Interstellar extinction curve variations towards the inner Milky Way: a challenge to observational cosmology

David M. Nataf, 1, 2★ Oscar A. Gonzalez, 3 Luca Casagrande, 1, 2 Gail Zasowski, 2, 4† Christopher Wegg, 5 Christian Wolf, 1 Andrea Kunder, 6 Javier Alonso-García, 7, 8 Dante Minniti, 8, 9, 10 Marina Rejkuba, 11, 12 Roberto K. Saito, 13 Elena Valenti, 11 Manuela Zoccali, 11, 14 Radosław Poleski, 15, 16 Grzegorz Pietrzyński, 15, 17 Jan Skowron, 15 Igor Soszyński, 15 Michał K. Szymański, 15 Andrzej Udalski, 15 Krzysztof Ulaczyk, 15, 18 and Łukasz Wyrzykowski 15

Affiliations are listed at the end of the paper

Accepted 2015 December 2. Received 2015 November 30; in original form 2015 July 26

ABSTRACT

We investigate interstellar extinction curve variations towards ∼4 deg$^2$ of the inner Milky Way in VIK$_{\text{S}}$ photometry from the OGLE-III (third phase of the Optical Gravitational Lensing Experiment) and VVV (VISTA Variables in the Via Lactea) surveys, with supporting evidence from diffuse interstellar bands and F435W, F625W photometry. We obtain independent measurements towards ∼2000 sightlines of $A_I$, $E(V - I)$, $E(I - J)$ and $E(J - K_S)$, with median precision and accuracy of 2 per cent. We find that the variations in the extinction ratios $A_I/E(V - I)$, $E(I - J)/E(V - I)$ and $E(J - K_S)/E(V - I)$ are large (exceeding 20 per cent), significant and positively correlated, as expected. However, both the mean values and the trends in these extinction ratios are drastically shifted from the predictions of Cardelli and Fitzpatrick, regardless of how $R_V$ is varied. Furthermore, we demonstrate that variations in the shape of the extinction curve have at least two degrees of freedom, and not one (e.g. $R_V$), which we confirm with a principal component analysis. We derive a median value of $\langle AV/\text{AKs} \rangle = 13.44$, which is ∼60 per cent higher than the ‘standard’ value. We show that the Wesenheit magnitude $W_I = I - 1.61(I - J)$ is relatively impervious to extinction curve variations. Given that these extinction curves are linchpins of observational cosmology, and that it is generally assumed that $R_V$ variations correctly capture variations in the extinction curve, we argue that systematic errors in the distance ladder from studies of Type Ia supernovae and Cepheids may have been underestimated. Moreover, the reddening maps from the Planck experiment are shown to systematically overestimate dust extinction by ∼100 per cent and lack sensitivity to extinction curve variations.

Key words: dust, extinction – ISM: lines and bands.

1 INTRODUCTION

1.1 Interstellar extinction curve towards the inner milky way

The interstellar extinction curve towards the inner Milky Way has long been argued to be ‘non-standard’. This was first suggested from observations of planetary nebulae. Stasińska et al. (1992) analysed data on the Balmer decrement and the ratio of radio to H$\beta$, and argued for an extinction curve that declines more rapidly with increasing wavelength (i.e. steeper) than the $R_V = A_V/E(B - V) = 3.1$ extinction curve, either $R_V = 2.0$ or $R_V = 2.7$ depending on the choice of parametrization, a conclusion supported by Tylenda et al. (1992) using a similar method. Pottasch & Zijlstra (1994) confirm a systematic difference in extinction derived from the radio/H$\beta$ flux ratio and the Balmer decrement using additional measurements. Ruffle et al. (2004) obtain additional measurements for a set of lines ([O III], H$\alpha$, etc.) and estimate a mean extinction curve towards the inner Galaxy of $(R_V) = 2.0$.

Separately, and independently, the anomalous dereddened colours of standard crayons in and near the Galactic bulge such as...
RR Lyrae stars have also suggested a non-standard extinction curve (Stutz, Popowski & Gould 1999). The evidence has since accumulated from measurements of magnitude–colour slope of red clump (RC) centroids in optical photometry (Udalski 2003b; Natal et al. 2013b, following the work of Stanek 1996 and Woźniak & Stanek 1996) from the Optical Gravitational Lensing Experiment (OGLE), in optical Hubble Space Telescope (HST) photometry (Revnivtsev et al. 2010), in ground-based near-infrared (IR) photometry (Nishiyama et al. 2009), with a combination of ground-based near-IR and space-based mid-IR photometry from the Spitzer Space Telescope (Zasowski et al. 2009), in optical photometry of RR Lyrae stars (Pietrukowicz et al. 2012, 2015), with photometry of individual red giant stars in near-IR photometry (Gosling, Bandyopadhyay & Blundell 2009), and from line-emission ratios towards the Galactic Centre (Fritz et al. 2011). For dissenting analyses, see Kundu et al. (2008) and Pottasch & Bernard-Salas (2013).

The mean shape and variation of the extinction curve towards the bulge are of concern to the fields of microlensing (e.g. Bachele et al. 2012; Henderson et al. 2014; Yee et al. 2015), extremely-metal-poor stars (Howes et al. 2014, 2015), Galactic globular clusters (Massari et al. 2012; Saracino et al. 2015) and Galactic structure (Cao et al. 2013; Wegg, Gerhard & Portail 2015), among others. Minniti et al. (2014) argued that a large fraction of the interstellar dust towards the bulge is located in a single ‘great dark lane’. The level of interest and controversy has thus been amply justified and driven by the research needs.

Natal et al. (2013b) followed up on the issue of extinction curve variations by combining measurements of the optical reddening $E(V-I)$ from $\sim100$ deg$^2$ of photometry from OGLE-III with the corresponding measurements of the near-IR reddening $E(J-K_s)$ from the VISTA Variables in the Via Lactea (VVV) survey (Gonzalez et al. 2012). The mean extinction ratios measured were $A_V/E(V-I) \approx 1.22$ and $E(J-K_s)/E(V-I) \approx 0.34$, both with statistically significant $1\sigma$ scatter of $\sim10$ per cent and both approximately consistent in the mean with the $R_V = 2.5$ interstellar extinction curve from Cardelli, Clayton & Mathis (1989), but not consistent with the mean value of $A_V/E(V-I) = 1.44$ measured towards the Large Magellanic Cloud (LMC; Udalski 2003b). One of the results from Natal et al. (2013b), that the optical extinction could be parameterized as $A_V = 0.7465E(V-I) + 1.3700E(J-K_s)$, has since been confirmed by Pietrukowicz et al. (2015) and also used by Kundu et al. (2015), in their studies of bulge RR Lyrae stars.

A legitimate concern that one can level against the findings of Natal et al. (2013b) is that the argument is dependent on assuming that the ‘standard’ extinction curve is that of Cardelli et al. (1989). In contrast, both Fitzpatrick (1999) and Fitzpatrick & Massa (2007) find substantially steeper mean Galactic extinction curves for $\lambda \gtrsim 6000$ Å even if one fixes $R_V \approx 3.1$. Indeed, the conclusion of Kundu et al. (2008) that the interstellar extinction towards the bulge is standard uses the curve of Fitzpatrick (1999) to anchor what ‘standard’ means – if Kundu et al. (2008) had relied upon the analysis of Cardelli et al. (1989), their conclusion would have been reversed. Schlafly & Finkbeiner (2011) have recently demonstrated that the $R_V = 3.1$ curve from Fitzpatrick (1999) yields a much better fit to optical photometry of main-sequence turnoff stars in the northern Galactic halo than that of Cardelli et al. (1989), a conclusion that Wolf (2014) confirmed using quasi-stellar objects (QSOs) as standard crayons. Babusiaux et al. (2014) recently derived arguably self-consistent distances to Galactic bulge stars by assuming the extinction curve of Fitzpatrick & Massa (2007).

This continuing discrepancy is thus inevitable due to the large number of degrees of freedom. When one can vary not just the parameter $R_V$ but also the choice of parametrization (Cardelli et al. 1989; Fitzpatrick 1999, etc), one has a lot of flexibility with which to fit extinction data. A compelling option with which to resolve this discrepancy is to acquire more photometry and thus more colours, and that is what we do in this investigation. Similarly to Natal et al. (2013b), our combination of OGLE-III and VVV photometry allows us to measure $A_V/E(V-I)$ and $E(U-K_s)/E(V-I)$. What we also do, by matching sources between the two catalogues, is measure the reddening ratio $E(U-J)/E(V-I)$, for which the different parametrizations and different values of $R_V$ yield specific and distinct predictions.

The structure of this paper is as follows. In Section 1.2, we discuss the relevance of the inner Milky Way extinction curve to cosmology. In Section 2, we describe the raw data used in this investigation. We explain our methodology in Section 3, the expectations from theory are stated in Section 4, present our results in Section 5, we compare our results to select other investigations in Section 6, and our conclusions in Section 7.

### 1.2 Extinction towards Type Ia supernovae and extragalactic Cepheids

Natal et al. (2013b) argued that there may be a link between the extinction curve variations observed towards the inner Milky Way and observations of Type Ia supernovae (SN Ia), towards which low values of $R_V$ are common (e.g. Chotard et al. 2011; Goobar et al. 2014). The Carnegie Supernova Project (Burns et al. 2014) has measured densely sampled light curves in 8–10 bandpasses covering the optical and near-IR wavelength regime, and confirm steeper-than-standard extinction curves as common – they report a mean $R_V = 2.15 \pm 0.16$ towards their sample. Similarly, Rigault et al. (2015) find that the distance dispersion towards SNe Ia is minimized if they assume $R_V = 1.7$ as the mean of their sample, though Jones, Riess & Scolnic (2015) have demonstrated that this result may emerge from the assumption that star-forming and quiescent galaxies have identical interstellar extinction curves. Phillips et al. (2013) have shown that this extinction is likely interstellar rather than circumstellar extinction, as the ratio of the equivalent width of the diffuse interstellar band (DIB) at 5780 Å to the inferred extinction in the V band is consistent with the value for the Milky Way interstellar medium (ISM). Burns et al. (2014) found that many of the SNe Ia with low values of $R_V$ are also high-velocity events. In turn, Wang et al. (2013) argue that high-velocity SNe Ia are substantially more concentrated in the inner and brighter regions of host galaxies. Thus, the findings of a steeper extinction curve towards the inner Milky Way (also recently demonstrated for the Andromeda galaxy; see Dong et al. 2014) may be consistent with what is found from SN Ia host galaxies.

These findings continue to be controversial, as the $R_V$ values seemingly ubiquitous towards SNe Ia are believed to be rare in the Milky Way, and some have argued that they emerge due to a degeneracy between extinction curve variations and intrinsic colour variations of SNe Ia (Scolnic et al. 2014). As manifestly plausible as the argument of colour variations clearly is, we suggest that some of the discrepancy is due to the following two reasons.

(i) There is a greater range of extinction curve variations within the Milky Way than commonly believed. In particular, $R_V = 3.1$ may simply be the most common value for the solar neighbourhood, rather than the sharply peaked mode for the diffuse ISM throughout the Galaxy.

MNRA456, 2692–2706 (2016)
The anonymous referee notes that the investigation of Fitzpatrick & Massa (2007) already addressed some of these issues. We quote directly from the referee:

“Fitzpatrick & Massa (2007) have demonstrated that interstellar extinction is, in general, far more complex than implied by the CCM [re: Cardelli et al. 1989 relations and that the apparent CCM relations are largely the result of correlated errors. Even CCM never intended the relationships to be considered a ‘law’, but rather a means to account for a large component of the observed variation. Further, Fitzpatrick & Massa (2009) have shown that optical-NIR extinction curves in the local ISM require at least 2 parameters to explain the observed variations.” – The anonymous referee.

Uncertainties in the mean value of the interstellar extinction curve are also emerging as an issue in the determination of the Cepheid distance scale, which anchors Hubble’s constant $H_0$. Altavilla et al. (2004) estimate $R_V = 2.5$ as the best-fitting extinction curve parameter towards an archival sample of extragalactic Cepheid light curves. Fausnaugh et al. (2015) used $BVRJ$ photometry of Cepheids in the maser-host galaxy NGC 4258, and found that $R_V = 4.9$ provided the best fit. Nataf (2015) recently showed that the extinction towards the Cepheids in M101 (the Pinwheel galaxy) was better characterized by $A_I/E(V - I) \approx 1.15$ rather than the canonical value of $A_I/E(V - I) \approx 1.47$ (Cardelli et al. 1989). The situation is such that Riess et al. (2011) and Riess, Fili & Valls-Gabaud (2012) have shifted to using $H$-band observations of Cepheids to infer distances, for which the extinction is believed to be a smaller uncertainty. However, this comes at the cost of a broader point spread function (PSF; since $HST$ photometry is nearly diffraction-limited) and thus greater blending in the $H$ band, in addition to greater relative flux contributions from colder red giant branch and asymptotic giant branch stars.

In light of these factors, we consider plausible the idea that better characterization of the extinction curve towards the inner Milky Way – an independently interesting and tractable scientific endeavour – may facilitate superior understanding of the extinction towards both Cepheids and SNe Ia. Though the inner Milky Way may be dismissed as less than 2 per cent of the sky in surface area, it represents no less than ~25 per cent of the stellar mass of the Milky Way (Dwek et al. 1995; Nataf et al. 2013b; Portail et al. 2015), a number which accounts for the bulge alone and does not include the inner disc in which this dust is likely located. Indeed, from Bovy & Rix (2013), we can estimate that the Milky Way’s disc contains three times as much stellar mass in the Galactocentric range $4 \leq R_{GC} \leq 8$ as it does in the outer disc $8 \leq R_{GC} \leq 20$. Thus, the dust extinction work in this study may be that which characterizes how the Milky Way would appear to outside observers, much more so than the dust extinction curve of the solar neighbourhood, and is thus pertinent to better interpreting extragalactic stellar populations. We note of a recent pre-print posted on astro-ph, which measured a mean extinction curve parameter $R_V = 2.4$ towards a sample of 16 intermediate-redshift quasars.

We are aware of uncertainties in the 3D distribution of dust. Neckel, Klar & Sarcander (1980) report $2.6 \leq A_V \leq 3.3$ towards the bulk of our sightlines within 1 kpc of the Sun, which would contribute most of the extinction measured in our work and thus contradict the statement above concerning the Galactic distribution of dust. On the other hand, Schultheis et al. (2014) find an extinction excess towards the Galactic centre located ~6 kpc from the Sun, which Minniti et al. (2014) interpret as a ‘Great Dark Lane’ for the Milky Way. Regardless of the details of how the dust is distributed, we expect the integrated sum of extinction to span a range of Galactocentric radii.

2 DATA

OGLE-III, the third phase of the Optical Gravitational Lensing Experiment, produced photometric maps of the Galactic bulge, parts of the Galactic disc, and the Magellanic Clouds. Observations were taken with the 1.3 m Warsaw Telescope, located at the Las Campanas Observatory. The camera had eight $2048 \times 4096$ detectors, with a scale of approximately 0.26 arcsec pixel$^{-1}$ yielding a combined field of view $0.6 \times 0.6$. More detailed descriptions of the instrumentation, photometric reductions and astrometric calibrations are available in Udalski (2003a), Udalski et al. (2008) and Szymański et al. (2011). The photometry is in the $V$ optical filters as calibrated by Landolt standard stars (Landolt 1992). OGLE-III photometry and reddening maps are available for download from the OGLE webpage.1 The fourth phase of the OGLE project (OGLE-IV; Udalski, Szymański & Szymański 2015) has actually been underway since 2010, with photometric coverage of about 2100 square degrees of the Galactic bulge and disc. However, we use OGLE-III photometry in this study for consistency with prior investigations.

The VVV ESO public survey (Minniti et al. 2010) is a near-IR photometric survey covering 560 deg$^2$ of the Galactic bulge and southern disc. Observations were carried with the VISTA Infrared Camera (VIRCAM; Dalton et al. 2006; Emerson & Sutherland 2010), mounted at the 4.1 m telescope VISTA (Visible and Infrared Survey Telescope for Astronomy) telescope (Sutherland et al. 2015), located in its own peak at the ESO Cerro Paranal Observatory in Chile. VIRCAM has a mosaic of 16 Raytheon VIRGO $2048 \times 2048$ pixel detectors, with a scale of 0.339 arcsec pixel$^{-1}$. VVV observations use the stack of two slightly differed images to produce a stacked image known as a ‘pawprint’. A sequence of six offset ‘pawprints’ is used to cover the gaps between the detectors to produce a full, nearly uniform sky coverage of 1.50 deg$^2$ known as a ‘tile’. Images are reduced, astrometrized and stacked at the Cambridge Astronomy Survey Unit (CASU) using the VISTA Data Flow System pipeline (Emerson et al. 2004; Hambly et al. 2004; Irwin et al. 2004). VVV photometric catalogues at the ‘tile’ level are also produced at CASU. A detailed description of these VVV catalogues can be found in Saito et al. (2012). For this work, we use catalogues obtained using PSF photometry based on DOPHOT (Schechter, Mateo & Saha 1993; Alonso-García et al. 2012) measurements from pawprint-stacked $J$- and $K_s$-band images. CASU photometric catalogues are then used to calibrate the PSF photometry. Final magnitudes are therefore based on the VISTA photometric system. Details on the construction of VVV PSF catalogues can be found in Alonso-García et al. (2015). IR $E(J - K_s)$ reddening maps measured with VVV photometry can be found on the online BEAM calculator.2 We use observations from a subset of OGLE-III fields that were selected such that our study would span the range of reddening curve parameters $E(J - K_s) / E(V - I)$ towards the bulge, as measured by Nataf et al. (2013b). The photometric coverage used in this work is

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1 http://ogle.astrouw.edu.pl/
and do not transform to 2MASS (Skrutskie et al. 2006). The total we leave the near-IR photometry in the original VISTA filter system 2004; Cordier et al. 2007) for a 12-Gyr-old, [Fe/H] ≤ 0.35 and +0.40 yields a derivative of d(V − I)$_{RC, 0}$ ≈ 0.29 mag dex$^{-1}$ (Nataf et al. 2014, table 1), with the effect of variations in age or [α/Fe] being negligible. However, the total range in the mean metallicity across our fields is no greater than +0.10 dex (Gonzalez et al. 2013, see also Rich, Origlia & Valenti 2012) and thus the effect is negligible. This emerges due to our choice of sightlines, which are relatively similar in direction, do not span the whole bulge and thus do not probe a significant spread in mean metallicity.

We use model atmospheres to estimate the intrinsic colours of the bulge RC in the full range of filters used in this work, as well as others that may be of interest to future studies, which we show in Table 1. The atmospheric parameters log g = 2.2 and [Fe/H] = 0 are typical of the RC (Ness et al. 2013), and $T_{\text{eff}} = 4650$ K is chosen to agree with the intrinsic colour (V − I)$_{RC, 0}$ = 1.06. The remaining synthetic colours were computed interpolating over a grid of MARCS model atmosphere (Gustafsson et al. 2008) at the $T_{\text{eff}}$, log g and [Fe/H] quoted above, and appropriate filter transmission curves (see details in Casagrande & VandenBerg 2014).

Synthetic optical and 2MASS $JHK_s$ magnitudes were transformed into the VISTA system.$^3$ The remaining colours emerge from a

3 See http://casu.ast.cam.ac.uk/surveys-projects/vista/technical/photometric-properties for more information on the VISTA photometric system.

Figure 1. Subset of OGLE-III subfields shown in red overlaid on an optical image of the Galactic bulge with Galactic coordinate system shown as well. The subfields used in this work, for which we also use the matching VVV photometry, are shown in green.

3 METHODOLOGY

3.1 Photometric zero-points of the RC

The mean intrinsic (dereddened) colour of the Galactic bulge RC is measured (Bensby et al. 2013) to be

$$(V − I)_{RC, 0} = 1.06.$$ (1)

That is the value adopted by our investigation as the zero-point; it is derived by equating the photometric colours of the main-sequence turnoff and subgiant branch stars studied by Bensby et al. (2013) with their spectroscopic temperatures, as derived from high-signal-to-noise, high-resolution spectra. This derivation is consistent with several other determinations.

(i) The prediction from the BaSTI isochrones (Pietrinferni et al. 2004; Cordier et al. 2007) for a 12-Gyr-old, [Fe/H] = 0 stellar population is (V − I)$_{RC, 0}$ = 1.06 (Nataf, Cassisi & Athanassoula 2014, table 1).

(ii) Based on empirically calibrated population parameters (Thompson et al. 2010; Brogaard et al. 2011, 2012; Bragaglia et al. 2014; Cunha et al. 2015) and observed photometry (Stetson, Bruntt & Grundahl 2003; Sarajedini et al. 2007) from 47 Tuc and NGC 6791, and assuming Baade’s window metallicity distribution function (Hill et al. 2011), the Galactic bulge RC value is (V − I)$_{RC, 0}$ ≈ 1.07. This is marginally lower than that estimated by Nataf et al. (2013b) using the same method due to the increased best-fitting metallicity for NGC 6791.

(iii) The Galactic bulge RC is 0.55 mag redder than Galactic bulge ab-type RR Lyrae (Nataf et al. 2013b), where throughout this discussion we only refer to ab-type RR Lyrae. The Fourier coefficients of the RR light curves yield a mean intrinsic RR Lyrae colour of (V − I)$_{RR}$ = 0.49 (Pietrukowicz et al. 2015), for an estimated mean intrinsic RC colour of (V − I)$_{RC, 0}$ = 1.04. Alternatively, we have selected 2301 RR Lyrae from OGLE-III with four or more V-band observations near minimum light (phase 0.50 ≤ φ ≤ 0.78), and contrasted them to the nearest RC reddening measured by Nataf et al. (2013b). We kept the data points where the reddening agreed to be better than 0.20 mag in (E(V − I)) to remove spurious outliers, and for which (V − I)$_{RC}$ ≤ 3.30, leaving 1987 RR Lyrae. We find that the ratio of reddening agrees to better than 1 per cent in the mean, and that the difference in reddening is 0.02 mag when we regress the offset to (V − I)$_{RC, 0}$ = 0 − exactly equivalent to the above inferred value of (V − I)$_{RC, 0}$ = 1.04.

(iv) The near-IR colour (J − K)$_{RC, 0}$ = 0.68 was derived by Gonzalez et al. (2011), based on a determination of (E(B − V) = 0.55 towards Baade’s window by Zoccali et al. (2008). Applying the conversions from Table 1, which is explained below, this corresponds to (V − I)$_{RC, 0}$ ≈ 1.07.

(v) The reddening measurements of Gonzalez et al. (2011) were in turn tested by comparing spectroscopic temperatures and photometric temperatures (Rojas-Arriagada et al. 2014). The discrepancy in the zero-point is measured to be no greater than ΔE(J − K) = −0.006 ± 0.026 – consistent with zero.

Our assumed mean colour (V − I)$_{RC, 0}$ = 1.06 ± 0.03 is thus well supported by a diverse array of inferential methods, where the 0.03 mag is a conservative estimate of the 1σ error from the arguments presented above. A possible concern is that the effect of metallicity variations can be significant: interpolating between the BaSTI-predicted values for [Fe/H] = −0.35 and +0.40 yields a derivative of d(V − I)$_{RC, 0}$/d[Fe/H] ≈ 0.29 mag dex$^{-1}$ (Nataf et al. 2014, table 1), with the effect of variations in age or [α/Fe] being negligible. However, the total range in the mean metallicity across our fields is no greater than +0.10 dex (Gonzalez et al. 2013, see also Rich, Origlia & Valenti 2012) and thus the effect is negligible. This emerges due to our choice of sightlines, which are relatively similar in direction, do not span the whole bulge and thus do not probe a significant spread in mean metallicity.

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Table 1. Estimated mean photometric colours for the Galactic bulge RC.

In the second column (Colour 1), we assume a model atmosphere with log $g = 2.2$, [Fe/H] = 0.0, [$\alpha$/Fe] = 0.0 and $T_{\text{eff}} = 4650$ K, which yield the intrinsic RC colours adopted in this work. In the third column (Colour 2), we assume log $g = 2.2$, [Fe/H] = $-0.30$, [$\alpha$/Fe] = +0.10 and $T_{\text{eff}} = 4800$ K. Colours are computed using the methodology of Casagrande & VandenBerg (2014), where the accuracy of synthetic colours is also discussed (in particular the shortcomings at blue and ultraviolet wavelengths). UBVRI and WFC3 magnitudes are in the Vega system, SDSS magnitudes are in the AB system. Some of the colours are thus in a composite Vega–AB system.

<table>
<thead>
<tr>
<th>Index</th>
<th>Colour 1</th>
<th>Colour 2</th>
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<td>-0.522</td>
</tr>
<tr>
<td>$(V - J_{\text{VISTA}})$</td>
<td>0.004</td>
<td>0.007</td>
</tr>
<tr>
<td>$(V - J_{\text{VISTA}})$</td>
<td>-0.103</td>
<td>-0.092</td>
</tr>
<tr>
<td>$(V - J_{\text{VISTA}})$</td>
<td>0.271</td>
<td>0.248</td>
</tr>
<tr>
<td>$(V - J_{\text{VISTA}})$</td>
<td>0.491</td>
<td>0.451</td>
</tr>
<tr>
<td>$(V - J_{\text{VISTA}})$</td>
<td>0.993</td>
<td>0.935</td>
</tr>
<tr>
<td>$(V - J_{\text{VISTA}})$</td>
<td>1.070</td>
<td>1.006</td>
</tr>
<tr>
<td>$(V - J_{\text{VISTA}})$</td>
<td>1.284</td>
<td>1.205</td>
</tr>
<tr>
<td>$(V - J_{\text{VISTA}})$</td>
<td>1.380</td>
<td>1.292</td>
</tr>
<tr>
<td>$(V - J_{\text{VISTA}})$</td>
<td>1.657</td>
<td>1.549</td>
</tr>
<tr>
<td>$(V - J_{\text{VISTA}})$</td>
<td>1.845</td>
<td>1.727</td>
</tr>
<tr>
<td>$(V - J_{\text{VISTA}})$</td>
<td>2.070</td>
<td>1.946</td>
</tr>
<tr>
<td>$(V - J_{\text{VISTA}})$</td>
<td>2.279</td>
<td>2.152</td>
</tr>
<tr>
<td>$(I - J_{\text{VISTA}})$</td>
<td>0.833</td>
<td>0.779</td>
</tr>
<tr>
<td>$(I - J_{\text{VISTA}})$</td>
<td>0.653</td>
<td>0.582</td>
</tr>
</tbody>
</table>

For the dereddened apparent magnitude of the RC, we use the equation

$$I_{\text{RC,0}} = 14.3955 - 0.0239 * l + 0.0122 * |b|,$$

(2)

where the zero-point is taken from Nataf et al. (2013b), and the derivatives are derived by fitting to the data of Wegg & Gerhard (2013) within the coordinate range $-2.00 \leq l \leq 4.0$, $|b| \leq 4.5$. The gradients emerge due to projection effects, as stars towards the Galactic bulge are distributed as a bar. The standard deviation between the fit and the data is 0.018 mag. We have verified that there is no significant evidence for cross-terms or higher order terms in longitude or latitude for the apparent magnitude. Our zero-point assumption of the apparent magnitude of the RC is, as per the work of Cao et al. (2013), equivalent to assuming

$$M_{I,\text{RC}} = -0.12 - 5 \log (R_0/8.13),$$

(3)

where $R_0 = R_{\text{GC,⊙}}$ is the distance between the Sun and the Galactic Centre. Equation (3) is consistent with the canonical Galactocentric distance of 8.33 ± 0.11 kpc (Chatzopoulos et al. 2015) if $M_{I,\text{RC}} = -0.17 ± 0.03$. From these arguments, the zero-point systematic error (bias) in our extinctions is exactly 0 if $M_{I,\text{RC}} = -0.17$ and $R_0 = 8.33$. However, Nataf et al. (2013a) estimated $M_{I,\text{RC}} = -0.12$ by means of an empirical calibration, and Stanek & Garnavich (1998) measured $M_{I,\text{RC}} = -0.23$ for the solar neighbourhood RC. We thus assume a $\sigma$ systematic error of 0.05 mag in the values of $A_I$.

3.2 The measurement errors in the reddening and extinction

From the arguments in the preceding section, the zero-point (systematic) errors in $A_I$, $E(V - I)$, $E(I - J)$ and $E(J - K_s)$ are no greater than 0.05, 0.03, 0.03 and 0.02 mag, respectively, where the errors on $E(I - J)$ and $E(J - K_s)$ are derived from the error in $E(V - I)$ and the vectors defined by Table 1. These are errors which will shift the entire scale, with the errors in $E(V - I)$, $E(I - J)$ and $E(J - K_s)$ being positively correlated. The total colour error due to mean metallicity variations in our sample goes as $\sim 1/\sqrt{12}\times$ the spread due to metallicity if we use the uniform distribution as a probabilistic proxy, and thus the error from metallicity variations is 0.006, 0.006, 0.005 and 0.007 mag, respectively. The systematic errors in the colours are positively correlated with each other, and negatively correlated with the error in the brightness, since the RC becomes redder and dimmer with increasing metallicity.

The statistical errors in the fit, due to the finite number of RC stars per sightline, average 0.038 mag in $A_I$ and 0.011 mag in $E(V - I)$. The corresponding errors in $E(I - J)$ and $E(J - K_s)$ are likely smaller than and positively correlated with the error in $E(V - I)$, since both the reddening and the dispersion due to temperature are smaller at these longer wavelengths. The correlation in the statistical error with $A_I$ is virtually zero, as the red giant branch is nearly vertical in the $V$ colour–magnitude diagram (CMD) at the location of the RC.

These errors are small relative to both the mean values and sample dispersions of $A_I = 1.91 ± 0.67$, $E(V - I) = 1.61 ± 0.60$, $E(I - J) = 1.18 ± 0.40$ and $E(J - K_s) = 0.47 ± 0.17$ in our sample, where the measured sample means and sample dispersions are discussed in Section 5. We thus measure highly significant extinction and reddening values, with mean accuracies no worse than 3, 2, 3 and 4 per cent, respectively, and mean precisions no worse than 2, 1 and 2 per cent, respectively. Given that the errors in our reddening values are positively correlated with one another, the errors in the fractions $E(I - J)/E(V - I)$ and $E(J - K_s)/E(V - I)$ will be even smaller than the errors in the constituent parts.
3.3 Fitting for the RC magnitude and colours

The iterative fit for the RC apparent magnitude assumes the same luminosity function as used by Nataf et al. (2013b):

\[
N(I)dI = A \exp \left[ B(I - I_{RC}) \right] \\
+ \frac{N_{RC}}{\sqrt{2\pi} \sigma_{RC}} \exp \left[ -\frac{(I - I_{RC})^2}{2\sigma_{RC}^2} \right] \\
+ \frac{N_{RGBB}}{\sqrt{2\pi} \sigma_{RGBB}} \exp \left[ -\frac{(I - I_{RGBB})^2}{2\sigma_{RGBB}^2} \right] \\
+ \frac{N_{AGBB}}{\sqrt{2\pi} \sigma_{AGBB}} \exp \left[ -\frac{(I - I_{AGBB})^2}{2\sigma_{AGBB}^2} \right].
\]

Further discussion and details on the issues pertaining to this fit, such as how to account for the red giant branch bump (RGBB) and asymptotic giant branch bump (AGBB), are now well documented in the literature (Clarkson et al. 2011; Gonzalez et al. 2011, 2012; Nataf et al. 2011, 2014, 2015; Wegg & Gerhard 2013; Wegg et al. 2015). We assume the same stellar parameters for the RGBB and AGBB as Nataf et al. (2013b).

We require a dual-colour cut and a magnitude cut on our sample to select the red giant branch:

\[
(V - I) \geq (V - I_{RC}) - 0.30, \\
(J - K_s) \geq (J - K_{s,RC}) - 0.30, \\
|I - I_{RC}| \leq 1.50,
\]

where the colour cuts are only applied if the colours are measured. The fit is repeated until the guessed parameters agree with the output parameters to 0.03 mag in \((V - I)\) and 0.10 mag in \(I\). The relatively weak Gaussian priors in the parameters \(A, B\) and \(N_{RC}\):

\[
B \sim N(0.55, 0.03) \\
N_{RC}/A \sim N(1.17, 0.07),
\]

are imposed to increase stability of the fit and, thus, reduce the scatter in the derived value of \(I_{RC}\). The three RC colours \((V - I_{RC}), (J - K_s)_{RC}\) and \((J - K_s)_{RC}\) are determined independently, by picking the colour that minimizes the dispersion in colour at the luminosity of the RC. A demonstration of the colour and magnitude determinations is shown in Fig. 2. We show four fields with four different reddening values in Fig. 3.

A difficulty with our study, not shared by most previous photometric bulge studies, is the different sensitivities of the VVV and OGLE-III data sets. The near-IR data set probes further down the luminosity function in highly reddened fields. Nataf et al. (2013b) excluded sightlines with \((V - I_{RC}) \geq 3.30\) for that reason, as \(V\) magnitudes of stars located in the CMDs close to the RC were at or below the detection limit in OGLE data set for reddening values \((V - I_{RC}) \geq 3.30\). In this investigation, in order to be able to also include such highly reddened sightlines, we fit the \((V - I)\) versus \((I - K_s)\) colour–colour relations for stars slightly brighter and redder than the RC, satisfying \(0.50 \leq I_{RC} - I \leq 2.0\) and \(0 \leq (I - K_s) - (I - K_s)_{RC} \leq 0.70\). The fit is only applied to stars redder than the RC (observationally, not intrinsically), to avoid contamination from foreground disc stars that would have lower mean reddening, and thus shifted colour–colour terms. The intercept to the colour–colour relations is used for sightlines where the intercept satisfied

\[
(V - I_{RC}) \geq 3.20.
\]

\footnote{The condition is relaxed to 0.04 mag if \((V - I_{RC}) \geq 3.20\).}

\[(V - I_{RC}) \geq 3.20.\] There is a systematic shift of 0.0413 mag between the intercept to the colour–colour relations and the RC colour determined in the standard way, plausibly due to a gravity term in the colour–colour relations. This shift is measured from sightlines where \(2.40 \leq (V - I_{RC}) \leq 3.20\), and applied to more reddened sightlines, \((V - I_{RC}) \geq 3.20\).

\[\text{Figure 3. } I \text{ versus } (V - I) \text{ CMDs of four red giant branches with RC colours of } (V - I_{RC}) = 1.76, 2.25, 2.75 \text{ and } 3.22. \text{ Reddening and extinction are clearly quantities that can be precisely measured.}\]
would measure in a narrow-band filter placed at 5500 Å (very close but not identical to the Landolt V-band filter), a definition chosen to avoid ambiguities with respect to $A_V$ or $E(B - V)$. We find that what affect the extinction coefficients a great deal are the underlying parametrization (i.e. the chosen reference) and the choice of $R_V$ value. The convolution with the extinction curve itself has little impact, and thus extinction coefficients can be assumed to be independent of extinction. The ratio $E(V - J)/A_{5500}$, where $A_{5500}$ is the extinction at 5500 Å, is predicted to shrink by $\sim 0.75$ per cent or 0.003 as $A_V$ is doubled. Though there are contexts where this will matter, an offset of $\sim 0.01$ mag in $E(V - I)$ is too small to affect any of the conclusions reached in this work. We also list the predicted extinction coefficients for the spectra of a typical RR Lyrae star, they are nearly identical to those of RC stars, and thus studies of RR Lyrae stars and RC stars should yield consistent answers for the photometric filters used here.

5 RESULTS

We show in Fig. 4 the distribution of the extinction $A_V$, and the reddening ratios $A_J/E(V - I)$ and $E(J - K_s)/E(I - J)$ as measured towards our 1854 sightlines satisfying each of the photometric completeness criteria $(V - I)_{RC} \leq 4.30$, and the two differential reddening criteria $\sigma_{\delta, RC} \leq 0.30$ and $\sigma_{V - I, RC} \leq 0.18$. We obtain a broad distribution in each of these parameters, demonstrating our sensitivity to variations in the input parameter space. Specific findings are discussed below. The full list of values derived is available in Table 3.

5.1 The extinction curve is variable

We confirm a result of Udalski (2003b), Gosling et al. (2009) and Nataf et al. (2013b), that the extinction curve towards the inner Milky Way is variable.

In the left-hand panel Fig. 5, we show the CMDs for two sightlines which have $E(V - I)$ values that agree to $\sim 0.02$ mag, suggesting that they have the `same reddening'. They do not, the similar values of $E(V - I)$ are due to a fortuitous cancellation between the types and quantities of dust towards those two sightlines. Though the $E(V - I)$ values agree, the $E(J - K_s)$ values differ by 0.28 mag.
Extinction curve variations in VIJK

Figure 5. CMDs of two bulge sightlines demonstrating the variable extinction curve. The \((V - I)_{RC}\) colours of the two sightlines agree to \(\sim 0.02\) mag, yet their \((J - K_s)_{RC}\) differ by \(\sim 0.28\) mag, thus demonstrating a variation in the extinction curve. The extinction towards one sightline goes as 
\[E(J - K_s)/E(V - I) = 0.26\] (orange dots), whereas that towards the other goes as 
\[E(J - K_s)/E(V - I) = 0.41\] (blue dots).

Figure 6. CMDs of two bulge sightlines demonstrating the variable extinction curve. The two sightlines are selected to have reddening 
\(E(I - J)\) that agrees to 0.01 mag. However, the sightline with greater 
reddening \(E(V - I)\) by 0.17 mag (blue dots) has higher reddening 
\(E(J - K_s)\) by 0.21 mag.

Figure 7. Scatter plots of \(A_J/E(V - I), E(I - J)/E(V - I)\) and \(E(J - K_s)/E(V - I)\) versus one another. The extinction curves of Cardelli et al. (1989), Fitzpatrick (1999) and Fitzpatrick & Massa (2007) are poor fits to the data both in the mean and in the trend, regardless of how \(R_V\) is varied, with the large blue, green and magenta symbols referring to the predicted \(R_V = 3.1, 3.1, 3.0\) cases, respectively.

We present the same idea in a different manner in Fig. 6. These two sightlines are selected to have nearly equal values of 
\(E(I - J)\), but the sightline with greater \(E(J - K_s)\), by 0.21 mag, has an 
\(E(V - I)\) value that is 0.17 mag lower.

5.2 The extinction curve is non-standard

We show in Fig. 7 the scatter of \(A_J/E(V - I), E(I - J)/E(V - I)\) and 
\(E(J - K_s)/E(V - I)\) relative to one another. These extinction coefficients vary in a correlated manner. We also show the predictions of Cardelli et al. (1989), Fitzpatrick (1999) and Fitzpatrick & Massa (2007), which are obtained by convolving their extinction curves with a synthetic RC atmospheric spectrum and 4 mag of extinction at 5500 Å, typical of the sightlines investigated in this work.

The comparison to predictions leads to a conclusion that are entirely new to this investigation. Not only the \(R_V = 3.1\) curves of Cardelli et al. (1989) and Fitzpatrick (1999, delineated by the blue and green circles, respectively) are poor fits to the data over the entire span of extinction curves measured towards the bulge, but these parametrizations actually fail to intersect the bulge extinction trends regardless of how \(R_V\) is varied. Nataf et al. (2013b) claimed that the \(R_V \approx 2.5\) extinction curve from Cardelli et al. (1989) was a good fit, as it nearly fits the mean values of 
\(A_J/E(V - I)\) and 
\(E(J - K_s)/E(V - I)\), see the bottom-right panel of Fig. 7. However,
the addition of the measurement $E(I - J)$ shows that the extinction curves of Cardelli et al. (1989) and Fitzpatrick (1999) fail for all values of $R_V$, not just in the mean, but they fail completely. The blue and green lines never intersect the cloud of red points.

A possible explanation for this is that the extinction curve towards the inner Milky Way is in fact standard, but the ‘standard’ is not accurately characterized by the works of Cardelli et al. (1989) and Fitzpatrick (1999), and that studies of the bulge should instead use the mean Galactic extinction curve of Fitzpatrick & Massa (2007) – effectively zero. As with Fig. 7, the observed distribution of extinction points is well fitted by the mean Galactic extinction curve of Fitzpatrick & Massa (2007) never intersects the trend spanned by the red points, and the offsets will clearly often be larger than the offset to the mean.

Thus, the extinction towards the inner Milky Way, both the mean curve and the dominant trends in the curve, is not well fitted by the works of Cardelli et al. (1989), Fitzpatrick (1999) and Fitzpatrick & Massa (2007), even allowing for variations in $R_V$.

5.3 Whither $R_V$? the shape of the extinction curve has at least two degrees of freedom

We show in Fig. 8 the distribution of $E(J - K_s)/E(I - J)$ versus $A_I/E(V - I)$ – they appear uncorrelated. A Pearson coefficient for 1854 measurements satisfying the criteria $(V - I)_{BC} \leq 4.30$, $\sigma_{I/(V - I)} \leq 0.18$, $R_J \leq 1.45$ and $\sigma_{J/(V - I)} \leq 0.30$ yields $\rho = -0.0274$ – effectively zero. As with Fig. 7, the observed distribution of extinction parameters lies off the relations predicted by Cardelli et al. (1989), Fitzpatrick (1999) and Fitzpatrick & Massa (2007).

That these two ratios have uncorrelated variations disproves the canonical expectation that variations in the shape of the optical+near-IR extinction curve can be explained by a single parameter, $R_V$. There are at least two independent degrees of freedom in the optical+near-IR wavelength regime, and the fact that the maximum we can possibly measure with four photometric bandpasses is three degrees of freedom suggests that there may be more.

In Fig. 9, we show the distributions of $E(J - K_s)/E(V - I)$ and $A_I/E(V - I)$ as a function of direction. In both cases, adjacent sightlines tend to have similar values of the extinction coefficient, which robustly suggests that the measurements and their variations are significant. The distinct distributions in the left- and right-hand panels clearly demonstrate that the variations are largely uncorrelated.

Of interest in Fig. 8 is a sparse cloud of outliers with much higher values of $E(J - K_s)/E(I - J)$. These points appear spurious at first glance, but they turn out to be legitimate. In Fig. 10, we show the CMDs for a sightline towards $(l, b) = (1.68, -3.58)$, with measured extinction coefficient $E(J - K_s)/E(I - J) = 0.66$, vastly higher than the sample mean of $E(J - K_s)/E(I - J) = 0.40$. The CMDs reveal that the sightline looks fine; there is no confounding issue such as neglected removal of globular cluster contamination, differential reddening or failed colour selection. We also verify the photometry by comparing the VISTA photometry to the 2MASS photometry for some of the brighter points, to rule out any potential issues with calibration or observational factors such as the passage of small clouds or bright Solar system bodies during the VISTA observations. In both $J$ and $K_s$, the differences are usually less than 0.10 mag, and thus the measurements are deemed reliable.
We compute the principal components over three variables, 0.8352 × \( A_J/E(V - I) - 1.1973 \), 1.3406 × \( (E(I - J)/E(V - I) - 0.7459) \) and 3.3822 × \( (E(J - K_s)/E(V - I) - 0.2965) \). The coefficients \( 0.8352, 1.3406, 3.3822 \) are chosen such that each input dimension has the same mean value, otherwise the first principal component will be nearly parallel to the largest vector, whereas we are interested in diagnosing extinction curve variations consistently over the entire wavelength regime. Principal component decomposition automatically subtracts the means of the three vectors: \( \{ 1.1973, 0.7459, 0.2965 \} \).

The three eigenvalues of the principal component decomposition are 0.0117, 0.0034 and 0.0008, corresponding to standard deviations along the axes of \( \sim(11, 6, 3) \) per cent in the three rotated reddening ratios. The three principal components derived contribute 73, 22 and 5 per cent of the total variance, consistent with the claim made in the prior section that we find two degrees of freedom to the extinction curve in our data set. The projection of the reddening vectors on to the principal component space is shown in Fig. 11. The first two principal components are equal to

\[
PC_1 = 0.6340(A_I/E(V - I) - 1.1973) \\
+ 0.3555(E(I - J)/E(V - I) - 0.7459) \\
+ 0.2081(E(J - K_s)/E(V - I) - 0.2965) \\
(7)
\]

\[
PC_2 = -0.5088(A_I/E(V - I) - 1.1973) \\
- 0.4229(E(I - J)/E(V - I) - 0.7459) \\
+ 0.2092(E(J - K_s)/E(V - I) - 0.2965). \\
(8)
\]

5.5 Critical boundary value: relative extinction in \( V \) and \( K_s \)

We can estimate extinction in different filters with a conversion such as the following:

\[
A_{K_s} = A_V - E(I - J) - E(J - K_s). \\
(9)
\]

Though equation (9) has the advantage of being analytically exact, it has the disadvantage of producing extinction measurements with correlated errors, as the error in \( A_I \) enters linearly into \( A_{K_s} \), which is why the majority of the analysis in this paper focuses on the (independent) measurements of the colour excesses.

Regardless, the ratios should still be reliable in the median, for which we measure

\[
| \frac{A_V}{A_{K_s}} | = 13.44. \\
(10)
\]

That is a considerably greater ratio than the canonical value of 8.25 (Cardelli et al. 1989). The three median ratios measured in this work, \( A_V : A_I : A_J : A_{K_s} \), are 1: 1.85: 4.84: 13.44. We report the median rather than the mean as the mean is distorted by a small number of sightlines with considerable errors, leading to unphysically small values of \( A_{K_s} \).

5.6 Construction of better Wesenheit functions to minimize the effects of extinction

In various fields of astronomy such as the cosmological distance ladder, Wesenheit\(^5\) functions are used to minimize the dependence of extinction on apparent magnitudes and thus distances (Madore 1982; Majaess, Turner & Gieren 2011; Shappee & Stanek 2011; Wagner-Kaiser et al. 2015). This is done by subtracting from the apparent magnitude a colour term where the slope is believed to be the average total-to-selective extinction ratio, for example:

\[
W_l = I - 1.45(V - I) \\
(11)
\]

is commonly used, and has some empirical support towards sightlines such as the LMC (Udalski 2003b; Pejcha & Kochanek 2012). Though equation (11) no doubt performs very well over large swaths of the sky, we have demonstrated in this work that it fails spectacularly towards the inner Milky Way. We have also demonstrated that there is no single universal extinction curve for this Galaxy, and thus it is safe to assume that the same applies to other galaxies. Thus, such simple Wesenheit functions should usually be done away with in this era of precision cosmology.

An alternative, as per the fits seen in Fig. 7, the use of \( \textit{uber} \)
\(^6\)-Wesenheit functions, such that the apparent magnitude, is insensitive to not only variations in extinction assuming a mean extinction

\(^5\) ‘Wesenheit’ is the German word for ‘essence’ or ‘nature’.

\(^6\) ‘uber’ is the German word for ‘above’ or ‘at a higher level’.
curve, but also the dominant first-order variations in the extinction curve. We remove sources with high differential reddening ($\sigma_{V-I}_{\text{diff}} \geq 0.18$), poor fits ($\sigma_{\text{BC}} \geq 0.30$), very high reddening values that increase the odds of potential systematics such as incompleteness ($(V-I)_{\text{BC}} \geq 4.30$) and extreme values of the extreme coefficients ($A_I/E(V-I) \geq 1.45, E(J-K_s)/E(I-J) \geq 0.46$). We recursively remove 3σ outliers and obtain the following relation on the VII plane:

$$A_I = 0.1333E(V-I) + 1.4254E(I-J).$$

(12)

It is a tighter relation, with a correlation coefficient $\rho = 0.8194$, and can also be discerned from Fig. 7. A serendipitous result emerges: the coefficient of $E(V-I)$ is very small, only 9 per cent the size of the coefficient of $E(I-J)$. In practice, it turns out that the total-to-selective extinction ratio $A_I/E(I-J)$ has very little dependence on extinction curve variations. The mean value is given by $1/(A_I/E(I-J)) = 1.6063$ and the 1σ scatter by $\sigma_{A_I/E(I-J)} = 0.066$, or 4.1 per cent. In contrast, the scatter we expect just from the statistical measurement error in $I_{\text{BC}}$ is 2.3 per cent, and thus the intrinsic scatter in $A_I/E(I-J)$ is as small as 3.4 per cent in our sample.

We thus suggest

$$W_I = I - 1.61(I-J),$$

(13)
as a surprisingly robust Wesenheit magnitude. We note that the predicted extinction coefficients of $A_I/E(I-J)$ from Cardelli et al. (1989), Fitzpatrick (1999) and Fitzpatrick & Massa (2007) are 1.93, 1.89 and 1.79 respectively.

### 6 COMPARISONS TO PRIOR INVESTIGATIONS

#### 6.1 Shorter wavelength photometry

In principle, it would be interesting to map extinction curve variations over the broadest possible wavelength, which should become possible over time as more photometry of the Galactic bulge is taken.

One study available for comparison is that of Revnivtsev et al. (2010), who measured photometry of the ‘Chandra bulge field’ (towards $(l, b) = (0:11, -1:43)$) in a diverse array of filters with HST’s Advanced Camera for Surveys (ACS). Unfortunately, we cannot make a direct comparison as they did not publish their input data, only their final results. They report $A_{F625W, ACS}/(A_{F435W, ACS} - A_{F625W, ACS}) = 1.25 \pm 0.09$. Their error was the uncertainty on the regression, which emerges due to both measurement errors and the extreme values of the extreme coefficients of $A/I$ to $E(I-J)$ from Cardelli et al. (1989) or $R_V = 2.46$ (Fitzpatrick 1999).

The typical extinction coefficients we measure towards those sightlines are $A_I/E(V-I) = 1.10, E(I-J)/E(V-I) = 0.70$ and $E(J-K_s)/E(V-I) = 0.25$. Interestingly, we cannot find an $R_I$ match even if we restrict the fit to the optical filters. Fitting $A_I/E(V-I) = 1.10$ requires $R_I \approx 2.20$ in either the parametrization of Cardelli et al. (1989) or that of Fitzpatrick (1999). The resulting predicted values of $A_{F625W, ACS}/(A_{F435W, ACS} - A_{F625W, ACS})$ are 1.39 and 1.09, respectively, both failing to match the result of Zasowski et al. (2010), with equal errors of opposite signs. We list all of the implied values of $R_V$ towards this sightline in Table 4.

### 6.2 Measurements of DIBs

The correlation between the DIB located at $\lambda_0 = 15\,272.42$ Å and interstellar reddening was measured by Zasowski et al. (2015):

$$(W_{\text{DIB}}) = 12.2572 * (E(H-[4.5\mu])),$$

where $W_{\text{DIB}}$ is the diffuse interstellar band equivalent width in milli-angstroms, and the reddening $E(H-[4.5\mu])$ is taken from Nidever, Zasowski & Majewski (2012). The function reported by Zasowski et al. (2015) is in terms of $A_V$, which was extracted from the measurements of Majewski, Zasowski & Nidever (2011) with the conversion factors $A_V = 8.8A_k, A_k = 0.918E(H-[4.5\mu])$.

We match our catalogue of reddening and extinction curve variations with that of $W_{\text{DIB}}$ and $E(H-[4.5\mu])$ measurements satisfying $(|l| \leq 5, |b| \leq 5^\circ)$ from Zasowski et al. (2015). We obtain a paltry 23 matches, due to poor spatial overlap. In order to expand the sample, we match the DIB catalogue of Zasowski et al. (2015) to the $E(J-K_s)/E(V-I)$ catalogue of Nataf et al. (2013b), which is the extinction ratio most reliably measured in that work. This yields 137 matches.

Then, from each $W_{\text{DIB}}$ measurement, we subtract the predicted measurement to obtain a residual:

$$\delta W_{\text{DIB}} = W_{\text{DIB}} - 12.2572 * (E(H-[4.5\mu])).$$

(15)

One might expect the residuals to be randomly distributed and have a mean of zero. However, we instead find a correlation of $\rho = +0.34$ between $\delta W_{\text{DIB}}/W_{\text{DIB}}$ and $E(J-K_s)/E(V-I)$ (Fig. 12). The $p$-value for the correlation is $4.6 \times 10^{-5}$ – the odds of deriving this correlation by chance are $\sim 2000:1$.

Interestingly, the mean value in $\delta W_{\text{DIB}}/W_{\text{DIB}}$ is $-0.26$. This is extremely unlikely to be due to chance as the scatter measured by Zasowski et al. (2015) was $\sim 50$ per cent per star, and thus our sample mean is an $-6.2\sigma$ outlier. This offset is consistent with the accumulating evidence that the ISM towards the inner Milky Way has systematically different properties from that elsewhere in the Galaxy. The correlation between $E(J-K_s)/E(V-I)$ and $\delta W_{\text{DIB}}/W_{\text{DIB}}$ implies that a ‘standard’ value of $\delta W_{\text{DIB}}/W_{\text{DIB}}$ would be reached in the mean if a ‘standard’ value of $E(J-K_s)/E(V-I)$ is also reached in the mean.

This suggests that the ratio between DIB strength and interstellar extinction may depend on the properties of ISM. This is not surprising, given observations of sightline dependence for other DIBs (Kos & Zwitter 2013), though it is the first demonstration for the $\lambda_0 = 15\,272.42$ Å DIB. This issue warrants further investigation.

We point to the recent measurement of five distinct DIB equivalent widths towards the SN Ia 2014J by Jack et al. (2015), which is

<table>
<thead>
<tr>
<th>Index</th>
<th>Value</th>
<th>$R_{V,C99}$</th>
<th>$R_{V,F99}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{F625W}/(A_{F435W} - A_{F625W})$</td>
<td>1.25</td>
<td>1.97</td>
<td>2.46</td>
</tr>
<tr>
<td>$A_I/E(V-I)$</td>
<td>1.10</td>
<td>2.20</td>
<td>2.21</td>
</tr>
<tr>
<td>$E(J-K_s)/E(V-I)$</td>
<td>0.70</td>
<td>2.72</td>
<td>3.07</td>
</tr>
<tr>
<td>$E(J-K_s)/E(I-J)$</td>
<td>0.25</td>
<td>1.97</td>
<td>0.98</td>
</tr>
<tr>
<td>$E(J-K_s)/E(I-J)$</td>
<td>0.36</td>
<td>2.69</td>
<td>3.67</td>
</tr>
</tbody>
</table>
located behind anomalous dust (Goobar et al. 2014), as an example of potential future applications.

6.3 The Planck reddening maps

We compare our reddening measurements to the version 1.1 $E(B-V)$ all-sky maps from Planck and the version 1.2 maps (Planck Collaboration XI 2014). The Planck maps report $E(B-V)$, which we can compare to our measurements of $A_V$ to obtain a fiducial $R_V$. We show various diagnostics in Fig. 13. We find that neither reddening map works well, both have unexplained scatter, but the bias in the version 1.2 maps relative to the measured reddening reaching a catastrophic and colossal ~100 per cent.

The version 1.1 maps (left-hand panels) yield a mean and standard deviation of $R_V = 2.55 \pm 0.25$, both plausible given the other measurements in this work. However, in the bottom panel, we see that the suggested $R_V$ is uncorrelated with $A_V/E(V - I)$ ($\rho = 0.25$), with the correlation dropping to $\rho = 0.20$ and 0.03 for $E(I - J)/E(V - I)$ and $E(J - K)/E(V - I)$, respectively. In contrast, the correlations between $A_V/E(V - I)$ and $E(I - J)/E(V - I)$ and $E(J - K)/E(V - I)$ were $\rho = 0.80$ and 0.57, respectively, see Fig. 7. The fact that all of these correlations are small suggests that extinction curve variations are not responsible for the offset between reddenings inferred from IR emission and that measured from stellar colours, and that there is another source of ‘error’ at play.

The version 1.2 maps (right-hand panels of Fig. 13) yield a mean and standard deviation of $R_V = 1.33 \pm 0.22$, which is not plausible given the other measurements in this work, and suggests that reddening in the version 2 maps is overestimated by a factor of 2. In the bottom panel, we see that the suggested $R_V$ is uncorrelated with $A_V/E(V - I)$ ($\rho = 0.20$), which seems unlikely, though it would be beneficial to obtain $B$-band photometry of the bulge in order to be

\[ \rho \left( \frac{E(V - I)}{E(B - V)}_{\text{Planck v1.2}} \right) = +0.052. \]  

The Planck correlation coefficient if we instead use the Planck v1.2 maps is $\rho = -0.147$. If background emission was a significant source of error in the analysis, then the ratio of measured to expected reddening would drop rapidly with decreasing absolute latitude; in other words, there would be a strong positive correlation. We do not find a large, positive value of $\rho$ with either map, in agreement with our expectation that the systematic error from background emission is small.

The non-linearities that Wolf (2014) identified when comparing the Planck reddening maps to photometry of QSOs are not present in our comparison, furthering the argument that they are due to zero-point calibrations. Our methodology will necessarily be less sensitive to zero-point calibrations, as the reddening values probed in this work are $\sim 10 \times$ higher than those probed by Wolf (2014). What is consistent between our two works is that the version 1.1 map is accurate in the mean whereas the version 1.2 map overestimates reddening by a factor $\sim 2$. This consistency is impressive given the different methodology; Wolf (2014) used $ugriz$ photometry to study reddening towards QSOs in halo sightlines spanning $\sim 10,000$ deg$^2$, and thus probed dust predominantly from the solar neighbourhood, with a normalization of $E(B - V) \lesssim 0.20$. 

\[ \delta W_{\text{dif}} = W_{\text{obs}} - W_{\text{mod}} \]  

Figure 12. Scatter of residual of DIB strength relative to predictions of Zasowski et al. (2015), versus the $E(J - K)/E(V - I)$ measurements from Nataf et al. (2013b), shown as red points. There is a slight, positive correlation, $\rho = +0.34$. The thick black lines denote the mean values of the two variables for the 137 data points. The mean $1\sigma$ measurement error in $\delta W_{\text{dif}}/W_{\text{dif}}$ is 0.27.

Figure 13. Planck determinations of $E(B - V)$ from the version 1.1 maps (left-hand panels) and 1.2 maps (right-hand panels) as a function of $A_V$. The version 1.1 maps do better in the mean than the version 1.2 maps. Neither version appears sensitive to extinction curve variations.
These discrepancies will ultimately require more resolution, more wavelength coverage and superior comparison with models to resolve. Of possible interest may be the dust model of Jones et al. (2013), which incorporate different distributions of small carbon grains and larger silicate/iron grains.

7 SUMMARY

In this investigation, we have combined $VIJ\alpha$ photometry from the OGLE-III and VVV surveys to make nearly 2000 independent measurements of each of $A_I, E(V-I), E(I-J)$ and $E(I-K_s)$ towards the bulge. We have done so over a range of coordinates within which metallicity variations are small, and for which distance effects due to the Galactic bar can be accounted for.

We confirm previous reports that the extinction curve towards the inner Milky Way is variable and non-standard (Udalski 2003b; Gosling et al. 2009; Nataf et al. 2013b). Furthermore, not only is the RV regardless of how $R_v$ is varied. The mean Galactic extinction curve of Fitzpatrick & Massa (2007) is also a poor fit. These fits are poor both with respect to the mean of the Galactic bulge extinction curve and the fact they never intersect the variations thereof.

We find that the shape of the interstellar extinction has at least two degrees of freedom, as the variations in $A_I/E(V-I)$ and $E(J-K_s)/E(I-J)$ are uncorrelated. We use PCA to confirm the presence of two significant independent degrees of freedom in our data. This suggests a relatively large, and completely undiagnosed, source of systematic errors in cosmological investigations of Cepheids and SNe Ia.

We look forward to extending our investigations over a broader range of wavelengths, for example by incorporating photometry from the Dark Energy Camera (DePoy et al. 2008) and Pan-STARRS (Tonry et al. 2012). Further insights may be gleaned by comparison to measurements of DIBs from surveys such as APOGEE (Zasowski et al. 2015), GALAH (De Silva et al. 2015) and Gaia-ESO (Puspitarini et al. 2015).

ACKNOWLEDGEMENTS

We thank the referee for a helpful review of the manuscript.

We also thank Andrew Gould, Carine Babusiaux, Albert Zijlstra and Edward Schlafly for helpful discussions.

DM was primarily supported by the Australian Research Council grant FL110100012. This research was supported in part by the National Science Foundation under Grant No. NSF PHY11-25915. The OGLE project has received funding from the National Science Centre, Poland, grant MAESTRO 2014/14/A/ST9/00121 to AU.

We gratefully acknowledge the use of data from the ESO Public Survey programme ID 179.B-2002 taken with the VISTA telescope, data products from the Cambridge Astronomical Survey Unit. Support for the authors is provided by the BASAL CATA Center for Astrophysics and Associated Technologies through grant PFB-06, and the Ministry for the Economy, Development, and Tourism’s Programa Iniciativa Científica Milenio through grant IC120009, awarded to Millennium Institute of Astrophysics (MAS). DM and MZ acknowledge support from FONDECYT Regular grant no. 1130196 and 1150345, respectively. RKS acknowledges support from CNPq/Brazil through projects 310636/2013-2 and 481468/2013-7. JA-G acknowledges support from Fondecyt Post-doctoral project 3130552 and FIC-R Fund project 30321072.

This publication makes use of data products from the Two Micron All Sky Survey, which is a joint project of the University of Massachusetts and the Infrared Processing and Analysis Center/California Institute of Technology, funded by the National Aeronautics and Space Administration and the National Science Foundation.

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