Viscosity structure of Earth’s mantle inferred from rotational variations due to GIA process and recent melting events

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SUMMARY
We examine the geodetically derived rotational variations for the rate of change of degree-two harmonics of Earth’s geopotential, \( \dot{J}_2 \), and true polar wander, combining a recent melting model of glaciers and the Greenland and Antarctic ice sheets taken from the IPCC 2013 Report (AR5) with two representative GIA ice models describing the last deglaciation, ICE5G and the ANU model developed at the Australian National University. Geodetically derived observations of \( \dot{J}_2 \) are characterized by temporal changes of \(- (3.7 \pm 0.1) \times 10^{-11} \text{yr}^{-1}\) for the period 1976–1990 and \(- (0.3 \pm 0.1) \times 10^{-11} \text{yr}^{-1}\) after ~2000. The AR5 results make it possible to evaluate the recent melting of the major ice sheets and glaciers for three periods, 1900–1990, 1991–2001 and after 2002. The observed \( \dot{J}_2 \) and the component of \( \dot{J}_2 \) due to recent melting for different periods indicate a long-term change in \( \dot{J}_2 \) — attributed to the Earth’s response to the last glacial cycle — of \(- (6.0–6.5) \times 10^{-11} \text{yr}^{-1}\), significantly different from the values adopted to infer the viscosity structure of the mantle in most previous studies. This is a main conclusion of this study. We next compare this estimate with the values of \( \dot{J}_2 \) predicted by GIA ice models to infer the viscosity structure of the mantle, and consequently obtain two permissible solutions for the lower mantle viscosity (\( \eta_{\text{lm}} \)), \( \sim 10^{22} \) and \( (5–10) \times 10^{22} \text{Pa.s} \), for both adopted ice models. These two solutions are largely insensitive to the lithospheric thickness and upper mantle viscosity as indicated by previous studies and relatively insensitive to the viscosity structure of the \( D^* \) layer. The ESL contributions from the Antarctic ice sheet since the last glacial maximum (LGM) for ICE5G and ANU are about 20 and 30 m, respectively, but glaciological reconstructions of the Antarctic LGM ice sheet have suggested that its ESL contribution may have been less than \( \sim 10 \text{ m} \). The GIA-induced \( \dot{J}_2 \) for GIA ice models with an Antarctic ESL component of \( \sim 10 \text{ m} \) suggests two permissible lower mantle viscosity solutions of \( \eta_{\text{lm}} \sim 2 \times 10^{22} \) and \( \sim 5 \times 10^{22} \text{Pa.s} \) or one solution with \( (2–5) \times 10^{22} \text{Pa.s} \). These results suggest that the effective lower mantle viscosity is larger than \( \sim 10^{22} \text{Pa.s} \) regardless of the uncertainties for an Antarctic ESL component. We also examine the polar wander due to recent melting and GIA processes, suggesting that the observed polar wander may be significantly attributed to convection motions in the mantle and/or another cause, particularly for permissible lower mantle viscosity solution of \( (5–10) \times 10^{22} \text{Pa.s} \).

Key words: Earth rotation variations; Dynamics of lithosphere and mantle; Rheology: mantle.

1 INTRODUCTION
Rotational variations for degree-two deformation of the Earth, described by true polar wander (TPW) and the rate of change of degree-two zonal harmonic of the Earth’s geopotential, \( \dot{J}_2 \), are caused by perturbations of inertia elements due to mass redistribution on and/or within the Earth (Munk & MacDonald 1960; Lambeck 1980). In practice, glacial isostatic adjustment (GIA) processes due to the last glacial cycle contribute significantly to the geodetically observed variations and the GIA component of the rotational variations are highly sensitive to lower mantle viscosity (e.g. O’Connell 1971; Nakiboglu & Lambeck 1980; Sabadini & Peltier 1981; Yuen et al. 1982; Yoder et al. 1983; Rubincam 1984; Wu & Peltier 1984; Spada et al. 1992; Milne & Mitrovica 1996; Vermeersen & Sabadini 1996; Vermeersen et al. 1996, 1997; Mitrovica & Milne 1998; Johnston & Lambeck 1999; Nakada 2002;
Nakada & Okuno 2003; Mitrovica et al. 2005; Peltier 2007; Cambiotti et al. 2010; Morrow et al. 2013) and to the characteristics of the low viscosity $D''$ layer (Peltier & Drummond 2010; Nakada & Okuno 2013). GIA-induced rotational variations therefore provide an important constraint on the viscosity structure of the deep mantle, a crucial quantity in discussing mantle dynamics (e.g. Forte 2007).

The observations are also affected by recent melting of glaciers and the Greenland and Antarctic ice sheets (e.g. Lambeck 1980; Gasperini et al. 1986; Yuen et al. 1987; Peltier 1988; Sabadini et al. 1988; Mitrovica & Peltier 1993; Trupin 1993; James & Ivins 1997; Nakada & Okuno 2003; Mitrovica et al. 2006; Nerem & Wahr 2011; Morrow et al. 2013). For example, Peltier (1988) suggested that the melting of mountain glaciers tabulated by Meier (1984) is important in modelling $J_2$ and polar wander. Lemke et al. (2007) undertook an extensive review of recent melting of these ice masses in the Fourth Assessment Report of the Intergovernmental Panel on Climate Change (AR4), which has subsequently been updated and extended by Vaughan et al. (2013) in the Fifth Assessment Report of the Intergovernmental Panel of Climate Change (AR5).

The latter study comprehensively discussed recent melting of the glaciers and major ice sheets for separate periods after 1900 based on sharp accelerations in the rate of melting. From these results, it is possible to evaluate the rotational variations for three broad periods, 1900–1990, 1991–2001 and after 2002, caused by the melting of glaciers and the Greenland and Antarctic ice sheets separately.

McCarthy & Luzum (1996) inferred TPW of $\sim 1$ Myr$^{-1}$ towards Hudson Bay and an estimate of $J_2$ by Nerem & Klosko (1996) based on observed $J_2$ for two decades after 1976 is $-(2.5-3.0) \times 10^{-11}$ yr$^{-1}$. However, recent observations of rotational variations indicate an abrupt change in polar wander direction around 2005 (Chen et al. 2013) and a gradual deceleration in the rate of the decrease in $J_2$ after $\sim 1990$ (e.g. Cheng et al. 2013). These observations provide important constraints on the rate of the surface mass redistribution from melting of the glaciers and both polar ice sheets. Nerem & Klosko (1996) studied two decades of observations of $J_2$ and noted no temporal dependence, suggesting that the changes observed in the more recent time series are due to the effects of subsequent surface mass redistributions (e.g. Nerem & Wahr 2011; Roy & Peltier 2011; Cheng et al. 2013), and that the contribution from recent melting and GIA processes may be separable if we incorporate the modern melting history developed by Vaughan et al. (2013).

In this study, the observationally derived estimates of both the recent melting of glaciers and polar ice sheets, and the concomitant rotational variations of $J_2$, are used to extract the GIA-induced rotational component of $J_2$ from the observed variations. The extraction is possible principally because the observationally derived $J_2$ is almost insensitive to the convective processes in the mantle (Greff-Lefftz et al. 2010). This is a principle goal of this study. The inferred GIA-induced $J_2$ is then used to constrain the rheological parameters of the mantle, particularly for the lower mantle, and the GIA ice models describing the melting histories for the last deglaciation. In most studies (e.g. Spada et al. 1992; Peltier 2007; Peltier & Drummond 2010), the lower mantle viscosity has been inferred from the observed $J_2$ of $\sim 3 \times 10^{-11}$ yr$^{-1}$. It is, however, necessary to adopt GIA-induced $J_2$ in inferring the viscosity structure of the mantle. The mathematical method adopted here is essentially the same as that by Nakada & Okuno (2013), but we evaluate the rotational variations by incorporating the Earth’s rotational feedback (Milne & Mitrovica 1998) into the sea level calculation of Okuno & Nakada (2001). An evaluation of the rotational variation is based on the formulation of Nakada (2009) incorporating the effect of convectively supported excess ellipticity first indicated by Mitrovica et al. (2005) (see also Matsuyama et al. 2010).

We also briefly discuss the observed TPW, our predictions involving the effects of recent melting and GIA processes and the misfit of both quantities. The misfit may constrain the potential impact of convective motions in the mantle on TPW. This phenomenon was first quantitatively discussed by Steinberger & O’Connell (1997) who estimated TPW, at a rate of 0.37″ Myr$^{-1}$ and direction of 24° W, due to material flow induced by mantle density heterogeneities inferred from seismic tomography data (see also Ricard & Saba- dini 1990; Nakada 2008; Cambiotti et al. 2011; Chan et al. 2011; Creveling et al. 2012).

The paper is organized as follows. In Section 2, we explain the viscosity models of the mantle, and recent melting and GIA ice models adopted in this study. We show the results for GIA-induced $J_2$ derived from recent melting model and observationally derived $J_2$ in Section 3, and infer the viscosity structure by comparing the GIA-induced $J_2$ with the predicted $J_2$ based on GIA ice models in Section 4. In Section 5, we discuss the polar wander involving the effects of recent melting and GIA processes (see also Appendix A). In Section 6, we summarize the results obtained in this study.

## 2 Models Adopted in This Study

### 2.1 Earth models

We adopt the seismological model PREM (Dziewonski & Anderson 1981) to calculate the density and elastic constants as a function of depth. The rheological models adopted here are characterized by lithospheric thickness ($H$) (which is assumed to be effectively elastic), upper mantle viscosity above 670 km depth ($\eta_{um}$), lower mantle viscosity above the $D''$ layer ($\eta_{um}$), and viscosity structure of the $D''$ layer with 300 km thickness. The adopted ranges of values for $H$, $\eta_{um}$ and $\eta_{lm}$ are: $H = 65$ and $100$ km, $10^{20} \leq \eta_{um} \leq 10^{21}$ Pa s and $10^{21} \leq \eta_{lm} \leq 10^{23}$ Pa s. We have two baseline models for lithospheric thickness and upper mantle viscosity. The first uses values of $H = 65$ km and $\eta_{um} = 4 \times 10^{20}$ Pa s, and is derived from the inversion of observed relative sea level (RLS) changes due to the melting of the British and Scandinavian ice sheets (e.g. Lambeck et al. 1990; Lambeck & Johnston 1998). The second model uses values of $H = 65$ km and $\eta_{um} = 2 \times 10^{21}$ Pa s given by the inversion of far-field RLS changes (Nakada & Lambeck 1989; Lambeck & Nakada 1990; Lambeck et al. 2014). The range of the lower mantle viscosity is based on the results by RSL changes for the Australian region (Nakada & Lambeck 1989; Lambeck & Nakada 1990), the flow models derived from global long geoid anomalies (e.g. Hager et al. 1985) and the joint inversions of GIA and convection data sets (e.g. Mitrovica & Forte 2004) summarized by Forte (2007).

The sensitivity to the low viscosity $D''$ layer has been rigorously discussed by Nakada & Okuno (2013) (see also Peltier & Drummond 2010). In this study, we adopt three types of viscosity models for the $D''$ layer, LVN, LV50 and LV2L (Table 1). The LVN model has no low viscosity $D''$ layer as is usually assumed in GIA studies (e.g. Wu & Peltier 1984) and the LV50 model has a low viscosity $D''$ layer (300 km thickness) of $5 \times 10^{19}$ Pa s. The LV2L model has a low viscosity $D''$ structure with the upper 200 km section having a viscosity of $5 \times 10^{19}$ Pa s and the lower 100 km section having a viscosity of $10^{17}$ Pa s. The viscosity models of LV50 and LV2L were inferred from both the decay time of the Chandler wobble and tidal deformations (Nakada & Karato 2012; Nakada et al. 2012), and the LV2L model is the preferred model of Nakada & Karato (2012).
2.2 Ice models

Two ice models describing the melting histories of glaciers and polar ice sheets are used to evaluate the rotational variations for the past \( \sim 100 \) yr. One is the GIA ice model describing the melting histories for the last deglaciation, and the other is that for the melting of glaciers and the Greenland and Antarctic ice sheets after \( \sim 1900 \).

Here we adopt two typical GIA ice models: ICE5G (Peltier 2004) and the ANU model used as the initial model in the inversion study for far-field sea level data by Lamb et al. (2014). ICE5G is an updated version of global ice coverage developed by Peltier’s group, which has been adopted in many GIA studies. These ice models were constructed mainly based on the comparison of late Pleistocene and Holocene sea level observations with GIA model predictions. Consequently, there is a trade-off between the viscosity and GIA ice models. In fact, the preferred lower mantle viscosity is \((2–3) \times 10^{21} \) Pa s, VM2 for Peltier (2004), and \((1–5) \times 10^{22} \) Pa s for ANU ice model (e.g. Lamb & Johnston 1998). Lamb et al. (2014) discuss this ambiguity seen in their inversions of far-field sea level data and we return to this point in Section 4.

The ANU ice model gives the melting histories of the major ice sheets for the past 250 kyr (the last two glacial cycles). The ICE5G model, however, describes the histories after the last glacial maximum (LGM) at \( \sim 20 \) kyr BP and we reconstruct the melting histories during the past 240 kyr (two glacial cycles) by adopting the ice volume change converted from smoothed oxygen isotopic data by Lisiecki & Raymo (2005). The equivalent sea level (ESL) curve is then converted to an ice sheet history by assuming that the geometry of the ice sheet is identical to the one for the ICE5G whenever the ESL matches a snapshot of ESL in the ICE5G. The GIA components of modern rotational variations are almost entirely due to the effects of the last deglaciation, and we confirmed that the melting histories before 240 or 250 kyr BP have no impact on the GIA-related rotational variations.

Table 1. Viscosity models for the \( D^2 \) layer adopted in this study. The bottom of the \( D^2 \) layer is 2891 km depth. In all models, the lithospheric (elastic) thicknesses (\( H \)) are 65 and 100 km, upper mantle viscosities (\( \eta_{100} \)) above 670 km depth are \((1, 2, 3, 4, 5, 7, 10) \times 10^{20} \) Pa s and lower mantle viscosities (\( \eta_{70} \)) (except for the \( D^2 \) layer) are \((1, 2, 5, 10, 20, 50, 100) \times 10^{21} \) Pa s.

<table>
<thead>
<tr>
<th>Model name</th>
<th>Viscosity structure of the ( D^2 ) layer</th>
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<tbody>
<tr>
<td>LVN</td>
<td>No low viscosity ( D^2 ) layer</td>
</tr>
<tr>
<td>LV50</td>
<td>( 5 \times 10^{19} ) Pa s (300 km thickness)</td>
</tr>
<tr>
<td>LV2L</td>
<td>( 5 \times 10^{19} ) Pa s for the upper 200 km thickness and ( 10^{21} ) Pa s for the lower 100 km thickness</td>
</tr>
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</table>

3 \( J_2 \) DUE TO RECENT MELTING AFTER \( \sim 1900 \) AND GIA-INDUCED \( J_2 \)

Temporal variations of the \( J_2 \) gravitational potential coefficient are insensitive to the convective processes in the mantle (Greb-Lefftz et al. 2010), and therefore, the secular rate of \( J_2 \) is given by:

\[
\frac{\Delta J_2}{\Delta t} = \frac{\Delta J_2^{GM}}{\Delta t} + \frac{\Delta J_2^{SM}}{\Delta t}
\]

In this study, we discuss \( J_2^{SM} \) for two periods: 1900–1990 and 2002–2011. We consider these periods separately because the rate of ice sheet melting and sea level rise have accelerated with time since \( \sim 1900 \) (e.g. Cazenave & Nerem 2004; Church et al. 2013; Vaughan et al. 2013). This is reflected in the model of global ice volume developed by Vaughan et al. (2013) and the corresponding rate for the period 2003–2009 from Gardner et al. (2013). The normalized mass change rates for each disk (the upper for the MG1 and the lower for the MG2) are also shown in Fig. 1(a). Positive values indicate melting while negative values indicate growth. The estimates of equivalent sea level rise (ESLR) for three periods, 1900–1990, 1991–2001 and 2002–2011, are estimated based on the results of tables 4.5 and 4.6 by Vaughan et al. (2013), which are also used in estimating the ESLR values for ice models discussed below (Table 2).
Figure 1. Distributions of disk loads for recent melting models of glaciers and the Greenland and Antarctic ice sheets adopted in this study derived from the results by Vaughan et al. (2013) in the AR5: (a) for glaciers excluding peripheral glaciers around the Greenland and Antarctic ice sheets (MG1 for the period 1900–1990 and MG2 for 2002–2011), (b) for glaciers peripheral to the Greenland ice sheet describing recent melting for 1900–1990, (c) for glaciers peripheral to the Antarctic ice sheet describing recent melting for 1900–1990, (d) for the Greenland ice sheet for the period 2002–2011 and (e) for the Antarctic ice sheet for the period 2002–2011. Model names are shown in each figure, and detailed descriptions of these melting models are explained in the text. The numbers shown for (a), (d) and (e) indicate the normalized mass change rates for each disk load, and the upper and lower ones for (a) are for MG1 and MG2, respectively. The rates for (b) and (c) are assumed to be constant for all disks.
values of ESLR calculated for each period. It is also apparent in the observationally derived \( J_2 (t) \) for recent melting models (see Section 2.2) with an assumed ESLR value of 1 mm yr\(^{-1}\) based on viscosity models of LVN and LV50 (Table 1), and evaluate the \( J_2 (t) \) for two periods using these normalized values. The results for the LV2L are nearly similar to those for the LV50. Fig. 2 shows \( J_2 (t) \) after the onset of the melting based on viscosity models with lithospheric thickness \( H \) of 65 km, upper mantle viscosity \( (\eta_{um}) \) of \( 4 \times 10^{20} \) Pa s and lower mantle viscosity \( (\eta_m) \) of \( 10^{23} \) Pa s. The prediction is insensitive to the values of \( H \) and \( \eta_{um} \) adopted in this study as previously noted (Vermeersen \& Sabadini 1996; Mitrovica \& Milne 1998; Johnston \& Lambeck, 1999). The \( J_2 (t) \) for LV50 decreases in the initial phase \(( t < 20 \) yr after the commencement of melting) because of the low viscosity \( D' \) layer, but the trend after \( t > 20 \) yr is nearly identical for the two viscosity models. That is, the rapid initial decrease due to \( D' \) layer may be important in estimating \( J_2 (t) \) due to the melting for the period of 2002–2011.

The \( J_2 (t) \) values for mountain glacier ice models MG1 and MG2 indicate that the differences between the predictions for both ice models are negligible. That is, \( J_2 (t) \) is only sensitive to the ESLR of the mountain glacier mass balance. The ANT melting model is more effective in exciting the \( J_2 (t) \) than other models, and the \( J_2 (t) \) for MG1 and MG2 is \( \sim 10^{-11} \) yr\(^{-1}\), much smaller than the contributions from Antarctica and Greenland. We also note that the predictions for GREENg and ANTg models, melting models for peripheral glaciers of both polar ice sheets, are similar to those for GREEN and ANT, respectively.

To examine the sensitivity of \( J_2 \) for the period 1900–1990 to recent melting models, we consider \( J_2 (t) \) at \( t = 45 \) yr after the commencement of the melting (see Fig. 2). Fig. 3 shows the predicted \( J_2 (t) \) as a function of lower mantle viscosity for ice models MG1, GREEN and ANT with an assumed ESLR value of 1 mm yr\(^{-1}\). The predictions for MG2 are almost the same as those for the MG1 as inferred from the results shown in Fig. 2. Though Vaughan et al. (2013) state that the meltwater contribution from the Greenland and Antarctic ice sheets is negligible for the period 1900–1990, we also show the results for ice models GREEN and ANT. The predicted \( J_2 \) for the GREENg is nearly the same as that for GREEN as inferred from the results shown in Fig. 2. Although we do not show it here, we also evaluated the \( J_2 \) based on melting models for both polar ice sheets by considering the uncertainties such as mass change rate of each disk load and the melting regions. In particular, we examined the \( J_1 \) for the melting model of the Antarctic ice sheet with melting disk loads limited to the West Antarctica (red circles in Fig. 1e). However, the difference between the predictions for such models and those for ANT in Fig. 3 is at most \( 1.5 \times 10^{-12} \) yr\(^{-1}\). The ice model sensitivities shown in Fig. 3 are also applicable to the predictions at \( t \sim 5 \) yr (see Fig. 2), which is used in evaluating \( J_2 (t) \) for the period 2002–2011. We therefore adopt the predictions based on ice models of MG1, GREEN and ANT in discussing \( J_2 (t) \) for both periods 1900–1990 and 2002–2011.

The AR5 states that ocean thermal expansion and glacier melting, both mountain glaciers (MG1) and those peripheral to the Greenland ice sheet (GREENg), have been the dominant contributors to global sea level rise for the period 1900–1990 (Church et al. 2013; Vaughan et al. 2013). The estimates of ESLR with uncertainty range of 90 per cent, for the alpine glaciers and those peripheral to the Greenland ice sheet, are \( 0.54 \pm 0.07 \) and \( 0.15 \pm 0.05 \) mm yr\(^{-1}\), respectively (see Table 2). We evaluate \( J_2 (t) \) for these estimates using the normalized predictions with ESLR = 1 mm yr\(^{-1}\) in Fig. 3 and show the results in Table 3. The values of \( J_2 (t) \) for 1945 for a combined ice model of MG1 and GREENg estimated using the results at \( t = 45 \) yr (Fig. 3), are \((2.1 \pm 0.4) \times 10^{-11} \) and \((2.0 \pm 0.4) \times 10^{-11} \) yr\(^{-1}\) for viscosity models of LVN and LV50, respectively. These predictions are nearly insensitive to the lower

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**Table 2.** Values of equivalent sea level rise (ESLR) for recent melting models used to evaluate the rotational variations shown in Table 3 and Fig. 9. The estimates of ESLR are based on the results with uncertainty range of 90 per cent in Tables 4.5 and 4.6 by Vaughan et al. (2013) in the IPCC 2013 Report (AR5). The distribution of the disk loads for each model is shown in Fig. 1.

<table>
<thead>
<tr>
<th>Period (year)</th>
<th>Model name</th>
<th>ESLR (mm yr(^{-1}))</th>
<th>Model name</th>
<th>ESLR (mm yr(^{-1}))</th>
<th>Model name</th>
<th>ESLR (mm yr(^{-1}))</th>
<th>Total ESLR (mm yr(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1900–1990</td>
<td>MG1</td>
<td>0.54 ± 0.07</td>
<td>GREENg</td>
<td>0.15 ± 0.05</td>
<td>ANT</td>
<td>0.08 ± 0.19</td>
<td>0.69 ± 0.12</td>
</tr>
<tr>
<td>1991–2001</td>
<td>MG1</td>
<td>0.73 ± 0.37</td>
<td>GREEN</td>
<td>0.09 ± 0.11</td>
<td>ANT</td>
<td>0.08 ± 0.19</td>
<td>0.90 ± 0.67</td>
</tr>
<tr>
<td>2002–2011</td>
<td>MG1 (MG2)</td>
<td>0.80 ± 0.37</td>
<td>GREEN</td>
<td>0.59 ± 0.16</td>
<td>ANT</td>
<td>0.40 ± 0.21</td>
<td>1.79 ± 0.74</td>
</tr>
</tbody>
</table>

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**Figure 2.** Predicted \( J_2 (t) \) after the commencement of melting for recent melting models, MG1, MG2, GREEN (Greenland ice sheet), GREENg (peripheral glaciers of the Greenland ice sheet), ANT (Antarctic ice sheet) and ANTg (peripheral glaciers of the Antarctic ice sheet), with an assumed ESLR value of 1 mm yr\(^{-1}\): (a) for viscosity model LVN and (b) for LV50 (see Table 1). The predictions are based on the viscosity models with lithospheric thickness \( H \) of 65 km, upper mantle viscosity \( (\eta_{um}) \) of \( 4 \times 10^{20} \) Pa s and lower mantle viscosity \( (\eta_m) \) of \( 10^{22} \) Pa s.
mantle viscosity as inferred from the results shown in Fig. 3. Then, the GIA-induced $J_2$ values, $J_2^ {GI A}$ based on MG1 and GREEN, and observationally derived $J_2$ of $-(3.7 \pm 0.1) \times 10^{-11} \text{yr}^{-1}$, are $-(5.8 \pm 0.5) \times 10^{-11}$ and $-(5.7 \pm 0.5) \times 10^{-11} \text{yr}^{-1}$, respectively (Table 3). Both viscosity models give effectively the same answer for $J_2^ {GI A}$. The reason being that the Earth response is essentially elastic on these time scales. The magnitude for the estimates of $J_2^ {GI A}$ may be a lower limit if there is also a recent melting component from the Antarctic ice sheet (e.g. Nakada et al. 2013).

We now consider $J_2$ for the period 2002–2011 using ESLR values of MG1, GREEN and ANT ice models (Table 2) based on the results by Vaughan et al. (2013), namely:

(1) $0.73 \pm 0.37 \text{mm yr}^{-1}$ for 1991 $\leq t \leq 2001$ and
(2) $0.80 \pm 0.37 \text{mm yr}^{-1}$ for $t \geq 2002$ for MG1,
(3) $0.09 \pm 0.11 \text{mm yr}^{-1}$ for 1991 $\leq t \leq 2001$ and
(4) $0.59 \pm 0.16 \text{mm yr}^{-1}$ for $t \geq 2002$ for GREEN, and
(5) $0.08 \pm 0.19 \text{mm yr}^{-1}$ for 1991 $\leq t \leq 2001$ and
(6) $0.40 \pm 0.21 \text{mm yr}^{-1}$ for $t \geq 2002$ for ANT.

We then numerically evaluate $J_2^ {RM}$ and adopt the prediction at 2006 as $J_2^ {RM}$ for this period. The $J_2^ {RM}$ value is $(6.3 \pm 2.6) \times 10^{-11} \text{yr}^{-1}$ for viscosity model LN and $(6.0 \pm 2.5) \times 10^{-11} \text{yr}^{-1}$ for LV50 (Table 3). The values of $J_2^ {RM}$, estimated from these predictions and the observationally derived $J_2$ of $-(0.3 \pm 0.1) \times 10^{-11} \text{yr}^{-1}$, are $-(6.6 \pm 2.7) \times 10^{-11} \text{yr}^{-1}$ for the LN and $-(6.3 \pm 2.6) \times 10^{-11} \text{yr}^{-1}$ for the LV50. These results are summarized in Table 3.

The uncertainty ranges for the period 2002–2011 are considerably larger than those for the period 1900–1990, but the estimate for 2002–2011 is not inconsistent with that for 1900–1990. That is, the observed $J_2$ and the component of $J_2$ due to recent melting for two periods indicate a GIA-induced, $J_2^ {GI A}$, of $-(6.0 \pm 6.5) \times 10^{-11} \text{yr}^{-1}$.

This is a main conclusion of our study. In the next section, we examine the $J_2^ {GI A}$ by considering two GIA ice models and viscosity models.

### 4. J_2 due to GIA ice models and mantle viscosity

#### 4.1 Sensitivity of $J_2^ {GI A}$ to Northern and Southern Hemisphere ice sheets

The GIA-related rotational variations are equally sensitive to both the melting histories for the last deglaciation and the viscosity structure of the mantle. Here we briefly explain the melting histories for ICE5G and ANU ice models before discussing the viscosity structure inferred from $J_2^ {GI A}$ obtained in this study. In particular, we will examine the sensitivities of $J_2^ {GI A}$ to the northern and southern parts of the GIA ice model and their significance in determining the relationship between $J_2^ {GI A}$ and the GIA ice model (e.g. Wu & Peltier 1984).

Fig. 4 shows the spatial distributions of total melted ice thickness during the last deglaciation, and their equivalent sea level (ESL) as a function of time (ESL-histories). For reason of comparability of the two ice models, ESL is defined here as the change in meltwater volume divided by the surface of the ocean at the present-day. We stress that there is no melting after 1 kyr BP for these ice models. The Northern Hemisphere ice models are largely constructed using the RSL observations sensitive to viscosity structure of the mantle, and the preferred lower mantle viscosity is $(2-3) \times 10^{22} \text{Pa s}$ for ICE5G (Peltier 2004), and $(1-5) \times 10^{22} \text{Pa s}$ for ANU ice model (e.g. Lambeck & Johnston 1998; Lambeck et al. 2014). The GIA-related rotational variations (degree-two Earth
deformations) are only sensitive to the gross melting histories and we therefore discuss the general (overall) characteristics of the northern and southern (Antarctic) parts separately. However, we emphasize that there is a trade-off between the lower mantle viscosity and GIA ice models.

For the Northern Hemisphere, we notice that ICE5G and ANU models are quite similar in both the gross spatial distributions of total melted ice thicknesses (Figs 4a and c) and ESL-histories shown in Fig. 4e (the total ESL-value is ~103 m in both models). For the Antarctic ice sheet, the total ESL-values for the ICE5G and ANU models are ~20 and ~30 m, respectively. The ESL for the Antarctic ice sheet is less reliable compared with that for the northern hemisphere ice sheets because of scarcity of RSL observations for the Antarctic region. That is, the ESL component would highly

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**Figure 4.** Spatial distributions of total melted ice thicknesses since the last glacial maximum (LGM) for two representative GIA ice models: (a), (b) for ICE5G (Peltier 2004) and (c), (d) for ANU ice model (Lambeck et al. 2014). Contour intervals are 500 m for the northern hemisphere ice sheets and 200 m for the Antarctic ice sheet. (e) Equivalent sea level (ESL) rise as a function of time for GIA ice models of ICE5G and ANU and those for the northern, ICE5G(N) and ANU(N), and southern hemisphere (Antarctic) ice sheet models, ICE5G(S) and ANU(S). For reason of comparability of the two ice models, ICE5G and ANU, ESL is defined here as the change in meltwater volume of all grounded ice divided by the surface of the ocean at the present-day.
depend on RSL observations at the last glacial maximum (LGM) of \(\sim 20\) kyr BP. Barbados RSL observations for ICESG (Peltier 2004; Peltier & Fairbanks 2006) and Bonaparte RSL data for ANU ice model (Yokoyama et al. 2000). In fact, the Antarctic melting histories are controversial (e.g. Nakada & Lambeck 1988; Nakada et al. 2000; Whitehouse et al. 2012; Ivins et al. 2013; Morrow et al. 2013; Lambeck et al. 2014), and we discuss their impacts on the rotational variations in this section and Section 4.3.

We consider the GIA-related rotational variations for the northern and southern (Antarctic) parts separately to examine the sensitivity of the model predictions to the GIA ice models. Fig. 5 shows the \(J_2\) and polar wander rates based on a viscosity model of LVN with lithospheric thickness of 65 km and upper mantle viscosity of \(4 \times 10^{20}\) Pa s. Fig. 5(a) shows the \(J_2\) for the northern hemisphere (N) ice sheets. The predictions for both models are comparable as might be inferred from their ESL-histories. For a lower mantle viscosity \(\eta_{\text{lm}} = 2 \times 10^{22}\) Pa s, for example, the ANU and ICESG models give values of \(-4.3 \times 10^{-11}\) and \(-4.7 \times 10^{-11}\) yr\(^{-1}\) for \(J_2\), respectively. It is also interesting that the predictions of \(J_2\) for two ice models, ICESG(N) and ANU(N), are nearly similar in a viscosity range of \(10^{21} \leq \eta_{\text{lm}} \leq 10^{22}\) Pa s. This may imply that the \(J_2\) mainly depends on the gross melting histories (or ESL histories), at least, for the northern hemisphere GIA ice models.

As indicated by Wu & Peltier (1984), the melting of the Antarctic ice sheet is more efficient in inducing \(J_2\) than the northern hemisphere ice sheets and this conclusion is clearly confirmed by the results in Fig. 5(a) and the total ESL-values for both ice models in Fig. 4(e). The difference in \(J_2\)'s for the ANU(S) and ICESG(S) models is approximately explained by considering their different ESL-histories. The ESL-history for the ICESG is roughly synchronous with that for the ANU, and the total ESL-value for the ICESG is \(\sim 20\) m, about two-thirds of \(\sim 30\) m for the ANU (Fig. 4e). The minimum \(J_2\) for the ICESG at \(\eta_{\text{lm}} = 2 \times 10^{22}\) Pa s is also about two-thirds of that for the ANU model: \(-2.5 \times 10^{-11}\) yr\(^{-1}\) for the ICESG and \(-4.0 \times 10^{-11}\) yr\(^{-1}\) for the ANU (Fig. 5a). As also previously stated, the ESL history for the Antarctic ice sheet would be tightly restricted by both the ESL history for the northern hemisphere ice sheets and RSL observations at the LGM for Barbados and Bonaparte Gulf. That is, comparisons for the predicted \(J_2\) of ICESG and ANU and the inference of ESL of both models indicate that it is important to broadly constrain the ESL-history of the Antarctic ice sheet in accurately predicting the GIA-related \(J_2\) as discussed below.

Before examining the TPW for the northern and southern hemisphere components of the ice models (see Section 5 below), we briefly discuss the methods evaluating the GIA-induced TPW by Mitrovica et al. (2005) and Nakada (2009). Mitrovica et al. (2005) employ the ‘\(\beta\)-value’ describing the impact of convectively supported excess ellipticity on TPW, which may be applicable to an axisymmetric earth model. On the other hand, Nakada (2009) evaluated the TPW on a triaxial Earth [see eqs (12) to (14) in Nakada (2009) and Matsuyama et al. (2010)]. Here we evaluate the polar wander using the hydrostatic \(J_2\) by Chambat et al. (2010) and Stokes coefficients by Bruinsma et al. (2014). The ‘\(\beta\)-value’ by Mitrovica et al. (2005) is 0.009925 and the values required to estimate the TPW for a triaxial Earth are given in Appendix A (eqs A7 and A8). Fig. 5(b) shows the results for the predicted rates using \(\beta = 0.009925\) [ICESG(T) and ANU(T)] and those for triaxial Earth by Nakada (2009) [ICESG(T) and ANU(T)] based on GIA ice models of ICESG and ANU. These predictions are based on a viscosity model of LVN with lithospheric thickness (\(H\)) of 65 km and upper mantle viscosity (\(\eta_{\text{lm}}\)) of \(4 \times 10^{20}\) Pa s.

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**Figure 5.** Predictions of (a) \(J_2\) and (c) polar wander rate for the northern parts of GIA ice models, ICESG(N) and ANU(N), and southern hemisphere ice sheet models, ICESG(S) and ANU(S). (b) Predicted polar wander rates using \(\beta = 0.009925\) by Mitrovica et al. (2005) [ICESG(T) and ANU(T)] and those for triaxial Earth by Nakada (2009) [ICESG(T) and ANU(T)] based on GIA ice models of ICESG and ANU. These predictions are based on a viscosity model of LVN with lithospheric thickness (\(H\)) of 65 km and upper mantle viscosity (\(\eta_{\text{lm}}\)) of \(4 \times 10^{20}\) Pa s.

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wander based on the method by Nakada (2009). The polar wander rates for the northern and southern hemisphere components of the ice models are shown in Fig. 5(c). The rates for the Antarctic part are significantly smaller than those for the northern hemisphere ice sheets regardless of the GIA ice models simply because the melting for the Antarctic ice sheet is confined to high latitudes.
4.2 Inference of mantle viscosity from the $J_2^{GIA}$ for GIA ice models

Predictions of $J_2$ for both GIA ice models (ICE5G and ANU) and viscosity models LVN, LV50 and LV2L (see Table 1) are shown in Fig. 6. We show the results for viscosity models with lithospheric thickness ($H$) of 65 and 100 km for the ANU model, and those for $H = 65$ km for the ICE5G. The predictions are insensitive to the lithospheric thickness as commonly accepted by the GIA community (e.g. Vermeersen & Sabadini 1996; Mitrovica & Milne 1998), and we therefore restrict our attention to the predictions with $H = 65$ km. The results in Fig. 6 indicate that the predicted $J_2$ is chiefly sensitive to the lower mantle viscosity ($\eta_{lm}$) in a range of $\eta_{lm}$-value smaller than $\sim 10^{22}$ Pa s, but that it is moderately sensitive to the upper mantle viscosity ($\eta_{um}$) for viscosity models with $\eta_{lm}$-value larger than $\sim 10^{22}$ Pa s. The results for the LV2L are nearly the same as those for a constant $D'$ layer viscosity of $10^{18}$ Pa s by Nakada & Okuno (2013). In a range of $\eta_{lm} < 10^{22}$ Pa s, however, the magnitude of $J_2$ for the LV2L is rather smaller than that for LVN and LV50 viscosity models.

The magnitude of $J_2$ for the ICE5G is $\sim 10^{-11}$ yr$^{-1}$ smaller than that for ANU model, which is related to the melting of the Antarctic ice sheet (Fig. 4). This is clearly confirmed from the comparison between the predicted $J_2$ of, for example, $-5 \times 10^{-11}$ yr$^{-1}$ for the ICE5G and $-6 \times 10^{-11}$ yr$^{-1}$ for the ANU. This point may be important in discussing the effects of the $J_2$ arising from the
uncertainties about the GIA ice model of the Antarctic ice sheet discussed in Section 4.3.

Here we tentatively adopt an observationally derived value of $J_2^{\text{GIA}} \sim -6 \times 10^{-11} \text{yr}^{-1}$ by considering the estimates from recent melting for the periods 1900–1990 and 2002–2012 (see Table 3). For a $D^\prime$ layer viscosity model of LV2L preferred by Nakada & Karato (2012), the permissible lower mantle viscosities ($\eta_{\text{lm}}$) are $(1-2) \times 10^{22} \text{Pa s}$ and $\sim 10^{23} \text{Pa s}$ for both GIA ice models, and these two solutions are nearly insensitive to the upper mantle viscosity. The permissible lower mantle viscosities for viscosity models of LVN and LV50 are nearly similar to those for the LV2L, but the solutions are slightly sensitive to the upper mantle viscosity. Here we examine the solutions for $\eta_{\text{lm}} = 2 \times 10^{20}$ and $4 \times 10^{20} \text{Pa s}$. In the LVN with no low viscosity $D^\prime$ layer generally used in previous studies (e.g. Wu & Peltier 1984; Peltier 2007), the permissible $\eta_{\text{lm}}$-values for $\eta_{\text{lm}} = 2 \times 10^{20} \text{Pa s}$ are $\sim 10^{22}$ and $\sim 8 \times 10^{22} \text{Pa s}$ for the ANU and $\sim 2 \times 10^{22}$ and $\sim 6 \times 10^{22} \text{Pa s}$ for the ICE5G. Those for $\eta_{\text{lm}} = 4 \times 10^{20} \text{Pa s}$ are $\sim 8 \times 10^{23}$ and $\sim 8 \times 10^{23} \text{Pa s}$ for the ANU and $(1-2) \times 10^{22}$ and $\sim 8 \times 10^{22} \text{Pa s}$ for the ICE5G. Similar solutions are also obtained for the LV50.

The differences between the permissible solutions for both ice models are approximately attributed to the total ESL-values for the Antarctic ice sheet as stated previously. $\sim 30 \text{ m}$ for the ANU and $\sim 20 \text{ m}$ for the ICE5G. However, it would be safe to state that there are two permissible solutions of $\eta_{\text{lm}} \sim 10^{22}$ and $(5-10) \times 10^{22} \text{Pa s}$ regardless of GIA ice models (ICE5G and ANU) and viscosity models for the $D^\prime$ layer.

Here we comment about the inferred lower mantle viscosity in most previous studies using viscosity model LVN with no low viscosity $D^\prime$ layer (e.g. Spada et al. 1992; Peltier 2007). They discussed the lower mantle viscosity based on the $J_2$ of $\sim 3 \times 10^{-11} \text{yr}^{-1}$, for example, observationally derived $J_2$ of $(2.5-3.0) \times 10^{-11} \text{yr}^{-1}$ by Nerem & Klosko (1996). If we adopt such $J_2$ value, then the inferred lower mantle viscosity is certainly $\sim 2 (2-3) \times 10^{22} \text{Pa s}$, VM2 in Peltier (2004). The inference for the ANU is slightly smaller than that for the ICE5G, which is related to the melting of the Antarctic ice sheet. However, we should adopt the GIA-induced $J_2$ of $\sim 6 (6-6.5) \times 10^{-11} \text{yr}^{-1}$ in inferring the lower mantle viscosity.

### 4.3 Effect of the melting of the Antarctic ice sheet on the $J_2^{\text{GIA}}$

The Antarctic melting histories are controversial as was stated earlier. The values of total equivalent sea level (ESL) for the melting of the Antarctic ice sheet are about 30 and 20 m for the ANU and ICE5G ice models, respectively whereas those by Whitehouse et al. (2012) and Ivins et al. (2013) are 8–10 and $\sim 8 \text{ m}$, respectively, but neither analysis considers the consequence of these lower values on the global ice-ocean mass balance. More recently, Morrow et al. (2013) indicated that the time rate of change of the degree four zonal harmonic of Earth’s gravitational potential, $J_4$, is dominantly sensitive to the melting of the Antarctic ice sheet. Then, they dealt directly with the issue raised by recent studies (Whitehouse et al. 2012; Ivins et al. 2013) in regard to the excess Antarctic ice volume at LGM being smaller than the global ICE5G and ANU models. The GIA-corrected $J_4$ over the 2000–2011 using the model by Whitehouse et al. (2012) seems to be consistent with the predicted $J_4$ for the recent ice mass flux.

Here we approximately examine the effects of the ESL-value of the Antarctic ice sheet on the $J_2$ by adopting constant scale parameters, $\alpha_s$ and $\alpha_s$, for the heights of the northern and southern ice sheets of ANU ice model. Table 4 summarizes a range of such scaling together with the corresponding ESL values at the LGM. The observational evidence for the change in ocean volume and hence the ESL function comes from the analyses of far-field sea level data corrected for GIA due to the changes in ice and water loading. Such analyses from ocean island and continental margin sites yield a value of $\sim 134 \pm 5 \text{ m}$ at $\sim 21 \text{ kyr BP}$ (Lambeck et al. 2014) and it is this value that constrains the total LGM grounded (included on shelves) ice volume of $\sim 5.2 \times 10^6 \text{ km}^3$ in the ANU model. By restricting the analysis to data from sites far from the former ice margins the GIA corrections to the individual observations are relatively insensitive to the source of the meltwater but they are sensitive to the total amount of meltwater added into the oceans and the global estimate is a result of an iterative solution between the far-field solutions and near-field analyses for ice volumes in the northern hemisphere with the discrepancy in total ice volume between the two distributed within Antarctica (Nakada & Lambeck 1988; Lambeck et al. 2014). Fig. 7 illustrates the corresponding $J_2$ for these ice-scaled models and viscosity models LVN and LV2L (Table 1) with $H = 65 \text{ km}$ and $\eta_{\text{lm}} = 2 \times 10^{20}$ and $4 \times 10^{20} \text{ Pa s}$. Although we do not show the results for LV50, the results are essentially the same as those for the LV2L. The prediction for ICE5G is similar to that for ANU(1, 2/3) because the ESL-history for ANU(1, 2/3) is also similar to that for the ICE5G (see Figs 4 and 5).

The magnitude of $J_2$ for the models with the same total ESL function, ANU(1.18, 1/3) and ANU(1.11, 2/3), is rather smaller than that for the ANU: 10 m scaling down of the Antarctic ice sheet compensated by a scaling up of the northern hemisphere ice sheet resulting in a change of magnitude in $J_2$ of $\sim (1-1.5) \times 10^{-11} \text{ yr}^{-1}$ at $\eta_{\text{lm}} = 2 \times 10^{22} \text{ Pa s}$ with the magnitude of $J_2$ effectively decreasing with decreasing total ESL-value of the Antarctic ice sheet (e.g. Wu & Peltier 1984). Consequently the lower value for the permissible $\eta_{\text{lm}}$ solutions increases in comparison with that for $(\alpha_s, \alpha_s) = (1, 1)$ and vice versa for the higher value.

If we adopt the Antarctic ice model with a total ESL value of $\sim 10 \text{ m}$ (Whitehouse et al. 2012; Ivins et al. 2013), then the northern hemisphere GIA ice model requires a total ESL value of $\sim 125 \text{ m}$ at the time of the LGM which is considerably greater than permitted by the regional northern hemisphere solutions. It is beyond the scope of the present study to discuss whether it is possible or not to increase the ESL-value for the melting of the northern hemisphere ice sheets. It is, however, noted that we get permissible solutions of $\eta_{\text{lm}} \sim 2 \times 10^{22}$ and $\sim 5 \times 10^{22} \text{ Pa s}$ or one solution with $(2-5) \times 10^{22} \text{ Pa s}$ for such GIA models with a total ESL value of $\sim 10 \text{ m}$ for the Antarctic ice sheet as inferred from the predicted $J_2^{\text{GIA}}$ for the ANU(1.18, 1/3).

### Table 4. Ice models to examine the effects of ESL-value on the $J_2$ characterized by $\alpha_s$ and $\alpha_s$ values describing the ESL at the LGM for the Northern Hemisphere (NH) and Southern Hemisphere (SH), respectively.

<table>
<thead>
<tr>
<th>Ice model</th>
<th>$\alpha_s$, $\alpha_s$</th>
<th>ESL at the LGM for NH (m)</th>
<th>ESL at the LGM for SH (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANU</td>
<td>1, 1</td>
<td>107.7</td>
<td>29.7</td>
</tr>
<tr>
<td>ANU (1, 0)</td>
<td>1, 0</td>
<td>103.7</td>
<td>0</td>
</tr>
<tr>
<td>ANU (1, 2/3)</td>
<td>1, 2/3</td>
<td>103.7</td>
<td>19.8</td>
</tr>
<tr>
<td>ANU (1.18, 1/3)</td>
<td>1.18, 1/3</td>
<td>123.4</td>
<td>10</td>
</tr>
<tr>
<td>ANU (1.11, 2/3)</td>
<td>1.11, 2/3</td>
<td>113.6</td>
<td>19.8</td>
</tr>
</tbody>
</table>

Downloaded from http://gji.oxfordjournals.org/ at The Australian National University on June 29, 2016
predictions, we adopt (i) ESLR-values of 0.54 and 0.15 mm yr\(^{-1}\) for MG1 (alpine glaciers in Fig. 1a) and GREENg (glaciers peripheral to the Greenland ice sheet in Fig. 1b) respectively describing recent melting for the period 1900–1990, and (ii) 0.80, 0.59 and 0.40 mm yr\(^{-1}\) for MG2 (see Fig. 1a), GREEN (Fig. 1d) and ANT (Fig. 1e) for the period 2002–2011, respectively (see Table 2). The rate for ANTG (glaciers peripheral to the Antarctic ice sheet in Fig. 1c) is negligible, and we do not show this contribution here. The prediction is almost elastic in character (insensitive to viscosity structure), and consequently only depends on the ESLR-value at the time when we evaluate the polar wander. The rate vector for the period 1900–1990 is evaluated from the predictions for MG1 and GREENg, and the rate is less than 0.1° Myr\(^{-1}\). That for 2002–2011 is described by the rate of ~0.7° Myr\(^{-1}\) and the direction of ~0° E (direction of Greenwich meridian).

Rate vectors derived from the ANU and recent melting models are shown in Fig. 9(b), in which we only show the predictions for a viscosity model of \(\eta_{\text{lm}} = 10^{22} \text{ Pa s}\) because the rate for \(\eta_{\text{lm}} \sim (5-10) \times 10^{22} \text{ Pa s}\) is less than 0.2° Myr\(^{-1}\) (Fig. 8) and significantly smaller than the observed one. The rate for the period 1900–1990 based on a viscosity model of LVN with no low viscosity \(D'\) layer (Table 1), denoted by ANU + MG1 + GREENg (LVN) in Fig. 9(b), is similar to the observed one. However, the rates for the LV2L with a low viscosity \(D'\) layer are ~0.4° Myr\(^{-1}\), suggesting additional sources such as convective motions in the mantle inducing additional polar wander with rate of ~0.5° Myr\(^{-1}\) and its direction towards the Hudson Bay (see Appendix A). Also, if the effective lower mantle is \(\eta_{\text{lm}} \sim (5-10) \times 10^{22} \text{ Pa s}\), one permissible solution derived from the GIA-induced \(J_{2,\text{GIA}}\) (Fig. 6), then the observed
polar wander for the period 1900–1990 would be almost attributed to convection motions in the mantle and/or another cause regardless of the viscosity structure of the D'' layer.

Interpretation of the rate vectors for the period 2002–2011 may be more complex than that for 1900–1990. Chen et al. (2013) indicated that the polar wander direction changed rapidly from the ∼70° W to the east around 2005 and suggested that the sudden change is due to accelerated melting of polar ice sheets and mountain glaciers, and related sea level rise. This is consistent with the polar wander direction due to the melting of the Antarctic ice sheet for ANT (see Fig. 9a and also the predictions for ANU + MG2 + GREEN + ANT in Fig. 9b). However, the observed rapid change cannot be explained by model predictions for ANT with ESLR of 0.40 ± 0.21 mm yr⁻¹, and reconciling model predictions with observations remains an outstanding issue for future study.

6 CONCLUDING REMARKS

We examined the geodetically derived rotational variations, $J_2$, TPW, and compared these with contributions from recent melting of alpine glaciers and of the Greenland and Antarctic ice sheets reviewed by Vaughan et al. (2013) in AR5 and from two typical GIA models describing the Earth response to the last glacial cycle: ICE5G and ANU. The geodetically derived $J_2$ is characterized by temporal changes of $- (5.6 \pm 0.1) \times 10^{-11}$ yr⁻¹ for the period 1976–1990 and $- (0.3 \pm 0.1) \times 10^{-11}$ yr⁻¹ after ∼2000 (Roy & Peltier 2011; Cheng et al. 2013). Vaughan et al. (2013) comprehensively discuss recent melting for separate periods after 1900 based on sharp accelerations in the rate of melting, and the results make it possible to evaluate the recent melting of the major ice sheets and glaciers for three broad periods, 1900–1990, 1991–2001, and after 2002.

We first evaluated the GIA-induced $J_2$, $\dot{J}_2^{\text{GIA}}$, by considering the observed $J_2$ and the component of $J_2$ due to recent melting model with an uncertainty range of 90 per cent. The derived values of $\dot{J}_2^{\text{GIA}}$ for the melting of the periods 1900–1990 and 2002–2011 are $- (5.8 \pm 0.5) \times 10^{-11}$ and $- (6.6 \pm 2.6) \times 10^{-11}$ yr⁻¹ for the viscosity model LVN with no low viscosity D'' layer, respectively (see Table 3). Similar values are obtained for viscosity models of LV50 and LV2L with low viscosity D'' layer. That is, the GIA-induced $J_2$ from these observations is estimated to be $- (6.0–6.5) \times 10^{-11}$ yr⁻¹, significantly different from $- 3 \times 10^{-11}$ yr⁻¹ adopted to infer the viscosity structure of the mantle in most previous studies (e.g. Spada et al. 1992; Peltier 2007). This is a major conclusion of this study.
We then compared this estimate with the predicted $J_2$ due to GIA processes using ICE5G and ANU ice models to infer the viscosity structure of the mantle. Through this process, we obtained two permissible solutions for the lower mantle viscosity ($\eta_{lm}$), $\sim 10^{22}$ and $(5–10) \times 10^{22}$ Pa s, for both adopted GIA ice models (Fig. 6). These two solutions are insensitive to lithospheric thickness and upper mantle viscosity as indicated by previous studies (e.g. Vermeersen & Sabadini 1996; Mitrovica & Milne 1998; Peltier 2007) and relatively insensitive to the viscosity structure of the $D'$ layer. The solution with $\eta_{lm} \sim 10^{22}$ Pa s seems to be consistent with the estimates derived from other geophysical observables and models (e.g. Forte 2007). The higher value of $\eta_{lm} \sim (5–10) \times 10^{22}$ Pa s may be consistent with the inferences from the sinking speed of subducted lithosphere by Čižková et al. (2012), geodynamic modelling for dynamic topography and geoid anomaly using recent seismic tomographic model of the South Pacific superswell (Adam et al. 2014) and more recent GIA studies using far-field sea level observations by Lambeck et al. (2014). GIA ice models in which the Antarctic ice volume change is restricted to $\sim 10$ m ESL (e.g. Whitehouse et al. 2012; Ivins et al. 2013) indicate two permissible solutions of $\eta_{lm} \sim 2 \times 10^{22}$ and $\sim 5 \times 10^{22}$ Pa s or one solution with $(2–5) \times 10^{22}$ Pa s (see Fig. 7). Although there may be uncertainties for the melting of both polar ice sheets, particularly for the Antarctic ice sheet, the effective lower mantle viscosity inferred from the GIA-induced $J_2$ seems to be larger than $\sim 10^{22}$ Pa s. The analysis for the $J_4$, dominantly sensitive to the melting of the Antarctic ice sheet...
(Morrow et al., 2013), would help to tightly constrain the lower mantle viscosity.

We also examined polar wander due to recent melting and GIA processes. For low viscosity $D^*$ layer models (LV50 and LV2L) with $\eta_m \sim 10^{22}$, we may require the impact of convective motions in the mantle to reproduce a polar wander rate of $\sim 0.5^\circ$ Myr$^{-1}$ towards Hudson Bay. However, such a forcing is not required for no low viscosity $D^*$ layer model (LVN) with $\eta_m \sim 10^{22}$. If the effective lower mantle is $\eta_m \sim 5 \times 10^{22}$ Pa s, the highest permissible solution derived from the GIA-induced $J_2$, then the observed polar wander for the period 1900–1990 would suggest a dominant contribution from convection motions in the mantle and/or another cause regardless of the viscosity structure of the $D^*$ layer.

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rotating Earth: first results from a gravitationally self-consistent sea-level


**APPENDIX A: NUMERICAL METHOD TO EVALUATE POLAR WANDER AND SOME REMARKS FOR THE IMPACTS OF CONVECTIVE PROCESSES IN THE MANTLE**

Here we explain a mathematical formulation to evaluate the GIA-related rotational variations on a convecting mantle (see also Nakada 2009)). The Earth’s rotational variations with no external torque are described by \( d\mathbf{H}/dt + \omega \times \mathbf{H} = 0 \), in which \( \mathbf{H} = (H_x, H_y, H_z) \) and \( \omega = (\omega_x, \omega_y, \omega_z) \) are angular momentum and angular velocity, respectively. The terms of \( dH_x/dt \) and \( dH_y/dt \) in the Liouville equation are safely neglected in evaluating the secular variations (e.g. Nakada 2009), and the governing equations are as follows:

\[
\begin{align*}
\omega_2 H_1 - \omega_1 H_2 &= 0 \quad (A1) \\
\omega_3 H_1 - \omega_1 H_3 &= 0 \quad (A2) \\
dH_x/dt &= 0, \quad (A3) \\
H \text{ is given by } H = J \cdot \omega \text{ using time-dependent second-degree inertia tensor } J \text{ and angular velocity } \omega, \text{ and a generalized form of the inertia tensor, } J_\omega(t), \text{ is given by } (Munk & MacDonald 1960): \\
J_\omega(t) &= I_\omega + \frac{\kappa_2^2(\omega^2)^2}{3G} \left[ (\omega(t)\omega(t) - \frac{1}{3} \omega^2(t)I_\omega) \right] \\
+ E_{ij}(t), \quad (A4)
\end{align*}
\]

\( I \) is the inertia tensor for a non-rotating Earth and given by \( I = (2A + C)/3 \) using equatorial and polar moments of inertia of \( C \) and \( A \) for a purely hydrostatic Earth rotating with mean angular velocity \( \Omega \) and \( \delta t \) is the Kroncker delta. \( \kappa_2^2(t) \) is degree-two tidal Love number (e.g. Peltier 1976), \( a \) is the mean radius of the Earth, \( G \) is the gravity constant, \( \omega^2(t) = \omega_x^2(t) + \omega_y^2(t) + \omega_z^2(t) \), and \( \delta(t) \) is the delta function and asterisk denotes a time convolution. The term of \( E_{ij}(t) \) includes the effects of convective processes in the mantle and surface mass redistributions related to the GIA processes.

Here we consider an ideal case that the Earth for \( t < 0 (-\infty < t < 0) \) is a purely hydrostatic state given by \( \omega = (0, 0, \Omega) \). The Earth for this period has no hydrostatic geoid due to mantle convection and no ice age cycle, and is characterized by \( E_{ij}(t) = 0 \) in eq. (A4). Then the integration constant of eq. (A3) for \( t < 0 \) is \( C \Omega \), and the form for \( t > 0 \) is also true for an arbitrary time \( \sim t < t \) in the case with no external torque. That is, we get:

\[
J_{\omega3}(t)\omega(t) + J_{\omega3}(t)\omega(t) + J_{\omega3}(t)\omega(t) = C \Omega. \quad (A5)
\]

We illustrate the rotational variations by usually used dimensionless quantities \( m_1, m_2 \) and \( m_3 \) defined by \( \tan(m_1) = \omega_1/\omega_3, \tan(m_2) = \omega_2/\omega_3 \) and \( \tan(m_3) = 1 + m_2 \). (Munk & MacDonald 1960).

The quantities \( m_1 \) and \( m_2 \) describe the displacements of the rotation axis in the directions 0° and 90° E, respectively, and \( m_3 \) describes the rate of change of the Earth’s rotation. The equations for \( |m_1| < 1 \) generally used for the GIA (e.g. Sabadini & Peltier 1981; Wu & Peltier 1984) are given by putting eq. (A4) into eqs (A1), (A2) and (A5) and using \( |m_1| \ll 1 \) [see eqs (8) and (9) in Nakada (2009)].

Inertia elements \( E_{ij}(t) \) are related to convective processes, \( D_{ij}(t) \), and surface mass redistributions associated with the GIA processes, \( P_{ij}(t) \), and given by:

\[
E_{ij}(t) = D_{ij}(t) + P_{ij}(t). \quad (A6)
\]

\( D_{ij}(t) \) may generally be described by two factors related to convective processes, (i) \( D_{ij}(t) \) for the non-hydrostatic geoid at the present-day and (ii) \( D_{ij}(t) \) for temporarily changing (steady) convective motions in the mantle. The rotational variations for case (i) require the impact of convectively supported excess ellipticity (Mitrovica et al. 2005; Nakada 2009; Matsuwaya et al. 2010), \( D_{ij}(t) = D_{ij} \), for the Quaternary ice age. We estimate the quantities using the hydrostatic \( J_\ast \) by Chambat et al. (2010) and Stokes coefficients by Bruinsma et al. (2014) [see also Lambeck (1988)]. Each component of \( D_{ij} \) (kg m⁻²) for a triaxial Earth (Nakada 2009; Nakada & Okuno 2013) is given by:

\[
D_{11}^* = -1.686 \times 10^{13}, \quad D_{22}^* = -1.590 \times 10^{12}, \\
D_{12}^* = 1.845 \times 10^{13}, \quad (A7)
\]

The ‘\( \beta \)-value for an axisymmetric Earth by Mitrovica et al. (2005) is estimated from \( \beta = 3G(D_{11} - D_{12}/a^2(\omega^2)^2 \) with \( D_{11} = D_{22} = -D_{12}/2 \) and \( D_{12} = 0 \) and given by:

\[
\beta = 0.009925. \quad (A8)
\]

On the other hand, the corresponding values for the triaxial Earth are given by \( \beta_{11} = 3G(D_{11} - D_{12}/a^2(\omega^2)^2 \) and \( \beta_{12} = 3G(D_{12} - D_{22}/a^2(\omega^2)^2 \) [see eq. (14) in Nakada (2009)] and those using the quantities of eq. (A7), which satisfy \( \beta = (\beta_{11} + \beta_{12})/2 \), are \( \beta_{11} = 0.01266 \) and \( \beta_{12} = 0.007187 \). In this study, we evaluate the GIA-induced polar wander using the quantities of eq. (A7) based on the method by Nakada (2009) and Nakada & Okuno (2013) (see Section 4.1).

Here we briefly discuss the misfit between the observed (McCarthy & Luzum 1996) and GIA-induced polar wander before \( \sim 2000 \) (see Fig. 9b) by employing the numerical method for case (ii). In case (ii), we first numerically build a background rotational state for convective motions before the Quaternary ice age with onset time of \( t = t_0 \) (Nakada 2008). The period for building the state \( (0 < t < t_o) \) is required to construct a steady state of the Earth’s rotation due to convective motions with rates of change in inertia elements, \( d_i \) (kg m² s⁻¹), constant for the Quaternary ice age with a period of \( \sim 1 \) Myr. Here we assume that \( D_{ij}(t) = 0 \) is \( D_{ij}(0) = D_{ij}^* \) for the present-day given by eq. (A7) and the rates of change for \( 0 \leq t \leq t_o/2 \) are \( -d_i \) and \( d_i \) for \( t \geq t_o/2 \). Then we solve the Liouville equations by imposing:

\[
D_{ij}(t) = D_{ij}^* - d_i t \quad (0 \leq t \leq t_o/2) \\
D_{ij}(t) = D_{ij}^* + d_i (t - t_o) \quad (t \geq t_o/2). \quad (A9)
\]

This numerical procedure makes it possible to build the following state at the onset of the Quaternary ice age \((t = t_o)\) \( D_{ij}(t_o) = D_{ij}^* \), \( m_1 \sim 0 \) and constant secular rotational variations \( dm_1/dt \sim \) constant. This state is obtained using \( t_o \sim 60 \) Myr according to the numerical experiments. For the Quaternary ice age, the values of \( D_{ij}(t) \) except for \( D_{ij}(t) \) and \( D_{ij}(t) \) should be nearly similar to those of \( D_{ij}^* \) for the present-day. That is, the solutions for case (ii) correspond to GIA-related rotational variations involving the impacts of convectively supported excess ellipticity and also convective motions in the mantle. Here we use \( d_1 = d_3 \times D_{11}/D_{13} \) and \( d_2 = d_3 \times D_{22}/D_{13} \).
using $D_{11}^\ast$, $D_{22}^\ast$ and $D_{33}^\ast$, and adopt $d_{12} = 0$. In evaluating the rotational variations for this case, we numerically check the solutions by solving the linearized equations for $|m_i| \ll 1$, and nonlinear ones for arbitrary $m_i$ (eqs A1, A2 and A5) using an iteration method adopted by Nakada (2007).

We only show the results for a viscosity model of LV2L with $H = 65$ km, $\eta_{um} = 4 \times 10^{20}$ Pa s and $\eta_{lm} = 10^{22}$ Pa s [see prediction by ANU + MG1 + GREENg (LV2L) in Fig. 9b]. Here we adopt convective-derived $d_{13}$ and $d_{23}$ with magnitude of $(10^{31} - 10^{32})$ kg m$^2$ Myr$^{-1}$ and $d_{33} = -10^{31}$ kg m$^2$ Myr$^{-1}$ by considering the results of Ricard et al. (1993) based on a model of present-day mantle density heterogeneities for Cenozoic and Mesozoic Plate motion reconstructions (see also Becker & Boschi 2002; Torsvik et al. 2006; Zhong et al. 2007; Conrad et al. 2013). The polar wander due to convective motions at the present-day is derived from the wander with the impacts of convective motions and GIA processes and that for GIA processes only. The rates of convective-derived polar wander at the present-day, $dm_1/dt$ and $dm_2/dt$ (° Myr$^{-1}$), are approximately proportional to the values of forcing elements $d_{13}$ and $d_{23}$ (kg m$^2$ Myr$^{-1}$), and given by $dm_1/dt \sim 1.1 \times 10^{-32}d_{13}$ and $dm_2/dt \sim 1.7 \times 10^{-32}d_{23}$ through the numerical experiments. The difference of the coefficients is attributed to the triaxiality of the Earth. Consequently, we get the polar wander with a rate of $\sim 0.9°$ Myr$^{-1}$ and direction of $\sim 70°$ W for a model with ANU ice model and forcing elements of $d_{13} \sim 1.0 \times 10^{31}$ and $d_{23} \sim -2.5 \times 10^{31}$ kg m$^2$ Myr$^{-1}$. 