on Fundamentals of Electronics, Communications and Computer Sciences

**VOL. E97-A NO. 9
SEPTEMBER 2014**

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A PUBLICATION OF THE ENGINEERING SCIENCES SOCIETY

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LETTER

Outage Probability of N -th Best User Selection in Multiuser Two-Way Relay Networks over Nakagami- m Fading

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SUMMARY We analyze the outage probability of the multiuser two-way relay network (TWRN) where the N -th best mobile user (MU) out of M MUs and the base station (BS) exchange messages with the aid of an amplify-and-forward relay. In the analysis, we focus on the practical unbalanced Nakagami- m fading between the MUs-relay link and the relay-BS link. We also consider both perfect and outdated channel state information (CSI) between the MUs and the relay. We first derive tight closed-form lower bounds on the outage probability. We then derive compact expressions for the asymptotic outage probability to explicitly characterize the network performance in the high signal-to-noise ratio regime. Based on our asymptotic results, we demonstrate that the diversity order is determined by both Nakagami- m fading parameters, M , and N when perfect CSI is available. When outdated CSI is available, the diversity order is determined by Nakagami- m fading parameters only. In addition, we quantify the contributions of M , N , and the outdated CSI to the outage probability via the array gain.

key words: multiuser, two-way relay networks, outdated channel state information, Nakagami- m fading

1. Introduction

The deployment of relay nodes has been recognized as one of the core ingredients to provide flexible and reliable wireless data transmission between the base station and the remotely located mobile users in multiuser cellular networks [1], [2]. In order to mitigate the spectral-efficiency loss caused by the standard one-way relaying, the two-way relay network (TWRN) was proposed to complete the bidirectional information exchange between two nodes within two time slots [3]. From the engineering design perspective, the amplify-and-forward (AaF) protocol has been widely considered in TWRNs due to its low implementation complexity and low signal processing cost [4], [5]. The performance of TWRNs has been examined extensively in a point-to-point context where a single base station (BS) exchanges its information with a single mobile user (MU) via a bidirectional relay, e.g., [6], [7].

Motivated by the growing use of multiuser communications, the performance of TWRNs have recently been addressed in a point-to-multipoint context where a single BS exchanges its information with multiple MUs via a bidirectional relay [8]–[14]. In order to exploit the multiuser benefits, the user selection technique has been adopted to exploit the independent channel variations across multiple users. Of course, the selection of the best user with the highest channel quality achieves multiuser diversity [8]. However, such a selection cannot be always guaranteed, which might be due to load balancing schemes [9]. In this case, the N -th best users might be selected for transmission. The performance of the multiuser TWRN with N -th best user selection with perfect feedback has been studied in [10], [11]. Specifically, [10] examined the performance in Rayleigh fading and [11] examined the performance in the mixed Rayleigh and Rician fading. In practise, if the system operates over a time-varying channel, the user selection is probably performed based on outdated channel state information (CSI) [12], [13]. The performance of the multiuser TWRN with N -th user selection with outdated CSI was analyzed in [14] for Rayleigh fading channels. In this work, we focus on the performance of the multiuser TWRN over general Nakagami- m fading by taking both perfect CSI and outdated CSI into consideration. Nakagami- m fading has been demonstrated to be preferably fitting with a wide variety of realistic line-of-sight/non line-of-sight fading channels [15], [16].

In this letter, we derive tight and closed-form lower bounds on the outage probability of the multiuser TWRN. In the derivation, we consider practical unbalanced Nakagami- m fading channels between the MUs-relay link and the relay-BS link. We also derive compact expressions for the asymptotic outage probability which enables us to characterize the diversity order and the array gain, both of which offer valuable insights into the network behavior in the high signal-to-noise (SNR) regime. In the analysis, we examine both perfect and outdated CSI in the MUs-relay link. Using the asymptotic results, we demonstrate that the diversity order is $\min(m_1(M - N + 1), m_2)$ in the presence of perfect CSI, where M is the number of MUs, m_1 is the Nakagami- m fading parameter of MUs-relay link, and m_2 is the Nakagami- m fading parameter of relay-BS link. In the presence of outdated CSI, we demonstrate that the diversity order is $\min(m_1, m_2)$, which is independent of N and M . We also explicitly quantify the impact of N , M , and the correlation coefficient of outdated CSI on the performance through the array gain. Numerical results are presented to verify the

Manuscript received April 9, 2014.

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DOI: 10.1587/transfun.E97.A.1987

accuracy of our analysis.

2. System Model

We consider the multiuser two-way relay network (TWRN) where the mobile users (MUs) and the base station (BS) communicate via an intermediate relay R. We assume that the MUs, the BS, and the relay are equipped with a single antenna. The direct link between the MUs and the BS are assumed to be in absence, which is valid for the practical scenario where the MUs are located far away from the BS or within heavily shadowed areas. We consider unbalanced Nakagami- m fading between the MUs-R link with fading parameter m_1 and the R-BS link with fading parameter m_2 .

In the multiuser TWRN, each information exchange takes place over two consecutive time slots. In the first time slot, the selected N -th best user MU_N and the BS transmit s_1 and s_2 concurrently to R, respectively. Here, we assume that s_1 and s_2 have zero-mean and unit variance such that $E[|s_1|^2] = E[|s_2|^2] = 1$, where $E[\cdot]$ is the expectation. Prior to data transmission, the user selection is conducted at R such that R selects MU_N from all the MUs based on their instantaneous SNRs from R [14], [17]. When the duration of user selection is longer than the coherence time of the channel, the selected user is based on the outdated channel state information (CSI) rather than the precise CSI since the channel condition changes when data transmission commences. As such, we denote $h_{SR_N}^T$ as the delayed channel coefficient of the MU_N -R link during user selection. In the first time slot, the received signal at R is given by

$$z = \sqrt{P_1} h_{SR_N} s_1 + \sqrt{P_2} g_{RD} s_2 + n_R, \quad (1)$$

where P_1 and P_2 denote the transmit power at MU_N and the BS, respectively, h_{SR_N} denotes the channel coefficient of the MU_N -R link, g_{RD} denotes the channel coefficient of the R-BS link, and n_R is the additive white Gaussian noise (AWGN) component with unit variance $\sigma_n^2 = 1$ at R.

In the second time slot, the relay adopts the AaF relaying to amplify the received signal by the gain $\kappa = 1 / \sqrt{P_1 |h_{SR_N}|^2 + P_2 |g_{RD}|^2 + 1}$ and then broadcasts the resultant signal to MU_N and the BS with power P_R . The received signals at MU_N and the BS are given by

$$\begin{aligned} z_1 = & \kappa \sqrt{P_1 P_R} |h_{SR_N}|^2 s_1 + \kappa \sqrt{P_2 P_R} g_{RD} h_{SR_N} s_2 \\ & + \kappa \sqrt{P_R} h_{SR_N} n_R + n_1 \end{aligned} \quad (2)$$

and

$$\begin{aligned} z_2 = & \kappa \sqrt{P_2 P_R} |g_{RD}|^2 s_2 + \kappa \sqrt{P_1 P_R} g_{RD} h_{SR_N} s_1 \\ & + \kappa \sqrt{P_R} g_{RD} n_R + n_2, \end{aligned} \quad (3)$$

respectively, where n_1 and n_2 are the AWGN components with unit variance $\sigma_n^2 = 1$ at MU_N and the BS, respectively. Subtracting the self-interference $\kappa \sqrt{P_1 P_R} |h_{SR_N}|^2 s_1$ and $\kappa \sqrt{P_2 P_R} |g_{RD}|^2 s_2$ from z_1 and z_2 , respectively, the instantaneous SNRs at MU_N and the BS are obtained as

$$\gamma_1 = \frac{\alpha \gamma_{SR} \gamma_{DR}}{(1 + \alpha) \gamma_{SR} + \gamma_{RD} + 1} \quad (4)$$

and

$$\gamma_2 = \frac{\beta \gamma_{SR} \gamma_{DR}}{\gamma_{SR} + (1 + \beta) \gamma_{RD} + 1}, \quad (5)$$

respectively, where $\alpha = P_R/P_1$ and $\beta = P_R/P_2$. In (4) and (5), we define $\gamma_{SR} = P_1 |h_{SR_N}|^2$ as the instantaneous SNR of the MU_N -R link and $\gamma_{RD} = P_2 |g_{RD}|^2$ as the instantaneous SNR of the R-BS link. The average SNRs of the MU_N -R link and the R-BS link are given by $\bar{\gamma}_{SR} = E\{|h_{SR_N}|^2\} P_1$ and $\bar{\gamma}_{RD} = E\{|g_{RD}|^2\} P_2$, respectively.

3. Outage Probability Analysis

In the multiuser TWRN, the instantaneous rate at MU_N and the BS are written as $R_{MU_N} = \frac{1}{2} \log_2(1 + \gamma_1)$ and $R_{BS} = \frac{1}{2} \log_2(1 + \gamma_2)$, respectively. In such a network, the outage probability is defined as the probability that either R_{MU_N} or R_{BS} is falls below the predetermined threshold rate R_0 . According to definition, the outage probability, P_{out} , is formulated as

$$\begin{aligned} P_{out} &= \Pr[\min(R_{MU_N}, R_{BS}) < R_0] \\ &= \Pr[\min(\gamma_1, \gamma_2) < \gamma_{th}] \\ &= 1 - \Pr[\min(\gamma_1, \gamma_2) \geq \gamma_{th}], \end{aligned} \quad (6)$$

where $\gamma_{th} = 2^{2R_0} - 1$. We note that it is mathematically intractable to derive an exact closed-form expression for P_{out} . Therefore, we turn to derive a tight lower bound on P_{out} .

In order to obtain P_{out} , the probability density functions (PDFs) of γ_{SR} and γ_{RD} are required. We first derive the PDF of γ_{SR} . In this work, the Jakes' model is adopted to characterize the outdated CSI. Specifically, the correlation coefficient between $|h_{SR_N}|^2$ and $|h_{SR_N}^T|^2$ is given by $\rho = J_0^2(2\pi f_d T_{D_k})$ for $\rho \in [0, 1]$ [14], where f_d is the Doppler frequency, T_{D_k} is the time difference between the user selection and data transmission, and $J_0(\cdot)$ is the zero-order Bessel function of the first kind. We clarify that $\rho = 0$ corresponds to the case of completely outdated CSI and $\rho = 1$ corresponds to the case of perfect CSI.

With the aid of the result in [9], we first express the PDF of $\gamma_{SR}^T = P_1 |h_{SR_N}^T|^2$ as

$$\begin{aligned} f_{\gamma_{SR}^T}(x) = & M \binom{M-1}{N-1} f_{\gamma_i}(x) [F_{\gamma_i}(x)]^{M-N} \\ & \times [1 - F_{\gamma_i}(x)]^{N-1}. \end{aligned} \quad (7)$$

In (7), the PDF and CDF of the received SNR of the i th MU -R link, γ_i , are given by

$$f_{\gamma_i}(x) = \frac{m_1^{m_1} x^{m_1-1}}{\Gamma(m_1) \bar{\gamma}_i^{m_1}} e^{-\frac{m_1}{\bar{\gamma}_i} x} \quad (8)$$

and

$$F_{\gamma_i}(x) = 1 - \frac{\Gamma(m_1, \frac{m_1}{\bar{\gamma}_i} x)}{\Gamma(m_1)}, \quad (9)$$

respectively. Substituting (8) and (9) into (7), we obtain $f_{\gamma_{SR}^T}(x)$ as

$$f_{\gamma_{SR}^T}(x) = \sum_{q=0}^{M-N} \sum_{k=0}^{\ell} \Psi_1 \xi_1^{m_1+k} x^{m_1+k-1} e^{-\frac{\xi_1}{\Theta_1} x}, \quad (10)$$

where $\ell = (q + N - 1)(m_1 - 1)$, $\xi_1 = m_1/\gamma_{SR}$, $\Theta_1 = 1/(q + N)$, and $\Psi_1 = \delta/\Gamma(m_1)$. In Ψ_1 , we define $\delta = (-1)^q M \binom{M-1}{N-1} \binom{M-N}{q} \zeta_{k(q+N-1)}$, where

$$\begin{aligned} \zeta_{kn} &= \sum_{i=k-m_1+1}^k \frac{\zeta_{i(n-1)}}{(k-i)!} I_{[0, (n-1)(m_1-1)]}(i), \\ \zeta_{00} &= \zeta_{0n} = 1, \\ \zeta_{k1} &= \frac{1}{k!}, \\ \zeta_{1n} &= n, \end{aligned}$$

and

$$I_{[a,b]}(n) = \begin{cases} 1, & a \leq n \leq b \\ 0, & \text{otherwise.} \end{cases}$$

For outdated CSI with $0 \leq \rho < 1$, we obtain the conditional PDF $f_{\gamma_{SR}|\gamma_{SR}^T}(x|y)$ with the aid of [19] as

$$\begin{aligned} f_{\gamma_{SR}|\gamma_{SR}^T}(x|y) &= \frac{m_1}{(1-\rho)\gamma_{SR}} \left(\frac{x}{\rho y} \right)^{\frac{m_1-1}{2}} e^{-\frac{m_1(x+\rho y)}{(1-\rho)\gamma_{SR}}} \\ &\quad \times I_{m_1-1} \left(\frac{2m_1 \sqrt{xy\rho}}{(1-\rho)\gamma_{SR}} \right), \end{aligned} \quad (11)$$

where $I_\nu(\cdot)$ is the ν -order modified Bessel function of the first kind. Using (10) and (11), we derive $f_{\gamma_{SR}}(x)$ as

$$\begin{aligned} f_{\gamma_{SR}}^{\dagger}(x) &= \int_0^{\infty} f_{\gamma_{SR}|\gamma_{SR}^T}(x|y) f_{\gamma_{SR}^T}(y) dy \\ &= \sum_{q=0}^{M-N} \sum_{k=0}^{\ell} \sum_{p=0}^k \Psi_2 \Theta_2^{-(m_1+k+p)} \xi_1^{m_1+p} \\ &\quad \times x^{m_1+p-1} e^{-\frac{\xi_1 x}{\Theta_2}}, \end{aligned} \quad (12)$$

where $\Theta_2 = 1 - \rho + \frac{\rho}{q+N}$ and

$$\Psi_2 = \frac{\delta \binom{k}{p} \rho^p (1-\rho)^{k-p} \Gamma(m_1+k)}{\Gamma(m_1) \Gamma(m_1+p) (q+N)^{m_1+k+p}}.$$

The integral in (12) is solved using [18, Eq. (6.643.2)].

For perfect CSI with $\rho = 1$, we find that $|h_{SR_N}|^2 = |h_{SR_N}^T|^2$. Therefore, we obtain the PDF of γ_{SR} as

$$f_{\gamma_{SR}}^{\ddagger}(x) = \sum_{q=0}^{M-N} \sum_{k=0}^{\ell} \Psi_1 \xi_1^{m_1+k} x^{m_1+k-1} e^{-\frac{\xi_1 x}{\Theta_1}}. \quad (13)$$

We next obtain the PDF of γ_{RD} as

$$f_{\gamma_{RD}}(x) = \frac{\xi_2^{m_2} x^{m_2-1}}{\Gamma(m_2)} e^{-\xi_2 x}, \quad (14)$$

where $\xi_2 = m_2/\gamma_{RD}$ and $\Gamma(\cdot)$ is the gamma function [18, Eq. (8.310.1)].

3.1 Exact Analysis

In this subsection, we use (12), (13), and (14) to derive the tight lower bound on the outage probability, $P_{\text{out}}^{\text{LB}}$.

3.1.1 Outdated CSI with $0 \leq \rho < 1$

In this case, we derive the lower bound on the outage probability, $P_{\text{out}}^{\text{LB}\dagger}$, as

$$\begin{aligned} P_{\text{out}}^{\text{LB}\dagger} &= 1 - \sum_{q=0}^{M-N} \sum_{k=0}^{\ell} \sum_{p=0}^k \sum_{i_1=0}^{m_2-1} \left(\frac{\eta_2}{\eta_1} \right)^{i_1} \frac{\xi_1^{m_1+p} \xi_2^{i_1}}{i_1!} \\ &\quad \times \Theta_2^{-(m_1+k+p)} \left(\frac{\xi_1}{\Theta_2} + \frac{\xi_2 \eta_2}{\eta_1} \right)^{-(m_1+p-1)} \\ &\quad \times \Psi_2 \Gamma \left(m_1 + p + i_1, \left(\frac{\xi_1}{\Theta_2} + \frac{\xi_2 \eta_2}{\eta_1} \right) \frac{\gamma_{\text{th}}}{\eta_2 P_R} \right) \\ &\quad - \sum_{q=0}^{M-N} \sum_{k=0}^{\ell} \sum_{p=0}^k \sum_{i_2=0}^{m_1+p-1} \left(\frac{\eta_1}{\eta_2} \right)^{i_2} \frac{\xi_2^{m_2} \xi_1^{i_2}}{i_2!} \\ &\quad \times \frac{\Gamma(m_1+p) \Theta_2^{-k-i_2}}{\Gamma(m_2)} \left(\frac{\xi_1 \eta_1}{\Theta_2 \eta_2} + \xi_2 \right)^{-(m_2+i_2)} \\ &\quad \times \Psi_2 \Gamma \left(m_2 + i_2, \left(\frac{\xi_1 \eta_1}{\Theta_2 \eta_2} + \xi_2 \right) \frac{\gamma_{\text{th}}}{\eta_1 P_R} \right), \end{aligned} \quad (15)$$

where $\eta_1 = \min(\frac{1}{P_2}, \frac{1}{P_1+P_R})$ and $\eta_2 = \min(\frac{1}{P_1}, \frac{1}{P_2+P_R})$.
Proof: See Appendix A.

3.1.2 Perfect CSI with $\rho = 1$

In this case, we derive the lower bound on the outage probability, $P_{\text{out}}^{\text{LB}\ddagger}$, as

$$\begin{aligned} P_{\text{out}}^{\text{LB}\ddagger} &= 1 - \sum_{q=0}^{M-N} \sum_{k=0}^{\ell} \sum_{i_1=0}^{m_2-1} \left(\frac{\eta_2}{\eta_1} \right)^{i_1} \frac{\xi_1^{m_1+k} \xi_2^{i_1}}{i_1!} \\ &\quad \times \Psi_1 \left(\frac{\xi_1}{\Theta_1} + \frac{\xi_2 \eta_2}{\eta_1} \right)^{-(m_1+k+i_1)} \\ &\quad \times \Gamma \left(m_1 + k + i_1, \left(\frac{\xi_1}{\Theta_1} + \frac{\xi_2 \eta_2}{\eta_1} \right) \frac{\gamma_{\text{th}}}{\eta_2 P_R} \right) \\ &\quad - \sum_{q=0}^{M-N} \sum_{k=0}^{\ell} \sum_{i_2=0}^{m_1+k-1} \left(\frac{\xi_1 \eta_1}{\Theta_1 \eta_2} + \xi_2 \right)^{-(m_2+i_2)} \\ &\quad \times \frac{\Gamma(m_1+k) \Theta_1^{m_1+k-i_2}}{\Gamma(m_2)} \left(\frac{\eta_1}{\eta_2} \right)^{i_2} \frac{\xi_2^{m_2} \xi_1^{i_2}}{i_2!} \\ &\quad \times \Psi_1 \Gamma \left(m_2 + i_2, \left(\frac{\xi_1 \eta_1}{\Theta_1 \eta_2} + \xi_2 \right) \frac{\gamma_{\text{th}}}{\eta_1 P_R} \right). \end{aligned} \quad (16)$$

Proof: Following the procedure outlined in Appendix A, we obtain (16) by using (13) and (14).

Summarizing the results presented in (15) and (16), the

tight lower bound on the outage probability is given by

$$P_{\text{out}}^{\text{LB}} = \begin{cases} P_{\text{out}}^{\text{LB}^\dagger}, & 0 \leq \rho < 1 \\ P_{\text{out}}^{\text{LB}^\ddagger}, & \rho = 1. \end{cases} \quad (17)$$

3.2 Asymptotic Analysis

In order to gain valuable insights into the network behavior, we focus on the asymptotic outage probability in the high SNR regime in this subsection.

3.2.1 Outdated CSI with $0 \leq \rho < 1$

In this case, the asymptotic outage probability at high SNRs is derived as

$$P_{\text{out}}^{\dagger\infty} \approx \begin{cases} \frac{\Delta}{m_1} \vartheta_1^{m_1}, & m_1 < m_2 \\ \frac{\Delta}{m} \vartheta_1^m + \frac{\xi_2^m}{m!} \vartheta_2^m, & m_1 = m_2 = m \\ \frac{\xi_2^{m_2}}{m_2!} \vartheta_2^{m_2}, & m_1 > m_2, \end{cases} \quad (18)$$

where $\vartheta_1 = \gamma_{\text{th}}/\eta_1 P_R$, $\vartheta_2 = \gamma_{\text{th}}/\eta_2 P_R$, and

$$\Delta = \sum_{q=0}^{M-N} \sum_{k=0}^{\ell} \frac{\delta \xi_1^{m_1} \Theta^{-(m_1+k)} (1-\rho)^k \Gamma(m_1+k)}{(q+N)^{m_1+k} \Gamma(m_1)^2}.$$

Proof: See Appendix B.

3.2.2 Perfect CSI with $\rho = 1$

In this case, the asymptotic outage probability at high SNRs is derived as

$$P_{\text{out}}^{\ddagger\infty} \approx \begin{cases} \frac{\Lambda}{m_1 \varrho} \vartheta_1^{m_1 \varrho}, & m_1 \varrho < m_2 \\ \frac{\Lambda}{m \varrho} \vartheta_1^m + \frac{\xi_2^m}{m!} \vartheta_2^m, & m_1 \varrho = m_2 = m \\ \frac{\xi_2^{m_2}}{m_2!} \vartheta_2^{m_2}, & m_1 \varrho > m_2, \end{cases} \quad (19)$$

where $\varrho = M - N + 1$ and

$$\Lambda = M \binom{M-1}{N-1} \frac{\xi_1^{m_1 \varrho}}{m_1!^{(M-N)} \Gamma(m_1)}.$$

Proof: We first obtain the first order expansion of $f_{\gamma_{SR}}(x)$ for $\rho = 1$ as

$$f_{\gamma_{SR}}^{\ddagger\infty}(x) \approx \Lambda x^{m_1 \varrho - 1}. \quad (20)$$

Using (20) and following the procedure outlined in Appendix B, we obtain (19).

Summarizing the results presented in (18) and (19), the asymptotic outage probability is obtained as

$$P_{\text{out}}^{\infty} = \begin{cases} P_{\text{out}}^{\dagger\infty}, & 0 \leq \rho < 1 \\ P_{\text{out}}^{\ddagger\infty}, & \rho = 1. \end{cases} \quad (21)$$

The asymptotic result in (21) offers several valuable insights into the network behavior in the high SNR regime, which are presented as follows:

1. The diversity order is $\min(m_1, m_2)$ for outdated CSI. Notably, this diversity order is only determined by Nakagami- m fading parameters but independent of M and N . This result is not surprising since user selection cannot correctly select the N -th best user when the CSI of the MU-R link is outdated. As such, the benefits of user selection cannot be exploited and the diversity order of the MU-relay link is m_1 , which is not affected by M or N .
2. The diversity order is $\min(m_1(M - N + 1), m_2)$ for perfect CSI. This diversity order indicates that when $\rho = 1$, the impact of M and N on the diversity order is reflected via $(M - N + 1)$ in the MU-R link.
3. The contribution of the MU_N-R link on the array gain is reflected via Λ/m_1 in outdated CSI and $\Delta/m_1(M - N + 1)$ in perfect CSI. Moreover, the contribution of the R-BS link on the array gain is reflected via $\xi_2^{m_2}/m_2!$.

4. Numerical and Simulation Results

Numerical results are presented in this section to examine the impact of M , N , and ρ on the outage probability of multiuser TWRNs. Throughout this section, we normalize the channel gains to unity such that $E[|h_{SR_N}|^2] = E[|g_{RD}|^2] = 1$. Without the loss of generality, we set the individual transmit power as $P_1 = P_2 = 0.2P_S$, $P_R = 0.6P_S$, where P_S is the total transmit power. The SNR threshold is set to be $\gamma_{\text{th}} = 0$ dB. It is observed from Figs. 1, 2 and 3 that our lower bounds on the outage probability obtained from (15) and (16) match precisely with Monte Carlo simulations, especially in the medium and high SNR regimes. In addition, the asymptotic curves obtained from (18) and (19) accurately predict the diversity order and the array gain at high SNRs.

Figure 1 plots the outage probability versus P_S for different values of ρ . In Fig. 1, we observe that the outage probability deteriorates with smaller ρ . For example, we observe that $\rho = 0.75$ outperforms $\rho = 0.25$ by approximately 6.2 dB at the outage probability of 10^{-3} . We also observe that the diversity order remains unchanged when $0 \leq \rho < 1$, as predicted by our asymptotic result in (18). We further observe that the diversity order increases to 3 when $\rho = 1$. This is due to the fact that the diversity order is $\min(m_1(M - N + 1), m_2)$ for perfect CSI.

Figure 2 plots the outage probability versus P_S for different values of ρ and N . Figure 2 clearly indicates that the outage probability improves as N decreases for a given ρ . As expected, the figure shows that the diversity order does not increase with N when $0 \leq \rho < 1$. Moreover, for perfect CSI, the diversity order increases from $N = 4$ to $N = 3$ and does not increase for $N = 3$ to $N = 2$. This is due to the fact that the diversity order is determined by $\min(m_1(M - N + 1), m_2)$, as indicated by (19).

Fig. 1 Outage probability for different values of ρ with $m_1 = 1$, $m_2 = 3$, $M = 3$, and $N = 1$.

Fig. 3 Outage probability versus P_S for different values of ρ and M with $m_1 = 1$, $m_2 = 3$, and $N = 2$.

Fig. 2 Outage probability versus P_S for different values of ρ and N with $m_1 = 1$, $m_2 = 3$, and $M = 5$.

Figure 3 plots the outage probability versus P_S for different values of ρ and M . It is evident from Fig. 3 that a decreasing outage probability is brought by an increasing M for a given ρ . For example, when $\rho = 1$ and $P_S = 20$ dB, increasing M from 3 to 5 decreases the outage probability by a factor of 10. As expected, the figure shows that the diversity order does not increase with M when $0 \leq \rho < 1$. Moreover, the diversity order increases from $M = 3$ to $M = 4$ and does not increase for $M = 4$ to $M = 5$ for perfect CSI, which is due to the diversity order of $\min(m_1(M - N + 1), m_2)$.

5. Conclusion

We examined the impact of the imperfectness of CSI on the outage probability of the multiuser TWRN with the N -th

best user selection over Nakagami- m fading channels. We first derived the new and tight lower bounds on the outage probability. To gain further insights, we derived compact expressions for the asymptotic outage probability at high SNRs. We proved that in the presence of perfect CSI, the diversity order of the network is $\min(m_1(M - N + 1), m_2)$. However, in the presence of outdated CSI, the diversity order reduces to $\min(m_1, m_2)$. Numerical results were provided to demonstrate the accuracy of our analysis.

References

- [1] R. Zhang, L. Wang, G. Parr, O.G. Aliu, B. Awoseyila, N. Azarmi, S. Bhatti, E. Bodanese, H. Chen, M. Dianati, A. Dutta, M. Fitch, K. Giridhar, S. Hailes, K.V.S. Hari, M.A. Imran, A.K. Jagannatham, A. Karandikar, S. Kawade, M.Z. Ali Khan, S.C. Kompalli, P. Langdon, B. Narayanan, A. Mauthe, J. McGeehan, N. Mehta, K. Millet, K. Moessner, R. Rajashekar, B. Ramkumar, V. Ribeiro, K. Vasudevan, L. Hanzo, and J. Bigham, "Advances in base- and mobile-station aided cooperative wireless communications: An overview," *IEEE Veh. Technol. Mag.*, vol.8, no.1, pp.57–69, March 2013.
- [2] Z. Sheng, K.K. Leung, and Z. Ding, "Cooperative wireless networks: From radio to network protocol designs," *IEEE Commun. Mag.*, vol.49, no.5, pp.64–69, May 2011.
- [3] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Sel. Areas Commun.*, vol.25, no.2, pp.379–389, Feb. 2007.
- [4] J. Tian, Q. Zhang, and F. Yu, "Non-coherent detection for two-way AF cooperative communications in fast Rayleigh fading channels," *IEEE Trans. Commun.*, vol.59, no.10, pp.2753–2762, Oct. 2011.
- [5] D. Benevides da Costa, H. Ding, M.D. Yacoub, and J. Ge, "Two-way relaying in interference-limited AF cooperative networks over Nakagami- m fading," *IEEE Trans. Veh. Technol.*, vol.61, no.8, pp.3766–3771, Dec. 2012.
- [6] H. Guo, J. Ge, and H. Ding, "Symbol error probability of two-way amplify-and-forward relaying," *IEEE Commun. Lett.*, vol.15, no.1, pp.22–24, Jan. 2011.
- [7] V.T. Muralidharan and B.S. Rajan, "Performance analysis of adaptive physical layer network coding for wireless two-way relaying," *IEEE Trans. Wireless Commun.*, vol.12, no.3, pp.1328–1339, March 2013.

- [8] L. Yang, M.-S. Alouini, and K. Qaraqe, "On the performance of spectrum sharing systems with two-way relaying and multiuser diversity," *IEEE Commun. Lett.*, vol.6, no.8, pp.1240–1243, Aug. 2012.
- [9] S.S. Ikki and M.H. Ahmed, "On the performance of amplify-and-forward cooperative diversity with the N -th best-relay selection scheme," *Proc. IEEE ICC 2009*, pp.1–6, Dresden, Germany, June 2009.
- [10] Y.-U. Jang and Y.H. Lee, "Performance analysis of user selection for multiuser two-way amplify-and-forward relay," *IEEE Commun. Lett.*, vol.14, no.11, pp.1086–1088, Nov. 2010.
- [11] H. Ding, J. Ge, D.B. da Costa, and Y. Guo, "Outage analysis for multiuser two-way relaying in mixed Rayleigh and Rician fading," *IEEE Commun. Lett.*, vol.15, no.4, pp.410–412, April 2011.
- [12] M. Torabi and D. Haccoun, "Capacity analysis of opportunistic relay-ing in cooperative systems with outdated channel information," *IEEE Commun. Lett.*, vol.14, no.12, pp.1137–1139, Dec. 2010.
- [13] J.L. Vicario, A. Bel, J.A. Lopez-Salcedo, and G. Seco, "Opportunistic relay selection with outdated CSI: Outage probability and diversity analysis," *IEEE Trans. Wireless Commun.*, vol.8, no.6, pp.2872–2876, June 2009.
- [14] L. Fan, X. Lei, P. Fan, and R.Q. Hu, "Outage probability analysis and power allocation for two-way relay networks with user selection and outdated channel state information," *IEEE Commun. Lett.*, vol.16, no.5, pp.638–641, March 2012.
- [15] N. Yang, M. El-kashlan, and J. Yuan, "Outage probability of multiuser relay networks in Nakagami- m fading channels," *IEEE Trans. Veh. Technol.*, vol.59, no.5, pp.2120–2132, June 2010.
- [16] K. Yang, N. Yang, C. Xing, and J. Wu, "Relay antenna selection in MIMO two-way relay networks over Nakagami- m fading channels," *IEEE Trans. Veh. Technol.*, vol.63, no.5, pp.2349–2362, June 2014.
- [17] M. Chen, T.C.-K. Liu, and X. Dong, "Opportunistic multiple relay selection with outdated channel state information," *IEEE Trans. Veh. Technol.*, vol.61, no.3, pp.1333–1345, March 2012.
- [18] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series and Products*, 7th ed., Academic, San Diego, C.A., 2007.
- [19] J.L. Vicario and C.A. Haro, "Analytical assessment of multi-user vs. spatial diversity trade-offs with delayed channel state information," *IEEE Commun. Lett.*, vol.10, no.8, pp.588–590, Aug. 2006.

Appendix A: Proof of (15)

Applying the inequality of $\frac{xy}{x+y} < \min(x, y)$ for positive values of x and y , we obtain the upper bounds on γ_1 and γ_2 as

$$\gamma_1 < \frac{\alpha\gamma_{SR}\gamma_{DR}}{(1+\alpha)\gamma_{SR} + \gamma_{DR}} < P_R \min\left(\frac{\gamma_{SR}}{P_1}, \frac{\gamma_{DR}}{P_1 + P_R}\right) \quad (\text{A} \cdot 1)$$

and

$$\gamma_2 < \frac{\beta\gamma_{SR}\gamma_{DR}}{\gamma_{SR} + (1+\beta)\gamma_{DR}} < P_R \min\left(\frac{\gamma_{DR}}{P_2}, \frac{\gamma_{SR}}{P_2 + P_R}\right), \quad (\text{A} \cdot 2)$$

respectively. Substituting (A·1) and (A·2) into (6), we re-express the outage probability as

$$P_{\text{out}} = 1 - \Pr\left[\min(\eta_2\gamma_{SR}, \eta_1\gamma_{RD}) \geq \frac{\gamma_{\text{th}}}{P_R}\right]. \quad (\text{A} \cdot 3)$$

We then compute $\Pr\left[\min(\eta_2\gamma_{SR}, \eta_1\gamma_{RD}) \geq \frac{\gamma_{\text{th}}}{P_R}\right]$ as

$$\begin{aligned} & \Pr\left[\min(\eta_2\gamma_{SR}, \eta_1\gamma_{RD}) \geq \frac{\gamma_{\text{th}}}{P_R}\right] \\ &= \Pr\left[\eta_2\gamma_{SR} \geq \frac{\gamma_{\text{th}}}{P_R} \mid \eta_2\gamma_{SR} < \eta_1\gamma_{RD}\right] \\ &+ \Pr\left[\eta_1\gamma_{RD} \geq \frac{\gamma_{\text{th}}}{P_R} \mid \eta_1\gamma_{RD} < \eta_2\gamma_{SR}\right] \\ &= \int_{\frac{\gamma_{\text{th}}}{\eta_2 P_R}}^{\infty} \int_{\frac{\eta_2 \gamma_{SR}}{\eta_1}}^{\infty} f_{\gamma_{RD}}(x) dx f_{\gamma_{SR}}(\gamma_{SR}) d\gamma_{SR} \\ &+ \int_{\frac{\gamma_{\text{th}}}{\eta_1 P_R}}^{\infty} \int_{\frac{\eta_1 \gamma_{RD}}{\eta_2}}^{\infty} f_{\gamma_{SR}}(x) dx f_{\gamma_{RD}}(\gamma_{RD}) d\gamma_{RD}. \end{aligned} \quad (\text{A} \cdot 4)$$

Substituting (12) and (14) into (A·4), and using [18, Eq. (3.351.2)] to calculate the resultant integral, the desired lower bound on the outage probability, given in (15), is derived.

Appendix B: Proof of (18)

Applying [18, Eq. (1.211.1)] to expand the exponential function in $f_{\gamma_{SR}}(x)$ and $f_{\gamma_{RD}}(x)$, we retain the first order item of $f_{\gamma_{SR}}(x)$ and $f_{\gamma_{RD}}(x)$ and discard the high order items. As such, we obtain the first order expansion of $f_{\gamma_{SR}}(x)$ for $0 \leq \rho < 1$ and the first order expansion of $f_{\gamma_{RD}}(x)$ as

$$f_{\gamma_{SR}}^{\dagger\infty}(x) \approx \Delta x^{m_1-1} \quad (\text{A} \cdot 5)$$

and

$$f_{\gamma_{RD}}^{\infty}(x) \approx \frac{\xi_2^{m_2} x^{m_2-1}}{\Gamma(m_2)}, \quad (\text{A} \cdot 6)$$

respectively. Using (A·5) and (A·6), we obtain the first order expansion of the CDF of γ_{SR} for $0 \leq \rho < 1$ as

$$F_{\gamma_{SR}}^{\dagger\infty}(x) \approx \Delta \frac{x^{m_1}}{m_1}, \quad (\text{A} \cdot 7)$$

and obtain the first order expansion of the CDF of γ_{RD} as

$$F_{\gamma_{RD}}^{\infty}(x) \approx \frac{(\xi_2 x)^{m_2}}{\Gamma(m_2 + 1)}. \quad (\text{A} \cdot 8)$$

The first order expansion of $\Pr\left[\min(\eta_2\gamma_{SR}, \eta_1\gamma_{RD}) \geq \frac{\gamma_{\text{th}}}{P_R}\right]$ can be expressed as

$$\begin{aligned} & \Pr\left[\min(\eta_2\gamma_{SR}, \eta_1\gamma_{RD}) \geq \frac{\gamma_{\text{th}}}{P_R}\right] \\ &= \int_{\frac{\gamma_{\text{th}}}{\eta_2 P_R}}^{\infty} \left(1 - F_{\gamma_{RD}}^{\infty}\left(\frac{\eta_2 \gamma_{SR}}{\eta_1}\right)\right) f_{\gamma_{SR}}^{\dagger\infty}(\gamma_{SR}) d\gamma_{SR} \\ &+ \int_{\frac{\gamma_{\text{th}}}{\eta_1 P_R}}^{\infty} \left(1 - F_{\gamma_{SR}}^{\dagger\infty}\left(\frac{\eta_1 \gamma_{RD}}{\eta_2}\right)\right) f_{\gamma_{RD}}^{\infty}(\gamma_{RD}) d\gamma_{RD}. \end{aligned} \quad (\text{A} \cdot 9)$$

Substituting (A·5), (A·6), (A·7), and (A·8) into (A·9), we re-express (A·9) as a single polynomial term given by

$$\Pr\left[\min(\eta_2\gamma_{SR}, \eta_1\gamma_{RD}) \geq \frac{\gamma_{\text{th}}}{P_R}\right]$$

$$\approx \begin{cases} 1 - \frac{\Delta}{m_1} \vartheta_1^{m_1}, & m_1 < m_2 \\ 1 - \frac{\Delta}{m} \vartheta_1^m - \frac{\xi_2^m}{m!} \vartheta_2^m, & m_1 = m_2 = m \\ 1 - \frac{\xi_2^{m_2}}{m_2!} \vartheta_2^{m_2}, & m_1 > m_2. \end{cases} \quad (\text{A} \cdot 10)$$

Substituting (A·10) into (A·3), we obtain the desired asymptotic outage probability given in (18), which completes the proof.