Financial Frictions, Business Cycles and Optimal Monetary Policy

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Declaration

I certify that this thesis is my own original work. This thesis has not previously been submitted for a degree at this or any other university. To the best of my knowledge, it contains no other material previously published by another person, except where due reference is made in the text of this thesis.

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To my parents.
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Abstract

The great recession that started in 2007, has not only changed the perspective of the macroeconomic literature about the role of financial frictions within the canonical New Keynesian (henceforth, NK) monetary models but also has rekindled the debate about sources of business cycle fluctuations. This dissertation, comprising of three self-contained essays, makes theoretical and empirical contributions to the emerging strands of literature incorporating financial frictions in the NK monetary models.

The first essay (Chapter 2) of this dissertation extends traditional optimal monetary policy analysis to NK models with capital and financial frictions. It compares the performance of a standard inflation targeting monetary rule and the Taylor monetary policy rule with Ramsey monetary policy. In the case of a productivity shock, the chapter finds that: 1) a standard inflation targeting rule dominates the Taylor rule in the NK model with capital as it approximates the welfare level associated with the Ramsey policy; 2) in the NK model with capital and with financial frictions, which comprises balance sheet effects, the relative performance of the economy under standard inflation targeting is much better compared to alternative policies because it approximates Ramsey monetary policy. However, welfare analysis shows that maintaining price stability is not optimal in response to productivity shocks in the presence of financial frictions in a standard NK model. In the case of a financial shock, the chapter shows that the inflation targeting rule provides a welfare level that is close to the welfare level achieved under optimal monetary policy under commitment.

The second essay (Chapter 3) of this dissertation estimates the importance of financial shocks in business cycle fluctuations for the US economy using structural VAR models. In that chapter, financial and non-financial shocks are identified with a min-
imum set of sign restrictions based on the two competing NK models: the standard NK model augmented with a financial accelerator, in which financial frictions pertain to non-financial firms (demand side perspective) and the NK model augmented with financial intermediaries that are subject to a balance sheet constraint (supply side perspective). Estimation results show that a financial shock, emanating both from entrepreneur's net worth and financial intermediaries net worth, is prominent in explaining fluctuations in real output and interest rate spread. As far as the relative importance of these two financial shocks is concerned, the following results stand out. A financial shock related to the demand side is relatively the major driver of output fluctuations in both time horizons while financial shocks related to financial intermediaries explain a moderate variation in output fluctuations in both time horizons. In addition, financial shocks related to financial intermediaries account for a relatively larger share of interest rate spread fluctuations at both time horizons compared to a financial shock related to the demand side. A financial shock related to the supply side explains relatively larger variations in the price level over the shorter time horizons while the financial shock related to the demand side is the major driver in explaining prices fluctuations over the longer time horizon.

The third essay (Chapter 4) of this dissertation studies the interaction of durable goods, financial frictions and real economic activity by extending Gertler and Karadi's model (2011) into a two-sector setting. The Two-Sector Financial Accelerator model not only helps to incorporate the differences in the leverage ratios of commercial and investment banks but also introduces additional shocks that capture some features of the sub-prime financial crisis in the simulated economy. In addition to the importance of the financial sector in propagating shocks to the real economy, a shock to financial intermediaries' asset portfolio precipitates a financial crisis and hinders real economic activity in the simulated economy.
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Chapter 1

Introduction

1.1 Introduction

Since the great recession of 2007, central banks around the globe have increased their emphasis on financial conditions in the deliberation of monetary policy decisions. At the same time, this has also sparked interest in the analysis of the effects of financial frictions in the three distinct strands of the macroeconomics literature covering the theoretical and empirical aspects of financial frictions and shocks.

Specifically, the first strand of the macroeconomics literature, which has witnessed substantial changes following the great recession, is related to the incorporation of financial frictions within the canonical monetary New Keynesian (henceforth, NK) model. This contrasts with the literature that predates the financial crisis that downplayed the role of financial intermediaries by treating them as a veil. In response to the financial crisis, a number of authors have sought to incorporate financial frictions in the supply side of credit by focusing on the role of financial intermediaries (e.g., Gertler & Karadi (2011), Dib (2010), Gertler & Kiyotaki (2011)). The second strand of the macroeconomics literature, which attracted a lot of attention from both academics and policymakers, is related to optimal monetary policy design. Prior to the great recession, the literature on the optimal monetary policy was weaved around the NK monetary models which abstract from financial frictions and the financial intermediary sector. Since the great recession, there have been parallel efforts to strengthen

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the theoretical framework of optimal monetary policy (e.g., Taylor (2008), Carlstrom et al. (2010), Curdia & Woodford (2010), Woodford (2011), and Hansen (2011)). The third strand of the macroeconomics literature, which has generated a renewed interest, is related to the fundamental empirical issue about the sources of business cycle fluctuations. Specifically, the traditional view, which assigns a prominent role to a technology shock in explaining business cycle fluctuations, has been questioned by the growing literature giving an alternative view about sources of economic fluctuations.

1.2 Structure of the thesis

The main objective of this dissertation is to deepen our understanding about the effects of financial frictions on the design of optimal monetary policy in the NK models and to explore the relative importance of financial shocks for business cycles fluctuations. This dissertation, comprising of three self-contained papers, makes theoretical and empirical contributions to the aforementioned emerging strands of literature incorporating financial frictions in the NK monetary models.

Chapter 2, titled "Optimal Monetary Policy with Capital Formation and Financial Frictions", presents an analysis of optimal monetary policy for two NK models starting from the NK model with capital to the more elaborate Gertler & Karadi (2011) NK model with capital and financial frictions. The rationale for using Gertler and Karadi's (2011) model as benchmark model is that this model incorporates financial intermediaries sector with financial frictions in the standard monetary business cycle framework developed by Christiano et al. (2005) and Smets & Wouters (2007).
the Taylor rule and the strict inflation targeting rule. This chapter contributes to the literature on optimal monetary policy by analysing the following questions. First, how much of a difference do financial variables or frictions make for the conduct of optimal monetary policy? Second, how well does the optimal monetary policy respond to a non-financial shock and financial shock compared to alternative monetary policy regimes such as the Taylor rule and the strict inflation targeting rule?

The main findings of this paper for a productivity shock are as follows: first, an inflation targeting rule emerges as the best policy compared to the Taylor rule in the NK model with capital as it approximates the welfare level associated with the Ramsey policy, which is in line with the findings of Goodfriend & King (2001) and Woodford (2003); second, in the NK model with capital and with financial frictions, which comprises balance sheet effects, Ramsey policy outperforms alternative monetary policies by suggesting a relatively larger cut in the real interest rate; and third, the relative performance of the economy under an optimized monetary policy rule with interest rate smoothing and an inflation targeting rule is better compared to other alternative policies except for the Ramsey policy. However, welfare levels are somewhat sub-optimal to those level achieved under the Ramsey policy in response to a productivity shock. This shows that strict inflation targeting is sub-optimal in response to productivity shocks in the presence of financial frictions in a standard NK model as discussed by Carlstrom et al. (2010). In case of a financial shock, an optimized monetary policy rule without interest rate smoothing and an inflation targeting rule provide a welfare level which approximates the welfare level associated with an optimal monetary policy. Last but not least, Ramsey policy under commitment performs well compared to alternative monetary policy regimes by aggressively minimizing the impact of financial constraints on interest rate spread.

Chapter 3, titled "Financial Shock Identification with Model-based Sign Restrictions", quantifies the relative importance of financial shocks pertaining to non-financial firms and financial firms for the US business cycle fluctuations through structural
VARs identified with sign restrictions. The identifying sign restrictions are derived from the two competing NK models with financial frictions. The first model augments the standard NK model with a financial accelerator mechanism as suggested by Bernanke et al. (1999), in which financial friction pertains to non-financial firms (i.e., entrepreneurs) that gives a demand side perspective of credit. The alternative NK model, which provides a supply side perspective, incorporates financial frictions in the balance sheet of the financial intermediaries into the standard NK model as suggested by Gertler & Karadi (2011). The second contribution of this paper is the identification of a net worth shock (a wealth re-distribution shock from the financial intermediary to households) and a financial intermediary’s assets impairment (a capital quality shock), separately. To the best of our knowledge, this paper is the first attempt that directly provides empirical evidence for Gertler & Karadi (2011) and Gertler & Kiyotaki (2011) simulated models.

Using quarterly US data from 1980:Q1 to 2008:Q1, this paper identifies four shocks (supply, demand, monetary, and financial shock (entrepreneur’s net worth shock)) by using the sign restrictions based on the NK model augmented with a financial accelerator mechanism. The financial shock is then separated into a financial intermediary’s net worth shock and an assets impairment shock (a capital quality shock) with sign restrictions based on the NK model with financial intermediaries à la Gertler & Karadi (2011). It is pertinent to note that a net worth shock and an assets impairment shock follow the same mechanism to effect the simulated economy although the magnitudes differ. Specifically, a negative shock to net worth or an assets impairment tightens a balance sheet constraint which causes a rise in the interest rate.

5 The sign restrictions approach was initially pioneered by Faust (1998), Canova and de Nicolo (2002) and Uhlig (2005) and some significant extensions in the methodology were proposed by Peersman (2005) and Fry & Pagan (2011).

6 Bernanke, Gertler & Gilchrist (1999) incorporates a financial accelerator mechanism in the dynamic model through costly state verification and shows how fluctuations in borrowers’ net worth can amplify and propagate exogenous shocks. It is pertinent to note that most of the recent aforementioned empirical studies have augmented NK models with financial accelerator mechanism.

7 Gertler and Karadi’s (2011) model pertains to a class of NK models, in which, financial frictions are defined over the supply side of credit i.e., on the balance sheet of financial intermediaries. Specifically, financial frictions are embedded in the balance sheet of financial intermediaries through an agency problem between bankers and households that limits the amount of credit.
premium leading to a subsequent distress in investment and output. To disentangle a net worth shock from an assets impairment shock, this essay uses real consumption data. In contrast to the assets impairment shock, a negative net worth shock, which re-distributes wealth from the representative financial intermediary to households, increases real consumption for a few quarters in the simulated model devised by Gertler & Karadi (2011). However, real distress induced by a rise in interest rate premium subsequently reduces consumption expenditures in the simulated model.

The main four findings of this paper are as follows. First, a financial shock, emanating both from entrepreneurs and financial intermediaries, is prominent in explaining fluctuations in real output and real investment, interest rate spread, prices and real consumption. Second, the comparison of variance decomposition of real output suggests that a financial shock related to the demand side is relatively the major driver of output fluctuations at both time horizons while financial shocks related to financial intermediaries also explain a modest share of output fluctuations. Third, financial shocks related to financial intermediaries account for a relatively larger share of interest rate spread fluctuations at both time horizons compared to a financial shock related to the demand side. Fourth, a financial shock related to financial intermediaries explains relatively larger variations in short run price level compared to the demand side financial shock while the latter explains relatively larger variations in prices over the longer time horizons.

Chapter 4, titled "Financial Frictions, Durable Goods, and a Two-Sector Financial Accelerator Model", presents a Two-Sector New Keynesian Financial Accelerator model (henceforth, the Two-Sector FA model) by incorporating not only durable goods \(^8\) but also by taking into account the difference in leverage ratios of financial intermediaries in the US economy. The main results of this paper are as follows.

First, the Two-Sector FA model successfully captures some of the underlying mechanism of the great recession, particularly a modest increase in the interest rate spread

\(^8\)Durable goods, non-consumption goods and Housing goods are used as synonyms in this paper.
even at a higher leverage ratio contrary to Gertler and Karadi’s (2011) model. This necessitates the expansion of Gertler and Karadi’s (2011) model into the Two-Sector FA model. Second, the impact on consumption expenditure is also more pronounced in response to shocks in the Two-Sector FA model compared to the benchmark-FA model due to the presence of collateral effects. Third, the effect of a capital quality shock on real activity in the Two-Sector FA model is not amplified mainly due to a relatively smaller drop in the relative price of non-consumption goods. Fourth, a relatively stronger financial intermediary’s balance sheet in the Two-Sector FA model underpins a modest fallout of the monetary tightening shock despite the higher impact on nominal interest rates. Last but not least, the chapter also finds that the output recovery to trend would remain slow in the simulated economy as far as the relative price of non-consumption goods is not recovered to its trend.
Chapter 2

Optimal Monetary Policy with Capital Formation and Financial Frictions

2.1 Introduction

Since the great recession, central banks around the globe have increased their emphases on financial conditions in the deliberation of monetary policy decisions. At the same time, there have been parallel efforts to strengthen the theoretical framework of optimal monetary policy (e.g., Taylor (2008), Carlstrom et al. (2010), Curdia & Woodford (2010), Woodford (2011), and Hansen (2011)). More specifically, the literature on optimal monetary policy, prior to the global financial crisis, neither incorporates financial intermediation features nor financial frictions.1 Why is studying these two aspects simultaneously important? First, it is important because this gives us a theoretical framework to explore the role of financial conditions that, in turn, affect a central bank's interest rate decisions. Second, adverse shocks hitting the economy are propagated and amplified through financial variables such as a decrease in net worth and a rise in interest rate spreads (e.g., Gertler & Karadi (2011) and Gertler & Kiyotaki (2011)).

1The simplest contribution comes none other than from Taylor himself who advocated the use of the modified Taylor rule, see Taylor (2008). Similarly, McCulley & Toloui (2008) and Curdia & Woodford (2010) suggest a modified Taylor rule that incorporates credit spread in the Taylor rule.

2Prior to the global financial recession, the literature on the optimal monetary policy was weaved around the NK models which abstract from financial frictions. Prominent contributions such as Rotemberg & Woodford (1997), Woodford (2003), and Ereg & Levin (2006), etc.
This chapter extends the analysis of optimal monetary policy for two New Keynesian (henceforth, NK) models starting from the NK model with capital to the more elaborate NK model with capital and financial frictions: Gertler and Karadi's (2011) model. The main objective of this chapter is to answer the following questions pertaining to the effect of financial frictions on the optimal monetary policy. First, how much of a difference do financial variables or frictions make for the conduct of optimal monetary policy? Second, how well does optimal monetary policy respond to a non-financial shock and a financial shock compared to alternative monetary policy regimes such as the Taylor rule and the strict inflation targeting rule? This chapter uses the Ramsey optimal monetary policy approach, where monetary policy decisions are made in a timeless perspective, as a benchmark to compare alternative monetary policy regimes such as the Taylor rule and the strict inflation targeting rule for the NK model with capital, and the NK model with capital and with financial friction.

There has been a sustained literature that seeks to study the effects of financial frictions on the business cycle and that suggests some policy implications for the conduct of monetary policy. The most prominent finding of this literature suggests that the central bank has an incentive to deviate from strict inflation targeting in response to a productivity shock in the presence of credit market imperfections (see, Carlstrom et al. (2010)). Nonetheless, the marginal welfare gain associated with offsetting the credit imperfection is not substantial and, as Curdia and Woodford (2008) show, strict inflation targeting is still not far from optimal in that case. This chapter is related to an emerging literature on the evolution of monetary policy using NK models with financial frictions, including Hansen (2011) and Carlstrom et al. (2010). This chapter differs from this literature in two main ways. First, this chapter mainly focuses on a financial friction defined by the financial intermediaries' balance sheet while the aforementioned studies use financial frictions related to the credit channels of non-financial firms. Second, this chapter uses Ramsey monetary policy using the utility

3The rationale for using Gertler and Karadi's (2011) model is that this model incorporates financial intermediaries sector with financial frictions in the standard NK monetary models developed by Christiano et al. (2005) and Smets & Wouters (2007).
function of the household as the objective function instead of the Linear Quadratic approach that explicitly derives the loss function.\textsuperscript{4}

This chapter is also related to the strand of literature that incorporates endogenous capital formation as an additional channel through which monetary policy effects the overall economy in the NK model (see, Schmitt-Grohe & Uribe (2007), Hansen (2011) and Takamura et al. (2006)). These studies assume a perfect capital market while this chapter relaxes this assumption and incorporates financial intermediaries which are subject to financial frictions.

The main findings of this chapter for a productivity shock are as follows: first, the inflation targeting rule emerges as the best policy compared to the Taylor rule in the NK model with capital as it approximates the welfare level associated with the Ramsey policy, which is in line with the findings of Goodfriend & King (2001) and Woodford (2003); second, in the NK model with capital and with financial frictions, which comprises balance sheet effects, Ramsey policy suggests a larger cuts in the real interest rate relative to alternative monetary policies; and third, the relative performance of the economy under the optimized monetary policy rule without interest rate smoothing and the inflation targeting rule though is better compared to other alternative policies except Ramsey policy. However, the welfare levels which are achieved are somewhat sub-optimal compared to the level that achieved with Ramsey policy in response to a productivity shock. This shows that strict inflation targeting is not optimal in response to a productivity shock in the standard NK model with financial frictions. In the case of a financial shock, a non-optimized monetary policy rule without interest rate smoothing and an inflation targeting rule provide a welfare level which is close to the welfare level of the optimal monetary policy under commitment. Last but not least, the Ramsey policy under commitment performs well compared to alternative monetary policy regimes by aggressively minimizing the impact of financial constraints on interest rate spread.

\textsuperscript{4}The Linear Quadratic approach is not suitable for the NK model with capital and financial intermediaries sector due to the presence of state variable in the loss function derived from the model.
The rest of the chapter is structured as follows. Section 2.2 presents the description of the NK model with capital (with perfect capital market). Section 2.3 relaxes the assumption of a perfect capital market and incorporates financial intermediaries and financial friction by following Gertler and Karadi's (2011) model. Section 2.4 specifies the Ramsey policy problem which serves as the benchmark against which the performance of the alternative monetary policies are compared. Section 2.5 presents calibration, dynamic analysis, and welfare analysis of these models. Section 2.6 concludes and presents some implications for the conduct of monetary policy.

2.2 The New Keynesian model with Capital

Before presenting the main results, it is instructive to start with the basic setup of the New Keynesian model with capital, which comprises two standard distortions: monopolistic competition in goods markets and sticky prices à la Calvo pricing while the capital market is perfect. The NK model with capital is comprises of six agents: households, non-financial firms, capital goods-producing firms, retailers, government, and a central bank. Households consume final goods, hold one-period risk-free bonds and supply labor to the intermediate goods producing firms. Intermediate goods producing firms hire labor and capital from households and capital goods-producing firms, respectively and produce differentiated goods by using the Cobb-Douglas technology. These goods are sold in monopolistically competitive goods markets. Retailers convert these intermediate differentiated goods to final consumption goods subject to the nominal rigidities. Government has a simple rule in the model to impose a lump sum tax to finance its exogenous government expenditure. A central bank follows the Taylor rule while setting the nominal interest rate.

2.2.1 Households

There is a continuum of identical households of length unity. Each household supplies labor in the market, \( L_t \), purchases final consumption goods, \( C_t \), and stores its savings in one-period risk-free bonds \( D_t \). Households also own capital goods-
§2.2 The New Keynesian model with Capital

producing firms. Let $E_t$ denote the expectation operator conditional on information available at time period $t$, $h$ represent the external habit formation parameter, $\chi$ measure the dis-utility of labor supply in the utility function, and let $\phi$ denote the inverse Frisch elasticity of substitution of labor supply. The representative household utility function is defined over consumption with external habit formation and number of hours employed. The household objective is to maximize its expected discounted utility by choosing a sequence of $C_t, L_t, D_t$:

$$\max_{E_t} \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t - hC_{t-1}) - \frac{\chi L_t^{1+\phi}}{1 + \phi} \right]$$ (2.1)

subject to the following budget constraint as represented by Equation 2.2:

$$C_t + D_t = W_t L_t + R_{t-1} D_{t-1} + \Pi^t_t - T_t$$ (2.2)

where $D_t$ denotes the volume of deposits/bonds, which is a one-period risk-free debt, $W_t$ represents real wages in the goods producing sector, $R_{t-1}$ represents a gross real interest income to households on the one-period risk-free bonds, $\Pi^t_t$ shows the net transfers from non-financial and capital goods-producing firms and $T_t$ denotes lump sum taxes. The first order conditions (henceforth, FOCs) are as follows:

$$\lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h E_t \left( \frac{1}{C_{t+1} - hC_t} \right)$$ (2.3)

$$\lambda_t W_t = \chi t^{\phi}_{t}$$ (2.4)

$$E_t \beta \Lambda_{t,t+1} = 1$$ (2.5)

with $\Lambda_{t,t+1} = \frac{\lambda_{t+1}}{\lambda_t}$

In Equation 2.3, $\lambda_t$ measures the marginal utility of consumption when there is a habit formation in consumption. Equation 2.4 represents standard labor supply first order equilibrium conditions and measures the dis-utility to the household for work-
ing in the goods-producing sectors. Equation 2.5 represents the standard Euler equation, with $\Lambda_{t,t+1}$ representing a stochastic discount factor.

### 2.2.2 The role of capital formation

The capital accumulation not only incorporates an important channel for the transmission of aggregate disturbances in the model but also improves the determinacy properties of the standard NK models without capital (see Dupor (2001) and Carlstrom and Fuerst, 2010). In addition, this also enables us to introduce variable capital utilization in the NK models which has implications for the inflation dynamics (see Christiano et al. (2005)).

In the NK Model with capital, households own these capital goods-producing firms and the profits of these firms are returned to households. A continuum of competitive capital producers produces new capital through linear technology. These firms use a fraction of final goods purchased from retailers as investment goods, $I_t$, and the production of new capital is subject to a quadratic adjustment cost, $[A(I_t, I_{t-1})]$, which induces price volatility. In addition, these firms also purchase and refurbish the undepreciated capital from both intermediate goods producing firms at the end of the production period. It is assumed that the refurbishment of the undepreciated capital does not involve any adjustment cost.

### 2.2.3 Intermediate goods-producing firms

Within non-financial firms, there are two distinct types of firms: intermediate goods-producing firms and capital goods-producing firms. Intermediate goods-producing firms use labor, $L_t$, and capital, $K_t$, as inputs in Cobb-Douglas physical technology to produce intermediate goods:

$$Y_t(j) = A_t(U_t \xi_t K_t(j))^{a} (L_t(j))^{1-a}$$  \hfill (2.6)

$$\xi_t = \rho_{\xi} \xi_{t-1} + e^{t}_{\xi}$$  \hfill (2.7)
where $\zeta_t$ represents a capital quality shock and it follows a Markov process as in Gertler & Karadi (2011) and others. It is pertinent to note that the market price of capital is still endogenous in the model which is explained later in the capital producers' problem. The consumption goods-producing firms' profit maximization problem involves choosing a utilization rate, $U_t$, and labor, $L_t$, to satisfy the following equilibrium conditions:

$$ \frac{Y_t}{U_t} = \delta'(U_t) \zeta_t K_t $$

(2.8)

$$ W_t = (1 - \alpha) \frac{Y_t}{L_t} $$

(2.9)

Given these FOCs, firms pay the ex-post return to capital inputs. Accordingly, $R_{k,t+1}$ is given by:

$$ R_{k,t+1} = \frac{\alpha \frac{Y_{t+1}}{K_{t+1}} + Q_{t+1} - \delta(U_{t+1})}{Q_t} \zeta_{t+1} $$

(2.10)

It is pertinent to mention that there is no formal financial intermediary sector in the model economy. Households perform such intermediary roles in a friction-less environment in such a way that it satisfies the following no arbitrage equation:

$$ E_t \beta^t \Lambda_{t,t+1+i} (R_{k,t+1+i} - R_{t+1+i}) = 0 $$

(2.11)

### 2.2.4 Capital producers

Capital producers maximize the following discounted profit by choosing $l_t$ to solve:

$$ \text{Max}_{E_t} \sum_{T=t}^{\infty} \beta^T \Lambda_{t,T} \left\{ Q_T I_{N_T} - \left[ 1 + f\left( \frac{I_{N_T}}{I_{N_T-1}} \right) \right] I_{N_T} \right\} $$

(2.12)

The following FOC of the above profit maximization problem represents the standard Tobin's Q equation that relates the real price of capital to the marginal adjustment
cost of investment goods:

\[ E_t \beta' \Lambda_{t,T} \left[ Q_T - 1 - f\left( \frac{I_{N_T}}{I_{N_T-1}} \right) + I_{N_T} f'\left( \frac{I_{N_T}}{I_{N_T-1}} \right) \cdot \frac{1}{I_{N_T-1}} - E_t \beta' \Lambda_{t,T} \left( \frac{I_{N_T+1}}{I_{N_T}} \right)^2 f'\left( \frac{I_{N_T+1}}{I_{N_T}} \right) \right] = 0 \]  

(2.13)

Capital producers earn profit only outside the steady state, which in turn redistributes a lump sum to the households. As shown by the following equation, the net investment, \( I_N \), which these capital producers carry out, is total investment, \( I_t \), net of refurbishment of the un-depreciated capital purchased from goods-producing firms at the end of the production period:

\[ I_{N,t} = I_t - \delta(U_t)K_t \]  

(2.14)

The aggregate capital consists of undepreciated capital and investment subject to adjustment cost and evolves according to:

\[ K_{t+1} = K_t(1 - \delta) + A(I_t, I_{t-1}) \]  

(2.15)

2.2.5 Retailers

The retailers, which incorporate nominal rigidities in the model, re-package the intermediate goods purchased from intermediate goods producers into final goods at no cost and sell these goods in a monopolistically competitive goods market. The decision problem for a retail firm is static and involves choosing a sequence \( \{ Y_t, y_t(j) \}_{j=0}^1 \) to maximize its profit:

\[ \Pi_{t}^f = P_t \left[ \int_0^1 y_t(j)e^x dj \right]^{\frac{\varepsilon-1}{\varepsilon}} - \int_0^1 P_t(j)y_t(j) dj \]  

(2.16)

FOCs yield the following demand for each inputs and price:

\[ y_t^*(j) = \left( \frac{P_t(j)}{P_t} \right)^{(e)} - Y_t \]  

(2.17)

\[ P_t^x = \left[ \int_0^1 P_t(j)^{1 - e} dj \right]^{\frac{1}{1-e}} \]  

(2.18)
§2.2 The New Keynesian model with Capital

In a retail price setting of final goods, each intermediate goods-producing firm receives a random draw in every period from the Bernoulli distribution with probability \((1 - \Psi), \Psi \in (0, 1)\), and the firm receiving a successful draw can adjust its price. With probability \((\Psi)\), the firm gets an unsuccessful draw and cannot change its price. The retailer's pricing problem of final goods is to choose the optimal price \(P_t^*\) to solve:

\[
\text{Max } E_t \sum_{i=0}^{\infty} (\beta^\Psi)^i \Lambda_{t,i}; \quad \left[ \left( \frac{P_t(j)}{P_{t+i}} - \lambda_{t+i} \right) \left( \frac{P_t(j)}{P_{t+i}} \right)^{-e \Psi^{t+i}} \right]
\]

(2.19)

\[P_t^*(j) = \frac{E_t \sum_{i=0}^{\infty} (\beta^\Psi)^i \Lambda_{t,i} \lambda_{t+i} \Psi^{t+i} Y_{t+i}}{E_t \sum_{i=0}^{\infty} (\beta^\Psi)^i \Lambda_{t,i} \Psi^{t+i} Y_{t+i}}\]

(2.20)

The above equation shows that the intermediate goods-producing firms will set their prices as a constant mark up over the ratio of an expression related to its expected discounted nominal total cost to an expression to their expected discounted real output. There is no \(j\) involved in the above equation, which implies that when a firm has a chance to change its price in period \(t\) it will choose the same price:

\[P_t = \left( (1 - \Psi)(p_t^*)^{1-e} + \Psi (P_t)^{1-e} \right)^{1/e_t}\]

(2.21)

2.2.6 Government and the monetary policy rule

The role of the government in this model is very simple: it imposes lump sum taxes to finance exogenous government expenditure as shown in Equation 2.22.

\[G_t = T_t\]

(2.22)

\[i_t = (1 - \rho) \left[ i + \kappa_{i1} \Pi_t + \kappa_y (\log Y_t - \log Y^*_t) \right] + \rho i_{t-1} + \epsilon_t\]

(2.23)

Let \(i_t\) denote the nominal interest rate; \(\epsilon_t\) show the exogenous shock to monetary policy; \(i\) denote the steady state nominal interest rate; \(Y^*\) represent the natural level of output; and let \(0 < \rho < 1\) show an interest rate smoothing parameter. Equation 2.23 and Equation 2.24 define the monetary policy rule and a standard Fisher equation,
respectively.

\[ 1 + i_t = R_{t+1} \frac{P_{t+1}}{P_t} \]  \hspace{1cm} (2.24)

### 2.2.7 Equilibrium

The market equilibrium conditions in labor and goods markets are used to close the model. Labor market equilibrium conditions require that demand for labor equalizes labor supply as represented by Equation 2.25.

\[ (1 - \alpha) \frac{Y_t}{L_t} = \lambda_t \frac{Y_t}{L_t} \]  \hspace{1cm} (2.25)

Goods market equilibrium condition or resource constraint of the economy requires that:

\[ Y_t = C_t + [1 + f(\frac{L_t}{L_{t-1}})]L_t + G_t \]  \hspace{1cm} (2.26)

where \( f(\frac{L_t}{L_{t-1}}) \) reflects physical adjustment cost with \( f(1) = f'(1) = 0 \) and \( f''(\frac{L_t}{L_{t-1}}) > 0 \).

### 2.2.8 Shocks

Apart from a monetary policy shock, another shock in the NK model with capital is a total productivity shock as described in Equation 2.27.

\[ a_t = \rho_a a_{t-1} + e_t^a \]  \hspace{1cm} (2.27)

### 2.3 The role of financial intermediaries and financial frictions

This section extends the NK model with capital by incorporating a formal financial intermediary sector by following the Gertler & Karadi (2011) Financial Accelerator model. By using an agency problem, Gertler & Karadi (2011) incorporate a financial friction which endogenously determines a balance sheet constraint on financial intermediaries. This constraint imposes a limit on the volume of funds that financial intermediaries can collect from households in the form of deposits. The noticeable
difference in the extended model, the NK model with capital and with financial frictions, is as follows.

2.3.1 Households

With the introduction of the financial intermediary sector in the model, there are two types of members in each household at time t. The fraction \((1 - f)\) of household members serves as workers and supplies labor in exchange for wages. The rest of the members, \(f\), manage a financial intermediary and transfer their earnings back to households on their exit. It is assumed that there is perfect consumption insurance within the family.\(^5\) Individuals can change their occupation over time, a banker in \(t\)-period can become a worker in the next period with a probability \(\theta\), with the average survival time of a banker of \(\frac{1}{1-\theta}\). This assumption ensures that a banker does not become self-funded and also transfers the net worth of the banker to their respective households on exit.

2.3.2 Financial intermediaries

To carry out an intermediary role, each bank raises deposits, \(D_{jt+1}\), from households at the beginning of the period at \(R_{t+1}\). The balance sheet of the commercial bank is as follows:

\[
Q_tK_{jt} = NW_{jt} + D_{jt+1}
\]  

(2.28)

The left-hand side (LHS) of the above equation represents the assets side of the balance sheet while the right-hand side (RHS) represents the liability and inside equity/net worth. Let \(R_{kt+1}\) be stochastic return on state contingent lending to entrepreneurs and let \(R_{t+1}\) be a non-contingent gross return on deposit. Using equation 2.35, the evolution of the commercial banker's net worth can be expressed as:

\[
NW_{jt,t+1} = R_{kt+1}Q_tK_t - R_{t+1}D_{jt+1}
\]  

(2.29)

\(^5\)There is also an alternative approach in the literature which splits households into two distinct groups, borrowers and lenders, with having a varying degree of discount factor without any consumption insurance between these groups.
Let $\beta^i \Lambda_{t,t+i}$ be an augmented stochastic discount factor, a commercial banker’s object is to maximize the expected terminal net worth, given by:

$$V_{jt} = \max_{E_t} \sum_{t=0}^{\infty} \left(1 - \theta\right)^{\theta^j} \beta^{j+1} \Lambda_{t,t+1+i} (NW_{jt+1+i})$$

(2.30)

Following Gertler & Karadi (2011) and Gertler & Kiyotaki (2011), this essay uses an agency problem in the model to introduce financial frictions, which constrains the bank’s ability to obtain funds from households. In this environment, bankers choose to divert a fraction ($\lambda$) of total deposits and net worth for their personal benefit. Households recognise the fact that they may claim only $(1 - \lambda)$ of their funds in the event of a default, so they restrict the amount of funds they lend to commercial banks. Accordingly, this helps to define an incentive constraint for commercial banks to be in the intermediary business as follows:

$$V_{jt} \geq \lambda (D_{jt} + NW_{jt})$$

(2.31)

The LHS measures the loss of filing for bankruptcy whereas the RHS measures the gains to the banker from diverting money. $V_{jt}$ can also be expressed as:

$$V_{jt} = v_t (Q_t K_{jt}) + \eta_t NW_{jt}$$

(2.32)

with

$$v_t = E_t \left\{(1 - \theta) \beta \Lambda_{t,t+1} (R_{kt+1} - R_{t+1}) + \beta \Lambda_{t,t+1} \theta x_{t,t+1} v_{t+1}\right\}$$

(2.33)

$$\eta_t = E_t \left\{(1 - \theta) + \beta \Lambda_{t,t+1} \theta z_{t,t+1} \eta_{t+1}\right\}$$

(2.34)

where $x_{t,t+1} \equiv (Q_t K_{jt+1})/(Q_t K_{jt})$ represents the gross growth rate in assets and $z_{t,t+1} \equiv NW_{jt+1}/NW_{jt}$ represents the gross growth rate in net worth and $v_t$ measures the marginal expected discounted gain by expanding one unit of assets, with given net worth. Finally, $\eta_t$ measures the marginal expected discounted marginal gain by expanding one unit of net worth, holding bank’s assets constant. In the case of the binding incentive constraint, the banker’s issuance of assets depends directly with
Specification of the Ramsey policy problem

2.4 Specification of the Ramsey policy problem

Ramsey policy, which is defined as a monetary policy under commitment, uses the utility function of households as its objective function. A Ramsey policy maker solves the following optimal control problem:

\[
\max_{\{y_t\}_{t=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \beta^{\tau-t} \left[ \log(C_{\tau-t} - hC_{\tau-t-1}) - \frac{\lambda^{1+\phi}}{1 + \phi} \right]
\]

where \(\phi_t\) is the leverage ratio of the commercial bank. Holding the bank's net worth constant, expanding the assets of the bank raises the bankers' incentive to divert funds for personal use. The binding incentive constraint puts a limit on the leverage ratio and restricts the banker's ability to expand assets as the banker's incentive to divert money equates to the cost of a bankruptcy. Since the leverage ratio does not involve bank-specific factors, this can be expressed as an aggregate across individual commercial banks.

From the entry and exit of the bankers, an equation of motion for \(NW_t\) can be derived by adding up the net worth of existing commercial banker, \(NWE_{t-1}\), and the new banker's net worth, \(NWN_t\). A fraction \(\theta\) of banker's at \(t-1\) survive until the next period \(t\), and it is assumed that the household transfers the fraction of \(n/(1-\theta)\) of the final assets of the exiting bankers to a new banker each period as a start-up fund. The following equation describes the law of motion for \(NW_t\) as:

\[
NW_t = \frac{\theta \left[(R_{kt} - R_t)\phi_{t-1} + R_t\right] NW_{t-1} + n(Q_tK_{t-1})}{NWE_t, NWN_t}
\]

2.4 Specification of the Ramsey policy problem

Ramsey policy, which is defined as a monetary policy under commitment, uses the utility function of households as its objective function. A Ramsey policy maker solves the following optimal control problem:

\[
\max_{\{y_t\}_{t=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \beta^{\tau-t} \left[ \log(C_{\tau-t} - hC_{\tau-t-1}) - \frac{\lambda^{1+\phi}}{1 + \phi} \right]
\]

Under Ramsey policy, monetary policy decisions are made in a timeless perspective as described by Woodford (2003).
subject to

\[ E_T f(y_{\tau+1}, y_{\tau}, y_{\tau-1}, e_{\tau}) = 0 \] \hspace{1cm} (2.39)

Where \( f(.) \) comprises of model's first order conditions, equilibrium conditions, and shocks.

The Lagrangian of the above optimization problem is:

\[ L = E_T \sum_{T=0}^{\infty} \beta^{T-t} \left( \log(C_{T-t} - hC_{T-t-1}) - \frac{X^{T+t-1}}{1 + \phi} \right) - \mu'_{\tau}(y_{\tau+1}, y_{\tau}, y_{\tau-1}, e_{\tau}) \] \hspace{1cm} (2.40)

where \( y_{\tau} \) represents endogenous variables, \( e_{\tau} \) denotes stochastic shocks in the model and \( \mu'_{\tau} \) represents a vector of the discounted value of the Lagrange multipliers. In this chapter, the above control problem is used to characterize the properties of the optimal policy under commitment for the two NK models presented in this chapter (for more detail, see Appendix A).

### 2.5 Empirical results

#### 2.5.1 Calibration of the NK models

Table 2.1 presents parameter values which are used in the simulation of the aforementioned NK models. Most of these parameters are conventional except parameters related to the financial intermediaries. The current chapter uses conventional values for the discount factor \( \beta \), the depreciation rate \( \delta \), the share of capital in output \( \alpha \) and elasticity of substitution \( \varepsilon \). For other conventional parameters, the probability of not changing the prices of consumption goods, \( \Psi \), is 0.779 which reflects the fact where firms re-optimise prices of consumption goods once in every four quarters while price indexation, \( \Psi^p \), is 0.241.

Within the financial sector, the parameters related to the commercial banks, \( (\theta, \lambda, \varphi, \text{ and } n) \) are taken from Gertler & Karadi (2011). The values of these parameters are...
§2.5 Empirical results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Calibrated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.990</td>
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<td>$h$</td>
<td>Consumption habit parameter</td>
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<td>$\lambda$</td>
<td>Relative weight of labor in utility function</td>
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<tr>
<td>$\phi$</td>
<td>Inverse Frisch elasticity of substitution of labor supply</td>
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<tr>
<td>$e$</td>
<td>Elasticity of substitution for goods</td>
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<tr>
<td>$\Psi$</td>
<td>Probability of not changing prices of goods</td>
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</tr>
<tr>
<td>$\Psi_p$</td>
<td>Measure of price indexation of goods</td>
<td>0.241</td>
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<tr>
<td>$\rho_i$</td>
<td>Interest rate smoothing</td>
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<tr>
<td>$\kappa_1$</td>
<td>Inflation coefficient in monetary policy rule</td>
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<tr>
<td>$\kappa_2$</td>
<td>Markup coefficient (output gap proxy) in monetary policy rule</td>
<td>-0.125</td>
</tr>
</tbody>
</table>

Table 2.1: Parameters of the NK models

chosen by Gertler & Karadi (2011) to match a steady state interest rate spread of 100 basis points, a steady state leverage ratio of 4, and an average horizon of commercial bankers over a decade. Out of these data matching moment, the steady state interest rate spread of 100 basis points is equivalent to the average of three pre-2007 interest rate spreads pertaining to the spread between mortgage rates and government bonds, the spread between BAA corporate versus government bonds, and the spread between commercial papers and Treasury Bills rate.

2.5.2 Dynamic simulation

In order to gain further insight into the nature of optimal policy, this chapter simulates the effects of both financial and non-financial shocks in a calibrated version of the NK models for three monetary policy regimes: the ex-ante optimal policy with
commitment (the Ramsey policy), a strict inflation targeting monetary policy rule and a Taylor-type interest rate rule.

2.5.2.1 Non-financial shock: Total factor productivity shock

Figure 2.1 displays the impulse responses of selected variables to a negative 1 per cent productivity shock for the NK model with capital. In the absence of any financial frictions, output and consumption expenditures coincide under the alternative monetary regimes in the NK model with capital. This is in line with the findings of Christiano (2007) who shows that without wage rigidities the real impact of alternative policies coincide. Inflation in the case of the Taylor rule is higher because of the variable capital utilization which is in line with the findings of Christiano et al. (2005). It is also pertinent to note that inflation targeting policy performs better compared to the Taylor rule in combating inflation in the standard NK model which is in line with the findings of Goodfriend & King (2001) and Woodford (2003). In addition, there is a positive effect on labor which is mainly driven by a wealth effect. 7

The role of financial frictions

Figure 2.2 shows impulse responses of key variables of the NK model with capital and financial friction in response to 1 per cent negative productivity shock.

With financial intermediaries and financial frictions in the model, a negative productivity shock is propagated into the real economy through higher interest rate premium and the tightening of the balance sheet constraint of financial intermediaries. The fall in investment expenditure is higher in the NK model with financial frictions compared to the NK model with frictionless capital market across the monetary regime. Figure 2.2 also shows that the model economy performs better under Ramsey policy in response to a negative productivity shock by suggesting a larger real interest rate cut relative to alternative monetary policies. The relative performance of the model's economy under an inflation targeting regime is worse even when compared to the Taylor rule. Specifically, key financial variables such as the

---

7The separable utility function used in this chapter basically include wealth effect while the GHH preferences shut wealth effect on labor hours.
§2.5 Empirical results

Figure 2.1: The NK model with capital-1 per cent negative productivity shock
Figure 2.2: The NK Model with capital and with financial frictions- 1 per cent negative productivity shock
interest rate premium, net worth, leverage and asset prices have deteriorated more under the inflation targeting regime. The deterioration of the financial intermediary’s balance sheet combined with a higher interest rate premium has adversely affected investment and capital formation and thus real economy. This reflects the fact that maintaining price stability is not optimal in response to productivity shocks in the presence of financial frictions into the standard NK model. This is in line with the findings of Carlstrom et al. (2010) who show that an inflation targeting becomes sub-optimal in response to a productivity shock with the presence of credit distortion.

It is pertinent to note that there is volatility in the nominal interest rate and inflation under Ramsey policy at short horizons. This is driven by the two distortions: (i) sticky prices which lead to a variable mark-up, and (ii) agency costs which lead to an interest rate premium. The result shows that there is a tradeoff between inflation and interest rate premium at short horizons. In order to keep inflation constant, the model economy will have to face volatility of the interest rate premium. The Ramsey planner tolerates a little inflation (and thus mark-up variability) to lower the variability in the rate premium.

**Comparative analysis of Ramsey policy**

Figure 2.3 compares Ramsey policy for the NK models in response to 1 per cent negative productivity shock. The effect of this shock on output is propagated with the inclusion of capital formation and financial frictions. In the case of the basic NK model the real interest rate overshoot which has dampened the impact on consumption. In contrast, the undershooting of the interest rate in the NK model with capital and with financial frictions contains the adverse impact on balance sheet of the financial intermediaries. The stronger rebound in the investment expenditure and corresponding quicker rebuilding of the capital stock also weakens the adverse effects on output and consumption in the simulated economy.
Figure 2.3: The NK models with Ramsey policy- 1 per cent negative productivity shock
2.5.2.2 Financial shock

Figure 2.4 plots the responses of the selected variables from a capital quality shock for the NK model with capital and frictionless capital market. The magnitude of the capital quality innovation is negative one per cent with some degree of persistence as reflected by an autoregressive coefficient 0.66. The initial impact of this shock reduces the stock of capital available in the simulated economy. The initial impact on output leads to lower consumption and prompts households to work more. An increase in the interest rate, in turn, discourages investment expenditure and capital formation in the simulated economy.

The role of financial frictions

Figure 2.5 plots the responses of the key variables from a capital quality shock for the NK model with capital and financial frictions. In this model, this shock represents an exogenous source of an impairment of banks' assets which reduces the banks' net worth on impact. The magnitude of that impact is positively related to the leverage ratio. The initial contraction in net worth tightens the banks' endogenous balance sheet constraints which induces a fire sale of assets that further put downward pressure on asset values. The resulting financial distress then propagates into the real economy through a reduced capital stock and a contraction in investment.

It appears that the NK model with financial frictions explains a downturn which mimics a financial crisis with a varying degree of amplification effects. However, the contraction in real output and investment is relatively smaller under Ramsey policy due to a smaller uptick in the interest rate premium compared to an alternative monetary policy regime. In response to this shock, the loan portfolio of the financial intermediary deteriorates which has tightened the leverage constraint for financial intermediaries. The tightening of the leverage constraint causes a fire sale value of assets accompanied by an associated increase in the interest rate spread which, in turn, discourages investment and capital formation in the simulated economy.
Figure 2.4: The NK Model with capital and without financial frictions - 1 per cent capital quality shock
Comparing the performance of the model's economy under a monetary policy regime, Ramsey policy performs well by minimizing the impact of financial constraints and by reducing the interest rate spread considerably close to the steady state spread of 100 basis points. It is pertinent to mention a caveat about Ramsey policy in response to a large financial shock. A large financial shock (for instance, a 5 per cent shock to the capital quality in the NK model with capital and with financial frictions) will breach the non-negativity condition (zero lower bound) of the interest rate. Under the inflation targeting regime, the NK model with financial frictions performs relatively better compared to the Taylor rule with a smaller uptick in the interest rate spread increases. In addition, the impact on financial intermediaries' net worth and leverage is also relatively smaller in a strict inflation targeting regime compared to the Taylor rule. Consequently, these relative better financial conditions translate into a much weaker adverse effect on the real economy under the inflation targeting regime than it does under the Taylor rule. As shown by Figure 2.5, both financial intermediaries are slowly re-building their net worth throughout without any support from the central bank. It is also interesting to note that deep recession and high amplitude, emanating from financial frictions, causes households to work more because of wealth effect in comparison to Figure 2.4.

2.5.3 Welfare analysis

Let's define $V_0^{\text{ram}}$ as the welfare associated with the time-invariant equilibrium under the Ramsey policy conditional on a particular state of the economy in period 0 as:

$$
V_0^{\text{ram}} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log (C_t^{\text{ram}} - hC_{t-1}^{\text{ram}}) - \frac{\lambda L_t^{\text{ram},1+\phi}}{1+\phi} \right]
$$

where $C_t^{\text{ram}}$ and $L_t^{\text{ram}}$ are contingent plans for consumption and labor hours under Ramsey policy. Let's define $V_0^{\text{alt}}$ as the welfare associated with conditional welfare under alternative policy regimes as:
Figure 2.5: The NK model with capital and with financial frictions- 1 per cent negative capital quality shock
\[ V^\text{alt}_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t^\text{alt} - hC_{t-1}^\text{alt}) - \frac{\chi^{\text{alt},1+\phi}}{1+\phi} \right] \]

\[ = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t^\text{ram} - hC_{t-1}^\text{ram}) (1 - \lambda_c) - \frac{\chi^{\text{ram},1+\phi}}{1+\phi} \right] \] (2.42)

where \( \lambda_c \) measures conditional welfare cost, is the fraction of consumption under Ramsey policy \((C_t^\text{ram})\) that a household is willing to give up to attain the same conditional expectation of lifetime utility under an alternative policy regime \((C_t^\text{alt})\). A second-order measure of welfare cost \( \lambda_c \) is:

\[ \lambda_c = - (1 - \beta) \left[ g_v(x, 0) - g_v^\text{ram}(x, 0) \right] \star \left( \rho^2 \right) \star 100 \] (2.43)

Table 2.2: Welfare Analysis I: The NK model with capital

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<tr>
<th>K_\pi</th>
<th>K_y</th>
<th>Productivity Shock</th>
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<td>with Interest Rate Smoothing</td>
<td></td>
<td>1.59</td>
</tr>
<tr>
<td>without Interest Rate Smoothing</td>
<td></td>
<td>1.50</td>
</tr>
<tr>
<td>Non-Optimized Monetary Policy Rules</td>
<td></td>
<td>1.50</td>
</tr>
<tr>
<td>Taylor Rule</td>
<td></td>
<td>500</td>
</tr>
<tr>
<td>Simple Taylor Rule</td>
<td></td>
<td>1.50</td>
</tr>
<tr>
<td>Inflation Targeting Rule</td>
<td></td>
<td>500</td>
</tr>
</tbody>
</table>

Table 2.2 and Table 2.3 present the welfare analysis of five alternative monetary policy regimes along with Ramsey policy for the NK model with capital and the NK model with capital and financial frictions, respectively. Two of these policies are based on
the optimized interest rate rules and three are non-optimized monetary policy rules. These welfare losses are scaled by the variance of stochastic shocks, productivity and financial, that are calibrated as $\rho_a^2 = \rho_b^2 = (0.01)^2$. Column 4 and Column 6 exhibit raw conditional welfare scores for various monetary policy rules in response to a productivity shock and a financial shock, respectively. Column 5 and Column 7 on the other hand present the welfare cost, $\lambda_c$, for various monetary policy rules in response to a productivity and a financial shock, respectively.

As shown in by Column 4 of Table 2.2, the welfare associated with the inflation targeting rule in response to a productivity shock, under a perfect capital market assumption, the inflation targeting rule is the best policy compared to the Taylor rule in the standard NK model with capital in absence of financial frictions. The introduction of the financial distortion related to the financial intermediaries increases the welfare across all monetary policy regime with varying degree of magnitudes as shown by Column 4 in Table 2.3 compared to Column 4 in Table 2.2.

| Table 2.3: Welfare Analysis II: The NK model with capital and financial frictions |
|-------------------|-------------------|-------------------|
|                   | $K_x$  | $K_y$  | Productivity Shock | Financial Shock |
| Ramsey Policy     | -305.65 | 0 $\sigma_a^2$ | -306.50 | 0 $\sigma_f^2$ |
| Optimized Monetary Policy Rules |
| with Interest Rate Smoothing | 4.25 | -2.00 | -306.80 | 0.573$\sigma_a^2$ | -306.77 | 0.558$\sigma_f^2$ |
| without Interest Rate Smoothing | 4.90 | -1.68 | -306.74 | 0.543$\sigma_a^2$ | -306.70 | 0.523$\sigma_f^2$ |
| Non-Optimized Monetary Policy Rules |
| Taylor Rule      | 1.50 | -0.125 | -309.24 | 1.790$\sigma_a^2$ | -309.18 | 1.763$\sigma_f^2$ |
| Simple Taylor Rule | 1.50 | 0 | -312.12 | 5.232$\sigma_a^2$ | -312.04 | 5.193$\sigma_f^2$ |
| Inflation Targeting Rule | 500 | 0 | -306.75 | 0.548$\sigma_a^2$ | -306.72 | 0.533$\sigma_f^2$ |

As shown in Figure 2.3, optimized interest rate rules suggest an aggressive response to inflation compared to output in line with the findings of Schmitt-Grohe & Uribe
(2007). In terms of welfare cost, an optimized monetary policy rule with interest rate smoothing and an inflation targeting rule nevertheless perform well compared to other alternative policies and give a welfare level somewhat sub-optimal to those achieved under Ramsey policy under commitment in response to a productivity shock as shown in Table 2.3. In the case of a financial shock, an optimized monetary policy rule without interest rate smoothing and an inflation targeting rule provide a welfare level which is close to the welfare level of optimal monetary policy under commitment as reflected by the last two columns of Table 2.3.

2.6 Conclusion

Since the recent recession, the literature on financial frictions, business cycles and optimal monetary policy has undergone a major overhaul in various areas. This chapter has compared the performance of Ramsey monetary policy with alternative monetary policy regimes such as a strict inflation targeting monetary policy regime and the Taylor rule. In the case of a productivity shock, the main results of this chapter are: first, an inflation targeting rule emerges as the best policy compared to the Taylor rule in the standard NK model with capital as this policy approximates the welfare level associated with Ramsey policy, which is in line with the findings of Goodfriend & King (2001) and Woodford (2003); and second, in the NK model with capital and with financial frictions, which comprises of balance sheet effects, the relative performance of the model’s economy under optimized monetary policy rule with interest rate smoothing and an inflation targeting rule though is better compared to other alternative policies except Ramsey policy. However, welfare levels are somewhat sub-optimal compared to those achieved under Ramsey policy under commitment in response to a productivity shock, which shows that strict inflation targeting is sub-optimal in response to productivity shocks in the presence of financial frictions in a standard NK model as described by Carlstrom et al. (2010). In the case of a financial shock, the optimized monetary policy rule without interest rate smoothing and an inflation targeting rule provide a welfare level which approximates the welfare level associated with an optimal monetary policy. Last but not least, Ramsey policy under
commitment performs well, compared to alternative monetary policy regimes, by aggressively minimizing the impact of financial constraints on interest rate spread. However, the zero lower bound becomes binding with a large financial shock.
Chapter 3

Financial Shock Identification with Model-based Sign Restrictions

3.1 Introduction

The great recession that started in 2007 has not only changed the perspective of the macroeconomic literature about the role of financial intermediaries within the canonical New Keynesian (NK) monetary models but has also rekindled the debate about sources of business cycle fluctuations. More specifically, the traditional view, which assigns a prominent role to a technology shock in explaining business cycle fluctuations, has been questioned by the growing literature giving an alternative view about sources of economic fluctuations. This includes not only shocks that are related to the financial sector such as risk shocks (Christiano, Motto & Rostagno (2014)), credit supply shocks (Gilchrist & Zakrajsek (2012)), net worth shocks (such as: Kaihatsu & Kurozumi (2014), Graeve (2008), Christiano et al. (2014), Hirakata et al. (2011), CMR (2007), etc.), credit multiplier shocks (Peersman (2011)), financial sector efficiency shocks (Nolan & Thoenissen (2009) and Meh & Moran (2010)) but also shocks related to news, uncertainty and the marginal efficiency of investment (Justiniano et al. (2010)).

Within this context, the objective of this chapter is to quantify the relative importance of financial shocks pertaining to non-financial firms and financial firms for the US business cycle fluctuations using structural VARs identified with sign restrictions.\(^1\)

\(^1\)The sign restrictions approach was initially pioneered by Faust (1998), Canova and de Nicolo (2002)
Identifying sign restrictions are derived from the two competing NK models with financial frictions. The first model augments the standard NK model with a financial accelerator mechanism à la Bernanke et al. (1999), in which financial frictions pertain to non-financial firms (i.e., entrepreneurs) which gives a demand side perspective. The alternative model, which provides a supply side perspective, incorporates financial frictions in the balance sheet of the financial intermediaries into the standard NK model à la Gertler & Karadi (2011). The second contribution of this chapter is in the identification of a net worth shock (a wealth re-distribution shock from the financial intermediary to households) and a financial intermediary’s assets impairment (a capital quality shock), separately. This chapter is a first attempt to directly provide empirical evidence for the Gertler & Karadi (2011) and Gertler & Kiyotaki (2011) simulated models.

Using quarterly US data from 1980:Q1 to 2008:Q1, this chapter identifies four shocks: supply, demand, monetary and financial (entrepreneur’s net worth shock) by using the sign restrictions based on the NK model augmented with a financial accelerator mechanism. The financial shock is then separated into a financial intermediary’s net worth shock and an assets impairment shock (a capital quality shock) with sign restrictions based on the NK model with financial intermediaries. It is pertinent to note that a net worth shock and an assets impairment shock follow the same mechanism to affect the simulated economy although the magnitudes differ. Specifically, a negative shock to net worth or an assets impairment tightens the balance sheet constraint, which causes an increase in the interest rate premium, leading to subsequent distress in investment and output. To disentangle a net worth shock from an assets impairment shock, this chapter uses real consumption data. Basically, a negative net

\footnote{and Uhlig (2005) and further significant extensions of the methodology were proposed by Peersman (2005) and Fry & Pagan (2011).}

\footnote{Bernanke et al. (1999) incorporate a financial accelerator mechanism, through costly state verification, and show how fluctuations in borrowers’ net worth can amplify and propagate exogenous shocks. It is pertinent to note that most of the recent aforementioned empirical studies have augmented NK models with the financial accelerator.}

\footnote{Gertler and Karadi’s (2011) model pertains to a class of models in which financial frictions are defined over the supply side of credit through an agency problem between bankers and households that limits the amount of credit.
worth shock as described by Gertler and Karadi’s (2011) model is defined as a shock which re-distributes wealth from the representative financial intermediary to households. This in turn, increases real consumption for a few quarters in the simulated model devised by Gertler & Karadi (2011). However, real distress induced by a rise in interest rate premium subsequently reduces consumption expenditures.

The main three findings of this chapter are as follows. First, a financial shock, emanating both from entrepreneurs and financial intermediaries, is prominent in explaining fluctuations in real output and real investment, the interest rate spread, prices and real consumption. Second, the comparison of the variance decomposition of real output suggests that a financial shock related to the demand side is the major driver of short run output fluctuations while financial shocks related to financial intermediaries explain a relatively larger share of output fluctuations over the longer time horizon. Third, financial shocks related to financial intermediaries account for a relatively larger share of interest rate spread fluctuations at a shorter time horizon compared to a financial shock related to non-financial firms (the demand side of the credit), while a financial shock related to the demand side explains relatively larger variations in the interest rate spread across a longer time horizon.

This chapter is related to the recent strand of rich empirical literature which identifies shock(s) originating in the financial sector. Most prominent contributions in this area include Christiano et al. (2014), Kaihatsu & Kurozumi (2014), Gilchrist & Zakrajsek (2012) and Furlanetto, Ravazzolo & Sarferaz (2014). The first three papers estimate NK models which comprise of a broader array of shocks by using richer time series data for the US economy. Unlike a structural VAR identified with sign restrictions which is more flexible and imposes a minimum structure on the data, the Bayesian DSGE estimation approach imposes strong cross-equations restrictions. Like this current chapter, Christiano et al. (2014) augmented the NK model with a financial accelerator, however, their NK model comprises a richer shocks’ structures and, in turn, uses richer observables with sixteen time series in their data set. They identify two shocks related to the financial sector: a risk shock and a net worth shock. The
risk shock, which is similar to our financial shock (a capital quality shock), emerges as a prime driver of US business cycle fluctuations. Kaihatsu & Kurozumi (2014) also incorporate a financial accelerator in the NK model along with two distinct financial shocks: a finance premium shock and a net worth shock. By estimating their NK model with the Bayesian approach with 11 observables, Kaihatsu & Kurozumi (2014) show that the technology shocks dominantly explain output and investment fluctuations. On the other hand, a finance premium shock explains investment fluctuations, while a net worth shock surprisingly plays a negligible role in their paper. Using eight observables, including their novel survey based credit spread data, Gilchrist & Zakrajesk (2012) establish a prominent role for a net worth shock to explain output and investment fluctuations. This chapter is similar to that of Gilchrist & Zakrajesk (2012) in two respects: first, it also does not contain any investment shock in the NK model, and second, one of the NK models of the current chapter has also used a financial accelerator mechanism.

The current chapter is related to Furlanetto et al. (2014), who use a structural VAR approach instead of estimating the NK model using the Bayesian approach. Furlanetto et al. (2014) differ from this chapter in two aspects. First, Furlanetto et al. (2014) borrow sign restrictions from the recent NK models with financial factors. This chapter, specifically, incorporates a financial accelerator mechanism and a financial intermediary into the standard NK model. Second, Furlanetto et al. (2014) start from a baseline model with an identification of a financial shock and shows that the financial shock largely explains the output fluctuations. In addition to the baseline model, they also dis-entangle the financial shock into three different shocks: a housing shock, a credit shock and an uncertainty shock. In sharp contrast, this chapter dis-entangles the financial shock into a net worth shock and a financial intermediary assets impairment shock. The housing shock emerges as a prominent shock in their extended version of an estimated structural VAR.

This chapter is also related to the strand of literature which identifies a large number of shocks using the sign restrictions approach. Some notable contributions are made

The rest of the current chapter is structured as follows. Section 3.2 describes the two building blocks of this chapter in detail: a financial accelerator mechanism and financial intermediaries together with the description of the standard NK model. Section 3.3 presents a structural VAR model and sheds some light on the model-based sign restrictions identification scheme. This section also outlines the algorithm used in the estimation of the structural VAR models. Section 3.4 analyzes the impulse responses and variance decomposition of endogenous variables of the structural VAR models. Section 3.5 provides some robustness analysis and section 3.6 summarises the main findings of the chapter.

### 3.2 The Models

#### 3.2.1 Model 1: The NK model augmented with a financial accelerator

This section incorporates the financial accelerator mechanism of Bernanke et al. (1999) in the standard NK model. The standard NK model has the following standard features: habit formation in consumption; monopolistic competition in the goods market; capital adjustment costs; variable capital utilization; nominal rigidities in the spirit of Calvo staggered pricing; and price indexation to past inflation in line with the models of Christiano et al. (2005) and Smets & Wouters (2007).

Model 1 is inhabited by seven agents: households, non-financial firms, capital producers, entrepreneurs, retailers, government, and a central bank. Households comprise of workers and entrepreneurs, consume final goods and hold one-period risk-free bonds. Workers supply labor hours to the intermediate goods-producing firms and these firms hire capital along with labor hours to produce differentiated intermediate goods using a Cobb-Douglas production function. These goods are sold in monopolistically competitive markets to retailers who convert these intermediate goods into final consumption goods that are subject to nominal rigidities. Capital
producers not only produce new capital goods from investment which is subject to some adjustment cost but also refurbish undepreciated capital goods at no cost. Entrepreneurs purchase capital goods from the capital producers by partially using their net worth and borrowing by issuing state contingent securities. Government imposes a lump sum tax to finance its exogenous government expenditure. A central bank sets the nominal interest rate by following the Taylor rule. A complete model is presented in Table B1 of Appendix B.

3.2.1.1 Households

There is a continuum of households of length unity. Each household supplies labor hours, $L_t$, to the intermediate goods-producing firms and purchases final consumption goods, $C_t$, and stores its savings in one-period risk free bonds $D_t$. Let $E_t$ be the expectation operator conditional on information available at time period $t$, $h$ be an external habit formation parameter, $x$ be the dis-utility of labor supply in the utility function, and let $\phi$ be the inverse Frisch elasticity of substitution of labor supply. The representative household utility function is defined over consumption with external habit formation and number of hours employed. The household objective is to maximize its expected discounted utility by choosing a sequence of $C_t, L_t, D_t$:

$$
Max E_t \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t - hC_{t-1}) - \frac{X^{1+\phi}}{1+\phi} \right]
$$

subject to the following budget constraint as represented by Equation 3.2:

$$
C_t + D_t = W_t L_t + R_{t-1} D_{t-1} + \Pi^*_t - T_t
$$

where $D_t$ denotes the volume of bonds, which is one-period risk-free debt, $W_t$ denotes real wages in the goods-producing sector, $R_{t-1}$ represents a gross real interest rate on one-period risk-free bonds, $\Pi^*_t$ denotes the net transfers from non-financial
§3.2 The Models

firms and \( T_t \) denotes lump sum taxes. The FOCs are as follows:

\[
\lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h E_t \left( \frac{1}{C_{t+1} - hC_t} \right) \quad (3.3)
\]

\[
\lambda_t W_t = \chi L_t^\theta \quad (3.4)
\]

\[
E_t \beta \Lambda_{t,t+1} R_{t+1} = 1 \quad (3.5)
\]

with \( \Lambda_{t,t+1} = \frac{\lambda_{t+1}}{\lambda_t} \)

In Equation 3.3, \( \lambda_t \) measures the marginal utility of consumption when there is a habit formation in consumption. Equation 3.4 represents standard labor supply first order equilibrium condition and measures the dis-utility to the household for working in the goods-producing sectors. Equation 3.5 represents the standard Euler equation, with \( \Lambda_{t,t+1} \) representing a stochastic discount factor.

### 3.2.1.2 Non-financial firms: intermediate goods-producing firms

Non-financial firms comprise of two distinct types of firms: intermediate goods-producing firms and capital goods-producing firms. Intermediate goods-producing firms use labor, \( L_t \), and capital, \( K_t \), as inputs in the Cobb-Douglas technology to produce differentiated intermediate goods:

\[
Y_t(j) = A_t(U_t \xi_{t+1} K_t(j))^{\alpha}(L_t(j))^{1-\alpha} \quad (3.6)
\]

where \( \xi_t \) represents a capital quality shock and it follows a Markov process. This shock captures an exogenous source of a bank’s assets impairment shock. As Model 1 incorporates financial frictions pertaining to the non-financial firms, this shock is shut down in Model 1 while it will become operational in the following model.

\[
\xi_t = \rho \xi_{t-1} + \epsilon_t^\sigma \quad (3.7)
\]
The consumption goods-producing firms' profit maximization problem involves choosing a utilization rate, \( U_t \), and labor, \( L_t \), to satisfy the following equilibrium condition:

\[
\frac{Y_t}{U_t} - \delta'(U_t) = \frac{\alpha}{U_t} \xi_t K_t
\]  

(3.8)

\[
W_t = (1 - \alpha) \frac{Y_t}{L_t}
\]  

(3.9)

Given these FOCs, firms pay the ex-post return on capital inputs. Accordingly, \( R_{k,t+1} \) is given by:

\[
R_{k,t+1} = \frac{\alpha k_{t+1} + Q_{t+1} - \delta(U_{t+1})}{Q_t} \xi_{t+1}
\]  

(3.10)

### 3.2.1.3 Capital producers

A continuum of competitive capital-producing firms produces new capital, \( I_t \), using linear technology by using a fraction of final goods as inputs subject to a quadratic adjustment cost, \( [A(I_t, I_{t-1})] \). In addition to that, these firms also refurbish the undepreciated capital with no adjustment cost after purchasing from intermediate goods-producing firms at the end of the production period.

Capital producers maximize the following discounted profit by choosing \( I_t \) to solve:

\[
Max E_t \sum_{\tau=t}^{\infty} \beta^\tau \Lambda_{t,\tau} \left\{ Q_{\tau} I_{N\tau} - \left[ 1 + f \left( \frac{I_{N\tau}}{I_{N\tau-1}} \right) \right] I_{N\tau} \right\}
\]  

(3.11)

The following FOC of the above profit maximization problem represents the standard Tobin's Q equation that relates the real price of capital to the marginal adjustment cost of investment goods:

\[
E_t \beta^{\Lambda_{t,\tau}} \left[ Q_t - 1 - f \left( \frac{I_{N\tau}}{I_{N\tau-1}} \right) + I_{N\tau} f' \left( \frac{I_{N\tau}}{I_{N\tau-1}} \right) \frac{1}{I_{N\tau-1}} - E_t \beta^{\Lambda_{t,\tau}} \frac{I_{N\tau+1}}{I_{N\tau}} \right] f' \left( \frac{I_{N\tau+1}}{I_{N\tau}} \right) = 0
\]  

(3.12)

Capital producers earn profit only outside the steady state, which in turn redistributes a lump sum to the households. As shown by the following equation, the net investment, \( I_N \), which these capital producers carry out, is total investment, \( I_t \), net of
refurbishment of the un-depreciated capital purchased from goods-producing firms at the end of the production period:

\[ I_{N,t} = I_t - \delta(U_t)K_t \]  

(3.13)

The aggregate capital consists of undepreciated capital and investment subject to adjustment cost and it evolves according to the following equation:

\[ K_{t+1} = K_t(1 - \delta) + A(I_t, I_{t-1}) \]  

(3.14)

3.2.1.4 The financial accelerator mechanism

The following six equations serve as a standard way to introduce the Bernanke et al. (1999) financial accelerator in the standard NK model. Equation 3.15 describes how movements in net worth, \( N_{t+1} \), determine the cost of capital. More specifically, it shows that the interest rate firms pay on the capital financing, \( Q_t K_{t+1} \), which is determined by the function \( s(.) \) representing the ratio of the costs of external and internal finance. This equation shows that the cost of funds is an inverse function of firm's net worth, \( N_{t+1} \).

\[ E \left\{ R_{t+1}^k \right\} = -s \left( \frac{N_{t+1}}{Q_t K_{t+1}} \right) R_{t+1} \]  

(3.15)

Let \( V_t \) represent the actually accumulated profits or value of the firms and let \( \mu \int_0^{\omega_t} \omega R_t^k Q_{t-1} K_t dF(\omega) / Q_{t-1} K_t - N_{t-1} \) denote the ratio of default costs to quantity borrowed that in turn reflects an external finance premium. The external finance premium, in turn, depends on a monitoring cost, \( \mu \), a cut-off rate for defaults, \( \omega \), and \( F(\omega) \), a quarterly business failure rate. The value of the entrepreneur can be defined as follows:

\[ V_t = R_t^k Q_{t-1} K_t - \left( R_t + \mu \int_0^{\omega_t} \omega R_t^k Q_{t-1} K_t dF(\omega) / Q_{t-1} K_t - N_{t-1} \right) Q_{t-1} K_t - N_{t-1} \]  

(3.16)
\[ N_{t+1} = \Phi^{nw}_t \gamma V_t \]  

(3.17)

where \( \gamma \) denotes the probability of survival of the entrepreneur and \( \Phi^{nw}_t \) represents a financial/net worth shock, to the representative entrepreneur and it follows an AR(1) process. This shock also refers as an efficiency shock to financial intermediation as in Nolan & Thoenissen (2009).

\[ \Phi^{nw}_t = \rho_{nw} \Phi_{t-1} + \epsilon^{nw}_t \]  

(3.18)

Substituting \( V_t \) in the above equation yields the following difference equation for \( N_{t+1} \) that characterizes the endogenous variation in net worth:

\[ N_{t+1} = \Phi^{nw}_t \gamma \left[ R_t^k Q_{t-1} K_t - \left( R_t + \frac{\int_0^\infty \omega R_t^k Q_{t-1} dF(\omega)}{Q_{t-1} K_t - N_{t-1}} \right) Q_{t-1} K_t - N_{t-1} \right] \]  

(3.19)

Finally, the last equation of the financial accelerator is an equation representing entrepreneur consumption, \( C_f \), which is a small fraction of the aggregate net worth and varies proportionately with aggregate net worth as shown by the following equation:

\[ C_f = (1 - \gamma) V_t \]  

(3.20)

3.2.1.5 Retailers

Retailers, which incorporate nominal rigidities in the model, re-package the intermediate goods into final goods at no cost and sell these goods in a monopolistically competitive goods market. The decision problem for a retail firm is static and involves choosing a sequence \( \{ Y_t, y_t(j) \}_{j=0}^1 \) to maximize its profit:

\[ \Pi_f = P_t \left[ \int_0^1 y_t(j)^{\varepsilon_k} dj \right]^{\frac{1}{\varepsilon_k}} - \int_0^1 P_t(j) y_t(j) dj \]  

(3.21)

FOCs yield the following demand for each input and price:

\[ y^*_t(j) = \left( \frac{p^*_t(j)}{P_t} \right)^{(c)} - Y_t \]  

(3.22)
\[ P_t^* = \left[ \int_0^1 P_t(j)^1 - edj \right]^{-\frac{1}{1-e}} \] (3.23)

In a retail price setting of final goods, in every period, each intermediate goods-producing firm receives a random draw from the Bernoulli distribution with probability \((1 - \Psi)\), \(\Psi \in (0,1)\), and the firm receiving a successful draw can adjust its price. With probability \((\Psi)\), the firm gets an unsuccessful draw and cannot change its price. The retailers' pricing problem of final goods is to choose the optimal price \(P_t^*\) to solve:

\[
\max_{\Psi} \mathbb{E} \left[ \sum_{i=0}^{\infty} \beta^i \left( \frac{P_t(j)}{P_{t+i}} \right)^{Y_{t+i}} \right] \] (3.24)

\[
P_t^*(j) = \frac{\epsilon}{\epsilon - 1} \left[ \frac{E_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t+i} \lambda_{t+i} \lambda_{t+i} \epsilon_{t+i} \epsilon_{t+i}^{Y_{t+i}}} {E_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t+i} \lambda_{t+i} \epsilon_{t+i}^{Y_{t+i}}} \right] \] (3.25)

The above equation shows that the intermediate goods-producing firms will set their prices as a constant mark up over the ratio of an expression related to expected discounted nominal total cost to an expression to expected discounted real output. There is no \(j\) involved in the above equation, which implies that when a firm has a chance to change its price in period \(t\) it will choose the same price:

\[
P_t = \left[ (1 - \Psi)(P_t^*)^{1-e} + \Psi(P_t)^{1-e} \right]^{-\frac{1}{1-e}} \] (3.26)

### 3.2.1.6 Government and the monetary policy rule

The role of the government in this model is very simple: it imposes lump sum taxes to finance exogenous government expenditure as shown in Equation 3.27.

\[ G_t = T_t \] (3.27)

\[
i_t = (1 - \rho) \left[ i + \kappa_{\Pi} \Pi_t + \kappa_Y (\log Y_t - \log Y_{t}^*) \right] + \rho i_{t-1} + \epsilon_t^i \] (3.28)
Let $i_t$ be the nominal interest rate, $e_t$ be the exogenous shock to monetary policy, $i$ be the steady state nominal interest rate, $Y^*$ be the natural level of output and let $0 < \rho < 1$ be the interest rate smoothing parameter. Equation 3.28 and Equation 3.29 define the monetary policy rule and a standard Fisher equation, respectively.

$$1 + i_t = R_{t+1} \frac{P_{t+1}}{P_t} \quad (3.29)$$

### 3.2.1.7 Equilibrium

To close the model, apart from a labor market equilibrium, the following revised version of the goods market equilibrium condition is used to satisfy the resource constraint:

$$Y_t = C_t + C^*_t + [1 + f(\frac{L_t}{L_{t-1}})]L_t + G_t \quad (3.30)$$

where $f(\frac{L_t}{L_{t-1}})$ reflects physical adjustment cost with $f(1) = f'(1) = 0$ and $f''(\frac{L_t}{L_{t-1}}) > 0$.

### 3.2.1.8 Shocks

Apart from a monetary policy shock and a net worth shock defined earlier, there are two additional shocks in Model 1. Equation 3.31 and Equation 3.32 describe a total productivity shock and a government expenditure or demand shock, respectively:

$$a_t = \rho_a a_{t-1} + \epsilon_t^a \quad (3.31)$$

and

$$g_t = \rho_g g_{t-1} + \epsilon_t^g \quad (3.32)$$

### 3.2.2 Model 2: The NK model with financial intermediaries

Model 2 augments the baseline NK model with financial intermediaries in line with Gertler and Karadi’s (2011) model instead of the financial accelerator mechanism. Gertler & Karadi (2011) incorporate a financial friction through an agency problem.
that endogenously determines a balance sheet constraint on financial intermediaries. This constraint imposes a limit on the volume of funds which financial intermediaries can raise from the households in the form of deposits. For brevity, only the augmented component of Model 2, financial intermediaries, are described in detail in the following. The outline of the complete Model 2 is presented in Table B2 of Appendix B.

With the presence of financial intermediaries, the representative household now comprises two types of members at time $t$. The fraction, $(1 - f)$, of household members are workers and they supply labor in exchange for wages. The rest of the members, $f$, manage a financial intermediary and transfer their earnings back to households on their exit. It is assumed that there is perfect consumption insurance within the family. Individuals can change their occupation over time, a banker in $t$-period can become a worker in the next period with a probability $\theta$, with the average survival time of a banker of $\frac{1}{1-\theta}$. This assumption ensures that a banker does not become self-funded and also transfers the net worth of the banker to their respective households on exit.

### 3.2.2.1 Financial intermediaries

In Model 2, financial intermediaries have incorporated as described by Gertler & Karadi (2011) instead of the financial accelerator mechanism used in Model 1. To carry out an intermediary role, each bank raises deposits $D_{t+1}$ from households at the beginning of the period at $R_{t+1}$. The balance sheet of the commercial bank is as follows:

$$Q_t K_t = NW_t + D_{t+1}$$  \hspace{1cm} (3.33)

The left-hand side (LHS) of the above equation represents the assets side of the bal-

---

4There is also an alternative approach in the literature which splits households into two distinct groups, borrowers and lenders, with a varying degree of discount factor without any consumption insurance between these groups.
ance sheet while the right-hand side (RHS) represents the liability and inside equity/net worth. Let $R^k_{t+1}$ be stochastic returns on state-contingent lending to entrepreneurs and $R_{t+1}$ be a non-contingent gross return on deposit. Using Equation 35, the evolution of the commercial banker’s net worth can be expressed as:

$$NW_{j,t+1} = R_{kt+1}Q_tK_t - R_{t+1}D_{jt+1}$$  \hspace{1cm} (3.34)

Let $\beta^i\Lambda_{t,t+i}$ be an augmented stochastic discount factor, then a commercial banker’s object is to maximize the expected terminal net worth, given by:

$$V_{jt} = \text{Max} E_t \sum_{\tau=0}^{\infty} (1 - \theta)\theta^\tau \beta^{i+1}\Lambda_{t,t+1+i}(NW_{jt+1+i})$$  \hspace{1cm} (3.35)

The current chapter uses an agency problem in the model following Gertler & Karadi (2011) and Gertler & Kiyotaki (2011) to introduce financial frictions, which constrains the bank’s ability to obtain funds from households. In this environment, bankers choose to divert a fraction ($\lambda$) of total deposits and net worth for their personal benefit. Households recognise the fact that they may claim only $(1 - \lambda)$ of their funds in the event of a default, so they restrict the amount of funds they lend to commercial banks. Accordingly, this helps to define an incentive constraint for commercial banks to be in the intermediary business as follows:

$$V_{jt} \geq \lambda(D_{jt} + NW_{jt})$$  \hspace{1cm} (3.36)

The LHS measures the loss of filing for bankruptcy whereas the RHS measures the gains to the banker from diverting money. $V_{jt}$ can also be expressed as:

$$V_{jt} = v_t(Q_tK_{jt}) + \eta_tNW_{jt}$$  \hspace{1cm} (3.37)

with

$$v_t = E_t \{(1 - \theta)\beta\Lambda_{t,t+1}(R_{kt+1} - R_{t+1}) + \beta\Lambda_{t,t+1}\theta x_{t,t+1}v_{t+1}\}$$  \hspace{1cm} (3.38)

$$\eta_t = E_t \{(1 - \theta) + \beta\Lambda_{t,t+1}\theta z_{t,t+1}\eta_{t+1}\}$$  \hspace{1cm} (3.39)
where $x_{t,t+1} \equiv \left( Q_t K_{j,t+1} / Q_t K_{j,t} \right)$ denotes the gross growth rate of the assets and $z_{t,t+1} \equiv NW_{j,t+1} / NW_{j,t}$ represents the gross growth rate in net worth. Let $v_t$ measure the marginal expected discounted gain by expanding one unit of assets, with given net worth. Finally, $\eta_t$ measures the marginal expected discounted marginal gain by expanding one unit of net worth, holding the bank’s assets constant.

If the incentive constraint binds, then the banker’s issuance of assets depends directly on banker’s net worth:

$$Q_t K_{j,t} = \frac{\eta_t}{\lambda - v_t} NW_{j,t}$$

(3.40)

$$Q_t K_t = \varphi_t NW_t$$

(3.41)

where $\varphi_t$ denotes the leverage ratio of the commercial bank. Holding the bank’s net worth constant, expanding the assets of the bank raises the bankers’ incentive to divert funds for personal use. The binding incentive constraint puts a limit on the leverage ratio and restricts the banker’s ability to expand assets as the banker’s incentive to divert money equates to the cost of a bankruptcy. Since the leverage ratio does not involve bank-specific factors, this can be expressed as an aggregate across individual commercial banks by dropping the bank-specific $j$-term.

From the entry and exit of the bankers, an equation of motion for $NW_t$ can be derived by adding up the net worth of the existing commercial banker, $NWE_t$, and the new banker’s net worth, $NWN_t$. A fraction $\theta$ of bankers survives at $t - 1$ until the next period $t$, and it is assumed that the household transfers the fraction of $n/(1 - \theta^b)$ of the final assets of the exiting bankers to a new banker each period as a start-up fund. The following equation describes the law of motion for $NW_t$ as:

$$NW_t = \theta \left[ (R_{st} - R_t) \varphi_{t-1} + R_t \right] NW_{t-1} + n(Q_t K_{t-1})$$

(3.42)
3.2.3 Theoretical impulse responses

Table 3.1 presents the parameter values used to derive the theoretical impulse responses from the simulated models as shown in Figures 3.1-3.2. Most of these parameters are conventional except for the parameters related to the financial accelerator mechanism and financial intermediaries which are taken from Bernanke et al. (1999) and Gertler & Karadi (2011), respectively. On the basis of the theoretical impulse responses shown in Figure 3.1 and Figure 3.2, the signs restrictions of select variables to various shocks are reported in Table 3.2. Panel (a) exhibits sign restrictions of four shocks from Model 1: a TFP/technology shock $e^a$, a demand shock $e^s$, a nominal shock $e^i$ and a financial shock $e^\Phi$. These restrictions are subsequently used in the empirical SVAR(1) model to identify the structural shocks for the US economy. Panel (b) presents sign restrictions of five shocks derived from Model 2 including an additional capital quality/bank's assets impairment shock, $e^e$.7

Not surprisingly, these model-based sign restrictions are broadly in line with the standard literature related to the two NK models.8 As shown in panel (a), a total factor productivity shock is identified by imposing sign restrictions on the impulse responses of real output and prices. A negative TFP shock reduces output and causes an increase in the price level. A demand shock is identified as a shock which has a positive effect on output, interest rate and prices. An unanticipated monetary policy shock is identified as a shock that has a positive effect on the interest rate spread.

---

5The aforementioned NK models are log-linearized around the steady state and the resulting log-linearized models are simulated in dynare by using parameters values which are calibrated for the US economy.

6The values of non-standard parameters are calibrated by Gertler & Karadi (2011) to match three targets: a steady state interest rate premium of 100 basis points, a steady state leverage ratio of 4, and an average horizon of commercial bankers over a decade.

7Risk shock, MEI, and Investment-specific technology-IST shocks, used in the literature, are basically based on the DSGE models with financial accelerator which are related to the demand side of the financial frictions. In contrast, this chapter has used a net worth shock in the spirit of BGG1999 model in which non-financial firms' net worth plays an important role in the interest rate spread in turn investment. Essentially, I have used both investment and interest rate spread data to identify this shock. If I include ICT shock or risk shock, then it is difficult to dis-entangle the net worth shock from the risk shock, MEI, or ICT shock.

8Sign restrictions used in the chapter are robust to wide range of parameter combination.
Figure 3.1: Identification signs from Model 1
Figure 3.2: Identification signs from Model 2
The Models

### Table 3.1: Parameters of the NK models

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Calibrated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.990</td>
</tr>
<tr>
<td>$h$</td>
<td>Consumption habit parameter</td>
<td>0.815</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Relative weight of labor in utility function</td>
<td>3.409</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Inverse Frisch elasticity of substitution of labor supply</td>
<td>0.276</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Elasticity of substitution for goods</td>
<td>4.167</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Probability of not changing prices of goods</td>
<td>0.779</td>
</tr>
<tr>
<td>$\Psi_p$</td>
<td>Measure of price indexation of goods</td>
<td>0.241</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Interest rate smoothing</td>
<td>0.800</td>
</tr>
<tr>
<td>$\kappa_{II}$</td>
<td>Inflation coefficient in monetary policy rule</td>
<td>1.500</td>
</tr>
<tr>
<td>$\kappa_x$</td>
<td>Markup coefficient in monetary policy rule</td>
<td>-0.125</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Share of capital in total output</td>
<td>0.330</td>
</tr>
<tr>
<td>$U(\delta)$</td>
<td>Steady state capacity utilization</td>
<td>1.000</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Elasticity of depreciation w.r.t. utilization</td>
<td>7.200</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Inverse elasticity of net investment to the price of capital</td>
<td>1.725</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Coefficient of capital adjustment cost</td>
<td>0.500</td>
</tr>
<tr>
<td>$\rho_e$</td>
<td>Persistence of TFP shock</td>
<td>0.90</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>St. deviation of TFP shock</td>
<td>0.010</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>Persistence of capital quality shock</td>
<td>0.660</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>St. deviation of capital quality shock</td>
<td>0.010</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Persistence of government expenditure shock</td>
<td>0.900</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>St. deviation of government expenditure shock</td>
<td>0.010</td>
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</table>

**Related to Financial Accelerator in Model 1**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Calibrated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>Proportion of output lost to monitoring</td>
<td>0.02</td>
</tr>
<tr>
<td>$QK/N$</td>
<td>Capital to net worth ratio</td>
<td>2.0</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Survival probability of entrepreneurs</td>
<td>0.975</td>
</tr>
<tr>
<td>$F(\omega)$</td>
<td>Quarterly business failure rate</td>
<td>0.007</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Cut-off rate for default</td>
<td>0.487</td>
</tr>
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</table>

**Related to Financial Intermediaries in Model 2**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Calibrated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>Share of capital and deposits diverted by commercial banks</td>
<td>0.382</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Leverage ratio of commercial bank</td>
<td>4.0</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Survival rate of the commercial bankers</td>
<td>0.972</td>
</tr>
<tr>
<td>$n$</td>
<td>Proportional transfer to new banker</td>
<td>0.002</td>
</tr>
</tbody>
</table>

A negative effect on prices and real output. Turning to a financial shock, a negative shock to an entrepreneur’s net worth is identified as a shock that has a negative effect on investment due to a rise in interest rate spread which eventually leads to a reduction in output.

Panel (b) presents the sign restrictions derived from Model 2 to identify five shocks. It is pertinent to note that there are two financial shocks: a net worth shock, which represents a wealth re-distribution from financial intermediaries to households, and
Table 3.2: The Models-based sign restrictions

<table>
<thead>
<tr>
<th>Variables</th>
<th>$e^a$</th>
<th>$e^\ell$</th>
<th>$e^n$</th>
<th>$e^{nw}$</th>
<th>$e^z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel a: Model 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Output</td>
<td>$\geq 0$</td>
<td>$\geq 0$</td>
<td>$\leq 0$</td>
<td>$\leq 0$</td>
<td></td>
</tr>
<tr>
<td>Real Investment</td>
<td>?</td>
<td>$\geq 0$</td>
<td>$\geq 0$</td>
<td>$\leq 0$</td>
<td></td>
</tr>
<tr>
<td>Prices</td>
<td>$\leq 0$</td>
<td>?</td>
<td>$\leq 0$</td>
<td>$\leq 0$</td>
<td></td>
</tr>
<tr>
<td>Interest Rate Spread</td>
<td>?</td>
<td>$\geq 0$</td>
<td>$\leq 0$</td>
<td>$\geq 0$</td>
<td></td>
</tr>
<tr>
<td>Panel b: Model 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Output</td>
<td>$\geq 0$</td>
<td>$\geq 0$</td>
<td>$\leq 0$</td>
<td>$\leq 0$</td>
<td></td>
</tr>
<tr>
<td>Interest Rate</td>
<td>?</td>
<td>$\geq 0$</td>
<td>$\geq 0$</td>
<td>$\leq 0$</td>
<td></td>
</tr>
<tr>
<td>Real Consumption</td>
<td>?</td>
<td>$\geq 0$</td>
<td>?</td>
<td>$\geq 0$</td>
<td>$\leq 0$</td>
</tr>
<tr>
<td>Prices</td>
<td>$\leq 0$</td>
<td>?</td>
<td>$\leq 0$</td>
<td>$\leq 0$</td>
<td></td>
</tr>
<tr>
<td>Interest Rate Spread</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>$\geq 0$</td>
<td>$\geq 0$</td>
</tr>
</tbody>
</table>

1 Where $e^a$ denotes TFP shock, $e^\ell$ denotes demand shock, $e^n$ stands for nominal shock, $e^{nw}$ represents net worth shock, and $e^z$ represents financial shock.

2 Where $\geq 0$ denotes a positive or zero impact, $\leq 0$ denotes a negative effect or zero impact to a change in a shock and ? denotes that the impact on the variable is left unrestricted.

As shown in Figure 3.2, a net worth shock and a capital quality shock produce a downturn in the simulated economy which mimics a financial crisis with varying degrees of amplification effects. Specifically, both a net worth shock and a capital quality shock tighten the leverage constraint accompanied by an associated increase in the interest rate spread. A rise in the interest rate premium reduces output due to the lower capital formation in the simulated economy. It is pertinent to note that a rise in interest rate spread and its consequent real effect on real economy in response to a capital quality shock is relatively stronger compared to a net worth shock. Moreover, these two shocks follow the same underlying mechanism and have more or
less the same directional impact on most of the variables in the model. This poses a challenge as regards to how to dis-entangle these two shocks while applying the sign restrictions methodology.

A close inspection of theoretical impulse responses helps us to identify consumption expenditures as a potential data set to dis-entangle these two shocks. Specifically, a negative shock to a financial intermediary’s net worth produces a positive impact on consumption at a shorter horizon despite the tightening of balance sheet constraints and a rise in the interest rate premiums. On the other hand, a capital quality shock results in a reduction in consumption expenditures. This suggests using real consumption as an endogenous variable in the structural VAR model instead of using real investment. Therefore, a net worth shock is identified as a shock that reduces consumption at a short horizon, increases interest rate spread due to a balance sheet constraint and produces real distress with disinflation in the simulated economy. Finally, a capital quality shock is identified as a shock that increases interest rate spread, reduces output and consumption, and produces disinflation.

3.3 SVAR models with model-based sign restrictions

The VAR model is defined as follows:

$$B_0Y_t = c + A(L)Y_{t-1} + \varepsilon_t$$  \hspace{1cm} (3.43)

where $B_0$ represents the NxN matrix of contemporaneous coefficients, $Y_t$ is a vector of endogenous variables, $c$ is a Nx1 constants vector, $A(L)$ is a polynomial, and $\varepsilon_t$ is the NxT normally distributed structural errors with zero mean and $\Sigma$ is NxN variance-covariance matrix.

$$Y_t = BY_{t-1} + u_t$$  \hspace{1cm} (3.44)

where $B = A(L)B_0^{-1}, \quad u_t = B_0^{-1}\varepsilon_t \sim N(0,1) \quad \Sigma = E[u_t^t]u_t$
The main task is to map the reduced form error terms, $u_t$, in equation 44 back into the structural disturbances, $e_t$, in equation 43 using the sign restrictions approach. Sign restrictions which are used to uniquely dis-entangle and identify various structural shocks, are based on the competing NK models as exhibited by Table 3.2. Two structural VAR models are estimated in levels with four lags by using the US quarterly data from 1980:Q1 to 2008:Q1. The SVAR(1) model contains four endogenous variables: the log of real GDP, the log of personal expenditure (henceforth, PCE) data, the log of real investment, and the interest rate spread which is computed as the yield difference between BAA and AAA corporate bonds. This model identifies four shocks (supply shock, net worth shock, demand shock and monetary policy shock) with model-based sign restrictions based on model 1. The SVAR(2) model comprises five endogenous variables: the log of real GDP, the log of PCE, the federal fund rate, the log of real consumption and interest rate spread between BAA and AAA bonds yield. This model identifies five shocks (supply shock, capital quality shock, net worth shock, demand shock and monetary policy shock) with sign restrictions based on Model 2.

Given the large numbers of shocks identified, this chapter has used the householder-transformation methodology which shows that for a given value of $(B, \Sigma)$, the orthogonal matrix of NxN size is randomly drawn from a uniform distribution with QR decomposition. The algorithm used to identify various structural shocks is as follows:

**Step 1:** Estimate the VAR as represented by Equation 3.44.

**Step 2:** Estimate the BVAR by using $(B, \Sigma)$ with the conjugate prior belonging to a Normal Inverted-Wishart family.

**Step 3:** Draw 50000 $(B$ and $\Sigma)$ from multi-normal distribution and wishart random numbers posterior, respectively. Discard the first 25000 draws.
Step 4: Use the Cholesky decomposition to compute impulse response functions for the remaining 25000 draws left after burn in.

Step 5: Draw a NxN matrix of A from a standard normal distribution and compute the QR decomposition of matrix A.

Step 6: Generate candidate impulse responses by taking a joint draw from Cholesky impulse response functions and decomposition implied by Q.

Step 7: If the sign restrictions, described in Table 3.2, over the selected horizon are satisfied, store the impulse response functions. Otherwise, discard and iterate again step 5-7 until sign restrictions are satisfied.

Step 8: Repeat Steps 5-7, 1000 times and compute median responses together with 16th and 84th percentile of these retained draws.

3.4 Empirical results

3.4.1 Impulse responses analysis

Figure 3.3 exhibits the dynamic response of endogenous variables to various shocks in the SVAR(1) model. The solid line represents the median response of the 1000 successful draws together with the 68 per cent posterior confidence interval with two lines around the median response representing the 84 per cent and the 16 percent posterior mean, respectively.

As shown in Row 1, a positive productivity shock has a humped shape impact on real investment and output. This, in turn, reduces the price level as consistent with the sign restrictions. Impulse responses in Row 2 and Row 3 exhibit responses of output, investment, prices and interest rate spread to a demand shock and to a monetary policy shock, respectively. By construction, an unanticipated monetary policy softening leads to increase in both real output and inflation in line with the sign restrictions imposed to identify these shocks. A negative demand shock induces a
reduction in output, prices level and interest rate spread. The reduction in the interest rate spread results into an increase in aggregate investment in the economy. Row 4 in the figure presents the impact of endogenous variables to a positive shock to an entrepreneur’s net worth. This shock drives the interest rate spread down and a consequent increase in real output and investment in the economy.

Figure 3.4 exhibits the median impulse response of endogenous variables to various shocks in the SVAR(2) model together with a 68 per cent posterior confidence interval of the 1000 successful draws. Row 1 in Figure 3.4 presents the impact of a positive productivity shock on endogenous variables. A positive TFP shock is propagated into the real economy through a lower interest rate premium and the relaxing of the balance sheet constraint of the financial intermediaries, which increases real output and consumption, and leads to a reduction in the price level. An unanticipated monetary policy shock and a demand shock also produce effects on endogenous variables in line with the imposed sign restrictions as exhibited by Row 2 and Row 3, respectively.

As shown in Row 4 of Figure 3.4, impairment of banks’ assets increases the interest rate spread on impact. A rise in interest rate spread adversely affects capital formation in the economy amid higher borrowing cost. This results in a lower output and deflation in the economy. Row 5 of Figure 3.4 shows the effects of a negative net worth shock. A redistribution of wealth away from the financial intermediary sector produces real distress due to a rise in interest rate spread with a positive effect on consumption over the shorter horizon in line with the sign restrictions. It is pertinent to note that impulse responses of endogenous variables in response to various shocks closely follow the theoretical impulse responses despite the fact that sign restrictions are only used over a short horizon of 3 quarters.
Figure 3.3: SVAR(1) model with sign restrictions
Figure 3.4: SVAR(2) model with sign restrictions
3.4.2 Median impulse responses and orthogonality condition

A posterior median, a summary measure of all the feasible draws satisfying sign restrictions, reported in the last two figures does not satisfy the non-orthogonality condition as pointed out by Fry & Pagan (2011). The violation of the orthogonality condition makes the posterior median unfeasible to use for the variance decomposition analysis. In order to satisfy the orthogonality condition, Fry & Pagan (2011) have suggested the median threshold approach which aims at to find a unique model/draw from all the posterior draws whose impulse responses are closed to a summary measure of the posterior.

Figure 3.5 and Figure 3.6 exhibit the posterior median along with the optimal posterior median for the SVAR models presented earlier. The optimal posterior median basically represents a single model which minimizes the standardized squared distance of each feasible draw from the posterior median in the spirit of Fry & Pagan’s (2011) suggestion. As shown in Figure 3.5 and Figure 3.6, these two summary measures of the posterior draws are identical except for a few endogenous variables. The optimal posterior median of the SVAR models is used in the computation of the variance decomposition because it fulfills the orthogonality condition.

3.4.3 Variance decomposition analysis

Column 1 in Figure 3.7 presents the variance decomposition analysis of the endogenous variables to four shocks from the SVAR(l). A close inspection of Column 1 in Figure 3.7 suggests that a financial innovation emerges as a main source of variation in output, investment, prices and the interest rate spread for the US over the business cycle frequencies. Specifically, this shock accounts sizable variation at around 35 per cent and 30 per cent in output and investment respectively, at the shorter horizon. At the longer horizon, the relative importance of the financial (net worth) shock in explaining variation in output is higher than it is in the short run contribution at around 47 per cent. On the other hand, the net worth shock explains 11 per cent variation in investment over the longer time horizon.
Figure 3.5: SVAR(1): Posterior median and optimal posterior median
Figure 3.6: SVAR(2): Posterior median and optimal posterior median
The substantial contribution of a financial shock in output fluctuations echoes the marginal efficiency to investment (MEI) shock of Justiniano et al. (2010), which also explains 50-60 per cent variation in output for the US economy over the business cycle frequencies. Since an MEI shock is not identified in this chapter, it appears that a financial shock in this chapter has inherited the contribution from the MEI shock. In addition, the financial shock also explains a significant variation in the interest rate spread and prices. On the other hand, a technology shock explains 23 per cent variation in output in the short run and 25 per cent variation in the long run. This is in line with the SVAR literature that finds technology shocks account for a quarter of output fluctuations over the US business cycle. As for prices, the real effect of a monetary policy shock is very small in the short run. Demand shock, productivity shock and financial shock explain sizable variations in the prices both in the short run and the long run over the US business cycle.

Column 2 of Figure 3.7 shows a forecast error variance decomposition of the endogenous variables to five shocks identified in the SVAR(2) model. A net worth shock loses its prominence as a primary source of variation in the output and interest rate spread fluctuations to an additional financial shock, a capital quality shock. Specifically, a capital quality shock explains more than 30 per cent variations in the forecast error of the variances at a shorter horizon and 19 per cent variations in output over a longer time horizon. Overall, these two financial shocks jointly account for 36 per cent and 21 per cent variations in output fluctuations over the shorter horizon and longer horizon, respectively. In addition, the contribution of the TFP shock in explaining output fluctuations has increased substantially with the inclusion of a capital quality shock compared to the SVAR(1) model. Moreover, the combined effect of these two shocks in explaining inflation variation is also significant in explaining 55 per cent variations in prices over the longer horizon and around 48 per cent variations over the shorter time horizon. Overall, financial shocks drive the variations in all real endogenous variables in the model. A substantial contribution of these financial shocks in explaining a larger share endogenous variables fluctuation pro-
Figure 3.7: SVAR models with sign restrictions: Variance decomposition
vides empirical credence to the simulated NK models: Gertler and Karadi's (2011) model and Gertler, Kiyotaki and Queralto's (2011) model. These simulated models use a capital quality shock to produce real distress that mainly operates through the interaction of the balance sheet constraints and the consequent increase in the interest rate spread.

The comparison of the variance decomposition of output, the interest rate spread and prices in Column 1 and Column 2 in Figure 3.7 reveals the relative importance of financial shocks relating to the demand side and the supply side in explaining variation in these variables. Three findings stand out. First, financial shock related to the demand side is the major driver of output fluctuations both in the shorter and longer time horizon compared to the financial shocks related to financial intermediaries that explain a moderate variations in output fluctuations. Second, financial shocks related to financial intermediaries account for the relatively larger share of interest rate spread fluctuations at the shorter time horizon compared to a financial shock related to the demand side. The former financial shock explains a decent (27 per cent) variations in interest rate spread over a longer time horizon. Third, financial shocks related to the financial intermediaries explain the larger variations in prices over the shorter time horizons relative to a financial shock related to the demand side. On the other hand, financial shock related to the demand side explains relatively larger variations over the longer time horizon.

3.4.4 Sensitivity analysis

In order to ensure that the main findings of this chapter are robust, the following sensitivity analysis is carried out: (i) the estimation of the SVAR models with lower lag lengths of size 3 compared to the SVAR models estimated in the last section with lag lengths of size 4, (ii) an alternative definition of the interest rate spread (the difference between 20 years government bonds yield and effective federal fund rate) compared to the interest rate spread (the yield difference between BAA and AAA corporate bonds) used in the SVAR models presented in the last section, (iii) a fewer number
of quarters that the sign restrictions bind, and (iv) an alternative usage of diffuse priors for the Bayesian VAR estimation compared to the SVAR models presented in the last section which are estimated with conjugate priors assumption. The results of all these robustness checks except with diffuse priors assumption are reported in Table B.3 in Appendix B3. The findings of the sensitivity analysis broadly suggest that the main results remain robust except with a robustness check with less stringent sign restrictions bindings for two quarters. As shown by variance decomposition analysis in Table B.3, there is a moderation in explanation of variation in output, consumption and interest rate spread fluctuations of various shocks as compared to the baseline. The softening of the sign restrictions in terms of time horizon results in an acceptance of a less admissible set of the impulse responses which was not possible otherwise. Nonetheless, the overall results still hold and the financial shocks continue to explain a larger variations in output, prices, investment, consumption, and interest rate spread.

3.5 Conclusion

This chapter has quantified the importance of financial factors in economic fluctuations for the US economy from 1980:Q1 to 2008:Q1 through SVAR models identified with theoretical model-based sign restrictions. The main findings of this chapter are as follows. First, a financial shock emanating both from entrepreneurs and financial intermediaries is prominent in explaining fluctuations in real output, interest rate spread and prices. Second, the inclusion of financial shocks related to the financial intermediaries substantially increases the contribution of the technology shock in explaining output variations in both time horizons compared to the structural VAR model with a financial shock related to non-financial firms. Third, the real effect of an unanticipated monetary policy shock on prices is much smaller relative to the demand shock over the US business cycle. Fourth, a financial shock related to the demand side is relatively a major driver of output fluctuations over both time horizons.

9Results of variance decomposition of the SVAR model with diffuse priors are not reported because the change in the assumption of Bayesian priors from conjugate prior to diffuse priors makes no difference to the variance decomposition of endogenous variables.
while financial shocks related to the financial intermediaries also explain a modest share of output fluctuations. Fifth, financial shocks related to financial intermediaries account for a relatively larger share of interest rate spread fluctuations over both time horizons compared to a financial shock related to non-financial firms. Last but not least, a financial shock related to financial intermediaries explains relatively larger short run prices fluctuations variations compared to a financial shock related to the demand side while the latter explains relatively larger variations in prices over the longer time horizons.
Financial Frictions, Durable Goods, and a Two-Sector Financial Accelerator Model

4.1 Introduction

The great recession of 2007 has sparked interest in the analysis of the effects of financial frictions on the real economy within the canonical New Keynesian monetary models. In particular, the link between financial frictions and the business cycle had attracted voluminous amounts of literature even well before the recent financial crisis. However, the focus of the literature on this issue predates the financial crisis was confined to non-financial firms only that downplayed the role of financial intermediaries by treating them as a veil. In response to the crisis, a number of authors have sought to incorporate financial frictions in the supply side of credit by focusing on the role of financial intermediaries (e.g., Gertler & Karadi (2011), Dib (2010), Gertler & Kiyotaki (2011)).

1Prominent studies that incorporate financial frictions in the DSGE models include those developed by Gertler & Karadi (2011), Curdia & Woodford (2009) Christiano, Motto & Rostagno (2009), Gilchrist, Yankov & Zakrajsek (2009), and Negro, Eggertsson, Ferrero & Kiyotaki (2010).
2See, Bernanke, Gertler & Gilchrist (1999) for a brief review of much of the earlier work and Brunnermeier, Eisenbach & Sannikov (2011) for a survey of recent literature on the macro economy with financial frictions.
3Gertler & Karadi (2011) have extended the canonical monetary DSGE models proposed by Christiano, Eichenbaum & Evans (2005) and Smets & Wouters (2007) due to its better empirical fit of the US data and its rich representation of various features and with multiple shocks.
Although the micro-foundation of a financial intermediary has improved in the aforementioned models within a one-sector setting, these models relied on a counter-factual low leverage financial intermediary for capturing some of the dynamics of the great recession. In addition, these aforementioned models do not explain the collapse of durable goods spending (particularly, housing purchases) and the consequent sustained contraction in the US economy following the financial crisis. In addition, these models do not take into account two important stylized facts: (i) the dominant role of market-based financial institutions or investment banks in the overall credit extension to the US economy since the early 1990s' and (ii) the difference between the leverage ratio of conventional commercial banks and investment banks preceding the financial crisis. There is a strand of literature which incorporates a housing sector in general equilibrium models; however, it does not provide a framework for studying financial crises as these models do not incorporate the supply side of the financial intermediaries (see Davis & Heathcote (2005), Iacoviello & Neri (2010) and Forlati & Lambertini (2011)). Furthermore, with a parsimonious representation of financial intermediaries in Gertler & Karadi (2011), Dib (2010), Gertler & Kiyotaki (2011), these models do not take into account two important stylized facts: (i) the dominant role of market-based financial institutions or investment banks in the overall credit extension to the US economy since early 1990s (as shown by Figure 4.1); and (ii) the difference between leverage ratio of conventional commercial banks and investment banks which ranged between 15-20 and 20-30, respectively at the time of financial crisis. As Gertler & Karadi (2011) point out "...Ideally, one would like to extend the model to a multi-sector setting which accounts for the differences in leverage ratios. In the interest of tractability however, we stick with our one sector setting and choose a leverage ratio of four, which roughly captures the aggregate data".

Within this context and building on the work of Gertler & Karadi (2011), Gertler, Kiyotaki & Queralto (2012) and Gertler & Kiyotaki (2011), this chapter presents a Two-Sector New Keynesian Financial Accelerator model (henceforth, the Two-Sector FA model) by incorporating durable goods⁴, which not only takes into account the

⁴Durable goods, non-consumption goods and housing goods are used as synonyms in this chapter.
difference in leverage ratios of financial intermediaries but also extends the financial intermediaries sector. The Two-Sector FA model has the following additional features: (1) a non-consumption goods/housing sector is introduced into the model along with a consumption goods sector whereby households get utility from the housing services. Households secure mortgage financing from the financial intermediaries subject to a borrowing constraint, one of the financial frictions which induces a financial accelerator mechanism; (2) besides commercial banks, this chapter also incorporates investment banks into the model and both financial intermediaries’ balance sheets are subject to an endogenous constraint. The role of these financial intermediaries has increased immensely in the provision of credit to the US economy during the last two decades. The drying up of overall credit to the US economy would have been missed if one keeps analysing conventional commercial banks only. The investment banks in the model use their mortgage loans portfolio to originate securities and also raise equity from households.
The main results of this chapter are as follows. First, the model proposed by Gertler & Karadi (2011) (henceforth, the benchmark-FA model) fails to capture an underlying mechanism of the financial crisis at a high-leverage, particularly a modest increase in the interest rate spread. In addition, the Gertler & Karadi’s model because of its one sector setting also unable to explain for the differences in leverage ratios for commercial banks and investment banks. This necessitates the expansion of the benchmark-FA model into the Two-Sector FA model. Second, the impact on consumption expenditure is also more pronounced in the Two-Sector FA model compared to the benchmark-FA model due to the presence of collateral effects. Third, a relatively stronger financial intermediary’s balance sheet in the Two-Sector FA model underpins modest fallout of the monetary tightening shock despite the higher impact on the nominal interest rates. Fourth, it is pertinent to mention that the Two-Sector FA model also accords with the literature on a number of issues, particularly the behaviour of the relative price of durable goods, the expenditure on durable goods and the price puzzle following the monetary policy tightening shock. Last but not least, the chapter finds that the output recovery to trend would remain slow in our simulated economy as long as the relative price of non-consumption goods has not recovered to its trend.

This chapter is related to a vein of literature that studies housing sector spill over to the real economy (see Aoki, Proudman and Vlieghe (2004), Davis & Heathcote (2005), and Iacoviello & Neri (2010)). This chapter differs from this literature by mainly focusing on financial frictions related to a financial intermediaries’ balance sheet. This chapter also shares some features of the strand of literature on durable goods and collateral constraints (Erceg & Levin (2006) and Monacelli (2009)). As mentioned earlier, this chapter is closely related to the emerging literature which incorporates supply side financial frictions related to financial intermediaries into the general equilibrium model (see Gertler & Karadi (2011), Gertler, Kiyotaki & Queralto (2012) and Gertler & Kiyotaki (2011)).

The rest of this chapter is structured as follows, section 4.2 outlines the description
of the Two-Sector FA model with an enriched financial intermediary representation. Section 4.3 presents the calibration and dynamic analysis of the model. Section 4.4 presents the robustness analysis and section 4.5 concludes.

4.2 The Two-Sector FA model

The model economy is inhabited by six agents: households, non-financial firms, financial intermediaries, retailers, government, and a central bank. Within non-financial firms, there are three distinct types of firms: two intermediate goods producers (consumption goods and non-consumption/durable goods-producing firms) and capital goods-producing firms. Retailers comprise of two final goods producers: one is converting intermediate consumption goods into final consumption goods while the other retailer is converting intermediate non-consumption goods into final durable goods and also performing a mortgage issuer role at no cost. Two financial frictions are incorporated in the model: (i) a borrowing constraint on the household which limits the amount of mortgage financing to the expected value of non-consumption goods, and (ii) an endogenously determined balance sheet constraint on financial intermediaries. This constraint imposes a limit on the volume of funds which financial intermediaries can raise from households. Financial intermediaries in the model comprise of commercial banks (cb), investment banks (ib) and the frictionless interbank money market (ibm). To motivate the role of the interbank money market in the model, this chapter incorporates an idiosyncratic liquidity shock, following Kiyotaki & Moore (2008) and Kiyotaki & Moore (1997).

4.2.1 Households

There is a continuum of households of length unity. Each household supplies labor in the market, $L_t$, which is a constant elasticity of substitution (CES) aggregation of sector-specific labor inputs. Each household consumes final consumption goods, $C_t$, and utilizes services from non-consumption/housing goods, $H_t$, by taking mortgages, $B^H_t$, from financial intermediaries. A representative household stores its sav-
ings in deposits, $D_t$, held with commercial banks and in equity, $q_t e_t^b$, in investment banks. At time $t$, there are two types of members in the household. The fraction, $(1 - f)$, of household members are workers and they supply labor in exchange for wages. The rest of the members, $f$, manage a financial intermediary and transfer their retained earnings to households on exit. It is assumed that there is perfect consumption insurance within the family.\(^5\) Individuals can change their occupation over time and a banker in $t$-period can become a worker in the next period with a probability $\theta$, with the average survival time of a banker of $\frac{1}{1-\theta}$. This assumption ensures that a banker does not become self-funded and also transfers the net worth of the banker to their respective households on exit.

Let $E_0$ denote the expectation operator conditional on information available at time period $t$, $h$ denote external habit formation parameter, $j_t$ represent a demand preference shock of non-consumption goods, $\chi$ measure the dis-utility of labor supply in the utility function, $\phi$ denote the inverse Frisch elasticity of substitution of labor supply and $\zeta$ represent the sectoral elasticity of substitution of labor. Labor inputs are considered a perfect substitute if the value of $\zeta$ is zero while the positive value of that coefficient indicates that labor inputs between the two sectors are imperfect substitutes. The household preference representation allows external habit formation in consumption and also introduces housing services, $H_t$, à la Iacoviello (2005) and Iacoviello & Neri (2010).

$$\max E_0 \sum_{t=0}^\infty \beta^t \left[ \log(C_t - hC_{t-1}) + j_t \log H_t - \frac{\chi}{1 + \phi} \left( (L_t^C)^{1+\zeta} + (L_t^H)^{1+\zeta} \right)^{\frac{1+\zeta}{1+\zeta}} \right]$$ (4.1)

The household objective is to maximize its expected discounted utility by choosing a sequence of $C_t, L_t, D_t, q_t e_t^b, B_t^H, H_t$ subject to the following budget constraint and the

\(^5\)There is also an alternative approach in the literature which splits households into two distinct groups, borrowers and lenders, with a varying degree of discount factor without any consumption insurance between these groups.
The Two-Sector FA model

borrowing constraint as represented by Equation 4.2 and Equation 4.3:

\[ C_t + q_t^H \Delta H_t + R_{t-1}^H B_{t-1}^H + D_t + q_t^{ib} e_t^b = W_t^i L_t^i + R_{t-1} D_{t-1} + R_{t-1}^b q_{t-1}^{ib} e_{t-1}^b + B_t^H + \Pi_t^l - T_t \]

(4.2)

where \( q_t^H \) represents a relative price of non-consumption goods in terms of consumption goods, \( \Delta H_t \) represents a change in the household's housing wealth, \( R_{t-1}^H \) denotes the gross interest rate on a t-1 period mortgage debt \( B_{t-1}^H \), \( D_t \) denotes the volume of deposits, which is one-period riskless debt issued by commercial banks, \( q_t^{ib} \) denotes the unit price of equity \( e_t^b \) issued by investment banks to households, \( W_t^i \) denotes wages in the goods producing sectors, where \( i \in (C, H) \), \( R_{t-1} \) is a gross real interest rate on the deposit, \( R_{t-1}^b \) is the gross equity return, \( \Pi_t^l \) represents the net transfers from financial and non-financial firms and \( T_t \) denotes lump sum taxes.

Let \( m \) denote the loan-to-value (LTV) ratio for the mortgage and \( q_t^H = \frac{P_{mf}^H}{P_m} \) describe the relative price of non-consumption goods \( P_{mf}^H \) in terms of consumption goods \( P_m^C \). A representative household also faces the following borrowing constraint, which shows that the expected value of the housing stock must guarantee a repayment of mortgage and interest. \(^6\)

\[ R_t^H B_t^H \leq m_t E_t[q_{t+1}^H H_t] \]

(4.3)

The FOCs are as follows:

\[ \lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h E_t(\frac{1}{C_{t+1} - hC_t}) \]

(4.4)

\[ \lambda_t q_t^H = \frac{j_t}{H_t} + \beta E_t(\lambda_{t+1} q_{t+1}^H) + \mu_t \lambda_t m \beta E(q_{t+1}^H) \]

(4.5)

\(^6\)The borrowing constraint will always binding at the steady state under the reasonable parameters values for the US economy. But if one uses un-reasonable parameters values such as relatively lower or higher values for LTV ratio, this constraint will not bind.
\[
\lambda_t W_t^i = \lambda (L_t^C)^{1+\xi} + (L_t^H)^{1+\xi} L_t^\xi
\]  
\begin{equation}
\mu_t R_t^H = 1 - E_t \beta \lambda_{t+1} R_t^H
\end{equation}

\begin{equation}
E_t \beta \Lambda_{t,t+1} R_{t+1} = 1
\end{equation}

\begin{equation}
E_t \beta \Lambda_{t,t+1} R_t^f = 1
\end{equation}

with

\[
\Lambda_{t,t+1} = \frac{\lambda_{t+1}}{\lambda_t}
\]

and \( R_t^f = \frac{[Z_t + (1-\delta)q_{t+1}'\xi]}{q_t} \)

In Equation 4.4, \( \lambda_t \) measures the marginal utility of consumption when there is a habit formation in consumption. Equation 4.5 represents the marginal utility of housing services and that requires the household to equate the marginal utility of consumption to the shadow price of non-consumption goods. The shadow value of the non-consumption goods consists of three elements as shown by the right-hand side of Equation 4.5: (i) the direct utility to the household by consuming one additional unit of the non-consumption goods, (ii) the expected utility from the capital gain/loss on the account of the non-consumption goods purchased in the previous period, and (iii) the marginal utility to the household from the relaxation of the borrowing constraint. Equation 4.6 represents two standard first-order labor supply equilibrium conditions and measures the dis-utility to the household for working in the consumption and non-consumption goods-producing sectors. Equation 4.7 exhibits the modified Euler equation for the mortgage loan. In the case of \( \mu_t = 0 \), this equation reduces to a standard Euler equation.  

7 Equations 4.8 and 4.9 represent the standard Euler equations for deposits, and household’s equity to investment banks, respectively, with \( \Lambda_{t,t+1} \) representing a stochastic discount factor. These two equations also show an arbitrage relation between the gross returns on deposit and
4.2.2 Non-financial firms

Non-financial firms comprise of intermediate goods producers (consumption goods, \(Y^C_f\), and non-consumption goods, \(Y^H_f\)) and capital producers. Intermediate goods-producing firms sell their output to retailers of final consumption goods and final non-consumption goods (mortgage issuers).

4.2.2.1 Intermediate consumption goods-producing firms

Intermediate consumption goods-producing firms use labor, \(L^C_f\), and capital, \(K^C_f\), specific to consumption goods production as inputs in Cobb-Douglas technology to produce consumption goods:

\[
Y^C_f(j) = A_t(U^C_f(j)K^C_f(j))^{\alpha}(L^C_f(j))^{1-\alpha}
\]

(4.10)

where \(\xi_t\) represents a capital quality shock and it follows a Markov process in line with Gertler & Karadi (2011), Merton (1973), Gertler, Kiyotaki & Queralto (2012) and others. The consumption goods-producing firms' profit maximization problem involves choosing a utilization rate, \(U^C_f\), and labor, \(L^C_f\), to satisfy the following equilibrium condition:

\[
\frac{\alpha Y^C_f}{U^C_f} = \delta'(U^C_f)\xi_tK^C_f
\]

(4.11)

\[
W^C_f = (1 - \alpha) \frac{Y^C_f}{L^C_f}
\]

(4.12)

Given these FOCs, firms pay the ex-post return to capital inputs. Accordingly, \(R^C_{k,t+1}\) is given by:

\[
R^C_{k,t+1} = \left[\alpha \frac{Y^C_{t+1}}{\xi_{t+1}K^C_{t+1}} + Q_{t+1} - \delta(U^C_{t+1})\right] \xi_{t+1} \frac{Q_t}{Q_{t+1}}
\]

(4.13)
4.2.2.2 Intermediate non-consumption goods-producing firms

Firms in the non-consumption goods sector produce houses, $H_t$, by using capital specific to non-consumption goods, $K_t^H$, $L_t^H$, and land, $LD_t$, as inputs through Cobb-Douglas physical technology:

$$Y_t^H(j) = A_t(U_t^H \xi_t(j) K_t^H(j))^a (LD_t(j))^b (L_t^H(j))^{1-a-b} \delta_t$$  \hspace{1cm} (4.14)

As discussed earlier, \( \xi_t \) denotes a capital quality shock. A firm profit maximization problem involves choosing a utilization rate, $U_t^H$, labor, $L_t^H$, and a demand for land, $LD_t$, to satisfy the following equilibrium condition:

$$q_t^H U_t^H Y_t^H = \delta_t(U_t^H) \xi_t K_t^H$$  \hspace{1cm} (4.15)

$$R_t^{LD} = q_t^H \alpha_{ld} \frac{Y_t^H}{LD_t}$$  \hspace{1cm} (4.16)

$$W_t^H = q_t^H (1 - \alpha - \alpha_{ld}) \frac{Y_t^H}{L_t^H}$$  \hspace{1cm} (4.17)

For wage equalization across sectors, wages in each sector must satisfy the following equation:

$$(1 - \alpha) \frac{Y_t^C}{L_t^C} = q_t^H (1 - \alpha - \alpha_{ld}) \frac{Y_t^H}{L_t^H}$$  \hspace{1cm} (4.18)

Given these FOCs, firms pay the ex-post return to capital inputs. Accordingly, $R_{k,t+1}^H$ is given by:

$$R_{k,t+1}^H = \left[ q_{t+1}^H \alpha \frac{Y_t^H}{\xi_{t+1} K_t^{H+1}} + Q_{t+1} - \delta_t(U_t^{H+1}) \right] \xi_{t+1}$$  \hspace{1cm} (4.19)

Through arbitrage, the return on capital across sectors equalizes and each sector generates the same expected return on capital as shown by the following equation:

$$E_t \left\{ \left[ R_{k,t+1}^C - R_{k,t+1}^H \right] \beta A_{t+1} \right\} = 0$$  \hspace{1cm} (4.20)
The intermediate output $Y^H_t$ is also considered as an investment in non-consumption goods and supplements the total non-consumption goods stock which evolves as follows:

$$H_{t+1} = (1 - \delta)H_t + Y^H_t \quad (4.21)$$

### 4.2.2.3 Capital producers

A continuum of competitive capital goods-producing firms produce new capital, $I_t$, using linear technology by using a fraction of final goods as inputs subject to a quadratic adjustment cost, $[A(I_t, I_{t-1})]$. In addition to that, these firms also refurbish the undepreciated capital with no adjustment cost by purchasing it from intermediate goods-producing firms at the end of the production period.

Capital producers maximize the following discounted profit by choosing $I_t$ to solve:

$$\text{Max} \sum_{\tau=t}^{\infty} \beta^\tau \cdot \frac{Q_t I_N}{I_{N_t}} - \left[ 1 + f\left( \frac{I_{N_t}}{I_{N_{t-1}}} \right) \right] I_{N_t} \quad (4.22)$$

The following FOC of the above profit maximization problem represents the standard Tobin's Q equation that relates the real price of capital to the marginal adjustment cost of investment goods:

$$E_t \beta^\tau \cdot \left[ Q_t - 1 - f\left( \frac{I_{N_t}}{I_{N_{t-1}}} \right) + I_{N_t} f'\left( \frac{I_{N_t}}{I_{N_{t-1}}} \right) \frac{1}{I_{N_{t-1}}} - E_t \beta^\tau \cdot \left( \frac{I_{N_{t+1}}}{I_{N_t}} \right)^2 f'\left( \frac{I_{N_{t+1}}}{I_{N_t}} \right) \right] = 0 \quad (4.23)$$

Capital producers earn profits only outside the steady state, which in turn redistributes a lump sum to the households. As shown by the following equation, the net investment, $I_N$, which these capital producers carry out, is total investment, $I_t$, the net of refurbishment of the un-depreciated capital purchased from goods-producing firms at the end of the production period:

$$I_{N,t} = I_t - \delta(U_t) \left[ K_t^C + K_t^H \right] \quad (4.24)$$
The aggregate capital stock evolves according to the following equation:

$$K_{t+1} = K_t(1 - \delta) + A(I_t, I_{t-1})$$  \hspace{1cm} (4.25)

### 4.2.3 Retailers

The retailers, which incorporate nominal rigidities in the model, re-package the intermediate goods into final goods at no cost and sell these goods in a monopolistically competitive goods market. There are two types of retailers: the first deals with the retail sales of final consumption goods and the other performs a mortgage-issuer role. A mortgage issuer in the model purchases the non-consumption intermediate goods from the intermediate producers and re-packages these intermediate goods into mortgage assets. They also intermediate between households and financial intermediaries for mortgage loans at no cost. The decision problem for a retail firm in each sector is static and involves choosing a sequence $\{Y_t, y_t^i(j)\}_{i=0}^1$ to maximize its profit:

$$\Pi_t^i = P_t^l \left[ \int_0^1 y_t^i(j) \frac{e^{j \lambda_i}}{\lambda_i} dj \right]^{\delta^{-1}} - \int_0^1 P_t^l(j) y_t^i(j) dj$$  \hspace{1cm} (4.26)

where $i \in (c, h)$. FOCs yield the following demand for each input and price:

$$y_t^i(j) = \left( \frac{P_t^i(j)}{P_t^l} \right)^{\gamma_i} - Y_t^i$$  \hspace{1cm} (4.27)

$$P_t^i *= \left[ \int_0^1 P_t^l(j)^{1 - \epsilon^i} dj \right]^{-\epsilon^i}$$  \hspace{1cm} (4.28)

In a retail price setting of final goods, there are nominal rigidities in both sector; however, non-consumption goods are assumed to have relatively more flexible prices in line with what has been proposed by Barsky et al. (2007) and Iacoviello & Neri (2010). Specifically, each intermediate goods-producing firm receives a random draw from the Bernoulli distribution in every period with probability $(1 - \Psi)$, $\Psi \in (0, 1)$ and the firm receiving a successful draw can adjust its price. With probability $(\Psi)$, the firm gets an unsuccessful draw and cannot change its price. The retailer’s pricing
problem of final goods is to choose the optimal price $P^*_t$ to solve:

$$\begin{align*}
\text{Max} & \sum_{i=0}^{\infty} (\beta \Psi)^i \Lambda_{t+i} \left[ \frac{P^i(j)}{P^i_{t+i}} - \lambda^i_{t+i} \right] (\frac{P^i(j)}{P^i_{t+i}})^{-e} Y^i_{t+i} \\
& = \left[ \frac{E_t \sum_{i=0}^{\infty} (\beta \Psi)^i \Lambda_{t+i} \lambda_{t+i} P^i_{t+i} Y^i_{t+i}}{E_t \sum_{i=0}^{\infty} (\beta \Psi)^i \Lambda_{t+i} Y^i_{t+i}} \right]^{1-\epsilon}
\end{align*}$$

(4.29)

(4.30)

The above equation shows that the intermediate-goods producing firms will set their prices as a constant mark up over the ratio of an expression related to its expected discounted nominal total cost to an expression to their expected discounted real output. There is no $j$ involved in the above equation, which implies that when a firm has a chance to change its price in period $t$ it will choose the same price:

$$P^i_t = \left[ (1 - \Psi)(P^*_t)^{1-\epsilon} + \Psi(P^i_t)^{1-\epsilon} \right]^{1/\epsilon}$$

(4.31)

### 4.2.4 Financial intermediaries

This section contains the second major value addition of this chapter and incorporates a rich financial intermediary sector that comprises of commercial banks (cb), investment banks (ib) and a frictionless interbank money market (ibm). To motivate the role of an interbank money market in our model, an idiosyncratic liquidity shock is introduced in line with Kiyotaki & Moore (2008).

#### 4.2.4.1 Commercial banks

To carry out an intermediary role, each commercial bank raises deposits, $D_{jt+1}$, from households at the beginning of the period at $R_{t+1}$. After this, investment opportunities, $\gamma^{cb}$ and $\Xi^{cb}$, arise randomly in capital financing, $Q_i K_{jt}$, and mortgage financing, $m_{jt}^{H} Y_{jt}^{H}$, respectively. These idiosyncratic liquidity shocks in turn determine the volume of interbank money market borrowing and lending $L_{jt}^{ibm}$. For simplicity and tractability of the model, the calibration of $\gamma^{cb}$ and $\Xi^{cb}$ values implies that the commercial banks are more likely to receive higher investment opportunities for capital
financing than mortgage financing investment opportunities. An alternative explanation for this assumption is that the commercial banks are more efficient in evaluating and monitoring capital loans than mortgage loans. The balance sheet of the commercial bank is as follows:

\[ \gamma^{cb} Q_t K_t + \sum^{cb} m q^H_t H_t + \omega^{ibm} Ln_{jt}^{ibm} = NW_{jt} + D_{jt+1} \]  \hspace{1cm} (4.32)

The left-hand side (LHS) of the above equation represents the assets side of the balance sheet while the right-hand side (RHS) represents the liability and inside equity/net worth. Let \( R_{kt+1} \) be stochastic return on state contingent lending to entrepreneurs and \( R_{t+1}^{ibm} \) be the gross return on non-state contingent lending in the interbank money market; \( R_{t+1} \) be a non-contingent gross return on deposit; and \( R_{kt+1}^{ib} \) be a stochastic returns on state contingent mortgage lending to the household. Using Equation 4.32, the evolution of the commercial banker’s net worth can be expressed as:

\[ NW_{jt+1} = R_{kt+1} (\gamma^{cb} Q_t K_t + \sum^{cb} m q^H_t H_t) + R_{t+1}^{ibm} Ln_{jt}^{ibm} - R_{t+1} D_{jt+1} \]  \hspace{1cm} (4.33)

Let \( \beta^i \Lambda_{jt+1} \) be an augmented stochastic discount factor. Following this, a commercial banker’s object is to maximize the expected terminal net worth as given by:

\[ V_{jt} = Max_{E_t} \sum_{t=0}^{\infty} (1 - \theta) \beta^i \beta^{i+1} \Lambda_{jt+1+i} (NW_{jt+1+i}) \]  \hspace{1cm} (4.34)

Following Gertler & Karadi (2011) and Gertler & Kiyotaki (2011), this chapter uses an agency problem in the model to introduce a financial friction, which constrains the bank’s ability to obtain funds from households. In this environment, bankers choose to divert a fraction (\( \lambda \)) of total deposits and net worth for their personal benefit. Households recognize the fact that they may claim only \( (1 - \lambda) \) of their funds in the event of a default, so they restrict the amount of funds they lend to commercial banks. Accordingly, this helps to define an incentive constraint for commercial banks.
to be in the intermediary business as follows:

$$V_{jt} \geq \lambda(D_{jt} + NW_{jt}) \tag{4.35}$$

The LHS measures the loss of filing for bankruptcy whereas the RHS measures the gains to the banker from diverting money. $V_{jt}$ can also be expressed as:

$$V_{jt} = v_{t}^{cb}(\gamma^{cb}Q_{t}K_{jt} + \omega^{ibm}Ln_{jt}^{ibm} + \Sigma^{cb}m_{jt}^{H}H_{jt}) + \eta_{t}^{cb}NW_{jt} \tag{4.36}$$

with

$$v_{t}^{cb} = E_{t}\{ (1 - \theta)\beta \Lambda_{t,t+1}(R_{kt+1} - R_{t+1}) + \beta \Lambda_{t,t+1}\theta x_{t,t+1}^{cb} + \eta_{t+1}^{cb} \} \tag{4.37}$$

$$\eta_{t}^{cb} = E_{t}\{ (1 - \theta) + \beta \Lambda_{t,t+1}\theta z_{t,t+1}^{cb} + \eta_{t+1}^{cb} \} \tag{4.38}$$

where $x_{t,t+1} = (\gamma^{cb}Q_{t}K_{jt} + Ln_{jt}^{ibm} + \Sigma^{cb}m_{jt}^{H}H_{jt})/(\gamma^{cb}Q_{t}K_{jt} + Ln_{jt}^{ibm} + \Sigma^{cb}m_{jt}^{H}H_{jt})$ represents the gross growth rate in assets and $z_{t,t+1} = NW_{jt+1}/NW_{jt}$ represents the gross growth rate in net worth. $v_{t}^{cb}$ measures the marginal expected discounted gain by expanding one unit of assets with given net worth. Finally, $\eta_{t}^{cb}$ measures the marginal expected discounted marginal gain by expanding one unit of net worth, holding a bank’s assets constant.

If the incentive constraint binds, then the banker’s issuance of assets depends directly on net worth:

$$\gamma^{cb}Q_{t}K_{jt} + \omega Ln_{jt}^{ibm} + \Sigma^{cb}m_{jt}^{H}H_{jt} = -\frac{\eta_{t}^{cb}}{\lambda - v_{t}^{cb}}NW_{jt} \tag{4.39}$$

$$\gamma^{cb}Q_{t}K_{t} + \omega Ln_{t}^{ibm} + \Sigma^{cb}m_{t}^{H}H_{t} = \varphi_{t}^{cb}NW_{t} \tag{4.40}$$

where $\varphi_{t}^{cb}$ is the leverage ratio of the commercial bank. Holding the bank’s net worth constant, expanding the assets of the bank raises the bankers’ incentive to divert funds for personal use. The binding incentive constraint puts a limit on the leverage.
ratio and restricts the banker's ability to expand assets as the banker's incentive to divert money equates to the cost of a bankruptcy. Since the leverage ratio does not involve bank-specific factors, it is expressed as an aggregate across individual commercial banks.

From the entry and exit of the bankers, an equation of motion for $NW_{Icb}^t$ can be derived by adding up the net worth of the existing commercial banker, $NW_{Ecb}^t$, and a new banker's net worth, $NWN_{cb}^t$. A fraction $\theta_{cb}$ of bankers survives at $t - 1$ until the next period $t$, and it is assumed that the household transfers the fraction of $n/(1 - \theta_{cb})$ of the final assets of the exiting bankers to a new banker each period as start-up fund. The following equation describes the law of motion for $NW_{Icb}^t$ as:

$$NW_{Icb}^t = \theta_{cb} \left[ (R_{Ht} - R_t)q_{Icb}^{cb} + R_t \right] NW_{Ecb}^{t-1} + n(\gamma_{cb}Q_tK_{t-1} + \omega L_n^{b1} + \Xi^{cb}Q_I^{h}H_{t-1})$$

(4.41)

4.2.4.2 Investment banks

A representative investment commercial bank in the model is funded by households through equity, $q_t^{cb}e_{lt}$, and net worth (retained earnings), $NW_{fb}^t$. Given the random arrival of investment opportunities for capital and mortgage financing, the represented investment bank underwrites $(1 - \gamma_{cb})$ of capital to entrepreneurs, $Q_tK_t$. Moreover, a representative investment bank in the model is engaged in a simple securitization of the pool of mortgage loans $S_{Icb}^{b1} = (1 - \Xi^{cb})m_{q_I^{H}}H_t$ financed from their net worth, equity and a borrowing from the interbank money market. Moreover, a representative investment bank also borrows from commercial bank $Ln_{fb}^{b1}$ at a gross rate of return $R_{Icb}^{b1}$.

The balance sheet of a representative investment bank is as follows:

$$S_{Icb}^{b1} + (1 - \gamma_{cb})Q_tK_t = NW_{fb} + q_t^{cb}e_{lt} + \omega L_n^{b1}$$

(4.42)
where the LHS shows the asset side and the RHS shows the liability side of the investment bank balance sheet and $S^b$ is the number of total securities issued from the mortgage financing and $L^{ibm}$ is the volume of interbank money market borrowing. Let $R_{k+1}$ be a stochastic return on state-contingent lending to entrepreneurs, $R_{e+1}$ be a state contingent gross return on equity, and $R^{lbm}$ be a non-state contingent interest rate on borrowing from the interbank money market. The evolution of the investment banker’s net worth can be written as:

$$NW_{j,t+1} = R_{k+1}^b + (1 - \gamma^b)Q_tK_t - R^{ibm}L_t^{ibm} - R_{e+1}^b\eta_{e+1}^{ib}$$  \hspace{1cm} (4.43)

where $\beta^b\Lambda_{t+1}$ represents a stochastic discount factor, the investment banker’s object is to maximize the expected terminal net worth, given by:

$$V_{j,t}^{ib} = \text{Max}_{E_t} \sum_{0}^{\infty} (1 - \theta^b)(\theta^b)^i \beta^{i+1} \Lambda_{t,i+1}^{ib}NW_{j,t+1+i}$$  \hspace{1cm} (4.44)

In line with the commercial bank representation discussed earlier, an investment bank also faces an agency problem and the investment banker chooses to divert a fraction $(\lambda^b)$ of net worth and equity. Accordingly, an incentive constraint for an investment banker can be defined as follows:

$$V_{j,t}^{ib} \geq \lambda^b (\eta_{e,t}^{ib} + NW_{j,t}^{ib})$$  \hspace{1cm} (4.45)

where the LHS measures the loss of filling a bankruptcy whereas the RHS measures the gains from diverting money. $V_{j,t}^{ib}$ can be written as:

$$V_{j,t}^{ib} = v_t^{ib}(S_{j,t}^{ib} + (1 - \gamma^b)Q_tK_t - \omega L^{ibm}_{j,t}) + \eta_t^{ib}NW_{j,t}$$  \hspace{1cm} (4.46)

with

$$v_t^{ib} = E_t \left\{ (1 - \theta^b)\beta\Lambda_{t+1}^{ib}R_{k+1}^{ib} - R^{ibm}_{j,t+1} \right\}$$  \hspace{1cm} (4.47)

$$\eta_t^{ib} = E_t \left\{ (1 - \theta^b) + \beta\Lambda_{t+1}^{ib} \right\}$$  \hspace{1cm} (4.48)
where $R_{t+1}^{ibm}$ represents the average funding cost, $x_{t+1}^{ib} = (S_{j,t+1}^{ib} + (1 - \gamma^{cb})Q_tK_{t+1}) / (S_{j,t}^{ib} + (1 - \gamma^{cb})Q_tK_t)$ represents a gross growth rate of assets and $z_{t+1}^{ib} = NW_{j,t+1}^{ib} / NW_{j,t}^{ib}$ denotes the gross growth rate of net worth. Where $v_{t}^{ib}$ measures the marginal expected discounted marginal gain by expanding one unit of assets, with given net worth and $\eta_{t}^{ib}$ measures the marginal expected discounted marginal gain by expanding one unit of net worth, holding the bank’s assets constant.

As discussed in the case of a commercial bank, the size of the assets of an investment banker will depend positively on his/her equity capital when the incentive constraint binds:

\[
S_{j,t}^{ib} + (1 - \gamma^{cb})Q_tK_t - \omega LN_{j,t}^{ibm} = \frac{\eta_{t}^{ib}}{v_{t}^{ib}}NW_{j,t}
\]

\[
S_{t}^{ib} + (1 - \gamma^{cb})Q_tK_t - \omega LN_{t}^{ibm} = \varphi_{t}^{ib}NW_{t}^{ib}
\]

where $\varphi_{t}^{ib}$ is the leverage ratio of the investment bank. The incentive constraint also puts a limit on the leverage ratio of an investment bank and an agency problem leads to an endogenous capital which constrains the investment bank’s ability to expand assets. Similar to a commercial bank, the leverage ratio of an investment bank also does not involve firm-specific factors and can be expressed as aggregated across individual investment banks as shown by the above equation.

An equation of motion for $NW_{t}^{ib}$ comprises the net worth of existing investment bankers, $NWE_{t}^{ib}$, and the new banker, $NWN_{t}^{ib}$. Following Gertler & Karadi (2011), it is assumed that the household in each period transfers a fraction of $n / (1 - \theta^{ib})$ of terminal assets of the exiting investment banker to a new investment banker as a start up fund. An equation of motion for $NW_{t}^{ib}$ can be written as:
\[ NW_{t}^{ib} = \frac{\theta^{ib} \left[ (R_{kt} - R_{t}) q_{t-1}^{ib} + R_{t} \right] NW_{t-1}^{ib}}{NW_{t-1}^{ib}} + n \left( \gamma_{t-1}^{ib} + (1 - \gamma^{cb}) Q_{t} K_{t-1} - \omega L_{t}^{ibm} \right) \]

(4.51)

4.2.4.3 Interbank bank market

Given the random arrival of investment opportunities of both capital and mortgage financing for financial intermediaries, it is assumed that commercial banks are the net lenders while investment banks are net borrowers in the interbank market. In line with Gertler, Kiyotaki & Queralto (2012), \( \omega \in (0, 1] \) measures the efficiency of the interbank market. With \( \omega = 1 \), the interbank market operates frictionlessly in the sense that a borrowing investment bank cannot divert assets financed from interbank money market borrowing. For tractability of the model, it is assumed that there is a frictionless interbank money market as \( \omega = 1 \) in the model.\(^8\)

4.2.5 Government and the monetary policy rule

The role of the government in this model is very simple: it imposes lump sum taxes to finance exogenous government expenditure as shown in Equation 4.52:

\[ G_{t} = T_{t} \]

(4.52)

\[ i_{t} = (1 - \rho) \left[ i + \kappa_{\Pi} \Pi_{t} + \kappa_{y} (\log Y_{t} - \log Y^{*}) \right] + \rho i_{t-1} + \epsilon_{t} \]

(4.53)

where \( i_{t} \) represents nominal interest rate, \( \epsilon_{t} \) denotes the exogenous shock to monetary policy, \( i \) denotes the steady state nominal interest rates, \( Y^{*} \) denotes the natural level of output and \( 0 < \rho < 1 \) is the interest rate smoothing parameter. It is pertinent to note that the monetary policy rule will be abandoned in response to a financial

\(^80 < \omega < 1 \) shows friction in the interbank market. The lower the value of \( \omega \) from one, the higher will be the friction in the interbank money market (for more detail, see Gertler, Kiyotaki & Queralto (2012)).
criterion. In a financial crisis, the central bank uses various un-conventional monetary policies including credit policies as recently adopted by the US Federal Reserve to support the slow economic recovery. Equation 4.53 and Equation 4.54 define the monetary policy rule and a standard Fisher equation, respectively.

\[
1 + i_t = R_{t+1} \frac{P_{t+1}}{P_t}
\]  

(4.54)

4.2.6 Equilibrium

The following market equilibrium conditions in labor, goods markets, deposits, equity, and interbank markets are used to close the model. Setting sectoral elasticity of substitution of capital \( \zeta = 0 \), labor market equilibrium conditions require that demand for labor in each sector equalizes labor supply as represented by the following equations:

\[
(1 - \alpha) \frac{Y^C_t}{L^C_t} \lambda_t = \chi L^p_{C,t}
\]

(4.55)

\[
q^H_t (1 - \alpha - \alpha_{id}) \frac{Y^H_t}{L^C_t} \lambda_t = \chi L^p_{H,t}
\]

(4.56)

Goods market equilibrium condition or a resource constraint of the economy requires to satisfy the following equation:

\[
Y_t = Y^C_t + Y^H_t = C_t + [1 + f(\frac{I_t}{I_{t-1}})]I_t + G_t
\]

(4.57)

where \( f(\frac{I_t}{I_{t-1}}) \) reflects a physical adjustment cost with \( f(1) = f'(1) = 0 \) and \( f''(\frac{I_t}{I_{t-1}}) > 0 \).

The equilibrium for the outside equity market requires that the demand by households of these equities equals the supply issued by the investment bank:

\[
q_t \tilde{d}_t = x_t \left[ S^b_t + (1 - \gamma^c_b) Q_t K_t - \omega^b m L^n_{t}^b \right]
\]

(4.58)

where \( x_t \) measures the outside equity to assets ratio of the investment banks.
4.2.7 Shocks

Apart from a monetary policy shock, there are four additional shocks in the model. As discussed earlier in the chapter, the capital quality shock is defined in Equation 4.59. Equation 4.60 and Equation 4.61 define the mortgage portfolio valuation (mpv) shock and non-consumption goods demand preference shock, respectively:

\[ \xi_t = \rho \xi_{t-1} + \epsilon_t^\xi \]  

(4.59)

\[ mpv_t = (\rho_{mpv})_{t-1} + \epsilon_{t}^{mpv} \]  

(4.60)

\[ j_t = \rho_j j_{t-1} + \epsilon_t^j \]  

(4.61)

4.3 Quantitative Results

4.3.1 Calibration of the FA models

Table 4.1 presents the parameter values which are used in the simulation of the aforementioned models. Most of these parameters are conventional except the parameters related to the financial intermediaries. The current chapter uses conventional values for the discount factor \( \beta \), the depreciation rate \( \delta \), the share of capital in output \( \alpha \) and elasticity of substitution \( \epsilon^l \). For other conventional parameters, the probability of not changing the prices of consumption goods, \( \Psi_C \), is 0.779 which reflects the fact that prices of consumption goods are re-optimised once every four quarters while price indexation, \( \Psi_C^p \), is 0.241.

Within the financial sector, the values of non-standard parameters related to the commercial banks, \( (\theta^{cb}, \lambda^{cb}, \phi^{cb}, \text{ and } \mu^{cb}) \), are calibrated by Gertler & Karadi (2011) to match three targets: a steady state interest rate premium of 100 basis point, a steady state leverage ratio of 4, and an average horizon of commercial bankers over a decade. The steady state spread reflects the pre-2007 average of three interest rate
Table 4.1: Parameters of the Two-Sector FA Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Calibrated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.990</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Intertemporal elasticity of substitution</td>
<td>1.000</td>
</tr>
<tr>
<td>$h$</td>
<td>Consumption habit parameter</td>
<td>0.815</td>
</tr>
<tr>
<td>$X$</td>
<td>Relative weight of labor in utility function</td>
<td>3.409</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Inverse Frisch elasticity of substitution of labor supply</td>
<td>0.276</td>
</tr>
<tr>
<td>$m$</td>
<td>Loan to value ratio</td>
<td>0.900</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of capital in consumption and Non-consumption output</td>
<td>0.300</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Share of Land in Non-consumption output</td>
<td>0.100</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Sectoral elasticity of substitution of capital</td>
<td>0.000</td>
</tr>
<tr>
<td>$U(\delta)$</td>
<td>Steady state capacity utilization</td>
<td>1.000</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\epsilon_c$</td>
<td>Elasticity of depreciation wrt utilization</td>
<td>7.200</td>
</tr>
<tr>
<td>$i_i$</td>
<td>Inverse elasticity of net investment to the price of capital</td>
<td>1.725</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Coefficient of capital adjustment cost</td>
<td>0.500</td>
</tr>
<tr>
<td>$e^c$</td>
<td>Elasticity of substitution for consumption goods</td>
<td>4.167</td>
</tr>
<tr>
<td>$e^H$</td>
<td>Elasticity of substitution for non-consumption goods</td>
<td>4.167</td>
</tr>
<tr>
<td>$\Psi_C$</td>
<td>Probability of not changing prices of consumption goods</td>
<td>0.779</td>
</tr>
<tr>
<td>$\Psi_P$</td>
<td>Measure of price indexation of consumption goods</td>
<td>0.241</td>
</tr>
<tr>
<td>$\Psi_H$</td>
<td>Probability of not changing prices of non-consumption goods</td>
<td>0.330</td>
</tr>
<tr>
<td>$\Psi_{ph}$</td>
<td>Measure of price indexation of non-consumption goods</td>
<td>0.670</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Interest rate smoothing</td>
<td>0.800</td>
</tr>
<tr>
<td>$\kappa_{II}$</td>
<td>Inflation coefficient in monetary policy rule</td>
<td>1.500</td>
</tr>
<tr>
<td>$\kappa_x$</td>
<td>Markup coefficient in monetary policy rule</td>
<td>-0.125</td>
</tr>
</tbody>
</table>

Parameters related to Shocks

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Calibrated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_j$</td>
<td>Non-consumption goods demand preference shock</td>
<td>0.900</td>
</tr>
<tr>
<td>$\sigma_j$</td>
<td>St. deviation of demand preference shock</td>
<td>0.050</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>Capital quality shock</td>
<td>0.660</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>St. deviation of capital quality shock</td>
<td>0.050</td>
</tr>
<tr>
<td>$\rho_{mpv}$</td>
<td>Mortgage Portfolio Valuation</td>
<td>0.660</td>
</tr>
<tr>
<td>$\sigma_{mpv}$</td>
<td>St. deviation of Mortgage Portfolio Valuation shock</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Parameters related to Financial Intermediaries

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Calibrated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^{cb}$</td>
<td>Share of capital financing by commercial bank</td>
<td>0.800</td>
</tr>
<tr>
<td>$\gamma^{cb}$</td>
<td>Share of mortgage financing by commercial bank</td>
<td>0.250</td>
</tr>
<tr>
<td>$\lambda^{cb}$</td>
<td>Share of capital and deposits diverted by commercial banks</td>
<td>0.382</td>
</tr>
<tr>
<td>$\phi^{cb}$</td>
<td>Leverage ratio of commercial bank</td>
<td>8.000</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Survival rate of the commercial bankers</td>
<td>0.972</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Proportional transfer to new banker</td>
<td>0.002</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Frictionless interbank market</td>
<td>1.000</td>
</tr>
<tr>
<td>$\theta^{ib}$</td>
<td>Survival rate of the investment bankers</td>
<td>0.950</td>
</tr>
<tr>
<td>$\lambda^{ib}$</td>
<td>Share of capital and equity diverted by investment banks</td>
<td>0.404</td>
</tr>
<tr>
<td>$\phi^{ib}$</td>
<td>Leverage ratio of investment banks</td>
<td>15.00</td>
</tr>
</tbody>
</table>
spreads pertaining to the spread between mortgage rates and government bonds, the spread between BAA corporate versus government bonds, and the spread between commercial papers and the Treasury Bills rate. The parameter values of $\gamma^{cb}$ and $\Xi^{cb}$ for commercial banks are set at 0.80 and 0.25 to show that more than three capital financing investment opportunities and one mortgage financing opportunity arrives on average each year for commercial banks. The parameters related to the investment banks ($\theta^{ib}$, $\lambda^{ib}$, $\varphi^{cb}$, and $\varphi^{ib}$), specific to the Two-Sector FA model, are calibrated to match a steady state interest rate spread of 100 basis point, a relatively higher leverage ratio at 15 and an average life of 5 years for investment banks.\footnote{In light of a sizable exposure of the investment banks to mortgage financing, the leverage ratios of these financial institutions is kept high while the average survival period is kept low.}

That leaves five parameters related to the non-consumption goods sector that need to be specified. The loan-to-value (LTV) ratio, $m$, and the value for the share of land in non-consumption goods output, $\alpha^{ld}$, are taken from Iacoviello (2005). The parameters values for the probability of not changing the prices of non-consumption goods, $\Psi_{H}$, is set at 0.33 and the price indexing parameter, $\Psi_{P}$, at 0.67 to show that that prices of non-consumption goods are relatively more flexible and are re-optimised in every 1.5 quarters. For the tractability of the model, sectoral elasticity of substitution of capital, $\zeta$, is set at zero to reflect the fact that capital employed in both goods-producing sectors is a perfect substitute.

### 4.3.2 Steady state of the NK models

Table 4.2 lists some important steady state ratios of the benchmark-FA model and the Two-Sector FA model. The area or parcel of land, normalized to 1, is released in each period. The limited supply of land in each period is tantamount to introducing an adjustment cost to a housing production. As shown by Ellis (2010), the assumption about the release of a limited supply of land in each period helps to match the sluggishness of residential investment to changes in housing demand. The steady state of the model matches some of the key aggregate ratios and are presented in Table 4.2.
Table 4.2: Some important steady state ratios

<table>
<thead>
<tr>
<th>Description</th>
<th>The Benchmark-FA Model</th>
<th>The Two-Sector FA Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption to GDP ratio</td>
<td>0.63</td>
<td>0.58</td>
</tr>
<tr>
<td>Government expenditure to GDP ratio</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>Investment to GDP ratio</td>
<td>0.17</td>
<td>0.22</td>
</tr>
<tr>
<td>Capital Stock to GDP ratio</td>
<td>6.67</td>
<td>8.78</td>
</tr>
<tr>
<td>Sectoral Capital Stock ratio</td>
<td>-</td>
<td>4.00</td>
</tr>
<tr>
<td>Housing wealth to GDP ratio</td>
<td>-</td>
<td>10.00</td>
</tr>
<tr>
<td>Housing Stock to Capital Stock</td>
<td>-</td>
<td>1.14</td>
</tr>
<tr>
<td>Consumption output to GDP ratio</td>
<td>0.56</td>
<td>0.64</td>
</tr>
<tr>
<td>Non-consumption output to GDP ratio</td>
<td>-</td>
<td>0.14</td>
</tr>
<tr>
<td>Leverage ratio-Commercial Bank</td>
<td>4.00</td>
<td>8.50</td>
</tr>
<tr>
<td>Leverage ratio-Investment Banks</td>
<td>-</td>
<td>16.00</td>
</tr>
<tr>
<td>Steady State Normalized Value of Land</td>
<td>-</td>
<td>1.00</td>
</tr>
<tr>
<td>Steady State Labor</td>
<td>0.333</td>
<td>0.333</td>
</tr>
</tbody>
</table>

4.3.3 Dynamic simulation

To gauge the importance of using the Two-Sector FA model in explaining the underlying mechanism of financial crisis, this section presents the dynamic simulation of the Two-Sector FA model along with Gertler and Karadi’s (2011) model as the benchmark-FA model. The log-linearized version of the Two-Sector FA model around the steady state is presented in Appendix C. Before going on to discuss the dynamic analysis of these models, it is instructive to present an experiment that compares the performance of the benchmark-FA model to a capital quality shock under a low-leverage case, 4.0, as used by Gertler & Karadi (2011) with a high-leverage case, 9.5, which is more than twice the leverage ratio used in the benchmark-FA model.

4.3.3.1 Capital quality shock

Figure 4.2 displays the response of key variables of the benchmark FA model to capital quality shock with a low-leverage case and a high-leverage case. This shock represents an exogenous source of an impairment of banks' assets which reduces the banks’ net worth on impact. An impact of that magnitude is positively related to the leverage ratio. The initial contraction in net worth tightens the bank’s endogenous balance sheet constraint, which in turn induces a fire sale of assets that further depresses asset values. The resulting financial distress then feeds into the real economy through a contraction in investment and a reduced capital stock.
Figure 4.2: Capital quality shock: The benchmark-FA model (high leverage and low leverage)
The magnitude of the capital quality innovation is negative 5 per cent with some degree of persistence with autoregressive coefficient 0.66. Surprisingly, the benchmark-FA model fails to capture an underlying mechanism of the financial crisis at a high-leverage case. Specifically, the impulse responses of all these key variables exhibit stark differences in terms of their deviation from the steady state in these two cases. The more pronounced differences are evident in the interest rate premium, investment, output and in a deleveraging process. The net worth of financial intermediaries in a high-leverage case is thin and a loss of capital quality by the same magnitude causes a relatively stronger negative impact on the bank’s net worth as expected compared to banks in a low-leverage case. Despite the larger depletion of the banks’ net worth in a high-leverage case, the interest rate premium, on impact, has registered a smaller increase compared to a low-leverage case. The probable reason for a smaller increase in interest spread is a smaller adjustment in the nominal interest rate emanating from a smaller output gap compared to a low-leverage case driven by a wealth effect that causes household to work more. In a high-leverage case, a modest increase in the interest rate premium induces a sharp reversal in investment from the second quarter following the shock that helps to rebuild the capital stock more quickly. This leads to a consequent smaller drop in output and a subsequent quicker recovery starts from the beginning of the second year. A smaller interest rate premium also leads to a slower deleveraging in a high-leverage case compared to a low-leverage case model. A slower pace of deleveraging and an increase in investment also cause a modest decline in asset prices in a high-leverage case compared to a much larger decline in asset prices in a low-leverage case.

This experiment underlines the need to enrich the benchmark-FA model into the Two-Sector FA model. The Two-Sector FA model presented in this chapter not only introduces non-consumption goods along with consumption goods but also covers high-leveraged financial intermediaries. This provides an opportunity to analyze additional shocks such as mortgage portfolio valuation shock and demand preference shock of non-consumption goods.
Figure 4.3: Capital quality shock: The NK models

Figure 4.3 plots the responses of the key variables from a capital quality shock for the Two-Sector FA model juxtaposed with the benchmark-FA model with high and low leverage ratios. In response to this shock, it appears that the Two-Sector FA model follows the benchmark-FA model with low leverage, even in the presence of substantially higher leverage ratios, in explaining the downturn, which mimics a financial crisis with a varying degree of amplification effects. However, the contraction in real output and investment is relatively smaller with the Two-Sector FA model as compared to the benchmark-FA model. This smaller effect on real activity arises due to a smaller uptick in the interest rate premium compared to the benchmark-FA model. The initial impact of this shock reduces the value of the capital portfolio of financial intermediaries due to a reduction in the effective quantity of capital available in the
simulated economy by the same magnitude across both models. The second-round
effect of this shock on the assets values of financial intermediaries in the Two-Sector
model depends on the price of capital, $Q_t$, and the relative price of non-consumption
goods and the composition of capital and mortgage assets.

In the Two-Sector FA model, the effect of a capital quality shock on real activity is not
amplified mainly due to wealth effect induced positive response on labor hours and
a relatively small fall in the relative price of non-consumption goods which offsets
the impact of falling mortgage stock to some extent. In addition, the reduction in
the net worth of commercial banks in the Two-Sector FA model is much lower rela-
tive to the net worth of commercial banks in the Two-Sector FA model, which means
that commercial banks’ losses are somewhat offset by a relatively better performance
of mortgage portfolios. This leads to a relatively lower reduction of the net worth
of commercial banks and, in turn, translates into a quick rebound of investment
expenditures and a capital stock. The net worth of the investment banks declines
even though the losses on the capital destruction are somewhat compensated for by
a quick response of assets prices. In addition, the rebuilding of capital stock also
leads to a relatively lower reduction of real output in both sectors thereby leading to
a quick rebound of relative prices of non-consumption goods. As shown in Figure
4.3, both financial intermediaries are slowly re-building their net worth throughout
without any support from the central bank. Moreover, the impact on consumption
expenditure is also more pronounced in the Two-Sector FA model compared to the
benchmark-FA model due to the presence of a collateral effect on consumption ema-
nating from the fall in the relative price of non-consumption goods and the decline
in the stock of non-consumption goods. The comparison of the Two-Sector FA model
with the benchmark-FA model with high leverage reveals that both models induce
households to work more because of wealth effect and create investment boom. How-
ever, the benchmark-FA model is non-inflationary and also has a smaller impact on
the interest rate premium at a higher leverage.

10Given the steady state capital stock to housing stock ratio, the balance sheet of an investment bank
in the Two-Sector FA model is slightly dominated by mortgage assets where as the balance sheet of a
commercial bank in the benchmark-FA model comprises of capital assets only.
4.3.3.2 Monetary policy shock

Figure 4.4 exhibits the responses of key variables of the FA models emanating from a 25 basis point unanticipated increase in policy interest rate with a quarterly autoregressive parameter of 0.80. This translates into a 18 and a 22 basis points uptick in the nominal interest rate, on impact, in the benchmark-FA model and the Two-Sector FA model, respectively. As shown in Figure 4.4, there is a plausible impact on a financial intermediary’s balance sheets, asset prices and aggregate demand variables in both models, with the benchmark-FA Model showing a pronounced impact compared to the Two-Sector FA model. This is mainly due to a higher increase in interest rate premium, on impact, and a drop in the financial intermediary’s net worth and uptick in the financial intermediary’s leverage from the corresponding trend values in the benchmark-FA model compared to the Two-Sector FA model.

On impact, the drop in the net worth of a commercial bank in the benchmark-FA model is 7 per cent from trend value, which is much higher than in the Two-Sector FA model due to a relatively higher negative both quantum and price impact of capital portfolio assets on the part of investment banks which are the main provider of mortgage loans in the model’s economy. The lower demand for durables goods following the monetary tightening reduces returns on capital in housing sector and induces the shifting of capital and labor inputs from housing sector to consumption goods sector. It is pertinent to note that there are kinks in the interest rate premium, inflation, net worth and leverage of investment banks because of the initial impact on inputs transfer across sectors. These kinks are mainly driven by the initial wealth effect of loss in dividend income on labor dissipates and initial inter-sectoral movements of labor and capital inputs. Once the economy settles from these effect, the kinks disappear within initial few quarters following the shock.

In the case of the benchmark-FA model, the fall in investment arising from an uptick in interest premium, the fire sale of assets due to the tightening of leverage constraint, and the consequent persistent deleveraging of net worth contributed to a fall
Figure 4.4: Monetary policy shock
in the capital stock for a prolong period (i.e., for 13 quarters, following the monetary tightening shock). Even though the capital stock bottoms out at the beginning of the fourth year, the build up of capital stock, nonetheless, remains slow reflecting a continued deleveraging of the financial intermediary’s net worth.

In the case of the Two-Sector FA model, the impact on the fire sale value of capital is not only much lower but also dies out more quickly as the leverage constraint is not as tight as the case in the benchmark-FA model. A relatively stronger financial intermediary’s balance sheet in the Two-Sector model underpins modest fallout of the monetary tightening shock despite the higher impact on the nominal interest rate. Specifically, the fall in the capital stock is only 0.12 per cent from the steady state value, which is very low compared to a fall of 0.64 per cent in the benchmark-FA model. Consequently, the drop in the output in the benchmark-FA model is more than four times the drop of output in the Two-Sector model from the steady state. Even though consumption has dropped in both models, the fall in the Two-Sector model has outpaced the decline in consumption within a year despite a smaller fall in output due to the collateral constraint effect. Since prices of durable goods are relatively more flexible than consumption goods, a tightening of monetary policy induces a fall in the relative price of durable goods. The collateral constraint effect accompanied by a fall in the relative price of durable goods explain the wealth effect on consumption expenditure in line with a number of studies incorporating borrowing constraints in the DSGE models (e.g., Gertler & Karadi (2011), Dib (2010), Gertler & Kiyotaki (2011), Campbell and Cocco, 2003, Calza, Monacelli and stracca, 2009). In addition, the fall in the relative pricing following the monetary policy shock adversely affected the net worth of investment banks which in turn causes increase in labor hour because of the wealth effect.

It is pertinent to mention that the Two Sector-FA model also accords with the literature on a number of issues related to the impact of monetary policy shock. First, the relative price of durable goods drops as expected as monetary tightening affected spending on durables. Second, investment or expenditure on durable goods
declines, and third, the price puzzle associated with nominal shock is present in the Two-Sector FA model as inflation increases initially to monetary policy tightening, but falls subsequently over the course of time.

4.3.3.3 Mortgage portfolio valuation shock

The prime factor which triggered the financial crisis in the US economy was a sharp decline in housing prices which impaired the mortgage assets on the financial intermediaries. The Two-Sector FA model allows us to investigate this phenomenon through two additional shocks: one is related to a financial shock-mortgage portfolio valuation shock and other is related to a non-financial shock-a demand preference shock of non-consumption goods.

Figure 4.5 plots the responses of key variables emanating from a mortgage portfolio valuation shock for the Two-Sector FA model. The size of the disturbance is a 1 percent decrease in mortgage portfolio valuation with a quarterly autoregressive parameter of 0.66. This shock traces the impact of the deterioration of mortgage portfolio valuation on the value of financial intermediaries’ assets and a resulting downturn that captures some features of financial crisis in the simulated economy.

The initial impact of this disturbance on the relative price of durable goods is magnified through two channels within the simulated economy. The first channel is the leverage channel, in which the decline in mortgage assets values affects the net worth of both commercial and investment banks. The impact on the investment bank’s net worth is more pronounced compared to commercial banks due to its higher degree of leverage and more exposure to mortgage financing. In this channel, the drop in net worth tightens the financial intermediaries’ borrowing constraint from households, which induces a fire sale of these financial intermediaries’ assets that depresses asset values and shrinks intermediary balance sheets further.
Figure 4.5: Mortgage portfolio valuation shock
The distress in the financial intermediaries’ balance sheets then feeds into the real activity primarily due to a rise in interest rate premium/spread as evident from the falling investment, capital stock, housing stock and total output shown in Figure 4.5. A decline in the consumption expenditure is more pronounced because of the tightening of households’ borrowing constraint emanating from a drop in the relative price of non-consumption goods. Figure 4.5 also shows that as far as the relative price of non-consumption goods is not recovered to its trend, the output recovery to trend would remain slow in our simulated economy. As shown by Figure 4.5, both financial intermediaries are slowly re-building their net worth throughout, thus effectively portraying a gradual recovery.

As in case of monetary policy shock, there are also kinks in some of the variables such as interest rate premium, inflation net worth and leverage of both commercial banks and investments banks. The decrease in the relative price of durables goods following this shock tightening balance sheets of investment banks and also witnessed the shifting of capital and labor inputs from housing sector to consumption goods sector. These factors also induces kinks in the interest rate premium, inflation, net worth and leverage of both commercial banks investment banks. Once the economy settles from these effect, the kinks disappear within initial few quarters following the shock.

4.3.3.4 Demand preference shock of non-consumption goods

Figure 4.6 exhibits impulse responses to a negative housing preference shock. The purpose of this shock is to create a scenario where a persistent negative house preference shock, with an autocorrelation coefficient of 0.80, generates a drop in the relative price of non-consumption goods. This in turn triggers a downturn in the simulated economy. Specifically, the drop in the relative price of non-consumption by around 4 percent in response to this shock reduces the returns on housing investment, which leads to a reduction in the investment in non-consumption goods sector by 30 per cent. The impact on investment in non-consumption goods is more than
Figure 4.6: Demand preference shock: non-consumption goods
twice the impact as observed by Iacoviello & Neri (2010) and Topel and Rosen (1988). The is mainly driven by the interaction of borrowing and an endogenous financial intermediaries' balance sheet constraints while the aforementioned studies have incorporated only a borrowing constraint in their models.

The falling relative price of non-consumption goods adversely affects the balance sheets of both commercial and investment banks. The tightness of the endogenous constraints of these financial institutions results in a fire sale value of both capital and mortgage assets by 4 per cent and 0.8 per cent, respectively. With the rise in the interest premium, the consequent negative effect on investment causes a persistent decline in capital stock from its steady state. This leads to a decline in output from its trend values. In addition, the fall in the relative prices of non-consumption goods leads to a decline of 6 per cent and 12 per cent in the net worth of commercial and investment banks, respectively. The fall in the net worth of the latter compared to the former is relatively low due to a smaller exposure of commercial banks in mortgage financing compared to investment banks in the model. A continued build-up of net worth and the resulting deleveraging on the part of financial intermediaries is adversely affecting the capital accumulation in our simulated economy and investment activity.

4.4 Robustness analysis

The results of the Two-Sector FA model are robust to a lower LTV ratio (m) and a higher degree of price flexibility of non-consumption goods compared to the baseline calibrated values. In this section, the results of the two shocks, a capital quality shock and a demand preferences shock, are presented.

Column 1 of Figure 4.7 presents the impulse responses of key variables of the baseline Two-Sector FA model (m=0.90), the Two-Sector FA model with a lower LTV ratio (m=82) and the Two-Sector FA model with flexible durable goods prices to a capital
Figure 4.7: Robustness analysis I
quality shock. The tightening of the mortgage loan standards by lowering the LTV value has a more pronounced adverse effect on output and consumption compared to the baseline model. This has effectively dragged output and consumption recovery following the capital quality shock. A relative tightening of leverage constraint for financial intermediaries, in the alternative models, results in a slightly higher interest premium for the first few quarters than in the baseline Two-Sector FA model. This induces a higher reduction in investment compared to the baseline model during this time period. The difference in the interest rate premium among these models disappears once the leverage constraints ease; however, the lower investment in the initial period results in a slightly larger persistent decline in capital stock throughout and this induces a relative decline in output in the alternative models, the Two-Sector FA model with a lower LTV ratio and the Two-Sector FA model with flexible durable goods prices. It is also pertinent to note that consumption shows a larger decrease during the first few years despite a relatively lower output reduction in the first year in the Two-Sector FA model with flexible durable goods prices. This is mainly driven by a relatively larger increase in the relative price of non-consumption goods followed by a smaller drop compared to the baseline Two-Sector FA model.

Column 2 of Figure 4.7 presents the impulse responses of key variables of the baseline Two-Sector FA model (m=0.90), the Two-Sector FA model with a lower LTV ratio (m=0.82) and the Two-Sector FA model with flexible durable goods prices to a demand preference shock. The results of the Two-Sector FA model with a lower LTV ratio show that there is a relatively larger output and consumption decline compared to the baseline Two-Sector FA model. A lower LTV ratio accompanied by a slightly larger drop in the relative price of non-consumption goods initial period explains a more sustained reduction in consumption expenditures. The effect on output is due to a larger decline in capital stock emanating from a relative tightening of leverage constraints. In the case of the Two-Sector FA model with flexible durable goods prices, the demand preference shock induces a larger drop in relative prices of non-consumption goods in the alternative model compared to the baseline Two-Sector FA model. This is accompanied by a larger drop in the asset price of capital results in a
further tightening of leverage constraints that, in turn, lead to a larger reduction in consumption and output relative to the baseline Two-Sector FA model.

### 4.4.1 The Two-Sector FA model with a representative Universal Bank

Another robustness check regarding the Two-Sector FA model is to dismantle the distinction between a commercial bank and an investment bank by focusing on a representative universal bank which caters for both capital and mortgage financing in their entirety. As shown in Figure 4.8, the impulse responses of key variables to the capital quality shock for the Two-Sector model with the universal bank show a much larger amplification effect compared to the Two-Sector model even at a five times smaller shock. This is mainly because of multiple assets both mortgage and capital financing in the universal bank's portfolio. Loss on one assets' class tightens the overall balance sheet of the banks and thus has a bearing on overall business of
the universal bank. For instance, in case of the capital quality shock, the universal bank has registered a larger decline in net worth by 44 per cent to the steady state level compared to a decline of 32 per cent in the net worth of commercial banks in the Two-Sector FA model. The net worth of an investment bank, as a part of the Two-Sector FA model, also suffers a decline of 60 per cent on impact in net worth from trend. The resulting tightness of leverage constraint leads to a much larger increase in interest rate spread precipitating a much larger downturn in the real economy in the Two-Sector FA model with a universal bank. The contraction in real output is mainly found in the consumption sector as the housing stock fall only slightly, which induces a much larger drop in the relative price of non-consumption goods on impact in the Two-Sector FA model with a universal bank. Moreover, the recovery in the relative price is much slower from the trough compared to the Two-Sector FA model. However, the reduction in consumption expenditure is much larger in the Two-Sector FA model because of a larger drop in the housing stock compared to the FA model with a universal bank.11

4.5 Conclusion

This chapter has presented the Two-Sector FA model augmented with an enriched financial intermediaries sector within a canonical NK monetary model. The Two-Sector FA model has the following additional features: (1) non-consumption goods has been introduced into the model with a representative household getting utility from the housing services by getting a mortgage from the financial intermediaries, (2) the model has incorporated the market-based banking system through investment banks along with commercial banks, and (3) a frictionless interbank market.

The mains results of this chapter are as follows. First, the chapter has found that the benchmark-FA model is unable to capture an underlying mechanism of the 2008 financial crisis at a high-leverage. Second, it has also found that the effect of a capi-

11 The Two-Sector FA model with a universal bank also generates huge amplification effects relative to the Two-Sector FA model in response to a demand preference shock of non-consumption goods.
tal quality shock on real activity is not amplified in the Two-Sector FA model mainly due to a relatively smaller drop in the relative price of non-consumption goods which offsets the impacts of falling mortgage asset stock to some extent. Third, the impact on consumption expenditure is also more pronounced in the Two-Sector FA model compared to the benchmark-FA model due to the presence of a collateral effect emanating from the fall in the relative price of non-consumption goods and the decline in non-consumption goods stock. Fourth, a relatively stronger financial intermediary's balance sheet in the Two-Sector model underpins a modest fallout of the monetary tightening shock despite the higher impact on the nominal interest rates. Last but not least, the output recovery to trend would remain slow in our simulated economy as the relative price of non-consumption goods is not recovered to its trend.
Conclusion

The main objective of this dissertation has been to deepen our understanding of the effects of financial frictions on the design of optimal monetary policy in the New Keynesian models and to explore the relative importance of financial shocks for business cycles fluctuations. This dissertation, consisting of three self-contained papers, has also tried to improve our understandings of financial frictions by making some theoretical and empirical contributions to the emerging strands of literature incorporating financial frictions in the NK monetary models. It has also opened up some interesting questions for further research.

5.1 Summary of main findings

Using the NK models, Chapter 2 compared the performance of Ramsey monetary policy with alternative monetary policy regimes such as a strict inflation targeting monetary policy regime and the Taylor rule. In the case of a productivity shock, this chapter found that an inflation targeting rule emerges as the best policy compared to the Taylor rule in the standard NK model with capital as it approximates the welfare level associated with Ramsey policy. However, the welfare level is somewhat sub-optimal in the case of the NK model with capital and with financial frictions compared to the level achieved under the Ramsey policy under commitment. This suggests that a strict inflation targeting is sub-optimal in response to productivity shocks in the presence of financial frictions in a standard NK model which is in line with the results of Carlstrom et al. (2010). In the case of a financial shock, the
non-optimized monetary policy rule without interest rate smoothing and an inflation targeting rule provide a welfare level which approximates the welfare level achieved under the optimal monetary policy under commitment. This chapter also found that Ramsey policy under commitment advocates an aggressive interest rate cut following a financial shock in order to minimize the adverse effects on financial variables including the interest rate spread and thus real output.

Chapter 3 quantified the importance of financial factors in economic fluctuations for the US economy from 1980:Q1 to 2008:Q1 through SVAR models identified with model-based sign restrictions. The empirical results of this chapter using the US data provide the following insights about the role of financial shock in the business cycle fluctuations. The first insight is that the financial shocks emanating both from entrepreneurs and financial intermediaries are prominent in explaining fluctuations in real output, interest rate spread and prices. The second insight is related to the relative importance of financial shocks arising in non-financial firms to the financial shocks originating in financial firms. The analysis found that a financial shock related to the demand side is the major driver of output fluctuations in both time horizons while financial shocks related to financial intermediaries also explain a modest variation of output fluctuations over both time horizons. In addition, financial shocks related to financial intermediaries account for a relatively larger share of interest rate spread fluctuations over both time horizons compared to a financial shock related to non-financial firms. The financial shocks related to financial intermediaries explain relatively larger short run prices fluctuations variations compared to a financial shock related to the demand side while the latter explains relatively larger variations in prices over the longer time horizons.

Chapter 4 analysed the interaction of durable goods, financial frictions and real economic activity by extending the benchmark FA model into two-sector. The Two-Sector FA model developed in Chapter 4 not only matches the leverage ratio data well but also captures some underlying mechanism of the financial crisis at the higher leverage ratio compared to the benchmark model. The chapter also found
that the effect of capital quality shock on real activity is not amplified mainly due
to a relatively smaller drop in the relative price of non-consumption goods which
offset the impacts of falling mortgage asset stock to some extent. It also found that
the impact on consumption expenditure is more pronounced in the Two-Sector FA
model compared to the benchmark model due to the presence of collateral effects.
This chapter also found that the output recovery to trend would remain slow in the
simulated economy as far as the relative price of non-consumption goods has not
recovered to its trend. It is also pertinent to mention that the Two-Sector FA model
also accords with the literature on a number of issues, particularly the behaviour of
relative price of durable goods, the expenditure on the durable goods, and the price
puzzle following the monetary policy tightening shock.

5.2 Future research directions

By deepening our understandings of the effects of financial frictions, the thesis sug-
gests a number of future research directions.

In Chapter 2, the particular definition of financial frictions is used while investigating
the impact of financial frictions on the design of optimal monetary policy. It would be
interesting to generalize these results to the various definitions of the financial fric-
tions incorporated in the various NK models in the literature. It would be interesting
to know how the design of optimal monetary policy is sensitive to the particular
definition of financial frictions. It would also be interesting to incorporate a zero
lower bound in the NK models presented in chapter 2. Another research direction
related to this area would be to analyze the effects of financial frictions on an optimal
monetary policy design by quantifying the welfare gain of various non-conventional
monetary policies (QE1, QE2 and QE3) as implemented by the US Federal Reserve
to support the intermediation process of financial institutions following the financial
An interesting future research direction related to Chapter 3 would be to derive model-based sign restrictions from a NK model which incorporates financial frictions in both financial and non-financial firms. Another important direction for future research is to investigate the empirical implications of the interaction of durable goods, financial frictions and economic fluctuations.

Chapter 4 also points to some future research directions. Specifically, it would be worth investigating various interesting financial phenomenon particularly involving non-linearities in the solution of the benchmark and the Two-Sector FA models. This requires the solution of these models for a full dynamics equilibrium rather than an analysis of a log-linearized model near the steady state. The easiest first building block in this direction would be to incorporate recursive preferences and stochastic volatility in these models by incorporating Epstein and Zin preferences in the specification of household’s preferences. Another extension in these models would be to incorporate asymmetric information in the model by incorporating risky debt contract in the interbank market between banks. These questions are left for future research.
Appendices
Table A.1: Optimal monetary policy with the NK model with capital and financial frictions

1. Household's utility function
   \[ \text{Max } E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t - h(C_{t-1})) - \frac{C_t}{2\delta} \right] \]

subject to:
   \[ E_t[f(y_{t+1}, y_t, y_{t-1}, \delta_t)] = 0 \]

Where the above constraint comprises of the following equations:

2. Marginal utility of consumption:
   \[ \lambda_t = \frac{1}{C_t - h(C_{t-1})} + \beta E_t \left[ \frac{C_{t+1}}{C_t} \right] = 0 \]

3. Labor market equation:
   \[ \lambda_t W_t - (1 + \delta) L_t^f = 0 \]

4. Euler equation:
   \[ E_t [\beta L_{t+1} R_{t+1} - 1] = 0 \]

5. Production function of intermediate good:
   \[ Y_t = A_t Z_t K_t^\rho \]

6. Equilibrium wage equation:
   \[ W_t - (1 + \delta) L_t^f = 0 \]

7. Optimal capacity utilization:
   \[ \frac{K_{t+1}}{K_t} = \delta (U_t)^{\rho+1} K_t = 0 \]

8. Capital return:
   \[ r_t \rho - R_{t+1} = 0 \]

9. Marginal value of bank's capital:
   \[ v_t = \beta \Delta E_t \left[ (1 - \rho) \beta A_t L_{t+1} R_{t+1} \right] = 0 \]

10. Marginal value of bank's net worth:
    \[ \frac{\Delta E_t}{\rho} \left[ (1 + \delta) \beta A_t L_{t+1} R_{t+1} \right] = 0 \]

11. Total factor productivity (non-financial) shock:
    \[ \alpha_t \sim \mu_{t+1} \]

12. Capital quality (financial) shock
    \[ \delta_t \sim \mu_{t+1} \]

Supplementary Equations for The NK Model with capital and financial frictions

24. Bank's balance sheet:
    \[ Q_t K_t = NW_t - D_{t+1} = 0 \]

25. Aggregate capital:
    \[ Q_t K_t = \phi_t N \]

26. Optimal leverage:
    \[ \phi_t = \frac{N}{K_t^\delta} = 0 \]

27. Marginal value of bank's capital:
    \[ \eta_t - E_t \left\{ (1 - \theta) \beta A_t L_{t+1} (R_{t+1} - \theta x_{t+1}) - \beta A_t L_{t+1} \theta x_{t+1} \right\} = 0 \]

28. Marginal value of bank's net worth:
    \[ \eta_t - E_t \left\{ (1 - \theta) + \beta A_t L_{t+1} \theta x_{t+1} \right\} = 0 \]

29. Net worth of the existing banker:
    \[ NW_t - \theta (R_t - R_{t-1}) \phi_t + R_t NW_{t-1} = 0 \]

30. Net worth of the new banker:
    \[ NW_t - \phi_t Q_t K_t - 1 = 0 \]

31. Aggregate net worth:
    \[ NW_t = NW_{t-1} - N (Q_t K_t) = 0 \]
Appendix to Chapter 3
Table B.1: Model 1-The NK model augmented with a financial accelerator

1. Marginal utility of consumption: 
   \[ 
   \lambda_t = \frac{c_t}{c_{t+1}^{\delta}} - \beta h E_t \left( \frac{1}{c_{t+1}^{\delta}} \right) 
   \]

2. Labor market equation: 
   \[ 
   \lambda_t W_t = X T_t^p 
   \]

3. Euler equation: 
   \[ 
   E_t \beta \Lambda_{t+1} R_{t+1} = 1 
   \]

4. Production function of intermediate good: 
   \[ 
   Y_t(j) = A_t \left( \delta \left( U_t \right) K_t \right)^{(1/\beta)} 
   \]

5. Equilibrium wage equation: 
   \[ 
   W_t = (1 - \alpha) W_t^0 
   \]

6. Optimal capacity utilization: 
   \[ 
   K^* = \delta \left( U_t \right) K_t 
   \]

7. Capital return: 
   \[ 
   R_{t+1} = R_t + \delta K_t - \delta \left( U_t \right) K_t 
   \]

8. External finance premium: 
   \[ 
   e_{f, t+1} = R_{t+1} - R_t 
   \]

9. The standard Tobin’s Q equation: 
   \[ 
   E_t \Lambda_{t+1} \left[ Q_t = 1 + f \left( \frac{Y_t}{K_t} \right) + I_{t+1} \left( f \left( \frac{Y_t}{K_t} \right) - \frac{K_{t+1}}{K_t} \right) - E_t \beta^2 \Lambda_{t+1} \left( \frac{K_{t+1}}{K_t} \right)^2 f \left( \frac{Y_t}{K_t} \right) \right] 
   \]

10. Net investment: 
    \[ 
    I_{t+1} = \lambda_t + \delta \left( U_t \right) K_t 
    \]

11. Depreciation rate: 
    \[ 
    \delta \left( U_t \right) K_t = \delta_t + \frac{1}{1 + \delta} \frac{U_{t+1}}{U_t} 
    \]

12. Capital accumulation: 
    \[ 
    K_{t+1} = K_t \left( 1 - \delta \right) 
    \]

13. Price dispersion: 
    \[ 
    D_t = \frac{1}{n_t} \sum_{i=1}^{n_t} \left( P_t^i - P_t^d \right) 
    \]

14. Recursive formulation of optimal prices I: 
    \[ 
    \Pi_t = \left( \frac{\Pi_{t+1}}{\Pi_t} \right)^{\gamma} \frac{\Pi_t}{\Pi_t} 
    \]

15. Recursive formulation of optimal prices II: 
    \[ 
    F_t^1 = Y_t P_{t+1} + \frac{\Pi_t \Lambda_{t+1}}{\Pi_{t+1}} \left( \frac{Y_t}{\Pi_t} \right) F_{t+1}^1 
    \]

16. Recursive formulation of optimal prices III: 
    \[ 
    F_t^2 = Y_t + \frac{\Pi_t \Lambda_{t+1}}{\Pi_{t+1}} \left( \frac{Y_t}{\Pi_t} \right) F_{t+1}^2 
    \]

17. Inflation: 
    \[ 
    \Pi_t = \left( \frac{1}{\gamma} \right) \left( \frac{\gamma}{\Pi_t} \right)^{1-\gamma} \frac{\gamma}{\Pi_t} \frac{\Pi_{t+1}}{\gamma} 
    \]

18. Wholesale, retail output: 
    \[ 
    Y_t = D_t Y_{t+1} 
    \]

19. Fisher equation: 
    \[ 
    1 + \delta_t = R_{t+1} \delta_t 
    \]

20. Taylor Rule: 
    \[ 
    i_t = (1 - \rho) \left[ k + \rho \left( \log Y_t - \log Y_{t+1} \right) \right] + \rho \delta_{t-1} + \epsilon_t 
    \]

21. Aggregate resource constraint: 
    \[ 
    Y_t = C_t + G_t + \delta_t \left[ 1 + f \left( \frac{Y_t}{\Pi_t} \right) \right] I_t + G_t 
    \]

22. Total factor productivity shock: 
    \[ 
    \delta_t = \rho \delta_{t-1} + \epsilon_t 
    \]

23. Net worth shock: 
    \[ 
    \Phi_{t+1} = \rho \phi \Phi_{t-1} + \epsilon_{t+1} 
    \]

24. Demand shock: 
    \[ 
    \zeta_t = \rho \phi \gamma_{t-1} + \epsilon_t 
    \]

Supplementary equations for the financial accelerator mechanism

25. Supply of investment fund: 
    \[ 
    E_t \left( \frac{R_{t+1}}{R_t} \right) = \left( \frac{\Phi_{t+1}}{\Phi_{t+1}} \right) R_{t+1} 
    \]

26. Value of the entrepreneur firms: 
    \[ 
    V_t = \frac{1}{Q_t} \left[ Q_{t+1} - K_t \right] - \left( R_t + \frac{E_t \left( \frac{Q_{t+1}}{Q_t} \right) K_{t+1}}{Q_t} - N_t \right) Q_{t+1} K_t - N_{t+1} 
    \]

27. Evolution of net worth: 
    \[ 
    N_{t+1} = \Phi_{t+1} \gamma \left[ \frac{1}{Q_t} \left( Q_{t+1} - K_t \right) - \left( R_t + \frac{E_t \left( \frac{Q_{t+1}}{Q_t} \right) K_{t+1}}{Q_t} - N_t \right) Q_{t+1} K_t - N_{t+1} \right] 
    \]

28. Entrepreneur’s consumption: 
    \[ 
    C_t^* = (1 - \gamma) V_t 
    \]
Table B.2: Model 2-The NK model with financial intermediaries

1. Marginal utility of consumption: \( \lambda_1 = \frac{h_{Ct-1}}{h_{Ct}} - \beta E_t \left( \frac{h_{Ct+1}}{h_{Ct}} \right) \)

2. Labor market equation: \( \lambda_t W_t = \lambda_t T_t^P \)

3. Euler equation: \( E_t \beta \Lambda_{t+1} R_{t+1} = 1 \)

4. Production function of intermediate good: \( Y_t(j) = A_t \left( \theta_t(j) K_t(j) \right)^{\lambda_t} \)

5. Equilibrium wage equation: \( W_t = \left( 1 - \alpha \right)^{h_t} \)

6. Optimal capacity utilization: \( \frac{\alpha_t}{K_t} = \delta \left( \log L_t \right) K_t \)

7. Capital return: \( r_t P_t = R_{t+1} - R_t \)

8. External finance premium: \( \epsilon^{EF} = R_k, t+1 - R_k, t \)

9. The standard Tobin’s Q equation: \( E_t \beta^{EF} A_t = (1 + f(\Delta L_t))^2 f^2(\Delta K_t) \)

10. Net investment: \( I_t = I_t + S(U_t) K_t \)

11. Capital accumulation: \( K_{t+1} = K_t (1 - \delta) - I_t \)

12. Price dispersion: \( D_t = \frac{\gamma_t}{1 - \gamma_t} \left[ \theta_t - \left( 1 - \gamma_t \right) \Delta Y_t \right] \)

13. Recursive formulation of optimal prices I: \( \Pi_t = \left( 1 - \gamma_t \right) \Pi_t \)

14. Recursive formulation of optimal prices II: \( F_t^1 = Y_t P_t + \gamma_t \beta^{EF} A_t \left( \frac{\Delta Y_t}{\Delta K_t} \right) F_t^2 \)

15. Recursive formulation of optimal prices III: \( F_t^2 = Y_t + \gamma_t \beta^{EF} A_t \left( \frac{\Delta Y_t}{\Delta K_t} \right) F_t^3 \)

16. Inflation: \( \Pi_t = \left( \frac{1}{1 - \gamma_t} \right) \Pi_t \)

17. Fisher equation: \( 1 + \theta_t = R_{t+1} \frac{P_{t+1}}{P_t} \)

18. Fisher equation: \( 1 + \theta_t = R_{t+1} \frac{P_{t+1}}{P_t} \)

19. Taylor Rule: \( i_t = (1 - \rho) \left[ \log \left( \frac{Y_t}{\gamma_t} \right) \right] + \rho i_{t-1} + \epsilon_i^t \)

20. Aggregate resource constraint: \( Y_t = C_t + \left[ (1 + f(\Delta Y_t)) L_t \right] + G_t \)

21. Total factor productivity shock: \( a_t = \rho_a \epsilon_a \)

22. Aggregate capital: \( Q_t K_t = \gamma_t \gamma_t \)

23. Aggregate capital: \( Q_t K_t = \gamma_t \gamma_t \)

24. Aggregate capital: \( \theta_t = \theta_t \gamma_t \)

25. Marginal value of one unit of bank’s capital: \( v_t = E_t \left( (1 - \theta) \beta \Lambda_{t+1} (R_{t+1} - R_t) + \beta \Lambda_{t+1} \delta \epsilon_{t+1} \right) \)

26. Marginal value of bank’s net worth: \( \eta_t = E_t \left( (1 - \theta) + \beta \Lambda_{t+1} \delta \epsilon_{t+1} \right) \)

27. Net worth of the existing banker: \( NW_{t+1} = \theta_t \left[ (R_t - R_t) \gamma_t + R_t \right] \)

28. Net worth of the new banker: \( NW_{t+1} = \eta_t \left[ (R_t - R_t) \gamma_t + R_t \right] \)

29. Aggregate net worth: \( N_t = NW_t + NW_{t+1} \)
Table B.3: Robustness Analysis

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<th>Variables</th>
<th>Time</th>
<th>Baseline</th>
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<th>Robustness check II</th>
<th>Robustness check III</th>
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Appendix to Chapter 4

C.1 The Two-Sector FA model: Log-linearized model

In the following model, the variables with cap represent deviation from its respective steady state values and variables with bar represent steady state values of the model.

\[ \lambda_t = - \left[ (1 - \beta h)(1 - h) \right]^{-1} \left[ \tilde{e}_t - h \tilde{e}_{t-1} - \beta h(\tilde{e}_{t+1} - h \tilde{e}_t) \right] \]  

(C.1)

\[ q \bar{r} = -1 \lambda \bar{H}^{-1} \left[ \dot{\lambda}_t + \hat{d}_t \right] + \beta q \bar{r}(q \bar{r} + \dot{\lambda}_t + \hat{d}_t) + \bar{f} \bar{H}^{-1} \]  

(C.2)

\[ \dot{\lambda}_t = \dot{\lambda}_t - \dot{\lambda}_{t-1} \]  

(C.3)

\[ \dot{\lambda}_{t+1} + \dot{r}_{t+1} = 0 \]  

(C.4)

\[ \dot{\lambda}_{t+1} + \dot{r}_{t+1} = 0 \]  

(C.5)

\[ q \bar{r} + y \bar{m}^h_t - \dot{d}_t = \bar{r} d_t \]  

(C.6)

\[ - \bar{R} k^{-1} \bar{r}^h_{t+1} = \beta \dot{\lambda}_{t+1} \bar{A} \]  

(C.7)

\[ q \bar{r} = p \bar{m}_t^h - p \bar{m}_t^c + \epsilon_s \]  

(C.8)

\[ \bar{y} \bar{r}_t \bar{y}_t = \beta(1 - \theta) \left[ \bar{R} k \bar{k}^c_t - \bar{R} p_t \right] + \beta \theta \bar{X}_t \bar{y}_t \bar{y}_t \left[ \dot{\lambda}_{t+1} + \dot{\lambda}_{t+1} \right] \right] \]  

(C.9)

\[ \bar{y}_t \bar{y}_t = \beta(1 - \theta) \bar{A} \bar{R} \left[ \dot{\lambda}_{t+1} + \dot{r}_t \right] + \beta \theta \bar{Z}_t \bar{y}_t \bar{y}_t \left[ \dot{\lambda}_{t+1} + \dot{\lambda}_{t+1} \right] \]  

(C.10)
\[ \lambda \Phi_t \phi_t^b - \Phi_t v_t^b \left[ \phi_t^b + v_t^b \right] = \eta_t^b \]  

(C.11)

\[ Z_{cb,t} \delta_t = \Phi_t \left[ R_k k_{t+1} + R_f f_t \right] + \Phi_{t-1} \left[ R_k K - K \right] + R f - 1 \]  

(C.12)

\[ \dot{\phi}_t^b = \phi_t^b + \dot{z}_t^b \]  

(C.13)

\[ \overline{NW}_{cb} n \omega_t^b = \overline{NW}_{cb} n \omega_t^b + \overline{NW}_{cb} n \omega_t^b \]  

(C.14)

\[ \overline{n \omega_t^b} = z_t^b + \overline{n \omega_t^b} \]  

(C.15)

\[ \overline{NW}_{cb} n \omega_t^b = n \gamma \overline{QK} \left[ \dot{q}_t + \dot{k}_t \right] + n \overline{m \overline{qm} \overline{qh} \overline{H} (q^p r_i + h h + m p v_t) \]  

(C.16)

\[ \overline{\phi}_t^b \phi_t^b = -n \overline{NW}_{cb}^{-1} \left[ \gamma \overline{QK} + \overline{m \overline{qm} \overline{qh} \overline{H} (q^p r_i + h h + m p v_t) \right] + \overline{m \overline{qm} \overline{qh} \overline{H} \overline{NW}_{cb}^{-1} (q^p r_i + h h + m p v_t) \]  

(C.17)

\[ \overline{v}_{ib} v_{ib}^t = \beta (1 - \theta_{ib}) \left[ R k k_{i+1} - R f f_t \right] + \beta \theta_{ib} X_{ib} V_{ib} \left[ \hat{\lambda}_{i+1} + \hat{z}_{ib} + \hat{v}_{ib} + \right] \]  

(C.18)

\[ \overline{\eta}_{ib} \eta_{ib}^t = \beta (1 - \theta_{ib}) \Lambda R \left[ \hat{\lambda}_{i+1} + \hat{r}_{ib} \right] + \beta \theta_{ib} Z_{ib} \bar{\eta}_{ib} \left[ \hat{\lambda}_{i+1} + \hat{z}_{ib} + \hat{\eta}_{ib} \right] \]  

(C.19)

\[ \Lambda_{ib} \bar{\eta}_{ib} \phi_{ib} - \bar{\eta}_{ib} \bar{\phi}_{ib} \left[ \phi_{ib} + \bar{v}_{ib} \right] = \eta_{ib} \eta_{ib}^t \]  

(C.20)
Appendix to Chapter 4

\[ Z_{ib}z_{ib} = \bar{\varphi}_{ib}\left[R\bar{k}_{t+1} - \bar{R}\hat{\gamma}_t\right] + \bar{\varphi}_{ib}\hat{\phi}_{i-1}^{ib}\left[R\bar{k} - \bar{R}\right] + \bar{R}\hat{\gamma}_{t-1} \]  
(C.21)

\[ \hat{x}_{i}^{ib} = \hat{\phi}_{i}^{ib} - \hat{\phi}_{i-1}^{ib} + \bar{z}_{i}^{ib} \]  
(C.22)

\[ \bar{N}\bar{W}_{ib}n\omega_{i}^{ib} = \bar{N}\bar{W}\bar{E}_{ib}n\bar{\omega}_{i}^{ib} + \bar{N}\bar{W}n\bar{w}_{i}^{ib} \]  
(C.23)

\[ n\bar{\omega}_{i}^{ib} = \bar{z}_{i}^{ib} + \bar{n}\omega_{i}^{ib} \]  
(C.24)

\[ \bar{N}\bar{W}\bar{N}_{ib}\bar{n}\bar{w}_{i}^{ib} = n(1 - \gamma)\bar{Q}\bar{K}\left[q_{i} + \bar{k}_{i}\right] + n(1 - \Xi)m\bar{q}\bar{r}\bar{p}\bar{H}(q\bar{p}_{t} + \bar{h}_{t} + m\bar{p}\bar{n}_{t}) \]  
(C.25)

\[ \bar{\varphi}_{ib}\hat{\phi}_{i}^{ibc} = -\bar{N}\bar{W}_{ib}^{-1}\left[(1 - \gamma)\bar{Q}\bar{K} + (1 - \Xi)\bar{q}\bar{r}\bar{p}\bar{H}\right]n\omega_{i}^{ib} + \bar{Q}\bar{K}\bar{N}\bar{W}_{ib}^{-1}\left(q_{i} + \bar{k}_{i}\right) + 
\]  
\[ (1 - \Xi)\bar{q}\bar{r}\bar{p}\bar{H}\bar{N}\bar{W}_{ib}^{-1}(q\bar{p}_{t} + \bar{h}_{t}) \]  
(C.26)

\[ \bar{\gamma}_{i}^{ib} = -\bar{e}\bar{E}\bar{F}\bar{P}e^{\bar{f}}p_{t} \]  
(C.27)

\[ \hat{\phi}_{i}^{ibc} + \bar{\gamma}_{i} = \hat{\phi}_{i}^{ib} \]  
(C.28)

\[ \bar{R}\bar{k}(\bar{Q}\bar{K}_{c})\left[\bar{R}\bar{k}_{t+1} + \bar{q}_{t} + \bar{k}_{t}\right] = \alpha_{c}\bar{Y}_{c}\bar{y}_{m_{t}} + \bar{K}_{c}(\bar{q}_{t} + \bar{k}_{t}) - \delta\bar{K}_{c}(\bar{q}_{t} + \bar{k}_{t}) \]  
(C.29)

\[ \bar{R}\bar{k}(\bar{Q}\bar{K}_{c})\left[\bar{R}\bar{k}_{t+1} + \bar{q}_{t} + \bar{k}_{t}\right] = \alpha_{c}\bar{q}\bar{r}\bar{p}\bar{Y}_{h}\bar{y}_{m_{t}} + \bar{K}_{h}(\bar{q}_{t} + \bar{k}_{t}) - \delta\bar{K}_{h}(\bar{q}_{t} + \bar{k}_{t}) \]  
(C.30)
\( \ddot{r}_k(r_{k_c} - r_{k_h}) = 0 \) \hspace{1cm} (C.31)

\[ \dot{i}_t = \dot{r}_t + \dot{r}_{t+1} \] \hspace{1cm} (C.32)

\[ \dot{i}_t = \rho_t \dot{i}_{t-1} + (1 - \rho_t)(\kappa_{r_t} \dot{r}_t + \kappa_y \dot{y}_t) + \varepsilon_t \] \hspace{1cm} (C.33)

\[ e\dot{p}_t^c = r_{k_{t+1}} - \dot{r}_t \] \hspace{1cm} (C.34)

\[ e\dot{p}_t^h = r_{k_{t+1}} - \dot{r}_t \] \hspace{1cm} (C.35)

\[ \dot{e}r_{k_t}^c = r_{k_{t+1}} \] \hspace{1cm} (C.36)

\[ \dot{e}r_{k_t}^h = r_{k_{t+1}} \] \hspace{1cm} (C.37)

\[ \chi^t_{\dot{t}_t} - \dot{\theta}_t = q^t p_t + \dot{g}_t^c - \ddot{p}_t^h \] \hspace{1cm} (C.38)

\[ \bar{L}_t = \bar{L}_c \dddot{L}_c + \bar{L}_h \dddot{L}_h \] \hspace{1cm} (C.39)

\[ y\dot{m}_t^c = \dot{a}_t + \alpha_c(u_{t_c} + \dot{\xi}_t^c + \dot{k}_t^c) + (1 - \alpha_c)\dddot{\ell}_t^c \] \hspace{1cm} (C.40)

\[ y\dot{m}_t^h = \dot{a}_t + \alpha_c(u_{t_h} + \dot{\xi}_t^h + \dot{k}_t^h) + (1 - \alpha_c - \alpha_{ld})\dddot{\ell}_t^h + \alpha_{ld}\dddot{\ell}_t \] \hspace{1cm} (C.41)

\[ \dddot{k}_t = \dddot{k}_c^c + \dddot{k}_h^h \] \hspace{1cm} (C.42)
\[
\frac{\bar{c}}{(\bar{c} - \bar{c}^c)} \delta_t^c = (1 + \zeta) \hat{u}_c
\]  
(C.43)

\[
\bar{\gamma} \hat{y}_t = \bar{c} \hat{c}_t + \bar{I}_t
\]  
(C.44)

\[
p \hat{m}_t^c = p \hat{m}_t^c - \hat{I}_t
\]  
(C.45)

\[
p \hat{m}_t^h = p \hat{m}_t^h - \hat{I}_t^h
\]  
(C.46)

\[
m \hat{c}_t^c = -p \hat{m}_t^c
\]  
(C.47)

\[
m \hat{c}_t^h = -p \hat{m}_t^h
\]  
(C.48)

\[
\bar{\gamma} \hat{y}_t = \bar{\gamma} \hat{c}_t + \bar{\gamma} \hat{y}_t
\]  
(C.49)

\[
y \hat{m}_t^c + - \hat{u}_c = \zeta \hat{u}_c + \hat{\xi}_t + \hat{k}_t^c
\]  
(C.50)

\[
y \hat{m}_t^h + q \hat{p}_t - \hat{u}_h = \zeta \hat{u}_h + \hat{\xi}_t + \hat{k}_t^h
\]  
(C.51)

\[
\hat{m}_t = \bar{I}_t - \bar{\delta} \bar{K}(\hat{k}_{t-1} + \hat{\xi}_t + \delta c_t)
\]  
(C.52)

\[
\hat{q}_t = \kappa(\hat{m}_t - \hat{m}_{t-1})T^{-1} - \beta \kappa(\hat{m}_{t+1} - \hat{m}_t)T^{-1}
\]  
(C.53)

\[
\bar{K} \hat{k}_{t+1} = \bar{K}(\hat{k}_t + \hat{\xi}_t)(1 - \delta_t^c) + \hat{m}_t
\]  
(C.54)
The Two-Sector FA model: Log-linearized model

\[ \bar{H} \hat{h}_{t+1} = \bar{H} \hat{h}_t (1 - \delta^h_t) + Y \bar{y} \hat{h}_t \]  
(C.55)

\[ \delta_t = \rho \delta_{t-1} - \epsilon^\delta_t \]  
(C.56)

\[ m \hat{p}_t = \rho_{mp} m \hat{p}_{t-1} - \epsilon^m_{t} \]  
(C.57)

\[ \hat{j}_t = \rho_{j} \hat{j}_{t-1} - \epsilon^j_t \]  
(C.58)

\[ \hat{\xi}_t = \rho_{\xi} \hat{\xi}_{t-1} - \epsilon^\xi_t \]  
(C.59)

\[ Dc d c_t = \Psi Dc d c_t^{(1-\Psi_p)} (-\Psi_p e c_t \hat{\Gamma}_t + e c_t \hat{\Gamma}_t + \hat{d} c_{t-1}) + \frac{\epsilon_c}{1 - \Psi} \left[ \frac{((1 - \Psi P I^{(\gamma-1)(1-\Psi_p)})}{(1 - \Psi)} \right]^{\frac{\epsilon_c}{(1-\Psi)}} \]  
(C.60)

\[ \Psi P I^{(\gamma-1)(1-\Psi_p)} ((\Psi - 1) \hat{\Gamma}_t + \Psi P (1 - \Psi) \hat{\Gamma}_{t-1}) \]

(C.61)

\[ F c f c_t = \bar{Y} c P m c (y m c_t + p m c_t^\gamma) + \beta \Psi \bar{\lambda} F c (\hat{\lambda}_{t+1} - \Psi_p e c_t \hat{\Gamma}_t + e c_t \hat{\Gamma}_{t+1} + f c_{t+1}) \]  
(C.62)

\[ F c \bar{f} c_t = \bar{Y} c y m c_t + \beta \Psi \bar{\lambda} F c^{(\gamma-1)} F c \bar{f} c_{t+1} + \Psi_p (1 - e c_t) \hat{\Gamma}_t - (1 - e c_t) \hat{\Gamma}_{t+1} + f c_{t+1} \]

(C.63)

\[ \hat{\Gamma}_{c,t} = f c_t - \bar{f} c_t + \hat{\Gamma}_t \]

(C.64)

\[ \hat{\Gamma}_{h,t} = f h_t - \bar{f} h_t + \hat{\Gamma}_t \]
\[ \hat{\Pi}_{c,t} = \Psi \hat{\Pi}_{c,t} + (1 - \Psi) \hat{\Pi}_{c,t} \]  
\[ \hat{\Pi}_{h,t} = \Psi_h \hat{\Pi}_{h,t} + (1 - \Psi_h) \hat{\Pi}_{h,t} \]  
\[ \bar{D}h_d h_t = \Psi_h \bar{D}h D^{\Psi_h(1-\Psi_h)} (-\Psi_{ph} \epsilon_h h_{t-1}^h + \epsilon_h h_{t}^h + d_{t-1}) \]  
\[ + \frac{\epsilon_h}{1 - \Psi_h} \left[ \frac{((1 - \Psi_h D (\Psi_h - 1)(1 - \Psi_{ph}))}{(1 - \Psi)} \right]^{\frac{\Psi_h}{1 - \Psi_h}} \]  
\[ \Psi_{ph} D (\Psi_h - 1)(1 - \Psi_{ph}) ((\Psi_h - 1) h_{t}^h + \Psi_{ph}(1 - \Psi_h) h_{t-1}) \]  
\[ F_{h} h_t = \Psi_{ph} m_{t}(y_{m} h_t + p_{m} h_t) + \beta \Psi_h F_h (\lambda_{t+1} - \Psi_{ph} \epsilon_h h_t + \epsilon_h h_{t+1} + f_{t+1}) \]  
\[ \bar{F} \hat{h} h_t = \Psi_{ph} \bar{F}(\Psi_{ph} - 1) \bar{F} h (\lambda_{t+1} + \Psi_{ph}(1 - \epsilon_h) h_{t}^h - (1 - \epsilon_h) h_{t+1} + f_{t+1}) \]
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