech, and J. Bloch, All-optical phase modulation in a cavity-polariton Mach-Zehnder interferometer, *Nat. Comm.* **5**, 3278 (2014).
 - [10] T. Gao, P. S. Eldridge, T. C. H. Liew, S. I. Tsintzos, G. Stavrinidis, G. Deligeorgis, Z. Hatzopoulos, and P. G. Savvidis, Polariton condensate transistor switch, *Phys. Rev. B* **85**, 235102 (2012).
 - [11] D. Ballarini, M. De Giorgi, E. Cancellieri, R. Houdré, E. Giacobino, R. Cingolani, A. Bramati, G. Gigli, and D. Sanvitto, All-optical polariton transistor, *Nat. Commun.* **4**, 1778 (2013).
 - [12] J. Schmutzler, P. Lewandowski, M. Aßmann, D. Niemietz, S. Schumacher, M. Kamp, C. Schneider, S. Höfling, and M. Bayer, All-optical flow control of a polariton condensate using nonresonant excitation, *Phys. Rev. B* **91**, 195308 (2015).
 - [13] F. Manni, K. G. Lagoudakis, T. C. H. Liew, R. André, and B. Deveaud-Plédran, Spontaneous Pattern Formation in a Polariton Condensate, *Phys. Rev. Lett.* **107**, 106401 (2011).
 - [14] A. Askitopoulos, H. Ohadi, A. V. Kavokin, Z. Hatzopoulos, P. G. Savvidis, and P. G. Lagoudakis, Polariton condensation in an optically induced two-dimensional potential, *Phys. Rev. B* **88**, 041308(R) (2013).
 - [15] A. Askitopoulos, T. C. H. Liew, H. Ohadi, Z. Hatzopoulos, P. G. Savvidis, and P. G. Lagoudakis, Robust platform for engineering pure-quantum-state transitions in polariton condensates, *Phys. Rev. B* **92**, 035305 (2015).

- [16] R. Dall, M. D. Fraser, A. S. Desyatnikov, G. Li, S. Brodbeck, M. Kamp, C. Schneider, S. Höfling, and E. A. Ostrovskaya, Creation of Orbital Angular Momentum States with Chiral Polaritonic Lenses, *Phys. Rev. Lett.* **113**, 200404 (2014).
- [17] L. Ge, A. Nersisyan, B. Oztop, and H. E. Tureci, Pattern formation and strong nonlinear interactions in exciton-polariton condensates, [arXiv:1311.4847](#) (unpublished).
- [18] A. Dreismann, P. Cristofolini, R. Balili, G. Christmann, F. Pinsker, N. G. Berloff, Z. Hatzopoulos, P. G. Savvidis, and J. J. Baumberg, Coupled counterrotating polariton condensates in optically defined annular potentials, *Proc. Natl. Acad. Sci. USA* **111**, 8770 (2014).
- [19] V. K. Kalevich, M. M. Afanasiev, V. A. Lukoshkin, D. D. Solnyshkov, G. Malpuech, K. V. Kavokin, S. I. Tsintzos, Z. Hatzopoulos, P. G. Savvidis, and A. V. Kavokin, Controllable structuring of exciton-polariton condensates in cylindrical pillar microcavities, *Phys. Rev. B* **91**, 045305 (2015).
- [20] Y. Sun, Y. Yoon, S. Khan, L. Ge, L. N. Pfeiffer, K. West, H. E. Tureci, D. W. Snoke, and K. A. Nelson, Stable switching among high-order modes in polariton condensates, [arXiv:1602.03024](#) (unpublished).
- [21] M. Wouters, T. C. H. Liew, and V. Savona, Energy relaxation in one-dimensional polariton condensates, *Phys. Rev. B* **82**, 245315 (2010).
- [22] P. R. Eastham, Mode locking and mode competition in a nonequilibrium solid-state condensate, *Phys. Rev. B* **78**, 035319 (2008).
- [23] M. Galbiati, L. Ferrier, D. D. Solnyshkov, D. Tanese, E. Wertz, A. Amo, M. Abbarchi, P. Senellart, I. Sagnes, A. Lemaître, E. Galopin, G. Malpuech, and J. Bloch, Polariton Condensation in Photonic Molecules, *Phys. Rev. Lett.* **108**, 126403 (2012).
- [24] D. Tanese, H. Flayac, D. Solnyshkov, A. Amo, A. Lemaître, E. Galopin, R. Braive, P. Senellart, I. Sagnes, G. Malpuech, and J. Bloch, Polariton condensation in solitonic gap states in a one-dimensional periodic potential, *Nat. Commun.* **4**, 1749 (2013).
- [25] V. G. Sala, D. D. Solnyshkov, I. Carusotto, T. Jacqmin, A. Lemaître, H. Tercas, A. Nalitov, M. Abbarchi, E. Galopin, I. Sagnes, J. Bloch, G. Malpuech, and A. Amo, Spin-Orbit Coupling for Photons and Polaritons in Microstructures, *Phys. Rev. X* **5**, 011034 (2015).
- [26] I. A. Shelykh, A. V. Kavokin, and G. Malpuech, Spin dynamics of exciton polaritons in microcavities, *Phys. Stat. Solid. B* **242**, 2271 (2005).
- [27] F. Manni, Y. Léger, Y. G. Rubo, R. André, and B. Deveaud, Hyperbolic spin vortices and textures in exciton-polariton condensates, *Nat. Commun.* **4**, 2590 (2013).
- [28] G. Li, T. C. H. Liew, O. A. Egorov, and E. A. Ostrovskaya, Incoherent excitation and switching of spin states in exciton-polariton condensates, *Phys. Rev. B* **92**, 064304 (2015).
- [29] M. Wouters and I. Carusotto, Excitations in a Nonequilibrium Bose-Einstein Condensate of Exciton Polaritons, *Phys. Rev. Lett.* **99**, 140402 (2007).
- [30] E. Kammann, T. C. H. Liew, H. Ohadi, P. Cilibrizzi, P. Tsotsis, Z. Hatzopoulos, P. G. Savvidis, A. V. Kavokin, and P. G. Lagoudakis, Nonlinear Optical Spin Hall Effect and Long-Range Spin Transport in Polariton Lasers, *Phys. Rev. Lett.* **109**, 036404 (2012).
- [31] M. Vladimirova, S. Cronenberger, D. Scalbert, K. V. Kavokin, A. Miard, A. Lemaître, J. Bloch, D. Solnyshkov, G. Malpuech, and A. V. Kavokin, Polariton-polariton interaction constants in microcavities, *Phys. Rev. B* **82**, 075301 (2010).
- [32] The physical values of the parameters that we use are the following: polariton mass $m_p = 10^{-4}m_e$, where m_e is free electron mass; the coherent polaritons relaxation rate $\Gamma_c = 0.05 \text{ ps}^{-1}$; the stimulated scattering rate $R = 0.01 \mu\text{m}^2/\text{ps}$; the effective nonlinearity (blue shift caused by the coherent polaritons) $g_c = 6 \times 10^{-3} \text{ meV} \mu\text{m}^2$; the characteristic time scale is $\tau = 1/\Gamma_c = 20 \text{ ps}$; and the length scale $L = 3.4 \mu\text{m}$. The value of the linear coupling strength used in the numerical modeling is $\sigma = 0.01 \text{ meV}$ in physical units.
- [33] K. Winkler, O. A. Egorov, I. G. Savenko, X. Ma, E. Estrecho, T. Gao, S. Müller, M. Kamp, T. C. H. Liew, E. A. Ostrovskaya, S. Höfling, and C. Schneider, Collective state transitions of exciton-polaritons loaded into a periodic potential, *Phys. Rev. B* **93**, 121303(R) (2016).
- [34] Y. G. Rubo, Half Vortices in Exciton Polariton Condensates, *Phys. Rev. Lett.* **99**, 106401 (2007).
- [35] K. G. Lagoudakis, T. Ostatnick, A. V. Kavokin, Y. G. Rubo, R. André, and B. Deveaud-Plédran, Observation of half-quantum vortices in an exciton-polariton condensate, *Science* **326**, 974 (2009).
- [36] G. Liu, D. W. Snoke, A. Daley, L. N. Pfeiffer, and K. West, A new type of half-quantum circulation in a macroscopic polariton spinor ring condensate, *Proc. Natl. Acad. Sci. USA* **112**, 2676 (2015).
- [37] M. R. Dennis, K. O'Holleran, and M. J. Padgett, Singular optics: optical vortices and polarization singularities, *Prog. Opt.* **53**, 293 (2009).
- [38] Y. V. Izdebskaya, A. S. Desyatnikov, G. Assanto, and Y. S. Kivshar, Dipole azimuthons and vortex charge flipping in nematic liquid crystals, *Opt. Express* **19**, 21457 (2011).
- [39] A. S. Desyatnikov, A. A. Sukhorukov, and Y. S. Kivshar, Azimuthons: Spatially Modulated Vortex Solitons, *Phys. Rev. Lett.* **95**, 203904 (2005).
- [40] V. M. Lashkin, E. A. Ostrovskaya, A. S. Desyatnikov, and Y. S. Kivshar, Vector azimuthons in two-component Bose-Einstein condensates, *Phys. Rev. A* **80**, 013615 (2009).
- [41] G. Li, Azimuthons and pattern formation in annularly confined exciton-polariton Bose-Einstein condensates, *Phys. Rev. A* **93**, 013837 (2016).
- [42] L. Dominici, G. Dagvadorj, J. M. Fellows, D. Ballarini, M. De Giorgi, F. M. Marchetti, B. Piccirillo, L. Marrucci, A. Bramati, G. Gigli, M. H. Szymańska, and D. Sanvitto, Vortex and half-vortex dynamics in a nonlinear spinor quantum fluid, *Sci. Adv.* **1**, e1500807 (2015).