Centroid–moment tensor inversions using high-rate GPS waveforms

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SUMMARY
Displacement time-series recorded by Global Positioning System (GPS) receivers are a new type of near-field waveform observation of the seismic source. We have developed an inversion method which enables the recovery of an earthquake’s mechanism and centroid coordinates from such data. Our approach is identical to that of the ‘classical’ Centroid–Moment Tensor (CMT) algorithm, except that we forward model the seismic wavefield using a method that is amenable to the efficient computation of synthetic GPS seismograms and their partial derivatives. We demonstrate the validity of our approach by calculating CMT solutions using 1 Hz GPS data for two recent earthquakes in Japan. These results are in good agreement with independently determined source models of these events. With wider availability of data, we envisage the CMT algorithm providing a tool for the systematic inversion of GPS waveforms, as is already the case for teleseismic data. Furthermore, this general inversion method could equally be applied to other near-field earthquake observations such as those made using accelerometers.

Key words: Satellite geodesy; Earthquake ground motions; Earthquake source observations; Computational seismology; Theoretical seismology; Early warning.

1 INTRODUCTION
The determination of earthquake source parameters is a fundamental problem in seismology. The Centroid–Moment Tensor (CMT) method of Dziewonski et al. (1981) is one of the most robust techniques that has been developed for obtaining the location and mechanism of a seismic event; it has been routinely applied to the inversion of teleseismic waveforms for over 30 years, and underpins the ongoing Global CMT project (www.globalcmt.org). Recently, a new type of near-field waveform data has become available to seismologists: high-rate Global Positioning System (HRGPS) measurements of ground displacements (Larson et al. 2003). In this paper, we investigate the possibility of using 1 Hz GPS data in CMT inversions to recover the moment tensor and spatio-temporal location of an earthquake.

The present work is made possible by the increasing availability of high-quality GPS data to seismologists. The versatility of GPS as a tool to measure Earth’s deformation at timescales ranging from seconds to years has meant that permanent GPS networks are now routinely deployed in tectonically active regions. Such networks, typically operating at 1 Hz sampling rates, will capture local seismicity regardless of their original observational purpose. Furthermore—partly in response to the demands of seismologists (Hammond et al. 2010)—data acquisition, storage and processing systems have become more sophisticated over time so that state of the art GPS networks are now able to stream 10 Hz data, yielding displacement waveforms with millimetre-level accuracy in real-time (Bock et al. 2011). This potentially rich data resource is yet to be fully exploited, and so there is a need to develop methods for the routine analysis of HRGPS waveforms; the CMT algorithm is a strong candidate since it is well understood and accepted for teleseismic and regional data (e.g. Dziewonski et al. 1983; Pondrelli et al. 2002).

GPS networks have been used to make many observations of dynamic displacements caused by earthquakes (e.g. Nikolaids et al. 2001; Kouba 2003; Davis & Smalley 2009; Shi et al. 2010; O’Toole & Woodhouse 2011; Bock et al. 2011). However, such data have only been used to construct source models for a handful of events: the 2003 Tokachi–Oki earthquake (Miyazaki et al. 2004), the 2003 San Simeon earthquake (H et al. 2004), the 2005 Fukuoka earthquake (Kobayashi et al. 2006), the 2008 Iwate–Miyagi earthquake (Yokota et al. 2009), the 2008 L’Aquila earthquake (Avalone et al. 2011), the 2010 Maule earthquake (Delouis et al. 2010; Vigny et al. 2011) and the 2011 Tohoku earthquake (Ammon et al. 2011; Yue & Lay 2011). All of these studies inverted HRGPS data to obtain a kinematic rupture model of the earthquake on a finite fault of assumed location, size and orientation. The application of GPS data presented here differs from these earlier studies as we solve for the optimal long-period description of an earthquake without making any a priori assumption about the location or mechanism.

Several characteristics of HRGPS time-series make them suitable for inversion using the CMT algorithm. The fact that GPS receivers measure displacement directly means that they record waveforms
that are inherently rich in long-period information. Indeed since
GPS receivers capture the permanent ‘static’ displacement caused
by an earthquake, they are actually sensitive to frequencies down to
0 Hz. Furthermore, Bock et al. (2004) showed that GPS receivers
can detect arbitrarily large ground displacements without clipping.
As a consequence GPS receivers are, in effect, long-period–strong-
motion displacement seismometers. Since the CMT method re-
quires long-period, long-wavelength data to justify its parameteri-
zation of an earthquake as a point source, HGRGS data should be
well suited to an inversion within the CMT framework. Of course,
when compared to teleseismic observations, finite fault effects will
be more significant in the near-field. However for the earthquakes
examined in this report, we find that rupture complexity does not
preclude the stable recovery of a CMT solution from low-pass fil-
tered HGRGS waveforms.

Ultimately, we would like to improve our sampling of the seismic
displacement field in both space and frequency through simulta-
neous inversion of seismic and geodetic earthquake observations
(e.g. Lay 2011). The CMT inversion formulation is well suited to
a straightforward implementation of joint inversions, and so a first
step towards this aim is to carry out inversions of HGRGS wave-
forms alone. Furthermore, since the CMT approach is general it
may also be applied to other near-field seismic observations, pro-
vided that instruments remain on-scale during an earthquake and are
sensitive to long-period ground motions. The use of these new and
independent types of data within the CMT inversion framework may
be of great value in quantifying and reducing uncertainties in earth-
quake source parameters determined using long-period teleseismic
data.

We follow the approach of Dziewonski et al. (1981) to retrieve the
six independent components of the moment tensor plus the spatio-
temporal position of the earthquake’s centroid from HGRGS data.
Starting at a reference source time and location—the hypocentre
obtained from P-wave arrival times, for example—we invert for
an initial moment tensor. Subsequently, we solve iteratively for
the correction to this original source mechanism, location and origin
time which minimizes the least-squares misfit between HGRGS data
and corresponding synthetics. At each iteration, the value of the
perturbation is evaluated using so-called ‘kernel functions’: partial
derivatives of the synthetic waveforms with respect to the 10 source
parameters.

In this paper, we extend the work of O’Toole & Woodhouse
(2011) to compute accurate derivative kernels for synthetic GPS
seismograms, allowing us to invert HGRGS data for an earthquake’s
CMT solution. To validate our approach, we perform test inversions
of synthetic 1 Hz GPS data sets. Using this new method, we obtain
CMT solutions for two moderate sized earthquakes that were well
observed by GPS instruments in Japan.

2 THEORY: CMT INVERSIONS

For completeness, we begin by briefly summarizing the CMT al-
gorithm; for a detailed description of the method as originally con-
ceived for long-period teleseismic waveform data, see Dziewonski
et al. (1981), Dziewonski & Woodhouse (1983a) and Dziewonski
& Woodhouse (1983b). In the following, we use the word ‘seismo-
gram’ generally to describe any functional of the seismic source
for which observations and forward modelling methods exist since
any such data could, in principle, be used in inversions for earth-
quake source parameters; in the present context a seismogram is a
high-rate GPS waveform.

2.1 Linear inversion: the moment tensor at an assumed
centroid location

An indigenous source such as an earthquake is required to exert no
net force or torque on the Earth. Supposing that the location of a
seismic event is known in time t and space x, we can completely
specify the source, f, in terms of the six independent elements of
a symmetric moment tensor M (e.g Dziewonski & Woodhouse
1983b)

\[
f = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}^T.
\]  

(1)

The forward problem for the predicted waveforms arising from such
a source located at x, can be written (Gilbert & Dziewonski 1975)

\[
u_k(x, t) = \sum_{i=1}^{6} \psi_i(x, x, t)f_i.
\]  

(2)

The kth seismogram in the data set, u_k, is given by a linear combina-
tion of the so-called excitation kernels, \( \psi_i \), weighted by the source vector. These kernel functions, the partial derivatives of the seismo-
gram with respect to the components of f, represent the response
excited by unit sources corresponding to each independent compo-
ent of the moment tensor, and so also depend on the assumed earth
model.

Our problem is then to estimate f, given a set of observed seis-
mograms d. From (2), we must solve a linear inverse problem of the form

\[
A f = d,
\]  

(3)

where A is a matrix of excitation kernels calculated for all the
seismograms contained in the data vector. Generally we have many
more data than the six unknown components of f, so we seek the
source that minimizes the least-squares misfit, m^2, between data and
the corresponding synthetic seismograms s

\[
m^2 = \sum_{i=1}^{6} (d_i - s_i)^2.
\]  

(4)

which can be shown (see, e.g. Menke 1989) to lead to the solution

\[
f = (A^T A)^{-1} A^T d.
\]  

(5)

Given some estimate of the source location—for example, the
hypocentre as determined from P-wave arrival times—we can use
this linear inversion scheme to obtain the moment tensor. Within
the limitations of this source representation, the solution is exact,
and convergence is reached after a single iteration.

2.2 Non-linear inversion: retrieval of all 10 source
parameters from waveform data

Since the source location assumed in Section 2.1 may not corre-
spond to the position of the earthquake’s centroid, we should ac-
tually determine the location of the earthquake in space and time
in addition to the moment tensor, which requires the solution of a
non-linear inverse problem. In this case, the source vector f has 10
components

\[
f = \begin{pmatrix} M_{11} & M_{12} & M_{13} & \theta_1 & \phi_1 & \zeta_1 & t_1 \\ M_{21} & M_{22} & M_{23} & \theta_2 & \phi_2 & \zeta_2 & t_2 \end{pmatrix}^T,
\]  

(6)

where the centroid latitude, longitude and depth are given by (\( \theta_i \),
\( \phi_i \), \( \zeta_i \)) and \( t_i \) is the centroid time. Since non-linearity is weak, we
can linearize the problem and perform an iterative least-squares
inversion for the best-fitting point source via

\[
f_{i+1} = f_i + (A_i^T A_i)^{-1} A_i^T (d_i - s_i),
\]  

(7)
CMT inversions using high-rate GPS waveforms

2.3 Calculation of synthetic seismograms and their partial derivatives

The calculation of excitation and location kernels appropriate to a particular type of data is a prerequisite for applying the foregoing theory to the inversion of those data. The CMT algorithm uses normal mode summation to compute long-period teleseismic waveform synthetics and their partial derivatives. While we could also use normal mode summation to compute synthetic GPS waveforms, it is unlikely to be the best choice for the application presented here. Apart from the well-known problem of computing mode catalogues at frequencies higher than about 100 mHz (e.g. Yang et al. 2010), the construction of whole earth eigenfunctions is computationally expensive for near-field data. Furthermore, we would like to compute kernels using the local crustal model at the centroid location; to use mode summation for the same calculation would require the recalculation of mode catalogues every time we encountered a new crustal structure.

Instead, we follow O’Toole & Woodhouse (2011) and compute accurate GPS waveforms for a plane layered elastic medium using the minor vector theory of Woodhouse (1980). This approach ensures numerical stability at all frequencies, including 0 Hz, as is required if we wish to recover the static displacement as part of the synthetic time-series. Furthermore, the method can be straightforwardly adapted to enable the efficient computation of excitation and location kernels appropriate to GPS data, as shown in the Appendix.

The excitation kernels, being the partial derivatives of a synthetic seismogram with respect to the elements of the moment tensor, are simply the displacements arising from six elementary sources corresponding to each component of the moment tensor. Fig. 1 shows an example set of excitation kernels computed for a GPS receiver at a distance of 39.6 km and azimuth 118.2° for a source with a depth of 35 km. The earth model used in the computation of these kernels is given in Table 1. The relatively small amplitudes of the $M_{rr}$, $M_{θθ}$ and $M_{φφ}$ kernels indicate that this location is not

Table 1. Crustal model used for the computations shown in Figs. 1 and 2, and in the inversion of GPS waveforms for the 2005 Fukuoka earthquake described in Section 4.1. This earth model was used by Kobayashi et al. (2006) in their finite fault inversions of HRGPS data for this event.

<table>
<thead>
<tr>
<th>Thickness (km)</th>
<th>$v_p$ (km s$^{-1}$)</th>
<th>$v_s$ (km s$^{-1}$)</th>
<th>$ρ$ (kg m$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>3.20</td>
<td>2.00</td>
<td>2.10</td>
</tr>
<tr>
<td>1.90</td>
<td>5.15</td>
<td>2.85</td>
<td>2.50</td>
</tr>
<tr>
<td>3.00</td>
<td>5.50</td>
<td>3.20</td>
<td>2.60</td>
</tr>
<tr>
<td>13.00</td>
<td>6.00</td>
<td>3.46</td>
<td>2.70</td>
</tr>
<tr>
<td>14.00</td>
<td>6.70</td>
<td>3.87</td>
<td>2.80</td>
</tr>
<tr>
<td>∞</td>
<td>7.70</td>
<td>4.30</td>
<td>3.30</td>
</tr>
</tbody>
</table>

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very sensitive to explosive sources. Furthermore, it is clear that these moment tensor components do not generate significant static displacements, and so if any coseismic offset were observed at this station it would require the excitation of the off-diagonal elements of the moment tensor.

Example location kernels are shown in Fig. 2; these were calculated using the same crustal model and source–receiver geometry as in Fig. 1. These kernels are scaled so that the $\frac{\partial}{\partial \theta_c} u_k$, $\frac{\partial}{\partial \phi_c} u_k$ and $\frac{\partial}{\partial z_c} u_k$ kernels represent the change in the seismogram with respect to a 10 km perturbation in each hypocentral coordinate; similarly the time kernel, $\frac{\partial}{\partial t_c} u_k$, corresponds to a 2 s shift in origin time. Here, the variety of waveforms suggests that high-rate GPS data should allow us to resolve the location of the centroid in space and time, as we require. The shape of these kernels depends on the moment tensor, which is given in Table 2 (‘Iteration 0’ column); the synthetic HRGPS time-series corresponding to this source and station are also shown in Fig. 2 for comparison to the location kernels.

Table 2. The results of a numerical test of the CMT method using synthetic GPS waveforms. Synthetic data were computed for a strike-slip earthquake superimposed on an explosion (‘Input’ column); the initial location is 15 km away from the actual centroid. The CMT algorithm converges to the true source parameters after five iterations of the non-linear inversion scheme described in Section 2.2. The origin time is expressed relative to some reference value such as the hypocentral time determined from body wave arrivals; here the centroid of moment release occurred two seconds after this time. The moment tensors are given in units of $10^{19}$ Nm.

<table>
<thead>
<tr>
<th>Location:</th>
<th>Input</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>origin time (s)</td>
<td>2.00</td>
<td>0.00</td>
<td>1.80</td>
<td>2.45</td>
<td>1.88</td>
<td>2.01</td>
<td>1.99</td>
</tr>
<tr>
<td>longitude</td>
<td>34.00° N</td>
<td>33.90° N</td>
<td>33.90° N</td>
<td>34.02° N</td>
<td>33.99° N</td>
<td>34.00° N</td>
<td>34.00° N</td>
</tr>
<tr>
<td>depth (km)</td>
<td>35.00</td>
<td>30.00</td>
<td>31.76</td>
<td>32.05</td>
<td>35.86</td>
<td>34.99</td>
<td>35.00</td>
</tr>
<tr>
<td>Moment tensor:</td>
<td>M_{rr}</td>
<td>M_{\theta\theta}</td>
<td>M_{\phi\phi}</td>
<td>M_{r\theta}</td>
<td>M_{r\phi}</td>
<td>M_{\theta\phi}</td>
<td></td>
</tr>
<tr>
<td>M_{rr}</td>
<td>1.0000</td>
<td>0.3406</td>
<td>0.4638</td>
<td>0.5376</td>
<td>1.0565</td>
<td>0.9938</td>
<td>1.0000</td>
</tr>
<tr>
<td>M_{\theta\theta}</td>
<td>1.0000</td>
<td>0.7798</td>
<td>0.8958</td>
<td>0.8990</td>
<td>1.0398</td>
<td>0.9991</td>
<td>0.9999</td>
</tr>
<tr>
<td>M_{\phi\phi}</td>
<td>1.0000</td>
<td>0.6549</td>
<td>0.7844</td>
<td>0.9232</td>
<td>1.0310</td>
<td>0.9994</td>
<td>1.0001</td>
</tr>
<tr>
<td>M_{r\theta}</td>
<td>0.0000</td>
<td>0.0005</td>
<td>-0.0281</td>
<td>0.0040</td>
<td>0.0143</td>
<td>0.0013</td>
<td>0.0000</td>
</tr>
<tr>
<td>M_{r\phi}</td>
<td>0.0000</td>
<td>0.1610</td>
<td>0.1638</td>
<td>0.1457</td>
<td>0.0102</td>
<td>0.0006</td>
<td>0.0000</td>
</tr>
<tr>
<td>M_{\theta\phi}</td>
<td>1.0000</td>
<td>0.1430</td>
<td>0.1736</td>
<td>0.5333</td>
<td>0.9308</td>
<td>0.9983</td>
<td>1.0000</td>
</tr>
<tr>
<td>Relative RMS</td>
<td>0.6023</td>
<td>0.4336</td>
<td>0.1463</td>
<td>0.0105</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
3 A TEST OF THE CMT METHOD USING SYNTHETIC GPS WAVEFORMS

To validate our implementation of GPS data in the CMT algorithm, we performed an inversion of a synthetic 1 Hz GPS waveform data set. We computed three component GPS seismograms for the point source given in Table 2 (‘Input’ column), which represents the superposition of a strike-slip earthquake with an explosion. Synthetics were calculated using an appropriate local crustal model as described in Table 1. For this experiment, we used a realistic distribution of stations, shown in Fig. 3, which corresponds to GPS receivers that observed the 2005 Fukuoka earthquake; we invert the actual GPS data for this earthquake in Section 4.1. Since we are interested in characterizing seismic sources at low frequencies, we applied a cosine low-pass filter with corner frequencies at 0.05 and 0.2 Hz to the synthetic data set and derivative kernels. The starting coordinates for the inversion are 15 km away from those of the true source, and the initial origin time is 2 s earlier than the correct value. We inverted this data set using the method set out in Section 2; ideally we should recover the source parameters used to calculate the synthetic data.

Table 2 shows the results of this synthetic test at each inversion iteration. The initial mislocation of the source means that the zeroth iteration produces a moment tensor that is far removed from the actual solution. Subsequently, the centroid moves towards its true position, resulting in incremental improvements in the values of the moment tensor. There is some trade-off between the origin time and depth; hence these parameters overshoot their true values during the second and third iterations, respectively. We recover the correct source parameters after five iterations of the inversion algorithm; this number is broadly comparable to our experience with inversions of teleseismic data. The success of this experiment demonstrates that our implementation of GPS data into the CMT algorithm is sound as it gives self-consistent results.

Since GPS measurement errors are typically largest in the vertical coordinate, in reality we may wish to ignore these data and invert just the horizontal component waveforms. To simulate one such situation, we performed a second synthetic inversion using horizontal displacements only. The setup of this experiment is much the same as before, except that synthetic HRGPS waveforms were computed for a $M_w 6.6$ earthquake occurring on a fault with strike $30^\circ$, dip $80^\circ$ and rake $20^\circ$. In CMT inversions of real earthquake data it is generally assumed that there is no volume change associated with the seismic event, and so we also impose the additional constraint that the trace of the moment tensor is zero in this synthetic test. The results of this experiment are shown in Table 3, and indicate that the inversion algorithm performs well even in the absence of any vertical displacement data being available.

The ‘radius of convergence’ of the algorithm depends on the local form of the misfit function; in other synthetic experiments carried out under ideal conditions we were able to recover the true source parameters even when the initial location was in error by as much as 70 km and 25 s. In fact, for both of the earthquakes we examine later...
in this study, the USGS Preliminary Determination of Epicenters (PDE) hypocentre, which we assume for the zeroth iteration, is close enough to the centroid for our inversions of GPS data to converge.

Non-linearities encountered when a station lies in-between the inversion’s starting location and the true centroid position may cause the inversion to become unstable. In practice this problem can be avoided by ignoring all stations within a certain distance of the initial epicentre. This approach obviously decreases the quantity of data available for inversion, however the loss of data is acceptable since it actually helps reduce the risk of the data set being contaminated by finite source effects. For the earthquakes we examine in Section 4—both of which have $M_w < 7.0$—we find a threshold distance of 25 km is enough to prevent instabilities and finite source effects hampering the inversion.

4 CMT INVERSIONS USING HRGPS WAVEFORMS

As a proof of concept, we now attempt to perform CMT inversions using real 1 Hz GPS data obtained by Japan’s GEONET for two recent events: the 2005 $M_w 6.6$ Fukuoka earthquake, and the 2008 $M_w 6.9$ Iwate–Miyagi earthquake. Rupture histories for these events have already been determined using these GPS waveforms by Kobayashi et al. (2006) and Yokota et al. (2009), respectively; interested readers should refer to these papers for details of the GPS processing methodologies employed. The performance of our inversion scheme can be assessed by comparing our CMT solutions to the first-order features of the finite fault slip models determined in these earlier studies, although one should bear in mind the differences in the inversion approaches which will be reflected in the results.

4.1 The 2005 $M_w 6.6$ Fukuoka, Japan earthquake

Fig. 4 shows the distribution of GEONET stations (triangles) that recorded 1 Hz GPS displacement waveforms for the 2005 March 20, $M_w 6.6$ Fukuoka earthquake. Kobayashi et al. (2006) inverted these data to infer the source process of this event, assuming that the earthquake occurred on a near-vertical fault (Fig. 4, rectangle), with a left-lateral strike-slip mechanism. They recovered a simple rupture history characterized by a single slip patch. Consequently, this event provides a good test of the CMT method applied to GPS data since we should recover a centroid location and mechanism close to that obtained by Kobayashi et al. (2006).

Apart from the obvious difference between our source parameterization and that employed by Kobayashi et al. (2006), we aim to emulate their inversion as closely as possible; we use their crustal structure, shown in Table 1, and extract identical minute-long windows from each displacement seismogram. One important difference is that we invert horizontal displacements only, as the vertical channels appear to be dominated by noise. Also, to mitigate against finite source effects contaminating the data set, we omit station 1062 (Fig. 4, red triangle) from our inversion as it lies within 25 km of the PDE hypocentre. Kobayashi et al. (2006) present an inversion that also ignores this station; in the following, any comparisons that we make relate to this particular solution.

As with our synthetic test, we apply a cosine low-pass filter to the data and synthetics, with corner frequencies at 0.05 and 0.2 Hz. Following the Global CMT algorithm, we constrain the trace of the moment tensor to be zero (i.e. $M_{rr} + M_{θθ} + M_{φφ} = 0$), so that there is no explosive component to the source. We take the starting location for the inversion from the PDE catalogue (Fig. 4, red star).
The result of our CMT inversion is given in Table 4, and shown as the red beachball in Fig. 4. Our moment tensor resembles the Global CMT mechanism (Fig. 4, black beachball), and is positioned about 7 km east of the catalogue solution, which is reasonable given the expected errors in Global CMT locations that arise from unmodelled 3-D earth structure (e.g. Valentine & Woodhouse 2010). Our centroid, located at 130.21° E, 33.71° N with depth 6.82 km, is consistent with the slip patch obtained by Kobayashi et al. (2006) from the same HRGPS data, which has approximate centroid coordinates of 130.23° E, 33.72° N and a depth of 6 km (Fig. 4, green circle).

Since Kobayashi et al. (2006) assumed that the earthquake occurred purely as slip on a plane, it is instructive to compare our best double-couple solution to the orientation of their assumed fault. As Table 5 shows, our best double-couple solution corresponds to a fault mechanism that is almost identical to the one adopted by Kobayashi et al. (2006), and also compares favourably to the Global CMT best double-couple solution. Thus the mechanism and location that we recover appear to be consistent with those previously determined for this earthquake.

The non-double-couple component of a moment tensor can be expressed by the parameter \( \epsilon = \frac{\lambda_{\text{min}}}{\lambda_{\text{max}}} \) where \( \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \) are respectively the smallest and largest absolute values of the moment tensor’s eigenvalues, with \( \epsilon = 0 \) corresponding to a pure double-couple. Values of \( \epsilon \) for each solution are also given in Table 5. These \( \epsilon \) values indicate that the Global CMT solution is almost pure double-couple whereas our CMT solution from HRGPS waveforms has a significant non-double-couple component, as is also apparent from a visual comparison of the beachballs plotted in Fig. 4. This difference likely arises due to unmodelled complexities in source and structure, which will more strongly affect the near-field data. Regardless of the similarity of our result to earlier solutions, the excellent agreement between the observed GPS waveforms and those calculated for our final source, as shown in Fig. 5, suggests that it is possible to recover a reliable CMT solution from GPS waveform data.

### 4.2 2008 Mw 6.9 Iwate–Miyagi, Japan earthquake

To further validate the CMT method as applied to HRGPS data, we inverted a second set of 1 Hz GPS waveforms that were obtained for the 2008 June 13 Mw 6.9 Iwate–Miyagi earthquake. Fig. 6 shows the distribution of GEONET GPS receivers (triangles) that recorded this event. Yokota et al. (2009) inverted these data to infer the source process of this event, assuming a westward dipping thrust fault (Fig. 6, rectangle); they found that the rupture was best described by a single main slip patch with approximate centroid coordinates 140.88° E, 38.96° N and a depth of 3.6 km. The first-order features of this solution provide a useful check on the reasonableness of our CMT solution derived from the same GPS data set. However it should be kept in mind that, apart from the obvious differences in our source parameterizations, Yokota et al. (2009) used station-specific crustal models which we have not implemented, and—since three stations lie within 25 km of the hypocentre—we invert only a subset of the data that they used. We use the same filter parameters as before, and again invert only the horizontal GPS waveforms as the vertical displacements appear to be dominated by noise. The kernel functions are computed in the local earth model taken from Crust 2.0 (http://igppweb.ucsd.edu/~gabi/rem.html).

The result of our CMT inversion is given in Table 6, and shown as the red beachball in Fig. 6. Our centroid and moment tensor are similar to the Global CMT solution (Fig. 6, black beachball). Furthermore, our centroid is located close to that inferred from the slip distribution of Yokota et al. (2009, Fig. 6 green circle); the level of agreement between these locations is reasonable given the differences in earth models used. As Table 7 shows, our best double-couple mechanism is almost identical to the fault plane that Yokota et al. (2009) used, and also compares well to the best double-couple solution from the Global CMT catalogue. The \( \epsilon \) values corresponding to these different results, also shown in Table 7, indicate that both our CMT solution and the Global CMT solution have a non-double-couple component, although the deviation from a pure double-couple is larger for our near-field solution. So the CMT solution that we obtain from GPS data for this earthquake is consistent with independently determined mechanisms and locations. Furthermore the excellent agreement between the observed and synthetic GPS waveforms, as Fig. 7 shows, demonstrates that our CMT solution explains the data very well and implies that—at low frequencies—a point moment tensor source is a sufficient model for this earthquake.

### 5 DISCUSSION

Our inversions of synthetic and real GPS waveforms clearly demonstrate the potential value of HRGPS data for determining the first-order source parameters of an earthquake. The good agreement between our CMT solutions and other independent source models strongly suggests that the data selection, processing and inversion schemes described in this paper are robust. For both of the earthquakes that we have analysed our CMT solution explains the GPS data very well, with a variance reduction of over 90 per cent.

---

**Table 4.** The results of our CMT inversion of GPS waveforms recorded for the 2005 Mw 6.6 Fukuoka earthquake. The moment tensor is given in units of 10^{19} Nm.

<table>
<thead>
<tr>
<th>Location</th>
<th>Initial</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>Origin time (UT)</td>
<td>01:53:41.80</td>
<td>01:53:48.04</td>
</tr>
<tr>
<td>Latitude</td>
<td>33.81° N</td>
<td>33.71° N</td>
</tr>
<tr>
<td>Longitude</td>
<td>130.13° E</td>
<td>130.21° E</td>
</tr>
<tr>
<td>Depth (km)</td>
<td>10.00</td>
<td>6.82</td>
</tr>
</tbody>
</table>

| Moment tensor: | | |
|----------------|---------------------|
| \( M_{xx} \)   | 0.1365              |
| \( M_{yy} \)   | 0.4420              |
| \( M_{zz} \)   | -0.5785             |
| \( M_{xy} \)   | 0.0161              |
| \( M_{xz} \)   | 0.0151              |
| \( M_{yz} \)   | 0.2097              |
| Relative RMS   | 0.0895              |

**Table 5.** A comparison of double-couple mechanisms for the 2005 Mw 6.6 Fukuoka earthquake. Our solution is very similar to the 1 Hz catalogue solution of Kobayashi et al. (2006) and the best double-couple mechanism from the Global CMT catalogue. The parameter \( \epsilon \), defined in Section 4.1, describes the deviation of the solution from a pure double-couple, which has \( \epsilon = 0 \).

<table>
<thead>
<tr>
<th>Strike</th>
<th>Dip</th>
<th>Rake</th>
<th>Moment</th>
<th>( \epsilon )</th>
<th>Best Double-Couple</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(×10^{19} Nm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>This study</td>
<td>124</td>
<td>87</td>
<td>+2</td>
<td>0.55</td>
<td>0.22</td>
</tr>
<tr>
<td>Kobayashi et al. (2006)</td>
<td>123</td>
<td>88</td>
<td>-1</td>
<td>0.88</td>
<td>0.00</td>
</tr>
<tr>
<td>Global CMT catalogue</td>
<td>302</td>
<td>88</td>
<td>-14</td>
<td>0.92</td>
<td>0.02</td>
</tr>
</tbody>
</table>

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This level of agreement suggests that a point moment tensor source is a good model for these moderate magnitude earthquakes, even for near-field data.

Although we have only analysed two events, some features of these preliminary results are worthy of further discussion. For both earthquakes, we find that the origin time perturbation is positive, that is, the centroid time is later than the hypocentral time, which is expected for a rupture of finite duration and follows from the definition of the centroid as opposed to the hypocentre. The larger earthquake required a greater shift in origin time, which is consistent with the observation that, for simple ruptures, the time perturbation is analogous to the earthquake’s half-duration (Dziewonski & Woodhouse 1983a).

Both of the HRGPS CMT solutions that we have determined have non-double-couple components that are larger than those in the corresponding Global CMT solutions. These could arise from source complexities such as curved or multiple faults, and unmodelled 3-D structural effects that impact the near-field data more heavily than the longer-period teleseismic waveforms used to construct Global CMT solutions.

We also find that both of our data inversions yield lower moment estimates than the Global CMT solutions. Some of this disagreement can be explained by the shallow centroid depths that we recover relative to the catalogue solutions, which are fixed to a depth of 12 km during the inversion. Since our focal depths agree with those obtained from finite fault slip inversions we believe that the shallow depths we recover are real and can be trusted. It is interesting that we can obtain stable moment tensors for earthquakes as shallow as 3 km using GPS waveforms, given the well known problem of determining the $M_{\theta\theta}$ and $M_{\phi\phi}$ components of shallow events from long-period teleseismic data. High-rate GPS waveforms from more earthquakes obviously must be analysed to assess the general validity of this observation.

The scope of our analysis has so far been limited to only two events principally because seismologists’ access to GPS waveforms is on a somewhat ad hoc basis. Melgar et al. (2011) estimate that since 2002 Japan’s GEONET has recorded 1 Hz GPS data for 29 earthquakes larger than magnitude 7.0. As noted by Larson (2009) it is desirable for further HRGPS waveform data sets to be made publicly available in seismological repositories. With better access to data, it is possible that the method outlined in this paper could be applied systematically to determine earthquake source parameters from GPS waveforms. Such an undertaking may be useful for improving the completeness of seismic catalogues in the immediate

Figure 5. GPS waveforms (black traces) observed at the stations shown in Fig. 4 after the 2005 $M_w$ 6.6 Fukuoka earthquake, and used in our CMT inversion. Synthetic seismograms (red traces) computed for our recovered CMT solution fit the data well in both phase and amplitude, with an overall variance reduction of 91 per cent. Time is relative to the PDE hypocentral time.
CMT inversions using high-rate GPS waveforms

To fully exploit the available GPS data resource, we need to validate our method for larger earthquakes. Thus far we have assumed that the source acts instantaneously, which is only reasonable if the data is long-period relative to the duration of the earthquake. For larger events, in addition to filtering the data at lower frequencies, we will need to implement a source time function of realistic duration. As the spatial dimensions of the source grow, we modify our strategy to minimize finite fault effects—including data from stations that are less than 25 km away from the source—will also need to be refined. Neither of these implementation issues fundamentally prohibits CMT inversions of HRGPS waveforms from larger earthquakes, and existing scaling laws (e.g. Kanamori & Anderson 1975) suggest that simple rules can be applied to accommodate the finite spatial and temporal extent of the earthquake source.

There is growing interest in using GPS data to rapidly characterize seismic sources for the purpose of earthquake early warning (e.g. Crowell et al. 2009). The inversion method described in this paper could be used to determine the location, mechanism and moment of earthquakes from HRGPS waveforms in real time. So far, rapid source inversions of GPS data streams have exploited only the static displacement field (Allen & Ziv 2011; Melgar et al. 2011); additional information about the source could be gained by including frequencies other than 0 Hz in the inversion. Furthermore, the CMT algorithm is amenable to real-time implementation since the data can be windowed automatically and the only a priori information needed is a starting source location, such as the hypocentre determined from P-wave arrival times, and an earth model, which—in the absence of a more appropriate local alternative—can be found from Crust 2.0. The rate determining step in the inversion is the rate of the source acting instantaneously, which is only reasonable if the data is long-period relative to the duration of the earthquake. For larger events, in addition to filtering the data at lower frequencies, we will need to implement a source time function of realistic duration. As the spatial dimensions of the source grow, we need to implement a source time function of realistic duration. As the spatial dimensions of the source grow, we need to implement a source time function of realistic duration. As the spatial dimensions of the source grow, we need to implement a source time function of realistic duration.

For larger events, in addition to filtering the data at lower frequencies, we will need to implement a source time function of realistic duration. As the spatial dimensions of the source grow, we need to implement a source time function of realistic duration. As the spatial dimensions of the source grow, we need to implement a source time function of realistic duration. As the spatial dimensions of the source grow, we need to implement a source time function of realistic duration. As the spatial dimensions of the source grow, we need to implement a source time function of realistic duration. As the spatial dimensions of the source grow, we need to implement a source time function of realistic duration.
Using this method, we have inverted 1 Hz GPS data sets pertaining to two recent earthquakes in Japan. The CMT solutions that we recover are consistent with finite source models determined from the same GPS data sets, and also with the Global CMT catalogue solutions obtained from long-period teleseismic waveform data. This agreement suggests that we can recover reliable source parameters from GPS data. Our CMT solutions explain the GPS data very well, implying that a point moment tensor source is a good model for these earthquakes even in the seismic near-field. There is much scope for further development of the method presented in this study, particularly in the context of earthquakes larger than magnitude 7.0. By providing a framework for the inversion of HRGPS waveforms that mirrors the one used for CMT inversions of teleseismic data, the present work should allow direct comparison and amalgamation of information from diverse data sets, yielding an improved understanding of earthquake source models and their uncertainties.

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APPENDIX: DIRECT CALCULATION OF ACCURATE LOCATION KERNELS

Here, we derive expressions for the location kernels which we use in our CMT inversions of GPS waveforms. We have verified our implementation of the theory presented here through comparison to derivative kernels calculated using the method of finite differences. Following O’Toole & Woodhouse (2011), we begin by defining a right-handed Cartesian coordinate system (x, y, z) related to cylindrical polar coordinates (r, φ, z) by

\[ x = x_1 = r \cos \phi, \]  
\[ y = y_2 = r \sin \phi, \]  
\[ z = x_3, \]

where the positive z-axis is directed upwards.

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A1 Centroid time kernel

The centroid time kernel is the negative of the time derivative of the synthetic seismogram and is straightforward to calculate in the frequency domain via multiplication of the synthetic spectra by $-i\omega$.

A2 Epicentral location kernels

Consider a seismic source located at $x_c = (x_c, y_c, z_c)$. We seek the derivative of the displacement field, $u(r, \phi, t)$, with respect to the epicentral location. Using the chain rule, we can write

$$\frac{\partial u}{\partial x_c} = \frac{\partial u}{\partial r} \frac{\partial r}{\partial x_c} + \frac{\partial u}{\partial \phi} \frac{\partial \phi}{\partial x_c},$$  \hspace{1cm} (A4)

$$\frac{\partial u}{\partial y_c} = \frac{\partial u}{\partial r} \frac{\partial r}{\partial y_c} + \frac{\partial u}{\partial \phi} \frac{\partial \phi}{\partial y_c},$$  \hspace{1cm} (A5)

with

$$\frac{\partial r}{\partial x_c} = -\cos \phi, \quad \frac{\partial \phi}{\partial x_c} = \frac{\sin \phi}{r},$$  \hspace{1cm} (A6)

$$\frac{\partial r}{\partial y_c} = -\sin \phi, \quad \frac{\partial \phi}{\partial y_c} = \frac{-\cos \phi}{r}.$$  \hspace{1cm} (A7)

From eqs (27)–(29) of O’Toole & Woodhouse (2011), we can write the displacement field as an integral over angular frequency $\omega$ and horizontal wavenumber $k$

$$u_r(r, \phi, t) = \int \frac{d\omega}{2\pi i\omega} \sum_{m=-2}^{2} e^{i\omega \phi} K_n^m(r, \omega)$$  \hspace{1cm} (A8)

$$u_\phi(r, \phi, t) = \int \frac{d\omega}{2\pi i\omega} \sum_{m=-2}^{2} e^{i\omega \phi} K_n^m_r(r, \omega)$$  \hspace{1cm} (A9)

$$u_t(r, \phi, t) = \int \frac{d\omega}{2\pi i\omega} \sum_{m=-2}^{2} e^{i\omega \phi} K_n^m(r, \omega)$$  \hspace{1cm} (A10)

with

$$K_n^m(r, \omega) = \frac{1}{2\pi} \int_0^\infty \left[ k V_m J_m(kr) + imr^{-1} W_m J_m(kr) \right] dk$$  \hspace{1cm} (A11)

$$K_n^m_r(r, \omega) = \frac{1}{2\pi} \int_0^\infty \left[ imr^{-1} V_m J_m(kr) - kW_m J_m(kr) \right] dk$$  \hspace{1cm} (A12)

$$K_n^m_t(r, \omega) = \frac{1}{2\pi} \int_0^\infty k U_m J_m(kr) dk,$$  \hspace{1cm} (A13)

where $J_m(kr)$ is a Bessel function and a prime denotes differentiation with respect to its argument. $U_m$, $V_m$, and $W_m$ are the displacement elements of the stress-displacement vectors. Differentiating expressions (A8)–(A10) with respect to $\phi$ we obtain

$$\frac{\partial u_r}{\partial \phi} = \int \frac{d\omega}{2\pi i\omega} \sum_{m=-2}^{2} e^{i\omega \phi} \frac{\partial K_n^m}{\partial r}.$$  \hspace{1cm} (A14)

The derivatives of (A8)–(A10) with respect to the radial coordinate are of the form

$$\frac{\partial u_r}{\partial r} = \int \frac{d\omega}{2\pi i\omega} \sum_{m=-2}^{2} \frac{\partial K_n^m}{\partial r}.$$  \hspace{1cm} (A15)

From (A11)–(A13) we write

$$\frac{\partial K_n^m}{\partial r} = \frac{1}{2\pi} \int_0^\infty \left[ k V_m \frac{\partial}{\partial r} J_m(kr) - \frac{im}{r^2} W_m J_m(kr) + \frac{im}{r} W_m k J_m(kr) \right] dk$$  \hspace{1cm} (A16)

$$\frac{\partial K_n^m_r}{\partial r} = \frac{1}{2\pi} \int_0^\infty \left[ \frac{im}{r^2} V_m J_m(kr) + \frac{im}{r} V_m k J_m(kr) - kW_m \frac{\partial}{\partial r} J_m(kr) \right] dk$$  \hspace{1cm} (A17)
\[ \frac{\partial K^m}{\partial r} = \frac{1}{2\pi} \int_0^\infty k^2U_m J_m(kr) \, dk. \]  

(A18)

To improve computational efficiency, we seek an expression for the second derivative of the Bessel function

\[ \frac{\partial}{\partial r} J_m(kr) = k J'_m(kr) \]  

(A19)

in terms of its lower derivatives, which we have already calculated. Rearranging Bessel’s equation we obtain

\[ \frac{\partial}{\partial r} J_m(kr) = \frac{1}{kr} \left[ -k r J_m(kr) - (k^2 r^2 - m^2) J_m(kr) \right] \]  

(A20)

which we use in (A16)–(A18) to calculate the radial derivatives of the displacement.

### A3 Source depth kernel

The algorithm of O’Toole & Woodhouse (2011) allows us to solve systems of inhomogeneous ordinary differential equations of the form

\[ \frac{\partial b}{\partial z} = Ab + f. \]  

(A21)

where the stress-displacement vector \( b \), the system matrix \( A \) and the source representation \( f \) are functions only of the vertical coordinate, \( z \). Assuming that the source does not lie on a structural discontinuity, we can differentiate (A21) with respect to the source depth \( z_s \) to obtain

\[ \frac{\partial}{\partial z_s} \frac{\partial b}{\partial z} = A \frac{\partial b}{\partial z_s} + \frac{\partial f}{\partial z_s}. \]  

(A22)

Since this expression is of the same form as (A21), we can use the same method of solution to obtain the source depth kernel by replacing the force term with its derivative with respect to source depth. The action of the source is implemented as a discontinuity in the solution at the source depth. After Al-Attar & Woodhouse (2008), who presented a similar result for the equivalent problem in a spherical geometry, we convert the force term into a slip condition at the source depth as follows.

A trial solution of (A21) is

\[ b(z) = P(z, z_1) v(z), \]  

(A23)

where \( v(z) \) is to be determined; \( P(z, z_1) \) is the propagator matrix (Gilbert & Backus 1966) satisfying

\[ \frac{\partial}{\partial z} P(z, z_1) = A(z) P(z, z_1), \quad P(z_1, z_1) = I \]  

(A24)

and \( I \) is the identity matrix. Differentiating (A23) with respect to \( z \), we obtain

\[ \frac{\partial}{\partial z} v(z) = P^{-1}(z, z_1) f(z). \]  

(A25)

Integrating this expression upwards from some arbitrary depth \( z_1 \) below the source (i.e. \( z_1 < z_s \)) we find a particular integral of (A21) is

\[ v(z) = \int_{z_1}^z P^{-1}(z', z_1) f(z') \, dz' + v(z_1) \]  

(A26)

and recalling the definition (A23) we can express the stress-displacement vector as

\[ b(z) = \int_{z_1}^z P(z, z') f(z') \, dz' + P(z, z_1) v(z_1), \]  

(A27)

where we have also used the property of the propagator matrix \( P(z_1, z_1) = P^{-1}(z_1, z_1) \). Since the solution to the homogeneous equation is a continuous function of depth, the discontinuity at the source depth comes from the consideration of this particular integral alone. The force term is given by

\[ f = a \delta(z - z_s) + c \delta'(z - z_s), \]  

(A28)

where \( \delta(z - z_s) \) is the Dirac delta function and the prime denotes its derivative. Using this expression in (A27), the solution above the source is

\[ b(z) = P(z, z_1) a(z_1) - \int_{z_1}^z \frac{\partial}{\partial z} P(z, z') \delta(z' - z_s) \, dz' + P(z, z_1) v(z_1), \]  

(A29)

where we have used the result (Riley et al. 2007)

\[ \int_{z} f(z) \delta''(z) \, dz = (-1)^{\prime} \int_{z} f''(z) \delta(z) \, dz \]  

(A30)
for the \(n\)th derivative of the Dirac delta function in the interval \(I\). We obtain the derivative of the propagator matrix with respect to its second argument by considering

\[
\frac{\partial}{\partial z}(P(z, z')P^{-1}(z, z')) = 0
\]

(A31)

which leads to

\[
\frac{\partial}{\partial z}P(z, z') = -P(z, z')A(z').
\]

(A32)

Using this result in (A29), we can write the solution as

\[
b(z) = \begin{cases} 
  P(z, z)v(z) & z \in [z_2, z_1) \\
  P(z, z)v(z) + P(z, z_1)a(z_1) + P(z, z_1)c(z_1) & z \in (z_1, z_2].
\end{cases}
\]

(A33)

It follows that the point source \((A28)\) produces a discontinuity, \(s\), in the solution vector at the source depth, given by

\[
s = \lim_{\epsilon \to 0} \left[ b(z_0 + \epsilon) - b(z_0 - \epsilon) \right] = a(z_0) + A(z_0)c(z_0).
\]

(A34)

Applying the same argument to \((A22)\), we find the derivative of the force term \((A28)\) with respect to the source depth is

\[
\frac{\partial f}{\partial z_0} = -a\delta(z - z_0) - c\delta(z - z_0)
\]

(A35)

which corresponds to a discontinuity in the solution vector at the source depth given by

\[
s = \lim_{\epsilon \to 0} \left[ \frac{\partial b(z_0 + \epsilon)}{\partial z_0} - \frac{\partial b(z_0 - \epsilon)}{\partial z_0} \right] = -A(z_0)a(z_0) - A(z_0)c(z_0).
\]

(A36)

Thus we can easily calculate the derivative with respect to source depth by changing the discontinuity vector at the source depth to that given in \((A36)\). Using the same notation as O'Toole & Woodhouse (2011), we write the discontinuity vectors explicitly as

\[
s^{SV}_m = \begin{cases} 
  \frac{F_2}{n} & \text{if } m = 0 \\
  \frac{-k^2(M_{11} + M_{22}) + \frac{(2\lambda + \mu\sigma)(M_{13})}{\sigma}}{kF_2} & \text{if } m = \pm 1 \\
  \frac{1}{\sigma} \left[ k^2\lambda(M_{13} + iM_{23}) + \frac{1}{\sigma}(M_{13} - iM_{23})(k^2(\lambda\mu - (\gamma + \mu)\sigma + \rho\sigma\omega^2)) \right] & \text{if } m = \pm 2 \\
  0 & \text{otherwise}
\end{cases}
\]

(A37)

and

\[
s^{SH}_m = \begin{cases} 
  \frac{1}{\sigma}(M_{12} + M_{21}) & \text{if } m = 0 \\
  \frac{1}{\sigma}((\gamma F_2 - iF_1) & \text{if } m = \pm 1 \\
  \frac{1}{\sigma}(k^2\mu - \rho\omega^2)(\pm M_{23} + iM_{13}) & \text{if } m = \pm 2 \\
  0 & \text{otherwise}
\end{cases}
\]

(A38)