PARAMETER EFFICIENT PREDICTION OF UNCONFINED GROUNDWATER LEVELS AND STREAMFLOW

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March 2005

A thesis submitted for the degree of Doctor of Philosophy of
The Australian National University
This thesis represents independent and original research and is my own work except as indicated in the acknowledgements.

Damian Jellett

30 March 2005
ACKNOWLEDGEMENTS

The author wishes to express thanks to his supervisors Professor Ian White and Professor Michael Hutchinson at the Centre for Resource and Environmental Studies (CRES), Australian National University; Dr. Mirko Stauffacher and Dr. Lu Zhang at the Commonwealth Scientific and Industrial Research Organization (CSIRO) Division of Land and Water; and Dr. Alan Wade formerly at ActewAGL and now at CRES.

Thanks are due to Dr. Jason Sharples at CRES for advice. Thanks are due to Mr. Garry Newton at Ecwise Environmental for help obtaining data. Thanks are also conveyed to the staff of CRES for logistical support.

The author wishes to express thanks to his parents, siblings and friends for support during this research.
ABSTRACT

The aims of this research were to critically examine different approaches to predicting unconfined groundwater elevations and streamflow; to develop and test a parameter efficient model for the prediction of groundwater elevations and streamflow given spatially varying climatic variables; and to use the model to investigate possible effects of climate change on groundwater levels and streamflow in the Australian Capital Territory (ACT). A range of available techniques, process-based, empirical and conceptual, for groundwater level predictions together with water balance models were critically examined. Conceptual and empirical models were selected for examination.

The data used in this research included: monthly groundwater level records from five to thirty years in length for thirty two bores in consolidated alluvial soils in the ACT and one bore in New South Wales (NSW); and monthly rainfall and pan evaporation data for south eastern Australia. Elevation dependent tri-variate thin-plate smoothing splines were used to spatially interpolate the monthly rainfall and pan evaporation data to allow modelling of groundwater levels in bores where there were no nearby rainfall or pan evaporation gauges. The square-root transformation was used to normalise both the rainfall and pan evaporation data when applying the splines. This ensured more reliable statistical uncertainty estimates and allowed for a procedure for detecting erroneous data to be established.

Investigations were carried out on an empirical model based on accumulated deviations from average rainfall. The model was developed to identify trends in monthly and quarterly average groundwater levels. Model fits were improved using monthly pan evaporation data. A lumped parameter water balance model, previously used for weekly soil moisture index prediction for crop growth, was extended to predict groundwater levels. This model was compared with a lumped parameter monthly water balance model used for streamflow prediction that was extended to predict groundwater levels. In testing the models, $h$-$v$-block cross-validation was used. The moving-block bootstrap procedure and also uniform grid-search were employed to calculate estimates of uncertainty in the modelling. The extended
streamflow prediction model was found to be superior and was efficiently parameterised to accurately model the dynamics of groundwater levels.

The model was found to work accurately in different applications including: determination of recharge patterns over time; use of weekly data and data of different configurations of spatial and temporal interpolations; month-ahead prediction of groundwater levels; prediction of groundwater levels after calibration on streamflow data; prediction of streamflow after calibration on groundwater level data. In addition the model was applied to calculate possible effects of climate change on groundwater level and streamflow.

The CSIRO Mk2 Global Climate Model (Hirst et al., 1996; Hirst et al., 1999) was used to estimate the effect of projected climate change on rainfall and pan evaporation in the ACT up to 2030AD under different scenarios. Applying the tested model to the projected climate changes yielded a range of possible groundwater level and streamflow changes in the Orroral Valley, ACT, up to 2030AD. The calculations suggested that there would be no significant change in Orroral Valley groundwater levels or streamflows by 2030AD due to climate change. Suggestions for future research are also discussed.
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INTRODUCTION

1.1 Water and International Priorities

Life as we know it depends on water. Over the past two decades there has been increasing recognition that both the use of water by humans and their impacts on its quality pose threats to whole communities. The UN General Assembly Millennium Declaration in 2000 resolved:

"To halve by the year 2015 the proportion of the world’s population who are unable to reach or afford safe drinking water" and "to stop the unsustainable exploitation of water resources."

The Stockholm International Water Institute has further considered the UN Millennium Target. In its 2002 Stockholm Statement, it concluded:

"There is an urgent need for all governments, inter-governmental and non-governmental organizations, other policy and decision-making bodies and actual water users to take immediate action to ensure that water security, in its broadest sense, becomes a reality during the next generation. Such action should be based on four principles: water users must be involved in the governance of water resources; the link between economic growth and water degradation must urgently be broken; urban water services are crucial for urban stability and security; and policy, planning and implementation must be based on integrated solutions."

Water was a major concern at the World Summit on Sustainable Development in Johannesburg in 2002. The Summit’s Implementation Plan reiterated the Millennium Development Goal on water and also set a new target:

"To halve the proportion of people who do not have access to basic sanitation by 2015."
As a way of achieving both water supply and sanitation goals, the Summit recommended to improve the efficient use of water resources and promote their allocation among competing uses in a way that gives priority to the satisfaction of basic human needs and balances the requirement of preserving or restoring ecosystems and their functions, in particular fragile environments, with human domestic, industrial and agricultural needs, including safeguarding drinking water quality. These goals recognise the key role of water in alleviating poverty and in the areas of agriculture, energy, health, biodiversity and ecosystems. European nations have been generous and energetic supporters of international efforts to use and share water more effectively.

These calls for action and their urgency were reiterated at the Third World Water Forum in Kyoto in March 2003, where the interlinking of societies, economies, land uses and ecosystems by water was explored. Central issues at the Forum were: the sustainability of water supply systems and their governance; the impact of water supply systems on human settlements and ecosystems; irrigation and poverty alleviation; and transboundary water sharing. The Forum also highlighted the tensions between the concepts of water as a commodity and water as a human and ecological necessity. The impact of water resource development and pricing on poor and indigenous communities was of particular concern.

Internationality priorities of necessity focus on areas of most urgent needs. Priorities within regions and countries vary depending on needs.
1.2 Regional Priorities

Australia has long-acknowledged interests and responsibilities in the Asia-Pacific Region. Problems in water management in this region are amongst the most severe in the world. Limited resources, rapid population growth, the development of megacities, extreme events of both major inundations and severe drought, and limited capacity provide major challenges. Australia has recognised its responsibilities in the region and published in 2003 its water aid policy for the region “Making every drop count. Water and Australian Aid” (AusAID, 2003). The aims of Australia’s water aid policy are to help reduce poverty and raise living standards in developing countries through promotion of efficient, equitable and sustainable use of water resources. Australia’s water assistance will focus on the two central themes of water governance and delivery systems. In many of the island nations within Australia’s sphere of influence, and particularly small island nations, groundwater is vitally important.
1.3 Australian Priorities

Australia is the driest inhabited continent after Antarctica, yet it is amongst the highest per capita water consuming nations (Smith, 1998). Local problems together with the international focus on freshwater have helped raise water awareness to a high level in Australia. Problems include droughts in Eastern Australia, increasing salinity, climate change, particularly in Western Australia and the impacts of water reforms.

The adoption by Council of Australian Governments (COAG) of both the 1992 National Strategy for Ecologically Sustainable Development and the 1993 Hilmer report on national competition policy have led to the greatest changes in water policy and strategy since federation. The 1995 COAG water reforms had two central themes: the need for pricing mechanisms to reflect the true costs in supplying water; and the need to better balance water allocation between consumers and the environment.

All state and territory governments embraced these reforms, although current water markets are more restricted than envisaged initially by COAG. The reforms are aimed at reducing inefficient or low-return water use and at introducing appropriate rates of depreciation of the investment, nationally estimated to be $90 billion, in water-supply infrastructure. These reforms have significant implications for rural restructuring.

To provide for environmental flows, the Murray-Darling Basin Commission (MDBC) in 1997 imposed a voluntary cap on further diversions of surface waters in the Murray-Darling Basin. The cap, which fixed seasonally adjusted diversions at the 1994 levels, has not yet been met by all states. The cap did not include groundwater and led to major increases in groundwater abstraction. In order to recognise the costs to the environment associated with removing water, the ACT government has imposed a water abstraction fee, levied on all consumers. This is one of the first examples in the world of the use of such an economic instrument that evaluates the costs associated with water abstraction and passes the costs on to consumers.
Transboundary water sharing is central to the MDBC’s mission. Catchment water allocations vary significantly between states. Implementation of the cap raises the problematic and controversial claw-back of water licenses or allocations to permit increased allocations to environmental flows. These changes have led water users to lobby for water rights.

The temporal variability of rainfall in Australia, with its generally old, weathered soils and stores of salinity pose particular problems. The Australian State of the Environment Advisory Council (SEAC, 1996) concluded that the long-recognised deterioration of water and related land resources continues unabated. Problems identified include:

- Equitable sharing of freshwater between competing sectors (including the environment).
- Inadequacy of and a lack of commitment to monitoring.
- Over allocation of water for irrigation in substantial areas of the Murray-Darling Basin.
- Water quality and health in indigenous and rural communities.
- The price of water not encouraging conservation nor reflecting the cost of supply and downstream environmental costs.
- Impacts of land use (high groundwater levels, sediment loads, salinity and soil acidity) on streams, estuaries and groundwater.
- Toxic algal blooms posing threats to domestic water supplies.
- River regulation through dams and weirs dramatically altering the ecology of streams and floodplains, with severe impacts on native biota such as fish.
- Half of Australia’s wetlands having been lost with others under threat.
- Land use changes in domestic water supply catchments posing problems for drinking water quality for urban communities.
- Increasing water demands of large cities and their downstream impacts together with increased waste water discharge.
- Obsolescence and deterioration of water storage and reticulation infrastructure valued at over $90 billion.
- Inadequacy of institutions and policy.
The National Water Initiative

In order to address these and other issues COAG released a wide-ranging and ambitious water policy package called the National Water Initiative in June 2004. The broad aims of this initiative are to establish tradeable water rights, to provide for environmental flows, to develop institutions and improve intergovernmental coordination and to establish intensive information systems. The envisaged outcomes flowing from these decisions will be a nationally compatible water market with a regulatory and planning based system of managing surface and groundwater resources for rural and urban use that optimises economic, social and environmental outcomes.

The National Water Initiative has the following ten key objectives:

- Clear and nationally-compatible characteristics for secure water access entitlements.
- Transparent, statutory-based water planning.
- Statutory provision for environmental and other public benefit outcomes, and improved environmental management practices.
- The return of all currently over allocated or overused systems to environmentally-sustainable levels of extraction.
- Progressive removal of barriers to trade in water and meeting other requirements to facilitate the broadening and deepening of the water market, with an open trading market to be in place.
- Clarity around the assignment of risk arising from future changes in the availability of water for the consumptive pool.
- Water accounting which is able to meet the information needs of different water systems in respect to planning, monitoring, trading, environmental management and on-farm management.
- Policy settings which facilitate water use efficiency and innovation in urban and rural areas.
- Addressing future adjustment issues that may impact on water users and communities.
- Recognition of the connectivity between surface and groundwater resources and connected systems managed as a single resource.
A National Water Commission (NWC) is in the process of being established. The NWC will be responsible for providing advice to COAG on national water issues and to assist in the implementation of the National Water Initiative.

Knowledge gaps identified in the National Water Initiative that hindered water reform implementation included knowledge of water budgets and assessments of water availability through time and across catchments; changes to water availability through climate change and land use change; and interaction between surface and groundwater components.

The water situation in Australia, and Australia’s international obligations indicate that there is a need to develop research-based expertise to address the issues identified above.
1.4 Unconfined Groundwater

For many of the most water stressed countries, groundwater is a major source of freshwater. It is estimated that around 99% of the liquid fresh water on Earth is groundwater (Freeze and Cherry, 1979). Unconfined groundwater is a particularly important water source. It is generally more readily accessible and has better yields than confined groundwater. Unconfined groundwater levels fluctuate over time. These fluctuations can range up to ten metres in a year and occur when recharge to the groundwater aquifer is not equal to aquifer discharge and extraction (Olin and Svensson, 1992). Changes in water levels are caused by variability in precipitation, evaporation, temperature; atmospheric pressure especially in the case of swelling soils; transpiration by vegetation; tidal effects near the sea (Shih and Lin, 2002); earthquakes (Wang et al., 2004); artificial recharge; and extractions through pumping.

The groundwater level in a consolidated, unconfined aquifer is the elevation of the surface at which the pressure of the water is equal to the atmospheric pressure. Recharge is inflow of water to an aquifer while discharge is defined as outflow of water from an aquifer. Figure 1.1 shows the groundwater level of an unconfined aquifer, or water table above an impermeable layer.

![Vertical soil profile showing different layers](image-url)

**Figure 1.1** Vertical soil profile showing different layers
Prediction of groundwater levels is of critical importance for a number of reasons. Settlements, agricultural communities and businesses that depend on groundwater need to know about the sustainability of their resources (Wolfgang, 2000); waste facilities need an indication of how likely it is that groundwater will impinge on buried waste storage sites (Czarnecki, 1989; Chekuri et al., 1994); local governments need to investigate the possibility of subsidence of land caused by decreases in groundwater levels (White et al., 2001; Leake, 2003); state authorities can use groundwater level as a drought indicator (Hoffman and Domber, 2003). In addition, there is broad concern over increases in shallow groundwater levels and their influence on dryland salinity, particularly in Australia (Ghassemi et al., 1995). In the Murray-Darling Basin, assessment of groundwater resources has been of particular concern as the cap on surface water diversions in the basin has increased demands on groundwater resources (COAG, 2004). Being able to predict changes in groundwater levels increases our understanding of the water cycle and human impacts on it.
1.5 This Research

The assessment of groundwater resources has often been hindered by inadequate data for parameter identification in complex models (Chapman, 1990). Simple, parameter efficient models offer the potential to be parameterised using the available data and allow for applications such as land use change assessment and climate change predictions. Most available hydrological models are over-parameterised (Jakeman and Hornberger, 1993) leading to interactions between parameters and correlation between parameter estimates.

The main aim of this research was to critically examine different approaches to predicting groundwater levels and streamflow and to develop a parameter efficient model that could accurately predict groundwater levels and streamflow in the Australian Capital Territory using the available data. The hypothesis being tested is that a simple model with few parameters is sufficient to accurately predict groundwater levels and streamflow at the monthly time scale.

A secondary aim of this research was to allow the prediction of groundwater levels at locations where there were no meteorological stations nearby. This aim involves the use of spatial interpolation techniques and data transformations to normalise meteorological data.

Other aims of this research were to compare the performance of simple models while avoiding violation of statistical assumptions and to quantify uncertainties in parameter estimates and uncertainties in predictions using the most appropriate statistical techniques.

A final objective of this work was to test the most efficient model in applications including: identification of groundwater recharge and seasonal recharge patterns; prediction of groundwater levels at different temporal and spatial scales; prediction of groundwater levels after calibration on streamflow data; prediction of streamflow after calibration of groundwater level data. The model was also required to be robust
1.5 Introduction: This Research

enough to allow long term predictions to investigate possible effects of climate change on the groundwater levels and streamflow.

Thesis Outline

Chapter 1 presents the introduction to this research giving an indication of how this research relates to broader international, regional and national concerns and priorities in water resources.

Chapter 2 presents a review of techniques of groundwater level prediction and water balance models. This identified contemporary approaches to streamflow and groundwater level measurement and prediction. When data are limited, either empirical models or conceptual water balance models appear most suited. An empirical model and two conceptual water balance models were selected for modification and extension.

Chapter 3 describes the study region and available data and the processing required. Available data include monthly rainfall and pan evaporation measurements at many locations in Australia, along with groundwater level data for thirty two bores in the ACT and one bore in Wagga Wagga, NSW. Streamflow data were also available for four streams located in the ACT. The rainfall and pan evaporation data were spatially interpolated using elevation dependent tri-variate thin-plate smoothing splines. This allowed estimation of rainfall and pan evaporation at sites where there were no gauges.

Chapter 4 describes the models examined. These models were chosen for their reported good performance. Proposed extensions and modifications are discussed, including the addition of recharge equations to models designed only for streamflow prediction.

Chapter 5 details the model testing process and the model performances on groundwater level prediction in the ACT. The procedure used for the testing was \( hv \)-block cross-validation which was able to handle model residuals that were autocorrelated. The \( hv \)-block cross-validation procedure is asymptotically equivalent to the Bayesian Information Criterion and rewards model simplicity along with
model predictive performance (Racine, 2000). An extension of a conceptual model was found to be the best performing model and became the focus of the remainder of this research.

Chapter 6 investigates model uncertainty and presents the parameter uncertainties and prediction uncertainties for the best performing model from Chapter 5. While the model uncertainty was implicitly calculated in the model testing in Chapter 5, the model uncertainty is explicitly presented in Chapter 6 along with different techniques for investigating model uncertainty. Global sensitivity analysis methods, the uniform grid-search and the moving-block bootstrap procedure were used. The parameter values are also interpreted along with model internal variables in this chapter.

Chapter 7 presents some applications of the model. These include indication of recharge patterns; accurate month-ahead prediction of groundwater levels; model operation at different temporal and spatial scales; prediction of streamflow after calibration on groundwater level data; and prediction of groundwater levels after calibration on streamflow data. The model was also used to predict changes in groundwater level and streamflow in the Orroral Valley, ACT due to projected climate change up to 2030AD. Estimates of changes in rainfall and pan evaporation due to climate change were calculated using the CSIRO Mk2 model (Hirst et al., 1996; Hirst et al., 1999) under different scenarios of greenhouse gas emissions.

Chapter 8 presents the conclusions. The main conclusion was that an accurate, simply parameterised groundwater level and streamflow prediction model had been developed as an extension of a streamflow prediction model. This model performed better than the other tested models and was able to accurately predict groundwater level and streamflow simultaneously. The climate change investigation found that groundwater levels and streamflow in the Orroral Valley would not change significantly due to climate change by the year 2030. Suggestions for future work are also given including using the model to evaluate the effects of the vegetation changes in the Orroral Valley caused by the January 2003 bushfire.
Chapter 2 presents a review of techniques for unconfined groundwater level prediction and water balance models. Water balance models used to predict streamflow were investigated with a view to their extension to predict groundwater levels. The models reviewed can be broadly classified as process-based, conceptual or empirical.

2.1 Overview

Unconfined groundwater elevations and groundwater recharge may be measured directly, estimated from chemical or isotopic tracer data or estimated from climatic data. The direct measurement of groundwater levels uses piezometers or float mechanisms for continuous measurements or manual readings, as discussed in Chapter 3. Other measuring devices include weighing lysimeters, to measure changes in soil moisture storage, such as used by Chapman and Malone (2002). Changes in soil water storage can be used to infer groundwater recharge. Weighing lysimeters, while accurate for small areas, are costly to construct and operate. The direct measurement of groundwater levels is absolutely essential in order to understand groundwater systems and to test the predictive capability of models.

Estimation of groundwater recharge using tracer data includes the use of chemical tracers such as chloride and radioisotope tracers such as Tritium (Allison et al., 1994). The chloride mass-balance approach, which relies on the progressive changes of chloride concentration from rain to the soil water phase and to groundwater, provides a measure of the evapotranspirative loss from the system. This technique is particularly attractive because of its accuracy, simplicity and relatively low expense (Allison et al., 1994).

Modelling of groundwater levels allows simulations of different scenarios such as investigation of the effects of land use or climate change. Field measurements alone
are limited in such applications because it is expensive to trial different land use options with replicated measurements and often long time scales are involved. Modelling allows land use and climate change to be simulated, but the veracity of model predictions must be determined by comparison with actual measurements.

In this research, tracer data were not available, so modelling was investigated to determine its usefulness in estimating groundwater recharge and in predicting groundwater levels.
2.2 Process-Based Models

Process-based models consist predominantly of the relevant physical laws governing water flow into and through soil, with minimal reliance on empirical relations. The hydrological models identified in this review all had empirical components, so the classification of process-based models was used here rather than the classification of physical models. The advantage of process-based models is that they use our understanding of the key processes involved thus allowing for versatile extrapolative predictive capabilities (Evans, 2000). A disadvantage of process-based models is that it is often not feasible to measure all of the spatially-dependent parameters. Indeed the limited quantity and quality of available data may preclude parameter estimation. The process-based models examined here were mostly designed for fine spatial scales such as paddocks or plots and short temporal scales of one day or less.

2.2.1 Processes

The processes considered here are components of the hydrological cycle. The hydrological cycle is the process of evaporation, condensation and transport that controls the distribution of the Earth’s water (Freeze and Cherry, 1979). Figure 2.1 shows the flows of water that occur at various stages in the hydrological cycle.

![Figure 2.1 Flows of water in the hydrological cycle](image-url)
As Figure 2.1 shows, water precipitates from the atmosphere and falls to the ground as rain or snow. Some of this precipitated water recharges underground aquifers; some runs off into streams, reservoirs and the ocean; some evaporates; and some is transpired by vegetation. The evaporated and transpired water then precipitates and the cycle continues.

Figure 2.2 shows an idealised vertical profile of a catchment indicating surface and groundwater flows. The rainfall that infiltrates into the unsaturated soil layer can drain further and recharge the unconfined groundwater thus increasing the groundwater level. When groundwater is discharged into the stream as base flow the groundwater level decreases (Lee and Risley, 2002). The word evapotranspiration in Figure 2.2 refers to the sum of the water evaporated from the soil and open water and the water transpired by vegetation. The components of the catchment water balance in Figure 2.2 may be modelled, measured on site or estimated remotely (Marsily, 1986).

**Figure 2.2** Idealised vertical profile of a catchment showing flows of water and the groundwater level at a bore
Process-based hydrological models usually operate at daily or shorter time scales or may operate as event-based models, particularly for flood forecasting. These models use water balance data such as rainfall and pan evaporation rates. In addition, process-based models often use energy balance data such as temperature, solar radiation and wind velocity to estimate actual evapotranspiration. Process-based models also often employ detailed land use as well as vegetation data such as leaf area index. In the following we consider in detail the hydrologic components described in process models.

### 2.2.2 The WAVES Model

The WAVES model was developed by Zhang and Dawes (1998) as an integrated energy balance and water balance model. The model is one-dimensional and uses a daily time-step to simulate the vertical fluxes of mass and energy between the atmosphere, vegetation and soil systems. Typically, it is applied to paddock or plot scale observations.

It is assumed in the WAVES model are that the soil is rigid, isothermal, non-hysteretic (the soil response depends on the current situation rather than previous situations), with no soil airflow, and with preferred pathways and macropores not explicitly modelled. Solute is preserved, soil properties are stationary over time and rainfall intensity is constant over each rainfall event.

The WAVES model uses the one-dimensional water continuity equation as the water balance equation:

\[
\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z} \tag{2.1}
\]

where \( \theta \) is water content \((m^3 m^{-3})\), \( t \) is time \((s)\), \( q \) is the water flux \((m \ s^{-1})\) and \( z \) is the depth \((m)\) with the positive direction downwards. This states that the change in water content with time is equal to the change in the rate of water flow with depth.
The WAVES model also uses Darcy's Law (Darcy, 1856) to describe soil water flow in one-dimension:

$$q = -K \frac{\partial h}{\partial z} \tag{2.2}$$

where $K$ is the water content dependent hydraulic conductivity (m s$^{-1}$); $h$ is the hydraulic head (m); and $z$ is the depth (m) with the positive direction downwards. It states that the rate of water flow through a porous media such as sand is proportional to the change in hydraulic head $\partial h$ (m) and inversely proportional to the change in depth $\partial z$ (m).

The hydraulic head $h$ (m) is equal to the sum of the elevation head in the gravitational field $-z$ (m) (the minus sign makes the upwards direction positive) and the hydraulic potential $\psi$ (m) as shown:

$$h = -z + \psi \tag{2.3}$$

Partially differentiating Equation 2.3 with respect to $z$ gives:

$$\frac{\partial h}{\partial z} = -1 + \frac{\partial \psi}{\partial z} \tag{2.4}$$

Substituting Equation 2.4 into Equation 2.2 gives Equation 2.5:

$$q = K \left( 1 - \frac{\partial \psi}{\partial z} \right) \tag{2.5}$$

which is the one-dimensional form of Darcy's Law used in the WAVES model. Although Darcy's Law was originally found empirically, but it has been derived from the Navier-Stokes equations for fluid flow in homogenous porous media (Bear and Bachmat, 1991).
Richards’ Equation (Richards, 1931) for flow in porous media follows by substituting Darcy’s Law Equation 2.5 into the continuity Equation 2.1 to give:

$$\frac{\partial \theta}{\partial t} = -\frac{\partial}{\partial z} K \left( 1 - \frac{\partial \psi}{\partial z} \right)$$  \hspace{1cm} (2.6)

In the WAVES model, Equation 2.6 is then solved numerically for \( q \) with different initial and boundary conditions for different soil layers under investigation.

The WAVES model uses the energy balance equation:

$$R_n = P_x + \lambda E + H + A_h + G + S$$  \hspace{1cm} (2.7)

where \( R_n \) is the net radiation (W m\(^{-2}\)), \( P_x \) is the energy absorbed in photosynthesis (W m\(^{-2}\)), \( \lambda E \) is the energy used for evapotranspiration (W m\(^{-2}\)), \( \lambda \) is the latent heat of vaporisation of water, \( E \) is the evapotranspiration rate, \( H \) is the sensible heat (W m\(^{-2}\)), \( A_h \) is the advected energy (W m\(^{-2}\)), \( G \) is the soil heat flux (W m\(^{-2}\)) and \( S \) is the stored energy (W m\(^{-2}\)). Each term in the energy balance equation is calculated using equations containing variables representing measurable quantities of soil and vegetation properties.

The Penman-Monteith combination equation (Penman, 1948; Monteith, 1981) is used to model evapotranspiration:

$$\lambda E = \frac{\Delta R_n + \rho c_p A \frac{1}{r_a}}{\Delta + \gamma (1 + \frac{r_c}{r_a})}$$  \hspace{1cm} (2.8)

where \( \rho \) is the air density, \( c_p \) is a constant, \( \gamma \) is the psychrometric constant, \( r_c \) is the bulk canopy resistance (s m\(^{-1}\)), \( r_a \) is the aerodynamic resistance (s m\(^{-1}\)) (10 s m\(^{-1}\) for smooth canopies, 30 s m\(^{-1}\) for rough canopies), \( \Delta \) is the slope of the saturation vapour pressure curve, \( \lambda \) is the latent heat of vaporisation of water.
2.2 Literature Review: Process-Based Models

The WAVES model estimates parameters in Equation 2.8 are using the empirical relations:

\[ e_a = 6.1078 \exp \left( \frac{17.269 T_a}{T_a + 237.16} \right) \]  

(2.9)

\[ \gamma = 0.646 + 0.0006 T_a \]  

(2.10)

\[ \Delta = e_a (T_a + 0.5) - e_a (T_a - 0.5) \]  

(2.11)

\[ \rho = 1.292 - 0.00428 T_a \]  

(2.12)

\[ \lambda = 2501000 - 2400 T_a \]  

(2.13)

where \( T_a \) is the average of daily minimum and maximum air temperature (K) and \( e_a \) is the water vapour pressure. The use of empirical equations to calculate parameters of the Penman-Monteith combination equation is an example of a process-based model with many empirical components.

The Penman-Monteith Equation 2.8 is solved numerically for evapotranspiration for the different layers of atmosphere, canopy and soil, each with different boundary conditions.

Similarly empirical equations are used to relate each of the terms in the energy balance Equation 2.7 to measurable quantities. Parameters used in the energy balance equation include plant canopy leaf area index, surface albedo, light extinction coefficient and other vegetation properties.

The WAVES model also contains a solute (sodium chloride) balance equation to allow for salinity studies. In addition, a carbon balance equation is incorporated into the WAVES model to allow for the prediction of plant growth responses and recharge under different land use and vegetation covers. The many linked equations containing many parameters is typical of process-based models. Most of the meteorological parameters used are point measurements taken at a single location.
2.2 Literature Review: Process-Based Models

The WAVES model has been used to estimate recharge under different land uses of crop, pasture and fallow in the Mallee region of Victoria (Zhang et al., 1999). The recharge was inferred from neutron scattering measurements of soil moisture. The model simulations of recharge compared well with measurements of recharge. The main drawback of the WAVES model is that a large amount of data are required to operate the model.

2.2.3 Other Process-Based Models

Other examples of process-based models for predicting groundwater recharge and streamflow include the NTRM model (Shaffer and Pierce, 1982; Shaffer and Larson, 1987), the SHE model (Abbot et al., 1986), the WSHS model (Al-Soufi, 1987), the PERFECT model (Littleboy et al., 1989), the THALES model (Grayson et al., 1992) and the SWAT model (Arnold et al., 1993). Typically, these models use Richards' Equation for subsurface flows and a kinematic wave equation for surface flow. These models mainly differ through data requirements, different assumptions and parameterisations or involve different processes such as the presence of snow. In theory the parameters of these models have physical meaning and can be measured.

The NTRM (Nitrogen Tillage – Residue Management) model (Shaffer and Pierce, 1982; Shaffer and Larson, 1987) is a simulation model used to examine soil-plant interactions. Uses of the model include determination of soil management strategies and the effect of climate change on soil water, crop yields and nitrate leaching. The model combines process equations for water flow, solute transport, plant growth, tillage, climate interactions, nutrient effects and soil temperature. The required data are daily weather data, soil and vegetation measurements for the field plot. The model predicts plant growth, water flows and soil temperatures. The NTRM model has been used by Radke et al. (1991) to simulate a managed corn field in Pennsylvania, USA. It predicted daily soil water, nitrogen, and temperature in the soil profile; daily biomass and leaf area; and final grain yield. Model runs gave accurate soil temperature and plant biomass predictions but poor soil water predictions compared to field measurements.
The PERFECT (Productivity, Erosion and Runoff Functions to Evaluate Conservation Techniques) model (Littleboy et al., 1989) simulates the effects of land management and environment to predict runoff, soil loss, soil water, drainage, crop growth and yield in agricultural systems. Parameters are included for soil and crop type, climate, evaporation functions, rotation and management strategy. The weather data required are daily rainfall, pan evaporation, solar radiation, temperature and daily runoff is a model output. Thomas et al. (1995) successfully applied the PERFECT model on six different soil types in Maranoa, Queensland.

The SWAT (Soil and Water Assessment Tool) model (Arnold et al., 1993) is a daily model designed to predict the effects of climate change, vegetation change, and groundwater withdrawal on runoff and groundwater (Arnold et al., 1993). The SWAT model is a distributed model with the catchment modelled being divided into homogeneous units. The empirical Soil Conservation Service (SCS) runoff curves (USDA, 1986) are used for the computation of runoff in the United States. Other data used includes soil type, texture and depth along with daily precipitation, temperature and solar radiation data. In the SWAT model, soil profiles are divided into ten layers. Infiltration moves into the soil profile where it is routed through the soil layers. A storage routing flow coefficient is used to predict flow through each soil layer, with flow occurring when a layer exceeds field capacity. When water percolates past the bottom layer, it enters the shallow aquifer zone (Arnold et al., 1993). Flow to deep aquifer systems is also modelled. The SWAT model has been applied to catchments worldwide. Chu and Shirmohammadi (2004) used the SWAT model to simulate runoff from a 1 km$^2$ catchment in Maryland, USA. The SWAT model underestimated subsurface flow and total streamflow, especially during wet periods. The study concluded that the SWAT model was able to perform acceptably for prediction of long term runoff for management purposes, but failed produce acceptable short term runoff predictions.

Finch (1998) developed a model to evaluate the sensitivity of groundwater levels to land surface parameters. The model divides the root zone into four layers as an approximation to the continuous distribution of roots with depth. Land cover was classified into three groups: permanent short vegetation; annual short vegetation; and coniferous forest. The model predicts daily streamflow and groundwater recharge.
given inputs of daily rainfall, temperature, solar radiation and wind speed. In the study conducted by Finch (1998) in the UK, the model was able to perform acceptably at estimating groundwater recharge.

None of the process-based models were tested in this research because of the lack of the extensive data required to run these models.
2.3 Conceptual Models

This section reviews conceptual models used for groundwater level or streamflow prediction. Conceptual models use concepts of the physical processes involved together with empirical relations. Conceptual models have the advantage of being simple to understand (Wheater et al., 1993). Most of the conceptual models investigated here are lumped parameter, one-dimensional models. The one dimension being modelled is the vertical dimension (Alley, 1984). A lumped parameter model uses one parameter set to encapsulate the processes and represent the spatial regions being modelled. Distributed parameter models employ different parameter sets to represent smaller areas within the total region being modelled. A feature of conceptual models is that many physical processes can be incorporated into a single equation. The spatial scale of the conceptual models often varies but is often the scale of a catchment, of order 100km² or larger. Temporal scales for many conceptual models vary from daily to monthly depending on the application. Simplifications often follow the move from shorter to longer time scales.

2.3.1 Water Balance Models

Water balance models are a subset of conceptual hydrological models where the amount of water going into a catchment, being stored in a catchment and flowing out of a catchment is explicitly conserved using a water balance equation. Monthly water balance models are frequently essential in water resource management in catchments (Makhlouf and Michel, 1994). Water balance models are useful for forecasting catchment water balance components including groundwater levels (Walker and Zhang, 2002). It is standard in one-dimensional water balance models to express all of the catchment water quantities in volume per unit area or length units, frequently millimetres. This is in preference to volume units so that properties of catchments with different areas can be compared.

The water balance equation is often modified to include different storages such as snow pack (Mimikou et al., 1991), lakes and dams (Crapper et al., 1996) as well as to include outflows such as groundwater abstraction and discharge; and flows such as interflow, the lateral flow of water above the water table. The choice of what
processes to include in the water balance equation depends on the spatial and temporal scale being modelled, the application being considered, as well as the data available.

Figure 2.3 shows the processes in a simple water balance model at the Earth’s surface. Precipitation $P$ (mm) falls on the catchment. Some water is lost through evapotranspiration $E$ (mm) while some water leaves the catchment as runoff $Q$ (mm). The remaining water infiltrates into the soil causing an increase in soil moisture storage $\Delta S$ (mm).

![Water Balance Model](image)

**Figure 2.3** The partitioning of precipitation at the ground surface into runoff, evapotranspiration and change in the soil moisture store.

The water balance equation for the system shown in Figure 2.3 is:

$$P = E + Q + \Delta S$$  \hspace{1cm} (2.14)

or in discrete time intervals, $t$

$$P_t = E_t + Q_t + S_t - S_{t-1}$$  \hspace{1cm} (2.15)
2.3 Literature Review: Conceptual Models

2.3.2 Threshold Tipping-Bucket Models

Simple water balance models were initially developed by Thornthwaite (1948) and later revised by Thornthwaite and Mather (1955). Monthly precipitation and monthly potential evapotranspiration were used as input data. Soil moisture capacity, and surplus surface water remaining fraction were the model’s two parameters, while monthly runoff was the model output. The Thornthwaite model conceptualised the recharge process with the soil profile as a single store represented by a bucket that is filled by infiltration and emptied by evapotranspiration as shown in Figure 2.4. Runoff occurs when the bucket is full and rainfall exceeds evapotranspiration. The model contained no explicit groundwater component.

![Figure 2.4](image)

**Figure 2.4** One soil layer tipping-bucket model

In this model the soil moisture content is identified with the plant available water capacity. This is the amount of water held between the ground surface and the lowest point in the root zone. The soil moisture capacity is defined as the water content at which internal drainage ceases. The lowest soil moisture content in the root zone is known as the permanent wilting point, a content beyond which plant roots cannot extract water from the soil.

The Palmer model (Palmer, 1965) is a tipping bucket water balance model developed to provide an index of meteorological drought. The soil was divided into two layers, as shown in Figure 2.5, each with its own soil moisture capacity as the model’s two parameters. Recharge to the lower layer only occurred when the upper layer had reached field capacity. Input data were monthly precipitation and monthly potential...
evapotranspiration. The output was monthly runoff and the drought index. This model is mainly used in the United States to assess agricultural drought.

Both the Thornthwaite and Palmer models used the concept that runoff or recharge does not occur until the soil moisture capacity threshold is reached. This assumption leads to the tendency to underestimate runoff during summer and autumn (Alley, 1984), as runoff can still occur over a range of soil water contents.

The Watbal model (Keig and McAlpine, 1974) is a water balance model used in the packages GROWEST (Nix et al., 1977; Hutchinson et al., 2002) and ANUCLIM (Houlder et al., 2000) to predict crop growth and ecological responses to climate. The model is a one soil layer model with weekly precipitation and weekly pan evaporation as inputs. Soil moisture capacity and soil type are the model’s two parameters. The output of the model is the soil moisture index and runoff. The soil moisture index is used in calculation of plant growth indices. The Watbal model was also used by Crapper et al. (1996) to predict lake levels. This model is described in detail in Chapter 4.
The four-parameter *abcd*-model (Thomas, 1981; Thomas et al., 1983) uses inputs of monthly rainfall and pan evaporation. Groundwater recharge and discharge were both explicitly modelled along with runoff. The parameter *a* represented the propensity of runoff to occur before the soil is fully saturated. The parameter *b* is an upper limit on the sum of evapotranspiration and soil moisture content. The parameter *c* is related to the fraction of mean runoff that comes from groundwater. The parameter *d* is the reciprocal of the groundwater residence time.

Alley (1984) reviewed the Thornthwaite, Palmer and *abcd* models and compared the models at prediction of monthly streamflow for fifty years of monthly rainfall, pan evaporation and streamflow data from New Jersey. It was concluded that predictive errors were similar for these models. This indicated that the simpler two parameter models were as successful as the four parameter model.

Steenhuis and Van Der Molen (1986) modified the Thornthwaite model to predict groundwater recharge at the daily time scale. The model performed well at predicting daily groundwater recharge in Long Island, USA, given daily rainfall and pan evaporation data. This extended model had six parameters including the length of soil zone being modelled, a pan evaporation factor, saturated hydraulic conductivity of the soil, dry soil moisture content, saturated moisture content and a recharge parameter. These parameters pose difficulties in estimation in large, spatially varying catchments.

Since the more simply parameterised water balance approaches appear to work as effectively as more complex models at a monthly time scale, the Watbal model was chosen from threshold tipping-bucket type of model for modification and testing. The Watbal model is still used regularly in the GROWEST package (Hutchinson et al., 2002) to predict runoff and soil moisture index but not groundwater levels. Hence the Watbal model required the addition of a recharge model in order to predict groundwater levels. Different recharge models proposed in this study for use with the Watbal model are examined in Section 4.2.
2.3.3 Non-Threshold Models

A set of monthly water balance models for streamflow prediction for large catchments was produced by Vandewiele (1992). Input data were monthly rainfall and pan evaporation. These models had three or four parameters and modelled both slow flow and fast runoff. These models were calibrated and tested on data from Belgium, China and Burma with good results. Vandewiele et al. (1992) used the square-root transformation to normalise the streamflow data before calibration. One problem faced in hydrology is the problem of model calibration in ungauged catchments. Vandewiele and Elias (1995) considered this problem by fitting a spatial surface to the parameter values of the model at different gauged locations. The procedure used to fit the spatial surface was kriging (Cressie, 1991) and this allowed the estimation of parameter values in ungauged catchments.

A monthly water balance model with five parameters was developed by Guo (1992) to predict monthly streamflow for large regions. The parameter set consisted of a pan evaporation factor, soil moisture capacity, a runoff parameter, an interflow parameter and a groundwater flow parameter. The interflow (lateral flow in the unsaturated layer) and groundwater flow each explicitly contributed to streamflow with a time lag. After calibration, the model performed well at predicting monthly streamflow for catchments in China given monthly rainfall and pan evaporation data.

The GR2M model (Makhlouf and Michel, 1994) is a two parameter monthly water balance model used to predict monthly streamflow using monthly rainfall and pan evaporation as inputs. The GR2M model treats the soil as two layers with the soil moisture capacity of the upper layer fixed at 200mm. The model's two parameters are a runoff parameter and a parameter to adjust the rainfall and pan evaporation values. The model was tested on data from France with reasonable success (Makhlouf and Michel, 1994).

Xiong and Guo (1999) developed a two parameter monthly water balance model and used it to simulate the streamflow in seventy catchments in the south of China. They found that this two parameter model performed as well as the five-parameter model of Guo (1992) using input data of monthly rainfall and pan evaporation. Guo et al. (2002) used this model to estimate effects of climate change on streamflow in China.
The model, termed here the XG model, was selected for testing in this research because of the demonstrated good performance while only requiring two parameters.

2.3.4 Daily Conceptual Models

The HBV model (Bergström, 1992) is a daily streamflow model that includes the snow pack and lake water storage as well as upper and lower groundwater stores. The input data are daily rainfall, air temperature and potential evapotranspiration. Air temperature data are used for calculations of snow accumulation and melt. A threshold temperature is used to distinguish rainfall from snowfall. The fourteen parameters of the HBV model include five snow storage parameters, three soil type parameters and six groundwater storage parameters, all of which are estimated by calibration. Although the HBV model has produced accurate predictions in many catchments throughout the world, Seibert (1997) found that the model is over-parameterised with the model predictions being insensitive to large changes in parameter values.

The IHACRES model (Identification of unit Hydrographs And Component flows from Rainfall, Evaporation and Streamflow data) (Jakeman et al., 1990; Jakeman and Hornberger, 1993) is a systems based model that has seven parameters and predicts streamflow given rainfall and temperature data. The supported time steps are minutes, hours and days. The subsurface flows are explicitly modelled. Chapman and Malone (2002) tested the daily groundwater recharge equations used in the IHACRES model using weighing lysimeter data with good results. Croke et al. (2002) investigated the use of groundwater discharge equations in IHACRES involving from one to three parameters. IHACRES has been successfully applied to catchments of different sizes and under different climate conditions from 1km² experimental catchments for the Thames River in the UK to 100,000km² catchments for the Avon River in Western Australia.

The AWBM model (Australian Water Balance Model) (Boughton, 1993) was developed as an eight parameter catchment water balance model that relates runoff to rainfall at either daily or hourly time scales. The input data required are monthly pan evaporation, and daily or hourly rainfall. The model was found to perform satisfactorily at daily streamflow prediction in Australia (Boughton, 1993).
The SIMHYD model was developed by Chiew et al. (2002) who found that it performed satisfactorily for streamflow prediction in 300 catchments across Australia. The SIMHYD model has seven parameters and requires input data of daily rainfall and daily areal potential evapotranspiration to predict daily runoff.

Since the simply parameterised XG model appeared to work as effectively as more complex models at a monthly time scale, the XG model was chosen from non-threshold type of model for modification and testing. The XG model has been used to predict runoff but not groundwater levels. Hence the XG model required the addition of a recharge model in order to predict groundwater levels. Different recharge models proposed in this study for use with the XG model are examined in Section 4.3. In addition, the exponential recharge equation found to work at the daily time scale by Chapman and Malone (2002) was selected for testing.
2.4 Empirical Models

Empirical models involve relationships between inputs and outputs that do not represent the physical laws governing the key processes. Empirical models can only predict under the same conditions that existed in the historical record used for calibration (Evans, 2000). The advantage of empirical models is that they are simple and usually easily programmed. Their main disadvantage is that they are limited in extrapolative predictive capability. The empirical approach to groundwater level estimation includes simple linear regression, autoregressive time series models, transfer function models and neural network models.

2.4.1 Simple Linear Regression

The simplest model of groundwater levels assumes that the groundwater level stays constant over time. Equation 2.16 shows this model

\[ G_t = \beta_0 + \epsilon_t \]  

(2.16)

where \( G_t \) is the groundwater level (m) at time \( t \), \( \beta_0 \) is the mean groundwater level parameter (m) and \( \epsilon_t \) is the residual (m) at time \( t \). This model has been used in nearly all of the groundwater level prediction literature as the benchmark for the performance of other models. This model was tested in this research in Chapter 5.

Another simple linear regression model of groundwater level is the Time Trend model given by Equation 2.17.

\[ G_t = \beta_1 + \beta_2 t + \epsilon_t \]  

(2.17)

where \( \beta_1 \) is the initial groundwater level parameter (m), \( \beta_2 \) is the trend parameter (m [time]^{-1}). This is appropriate when groundwater levels change consistently over time. Ferdowsian and Pannel (2001) tested this model on groundwater bores in Western Australia. This model was tested in this research.
2.4 Literature Review: Empirical Models

Ferdowsian et al. (2001) extended the Time Trend model to include the influence of rainfall. The HARTT (Hydrograph Analysis: Rainfall and Time Trends) model (Ferdowsian et al., 2001) involved regression between accumulated residual rainfall and groundwater level at the monthly time scale. They tested this model with data from Western Australia with a Mediterranean climate. Because of the apparent good performance, the HARTT model was tested in this research.

A disadvantage of simple linear regression models used in groundwater level modelling is that the residuals are usually autocorrelated. This is because the current time-period’s groundwater level depends on the previous time-period’s groundwater level. This autocorrelation means that Neyman-Pearson hypothesis testing may not be performed because the independence assumption is violated. In an effort to overcome autocorrelated residuals, the use of quarterly data rather than monthly data has been tested by Holtschlag and Sweat (1999) and Ferdowsian et al. (2001) with varied success. Autocorrelation of residuals is specifically addressed in this work in Chapter 5.

2.4.2 Autoregressive Time Series Models

To overcome the autocorrelation of residuals, autoregressive models have been used (Kothyani et al., 1993; Bierkens et al., 2001). Bierkens et al. (2001) used an autoregressive lag 1 model with rainfall as an exogenous input (ARX model) given by

\[ G_t - \beta_1 = \beta_2 (G_{t-1} - \beta_1) + \beta_3 P_t + \epsilon_t \]  

(2.18)

where \( \beta_1 \) is the mean groundwater level parameter (m), \( \beta_2 \) is an autocorrelation parameter and \( \beta_3 \) is the rainfall influence parameter.

In essence, the conceptual models discussed in Section 2.3 may be interpreted as autoregressive models because in the water balance equation used in conceptual models, the current time-period’s soil moisture content is a function of the previous time-period’s soil moisture content.
2.4.3 Nonparametric Regression

Autoregressive moving average (ARMA) models have been used in hydrological modelling (McLeod et al., 1977; Bras and Rodriguez-Iturbe, 1985). The ARMA model describes a time series with trend removed and is used to smooth the data. Adamowski and Feluch (1991) used nonparametric regression to estimate groundwater levels given streamflow with good results.

2.4.4 Transfer Function Models

Linear transfer function models (Box et al., 1994) have been used for monthly streamflow predictions with some success in Australian catchments (Norton and Chanat, 2003).

2.4.5 Artificial Neural Network Models

The GR3J and GR4J artificial neural network models have been used by Anctil et al. (2004) to model daily streamflow given rainfall and pan evaporation data. The artificial neural network model used a large set of linear equations to represent the system and was tested on catchment data from France with some success. The large set of parameters in the linear equations were estimated using a training set of the data and the parameter values of the hidden layer linear equations did not have physical interpretations.

2.4.6 Fractal Soil Models

The use of fractal scaling to model soil particle-size distributions (Tyler and Wheatcraft, 1992) may be more realistic than the assumption of separate homogeneous soils layers used in many hydrological models. However, Bird and Dexter (1997) found that the derivation of pore size distributions from the soil water retention characteristic is complicated by the influence of the connectivity of the pore space on drainage. They used a random fractal pore network model and found that it was not possible to obtain an accurate measure of the pore size distribution from the water retention data alone because of the connectivity between pores.
Because of the long term use and their practical implications the Mean model and the Time Trend model were selected for closer examination. The simplicity of the HARTT model also made it attractive for examination and modification. These models are described in detail in Chapter 4.
2.5 Model Selection

2.5.1 Scale

It is clear that groundwater level and streamflow may be modelled at different temporal and spatial scales. The appropriate scale depends on the scale of the available data and the desired scale of the model output and the particular applications. The choice of time scale affects the type of model that may be used. The following example illustrates the effect of different temporal scales on model structure. At the hourly time scale, vegetation water storage in large woody plants is a significant variable. This is because some plants sustain transpiration during the day using water that was absorbed during the night (Goldstein et al., 1998). At time scales of one day or greater, vegetation water storage is relatively constant, except when an extreme event such as a bushfire occurs. Hence, at longer time scales vegetation water storage can be neglected.

Spatial scale also affects the type of model that may be used. In regions at broad spatial scales of around 1000 km$^2$, the net groundwater flow is of smaller magnitude than net surface flow. Therefore groundwater flow is often not modelled in broad spatial scale models. At finer spatial scales of around 10 km$^2$, groundwater flow is a significant component of the water balance and must be considered.

Another example of the importance of spatial scale is shown in Figure 2.6. When catchment runoff is being estimated and the catchment is divided into smaller spatial units, runoff routing is required. When a lumped catchment is considered, runoff routing is irrelevant, particularly at longer time scales. This leads to the conclusion that models based on longer time scales and broader spatial scales are simpler than those based on shorter time scales and finer spatial scales. The scale used for a given task is often a compromise between simplicity of the model and the required output of the model.
In this research, the temporal scale of interest was monthly or weekly and the spatial scale was catchment-based with areas with order of magnitude 10km$^2$ to 100km$^2$. This scale and the available data dictated the use of conceptual and empirical models rather than process-based models.

**Figure 2.6** Diagram showing spatial units and runoff routes for the a. fine scale and b. lumped catchment broad scale
2.5.2 Model Uncertainty

Uncertainty in model prediction is the sum of uncertainty caused by model structural errors and uncertainty caused by errors in the measured input data. The component of output uncertainty of process-based models due to model structural error should theoretically be small. However, the component of output uncertainty of process-based models due to input data uncertainty may be large (Jakeman and Hornberger, 1993). This is because more parameters and more input data mean more input uncertainties. This suggests the use of conceptual models with fewer input data requirements. None of the process-based models identified in this literature review were tested in this research because of the lack of data required for their use.

Detailed descriptions of the statistical techniques of model selection based on model uncertainty are given in Chapter 5.
2.6 Knowledge Gaps

2.6.1 Hydrological Models

Most of the hydrological models discussed above were designed for the purpose of streamflow prediction rather than groundwater level prediction. Many of the models identified have not been tested on groundwater level data. This may be because the models were designed for broad spatial scale catchments where the groundwater flow was very slow compared to the streamflow. Also, streamflow data are generally much more readily available than groundwater level data. The review of process-based hydrological models revealed that even the most detailed models contained empirical relations. Many models have been designed for the same task and with the same input data requirements, but there is no consensus as to which model is the best for a given task (Beven, 2001).

It appears that no model has been tested on streamflow and groundwater level data simultaneously within the same catchment. A model that could be calibrated on streamflow data but accurately predict unconfined groundwater level data at the same time would be very useful in places where streamflow data is available but groundwater levels are not monitored.

2.6.2 Spatial Interpolation

The climatic data used in most hydrological studies is usually not spatially interpolated. Instead the rainfall and pan evaporation data nearest to the streamflow gauge or groundwater piezometer have been used to represent the whole catchment. In some cases, where there were multiple gauges, the rainfall and pan evaporation data have been averaged within a catchment. In catchments with significant orographic effects this can lead to considerable underestimation of inputs. The use of thin-plate smoothing splines, used previously for monthly scale climate studies, is suitable for finding catchment rainfall and pan evaporation particularly in ungauged catchments. It was decided that this spatial interpolation technique would be used in this research.
2.6.3 Model Testing
It was found in this review of hydrological models that much of the model testing was statistically inadequate. Statistical assumptions were violated and potentially biased procedures were used such as split-sample validation. A review of the methods of model testing appears in Chapter 5. The poor model testing procedures used has led to an abundance of over-parameterised models. Use of inadequate data by an over-parameterised model gives rise to many difficulties in parameter estimation and statistical testing (Jakeman and Hornberger, 1993). In this research, statistically sound model testing procedures were used to find the most parameter efficient model.

2.6.4 Model Uncertainty
Most of the hydrological literature reviewed did not include estimates of prediction or parameter uncertainties. A review of methods of prediction and parameter uncertainty estimation appears in Chapter 6. Both prediction and parameter uncertainties are estimated in this research, using the most appropriate techniques.

2.6.5 Models Selected for Further Study
In this research, the available data were monthly rainfall, pan evaporation, groundwater elevation and streamflow as discussed in Chapter 3. This dictated the selection of empirical and conceptual models. The empirical HARTT model was chosen for modification and testing. The threshold based Watbal model was also selected for extension and testing. This model had been designed to predict a soil moisture index and not groundwater levels so recharge models are proposed in Chapter 4 to extend the Watbal model to predict groundwater levels. The non-threshold based XG model was also chosen for extension and testing. This model had been designed to predict streamflow so recharge models are proposed in Chapter 4 to extend the XG model to predict groundwater levels. These models were selected because they performed well on the data they were tested on and each of these models was different enough to represent a broad span of the available and useable models.
CHAPTER 3

THE STUDY REGION AND CATCHMENT DATA

This chapter provides a description of the study region and the available data as well as the data processing that was performed. The quality of the data is also discussed. The quality and availability of the data dictated what models could be tested in this research. The uncertainty in spatially interpolating the rainfall and pan evaporation data is also examined in this chapter.

3.1 General Description

In this study meteorological data from south eastern Australia and groundwater elevation data, mostly from the ACT were used. The following presents a brief general description of the study region and its water resources.

There is evidence that the Aboriginal people were living in the interior, south eastern portion of Australia from at least 21,000 years ago. Bogong moths in the high country were a favoured summer food source. European explorers first visited the “Limestone Plains” in 1820. Pastoralists commenced settlement in river flats in the 1830’s and used higher country for summer grazing where they annually burnt off to encourage pasture growth. Lower catchments, to the west and south of Canberra, were partly cleared for grazing around 1890 and severe erosion followed the rabbit invasion in 1925. Development of the national capital, Canberra, commenced in 1913 and accelerated in the 1960’s and 1970’s. To protect its cultural, flora and fauna values, an area of 1059 km², south-west of Canberra was declared firstly Gudgenby Nature Reserve in 1979 then Namadgi National Park in 1984. It is now used by small numbers of bush walkers and hikers.

Operation of Canberra’s first dam on the Lower Cotter River in the western portion of the ACT, with a capacity of about 1.8 gigalitres, commenced in 1918. The dam wall was raised in 1951, increasing storage in the Lower Cotter to its present 4.7 GL. Water is currently not drawn from the Lower Cotter because of turbidity levels due to forestry practices. In 1961 a dam on the upper Cotter, Bendora Dam, was
completed, providing storage of 10.7 GL. The rapid development of the ACT required a second dam, Corin Dam, upstream of Bendora. The 75.4 gigalitre dam and associated main was completed in 1968. Mt Stromlo Water Treatment Plant, opened in June 1967, enabled settling and disinfection of water before distribution to Canberra and Queanbeyan. Because of the then near-pristine nature of the upper Cotter catchment, water from the Bendora main required only disinfection, fluoridation and minimal pH adjustment. This, together with the fact that water flows from Bendora to Mt Strololo under gravity, enabling power generation from a small hydro plant, made the Upper Cotter the water supply of choice and 60% of the Canberra water supply flows through the Bendora-Mt Stromlo main.

The Upper Cotter catchment lies almost totally within Namadgi National Park. Streamflow records suggest that, despite the steepness of the catchment, groundwater discharge is a major contributor to streamflow (N. Mueller, Ecwise Environmental, pers. Comm., August, 2004).

With the rapid expansion of Canberra, shallow groundwater monitoring bores were installed progressively from the late 1960s in many planned urban areas to monitor the impact of development on groundwater. Three reference sites within Namadgi National Park were also monitored. Besides shallow groundwater, there are fractured rock aquifers in the mountains to the south of Canberra. Apart from farms, where windmills were used for stock watering, little use has been made of groundwater in the region. During the 2002-2004 drought, domestic bores were licensed in southern Canberra for garden watering.

Organisational changes in water management in the ACT, including corporatisation and privatisation, saw the abandonment of monitoring of most shallow groundwater monitoring bores in 1987. The reason given was that, like stormwater, there were no customers for groundwater. Despite this monitoring has continued in the Orroral Valley in Namadgi National Park. Orroral Valley was cleared for grazing and cropping around 1836 and was also the site of the Orroral Valley Tracking Station. Since the declaration of Gudgenby Nature Reserve, pasture in the vicinity of the groundwater piezometer has been slowly returning to bush land although fires have kept the area generally open.
3.2 Groundwater Levels

Continuous groundwater level data were obtained from Ecowise Environmental for gauges in the Australian Capital Territory. In addition, data from a bore in Wagga Wagga, New South Wales was obtained from the Wagga Wagga local government. Figure 3.1 shows Orroral Valley groundwater level gauge 000601 in the Namadgi National Park, ACT.

Groundwater level measurements in each bore were made by Ecowise Environmental using a float with a counter-weight attached by a line wrapped around a revolution-counter at the top of the bore. A data logger connected to the revolution-counter stored the time of each change in groundwater level together with the groundwater level. Figure 3.2 shows a typical groundwater float with the smaller counter-weight by its side.
A possible source of uncertainty in groundwater level measurements includes the float getting caught on the inside of the bore because of rust or silt. Another possible source of uncertainty includes air entrapment within groundwater that may occur after an extreme rainfall event. Air may become trapped between the new recharge and the water table and cause higher pressures driving water into the bore and giving falsely high water table level measurements (Freeze and Cherry, 1979).

The bore dictionary in Table 3.1 details bores with known location and more than five years of data.
The Study Region and Catchment Data: Groundwater Levels

### Bore Dictionary

<table>
<thead>
<tr>
<th>BoreID</th>
<th>Name</th>
<th>Longitude (°E)</th>
<th>Latitude (°N)</th>
<th>Elevation (m AHD)</th>
<th>Land Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>000601</td>
<td>Orroral Bore</td>
<td>148.950</td>
<td>-35.627</td>
<td>940</td>
<td>National Park</td>
</tr>
<tr>
<td>000605</td>
<td>Charnwood North</td>
<td>149.044</td>
<td>-35.189</td>
<td>619</td>
<td>Modified Urban</td>
</tr>
<tr>
<td>000606</td>
<td>Charnwood South</td>
<td>149.042</td>
<td>-35.192</td>
<td>603</td>
<td>Modified Urban</td>
</tr>
<tr>
<td>000607</td>
<td>Gungahlin off Wells Rd</td>
<td>149.142</td>
<td>-35.188</td>
<td>600</td>
<td>Modified Urban</td>
</tr>
<tr>
<td>000608</td>
<td>Horsepark Homestead</td>
<td>149.132</td>
<td>-35.159</td>
<td>644</td>
<td>Modified Urban</td>
</tr>
<tr>
<td>000611</td>
<td>Gungahlin Gundaroo Rd</td>
<td>149.100</td>
<td>-35.207</td>
<td>602</td>
<td>Modified Urban</td>
</tr>
<tr>
<td>000612</td>
<td>Yarralumla at School</td>
<td>149.094</td>
<td>-35.314</td>
<td>563</td>
<td>Modified Urban</td>
</tr>
<tr>
<td>000613</td>
<td>Campbell at BMR</td>
<td>149.133</td>
<td>-35.299</td>
<td>562</td>
<td>Modified Urban</td>
</tr>
<tr>
<td>000614</td>
<td>Fyshwick behind BMR</td>
<td>149.174</td>
<td>-35.344</td>
<td>578</td>
<td>Modified Urban</td>
</tr>
<tr>
<td>000615</td>
<td>Red Hill near School</td>
<td>149.124</td>
<td>-35.333</td>
<td>597</td>
<td>Modified Urban</td>
</tr>
<tr>
<td>000616</td>
<td>Turner at McCaughey</td>
<td>149.117</td>
<td>-35.267</td>
<td>561</td>
<td>Modified Urban</td>
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<tr>
<td>000617</td>
<td>Corin Dam at Smokers</td>
<td>148.892</td>
<td>-35.525</td>
<td>1239</td>
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<tr>
<td>000618</td>
<td>Kowen Forest</td>
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<td>-35.317</td>
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<td>Pine Forest</td>
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<td>000619</td>
<td>Tuggeranong near Ser</td>
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<td>-35.411</td>
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<td>000620</td>
<td>Tuggeranong near Exc</td>
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<tr>
<td>000621</td>
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<td>000622</td>
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Table 3.1  Bore names, locations relative to the Australian Geodetic Datum 1966 (AGD66), elevations relative to the Australian Height Datum (AHD) and principal land uses

Bore drill logs were not available but a geological map of the ACT showed that almost all of these bores were in consolidated alluvial soil. Table 3.2 lists the periods over which the bores were monitored, along with the mean depth of groundwater below the ground surface and the standard deviation of the groundwater levels.
### Bore Details

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<th>Mean Groundwater Level (m)</th>
<th>Mean Depth (m)</th>
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**Table 3.2** Period of bore monitoring and the mean and standard deviation of groundwater depths below the ground surface

Figures 3.3, 3.4 and 3.5 show the location of the bores within south eastern Australia. The most westerly bore in Figure 3.4 is South Wagga Wagga bore 000016 and the second most westerly bore is Southwell Park Number 1 bore 000656.
3.2 The Study Region and Catchment Data: Groundwater Levels

Figure 3.3  Image of Australia with a rectangle containing the Australian Capital Territory

Figure 3.4  Elevation map of the area containing the rectangle of Figure 3.3 with bore locations marked
As Table 3.2 and Figure 3.5 shows, the bores were at elevations ranging from 182m to 1240m above the Australian Height Datum (AHD) and, as a result, were subject to different rainfall and evaporation rates.

3.2.1 Data Preparation

The groundwater level data for the Australian Capital Territory was extracted from data stored in the Hydysys software package. Groundwater levels were stored in
metres above the Australian Height Datum (AHD) and times of changes in groundwater level were stored in Australian Eastern Standard Time (GMT + 10).

The first step in data preparation was to exclude all of the bores with less than five years of data. This corresponds to sixty months of data and was determined to be too small to allow useful statistical testing. Table 3.1 and 3.2 list only bores with more than five years of data.

The next step in the data preparation was to examine Table 3.1 for obvious errors. It was noticed that Southwell Park Number 1 bore 000656 was at a location nearly one hundred kilometres west of the adjacent Southwell Park Number 2 bore 000657 and Southwell Park Number 4 bore 000659. There is clearly an error in the location of Southwell Park Number 1 bore 000656. The location of bore 000656 was replaced with the midpoint between bore 000657 and bore 000659.

Table 3.2 was also examined for obvious errors and it was noticed that Gungahlin bore 000607 had a negative groundwater depth. This indicated an error in the bore elevation. Gungahlin bore 000607 was still included in the study without changing the elevation because the changes in groundwater elevation could still be modelled in this research.

Data were extracted as text from Hydsys. The groundwater level data was then averaged to mean monthly and mean weekly data. In addition, instantaneous groundwater levels at the end of each month were calculated. The quality codes accompanying the data were used to exclude poor quality data. Quality code 1 indicated good quality continuous measurement. Quality code 26 indicated good quality daily measurements. Quality code 30 indicated good irregular time interval data. A quality code of 30 was used for the cut-off for the data because this ensured that the quality was good and that there was still enough data to use from each bore.

An example of the results of conversion of the measured data to monthly data is shown in Figure 3.6 for the Barney's Hill West bore 000621. The quality code of the data was 30 and there was usually only one groundwater level reading made on one day irregularly chosen during each month.
Figure 3.6  Groundwater level a. as measured and b. temporally interpolated for Barney’s Hill West bore 000621

For the weekly data, the standard week (Keig and McAlpine, 1969) was used to solve the problem of leap years being longer than normal years. The standard week is defined such that 01 Jan is always the first day of week 1 and 31 Dec is the last day of week 52. To accomplish this, week 30 has one day extra and week 9 has one day extra in leap years.

The next step in the data preparation process was to plot the groundwater levels for every bore and inspect for obvious errors. As Figure 3.7 shows, Campbell bore 000613 had two distinctly low outliers. The original data was investigated and the quality code of the two distinctly low outliers was 30. These two data points were removed from the data set.
3.2 The Study Region and Catchment Data: Groundwater Levels

Figure 3.7  Groundwater level for Campbell bore 000621 showing two distinctly low outliers

Figure 3.8  a. Groundwater level, b. rainfall and c. pan evaporation for Downer bore 000634
As Figure 3.8 shows, Downer bore 000634 experienced an unexplained maintained drop of 10m after a one-year break in record during 1982. It was decided that this drop in groundwater elevation was probably due to a redefinition of station elevation. Since the correct station elevation was unknown and the record was not particularly long, Downer bore 000634 was removed from the analysis.

The Orroral Valley bore 000601 has the largest record of measurement. Figure 3.9 shows the groundwater level, rainfall and pan evaporation for this bore. The Orroral Valley bore 000601 is used as the example bore in many of the analyses in this research because it had high quality code 1 data and a long record length. It is also in the Namadgi National Park and hence there was no artificial recharge or extraction of groundwater during the measurement period.

Groundwater level data from the bores in Table 3.1 and 3.2 were prepared as: mean monthly groundwater level to represent mid-month groundwater level; instantaneous groundwater level at the end of each month; and mean weekly groundwater level to represent mid-week groundwater level.
Figure 3.9  Monthly values of a. Groundwater level, b. rainfall and c. pan evaporation data for Orroral Valley bore 000601
3.3 Streamflow

Continuous stream stage height data were obtained for gauging stations in the Namadgi National Park, ACT, from Ecowise Environmental. Figure 3.10 shows Licking Hole Creek stream gauge 410776 in the upper Cotter Catchment.

Figure 3.10 Gauging station at Licking Hole Creek stream gauge 410776 in the upper Cotter Catchment, ACT

The stream stage height recorder shown in Figure 3.10 operated on a similar principle to the groundwater level gauge with a float inside a pipe to measure the height of the water in the stream. The cross sectional area of the stream and streamflow rates had been measured periodically by Ecowise Environmental to generate ratings tables correlating stream height with stream discharge. A data logger stored the time of each change in stream level and the stage height.

The streamflow \((m^3 \, s^{-1})\) was calculated from the stream level \((m)\) using the appropriate ratings tables. Mean monthly streamflow data were prepared from the instantaneous streamflow values by numerical integration using Simpson’s Rule. Only streamflow values of good quality code 1 were used. The mean monthly streamflow data represents the mid-month streamflow.
The Orroral Creek stream gauge 410736 whose location is shown in Figure 3.11 was used in this research. This gauge is at the lower end of the Orroral Valley at position 148.988°E, -35.666°N, and an elevation 870m AHD. The catchment area for the water discharging through the gauge is 89.6km².

Figure 3.11 Catchment boundary for the Orroral Creek stream gauge 410736 and Orroral Valley groundwater bore 000601

The data obtained from the Orroral Creek stream gauge 410736 is depicted in Figure 3.12.
Figure 3.12 Monthly values of a. Streamflow, b. rainfall and c. pan evaporation data for Orroral Creek stream 410736
3.4 Rainfall and Pan Evaporation

3.4.1 Rainfall

Monthly totals of rainfall data were obtained from the Bureau of Meteorology for all of their Australian rainfall gauges up to December 1999. Continuous rainfall data were also acquired from Ecwise Environmental for gauges in the Namadgi National Park, Australian Capital Territory. Figure 3.13 shows the tipping bucket rainfall gauge 570999 in the ACT at Middle Creek in the Orroral Valley.

![Figure 3.13 Rainfall gauge 570999 at Middle Creek in the Orroral Valley](image)

The diameter of the collector funnel was a standard 200mm and the small bucket inside the gauge tipped to empty itself each time the equivalent of 0.2mm of precipitation had entered the gauge. The data logger stored the time of each 0.2mm increment in rainfall.
Possible sources of uncertainty in the rainfall data includes wind effects such as rain falling non-vertically (Sharon, 1980), snow blowing out of the gauge and water evaporating within the gauge. These typically involve errors of the order of 10%.

The Bureau of Meteorology rainfall data was converted to a text file. The rainfall data for the Namadgi National Park was aggregated to monthly values and put into the same text file as the Bureau of Meteorology data. In addition to the monthly data, the Namadgi National Park rainfall data was also aggregated into weekly values. A table showing the details of the hundreds of rainfall gauges used appears on the Appendix CD (file:///D:/Ch3Data/DataPrep/RainAndEvapDictionary.txt).

Monthly accumulation of rainfall had the effect of averaging out the error in the collection of the rainfall data but information was lost about whether the rain all fell during one day or uniformly during the month. Hutchinson (1995a) has suggested that monthly data, which are reasonably well determined from standard meteorological networks, are sufficient to resolve much ecological and hydrological behaviour, particularly at spatial resolutions of a few kilometres. The monthly total rainfall values were used to represent the end-of-month rainfall.

3.4.2 Pan Evaporation

Monthly totals of pan evaporation were obtained from the Bureau of Meteorology for all of their Australian pan evaporation gauges up to December 1999. Continuous pan evaporation data were also supplied by Ecwise Environmental for the Orroral Valley, Australian Capital Territory. Figure 3.14 shows Orroral Valley pan evaporimeter 570928.
The standard Class A Evaporation Pan shown in Figure 3.14 had a diameter of 1.2065m with a depth of 254mm. The wire mesh over the evaporation pan in Figure 3.14 prevented animals and birds from drinking the water. The pan evaporimeter works by measuring the amount of water required to keep the pan full in 0.2mm increments. Rainfall is not a problem because it overflows out of the pan. The white water tank used to top up the pan in Figure 3.14 is three metres from the evaporation pan.

The Bureau of Meteorology pan evaporation data was converted to a text file. The pan evaporation data for the Namadgi National Park were aggregated to monthly values and put into the same text file as the Bureau of Meteorology data. In addition to the monthly data, the Namadgi National Park pan evaporation data were aggregated into weekly values. A table showing the dictionary details of the hundreds of pan evaporation gauges used in this work appears on the Appendix CD.
The Study Region and Catchment Data: Rainfall and Pan Evaporation

(file:///D:/Ch3Data/DataPrep/RainAndEvapDictionary.txt). The monthly total pan evaporation values were used to represent the end-of-month pan evaporation.

3.4.3 Rainfall and Pan Evaporation Correlation

The correlation between the rainfall and pan evaporation for the Orroral Valley was examined. This allowed evaluation of the possibility of model calibration problems that can arise if predictors (rainfall and pan evaporation) are correlated. Figure 3.15 shows the correlation between rainfall and pan evaporation.

![Figure 3.15](image)

**Figure 3.15** The relationship between monthly rainfall from gauge 570980 and pan evaporation from gauge 570928 from 1972 to 2000 in the Orroral Valley

The correlation coefficient between monthly rainfall and pan evaporation in the Orroral Valley was found to be \(-0.2142\). This correlation was found to be significantly different from 0.0 at the 5% level of significance. This means that the rainfall and pan evaporation data contained some of the same information. The coefficient of determination was \(R^2 = 0.0459\) indicating that only 4.59% of the variability in the pan evaporation was explained by variability in rainfall. It was decided that the correlation was small enough to be ignored in this research.
3.5 Spatial Thin-Plate Smoothing Spline Interpolation

In order to estimate the rainfall and pan evaporation over the bores, each month of rainfall and pan evaporation data were spatially interpolated. Monthly rainfall and pan evaporation data are suitable for spatial interpolation because rainfall and evaporation processes are spatially consistent at the monthly time scale (Hutchinson, 1995a).

Thin-plate smoothing splines (Wahba, 1990) were used rather than kriging (Cressie, 1991) for the spatial interpolation. Kriging is a local interpolation procedure, using only points within a specified range to calculate the value at each interpolated point. Splines provide a global interpolation procedure using all of the data given to calculate the value at each interpolated point. Spatial correlation results from points that are spatially close being dependent. This violates the statistical assumption of independence of observations. If spatial correlation is present then it either requires modelling using a semi-variogram (Cressie, 1991); or removal using knots.

A thin-plate smoothing spline (Wahba, 1990) is a surface (thin-plate) that does not pass exactly (smoothes) through the data points that it is fitted to. In this research, tri-variate splines were used with the three spatial coordinates longitude, latitude and elevation as predictors. The software used to perform the spline fitting procedure was ANUSPLIN (Hutchinson, 2002). This software provided detection of possibly erroneous data by flagging values that were more than 3.6 standard deviations from the spline. This procedure had the requirement that the spline residuals be normally distributed (Sharples et al., 2005).

3.5.1 Data Transformations

Because rainfall and pan evaporation data cannot be negative, their frequency distributions tend to be positively skewed. This causes rainfall and pan evaporation spline residuals to be skewed. In order to ensure that the spline residuals were normally distributed, both rainfall and pan evaporation were transformed to be normally distributed before fitting splines. Figures 3.16, 3.17 and 3.18 show statistical plots for indicating the usefulness of the square-root transformation over
the log transformation in normalising monthly rainfall data. The quantile-quantile plot (QQ-plot) (Neter et al., 1996) is a plot of expected normal values versus observed values. Normality is indicated by a straight line. The expected normal value set was generated with the same number of elements, the same mean and the same standard deviation as the observation set. The expected normal value set and the observation set are ordered and paired to produce the QQ-plot.

**Figure 3.16** Orroral Valley rainfall gauge 570980 monthly rainfall histogram and normal QQ-plot

**Figure 3.17** Orroral Valley rainfall gauge 570980 log monthly rainfall histogram and normal QQ-plot
3.5 Spatial Thin-Plate Smoothing Spline Interpolation

Histogram of Sqrt(Rainfall (mm))

Normal Q-Q Plot of Sqrt(Rainfall (mm))

The Kolmogorov-Smirnov test for normality (Neter et al. 1996) gave a p-value of 0.000 for both the untransformed rainfall and the log rainfall. This indicates that the untransformed and log transformed rainfall data at gauge 570980 significantly deviated from normality. For the square-root rainfall, the p-value was 0.200 which is greater than 0.05. Hence the square-root rainfall did not significantly deviate from normality at the 5% level of significance.

Figures 3.19, 3.20 and 3.21 show statistical plots for indicating the usefulness of the square-root transformation over the log transformation in normalising monthly pan evaporation data.

Figure 3.18 Orroral Valley rainfall gauge 570980 square-root monthly rainfall histogram and normal QQ-plot
Figure 3.19  Orroral Valley gauge 570928 monthly pan evaporation histogram and normal QQ-plot

Figure 3.20  Orroral Valley gauge 570928 log monthly pan evaporation histogram and normal QQ-plot
The test for normality gave a p-value of 0.000 for both the untransformed pan evaporation and log pan-evaporation data. This indicates that the untransformed and log transformed pan evaporation data at gauge 570928 significantly deviated from normality. For the square-root pan evaporation, the p-value was 0.058 which is greater than 0.05. Hence the square-root pan evaporation did not significantly deviate from normality at the 5% level of significance.
3.5.2 Spline Equations

Equation 3.1 shows the tri-variate spline model used in this research for the rainfall

\[ P_k^{\frac{1}{2}} = f(x_k, y_k, h_k) + \varepsilon_k, \quad k = 1, \ldots, n \]  

(3.1)

where \( P_k^{\frac{1}{2}} \) is the square-root rainfall data (\( \sqrt{\text{mm}} \)), \( f \) is the spline function, \( x_k \) is the longitude (°), \( y_k \) is the latitude (°), \( h_k \) is the elevation (km), \( \varepsilon_k \) is the residual (\( \sqrt{\text{mm}} \)), \( k \) is the spatial index that references a particular month of data from different rainfall gauges and \( n \) is the number of data points.

The elevation was in units of kilometres and the longitude and latitude in degrees. This was necessary because Hutchinson (1995b) found that a change in elevation would cause a change rainfall by approximately 100 times what the same change in a horizontal direction would cause. One degree equates to approximately 100 kilometres.

The objective function that was minimised to fit the rainfall splines was:

\[ \sum_{k=1}^{n} (P_k^{\frac{1}{2}} - f(x_k, y_k, h_k))^2 + \rho J_2(f) \]  

(3.2)

where \( \sum_{k=1}^{n} (P_k^{\frac{1}{2}} - f(x_k, y_k, h_k))^2 \) is the rainfall spline residual sum of squares indicating the goodness-of-fit, \( \rho \) is the smoothing parameter and \( J_2(f) \) is the bending energy or second order roughness penalty calculated using:

\[ J_2(f) = \iint_{\mathbb{R}^3} (f_{xx}^2 + f_{yy}^2 + f_{hh}^2 + 2 f_{xy}^2 + 2 f_{yh}^2 + 2 f_{yh}^2) \, dx \, dy \, dh \]  

(3.3)

Using these, the spline is the surface that fits the data well and with simultaneous small bending energy. The smoothing parameter \( \rho \) was found by minimising the generalised cross-validation score. Cross-validation is discussed in Chapter 5. In this research, the order of the spline functions was set to 2.
The tri-variate spline model used in this research for the pan evaporation data is similar to the above:

$$E_k^{\frac{1}{2}} = f(x_k, y_k, h_k) + \varepsilon_k, \ k = 1, ..., n$$

(3.4)

where $E_k^{\frac{1}{2}}$ is the square-root pan evaporation data ($\sqrt{\text{mm}}$), $f$ is the spline function, $x_k$ is the longitude (°), $y_k$ is the latitude (°), $h_k$ is the elevation (km), $\varepsilon_k$ is the residual ($\sqrt{\text{mm}}$), $k$ is the spatial index that references a particular month of data from different pan evaporation gauges and $n$ is the number of data points.

The objective function that was minimised to fit the pan evaporation splines was similarly

$$\sum_{k=1}^{n} (E_k^{\frac{1}{2}} - f(x_k, y_k, h_k))^2 + p J_2(f)$$

(3.5)

where $\sum_{k=1}^{n} (E_k^{\frac{1}{2}} - f(x_k, y_k, h_k))^2$ is the pan evaporation spline residual sum of squares indicating the goodness-of-fit.

Bi-variate splines with the elevation $h$ were also tested for the rainfall and pan evaporation data but the tri-variate splines performed better as indicated by lower generalised cross-validation scores.

The spline residuals $\varepsilon_k$ ($\sqrt{\text{mm}}$) for both the rainfall and pan evaporation splines were normally distributed as a result of the square-root transformation. An added benefit of using the square-root transformation was that, after the surface values were squared to return to rainfall or pan evaporation, the result was only positive values. This is in contrast to the log transformation that cannot handle zero values and negatively skews both rainfall and pan evaporation data.

Figure 3.22 shows the region of data used to construct the rainfall splines comprising longitude from 144.000° to 151.000° and latitude from –39.000° to –34.000°.
3.22 also shows the region of data used to construct the pan evaporation splines comprising longitude from 140.000° to 154.000° and latitude from −40.000° to −24.000°.

![Image of Australia showing the areas used for spline surface fits for rainfall and pan evaporation](image)

**Figure 3.22** Image of Australia showing the areas used for spline surface fits for **a.** rainfall and **b.** pan evaporation

Different areas were chosen for rainfall and pan evaporation to include a large enough number of data points to give good results but a small enough number of data points to make the spline fitting procedure take a reasonable amount of time. The density of rainfall and pan evaporation stations differs in south eastern Australia.

The spline fitting procedure involved many calculations using very large matrices. In order to increase the speed of the spline fitting process, knots were used (Bates and Wahba, 1982; Hutchinson and Bischof, 1983; Hutchinson, 2002). The knots were data points selected as the most even spread of that particular number of points within the total data set. When using knots, all of the data were still used to calculate the spline but the spline was based at the knots. This had the added benefit of removing short-range spatial correlation between the closest meteorological stations.
Figure 3.23 shows that the number of meteorological stations measuring rainfall in area a has been decreasing since 1972.

The number of knots used in fitting the rainfall surfaces was 400 for surfaces fit from 1951 to 1972, 500 for 1973 to 1975 and 400 for 1976-1999.

Figure 3.24 shows how the number of pan evaporation stations in area b has changed since 1951.
The number of knots used in fitting the pan evaporation surfaces was the number of points found in Figure 3.24 minus 5 and surfaces were only fitted to data from 1971-1999. The use of this number of knots allowed a maximum of 5 missing gauges for any given month. There were fewer pan evaporation gauges than rainfall gauges but this was not a problem as evaporation processes are more spatially consistent than rainfall at the monthly time scale.

Splines were fitted to each month of square-root rainfall and square-root pan evaporation data. A flagging procedure of possibly erroneous points was used and every month of flagged data was inspected and compared with the data at the nearest neighbour meteorological stations. In this research, a program entitled Nearmet was written in Fortran 90 to retrieve the data from the nearest neighbours of the stations with flagged values. Each time a month of data was removed from a station, the spline was fitted again. This process was iterated until there were no new un inspected flagged months of data. In this way, many stations with missing values incorrectly stored as 0.0 were removed. Table 3.3 shows a missing value incorrectly stored as 0.0 for rainfall gauge 070116 in January 1975.

<table>
<thead>
<tr>
<th>ID</th>
<th>Year</th>
<th>Longitude (°E)</th>
<th>Latitude (°N)</th>
<th>Elevation (m AHD)</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
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<tr>
<td>070116</td>
<td>1975</td>
<td>149.247</td>
<td>-34.446</td>
<td>770</td>
<td>0.0</td>
<td>27.0</td>
<td>13.2</td>
<td>113.5</td>
<td>21.9</td>
<td>27.4</td>
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<td>070111</td>
<td>1975</td>
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<td>-34.563</td>
<td>705</td>
<td>36.6</td>
<td>46.4</td>
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<td>45.8</td>
<td>28.0</td>
<td>91.0</td>
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<tr>
<td>070047</td>
<td>1975</td>
<td>149.124</td>
<td>-34.402</td>
<td>560</td>
<td>32.2</td>
<td>43.1</td>
<td>62.0</td>
<td>52.9</td>
<td>21.8</td>
<td>69.4</td>
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<tr>
<td>063119</td>
<td>1975</td>
<td>149.265</td>
<td>-34.235</td>
<td>720</td>
<td>45.4</td>
<td>67.8</td>
<td>47.7</td>
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<td>38.2</td>
<td>75.2</td>
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<tr>
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<td>1975</td>
<td>149.450</td>
<td>-34.533</td>
<td>920</td>
<td>65.1</td>
<td>77.0</td>
<td>65.7</td>
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<tr>
<td>070025</td>
<td>1975</td>
<td>149.469</td>
<td>-34.457</td>
<td>887</td>
<td>71.0</td>
<td>62.6</td>
<td>68.8</td>
<td>53.8</td>
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<td>156.6</td>
</tr>
<tr>
<td>070213</td>
<td>1975</td>
<td>149.394</td>
<td>-34.650</td>
<td>825</td>
<td>41.8</td>
<td>59.2</td>
<td>43.8</td>
<td>50.8</td>
<td>10.2</td>
<td>121.3</td>
</tr>
</tbody>
</table>

Table 3.3 Example of Nearmet output for Wheeo (St. Elmo) NSW rainfall gauge 070116 January 1975 showing neighbouring gauge rainfall values

The spline statistics were inspected to ensure that the spline provided an accurate fit with a low generalised cross-validation score. The fit of the splines was improved as a result of removing the erroneous data.
Surfaces were generated from each spline by varying longitude $x$ (°) and latitude $y$ (°) in a grid of spacing 0.025°. Elevation $h(x, y)$ (km) was retrieved from the Hutchinson and Dowling (1991) digital elevation model (DEM) of horizontal resolution 0.025° and the surface value $(f(x, y, h))^2$ was calculated for each grid point. The spline value was squared to undo the square-root transformation and produce a surface of rainfall or pan evaporation values rather than square-root rainfall or square-root pan evaporation. ANUSPLIN also corrected for the small bias introduced by the square-root transformation (Hutchinson, 2002).

Figure 3.25 shows the high resolution of an example rainfall surface and also shows the high concentration of rainfall gauges in the urban area of the ACT.
Figure 3.26 shows the high resolution of an example pan evaporation surface and also shows the high concentration of pan evaporation gauges in the ACT.

Figures 3.27 and 3.28 show a comparison of the actual gauge and interpolated surface values of monthly rainfall and pan evaporation for the Orroral Valley, ACT. The coefficient of determination $R^2$ value for the rainfall was 95.45% and the $R^2$ value for the pan evaporation was 81.22%. The pan evaporation result was not as good as that for the rainfall because the pan evaporation gauge network is less dense and pan evaporation is more difficult to measure. Additionally, there are many gauges located at airports where, because of their open location, the wind is greater
than in surrounding areas. However both the rainfall and pan evaporation surfaces fit the point data reasonably well.

**Figure 3.27** Spatially interpolated monthly rainfall versus measured monthly rainfall for Orroral Valley rain gauge 570980 between 1972 and 2000

**Figure 3.28** Spatially interpolated monthly pan evaporation versus measured monthly pan evaporation for Orroral Valley pan evaporimeter 570928 between 1972 and 2000
3.5 Spatial Thin-Plate Smoothing Spline Interpolation

Monthly rainfall and pan evaporation values were calculated for each groundwater bore using the surfaces and bore locations from Table 3.2. Weekly values were also prepared for the Orroral Valley by temporal interpolation of the monthly spatially interpolated values using cubic Bessel function interpolation as described by De Boor (1978).

3.4.3 Conclusions about Spatial Thin-Plate Smoothing Spline Interpolation

The spline fitting procedure was found to be robust and allowed for detection of erroneous data that could be removed (Sharple et al., 2005). To allow for the error detection procedure, the data required transformation to be normally distributed. The square-root transformation was found to normalise the monthly rainfall and pan evaporation data whereas the log transformation was unable to normalise the data and changed the direction of the distribution skew. The pan evaporation data were of poorer quality and consisted of fewer stations than the rainfall data. However, both rainfall and pan evaporation surfaces fitted the data well when latitude, longitude and elevation were used as predictors.
3.6 Computer Programming for Data Preparation

Routines were coded into Fortran 90 to convert between different formats and to create appropriate text-file databases. All of the data were converted to ASCII text files because this allowed simple and portable management of the data using programs written in Fortran 90.

Two separate software packages were written for data preparation, one for the Bureau of Meteorology rainfall and pan evaporation data and the other for the Ecowise Environmental data. Missing values were stored as -9.0 because rainfall, pan evaporation, groundwater level and streamflow are all positive quantities. Extensive testing was performed on the data preparation software to ensure the absence of programming errors. The Fortran 90 source code written for this study does not appear as a printed appendix because the length is greater than 300 pages. The source code is given in the Appendix CD (file:///D:/SourceCode/) along with the executables (file:///D:/Executables/PC/).

3.6.1 Data Preparation of Bureau of Meteorology Rainfall and Pan Evaporation Data

The Appendix CD contains source code and descriptions of the following modules and programs:

**Modules**

<table>
<thead>
<tr>
<th>StationCollection</th>
<th>Contains an array of Station objects, sort routines, read and write routines for different formats and dictionary manipulation routines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station</td>
<td>Contains dictionary details and one year of monthly data (rainfall or pan evaporation) for one meteorological station</td>
</tr>
</tbody>
</table>


3.6 Main Programs

DataPrepRain: Reads in monthly rainfall data from different sources, sorts by station ID and year and then saves the data into different text files.

DataPrepEvap: Reads in monthly pan evaporation data from different sources, sorts by station ID and year and then saves the data into different text files.

GenDiction: Reads different data files and combines the dictionary details and saves the dictionary.

GenRainSEAusCount: Counts the number of active rainfall gauges in the South East Australia region for each year.

Nearmet: Reads in a flag file and finds the six nearest neighbours of the flagged stations and displays them for error detection.

3.6.2 Data Preparation of Ecowise Environmental Data

Modules

MthDataYr: Contains station ID and one year of monthly values.

MthData: Contains an array of MthDataYr objects and routines to read and write the MthData format.

WkDataYr: Contains station ID and one year of weekly values.

WkData: Contains an array of WkDataYr objects and routines to read and write the WkData format.

StnData: Contains an array of either weekly or monthly values and routines to read and write the StnData format.

Main Programs

C2S: Converts .CSV (comma separated values) file to StnData file.

CI2S: Converts Combined Intgrd output file to StnData file.

CL2S: Converts Combined Lapgrd output file to StnData file.
FixMiss

Loads two StnData files and fills the missing values in the first file with the present values in the second file.

H2S

Converts Hydsys output to StnData file

H2S2

Converts Hydsys output to StnData file

M2S

Converts MthData file to StnData file

Mth2Wk

Converts MthData file to WkData file using Cubic Bessel function interpolation

S2MW

Converts StnData file to either MthData file or WkData file whichever is appropriate

SL2SF

Converts Hydsys output Stream Level to StnData Streamflow file
CHAPTER 4
DEVELOPMENT OF PARAMETER EFFICIENT MODELS FOR GROUNDWATER LEVEL PREDICTION

This chapter presents descriptions of the models chosen for testing and discusses improvements and extensions of the models. These models were selected based on using few parameters while reportedly performing well at groundwater level prediction for the first model and streamflow prediction for the other two models.

The choice of models for testing in this study was based on the required characteristics of the output data and the characteristics of available input data. As discussed in Chapter 3, the available model input data were monthly totals of rainfall and pan evaporation. The required model output was monthly mean groundwater level. Figure 4.1 shows the variable symbols used in the models that follow.

![Diagram](image)

**Figure 4.1** Vertical profile of a catchment showing flows of water and the symbols used for different variables
4.1 The HARTT Model

The Hydrograph Analysis: Rainfall and Time Trends (HARTT) model was developed by Ferdowsian and Pannel (2001) and also Ferdowsian et al. (2001) who studied groundwater levels in southern coastal Western Australia. In the model, the effects of atypical rainfall events were separated from the underlying time trend and the time lag between rainfall and its impact on groundwater was explicitly represented. Ferdowsian et al. (2001) also included land use change using a categorical (0 or 1) variable.

The HARTT model is an empirical groundwater model that uses rainfall data to predict groundwater depth at the monthly time scale. The parameters are approximate initial groundwater depth, a rainfall influence parameter, a trend parameter and time lag. The output is predicted groundwater depth. The basic assumption in this approach is that all of the previous rainfall events have an effect on the current month’s depth to groundwater.

The assumptions of the HARTT model are that the soil is rigid with preferred pathways and macropores not explicitly modelled, soil and vegetation properties are stationary over time, rainfall intensity is uniform over the entire month and the current groundwater level depends on all of the previous rainfall events.

The rainfall in this model is converted to the accumulated monthly residual rainfall $X_{MPt}$ (mm) at time $t$ using

$$X_{MPt} = \sum_{i=1}^{I} \left( P_i - \bar{P}_{j(i)} \right)$$

(4.1)

where $P_i$ is the precipitation at time $I$ of the data set, $j(i)$ is the Gregorian month (1 to 12) of time $I$ of the data set, and $\bar{P}_{j(i)}$ is the mean precipitation of the $j(i)$-th Gregorian month.

Ferdowsian et al. (2001) found that the accumulated monthly residual rainfall $X_{MPt}$ had relatively low within-year variability because the fluctuations in actual rainfall
were offset by the seasonal variation in average monthly rainfall in Western
Australia. In order to obtain better fits with generally shallow water table data, the
accumulated annual residual rainfall $X_{APt}$ (mm) in Equation 4.2 was introduced that
had higher within-year variability.

$$X_{APt} = \sum_{i=1}^{t} (P_i - \overline{P})$$  \hspace{1cm} (4.2)

where $\overline{P}$ is one twelfth of the mean annual precipitation.

The HARTT model then calculates groundwater depth $G_{Di}$ (m, upward direction is
positive) is using the linear equation

$$G_{Di} = \beta_{D0} + \beta_1 t + \beta_2 X_{MPt-L} + \varepsilon_t$$  \hspace{1cm} (4.3)

where $\beta_{D0}$ is the approximate initial groundwater depth parameter (m), $\beta_1$ is the
trend parameter (m mth$^{-1}$), $\beta_2$ is the rainfall influence parameter (m mm$^{-1}$) and $L$ is
the time lag (mths). Equation 4.3 states that the depth to groundwater is equal to the
approximate initial depth plus the trend plus the influence of rainfall. Ferdowsian et
al. (2001) found that this form was more accurate in identifying trends than a time-
trend only model.

A plot of the values of $X_{MPt}$ and $X_{APt}$ in the Orroral Valley, ACT is shown in Figure
4.2. There is little difference between the two functions because there is little
seasonality so that mean rainfalls for each Gregorian month are approximately equal
in the Orroral Valley. The mean value of the two functions shown in Figure 4.2 is
not zero because the series shown was calculated using rainfall data beginning in
1951 but only data from 1971 to 1999 are displayed.
4.1 Extensions of the HARTT Model

In this research, it was decided to run the HARTT model on groundwater level (elevation) rather than groundwater depth. This necessitated a change in Equation 4.3 of the HARTT model. The modification relates groundwater depth to groundwater level and gauge elevation

\[ G_{D_t} = G_t - \beta_H \]  

(4.4)

where \( G_t \) is the groundwater elevation (m AHD) and \( \beta_H \) is the gauge elevation (m AHD).

Substituting Equation 4.4 into Equation 4.3 gives

\[ G_t = (\beta_{D0} + \beta_H) + \beta_t t + \beta_2 X_{MP_t-L} + \varepsilon_t \]  

(4.5)
This simplifies to the model used in this research:

\[ G_t = \beta_0 + \beta_1 t + \beta_2 X_{MPt-L} + \epsilon, \]  

(4.6)

where \( \beta_0 \) is the approximate initial groundwater level parameter (m).

The HARTT model was developed for an area with marked seasonal rainfall and a Mediterranean climate. It does not use data for evapotranspiration which is a key driver of groundwater recharge processes. In this research, the HARTT model was extended to include pan evaporation as input data.

Accumulated monthly residual pan evaporation \( X_{ME_{pan,t}} \) (mm) was calculated using:

\[ X_{ME_{pan,t}} = \sum_{i=1}^{t} (E_{pan_i} - \overline{E}_{pan_j(i)}) \]  

(4.7)

where \( E_{pan_i} \) is the pan evaporation at time \( i \) of the data set, \( j(i) \) is the Gregorian month (1 to 12) of time \( i \) of the data set, and \( \overline{E}_{pan_j(i)} \) is the mean precipitation of the \( j(i) \)-th Gregorian month. In addition, the accumulated annual residual pan evaporation \( X_{AE_{pan,t}} \) (mm) was calculated using Equation 4.8

\[ X_{AE_{pan,t}} = \sum_{i=1}^{t} (E_{pan_i} - \overline{E}_{pan}) \]  

(4.8)

where \( \overline{E}_{pan} \) is one twelfth of the mean annual pan evaporation.

A plot of the values taken by \( X_{ME_{pan,t}} \) and \( X_{AE_{pan,t}} \) in the Orroral Valley, ACT is shown in Figure 4.3. There is a noticeable difference with the evaporation because the mean evaporation differs for each Gregorian month in the Orroral Valley. The two functions were calculated from pan evaporation values from 1969 to 1999.
Equation 4.9 shows the HARTT Plus Evaporation model

$$G_t = \beta_0 + \beta_1 t + \beta_2 X_{MP_{t-L}} + \beta_3 X_{ME_{pan-t-L}} + \epsilon_t$$  \hspace{1cm} (4.9)$$

where $\beta_3$ is the pan evaporation influence parameter (m mm$^{-1}$).
4.1 The HARTT Model

4.1.2 Summary

A range of modifications and extensions of the HARTT model were tested in this study. These are:

**Mean Model**

\[ G_t = \beta_0 + \epsilon_t \]

This is the simplest model which assumes that groundwater levels remain constant over time.

**Time Trend Model**

\[ G_t = \beta_0 + \beta_1 t + \epsilon_t \]

This model assumes that the groundwater level changes by a fixed amount with each time step.

**HARTT Model**

\[ X_{MP_t} = \sum_{i=1}^{t} (P_i - \overline{P}_{j(i)}) \]

\[ G_t = \beta_0 + \beta_1 t + \beta_2 X_{MP_{t-L}} + \epsilon_t \]

This is the original model using accumulated monthly residual rainfall.

**HARTT Minus Trend Model**

\[ X_{MP_t} = \sum_{i=1}^{t} (P_i - \overline{P}_{j(i)}) \]

\[ G_t = \beta_0 + \beta_2 X_{MP_{t-L}} + \epsilon_t \]

This is the HARTT model with the time trend removed.
HARTTA Model

\[ X_{APt} = \sum_{i=1}^{t} (P_i - \bar{P}) \]

\[ G_t = \beta_0 + \beta_1 t + \beta_2 X_{APt-L} + \epsilon_t \]

This is the original model using accumulated annual residual rainfall.

HARTT Plus Evap Model

\[ X_{MPt} = \sum_{i=1}^{t} (P_i - \bar{P}_{j(i)}) \]

\[ X_{ME_{pan}t} = \sum_{i=1}^{t} (E_{pan,i} - \bar{E}_{pan,j(i)}) \]

\[ G_t = \beta_0 + \beta_1 t + \beta_2 X_{MPt-L} + \beta_3 X_{ME_{pan}t-L} + \epsilon_t \]

This is a proposed improvement to the HARTT model with the effect of evaporation included as accumulated monthly residual pan evaporation.

HARTT Plus Evap Minus Trend Model

\[ X_{MPt} = \sum_{i=1}^{t} (P_i - \bar{P}_{j(i)}) \]

\[ X_{ME_{pan}t} = \sum_{i=1}^{t} (E_{pan,i} - \bar{E}_{pan,j(i)}) \]

\[ G_t = \beta_0 + \beta_2 X_{MPt-L} + \beta_3 X_{ME_{pan}t-L} + \epsilon_t \]

This is a proposed improvement to the HARTT model with the effect of evaporation but not time trend included in the model.
HARTTA Plus EvapA Model

\[ X_{APt} = \sum_{i=1}^{t} \left( P_i - \bar{P} \right) \]

\[ X_{AE_{pan}} = \sum_{i=1}^{t} \left( E_{pan_i} - \bar{E}_{pan} \right) \]

\[ G_t = \beta_0 + \beta_1 t + \beta_2 X_{APt-L} + \beta_3 X_{AE_{pan}} + \epsilon_t \]

This is a proposed improvement to the HARTTA model with the effect of evaporation included as accumulated annual residual pan evaporation.

HARTTA Plus EvapA Minus Trend Model

\[ X_{APt} = \sum_{i=1}^{t} \left( P_i - \bar{P} \right) \]

\[ X_{AE_{pan}} = \sum_{i=1}^{t} \left( E_{pan_i} - \bar{E}_{pan} \right) \]

\[ G_t = \beta_0 + \beta_1 t + \beta_2 X_{APt-L} + \beta_3 X_{AE_{pan}} + \epsilon_t \]

This is a proposed improvement to the HARTTA model with the effect of evaporation but not time trend included in the model.

Where the variables are:

- \( E_{pan_i} \) pan evaporation (mm)
- \( \bar{E}_{pan} \) mean annual pan evaporation divided by 12 (mm)
- \( \bar{E}_{pan j(i)} \) mean monthly pan evaporation of the \( j(i) \)-th Gregorian month (mm)
- \( \epsilon_t \) model residual (m)
The HARTT Model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_t )</td>
<td>groundwater level</td>
<td>(m)</td>
</tr>
<tr>
<td>( P_t )</td>
<td>rainfall</td>
<td>(mm)</td>
</tr>
<tr>
<td>( \bar{P}_{j(i)} )</td>
<td>mean monthly rainfall of the ( j(i) )-th Gregorian month</td>
<td>(mm)</td>
</tr>
<tr>
<td>( \bar{P} )</td>
<td>mean annual rainfall divided by 12</td>
<td>(mm)</td>
</tr>
<tr>
<td>( t )</td>
<td>time</td>
<td>(mths)</td>
</tr>
<tr>
<td>( X_{AEt} )</td>
<td>accumulated annual residual pan evaporation</td>
<td>(mm)</td>
</tr>
<tr>
<td>( X_{APt} )</td>
<td>accumulated annual residual rainfall</td>
<td>(mm)</td>
</tr>
<tr>
<td>( X_{MEt} )</td>
<td>accumulated monthly residual pan evaporation</td>
<td>(mm)</td>
</tr>
<tr>
<td>( X_{MPt} )</td>
<td>accumulated monthly residual rainfall</td>
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and the parameters are:

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<thead>
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<th>Symbol</th>
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<th>Unit</th>
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<td>rainfall influence parameter</td>
<td>(m mm(^{-1}))</td>
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<tr>
<td>( \beta_3 )</td>
<td>pan evaporation influence parameter</td>
<td>(m mm(^{-1}))</td>
</tr>
<tr>
<td>( L )</td>
<td>time lag</td>
<td>(mths)</td>
</tr>
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</table>
4.2 The Watbal Model

The Watbal model was developed by Keig and McAlpine (1974) as a conceptual weekly water balance model to predict the soil moisture index and runoff. The input data are rainfall and pan evaporation. The parameters are a soil type parameter and soil moisture capacity. This uses a one layer model of the soil moisture store. The Watbal model has been used in the Growest software package (Nix et al., 1977; Nix, 1981; Hutchinson et al., 2002) to calculate the soil moisture index which is then used in the estimation of vegetation growth indices. As part of this research, the Watbal model was extended to predict groundwater level.

The assumptions of the Watbal model are that the soil is rigid with preferred pathways and macropores not explicitly modelled. Soil and vegetation properties are stationary over time, rainfall intensity is uniform over the entire time week and the current soil moisture content depends on the previous soil moisture content. Catchment properties are lumped spatially and temporally.

The Watbal model is:

\[
M_t = \frac{1 - \exp \left( -\beta_s \min \left( 1, \frac{S_{t-1} + \frac{1}{2} P_t}{\beta_c} \right) \right)}{1 - \exp(-\beta_s)} \tag{4.10}
\]

\[
E_t = M_t E_{\text{pan}} \tag{4.11}
\]

\[
Q_t = \max (S_{t-1} + P_t - E_t - \beta_c, 0) \tag{4.12}
\]

\[
S_t = \min (\max (S_{t-1} + P_t - E_t, 0), \beta_c) \tag{4.13}
\]

where \(M_t\) is the soil moisture index, \(\beta_s\) is the soil type parameter, \(S_t\) is the unsaturated layer soil moisture content (mm), \(P_t\) is the rainfall (mm), \(E_t\) is the actual evapotranspiration (mm), \(E_{\text{pan}}\) is the pan evaporation (mm), \(Q_t\) is the runoff (mm) and \(\beta_c\) is the soil moisture capacity parameter (mm). The unsaturated layer soil
moisture content $S_t$ (mm) is the equivalent depth of water in the unsaturated soil layer and is not to be confused with the volumetric moisture content (Freeze and Cherry, 1979) which is a dimensionless quantity calculated as the volume of water per unit volume of dry soil.

Equation 4.10 calculates the soil moisture index $M_t$ given the soil type parameter $\beta_s$, the soil moisture capacity parameter $\beta_c$, the previous time period's soil moisture content $S_{t-1}$ and the rainfall $P_t$. The min( , ) function returns the minimum value out of the two arguments.

The soil moisture index $M_t$ is the ratio of actual evapotranspiration to potential evapotranspiration with a range of $0 \leq M_t \leq 1$. A value of $M_t = 0$ indicates that the soil is dry and a value of $M_t = 1$ indicates that the soil is saturated with water.

The soil type parameter $\beta_s$ is a positive number. In the Growest software package (Hutchinson et al., 2002) the soil type parameter can only take a value of 7.5 for sandy loam, 3.5 for clay loam or 1.5 for clay. In this research, any positive value of $\beta_s$ was allowed. The soil moisture capacity parameter $\beta_c$ is an upper bound for the soil moisture content $S_t$.

Figure 4.4 shows the soil moisture index as a function of the fractional soil moisture content. The graph is non-linear and monotonically increasing indicating that as the fractional soil moisture content increases, the soil moisture index approaches 1 at a rate depending on the soil type.
\[ M_t = \frac{1 - \exp\left(-\beta_s \min\left(1, \frac{S_{t-1} + \frac{1}{2} P_t}{\beta_c}\right)\right)}{1 - \exp(-\beta_s)} \]

Figure 4.4  Soil moisture index versus the fractional soil moisture content after half the rain has fallen for soil type parameter values of 7.5 for sandy loam, 3.5 for clay loam and 1.5 for clay

Equation 4.11 calculates the actual evapotranspiration \( E_t \) given the soil moisture index \( M_t \) and the pan evaporation \( E_{\text{pan}} \). It assumes that the potential evapotranspiration is equal to the pan evaporation. The equation ensures that there is less actual evapotranspiration from dry soil than from wet soil.

Equation 4.12 calculates the runoff \( Q \) using the water balance equation. The \( \max(, ) \) function returns the maximum value out of the two arguments. Equation 4.13 calculates the soil moisture content \( S_t \) using the water balance equation.
4.2 The Watbal Model

The water balance is maintained if \( S_{t-1} + P_t - E_t \) (the previous time period's soil moisture content plus the rainfall minus what has evapotranspired) is positive, otherwise the water balance is not maintained. The proof of this follows.

The soil moisture capacity is a positive number

\[ \beta_c \geq 0 \]

In the case that \( S_{t-1} + P_t - E_t \geq \beta_c \),

\[ Q_t = S_{t-1} + P_t - E_t - \beta_c \]
\[ S_t = \beta_c \]

Therefore

\[ P_t = S_t - S_{t-1} + E_t + Q_t \]

which is the water balance equation.

In the case that \( 0 \leq S_{t-1} + P_t - E_t \leq \beta_c \),

\[ Q_t = 0 \]
\[ S_t = S_{t-1} + P_t - E_t \]

Therefore

\[ P_t = S_t - S_{t-1} + E_t + Q_t \]

which is the water balance equation.

In the case that \( S_{t-1} + P_t - E_t \leq 0 \),

\[ Q_t = 0 \]
\[ S_t = 0 \]

which is not the water balance equation.

Therefore the water balance is maintained if \( S_{t-1} + P_t - E_t \) is positive and otherwise the water balance is not maintained. This is because when \( S_{t-1} + P_t - E_t \) is negative, the soil moisture content must be negative to maintain the water balance, but the Watbal model sets the soil moisture content to zero.
4.2.1 Extensions of the Watbal Model

The Watbal model was designed to predict soil moisture index and streamflow. In order to predict groundwater levels, different equations linking soil moisture content to recharge were examined.

**Recharge Model 1**

The first recharge model proposed is assumes that the recharge to the saturated layer is a fraction of the infiltration to the unsaturated layer:

\[ \Delta G_{t+L} = \frac{1}{\beta_G} \Delta S_t \]  

(4.14)

where \( \beta_G \) is the groundwater recharge parameter (mm \( m^{-1} \)) that depends on both the soil porosity and the fraction of water that moves between the unsaturated and saturated soil layers. \( L \) is the time lag. Equation 4.14 linearly correlates the change in groundwater level with the change in soil moisture content \( L \) time-steps ago, so that \( L \) is the time lag in recharge. The saturated soil moisture content is not explicitly calculated here since the soil porosity relating the groundwater level to the saturated soil moisture content is not known.

Equation 4.14 was expanded using \( \Delta G_{t+L} = G_{t+L} - G_{t+L-1} \) and \( \Delta S_t = S_t - S_{t-1} \) to give:

\[ G_{t+L} = G_{t+L-1} + \frac{1}{\beta_G} (S_t - S_{t-1}) \]  

(4.15)

**Recharge Model 2**

Recharge Model 2 explicitly models two soil layers: the unsaturated soil layer and the saturated soil layer. As a result, a different formula for the unsaturated soil moisture content was used so the notation of Equation 4.13 was changed to

\[ S_{BRt} = \min(\max(S_{r-1} + P_r - E_r, 0), \beta_C) \]  

(4.16)

where \( S_{BRt} \) is the unsaturated soil moisture content before recharge to the saturated soil layer has occurred.
The second recharge model was derived as follows. It was assumed that a change in the unsaturated soil moisture content would produce a change in the saturated soil moisture content \( L \) time periods later:

\[
\Delta S_{S_{t+L}} = \beta_F (S_{BRt} - S_{t-1}) \tag{4.17}
\]

where \( S_{S_t} \) is the saturated soil moisture content and \( \beta_F \) represents the fraction of recharge to the unsaturated soil layer that recharges the saturated soil layer. Here discharge is modelled as negative recharge that flows away as interflow and is grouped with runoff. The change in saturated soil moisture content results in a corresponding change in groundwater level depending on soil porosity, as shown in Equation 4.18.

\[
\Delta G_{t+L} \beta_P = \Delta S_{S_{t+L}} \tag{4.18}
\]

where \( \beta_P \) is the soil porosity parameter (\( \text{mm m}^{-1} \)) and also converts the units between saturated soil moisture capacity in millimetres to groundwater level in metres.

Substituting Equation 4.17 into Equation 4.18 gives:

\[
\Delta G_{t+L} = \frac{\beta_F}{\beta_P} (S_{BRt} - S_{t-1}) \tag{4.19}
\]

The unsaturated soil moisture content was then updated after it had recharged the saturated soil moisture layer:

\[
S_t = S_{t-1} + (1 - \beta_F)(S_{BRt} - S_{t-1}) \tag{4.20}
\]

Appendix 1 shows the proof that the range of the unsaturated soil moisture content calculated using Equation 4.19 is \( 0 \leq S_t \leq \beta_C \). In addition it is shown that the water balance, including the saturated soil layer given by Equation 4.21
\[ P_t = \Delta S_t + \Delta S_{S_{t+L}} + E_t + Q_t \] (4.21)

is maintained if \( S_{t-1} + P_t - E_t \geq 0 \) but not otherwise. This is the same criterion required for the original Watbal model to maintain the water balance.

Equation 4.19 was re-parameterised to reduce correlation of parameter estimates

\[ G_{t+L} = G_{t+L-1} \frac{1}{\beta_G} (S_{Brt} - S_{t-1}) \] (4.22)

where \( \beta_G \) is the groundwater recharge parameter (mm m\(^{-1}\)) and is equal to \( \frac{\beta_P}{\beta_F} \).

When the groundwater fraction parameter \( \beta_F \) is equal to zero, Recharge Model 2 is identical to Recharge Model 1 and the value of the soil porosity parameter \( \beta_P \) cannot be determined.
4.2.2 Summary

A summary of the models tested that were extensions of the Watbal model follows:

**Watbal Recharge 1 Model**

\[
M_t = 1 - \exp\left( -\beta_S \min\left( 1, \frac{S_{t-1} + \frac{1}{2} P_t}{\beta_C} \right) \right) \frac{1}{1 - \exp(-\beta_S)}
\]

\[
E_t = M_t E_{\text{pan}_t}
\]

\[
Q_t = \max(S_{t-1} + P_t - E_t - \beta_C, 0)
\]

\[
S_t = \min(\max(S_{t-1} + P_t - E_t, 0), \beta_C)
\]

\[
G_{t+L} = G_{t+L-1} + \frac{1}{\beta_G}(S_t - S_{t-1})
\]

This is the proposed extension of the Watbal model to predict groundwater levels with one extra parameter using the proposed Recharge Model 1.

**Watbal Recharge 2 Model**

\[
M_t = 1 - \exp\left( -\beta_S \min\left( 1, \frac{S_{t-1} + \frac{1}{2} P_t}{\beta_C} \right) \right) \frac{1}{1 - \exp(-\beta_S)}
\]

\[
E_t = M_t E_{\text{pan}_t}
\]

\[
Q_t = \max(S_{t-1} + P_t - E_t - \beta_C, 0)
\]

\[
S_{\text{Br}_t} = \min(\max(S_{t-1} + P_t - E_t, 0), \beta_C)
\]

\[
S_t = S_{t-1} + (1 - \beta_F)(S_{\text{Br}_t} - S_{t-1})
\]

\[
G_{t+L} = G_{t+L-1} \frac{1}{\beta_G}(S_{\text{Br}_t} - S_{t-1})
\]
This is the proposed extension of the Watbal model to predict groundwater levels with two extra parameters using the proposed Recharge Model 2.

Where the variables are:

\[ E_t \quad \text{actual evapotranspiration} \quad (\text{mm}) \]
\[ E_{pom} \quad \text{pan evaporation} \quad (\text{mm}) \]
\[ G_t \quad \text{groundwater level} \quad (\text{m}) \]
\[ M_t \quad \text{soil moisture index} \]
\[ P_t \quad \text{rainfall} \quad (\text{mm}) \]
\[ Q_t \quad \text{runoff} \quad (\text{mm}) \]
\[ S_t \quad \text{unsaturated layer soil moisture content} \quad (\text{mm}) \]
\[ S_{BRt} \quad \text{unsaturated layer soil moisture content before recharge to the saturated soil layer has occurred} \quad (\text{mm}) \]
\[ t \quad \text{time} \quad (\text{mths}) \]

and the parameters are:

\[ \beta_C \quad \text{soil moisture capacity parameter} \quad (\text{mm}) \]
\[ \beta_F \quad \text{groundwater fraction parameter} \]
\[ \beta_G \quad \text{groundwater recharge parameter} \quad (\text{mm m}^{-1}) \]
\[ \beta_S \quad \text{soil type parameter} \]
\[ L \quad \text{time lag} \quad (\text{mths}) \]
4.3 The XG Model

The XG model (Xiong and Guo, 1999) is a conceptual model that was designed as a monthly water balance model to predict monthly runoff from large catchments. The input data are monthly rainfall and pan evaporation and the parameters are an evapotranspiration parameter and a runoff parameter. The XG model has a one layer soil moisture store. Guo et al. (2002) have used this model on large catchments in China to predict streamflow. In this research, the XG model was extended to predict groundwater levels.

The assumptions of the XG model are that the soil is rigid with preferred pathways and macropores not explicitly modelled. Soil and vegetation properties are stationary over time, rainfall intensity is uniform over the entire month event and the current soil moisture content depends on the previous soil moisture content. Catchment properties are lumped spatially and temporally.

The XG model can be written as

$$E_t = \beta_E E_{pan} \tanh\left(\frac{P_t}{E_{pan}}\right)$$

(4.23)

$$Q_t = (S_{t-1} + P_t - E_t) \tanh\left(\frac{S_{t-1} + P_t - E_t}{\beta_Q}\right)$$

(4.24)

$$S_t = S_{t-1} + P_t - E_t - Q_t$$

(4.25)

where $E_t$ is the actual evapotranspiration (mm), $\beta_E$ is the evapotranspiration parameter, $E_{pan}$ is the pan evaporation (mm), $P_t$ is the rainfall (mm), $Q_t$ is the runoff (mm), $S_t$ is the unsaturated layer soil moisture content (mm) and $\beta_Q$ is the runoff parameter (mm).
Equation 4.23 calculates the actual evapotranspiration $E_r$ given the evapotranspiration parameter $\beta_E$, the pan evaporation $E_{\text{pan}}$ and the rainfall $P_t$. The evapotranspiration parameter $\beta_E$ is a positive number that takes values around 1. It is used to convert pan evaporation to potential evapotranspiration. Xiong and Guo (1999) identified values ranging from 0.7 to 1.3 for catchments in China. The actual evapotranspiration is found by multiplying the potential evapotranspiration by the moisture index $\tanh \left( \frac{P_t}{E_{\text{pan}}} \right)$.

The dependence of the moisture index $M_t$ on the ratio of rainfall to pan evaporation is shown in Figure 4.5. It is similar to the Watbal model moisture index shown in Figure 4.4.

![Figure 4.5](image)

**Figure 4.5** Moisture index versus rainfall divided by pan evaporation for the XG model
Equation 4.24 estimates the runoff $Q_t$ given the previous time period’s soil moisture content, the rainfall, the pan evaporation and the runoff parameter $\beta_Q$. The runoff parameter $\beta_Q$ is a positive number with units of millimetres. It is proportional to the soil moisture capacity. Xiong and Guo (1999) identified values ranging from 500mm to 2000mm for catchments in China. If the quantity $S_{t-1} + P_t - E_t$ is negative, the runoff $Q_t$ is positive. This is because the $\tanh()$ function is an odd function and the same quantity $S_{t-1} + P_t - E_t$ appears inside the $\tanh()$ function. This is in contrast to the W atbal model which sets runoff to zero when $S_{t-1} + P_t - E_t$ is negative.

$$Q_t = (S_{t-1} + P_t - E_t) \tanh \left( \frac{S_{t-1} + P_t - E_t}{\beta_Q} \right)$$

Figure 4.6 Runoff versus $S_{t-1} + P_t - E_t$ for runoff parameter values of 200mm, 2000mm and 20,000mm

Figure 4.6 shows that when $\beta_Q > 20,000mm$, the calculated runoff is very low. Therefore any value of $\beta_Q > 20,000mm$ is approximately equivalent in action to $\beta_Q = 20,000mm$. 
Equation 4.25 calculates the soil moisture content $S_i$ using the water balance equation. The soil moisture content $S_i$ takes physically impossible negative values if $S_{i-1} + P_i - E_i < 0$ but the water balance is maintained. This is in contrast to the Watbal model which changes negative soil moisture content values to zero and thus violates the water balance equation. In the Appendix 5, the XG model was tested with a modification to set $S_i = 0$ and $Q_i = 0$ when $S_{i-1} + P_i - E_i < 0$. The modification did not have a large effect because it was very rare for $S_{i-1} + P_i - E_i < 0$ since the rainfall minus the actual evapotranspiration is limited in practise.

Guo et al. (2002) demonstrated that the soil moisture content $S_i$ in the XG model cannot exceed $0.278\beta_Q$ hence the runoff parameter $\beta_Q$ does not represent the soil moisture capacity but is still related to it. This result was determined by making $Q$ the subject of Equation 4.25 and substituting into Equation 4.24 to give:

$$S_{i-1} + P_i - E_i - S_i = (S_{i-1} + P_i - E_i) \tanh \left( \frac{S_{i-1} + P_i - E_i}{\beta_Q} \right)$$

Dividing both sides by $\beta_Q$ and changing the subject to $\frac{S_i}{\beta_Q}$:

$$\frac{S_i}{\beta_Q} = (S_{i-1} + P_i - E_i) \left( 1 - \tanh \left( \frac{S_{i-1} + P_i - E_i}{\beta_Q} \right) \right)$$

The maximum value of this function is 0.278, therefore the soil moisture content $S_i$ cannot exceed $0.278\beta_Q$. This maximum occurs when $\frac{S_{i-1} + P_i - E_i}{\beta_Q} = 0.64$. So when $\frac{S_{i-1} + P_i - E_i}{\beta_Q} > 0.64$, the unrealistic scenario is obtained whereby more rainfall minus actual evapotranspiration leads to a decrease in soil moisture content. It was found to be rare for $\frac{S_{i-1} + P_i - E_i}{\beta_Q} > 0.64$ both in this research and by Guo et al. (2002) because the fitted value of $\beta_Q$ becomes large to prevent this.
4.3.1 Soil Moisture Discharge

In the XG model, the mechanism for loss of soil moisture is through the runoff term as discharge into a stream. This is demonstrated by considering the case when rainfall is zero. Substituting $P_t = 0$ into the evapotranspiration Equation 4.23 gives $E_t = 0$. Substituting $P_t = 0$ and $E_t = 0$ into the runoff Equation 4.24 gives

$$Q_t = (S_{t-1}) \tanh \left( \frac{S_{t-1}}{\beta_Q} \right)$$

and the soil moisture Equation 4.25 becomes

$$S_t = S_{t-1} - Q_t$$

$$= S_{t-1}(1 - \tanh \left( \frac{S_{t-1}}{\beta_Q} \right))$$

which is the discharge-only decay equation used in the XG model as shown in Figure 4.7. The discharge from the soil moisture is the runoff and is fed into the catchment stream as baseflow.

Figure 4.7  Soil moisture content versus time using the discharge-only decay equation of the XG model with $S_0 = 200$ mm and $\beta_Q = 1000$ mm
4.3.2 Extensions to the XG Model

The XG model was designed to predict streamflow. In order to predict groundwater levels, different equations linking soil moisture content to recharge were examined. The two recharge models used to extend the Watbal model were used here as well as a third recharge model.

Recharge Model 1

Similarly to the proposed Watbal model extension, the first recharge model assumes that the recharge to the saturated layer is a fraction of the infiltration to the unsaturated layer:

\[ \Delta G_{t+L} = \frac{1}{\beta_G} \Delta S_t \]  

(4.26)

where \( \beta_G \) is the groundwater recharge parameter (mm m\(^{-1}\)) that depends on both the soil porosity and the fraction of water that moves between the unsaturated and saturated soil layers. \( L \) is the time lag. Equation 4.26 linearly correlates the change in groundwater level with the change in soil moisture content \( L \) time-steps ago. The saturated soil moisture content is not explicitly calculated here since the soil porosity relating the groundwater level and saturated soil moisture content is not known.

Changing the subject of Equation 4.25 gives Equation 4.27

\[ \Delta S_t = P_t - E_t - Q_t \]  

(4.27)

Substituting Equation 4.27 into Equation 4.26 gave Recharge Model 1:

\[ G_{t+L} = G_{t+L-1} + \frac{1}{\beta_G} (P_t - E_t - Q_t) \]  

(4.28)

Recharge Model 2

Similarly to the proposed Watbal model extension, Recharge Model 2 explicitly models two soil layers: the unsaturated soil layer and the saturated soil layer. The second recharge model was derived as follows. It was assumed that a change in the
unsaturated soil moisture content would produce a change in the saturated soil moisture content $L$ time periods later as shown in Equation 4.29.

$$\Delta S_{S_{t+L}} = \beta_F (P_t - E_t - Q_t)$$

(4.29)

where $S_{S_t}$ is the saturated soil moisture content and $\beta_F$ is a number representing the fraction of recharge to the unsaturated soil layer that recharges the saturated soil layer. Here the discharge of groundwater is modelled as negative recharge that flows away as interflow and is grouped with the runoff term. The change in saturated soil moisture content results in a corresponding change in groundwater level depending on soil porosity:

$$\Delta G_{t+L} \beta_p = \Delta S_{S_{t+L}}$$

(4.30)

where $\beta_p$ is the soil porosity parameter (mm m$^{-1}$) and also converts the units between saturated soil moisture capacity in millimetres to groundwater level in metres.

Substituting Equation 4.29 into Equation 4.30 gives:

$$\Delta G_{t+L} = \frac{\beta_F}{\beta_p} (P_t - E_t - Q_t)$$

(4.31)

The unsaturated soil moisture content was then updated after it had recharged the saturated soil moisture layer as shown in Equation 4.32:

$$S_t = S_{t-1} + (1 - \beta_F)(P_t - E_t - Q_t)$$

(4.32)

Here the soil moisture $S_t$ is still positive because the maximum runoff is $S_{t-1} + P_t - E_t$, in which case the soil moisture content is:

$$S_t = S_{t-1} + (1 - \beta_F)(-S_{t-1})$$

when $\beta_F = 0$

$$S_t = 0$$
when \( \beta_p = 1 \)

\[ S_t = S_{t-1} \]

Therefore the water balance, including the saturated soil layer

\[ P_t = \Delta S_t + \Delta S_{S_t+L} + E_t + Q_t \]

is maintained.

Equation 4.31 was re-parameterised (as discussed in Chapter 6) in order to reduce correlation of parameter estimates:

\[ G_{t+L} = G_{t+L-1} \frac{1}{\beta_G} (P_t - E_t - Q_t) \quad (4.33) \]

where \( \beta_G \) is the groundwater recharge parameter (mm m\(^{-1}\)) and is equal to \( \frac{\beta_p}{\beta_F} \).

When the groundwater fraction parameter \( \beta_F \) is equal to zero, Recharge Model 2 is identical to Recharge Model 1 and the value of the soil porosity parameter \( \beta_p \) cannot be determined.

**Recharge Model 3**

Chapman and Malone (2002) investigated 13 different recharge equations relating groundwater recharge to soil moisture content. They found that an exponential function of soil moisture content performed well on daily data from a 2.4 m deep weighing lysimeter in a grass field in Ohio, USA.

The exponential recharge equation used by Chapman and Malone (2002) was re-parameterised from three to two parameters:

\[ R_t = \beta_4 e^{\beta_5 S_{S_t+L}} \quad (4.34) \]

where \( \beta_4 \) (mm) and \( \beta_5 \) (mm\(^{-1}\)) both depend on the field capacity and influence the rate at which recharge depends on the soil moisture content.
Equation 4.34 allowed only positive recharge values, since the equation is exponential, so groundwater discharge required a separate equation. After inspecting Figure 4.8a and noticing the exponential decay pattern, it was proposed that a discrete exponential decay function be tested for the saturated soil moisture discharge equation.

![Figure 4.8](image)

**Figure 4.8** a. Groundwater level, b. rainfall and c. pan evaporation data for Orroral Valley bore 000601 showing the early 1980s drought

The differential equation for the exponential decay of the saturated soil layer moisture content for modelling discharge only is:

\[
\frac{dS_s(t)}{dt} = -\beta S_s(t)
\]  

(4.35)

Equation 4.35 states that the more water that is present in the saturated soil moisture store, the faster it flows out of the store as discharge. A continuous solution to Equation 4.35 is:
4.3 The XG Model

\[ S_s(t) = S_s(0)e^{-\beta_6} \]  
(4.36)

Approximating Equation 4.35 using first forward differences gives a difference equation:

\[ \Delta S_{s_t} = -\beta_6 S_{s_t} \]  
(4.37)

where \( \beta_6 \) is a number that controls the rate of discharge. Equation 4.37 was used as the discharge equation. The change in saturated soil moisture content is equal to the recharge minus the discharge:

\[ \Delta S_{s_t} = \beta_4 e^{\beta_6 S_{s_{t-1}}} - \beta_5 S_{s_t} \]  
(4.38)

The change in saturated soil moisture content was then converted to a change in groundwater level depending on porosity using:

\[ \Delta G_t \beta_p = \Delta S_{s_t} \]  
(4.39)

The soil moisture content was then updated assuming that discharge from the saturated soil layer flows away and in grouped with the runoff term. In this way, the saturated soil layer discharge leads to streamflow.

\[ S_t = S_{t-1} - \Delta S_{s_t} \]  
(4.40)

4.3.3 XG Recharge 1 Model Expansion

The XG Recharge 1 model was examined further to express the groundwater level in terms of the initial groundwater level rather than the previous month’s groundwater
level. When $t = 1$ is substituted into the Recharge 1 model Equation 4.28, the result is:

$$G_{t+1} = G_t + \frac{1}{\beta_G} (P_t - E_t - Q_t)$$  \hspace{1cm} (4.41)

When $t = 2$ is substituted into the Recharge 1 model Equation 4.28, the result is:

$$G_{2+1} = G_{1+1} + \frac{1}{\beta_G} (P_2 - E_2 - Q_2)$$  \hspace{1cm} (4.42)

Substituting Equation 4.41 into Equation 4.42 gives

$$G_{2+1} = G_1 + \frac{1}{\beta_G} (P_1 - E_1 - Q_1) + \frac{1}{\beta_G} (P_2 - E_2 - Q_2)$$  \hspace{1cm} (4.43)

The pattern was continued and leads to the formula for $G_{t+1}$

$$G_{t+1} = G_t + \frac{1}{\beta_G} \sum_{k=1}^{t} (P_k - E_k - Q_k)$$  \hspace{1cm} (4.44)

But in Equation 4.44, the runoff $Q_k$ values are dependent, so an equation was required for the runoff in terms of the initial soil moisture.

Similarly to the groundwater level, soil moisture content may also be expressed in terms of the initial soil moisture content:

$$S_t = S_0 + \sum_{k=1}^{t} (P_k - E_k - Q_k)$$  \hspace{1cm} (4.45)

When $t = 1$ is substituted into the runoff Equation 4.24, we find

$$Q_1 = (S_0 + P_1 - E_1) \tanh \left( \frac{S_0 + P_1 - E_1}{\beta_Q} \right)$$  \hspace{1cm} (4.46)
When \( t = 2 \) is substituted into the runoff Equation 4.24, the result is

\[
Q_2 = (S_1 + P_2 - E_2) \tanh \left( \frac{S_1 + P_2 - E_2}{\beta_Q} \right)
\]

(4.47)

Substituting Equation 4.45 into Equation 4.47 gives

\[
Q_2 = \left( (S_0 + P_1 - E_1 - Q_1) + P_2 - E_2 \right) \tanh \left( \frac{(S_0 + P_1 - E_1 - Q_1) + P_2 - E_2}{\beta_Q} \right)
\]

(4.48)

Substituting Equation 4.46 into Equation 4.48 yields

\[
Q_2 = \left( (S_0 + P_1 - E_1 - (S_0 + P_1 - E_1)) \tanh \left( \frac{S_0 + P_1 - E_1}{\beta_Q} \right) + P_2 - E_2 \right)
\]

\[
\tanh \left( \frac{(S_0 + P_1 - E_1 - (S_0 + P_1 - E_1)) \tanh \left( \frac{S_0 + P_1 - E_1}{\beta_Q} \right) + P_2 - E_2}{\beta_Q} \right)
\]

\[
= \left( S_0 + (P_1 - E_1) + (P_2 - E_2) - (S_0 + P_1 - E_1) \tanh \left( \frac{S_0 + P_1 - E_1}{\beta_Q} \right) \right)
\]

\[
\tanh \left( \frac{S_0 + (P_1 - E_1) + (P_2 - E_2) - (S_0 + P_1 - E_1) \tanh \left( \frac{S_0 + P_1 - E_1}{\beta_Q} \right)}{\beta_Q} \right)
\]

(4.49)

The pattern continues with the size of the equation growing exponentially with each time step. Hence it was not appropriate to express the runoff or the groundwater level only in terms of the parameters, initial values and input data. This meant that derivative based parameter search and sensitivity analysis methods could not be used so alternative approaches were required as discussed in Chapters 5 and 6.
4.3.4 Summary

A summary of the models tested that were extensions of the XG model follows:

**XG Recharge 1 Model**

\[ E_t = \beta_E E_{pan_t} \tanh \left( \frac{P_t}{E_{pan_t}} \right) \]

\[ Q_t = (S_{t-1} + P_t - E_t) \tanh \left( \frac{S_{t-1} + P_t - E_t}{\beta_Q} \right) \]

\[ S_t = S_{t-1} + P_t - E_t - Q_t \]

\[ G_{t+1} = G_{t+1} + \frac{1}{\beta_G} (P_t - E_t - Q_t) \]

This is the proposed extension of the XG model to predict groundwater levels with one extra parameter using the proposed Recharge Model 1.

**XG Recharge 2 Model**

\[ E_t = \beta_E E_{pan_t} \tanh \left( \frac{P_t}{E_{pan_t}} \right) \]

\[ Q_t = (S_{t-1} + P_t - E_t) \tanh \left( \frac{S_{t-1} + P_t - E_t}{\beta_Q} \right) \]

\[ S_t = S_{t-1} + (1 - \beta_e)(P_t - E_t - Q_t) \]

\[ G_{t+1} = G_{t+1} + \frac{1}{\beta_G} (P_t - E_t - Q_t) \]

This is the proposed extension of the XG model to predict groundwater levels with two extra parameters using the proposed Recharge Model 2.
XG Recharge 3 Model

\[ E_t = \beta E_{pan_t} \tanh \left( \frac{P_t}{E_{pan_t}} \right) \]

\[ Q_t = (S_{s-1} + P_t - E_t) \tanh \left( \frac{S_{s-1} + P_t - E_t}{\beta Q} \right) \]

\[ S_{BRt} = S_{t-1} + P_t - E_t - Q_t \]

\[ S_{St} = S_{s-1} + \beta_4 e^{\beta_5 S_{BRt-1}} - \beta_6 S_{St} \]

\[ S_t = S_{t-1} - (S_{St} - S_{St-1}) \]

\[ G_t = G_{t-1} + \frac{1}{\beta P} (S_{St} - S_{St-1}) \]

This is the proposed extension of the Watbal model to predict groundwater levels with four extra parameters using the proposed Recharge Model 3.

Where the variables are:

- \( E_t \) actual evapotranspiration (mm)
- \( E_{pan_t} \) pan evaporation (mm)
- \( G_t \) groundwater level (m)
- \( P_t \) rainfall (mm)
- \( Q_t \) runoff (mm)
- \( S_t \) unsaturated layer soil moisture content (mm)
- \( S_{BRt} \) unsaturated layer soil moisture content before recharge to the saturated soil layer has occurred (mm)
- \( S_{St} \) saturated layer soil moisture content (mm)
- \( t \) time (mths)
and the parameters are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_E$</td>
<td>evapotranspiration parameter</td>
<td></td>
</tr>
<tr>
<td>$\beta_F$</td>
<td>groundwater fraction parameter</td>
<td></td>
</tr>
<tr>
<td>$\beta_G$</td>
<td>groundwater recharge parameter</td>
<td>(mm m$^{-1}$)</td>
</tr>
<tr>
<td>$\beta_P$</td>
<td>soil porosity parameter</td>
<td></td>
</tr>
<tr>
<td>$\beta_Q$</td>
<td>runoff parameter</td>
<td>(mm)</td>
</tr>
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<td>$\beta_A$</td>
<td>first exponential recharge parameter</td>
<td>(mm)</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>second exponential recharge parameter</td>
<td>(mm$^{-1}$)</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>discharge parameter</td>
<td>(m mm$^{-1}$)</td>
</tr>
<tr>
<td>$L$</td>
<td>time lag</td>
<td>(mths)</td>
</tr>
</tbody>
</table>
CHAPTER 5 TESTING OF THE MODELS

This chapter presents a discussion of model parameter estimation and model testing or selection. No previous hydrological studies have used hv-block cross-validation for model testing and this is an original contribution of this research. A discussion of the Fortran 90 computer programming of model testing procedures is also included. A comparison of the performances of different model on the same data is given. This resulted in the choice of a model for the uncertainty investigation in Chapter 6 and applications in Chapter 7.

5.1 Overview

5.1.1 Parameter Estimation
In this research, model parameters were not measured but were estimated from the available response data (groundwater level, streamflow) and input predictor data (rainfall, pan evaporation) using a calibration procedure. The calibration procedure involved numerous executions of the model with different combinations of parameter values to arrive at the parameter set that gave the best agreement between observed response data and predicted response data.

Goodness-of-fit
In order to objectively determine the level of agreement between observed response data and predicted response data, a goodness-of-fit index was required. The coefficient of determination or $R^2$ value was used in this research as it has been widely used in statistical regression and in hydrological literature (Nash and Sutcliffe, 1970).

The definition of $R^2$ (Neter et al., 1996) is

$$R^2 = 1 - \frac{SSE}{SST}$$

(5.1)
where \( SSE \) is the Residual Sum of Squares:

\[
SSE = \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2
\]  

(5.2)

and \( SST \) is the Total Sum of Squares:

\[
SST = \sum_{i=1}^{N} (Y_i - \bar{Y})^2
\]  

(5.3)

where \( N \) is the length of the data record; \( Y_t \) is the observed response data at time \( t \); \( \hat{Y}_t \) is the predicted response data at time \( t \); and \( \bar{Y} \) is the mean of the observed response data.

The model can be written as:

\[
Y_t = f_t(X_t; \beta) + \varepsilon_t
\]  

(5.4)

where \( f_t \) is a prescribed function of the independent predictors \( X_t \) and the model parameters \( \beta \). The residuals \( \varepsilon_t \) contain both model structural errors and data measurement errors. Once the model parameters have been estimated as \( \hat{\beta} \), the predicted response is given by:

\[
\hat{Y}_t = f_t(X_t; \hat{\beta})
\]  

(5.5)

For a linear model \( (f_t = f \text{ linear in } \beta) \), the range of \( R^2 \) is \( 0 < R^2 \leq 1 \) and this is equal to the squared correlation between observed and predicted response data, as shown in the following. The coefficient of correlation is defined as (Neter et al., 1996):

\[
r = \frac{\sum (\hat{Y}_t - \bar{Y})^2}{\sqrt{\sum (Y_t - \bar{Y})^2}}
\]  

(5.6)

For a linear model with an intercept, the total sum of squares is equal to the regression sum of squares plus the residual sum of squares (Neter et al., 1996):
\[
\sum (Y_i - \bar{Y})^2 = \sum (\hat{Y}_i - \bar{Y})^2 + \sum (Y_i - \hat{Y}_i)
\]  
(5.7)

Rearranging Equation 5.7:

\[
\sum (\hat{Y}_i - \bar{Y})^2 = \sum (Y_i - \bar{Y})^2 - \sum (Y_i - \hat{Y}_i)
\]  
(5.8)

Substituting Equation 5.8 into the Equation 5.6 gives

\[
\begin{align*}
R_{\text{linear}} &= \sqrt{\frac{\sum (Y_i - \bar{Y})^2 - \sum (Y_i - \hat{Y}_i)}{\sum (Y_i - \bar{Y})^2}} \\
&= \sqrt{1 - \frac{\sum (Y_i - \hat{Y}_i)}{\sum (Y_i - \bar{Y})^2}} \\
&= \sqrt{1 - \frac{\text{SSE}}{\text{SST}}} \\
&= \sqrt{R^2}
\end{align*}
\]  
(5.9)

So the \( R^2 \) value of a linear model is equal to the squared correlation between observed and predicted response data.

For a non-linear model the total sum of squares is not equal to the regression sum of squares plus the residual sum of squares (Neter et al., 1996):

\[
\sum (Y_i - \bar{Y})^2 \neq \sum (\hat{Y}_i - \bar{Y})^2 + \sum (Y_i - \hat{Y}_i)
\]  
(5.10)

Therefore the \( R^2 \) value of a non-linear model is not equal to the squared correlation between observed and predicted response data:

\[
\begin{align*}
R_{\text{nonlinear}} &= \sqrt{\frac{\sum (Y_i - \bar{Y})^2 - \sum (Y_i - \hat{Y}_i)}{\sum (Y_i - \bar{Y})^2}} \\
&\neq \sqrt{R^2}
\end{align*}
\]  
(5.11)
For a non-linear model the range of $R^2$ is $-\infty < R^2 \leq 1$ and is not equal to the squared correlation between observed and predicted response data. An $R^2$ of 1 indicates a perfect fit between the observed and predicted response data. A negative $R^2$ indicates that the fit is worse than predicting the observed response data using its mean value.

Maximising $R^2$ is equivalent to the ordinary least squares procedure of minimising the residual sum of squares $SSE$ except that maximising $R^2$ can be used when the total sum of squares $SST$ changes during calibration. This feature was required in this research as time lag was a parameter of the models. When the time lag is increased, the number of points available for calibration is decreased; hence the total sum of squares is decreased.

The root mean square error $RMSE$ (Hjorth, 1994) was also used in this research:

$$RMSE = \sqrt{\frac{SSE}{N}}$$

(5.12)

where $N$ is the number of data points and $SSE$ is the residual sum of squares. The range is $0 \leq RMSE < \infty$ with a value of 0 indicating a perfect fit. The $RMSE$ was used in this research in preference to the standard deviation because the $RMSE$ is used for cross-validation scores in the statistical literature (Hjorth, 1994).

The relative bias (Neter et al., 1996) is also estimated in this study:

$$b = \frac{\sum_{i=1}^{N} (Y_i - \hat{Y}_i)}{\sum_{i=1}^{N} Y_i}$$

(5.13)

The range of the relative bias is $-\infty < b < \infty$. This gives an indication of the extent to which the model is consistently over-predicting or under-predicting. A value of 0 indicates neither.
5.1 Testing of the Models: Overview

**Automatic Calibration Procedure**

The calibration procedure used to locate the parameter set with the maximum $R^2$ value was made automatic with the use of a search algorithm.

The choice of search algorithm was constrained because $R^2$ functions of hydrological models often contain discontinuities and multiple local maxima (Xiong and O'Connor, 2000). This means that Newton and quasi-Newton search algorithms that rely on derivatives may not be appropriate for hydrological models (Hendrichson et al., 1988) and hence were not used in this study. Direct search methods were investigated instead because they do not require calculation of the derivatives. Direct search methods include the Downhill Simplex Method (Nelder and Mead, 1965), the Adaptive Random Search (Brazil and Krajewski, 1987), the Genetic Algorithm (Wang, 1991) and the Shuffled Complex method (Sorooshian et al., 1993).

The search algorithm used in this research was the Downhill Simplex Method (Nelder and Mead, 1965; Press et al., 1992) because of the small number of parameters in the models being tested and because this method has frequently been used in hydrological research, for example, by Xiong and Guo (1999). A simplex is defined as a figure of $k+1$ vertices in the $k$-dimensional search space ($k$ parameters). A starting simplex is specified and then the simplex is expanded, contracted and reflected according to simple rules until it is within a specified distance from the maximum or until a specified number of function evaluations have been exceeded.

Once the search algorithm located an optimum, it was started again with a simplex around that point to confirm that the point was an optimum. In addition, multiple starting parameter sets were used to increase the confidence that the optimal parameter set found was global rather than local.

It was necessary in the calibration of some models to place bounds on the parameter values to prevent overflow errors, that is, numbers becoming too large to be represented in the computer's math processor.
5.1 Testing of the Models: Overview

5.1.2 Model Testing

Ockham’s Razor states that “entities are not to be multiplied beyond necessity” (Ockham, 1347). That is, models must be able to fit data adequately but not have too many parameters. This is the basis of the model selection methods: Neyman-Pearson hypothesis testing (Neter et al., 1996); Akaike’s Information Criterion testing (Akaike, 1977); Bayesian Information Criterion testing (Schwarz, 1978); and Cross-validation (Forster, 2000). Each of these methods is useful in different circumstances as each has its own set of assumptions.

Neyman-Pearson hypothesis testing assumes that the residuals \( e_t = Y_t - \hat{Y}_t \) are normally, independently, identically distributed. The assumption of independence means that autocorrelation should be zero. In groundwater models this is often not the case as each month’s groundwater level depends on the previous month’s groundwater level and this dependence is often carried through to the residuals. In streamflow models, the assumption that the residuals are identically distributed (homoscedastic, constant variance) is often violated because streamflow data are skewed. Higher streamflow values have higher variances. One way around this problem is to use the square root transformation to make the residuals

\[
e_t = \sqrt{Y_t} - \sqrt{\hat{Y}_t} \quad \text{normally distributed} \quad \text{(Vandewiele et al., 1992).}
\]

Similarly the Box-Cox transformation (Neter et al., 1996) which allows an index other than the 0.5 index of the square root transformation may be used to normalise streamflow model residuals. Kuczera (1998) used the Box-Cox transformation successfully with an index of 0.25 for a catchment in the Australian Capital Territory.

The Akaike Information Criterion involves minimisation of the Kullback-Liebler distance of the selected density function from the true density function (Akaike, 1977). Akaike’s Theorem gave an asymptotic formula for the Kullback-Liebler distance that has the number of parameters as a negative influence on the distance (Forster, 2000). Under certain assumptions this method is asymptotically biased, in the sense that it will tend to select models with too many parameters.

The Bayesian Information Criterion is similar to the Akaike Information Criterion but has a stronger penalty for large models and avoids the bias of the Akaike
Information Criterion. Marshall et al. (2003) give an explanation of the use of the Bayesian Information Criterion in calculating the best rainfall-runoff model for the Bass River Catchment, Victoria. They used the Adaptive Metropolis algorithm for Markov Chain Monte Carlo simulations to obtain the Bayesian Information Criterion for each of the models tested. Results obtained using the Bayesian Information Criterion are sensitive to choice of prior distributions of parameter estimates.

Split-sample validation has been used extensively in hydrological research (Guo et al., 2002). The use of the word validation is not intended to imply that a model is a true representation of reality. The word verification is often used in place of validation (Makhlouf and Michel, 1994; Xiong and Guo, 1999). In split-sample validation, data records are split into a calibration period and a validation period as in Figure 5.1. The model is calibrated over the calibration period and, using the parameter values thus obtained, the goodness-of-fit is evaluated over the validation period. The best model is the one with the highest validation goodness-of-fit. This tests the persistence of model performance and gives an indication of the stationarity of model structure and the stationarity of the parameter values obtained during calibration. A problem with split-sample validation is that the choice of calibration and validation periods is left to the user who may introduce bias by choosing only the calibration and validation sets that give the highest goodness-of-fit values.

Cross-validation removes the bias of manual validation-period selection by calibrating and validating on many different subsets of the data. The simplest cross-validation method for independent data is leave-one-out cross-validation. In this, a point of data is left out of the calibration procedure and then the value at that point is predicted and compared with the observed value. This is performed for each point in turn in the data set. A cross-validation statistic is then calculated from these differences. Usually the root mean square error (RMSE) is the statistic calculated. As
the number of observations becomes large, the leave-one-out cross-validation technique approaches equivalence to the Akaike Information Criterion technique (Mason and Tippett, 2004). Under certain assumptions leave-one-out cross-validation is asymptotically biased and tends to select models with too many parameters (Forster, 2000). Leave-k-out cross-validation solves this problem and is asymptotically equivalent to the Bayesian Information Criterion for appropriately chosen k. Racine (2000) extended these cross-validation procedures to hv-block cross-validation which allows for consistent cross-validation using dependent data.

The hv-block cross-validation procedure (Racine, 2000) involves breaking the data into consecutive time blocks of data of size $B$ with the configuration shown in Figure 5.2. The model is calibrated on the data outside of the chosen block of size $B$ and validated on the validation data block of size $B-2h$ within the block of size $B$. Validating on blocks of data results in dependent error structures being preserved (Racine, 2000). Leaving a buffer of size $h$ each side of the validation data block removes dependence between the calibration and validation period. The hv-block cross-validation procedure is asymptotically consistent for appropriately chosen block sizes, similarly to leave-k-out cross-validation (Racine, 2000).

![Figure 5.2 hv-block cross-validation configuration of calibration, buffer and validation periods for dependent data](image)

The blocks selected for analysis may overlap. For large data sets, the blocks may be chosen randomly with enough blocks chosen so as to generate useful statistics. For small data sets, every block of a particular size may be chosen. In this case the number of blocks is equal to the size of the data set $N$ minus the block size $B$ plus 1.
The way cross-validation rewards simpler models is that an over-parameterised
model will overfit in the calibration period, that is, random errors are modelled.
When the model is validated, the random errors will be different from the calibration
random errors and hence a poor validation goodness-of-fit will result. In this way a
balance between goodness-of-fit and model simplicity is found.

In this research, $hv$-block cross-validation was employed for three reasons: because
of the possibility of dependent residuals; because of its versatility; and because it
was simple to program in Fortran 90.
5.2 Computer Programming for Model Testing

The models, calibration and verification procedures, Monte Carlo (numerical simulation) procedures, and data preparation procedures were all programmed in Fortran 90 using an object-oriented approach. Programming in Fortran 90 ensures that the source code is portable to many different platforms including supercomputers, and the object-oriented approach allows for upgrades to be implemented simply. Because of the length of the source codes they are all contained in the Appendix CD (file:///D:/SourceCode/ModelsAndDataPrep/) along with the executables (file:///D:/Executables/PC/ModelTesting).

Fortran 90 was chosen as the programming language rather than a statistical package such as S-Plus, GAMS, Matlab or Maple V because of the speed required. The bootstrapping and cross-validation procedures run the model hundreds of thousands of times and take hours on a 3 GHz Pentium 4 or a 1 GHz Alpha. Fortran 90 was chosen over other programming languages such as C++ because of the array handling capabilities.

Extensive testing was performed to ensure the absence of errors. This included programming of some of the models as Visual Basic Scripts into Excel. The Appendix CD contains source code and descriptions of the following modules and programs:

**Modules**

- **MthDataYr**: Contains station ID and one year of monthly values
- **MthData**: Contains an array of MthDataYr objects and routines to read and write the MthData format
- **WkDataYr**: Contains station ID and one year of weekly values
- **WkData**: Contains an array of WkDataYr objects and routines to read and write the WkData format
- **StnData**: Contains an array of either weekly or monthly values and routines to read and write the StnData format
GWSYSTEM contains StnData objects for rainfall, pan evaporation, groundwater level and streamflow; models

**Main Program**

Fitwbm reads in data and runs the desired model on the data and saves the results.

Fitwbm has options allowing for monthly or weekly modes; different models; calibration on bore levels, bore level differences, streamflow and combinations of these; different starting values for optimal parameter search; different validation options including hypothesis testing, split-sample validation, cross-validation and bootstrapping. A sample Fitwbm log file appears in Appendix 2.
5.3 Model Comparisons

The groundwater level data used in this section was mean monthly groundwater level. The rainfall and pan evaporation data used in this section were both spatially interpolated to the particular bore site using tri-variate smoothing splines. Tests of point rainfall and pan evaporation data, at monthly and weekly time scales, are examined in Section 7.2.

Each model in Chapter 4 was calibrated and cross-validated on groundwater level data for each bore. Table 5.1 shows the record lengths and the block sizes used for the cross-validation.

The block sizes in Table 5.1 were calculated using

\[ B = n - \frac{n}{\ln(n) - 1} \]  

(5.14)

where \( B \) is the total block size and \( n \) is the number of monthly measurements of data. This satisfied the consistency criteria of \( hv \)-block cross-validation (Racine, 2000). The size of the buffer \( h \) was calculated from

\[ h = 0.05B \]  

(5.15)

which calculates \( h \) as a fraction of the total block size (Racine, 2000). Since the data sets tested in this study were not particularly large, every block of the particular block size was included in the analysis, rather than randomly sampling the blocks. Each block was assigned an index \( j \) which was equal to the index \( i \), within the total data set, of the first element of the block.
## Block Sizes

<table>
<thead>
<tr>
<th>BoreID</th>
<th>Start Date (yyy mm)</th>
<th>End Date (yyy mm)</th>
<th>Total Length N (mths)</th>
<th>Non-Missing (mths)</th>
<th>Block Length B (mths)</th>
<th>Buffer Length h (mths)</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1971 01</td>
<td>1999 12</td>
<td>348</td>
<td>292</td>
<td>209</td>
<td>10</td>
<td>57</td>
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<td>1987 07</td>
<td>192</td>
<td>123</td>
<td>71</td>
<td>3</td>
<td>70</td>
</tr>
<tr>
<td>000606</td>
<td>1971 08</td>
<td>1987 07</td>
<td>192</td>
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<td>89</td>
<td>4</td>
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### Table 5.1
Start and end dates; record lengths; non-missing lengths; cross-validation block sizes; and the number of cross-validation blocks for each of the groundwater bores

## 5.3.1 Determination of Initial Values
Makhlfouf and Michel (1994) used a spin-up period of two years to obtain estimates of the initial soil moisture content $S_0$. It was decided that use of a spin-up period was not suitable for cross-validation because the data in the spin-up period is not used to its full. Another method considered was to start the calibration during a severe
drought and set \( S_0 = 0 \) but this was not feasible for cross-validation. Xiong and Guo (1999) took \( S_0 \) as the average of that Gregorian month’s soil moisture contents. After considerable thought and experimentation it was decided to calibrate the initial groundwater level \( G_0 \) and initial soil moisture content \( S_0 \).

### 5.3.2 Determination of Parameter Starting Values

As shown in the sample Fitwbm log file in Appendix 4, appropriate parameter starting values were guessed and used as the first vertex of the starting simplex in the parameter optimisation search algorithm. Starting distances to search around each parameter were also estimated and these determined the other vertices in the starting simplex. The number of times to start the model parameter optimisation search algorithm over again with a new starting simplex was also specified and a range for the time lag was selected. The values used for the parameter starting values for each model and each bore appears in the model command files on the Appendix CD (For example file://D:/Ch5ModelTesting/HARTT/fitHpEGM.cmd.txt is the command file to fit the HARTT Plus Evap model to the groundwater level data at the monthly time scale). It was found that the identified optimal parameter values were not sensitive to initial estimates. After calibration using the whole data set, the time lag was fixed at the optimal value and the optimal parameter values were used as starting values for the calibration periods in the cross-validation procedure.

### 5.3.3 Bounds on Parameter Estimates

Bounds were placed on some parameters when calibrating the Watbal model and the XG model to force some parameters to be positive. This ensured the absence of 64-bit numerical overflows and also ensured that the models made physical sense. The bounds were programmed in the computer code by making the \( R^2 \) function return a value of \(-9999.0\) when a parameter value to be tested was negative. Bounds were able to be placed on the parameters because the downhill simplex optimisation method used did not rely on function derivatives.
5.3.4 Values of the Cross-Validation Root Mean Square Error and Total Values

In the tables that follow, the cross-validation root mean square error \( RMSE(C.V.) \) is the number used to compare between models. The best model is determined as the model with the smallest \( RMSE(C.V.) \) value.

The cross-validation root mean square error \( RMSE(C.V.) \) for each groundwater bore in Table 3.1 was calculated from:

\[
RMSE(C.V.) = \sqrt{\frac{\sum_{i,j}^{} SSE_{i,j}}{\sum_{j}^{} n_{i,j}}} \tag{5.16}
\]

where \( i \) is the groundwater bore index, \( j \) is the cross-validation block index; and \( n_{i,j} \) is the number of non-missing data points in groundwater bore \( i \), cross-validation block \( j \) minus the time lag.

The total \( R^2 \) and root mean square error values for all bores appear in the bottom line of the results tables. The total \( R^2 \) for all bores was calculated from:

\[
R^2_{Total} = 1 - \frac{\sum_{i}^{} SSE_{i}}{\sum_{i}^{} SST_{i}} \tag{5.17}
\]

The definition of the total \( R^2 \) in Equation 5.17 is effectively a weighted average of each groundwater bore's \( R^2 \) value where the weight is greater for gauges with more measurements and also with greater variance in those measurements.

The total calibration root mean square error for all bores was calculated using:

\[
RMSE_{Total}(Cal) = \sqrt{\frac{\sum_{i}^{} SSE_{i}}{\sum_{i}^{} n_{i}}} \tag{5.18}
\]

where \( i \) is the groundwater bore index and \( n_{i} \) is the number of present points of groundwater bore \( i \) minus the time lag.
The total \( h \)-block cross-validation root mean square error for all bores was calculated from:

\[
RMSE_{\text{Total (C.V.)}} = \sqrt{\frac{\sum_{i} \sum_{j} SSE_{i,j}}{\sum_{i} \sum_{j} n_{i,j}}}
\]

(5.19)

where \( i \) is the groundwater bore index, \( j \) is the cross-validation block number; and \( n_{i,j} \) is the number of present points of groundwater bore \( i \), cross-validation block \( j \) minus the time lag.

**Mean Model**

The time lag \( L \) was fixed to 0 for the Mean model because the calibration procedure tends to set the time lag to its maximum value. Table 5.2 shows the results of modelling the groundwater level using the Mean model. The calibration \( R^2 \) values listed in Table 5.2 are all 0.00 as expected. The results of the Mean model were used to place all of the other tested models into perspective. If a tested model produced larger \( RMSE(\text{C.V.}) \) values than the Mean model this indicated that the tested model was significantly worse than the Mean model at explaining the data and hence was an ineffective model.
### Mean Model

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<th>$RMSE$ (m)</th>
<th>Cross-Validation $RMSE$ (m)</th>
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**Table 5.2** Calibration parameters and statistics; and cross-validation statistics of the Mean model

The mean groundwater level and calibration $RMSE$ values in Table 5.2 are for the selected time periods shown in Table 5.1. These values are slightly different to the values in Table 3.2 where the whole record of each bore was used, including data after the year 2000. The cross-validation $RMSE$ values provide better estimates of the groundwater level standard deviations because the residuals were correlated. Figure 5.3 shows the fit of the Mean model to the Orroral Valley bore 000601.
Figure 5.3  Orroral Valley 000601 a. observed • and fitted — groundwater level using the Mean model; and b. modelled groundwater level showing an example of the calibration , buffer — and validation — blocks used for cross-validation
Testing of the Models: Model Comparisons

### Table 5.3
Calibration parameters and statistics; and cross-validation statistics of the Time Trend model

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Total 0.15 1.47 1.99

Table 5.3 shows that the Time Trend model total $RMSE$(C.V.) is greater than the Mean model total $RMSE$(C.V.). Therefore the Time Trend model was significantly worse than the Mean model overall at predicting ACT groundwater levels. This implies that overall, the groundwater level in the ACT has shown no consistent tendency to increase or decrease over long time periods.
Table 5.4  Calibration parameters and statistics; and cross-validation statistics of the HARTT Minus Trend model

Table 5.4 lists results for the HARTT Minus Trend model. The total \( RMSE(C.V.) \) is less than the Mean model total \( RMSE(C.V.) \) which shows that this model fits the data better than the Mean model. Note that time lags varied from 0 to 12 months.
### Table 5.5 Calibration parameters and statistics; and cross-validation statistics of the HARTT model

Table 5.5 lists results for the HARTT model. The RMSE(C.V.) for Southwell Pk #4 bore 000659 is missing because the time lag of 9 months was greater than the block size of 7 months. The original, empirical HARTT model clearly fits the data better overall than the HARTT Minus Trend modification.
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Total: 0.60 1.00 1.73

Table 5.6 Calibration parameters and statistics; and cross-validation statistics of the HARTT Minus Trend Plus Evap model

Table 5.6 lists results for the HARTT Minus Trend Plus Evap model. The RMSE(C.V.) for Southwell Pk #4 bore 000659 is missing because the time lag of 6 months left only 1 point in the cross-validation block of 7 months and 1 point is not enough to calibrate three parameters. Incorporation of pan evaporation data has improved the fit overall of the HARTT Minus Trend model (Table 5.3), but it is still not as good as the fit for the original HARTT model.
### HARTT Plus Evap Model

**Table 5.7** Calibration parameters and statistics; and cross-validation statistics of the HARTT Plus Evap model

Table 5.7 lists results for the HARTT Plus Evap model. The total RMSE(C.V.) over all the bores is lower than all of the other modifications of the HARTT model. This indicates that inclusion of evaporation has improved the HARTT model.
The relative bias, as calculated by Equation 5.13, given by each HARTT model modification for each bore was 0.0000. This indicates that there was no systematic over-prediction or under-prediction of groundwater levels by the HARTT model modifications. The results for the HARTTA model and its modifications appear on the Appendix CD (file:///D:/Ch5ModelTesting/HARTT) and are very similar to those for the HARTT model.

Figures 5.4 compares predicted groundwater levels from the fitted HARTT Plus Evap model using all of the measured data for calibration. Figure 5.5 shows the fit when only part of the record is used for calibration. Although the HARTT Plus Evap model performs well overall, Figure 5.5 clearly shows that the HARTT Plus Evap model does not validate well for the Orroral Valley bore 000601. This is in accordance with the Mean model $RMSE_{000601}(C.V.)$ value of 1.20m being smaller than the HARTT Plus Evap model $RMSE_{000601}(C.V.)$ value of 1.56m. This is because the model was unable to determine the influence of rainfall and pan evaporation correctly given subsets of the Orroral Valley bore 000601 data.

**Trends in Groundwater Levels**

The time trend of the bores overall was found to be insignificant. Individually, the time trend of twenty bores was found to be insignificant. Dryland salinity studies for New South Wales (Cook et al., 1999) have suggested that groundwater levels are rising almost everywhere as a result of land use change from native vegetation to pasture having increased recharge. This is not the case for some of the bores in the Australian Capital Territory where land use change has taken place. A discussion of these results for the HARTT model and extensions appears in Jellett et al. (2005).
Figure 5.5 Orroral Valley 000601 observed and modelled groundwater level using the HARTT Plus Evap model showing an example of the calibration, buffer and validation blocks used for cross-validation. Here $R^2(\text{Cal}) = 0.50$ and $R^2(\text{Val}) = -7.81$
## Table 5.8 Calibration parameters and statistics; and cross-validation statistics of the Watbal Recharge 1 model

Table 5.8 shows that the Watbal Recharge 1 model performed with a total RMSE(C.V.) of 1.40m. This shows that overall the Watbal Recharge 1 model performs better than the HARTT Plus Evap model. We note here that the time lag has also been reduced to between 0 to 4 months.
### Table 5.9  Calibration parameters and statistics; and cross-validation statistics of the Watbal Recharge 2 model

The Watbal Recharge 2 model in Table 5.9 performed with a total $RMSE$(C.V.) of 1.17m, which is better than the Watbal Recharge 1 model, suggesting that the additional structure in the Watbal Recharge 2 model is more consistent with the actual processes.
As was the case with the HARTT model modifications, the relative bias, as calculated by Equation 5.13, given by the Watbal Recharge 1 and 2 models for each bore was 0.0000. This indicates that there was no systematic over-prediction or under-prediction of groundwater levels by the Watbal Recharge 1 and 2 models.

In Table 5.9, the soil porosity was calculated using:

$$\text{Porosity} = \frac{\hat{\beta}_G \hat{\beta}_F}{1000}$$  \hspace{1cm} (5.20)

where $\hat{\beta}_G$ is the estimate of the groundwater recharge parameter and $\hat{\beta}_F$ is the estimate of the groundwater fraction parameter. When the groundwater fraction parameter estimate $\hat{\beta}_F$ is 0.000 the Watbal Recharge 2 model is identical to the Watbal Recharge 1 model and soil porosity cannot be separated from the groundwater recharge parameter. An example of this is for the Orroral Valley bore 000601 where the groundwater fraction parameter estimate $\hat{\beta}_F$ is 0.000 and accordingly the porosity value is missing.

In Table 5.9, Campbell bore 000613 and Macarthur Ave bore 000630 have unrealistic soil porosity values above 100%. Bores 000605, 000618, 000625, 000657 and 000016 have porosities less than or equal to 4%. The other soil porosity values are more realistic ranging from 6% to 32%.

Figure 5.6 shows the agreement between the observations and predictions for the Watbal Recharge 1 model for the Orroral Valley bore 000601. Here the Watbal Recharge 1 and Watbal Recharge 2 models are identical.

Figure 5.7 shows the comparison between predicted and measured groundwater levels for an example cross-validation block for the Orroral Valley bore 000601. The superior performance of the Watbal Recharge 1 model over the HARTT Plus Evap model is apparent.
A feature of Figure 5.6 is that the predicted groundwater level did not exceed 933.90m. This corresponds to the soil moisture capacity $\hat{\beta}_c$ threshold being reached ($S_i = \hat{\beta}_c$). The automatic calibration procedure had located the soil moisture capacity that gave the highest $R^2$ value. Manually increasing the soil moisture capacity parameter to allow for higher groundwater levels, decreases the $R^2$ value.
Figure 5.6  Orroral Valley 000601 observed • and fitted — groundwater level using the Watbal Recharge 2 model
Figure 5.7  Orroral Valley 000601 observed • and modelled groundwater level using the Watbal Recharge 2 model showing an example of the calibration —, buffer — and validation — blocks used for cross-validation. Here $R^2$(Cal) = 0.63 and $R^2$(Val) = 0.57.
### Table 5.10 Calibration parameters and statistics and; cross-validation statistics of the XG Recharge 1 model

The XG Recharge 1 model in Table 5.10 has the lowest total RMSE(C.V.) so far. Four bores 000613, 000615, 000630 and 000632 have a pan evaporation parameter $\hat{\beta}_E$ value of 0.000. This indicates that the actual evapotranspiration was modelled as unimportant in determining groundwater levels at these bores. A large runoff parameter $\hat{\beta}_Q$ and small initial soil moisture content $\hat{S}_0$ indicate that runoff was modelled as unimportant in determining groundwater levels.
### Table 5.11 
Calibration parameters and statistics; and cross-validation statistics of the XG Recharge 2 model

In Table 5.11 the XG Recharge 2 model total RMSE(C.V.) is 1.16m which is larger than the XG Recharge 1 model’s value of 1.13m. The XG Recharge 2 model calibration $R^2$ value of 0.77 is greater than the XG Recharge 1 model’s value of 0.75. This indicates that the $h_v$-block cross-validation procedure has detected that the XG Recharge 2 model is over-parameterised with respect to the XG Recharge 1 model.
### Table 5.12  Calibration parameters and statistics; and cross-validation statistics of the XG Recharge 3 model

The missing values in Table 5.12 indicate where a 64-bit numerical overflow occurred and hence the model could not be calibrated or cross-validated. The XG Recharge 3 model total \( \text{RMSE}(\text{C.V.}) \) is larger than for the XG Recharge 1 and 2 models indicating that this model is significantly worse at modelling the groundwater level data. Again the range of the time lags is from 0 to 4 months.
As was the case with the HARTT model modifications and the Watbal Recharge 1 and 2 models, the relative bias, as calculated by Equation 5.13, given by the XG Recharge 1, 2 and 3 models for each bore was 0.0000. This indicates that there was no systematic over-prediction or under-prediction of groundwater levels by the XG Recharge 1, 2 and 3 models.

In Table 5.11, Tuggeranong bore 000620 and Barney’s Hill East bore 000622 have physically impossible soil porosity values above 100%. Many other bores have extremely low porosity values in the range 0.1% to 4.8%. For shallow groundwater systems these appear to be physically improbable. This suggests that the XG Recharge 2 model is less consistent with some of the physical processes than the Watbal Recharge 2 model which had a greater number of realistic porosity values. Missing porosity values correspond to when the groundwater fraction parameter estimate $\hat{\beta}_f$ is 0.000 and the XG Recharge 2 model is identical to the XG Recharge 1 model.

The XG Recharge 3 model in Table 5.12 that used the most detailed exponential equations for recharge did not perform as well as the XG Recharge 1 model. This may be because the exponential recharge equation assumptions that were appropriate for daily data are not appropriate for monthly data. Makhlouf and Michel (1994) encountered similar results when testing daily models on monthly data.

Figures 5.8 and 5.9 show the XGR1 model predicted groundwater levels for the whole Orroral Valley bore 000601 data set as well as a single cross-validation block.

Figure 5.8 also shows that the groundwater level was under-predicted in 1971 and 1972. This may be due to the fact that the Orroral Valley rainfall gauge 570980 only came into operation in December 1972. Before this the rainfall was estimated using the spatially interpolated rainfall values with the nearest gauge 10km away.

Figure 5.8 indicates that the groundwater level was over-predicted in the drought years of 1981 and 1983. The original rainfall data of Orroral Valley rainfall gauge 570980 were investigated and it was noticed that the quality code of the data was
101 in February and March 1980 indicating poor quality data. In addition, the monthly recharge may have been overestimated when a month had an abnormally high number of small rainfall events of less than 2mm per day. In this case most of the rainfall would have evaporated on the day it fell.

In Figure 5.8, the groundwater level in May 1989 is also over-predicted. The Orroral Valley rainfall gauge 570980 data for April 1989 was found to be of quality code 101 which may have caused the following month’s over-prediction. The frequency of poor quality rainfall data was on average two poor quality months per decade.
Figure 5.8  Orroral Valley 000601 observed • and fitted — groundwater level using the XGR1 model
Figure 5.9  Orroral Valley 000601 observed • and modelled groundwater level using the XGR1 model showing an example of the calibration —, buffer — and validation — blocks used for cross-validation. Here $R^2$(Cal) = 0.77 and $R^2$(Val) = 0.70
5.3.5 Comparison of Model Performance

Table 5.13 shows model name abbreviations and the number of initial values and parameters for each model.

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Table 5.13  Model name abbreviations and the number of initial values and parameters for each model

Tables 5.14, 5.15 and 5.16 show the comparison between the models values for calibration coefficient of determination $R^2$, calibration root mean square error and cross-validation root mean square error.
### Table 5.14

Calibration $R^2$ values for each model and the whole data set of each bore. The highest value in each row is bold indicating the best calibration score for each bore.

Table 5.14 indicates that the model with the overall best calibration is the XG Recharge 2 model.
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**Table 5.15** Calibration RMSE values for each model and the whole data set of each bore. The lowest value in each row is bold indicating the best calibration score for each bore.

Similarly to Table 5.14, Table 5.15 indicates that the model with the overall best calibration is the XG Recharge 2 model.
Table 5.16 Cross-validation RMSE values for each model and the whole data set of each bore. The lowest value in each row is bold indicating the best performing model for each bore.

Table 5.16 is the main results table of this research and was used to determine which model performed the best at each site and which model performed the best overall. The XG Recharge 1 (XGR1) model is the best performing model overall because it has the smallest total RMSE(C.V.) value of 1.13m. This indicates that on average the groundwater level predictions of the XG Recharge 1 model were closer to the groundwater level observations than any other tested model, over the whole record.
of available groundwater level data. The total $RMSE(C.V.)$ value takes into account the length of record of each bore (Equation 5.19) and leads to the XG Recharge 1 model being selected as the best model even though the Watbal Recharge 2 model (WR2) performed better than the XG Recharge 1 model at more bores but with shorter record lengths. Even though the differences between $RMSE(C.V.)$ values is small, any difference at all is statistically significant as the $RMSE(C.V.)$ is itself the statistic being used to determine model performance. The empirical HARTT model, in all of its various modifications was the poorest at modelling the data. The Watbal Recharge 2 model was a distinct improvement over the Watbal Recharge 1 model.

The power of the $hv$-block cross validation procedure to detect over-parameterisation was demonstrated when it selected the XG Recharge 1 model over the XG Recharge 2 model even though the XG Recharge 1 model is nested within the XG Recharge 2 model (the XG Recharge 2 model with $\beta_F = 0$ is the XG Recharge 1 model).

Figures 5.10, 5.11 and 5.12 show example cross-validation blocks for the WR1, WR2, XGR1 and XGR2 models for different bores. The purpose of examining these graphs is to determine the behaviour of these models and understand why the XGR1 model was the best overall.
5.3 Testing of the Models: Model Comparisons

**Fyshwick bore 000614**

![Graph showing groundwater levels over time for different models](image)

**Figure 5.10** Fyshwick 000614 observed and modelled groundwater level using the WR1, WR2, XGR1 and XGR2 models showing an example of the calibration, buffer and validation blocks used for cross-validation.

In Figure 5.10, the best performing model is the XGR1 model ($RMSE(C.V.) = 0.56m$) and the worst performing model is the WR1 model ($RMSE(C.V.) = 0.79m$). Here all of the models optimised the time lag to 1 month.
In Figure 5.11, the WR1 model obviously exhibits the worst performance \((RMSE(C.V.) = 5.43m)\). Here the soil moisture content is modelled as zero most of the time with short time periods above this. The WR1 model optimised the time lag to 4 months where the other models found 1 month was the optimal time lag. Here the best performing model is the XGR1 model \((RMSE(C.V.) = 2.60m)\). Figure 5.11 also shows the large range of groundwater levels from 704m to 715m AHD, so the large \(RMSE(C.V.)\) value is not discouraging.
In Figure 5.12, the WR2 model is the best performing model \((RMSE(C.V.) = 0.85m)\) with an optimised the time lag of 3 months. The other models identified 2 months as the optimal time lag and the worst performing model was the XGR1 model \((RMSE(C.V.) = 1.28m)\). The jagged graph of WR2 model predictions indicates larger recharge events that occur less frequently than in the other models.
5.3 Testing of the Models: Model Comparisons

5.3.6 XGR1 Model Time Lag Values

Figure 5.13 shows the model time lag versus mean water table depth for each bore using the XGR1 model.

The correlation between the fitted time lag and mean water table depth was 0.245 and was not significantly different from 0.0 at the 5% level. This may be because other influences on model time lag are not included in Figure 5.13, that is, the influence of catchment size and underground flow rates.
5.3.7 Statistical Tests

Statistical tests of normality, autocorrelation and constant variance for the XG Recharge 1 model residuals for the Orroral Valley bore 000601 are presented here. These statistical test results are used justify the use of hv-block cross-validation as the model selection procedure rather than hypothesis testing.

Figure 5.14 shows the XG Recharge 1, Orroral Valley bore 000601 groundwater level residuals histogram and QQ-Plot. The Shapiro-Wilk test for normality gave a p-value of 0.01 for the residuals. This indicated that the residuals significantly deviated from normality at the 5% level of significance. At the 1% level of significance however, the residuals did not significantly deviate from normality.

![Histogram of Residuals](image)

![Normal Q-Q Plot of Residuals](image)

**Figure 5.14** Orroral Valley 000601 a. histogram and b. normal QQ-plot of the residuals of the XGR1 model
Figure 5.15 shows that the autocorrelation of lag 1 is 0.603 for the XGR1 model residuals for the Orroral Valley bore 000601. This is outside the Durbin-Watson confidence limits. This has a p-value of 0.000 and is therefore an autocorrelation significantly different from 0.0 at the 5% level of significance.

This autocorrelation is due to the property that the groundwater level of the current month was dependent on the groundwater level of the previous month. The coefficient of determination $R^2$ was $(0.603)^2 = 36.4\%$. Therefore 36.4% of the variability in the current residual was explained by variability in the previous residual.
Figure 5.16 shows the residuals as a function of predicted groundwater levels for the XG Recharge 1 model for the Orroral Valley bore 000601. The figure indicates that the variance of the residuals is constant.

These tests indicated that the residuals are normal, not independent and they are identically distributed. The presence of autocorrelation in the residuals justified the use of \( h \)-block cross-validation to test which model was significantly better performing than the other models on the given data.

Here it has been shown that overall, the XG model with the proposed Recharge Model 1 model performed significantly better than all of the other models tested on the data given. The additional structure of the XG Recharge 2 model gave poorer performance than the XG Recharge 1 model because of over-parametrisation. The Watbal Recharge 2 model performed well, although less well than the XG Recharge 1 and 2 models. The Watbal Recharge 2 model gave more meaningful estimates of soil porosity than the XG Recharge 2 model. Unlike the case for the XG model, the extra structure of Recharge Model 2 in the Watbal model distinctly improved performance over the Recharge Model 1. The XG Recharge 3 model that used the most detailed exponential equations for recharge did not perform as well as the XG Recharge 1 or 2 models. This may be because the exponential recharge equation assumptions that were appropriate for daily data are not appropriate for monthly
data. After the XG Recharge 1 model gave the best performance, different modifications of the model shown in Appendix 5 were tested in an attempt to improve the model. The results in Appendix 5 show that none of the modifications improved on the XG Recharge 1 model. Traditional statistical Neyman-Pearson hypothesis testing was not used to determine the significance of model performances, rather \( hv \)-block cross-validation was used which is just as valid a technique and required less assumptions to be met.
CHAPTER 6
UNCERTAINTY IN THE XG RECHARGE 1 MODEL

This chapter provides estimates of parameter uncertainty and prediction uncertainty using Monte Carlo methods for the XGR1 model that was determined to be the best performing model in Chapter 5.

6.1 Overview

An investigation of parameter uncertainty and prediction uncertainty was essential in order to gauge the usefulness of the XGR1 model. Uncertainty arises because of errors in model structure, interactions between parameters and errors in observational data (Seibert, 1997).

There are always uncertainties associated with the estimated optimal parameter set of a calibrated model (Kuczera, 1997). Investigation of parameter uncertainty is called sensitivity analysis (Freer et al., 1996). Local sensitivity analysis involves the use of partial derivatives of the objective function (likelihood function or $R^2$) with respect to the parameters being investigated. This method is convenient when Newton or quasi-Newton parameter search algorithms are used to locate the optimal parameter set. This is because the partial derivatives are calculated in Newton and quasi-Newton parameter search algorithms. As discussed in Chapter 5, derivative-based techniques are often not appropriate for hydrological modelling because the objective functions of hydrological models often contain discontinuities and multiple local maxima (Sorooshian and Arfi, 1982). An assumption of this method of sensitivity analysis is that parameter estimates are uncorrelated (Hendrichson et al., 1988).

More recent global methods of parameter uncertainty investigation have been used (Saltelli et al., 1999) that do not assume independence of parameter estimates. These methods include Monte Carlo computer simulation techniques. The uniform grid-search is an example of a Monte Carlo method. Uniform grid-search of the parameter space involves testing many different parameter sets at uniform intervals.
(Hornberger et al., 1986). Xiong and O’Connor (2000) found that the uniform grid-search was not as efficient as multivariate normal distribution sampling or Markov Chain Monte Carlo sampling. However, in this research, all models and related algorithms were programmed into Fortran 90 and could be executed very quickly, therefore a uniform grid-search could be performed within a reasonable time.

Bootstrapping is also a global method of parameter uncertainty investigation. Bootstrapping involves a resampling scheme where either model residuals or data are resampled to generate new data sets to which the model is fitted. The resampling and model fitting process is repeated a large number of times and statistics are calculated to describe the different optimal parameter sets obtained.

In the case that model residuals are independently and identically distributed (IID), the appropriate bootstrapping technique is IID bootstrapping (Efron, 1982; Politis et al., 1999). IID bootstrapping can be classified as parametric or non-parametric IID bootstrapping. In parametric IID bootstrapping, a continuous distribution function is fitted to the model residuals and new residuals are generated from the continuous distribution, whereas in non-parametric IID bootstrapping, the discrete distribution function of model residuals is resampled (Hjorth, 1994).

When model residuals are not independent, moving-block bootstrapping is used (Lahiri, 2003). The moving-block bootstrapping procedure implements resampling of blocks of data in contrast to IID bootstrapping which resamples single observations at a time. As a result, the dependence structure of the original observations is preserved within each block (Politis et al., 1999). Figure 6.1 shows an example of a block of data within a data set.
Data

Calibration Period

\[ B \]

Time

**Figure 6.1** Moving-block bootstrap configuration of calibration and buffer periods for dependent data

The blocks selected for the bootstrap procedure may overlap. For large data sets, the blocks may be chosen randomly with enough blocks chosen so as to generate useful statistics. For small data sets, every block of a particular size may be chosen. Efron (1982) showed that, for appropriately chosen block sizes, parameter estimate uncertainties obtained from bootstrapping are asymptotically equivalent to those obtained using the maximum likelihood technique.
6.2 Parameter Uncertainty

Here two methods of parameter uncertainty identification are used. The uniform grid-search and the moving-block bootstrap procedure. The resulting parameter uncertainty estimates from these two methods are compared.

6.2.1 Uniform Grid-Search

A uniform grid-search of the parameter space was performed to give an indication of the shape of the $R^2$ value as a function of parameter values for the Orroral Valley 000601 data set from 1971 to 1999.

Figure 6.2 indicates the range of parameters and interdependencies of parameters within the parameter sets that gave $R^2$ values greater than 0.75.

Table 6.1 shows the correlation of the parameter values within the parameter sets that gave $R^2$ values greater than 0.75.
Table 6.1  Correlation matrix of the uniformly sampled parameter sets that gave $R^2$ values greater than 0.75 for the XGR1 model for the Orroral Valley bore 000601.
All of the parameter value correlations in Table 6.1 are significant. The presence of significant correlations between parameters justifies the use of the global sensitivity analyses used in this research. Figures 6.3 to 6.7 show projections onto two dimensions of the $R^2$ value as a function of each initial value and parameter value. The purpose of investigating these figures was to determine the smoothness of the $R^2$ function to obtain an indication of how well defined the maximum was.

**Figure 6.3** $R^2$ versus uniformly sampled values of the initial groundwater level $G_0$ that gave $R^2$ values greater than 0.75 for the XGR1 model for the Orroral Valley bore 000601
6.2 Parameter Uncertainty

**Figure 6.4** $R^2$ versus uniformly sampled values of the initial soil moisture content $\hat{S}_0$ that gave $R^2$ values greater than 0.75 for the XGR1 model for the Orroral Valley bore 000601.

**Figure 6.5** $R^2$ versus uniformly sampled values of the runoff parameter $\hat{\beta}_0$ that gave $R^2$ values greater than 0.75 for the XGR1 model for the Orroral Valley bore 000601.
Figure 6.6 $R^2$ versus uniformly sampled values of the evapotranspiration parameter $\hat{\beta}_E$ that gave $R^2$ values greater than 0.75 for the XGR1 model for the Orroral Valley bore 000601.

Figure 6.7 $R^2$ versus uniformly sampled values of the groundwater recharge parameter $\hat{\beta}_G$ that gave $R^2$ values greater than 0.75 for the XGR1 model for the Orroral Valley bore 000601.
Figures 6.3 to 6.7 show the desirable feature that the curves containing the points in each graph were smooth with a single, well-defined maximum. This increased confidence that the downhill simplex method used to locate optimal parameter sets in the automatic calibration procedure would have succeeded in locating the global optimal parameter set.

Tables 6.2 and 6.3 give an indication of the uncertainty of parameter estimates for the XGR1 model for the Orroral Valley bore 000601 data.

<table>
<thead>
<tr>
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<th>̂βₑ</th>
<th>̂β₉₀</th>
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<td>StDev</td>
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**Table 6.2** Mean and standard deviation of the uniformly sampled parameter sets that gave R² values greater than 0.75 for the XGR1 model for the Orroral Valley bore 000601

<table>
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<td>̂β₉₀</td>
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**Table 6.3** Correlation matrix of the uniformly sampled parameter sets that gave R² values greater than 0.75 for the XGR1 model for the Orroral Valley bore 000601

Table 6.3 is an extract of Table 6.1 and indicates relationships between parameter estimates. Statistics of the initial groundwater level estimate ̂G₀ and the initial soil moisture content estimate ̂S₀ are not given here as these are expected to change when the model is initialised at a different time. Table 6.3 indicates that the larger the runoff parameter estimate ̂β₀, the smaller the evapotranspiration parameter estimate ̂βₑ. Similarly, the larger the runoff parameter estimate ̂β₀, the larger the groundwater recharge parameter estimate ̂β₉₀. Also, the larger the evapotranspiration
parameter estimate $\hat{\beta}_E$, the smaller the groundwater recharge parameter estimate $\hat{\beta}_G$.

The uniform grid-search took 10 hours to test 68,000,000 parameter sets on a 3GHz Pentium 4. The uniform grid-search also required a lot of user intervention to set the parameter bounds and needed to be run about five times to ensure that the parameter sets that gave $R^2$ values greater than 0.75 were contained in the search.
6.2 Parameter Uncertainty

6.2.2 Moving-Block Bootstrap

The block size used for data from the Orroral Valley bore 000601 was 114 months and was calculated using:

\[ B = 0.5(N - \frac{N}{\ln(N) - 1}) \]  
(6.1)

where \( N \) is the number of non-missing months of data. This formula for block size was chosen because it satisfies the criteria specified by Lahiri (2003) that make the bootstrap estimates converge to the maximum likelihood estimates. Since the data set was not particularly large, every block of the particular block size was included in the bootstrap analysis, rather than randomly sampling the blocks. There were 114 months in a block and the total length of the Orroral Valley record was 348 months, therefore the number of blocks was 235 (calculated as 348-114+1).

![Figure 6.8](image)

Figure 6.8  \( R^2 \) versus block end date in the moving-block bootstrap procedure for the XGR1 model for Orroral Valley bore 000601

Figure 6.8 illustrates the importance of using all of the data for model testing. The maximum \( R^2 \) was 90.97% when the model was calibrated from April 1974 to April 1983 and the minimum \( R^2 \) was 72.97% when calibrated from April 1988 to April 1997. Using split-sample validation, it would be possible that one of these extreme \( R^2 \) values would have been displayed, rather than an average \( R^2 \) value. This justified the use of cross-validation for the model testing in Chapter 5.
Figure 6.9 shows that the root mean square error did not have the same pattern of variation as the $R^2$ values. This is because the total sum of squares SST in each block, used in the calculation of $R^2$, varies differently to the number to non-missing data points in each block, used in the calculation of the RMSE.

![RMSE versus block end date](image)

**Figure 6.9** RMSE versus block end date in the moving-block bootstrap procedure for the XGR1 model for Orroral Valley bore 000601

Figure 6.10 plots the optimal parameter sets for each block from the moving-block bootstrap procedure. This is similar to Figure 6.2 and gives an indication of the variability of and correlation between parameter estimates.

Table 6.4 shows that the parameter estimates are highly correlated.
Figure 6.10 Matrix scatter plot indicating optimal parameter sets for each block from the moving-block bootstrap procedure on the XGR1 model using the Orroral Valley bore 000601 groundwater levels from 1971 to 1999

Table 6.4 Correlation matrix of the moving-block bootstrap optimal parameter sets for the XGR1 model for the Orroral Valley bore 000601
The significant correlations between the parameter estimates in Table 6.4 justified the use of the cross-validation procedure in Chapter 5 and the moving-block bootstrap procedure in this chapter (Hjorth, 1994).

Figure 6.10 and Table 6.4 match Figure 6.2 and Table 6.1 with the significant correlations of the same sign. The same estimates of parameter correlations have been found using the two different techniques of uniform grid-search and the moving-block bootstrap technique.

Figures 6.11, 6.12 and 6.13 show the $R^2$ value versus parameter estimates for the different parameters. When comparing the moving-block bootstrap generated Figures 6.11, 6.12 and 6.13 to the uniform grid-search generated Figures 6.5, 6.6 and 6.7 respectively, it is the mean and standard deviation rather than the value that gave the peak $R^2$ that should be compared. Figures for the initial groundwater level estimate $\hat{G}_0$ and the initial soil moisture content estimate $\hat{S}_0$ are not displayed here because in the moving-block bootstrap procedure the initial values are not expected to remain the same for each block because each block starts at a different time.

![Figure 6.11](image-url)

**Figure 6.11** $R^2$ versus estimates of the runoff parameter $\hat{\beta}_Q$ obtained from the moving-block bootstrap procedure for the XGR1 model for the Orroral Valley bore 000601
Lahiri (2003) showed that the variance of a parameter is more accurately estimated as the moving-block bootstrap variance of that parameter multiplied by the block length divided by the total record length. This correction was used to calculate the
standard deviations in Table 6.5. Since the parameter estimates are correlated, the
correlation matrix also needs to be displayed with the mean and standard deviation
of the parameter estimates.

Tables 6.5 and 6.6 give an indication of the uncertainty of parameter estimates. Table
6.5 is an extract of Table 6.4.

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<th>$\hat{\beta}_E$ (mm m⁻¹)</th>
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<tr>
<td>StDev</td>
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<td>2.89</td>
</tr>
</tbody>
</table>

**Table 6.5** Mean and standard deviation of the moving-block bootstrap
optimal parameter sets for the XGR1 model for the Orroral Valley bore 000601

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\hat{\beta}_Q$</th>
<th>$\hat{\beta}_E$</th>
<th>$\hat{\beta}_G$</th>
</tr>
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<tr>
<td>$\hat{\beta}_E$</td>
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<td>-0.783</td>
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<tr>
<td>$\hat{\beta}_G$</td>
<td>0.835</td>
<td>-0.783</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 6.6** Correlation matrix of the moving-block bootstrap optimal
parameter sets for the XGR1 model for the Orroral Valley bore 000601

The parameter uncertainties obtained from the moving-block bootstrap procedure
compare well with the values obtained from the uniform grid-search of the parameter
space. The signs of the significant correlations are the same. However, the moving-
block bootstrap procedure executed much quicker than the uniform grid-search and
was much simpler to initialise, requiring only one starting parameter set simplex.
6.3 Interpretation of Parameter Values

6.3.1 Time Lag
The time lag that gave the best fit for Orroral Valley bore 000601 was 1 month. As introduced in Chapters 3, the monthly groundwater level data used was the mean monthly value. This implies that the groundwater levels represent the mid-month groundwater levels. Both the rainfall and pan evaporation data used are monthly total values which implies that these are end-of-month values. Hence the model time lag of 1 month corresponded to a real time lag of 0.5 months. Since the resolution of the time lag is 1 month, the associated uncertainty is 0.5 months. Therefore, on average it took between 0 and 1 month for the rainfall to affect the groundwater level at the Orroral Valley bore 000601.

6.3.2 Runoff Parameter
The value of the runoff parameter for the Orroral Valley was 1060mm. This was within the range of values obtained by Xiong and Guo (1999) for catchments in China when the XG model was calibrated on streamflow. The runoff parameter $\beta_Q$ had an uncertainty of 300mm.

6.3.3 Evapotranspiration Parameter
The evapotranspiration parameter for the Orroral Valley was 1.13 with an uncertainty of 0.07. This was within the range of values obtained by Xiong and Guo (1999) for catchments in China when the XG model was calibrated on streamflow. This suggests that the actual evapotranspiration could exceed the pan evaporation when the rainfall is much greater than the pan evaporation.

6.3.4 Groundwater Recharge Parameter
The value of the groundwater recharge parameter for the Orroral Valley was 43mm m$^{-1}$ with an uncertainty of 3mm m$^{-1}$. This suggests that a change of 43mm in the soil moisture content changed the groundwater level by 1m after an average of time lag of between 0 and 1 month. This groundwater parameter is proportional to the porosity and inversely proportional to the fraction of water that moves between the unsaturated and saturated soil layers.
6.4 Interpretation of Internal Variables

The XGR1 model estimated monthly mean actual evapotranspiration, soil moisture content and runoff. These values were examined to determine how realistic they were.

6.4.1 Actual Evapotranspiration Values

Average actual evapotranspiration values for the Australian Capital Territory were obtained from Bureau of Meteorology maps (BOM, 2001). Table 6.7 shows that the XGR1 model actual evapotranspiration predictions match the Bureau of Meteorology values most of the year.

| Source | Quantity | Month | | |
|--------|----------|-------|---|---|---|---|---|---|---|---|---|---|---|---|---|
|        |          | 1     | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
| BOM    | $\bar{E}_i$ (mm) | 70.0  | 50.0 | 50.0 | 40.0 | 30.0 | 20.0 | 20.0 | 30.0 | 40.0 | 80.0 | 90.0 | 70.0 |
| XGR1   | $\bar{E}_i$ (mm)  | 70.3  | 53.6 | 52.7 | 39.5 | 32.1 | 23.2 | 24.2 | 37.3 | 50.1 | 59.4 | 64.8 | 56.4 |
| XGR1   | $s(\bar{E}_i)$ (mm) | 6.4  | 6.3  | 5.8  | 3.2  | 2.0  | 1.2  | 1.3  | 2.0  | 3.1  | 3.9  | 4.8  | 6.4  |

Table 6.7 Orroral Valley comparison of mean actual evapotranspiration $\bar{E}_i$ values for each month from the Bureau of Meteorology and the XGR1 model with the standard error $s(\bar{E}_i)$ of the XGR1 estimates of the means

In Table 6.7, the match is poorest in October and November. This may be due to the fact that the Orroral Valley is of higher altitude than the most of the Australian Capital Territory and the Bureau of Meteorology maps are at the very coarse scale of 1:30,000,000.

Figure 6.14 shows the actual evapotranspiration values from the XGR1 model versus time. Rainfall and pan evaporation are also shown as these are used to calculate the actual evapotranspiration.
Figure 6.14 Orroral Valley 000601. a. modelled actual evapotranspiration using the XGR1 model, b. measured rainfall and c. measured pan evaporation.
6.4 Interpretation of Internal Variables

6.4.2 Unsaturated Soil Moisture Content

Table 6.8 shows the unsaturated soil moisture content values predicted by the XGR1 model. These values appear plausible for the Orroral Valley, with maximum water content in August and minimum content in March.

<table>
<thead>
<tr>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>XGR1</td>
<td>$\bar{S}_r$ (mm)</td>
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<td>153</td>
<td>151</td>
<td>156</td>
<td>164</td>
<td>175</td>
<td>187</td>
<td>192</td>
<td>187</td>
<td>185</td>
<td>173</td>
<td>161</td>
</tr>
<tr>
<td>XGR1</td>
<td>$s(\bar{S}_r)$ (mm)</td>
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<td>7.0</td>
<td>7.4</td>
<td>9.8</td>
<td>8.8</td>
<td>7.9</td>
<td>9.9</td>
<td>9.8</td>
<td>8.7</td>
<td>8.4</td>
<td>7.2</td>
<td>6.7</td>
</tr>
</tbody>
</table>

Table 6.8  Orroral Valley mean unsaturated soil moisture content $\bar{S}_r$ values for each month calculated by the XGR1 model with the standard error $s(\bar{S}_r)$ of the estimates of the means

Figure 6.15 plots the soil moisture content values from the XGR1 model versus time. The groundwater level is also shown for comparison.

6.5.3 Runoff

The runoff values predicted by the XGR1 model are shown to be realistic in Section 7.4 where they are used to predict streamflow and compared with the Orroral Valley stream gauge 410736 measured streamflow values.
Figure 6.15 Orroral Valley 000601 a. modelled soil moisture content using the XGR1 model and b. measured groundwater level
6.5 Prediction Uncertainty

The end of Section 5.3 showed that the XGR1 model residuals had constant variance and were normally distributed but autocorrelated in the Orroral Valley. Therefore, the root mean square error could be used for prediction uncertainty. Since the residuals were autocorrelated, it was necessary to use the cross-validation root mean square error for the prediction uncertainty. If these assumptions had not been met then the moving-block bootstrap simulation results would have been used to identify the prediction uncertainty.

The prediction uncertainty is given by the \( RMSE(\text{C.V.}) \) which for the Mean model is 1.20m in the Orroral Valley as indicated in Table 5.10. The prediction uncertainty of the XGR1 model is 0.60m in the Orroral Valley.

The upper 95% confidence limit \( G_{U_t} \) was calculated using:

\[
G_{U_t} = \hat{G}_t + z_{\alpha/2} (RMSE(\text{C.V.})) \\
= \hat{G}_t + z_{0.025} (RMSE(\text{C.V.})) \\
= \hat{G}_t + 1.96(RMSE(\text{C.V.}))
\]  
(6.2)

where \( \hat{G}_t \) is the predicted groundwater level, \( \alpha \) is the significance level which was set to 5%, \( z_{\alpha/2} \) is the value above which a fraction \( \alpha/2 \) of observations lie in the standardised normal distribution and \( RMSE(\text{C.V.}) \) is the root mean square error of cross-validation.

The lower 95% confidence limit \( G_{L_t} \) was calculated from:

\[
G_{L_t} = \hat{G}_t - 1.96(RMSE(\text{C.V.}))
\]  
(6.3)

Figure 6.16 shows the Mean model predictions for the Orroral Valley bore 000601 data and 95% confidence envelope. This contrasts with the Figure 6.17 graph of XGR1 model predictions and 95% confidence envelope.
Figure 6.16 Orroral Valley 000601 observed • and fitted — groundwater level with upper and lower 95% confidence limits of predicted groundwater level using the Mean model.
Figure 6.17 Orroral Valley 000601 observed • and fitted — groundwater level with upper and lower 95% confidence limits of predicted groundwater level using the XGRI model.
This chapter presents practical applications of the XGR1 model that was found to be the best performing model tested. The XGR1 model is shown to be versatile, allowing for: evaluation of recharge patterns; use at different temporal and spatial scales; month-ahead prediction; prediction of streamflow after calibration on groundwater level data; prediction of groundwater levels after calibration on streamflow data; and assessment of the effects of climate change.

### 7.1 Temporal Recharge Patterns

Here two methods of determining recharge patterns are shown. The first method uses the measured groundwater levels alone to determine when recharge events occurred. The second method relies on the XG Recharge 1 model predictions of change in soil moisture content which are calculated by the model from monthly rainfall and pan evaporation data.

#### 7.1.1 Discharge-Only Model

A discharge-only model was proposed in this study to indicate recharge given groundwater level data. Recharge is indicated by positive residuals. The discharge-only exponential decay model is

\[
\frac{dG(t)}{dt} = -k(G(t) - G_E)
\]  

(7.1)

where \(G(t)\) is the groundwater level, \(k\) is the decay parameter and \(G_E\) is a parameter representing the lowest groundwater level when the unconfined aquifer is empty (m).
7.1 Recharge Estimation

Differential Equation 7.1 is solved discretely using the first forward difference approximation to give:

\[ \Delta G_t = -k(G_t - G_E) \]
\[ G_t - G_{t-1} = -k(G_t - G_E) \]
\[ G_t = \frac{1}{1+k} G_{t-1} + \frac{k}{1+k} G_E \]

(7.2)

Hence, the discharge-only exponential decay model can be fitted as the autoregressive model:

\[ G_t = \beta_0 + \beta_1 G_{t-1} + \epsilon_t \]

(7.3)

where \( \beta_0 = \frac{k}{k+1} G_E \) and \( \beta_1 = \frac{1}{k+1} \). The decay parameter is given by \( k = \frac{1}{\beta_1} - 1 \)

and the empty groundwater level parameter is given by \( G_E = \beta_0 \frac{1}{1-\beta_1} \).

The discharge-only model was fitted to the Orroral Valley monthly groundwater levels. The fitted value of \( \beta_0 \) was 95.34m and \( \beta_1 \) was 0.8978, hence \( k \) was 0.1139 and \( G_E \) was 932.58m. The fitted groundwater level when the soil store is empty was greater than the actual groundwater level at its lowest content to compensate for the lack of a recharge term in the model and the fitting process keeping the mean of the residuals at 0.0. Figure 7.1a shows the residuals of the discharge-only model and the recharge events are indicated as positive residuals.

7.1.2 XG Recharge 1 Model

The change in soil moisture content \( \Delta S_i \) (mm) predicted by the XGR1 model indicates recharge when positive and discharge when negative. Figure 7.1b shows the change in soil moisture content calculated by the XGR1 model for the Orroral Valley and is compared with the recharge events found using the discharge-only model in Figure 7.1a.
Figure 7.1  a. Discharge-Only model residuals and b. change in soil moisture content predicted by XGR1 model for the Orroral Valley bore 000601
The correlation between the discharge-only model residuals and the change in the soil moisture content $\Delta S_i$ (mm) was 0.678 indicating that the two techniques of recharge identification are in reasonable agreement about when the major recharge events occurred. Figure 7.1 shows that the extreme recharge events occurred episodically. These extreme recharge events probably occur during large rainfall events when the ground surface is saturated and groundwater is being recharged at the maximum possible rate.

Figure 7.2 shows that the XGR1 model predicted that on average most recharge occurs in late autumn to early spring in the Orroral Valley. This is entirely consistent with the mean pattern of rainfall and pan evaporation for the Orroral Valley in Figure 7.3.

![Figure 7.2](image)

**Figure 7.2** Mean monthly change in unsaturated soil moisture content for the Orroral Valley using the XGR1 model
The dependence of monthly change in soil water store on monthly rainfall is shown in Figure 7.4.

**Figure 7.3** Mean monthly rainfall, pan evaporation and actual evapotranspiration for the Orroral Valley from 1971-1999

**Figure 7.4** Change in unsaturated soil moisture using the XGR1 model versus Rainfall for the Orroral Valley from 1971-1999
Figure 7.4 shows that groundwater discharge is greater than recharge in months with less than 35mm rainfall. For a monthly rainfall of 150mm, the change in soil moisture will be between 0mm and 100mm depending on the amount of evaporation, the amount of soil moisture already present and the temporal pattern of the rainfall within the month.

In summary, two methods of determining the temporal recharge patterns have been examined. The first method used the measured groundwater levels alone to determine when recharge events occurred. The second method used the XG Recharge 1 model predictions of change in soil moisture content which are calculated by the model from monthly rainfall and pan evaporation data. The results of these two methods essentially agreed and showed that the recharge in the Orroral Valley occurs episodically. In addition, the XG Recharge 1 model showed that on average most recharge occurs in late autumn to early spring in the Orroral Valley, as expected.
7.2 Temporal and Spatial Scales

The purpose of this section is to demonstrate that the XGR1 model may be used at different temporal spatial scales. This section is an extract of Jellett et al. (2004). Here a comparison is given between model performance using monthly data, weekly data and temporally interpolated weekly data. Spatially interpolated (Chapter 3.5) rainfall and pan evaporation data and uninterpolated point measurements are also compared for the Orroral Valley bore 000601.

Table 7.1 shows the parameter values and statistics for the XGR1 model at monthly and weekly time scales; uninterpolated (U), spatially interpolated (S) and temporally interpolated [to weekly from monthly uninterpolated data (TU); or to weekly monthly spatially interpolated data (TS)].

The data used in this section were from the Orroral Valley piezometer 000601, and the uninterpolated data come from Orroral Valley rainfall gauge 570980 and pan evaporimeter 570928 from 1972 to 1999. Weekly values temporally interpolated from monthly values were calculated using Bessel functions as shown in the WkData.f90 program module on the Appendix CD (file://D:/SourceCode/ModelsAndDataPrep/Modules/WkData.f90).

The values in Table 7.1 for the monthly spatially interpolated data are different from the values in Table 5.10 because Table 5.10 used data from 1971 to 1999 whereas Table 7.1 used data from 1972 to 1999. This was because the weekly data for the Orroral Valley in 1971 was incomplete.
7.2 Temporal Scale and Spatial Scale

XGR1 Model

<table>
<thead>
<tr>
<th>Time</th>
<th>Interp.</th>
<th>( \hat{c}_0 ) (m)</th>
<th>( \hat{s}_0 ) (mm)</th>
<th>( \hat{\beta}_Q ) (mm)</th>
<th>( \hat{\beta}_E ) (mm m(^{-1}))</th>
<th>( R^2 )</th>
<th>RMSE (m)</th>
<th>RMSE (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mth</td>
<td>U</td>
<td>1 934.13</td>
<td>310.74</td>
<td>2011.4</td>
<td>0.903</td>
<td>55.1</td>
<td>0.73</td>
<td>0.59</td>
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<td>S</td>
<td>1 934.35</td>
<td>220.97</td>
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<td>1.132</td>
<td>47.2</td>
<td>0.79</td>
<td>0.52</td>
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<tr>
<td>Wk</td>
<td>U</td>
<td>2 934.44</td>
<td>491.38</td>
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<td>0.70</td>
<td>0.62</td>
</tr>
<tr>
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<td>9425.2</td>
<td>0.838</td>
<td>67.6</td>
<td>0.73</td>
<td>0.59</td>
</tr>
<tr>
<td>Wk</td>
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<td>2 933.88</td>
<td>275.32</td>
<td>7041.4</td>
<td>1.019</td>
<td>58.5</td>
<td>0.77</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Table 7.1 Calibration parameters and statistics; and cross-validation statistics of the XGR1 model run on monthly and weekly time scales and uninterpolated (U), spatially interpolated (S) and temporally interpolated (from monthly uninterpolated (TU) or monthly spatially interpolated (TS)) data from 1972 to 1999.

The best performing configuration with the smallest \( RMSE(C.V.) \), 0.57m, is the monthly spatially interpolated data. The poorest configuration with the largest \( RMSE(C.V.) \), 0.77m, is the weekly uninterpolated point data. It is possible that the monthly data performed better than the weekly data because not all of the water in the catchment has been transported to the groundwater bore has in a week. Whereas at the monthly scale, the catchment water transport to the bore may have occurred within the month. The spatially interpolated data performed better than the uninterpolated point data most probably because the spatially interpolated data are smoothed and more representative of the catchment region rainfall and pan evaporation.

7.2.1 Parameter Values

In weekly mode, the optimal time lag between rainfall and groundwater response was found to be 2 weeks, as opposed to 1 month in monthly mode. As discussed in Chapter 3 and Section 6.3, the groundwater level data are mid-month values whereas the rainfall and pan evaporation data are end-of-month values. The resolution of the time lag is 0.5 weeks in weekly mode. Hence the model time lag of 2 weeks indicates that on average it took between 1.5 and 2.5 weeks for the rainfall to affect
the groundwater level at the Orroral Valley bore 000601. This is consistent with the range estimated in monthly mode.

The initial groundwater level estimates \( G_0 \) appear to be similar in weekly and monthly mode. This is also the case for the initial soil moisture content estimates \( S_0 \), the pan evaporation parameter estimates \( \hat{P}_E \) and the groundwater recharge parameter estimates \( \hat{P}_G \).

The runoff parameter estimates \( \hat{P}_Q \) are larger in weekly mode than in monthly mode. This is a consequence of the way in which the XGR1 model calculates runoff:

\[
Q_t = (S_{t-1} + P_t - E_t) \tanh \left( \frac{S_{t-1} + P_t - E_t}{\beta_Q} \right)
\]  

(4.24)

Weekly rainfall and evapotranspiration are approximately one quarter of the monthly values. Evapotranspiration is subtracted from the rainfall in the coefficient of the tanh-function when calculating runoff in Equation 4.24. Since weekly soil moisture content is approximately equal to monthly soil moisture content, the quantity \((S_{t-1} + P_t - E_t)\) outside the tanh-function using weekly data is approximately the same as when using monthly data:

\[
S_{t-1} + P_{Wt} - E_{Wt} = S_{t-1} + P_{Mt} - E_{Mt}
\]

where a subscript of \( W \) indicates weekly and \( M \) indicates monthly data. Weekly runoff is approximately one quarter of the monthly runoff,

\[
Q_{Wt} = \frac{1}{4} Q_{Mt}
\]

Therefore \( \beta_Q \) needs to adjust so that the weekly value of the tanh-function will be approximately one quarter of its monthly value.

\[
\tanh \left( \frac{S_{t-1} + P_{Wt} - E_{Wt}}{\beta_{QW}} \right) = \frac{1}{4} \tanh \left( \frac{S_{t-1} + P_{Mt} - E_{Mt}}{\beta_{QM}} \right)
\]

Therefore

\[
\beta_{QW} > \beta_{QM}
\]
and the runoff parameter $\hat{\beta}_2$ values are larger in weekly mode than in monthly mode.

Figure 7.5 shows weekly observed and fitted groundwater levels for the Orroral Valley bore 000601. The XGR1 model can be seen to perform well at monthly and weekly time scales using either point or spatially interpolated rainfall and pan evaporation or temporally interpolated rainfall and pan evaporation. However, the XGR1 model performed best at a monthly time scale when spatially interpolated rainfall and pan evaporation data were used.

The success of using the spatially interpolated monthly climate data justifies its use in Chapter 5. An implication here is that interpolated rainfall and pan evaporation data can be used for estimating unconfined groundwater levels where no local climate data are available.
Figure 7.5  Orroral Valley 000601 observed • and fitted — groundwater level for the XGR1 model using weekly data temporally interpolated from monthly spatially interpolated rainfall and pan evaporation data
7.3 Month-Ahead Prediction

In this section, the ability of the XGRl model to predict future groundwater levels one month in advance is tested. Month-ahead prediction refers to the use of the \( k \)-th month of measured groundwater level as the initial groundwater level \( G_0 \) to predict the groundwater level at the \( (k+L) \)-th month for all months \( k \).

### XGRl in Month-Ahead Prediction Mode

<table>
<thead>
<tr>
<th>BoreID</th>
<th>Calibration</th>
<th>( L )</th>
<th>( \hat{S}_0 )</th>
<th>( \hat{\beta}_0 )</th>
<th>( \hat{\beta}_E )</th>
<th>( \hat{\beta}_G )</th>
<th>( R^2 )</th>
<th>RMSE</th>
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<td>( \hat{\beta}_0 )</td>
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<td></td>
<td>( \hat{\beta}_E )</td>
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<td></td>
<td>( \hat{\beta}_G )</td>
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</tr>
</tbody>
</table>

|         |          |        |        |        |
|         |          |        |        |        |

Table 7.2 Calibration parameter values and statistics of the XGRl model in month-ahead prediction mode
Table 7.2 shows that month-ahead prediction is very accurate with an overall $R^2$ value of 0.88. Figure 7.6 shows the predicted groundwater level for the Orroral Valley using the XGR1 model in month-ahead prediction mode. The parameter values are slightly different from those obtained using the XGR1 model using the predictions for the initial groundwater level but this is to be expected. The usefulness of month-ahead prediction of groundwater levels is in accurate short-term predictions at locations where the groundwater level is measured every month.
Figure 7.6  Orroral Valley 000601 observed • and fitted — groundwater level using the XGR1 model in month ahead prediction mode
7.4 Streamflow and Groundwater Level Prediction

The XG Recharge 1 model predicts runoff as well as groundwater level. From the runoff, streamflow predictions can be calculated. In this section, first the model is tested on both untransformed streamflow data and on square-root streamflow data.

Streamflow $W_t$ (m$^3$s$^{-1}$) was calculated from runoff $Q_t$ (mm) using Equation 7.4

$$W_t = \frac{\beta_A Q_t}{M_{L,j(t)}}$$

(7.4)

where $j(t)$ is the Gregorian month (1 to 12) of time $t$ of the data set, $M_{L,j(t)}$ (s) is the number of seconds in Gregorian month $j(t)$ and $\beta_A$ (m$^3$ mm$^{-1}$) is the streamflow parameter. The streamflow parameter is equal to the catchment area (km$^2$) multiplied by 1000 (to convert area from km$^2$ to m$^2$ and then runoff from mm to m) multiplied by the fraction of water leaving the catchment through the stream as opposed to that leaving the catchment as groundwater flow.

The fraction of water leaving the catchment through the stream required estimation because the Orroral Valley catchment is a small catchment (89.6 km$^2$) and a considerable amount of water leaves the catchment as groundwater discharge. The XG model was designed for larger catchments of area around 1000 km$^2$ where the amount of water leaving the catchment as groundwater was considered negligible.

In the following, the model is firstly used to predict streamflow and then the streamflow residuals are tested for normality, independence and constant variance. The residuals are shown to have increasing variance so the square-root transformation for streamflow was also examined.
7.4.1 Untransformed Streamflow

Table 7.3 shows the results of different configurations of calibration used for the XGR1 model. Calibration configuration \textbf{Str} denotes that all of the initial values and parameters were calibrated on streamflow data except the initial groundwater level \( G_0 \) and the groundwater recharge parameter \( \beta_G \) which can only be calibrated on groundwater level data. Calibration configuration \textbf{GW} denotes that all of the initial values and parameters were calibrated on groundwater level data except the streamflow parameter \( \beta_A \) which can only be calibrated on streamflow data. Calibration configuration \textbf{GW}+\textbf{Str} denotes that all of the initial values and parameter values were calibrated on groundwater level and streamflow simultaneously with the total \( R^2 \) calculated as the mean of the groundwater level \( R^2 \) and the streamflow \( R^2 \):

\[
R^2(\text{Total}) = \frac{1}{2}(R^2(\text{GW}) + R^2(\text{Str}))
\]  

(7.5)

which gives equal weight to the groundwater levels and streamflow data in determining the calibrated parameter estimates. Averaging the \( R^2 \) values allows for the groundwater level record and streamflow record to be different lengths with different variances.

The parameter estimates shown in Table 7.3 have different values depending on the calibration configuration. The \( R^2 \) and bias values in Table 7.4 indicate that the XGR1 model performed well at both groundwater level and streamflow prediction under calibration configurations \textbf{Str} and \textbf{GW}+\textbf{Str} but not as well under configuration \textbf{GW} using untransformed streamflow data.

<table>
<thead>
<tr>
<th>Calibration</th>
<th>( L ) (mths)</th>
<th>( \hat{G}_0 ) (m)</th>
<th>( \hat{S}_0 ) (mm)</th>
<th>( \hat{\beta}_G ) (mm)</th>
<th>( \hat{\beta}_E ) (mm m(^{-1}))</th>
<th>( \hat{\beta}_G ) (mm m(^{-1}))</th>
<th>( \hat{\beta}_A ) (m(^3) mm(^{-1}))</th>
<th>Stream Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textbf{Str}</td>
<td>1</td>
<td>933.75</td>
<td>319</td>
<td>2107</td>
<td>0.614</td>
<td>49.1</td>
<td>33211</td>
<td>0.371</td>
</tr>
<tr>
<td>\textbf{GW}</td>
<td>1</td>
<td>934.20</td>
<td>253</td>
<td>1581</td>
<td>0.997</td>
<td>47.9</td>
<td>49423</td>
<td>0.552</td>
</tr>
<tr>
<td>\textbf{GW}+\textbf{Str}</td>
<td>1</td>
<td>934.05</td>
<td>295</td>
<td>2023</td>
<td>0.890</td>
<td>50.7</td>
<td>44392</td>
<td>0.495</td>
</tr>
</tbody>
</table>

\textbf{Table 7.3} Parameters estimates for the XGR1 model calibrated using different calibration configurations for the Orroral Valley bore 000601 and stream gauge 410736 from 1971 to 1999.
Table 7.4  Statistics of the XGR1 model calibrated using different calibration configurations for the Orroral Valley using bore 000601 and stream gauge 410736 from 1971 to 1999

<table>
<thead>
<tr>
<th>Calibration</th>
<th>Groundwater $R^2$</th>
<th>Groundwater RMSE (m)</th>
<th>Groundwater Bias</th>
<th>Streamflow $R^2$</th>
<th>Streamflow RMSE (m$^3$s$^{-1}$)</th>
<th>Streamflow Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Str</td>
<td>0.72</td>
<td>0.59</td>
<td>0.00</td>
<td>0.78</td>
<td>0.13</td>
<td>0.01</td>
</tr>
<tr>
<td>GW</td>
<td>0.76</td>
<td>0.55</td>
<td>0.00</td>
<td>0.56</td>
<td>0.18</td>
<td>0.12</td>
</tr>
<tr>
<td>GW+Str</td>
<td>0.76</td>
<td>0.55</td>
<td>0.00</td>
<td>0.71</td>
<td>0.15</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Statistical tests of normality, autocorrelation and constant variance were used to test these properties for the streamflow data. Figure 7.7 shows the streamflow residual histogram and the QQ-Plot. The Shapiro-Wilk test for normality gave a p-value of 0.000 for the streamflow residuals. This indicated that the residuals significantly deviated from normality at the 5% level of significance.

Figure 7.7  Orroral Valley a. histogram and b. normal QQ-plot of the streamflow residuals of the XGR1 model
Figure 7.8 shows that the autocorrelation of time lag 1 is 0.345 and this is outside the Durbin-Watson confidence limits. This has a p-value of 0.000 and which indicates autocorrelation significantly different from 0.0 at the 5% level of significance.

**Figure 7.8** Orroral Valley partial autocorrelation of the streamflow residuals of the XGR1 model with 0.0 confidence limits shown

The streamflow residuals are autocorrelated because the next month’s streamflow depends on the current month’s soil moisture content which also depends on the previous month’s soil moisture content. Figure 7.9 indicates that the variance of the residuals is not constant but increases with predicted streamflow.
These tests showed that the streamflow residuals are not normally distributed, not independent and they are not identically distributed.

### 7.4.2 Square Root Streamflow

In an attempt to rectify the problem of no statistical assumptions being met, the square-root transformation was tested on the streamflow. Calibration configuration **RtStr** denotes that all of the initial values and parameters were calibrated on square-root streamflow data except the initial groundwater level $G_0$ and the groundwater recharge parameter $\beta_G$ which can only be calibrated on groundwater level data.

Calibration configuration **GW** denotes that all of the initial values and parameters were calibrated on groundwater level data except the streamflow parameter $\beta_A$ which can only be calibrated on streamflow data. Calibration configuration **GW+RtStr** denotes that all of the initial values and parameter values were calibrated on groundwater level and square-root streamflow simultaneously with the total $R^2$ calculated as the mean of the groundwater level $R^2$ and the streamflow $R^2$:

$$R^2(\text{Total}) = \frac{1}{2}(R^2(\text{GW}) + R^2(\text{RtStr}))$$

(7.6)
which gives equal weight to the groundwater levels and square-root streamflow data in determining the calibrated parameter estimates. The square-root streamflow residual sum of squares was calculated using:

\[ SSE = \sum_{t=1}^{N} (\sqrt{W_t} - \sqrt{\hat{W}_t})^2 \]  

(7.7)

where \( N \) is the length of the record. This was then used to calculate square-root streamflow \( R^2 \) and \( RMSE \) values.

The parameter estimates in Table 7.5 have less variation than those calculated without the square-root transformation in Table 7.3. The \( R^2 \) and bias values in Table 7.6 indicate that the XGR1 model performed well at both groundwater level and streamflow prediction under all of the calibration configurations \( RtStr, GW \) and \( GW+RtStr \) using the square-root streamflow data.

<table>
<thead>
<tr>
<th>Calibration</th>
<th>( L ) (mths)</th>
<th>( \hat{G}_0 ) (m)</th>
<th>( \hat{I}_0 ) (mm)</th>
<th>( \hat{P}_O ) (mm)</th>
<th>( \hat{P}_E ) (mm m(^{-1}))</th>
<th>( \hat{P}_G ) (m(^3) m(^{-1}))</th>
<th>( \hat{P}_A ) (m(^2) mm(^{-1}))</th>
<th>Stream Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>RtStr</td>
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<td>934.43</td>
<td>176</td>
<td>782</td>
<td>0.995</td>
<td>34.1</td>
<td>37930</td>
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<tr>
<td>GW</td>
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<td>934.21</td>
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<td>0.997</td>
<td>47.9</td>
<td>37467</td>
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<tr>
<td>GW+RtStr</td>
<td>1</td>
<td>934.29</td>
<td>238</td>
<td>1408</td>
<td>1.030</td>
<td>46.5</td>
<td>39889</td>
<td>0.445</td>
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**Table 7.5** Parameter estimates for the XGR1 model calibrated using different calibration configurations for the Orroral Valley bore 000601 and stream gauge 410736 from 1971 to 1999

<table>
<thead>
<tr>
<th>Calibration</th>
<th>Groundwater</th>
<th>RtStreamflow</th>
<th>Streamflow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R^2 )</td>
<td>( RMSE ) (m)</td>
<td>Bias</td>
</tr>
<tr>
<td>RtStr</td>
<td>0.73</td>
<td>0.58</td>
<td>0.00</td>
</tr>
<tr>
<td>GW</td>
<td>0.76</td>
<td>0.55</td>
<td>0.00</td>
</tr>
<tr>
<td>GW+RtStr</td>
<td>0.76</td>
<td>0.55</td>
<td>0.00</td>
</tr>
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</table>

**Table 7.6** Statistics of the XGR1 model calibrated using different calibration configurations for the Orroral Valley using bore 000601 and stream gauge 410736 from 1971 to 1999
Figure 7.10 shows the square-root streamflow residuals histogram and normal QQ-Plot. The Shapiro-Wilk test for normality gave a p-value of 0.01 for the residuals. This indicated that the residuals significantly deviated from normality at the 5% level of significance. At the 1% level of significance however, the residuals did not significantly deviate from normality.

![Histogram of Residuals](image1)

**Figure 7.10** Orroral Valley a. histogram and b. normal QQ-plot of the square-root streamflow residuals of the XGR1 model

Figure 7.11 shows that the autocorrelation of time lag 1 is 0.539 and this is outside the Durbin-Watson confidence limits. This has a p-value of 0.000 and is therefore an autocorrelation significantly different from 0.0 at the 5% level of significance.

![Autocorrelation Plot](image2)

**Figure 7.11** Autocorrelation of time lag 1
The coefficient of determination $R^2$ was $(0.552)^2 = 0.305$. Therefore 30.5% of the variability in the current residual was explained by variability in the previous residual.

Figure 7.12 indicates that the variance of the residuals is constant with the predicted square-root streamflow.

These tests indicate that the square-root streamflow residuals are normally distributed, not independent and are identically distributed. This justified the use of the square-root transformation on streamflow. Vandewiele et al. (1992) also used the square-transformation to normalise the streamflow residuals of rainfall-runoff models. An added benefit of the square-root transformation is that the predicted streamflow and uncertainties will always be positive.
Table 7.6 indicates that the bias of the streamflow predictions is 0.05. This means that on average the streamflow is under-predicted by 5%. Figure 7.13 shows the cumulative predicted streamflow volume versus the cumulative measured streamflow volume. This shows that the long term performance of the model is very good.

![Figure 7.13 Orroral Valley plot of cumulative measured streamflow volume versus cumulative predicted streamflow volume of the XGR1 model with all parameters calibrated on groundwater level except the streamflow parameter calibrated on square-root streamflow data from 1971 to 1999](image)

The stream fraction value in Table 7.5 ranges from 0.42 to 0.45. This indicates that between 42% to 45% of the total water leaving the Orroral Valley catchment each month is streamflow while the remainder is groundwater flow. Figure 7.14 gives an indication of the cumulative groundwater flow out of the Orroral Valley versus time.
Streamflow and Groundwater Level Prediction

Figure 7.14 Orroral Valley plot of cumulative groundwater flow out of the catchment versus time predicted by the XGR1 model with all parameters calibrated on groundwater level except the streamflow parameter calibrated on square-root streamflow data from 1971 to 1999.

Since the square-root streamflow residuals were normal and had constant variance, and the autocorrelation was low, the square-root streamflow RMSE was used to indicate square-root streamflow prediction uncertainty in the following confidence interval calculations.

The upper streamflow 95% confidence limit \( W_{Ut} \) was calculated using:

\[
W_{Ut} = \left( \hat{W}_t^{\frac{1}{2}} + z_{\alpha/2} (RMSE(RtStr)) \right)^2
\]

\[
= \left( \hat{W}_t^{\frac{1}{2}} + z_{0.025} (RMSE(RtStr)) \right)^2
\]

\[
= \left( \hat{W}_t^{\frac{1}{2}} + 1.96 (RMSE(RtStr)) \right)^2
\]  

(7.8)

where \( \hat{W}_t^{\frac{1}{2}} \) is the predicted square-root streamflow, \( \alpha \) is the significance level which was set to 5%, \( z_{\alpha/2} \) is the value above which a fraction \( \alpha / 2 \) of observations lie in
the standardised normal distribution and $RMSE(RtStr)$ is the square-root streamflow root mean square error.

The lower streamflow 95% confidence limit $W_{Lt}$ was calculated using:

$$W_{Lt} = \left(\hat{\mu} + 1.96(RMSE(RtStr))\right)^2$$  \hspace{1cm} (7.9)

Figure 7.15 shows the comparison between measured and predicted streamflow and the 95% confidence envelope after calibration on square-root streamflow data for Orroral Valley. Figure 7.16 plots the comparison between the measured and predicted streamflow and the 95% confidence envelope after calibration of groundwater level data. This compares very well with the graph in Figure 7.15. Figure 7.17 displays the comparison between the measured and predicted groundwater level and the 95% confidence envelope after calibration on square-root streamflow data for Orroral Valley.

It is concluded that the XG Recharge 1 model can be calibrated on groundwater level and square-root streamflow simultaneously, or calibrated on groundwater and used to predict streamflow or calibrated on square-root streamflow and used to predict groundwater level. The square-root transformation made the streamflow residuals normal with constant variance, hence confidence intervals could be estimated. The square-root transformation also tightened the parameter estimates. The ability of the XGR1 model to accurately predict groundwater levels after calibration on streamflow data is especially useful for groundwater level prediction because there are many locations where information about groundwater is required but only streamflow measurements are available.
Figure 7.15 Orroral Valley 410736 observed • and fitted — monthly streamflow with upper and lower 95% confidence limits using the XGRI model that was calibrated on square-root streamflow data.
Figure 7.16 Orroral Valley 410736 observed • and predicted — monthly streamflow with upper and lower 95% confidence limits using the XGR1 model that was calibrated on groundwater level data.
Figure 7.17 Orofıal Valley 000601 observed and predicted monthly groundwater level with upper and lower 95% confidence limits using the XGR1 model that was calibrated on square-root streamflow data.
7.5 Effect of Climate Change on Groundwater Levels and Streamflow in the Orroral Valley, ACT

Predictions of the effect of different scenarios of climate change on groundwater level and streamflow were investigated using the XGR1 model. The Intergovernmental Panel on Climate Change (IPCC, 2000) formed a set of Special Reports on Emissions Scenarios (SRES). The following descriptions of each scenario are from the IPCC (2000) document:

“SRES A1 describes a future world of very rapid economic growth, low population growth and rapid introduction of new and more efficient technology. Major underlying themes are economic and cultural convergence and capacity building, with a substantial reduction in regional differences in per capita income. In this world, people pursue personal wealth rather than environmental quality. The A1 scenario family develops into three groups that describe alternative directions of technological change in the energy system. The three A1 groups are distinguished by their technological emphasis: fossil intensive (A1FI), non-fossil energy sources (A1T), or a balance across all sources (A1B).”

“SRES A2 describes a very heterogeneous world. The underlying theme is that of strengthening regional cultural identities, with an emphasis on family values and local traditions, high population growth, and less concern for rapid economic development.”

“SRES B1 describes a convergent world with rapid change in economic structures, “dematerialization” and introduction of clean technologies. The emphasis is on global solutions to environmental and social sustainability, including concerted efforts for rapid technology development, dematerialization of the economy, and improving equity.”

“SRES B2 describes a world in which the emphasis is on local solutions to economic, social, and environmental sustainability. It is again a heterogeneous world with less rapid, and more diverse technological change but a strong emphasis on
7.5 Effect of Climate Change on Groundwater Levels and Streamflow

community initiative and social innovation to find local, rather than global solutions."

Each of these scenarios contained different numerical estimates of world population, energy use, greenhouse gas emissions and aerosol emissions every ten years from 1990 to 2100. Using these estimates, different Global Climate Models (GCMs) have been used to predict changes in atmospheric variables with Low, Mid and High climate change sensitivities. Table 7.7 shows predicted differences between 1990 global temperature and 2030 global temperature.

<table>
<thead>
<tr>
<th>Limit</th>
<th>SRES</th>
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</thead>
<tbody>
<tr>
<td>Low</td>
<td>0.65</td>
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<tr>
<td>Mid</td>
<td>0.85</td>
</tr>
<tr>
<td>High</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Table 7.7 Projected change from base global temperature in 2030 under different scenarios (SRES) of greenhouse gas emissions calculated by the CSIRO Mk2 model

In this research, the CSIRO Mk2 model (Hirst et al., 1996; Hirst et al., 1999) was used in the OzClim software package (CSIRO, 2004) to predict changes in rainfall and pan evaporation. The OzClim software produced monthly spatial grids at 25km resolution of Australia with predicted changes in rainfall and pan evaporation under each scenario with low, mid and high climate change sensitivities. The grids were spatially interpolated using thin-plate smoothing splines and the projected changes for each month in rainfall and pan evaporation for the Orroral Valley were obtained for the 2030 under each of the climate change scenarios.

The predicted changes in rainfall and pan evaporation data were used to generate rainfall and pan evaporation data for the year 2000 to 2030 for each scenario. This was undertaken by adding the 2030 monthly percentage changes to the corresponding months in the 1969 to 1999 data set and then relabelling each year. This procedure assumed that the trend in climate change has been linear and constant since 1969. This preserves dependence in the data including the structure of El Niño
events and other phenomena. The correlation between rainfall and pan evaporation was also preserved. In this research it was assumed that this would produce more reasonable results than if a stochastic weather generator was used. It has been assumed that the catchment properties will not change. That is, the model parameters retained the same values and uncertainties.

Table 7.8 shows the predicted change in rainfall for each month due to climate change. The base against which the change was measured was the 1961-1990 data.

<table>
<thead>
<tr>
<th>SRES</th>
<th>2030 ACT Rainfall Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Month</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Low</td>
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</table>

Table 7.8 Projected change from base rainfall in the Australian Capital Territory in 2030 under different scenarios of greenhouse gas emissions calculated by the CSIRO Mk2 model

Table 7.9 shows predictions that pan evaporation will increase under every scenario. This is in contrast to the finding of Roderick and Farquhar (2004) who found empirically by fitting a linear trend model that pan evaporation has been decreasing in Australia. The explanation given by Roderick and Farquhar (2004) is that this
decrease in pan evaporation is due to global dimming because of an increased number of clouds and humidity caused by rising global temperatures.

<table>
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</table>

Table 7.9  Projected change from base pan evaporation in the Australian Capital Territory in 2030 under different scenarios of greenhouse gas emissions calculated by the CSIRO Mk2 model

The XG Recharge 1 model is unable to take into account the complementary relationship between pan evaporation and actual evapotranspiration (Morton, 1983). This is based on there being less evaporative demand when the environment is wet but when the environment is dry there is more evaporative demand but less water to actually evaporate. This relationship holds for a region, not a point. In order to allow for this and the possibility that the predicted increases in pan evaporation by the CSIRO Mk2 model were incorrect, the model was also run with no change to pan evaporation but with change to the rainfall according to the greenhouse scenarios.
7.5 Effect of Climate Change on Groundwater Levels and Streamflow

7.5.1 Groundwater Level

Tables 7.10 and 7.11 show the predicted changes in groundwater level. The largest predicted change was a decrease in groundwater level of 0.15m under the A1T scenario with high sensitivity to climate change.

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Table 7.10 Predicted change of groundwater level at the Orroral Valley in 2030 calculated using the XGR1 model under different scenarios of greenhouse gas emissions with rainfall and pan evaporation change calculated by the CSIRO Mk2 model.

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Table 7.11 Predicted change of groundwater level at the Orroral Valley in 2030 calculated using the XGR1 model under different scenarios of greenhouse gas emissions with rainfall change calculated by the CSIRO Mk2 model and no pan evaporation change.

Under all of the climate change scenarios in Tables 7.10 and 7.11, the XGR1 model predicts that there will be a decrease in groundwater levels in the Orroral Valley. In order to determine if the projected changes in groundwater level were significant, confidence limits of the predictions of the A1T High scenario were calculated. Under this scenario, the projected mean change in groundwater level corresponded to a drop in groundwater level of 0.15m.
The mean square error $MSE$ of the predicted groundwater level changes is twice the $MSE$ of the XGR1 model groundwater level predictions:

$$MSE(GW \text{ Changes}) = 2MSE(GW) = 2(0.5955 \text{ m})^2$$

Therefore the root mean square error $RMSE$ of the predicted groundwater level changes is

$$RMSE(GW \text{ Changes}) = \sqrt{MSE(GW \text{ Changes})} = \sqrt{2(0.5955 \text{ m})^2} = 0.8422 \text{ m}$$

The upper 95% confidence limit $G(\text{Change})_{U_t}$ was calculated using:

$$G(\text{Change})_{U_t} = \hat{G}(\text{Change})_t + z_{\alpha/2} \cdot (RMSE(GW \text{ Changes}))$$

$$= \hat{G}(\text{Change})_t + z_{0.025} \cdot (RMSE(GW \text{ Changes}))$$

$$= \hat{G}(\text{Change})_t + 1.96 \cdot (RMSE(GW \text{ Changes})) \quad (7.10)$$

where $\hat{G}(\text{Change})_t$ is the predicted change in groundwater level due to climate change, $\alpha$ is the significance level which was set to 5%, $z_{\alpha/2}$ is the value above which a fraction $\alpha/2$ of observations lie in the standardised normal distribution and $RMSE(GW \text{ Changes})$ is the root mean square error of the predicted groundwater level changes.

The lower 95% confidence limit $G(\text{Change})_{L_t}$ was calculated from:

$$G(\text{Change})_{L_t} = \hat{G}(\text{Change})_t - 1.96 \cdot (RMSE(GW \text{ Changes})) \quad (7.11)$$

Figure 7.18 plots the predicted changes in groundwater level with confidence limits. The zero line is within the 95% confidence envelope of the predicted groundwater level change. Therefore, the predicted groundwater level change is not significantly different from zero at the 5% level.
Figure 7.18 Differences between groundwater level predictions under A1T High climate change and no climate change with 95% confidence limits for Orroral Valley bore 000601 using the XGR1 model.
7.5.2 Streamflow

Tables 7.12 and 7.13 show the predicted changes in streamflow for the climate change scenarios. The largest predicted change was a decrease in streamflow of 0.041 m$^3$s$^{-1}$ under the A1T scenario with high sensitivity to climate change. This corresponds to an annual flow decrease of 1.3 x 10$^6$ m$^3$.

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Table 7.12 Predicted change of streamflow in 2030 calculated using the XGR1 model under different scenarios of greenhouse gas emissions with rainfall and pan evaporation change calculated by the CSIRO Mk2 model.

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<th>A1F E0</th>
<th>A1T E0</th>
<th>A2 E0</th>
<th>B1 E0</th>
<th>B2 E0</th>
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<td>-0.021</td>
<td>-0.016</td>
<td>-0.016</td>
<td>-0.019</td>
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</table>

Table 7.13 Predicted change of streamflow in 2030 calculated using the XGR1 model under different scenarios of greenhouse gas emissions with rainfall change calculated by the CSIRO Mk2 model and no pan evaporation change

Under all of the climate change scenarios in Tables 7.12 and 7.13, the XGR1 model predicts that there will be a decrease in streamflow in the Orroral Valley. In order to determine if the projected changes in streamflow were significant, confidence limits of the predictions of the A1T High scenario were calculated. Under this scenario, the projected mean change in streamflow decrease in monthly mean streamflow of 0.041 m$^3$s$^{-1}$. 
In order to determine if the predicted changes in streamflow were significant, confidence limits of the predictions were calculated. The mean square error \( MSE \) of the predicted square-root streamflow changes is twice the \( MSE \) of the XGRL model square-root streamflow predictions:

\[
MSE(\text{RtStr Changes}) = 2MSE(\text{RtStr}) = 2(0.1416 \text{ m}^{3/2} \text{s}^{-1/2})^2
\]

The root mean square error \( RMSE \) of the predicted groundwater level changes is

\[
RMSE(\text{RtStr Changes}) = \sqrt{MSE(\text{RtStr Changes})} = \sqrt{2(0.1416 \text{ m}^{3/2} \text{s}^{-1/2})^2} = 0.2003 \text{ m}^{3/2} \text{s}^{-1/2}
\]

The upper 95% confidence limit \( W(\text{Change})_{U,t} \) was estimated using:

\[
W(\text{Change})_{U,t} = \hat{W}(\text{Change})_{t}^{\frac{1}{2}} + z_{a/2} \cdot (RMSE(\text{RtStr Changes})) = \hat{W}(\text{Change})_{t}^{\frac{1}{2}} + z_{0.025} \cdot (RMSE(\text{RtStr Changes})) = \hat{W}(\text{Change})_{t}^{\frac{1}{2}} + 1.96 \cdot (RMSE(\text{RtStr Changes})) \tag{7.12}
\]

where \( \hat{W}(\text{Change})_{t}^{\frac{1}{2}} \) is the predicted change in square-root streamflow due to climate change, \( \alpha \) is the significance level which was set to 5%, \( z_{a/2} \) is the value above which a fraction \( \alpha / 2 \) of observations lie in the standardised normal distribution and \( RMSE(\text{RtStr Changes}) \) is the root mean square error of the predicted square-root streamflow changes.

The lower 95% confidence limit \( W(\text{Change})_{L,t} \) was calculated from:

\[
W(\text{Change})_{L,t} = \hat{W}(\text{Change})_{t}^{\frac{1}{2}} - 1.96 \cdot (RMSE(\text{RtStr Changes})) \tag{7.13}
\]

Figure 7.19 shows the predicted changes in square-root streamflow with confidence limits. The zero line is within the 95% confidence envelope of the predicted square-
7.5 Effect of Climate Change on Groundwater Levels and Streamflow

root streamflow change. Therefore, the predicted square-root streamflow change and hence predicted streamflow change is not significantly different from 0.0 at the 5% level.
Figure 7.19 Differences between square-root streamflow predictions under A1T High climate change and no climate change with 95% confidence limits for Orroral Valley Streamflow Gauge 410736 using the XGR1 model.
The XGR1 model predicted a decrease in both groundwater level and streamflow when rainfall was decreased and when pan evaporation was increased. However, at the 5% level of significance, the results suggested climate change would not produce a significant change in groundwater level or streamflow in the Orroral Valley by 2030.

These results may also be interpreted as giving an indication of the sensitivity of XGR1 model to changes in rainfall and pan evaporation data. In this case, the model is not overly sensitive to uncertainties in rainfall and pan evaporation. The XGR1 model was also tested for sensitivity to large errors in rainfall in pan evaporation. When the Southwell Park Number 1 bore 000656 had an incorrect location, approximately 100km west of its actual location, and hence incorrect rainfall and pan evaporation values, the $R^2$ value was 0.5. When the location was corrected, the $R^2$ value was 0.8.
CHAPTER 8  CONCLUSIONS

This chapter presents the summary of findings of this research, a summary of the contributions of this work and suggestions for future research.

8.1 Summary of Findings and Contributions

The main aims of this research were to critically examine different approaches to predicting groundwater levels, to develop and test a parameter efficient model for the prediction of groundwater elevations and streamflow given spatially varying climatic input variables and to use the model to investigate possible effects of climate change on groundwater levels in the Australian Capital Territory (ACT). Additional aims of the work were to predict groundwater levels where there were no meteorological stations; to test the models and identify model uncertainties using sound statistical techniques; and to use the model to predict groundwater levels after calibration on streamflow data. The central hypothesis here was that a simple empirical or conceptual model with few parameters is sufficient to accurately predict unconfined groundwater levels and streamflow at the monthly time scale.

8.1.1 Previous Approaches

A range of different techniques for predicting the dynamics of groundwater levels and streamflow were reviewed. These ranged from empirical through conceptual to process-based approaches. While process-based models incorporate the processes involved in groundwater recharge and discharge, the number of physical properties required and their spatial variation often limit their application in the field, particularly when data are sparse. On the other hand, empirical models and some conceptual models offer simple parameterisations but can be obscure when there are changes in processes. Conceptual models provide some simplification in the number of parameters and also provide some insight into how land use changes may be incorporated into the model. An empirical model and two conceptual models were chosen for further investigation.
Most of the hydrological models identified were developed for streamflow prediction rather than groundwater level prediction. This may be because these models were designed for broad spatial scale catchments where the groundwater flow is very slow compared to the streamflow and is therefore considered negligible. Also, streamflow data are generally much more readily available than groundwater level data. The review of process-based hydrological models revealed that even the most detailed models contained empirical relations. Many models have been designed for the same task and with the same input data requirements, but there is no consensus as to which model is the best for a given task (Beven, 2001).

It was found in the review of hydrological models that much of the model testing was statistically inadequate. Statistical assumptions were violated and potentially biased procedures were used such as split-sample validation. The poor model testing procedures used has led to an abundance of over-parameterised models. Use of inadequate data by an over-parameterised model gives rise to many difficulties in parameter estimation and statistical testing (Jakeman and Hornberger, 1993). In this research, statistically sound model testing procedures were used to find the most parameter efficient model.

It appears that no model had previously been tested on streamflow and groundwater level data simultaneously within the same catchment. It was determined that a model that could be calibrated on streamflow data and accurately predict unconfined groundwater level data at the same time would be very useful in places where streamflow data is available but groundwater levels are not monitored.

### 8.1.2 Spatial Interpolation of Model Input Data

Many hydrological models of catchment streamflow rely on single point measurements of climatic variables as model inputs. The rainfall and pan evaporation data nearest to the streamflow gauge or groundwater piezometer are used to represent the whole catchment. In large catchments where orographic effects are important this can lead to relatively large errors. The use of thin-plate smoothing splines, used previously for monthly scale climate studies, is suitable for finding catchment rainfall and pan evaporation particularly in ungauged catchments. This spatial interpolation technique was used in this research. Here we compared model
predictions of groundwater fluctuations based on single point climatic inputs with those based on spatially interpolated climate variables using regional network data and thin-plate smoothing splines as an interpolation tool.

The data used in this research included monthly mean groundwater level records from five to thirty years in length for thirty piezometers in consolidated alluvial soils in the ACT. Monthly mean streamflow data were also used for one gauge in the Orroral Valley, ACT. Monthly rainfall and pan evaporation data for the ACT and south eastern Australia were spatially interpolated using tri-variate thin-plate smoothing splines with longitude, latitude and elevation as predictors. This meant that groundwater levels could be simulated accurately where there were no meteorological weather stations located at the groundwater bore sites.

The spline fitting procedure was found to be robust and allowed for detection of erroneous data that could be removed (Sharples et al., 2005). To allow for the error detection procedure, the data required transformation to be normally distributed. The square-root transformation was found to normalise the monthly rainfall and pan evaporation data whereas the log transformation was unable to normalise the data and changed the direction of the distribution skew. The pan evaporation data were of poorer quality and consisted of fewer stations than the rainfall data. However, both rainfall and pan evaporation surfaces fitted the data well when latitude, longitude and elevation were used as predictors.

In addition to interpolation of the input rainfall and pan evaporation data, the parameters of the model could be spatially interpolated. This would allow prediction of groundwater levels and streamflow at ungauged locations; however, this remains to be tested.

8.1.3 Model Development

In this research, the data available for monthly rainfall, pan evaporation, groundwater elevation and streamflow dictated the selection of empirical and conceptual models. The empirical Hydrograph Analysis: Rainfall and Time Trends (HARTT) model (Ferdowsian et al., 2001) was chosen for modification and testing. This model has been used to predict trend in groundwater levels given rainfall. One
of the proposed modifications was to include the influence of pan evaporation. The threshold based Watbal model (Keig and McAlpine, 1974) was also selected for extension and testing. This model had been designed to predict a soil moisture index and not groundwater levels so two recharge models were proposed to extend the Watbal model to predict groundwater levels. The non-threshold based conceptual XG model (Xiong and Guo, 1999) was also chosen for extension and testing. This model had been designed to predict streamflow so the same two recharge models were proposed to extend the XG model to predict groundwater levels. A third, more detailed recharge model was also incorporated into the XG model for testing. These models were selected because they performed well on the data they were tested on and each of these models was sufficiently different to represent the broad span of available and useable models.

8.1.4 Software

Over 300 pages of Fortran 90 code were developed for this research. The code was written using an object-oriented approach to make upgrade and expansion simple. The Fortran 90 code is very fast and portable, and is able to be compiled on different platforms including supercomputers. The code written allows for the conversion between different file formats and for model execution and testing. Programming the model testing routines in Fortran 90 was necessary because Fortran 90 is many times faster than statistical packages. Even so, some of the numerical simulations in this research took up to 10 hours using the Fortran 90 code.

8.1.5 Comparison of Models

In order to overcome potential bias introduced by split-sample calibration and validation, hv-block cross-validation (Racine, 2000) was used in this work to objectively compare the predictive performance of the different models on the same data. The hv-block cross-validation had not been used previously in hydrological studies. The technique was found to be well suited to hydrological model testing because of its ability to correctly handle models with autocorrelated residuals.

It was found that the model with the best hv-block cross-validation score was the XG Recharge 1 model. This was an extension of the two-parameter XG model (Xiong and Guo, 1999) with one extra parameter for the prediction of groundwater levels.
The $hv$-block cross-validation score indicated that the XG Recharge 1 model offered predictive capabilities significantly better than the other models tested. The power of the $hv$-block cross validation procedure to detect over-parameterisation was demonstrated when it selected this model over the XG Recharge 2 model even though the XG Recharge 1 model is nested within the XG Recharge 2 model.

The additional structure of the XG Recharge 2 model gave poorer performance than the XG Recharge 1 model because of over-parameterisation. The Watbal Recharge 2 model performed well, although not as well than the XG Recharge 1 and 2 models. The Watbal Recharge 2 model gave more realistic estimates of soil porosity than the XG Recharge 2 model. Unlike the XG model, the extra structure of Recharge Model 2 in the Watbal model distinctly improved performance over the Recharge Model 1. Traditional statistical Neyman-Pearson hypothesis testing was not used here to determine this significance. The $hv$-block cross-validation used is just as valid a technique and required less assumptions to be met.

The XG Recharge 3 model used the most detailed exponential equations for recharge but did not perform as well as the XG Recharge 1 model. This may be because the exponential recharge equation developed for daily data is not appropriate for monthly data. Makhlouf and Michel (1994) encountered similar results when testing daily models on monthly data.

The XG Recharge 1 model gave poorer results for groundwater level prediction for months when there were errors in the data. This indicates that the model may allow for detection of possibly erroneous data. The XGR1 model was also shown to perform exceptionally well at month-ahead prediction of groundwater levels.

### 8.1.6 Model Uncertainty

Most of the hydrological literature reviewed did not include estimates of uncertainty in prediction or parameter estimates. A review of methods of prediction and parameter uncertainty estimation was undertaken. Both prediction and parameter uncertainties were estimated in this research, using the most appropriate techniques.
The uncertainties of the XG Recharge 1 model parameter estimates were investigated using both uniform grid-search and the moving-block bootstrap procedure. Significant correlations between parameter estimates justified use of the \( hv \)-block cross-validation procedure and the moving-block bootstrap procedure. The parameter uncertainties obtained from the moving-block bootstrap procedure compared well with the values obtained from a uniform grid-search of the parameter space. The signs of the significant correlations were the same for both procedures. However, the moving-block bootstrap procedure was much quicker to execute than the uniform grid-search and was much simpler to initialise, requiring only one starting parameter set simplex. The moving-block bootstrap procedure was found to be suitable for hydrological testing because it could handle models with autocorrelated residuals.

For the XG Recharge 1 model predictions of groundwater level, 95% confidence envelopes were calculated. This gives an indication of the range of possible groundwater levels in months where the groundwater level data are missing. This can be used to fill in missing values in groundwater level data sets.

### 8.1.7 Temporal Recharge Patterns

Two methods of determining temporal recharge patterns were examined. The first method used the measured groundwater levels alone to determine when recharge events occurred. The second method used the XG Recharge 1 model predictions of change in soil moisture content which are calculated by the model from monthly rainfall and pan evaporation data. These two methods essentially agreed and showed that the recharge in the Orroral Valley occurs episodically. The XG Recharge 1 model showed that on average most recharge occurs in late autumn to early spring in the Orroral Valley, as expected.

### 8.1.8 Temporal and Spatial Scale

The XG Recharge 1 model was tested using different configurations of monthly or weekly data that was spatially interpolated, temporally interpolated and uninterpolated point data. The best performing configuration was the monthly spatially interpolated rainfall and pan evaporation data. The poorest configuration was produced when the weekly, uninterpolated point data were used. A possible
reason why the monthly data performed better than the weekly data is that not all of the catchment water transport to the bore has occurred. Whereas at the monthly scale, the catchment water transport to the bore may have occurred within the month. The spatially interpolated data performed better than the uninterpolated point data most probably because the spatially interpolated data are smoothed and more representative of the catchment rainfall and pan evaporation.

8.1.9 Streamflow and Groundwater Level Prediction
The XG Recharge 1 model was shown to be able to predict streamflow accurately after being calibrated on groundwater level data. In addition, the model was shown to be able to predict groundwater levels accurately after being calibrated on streamflow data. This is a particularly useful result suggesting that the model could be used to estimate recharge in areas where there was streamflow data but insufficient groundwater level data. The square-root transformation of the streamflow data improved the performance and allowed confidence limits to be calculated.

Groundwater in upland catchments of the Murray-Darling Basin is a significant contributor to streamflows. It was found that between 55% and 58% of the water leaving the 89.6km² catchment of the Orroral Valley, ACT, left as groundwater flow.

8.1.10 Possible Effects of Climate Change
The CSIRO Mk2 Global Climate Model (Hirst et al., 1996; Hirst et al., 1999) was used to estimate the effect of projected climate change on rainfall and pan evaporation in the ACT up to 2030AD under different greenhouse gas scenarios. Groundwater levels at a bore in the Orroral Valley, ACT, and also the streamflow values there were predicted up to 2030AD using the XG Recharge 1 model. The calculations suggested that there would not be a significant change, at the 5% level of significance, in Orroral Valley groundwater levels or streamflows by 2030AD due to climate change.

The XG Recharge 1 model is unable to take into account the complementary relationship between pan evaporation and actual evapotranspiration (Morton, 1983). This complementary relationship is based on there being less evaporative demand
when the environment is wet but more water to actually evaporate; while there is
more evaporative demand when the environment is dry but less water to actually
evaporate. This relationship holds for a region, not a point. In order to allow for this
and the possibility that the CSIRO Mk2 predicted increases in pan evaporation were
incorrect, the model was also run with no change to pan evaporation but with change
to the rainfall according to the greenhouse scenarios. These projections gave smaller
changes in groundwater levels and streamflow, still statistically insignificantly
different from no change.
8.2 Future Work

Several promising avenues for future research have become apparent through the course of this research. These include testing the XG Recharge 1 model predictions using other data such as chemical tracer data, and testing the recharge equations using weighing lysimeter data. Further testing of the model on other streamflow and groundwater level data simultaneously is another very useful avenue of future research. The XG Recharge 1 model needs to be further tested in other catchments and other countries.

Further work should be carried out to reparameterise the model so that the parameter estimates are no longer correlated. Further investigation into the possibility of calibrating on monthly data and simulating shorter time-step data would be useful.

A direct statistical test to compare the conceptual XG Recharge 1 model with a process-based model using \( h\)-block cross-validation would be extremely useful. It is possible that given the required data and associated measurement uncertainty, the XG Recharge 1 model may perform better with significantly lower prediction uncertainty than the process-based model.

Investigation of the impacts of land use change on model parameters is required. A possible application of this includes prediction of the effect of the January 2003 Canberra bushfires on groundwater recharge and streamflow, in the Cotter Water Supply Catchment of the ACT, where there are concerns about possible changes in catchment yield.

Predictions of increased dryland salinity throughout the Murray-Darling Basin have been based mostly on two point estimates of rising unconfined groundwater elevations. It would be useful to use the techniques developed here to identify rigorously the trends in groundwater.


References


References


References


Appendix 1 shows the ANUSPLIN log file for the thin-plate smoothing spline fitting to the monthly rainfall data. This output also appears on the Appendix CD (file:///D:/Ch3Data/DataPrep/RainSurfaces/final/rain1991.log.txt).

SPLINB VERSION 4.3 14/02/03
COPYRIGHT AUSTRALIAN NATIONAL UNIVERSITY

TITLE OF FITTED SURFACES (60 CHARs):
1991 SE Oz Rain (lat/long/elev + sqrt)

SURFACE VALUE UNITS CODE AND MISSING DATA VALUE:
0 - UNDEFINED
1 - METRES
2 - FEET
3 - KILOMETRES
4 - MILES
5 - DEGREES
6 - RADIANS
7 - MILLIMETRES
8 - MEGAJOULES
7 -9.000

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NUMBER OF INDEPENDENT COVARIATES (0 TO 7):
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NUMBER OF SURFACE SPLINE VARIABLES (0 TO 7):
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NUMBER OF SURFACE COVARIATES (0 TO 7):
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2 - X*A
3 - A*LOG(X + B)
4 - (X/B)**A
5 - A*EXP(X/B)
6 - A*TANH(X/B)
7 - ANISOTROPY ANGLE
8 - ANISOTROPY FACTOR

REFERENCE UNIT CODES
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4 - MILES
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7 - MILLIMETRES
8 - MEGAJOULES

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2 - SQUARE ROOT
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1 - COMMON SMOOTHING DIRECTIVE FOR ALL SURFACES
2 - DIFFERENT SMOOTHING DIRECTIVE FOR EACH SURFACE
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Appendix 2 shows the ANUSPLIN log file for the thin-plate smoothing spline fitting to the monthly pan evaporation data. This output also appears on the Appendix CD (file:///D:/Ch3Data/DataPrep/EvapSurfaces/fina/evap1991.log.txt).

SPLINE VERSION 4.3 14/02/03
COPYRIGHT AUSTRALIAN NATIONAL UNIVERSITY

TITLE OF FITTED SURFACES (60 CHARs):
1991 SE Oz Pan Evap (lat/long/elev + sqrt)

SURFACE VALUE UNITS CODE AND MISSING DATA VALUE:
0 - UNDEFINED
1 - METRES
2 - FEET
3 - KILOMETRES
4 - MILES
5 - DEGREES
6 - RADIANS
7 - MILLIMETRES
8 - MEGAJOULES
7 -9.000

INDEPENDENT VARIABLES
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NUMBER OF INDEPENDENT SPLINE VARIABLES (0 TO 10):
3
NUMBER OF INDEPENDENT COVARIATES (0 TO 7):
0
NUMBER OF SURFACE SPLINE VARIABLES (0 TO 7):
0
NUMBER OF SURFACE COVARIATES (0 TO 7):
0

TRANSFORMATION CODES
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0 - NO TRANSFORMATION
1 - X/A
2 - X*A
3 - A*LOG(X + B)
4 - (X/B)**A
5 - A*EXP(X/B)
6 - A*TANH(X/B)
7 - ANISOTROPY ANGLE
8 - ANISOTROPY FACTOR

REFERENCE UNIT CODES
-----------------------
0 - UNDEFINED
1 - METRES
2 - FEET
3 - KILOMETRES
4 - MILES
5 - DEGREES
6 - RADIANS
7 - MILLIMETRES
8 - MEGAJOULES

LOWER & UPPER LIMITS, TRANSF CODE, REF UNIT, MARGIN(S) FOR VARIABLE 1:
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0                5

LOWER & UPPER LIMITS, TRANSF CODE, REF UNIT, MARGIN(S) FOR VARIABLE 2:
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0                5

LOWER & UPPER LIMITS, TRANSF CODE, REF UNIT, MARGIN(S) FOR VARIABLE 3:
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1                1

ENTER 1 TRANSFORMATION COEFFICIENT(S):
1000.00

SURFACE DIRECTIVES
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DEPENDENT VARIABLE TRANSFORMATION:
0 - NO TRANSFORMATION
1 - NATURAL LOGARITHM
A2  ANUSPLIN Sample Pan Evaporation Log File

2 - SQUARE ROOT

ORDER OF SPLINE (AT LEAST 2):

2

NUMBER OF SURFACES (AT LEAST 1):

12

NUMBER OF RELATIVE VARIANCES (0, 1 OR 12):

0

OPTIMIZATION DIRECTIVE (NORMALLY 1):

0 - COMMON SMOOTHING PARAMETER FOR ALL SURFACES
1 - COMMON SMOOTHING DIRECTIVE FOR EACH SURFACE
2 - DIFFERENT SMOOTHING DIRECTIVE FOR EACH SURFACE

SMOOTHING DIRECTIVE (NORMALLY 1):

0 - FIXED SMOOTHING PARAMETER FOR EACH SURFACE
1 - MINIMIZE GCCV FOR EACH SURFACE
2 - MINIMIZE TRUE MEAN SQUARE ERROR FOR EACH SURFACE
3 - FIXED SIGNAL FOR EACH SURFACE

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/t32/djellett/mthdata/EcowiseAdded/mthevap/m0112/evap1991.txt

MAXIMUM NUMBER OF DATA POINTS (AT LEAST 4):

2000

NO. OF CHARACTERS IN SITE LABEL (0 TO 20):

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(a11,2f8.3,f6.0,12f7.1)

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/t32/djellett/surfevap/knots/evap1991.not

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OUTPUT BAD DATA FLAG FILE NAME (BLANK IF NOT REQUIRED):

evap1991.flagout.txt

OUTPUT LARGE RESIDUAL FILE NAME (BLANK IF NOT REQUIRED):

OUTPUT OPTIMIZATION PARAMETERS FILE NAME (BLANK IF NOT REQUIRED):

OUTPUT SURFACE COEFFICIENTS FILE NAME (BLANK IF NOT REQUIRED):

evap1991.sur

OUTPUT DATA LIST FILE NAME (BLANK IF NOT REQUIRED):

evap1991.lis

OUTPUT ERROR COVARIANCE FILE NAME (BLANK IF NOT REQUIRED):

VALIDATION DATA FILE NAME (BLANK IF NOT REQUIRED):

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NUMBER OF POINTS WITHIN LIMITS = 88
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PROGRAM SPLINE VERSION 4.3 DATE 24/02/2004 TIME 17.32.28
A3 Range of Soil Moisture Content in the Watbal Recharge 2 Model

Appendix 3 shows the proof that the range of the unsaturated soil moisture content is $0 \leq S_t \leq \beta_c$ in the Watbal Recharge 2 model. In addition it is proved that the water balance, including the saturated soil layer, is maintained if $S_{t-1} + P_t - E_t \geq 0$ but not otherwise.

The equations of the Watbal Recharge 2 model are:

$$M_t = \frac{1 - \exp\left(-\beta_S \min\left(1, \frac{S_{t-1} + \frac{1}{2}P_t}{\beta_C}\right)\right)}{1 - \exp(-\beta_S)}$$

$$E_t = M_t E_{\text{pan}_t}$$

$$Q_t = \max(S_{t-1} + P_t - E_t - \beta_C, 0)$$

$$S_{BRt} = \min(\max(S_{t-1} + P_t - E_t, 0), \beta_C)$$

$$S_t = S_{t-1} + (1 - \beta_F)(S_{BRt} - S_{t-1})$$

$$G_{t+L} = G_{t+L-1} \frac{1}{\beta_G} (S_{BRt} - S_{t-1})$$

A summary of the symbols appears in the last section of this thesis entitled Symbols.

The change in saturated soil moisture content is given by

$$\Delta S_{S_t+L} = \beta_F (S_t - S_{t-1})$$

The soil moisture capacity is a positive number

$$\beta_c \geq 0$$

The fraction of recharge to the unsaturated soil layer that recharges the saturated soil layer is between 0 and 1

$$-0 \leq \beta_F \leq 1$$
The previous time period’s unsaturated soil moisture content is between 0 and the soil moisture capacity

\[ 0 \leq S_{t-1} \leq \beta_C \]

In the case that \( S_{t-1} + P_t - E_t \geq \beta_C \),

\[ Q_t = S_{t-1} + P_t - E_t - \beta_C \]

\[ S_{BRt} = \beta_C \]

Therefore

\[ \Delta S_{S_{t+L}} = \beta_F (\beta_C - S_{t-1}) \]

\[ S_t = S_{t-1} + (1 - \beta_F)(\beta_C - S_{t-1}) \]

When \( \beta_F = 0 \), \( S_t = \beta_C \).

When \( \beta_F = 1 \), \( S_t = S_{t-1} \).

Therefore

\[ 0 \leq S_t \leq \beta_C \]

and

\[ \Delta S_t + \Delta S_{S_{t+L}} = (\beta_C - S_{t-1}) \]

\[ = P_t - E_t - Q_t \]

which is the water balance equation.

In the case that \( 0 \leq S_{t-1} + P_t - E_t \leq \beta_C \),

\[ Q_t = 0 \]

\[ S_{BRt} = S_{t-1} + P_t - E_t \]

Therefore

\[ \Delta S_{S_{t+L}} = \beta_F (P_t - E_t) \]

\[ S_t = S_{t-1} + (1 - \beta_F)(P_t - E_t) \]
When $\beta_F = 0$, $S_t = \beta_C$.

When $\beta_F = 1$, $S_t = S_{t-1}$.

Therefore

$$0 \leq S_t \leq \beta_C$$

and

$$\Delta S_t + \Delta S_{S_t + L} = (P_t - E_t)$$
$$= P_t - E_t - Q_t$$

which is the water balance equation.

In the case that $S_{t-1} + P_t - E_t \leq 0$,

$$Q_t = 0$$
$$S_{BRt} = 0$$

Therefore

$$\Delta S_{S_t + L} = 0$$
$$S_t = S_{t-1}$$

Therefore

$$0 \leq S_t \leq \beta_C$$

and

$$\Delta S_t + \Delta S_{S_t + L} = 0$$

which is not the water balance equation.

Hence the water balance, including the saturated soil layer, is maintained if $S_{t-1} + P_t - E_t \geq 0$ but not otherwise.
Appendix 4 shows the Fitwbm log file for testing of the XGR1 model on the Orroral Valley bore 000601. This output also appears on the Appendix CD as part of file://D:/Ch5ModelTesting/XG/X1GM.Log.txt.

### FITWBM Version 6.0 Copyright (c) 2003-2004 Damian Jellett
Fits a Water Balance Model to Water Balance Data

ID or -9 to Finish Data Entry:
000601

Data Time Scale (1=Monthly, 2=Weekly)

Model (1=XG, 2=Watbal, 3=HARTT, 4=HARTTPlusPanEvap, 5=HARTTPlusAct
6=FreeModel, 7=XGR1, 8=GR2M, 9=XGR3
11=HARTTA, 12=HARTTPlusPanEvap, 13=HARTTPlusAct
14=WatbalR1, 15=WatbalR2, 16=WatbalR3)

Fit Mode (1=Bore, 2=DBore, 3=Streamflow, 4=Bore and DBore
5=Bore and Streamflow, 6=DBore and Streamflow
7=Bore, DBore and Streamflow
8=No Fit)

Spin Up (Number of months at start of data not used in calibration)

<table>
<thead>
<tr>
<th>Data Time Scale</th>
<th>Model</th>
<th>Fit Mode</th>
<th>Spin Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 7 1 0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Starting Point (bore0 swat0 scParam etParam poParam frDown area):

932.500 100.000 500.000 1.000 50.000 0.000 52.000

Starting Point Perturbation (bore0 swat0 scParam etParam poParam frDown area):

(distance from starting point to search around the starting point)

(-9.0 for bore0/swat0 to be estimated directly from Bore Data):

5.000 50.000 500.000 0.100 100.000 0.000 0.000

Perturbation Divisions (bore0 swat0 scParam etParam poParam frDown area):

1 1 4 4 4 1 1

Bore, DBore and Streamflow

StartTimeLag FinishTimeLag:

0 4 1 1 1 1

Whole Period

StartYear StartMonth FinishYear FinishMonth:

1971 1 1999 12

Calibration and Validation Type

1=split-sample validation, calibration period specified as a fraction of the whole time period
2=split-sample validation, calibration period and validation period specified explicitely
3=hv-block cross-validation on whole period
4=moving-block bootstrap
5=IID bootstrap
6=no calibration or validation, load parameter-set file
7=uncertainty of parameters
8=calibrate on DBore then Bore, no validation)
Fraction of whole time period used for calibration

Validation Mode
(1=No Fit, 2=Fit bore0/swat0/swatS0)

Validation Type Fraction Validation Mode:
3 0.800 1

Calibration Period (May Be Blank If Validation Type is not 2)
StartYear StartMonth FinishYear FinishMonth:
1971 1 1988 12

Validation Period (May Be Blank If Validation Type is not 2)
StartYear StartMonth FinishYear FinishMonth:
1989 1 1999 12

Input Rain StnData Filename:
../../../Data/Rain/mthRain000601si.txt
600 elements read

Input Evaporation StnData Filename:
../../../Data/Evap/mthEvap000601si.txt
372 elements read

Input Bore StnData Filename (Blank If Not Required):
../../../Data/Groundwater/mthBore000601mm030.txt
402 elements read

Input Streamflow StnData Filename (Blank If Not Required):

Output Calibration GWSystem Filename (Blank If Not Required):
000601.XLGM.Cal.txt

Output Validation GWSystem Filename (Blank If No: Required):
000601.XLGM.Val.txt

348 elements

ID or -9 to Finish Data Entry:

-000009

Output Calibration Info Filename (Blank If Not Required):
XLGM.Cal.txt

Output Verification Info Filename (Blank If Not Required):
XLGM.Val.txt

Output Bootstrap Info Filename (Blank If Not Required):
XLGM.BS.txt
A5 XG Recharge 1 Model Modifications Tested

After the XG Recharge 1 model was found to be the best performing model overall, the modifications were made in an attempt to improve the model performance and reduce parameter estimate correlations. Table A5.1 shows that none of the model modifications tested performed as well as the unmodified XG Recharge 1 model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Calibration Total</th>
<th>C.V. Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R^2$</td>
<td>RMSE (m)</td>
</tr>
<tr>
<td>XGR1</td>
<td>0.7517</td>
<td>0.7951</td>
</tr>
<tr>
<td>XGR1A</td>
<td>0.7377</td>
<td>0.8171</td>
</tr>
<tr>
<td>XGR1B</td>
<td>0.7481</td>
<td>0.8008</td>
</tr>
<tr>
<td>XGR1C</td>
<td>0.5661</td>
<td>1.0529</td>
</tr>
<tr>
<td>XGR1D</td>
<td>0.4391</td>
<td>1.1983</td>
</tr>
<tr>
<td>XGR1E</td>
<td>0.1409</td>
<td>1.4816</td>
</tr>
</tbody>
</table>

Table A5.1 Calibration statistics; and cross-validation statistics of the XGR1 model and modification models

XG Recharge 1 Modification A Model

\[ E_t = \beta_E E_{pan_t} \tanh\left( P_t \frac{E_{pan_t}}{E_{pan_t}} \right) \]

IF \((S_{t-1} + P_t - E_t) < 0\) THEN

\[ Q_t = 0 \]

\[ S_t = 0 \]

ELSE

\[ Q_t = (S_{t-1} + P_t - E_t) \tanh\left( S_{t-1} + P_t - E_t \frac{1}{\beta_G} \right) \]

\[ S_t = S_{t-1} + P_t - E_t - Q_t \]

END IF

\[ G_{t+L} = G_{t+L-1} + \frac{1}{\beta_G}(P_t - E_t - Q_t) \]
This modification ensures that the soil moisture content $S_t$ cannot be negative but this is at the expense of not maintaining the water balance when $S_{t-1} + P_t - E_t < 0$, similarly to the Watbal model.

**XG Recharge 1 Modification B Model**

$$E_t = \beta_E E_{pan} \tanh\left(\frac{P_t}{E_{pan}}\right)$$

$$Q_t = (S_{t-1}) \tanh\left(\frac{S_{t-1}}{\beta_Q}\right)$$

$$S_t = S_{t-1} + P_t - E_t - Q_t$$

$$G_{t+1} = G_{t+L-1} + \frac{1}{\beta_G} (P_t - E_t - Q_t)$$

This modification takes runoff only from the previous month’s soil moisture content $S_{t-1}$.

**XG Recharge 1 Modification C Model**

$$E_t = \beta_E E_{pan} \tanh\left(\frac{P_t}{E_{pan}}\right)$$

$$Q_t = (P_t - E_t) \tanh\left(\frac{S_{t-1} + P_t - E_t}{\beta_Q}\right)$$

$$S_t = S_{t-1} + P_t - E_t - Q_t$$

$$G_{t+L} = G_{t+L-1} + \frac{1}{\beta_G} (P_t - E_t - Q_t)$$

This modification takes runoff only from the current month’s rainfall $P_t$ minus actual evapotranspiration $E_t$. 
**XG Recharge 1 Modification D Model**

\[ E_i = \beta_E E_{pan_i} \tanh\left( \frac{P_i}{E_{pan_i}} \right) \]

\[ Q_i = \beta_Q (P_i - E_i) \tanh(S_{i-1}) \]

\[ S_i = S_{i-1} + P_i - E_i - Q_i \]

\[ G_{i+L} = G_{i+L-1} + \frac{1}{\beta_G} (P_i - E_i - Q_i) \]

This modification has the runoff parameter \( \beta_Q \) as a multiplier outside of the runoff \( \tanh() \) function.

**XG Recharge 1 Modification E Model**

\[ E_i = \beta_E E_{pan_i} \tanh\left( \frac{P_i}{E_{pan_i}} \right) \]

\[ Q_i = S_{i-1} + (\beta_Q - S_{i-1}) \tanh\left( \frac{P_i - E_i}{P_i + E_i} \right) \]

\[ S_i = S_{i-1} + P_i - E_i - Q_i \]

\[ G_{i+L} = G_{i+L-1} + \frac{1}{\beta_G} (P_i - E_i - Q_i) \]

This modification has the runoff parameter \( \beta_Q \) outside of the runoff \( \tanh() \) function and only the rainfall \( P_i \) and actual evapotranspiration \( E_i \) inside the \( \tanh() \) function.
SYMBOLS

Variables

\( E_t \)  actual evapotranspiration (mm)

\( E_{\text{pan}} \)  pan evaporation (mm)

\( \bar{E}_{\text{pan}} \)  mean annual rainfall divided by 12 (mm)

\( \bar{E}_{\text{pan},j(i)} \)  mean monthly pan evaporation of the \( j(i) \)-th Gregorian month (mm)

\( \varepsilon_t \)  model residual (m)

\( G_t \)  groundwater level (m)

\( M_t \)  soil moisture index

\( M_{L,j(t)} \)  length of Gregorian month \( j(t) \) (s)

\( P_t \)  rainfall (mm)

\( \bar{P}_{j(i)} \)  mean monthly rainfall of the \( j(i) \)-th Gregorian month (mm)

\( \bar{P} \)  mean annual rainfall divided by 12 (mm)

\( Q_t \)  runoff (mm)

\( S_t \)  unsaturated layer soil moisture content (mm)

\( S_{BR,t} \)  unsaturated layer soil moisture content before recharge to the saturated soil layer has occurred (mm)

\( S_{St} \)  saturated layer soil moisture content (mm)

\( t \)  time (mths)

\( W_t \)  Streamflow (m³ s⁻¹)

\( X_{AE,t} \)  accumulated annual residual pan evaporation (mm)

\( X_{AP,t} \)  accumulated annual residual rainfall (mm)

\( X_{ME,t} \)  accumulated monthly residual pan evaporation (mm)

\( X_{MP,t} \)  accumulated monthly residual rainfall (mm)
Parameters

$\beta_A$ streamflow parameter \hspace{1cm} (m$^3$ mm$^{-1}$)

$\beta_C$ soil moisture capacity parameter \hspace{1cm} (mm)

$\beta_E$ evapotranspiration parameter

$\beta_F$ groundwater fraction parameter

$\beta_G$ groundwater recharge parameter \hspace{1cm} (mm m$^{-1}$)

$\beta_P$ soil porosity parameter

$\beta_Q$ runoff parameter \hspace{1cm} (mm)

$\beta_S$ soil type parameter

$\beta_0$ approximate initial groundwater level parameter \hspace{1cm} (m)

$\beta_1$ trend parameter \hspace{1cm} (m mth$^{-1}$)

$\beta_2$ rainfall influence parameter \hspace{1cm} (m mm$^{-1}$)

$\beta_3$ pan evaporation influence parameter \hspace{1cm} (m mm$^{-1}$)

$\beta_4$ first exponential recharge parameter

$\beta_5$ second exponential recharge parameter \hspace{1cm} (mm)

$\beta_6$ discharge parameter \hspace{1cm} (m mm$^{-1}$)

$L$ time lag \hspace{1cm} (mths)

Statistics

$R^2$ coefficient of determination

$MSE$ mean square error \hspace{1cm} (m^2)$

$RMSE$ root mean square error \hspace{1cm} (m)

$SSE$ residual sum of squares \hspace{1cm} (m)

$SST$ total sum of squares \hspace{1cm} (m)