Bistability phenomena in one-dimensional polariton wires

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We investigate the phenomena of bistability and domain-wall propagation in polaritonic systems with dissipation provided by interaction with an incoherent phonon bath. The results on temperature dependence of polariton bistability behavior and polariton neuron switching are presented.

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I. INTRODUCTION

The study of light-matter interactions is interesting and important from the point of view of both fundamental physics and device applications. In this context, the structure that attracted particular attention is a semiconductor microcavity with a quantum well (QW) embedded into it. Tuning the energy of the excitonic transition in resonance with the energy of the photonic cavity mode, one can reach a regime of strong coupling accompanied by the formation of hybrid quasiparticles called exciton polaritons. These half-light, half-matter particles exhibit a number of extraordinary properties. Due to their extremely small effective mass and bosonic nature, polaritons provide an opportunity to study various quantum collective phenomena, ranging from polariton BEC (Ref. 1) and Josephson effect2 to polariton-mediated superconductivity.3 Strong polariton-polariton interactions make it possible to observe several remarkable nonlinear effects such as superfluidity,4 bistability,5 and soliton-like propagation.6

Although free polaritons are two-dimensional (2D) particles, it was recently noted7 that it can be interesting to consider one-dimensional (1D) polariton wires (see setup in Fig. 1) provided by lateral confinement of the polaritons in one of the directions.8 Such wires have the potential to become basic building blocks in future spintronic devices, including polariton Berry-phase interferometers9 and polariton Datta and Das spin transistors.10 In the nonlinear regime, bistability effects in 1D polariton channels allow for the realization of logical circuits based on polariton neurons,11 in which a local switching between states propagates throughout the wire.

The above-mentioned phenomena require a proper theoretical basis for a description of all relevant processes, which is a nontrivial task. A successful theoretical consideration should include the effects of interaction with a reservoir of acoustic phonons which leads to polariton thermalization, and the effects of polariton-polariton scattering leading to blueshifts and nonlinearities. Additionally, it should be taken into account that polaritons have a finite lifetime, and the correct description of their dynamics should necessarily include the effects of pump and decay.

Currently, two main approaches have been pursued to describe the dynamics of interacting polaritons. First, with the assumption of full coherence, mean-field approximation gives the Gross-Pitaevskii equation (GPE) commonly used for the description of spatially inhomogeneous polariton condensates.12,13 However, while GPE includes polariton-polariton scattering, it does not describe interaction with a phonon reservoir. Assuming oppositely that the polaritons are completely incoherent, the dynamics in reciprocal space can be described by semiclassical Boltzmann equations.14–17 Unfortunately, this technique fails to describe the real-space dynamics of inhomogeneous systems.

Recently we proposed a formalism based on the full density-matrix approach to describe the dynamics of the interacting polariton system with dissipation in real space and time.18 This method was applied to study the propagation of 1D polariton droplets. However, we neglected the pumping terms, and thus consideration of the cw regime most relevant from the experimental point of view was not taken. This paper is devoted to bridging this evident gap. We investigate the phenomena of bistability and domain-wall propagation in polaritonic systems with dissipation provided by interaction with an incoherent phonon bath and present the results on temperature dependence of the hysteresis in the polariton system and polariton neuron switching.

II. FORMALISM

The general formalism for the time evolution of a spatially inhomogeneous bosonic system without account of the coherent pumping terms was developed in our previous paper.18 We refer to this paper for details and derivations and give here only a brief overview, adding detailed consideration of the pumping terms.

The state of the whole system is described by the combined polariton and phonon density matrix \( \rho = \rho_{ph} \otimes \rho_{pol} \) (factorization corresponds to Born approximation). The phonon part of the system is assumed to be time independent and thermalized, \( \rho_{ph} = \exp(-\beta H_{ph}) \), while we need to determine the time dependence of the single-particle polariton density matrix in real space. It can be conveniently represented using the polariton field operators \( \hat{\psi}(r,t), \hat{\psi}^\dagger(r,t) \):

\[
\rho(r,r',t) = \text{Tr}[\hat{\psi}^\dagger(r,t)\hat{\psi}(r',t)\rho] \equiv \langle \hat{\psi}^\dagger(r,t)\hat{\psi}(r',t) \rangle,
\]  

(1)

where the trace is performed by all the degrees of freedom of the system. A Fourier transform can be performed to work in reciprocal space, which makes the calculations easier.

\[
\rho(k,k',t) = (2\pi)^d/L^d \int e^{i(kr-k'r')} e^{i(kr-k'r')} \rho(r,r',t) dr dr',
\]  

(2)

where \( d \) is the dimensionality of the system (\( d = 2 \) for nonconfined polaritons, \( d = 1 \) for the polariton channel), \( L \) is its linear size, and \( a_k^\dagger a_k \) are the creation and annihilation operators.
operators of the polaritons with momentum $k$. If the time dependence of the density matrix in reciprocal space is determined, an inverse Fourier transform allows one to obtain the dynamics in real space straightforwardly.

The Hamiltonian of the system can be represented as a sum of the terms corresponding to various physically relevant processes in the system:

$$H = H_0 + H_{\text{pol}} + H_{\text{ph}} + H_{\text{cp}} + H_{\text{icp}},$$

where

$$H_0 = \sum_k E_k a_k^\dagger a_k$$

corresponds to free polariton propagation,

$$H_{\text{pol}} = U \sum_{k, k', p} a_{k'}^\dagger a_k a_{k+p} a_{k-p}$$

corresponds to polariton-polariton scattering,

$$H_{\text{ph}} = \sum_{k, q} D(q) a_k^\dagger a_{k-q} b_q + \text{H.c.}$$

corresponds to the polariton-phonon scattering, $H_{\text{cp}}$ corresponds to coherent laser pumping, and $H_{\text{icp}}$ corresponds to incoherent pumping and finite polariton lifetime (see expressions for these two terms below).

In the above formulas $E_k$ defines the dispersion of the free polaritons and the quantities $U$ and $D$ correspond to the polariton-polariton and polariton-phonon scattering. Their calculation is presented in Refs. 19–21.

The Hamiltonian (3) can be separated into the sum of the coherent part and the part introducing decoherence,

$$H = H_{\text{co}} + H_{\text{deco}},$$

$$H_{\text{co}} = H_0 + H_{\text{pol}} + H_{\text{cp}},$$

$$H_{\text{deco}} = H_{\text{ph}} + H_{\text{icp}}.$$  

The effects of the coherent and incoherent parts should be treated in different ways. As for the coherent processes in the system, they can be accounted for using the Liouville–von Neumann equation

$$i\hbar \langle \partial_t |\rho|_{\text{co}} \rangle = \{ \rho; H_{\text{co}} \}.$$  

On the contrary, the incoherent part of the evolution is described by the Lindblad equation, which reads

$$\partial_t \rho_{\text{deco}}(t) = -\int_{-\infty}^t dt' \{ H_{\text{deco}}(t'); [H_{\text{deco}}(t'), \rho(t)] \} = \delta_{\Delta E} \{ 2(H^+ \rho H^- + H^- \rho H^+) - (H^+ H^- + H^- H^+) \rho - \rho (H^+ H^- + H^- H^+) \},$$

where the coefficient $\delta_{\Delta E}$ denotes energy conservation and has dimensionality of inverse energy, and in the calculation is taken to be equal to the broadening of the polariton state. The terms $H^+$ and $H^-$ correspond to the processes when the thermal reservoir particle [phonon or other reservoir boson (see below)] is created or destroyed.

The effects of the terms corresponding to the polariton-phonon and polariton-polariton interactions were considered in detail in Ref. 18. For polariton-polariton scattering, these equations reproduce an analog of the Gross-Pitaevskii equation written for the density matrix, and for polariton-phonon interactions they are generalizations of the semiclassical Boltzmann equations. The corresponding equations for the elements of the density matrix and their derivation can be found there, and we reproduce them below after the introduction of a pump into the system (see below).

### A. Coherent pumping

We start from the case of a coherent pump. Its physical meaning is coupling to an electric field with a well-defined phase, provided, e.g., by an external laser beam. Mathematically, the corresponding Hamiltonian can be introduced as

$$\hat{H}_{\text{cp}} = \sum_{k'} p_k a_k^\dagger + \text{H.c.}$$

The coefficients $p_k$ are Fourier transforms of the pumping amplitudes in a real space $p(x, t)$, which in the case of a cw pump can be cast as

$$p(x, t) = P(x)e^{i k_p x - i \omega_p t},$$

where $P(x)$ is the pumping spot profile in real space, $k_p$ is an in-plane pumping vector resulting from the inclination of the laser beam with respect to the vertical, and $\omega_p$ is the pumping frequency of the single-mode laser.

Let us now check the effect of $\hat{H}_{\text{cp}}$ on the evolution of the polariton density matrix. Insertion of the Hamiltonian (10) into the Liouville–von Neumann equation yields the following result:

$$\partial_t \langle a_k^\dagger a_k \rangle = \frac{2}{\hbar} \text{Im} \{ p_k^* \langle a_k \rangle \},$$

$$\partial_t \langle a_k^\dagger a_k \rangle = \frac{i}{\hbar} \{ p_k \langle a_k \rangle - p_k^* \langle a_k^\dagger \rangle \}.$$
thus needs to obtain the expression for this quantity to close the system of the equations. Straightforward derivation gives
\[
\partial_t \langle a_k \rangle = \frac{i}{\hbar} \langle [E_k - \hbar \varepsilon_k] a_k \rangle - \frac{i}{\hbar} U \sum_{k_1, p} \rho(q_{k_1, k_2} - p) \langle a_{k_1} a_{k_2} \rangle
\]
\[
+ \sum_{q, E_k < E_{k+q}} W(q) \left[ \rho(k + q, k + q) - n_{q}^{ph} \right]
+ \sum_{q, E_k > E_{k+q}} W(q) \left[ -\rho(k + q, k + q) - n_{q}^{ph} - 1 \right] \langle a_k \rangle,
\]
where \( n_{q}^{ph} \) is the number of phonons with momentum \( q \), given by the Boltzmann distribution, and \( W(q) \) are the transition rates of phonon-assisted processes connected with matrix elements of the polariton-phonon interaction, \( W(q) = 2\pi D^2(q) \delta_{\Delta E}/\hbar \).

B. Incoherent pumping and lifetime

By incoherent pump we mean the exchange of particles between the polariton system and some incoherent bosonic reservoir whose nature depends on the pumping scheme. Usually, this will be an ensemble of incoherent excitons created by either an electrical pump or an incoherent optical excitation, or by a reservoir of external photonic modes providing leakage of the photons from the cavity. The corresponding Hamiltonian written in Dirac representation reads
\[
H_{\text{cp}} = \sum_{k, k'} K(k, k') e^{iE_k - E_{k'}}/\hbar \frac{i}{\hbar} a_k^{\dagger} b_{k'} + \text{H.c.}
\]
\[
= H_{\text{cp}}^+ + H_{\text{cp}}^-,
\]
where \( b_k \) is a secondary quantization operator corresponding to the bosonic reservoir in question, and \( K(k, k') \) are constants characterizing the coupling between the polariton system and the reservoir. The introduction of this Hamiltonian into the Lindblad equation leads to the standard terms whose derivation can be found elsewhere.\(^{22,24}\)

\[
\partial_t \langle a_k a_{k'} \rangle = i \hbar \delta_{kk'} - \frac{1}{2\hbar} (\gamma_k + \gamma_{k'}) (a_k a_{k'} + a_{k'} a_k),
\]
where the terms \( I_k \) and \( \gamma_k \) denote the intensity of the incoherent pump and broadening of the polaritonic levels connected with the lifetimes of the polariton states, \( \gamma_k = \hbar \tau_k^{-1} \). They are usually taken as phenomenological parameters but can be connected with the quantities entered in Hamiltonian (14):

\[
I_k = \frac{1}{\hbar} \sum_{k'} K(k, k')^2 \delta[\varepsilon(k) - \varepsilon(k')] n_{k'},
\]
\[
\gamma_k = \sum_{k'} K(k, k')^2 \delta[\varepsilon(k) - \varepsilon(k')],
\]
where \( n_{k'} \) are the occupancies of the bosonic reservoir.

In our further consideration we will consider only the case of the coherent pump, thus setting all \( I_k = 0 \) and retaining in the resulting equations, only the terms corresponding to the lifetime which corresponds to the case of an empty bosonic reservoir, \( n_{k} = 0 \) for all \( k \).

C. Final equations

To get a full system of equations for the dynamics of the polariton system with pump and decay one should combine the equations derived in Ref. 18 with expressions (12), (13), and (15). For the diagonal elements of the density matrix one gets

\[
\partial_t \rho(k, k) = \frac{2}{\hbar} \text{Im} \rho_k^* \langle a_k \rangle - \frac{\rho(k, k)}{\tau_k} + \frac{2}{\hbar} U \sum_{k_1, p} \text{Im} \rho_{k_1, k_2} (\rho_{k_1} - \rho_{k_2}) (\rho(k, k) + \rho(k, p) - \rho(p, k))
\]
\[
+ \sum_{q, E_k < E_{k+q}} W(q) \left[ \rho(k + q, k + q) (\delta_{q, q} - n_{q}^{ph}) (\rho(k, k + 1) - \rho(k, k) n_{q}^{ph} (\rho(k + q, k + q) + 1)}
+ \sum_{q, E_k > E_{k+q}} W(q) \left[ \rho(k + q, k + q) n_{q}^{ph} \left[ \rho(k, k) + 1 \right] \rho(k, k) (\rho(k + q, k + q) + 1) \right],
\]
where the first line corresponds to the coherent pumping and finite lifetime, the second one describes the polariton-polariton interaction, and the other lines refer to the polariton scattering with acoustic phonons.

For the off-diagonal part one has

\[
\partial_t \rho(k, k') = \frac{i}{\hbar} (E_k - E_{k'}) \rho(k, k') + \frac{i}{\hbar} \rho_k^* \langle a_k \rangle - \frac{1}{2\tau_k} \rho(k, k')
\]
\[
+ \frac{i}{\hbar} U \sum_{k, p} \rho(k, k - p) \rho(k - p, k') - \rho(k, k') - \rho(k, k) (\rho(k, k) + 1) \left[ \sum_{q, E_k < E_{k+q}} W(q) \left[ \rho(k + q, k + q) n_{q}^{ph} \right] \right]
+ \sum_{q, E_k > E_{k+q}} W(q) \left[ \rho(k + q, k + q) n_{q}^{ph} - 1 \right]
\]
\[
+ \sum_{q, E_k > E_{k+q}} W(q) \left[ \rho(k' + q, k' + q) n_{q}^{ph} - 1 \right] \right],
\]
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The resulting formalism is suitable for describing both 2D polaritons and polaritons confined in 1D channels. Consideration of the former case, however, needs powerful computing facilities, and in the present paper we focus only on consideration of the latter one.

III. RESULTS AND DISCUSSION

We considered a 2-\(\mu\)m-wide and 50-\(\mu\)m-long polariton channel in a microcavity with an active region based on InAlGaAs alloys with Rabi splitting 15 meV. The matrix elements of the polariton-polariton and polariton-phonon interactions were estimated using the standard formulas\(^{19-21}\) and were taken as \(U = 21 \, \mu\)eV and \(W(q) = 10 \, \text{neV}\), with the phonon interaction constant being \(q\) independent and estimated at a characteristic value of the exchange momentum. The boundary conditions are inherently periodic from the model, and the discretization length of the wire was \(a = L/100\). The maximum value in \(k\) space is thus \(k_{\text{max}} = \pi/a\) and the step size is \(k_{\text{step}} = 2\pi/L\).

The first phenomenon we modeled was the effect of thermalization in the polariton system provided by the polariton-phonon interaction on its bistable behavior. It is well known that if the polariton ensemble is fully coherent and its dynamics is described by the Gross-Pitaevskii equation containing coherent pumping and lifetime terms, and if the energy of the pumping laser lies slightly above the bottom of the lower polariton branch, the dependence of the concentration of the polaritons on pump intensity is described by an S-shaped curve, characteristic for systems revealing the effects of bistability and hysteresis.\(^{25}\) Such a behavior is due to the polariton-polariton interactions which introduce nonlinearity into the system.

On the other hand, in the approach based on the semiclassical Boltzmann equations corresponding to the limit of strong decoherence, bistability is absent and dependence of the occupancy of the ground state on the pump intensity is described by a single defined threshold function.\(^{15}\) One can expect that a transition between these two regimes should occur if one raises the temperature in the system, which leads to intensification of the polariton-phonon interactions and decoherence in the system. This was indeed observed in our calculations.

The computational results are presented in Fig. 2. The system is pumped with a spatially homogeneous laser beam oriented perpendicular to the QW (i.e., at \(k = 0\) in \(k\) space) at a slightly higher frequency than \(k = 0\) polariton frequency (0.5 meV detuning). At low pump intensity, the pump frequency is not in resonance with the condensate. Consequently, the condensate occupation remains fairly low. As the pumping is increased, the polariton energy is blueshifted into resonance with the pump and there is a sudden jump in the occupation of the \(k = 0\) polariton state at some characteristic pump intensity \(I_0\). If one then decreases the pump, the polariton occupancy jumps down to a different value \(I_1 < I_0\), which corresponds to the hysteresis behavior. However, the increase in temperature leads to intensification of the phonon scattering, and bistable behavior becomes less and less pronounced: the hysteresis area narrows and is quenched completely above some critical temperature \(T_c \approx 70\) K. This corresponds to the transition between the Gross-Pitaevskii and Boltzmann regimes in the polariton system. A similar phenomenon was earlier predicted for a microcavity-based terahertz-emitting device.\(^{26}\)

The effect of bistability can form a basis for creation of a variety of devices based on nonlinear polariton transport.\(^{27}\) Among them are polariton neurons,\(^{28}\) the building blocks of polariton-based optical integrated circuits\(^{11}\) utilizing the phenomenon of domain-wall propagation in bistable systems. The underlying idea is the following. Imagine that the polariton system is driven by a spatially homogeneous background cw pump with intensity corresponding to the bistable regime, and the steady-state occupancy of the system corresponds to the lower branch of the S-shaped curve. Then a short localized pulse is applied in the middle of the wire. Its intensity should be enough to send the condensate locally to the upper branch of the S-shaped curve. Due to diffusion, the polariton wave packet spreads to the neighboring regions and switches them to the upper branch. This way, the area of high occupancy steadily expands. This phenomenon is analogous to the propagation of the domain wall in ferromagnetic materials. It should be noted that although the polaritons have a finite lifetime, this does not limit the length of signal propagation in a polariton neuron, and the signal keeps propagating as long as the background cw pumping persists.

Temperature should have a strong effect on neuron behavior. Indeed, bistability switching strongly depends on it, as we have shown above. Moreover, the increase in temperature affects the dynamics of the polaritons in real space, which have a strong impact on the velocity of bistability switching, as we show below. Figure 3 represents the results of our calculations. At 0 K, the switching is clear and one can easily see the propagation of a sharp, well-defined domain wall. At 6 and 12 K (using the same pump intensity as before) the switching is still clearly visible, although the features are a bit smeared out.
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FIG. 3. (Color online) Neuron behavior and domain-wall propagation for (top to bottom) \( T = 0, 6, \) and 12 K. The plots show the polariton in units of \( \mu m^{-2} \). The system is near-resonantly pumped by a spatially homogeneous cw laser; at \( t = 25 - 35 \) ps a switching pulse arrives. At 0 K one sees the propagation of a distinct domain wall. It is still well visible at 6 and 12 K, although the features are a bit smeared out. In fact, the propagation speed of the domain wall increases with temperature, since the polaritons diffuse faster. Bottom plot: domain-wall propagation speed as a function of temperature. Above some critical temperature there is no bistability and therefore no population switching.

More importantly, the domain-wall propagation speed clearly increases with temperature, as is shown in the bottom plot of Fig. 3. This can be explained by the polaritons diffusing faster in space at higher temperature and could have a positive effect on functioning of the realistic polariton devices. Note, however, that increasing the temperature makes the domain wall less sharp, and at some critical temperature the switching behavior abruptly disappears.

Finally, the effect of phonon scattering on pure dephasing in the system was considered. We pumped the system with a coherent pulse having a Gaussian profile in the real space. For various temperatures of the system, we investigated in the steady state the spatial profiles of both the total polariton density and its coherent part determined as \( |\psi(x)|^2 \), where

\[
\psi(x) = \int_{-\infty}^{+\infty} \langle a_k \rangle e^{ikx} dk.
\]

The results are shown in Fig. 4 for temperatures 1, 10, and 50 K. Pumping at \( k = 0 \), no detuning. The solid lines show the total population, the dashed lines the coherent population, and the dotted lines the incoherent fraction. One can see that at low temperatures the coherent fraction is quite large at the center of the coherent pumping spot, but it dramatically decays outside of it. However, as the temperature is increased, the density profile gets more spread out over the wire, forming an almost constant background density made up of the decoherent population.

FIG. 4. Coherent fraction at (top to bottom) \( T = 1, 10, \) and 50 K. Pumping at \( k = 0 \), no detuning. The solid lines show the total population, the dashed lines the coherent population, and the dotted lines the incoherent population.

IV. CONCLUSION

In conclusion, we have considered the effects of coherent pumping and finite lifetime in a polaritonic system accounting for all physically relevant processes in the system. We applied our theory for the consideration of nonlinear polariton propagation in a 1D polariton wire. We have shown that the increase of temperature dramatically affects such processes as bistability switching and domain-wall propagation in polariton neurons.

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27. For a recent review on this subject see T. C. H. Liew, I. A. Shelykh, and G. Malpuech, *Physica E: Low Dimensional Systems and Nanostructures* **43**, 1543 (2011).