Hedging diversified equity portfolios using futures contracts

by

Katherine May Corrigan, BComm (Hons I) ANU

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A thesis submitted for the degree of Doctor of Philosophy of the Australian National University.
In compliance with the requirements relating to the examination and submission of theses for the degree of Doctor of Philosophy of the Australian National University, I hereby certify that, unless otherwise stated, the work which follows is my own and has not been submitted for a higher degree to any other institution or university.

K. Corrigan

Katherine Corrigan

April 2003
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This thesis examines the hedging of diversified equity portfolios of Australian investors. Four empirical studies are conducted to identify superior risk management strategies for hedging domestic and foreign equity risk using futures contracts written on the Australian All Ordinaries Share Price Index, the S&P 500, the Nikkei 225, the FTSE 100, and the Brazilian Bovespa Index. The application of superior hedging strategies should improve portfolio performance relative to unprotected portfolios which are subject to full risk, and relative to portfolios hedged using less effective risk management techniques. Hedge performance is measured out-of-sample using expected utility. Extensive sensitivity analysis is conducted with regard to equity portfolio construction, hedge ratio estimation method, hedge effectiveness measure, choice of hedging instrument, and time period.

The first study examines the cross-hedging of the foreign component of internationally diversified equity portfolios using single futures contracts, to determine whether any futures contract or hedge strategy achieves superior hedge effectiveness. The FTSE 100 and Nikkei 225 futures contracts generally provide the best hedges of the foreign equity portfolios, while S&P 500 futures do not generally provide good hedges. Comparing simple and complex hedge ratio estimation techniques, time-varying GARCH(1,1) ratios generally provide the highest expected utility relative to the other methods examined.

The second study examines the cross-hedging of Australian and foreign equity components of diversified portfolios using single futures contracts. A key issue is whether there is any benefit in accounting for “portfolio effects” by hedging the entire equity portfolio as a single spot series, or whether separate hedges of the Australian and foreign equity components are equally effective. Portfolio effects are the complex interactions between different components of multi-asset portfolios. For hedges of the entire equity portfolio using a single futures contract, Australian futures hedges provide the highest expected utility in periods of relatively low volatility of returns on the unhedged equity portfolio, but foreign futures hedges are superior in periods of high volatility. In the context of hedging equity risk using Australian futures contracts, accounting for portfolio effects generally improves hedge performance.

The third study extends the analysis of the impact of “portfolio effects” on hedge effectiveness to the case where Australian and foreign equity are hedged using pairs of futures contracts simultaneously. The findings indicate that hedge performance is improved through the selection of methods of hedge ratio estimation that incorporate portfolio effects, and in particular the
trivariate GARCH(1,1) model. However, that model has considerably greater computational complexity than the other models examined.

The final study draws together earlier findings and examines whether single or multiple futures contracts result in superior hedges, the impact of increasing the number of futures contracts on hedge ratio estimation model fit and hedge performance, and the performance of different combinations of futures contracts when hedging both Australian and foreign equity risk simultaneously. Hedges using small numbers of futures contracts are always preferred to hedges using larger numbers of futures contracts, and in many periods, hedges using single contracts are preferred over those employing multiple contracts. Managers may hedge effectively using one or a small number of futures contracts, and do not have to engage in more expensive and time-consuming management of a large number of contracts. In periods of relatively high volatility when hedging is most valuable, single foreign futures contracts generally provide better hedges than multiple futures contracts.

In summary, this thesis provides evidence on ways that managers of diversified equity portfolios may improve their risk management strategies. The use of superior hedging techniques and the resulting minimisation of portfolio losses may benefit investors through the provision of a more certain investment, which is important if the aim of their contribution is to support themselves financially during retirement, for instance.
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CHAPTER ONE

INTRODUCTION

1.1 Introduction

This thesis examines risk management strategies relevant to managers of diversified equity portfolios, and in particular the use of futures contracts to hedge both single and multiple risk exposures associated with those portfolios. In this chapter, the motivations and objectives of this research are discussed, the contribution of this work is highlighted, and an overview of the thesis structure is provided.

1.2 Research context

Australian investors benefit from holding diversified portfolios, as discussed by Watson and Dickinson (1981) and Izan et al (1991). Regardless of the style of management, the value of any investment portfolio is subject to risk. Equity is an important asset class, and is generally considered to be more risky than other investment classes. As such, this thesis focuses on the management of equity risk.

A basic method of managing risk is diversification, which involves the dispersion of capital to a variety of assets. Successful diversification requires investment in assets which have low correlation with other assets in the portfolio, where correlation measures how the returns on assets move together over time. However, simple international diversification strategies become less effective as the correlation between individual countries and regions increases due to globalisation. Further, as observed by Butler and Joaquin (2000) and Malevergne and Sornette (2002), correlations between equity markets are higher during bear markets, reducing the benefits of diversification when it is most advantageous.

Complex models of dynamic asset allocation aimed at decreasing downside risk have been proposed by Brennan et al (1997), Boyle and Yang (1997), Zhao and Ziemba (2000), Bielecki et al (2000), Stevenson (2001), and Zhao and Ziemba (2001). However, such models are highly computer intensive and difficult to implement in practice. As Collins and Fabozzi (1999, p 19)

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1 For instance, when the holdings of Australian managed funds are considered in aggregate, the value of equity exceeds the value of other assets.
argue, "diversification alone as a means to manage risk is not adequate in today's world of change characterized by rapidly advancing technology, deregulation, financial product proliferation, and volatile financial markets. New tools are required, and derivative instruments meet the requirement".

Derivative instruments, such as futures contracts, are financial securities whose value depends on the value of another asset (Hull (2000)). Derivative contracts may be employed to limit the potential loss associated with adverse movements in investment markets. Futures contracts bind two parties to trade a specific asset at a set date in the future, at a price determined at the formation of the contract. This thesis examines the use of futures contracts to hedge and cross-hedge the internationally diversified equity portfolios in an Australian context. Although the research is conducted using the portfolios of Australian managed funds, the results are relevant to all investors. The particular futures contracts studied are the Australian All Ordinaries Share Price Index futures, US S&P 500 futures, Japanese Nikkei 225 (SIMEX) futures, UK FTSE 100 futures, and Brazilian Bovespa index futures.

To hedge risk using futures contracts, it is necessary to determine an appropriate hedge ratio, which indicates the size of the futures position required to insure the underlying portfolio. Direct hedging is the use of a futures contract written on an underlying asset to hedge risk associated with that particular asset. For example, the equity risk associated with the Australian equity market, as represented by the Australian All Ordinaries Index, may be hedged directly using the Australian All Ordinaries Share Price Index Futures contract. In contrast, cross-hedging is the use of a futures contract written on one asset to reduce the risk associated with a different asset. Cross-hedging may be undertaken when there is no derivative written on the asset requiring protection, or when the market for the derivative written on the asset is illiquid or poorly developed such as in emerging markets. Many studies use a contract on a different asset to hedge risk, such as Benet (1990), Mun and Morgan (1997) and Aggarwal and DeMaskey (1997). The advantage of cross-hedging lies in the expansion of potential hedging instruments. Braga et al (1989, p 88) point out that by cross-hedging, a manager may choose a hedging instrument on the basis of transaction costs and market liquidity. This thesis identifies superior risk management strategies involving the hedging and cross-hedging of domestic and foreign equity risk using futures contracts. The application of superior strategies should improve the performance of a fund portfolio relative to unprotected portfolios which are subject to full risk, and relative to portfolios protected using less effective risk management techniques.
1.3 Research motivation

This thesis presents empirical evidence on futures hedging strategies that limit the risk of diversified equity portfolios. This is of practical importance for several reasons.

First, the context of this thesis is the risk management of equity portfolios, whereas prior research on futures hedging is conducted in other contexts, such as the risk management of multinational corporations exposed to foreign exchange risk, or farmers producing agricultural commodities. Consequently, the risks considered in this thesis differ from risks considered in the literature on cross-hedging and the simultaneous hedging of multiple risks. Analysis in this thesis is conducted from an Australian perspective using data on Australian investment portfolios, unlike prior research which is conducted from a foreign perspective using foreign data. Although this thesis employs data on managed fund portfolios, the results may apply to a variety of entities including banks, multinationals and private investors exposed to equity risks.

Second, the application of a superior risk management strategy should improve hedging performance relative to that of less effective hedging strategies. Risk management is an important issue facing portfolio managers and investors generally, and derivatives may be effective risk management tools. For instance, risk reduction is a prominent reason for the use of derivatives by Australian managed funds, according to a survey by Harris and Rosser (1993), and risk-averse investors who contribute to a fund using superior risk-management techniques have a relatively more certain investment, which is important if the aim of their contribution is to sustain themselves financially during retirement. Hedging may be of particular relevance to superannuation funds with defined benefits and fixed pension commitments. Generally, the relative performance of different hedging strategies is of interest to any investor or manager concerned with risk management using derivatives. A range of equity portfolios are analysed in this thesis, to accommodate diverse investment styles. Further, realistic portfolios containing multiple assets are used, in contrast to the majority of studies, which assume that a broad market index is a suitable proxy for a portfolio.\(^2\)

Third, a manager applying a superior risk management technique may improve their professional reputation by minimising portfolio losses, and receive greater remuneration commensurate with better relative performance of their portfolio. Arguably, all managers implicitly aim to limit the risk of loss, regardless of the stated management style. However, this does not necessarily translate into an argument that managers use derivatives to enhance their

\(^2\) An exception is Butterworth and Holmes (2001), who use portfolios of U.K. investment trust companies in the context of hedging using FTSE-100 futures and FTSE-Mid250 futures.
own compensation irrespective of the true needs of investors, as has been suggested in some literature, because risk management may also be achieved through dynamic asset allocation without the use of derivatives. A serious disadvantage of dynamic asset allocation strategies is their complexity and computational difficulties, and derivative contracts provide an alternative.

While all portfolio managers are likely to implicitly consider downside risk, investment style influences management decisions. Although hedging using derivatives is unlikely to be the most appropriate management strategy in all scenarios or for all portfolios, this thesis presents evidence to aid managers who choose to limit risk at some time. Further, the results are equally applicable to managers who fully hedge their portfolio and those who hedge only a portion of their portfolio. In short, the results are of interest to all managers considering equity hedges using futures contracts.

Another contribution of this thesis is the comparison of complex and simple techniques for hedge ratio estimation, and specifically the relative merits of methods that account for “portfolio effects” or complex interactions between the components of multi-asset portfolios. This is an issue of critical importance, for if the hedge ratio is inaccurate then the risk is either over-hedged or under-hedged, and may result in loss. Complex hedges are more difficult for managers to estimate and explain to investors and contributors, and evidence is mixed as to whether complex hedges produce superior hedge effectiveness in the context of agricultural commodities and currency risk. In short, there is no consensus in the literature on this issue. Further, this thesis applies out-of-sample performance evaluation, in contrast to prior research which relies on in-sample analysis which is founded on the unrealistic assumption of perfect foresight by the practitioner.

1.4 Contribution

This thesis examines the hedging of diversified equity portfolios using futures contracts. Prior literature on hedging using futures contracts is generally limited to comparisons of hedge ratio estimation techniques in the context of simple hedges involving single assets and single futures contracts. However, investors, funds and firms hold diversified portfolios that are subject to multiple risks. Further, a variety of futures contracts may be used as hedging tools, and multiple derivative contracts may be used simultaneously. This thesis extends the literature through four related empirical studies that investigate increasingly complex aspects of hedging risk using futures contracts.
The first study examines the cross-hedging of the foreign equity component of diversified equity portfolios using single futures contracts. The choice of futures contract is investigated to determine which, if any, single hedging instrument is superior, and whether Australian futures contracts are effective risk management tools against foreign equity risk. The use of a single contract is appealing because transaction costs are lower than if a portfolio of futures is used, and relatively less information and less management time are required. Extensive sensitivity analysis is conducted with respect to portfolio composition and construction, hedge ratio estimation technique, time period, and hedge effectiveness measurement. Of the five futures contracts, the FTSE 100 and Nikkei 225 futures contracts generally provide the best hedges of the foreign equity portfolios, while S&P 500 futures do not generally provide good hedges. Comparing simple and complex hedge ratio estimation techniques, time-varying GARCH(1,1) ratios generally provide the highest expected utility relative to the other methods examined. The results apply not only to investors considering a foreign equity portfolio, but may also be viewed as a study of partial hedging, where only the foreign portion of the portfolio is hedged and upside potential may be realised in the unhedged domestic component.

The second study examines the hedging of both the Australian and foreign components of diversified equity portfolios simultaneously using single futures contracts, a strategy that attracts lower costs than using multiple futures contracts. For hedges of the entire equity portfolio using a single futures contract, Australian futures hedges are found to provide the highest expected utility in periods of relatively low volatility of returns on the unhedged equity portfolio, but foreign futures hedges are superior in periods of high volatility. This is of importance because hedging is of most benefit given high volatility, as discussed in Chapter Two. Additional evidence is provided on the impact of “portfolio effects” on hedge performance, where portfolio effects refer to the complex interactions between components of multi-asset portfolios. A key issue is whether there is any benefit in accounting for portfolio effects through the hedge ratio estimation method or whether separate hedges of the Australian and foreign equity components provide an equally effective strategy. In the context of hedging equity risk using AFOI futures contracts, accounting for portfolio effects is found to generally improve hedge performance. Hence, managers may improve hedge effectiveness through the selection of hedge ratio estimation methods that account for portfolio effects.

The third study examines the hedging of equity risk given diversified portfolios containing Australian and foreign equity, using pairs of futures contracts. This is the most simple case of the use of multiple futures contracts to hedge multiple risks simultaneously. The investigation of portfolio effects in the second study is extended to the case of multiple futures contracts, and simple OLS regression models and complex hedge ratio estimation methods such as trivariate
GARCH(1,1) models are compared. The findings indicate that hedge performance is improved through the selection of methods of hedge ratio estimation that incorporate portfolio effects, and in particular the trivariate GARCH(1,1) model. However, that model has considerably greater computational complexity than the other models examined. This study can be distinguished from the wealth of prior empirical research comparing traditional hedge ratio models, in that the latter ignore portfolio effects, are generally limited to foreign (non-Australian) data, and are not conducted in the context of diversified fund portfolios. Further, this study can be differentiated from the work of researchers such as Gagnon et al (1998) on portfolio effects, given differences in data and the focus on different risks, and because this thesis employs out-of-sample measures of performance whereas prior work relies on in-sample analysis.

The fourth study investigates the use of single and multiple futures contracts to hedge the equity risk of diversified equity portfolios. It examines whether single or multiple futures contracts result in superior hedges, the impact of increasing the number of futures contracts on hedge ratio estimation model fit and hedge performance, and the performance of different combinations of futures contracts when hedging both Australian and foreign equity risk simultaneously. Using multiple futures contracts may improve the accuracy of the hedge, although hedges using small numbers of contracts are likely to attract lower transaction costs. Hedges using small numbers of futures contracts are always preferred to hedges using larger numbers of futures contracts, and in many periods, hedges using single contracts are preferred over those employing multiple contracts. Managers may hedge effectively using one or a small number of futures contracts, and do not have to engage in more expensive and time-consuming management of a large number of contracts. In periods of relatively high volatility when hedging is most valuable, single foreign futures contracts generally provide better hedges than multiple futures contracts.

In practical terms, the results in this thesis indicate ways in which managers of diversified equity portfolios may improve hedge performance. In the context of managed funds, minimising portfolio losses through superior hedging techniques may benefit both managers, whose reputation and remuneration depend on portfolio performance, and contributing investors who require a stable investment. More generally, the results are of interest to anyone wishing to hedge diversified equity portfolios using futures contracts.

1.5 Thesis structure

Relevant literature on Australian managed funds, risk exposures, and risk management is discussed in Chapter Two, to provide a context for this research. Chapter Three provides a more technical discussion of the literature on hedging using futures contracts. An overview of the
research questions and method is presented in Chapter Four. Chapter Five describes the data used, including analysis of characteristics which are drawn on in later chapters. The results of the empirical analysis are presented in Chapters Six to Nine. These chapters examine the hedging of foreign equity risk using single futures contracts, the hedging of Australian and foreign equity risk simultaneously using single futures contracts, the hedging of Australian and foreign equity risk simultaneously using pairs of futures contracts, and the hedging of Australian and foreign equity risk using more than two futures contracts, respectively. Finally, in Chapter Ten, conclusions are drawn, with discussion of implications and suggestions for further research.
CHAPTER TWO

INVESTMENT PORTFOLIOS AND RISK MANAGEMENT

2.1 Introduction

In this chapter, the investment strategies employed to construct diversified portfolios are described, and the choice of managed fund portfolios as the specific context for the research in this thesis is explained. The risks associated with investment portfolios are identified, and risk management strategies are discussed.

2.2 Investment portfolios

The investment decision common to all investors can be disaggregated into asset allocation and stock selection, which occur in the context of investment style. Asset allocation involves selecting an appropriate proportion of investment in broad asset classes such as stocks, bonds and cash. Stock selection is the process of choosing individual investments within a particular asset class. The management of the resulting portfolio may be active or passive.

2.2.1 Investment style

Investment style influences the construction and management of portfolios. Investment style may be indicated by organisational structure, such as the classification of Australian managed funds by the Reserve Bank of Australia as superannuation and approved deposit funds, cash management trusts, statutory funds of life offices, common funds, and public unit trusts. Alternatively, style may be described in terms of management characteristics, such as the identification of U.S. funds by Brown and Goetzmann (1997, p 381) as income, growth, growth-and-income, value, global timing, glamour, international, and metal funds. A simple and intuitively appealing method of identifying investment style is to consider the level of risk. For example, Viney (2000, pp 89-90) categorises funds from the most to the least risk-averse as capital guaranteed, capital stable, balanced growth, and managed growth. A similar categorisation is applied empirically by the data provider William M. Mercer Investment Consulting. Regardless of the particular style attributed to individual investors, common elements exist in the portfolio selection and management techniques of all investors. Specifically, asset allocation and stock selection determine the composition of all portfolios.
2.2.2 Asset allocation

Asset allocation is the apportionment of funds to different asset classes, including domestic equity, domestic fixed interest, global equity, global fixed interest, property, cash and precious metals. For instance, Figure 2.1 indicates the division of capital between various asset classes for Australian managed funds in the quarter ending in March 2002. The largest proportion of the capital of Australian managed funds is invested in equities and unit trusts. Australian equities and units in trust were valued at $A 239,663 million, which is 36.4% of all assets held by managed funds, while fixed interest securities account for 18.9% of total assets, and overseas investments including equity for 19.6%.

Figure 2.1: Investment of Australian managed funds in different asset classes, for the March quarter 2002

Constructed using data from the Australian Bureau of Statistics, Managed Funds, Document Number "5655.0". "Managed funds" include superannuation funds, life insurance offices, public unit trusts, friendly societies, common funds and cash management trusts. "Short term securities" and "long term securities" refer to fixed interest investments. "Placements" are defined as account balances with entities such as State government central borrowing authorities, which are not considered to be deposit-taking institutions. "Assets overseas" comprise both physical and financial assets, and are reported at market value.

3 These figures are calculated from data obtained from the Australian Bureau of Statistics, Managed Funds, Document Number "5655.0", March Quarter 2002. "Fixed interest" securities include the categories of short and long term fixed interest.
The specific choice of asset allocations may be explained in terms the Markowitz (1959) mean-variance theory of portfolio selection. Markowitz (1959) attributes investor behaviour to the motivating effects of risk and expected return. Under mean-variance theory, all investors share the objectives of maximising expected returns for a given level of risk, or alternatively minimising risk for a given expected return. Alternative theoretical frameworks used to explain asset allocation decisions include the scenario-based approaches of Koskosidis and Duarte (1997) and Grinold (1999), downside risk frameworks discussed by Harlow (1991), Jansen et al (2000), Stevenson (2001), and “behavioural portfolio theory” advanced by Shefrin and Statman (2000) and Fisher and Statman (1997).

The asset allocation decision may involve market timing, which is defined by Beebower and Varikooty (1991, p 79) as “moving into or out of broad asset classes so as to be invested in them during periods of high returns and out of them during periods of low returns”. Researchers including Merton (1981), Henriksson and Merton (1981), and Treynor and Mazuy (1966) have contributed to the extensive body of literature on the market timing ability of portfolio managers. Empirical tests conducted in the context of Australian fund managers do not generally support the existence of superior market timing ability. Specifically, Sinclair (1990) detects perverse market timing ability, but attributes this result to the stock market crash of 1987. Using a different sample, Hallahan and Faff (1999) find little evidence of market timing ability of Australian equity trust managers.

In addition to traditional risk-return considerations, asset allocations may be influenced by managerial incentives. For instance, Brown et al (1996) find that the managers of funds that perform poorly in the middle of the year tend to increase fund volatility in the latter part of the annual assessment period to a greater extent than “mid-year winners”. This effect has become more pronounced as industry growth and investor awareness of fund performance increases over time. Brown et al (1996) observe that current incentive structures encourage managers to adopt a short term rather than a long term focus. Similarly, Gendron and Genest (1990) investigate the impact of constraints on the proportion of assets invested in the market on managers’ performance. They emphasise the importance of an awareness of the practical constraints facing fund managers.

2.2.3 **Stock selection**

Subsequent to the asset allocation decision, stock selection is undertaken. Stock selection is the process by which individual assets are chosen from within each asset class for inclusion in the investment portfolio. According to Breen et al (1986, p 585), stock selection is the ability to form desirable return distributions using superior information regarding individual stocks. In an
Australian context, Sinclair (1990) finds that funds have positive security selection performance. Other empirical work by researchers including Grinblatt et al (1995) and Falkenstein (1996) investigates behavioural patterns among investors, such as momentum investing, which occurs when stocks are selected on the basis of past returns, and herding, which occurs when investors trade stock simultaneously.

However, Ellis (2000) suggests that arriving at appropriate asset allocations is a more reliable method of achieving good returns than attempting superior stock selection relative to the market. This is supported by the evidence of Blake et al (1999), who demonstrate that in the context of UK pension funds, the variations in portfolio returns are largely attributable to strategic asset allocation, while stock selection is "far less important" (p 429). Similarly, Brinson and Fachler (1986) show that the proportion of variability in portfolio returns that is attributable to the asset allocation decision exceeds ninety per cent.

2.2.4 Active and passive management

The frequency with which an investment portfolio is reassessed and re-balanced in accordance with long-term strategy determines whether the manager engages in active or passive management. Passive management entails the construction of a portfolio, which is not re-balanced until the end of the investment period. A "buy and hold" strategy typifies this approach. The main justification for passive management is that transaction costs outweigh the benefits from more frequent re-balancing of the portfolio. In contrast, active management involves frequent assessment of the portfolio's performance within the investment period, and re-balancing as required. For instance, Thomas (2000, p 25) points out that an active equity manager who is bullish on stock prices manipulates their portfolio to obtain a beta larger than the market beta, while a bearish manager re-balances to obtain a beta smaller than the market beta.

Ambachtsheer (1994, p 90) estimates that approximately 60% of stock and bond portfolios are actively managed. However, researchers such as Gruber (1996) have expressed surprise that active management is so common, given that actively managed funds generally under-perform index funds. Index funds aim to replicate the return on a market index. The rise of index funds has been discussed at length by researchers including Sorensen et al (1998), Grossman (1995), Miller and Meckel (1999), and Malkiel and Radisich (2001). Although index funds are generally categorised as passive investments, tracking an index closely involves considerable re-balancing and transaction costs because equity indices change continuously through time. Frino and Gallagher (2002) examine the performance of Australian equity index funds, and find a "significant tracking error" which is attributed to a variety of factors including market volatility,
transaction costs, fund cash flows and replication strategies. The distinction between an actively managed fund and an index fund is not as clear as discussions in some literature suggest.

In summary, managers construct investment portfolios through the processes of asset allocation and stock selection, in accordance with their investment style, and the resulting portfolio may be actively or passively managed depending on the frequency of re-balancing.

2.2.5 Importance of managed funds in Australia
Managed funds are institutions that receive financial contributions from individuals and make investments on their behalf. While the results in this thesis are of interest to any investor wishing to hedge diversified portfolios using equity derivatives, the analysis utilises managed fund portfolios in particular. Managed funds are of increasing social and economic importance in Australia, and provide readily identifiable portfolios for analysis.

The commercial importance of Australian managed funds is indicated by the growth in and dollar value of the assets managed by such funds. Figure 2.2 shows that the total value of assets under management has risen every year since 1988, and totaled A$757,186 million in June 2002. The growth in the dollar value of assets under the management of funds is evident not only in Australia, but in other developed countries. For instance, Fisher and Statman (1997, p 10) describe the growth of U.S. mutual funds since the early 1980s as “phenomenal”, and observe that “Americans have poured $235 billion into mutual funds in 1996, up from $22 billion in 1990.” Similarly, in a global context, Walter (1999, p 1) asserts that “[f]unds under institutional management are massive and growing rapidly, particularly as part of the resolution of pension pressures”, and that in 1997 the global total of assets under management was estimated at $30 trillion.

As the commercial importance of managed funds grows, the social impact of the activities of fund managers simultaneously increases. For example, the performance of superannuation funds impacts on the retirement incomes of aging populations in Australian and other developed countries and, in the words of Ambachtsheer et al (1998, p 15), “Baby Boomers around the world hope to enjoy their golden retirement years”. Guest and McDonald (2000) project that the baby boomer generation will retire between 2011 and 2031, and King et al (2001) observe that while the current Australian population is younger than most developed countries, it is expected to age rapidly, with a particular increase in the proportion of people living for 85 years or more. According to Perpetual Investments, it will become increasingly difficult for Australian taxpayers to support the Aged Pension through taxation alone, and the value of superannuation will exceed that of the family home for many Australians. Thus, the activities and performance
of managed funds providing retirement incomes is not merely of economic concern, but also of social significance.

The importance of the activities of fund managers generally is further indicated by the regulation of the fund management industry by the Commonwealth Government of Australia. The recent introduction of the *Managed Investments Act 1998* (Cth) demonstrates an explicit interest in the management of fund portfolios by the Australian government.

![Figure 2.2: Value of the assets of Australian managed funds (A$ million), from June 1988 to June 2002](image)

**Figure 2.2: Value of the assets of Australian managed funds (A$ million), from June 1988 to June 2002**

Date

- Superannuation funds
- Statutory funds of life insurance offices
- Public unit trusts
- Cash management trusts
- Common funds
- Friendly societies

constructed using data obtained from the Reserve Bank of Australia website, [www.rba.gov.au](http://www.rba.gov.au). June figures for each year are used.

### 2.3 Risk Management

Subsequent to the establishment of an investment portfolio, managers may adopt strategies to limit the risk that the value of the portfolio will decrease below a certain threshold. In this section, the risk exposures associated with diversified portfolios are identified, and risk management strategies are discussed.
2.3.1 Risk exposures

The value of an investment portfolio is subject to risk, which originates from a variety of sources. Market risk is the risk that an adverse movement in the market as a whole will reduce the value of an investment. Market risk is described by researchers such as Chance (1998), Collins and Fabozzi (1999) as the uncertainty arising from movements in interest rates, foreign exchange rates, equity prices or commodity prices. Equity market risk is the risk that a portfolio will lose value if the equity market falls. In the context of a diversified portfolio, seemingly independent risks that collectively constitute market risk have a potential impact on all components of the portfolio, due to the interrelationships between different components of the portfolio. Consider a simple portfolio containing equity and bonds. A change in interest rates has a direct influence on bond prices, but it also has indirect ramifications on the equity component of the portfolio because the relative riskiness and attractiveness of each asset class has altered. In short, a change in a particular risk exposure affects all parts of the portfolio. In many studies on risk management, the interdependence of risks is not considered, because of the tendency of prior researchers to examine techniques to control only single types of risk. However, this issue is addressed in this thesis.

Given that market risk affects all portfolios, fund managers may explicitly aim to limit such risk because they have an obligation to repay contributions at a future date. Arguably, all managers implicitly aim to limit the risk associated with fund portfolios because losses affect not only those contributing to the fund, but also the managers’ professional reputations and possibly their remuneration. Sawicki (2000) presents “unequivocal evidence” that the past performance of Australian managed funds influences the movement of assets-under-management. Her study was conducted over the period 1980 to 1995, using 124 funds. Similarly, Tufano (1996) presents evidence indicating that managerial incentives and managerial risk aversion influence corporate risk management policy. Fund managers may protect their portfolios through the use of asset allocation strategies or derivative contracts.

2.3.2 Management of risk through asset allocation

A basic method of managing risk using asset allocations is diversification. Diversification is the dispersion of capital to a variety of assets, as opposed to the concentration of capital in few assets. For instance, Figure 2.1 illustrates that Australian managed fund portfolios are diversified. Successful diversification requires investment in assets which have low correlation with other assets in the portfolio, where correlation measures how the returns on the assets move together over time. Including an asset with a high individual variance of returns may actually decrease the risk of the overall portfolio if the returns on that asset are negatively correlated with the returns on the other assets in the portfolio. A common method of diversifying a
domestic portfolio is to include foreign assets, and there is a large body of literature describing the benefits of international diversification, including Levy and Sarnat (1970), Solnik (1974), Solnik and Noetzlin (1982), and Jorion (1985). Izan et al (1991) find that Australian investors benefit from international diversification, even after controlling for estimation risk. This finding is consistent with prior Australian studies including Watson and Dickinson (1981).

However, simple international diversification strategies become less effective as the correlation between individual countries and regions increase due to globalisation. As Malevergne and Sornette (2002, p 129) observe, “diversification works when one does not really need it and may fail severely when it is most needed”. The increasing interrelationships between foreign markets is evident in the literature on volatility spill-overs across markets. For instance, Dekker et al (2001) find strong linkages not only between the U.S. equity market and equity markets in the Asia Pacific including Australia, but also between different markets within the Asia Pacific region. Ho et al (1999) and Hanna et al (1999) present empirical evidence that U.S. investors do not benefit materially from international diversification because the U.S. market constitutes a large proportion of the international equity market and is generally highly correlated with other markets. Further, Griffin (1997) and others observe a “home bias” where funds and investors generally invest more in domestic assets than the literature on international diversification advocates. However, Ueda (1999) suggests that this phenomenon is due to incomplete information about foreign assets, leading to caution about investing in a lesser known product or market. Butler and Joaquin (2000) find that correlations between international equity markets are higher during bear markets, suggesting that the gains from diversification are reduced when portfolio protection is most advantageous. In short, simple diversification strategies may not reduce risk sufficiently to provide adequate portfolio protection in an increasingly complex global market.

In contrast to simple diversification strategies, complex models of asset allocation aimed at controlling downside risk have emerged in recent literature. For instance, stochastic models for dynamic asset allocation with downside risk control have been proposed by Zhao and Ziemba (2000) and Zhao and Ziemba (2001). Similarly, Brennan et al (1997) and Boyle and Yang (1997) use stochastic optimal control to form a continuous-time strategic asset allocation model that incorporates intertemporal variation in expected returns of bonds, stock and cash. Stevenson (2001) applies downside risk methods of asset allocation in the context of emerging markets, while Bielecki et al (2000) demonstrate the use of a multi-period dynamic risk sensitive asset allocation decision criteria, applying it to Australian and US data.
The key disadvantages of dynamic asset allocation models as tools for risk management are their complexity and the computational difficulties associated with their practical application. Zhao and Ziemba (2000, p 98) explicitly acknowledge the difficulties in solving their model, and Brennan et al (1997, p 1380) describe the implementation of stochastic programming models as “highly computer intensive”. Further, trading individual assets in a portfolio in order to manage risk using either simple diversification strategies or complex asset allocation techniques can become expensive due to transaction costs, and time consuming due to the required management expertise and attention. Indeed, as Collins and Fabozzi (1999, p 19) argue, “diversification alone as a means to manage risk is not adequate in today’s world of change characterized by rapidly advancing technology, deregulation, financial product proliferation, and volatile financial markets. New tools are required, and derivative instruments meet the requirement”.

2.3.3 Management of risk through derivative contracts

Derivatives are financial securities “whose value depends on the values of other, more basic underlying variables”, according to Hull (2000, p 1). Derivative contracts such as options, futures, swaps, or exotic products may be employed to limit the potential loss associated with adverse movements in investment markets. However, the trust deeds governing some funds may restrict the use of derivatives, as noted by Harris and Rosser (1993). Further, the level of personal understanding and knowledge of derivatives by managers may also restrict the use of such tools. Ferry (2002, p 31) observes that “it is often difficult to convince treasury managers that it is in their best interest to implement a hedging strategy”. In the 1998 survey of derivatives use by U.S. institutional investors conducted by Levich et al (1999), it was found that 46% of all institutions and 63% of pension plans are permitted to use derivatives. They report that derivatives are most frequently used to manage the risk of foreign assets and foreign exchange risk. Guay and Kothari (2001) find that in a sample of 234 large nonfinancial U.S. institutions that use derivatives, “the magnitude of the derivatives position taken by most firms is economically small in relation to their average risk exposures” (p 27). This may be due to the existence of natural hedges, which reduce risk without recourse to derivatives. Other surveys on derivative use in the U.S. have been conducted by Berkman et al (1997), Bodnar et al (1996, 1998), Gay and Nam (1998) and Howton and Perfect (1998). Similar evidence in an Australian context is provided by Berkman et al (2002) and Nguyen and Faff (2002).

Protection from potential loss using derivatives attracts both explicit and implicit costs. Explicit costs include transaction costs and brokerage, which also apply to protection using dynamic asset allocation, whereas the loss in potential increases in returns associated with a hedge using futures contracts is an implicit cost of that strategy. Further, although derivatives may be used to
limit the risk of a portfolio, derivatives themselves entail risks. Risks associated with derivatives include credit risk, which is the risk that the other party to the contract will default, liquidity risk, which is the risk that the derivatives market will not be sufficiently liquid, and legal risk, which is the risk that the derivative contract will not be enforceable. Credit risk and legal risk, and to a certain extent liquidity risk, can be dealt with by using standard exchange traded derivatives, as opposed to over-the-counter instruments. A further category is model risk, namely that an inappropriate hedging model may be used. For instance, given an asset and its associated futures contract, a simple naïve hedge may fail to account for certain properties of the data, which results in an inaccurate hedge.

A further risk associated with derivatives is operational risk, the risk that derivatives may be misused or poorly managed in an organisation. Well known cases involving operational risk, such as Barings Bank and Metallgesellschaft, illustrate that major corporations have suffered serious financial losses as a consequence of derivatives programs. For instance, the $1.3 billion loss experienced by Metallgesellschaft through dealings with futures contracts has attained notoriety and received much attention in the literature from researchers including Edwards and Canter (1995), Culp and Miller (1995), and Pirrong (1997). Scholes (1996) lists other derivative disasters of similar economic magnitude. A sound understanding of the mechanics of futures trading and futures markets is essential to the avoidance of serious economic loss.

2.3.4. Decision to hedge using derivatives

Several determinants of the decision to hedge using derivatives have been discussed in the literature. In the context of firms, Smith and Stulz (1985) and Smith et al (1990) argue that hedging behaviour is determined by taxation conditions, the costs of financial distress, managerial incentives, and the risk aversion of ill-diversified investors. Similarly, Barton (2001) links derivative use by U.S. managers to agency costs, tax, information asymmetry and personal wealth. Managers of funds and firms may share common motivations, depending on factors such as fund style. For instance, motivations based on managerial incentives, and in particular remuneration and "reputational effects", arguably apply in both types of organisation. Although fund portfolios provide the specific empirical context in this thesis, the findings are of interest to anyone conducting an equity hedge using futures contracts. As such, the discussion of hedging motivations in this section is not restricted to funds, but also includes firms and individuals, in recognition of the general relevance of the findings in this thesis.

Tax effects

Smith and Stulz (1985) provide a theoretical demonstration that hedging may reduce expected tax liabilities under certain taxation conditions. Specifically, as the tax function becomes more
convex, firms have increased incentive to hedge because hedging will decrease the expected taxes demanded of the firm. Empirical evidence for U.S. institutions generally supports taxation as an incentive to hedge, including Nance et al (1993, p 280), Howton and Perfect (1998), and Graham and Smith (1999). Although Graham and Smith (1999, p 2241) criticise prior research for relying on surveys and regression analysis, their results concur with most prior findings. In contrast, Mian (1996) and Tufano (1996) find little evidence of the tax effect. In the absence of progressive tax rates, tax effects are not expected to have any relevance to Australian managed funds.

Costs of financial distress

Hedging may reduce expected costs associated with financial distress by reducing the fluctuations in firm value that would cause the probability of financial distress to increase, according to Smith and Stulz (1985) and Smith et al (1990). Evidence on this motivation for hedging using derivatives is mixed. Howton and Perfect (1998) and Whidbee and Wohar (1999) find that derivatives are used to reduce the costs of financial distress. Evidence presented by Mian (1996) and Tufano (1996) does not strongly support the financial distress argument.

Agency costs: Shareholders and bondholders

Hedging may reduce agency costs associated with the divergent interests of shareholders and bondholders. Nance et al (1993, p 270) observes that “taking a positive net present value project can reduce shareholders’ wealth if the gains accrue primarily to the debtholders”. This causes shareholders to under-invest by rejecting positive NPV projects, a problem identified by Myers (1977). Hedging alleviates this difficulty by reducing the chance of default on bond payments. Survey-based analysis presented by Nance et al (1993) indicates that organisations hedge to control such agency problems. In a theoretical paper, Bessembinder (1991, p531) argues that hedging by corporations can increase firm value because “hedges reduce agency costs” and “independent of effects on investment, hedging increases value by improving contracting terms.”

Mian (1996) offers mixed support for the agency costs argument, finding that firms in unregulated industries are more likely to hedge than those in regulated industries, which is consistent with the agency costs argument. However, Mian’s tests relating to growth opportunities indicate that “hedgers do not have higher market-to-book ratios” (p 431), which is inconsistent with the agency costs model. Geczy et al (1997) examine 372 of the Fortune 500 non-financial firms in 1990, about 41% of which use derivatives. Consistent with the agency cost hypothesis, they find a positive relationship between derivative use and firms exhibiting both high growth opportunities and low accessibility to financing.
Managerial incentives

Smith and Stulz (1985) identify two possible managerial incentives to hedge. First, managerial risk aversion caused by share ownership in the firm under management may induce managers to hedge more than otherwise. Second, managerial compensation contracts that cause concavity in managerial income as a function of firm value may induce managers to hedge more. In addition, DeMarzo and Duffie (1995) suggest that “reputational effects” may impact on the hedging decision. Specifically, managers at the beginning of their careers are likely to be most affected by reported profits because there is no prior performance to consider, and current profits determine future remuneration. They provide theoretical arguments that accounting disclosure of hedging activities affects the hedging behaviour of managers.

As with the other motivations to hedge, empirical evidence is varied, although the bulk of evidence lends support to the impact of managerial incentives on hedging using derivatives. For instance, in a study of U.S. gold mining firms, Tufano (1996) finds that managerial incentives influence the decision to hedge. Specifically, managers owning more wealth in common stock manage more price risk, and risk management levels are higher when senior financial managers have shorter job tenures. Similarly, Whidbee and Wohar (1999, p 251) find that the decision of US banks to use derivatives is influenced by “managerial incentives and external monitoring”. Their finding that managers with higher share ownership are less likely to use derivatives is inconsistent with prior evidence, but may be distinguished on the basis that prior research examines non-financial institutions, which do not have the high leverage or risk-shifting opportunities of banks. Their findings indicate that corporate governance and ownership-structure features impact on the likelihood that derivatives will be employed, even after controlling for size and risk exposures (p 274). Similarly, Koski and Pontiff (1999) find evidence consistent with the use of derivatives by U.S. mutual fund managers to decrease the impact of previous fund performance on fund risk.

However, Geczy et al (1997) find little difference between managerial share ownership between firms that hedge using currency derivatives and those that do not. However, they concede that firms using derivatives tend to have “greater analyst following and institutional ownership, and greater managerial option holdings” (p 1350), which is consistent with the view that managerial incentives and reputation may influence hedging behaviour.

Other explanations

Various other motivations for hedging have been postulated, including a positive relationship between hedging and the cost of external financing (Froot et al (1993), Howton and Perfect
Gay and Nam (1998)), research and development costs (Geczy et al (1997)), managing contractual relationships between firms (Pennings and Leuthold (2000)), and firm size and leverage (Berkman et al (2002)). An additional factor which may influence the decision to hedge is the increased availability of derivative products. The rapid development of derivatives markets in recent years has facilitated hedging by providing more flexible products such as futures, which are relatively inexpensive and provide a high degree of leverage.

Summary
Given the complexity and variety of factors potentially influencing the decision to hedge using derivatives, it seems unlikely that any one factor considered in isolation is solely responsible for the decision to hedge. This has been recognised in recent literature, which attempts to synthesise past models to explain hedging behaviour. For instance, Arias et al (2000) derive a new model of hedging behaviour where the motivations to hedge are “to reduce tax liabilities, bankruptcy costs, borrowing costs, and liquidity costs” (p 392). Their model combines these motivations, rather than considering them in isolation, and they find that these factors are sufficient to motivate firms to hedge, regardless of risk preference (p 393). Given the lack of consensus in the literature, this is an area for future research, although it is not pursued in this thesis. Although this thesis examines hedging in the context of fund portfolios, the findings are relevant to anyone hedging a diversified equity portfolio using futures contracts, regardless of their particular motivation.

2.3.5 Use of derivatives
The general consensus in the literature is that corporations use derivatives to reduce risk rather than to speculate. For instance, in the context of U.S. institutions, Guay (1999), Geczy et al (1997), and Allayannis and Weston (2001) find that derivatives are used to hedge rather than speculate, although Hentschel and Kothari (2001) find that U.S. firms do not appear to either be taking large risks or reducing risks greatly with derivatives. Tufano (1996, p 1097) refers to the Wharton/Chase Derivatives Survey (1995) which indicates that in excess of 70 percent of [U.S.] firms use derivatives as hedging vehicles. Similarly, Bodnar et al (1998) use 399 survey responses from U.S. non-financial firms, and find that 50% use derivatives in 1998, compared to 41% in 1995 and 35% in 1994 (p 71), illustrating that derivative use by U.S. firms is increasing over time.

Although few studies have been conducted in Australia, Australian and New Zealand evidence supports the U.S. findings. For instance, Berkman et al (1997) use 79 survey responses from New Zealand firms regarding derivative use, and 53.1% use derivatives, a higher percentage of respondents than in the U.S. in 1995 and 1994 in the study by Bodnar et al (1998). Similarly, a
A survey of 38 Australian pooled superannuation funds was conducted by Harris and Rosser (1993). Eighty-two percent of surveyed funds used futures, and eighty-five percent used options. A prominent reason given for using derivatives is risk reduction, and the Australian All Ordinaries Share Price Index futures contract and the interest rate futures are most widely used by Australian funds. Thus, fund managers do use derivatives to reduce risk associated with their portfolios. More recently, Berkman et al (2002) examine the derivative usage by Australian industrial and mining firms. They find that 52.8% of industrial firms and 61.5% of mining firms use derivatives.

Indirect evidence also supports the hypothesis that financial institutions use derivatives to hedge. For example, in a study of the S&P 500 stock index futures contract, Chang et al (2000) find that the demand for hedging increases as stock market volatility increases, implying that hedgers use futures contracts to control risk.

2.4 Conclusion

Diversified portfolios are exposed to market risk, which may consist of interest rate risk, equity price risk, foreign exchange risk, and commodity price risk. Risk exposures may be reduced using asset allocation strategies or derivatives. Because of the complexity and practical difficulties associated with dynamic asset allocation strategies, it is important to consider derivatives as an alternative tool for restructuring risk exposures. Several motivations for the use of derivatives for hedging have been suggested in the literature. Empirical evidence regarding these motivations is mixed, reflecting the complexity of the issue, and indicating that the existence of one single explanation is unlikely to apply in every case. However, the general consensus in empirical studies is that institutions use derivatives to control risk rather than for speculative purposes. Managers overseeing portfolios may use derivatives such as futures contracts to limit risk exposures associated with financial investments.

This chapter provides the context for the research. The following chapter presents more detailed technical information on hedging using futures contracts.
CHAPTER THREE

HEDGING USING FUTURES CONTRACTS

3.1 Introduction

This chapter presents information on hedging using futures contracts. First, the choice of futures contracts as hedging vehicles, as opposed to other derivative instruments, is discussed. Second, a theoretical framework for hedging using futures contracts is provided. Third, methods of calculating hedge ratios are discussed, and literature on hedging single and multiple risk exposures using futures contracts is reviewed.

3.2 Hedging risk using futures contracts

In this section, futures contracts are examined as tools to control risk exposures. Futures contracts are defined, their main characteristics are described, and the exclusive focus of this thesis on futures contracts as opposed to other derivatives such as options is justified with reference to the relative benefits of each derivative.

3.2.1 Futures contracts

A futures contract is defined as a legally binding agreement between two parties that they will trade a specific asset at a set date in the future, called the expiry date, at a price determined at the formation of the contract. The price fixed in the contract is the futures price, and the current price of the underlying asset is the spot price. The spot price is determined in the spot market, which is the market for the underlying asset.

Futures contracts exhibit several important characteristics. They are highly standardised with regard to the terms of the contract, such as expiry dates. Further, futures contracts are subject to conventions on trading procedures and quotation, and deposits are payable on initiation of the contract. Generally, futures positions are closed out prior to expiry by reversing the investor’s existing position.

---

4 In contrast, forward contracts have flexible terms, which are tailored to meet the needs of the particular investor. Unlike futures contracts, forward contracts are not traded on exchanges. Only futures contracts are examined in this thesis due to data availability and to enhance the generalisability of the findings.
A distinguishing feature of futures contracts is the process of marking to market. Marking to market affords protection against the risk of counter-party default by settling gains or losses on open futures positions at daily or intra-daily intervals. Unrealized gains or losses are determined, then margin calls are made. A margin call occurs when a party subject to unrealized loss is required to pay cash into their trading account to cover that amount. Similarly, any unrealized gains are added to the investor’s account. Failure to respond to a margin call results in automatic closure of that party’s position.

In Australia, futures contracts are traded via the Sydney Futures Exchange, currently the largest financial futures and options exchange in the Asia Pacific region. Prior to November 1999, day trading occurred under an open outcry system, and night trading took place through the Sydney Computerised Market (SYCOM), an electronic system introduced in 1989. Since 15 November 1999, all trading occurs electronically through SYCOM. The Sydney Futures Exchange Clearing House guarantees the financial performance of all contracts traded by enforcing the marking to market procedure, thus relieving contracting parties of the risk of default by the other party.

3.2.2 Futures contracts versus options
The two most common exchange traded derivatives are futures and options. This thesis focuses exclusively on futures contracts, for several reasons. Prior research indicates that futures outperform options as hedging vehicles. For instance, Battermann et al (2000, p 85) provide theoretical derivations demonstrating that futures are “unequivocally” preferred to options as hedging vehicles by a risk-averse exporting firm aiming to maximise expected utility, given both unbiased and mildly biased futures and options prices.

This theory is supported by a range of empirical evidence. For example, Benet and Luft (1995, p 715) examine the relative hedge effectiveness of stock index futures and options using out-of-sample tests, and conclude that futures provide superior performance relative to options, given practical costs and market frictions. This accords with the assertion of Hancock and Weise (1994, p 436) that transaction costs, contract features, margin requirements, and derivative risks influence the selection between different hedging instruments. Further, Hsin et al (1994, p 706) observe empirically that currency futures are superior hedging instruments relative to currency options, regardless of whether the options are compared in the form of synthetic futures or in delta/gamma hedges, and regardless of the risk preferences of investors. In a review of evidence comparing futures and options, Lien and Tse (2002, p 383) conclude that “empirical results are mostly in favour of futures”. Lien and Tse (2001, p 159) find that currency futures are superior
hedging instruments relative to currency options, when effectiveness is defined using lower partial moments.

In contrast, Frechette (2001) argues that for moderate and high levels of risk aversion, investors are “nearly indifferent” between an optimal hedge involving only futures or only options, although he points out that “the optimal futures-only strategy involved a lower hedge ratio than the optimal options-only hedge ratio” (p 710). However, all else equal, a hedger is likely to prefer the derivative associated with the smaller hedge ratio, because a smaller derivatives position would logically attract lower implementation costs.

In the context of equity risk, Ghosh (1993b, p 743) notes that generally, index futures are favoured as a hedging vehicle because of their relatively high liquidity and lower transaction costs. Australian futures may be preferred to futures options because Australian futures markets are considerably more liquid than Australian options markets. 5 This is evident in Figure 3.1, which depicts the annual levels of trading volume for futures and futures options written on the Australian All Ordinaries Share Price Index. The futures contracts have consistently higher trading volume than the futures options.

Figure 3.1: Annual trading volume for Australian All Ordinaries Share Price Index Futures and Futures Options, for the period 1983 to 2002

[Graph showing annual trading volume for SPI Futures and SPI Options from 1983 to 2002]

Construct from data provided by the Sydney Futures Exchange. “SPI Options” refer to futures options written on the SPI. References to the “SPI” include data for both the SPI and SPI 200.

5 Both Australian All Ordinaries Share Price Index futures contracts and futures options contracts are relatively new innovations, being introduced in 1983 and 1985 respectively.
Logically, futures should be less expensive than options, because options provide a more desirable payoff which should attract more valuable consideration. This postulation is supported in practice. For example, the Sydney Futures Exchange charges an exchange fee of A$0.99 per futures contract and a minimum charge of A$13 for short options, effective from 17/09/2001.

Studies on the empirical use of derivatives by institutions indicates that “futures are the most commonly used commodity hedging instruments and are among the most commonly used currency, interest rate and equity hedging instruments”, according to Ferguson and Leistikow (1999, p 125). Indeed, Stoll and Whaley (1997, p 140) describe stock index futures contracts as “the most successful financial innovation of the 1980s”.

In summary, the exclusive focus of this thesis on hedging using futures contracts as opposed to other derivatives is justified on the basis of the relatively high liquidity of Australian futures markets, the relatively small transaction costs associated with futures contracts, and prior empirical evidence which suggests that the hedging performance of futures contracts is superior to that of options.

### 3.3 Theoretical framework

To hedge the risk associated with an investment using futures contracts, it is necessary to determine an appropriate hedge ratio, which indicates the size of the derivatives position required to insure the underlying portfolio. In this section, a theoretical model of demand for futures contracts is developed, based on the work of Anderson and Danthine (1980, 1981) and Gagnon et al (1998). This model provides the theoretical framework for empirical tests conducted in this thesis.

Anderson and Danthine (1980) develop a single-period model of futures demand, given \( m \) spot assets and \( n \) futures contracts, and using unconditional variables. In the next year, the same researchers present a slightly different derivation of their earlier model. More recently, Gagnon et al (1998) extend the earlier analysis to conditional variables, but restrict the number of spot assets to equal the number of futures contracts. The following derivation synthesises prior work, particularly following Anderson and Danthine (1981). It is assumed that there are \( m \) spot assets, and \( n \) futures contracts, and that \( m \) does not necessarily equal \( n \).

---

6 Given a basic hedging scenario with one asset and one derivative, both futures and options contracts limit downside risk, but futures contracts also limit upside gains while options contracts allow upside gains to be realised.
Let the prices of each asset, \( s_{i,t} \), included in the investor's portfolio be represented in the \((m \times 1)\) column vector \( S_t \), where \( T \) indicates transpose:

\[
S_t = \begin{bmatrix} s_{1,t} & \cdots & s_{m,t} \end{bmatrix}^T
\]  

(3.1)

The portfolio weightings of the spot assets are given by the \((m \times 1)\) column vector \( \lambda_t \):

\[
\lambda_t = \begin{bmatrix} \lambda_{1,t} & \cdots & \lambda_{m,t} \end{bmatrix}^T
\]  

(3.2)

Similarly, let the prices of each futures contract \( f_{i,t} \), and the weightings of those contracts in the futures portfolio \( \gamma_{i,t} \), be defined as \((n \times 1)\) column vectors \( F_t \) and \( \gamma_t \) respectively:

\[
F_t = \begin{bmatrix} f_{1,t} & \cdots & f_{n,t} \end{bmatrix}^T
\]  

(3.3)

\[
\gamma_t = \begin{bmatrix} \gamma_{1,t} & \cdots & \gamma_{n,t} \end{bmatrix}^T
\]  

(3.4)

Short positions in futures contracts are indicated by negative \( \gamma \) weights, and long positions are shown as positive \( \gamma \) weights.

First, consider the portfolios of spot assets and futures contracts separately. Generally, the return on a portfolio of investments is \( R_{p,t} = \sum_{i=1}^{n} w_{i,t} R_{i,t} \), where \( R_{i,t} \) is the return on asset \( i \) at time \( t \), and \( w_{i,t} \) is the proportion of asset \( i \) included in the portfolio at time \( t \). Thus, the return\(^7\) on the spot portfolio, \( R_{s,t} \), is the sum of the weighted changes in prices of each individual asset in the portfolio, where the weights are the investment proportions:

\[
R_{s,t} = \begin{bmatrix} \lambda_{1,t-1} & \cdots & \lambda_{m,t-1} \end{bmatrix} \left( \begin{bmatrix} s_{1,t} \\ \vdots \\ s_{m,t} \end{bmatrix} - \begin{bmatrix} s_{1,t-1} \\ \vdots \\ s_{m,t-1} \end{bmatrix} \right) = \lambda_{t-1}^T (S_t - S_{t-1})
\]  

(3.5)

Similarly, the return on the futures portfolio, \( R_{f,t} \), is given by:

\[
R_{f,t} = \begin{bmatrix} \gamma_{1,t-1} & \cdots & \gamma_{n,t-1} \end{bmatrix} \left( \begin{bmatrix} f_{1,t} \\ \vdots \\ f_{n,t} \end{bmatrix} - \begin{bmatrix} f_{1,t-1} \\ \vdots \\ f_{n,t-1} \end{bmatrix} \right) = \gamma_{t-1}^T (F_t - F_{t-1})
\]  

(3.6)

The return on a portfolio of assets hedged using a portfolio of futures from time \( t-1 \) to time \( t \) is denoted \( H_t \), and is given by the sum of the return on the spot portfolio \( R_{s,t} \) and the return on the futures portfolio \( R_{f,t} \):

\[\text{For the purposes of empirical work, spot and futures prices are commonly transformed logarithmically, and the change in log price is a continuously compounded return.}\]
The covariance between different elements of a portfolio containing multiple assets and futures influences the number of futures contracts required to offset adverse movements in the returns of spot assets. For instance, given a portfolio of assets, natural hedges may arise in the absence of futures contracts, due to the interaction between the returns on individual assets. Similarly, if a portfolio of assets is protected using multiple futures contracts, the futures contracts may interact to enhance or reduce the benefits of the hedge. The covariance matrix captures all effects within both the stock and futures portfolios:

\[
\Sigma = \begin{bmatrix}
\Sigma_{s,s} & \Sigma_{s,f} \\
\Sigma_{s,f}^T & \Sigma_{f,f}
\end{bmatrix}
\]  

(3.8)

where

\[
\Sigma_{s,s} = \text{covariance matrix of spot prices (m x m)}
\]

\[
\Sigma_{f,f} = \text{covariance matrix of futures prices (n x n)}
\]

\[
\Sigma_{s,f} = \text{covariance matrix of spot and futures prices (n x m)}
\]

\[
\Sigma_{f,s}^T = \text{transpose of covariance matrix of spot and futures prices (m x n)}
\]

The variance-covariance matrix in equation (3.8) is an \((m+n) \times (m+n)\) matrix, and is consistent with that suggested by Anderson and Danthine (1980), but larger than the \((n+1) \times (n+1)\) matrices of Gagnon et al (1998) and Anderson and Danthine (1981). The latter two approaches treat the spot portfolio as a single asset, requiring only a single variance.

The variance of a portfolio return is defined generally as

\[
Var(P) = \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \text{cov}_{ij}
\]

where \(\text{cov}_{ij}\) is the covariance between assets \(i\) and \(j\). Thus, the variance of the hedged portfolio return is:

\[
Var(H_i) = \Sigma_{s,s} \lambda^2 + 2 \Sigma_{s,f} \lambda \gamma + \Sigma_{f,f} \gamma^2
\]  

(3.9)

As acknowledged by Kritzman (1993, p 95), “one of the most widely accepted principles of modern finance is that investors seek to maximize expected utility”. More specifically, Lien (2001, p 681) states that expected utility is generally used in the analysis of futures trading. Accordingly, the investor in this analysis is assumed to maximise expected utility, and an appropriate utility function is required. Under the Markowitz (1959) mean-variance theory, all investors share the objectives of maximising return for a given level of risk, or equivalently minimising risk for a given expected return. Although acknowledging that acceptance of the expected utility maxim does not necessarily imply acceptance of the use of “mean and variance
as criteria of portfolio selection" (p 209-10), Markowitz (1959) shows that when both models are combined, then the investor’s utility function is quadratic (p 288).8 Kolb and Okunev (1993) and Marshall and Herbst (1992) argue that quadratic utility functions may not provide accurate representations of investor preferences. However, Levy and Markowitz (1979) demonstrate empirically that preferences for different portfolios based on the maximisation of expected utility and the application of the mean-variance criteria are almost identical. Kroll et al (1984) confirm this result, finding that for a variety of utility functions and for a portfolio “chosen from any of the infinite number of portfolios of the standard constraint set the best mean-variance efficient portfolio has almost maximum obtainable expected utility” (p 59). Thus, for the purposes of this proof, expected utility is defined in terms of mean and variance. Specifically, the investor’s problem is:

$$\max_{\gamma} \left[ E(H_t) - \frac{\phi}{2} \text{Var}(H_t) \right]$$  \hspace{1cm} (3.10)

where the investor’s risk aversion is $\phi > 0$. Observe that this differs from Anderson and Danthine (1980, 1981), where they use a different function of net revenue. Expanding equation (3.10) gives the maximisation problem:

$$\max_{\gamma} \left[ \lambda_{t-1}^T (S_t - S_{t-1}) + \gamma_{t-1}^T (F_t - F_{t-1}) - \frac{\phi}{2} \left( \sum_{s} \lambda_s^2 + 2 \sum_{s,f} \gamma \lambda_s + \sum_{f,f} \gamma^2 \right) \right]$$  \hspace{1cm} (3.11)

While Anderson and Danthine (1980, 1981) maximise this function with respect to both the futures weights and the spot weights, the key choice variables of interest in this thesis are the futures weights. The focus of this thesis is the use of futures for hedging existing portfolios, and it is assumed that fund managers have already engaged in asset allocation and stock selection according to their own criteria. As in Gagnon et al (1998), only a single first order condition is obtained:

$$(F_t - F_{t-1}) - \phi \left( \sum_{f} \gamma + \sum_{s,f} \lambda_s \right) = 0$$  \hspace{1cm} (3.12)

This is one of the two first order conditions indicated by Anderson and Danthine (1980), but not in Anderson and Danthine (1981). This is because the net revenue functions differ, in that the returns on the futures and spot portfolios are added in the (1980) paper and subtracted in the (1981) paper. Rearranging further, the optimal futures position is obtained:

$$\gamma^* = \frac{1}{\phi} \sum_{f} \left( F_t - F_{t-1} \right) - \sum_{s,f} \lambda_s$$  \hspace{1cm} (3.13)

8 An alternative assumption is the normality of the return distribution. However, empirical evidence often indicates that financial return series are not normally distributed, particularly using high-frequency data. To illustrate, Alles and Spowart (1995) find that nonnormality of Australian equity returns is more pronounced for weekly returns rather than monthly returns.
Anderson and Danthine (1981) observe that the solution is unique iff $\Sigma_{f,f}$ is nonsingular. Equation (3.13) can be disaggregated into the hedge and speculative components of futures demand. The first term, $\frac{1}{\phi} \Sigma_{f,f}^{-1} \left( F_t - F_{t-1} \right)$, is the speculative component of the demand for futures. The demand for futures consists exclusively of the speculative component when futures prices follow a martingale, which means that $E(F_t) = F_{t-1}$, given that the model is a single-period model. The second term, $-\Sigma_{f,s}^{-1} \Sigma_{s,f} \lambda$, is the pure hedge component of futures demand.

That a hedge ratio consists of both a speculative component and a pure hedge component has been recognised in the literature by Johnson (1960, p 150), Anderson and Danthine (1981, p 1187), Duffie (1989, pp 91-93), Castelino (1992, p 189), Sutcliffe (1993, p 240), Kroner and Sultan (1993, p 537), and Shalen (1989, p 217). Despite this, the majority of empirical evidence on hedge ratios is restricted to the pure hedge component, to the exclusion of the speculative component. Individual hedgers are accorded different figures for the speculative component depending on their level of risk aversion, whereas each uses the same figure for the hedge component. The inclusion of the speculative component in empirical tests of hedge ratios creates practical estimation difficulties, such as determining an appropriate level of risk aversion for a particular hedger. An exception to the general trend of ignoring the speculative component of futures demand is the study by Glen and Jorion (1993), who provide empirical evidence on currency hedging. The following sections discuss empirical approaches to estimating the pure hedge component of futures demand, in the context of direct and cross-hedging single and multiple risk exposures.

3.4 Hedging single risks using single futures contracts

Various techniques have been employed to calculate the pure hedge component of futures demand, in the context of hedging single risk exposures with single types of futures contracts. In this section, traditional models such as the naive model and models based on ordinary least squares regression are compared with more recent models that incorporate error correction adjustments for cointegration, and generalised autoregressive conditional heteroskedasticity (GARCH) effects. The choice of hedge ratio estimation technique is critical to the effectiveness of the hedge. If an inappropriate technique is applied, an inaccurate hedge ratio will be obtained and the resulting hedge will either provide excessive protection (over-hedging) or insufficient protection (under-hedging). In the case of over-hedging, the original risk associated with the price of the underlying asset is neutralised, but a new risk is created due to over-exposure to movements in futures prices. In the case of under-hedging, the original risk exposure associated
with the underlying asset is not fully neutralised due to an insufficient number of futures contracts in the portfolio. In this section, hedge ratio estimation techniques are reviewed.

3.4.1 Naïve Model
The naïve model is the most simplistic approach to obtaining a hedge ratio. The naïve hedge ratio equals one, so that a naïve hedge entails the adoption of a position in futures contracts that is as large as the principal value of the underlying asset, but opposite in sign. This approach is founded on the highly questionable assumption that at the end of the hedge period, the spot and futures prices will have moved by precisely the same amount because they are perfectly positively correlated and their variances are identical. As observed by deJong et al (1997, p 818), this means that optimal reduction of risk using a naïve hedge can only be attained if basis risk does not exist. Basis is the price of the underlying asset less the price of the futures contract, and basis risk is the risk of unanticipated changes in that variable. The naïve model is not generally preferred to more complex models in empirical studies, as noted by researchers such as Kroner and Sultan (1993) and Anderson and Danthine (1981, p 1183), who reject the naïve strategy as too simplistic and generally “suboptimal”. Regression analysis provides an alternative to the naïve strategy.

3.4.2 OLS Regression Models
Castelino (1992, p 188) observes that while there is “no consensus regarding the appropriate method to estimate the minimum-variance hedge ratio, a distinct bias in favour of regression analysis exists”. Similarly, Ferguson and Leistikow (1998, p 851) note that the most commonly used method of calculating hedge ratios is ordinary least squares regression. Ghosh (1993b) identifies three versions of the regression method, namely regressing contemporaneous futures and spot price levels (price levels model), regressing the change in futures price against the change in spot price (price changes model), and regressing percentage changes in futures and spot prices (percentage change model). The most commonly applied model is the price changes regression model:

$$\Delta S_t = \alpha + \beta \Delta F_t + \epsilon_t$$

(3.14)

where

- $S_t =$ natural logarithm of the spot price level at time $t$
- $F_t =$ natural logarithm of the futures price level at time $t$
- $\beta =$ constant, equal to the hedge ratio
- $\alpha =$ constant
- $\epsilon_t =$ error term
For instance, Ederington (1979), Hill and Schneeweis (1981), Benet (1992) and Lindahl (1992) employ the price changes model. The price changes model is equivalent to a regression of continuously compounded futures returns against continuously compounded spot returns.

While favoured for their simplicity and ease of application, different versions of the basic regression model are subject to numerous criticisms. For instance, as Ghosh (1993b, p 744) observes, the price levels model ignores “short-run dynamics and amounts to spurious regression in the sense of Granger and Newbold (1974)”. Further, Lence (1995, p 388) demonstrates that that the beta of the (log of) price levels regression does not equal the optimal hedge ratio for all risk averse investors given unbiased futures prices. Thus, the hedge ratios obtained by researchers such as Dale (1981), who apply the price levels model, may be inaccurate. Similarly, the percentage change regression model is misspecified because it ignores lagged values, thus excluding short run dynamics: Ghosh (1993b, p 744). Evidence regarding the accuracy of the price changes model in equation (3.14) is more favourable. For instance, in an examination of hedge ratios for U.S. agricultural commodities, Myers and Thompson (1989) find empirical evidence indicating that price level regressions and returns regressions give “errors in optimal hedge ratio estimation”, while the price changes regression gives “reasonably accurate estimates” (p 858).

However, the accuracy of hedge ratios calculated using any version of OLS regression depends on assumptions of normality, homoskedasticity, and serial independence, as noted by Duarte and Mendes (1998, p 75). Recently, more complex models have been developed.

3.4.3 Error-correction models

One criticism of basic regression models in the context of hedge ratio estimation is that they ignore cointegration (Ghosh 1993b, p 744). The concept of cointegration was introduced by Granger (1981), and developed by Engle and Granger (1987). Cointegration exists when there is a relationship between variables such that they may deviate from each other in the short run, but will return to equilibrium in the long run. Engle and Granger (1987, p 254) demonstrate that cointegrated series have an error correction representation, meaning that a proportion of the disequilibrium in one period is expected to be corrected in the next period.

Wang and Yau (1994, p 458) argue that spot and futures markets should theoretically be cointegrated for reasons of market efficiency, and in particular because the efficient transmission of information between markets will be impaired in the absence of a close link between markets. Similarly, Geppert (1995, pp 511-512) argues that market efficiency, price convergence, and the stationarity of the cost of carry should ensure that the markets are
cointegrated, and that cointegration is a necessary condition for market efficiency in the absence of a risk premium.

These theoretical arguments are supported by empirical evidence which indicates a dynamic interaction between futures and spot markets. For instance, Stoll and Whaley (1990) examine the correlation of the returns on the Standard and Poor’s 500 (S&P 500) Index and futures contract, and the returns on the Major Market Index and futures contract. They conclude that futures and spot returns are largely contemporaneous, indicating a close link between the markets. Similarly, Kawaller et al (1987) examine the interaction between the S&P 500 index and futures, and find that the index and futures prices are “simultaneously related on a minute-to-minute basis throughout the trading day” (p 1327), which is “strong evidence that futures and spot prices move largely in unison” (p 1329). Consistent with such general findings, further empirical research confirms the existence of cointegration between share price index futures and spot markets. Specifically, Ghosh (1993a, 1993b), Wahab and Lashgari (1993), and Pizzi et al (1998) find that S&P 500 Index futures and spot markets are cointegrated. Lien and Luo (1993b) examine the Major Market Index, New York Stock Exchange Index and the S&P 500 Index, and find evidence of cointegration in each. Ghosh and Clayton (1996) find evidence of cointegration between the daily futures and stock index series of the French CAC 40, the U.K. FTSE 100, the German Dax, and the Japanese Nikkei (SIMEX) index. In an Australian context, Bhar (2001) finds evidence of cointegration between the All Ordinaries Index and the Share Price Index Futures contract, “despite a major structural event” (p 848), namely the re-scaling of the futures contract on 11 October 1993 to reduce the multiplier from $100 to $25, and increase the minimum tick size from 0.1 to 1 index point.9

The existence of cointegration between the futures and spot markets means that traditional regression models are misspecified because, as Wahab and Lashgari (1993, pp 712-713) observe, the levels, changes and percentage change specifications fail to account accurately for the dynamics in and between the futures and spot series. If the spot and futures rates are cointegrated, then the changes regression is misspecified because it involves “overdifferencing the data” and obscuring the true relationship between futures and spot prices, according to Kroner and Sultan (1993, p 536).

This misspecification may be corrected through the use of an error correction model, which provides an optimal hedge ratio that incorporates nonstationarity, long run equilibrium

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9 Brown (2001) analyses the impact of the re-denomination of the AOI share price index futures contract on the volume of trading, and concludes that the re-denomination was “successful” from the perspective of investors and exchange members.
relationship and short run dynamics of the futures and spot series. The optimal hedge ratio is obtained as the coefficient \( \beta \) from equation (3.15):

\[
\Delta S_t = c \Delta u_{t-1} + \beta \Delta F_t + \sum_{i=1}^{n} \delta_i \Delta F_{t-i} + \sum_{j=1}^{n} \phi_j \Delta S_{t-j} + \varepsilon_t
\]  

(3.15)

where

\( S_t = \) natural logarithm of spot asset price level at time \( t \)
\( F_t = \) natural log of futures price level at time \( t \)
\( \beta = \) hedge ratio
\( u_t = \) residuals from the price levels regression:

\[
S_t = \alpha + \gamma F_t + u_t
\]  

(3.16)

When the cointegrating relationship is ignored, an error correction variable is omitted, and the hedge ratios suffer from downward bias when the coefficient on the error correction variable is positive, according to Brenner and Kroner (1995, p 35). Other researchers including Lien (1996), Kroner and Sultan (1993), and Ghosh (1993b) have noted that when cointegration is not considered in the context of hedging, a smaller than optimal hedge ratio is obtained, resulting in relatively poor hedge performance. Similarly, using Nikkei 225 index futures, Lien and Tse (1999) find that the hedge ratios incorporating fractional cointegration are smaller than ratios incorporating full cointegration, and that the latter have superior out-of-sample performance. Chou et al (1996) also find that hedge ratios that incorporate cointegration are superior to the basic OLS ratios in the context of Nikkei index futures.

While most studies on futures hedging assume that the cost-of-carry is zero, Ferguson and Leistikow (1999) argue that the standard error correction model should be adjusted for cost-of-carry effects, and propose a Modified Regression Method (MRM). However, their empirical evidence comparing the standard methodologies and their proposed model produce very similar results, such that the relative performance between the models is not economically or statistically significantly different.

### 3.4.4 GARCH models

Another criticism of traditional regression approaches to hedge ratio estimation, made by researchers including Myers (1991), Myers and Thompson (1989), and Koutmos and Tucker (1996), is that they are based on the assumption that the covariance matrix of cash and futures prices is constant over time, implying that optimal hedge ratios are invariant over time. This has prompted researchers to apply more sophisticated techniques to calculate time-varying hedge ratios, such as generalised autoregressive conditional heteroskedasticity (GARCH) models.
Engle (1982) introduced an autoregressive conditional heteroskedasticity (ARCH) model, which allows for changing conditional volatility, where the conditional variance is expressed as a linear function of the first ‘p’ past squared innovations. Bollerslev (1986) developed a general version of the ARCH model (GARCH) by allowing the current conditional variance to depend on the first ‘q’ past conditional variances. Hedge ratio estimation techniques based on GARCH account for changing risk in the spot and futures markets, which may occur due to the arrival of new information. After an extensive review by Bollerslev et al (1992), they find that the GARCH(1,1) model specification is generally superior to more complex models.

The hedge ratio conditional on past information may be expressed as:

\[
\hat{h}_{t-1} = \frac{\text{Cov}(\Delta S_t, \Delta F_t | \Omega_{t-1})}{\text{Var}(\Delta F_t | \Omega_{t-1})}
\]

where

- \( \hat{h}_{t-1} \) = hedge ratio that minimises the conditional variance of the hedged portfolio return
- \( \Omega_{t-1} \) = information available at time t-1
- \( \text{Cov}(\Delta S_t, \Delta F_t | \Omega_{t-1}) \) = covariance of continuously compounded spot and futures returns, conditional on the information available at time t-1
- \( \text{Var}(\Delta F_t | \Omega_{t-1}) \) = variance of continuously compounded futures returns, conditional of information available at time t-1

To obtain the conditional hedge ratio, the conditional covariance matrix may be estimated using the BEKK parameterization of Engle and Kroner (1995), who extend the basic GARCH model to guarantee that the covariance matrix is positive-definite. This prevents the estimation of any negative variances. The conditional mean and variance that are jointly estimated are:

\[
y_t = \mu + \theta(f_{t-1} - s_{t-1}) + \varepsilon_t \tag{3.18}
\]

\[
\varepsilon_t | \Omega_{t-1} \sim N(0, H_t) \tag{3.19}
\]

\[
H_t = C'C + A'\varepsilon_{t-1}A + G'H_{t-1}G \tag{3.20}
\]

where

- \( y_t \) = vector of log-differenced spot and futures prices
- \( (f_{t-1} - s_{t-1}) \) = futures premium
- \( \mu \) = vector of means
- \( H_t \) = \((n \times n)\) conditional covariance matrix
- \( C, A, G \) = \((n \times n)\) parameter matrices
- \( \varepsilon_t \) = vector of error terms

GARCH models have been used to model the volatility of various equity indices and associated futures contracts, including the FTSE 100 Index (Antoniou and Holmes (1995)), the Australian All Ordinaries Index (Choudhry (1997), Bhar (2001)), the Nikkei 225 (SIMEX) Index (Lien and

Evidence as to whether GARCH models produce superior hedge ratios relative to traditional regression models is mixed. In an early application, Cecchetti et al (1988) apply an ARCH(3) model to Treasury bonds and T-bond futures, and find that optimal hedge ratios vary from 0.52 to 0.91. Park and Switzer (1995a) apply GARCH to S&P 500 stock index futures contracts and the Toronto 35 Index futures contracts, and detect an improvement in hedging performance relative to traditional models, even after transaction costs. Gagnon and Lypny (1995) apply a GARCH model to hedge interest rate risk associated with Canadian bankers acceptances, and find that the GARCH model outperforms naïve and OLS strategies. More recently, Gagnon and Lypny (1997) find that a GARCH(1,1) model provides superior hedge ratio estimates relative to OLS ratios in the context of hedging using the Toronto 35 Stock Index. In a different study of the Nikkei 225 stock index futures contract, Chen et al (1999) confirm that hedge performance depends “critically” on accounting for GARCH effects.

In contrast, other empirical studies challenge the superiority of GARCH hedge ratios. For example, Myers (1991) applies a GARCH model to hedge US wheat prices using wheat futures, and finds that the performance of the GARCH model is “only marginally better” than the performance of simple constant hedge ratios (p 52). He concludes that traditional OLS regression techniques that implicitly assume constant optimal hedge ratios may be “an adequate approximation” when hedging using US wheat futures (p 52). Similarly, Baillie and Myers (1991) apply a bivariate GARCH model to six commodities and their respective futures, and find that “the additional complexity of a GARCH model will be justified by superior hedging performance for some commodities but not others” (p 122). Also, Choudhry (2000) examines the Japanese, Hong Kong and Australian futures markets, and finds that within each market, GARCH hedge ratios outperform traditional ratios in some cases, but not in all cases. These findings are consistent with the view that the use of GARCH is appropriate only in some cases. Indeed, while Park and Switzer (1995a) correctly assert that their results indicate that GARCH produces superior hedge ratios for both assets examined, the GARCH ratio performs much better only for the TSE 35. However, care is required when comparing estimation techniques over a long hedge period, because a manager is unlikely to adopt a constant hedge ratio for an extensive time period without revision. A time-varying ratio would logically perform better than an unrevised constant ratio over long periods of time. However, it is possible that over shorter hedge periods, complex time-varying ratios are unnecessary. Thus, results like those of Choudhry (2000) may appear to support the superiority of time-varying ratios over constant ratios simply because the hedge is maintained for two years.
Dissatisfaction with the use of GARCH to calculate futures hedge ratios has been expressed more strongly by other researchers. For instance, after examining multi-period currency hedging decisions, Lien and Luo (1994, p 951) state that “notwithstanding the better statistical performance, the hedging performance of the GARCH hedge is not better than that of the constant or error correction hedges”. Further, “ex post criteria are not favourable to GARCH hedges at all”. Similarly, McNew and Fackler (1994, p 620) criticise the use of GARCH for estimating time-varying hedge ratios, describing it as “taxing to the practitioner”. McNew and Fackler (1994) find that in the context of corn futures and live cattle futures, a constant hedge ratio is a suitable assumption, and it is unnecessary to calculate complex hedge ratios. Also, Ferguson and Leistikow (1998, p 851) provide evidence that futures hedge ratios calculated using regressions are stationary and that out-of-sample hedge effectiveness is “not significantly improved by updating the hedge ratios”. Even if hedge ratios are found to vary over time in particular contexts, regression-based hedge ratios may be updated during the hedge period to achieve a time-varying effect. The frequency of revision of the hedge ratio would depend on the length of hedge period (the longer the hedge period, the more need for revision), and the cost of revision (the more frequent the adjustments, the more costly the hedge is to implement due to management and transaction costs).

Although GARCH models are complex, their use may be justified on the basis of increased accuracy of hedge ratios and hence superior “performance”, although given mixed empirical evidence, this apparently needs to be determined on a case-by-case basis.

### 3.4.5 Error Correction–GARCH Model

Drawing from the literature on cointegration between futures and spot markets and the literature on changing risk, researchers have proposed error-correction GARCH models to accommodate both effects in a single hedge ratio. For example, Kroner and Sultan (1993) apply a time-varying bivariate error correction model with a GARCH error structure to hedging foreign currency futures, and conclude that this model produces hedge ratios of superior risk reduction relative to conventional models. Park and Switzer (1995a) compare the GARCH EC model of Kroner and Sultan (1993) with a basic error correction model and basic OLS models, and find that GARCH EC is preferred on the basis of utility. They consider the S&P 500 and the TSE 35, but use only ninety-three data points in their regressions. Similarly, Park and Switzer (1995b) estimate risk-minimising futures hedge ratios for S&P 500 index futures, major market index futures and Toronto 35 Index futures, using a bivariate cointegration model with GARCH error structure. They find that their model is superior to conventional techniques, but they note that a disadvantage of the GARCH method is the frequent updating of the hedge to accommodate
changes in the optimal hedge ratio. In a comparison of several methods of hedge ratio calculation, Lien and Tse (1999) find that the error correction model that incorporates conditional heteroskedasticity improves hedging performance over hedge periods of 10 days or more. In summary, the GARCH EC model appears to be a superior technique for hedging single risks using single futures, although improvements in hedge effectiveness must be balanced against the increased complexity of the hedge ratio calculations and transaction costs.

3.4.6 Summary

In summary, a considerable body of literature exists on the estimation of futures hedge ratios in the context of the direct hedging of single risk exposures using single futures contracts. Hedge ratio estimation techniques range from the naïve model and simple OLS regression models to more complex models incorporating error-correction and GARCH effects. Empirical evidence is mixed as to whether the more complex models are superior, given the tradeoff between the accuracy of hedge ratios and computational complexity. The following section discusses cross-hedging using single futures contracts.

3.5 Cross-hedging single risks using single futures contracts

This section discusses theoretical and empirical research on the cross-hedging of single risks using single types of futures, a body of literature closely related to that addressed in the previous section. Cross-hedging is the process whereby the risk associated with a particular asset is reduced using a derivative written on a different asset. Anderson and Danthine (1981, pp 1187-88) state that “cross hedges are in order whenever the cash/futures correlation is a constant different from zero”, which illustrates the broad potential for cross-hedging. Cross-hedging is important because many assets do not have associated futures contracts. Australian managed funds hold diversified equity portfolios containing numerous assets, and the risks associated with a portfolio or components of a portfolio may not have associated futures contracts. In such circumstances, cross-hedging may be an appropriate risk management technique.

3.5.1 Theoretical work

The theoretical framework for cross-hedging single risks using single futures is a special case of the theory advanced by Anderson and Danthine (1980, 1981), discussed in Section 3.2. Other theoretical work on cross-hedging has been developed almost exclusively in the context of foreign exchange risk, by researchers including Broll et al (1995), Broll and Eckwert (1996), Broll (1997), Broll and Wahl (1998), Broll and Wong (1999), and Broll et al (1999). The main focus of these studies is cross-hedging by multinational corporations that export to different foreign markets. Some focus on cross-hedging foreign exchange risk in the absence of direct
hedging opportunities, such as Broll and Wahl (1998) and Broll et al (1995), while others dispute the desirability of full hedging that eliminates all foreign exchange risks, including Broll and Eckwert (1996), Broll (1997), Broll and Wong (1999), and Broll et al (1999).

3.5.2 Empirical evidence

Hedge ratio estimation techniques applied in direct hedging apply equally in cross-hedging, a point recognised by Anderson and Danthine (1981, p 1183). Cross-hedging single risks with single types of futures has been examined empirically in the context of agricultural commodities by Vukina and Anderson (1993), mortgaged-backed securities by Koutmos et al (1998), foreign currency risk in developed markets by Braga et al (1989) and Benet (1990), and foreign currency risk in emerging markets by Benet (1990), Mun and Morgan (1997), and Aggarwal and DeMaskey (1997). Two key issues addressed in prior research are the impact of choice of hedge ratio estimation technique on the effectiveness of cross-hedges, and the effectiveness of cross-hedges relative to direct hedges.

Evidence from the literature on simple direct hedging regarding the superiority of hedge ratios calculated using OLS regressions as opposed to naïve ratios is confirmed in the literature on cross-hedging. For instance, Braga et al (1989) use Deutsch Mark futures to cross-hedge the Lira/US Dollar exchange rate, and find that OLS regression hedge ratios are more effective than a naïve strategy, where performance is assessed using stochastic dominance. Similarly, Aggarwal and DeMaskey (1997) apply both naïve and price changes regressions to cross-hedge exchange rate risk associated with emerging Asian markets using futures and options from developed countries. Further, the prior finding that hedge ratios incorporating heteroskedasticity are superior to simple models in some cases is repeated in the research on cross-hedging.

Koutmos et al (1998) apply a bivariate error correction model with a GARCH error structure to cross-hedge fixed rate mortgage-backed securities using Treasury note futures. They find that the dynamic error correction-GARCH hedge ratios outperform traditional static regression hedge ratios, on the basis of in-sample and out-of-sample tests using variance-reduction and expected utility maximisation.

The second issue is whether cross-hedging provides advantages relative to other strategies, and specifically whether cross-hedges are superior to an unhedged position and a direct hedge. With respect to the first scenario, Mun and Morgan (1997) find that minimum-variance cross-hedges of single currencies with single futures, where the minimum variance position is established by regressing the return on spot against returns on futures\(^\text{10}\), provide superior performance relative

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\(^{10}\) This technique is equivalent to the price changes regression model where “prices” are taken to be the logarithmically transformed price levels of the spot and futures. In this case, the returns series used in the regression are continuously compounded.
to an unhedged position for all countries. Similarly, Aggarwal and DeMaskey (1997) find that cross-hedging risks associated with the currencies of emerging Asian markets using futures on the currencies of developed markets improve portfolio performance as measured by the Sharpe index, and this effect is stable over time. However, these studies fail to compare direct and cross-hedging, possibly because direct hedging is not possible in some cases.

Although it is likely that a direct hedge will outperform a cross-hedge, the latter may be preferred in cases where the transaction costs associated with direct hedging are relatively high or the market for the contract used in direct hedging is relatively illiquid. Empirical evidence comparing cross-hedging and direct hedging indicates that direct hedges are generally superior. For example, Eaker and Grant (1987) find that when hedging a single currency with a futures written on a different asset then the hedge is less effective than direct hedging, and that the effectiveness of cross-hedging “varies considerably” (p 96). Similarly, Braga et al (1989) find that while cross-hedging is less expensive than direct hedging on average (p 87), the effectiveness of cross-hedges varies more than that of direct hedges (p 98).

No clear indication has emerged as to the importance of the choice of cross-hedging instrument to the effectiveness of the hedge. The conflicting evidence may be illustrated in the context of hedging currency risk. On one hand, Eaker and Grant (1987) find that “commodity futures are not effective cross-hedges for currencies” (p 87). On the other, Benet (1990) demonstrates that commodity futures are an “equally effective, and perhaps more stable, alternative to currency contracts for hedging minor currency exposure” (p 302). “For example, a hedger should consider soybean futures just as seriously as pound or deutschmark futures when constructing a cross hedge ex ante against movements in the Brazilian cruizero” (p 302). Logically, there is no reason to expect currency futures to achieve greater hedge effectiveness than any other type of futures if the former are not as highly correlated with the spot asset. However, Benet’s (1990) result may no longer hold, because globalisation has increased the integration between foreign markets and increased the correlation between different currencies.

3.5.3 Summary

In summary, hedge ratio estimation techniques used in the direct hedging of single assets have been readily applied to cross-hedging single assets using single types of futures. While this basic extension has not presented major empirical difficulties, the results are mixed as to the benefits of cross-hedging. Empirical evidence indicates that cross-hedging currency risk is better than an unhedged position, but direct hedging appears preferable to cross-hedging. However, cross-hedging using futures contracts remains an important option because direct hedges are not always available.
3.6 Hedging multiple risks simultaneously

Prior research considered in the previous two sections focuses on the use of single futures contracts to hedge single risk exposures. However, diversified investors hold numerous assets and are exposed to numerous risks simultaneously. For instance, managed fund portfolios are diversified, as discussed in Chapter Two. Co-dependence between asset returns within a diversified portfolio plays a pivotal role in the determination of appropriate hedging strategies. For instance, natural hedges may arise within a portfolio, obviating or reducing the need for hedging using derivatives. To illustrate, the equity portfolios of managed funds are comprised of shares in various companies. If the returns on different shares in the portfolio are negatively correlated, they interact in such a way that the overall risk of the portfolio is reduced without the implementation of a futures hedge. However, if the returns on different shares are positively correlated, they move together such that the overall risk of the portfolio is enhanced relative to a portfolio where the returns are negatively correlated. In short, the interaction of portfolio components in multi-asset portfolios affects the appropriate hedging strategy.

This section discusses the theory and empirical evidence surrounding the complex problem of hedging multiple risk exposures simultaneously. Broadly, prior research may be divided into studies on hedging single assets using multiple futures contracts, and studies on hedging portfolios of assets using multiple futures contracts.

3.6.1 Theoretical work
Anderson and Danthine (1981) argue that hedgers may wish to trade in multiple futures contracts when there are no transaction costs and no perfect hedge. They argue that given transaction costs or "indivisibilities", the selection of an appropriate futures contract or portfolio of futures contracts to hedge or speculate is a question that may be answered empirically (p 1183), an issue addressed in this thesis. Anderson and Danthine (1981) propose that when estimating hedge ratios given multiple futures contracts, "the proportion of output that should be hedged in each contract is given by the coefficient of the theoretical multiple regression of cash prices on the coefficient of the theoretical multiple regression of cash prices on the n futures prices" (p 1188). This suggestion has been adopted throughout the literature to the extent that regression method dominates empirical analysis in this area.

3.6.2 Hedging single assets using multiple futures contracts
Hedging single assets with portfolios of futures has been investigated empirically in the context of foreign exchange risk by Eaker and Grant (1987), Mun and Morgan (1997), Benet (1990),
and DeMaskey (1997), commodity risk by Grant and Eaker (1989), and the risk of fixed-rate mortgaged-backed securities by Koutmos and Pericli (2000). An issue central to these studies is whether superior hedge effectiveness is obtained using a single type of futures contract or multiple futures contracts. As in studies on hedging with single contracts, comparisons between hedge ratio estimation techniques figure prominently in the literature on hedging using portfolios of futures.

Empirical evidence favouring the use of single futures contracts over multiple futures contracts is provided by Grant and Eaker (1989) in the context of commodities, and Koutmos and Pericli (2000), in the context of interest rate risk. Grant and Eaker (1989) hedge and cross-hedge oats, wheat and corn separately, using single and multiple futures contracts on those products. They conclude that complex hedges are not more effective than simple and naïve hedges, stating that “[n]aïve matching of spot and futures contracts in the same commodity reduces risk as effectively as simple regression estimates. Both of these methods perform as well or better than multivariate hedges that include futures in different commodities.” (p 26). Similarly, Koutmos and Pericli (2000) conduct out-of-sample tests comparing the effectiveness of single and multiple Treasury note futures contracts to hedge fixed-rate mortgaged-backed securities. They find that hedging using ten-year Treasury futures provides superior performance relative to hedging with multiple interest rate futures contracts simultaneously. Koutmos and Pericli (2000) suggest that the high correlation among the futures caused the hedge ratios to be inaccurate, which may explain their result.

In contrast, empirical evidence supporting the use of portfolios of futures to hedge single risks is presented by Eaker and Grant (1987) and DeMaskey (1997) in the context of foreign exchange risk, and Miller (1985) in the context of commodities. Eaker and Grant (1987) examine the cross-hedging of two groups of currencies, the first containing British, Canadian and Japanese currencies, and the second comprising the Italian, Greek, Spanish and South African currencies. Using both in-sample and out-of-sample measures of hedge effectiveness, Eaker and Grant (1987, p 90) find that when hedging a single currency using a portfolio of futures including a futures written on the particular asset, then there is no gain in using multiple futures relative to performance using direct hedging, which is “very effective”. When hedging a single currency with a futures portfolio that excludes futures written on the spot asset, then the hedge is less effective than direct hedging. However, when “cross-hedging is the only alternative multiple cross-hedges are more effective than simple cross-hedges” (p 87). The appeal of the findings of Eaker and Grant (1987) lies in their careful distinction between scenarios where direct hedging is and is not possible. Miller (1985) and DeMaskey (1997) similarly find that multiple hedges
are superior to single hedges in the contexts of commodities and European and Asian currency risks respectively.

The lack of consensus in the general literature is exemplified by the study of Mun and Morgan (1997), who use futures on British, Canadian, German, Japanese and Swiss currencies to cross-hedge the foreign exchange risk associated with emerging Asian markets. Measuring performance using the Sharpe index, they find that a portfolio of futures provides superior protection relative to single futures hedge for Indonesia, Singapore, and Thailand, while a single currency futures hedge is superior in the cases of Korea and Malaysia. In short, the results are country-specific, and not readily generalisable. Eaker and Grant (1987) may be distinguished from Mun and Morgan (1997) by the measures of effectiveness, the markets examined and the time period. However, the results may be reconciled if the scenarios examined by Mun and Morgan were divided according to when direct hedging is and is not possible.

In summary, prior evidence is mixed as to whether portfolios of futures outperform single futures as hedging instruments given the aim of controlling single risk exposures. Eaker and Grant (1987) find that the benefits of using multiple futures are contingent on whether the opportunity for direct hedging exists. If direct hedging is possible, there is no benefit from using multiple futures contracts, but in the absence of a direct hedge, multiple futures contracts appear to be superior to cross-hedges using single futures contracts.

The preference for single or multiple futures hedges is examined empirically in this thesis in the context of domestic and foreign equity risk associated with diversified managed fund portfolios. This is an important issue because, for example, the foreign equity component of a diversified fund portfolio has no single futures contract written on it, and direct hedging is unavailable. Prior evidence that cross-hedges using multiple contracts may outperform cross-hedges using any single contract may not necessarily be extrapolated to hedges of equity risk, because prior evidence is largely restricted to foreign exchange risk and also ignores the extra costs and required management expertise associated with the use of multiple contracts.

3.6.3 Hedging multiple risks using multiple futures contracts

A logical extension of hedging single risk exposures with portfolios of futures is the more complex situation of hedging multiple risk exposures simultaneously using multiple futures contracts. Researchers in this area, including Lypny (1988), Lien (1990), and Collins (2000), often refer to “portfolio effects”, which are the interactions between the components of spot and futures portfolios. For instance, Collins (2000, p 190) defines portfolio effects as “the extent to which price movements of the firm’s endowments are ‘self hedged’”. Similarly, Lien (1990, p
201) observes that "any futures contract serves both direct and cross hedging purposes", and argues that this must be considered when measuring hedge effectiveness. Specifically, the traditional effectiveness of a single futures contract can be thought of as its "marginal" contribution to the reduction of the overall risk.

An important issue in the research on hedging multiple risks simultaneously is how to accommodate portfolio effects in the calculation of hedge ratios. Lypny (1988, p 703) suggests that the portfolio effects are caused by relationships which are not stable over time, and Anderson and Danthine (1981) argue that the limited ability to identify stable moments is likely to be "the most serious constraint in developing very extensive multiple cross-hedges." (p 1196). This point is addressed by Gagnon et al (1998), who use trivariate GARCH to allow for time-varying covariability between all components of the system, and find that accounting for "portfolio effects" leads to efficiency and utility gains. Specifically, they compare constant hedge ratios calculated using regressions for each individual component of the portfolio, constant hedge ratios calculated from multivariate OLS, time varying hedge ratios calculated using a bivariate system ignoring portfolio effects, and time varying hedge ratios calculated using a multivariate system. Similarly, Fackler and McNew (1993) study U.S. soybean processors and find that hedge ratios from single commodity estimates which ignore portfolio effects are suboptimal, and that a multiple regression approach incorporating portfolio effects is superior in terms of reducing risk. Further support for the use of hedge ratio estimation techniques that incorporate portfolio effects is provided by Tzang and Leuthold (1990), who also study the U.S. soybean complex.

In contrast, other empirical studies support the use of simple estimation techniques rather than complex methodologies that incorporate portfolio effects, such as Lypny (1988), Eaker and Grant (1987) and Grant and Eaker (1989), Garcia et al (1995), and Collins (2000). For instance, Eaker and Grant (1987) and Grant and Eaker (1989) consider hedging portfolios of currencies and commodities respectively using multiple futures contracts. Both studies calculate naive, simple and complex hedge ratios to hedge two-asset and three-asset portfolios. Naïve positions are established by "matching the unit cash position in each currency with a unit futures position" (Eaker and Grant 1987, p 94), and "simple" hedges are calculated using traditional univariate regressions. Grant and Eaker (1989, p 24) describe the simple hedge as a "weighted average of the simple hedges of the individual spot positions" where "the weights are the proportions that each spot position represents in the portfolio". "Complex" hedge ratios are calculated using multivariate regressions, where the dependent variable is changes in the portfolio value and the independent variables are first differences of futures prices. Both Eaker and Grant (1987) and Grant and Eaker (1989) find that when hedging portfolios of assets using
multiple futures, naïve strategies are as effective as complex models on the basis of out-of-sample comparisons.

Similarly, Collins (2000) compares price changes OLS regression, a naïve strategy, and the methodologies of Anderson and Danthine (1980, 1981), Fackler and McNew (1993), and Tzang and Leuthold (1990), in the context of the U.S. soy complex. He concludes that "there is no evidence to suggest that any of the multivariate hedging models or the univariate risk-minimising model offer any risk-management advantages over a simple equal and opposite hedge" (p 202). However, the results presented by Eaker and Grant (1987), Grant and Eaker (1989) and Collins (2000) are surprising, given the assumptions underlying the naïve strategy, as discussed above in Section 3.4.1. Also, these findings are specific to risks associated with currencies and agricultural commodities, which have different characteristics from financial instruments such as equities. Hence, the results may not apply in the context of equity risk, and given other empirical evidence in favour of hedge ratio estimation methods that account for portfolio effects, this issue is examined in this thesis.

### 3.6.4 Summary

In summary, prior research on hedging multiple price risks using multiple futures focuses on hedge ratio estimation methods, and in particular on whether accounting for portfolio effects improves hedge performance, where portfolio effects are complex interactions between the components of a portfolio. As with much literature on hedge ratio estimation, there is no clear consensus as to whether complex techniques outperform simple methods. However, research on this subject indicating that simple naïve strategies perform as well as more complex hedges is surprising, and suggests the need for further investigation. With a view to extending the evidence, this thesis expands the work of Gagnon et al (1998) in several ways. Their $n$-asset, $n$-futures approach overlooks the common problem that futures are not written on every asset. In this thesis, this restriction is relaxed such that the number of futures contracts does not necessarily equal the number of individual assets forming the spot portfolio. In short, this thesis accommodates the $m$-asset, $n$-futures situation where $n$ does not necessarily equal $m$. This is an improvement because even if all assets in the portfolio had associated futures contracts, it may not be appropriate to hedge every single component of the portfolio because of the complex co-dependencies within the portfolio. Further, this thesis improves on the work of Gagnon et al (1998) by embracing an out-of-sample assessment of hedge performance, rather than relying exclusively on in-sample tests of hedge effectiveness as they do.
3.6.5 Summary of hedge ratio estimation literature

When the literature on hedge ratio estimation is considered from a general perspective, the shifting focus of the research as the complexity of the problem increases becomes apparent. In simple studies on the direct hedging of single risks with single futures, the choice of hedge ratio estimation technique is emphasised. Research on cross-hedging single risks using single futures focuses on whether cross-hedging is superior to direct hedging or not hedging. Where a single asset is hedged using multiple futures, and where multiple risks are hedged simultaneously using multiple futures, the main issue addressed is whether hedges using one derivative are better than those using multiple instruments. Themes of universal interest are the relative performance of different hedge ratio estimation techniques, and whether complex hedges are superior to simple hedges. This thesis examines some of these issues from an Australian perspective, as discussed in detail in the following chapter. Comparisons of hedge ratio estimation techniques require an understanding of the measurement of hedge effectiveness, and the factors which contribute to hedge effectiveness, as discussed in the following section.

3.7 Measuring hedge performance

3.7.1 Expected utility

The issue of measuring hedge performance is common to all studies on hedging. The dominant framework for performance measurement in the context of hedging is the mean-variance portfolio theory of Markowitz (1959), which emphasises risk and expected return. For instance, Peck (1975, p 412) states that “the Markowitz approach provides the traditional formulation of the decision criterion where both expected returns and risk are important”. As discussed in Section 3.3, it is assumed that investors aim to maximize expected utility, and this assumption may be reconciled with the mean-variance framework when the investor’s utility function is quadratic, according to Markowitz (1959, p 288). Hence, quadratic expected utility, $E(U)$, is employed to measure hedge performance in this thesis, and is calculated as:

$$E(U) = E(H_t) - \lambda \text{Var}(H_t)$$

where

$$\lambda = \text{risk aversion parameter}$$

$$R_{ut} = \text{return on unhedged equity portfolio}$$

$$R_{ft} = \text{return on futures position}$$

$$H_t = \text{return on the hedged portfolio}$$

This approach rests on the simplifying assumption that equities constitute the universe of investments, and treats the portfolio manager as the “investor” where the equity portfolio is their only asset. This allows the findings to be more general than a case study that considers
performance from the perspective of an individual contributor to the fund who may hold an infinite variety of other assets. The results in this thesis are of interest to investors hedging equity risk, and the treatment of portfolio managers as investors for the purposes of expected utility measurement is reasonable given that the same decisions confront individuals managing diversified portfolios in a world where equities are the only assets. The aim is to assess the relative performance of risk management techniques, and the performance measure is useful only to the extent that it is a ranking device. Expected utility is preferred to complex alternatives such as stochastic dominance and Value at Risk, which are discussed in detail below, because expected utility is a common choice in the literature on futures hedging and is relatively easy to implement and interpret.

An important distinction between all measures of hedge effectiveness is whether in-sample or out-of-sample analysis is required. In the context of hedging, in-sample tests of effectiveness rely on the same data set to estimate the hedge ratio and assess the effectiveness of that parameter. In contrast, out-of-sample tests utilise one data set to estimate the hedge ratio and a second separate data set from a subsequent period to test the effectiveness of the ratio. Out-of-sample tests are more realistic, because in-sample tests assume that the hedger has perfect foresight. Out-of-sample measures of hedge performance are employed in this thesis, in contrast to much of the literature on hedging.

3.7.2 Alternative measures of performance

Literature on futures hedging that does not rely on expected utility to assess hedge effectiveness generally depends on other methods that emphasise risk and expected return. For instance, Ederington (1979), Grant and Eaker (1989) and Eaker and Grant (1987) propose measures based on risk-minimisation. Other measures based on risk-return considerations have been advanced by Sharpe (1966, 1994), Howard and D’Antonio (1984, 1987), Chang and Shanker (1987), Gjerde (1987), Lindahl (1991), and Pennings and Meulenberg (1997). However, some of these models produce inconsistent results on empirical application. For instance, Brailsford et al (2001) demonstrate that the measures of effectiveness of Howard and D’Antonio (1987) and Lindahl (1991) do not produce consistently accurate results on empirical application to Australian data. Hedgers are required to exercise caution when selecting performance measures.

The widespread application of risk and expected return criteria in the measurement of hedge performance is attributable to their simplicity and intuitive appeal. However, alternative frameworks for analysing hedge effectiveness have emerged, including stochastic dominance, which considers the entire distribution of returns, and Value at Risk analysis, which considers the tails of the return distribution to determine the probability of losses.
Rules of stochastic dominance indicate the mathematical conditions under which one return distribution will be preferred to another, as discussed by Whitmore (1970), Levy (1992), Thistle (1993) and Conover and Dubofsky (1995). In the context of futures hedging, the Mean-Gini approach utilises stochastic dominance, as discussed by Kolb and Okunev (1992), Kolb and Okunev (1993), Lien and Luo (1993a), Shalit (1995), Lien and Shaffer (1999), Chen et al (2001). However, until the practical difficulties associated with the empirical application of Mean-Gini are resolved, it is unlikely to be preferred in practice relative to approaches based on mean-variance. Further, although stochastic dominance is promoted as imposing few restrictions on return distributions relative to mean-variance approaches, this feature is of greatest importance in the presence of non-normal distributions such as those heavily skewed by the inclusion of options. Futures contracts have symmetric payoffs, and therefore do not present the difficulty associated with options, reducing the incentive to adopt a stochastic dominance framework.

Other methods of assessing performance have been developed that focus on “downside” risk, which is the risk that loss on some investment will occur. Recently, for instance, Lien and Tse (1998), Van der Hoek and Sherris (2001), and Stevenson (2001) develop downside risk models. In classic papers, Roy (1952) and Arzac and Bawa (1977) use Value at Risk analysis to measure the downside risk faced by a “safety-first” investor. Value at Risk measures, applied by Baillie and DeGennaro (1990), Fong and Vasicek (1997), Brooks and Persand (2000), and Jansen et al (2000), may involve averages, exponentially weighted moving average, GARCH, or simulations. However, as Brooks and Persand (2000) note, a number of issues remain unresolved in Value at Risk analysis, including determining an appropriate sample size, lag length, method, and the impact of the positive relationship between volatility and correlation on the results. Downside risk measures such as Value at Risk are not applied in this thesis due to the uncertainty regarding their empirical application. The resolution of this uncertainty is a separate topic for further research. Further, extreme value measures such as Value at Risk focus exclusively on the "downside" risk of the portfolio, while investors also consider the upside potential, a factor that should be reflected in the measure of performance.

3.7.3 Summary

In summary, consistent with the theoretical framework for futures demand presented in Section 3.3, hedge performance in this thesis is measured using out-of-sample expected utility, which is a simple and intuitively appealing approach based on risk and expected return. This is consistent with the mean-variance theory, which is the dominant framework for measures of hedge effectiveness in the literature on futures hedging. Sensitivity analysis is conducted with regard
to the risk aversion parameter, and also using simple risk-return measures of performance that fall within the mean-variance framework. These methods are preferred to complex alternatives based on stochastic dominance and Value at Risk, which are relatively difficult to implement and interpret.

3.8 Factors affecting hedge construction and performance

Factors which may influence the effectiveness of a hedge include the hedge period, estimation period, timing of futures rollover, the maturity effect, thin trading, transactions costs, and taxes. Literature addressing each issue is discussed briefly in this section.

3.8.1 Hedge period

Malliaris and Urrutia (1991) examine the impact on hedge effectiveness of the length of hedge period, which is the length of time over which the hedge is undertaken. They conclude that for in-sample hedges, longer hedges are more effective. According to Geppert (1995), this result is not surprising given that it is generally accepted that in-sample hedging effectiveness increases as hedge period increases, based on the explanation that a reduction in noise occurs over the long term.

However, Malliaris and Urrutia (1991) also conduct out-of-sample tests, and conclude that shorter hedges (weekly) are more effective than longer hedging horizons (monthly). Similarly, Benet (1992, p 171) detects a general negative relationship between hedging period length and performance, and concludes that it is likely that “the intertemporal stability of hedge ratios is driving the unusual ex-ante results discovered”. Further, Geppert (1995) applies a “decomposition model” based on cointegration between spot and futures prices to investigate the relationship between hedge period and effectiveness. This is in contrast to Bennet (1992) and Malliaris and Urrutia (1991) who employ simple regression techniques. Geppert (1995, p 532) finds that out-of-sample hedging effectiveness tends to decrease as the hedge period increases. These studies are limited to the extent that they examine only a small number of hedge periods, and generally focus exclusively on short-term hedges.

3.8.2 Estimation period

A limited number of studies address the impact of the length of the period used to estimate the hedge ratio on the effectiveness of the hedge. Malliaris and Urrutia (1991) and Benet (1992) find that the length of estimation period is irrelevant to the effectiveness of foreign currency futures hedges.
3.8.3 Roll-over technique

When a futures contract which is being used to hedge a position in an underlying asset expires before the hedge period ends, the hedger must "roll" the hedge forward. Rollover is achieved by reversing out of the futures contract position currently held, and taking a position in a futures contract with a later maturity date.

There is surprisingly little empirical evidence regarding the impact of the timing of rollover on the effectiveness of hedge ratios. Most empirical studies explicitly or implicitly assume a rollover technique without discussion or sensitivity analysis. For instance, Kroner and Sultan (1993, p 540) choose a rollover of three weeks prior to expiry of the nearest futures contract, Park and Switzer (1995a, p 64) and Gagnon and Lypny (1995, p 772) assume a roll-over one week prior to expiry, and Lien and Tse (1999, p 463) use a roll-over ten days into the month, where expiry occurs on the third Wednesday of the contract month. The exception is provided by Ma et al (1992), who study the effect of choices regarding the roll-over of futures contracts on common empirical analyses of futures payoff distributions, serial dependence and day-of-the-week effects. Examining futures on gold, the S&P 500, T-bonds, Japanese yen, and soybeans, they find that the choice of roll-over technique biases results from the three tests performed (p 211), and suggest that the "selection of the "best" method may be contract specific" (p 216). They were unable to conclude which rollover technique is superior, and their study could only be described as preliminary.

Neuberger (1999) discusses the problems associated with adopting short-term oil futures contracts to hedge long-term risk. He provides evidence that for a hedge period of six years, it is preferable to use "medium-maturity" oil futures contracts, of up to eight months. However, this idea has not been investigated in the context of other futures contracts. Further, it is unlikely that such a concept will be appropriate in the share price index futures market because, as researchers including Shalen (1989) and Hancock and Weise (1994) note, the majority of hedgers prefer to use a contract with short time to maturity rather than one with a long time to maturity. This is because contracts with a long time to maturity exhibit low liquidity relative to contracts closest to expiry, a point recognised by Neuberger (1999, p 429). Indeed, Shalen (1989, p 215) suggests that a trade-off exists between liquidity and hedging effectiveness. For the purposes of this thesis, the hedger is assumed to employ the futures contract closest to maturity.

3.8.4 Maturity effect

An issue closely related to rollover technique is the "maturity effect", first discussed by Samuelson (1965). The maturity effect refers to the phenomenon of increasing volatility in the
futures market which is evident as maturity of the futures contract approaches. This occurs because market competition forces the spot and futures prices to converge at maturity. Galloway and Kolb (1996) describe the effect as a source of nonstationarity in futures prices which arises because prices of futures contracts close to expiration react more strongly to new information about the underlying commodity than do prices of futures contracts further from expiration.

Empirical evidence investigating the maturity effect is mixed. In an early study, Rutledge (1976) finds no evidence of the maturity effect for wheat and soybean oil futures, but silver and cocoa futures prices are more volatile just prior to expiry. Milonas (1986) and Khoury and Yourougou (1993) find evidence supporting the maturity effect in the context of agricultural, financial and metal futures. In contrast, Chen et al (1999) examine the maturity effect in the context of the Nikkei 225 stock index futures contracts using a GARCH model. They conclude that price volatility decreases as the futures contract approaches maturity, a result contradictory to Samuelson’s (1965) model. The lack of consensus in the literature suggests that the maturity effect may be contract-specific. Indeed, Rutledge (1976) found that the maturity effect is supported in the context of cocoa and silver futures contracts, but is rejected for wheat and soybean oil futures contracts. Similarly, Galloway and Kolb (1996) conduct an extensive empirical survey, and conclude that the maturity effect has a strong impact on futures prices for commodities subject to seasonal demand or supply, but is not important for commodities whose prices accord with the cost-of-carry model. However, the cost of carry model does not necessarily apply to stock index futures contracts. For instance, Heaney (1995) observes that the cost of carry model does not apply seamlessly to the Australian All Ordinaries Share Price Index Futures contract. Thus, the maturity effect may have an impact on hedges employing the All Ordinaries Share Price Index futures contract.

Stoll and Whaley (1997) examine other “expiration-day” effects associated with the All Ordinaries Share Price Index Futures contract. They consider both abnormal trading volume in the underlying index and price movements in the underlying index. Although they find some evidence that the trading volume of the index is higher around futures expiry, they do not detect systematic differences in index prices around expiry.

3.8.5 Thin trading

In addition to thin trading evident in futures contracts with long times to maturity, thin trading in spot assets, especially equities, may impact on calculated returns and hence on the assessment of hedged and unhedged portfolios. Theobald and Yallup (1997) observe that thin trading

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11 Nikkei 225 futures data are available from three sources, namely the Osaka Securities Exchange, the Singapore SIMEX and Chicago Mercantile Exchange. The significance of this is discussed in a later chapter. Chen et al (1999) obtain data from the Osaka Securities Exchange.
adjustments in the presence of linkages between the spot and futures markets will lead to biased hedging ratios. They conclude that adjustments to hedge ratios calculated using ordinary least squares regressions are only beneficial when the proportion of the index traded consecutively with the futures is below around 95%.

3.8.6 Transaction costs
Transaction costs impact on hedge performance, according to Hirshleifer (1988) and Frechette (2000). Transaction costs associated with hedging have been variously defined by researchers including Kritzman (1993), Collins and Fabozzi (1991) and Graham and Smith (1999) to encompass explicit costs such as management commissions, brokerage fees, execution costs, and implicit costs like the opportunity cost of management time, and the effect of the discount or premium of the forward contract (the implicit cost of the bid-ask spread).

3.8.7 Full and partial hedging
A practical decision influenced by transaction costs is whether a full or partial hedge is appropriate. Zilcha and Broll (1992, p 473) demonstrate theoretically that “unbiased futures market does not imply full-hedging by which the firm avoids price risk altogether”, in contrast to the full-hedging theory described by Holthausen (1979) and Kawai and Zilcha (1986). Similarly, Broll and Eckwert (1996, p 286) argue that in the context of cross-hedging exchange rate risk, “individually unbiased forward markets do not imply a full hedge, even if the firm has the possibility of hedging all risks. If the forward markets are jointly unbiased, then the firm fully hedges.” Froot et al (1993, p 1655) find that an implication of their theoretical work on why firms hedge is that “optimal hedging strategy does not generally involve complete insulation of firm value from marketable sources of risk.” More recently, Arias et al (2000, p 392) argue that “theoretical and empirical models usually suggest that farmers should hedge much more than they do”. The empirical fact that hedgers are taking positions smaller than “optimal” hedge ratios suggests the need for further research. In the context of equity risk, fund managers or investors may hedge only part of their portfolio, reducing risk and locking in a lower return on part of the investment, while simultaneously allowing higher risk and a potentially higher return on the remaining portion.

3.8.8 Summary
Factors identified in this section as potentially influencing hedge performance include the hedge period, the estimation period, assumptions regarding the roll-over of futures contracts, the maturity effect, thin trading, transaction costs, and the decision to hedge risk fully or partially. The specific assumptions adopted in this thesis are discussed and justified in later chapters, and relevant sensitivity analysis is performed.
3.9 Conclusion

For an investor who maximises expected utility, where expected utility is defined within the mean-variance framework, the demand for futures contracts consists of a pure hedge component and a speculative component. The majority of research on futures hedging, including this thesis, is restricted to the examination of the pure hedge component of futures demand. A considerable body of literature exists on the estimation of futures hedge ratios in the context of hedging single assets using single futures contracts. The techniques available include OLS regressions, error correction models, GARCH models, and models that combine error correction and GARCH features. The empirical application of these models has been studied in the context of direct hedging of risks using futures written on the particular asset being protected, and in the context of cross-hedging, where an asset is hedged using a derivative written on a different asset. Diversified portfolios contain multiple assets and hence are exposed to multiple sources of risk. Prior research on hedging multiple risks using multiple futures contracts simultaneously is limited. The lack of consensus in the literature with respect to the relative performance of simple and complex hedge ratio estimation techniques and the use of single or multiple futures contracts motivates further research in this area. The specific research questions addressed in this thesis are presented in the following chapter.
CHAPTER FOUR

RESEARCH QUESTIONS AND METHOD

4.1 Introduction

This chapter provides an overview of the research questions investigated in this thesis. Broadly, this thesis examines the use of futures contracts to hedge equity risks associated with diversified portfolios. The research is conducted from the perspective of Australian investors, and in particular Australian managed funds. This thesis presents four studies of hedging. In this chapter, the context for the research is briefly restated, an overview of equity portfolio construction is given, the treatment of foreign exchange risk is addressed, and the choice of particular futures contracts for use as hedging vehicles is discussed. The research topics and their motivations are identified, accompanied by a brief overview of the method. Detailed descriptions of method are given in the relevant subsequent chapters.

4.2 Research context

Recall from Chapter Two that diversified portfolios are exposed to numerous risks, and equity is generally considered to entail greater risk than other asset classes. In addition to traditional motivations for risk management and hedging discussed in the literature, it is arguable that all managers aim to prevent the value of their portfolio from decreasing below a certain limit, because portfolio losses impact on their professional reputation and remuneration. Hence, managers may wish to prevent the value of the equity component of their portfolios from decreasing, although this depends on investment style and does not imply the full hedging of all risks at all times.

Given the benefits of using futures contracts relative to options and dynamic asset allocation strategies, this thesis focuses on the use of futures contracts to hedge and cross-hedge the domestic and foreign equity risk associated with the equity component of internationally diversified portfolios. Foreign equity includes any equity investment that is not Australian. The measurement of hedge effectiveness is uniform throughout this thesis to facilitate cross-chapter comparisons.
4.3 Equity portfolios

This section provides an overview of the construction of equity portfolios, which is discussed at length in the following chapter. In recognition of the diversity of investment styles of Australian managed funds and other Australian investors, a range of portfolios are considered in this thesis. In order to construct portfolios, data indicating the asset allocations, or proportion of investment in different asset classes, is required along with data on assets that represent each asset class. Asset allocations for Australian managed funds are provided by William M. Mercer Investment Consulting, where funds are divided into several styles based on relative risk aversion. Equity investments are classed either as domestic Australian or foreign, and several allocations between these classes are employed to capture different fund styles. Domestic equity is represented using the Australian All Ordinaries Share Price Index. Foreign equity is represented in two ways: first, using the MSCI World excluding Australia Index, and second using a portfolio constructed using the regional allocations adopted by Templeton World Fund. A variety of diversified portfolios containing Australian and foreign equity are constructed.

4.4 Foreign exchange risk

Equity risk is the only risk under consideration in this thesis. However, the total risk associated with internationally diversified equity portfolios is a function of both equity risk and foreign exchange risk, given that foreign investments are traded in foreign currency. Foreign currency risk associated with internationally diversified equity portfolios is accounted for by converting all time series from foreign currency into Australian dollars. Specifically, returns on all foreign equity indices are expressed in Australian currency. Consider the general example given in Table 4.1, which shows the general form for the re-denomination of the value of a foreign futures position into Australian dollars. In this example, the currency is U.S. dollars, but this demonstration applies regardless of the particular foreign currency.

<table>
<thead>
<tr>
<th>Date</th>
<th>Settlement Price (USD)</th>
<th>Contract unit (USD)</th>
<th>Value (USD)</th>
<th>Exchange rate (USD:A$)</th>
<th>Value (A$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/mm/yy</td>
<td>A</td>
<td>D</td>
<td>AD</td>
<td>E</td>
<td>AD/E</td>
</tr>
<tr>
<td>2/mm/yy</td>
<td>B</td>
<td>D</td>
<td>BD</td>
<td>F</td>
<td>BD/F</td>
</tr>
<tr>
<td>3/mm/yy</td>
<td>C</td>
<td>D</td>
<td>CD</td>
<td>G</td>
<td>CD/G</td>
</tr>
</tbody>
</table>

Table 4.1: Conversion of the value of a U.S. futures position from U.S. dollars into Australian dollars
When the exchange rate is constant across time, \( E = F = G \) and the return at 2/mm/yy is:

\[
\ln \left( \frac{BD}{AD} \right) = \ln \left( \frac{BD}{AD} \right) = \ln \left( \frac{B}{A} \right) \tag{4.1}
\]

However, when the exchange rate changes across time, the return becomes:

\[
\ln \left( \frac{BD}{AD} \right) = \ln \left( \frac{BD}{AD} \right) = \ln \left( \frac{B}{A} \right) \tag{4.2}
\]

\[
= \ln \left( \frac{B}{F} \right) - \ln \left( \frac{A}{E} \right) \tag{4.3}
\]

\[
= \left[ \ln (B) - \ln (F) \right] - \left[ \ln (A) - \ln (E) \right] \tag{4.4}
\]

Rearranging (4.4):

\[
\ln \left( \frac{BD}{AD} \right) = \ln \left( \frac{B}{A} \right) - \ln \left( \frac{F}{E} \right) \tag{4.5}
\]

Comparing equations (4.1) and (4.5), it is clear that changes in the exchange rate affect the return calculation by the final term in equation (4.5). In summary, if the exchange rate is constant, there is no need to adjust for it in return calculations. However, if the exchange rate is changing over time, it is necessary to convert all prices into Australian currency. As the exchange rate is changing in practice, all price series in this thesis which are denominated in foreign currency are converted into Australian dollars prior to the calculation of returns. Thus, the price at any given date is expressed in Australian dollars using the exchange rate on that date, and the return series which are subsequently calculated are also denominated in Australian dollars. Accounting for foreign currency risk in this manner facilitates the isolation of equity risk for further analysis.

4.5 Futures contracts

Five futures contracts are selected as hedging vehicles, namely the Australian Share Price Index futures contract, the U.S. S&P 500 futures contract, the Japanese Nikkei 225 futures, the U.K. FT-SE Index futures contract, and the Brazilian Bovespa Index futures contract. These contracts are selected on the basis of region and correlation with the MSCI World index, as shown in Table 4.2. It is assumed that the MSCI World Index represents a fully diversified global portfolio for the purposes of this selection procedure. Co-movement of futures and spot series, as commonly measured by correlation, is an important variable when determining hedge ratios. It is expected that there is a high correlation between each futures and its respective equity market, so that the level of correlation between an equity market and the World Index is likely to be a good indication of the level of correlation between the futures on that equity and the
World Index. Within their respective regions, the U.S., U.K., Japanese and Brazilian equity markets have the highest correlation with the world equity index relative to other countries. The importance of these contracts is evidenced by their relatively high trading volume. The S&P 500 is the largest equity index contract traded on the Chicago Mercantile Exchange, and the Nikkei 225 is the second largest, as observed by Booth et al (1996).

Table 4.2: Correlation between the monthly continuously compounded returns on individual countries’ equity and the monthly continuously compounded returns on the MSCI World Price Index, where returns are denominated in Australian dollars

<table>
<thead>
<tr>
<th>Region</th>
<th>Country</th>
<th>Correlation with World Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>North America</td>
<td>USA</td>
<td>0.8265</td>
</tr>
<tr>
<td></td>
<td>Canada</td>
<td>0.7090</td>
</tr>
<tr>
<td></td>
<td>Mexico</td>
<td>0.3938</td>
</tr>
<tr>
<td>Europe</td>
<td>UK</td>
<td>0.7874</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>0.7300</td>
</tr>
<tr>
<td></td>
<td>Switzerland</td>
<td>0.7195</td>
</tr>
<tr>
<td></td>
<td>Spain</td>
<td>0.7161</td>
</tr>
<tr>
<td></td>
<td>Belgium</td>
<td>0.6904</td>
</tr>
<tr>
<td></td>
<td>Sweden</td>
<td>0.6628</td>
</tr>
<tr>
<td></td>
<td>Germany</td>
<td>0.6551</td>
</tr>
<tr>
<td></td>
<td>Denmark</td>
<td>0.6142</td>
</tr>
<tr>
<td></td>
<td>Italy</td>
<td>0.5853</td>
</tr>
<tr>
<td></td>
<td>Norway</td>
<td>0.5824</td>
</tr>
<tr>
<td></td>
<td>Finland</td>
<td>0.5017</td>
</tr>
<tr>
<td></td>
<td>Austria</td>
<td>0.4294</td>
</tr>
<tr>
<td>Asia</td>
<td>Japan</td>
<td>0.7479</td>
</tr>
<tr>
<td></td>
<td>Singapore</td>
<td>0.5784</td>
</tr>
<tr>
<td></td>
<td>Hong Kong</td>
<td>0.4924</td>
</tr>
<tr>
<td></td>
<td>Philippines</td>
<td>0.3590</td>
</tr>
<tr>
<td></td>
<td>Thailand</td>
<td>0.3214</td>
</tr>
<tr>
<td></td>
<td>Taiwan</td>
<td>0.3136</td>
</tr>
<tr>
<td></td>
<td>Korea</td>
<td>0.2758</td>
</tr>
<tr>
<td></td>
<td>Indonesia</td>
<td>0.1926</td>
</tr>
<tr>
<td>Latin America</td>
<td>Brazil</td>
<td>0.2805</td>
</tr>
<tr>
<td></td>
<td>Peru</td>
<td>0.2793</td>
</tr>
<tr>
<td></td>
<td>Colombia</td>
<td>0.1460</td>
</tr>
<tr>
<td></td>
<td>Venezuela</td>
<td>0.1245</td>
</tr>
<tr>
<td></td>
<td>Argentina</td>
<td>0.0796</td>
</tr>
<tr>
<td></td>
<td>Australia</td>
<td>0.3536</td>
</tr>
</tbody>
</table>

Return series are constructed using MSCI country price indices and the MSCI World price index, which are all obtained from Thompson Financial Datamonitor. The time periods over which the calculations are performed differ, due to the availability of MSCI data. The sample period for US, UK, France, Germany, Hong Kong, Japan, Singapore, Canada, Austria, Italy, Sweden, Switzerland, Denmark, Spain, Norway, Finland and Belgium is 31/01/1986 to 31/07/2001. The sample period for Indonesia, Korea, Philippines, Taiwan, Thailand, Brazil, Argentina, and Mexico is 31/01/1988 to 31/07/2001. The sample period for Peru, Colombia, and Venezuela is 31/01/1993 to 31/07/2001.

Nikkei 225 Index Futures contracts are traded on the Osaka Securities Exchange (OSE), the Singapore International Monetary Exchange (SIMEX), and the Chicago Mercantile Exchange (CME). SIMEX was the first to introduce the contract on 3 September 1986, followed by the OSE on 3 September 1988, and the CME on 25 September 1990. Trade in three separate markets may provide arbitrage opportunities, and Nikkei 225 futures have attracted the attention
of researchers including Bacha and Vila (1994), Booth et al (1996), and Ito and Lin (2001). The choice between data sources is not critical for the purposes of this thesis because, as Booth et al (1996, p 75) observe, "none of the three markets can be considered the main source of information flow, and each trading market is informationally efficient". Nikkei 225 futures data from SIMEX are employed in this thesis for three reasons. First, futures prices are available from SIMEX from an earlier date, providing the longest time series. Second, although the trading volume of Nikkei 225 futures is generally highest on the OSE, as shown in Figure 4.1, the lower transaction costs (commissions, price limits, and margins) and fewer trading restrictions of SIMEX make it an attractive alternative for investors. The erosion of the majority market share of the OSE began in 1991, when the OSE increased transaction costs while SIMEX simultaneously decreased costs. Third, Booth et al (1996, p 75) find weak evidence that of the three markets, SIMEX responds more quickly to new information.

Figure 4.1: Comparison of monthly trading volume of the Nikkei 225 Stock Index futures contracts on the Chicago Mercantile Exchange, the Osaka Securities Exchange and the Singapore International Monetary Exchange, for the period 01/10/87 to 01/10/2001

![Graph showing trading volume comparison](image)

[Graph constructed from data obtained from Thompson Financial Datastream.]

It is appropriate to consider Australian futures contracts in addition to foreign futures contracts because Australian managers are likely to be most familiar with domestic derivatives. Further, the Sydney Futures Exchange is the "largest futures and options exchange in Asia" according to the Sydney Futures Exchange and the Australian Bureau of Statistics\(^\text{12}\), so it is reasonable for an Australian investor to consider Australian futures contracts if choosing a single futures contract to cross-hedge equity risk.

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The research is presented in four chapters, which address increasingly complex hedging scenarios. The first examines the cross-hedging of the foreign equity component of diversified portfolios using single futures contracts. The second study examines the hedging of both domestic Australian and foreign equity simultaneously using single futures contracts, a strategy that attracts lower costs than using multiple futures contracts. The third study examines the hedging of Australian and foreign equity simultaneously using pairs of futures contracts, which is the most simple case of the use of multiple futures contracts to hedge multiple risks. The fourth study investigates the hedging of Australian and foreign equity simultaneously using up to five futures contracts simultaneously. The performance of single futures contracts relative to baskets of futures contracts is examined, and the performance of different numbers of contracts is assessed. These studies, which aim to identify ways in which managers may improve hedge performance, are described briefly in this section along with motivations for the research. Detailed methods are provided in each results chapter.

4.6.1 Hedging portfolios of foreign equity using single futures contracts

The first study, presented in Chapter Six, examines the hedging of the foreign component of internationally diversified equity portfolios using single futures contracts. In addition to providing a relatively simple introduction to the empirical issues involved in hedging, one aim is to determine whether any futures contract or hedge strategy achieves superior hedge effectiveness when used in isolation. This is important for several reasons. The use of a superior hedging instrument or strategy by managers would more effectively prevent losses on their portfolios and create tracking error relative to their performance benchmark. The use of a single futures contract is appealing because transaction costs are lower than if multiple contracts are used. Additionally, relatively less information and less management time are required when using a single contract. By comparing the performance of Australian and foreign futures contracts, the analysis also reveals whether there is any advantage to using foreign derivatives over domestic derivatives, given that no direct hedge is available for the portfolios. As discussed in the previous chapter, there is no consensus in prior literature as to whether more complex hedge ratio estimation techniques produce superior hedge ratios. For instance, Lien and Luo (1994, p 929) observe that much prior work on GARCH models fails to detect “significant improvement of GARCH hedging over conventional simple hedging”. Simple and complex techniques are compared when addressing this topic.

Thus, the foreign equity held by Australian managed funds is hedged using the Australian Share Price Index futures contract, the U.S. S&P 500 futures contract, the Japanese Nikkei 225
futures contract, the U.K. FTSE 100 index futures contract and the Brazilian Bovespa index futures contract, to determine whether the choice of hedging instrument affects hedge effectiveness when each futures is used in isolation. The results are examined for sensitivity to the construction of the equity portfolios, hedge ratio estimation technique, static and dynamic hedges, and time period. Hedge ratio estimation models used include the naïve model, price changes OLS regressions, and bivariate GARCH(1,1) models. Model specifications are provided in Chapter Six.

The results are applicable to managers and investors considering a global equity portfolio. This can be viewed as a study of partial hedging, where only the foreign component of the portfolio is hedged and upside potential may be realised in the remaining domestic component.

4.6.2 Hedging portfolios of Australian and foreign equity using single futures contracts

The second study, presented in Chapter Seven, examines the more complex scenario of cross-hedging both the Australian and foreign equity components of diversified portfolios using single futures contracts. A key issue is whether any particular hedging strategy provides superior performance when Australian equity is considered in addition to foreign equity. This issue is important because Australian equity constitutes a large proportion of fund portfolios, depending on style, a point demonstrated empirically in the following chapter. The motivations for using one futures contract to hedge multiple risks as opposed to multiple futures contracts include low transaction costs, ease of management, ease of re-balancing, and that less knowledge of different contracts is required. The performance of hedges using each of the five futures contracts is compared. Sensitivity analyses are performed with regard to hedge ratio estimation technique, time period, equity portfolio construction, and risk aversion parameter.

A second issue is whether accounting for portfolio effects improves hedge performance, where portfolio effects are the complex interactions between different components of many-asset portfolios. Hedge ratios that do not incorporate portfolio effects are easier to calculate than ratios that account for complex relationships between variables, but may not provide superior hedge effectiveness because the futures may react to other parts of the portfolio to either cancel out some of the protection, or protect unnecessarily given a natural hedge and create a new risk exposure. Thus, another research question is whether simple hedge ratio estimation techniques that ignore portfolio effects are equally effective as complex hedge ratio estimation techniques that incorporate portfolio effects. A preliminary investigation of this issue is conducted using the Australian All Ordinaries Share Price Index futures contract to hedge portfolios containing Australian and foreign equity. Traditional approaches to hedge ratio estimation that ignore portfolio effects, including price-changes OLS regressions and GARCH(1,1) models, are applied to domestic and foreign equity separately. The performance of the overall hedged
portfolio is then compared with that of hedges that incorporate portfolio effects between equity components. In the latter case, the total return series for the equity portfolio is used as a single spot asset, and interactions between domestic and foreign equity are accounted for in the single return series prior to the estimation of the hedge ratio. As in the previous chapter, a variety of sensitivity analysis is performed.

4.6.3 Hedging portfolios of Australian and foreign equity using pairs of futures contracts

The study in Chapter Eight extends the investigation of portfolio effects to the case where portfolios containing domestic and foreign equity are hedged using pairs of futures contracts simultaneously. This problem incorporates elements of direct hedging and cross-hedging, in that a futures contract used to directly hedge one risk will interact with other elements of the portfolio, which will impact on the overall return distribution. This is of interest to investors holding diversified portfolios who want to hedge the domestic and foreign parts simultaneously. For instance, an investor may choose to fully hedge their domestic and foreign equity using domestic and foreign futures contracts respectively. The hedging problem becomes complex as the returns on the foreign and domestic futures contracts interact not only with the equity components but also with each other. Prior empirical research on portfolio effects is limited, and this study contributes by using Australian data and considering equity risks. Prior studies are restricted to foreign exchange risks and commodity risks, using non-Australian data.

Price-changes OLS regression models and GARCH(1,1) models are extended to accommodate two futures contracts. An additional issue of interest is the relative performance of simple and complex techniques for hedging multiple exposures simultaneously. Evidence on the performance of bivariate GARCH hedge ratios is mixed and depends on the data set, as discussed in Chapter Three, justifying further study. The application of a trivariate GARCH(1,1) model using the BEKK parameterisation to hedge multiple risks has not been attempted in the context of equity risk. This is another contribution of this study.

4.6.4 Hedging multiple risk exposures simultaneously using multiple futures

The final study in Chapter Nine examines strategies for hedging diversified equity portfolios using multiple futures contracts. An important issue is whether hedging using a portfolio of futures is more effective than using an individual futures contract. Limiting the number of futures contracts will lower transaction costs and improve the ease of portfolio management. For instance, Giaccotto et al (2001, p 163) state that “intuitively, if one hedging instrument is a substitute for another, use only one, not both”. However, while monitoring and transaction costs may cast doubt on the practicality of many-futures hedges, some prior research indicates that multiple futures hedges out-perform single futures hedges. Using multiple futures may improve
hedge performance through greater “accuracy” or better matching of the spot and futures portfolios. Multivariate price-changes OLS regressions are used in the analysis, to the exclusion of multivariate GARCH models. The latter become too complex to implement in practice as the number of estimated parameters increases dramatically with the number of futures contracts. This study is of interest to any investor wishing to improve the effectiveness of their hedge.

4.7 Assumptions

Analysis in this thesis proceeds subject to a number of assumptions. A simplifying assumption is that when hedging multiple risks simultaneously, all hedge periods commence and finish at the same time. This is a common assumption in the literature. For instance, Lypny (1988, p 712) assumes this in the context of currency portfolios. This is reasonable, given that fund managers consider discrete reporting periods when calculating performance, so that in reality they consider the net position of their entire portfolio at certain points in time. The analysis is based around Australian reporting periods, namely each quarter.

Hedge performance is assessed out-of-sample, which assumes that time can be divided into two periods. The first chronological period is used to estimate the hedge ratio (the estimation period), and the second non-overlapping period is used to assess hedge effectiveness (the hedge period). This thesis utilises a two-period model in the sense that there are two time periods of particular interest, but the hedge ratio is calculated in one period only. Thus, it is assumed that the investor focuses on a single hedge period, as opposed to multiple consecutive hedge periods. Hedging over the long term can be viewed as a series of discrete single-period hedges that are reviewed at the end of each period. This avoids the complexity of multiperiod models such as that discussed by Lien and Luo (1994).

Transaction costs associated with trading assets and futures are ignored. Incorporation of explicit transaction costs is an extension for future research.

Hedge performance is assessed on the basis of expected utility, and sensitivity analysis is conducted with regard to the risk aversion parameter $\lambda$, and to compare the expected utility results with results using a simple risk reduction measure.

Analysis in this thesis is conducted to maximise the generalisability of the findings. For instance, instead of assuming that an investor or fund asset allocations are represented by some “average”, a range of portfolios are used, so that a particular manager or investor can identify the findings that are applicable to their portfolio, whether it be classified as growth, balanced or...
capital stable. A detailed explanation of equity portfolio construction is provided in the following chapter. Further assumptions regarding the manipulation of data are also discussed in the following chapter, and many assumptions are examined empirically through sensitivity analysis, and are therefore analysed in later results chapters.

4.8 Conclusion

This thesis examines the use of Australian and foreign equity futures contracts to manage the equity risks associated with diversified equity portfolios. Direct hedging and cross-hedging of single and multiple risk exposures will be examined, with a view to identifying superior strategies. Given the research topics outlined in this chapter, the following chapter provides a summary of the data used in this thesis.
CHAPTER FIVE

DATA

5.1 Introduction

This chapter describes the data used in this thesis. The construction of investment portfolios is discussed, followed by analysis of the futures contracts used to hedge the portfolios. The correlations between the returns on the futures and equity portfolios are examined to provide insight into the codependence between asset returns, an important factor in hedging. The sample period for all data extends from 1 January 1989 to 31 July 2001.

5.2 Equity Portfolios

In recognition of the diversity of investment styles of Australian managed funds and other Australian investors, a range of portfolios are considered in this thesis. The construction of fund portfolios requires data indicating the asset allocations, or proportion of investment in different asset classes, and data on assets that represent each asset class. The methods of portfolio construction are explained in this section, and the resulting portfolios are examined statistically.

5.2.1 Allocation Between Australian and Foreign Equity

Monthly asset allocations adopted by typical Australian growth funds, balanced funds and capital stable funds are provided by William M. Mercer Investment Consulting. Mercer Investment Consulting collects data on individual Australian funds’ asset allocations each month, and calculates a simple average. The results are reported as percentage investment in each asset class, with a total investment of 100% for each month.

Mercer Investment Consulting categorises investments as Australian equity, foreign equity, Australian fixed interest, foreign fixed interest, direct property, listed property, indexed bonds, cash, and other. Figure 5.1 indicates the average asset allocations for Australian balanced funds, capital stable funds and growth funds. Growth funds and balanced funds are more heavily

\[13 \text{ In this thesis, calculations for capital stable funds use Mercer’s “low-risk diversified funds” data. While Mercer provides other data on “capital stable funds”, that data merely combines data on low-risk diversified funds with data on a very small number of protected funds (e.g., in December 2000, two protected funds were included in the sample). The data on protected funds bears little resemblance to the rest of the data, and causes a distortion of the average. For these reasons, the data on low-risk diversified funds is used in this thesis to represent capital stable funds, to the exclusion of the protected fund data.} \]
invested in equity than the capital stable funds, which are most heavily invested in fixed interest and cash. Equity investments are generally considered to be more risky than cash and fixed interest. For instance, Canner et al (1997, p 181) observe that "popular" financial advisors recommend that investors with relatively higher risk-aversion adopt higher ratios of fixed interest securities to equities. Further, they present evidence that mean returns and standard deviations of returns on US equities are both higher than for US bonds from 1926 to 1992 (p 185). Similar evidence is available in Australia. For instance, over the period 1986 to 2000, Australian equity has a higher mean return and higher standard deviation than Australian government bonds.\footnote{Barksy (1989, p 1132) also draws attention to the "flight to quality" when investors withdraw from high risk equity and move into low risk bonds. Thus, the asset allocations indicated in Figure 5.1 are broadly consistent with the Markowitz (1959) mean-variance theory of portfolio selection, in that growth and balanced funds generally adopt higher risk investments with higher expected returns relative to capital stable funds, which invest mainly in lower risk investments with lower expected returns.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.1.png}
\caption{Average asset allocations for Australian growth funds, balanced funds and capital stable funds}
\end{figure}

\begin{tikzpicture}
\begin{axis}[
    width=\textwidth,
    height=\textwidth,
    ybar stacked,
    bar width=20pt,
    xtick=data,
    x tick label style={rotate=45, anchor=east},
    ytick={0,5,...,45},
    yticklabels={{0}, {5}, {10}, {15}, {20}, {25}, {30}, {35}, {40}, {45}},
    y label style={at={(axis description cs:0.5,0.5)},anchor=west},
    xticklabels={\textsc{aust equity}, \textsc{o' seas equity}, \textsc{direct prop}, \textsc{listed prop}, \textsc{aust fix int}, \textsc{o' seas indexed bonds}, \textsc{cash}, \textsc{other}},
    xticklabel style={align=center},
    x tick label style={text width=2cm, align=center},
    y label style={text width=2cm, align=center},
    y tick label style={text width=2cm, align=center},
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These asset allocation data are used to determine a range of appropriate divisions between Australian and foreign equity, strategies that are held constant over time in the analysis in this thesis. Real allocations are time-varying, and a fund manager changing allocations may still rely on the results in this thesis by choosing the most appropriate new scenario in the range of portfolios provided. In this way, a fund does not have to be “average” to benefit from the findings in this thesis. Thus, the findings are flexible because they may be applied to a range of investors without identifying any particular style, which may be problematic, and because the results apply regardless of whether a fund changes style over time.

The ratios of asset allocations between Australian and foreign equity for growth, balanced and capital stable funds are presented in Table 5.1. On average, growth funds invest the largest proportion of total equity in foreign securities, while capital stable funds invest the greatest proportion in Australian equity.

<table>
<thead>
<tr>
<th>Fund Style</th>
<th>Average Ratio of Australian Equity to Foreign Equity</th>
<th>Maximum percentage of total equity invested in Australian equity</th>
<th>Maximum percentage of total equity invested in foreign equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth</td>
<td>60:40</td>
<td>63%</td>
<td>46%</td>
</tr>
<tr>
<td>Balanced</td>
<td>69:31</td>
<td>72%</td>
<td>36%</td>
</tr>
<tr>
<td>Capital Stable</td>
<td>74:26</td>
<td>84%</td>
<td>37%</td>
</tr>
</tbody>
</table>

Calculated from data provided by Mercer Investment Consulting. Data on capital stable, balanced and growth funds commence in April 1991, January 1991 and March 1998 respectively, and all end in December 2000. The average ratios are calculated across the full sample periods for each fund style. Fund styles are determined by Mercer Investment Consulting.

Drawing on the results in Table 5.1, Table 5.2 indicates the investment ratios used in this thesis, and the explanation for each. The ratios chosen are average ratios for growth, balanced and capital stable funds, or form watersheds between types of funds meaning that they constitute the extremes of the investment range for a fund style. The greater the proportion of foreign equity, the greater the risk of the equity portfolio. Thus, the least risky portfolio held by the most risk-averse capital stable fund contains 85% Australian equity and 15% foreign equity. The most risky portfolio contains 55% Australian equity and 45% foreign equity, and represents the upper limit of risk tolerated by growth fund managers.

<table>
<thead>
<tr>
<th>Australian Equity to Foreign Equity</th>
<th>Importance of Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>85:15</td>
<td>Maximum Proportion of Australian Equity for Capital Stable Funds</td>
</tr>
<tr>
<td>75:25</td>
<td>Average Investment Proportions for Capital Stable Funds</td>
</tr>
<tr>
<td>70:30</td>
<td>Average Investment Proportions for Balanced Funds</td>
</tr>
<tr>
<td>65:35</td>
<td>Maximum Proportion of Australian Equity for Growth Funds, and</td>
</tr>
<tr>
<td></td>
<td>Minimum of Australian Equity for Balanced and Capital Stable</td>
</tr>
<tr>
<td>60:40</td>
<td>Average Investment Proportions for Growth Funds</td>
</tr>
<tr>
<td>55:45</td>
<td>Minimum Proportion of Australian Equity for Growth Funds</td>
</tr>
</tbody>
</table>
5.2.2 Representing Australian and Foreign Components of Equity Portfolios

To construct equity portfolios based on the asset allocations described in the previous section, Australian and foreign equity must be represented using readily identifiable investments, an issue addressed in this section.

Australian Equity

The Australian All Ordinaries share price index is used to represent investment in the Australian equity market. Within the sample period, the index is calculated using the market prices of approximately 300 of the largest companies listed on the Australian Stock Exchange. It is a value weighted index and covers over 95% of the total capitalisation of listed Australian stocks. Companies included in the index are required to show adequate turnover, as defined by the Australian Stock Exchange, and satisfy a minimum market capitalisation. The All Ordinaries index is a common choice for representing the Australian equity market. For instance, literature produced by the Sydney Futures Exchange asserts that the All Ordinaries Index “has long been the benchmark by which Australia’s professional money managers measure portfolio performance”.

In 2000, new indices representing the Australian equity market were introduced, including the S&P/ASX 200 Index, which is currently described as “Australia’s benchmark equity index” by the Sydney Futures Exchange\(^{15}\) and as the “primary index for the Australian market” by the Standard&Poors\(^{16}\). However, the introduction of the new indices occurred outside the sample period examined in this thesis. Further, because futures contracts written on the S&P/ASX 200 Index were listed on the Sydney Futures Exchange on 02/05/2000, it is not feasible to consider the new index and its associated contract in this thesis.

Foreign Equity

Representing foreign, or non-Australian, equity presents more of a challenge than representing Australian equity, because the former task potentially involves a large number of foreign equity markets in addition to large number of securities in individual markets. Two methods of representing foreign equity are applied. First, the MSCI World excluding Australia price index is taken to constitute a diversified foreign equity portfolio. The index incorporates MSCI country indices for Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United Kingdom, and the United States of America. Country indices achieve 60% coverage of total market capitalisation, while simultaneously “maintaining the

\(^{15}\) This statement is on the Sydney Futures Exchange website www.sfe.com.au, as at 15/04/02.

\(^{16}\) This statement is on the S&P website www.spglobal.com/indexmainasx.html as at 15/04/02.
overall risk structure of the market”. This is done by identifying all listed securities in the market, arranging the data according to industry groups, then selecting stocks on the basis of size, trading volume, cross-ownership and free float, where free float is the percentage of firm ownership that is available for trading on the stock market. Market capitalisation weights for each stock in the country index are then determined on the basis of free float. To ensure that each country’s weight in the world index is proportional to its weight in the total investable universe, market capitalisation weightings are used to calculate the world index.

Although the MSCI World excluding Australia price index may be viewed as a diversified foreign equity portfolio, it may not be representative of a fund portfolio. In contrast, the second method of foreign portfolio construction applies the asset allocations of Templeton World Fund to combinations of individual equity market indices used to represent foreign regions. The Templeton World Fund is a well-established fund, which commenced on 17 January 1978. The fund managers pursue an investment policy aimed at long-term capital growth, as distinct from replication of a world index. According to the current portfolio manager, Jeff Everett, they “search for values, look globally and do bottom-up research”.  \(^{17}\) As an indication of the relative performance of the fund, Morningstar rated Templeton World Fund’s return as “above average” and the risk as “below average” on 3 October 2001.  \(^{18}\) The asset allocations between various foreign equity markets adopted by Templeton are indicated in Table 5.3.

To apply the Templeton allocations, each geographic region is represented using a combination of individual equity market indices. Specifically, the U.S. S&P 500 index is used to represent the behaviour of North American equity because the U.S. equity market is the largest in that region and has a high positive correlation with the Canadian equity market. Similarly, Hong Kong, Japan and Singapore represent Asia, and the UK, France, Germany, and the Netherlands represent Europe. Argentina, Brazil, Chile, Turkey and South Africa represent arguably the riskiest investment regions of Latin America and the Middle East. The equity return for each foreign region is calculated as an equally weighted return on the individual markets, and all returns are converted into Australian currency for reasons discussed in detail in the following section. A small number of indices on individual markets are used in conjunction to represent each region, in preference to regional indices because a combination of the latter is more difficult and expensive to replicate in practice, given the management expertise, the time required and transaction costs. Further, regional indices do not necessarily have matching futures contracts. The use of individual indices with futures written on them, such as the U.S.

\(^{17}\) These quotes were extracted from the Franklin Templeton website, at [http://pub.franklintempleton.com](http://pub.franklintempleton.com), on 3 October 2001.

\(^{18}\) This rating was provided on Morningstar’s website [http://moneycentral.msn.com](http://moneycentral.msn.com).
Table 5.3: Percentage investments in the equity markets of North America, Europe, Asia, and Latin America and the Middle East, for the period August 1988 to August 2000

<table>
<thead>
<tr>
<th>Date</th>
<th>North America</th>
<th>Europe</th>
<th>Asia</th>
<th>Latin America, Middle East</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>31-Aug-88</td>
<td>85.85</td>
<td>13.63</td>
<td>0.39</td>
<td>0.13</td>
<td>100.00</td>
</tr>
<tr>
<td>31-Aug-89</td>
<td>77.45</td>
<td>19.36</td>
<td>2.39</td>
<td>0.80</td>
<td>100.00</td>
</tr>
<tr>
<td>31-Aug-90</td>
<td>58.72</td>
<td>35.47</td>
<td>5.47</td>
<td>0.35</td>
<td>100.00</td>
</tr>
<tr>
<td>31-Aug-91</td>
<td>53.93</td>
<td>37.31</td>
<td>8.50</td>
<td>0.25</td>
<td>100.00</td>
</tr>
<tr>
<td>31-Aug-92</td>
<td>47.14</td>
<td>41.31</td>
<td>11.54</td>
<td>0.00</td>
<td>100.00</td>
</tr>
<tr>
<td>31-Aug-93</td>
<td>44.74</td>
<td>27.68</td>
<td>16.85</td>
<td>10.73</td>
<td>100.00</td>
</tr>
<tr>
<td>31-Aug-94</td>
<td>41.22</td>
<td>37.07</td>
<td>12.32</td>
<td>9.39</td>
<td>100.00</td>
</tr>
<tr>
<td>31-Aug-95</td>
<td>39.77</td>
<td>37.31</td>
<td>11.14</td>
<td>11.79</td>
<td>100.00</td>
</tr>
<tr>
<td>31-Aug-96</td>
<td>32.48</td>
<td>41.42</td>
<td>12.62</td>
<td>13.48</td>
<td>100.00</td>
</tr>
<tr>
<td>31-Aug-97</td>
<td>35.24</td>
<td>38.10</td>
<td>10.79</td>
<td>15.86</td>
<td>100.00</td>
</tr>
<tr>
<td>31-Aug-98</td>
<td>35.79</td>
<td>41.09</td>
<td>11.42</td>
<td>11.70</td>
<td>100.00</td>
</tr>
<tr>
<td>31-Aug-99</td>
<td>33.16</td>
<td>36.76</td>
<td>20.59</td>
<td>9.49</td>
<td>100.00</td>
</tr>
<tr>
<td>31-Aug-00</td>
<td>39.04</td>
<td>30.30</td>
<td>21.20</td>
<td>9.46</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Construct from data collected from the annual reports of Templeton World Fund. Although only annual data is reported for the sake of brevity, semiannual data were also collected from the annual and semiannual reports of Templeton World Fund. The number of categories shown in the table have been scaled down from a larger number of categories listed in the official fund reports, including "Australia and New Zealand".

S&P 500 and the U.K. FTSE 100, allows for exact matching of futures contracts with a portion of the equity portfolio for hedging purposes. Only a small number of individual markets are required to represent each region because markets within a region tend to be closely correlated, diminishing the benefits of including a large number of markets.

Recall that foreign currency risk associated with internationally diversified equity portfolios is accounted for by converting all time series into Australian dollars. In short, the only type of risk under consideration is equity risk. This approach restricts the level of complexity of the hedging problem. A mathematical derivation of the necessity of expressing all returns in one common currency is provided and discussed in Section 4.4 of the previous chapter.

5.2.3 Data Sources and Portfolio Construction

Daily data for each asset class were collected over the period 01/01/1989 to 31/07/2001, from Thomson Financial Datastream. The series were converted automatically by Datastream into Australian currency, to neutralise the impact of foreign exchange risk. Extreme values in daily data series were cross-checked against alternative data sources where possible to ensure accuracy of the data. For instance, forty outliers in the Australian All Ordinaries Index series were compared with data published in the Australian Financial Review, Australia’s leading financial newspaper. Data points remain in the database if they are not caused by errors in data entry, because they are genuine readings that may occur again in the future. No data points in the All Ordinaries Index were removed.19

19 The data point on 18/06/1988 was reported in Datastream as 2608.1 and reported in the Financial Review as 2608.2. The difference of 0.1 points is negligible, and the data point was not altered. The other data points checked are consistent between the two data sources.
Weekly continuously compounded asset returns are calculated from daily asset price data by applying equation (5.1) to the Thursday prices from each week.

\[ R_{i,t} = \ln \left( \frac{P_{i,t}}{P_{i,t-1}} \right) \]  

(5.1)

where \( R_{i,t} \) = continuously compounded return on asset \( i \) at time \( t \)

\( P_{i,t} \) = price level of asset \( i \) at time \( t \)

\( \ln \) = natural logarithm

Continuously compounded returns are employed in preference to discrete returns, because the former are more likely to follow a normal distribution, and reduce problems associated with outliers, as observed by Praetz (1976). Continuously compounded returns reduce the volatility of returns caused by low priced investment securities, and are more consistent with the real process of return generation, which occurs through calendar time not merely through trading time.  

Weekly returns are used as opposed to daily returns because the distribution of the former is likely to more closely approximate the normal distribution, which is important when using OLS regressions to calculate hedge ratios. Weekly data generally do not exhibit the high autocorrelation associated with daily data. Further, the use of weekly data avoids the “day-of-the-week” effects documented in Australian equities by Ball and Bowers (1988) and Finn et al (1991). However, traders face such seasonalities in practice, and both the futures market and the equity market are affected simultaneously, so the last consideration is minor. Weekly portfolio returns are calculated from the weekly return series for individual assets and asset classes, using formula (5.2):

\[ R_p = \sum_{i=1}^{n} w_i R_i \]  

(5.2)

where \( R_p \) = continuously compounded portfolio return

\( w_i \) = asset allocation proportion for asset class \( i \)

\( R_i \) = continuously compounded return on asset class \( i \)

5.2.4  **Description of Fund Portfolios**

Statistical descriptions of Australian equity and foreign equity portfolios are provided in Table 5.4, where foreign equity portfolios do not contain any Australian equity. In addition to foreign

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20 When calculating the Bovespa index return series, zero returns are inserted on several dates because the price index level was reported at zero, making it impossible to calculate continuously compounded returns as the fraction in equation (5.1) becomes undefined. The dates are 19/01/1984, 26/01/1984, 13/12/1984, 20/12/1984, 27/12/1984, 03/01/1985, 21/09/1989, 28/09/1989, 13/07/1989, 20/07/1989, 27/07/1989.
portfolios based on the asset allocations of Templeton World Fund and the MSCI World excluding Australia index, a foreign portfolio is constructed to maximise expected utility for the purpose of comparison. Expected utility, \( E(U) \), is:
\[
E(U) = E(R_s) - \lambda \text{Var}(R_s)
\]  
(5.3)
where \( \lambda \) is the risk aversion parameter and \( R_s \) is the return on the equity portfolio. The expected-utility-maximising portfolio is constructed using the same asset classes used in the Templeton portfolio.

Table 5.4: Descriptive statistics for weekly continuously compounded returns on Australian equity and diversified foreign equity portfolios, for the period 05/01/1989 to 26/07/2001

<table>
<thead>
<tr>
<th></th>
<th>Australian All Ordinaries</th>
<th>Templeton World Fund</th>
<th>MSCI excl Australia</th>
<th>Maximise E(Utility)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0012</td>
<td>0.0023</td>
<td>0.0020</td>
<td>0.0030</td>
</tr>
<tr>
<td>Median</td>
<td>0.0016</td>
<td>0.0026</td>
<td>0.0021</td>
<td>0.0034</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.0178</td>
<td>0.0212</td>
<td>0.0220</td>
<td>0.0237</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.1049</td>
<td>-0.4122</td>
<td>-0.3044</td>
<td>-0.2926</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.4338</td>
<td>2.1646</td>
<td>2.3410</td>
<td>1.6380</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.0708</td>
<td>-0.1101</td>
<td>-0.1161</td>
<td>-0.1092</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0565</td>
<td>0.0901</td>
<td>0.1110</td>
<td>0.0914</td>
</tr>
<tr>
<td>Durbin Watson</td>
<td>2.0349</td>
<td>2.1748</td>
<td>2.1903</td>
<td>2.2738</td>
</tr>
<tr>
<td>Box-Pierce-Ljung (23 lags)</td>
<td>25.248</td>
<td>31.354</td>
<td>31.546</td>
<td>35.449</td>
</tr>
<tr>
<td>Jarque Bera Normality ( \chi^2 ) (2 d.f.)</td>
<td>6.050^*</td>
<td>143.555*</td>
<td>156.453*</td>
<td>80.730*</td>
</tr>
<tr>
<td>ARCH (1 d.f.)</td>
<td>3.346^*</td>
<td>9.998*</td>
<td>7.663*</td>
<td>9.513*</td>
</tr>
<tr>
<td>Annualised Mean</td>
<td>6.16%</td>
<td>12.17%</td>
<td>10.17%</td>
<td>15.61%</td>
</tr>
<tr>
<td>Annualised Median</td>
<td>8.32%</td>
<td>13.52%</td>
<td>10.92%</td>
<td>17.68%</td>
</tr>
</tbody>
</table>

* Significant at the 1% level.
^ Significant at the 5% level.

Thursday returns are used when calculating the weekly returns. The expected utility function used to determine optimal weightings is given in equation (5.3), where the risk aversion parameter is taken to equal one. All statistics are for weekly returns, except the annualised mean, which is calculated by multiplying the weekly mean by 52 and converting into percentage format. All returns are denominated in Australian dollars.

Australian equity has a lower expected return and lower risk as measured by standard deviation relative to diversified foreign portfolios regardless of the method of constructing the latter. In the context of foreign portfolios, the Templeton weightings provide a portfolio with a higher mean return and lower standard deviation than the MSCI World excluding Australia index. Thus, the foreign portfolio based on Templeton allocations is preferred to the MSCI portfolio under the Markowitz mean-variance theory. Given that the investment strategy of Templeton World Fund is promoted as being different from an index, it is expected that there is an important difference between the return distributions of the Templeton and MSCI foreign portfolios. An ANOVA test provides an F-statistic of 0.1036 (p-value 0.75), indicating no statistically significant difference between the means of the two foreign portfolios over the full sample period. The lack of statistical significance may be attributed to a number of data limitations. Although the MSCI represents broad investment covering all industry sectors in
each country while Templeton invests in specific sectors in each country, the available data on Templeton allocations are not sufficiently detailed to detect this difference. Similarly, while the MSCI index covers approximately 60% of market capitalisation within each country and Templeton invests in specific stocks, the assets used to proxy for the Templeton investment categories do not reflect this level of detail because they are relatively broad. Specifically, indices are used to represent countries and regions when constructing the Templeton portfolio, as opposed to individual stock, because data on investment in individual stock is not available. Another possible explanation for the lack of a statistically significant difference between the portfolio means is that the foreign component of the Templeton portfolio may be similar to the MSCI portfolio, but the investment allocations adopted by Templeton World Fund in the sectors amalgamated with other sectors, or not accounted for, may significantly improve the performance of the overall fund portfolio.

Arguably, the difference between portfolio returns has economic significance, despite the absence of statistical significance when weekly returns are analysed using ANOVA. When annual expected returns are calculated in Table 5.4, the Templeton portfolio provides a higher expected return than the MSCI portfolio by two percent per annum. Two percent of a large amount of money, such as that held by a managed fund, equates to an economically significant difference. Further, the annualised medians are different by about three percent, and other standard statistics in Table 5.4 indicate that the distributions of returns are different. Therefore, both foreign portfolios are considered in this thesis, for comparison.

Descriptive statistics in Table 5.4 indicate that all series exhibit negative skewness and leptokurtosis (peakedness) relative to the normal distribution. Under the Jarque-Bera Asymptotic LM Normality test, the null hypothesis is that residuals are distributed normally. All foreign equity return series are non-normal at the 1% significance level, and Australian equity is non-normal at the 5% level. Non-normality may be encompassed within the mean-variance framework through reliance on the alternative assumption that investors’ utility functions are quadratic.

The Durbin Watson statistic is applied to test for first-order autocorrelation in residuals. Zero autocorrelation is indicated by a Durbin Watson statistic close to two. None of the equity return series have significant autocorrelation. The Box-Pierce-Ljung statistic is constructed from the “squared autocorrelations of the estimated residuals”. Significant correlation is only associated with the portfolio constructed to maximise expected utility.
Heteroskedasticity occurs when the variance associated with the error terms is not constant. The presence of heteroskedasticity prevents the application of a regression model from yielding the best linear unbiased estimates. Under the null hypothesis of homoskedasticity, Engle’s (1982) ARCH test reveals significant ARCH effects in all equity portfolios.

While Australian and foreign equity are examined separately in Table 5.4, Table 5.5 presents descriptive statistics for complete portfolios constructed by combining these components, where foreign equity is represented using the Templeton portfolio.

Table 5.5: Descriptive statistics for weekly returns on diversified equity portfolios, where foreign equity is represented by the Templeton portfolio, for the period 05/01/1989 to 26/07/2001

<table>
<thead>
<tr>
<th></th>
<th>Australian Equity (%)</th>
<th>Foreign Equity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>85</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>55</td>
</tr>
<tr>
<td>Mean</td>
<td>0.00136</td>
<td>0.00147</td>
</tr>
<tr>
<td>Median</td>
<td>0.00195</td>
<td>0.00205</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.01666</td>
<td>0.01622</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.1924</td>
<td>-0.2701</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.5300</td>
<td>0.6816</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.0683</td>
<td>-0.0666</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0513</td>
<td>0.0496</td>
</tr>
<tr>
<td>Durbin Watson</td>
<td>2.0207</td>
<td>2.0201</td>
</tr>
<tr>
<td>Box-Pierce-Ljung (23 lags)</td>
<td>21.444</td>
<td>19.722</td>
</tr>
<tr>
<td>Jarque-Bera Normality $\chi^2$ (2d.f.)</td>
<td>11.332*</td>
<td>20.112*</td>
</tr>
<tr>
<td>ARCH (1 d.f.)</td>
<td>4.409^</td>
<td>5.083^</td>
</tr>
<tr>
<td>Annualised mean return</td>
<td>7.05%</td>
<td>7.66%</td>
</tr>
</tbody>
</table>

* Significant at the 1% level.
^ Significant at the 5% level.

All statistics are for weekly returns, except the annualised mean, which is calculated by multiplying the weekly mean by 52 and converting into percentage format. All returns are denominated in Australian dollars.

Table 5.5 indicates that complete equity portfolios constructed using the Templeton portfolio are negatively skewed and leptokurtic. Formal Jarque-Bera LM Normality tests confirm the non-normality of all portfolio return series at the 1% significance level. Durbin Watson statistics do not indicate significant first order autocorrelation and Box-Pierce-Ljung statistics fail to detect significant correlation in any series. ARCH effects are significant regardless of the percentage of foreign equity. Results using the MSCI index to represent foreign equity show similar trends. However, the MSCI portfolios do not exhibit statistically significant ARCH effects, and the range of annualised mean returns is 1.20% as opposed to 1.81% for the Templeton portfolios.

ANOVA indicates no statistically significant difference between the expected returns of the six Templeton portfolios, with an F-statistic of 0.0055 (p-value 1.00). Similarly, the F-statistic testing the equality of the means of the six MSCI portfolios is 0.0172 (p-value 1.00). Although
the difference between the portfolios in each series may hold economic significance, the benefit of a detailed analysis of all six is questionable given the small increments between the annual expected returns as the percentage of foreign equity increases. Analysis in the remainder of this thesis proceeds using the portfolios containing 85%, 70% and 55% Australian equity, utilising the extremes of investment behaviour and the middle approach.

5.3 Futures Contracts

Data on the futures contracts used to implement portfolio risk management strategies are described in this section. An explanation of the construction of the futures databases is provided, accompanied by statistical descriptions.

5.3.1 Data Sources

Daily data on Australian All Ordinaries Share Price Index Futures contracts are obtained from the Sydney Futures Exchange.21 Daily data on U.S. S&P 500 share price index futures and U.K. FTSE 100 index futures contracts are obtained from the Futures Industry Institute, while the Commodity Research Bureau provides data on Japanese Nikkei 225 (SIMEX) futures. Data on Brazilian Bovespa index futures spanning the period 1990 to 1997 are collected from the Futures Industry Institute, and data after 1997 are obtained from Datasync. In each case, data are collected on the contract closest to expiry, which is the most liquid, as evidenced by trading volume and open interest. All data series extend from 05/01/1989 to 31/07/2001, with the exception of the Bovespa data, which commence on 26/07/1990. These futures contracts are written on leading equity indices in their geographic regions, as discussed in the previous chapter. Contract specifications are similar, in that equity index futures must be cash settled, because there is no underlying asset to deliver.

As with the data used to construct the equity portfolios, the accuracy of extreme observations are verified where possible through comparison with data reported in the Australian Financial Review. For instance, thirty data points and their surrounding data were checked in the All Ordinaries Share Price Index Futures series, and no data points were changed.22

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21 The All Ordinaries SPI futures contract was delisted on 28/09/2001, but this occurred outside the sample period examined in this thesis.

22 Of the data points checked, 18 points matched exactly, 12 had a difference of one point due to the rounding up of decimals by the Australian Financial Review. Given that the Sydney Futures Exchange is a superior data source, the Australian data points that do not match exactly are not changed because they are not sufficiently different between data sources.
5.3.2 Construction of Futures Databases

The futures data are arranged according to the month in which each contract expires, then placed in chronological order. All the contracts expire in March, June, September and December of each year, with the exception of the Brazilian Bovespa futures, which expires in February, April, June, August, October, and December. All trading months are used for the Bovespa futures due to data limitations. The settlement price from day trading is used. Data on overnight futures trading are excluded because the hedger is assumed to operate in Australian time zones according to regular Australian business hours, without executing overnight futures trades.

Further, overnight futures data exhibit considerable thin trading.

Missing Data

The futures data are adjusted for missing observations. For instance, fourteen missing observations are included in the All Ordinaries Share Price Index futures series.23 The U.K. FTSE 100 futures data provided by the Futures Industry Institute were incomplete for the contracts expiring in June 1997 and March 2000. In total, eighty-four data points collected from the *Australian Financial Review* are included to complete the U.K. futures database.

Foreign Exchange Risk

The underlying value of a futures contract is given by the index points multiplied by a set amount of local currency. For instance, the underlying value of the Australian All Ordinaries Share Price Index futures contract is A$25 multiplied by the index level associated with the contract.24 Foreign exchange risk is controlled by converting all futures return series into Australian dollars, consistent with the treatment of equity portfolio returns. A simple demonstration that this is necessary is provided above in Section 4.4.

Logarithmic transformation

The data is subject to logarithmic transformation to obtain cleaner statistical results. As Duffie (1989) observes, the transformed data has several desirable properties. The change in logarithms of prices gives continuously compounded rates of returns, and logarithmic transformation reduces statistical problems associated with outliers. Further, much of the research on hedging relies on logarithmic transformations, such as Park and Switzer (1995a). While this fact in itself

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23 The All Ordinaries Share Price Index Futures was day-traded via SYCOM instead of floor trading on 24/12/1991, 03/08/1992, 05/10/1992, 24/12/1992, 02/08/1993, 04/10/1993, 24/12/1993, 03/10/1994, 23/12/1994, and 02/10/1995. These dates are Christmas eves, bank holidays and labour days in various years, which are not consistently recognised as annual holidays. In these cases, the SYCOM value is included in the database. On 25/07/1990, 23/06/1987, 31/12/1999, and 10/01/2000, the SPIF was not SYCOM or floor traded but the underlying index traded, so futures data entries were made at the previous days’ closing floor price.

24 The All Ordinaries SPI futures contract was downsized from $100 to $25 multiplied by the index level on 11/10/1993.
does not justify the use of transformations in this thesis, it is evidence that such a technique is generally accepted as necessary.

*Futures Roll-Over*

When estimating hedge ratios and implementing hedges over periods exceeding three months, futures contracts must be “rolled over”. For instance, at any given time, there are several All Ordinaries Share Price Index futures contracts trading on the Sydney Futures Exchange, each maturing at different times. Trading tends to concentrate in the contract with the shortest time to expiry. Hedgers who hold the contract closest to maturity and wish to maintain their hedged position after their contract expires must roll over into another contract before their current derivative expires. In short, the hedger must take up the same type of futures contract with a different expiry date.

A weekly database for each futures contract is constructed using Thursday price levels, consistent with the formation of the equity portfolios. The futures contracts are rolled over on the third Thursday prior to the last Thursday prior to expiry. The exact dates of rollover differ between contracts, because they expire on different dates and days. For instance, the June 2001 AOI futures contract expired on Friday 29/06/01, so that contract was rolled over on Thursday 14/06/01. Similarly, the June 2000 AOI futures contract expired on Friday 30/06/00, so that contract was rolled over on Thursday 15/06/00. The literature on futures hedging does not offer clear guidance as to the best rollover technique, as discussed in Chapter Three. The assumptions adopted here strike a balance between using the contract with the highest liquidity, while simultaneously limiting the impact of increasing volatility of futures prices as maturity approaches.

*Price Changes Across Futures Contract Series*

The weekly database accounts correctly for price changes across contract series at rollover. When futures price changes are calculated, they must occur in a single price series, otherwise artificial “jumps” in the return series are created. For example, to calculate a price change across a date when a March contract is rolled into a June contract, the change in price for the first entry of the June series is calculated as follows:

First futures price change in new series = \( \text{Price}_\text{June}, t - \text{Price}_\text{June}, t-1 \)

Not: First futures price change in new series = \( \text{Price}_\text{June}, t - \text{Price}_\text{March}, t-1 \)

In short, when calculating futures price changes at the point of roll over between two contract series, the prices at times \( t \) and \( t-1 \) must be taken from the same series. This requires an overlap of one data point between the expiring and the new futures series.
**Bovespa Data Issues**

Additional issues arise regarding the Bovespa futures data because they are gathered from two data sources. Data from the Futures Industry Institute allow full manipulation of contract rollover, but futures price data from *Datastream* are provided in a single continuous series which is automatically “rolled over” rather than as data on separate contracts in overlapping time periods. In short, *Datastream* futures data do not allow for control over the timing of rollover or for correct adjustment for price changes across contracts at rollover, which requires an overlap of prices between contracts of different expiry.

To gauge the magnitude of the effect, *Datastream* price changes series are compared with the Futures Industry Institute series prior to 1997. A small number of jumps in the *Datastream* price changes series which are absent in the Futures Industry Institute price changes confirm a “price changes across series” effect. Although it is possible that this effect impacts on the *Datastream* series after 1997, it is not possible to identify the exact rollover dates graphically because the Bovespa return series is volatile, and large jumps are common. Further, there is no statistically significant difference between the expected returns of the Bovespa price series obtained from both data sources, despite incongruity between the reported prices for the December 1994 contract. Specifically, an ANOVA test for the difference between the mean of weekly returns from each data source provides an F-statistic of 1.0625 (p-value 0.3033) using all available data over the period 13/01/1994 to 30/10/1997. Thus, the potential impact of the price changes effect on Bovespa futures after 1997 is noted, but is expected to be minor and is not adjusted for.

### 5.3.3 Description of Futures Contracts

Descriptive statistics on the futures contracts are provided in Table 5.6. All futures return series exhibit negative skewness, leptokurtosis, significant deviations from normality as measured by the Jarque Bera LM Normality test, and significant ARCH effects. Although the mean returns vary considerably between individual contracts, the standard deviations bear more similarities, with the exception of the Brazilian Bovespa futures. The Bovespa futures has the highest risk and the lowest return across the full sample period.

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25 If rollover dates could be identified, the artificial spike effect could be overcome by inserting the mean price-change over the prior three months for the return on the roll-over date. This approach only involves minimal smoothing.
Table 5.6: Descriptive statistics for weekly continuously compounded returns on futures contracts, for the period 05/01/1989 to 26/07/2001

<table>
<thead>
<tr>
<th></th>
<th>Australian AOI Futures</th>
<th>U.S.A S&amp;P 500 futures</th>
<th>U.K. FTSE 100 futures</th>
<th>Japanese Nikkei 225 futures</th>
<th>Brazilian Bovespa futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0006</td>
<td>0.0023</td>
<td>0.0013</td>
<td>-0.0011</td>
<td>-0.0177</td>
</tr>
<tr>
<td>Median</td>
<td>0.0015</td>
<td>0.0029</td>
<td>0.0005</td>
<td>-0.0003</td>
<td>-0.0076</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.0206</td>
<td>0.0243</td>
<td>0.0251</td>
<td>0.0360</td>
<td>0.0880</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.1976</td>
<td>-0.3144</td>
<td>-0.1683</td>
<td>-0.0222</td>
<td>-0.7097</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.8232</td>
<td>1.7287</td>
<td>0.5103</td>
<td>2.0880</td>
<td>3.4507</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.0877</td>
<td>-0.1138</td>
<td>-0.0903</td>
<td>-0.1570</td>
<td>-0.3972</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0675</td>
<td>0.0937</td>
<td>0.0787</td>
<td>0.1656</td>
<td>0.4096</td>
</tr>
<tr>
<td>Durbin Watson</td>
<td>2.1277</td>
<td>2.3012</td>
<td>2.1944</td>
<td>2.1570</td>
<td>1.9952</td>
</tr>
<tr>
<td>Box-Pierce-Ljung (23 lags)</td>
<td>24.156</td>
<td>34.983</td>
<td>36.759</td>
<td>33.195</td>
<td>67.785*</td>
</tr>
<tr>
<td>Jarque-Bera $\chi^2$ (2 d.f.)</td>
<td>22.085*</td>
<td>90.346*</td>
<td>9.842*</td>
<td>116.383*</td>
<td>326.658*</td>
</tr>
<tr>
<td>ARCH (1 d.f.)</td>
<td>3.317$^\ddag$</td>
<td>13.875*</td>
<td>5.324$^\ddag$</td>
<td>11.599*</td>
<td>19.028*</td>
</tr>
</tbody>
</table>

* Significant at the 1% level.
$^\wedge$ Significant at the 5% level.
$^\ddag$ Significant at the 10% level.

The Bovespa futures data extends from 26/07/1990 to 26/07/2001. All returns are denominated in Australian dollars. The Nikkei 225 Index futures data is SIMEX data.

A comparison of descriptive statistics in Tables 5.4 and 5.6 indicate that the All Ordinaries Index spot and futures contract share similar statistical characteristics. Returns on the All Ordinaries futures contract are more volatile than returns on the underlying asset, although the median returns are similar. The AOI futures has a lower mean return than the spot index, which is attributable to the lower minimum return and greater negative skewness of the AOI futures. Of all the futures contracts, the S&P 500 futures contract has statistical properties most closely resembling those of the foreign equity portfolios. For instance, the mean and median of S&P 500 futures returns shown in Table 5.6 are closest to those of the foreign portfolios, shown in Table 5.4. While the returns on the AOI futures, FTSE 100 futures and S&P 500 futures all have standard deviations similar to those of the foreign equity portfolios, the Nikkei and Bovespa futures are considerably more volatile than the foreign equity portfolios. Descriptive statistics associated with the Brazilian Bovespa futures offer the greatest contrast to the behaviour of the foreign equity portfolio returns.

5.4 Correlation between Assets

In the context of investment portfolios, correlation is a basic measure of linear co-dependence between asset returns. The correlations between Australian and foreign equity portfolios, and futures contracts are presented in Table 5.7. From an Australian perspective, domestic equity has a relatively low correlation with foreign equity portfolios. This suggests that Australian investors may gain from international diversification, in contrast to U.S. investors who find the benefits reduced due to a high correlation of the US market with global financial markets. The correlation between the foreign equity portfolios based on the MSCI and Templeton allocations...
is high, as expected given their statistical similarities shown in Table 5.4, but sufficiently low to justify using both for comparison.

Among the futures contracts, the Australian contract has low correlation with foreign futures. All the foreign futures have relatively low correlation with each other, except the S&P 500 and FTSE 100 contracts, which have a correlation of 0.63. Because of the low correlations, multicollinearity is not expected to present major difficulties in analysis involving the simultaneous use of several futures contracts to hedge risk. However, sensitivity analysis is performed in later chapters to assess the impact multicollinearity, and specifically the effect of using the U.S. and U.K. futures contracts in conjunction.

All correlations between the equity portfolios and futures contracts are positive, indicating that the futures contracts move together with the equity portfolios. The Australian All Ordinaries Index and its associated futures contract are highly correlated as expected. Of all the futures, the S&P 500 futures shows the highest correlation with both foreign equity portfolios. This may be attributable to the heavy weighting of U.S. equity in foreign investments and its influence on international financial markets. However, the correlations of the MSCI foreign equity portfolio with the futures contracts are more polarised towards large developed markets relative to the Templeton portfolio correlations, which do not tend to favour the large developed markets as strongly. This suggests that the Templeton foreign equity portfolio may be less sensitive to the choice of a particular futures contract when hedging foreign equity risk. This issue is examined empirically in the following chapter. The differences between the relationships of the MSCI and Templeton foreign equity portfolios and the futures contracts strengthens the motivation to analyse both portfolios.

**Table 5.7: Correlations between Australian equity, foreign equity portfolios, and futures contracts, for the period 05/01/1989 to 26/07/2001**

<table>
<thead>
<tr>
<th>Equity Portfolios</th>
<th>Futures Contracts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Ords Index</td>
</tr>
<tr>
<td>All Ords Index</td>
<td>1.00</td>
</tr>
<tr>
<td>MSCI World ex Au</td>
<td>0.37</td>
</tr>
<tr>
<td>Templeton</td>
<td>0.37</td>
</tr>
<tr>
<td>AOI Futures</td>
<td>0.96</td>
</tr>
<tr>
<td>S&amp;P 500 futures</td>
<td>0.30</td>
</tr>
<tr>
<td>FTSE 100 futures</td>
<td>0.31</td>
</tr>
<tr>
<td>Nikkei futures</td>
<td>0.27</td>
</tr>
<tr>
<td>Bovespa futures</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Correlations between Bovespa are between the period 26/07/1990 to 26/07/2001. Correlation is calculated as the covariance between the asset returns divided by the product of the standard deviations of each asset return series. All returns used in the calculations are denominated in Australian dollars.
This thesis examines the hedging of diversified equity portfolios using futures contracts over the period January 1989 to July 2001. A variety of equity portfolios are constructed using Australian and foreign equity, where foreign equity is represented using the MSCI World excluding Australia index and asset allocations from Templeton World Fund. Futures contracts used to implement portfolio insurance include the Australian All Ordinaries share price index futures, U.S. S&P 500 share price index futures, U.K. FTSE 100 share price index futures, Japanese Nikkei 225 (SIMEX) index futures, and Brazilian Bovespa share price index futures.
CHAPTER SIX

CROSS-HEDGING THE FOREIGN COMPONENT OF INTERNATIONALLY DIVERSIFIED PORTFOLIOS USING SINGLE FUTURES CONTRACTS

6.1 Introduction

This chapter presents empirical evidence on the hedging of diversified foreign equity portfolios using futures contracts. Specifically, the choice of a futures contract to hedge the foreign component of equity portfolios held by Australian investors is investigated to determine which, if any, single hedging instrument is superior, and whether Australian futures are effective risk management tools against foreign equity risk. These issues are important for several reasons. The use of a superior hedging instrument would more effectively prevent losses on the underlying portfolio. The use of a single contract is appealing because transaction costs are lower than if a portfolio of futures is used, and relatively less information and less management time are required. For instance, in order to monitor a futures position, the manager requires information such as specific contract terms and trading rules, in addition to current information on contract prices and exchange rates. As the number of futures contracts used to hedge the diversified equity portfolio increases, the complexity of hedge management increases, particularly as more foreign futures markets are included.

Another issue examined is the relative effectiveness of complex and simple hedge ratio estimation techniques. Although addressed in prior literature, the performance of complex GARCH hedges relative to simple OLS and naïve models remains unresolved, and is revisited in the context of hedging foreign equity portfolios.

Much prior literature examines the direct hedging of an underlying asset using a derivative contract written on that asset, or the cross hedging of individual assets using individual derivatives. In contrast, this chapter examines the cross-hedging of equity portfolios using an individual futures contract not written on those portfolios. The results are examined for sensitivity to time periods, the construction of the equity portfolios, and the risk aversion parameter adopted in the calculation of hedge effectiveness. First, the data and method are discussed, followed by the results, analysis, and conclusions.
The general method is to construct a variety of diversified equity portfolios, and cross-hedge the foreign component using each futures contract, employing a variety of hedge ratio estimation techniques. Foreign equity is represented by the MSCI World excluding Australia price index and a portfolio based on the asset allocations of Templeton World Fund, as described in the previous chapter. The choice of futures contracts used to hedge these portfolios is justified on the basis of their relatively high correlations with the world index compared to the correlations of other markets in their respective regions with the world index, as discussed in Chapter Four. Thus, the key issues addressed in this section are the hedge ratio estimation technique and the measurement of hedge effectiveness.

6.2.1 Hedge Ratio Estimation

Techniques employed to calculate futures hedge ratios in this chapter are the naive, price-changes ordinary least squares (OLS) regression, error correction, and bivariate GARCH(1,1) models, as appropriate. Under the naive model, the hedge ratio is always unity. This method is applied because it is the most simple method available.

The price-changes ordinary least squares regression model is:

\[ \Delta S_t = \alpha + h_t \Delta F_t + \epsilon_t \]  

where \( \Delta S_t \) is the continuously compounded return on equity and \( \Delta F_t \) is the continuously compounded return on the futures contract. Many studies on hedging rely on this technique, despite the non-normality of much financial data. While the data used in this thesis are shown to be non-normal in Chapter Five, the impact of outliers has been reduced through logarithmic transformation of the data, and the use of a weekly data frequency improves the statistical properties of the data set. The price-changes regression technique is included in this thesis for the purposes of comparison with prior literature, coupled with its simplicity and ease of application.

Literature discussed in Chapter Three indicates the existence of cointegration between some spot and futures price levels. However, those studies refer exclusively to matching futures and spot series. In this chapter, cross-hedging is examined, where equity portfolios are the spot assets, and individual futures contracts are not written on those portfolios. The first issue examined in the results section is whether it is necessary to adjust for cointegration between individual futures contracts and foreign equity portfolios. This is done using the commonly applied Engle and Granger (1987) method.
The final hedge ratio estimation technique is the bivariate GARCH(1,1) model, which accounts for heteroskedasticity and past information. The conditional mean and variance that are jointly estimated are:

\[ y_t = \mu + \theta(f_{t-1} - s_{t-1}) + \varepsilon_t \]  
\[ \varepsilon_t | \Omega_{t-1} \sim N(0, H_t) \]  
\[ H_t = C'C + A'\varepsilon_{t-1}A' + G'H_{t-1}G \]

where \( y_t \) = vector of log-differenced spot and futures prices  
\( (f_{t-1} - s_{t-1}) \) = futures premium  
\( \mu \) = vector of means  
\( H_t \) = \((n \times n)\) conditional covariance matrix  
\( C, A, G \) = \((n \times n)\) parameter matrices  
\( \varepsilon_t \) = vector of error terms

This model is estimated using the BEKK parameterization of Engle and Kroner (1995), who extend the basic GARCH model to guarantee that the covariance matrix is positive-definite. Consistent with Gagnon and Lypny (1995, p 774), Gagnon and Lypny (1998, p 204), and Park and Switzer (1995a, p 64), the differenced prices are expressed as percentages. The algorithm of Broyden, Fletcher, Goldfarb and Shanno (BFGS) is used for GARCH parameter estimation.\(^{26}\)

First, the matrix \( H_t \) is estimated using data in the estimation period, then the conditional hedge ratio is calculated by dividing the conditional covariance between futures and spot by the conditional variance of the futures series.

Given weekly data, it is necessary to determine an appropriate estimation period, which is the time period over which data are collected to calculate the hedge ratio. The literature offers little guidance as to the most appropriate estimation period. For example, in the context of calculating GARCH ratios, Cecchetti et al (1988), Gagnon and Lypny (1995, p 772), Gagnon and Lypny (1997, p 71) and Myers (1991, p 45) use 72, 161, 252, and 284 observations respectively for the estimation period. Park and Switzer (1995a, p 65) and Park and Switzer (1995b, p 135) both use estimation periods of 93 days. In this thesis, an estimation period of three years, or 156 weekly data points, is chosen as a reasonable balance between accuracy from including only the most recent information, and obtaining a sufficiently large sample.

The hedge period is the period of time over which the hedge position is maintained. Hedge periods of twenty trading days are used in this thesis, which is approximately one calendar month. This is an appropriate hedge period because portfolios are periodically re-balanced, and

\(^{26}\) This is the procedure used by RATS software. See RATS Version 5 User’s Guide (2000) Estima, USA.
an old hedge is unlikely to be maintained unadjusted for an extensive time. In addition to movements in the value of the equity portfolio, whether from active re-balancing or changes in the market price of various portfolio components, the frequency of revision of the hedge may alter the accuracy of the hedge. For instance, Baillie and Myers (1991) find that optimal hedge ratios are time varying, and that the usual assumption of constant hedge ratios is more costly for some commodities than for others (p 123). However, Ferguson and Leistikow (1998) find that regression approach futures hedge ratios are stationary, and out-of-sample hedging performance is not significantly improved by updating the hedge ratios. Transaction costs associated with frequent adjustment of the hedge position may outweigh the benefits of improved accuracy. In this chapter, constant hedge ratios are used on the assumption that they are accurate over relatively short hedge periods, and time-varying hedge ratios are also calculated for sensitivity analysis.

When the foreign equity portfolio return is treated as a single spot asset and only individual futures contracts are considered in portfolio protection strategies, a potentially complex hedging problem reduces to simple cross-hedging. To assess the intertemporal stability of the results, hedges are constructed for four hedge periods in each of 1992, 1995, 1998 and 2001. The hedge periods commence at the beginning of each quarter.

6.2.2 Measuring Hedge Effectiveness

Analysis of hedge effectiveness is conducted in an out-of-sample setting. Expected utility is used to measure hedge effectiveness, because it incorporates aspects of both risk and return. Further, expected utility allows added flexibility through potentially varying risk aversion, as opposed to effectiveness measures which are constant for all investors. The choice of expected utility as the measure of hedge effectiveness has been discussed in Chapter Three. Expected utility, $E(U)$, is calculated as:

$$E(U) = E(H_t) - \lambda \text{Var}(H_t)$$  \hspace{1cm} (6.5)

where $H_t = R_{us} - R_{ft}$ \hspace{1cm} (6.6)

$\lambda$ = risk aversion parameter

$R_{us}$ = return on unhedged equity portfolio

$R_{ft}$ = return on futures position

$H_t$ = return on the hedged portfolio

The analysis commences with a risk aversion parameter, $\lambda$, equal to three, and the sensitivity of the results to the risk aversion parameter is then assessed. Gagnon and Lypny (1995, p 780) and Gagnon and Lypny (1997, p 75) adopt a risk aversion parameter equal to three. Gagnon and

27 Only three hedge periods are examined in 2001 due to restrictions on the availability of data.
Lypny (1997, p 75) observe that the risk aversion parameter is usually between three and seven. For instance, literature that assumes that $\lambda$ is equal to four includes Koutmos et al (1998, p 46), Grossman and Shiller (1981), Kroner and Sultan (1993), and Park and Switzer (1995a, p 66). Gagnon and Lypny (1998, p 214) set $\lambda$ equal to one, but also examine a range between 0.25 and 2. Given the variety of assumptions in prior literature, in this chapter, $\lambda$ is initially set to three, and sensitivity analysis is then conducted.

6.3 Results and Analysis

The results are set out in four sections. First, a comparison of unhedged foreign equity portfolios is undertaken. Second, tests are conducted to determine whether it is appropriate to apply the error-correction method to calculate hedge ratios. Third, the effectiveness of constant hedge ratios is examined, followed by the effectiveness of time-varying ratios.

6.3.1 Comparison of Unhedged Foreign Equity Portfolios

Expected utilities from the unhedged foreign equity portfolios are reported in Table 6.1, where the risk aversion parameter, $\lambda$, is set equal to 0.5, 2, 3, 4, 5, and 10. The preferred portfolio construction under each risk aversion is indicated in bold. The rankings of the portfolios are unaffected by the risk aversion parameter in thirteen of the fifteen periods. In the two periods where differences in rankings arise, these are driven by very small differences in expected utility. For instance, in the period commencing 01/07/98, the expected utilities appear to be identical when reported at five decimal places in Table 6.1, and an examination of unrounded results was required to determine the appropriate ranking.

Two key findings are evident from Table 6.1. namely that the ranking of the portfolios appears robust to the value of the risk aversion parameter, and that neither foreign equity portfolio construction is clearly preferred across all time periods. The latter result likely reflects the impact of changing market conditions on the relative performance of the component markets, and hence their portfolio weightings. To illustrate, consider an equally weighted portfolio consisting of two assets, A and B, at time $t$. At time $t+1$, the conditions in market A are such that the value of asset A has increased, while the conditions in market B and the value of asset B are unchanged. Taking the portfolio as a whole at time $t+1$, the proportion of total wealth invested in asset A has increased, changing the portfolio from an equally weighted portfolio to a portfolio more heavily weighted in asset A than asset B. Returning to Table 6.1, the finding that neither portfolio construction is preferred in every time period is expected to the extent that performance of each portfolio depends on the changing performance and weightings of
Table 6.1: Comparison of the expected utilities associated with foreign equity portfolios represented by the MSCI world excluding Australia index and the Templeton World Fund portfolio, using several risk aversions, for periods between 1992 and 2001

<table>
<thead>
<tr>
<th>Risk Aversion, $\lambda$</th>
<th>0.5</th>
<th>0.5</th>
<th>2</th>
<th>2</th>
<th>3</th>
<th>3</th>
<th>4</th>
<th>4</th>
<th>5</th>
<th>5</th>
<th>10</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start Date</td>
<td>TWF</td>
<td>MSCI</td>
<td>TWF</td>
<td>MSCI</td>
<td>TWF</td>
<td>MSCI</td>
<td>TWF</td>
<td>MSCI</td>
<td>TWF</td>
<td>MSCI</td>
<td>TWF</td>
<td>MSCI</td>
</tr>
<tr>
<td>02/01/92 29/01/92</td>
<td>0.00082</td>
<td>-0.00020</td>
<td>0.00070</td>
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<td>0.00061</td>
<td>-0.00068</td>
<td>0.00053</td>
<td>-0.00087</td>
<td>0.00045</td>
<td>-0.00106</td>
<td>0.00045</td>
<td>-0.00202</td>
</tr>
<tr>
<td>01/04/92 30/04/92</td>
<td>0.00265</td>
<td>0.00177</td>
<td>0.00253</td>
<td>0.00145</td>
<td>0.00245</td>
<td>0.00124</td>
<td>0.00237</td>
<td>0.00102</td>
<td>0.00229</td>
<td>0.00081</td>
<td>0.00189</td>
<td>-0.00026</td>
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<tr>
<td>01/07/92 28/07/92</td>
<td>-0.00006</td>
<td>-0.00055</td>
<td>-0.00013</td>
<td>-0.00069</td>
<td>-0.00018</td>
<td>-0.00077</td>
<td>-0.00023</td>
<td>-0.00086</td>
<td>-0.00028</td>
<td>-0.00095</td>
<td>-0.00052</td>
<td>-0.00139</td>
</tr>
<tr>
<td>01/10/92 28/10/92</td>
<td>-0.00045</td>
<td>0.00064</td>
<td>0.00027</td>
<td>0.00044</td>
<td>0.0015</td>
<td>0.00031</td>
<td>0.00002</td>
<td>0.00219</td>
<td>0.00019</td>
<td>-0.00010</td>
<td>-0.00070</td>
<td>-0.00059</td>
</tr>
<tr>
<td>03/01/95 30/01/95</td>
<td>0.0034</td>
<td>0.00034</td>
<td>0.00025</td>
<td>0.00023</td>
<td>0.00019</td>
<td>0.00015</td>
<td>0.00013</td>
<td>0.00008</td>
<td>0.00007</td>
<td>0.00000</td>
<td>-0.00023</td>
<td>-0.00038</td>
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<tr>
<td>03/04/95 02/05/95</td>
<td>0.00207</td>
<td>0.00224</td>
<td>0.00201</td>
<td>0.00216</td>
<td>0.00197</td>
<td>0.00210</td>
<td>0.00193</td>
<td>0.00204</td>
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<tr>
<td>03/07/95 28/07/95</td>
<td>0.00022</td>
<td>0.00039</td>
<td>0.00017</td>
<td>0.00033</td>
<td>0.00014</td>
<td>0.00030</td>
<td>0.00011</td>
<td>0.00026</td>
<td>0.00008</td>
<td>0.00022</td>
<td>-0.00013</td>
<td>-0.00207</td>
</tr>
<tr>
<td>02/10/95 27/10/95</td>
<td>-0.00081</td>
<td>-0.00137</td>
<td>-0.00090</td>
<td>-0.00148</td>
<td>-0.00096</td>
<td>-0.00155</td>
<td>-0.00102</td>
<td>-0.00163</td>
<td>-0.00108</td>
<td>-0.00170</td>
<td>-0.00138</td>
<td>-0.000207</td>
</tr>
<tr>
<td>02/01/98 29/01/98</td>
<td>-0.00214</td>
<td>-0.00049</td>
<td>-0.00238</td>
<td>-0.00066</td>
<td>-0.00254</td>
<td>-0.00078</td>
<td>-0.00259</td>
<td>-0.00089</td>
<td>-0.00128</td>
<td>-0.00159</td>
<td>-0.00364</td>
<td>-0.00159</td>
</tr>
<tr>
<td>01/04/98 30/04/98</td>
<td>0.00107</td>
<td>0.00117</td>
<td>0.00093</td>
<td>0.00103</td>
<td>0.00085</td>
<td>0.00094</td>
<td>0.00076</td>
<td>0.00085</td>
<td>0.00067</td>
<td>0.00075</td>
<td>0.00022</td>
<td>0.00029</td>
</tr>
<tr>
<td>01/07/98 28/07/98</td>
<td>0.00075</td>
<td>0.00076</td>
<td>0.00056</td>
<td>0.00066</td>
<td>0.00056</td>
<td>0.00050</td>
<td>0.00054</td>
<td>0.00054</td>
<td>0.00048</td>
<td>0.00048</td>
<td>0.00019</td>
<td>0.00017</td>
</tr>
<tr>
<td>01/10/98 28/10/98</td>
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<td>0.00087</td>
<td>0.00023</td>
<td>0.00034</td>
<td>-0.00022</td>
<td>-0.00001</td>
<td>-0.00066</td>
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<td>-0.00110</td>
<td>-0.00071</td>
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<td>-0.00246</td>
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<tr>
<td>02/01/01 30/01/01</td>
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<td>0.00166</td>
<td>0.00191</td>
<td>0.00150</td>
<td>0.00183</td>
<td>0.00140</td>
<td>0.00176</td>
<td>0.00130</td>
<td>0.00169</td>
<td>0.00120</td>
<td>0.00133</td>
<td>0.00069</td>
</tr>
<tr>
<td>02/04/01 01/05/01</td>
<td>0.00114</td>
<td>0.00157</td>
<td>0.00067</td>
<td>0.00108</td>
<td>0.00036</td>
<td>0.00075</td>
<td>0.00005</td>
<td>0.00043</td>
<td>-0.00026</td>
<td>0.00010</td>
<td>-0.00181</td>
<td>-0.00153</td>
</tr>
<tr>
<td>02/07/01 27/07/01</td>
<td>-0.00158</td>
<td>-0.00096</td>
<td>-0.00175</td>
<td>-0.00114</td>
<td>-0.00185</td>
<td>-0.00127</td>
<td>-0.00196</td>
<td>-0.00139</td>
<td>-0.00207</td>
<td>-0.00152</td>
<td>-0.00260</td>
<td>-0.00214</td>
</tr>
</tbody>
</table>

Expected utility is calculated according to equation (6.5). The start and end dates coincide with the hedge periods used in the remainder of the analysis in this chapter. “TWF” represents the foreign portfolio based on the asset allocations of Templeton World Fund, and “MSCI” represents the foreign portfolio represented by the MSCI World Excluding Australia price index.
component markets. Because neither portfolio construction is obviously superior, results for both the MSCI and Templeton foreign portfolios are analysed throughout the remainder of this chapter. Because expected utility is generally not sensitive to the risk aversion parameter, \( \lambda \) is initially taken to equal three in subsequent analysis, although further sensitivity tests are conducted in the context of hedged portfolios.

### 6.3.2 Cointegration between Futures and Spot Assets

The method of Engle and Granger (1987) is applied to determine whether the futures and spot time series are cointegrated, and hence whether it is necessary to employ error correction method when calculating hedge ratios. The results of the application of the Augmented Dickey-Fuller and Phillips-Perron tests are presented in Table 6.2. Because cointegration relates to long run relationships, the full data set from 1989 to 2001 is tested. In the case of the Bovespa futures, the full available data set from 1990 to 2001 is used. The Augmented Dickey-Fuller and Phillips-Perron tests are conducted under the assumption that a trend exists, and the number of lags is determined automatically by the econometrics package SHAZAM.\(^{28}\) The null hypothesis associated with the tests is that no unit root exists. This hypothesis is rejected when the t-statistic is smaller than the critical value. Critical values reported by SHAZAM are taken from Davidson and MacKinnon (1993). Both the Augmented Dickey-Fuller and Phillips Perron tests indicate that the \( \text{Ln(TWF)}, \text{Ln(MSCI)}, \text{Ln(AOI F)}, \text{Ln(S&P 500 F)}, \text{Ln(FTSE 100 F)}, \text{Ln(NIKK F)} \) and \( \text{Ln(BOV F)} \) are nonstationary given the failure to reject the null hypothesis, and that their first differences are stationary at the 1% significance level. This suggests the existence of an I(1) relationship in each of the futures and spot price levels series, satisfying the first condition for cointegration to exist.

The second step in determining the presence or absence of cointegration is to test the residuals from the regression between the natural logarithms of the futures and spot price levels using the Augmented Dickey-Fuller and Phillips-Perron tests. The presence of cointegration is indicated if the critical value exceeds the test statistic. The results are presented in Table 6.3. Despite high R-squared statistics and low Durbin Watson statistics, both the Augmented Dickey-Fuller and Phillips Perron tests fail to reject the null hypothesis for regressions between each futures contract and the MSCI foreign equity portfolio, suggesting that there is no statistically significant cointegration between these series. Similarly, the Templeton foreign equity portfolio is not cointegrated with any futures.

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\(^{28}\) SHAZAM sets the lags at “the highest significant lag order from either the autocorrelation function or the partial autocorrelation function of the first differenced series” (User’s Reference Manual Version 8.0, p 166-7).
Table 6.2: Augmented Dickey-Fuller and Phillips-Perron tests for the presence of a unit root in various spot and futures price level series, for the period 05/01/1989 to 26/07/2001

<table>
<thead>
<tr>
<th>Start Date</th>
<th>End Date</th>
<th>Variables</th>
<th>Augmented Dickey-Fuller t-test</th>
<th>Phillips-Perron t-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>05/01/89</td>
<td>26/07/01</td>
<td>Ln(TWF)</td>
<td>-2.25</td>
<td>-2.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>First Difference Ln(TWF)</td>
<td>-6.03*</td>
<td>-28.61*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ln(MSCI)</td>
<td>-2.27</td>
<td>-2.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>First Difference Ln(MSCI)</td>
<td>-4.92*</td>
<td>-28.11*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ln(AOI F)</td>
<td>-2.88</td>
<td>-3.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>First Difference Ln(AOI F)</td>
<td>-6.68*</td>
<td>-27.27*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ln(S&amp;P 500 F)</td>
<td>-1.70</td>
<td>-1.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td>First Difference Ln(S&amp;P 500 F)</td>
<td>-5.36*</td>
<td>-29.74*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ln(FTSE 100 F)</td>
<td>-1.73</td>
<td>-2.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>First Difference Ln(FTSE 100 F)</td>
<td>-4.78*</td>
<td>-28.18*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ln(NIKKEI F)</td>
<td>-2.83</td>
<td>-2.78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>First Difference Ln(NIKKEI F)</td>
<td>-5.22*</td>
<td>-27.63*</td>
</tr>
<tr>
<td>26/07/90</td>
<td>26/07/01</td>
<td>Ln(TWF)</td>
<td>-2.45</td>
<td>-2.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>First Difference Ln(TWF)</td>
<td>-5.79*</td>
<td>-27.07*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ln(MSCI)</td>
<td>-1.96</td>
<td>-2.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>First Difference Ln(MSCI)</td>
<td>-5.21*</td>
<td>-26.55*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ln(BOV F)</td>
<td>-3.24</td>
<td>-2.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>First Difference Ln(BOV F)</td>
<td>-5.12*</td>
<td>-24.90*</td>
</tr>
</tbody>
</table>

* Significant at the 1% level. Tests are conducted assuming constant with trend. Weekly data is constructed from Thursday price levels. Ln(TWF) and Ln(MSCI) indicate the natural logarithms of the Templeton World Fund foreign equity portfolio and the MSCI world excluding Australia equity index respectively. Ln(AOI) indicates the natural logarithm of the Australian All Ordinaries Share Price Index futures price level. Ln(S&P 500 F) indicates the natural logarithm of the U.S. S&P 500 Share Price Index futures price level. Ln(FTSE 100 F) indicates the natural logarithm of the U.K. FTSE 100 Index futures price level. Ln(NIKKEI F) indicates the natural logarithm of the Nikkei 225 Index futures (SIMEX) price level. Ln(BOV F) indicates the natural logarithm of the Bovespa Index futures price level.

Table 6.3: Augmented Dickey-Fuller and Phillips-Perron tests for cointegration between futures and spot price series, for the period 05/01/1989 to 26/07/2001

<table>
<thead>
<tr>
<th>Augmented Dickey-Fuller</th>
<th>Phillips-Perron</th>
<th>R-squared</th>
<th>Durbin Watson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ln(TWF) and Ln(AOI F)</td>
<td>-2.0836</td>
<td>-2.4326</td>
<td>0.9417</td>
</tr>
<tr>
<td>Ln(TWF) and Ln(S&amp;P F)</td>
<td>-2.9715</td>
<td>-4.2681†</td>
<td>0.9925</td>
</tr>
<tr>
<td>Ln(TWF) and Ln(FTSE F)</td>
<td>-2.8401</td>
<td>-3.0969</td>
<td>0.9796</td>
</tr>
<tr>
<td>Ln(TWF) and Ln(NIKK F)</td>
<td>-1.7839</td>
<td>-2.2674</td>
<td>0.9347</td>
</tr>
<tr>
<td>Ln(TWF) and Ln(BOV F)</td>
<td>-2.4443</td>
<td>-2.6442</td>
<td>0.9341</td>
</tr>
<tr>
<td>Ln(MSCI) and Ln(AOI F)</td>
<td>-2.0774</td>
<td>-2.5393</td>
<td>0.9354</td>
</tr>
<tr>
<td>Ln(MSCI) and Ln(S&amp;P F)</td>
<td>-2.5157</td>
<td>-3.0876</td>
<td>0.9854</td>
</tr>
<tr>
<td>Ln(MSCI) and Ln(FTSE F)</td>
<td>-2.2071</td>
<td>-2.6029</td>
<td>0.9635</td>
</tr>
<tr>
<td>Ln(MSCI) and Ln(NIKK F)</td>
<td>-1.8866</td>
<td>-1.7291</td>
<td>0.9191</td>
</tr>
<tr>
<td>Ln(MSCI) and Ln(BOV F)</td>
<td>-1.9688</td>
<td>-2.7746</td>
<td>0.9420</td>
</tr>
</tbody>
</table>

* Significant at the 5% level.

The variables are as defined in Table 6.2. Tests involving the Bovespa futures contract are conducted over the period 26/07/1990 to 26/07/2001, which is the full available data set. Tests are conducted assuming constant with trend. Critical values are -3.78 (5%), and -3.50 (10%) for both the ADF tests and Phillips-Perron tests.
contract, with the exception of the S&P 500 futures which is cointegrated at the 5% significance level.

Given that major equity markets, such as the U.S. market, influence and generally move with international equity trends, coupled with the close association between the prices of equity futures contracts and their underlying equity market indices, it is reasonable to expect that futures contracts written on major equity market indices will move relatively closely with the MSCI and Templeton diversified foreign equity portfolios. Thus, the absence of statistically significant cointegration between futures on major equity markets and diversified foreign portfolios, with the exception of the S&P 500 futures and the Templeton portfolio, initially appears surprising. However, the results in Tables 6.2 and 6.3 are less surprising when Figures 6.1, 6.2 and 6.3 are considered. These figures depict the relationships between the natural logarithms of the price levels of the S&P 500 futures contract and the Templeton foreign equity portfolio, the natural logarithms of the price levels of the S&P 500 futures price levels and the MSCI world excluding Australia index, and the natural logarithm of the price levels of the FTSE 100 futures contract and the MSCI portfolio respectively. Using identical scales on the axes, Figures 6.1 and 6.2 show that the relationship between the S&P 500 futures and Templeton foreign equity portfolio follows a linear trend more closely than the relationship between the S&P 500 futures contract and the MSCI index, particularly for lower values. This supports the findings of the Augmented Dickey-Fuller and Phillips Perron tests. Similarly, linear relationships between other futures contracts and equity portfolios, such as that shown in Figure 6.3, are weaker relative to the relationship between the S&P 500 futures and Templeton portfolio. In short, although the futures contracts written on major equity markets are moving in association with the diversified foreign equity portfolios as expected, the finding that only the price levels of the U.S. futures contracts and the Templeton equity portfolio are cointegrated is reasonable.

In summary, statistical evidence indicates that the foreign equity portfolio represented by the MSCI Index is not cointegrated with any of the Australian All Ordinaries Share Price Index futures, S&P 500 index futures, FTSE 100 futures, Nikkei 225 (SIMEX) index futures or Bovespa index futures over the full sample period. Similarly, the Templeton foreign equity portfolio is not cointegrated with any futures contract except the S&P 500 futures. As such, the application of an error correction model is not appropriate in this chapter.
**Figure 6.1:** Relationship between the $\text{Ln}(\text{S&P 500 F})$ and $\text{Ln}(\text{TWF})$, for the period 05/01/89 to 26/07/01

$$\text{Ln}(\text{TWF})$$ and $\text{Ln}(\text{S&P 500 F})$ indicate the natural logarithms of the Templeton World Fund foreign equity portfolio and the S&P 500 futures contract respectively. A linear trendline is shown.

**Figure 6.2:** Relationship between $\text{Ln}(\text{S&P 500 F})$ and $\text{Ln}(\text{MSCI})$, for the period 05/01/89 to 26/07/01

$\text{Ln}(\text{MSCI})$ and $\text{Ln}(\text{S&P 500 F})$ indicate the natural logarithms of the MSCI World excluding Australia index and the S&P 500 futures contracts respectively. A linear trendline is shown.
Figure 6.3: Relationship between Ln(FTSE 100 F) and Ln(MSCI), for the period 05/01/89 to 26/07/01

Ln(MSCI) and Ln(FTSE 100 F) indicate the natural logarithms of the MSCI World excluding Australia index and the FTSE 100 futures contract respectively. A linear trendline is shown.

6.3.3 Cross Hedging using Constant Hedge Ratios

Hedge Ratios

This section describes the constant hedge ratios, prior to the discussion of hedge effectiveness in the following section. Tables 6.4 and 6.5 present constant hedge ratios for the Templeton foreign equity portfolio, calculated using price-changes regression and bivariate GARCH(1,1) methods respectively. All naïve hedge ratios equal one, and are therefore not reported. Comparing Tables 6.4 and 6.5, AOI futures hedge ratios exhibit the largest variation across time, regardless of hedge ratio estimation technique. Further, the ranges of the hedge ratios for all futures except the S&P 500 futures are greater under the GARCH estimation method. This is partly attributable to the presence of some negative GARCH hedge ratios associated with the Bovespa, Nikkei and AOI futures hedges. Recall that the GARCH hedge ratio is:

\[ h_{t-1} = \frac{\text{Cov}\{s_t, f_t | \Omega_{t-1}\}}{\text{Var}\{f_t | \Omega_{t-1}\}} \]  

(6.7)
where

\[ h_{t-1} = \text{hedge ratio that minimises the conditional variance of the hedged portfolio return} \]

\[ \Omega_{t-1} = \text{information available at time } t-1 \]

\[ Cov(s_t, f_t | \Omega_{t-1}) = \text{covariance of spot and futures returns respectively, conditional on the information available at time } t-1 \]

\[ Var(f_t | \Omega_{t-1}) = \text{variance of futures returns at time } t, \text{conditional of information available at time } t-1 \]

Covariance can be expressed as:

\[
Cov(s_t, f_t | \Omega_{t-1}) = \rho(s_t, f_t | \Omega_{t-1}) \sigma(s_t | \Omega_{t-1}) \sigma(f_t | \Omega_{t-1})
\]  

(6.8)

where

\[ \rho(s_t, f_t | \Omega_{t-1}) = \text{correlation between the spot and futures returns, conditional on information available at time } t-1 \]

\[ \sigma(s_t | \Omega_{t-1}) = \text{standard deviation of spot returns, conditional on information available at time } t-1 \]

\[ \sigma(f_t | \Omega_{t-1}) = \text{standard deviation of futures returns, conditional on information available at time } t-1 \]

Substituting the definition of covariance in equation (6.8) into equation (6.7), and reducing to the simplest form, the conditional hedge ratio becomes:

\[
h_{t-1} = \rho(s_t, f_t | \Omega_{t-1}) \times \frac{\sigma(s_t | \Omega_{t-1})}{\sigma(f_t | \Omega_{t-1})}
\]  

(6.9)

As acknowledged by Chou (1974, p 108), the standard deviation is the positive square root of the mean-square deviations of the data from the arithmetic mean. Hence, the ratio of standard deviations of the spot and futures returns in equation (6.9) is positive by definition.

However, correlation may vary between 1 and -1. Thus, negative hedge ratios can only be caused by the presence of a negative correlation between spot and futures returns. Over the long run, correlations between the individual futures and the equity portfolios are positive, as demonstrated in the previous chapter in Table 5.7. However, correlations fluctuate over time, responding to varying market conditions caused by the arrival of new information, and at particular points in time conditional correlations, and hence conditional hedge ratios, can be negative. In practical terms, given a long position in the spot asset, positive hedge ratios are associated with shorting or selling futures, while negative hedge ratios are associated with a long position in futures contracts.
Table 6.4: Constant hedge ratios for the foreign equity portfolio based on Templeton Fund investments, where hedge ratios are calculated using OLS regression, for periods between 05/01/1989 and 28/06/2001

<table>
<thead>
<tr>
<th>Estimation period Start date</th>
<th>Estimation period End date</th>
<th>AOI Futures</th>
<th>S&amp;P 500 Futures</th>
<th>FTSE 100 Futures</th>
<th>Nikkei 225 Futures</th>
<th>Bovespa Futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>05/01/89</td>
<td>26/12/91</td>
<td>0.4660</td>
<td>0.8663</td>
<td>0.5787</td>
<td>0.3735</td>
<td>0.0800</td>
</tr>
<tr>
<td>06/04/89</td>
<td>26/03/92</td>
<td>0.5408</td>
<td>0.8672</td>
<td>0.5684</td>
<td>0.3662</td>
<td>0.0693</td>
</tr>
<tr>
<td>06/07/89</td>
<td>25/06/92</td>
<td>0.5451</td>
<td>0.8690</td>
<td>0.5512</td>
<td>0.3441</td>
<td>0.0630</td>
</tr>
<tr>
<td>05/10/89</td>
<td>24/09/92</td>
<td>0.5356</td>
<td>0.8921</td>
<td>0.5628</td>
<td>0.3171</td>
<td>0.0671</td>
</tr>
<tr>
<td>09/01/92</td>
<td>29/12/94</td>
<td>0.1640</td>
<td>0.7508</td>
<td>0.4698</td>
<td>0.1583</td>
<td>0.0401</td>
</tr>
<tr>
<td>09/04/92</td>
<td>30/03/95</td>
<td>0.1644</td>
<td>0.7609</td>
<td>0.4886</td>
<td>0.1404</td>
<td>0.0467</td>
</tr>
<tr>
<td>09/07/92</td>
<td>29/06/95</td>
<td>0.1331</td>
<td>0.7595</td>
<td>0.5120</td>
<td>0.1315</td>
<td>0.0477</td>
</tr>
<tr>
<td>08/10/92</td>
<td>28/06/95</td>
<td>0.1319</td>
<td>0.7297</td>
<td>0.4983</td>
<td>0.1642</td>
<td>0.0385</td>
</tr>
<tr>
<td>05/01/95</td>
<td>25/12/97</td>
<td>0.3640</td>
<td>0.7058</td>
<td>0.5943</td>
<td>0.2603</td>
<td>0.0947</td>
</tr>
<tr>
<td>06/04/95</td>
<td>26/03/98</td>
<td>0.4161</td>
<td>0.7204</td>
<td>0.6016</td>
<td>0.2228</td>
<td>0.1768</td>
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<tr>
<td>06/07/95</td>
<td>25/06/98</td>
<td>0.3824</td>
<td>0.6838</td>
<td>0.5886</td>
<td>0.1746</td>
<td>0.2001</td>
</tr>
<tr>
<td>05/10/95</td>
<td>24/09/98</td>
<td>0.4369</td>
<td>0.6041</td>
<td>0.5311</td>
<td>0.1673</td>
<td>0.1779</td>
</tr>
<tr>
<td>08/01/98</td>
<td>28/12/00</td>
<td>0.6323</td>
<td>0.7005</td>
<td>0.7337</td>
<td>0.2447</td>
<td>0.1831</td>
</tr>
<tr>
<td>09/04/98</td>
<td>29/03/01</td>
<td>0.6360</td>
<td>0.7111</td>
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<td>0.1859</td>
</tr>
<tr>
<td>09/07/98</td>
<td>28/06/01</td>
<td>0.7773</td>
<td>0.7264</td>
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<td>0.1795</td>
</tr>
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<td>0.7297</td>
<td>0.5684</td>
<td>0.2447</td>
<td>0.0800</td>
</tr>
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<td>Range</td>
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<td>0.2880</td>
<td>0.3008</td>
<td>0.2420</td>
<td>0.1616</td>
</tr>
<tr>
<td>Minimum</td>
<td></td>
<td>0.1319</td>
<td>0.6041</td>
<td>0.4698</td>
<td>0.1315</td>
<td>0.0385</td>
</tr>
<tr>
<td>Maximum</td>
<td></td>
<td>0.7773</td>
<td>0.8921</td>
<td>0.7706</td>
<td>0.3735</td>
<td>0.2001</td>
</tr>
</tbody>
</table>

Table 6.5: Constant hedge ratios for the foreign equity portfolio based on Templeton Fund investments, where hedge ratios are calculated using a bivariate GARCH(1,1) model, for the period 05/01/1989 to 28/06/2001

<table>
<thead>
<tr>
<th>Estimation period Start date</th>
<th>Estimation period End date</th>
<th>AOI Futures</th>
<th>S&amp;P 500 Futures</th>
<th>FTSE 100 Futures</th>
<th>Nikkei 225 Futures</th>
<th>Bovespa Futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>05/01/89</td>
<td>26/12/91</td>
<td>0.2269</td>
<td>0.8584</td>
<td>0.6695</td>
<td>0.3488</td>
<td>0.0462</td>
</tr>
<tr>
<td>06/04/89</td>
<td>26/03/92</td>
<td>0.5680</td>
<td>0.8578</td>
<td>0.5877</td>
<td>0.3519</td>
<td>0.0031</td>
</tr>
<tr>
<td>06/07/89</td>
<td>25/06/92</td>
<td>0.2784</td>
<td>0.8246</td>
<td>0.5485</td>
<td>0.2168</td>
<td>-0.0089</td>
</tr>
<tr>
<td>05/10/89</td>
<td>24/09/92</td>
<td>0.2434</td>
<td>0.9028</td>
<td>0.2890</td>
<td>0.2158</td>
<td>0.0560</td>
</tr>
<tr>
<td>09/01/92</td>
<td>29/12/94</td>
<td>0.2831</td>
<td>0.7846</td>
<td>0.4865</td>
<td>0.1017</td>
<td>0.0308</td>
</tr>
<tr>
<td>09/04/92</td>
<td>30/03/95</td>
<td>0.1355</td>
<td>0.7776</td>
<td>0.5560</td>
<td>0.1245</td>
<td>0.1357</td>
</tr>
<tr>
<td>09/07/92</td>
<td>29/06/95</td>
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<td>0.8202</td>
<td>0.4587</td>
<td>0.2352</td>
<td>0.0413</td>
</tr>
<tr>
<td>08/10/92</td>
<td>28/09/95</td>
<td>0.2038</td>
<td>0.7788</td>
<td>0.5860</td>
<td>0.1543</td>
<td>0.0321</td>
</tr>
<tr>
<td>05/01/95</td>
<td>25/12/97</td>
<td>0.5255</td>
<td>0.7073</td>
<td>0.5010</td>
<td>0.2499</td>
<td>0.2218</td>
</tr>
<tr>
<td>06/04/95</td>
<td>26/03/98</td>
<td>0.3234</td>
<td>0.6542</td>
<td>0.5794</td>
<td>0.1653</td>
<td>0.1358</td>
</tr>
<tr>
<td>06/07/95</td>
<td>25/06/98</td>
<td>-0.1148</td>
<td>0.6714</td>
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<td>0.0259</td>
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</tr>
<tr>
<td>05/10/95</td>
<td>24/09/98</td>
<td>0.6764</td>
<td>0.6829</td>
<td>0.2236</td>
<td>-0.2461</td>
<td>0.1007</td>
</tr>
<tr>
<td>08/01/98</td>
<td>28/12/00</td>
<td>0.7826</td>
<td>0.8165</td>
<td>0.7862</td>
<td>0.0606</td>
<td>0.1596</td>
</tr>
<tr>
<td>09/04/98</td>
<td>29/03/01</td>
<td>0.9144</td>
<td>0.8887</td>
<td>0.5325</td>
<td>0.1249</td>
<td>0.1780</td>
</tr>
<tr>
<td>09/07/98</td>
<td>28/06/01</td>
<td>0.6992</td>
<td>0.6915</td>
<td>0.8894</td>
<td>0.4029</td>
<td>0.3073</td>
</tr>
<tr>
<td>Median</td>
<td></td>
<td>0.2831</td>
<td>0.7846</td>
<td>0.5485</td>
<td>0.1653</td>
<td>0.1007</td>
</tr>
<tr>
<td>Range</td>
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<td>1.0292</td>
<td>0.2486</td>
<td>0.6658</td>
<td>0.6490</td>
<td>0.3161</td>
</tr>
<tr>
<td>Minimum</td>
<td></td>
<td>-0.1148</td>
<td>0.6542</td>
<td>0.2236</td>
<td>-0.2461</td>
<td>-0.0089</td>
</tr>
<tr>
<td>Maximum</td>
<td></td>
<td>0.9144</td>
<td>0.9028</td>
<td>0.8894</td>
<td>0.4029</td>
<td>0.3073</td>
</tr>
</tbody>
</table>
A comparison of Tables 6.4 and 6.5 reveals that the OLS hedge ratios are positive in the periods where the GARCH ratios are negative. This occurs because the OLS ratios are calculated using the correlation over the full estimation period, while the GARCH ratios depend on the most recent correlation. To illustrate, Table 6.5 shows that the GARCH hedge ratio calculated using the Nikkei futures contract to hedge the Templeton portfolio in the estimation period commencing 05/10/95 is -0.2461, while the OLS hedge ratio is 0.1673. Figure 6.4 depicts the correlation between the Nikkei futures returns and the Templeton equity portfolio returns throughout the estimation period, and provides a comparison with the correlation over the full period. The correlation is generally positive, but becomes negative towards the close of the period. The GARCH hedge ratio that is applied in the out-of-sample hedge period following the estimation period is calculated using the most recent correlation, which Figure 6.4 shows to be negative. Hence, the GARCH hedge ratio in this case is negative. In contrast, the OLS ratio in this example is positive because the correlation over the full estimation period is positive. Hence, it is reasonable to obtain positive OLS ratios and negative GARCH ratios for the same spot and futures assets in the same time period, when correlation is negative at the close of the estimation period but positive over the full period.

Figure 6.4: Correlation between the Templeton portfolio and the Nikkei futures, in-sample for the estimation period 05/10/95 to 24/09/98

Constant correlation is the correlation over the full period, and is used in the OLS hedge ratio. Varying correlation indicates the correlation at each point in time, and the final figure is used in the GARCH hedge ratio.
Returning to Tables 6.4 and 6.5, the S&P 500 futures have the highest median hedge ratio, and the Bovespa futures have the lowest median hedge ratio for both OLS and GARCH hedge ratio estimation methods. Of the futures contracts analysed, the Bovespa futures is written on the least developed and most volatile underlying market. The high volatility of emerging markets relative to developed markets has been documented by researchers such as DeSantis and Imrohoroglu (1997). The finding that the lowest hedge ratios are associated with Bovespa futures hedges may be consistent with that contract’s ability to cover a highly volatile underlying market.

Simple ANOVA tests, presented in Table 6.6, reveal no statistically significant difference between the mean of hedge ratios calculated using the OLS regression and GARCH techniques. This implies that from a statistical perspective, there is no advantage to using the relatively complex GARCH method over simple OLS regressions when estimating hedge ratios. This is consistent with some prior literature, such as Thomas and Brooks (2001), Baillie and Myers (1991), and McNew and Fackler (1994), although those studies examine the hedging of individual assets only rather than portfolios of assets. However, while Table 6.6 supports the use of simple over complex techniques of hedge ratio estimation, in that there is no statistically significant difference in hedge ratios, the differences may still possess economic significance.

Table 6.6: ANOVA test statistics for equality of means of constant OLS regression and bivariate GARCH(1,1) hedge ratios, to hedge the Templeton foreign equity portfolio, in various estimation periods between 05/01/1989 and 26/07/2001

<table>
<thead>
<tr>
<th>ANOVA</th>
<th>AOI Futures</th>
<th>S&amp;P 500 Futures</th>
<th>FTSE 100 futures</th>
<th>Nikkei 225 futures</th>
<th>Bovespa futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-statistic</td>
<td>0.1497</td>
<td>0.6760</td>
<td>0.6236</td>
<td>2.6748</td>
<td>0.0014</td>
</tr>
<tr>
<td>p-value</td>
<td>0.7018</td>
<td>0.4179</td>
<td>0.4363</td>
<td>0.1131</td>
<td>0.9706</td>
</tr>
</tbody>
</table>

Comparison of Unhedged and Hedged Positions

This section compares the expected quadratic utility of hedged and unhedged portfolios to demonstrate the benefits of hedging, as a prelude to the comparison of hedging instruments and techniques in the following section. Table 6.7 provides expected quadratic utilities associated with hedged and unhedged positions, where hedge ratios are calculated using price-changes regressions and foreign equity is represented using the Templeton portfolio. The highest expected utility in each hedge period is shown in bold, indicating the most preferred strategy. In thirteen of a total of fifteen hedge periods, futures hedging provides utility gains over an unhedged position. This finding is similarly true when foreign equity is

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29 Over the period 05/01/89 to 26/07/01, the standard deviations of weekly continuously compounded returns on the Brazilian Bovespa index, Japanese Nikkei 225 index, UK FTSE 100 index, US S&P 500 index and Australian AOI Index are 0.0985, 0.0356, 0.0240, 0.0239 and 0.0178 respectively, where all returns are denominated in Australian dollars.
represented using the MSCI index, although the results are not reported. Further tests reveal the superiority of a hedged position over an unhedged position for different hedge ratio estimation techniques and frequencies of re-estimation of the hedge ratio, and the results apply to both Templeton and MSCI foreign equity portfolios. However, because the unhedged position is generally not associated with the lowest expected utility in each hedge period, it remains superior to poorly performing hedged positions. This shows the importance of examining which futures contracts, if any, produce superior hedge protection relative to other futures because all hedges do not perform equally, and while some are superior to an unhedged position, others are relatively inferior.

Table 6.7: Expected quadratic utility associated with unhedged and hedged Templeton foreign equity portfolios, where hedge ratios are calculated using OLS and are held constant over the hedge period, for periods between 1992 and 2001

<table>
<thead>
<tr>
<th>Hedge Period Start date</th>
<th>Hedge Period End date</th>
<th>Unhedged Hedge portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>TWF Futures hedge</td>
</tr>
<tr>
<td>02/01/92</td>
<td>29/01/92</td>
<td>0.000614</td>
</tr>
<tr>
<td>01/04/92</td>
<td>30/04/92</td>
<td>0.002451</td>
</tr>
<tr>
<td>01/07/92</td>
<td>28/07/92</td>
<td>-0.000182</td>
</tr>
<tr>
<td>01/10/92</td>
<td>28/10/92</td>
<td>0.000145</td>
</tr>
<tr>
<td>03/01/95</td>
<td>30/01/95</td>
<td>0.000194</td>
</tr>
<tr>
<td>03/04/95</td>
<td>02/05/95</td>
<td>0.001970</td>
</tr>
<tr>
<td>03/07/95</td>
<td>28/07/95</td>
<td>0.000141</td>
</tr>
<tr>
<td>02/10/95</td>
<td>27/10/95</td>
<td>-0.000199</td>
</tr>
<tr>
<td>02/01/98</td>
<td>29/01/98</td>
<td>-0.002535</td>
</tr>
<tr>
<td>01/04/98</td>
<td>30/04/98</td>
<td>0.000845</td>
</tr>
<tr>
<td>01/07/98</td>
<td>28/07/98</td>
<td>0.000600</td>
</tr>
<tr>
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<td>28/10/98</td>
<td>-0.000215</td>
</tr>
<tr>
<td>02/01/01</td>
<td>30/01/01</td>
<td>0.001833</td>
</tr>
<tr>
<td>02/04/01</td>
<td>01/05/01</td>
<td>0.000361</td>
</tr>
<tr>
<td>02/07/01</td>
<td>27/07/01</td>
<td>-0.001853</td>
</tr>
</tbody>
</table>

Expected utility is calculated using equation (6.5), and assuming that the risk aversion parameter $\lambda=3$. Bold figures indicated the highest expected utility and the most preferred scenario.

6.3.4 Effectiveness of Constant Hedge Ratios

A number of issues are relevant when determining whether any futures contract is preferred over other futures when hedging, such as the impact of hedge ratio estimation technique, equity portfolio construction, and the risk aversion parameter in the expected utility effectiveness measure.

Comparison of Australian and Foreign Futures Hedges

The relative benefits of hedging foreign equity portfolios with individual Australian or foreign futures is examined in Table 6.8. In any given hedge period, the expected utilities of different futures hedges are compared and rankings are accorded to each strategy, where a rank of one indicates the highest expected utility and is the most preferred option while a
rank of five indicates the lowest expected utility. Table 6.8 shows the rankings for Australian AOI futures relative to the other four foreign contracts, for a variety of hedge ratio estimation techniques. For example, in the hedge period commencing 02/01/92, under the OLS hedge ratio estimation method, the Australian AOI futures hedge is ranked second of the five futures hedges when foreign equity is represented using the Templeton portfolio. In the same hedge period, under the naïve hedge of the Templeton portfolio, the Australian futures hedge ranked second out of the five futures hedges examined.

Under the naïve approach, the Australian AOI futures is accorded the number one ranking in two of fifteen hedge periods, for both Templeton and MSCI foreign equity portfolios. This indicates that in two of fifteen periods, Australian futures provide better naïve hedges than foreign futures. Similarly, when constant hedge ratios are calculated using price changes regressions, Australian futures hedges are superior to all foreign futures hedges in two of fifteen periods for both equity portfolio constructions. When the bivariate GARCH approach is applied, Australian futures are ranked number one twice for the MSCI portfolio and three times for the Templeton portfolio.

Table 6.8: Rankings of Australian AOI futures hedges relative to the four foreign futures hedges under different hedge ratio estimation methods, using the Templeton and MSCI foreign equity portfolios, for hedge periods between 1992 and 2001

<table>
<thead>
<tr>
<th>Hedge period Start date</th>
<th>Hedge period End date</th>
<th>TWF</th>
<th>GARCH</th>
<th>MSCI</th>
<th>GARCH</th>
</tr>
</thead>
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<tr>
<td>02/01/92</td>
<td>29/01/92</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>01/04/92</td>
<td>30/04/92</td>
<td>3</td>
<td>3</td>
<td>3</td>
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<td>01/07/92</td>
<td>28/07/92</td>
<td>1</td>
<td>2</td>
<td>2</td>
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</tr>
<tr>
<td>01/10/92</td>
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<td>03/01/95</td>
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<td>2</td>
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</tr>
<tr>
<td>03/04/95</td>
<td>02/05/95</td>
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<td>3</td>
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<td>03/07/95</td>
<td>28/07/95</td>
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<tr>
<td>02/10/95</td>
<td>27/10/95</td>
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<td>4</td>
<td>1</td>
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<tr>
<td>02/01/98</td>
<td>29/01/98</td>
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<td>30/04/98</td>
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<td>27/07/01</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Ranks are based on expected utility, which is calculated using equation (6.5), assuming that the risk aversion parameter \( \lambda = 3 \). "TWF" indicates the foreign equity portfolio based on Templeton World Fund asset allocations. "MSCI" indicates the MSCI world excluding Australia index.

Naïve hedges using Australian futures are never ranked fifth, indicating that naïve Australian futures hedges outperform at least one naïve foreign hedge strategy in every hedge period. This is true for both equity portfolio constructions. Under the OLS approach, Australian
futures hedges are superior to at least one foreign futures hedge in all but one period for both the Templeton and MSCI portfolios. Finally, under the GARCH method, Australian futures outperform at least one foreign futures hedge in all but two cases.

To summarise, in approximately 13% of hedge periods, the use of Australian futures to hedge a foreign portfolio is more successful than the use of any of the foreign futures hedges examined. Further, Australian futures hedges outperform at least one foreign futures hedge in thirteen of fifteen hedge periods. This result is not generally sensitive to equity portfolio construction or hedge ratio estimation method. Australian fund managers are likely to be more familiar with domestic derivatives than foreign derivatives, and these results provide some support for the use of Australian equity futures to hedge internationally diversified equity portfolios.

Comparison of Individual Futures Hedges

Broadening this analysis to examine the relative effectiveness of each futures contract as a hedging vehicle, the frequency of number one rankings across time and the average ranking over all the time periods are both considered to arrive at a single overall ranking. The frequency of number one rankings is a first-past-the-post analysis where only the best ranking matters. In contrast, when using the highest average ranking across time, all rankings matter, so if a contract has high rankings in some periods but low rankings in other periods, relatively poor performance is also captured. The smaller the average rank, the more effective hedges using that futures contract are on average.

Consider, for example, the results in Table 6.9, which show the rankings associated with each futures hedge in each hedge period when constant hedge ratios are calculated using the GARCH(1,1) method and foreign equity is represented using the Templeton portfolio. Using average rankings, the most effective hedge is the Nikkei futures hedge, followed by the AOI and Bovespa futures hedges in second place, the FTSE futures hedge in third place, and the S&P 500 futures hedge last. Considering the frequency of number one rankings, the Nikkei futures hedge is best, the AOI and FTSE futures hedges rank jointly second, and the S&P 500 and Bovespa futures hedges are the least effective. However, all hedging vehicles attract a number one ranking in at least one hedge period. Taking these two aspects of performance into account, the Nikkei futures hedge is superior overall. This process is repeated for each method of hedge ratio estimation, and for both MSCI and Templeton foreign equity portfolios. The complete summary of results is presented in Table 6.10.
Table 6.9: Rankings of hedged portfolios on the basis of expected quadratic utility, where foreign equity is represented using the Templeton allocations, and constant hedge ratios are calculated using the GARCH(1,1) model

<table>
<thead>
<tr>
<th>Hedge period Start date</th>
<th>Hedge period End date</th>
<th>AOI futures hedge</th>
<th>S&amp;P 500 futures hedge</th>
<th>FTSE 100 futures hedge</th>
<th>Nikkei 225 futures hedge</th>
<th>Bovespa futures hedge</th>
</tr>
</thead>
<tbody>
<tr>
<td>02/01/92</td>
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<td>2</td>
<td>4</td>
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<td>2</td>
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<td>1</td>
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<tr>
<td>02/07/01</td>
<td>27/07/01</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Average</td>
<td>2.87</td>
<td>3.80</td>
<td>2.93</td>
<td>2.53</td>
<td>2.87</td>
<td></td>
</tr>
<tr>
<td>Frequency #1</td>
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<td>2</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Ranks are based on expected utility, which is calculated using equation (6.5), assuming that the risk aversion parameter \( \lambda = 3 \). A rank of one indicates the highest expected utility and the most preferred scenario, while a rank of five indicates the lowest expected utility and the least preferred option. The “Average” is a simple average of the rankings of each hedge ratio estimation method across all time periods for each futures contract. The “Frequency #1” shows the number of hedge periods in which the hedge using a particular futures contract was ranked best, or number one.

Table 6.10 illustrates the relative performance of all the constant futures hedges based on the average ranking and the frequency of number one rankings, where performance is measured by expected utility with \( \lambda \) equal to three. A rank of one indicates the most preferred scenario, while a rank of five indicates the least preferred option. For instance, the first row of Table 6.10 shows that on the basis of highest average expected utility ranking, the FTSE futures hedge performs best, followed by the AOI, Bovespa, Nikkei and S&P 500 futures hedges in that order, when the Templeton portfolio is naively hedged. The second row indicates that on the basis of the frequency of number one ranks, the FTSE and Bovespa futures hedges perform the best, and the S&P 500 futures hedge is worst.

When foreign equity is represented using the MSCI portfolio, constant FTSE futures hedges are superior for all methods of hedge ratio estimation under the best average rank criteria. Under the highest frequency of number one ranks, Nikkei futures hedges are superior when constant OLS and GARCH ratios are employed, and the FTSE and Bovespa futures hedges are jointly superior under the naive approach.
A rank of one indicates the most preferred scenario, while a rank of five indicates the least preferred option. Where two hedges are associated with the same level of preference, each attracts the same ranking, but the rank below is not allocated. For instance, in the second row, the FTSE and Bovespa futures both have the highest frequency of number one ranks, so both are ranked 1 under this criteria, and no "rank 2" is given because the top two hedges have already been accounted for. "Rank(Av rank)" indicates the order of preference for hedging vehicles based on the average rank across the fifteen hedge periods examined. "Rank(Freq #1)" indicates the order of preference for hedging vehicles based on the frequency with which the futures hedge is ranked best of the five possible hedges in each hedge period.

Results are generally more variable when foreign equity is represented using the Templeton portfolio. Under the best average rank criteria, FTSE futures hedges are best when using the naïve model, FTSE and Bovespa futures hedges are jointly superior under the OLS method, and Nikkei futures hedges are best when using constant GARCH. On the basis of the highest frequency of number one ranks, the Nikkei hedges are best for OLS and GARCH methods, and the FTSE and Bovespa futures perform jointly best under the naïve model.

S&P 500 futures hedges perform the worst, regardless of hedge ratio estimation technique. This occurs despite the relatively high correlation of the S&P 500 futures with the Templeton and MSCI equity portfolios, a finding demonstrated in Table 5.7 in the previous chapter, indicating that hedge effectiveness is not attributable entirely to simple linear correlation.

In summary, while the results are sensitive to the method of hedge ratio estimation and method of assessing hedge performance, generally hedging foreign equity portfolios using the FTSE and Nikkei futures contracts is more effective than hedging using the other futures contracts. The fact that different contracts perform better in different time periods, as shown for example in Table 6.9, suggests that no contract is redundant in the context of hedging foreign equity risk because they all have different features that make them valuable at
different times. However, the FTSE and Nikkei futures hedges generally outperform hedges using other futures when all hedge periods from 1992 to 2001 are considered together.

**Equity Portfolio Construction**

To assess the sensitivity of the performance of individual futures contracts to the construction of foreign equity portfolios, the performance of hedged portfolios are compared when foreign equity is represented using the portfolio based on Templeton allocations and using the MSCI World excluding Australia index. This is done by examining the rankings of individual futures hedges in a given hedge period when the Templeton portfolio is hedged and comparing this to the rankings in that hedge period when the MSCI portfolio is hedged, repeating this procedure for every hedge period. Given simple naïve hedge ratios, there is only one hedge period of a total of fifteen where the rankings of each futures contract are not identical regardless of portfolio construction. In short, the relative effectiveness of naïve hedges using the AOI, S&P 500, FTSE 100, Nikkei 225 and Bovespa futures contracts is unaffected by foreign equity portfolio construction, in all but one time period. When hedge ratios are calculated using price changes OLS regressions, differences in the ranks of individual futures contracts occur in three time periods. Under the GARCH method, which is the most complex hedge ratio estimation technique, ranks differ in eight of fifteen cases. In summary, as the complexity of the hedge ratio estimation technique increases, hedge effectiveness becomes more sensitive to the method of construction of the equity portfolio.

**Comparison of Hedge Ratio Estimation Techniques**

Given that the hedge effectiveness of individual futures contracts depends to some extent on the hedge ratio estimation method, this section examines whether one constant hedge ratio estimation method is preferred to another in the context of hedging foreign equity risk. The choice of hedge ratio estimation technique has received considerable attention in prior literature on direct hedging and cross-hedging, and remains an unresolved issue, as discussed at length in Chapter Three. Rankings of constant hedges estimated using different estimation methods for the Templeton portfolios are provided in Table 6.11. Table 6.11 illustrates that, on the basis of average ranking, the OLS model gives hedge ratios of the greatest expected utility for the AOI, Nikkei and Bovespa futures hedges, while the GARCH model is preferred when using S&P 500 futures. The OLS and GARCH models are jointly preferred over the naïve model when using the FTSE futures. Similarly, when the MSCI equity...
Table 6.11: Comparison of expected utility from constant hedge ratios calculated using the naïve, price-changes regression, and GARCH(1,1) approaches, for a variety of futures contracts, where the foreign equity portfolio is the Templeton portfolio.

<table>
<thead>
<tr>
<th>Hedge Period</th>
<th>Start Date</th>
<th>End Date</th>
<th>AOI futures Naïve</th>
<th>OLS</th>
<th>GARCH</th>
<th>S&amp;P futures Naïve</th>
<th>OLS</th>
<th>GARCH</th>
<th>FTSE futures Naïve</th>
<th>OLS</th>
<th>GARCH</th>
<th>Nikkei futures Naïve</th>
<th>OLS</th>
<th>GARCH</th>
<th>Bovespa futures Naïve</th>
<th>OLS</th>
<th>GARCH</th>
</tr>
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<tr>
<td></td>
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<td>02/01/92</td>
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<tr>
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<td>3</td>
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<tr>
<td>Rank(Av rank)</td>
<td></td>
<td></td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>3</td>
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<td>3</td>
</tr>
</tbody>
</table>

A rank of one indicates the highest expected utility and the most preferred scenario, while a rank of three indicates the lowest expected utility and the least preferred option. "Rank(Av rank)" indicates the order of preference for hedging vehicles based on the average rank across the fifteen hedge periods examined.
portfolio is hedged, the results for the AOI, S&P 500, and Bovespa futures hedges are the same. However, when hedging the MSCI portfolio using FTSE futures, the OLS method is best, and when using the Nikkei futures, the GARCH method achieves the best performance. The superiority of hedge ratio estimation method appears to be contract-specific, which may explain conflict in prior research over the relative performance of different hedge ratio estimation techniques. For instance, contrast Gagnon and Lypny (1997), Gagnon and Lypny (1995), and Park and Switzer (1995a), who find that GARCH models outperform naive and OLS models, with Lien and Luo (1994), Baillie and Myers (1991), and Myers (1991) who find evidence favouring more simple models.

While there is no statistically significant difference between the expected utilities associated with each hedge ratio estimation technique, as shown in Table 6.12, earlier arguments for economic significance remain. The lack of statistically significant differences between hedge ratios calculated using different techniques is not unique to this study. For instance, in an examination of hedge ratio estimation techniques in the context of hedging the Australian All Ordinaries Index with the futures contract written on that index, Thomas and Brooks (2001) find no significant difference between the effectiveness of GARCH and OLS hedge ratios.

Table 6.12: ANOVA tests for significant difference between the expected utility from each hedge ratio estimation method, for each futures contract, using the Templeton and MSCI foreign equity portfolios

<table>
<thead>
<tr>
<th>Equity portfolio</th>
<th>ANOVA</th>
<th>AOI futures</th>
<th>S&amp;P 500 futures</th>
<th>FTSE 100 futures</th>
<th>Nikkei futures</th>
<th>Bovespa futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWF</td>
<td>F-statistic</td>
<td>0.1872</td>
<td>0.4862</td>
<td>0.0956</td>
<td>1.8280</td>
<td>1.1427</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>0.8299</td>
<td>0.6184</td>
<td>0.9091</td>
<td>0.1733</td>
<td>0.3287</td>
</tr>
<tr>
<td>MSCI</td>
<td>F-statistic</td>
<td>0.3899</td>
<td>0.6855</td>
<td>0.1183</td>
<td>2.0386</td>
<td>1.1055</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>0.6796</td>
<td>0.5094</td>
<td>0.8887</td>
<td>0.1429</td>
<td>0.3405</td>
</tr>
</tbody>
</table>

Sensitivity to Risk Aversion Parameter

For all previous hedging results, the risk aversion parameter, $\lambda$, in the expected utility calculation is taken to equal three. To assess the sensitivity of the results to the risk aversion parameter, expected utility is recalculated using various risk aversions between 0.1 and 10. For instance, to quantify the impact of the risk aversion parameter on the OLS regression hedges of the Templeton portfolio, a new $\lambda$ is chosen, the expected utilities for all contracts in all time periods are recalculated, and the new ranks of the five futures hedges in each time period are derived. The new ranking pattern within each hedge period is then compared with the ranking pattern obtained when $\lambda$ equals three. The percentage of cases where changing the risk aversion parameter alters the ranking pattern of the futures hedges is calculated by dividing the number of hedge periods where the five futures hedges are ranked differently from the results where $\lambda$ equals three by the total hedge periods examined (fifteen), and
multiplying the result by 100. This procedure is repeated to compare each set of results using different risk aversions with the results where the risk aversion parameter equals three. Consider, for example, the results for the price-changes regression hedges on the Templeton equity portfolio in Figure 6.5. As the risk aversions become larger or smaller than the default of three, the percentage of hedge periods where the five futures are ranked differently increases, as expected. However, even when \( \lambda \) is set at the extremes of 0.1 and 10, the rankings in the majority of hedge periods remain unaffected by the changes in \( \lambda \). These findings are equally true for the MSCI portfolio. This indicates that the results are robust to the risk aversion parameter, which is assumed to equal three for the remainder of this chapter.

![Figure 6.5: Sensitivity of the rankings of futures hedges within each hedge period to the risk aversion parameter \( \lambda \), where constant OLS hedge ratios are applied to the Templeton foreign equity portfolio](image)

### 6.3.5 Cross-Hedging using Time-Varying Hedge Ratios

A criticism of the use of constant hedge ratios is that the arrival of new information over time may change the relationship between the futures and spot assets, introducing error into the hedge. To examine the impact of assuming constant hedge ratios on the analysis in this chapter, time varying hedge ratios are calculated for every hedge period. Given a weekly data frequency and a hedge period of one calendar month, five ratios are calculated in each hedge period. The first ratio is identical to the ratio held constant for all previous analysis, and the other four ratios are calculated throughout the hedge period by including the most recent data point and deleting the oldest data point so that the length of estimation period remains constant at three years of data. As with the constant hedge ratios, time-varying ratios are calculated using price-changes regression and bivariate GARCH(1,1) methods.
Generally, the OLS hedge ratios vary less than the GARCH ratios as the estimation period is rolled forward within a given hedge period.

**Comparison of Futures Contracts**

Using the same procedure as in Table 6.10, Table 6.13 summarises the results based on the expected utilities associated with dynamic hedges where \( \lambda \) equals three. On the basis of both the largest frequency of first ranks and the best average rank, the FTSE futures hedges are superior to other futures hedges for both the Templeton and MSCI portfolios, using the price changes OLS regression method. When the dynamic GARCH method is used to calculate hedge ratios for the Templeton portfolio, the Nikkei futures perform best on the basis of both the frequency of number one ranks and average ranks. When the dynamic GARCH method is applied to the MSCI portfolio, the Nikkei futures hedges exhibit superior performance based on the frequency of number one rankings, and the FTSE futures hedge is superior given average rankings.

**Table 6.13: Comparison of time-varying futures hedges based on the average rank and the frequency of number one ranks across time**

<table>
<thead>
<tr>
<th>Equity Portfolio</th>
<th>Hedge ratio estimation method</th>
<th>Ranking criteria</th>
<th>AOI futures hedge</th>
<th>S&amp;P 500 futures hedge</th>
<th>FTSE 100 futures hedge</th>
<th>Nikkei futures hedge</th>
<th>Bovespa futures hedge</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWF</td>
<td>Varying OLS</td>
<td>Rank (Average)</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rank (Freq #1)</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Varying GARCH</td>
<td>Rank (Average)</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rank (Freq #1)</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>MSCI</td>
<td>Varying OLS</td>
<td>Rank (Average)</td>
<td>2</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rank (Freq #1)</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Varying GARCH</td>
<td>Rank (Average)</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>4</td>
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<tr>
<td></td>
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<td>Rank (Freq #1)</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

A rank of one indicates the most preferred scenario, while a rank of five indicates the least preferred option. “Rank(Av rank)” indicates the order of preference for hedging vehicles based on the average rank across the fifteen hedge periods examined. “Rank(Freq #1)” indicates the order of preference for hedging vehicles based on the frequency with which the futures hedge is ranked best of the five possible hedges in each hedge period. “TWF” indicates the foreign equity portfolio based on Templeton World Fund allocations, and “MSCI” indicates the MSCI world excluding Australia index.

The results for the constant and time varying hedge ratios are summarised in Table 6.14. From Table 6.14, it is clear that the Nikkei and FTSE 100 futures contracts provide the most effective hedges of foreign equity portfolios, although the preferred contract depends on the method of hedge ratio estimation.
Table 6.14: Futures contracts associated with the most effective hedges of foreign equity portfolios

<table>
<thead>
<tr>
<th>Hedge ratio estimation method</th>
<th>Templeton Portfolio</th>
<th>MSCI Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naive</td>
<td>FTSE/Bovespa</td>
<td>FTSE</td>
</tr>
<tr>
<td>Constant OLS</td>
<td>Nikkei</td>
<td>FTSE/Bovespa</td>
</tr>
<tr>
<td>Constant GARCH</td>
<td>Nikkei</td>
<td>Nikkei</td>
</tr>
<tr>
<td>Time varying OLS</td>
<td>FTSE</td>
<td>FTSE</td>
</tr>
<tr>
<td>Time varying GARCH</td>
<td>Nikkei</td>
<td>FTSE</td>
</tr>
</tbody>
</table>

"FTSE/Bovespa" indicates that the FTSE and Bovespa futures are both preferred to all other futures in the given scenario but not to each other.

Comparison between Constant and Time Varying ratios

A comparison of constant and time varying hedge ratio estimation methods is undertaken for each futures contract. For example, consider the results for FTSE futures hedges in Table 6.15. Naive hedge ratios are most frequently ranked number one relative to other methods, but even more frequently receive the worst ranking. This illustrates the importance of considering the average ranking as well as how many number one rankings are obtained. On average, the time-varying GARCH ratios provide the highest expected utility when hedging the Templeton portfolio using the FTSE futures contract. This procedure is repeated for each futures contract to obtain the results comparing constant and dynamic hedge ratio estimation methods, which are summarised in Table 6.16.

Table 6.15: Rankings for hedges of the Templeton equity portfolio, using FTSE 100 futures contracts, where rank is based on expected quadratic utility

<table>
<thead>
<tr>
<th>Hedge period Start date</th>
<th>Hedge period End date</th>
<th>Naive</th>
<th>Constant OLS</th>
<th>Constant GARCH</th>
<th>Varying OLS</th>
<th>Varying GARCH</th>
</tr>
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<tbody>
<tr>
<td>02/01/92</td>
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</table>

Rank(Average) 5 2 2 2 1 1
Rank(# freq) 2 3 4 4 1

Expected utility is calculated using equation (6.5), where $\lambda = 3$. 

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Table 6.16 indicates that the naïve model is never preferred on the basis of highest average ranking. Although the time-varying GARCH method is preferred most frequently, no method is obviously preferred in all cases. In short, the results appear to be contract-specific. This confirms the conclusion of Baillie and Myers (1991, p 122), who find that in the context of commodity hedges, “the additional complexity of a GARCH model will be justified by superior hedging performance for some commodities but not others”.

<table>
<thead>
<tr>
<th>Futures</th>
<th>Templeton foreign portfolio</th>
<th>MSCI foreign portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOI futures</td>
<td>Constant OLS</td>
<td>Time varying GARCH</td>
</tr>
<tr>
<td>S&amp;P 500 futures</td>
<td>Time varying GARCH</td>
<td>Constant GARCH</td>
</tr>
<tr>
<td>FTSE 100 futures</td>
<td>Time varying GARCH</td>
<td>Constant OLS</td>
</tr>
<tr>
<td>Nikkei 225 futures</td>
<td>Time varying GARCH</td>
<td>Constant GARCH</td>
</tr>
<tr>
<td>Bovespa futures</td>
<td>Time varying OLS</td>
<td>Time varying GARCH</td>
</tr>
</tbody>
</table>

ANOVA tests indicate that differences between the dynamic and constant hedge ratio estimation methods are not statistically significant, a finding similar to those of McNew and Fackler (1994) and Ferguson and Leistikow (1998). Despite the lack of statistical significance in the difference between hedge ratio estimation methods and futures contracts, arguably the results have economic significance. The dollar value of portfolios held by Australian managed funds may be large, and even small improvements in hedge effectiveness may have a large economic impact. For comparison, Templeton World Fund had net assets to the value of approximately US$8,470 million as at 28 February 2001.30

6.4 Conclusion

The results of this chapter apply not only to investors considering a foreign equity portfolio, but may also be viewed as a study of partial hedging, where only the foreign portion of the portfolio is hedged and upside potential may be realised in the unhedged domestic component. Engle and Granger (1987) tests for cointegration are consistent with the absence of cointegration between the futures and equity portfolio series. Thus, prior literature on error correction hedge ratio estimation conducted in the context of perfectly matched spot and futures assets is not relevant in this chapter, which deals with the cross-hedging of portfolios with futures written on different underlying assets.

A comparison of the unhedged position with all hedges using each futures contract indicates that a hedged position outperforms the unhedged position on the basis of expected utility.

This result is robust to foreign equity portfolio construction, hedge ratio estimation technique, frequency of re-estimation of the hedge ratio, and time period. However, because the unhedged position is generally not associated with the lowest expected utility in each hedge period, it remains superior to poorly performing hedged positions. This reveals the importance of examining which futures contracts, if any, produce superior hedge protection relative to other futures contracts because all hedges do not perform equally. While some are superior to an unhedged position, others are relatively inferior. Thus, the examination of hedging techniques, including the relative merits of different futures contracts, in this chapter is important because hedging may improve expected investor utility.

In a comparison of futures contracts, the FTSE 100 and Nikkei 225 futures contracts generally provide the best hedges of the foreign equity portfolios, depending on the hedge ratio estimation method. Although Australian futures are not generally the most effective tools for hedging foreign equity risk, the performance of Australian futures relative to foreign futures improves as the complexity of hedge ratio estimation method increases. S&P 500 futures do not generally provide good hedges of the foreign equity portfolios in terms of expected utility, indicating that simple linear correlation does not fully explain hedge effectiveness. This issue is addressed further in later chapters. Time varying GARCH(1,1) ratios generally provide the highest expected utility for individual futures contracts, although this result depends on the equity portfolio.

Overall, the results point to the importance of an awareness of the interaction between the choices facing Australian fund managers when hedging foreign equity risk, particularly the hedge ratio estimation technique, the futures contract, and the construction of the equity portfolio.

While this chapter examines the hedging of the foreign equity component of diversified equity portfolios, the following chapter extends the analysis to examine the hedging of diversified equity portfolios consisting of both Australian and foreign equity, where portfolio effects complicate the hedging process.
CHAPTER SEVEN

HEDGING INTERNATIONALLY DIVERSIFIED EQUITY PORTFOLIOS USING SINGLE FUTURES CONTRACTS

7.1 Introduction

This chapter extends the work in the previous chapter by identifying which futures contract provides the best hedge from the perspective of an Australian investor holding a combination of Australian and foreign equity, and who wants to hedge both domestic and foreign equity risks simultaneously. Recall that Chapter Six examined the choice of a futures contract to hedge the foreign component of equity portfolios held by Australian managed funds. Australian equity constitutes a considerable proportion of total equity held by Australian managed funds regardless of fund style. As such, an important issue is whether to hedge the domestic and foreign components of an equity portfolio separately, or whether to hedge the whole portfolio.

Hedging components separately may not be optimal given that “portfolio effects”, or the interaction between different components of the portfolio, may change the level of hedging required. For instance, a natural hedge may occur if the returns on the domestic and foreign components of the portfolio have a negative association, reducing the need for hedging. These issues are examined in this chapter. The relative effectiveness of each hedging instrument is examined, when both Australian and foreign equity risks are considered, and the results are tested for sensitivity to equity portfolio construction, time period, hedge ratio estimation method, and level of risk aversion.

Following descriptions of the method and data, results where Australian and foreign equity are hedged as a single portfolio using a single futures contract are discussed. A preliminary analysis of portfolio effects is undertaken in the context of hedges using the Australian All Ordinaries Share Price Index futures contract, by comparing combined and separate hedges. The analysis is extended to the case of multiple futures contracts in the next chapter.
7.2 Research Method and Data

Australian and foreign equity are combined into an equity portfolio and cross-hedged as a single spot asset using a single futures contract. The expected return on the unhedged portfolio is:

\[ E(R_{UP,t}) = w_D E(R_{UD,t}) + w_F E(R_{UF,t}) \]  

(7.1)

where

\[ E(R_{UP,t}) = \text{expected return on the unhedged equity portfolio at time } t \]

\[ E(R_{UD,t}) = \text{expected return on the unhedged domestic Australian equity component at time } t \]

\[ E(R_{UF,t}) = \text{expected return on the unhedged foreign equity component at time } t \]

\[ w_D = \text{weighting of domestic Australian equity in the equity portfolio} \]

\[ w_F = \text{weighting of foreign equity in the equity portfolio} \]

\[ w_D + w_F = 1, \quad w_D \geq 0, \quad w_F \geq 0 \]

The determination of investment proportions, \( w_D \) and \( w_F \), is discussed in detail below. The expected return on the hedged portfolio is calculated as the expected return of the unhedged portfolio less the expected return on the futures position:

\[ E(R_{HP,t}) = E(R_{UP,t}) - h E(R_{f,t}) \]  

(7.2)

where

\[ E(R_{HP,t}) = \text{expected return on the hedged equity portfolio at time } t \]

\[ E(R_{f,t}) = \text{expected return on the futures contract at time } t \]

\[ h = \text{hedge ratio} \]

Under this method, a single futures contract is used to hedge the entire portfolio. For instance, the FTSE 100 futures contract may be used to hedge the entire equity portfolio containing both Australian and foreign equity. This is a simple approach, which allows for the interaction between Australian and foreign equity returns before any hedge is implemented. Although the relative performance of each futures contract depends in part on the regional weighting in the portfolio, this is not the only factor determining performance. For example, Table 5.3 in Chapter Five shows that in nine of thirteen years, the US market has a higher portfolio weighting than the European market, but the results in Chapter Six on the hedging of foreign equity indicate that the FTSE futures contract is generally preferred to the S&P 500 futures contract. Further, portfolios using different weightings are employed in this chapter, for sensitivity analysis. Thus, regional portfolio weightings are not expected to predetermine the hedging performance results. This method is employed using each of the
Australian and foreign futures contracts to determine whether any contract is preferred as a hedging instrument.

In order to examine portfolio effects in the context of single futures hedges, Australian and foreign equity are also hedged separately, and the returns on the individual hedged portfolios are then combined for performance measurement. Specifically, the components are hedged separately, and hedged returns are calculated:

\[
E(R_{HD,t}) = E(R_{UD,t}) - h_D E(R_{f,t}) \tag{7.3}
\]
\[
E(R_{HF,t}) = E(R_{UF,t}) - h_F E(R_{f,t}) \tag{7.4}
\]

where

\[
E(R_{HD,t}) = \text{the hedged return on Australian equity component at time } t
\]
\[
E(R_{HF,t}) = \text{hedged return on foreign equity component at time } t
\]
\[
E(R_{f,t}) = \text{expected return on equity futures contract at time } t
\]
\[
h_D = \text{hedge ratio for the Australian equity futures hedge}
\]
\[
h_F = \text{hedge ratio for the foreign equity futures hedge}
\]

The returns on the hedged portfolio are:

\[
E(R_{HP,t}) = w_D \left( E(R_{UD,t}) - h_D E(R_{f,t}) \right) + w_F \left( E(R_{UF,t}) - h_F E(R_{f,t}) \right) \tag{7.5}
\]

Rearranging equation (7.5):

\[
E(R_{HP,t}) = E(R_{UP,t}) - \left( w_D h_D + w_F h_F \right) E(R_{f,t}) \tag{7.6}
\]

Portfolio effects are examined by comparing results calculated using equation (7.2) where the Australian equity futures contract is used to hedge the entire equity portfolio, and equation (7.6) where foreign equity and Australian equity are hedged separately with Australian futures contracts. Hedging the foreign equity component of the portfolio using single futures contracts was examined in detail in the previous chapter, and although Australian futures are not generally the most effective tools for hedging foreign equity risk, the performance of Australian futures relative to foreign futures improves as the complexity of hedge ratio estimation method increases. The impact of interactions between different components of the portfolio is further explored in later chapters, where multiple futures contracts are employed to hedge multiple risks.

Sensitivity analysis is conducted to examine the impact of hedge ratio estimation method, including a comparison of OLS and GARCH hedges and a comparison of static and dynamic hedges. The price-changes OLS regression model used to calculate OLS hedge ratios is given in Chapter Six in equation (6.1), where the definitions of the variables are flexible to
allow sensitivity analysis. The GARCH(1,1) model is estimated using the BEKK parameterization of Engle and Kroner (1995), as defined by the system of equations (6.2), (6.3) and (6.4).

As in Chapter Six, hedge effectiveness is measured by expected quadratic utility, \( E(U) \), given in equation (6.5). The sensitivity of the results to the risk aversion parameter is also examined. The sensitivity of results to equity portfolio construction is assessed in two ways. First, the impact of the construction of foreign equity is examined by comparing results using the Templeton and MSCI foreign equity portfolios, as described in Chapter Six. Second, portfolios are constructed using different proportions of Australian and foreign equity, where the proportions are based on Australian managed fund practice as reported by Mercer Investment Consulting and discussed in detail in Chapter Five. Specifically, three portfolios are constructed such that the ratios of investment in Australian to foreign equity are 85:15, 70:30 and 55:45. Together, these proportions represent the range of asset allocations for Australian capital stable, balanced and growth funds. It is assumed that each portfolio is re-balanced through time to maintain the stated target asset allocations.31 The use of a variety of weights provides a test of the sensitivity of the results to a variety of managed fund investment styles. All data employed in this study are described in Chapter Five.

### 7.3 Hedging the entire portfolio as a single asset

In this section, the hedge of the complete portfolio is accomplished using the total portfolio return consisting of both domestic Australian and foreign equity, such that hedge ratio estimation is undertaken using a single spot asset. An advantage of treating the equity portfolio as a single asset is the simplicity of hedge ratio estimation, which is reduced to a single calculation. Further, this method accounts for some portfolio effects, as the co-movement between the Australian and foreign equity components is captured in the overall equity portfolio return. In practice this strategy is expected to have lower transaction costs and monitoring costs than the strategy of hedging the Australian and foreign equity separately because only one futures contract is used to hedge the whole portfolio. However, these costs are not accounted for explicitly in this analysis due to the complexity that would arise from costs in different currencies and the resulting foreign exchange risk.

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31 An alternative assumption is that initial investment weights are chosen such that at the end of the estimation period, the final weights are as stated (e.g., 85:15). However, this presupposes that prices are predictable, in which case there would be no demand for hedging. To avoid this line of reasoning, re-balancing of the portfolio to maintain target weights is assumed.
A disadvantage of hedging the entire portfolio using a single futures contract is the possible decrease in hedge effectiveness due to less accurate isolation of the component causing the most volatility. If either the Australian component or the foreign component of the equity portfolio is dominant, a hedge tailored specifically to that component may be more effective than a blanket hedge of the entire portfolio using the same futures contract. For instance, if returns on domestic Australian equity are relatively stable, but returns on the foreign equity component are highly volatile, a hedge tailored to the foreign component may be more effective than a hedge of the entire portfolio, although this depends on the interaction between Australian and foreign equity.

7.3.1 Hedging the entire equity portfolio

As in the previous chapter, static and dynamic hedge ratios are estimated using price-changes OLS regressions and GARCH(1,1) models, where the dependent variable is the return on the entire equity portfolio and the independent variable is the return on the futures contract. The expected utility associated with each hedge in each period is calculated using equation (6.5). Hedges using different futures contracts are then ranked within each period on the basis of expected utility. The best average rank over all hedge periods is calculated, along with the frequency with which a hedge using a given contract has the greatest expected utility relative to hedges using other contracts in that period. Preference for a hedge using a particular futures contract depends on the hedge period, so the purpose of these two measures is to examine whether any hedging strategy exhibits superior performance generally, when all hedge periods are considered.

Table 7.1 summarises the most effective futures hedges based on the best average rank and the greatest frequency of number one ranks over fifteen hedge periods from 1992 to 2001, where rankings are based on expected utility and the risk aversion parameter is three. When the frequency of best ranks is considered, AOI futures contracts provide the best hedges when the entire equity portfolio is treated as a single asset. The superior performance of AOI futures hedges is expected given that Australian equity is the largest constituent in every portfolio.

In contrast, when performance is assessed on the basis of the best average rank over all hedge periods, the FTSE 100 and Bovespa futures contracts generally provide the best hedges of diversified equity portfolios. Because these results are based on average rankings across fifteen separate hedge periods between 1992 and 2001, they do not indicate which contract performs best in any given hedge period. The performance of each futures hedge in a particular hedge period depends on the properties of each futures and spot market at that
Table 7.1: Summary of the most effective futures hedges, where performance is based on expected utility with a risk aversion of three, and where Australian and foreign equity are analysed as a single spot asset over hedge periods between 1992 and 2001.

<table>
<thead>
<tr>
<th>Hedge ratio estimation method</th>
<th>Australian equity %</th>
<th>Australian and TWF foreign equity</th>
<th>Australian and MSCI foreign equity</th>
<th>Australian and MSCI foreign equity</th>
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<tbody>
<tr>
<td>Constant OLS</td>
<td></td>
<td>BOV</td>
<td>AIF</td>
<td>FTSE/BOV</td>
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<td>85</td>
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<tr>
<td>70</td>
<td>BOV</td>
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<td>FTSE</td>
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<tr>
<td>Constant GARCH</td>
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<td>BOV</td>
<td>AIF</td>
<td>FTSE/BOV</td>
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<td>85</td>
<td>NIKK</td>
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<td>70</td>
<td>FTSE</td>
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<td>55</td>
<td>BOV</td>
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<tr>
<td>Varying OLS</td>
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<td>BOV</td>
<td>AIF</td>
<td>FTSE/BOV</td>
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<tr>
<td>85</td>
<td>FTSE</td>
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<tr>
<td>70</td>
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<td>Varying GARCH</td>
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<td>BOV</td>
<td>AIF</td>
<td>FTSE/BOV</td>
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<td>70</td>
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<td>55</td>
<td>FTSE</td>
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</table>

"TWF" indicates foreign equity portfolios based on the asset allocation of Templeton World Fund. "MSCI" indicates the foreign equity portfolios represented by the MSCI World Excluding Australia Index. "AOF" indicates the Australian All Ordinaries Share Price Index futures contract. "SP" indicates the U.S. S&P 500 Share Price Index futures contract. "FTSE" indicates the U.K. FTSE 100 Index futures contract. "NIKK" indicates the Japanese Nikkei 225 Index futures (SIMEX) contract. "BOV" indicates the Brazilian Bovespa Index futures contract. Ranks are based on expected utility, which is calculated according to equation (6.5), where $\lambda = 3$. Where two futures hedges perform equally best, both are included in the Table. For example, "FTSE/BOV" indicates that the FTSE futures hedge and Bovespa futures hedge perform jointly best.

point in time. The AOI futures did not perform as well under the “average rank” criteria as under the “highest frequency of number one ranks” criteria, because although it is frequently the best hedge strategy, it is also frequently among the worst. This occurs despite large proportion of Australian equity in every portfolio, and the resulting relatively high correlation between the equity portfolio returns and returns on the AOI futures contract. The poor performance of AOI futures hedges in some periods is understandable, because hedge performance is not determined exclusively by correlation, but is also influenced by the volatility of components of the equity portfolio and futures contracts and the average returns on the contracts.

Foreign equity returns are more volatile than Australian equity returns over the sample period, as shown by the standard deviations of Australian and foreign equity returns presented in Table 5.5. It is possible that in periods of relatively low volatility in global equity markets, Australian equity volatility dominates within the fund portfolios so that AOI futures contracts provide the best hedge, whereas in periods of high volatility in global equity, foreign equity volatility dominates and foreign futures contracts provide a better hedge of the portfolio. This suggestion is tested empirically later in this chapter. The finding that hedges of the entire portfolio using the Australian futures are not always the most effective strategy supports the examination of the relative performance of different futures.
contracts, even though the majority of each equity portfolio consists of Australian equity. Thus, Australian fund managers should not assume that the domestic futures contract will provide the best hedge of their portfolio simply because Australian equity forms the largest part of the entire portfolio.

At a more general level, Table 7.1 indicates that sets of results based on the performance criteria of average rank and frequency of first rankings must be interpreted with caution, because neither provides a comprehensive perspective of performance when considered in isolation. However, when used in conjunction, these methods provide a more accurate assessment of performance than when considered separately. Broadly, Table 7.1 illustrates that the choice of hedge performance measure has an important influence on findings and conclusions, a point which may explain some apparent contentions in prior literature on hedging.

The results are assessed for sensitivity to the risk aversion parameter, \( \lambda \) which was assumed to equal three for the above analysis. Tables 7.2 and 7.3 present the results of the analysis when the risk aversion is set at 0.1 and 10 respectively. When hedge performance is measured on the basis of the highest frequency of first rankings of expected utility, the preference for Australian AOI futures hedges is generally uniform for all equity portfolio constructions and risk aversion levels.

In contrast, when hedge performance is assessed by the average rank based on expected utility, the results are somewhat sensitive to the risk aversion parameter. Specifically, for portfolios containing MSCI foreign equity, the FTSE futures hedge is generally preferred when \( \lambda \) is 0.1, as opposed to the preference for FTSE and Bovespa futures hedges that is evident when \( \lambda \) equals 3. However, for portfolios containing Templeton foreign equity, the results do not vary greatly when \( \lambda \) is 0.1 or 3. When \( \lambda \) equals 10, the SPI futures hedges are generally favoured for all portfolio constructions, in contrast to the preference for FTSE and Bovespa futures hedges when \( \lambda \) equals 3. These findings indicate that for more risk averse investors, the AOI futures hedges are generally favoured on average when hedging the entire equity portfolio using a single futures contract, in contrast to the preference for foreign futures hedges by less risk averse investors. Further, the results in Table 7.3 reveal that under
Table 7.2: Summary of the most effective futures hedges, where performance is based on expected utility with a risk aversion of one tenth, and where Australian and foreign equity are analysed as a single spot asset over hedge periods between 1992 and 2001

<table>
<thead>
<tr>
<th>Hedge ratio estimation method</th>
<th>Australian equity %</th>
<th>Australian and TWF foreign equity</th>
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<td></td>
<td>70</td>
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<td>FTSE</td>
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<tr>
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<td>Varying GARCH</td>
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<td>AOIF</td>
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"TWF" indicates foreign equity portfolios based on the asset allocation of Templeton World Fund. "MSCI" indicates the foreign equity portfolios represented by the MSCI World Excluding Australia Index. "AOIF" indicates the Australian All Ordinaries Share Price Index futures contract. "SP" indicates the U.S. S&P 500 Share Price Index futures contract. "FTSE" indicates the U.K. FTSE 100 Index futures contract. "NIKK" indicates the Japanese Nikkei 225 Index futures (SIMEX) contract. "BOV" indicates the Brazilian Bovespa Index futures contract. Ranks are based on expected utility, which is calculated according to equation (6.5), where $\lambda = 3$. Where two futures hedges perform equally best, both are included in the Table. For example, "FTSE/BOV" indicates that the FTSE futures hedge and Bovespa futures hedge perform jointly best.

Table 7.3: Summary of the most effective futures hedges, where performance is based on expected utility with a risk aversion of ten, and where Australian and foreign equity are analysed as a single spot asset over hedge periods between 1992 and 2001

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<tr>
<td>Constant OLS</td>
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<td>Varying OLS</td>
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<td>AOIF/FTSE</td>
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a risk aversion of ten, only the managers of the riskiest equity portfolios containing 45% foreign equity prefer foreign futures contracts as hedging vehicles, and then only under the average rank criteria and only for certain hedging scenarios. A preference for domestic futures contracts by managers that have a relative dislike of risk makes intuitive sense, given that returns on foreign futures contracts are relatively more volatile, as demonstrated in Chapter 5 Table 5.7, and given that Australian managers are more likely to be familiar with domestic Australian derivatives than with foreign derivatives.

7.3.2 Volatility and Futures Hedge Performance

Analysis in the previous section is limited to an examination of the relative performance of each futures hedge. In this section, the analysis is extended to an empirical investigation of the relationship between the performance of different futures hedges and equity portfolio volatility using a three-step procedure. First, the volatility of the returns of each unhedged equity portfolio in each hedge period is classified in various ways. The volatilities of the returns of the unhedged equity portfolios in each of the fifteen distinct hedge periods between 1992 and 2001 are calculated as the standard deviations of returns over those periods. The “average” level of volatility associated with each unhedged equity portfolio is calculated as the standard deviation of returns on that portfolio over the full period from 02/01/90 to 31/07/01. Recall that there are six diversified equity portfolios: three constructed using Australian and Templeton foreign equity and three constructed using Australian and MSCI foreign equity, where the proportions of Australian to foreign equity are 85:15, 70:30 and 55:45. This analysis is therefore repeated six times.

Defining different states of volatility, any hedge period in which the standard deviation of returns is more than 15% above the average level of volatility is classed as a period of “high” volatility. Similarly, any hedge period in which the standard deviation of returns is more than 15% below the average level of volatility is classed as a period of “low” volatility. For example, the threshold volatility for an equity portfolio to be classified as highly volatile in a given period is \( \sigma + (0.15 \times \sigma) \), where \( \sigma \) is the standard deviation of returns on that portfolio over the sample period 02/01/90 to 31/07/01. Similarly, the threshold below which the volatility in a specific period is classed as “low” is \( \sigma - (0.15 \times \sigma) \). For instance, for portfolios containing 70% Australian and 30% Templeton foreign equity, the average volatility is 0.0068 over the full period from 1990 to 2001. Particular hedge periods in which volatility exceeds this level are classed as “exceeding average risk”. Particular periods in which volatility exceeds 0.0078 (which equals 0.0068+(0.15 x 0.0068)) are classed as periods in which risk “is more than 15% above average risk”. Note that there may be some overlap between these two classifications. Particular periods in which volatility is smaller
than 0.0058 (which equals 0.0068 – (0.15 x 0.0068)) are classed as periods in which risk “is more than 15% below average risk”. This classification system is illustrated in Figure 7.1, which is not drawn to scale.

**Figure 7.1: Conceptual representation of the different classifications of volatility, as measured by standard deviation of returns, used in the analysis of hedge performance**

The standard deviation of returns in each hedge period is compared to the average standard deviation of returns, and three separate series are constructed based on logic tests, such that if a period meets the volatility classification it is represented as 1 and if it does not then it is represented using 0. Three such series are used, one for high volatility, one for low volatility and one for volatility that is simply above the average level, as illustrated in Figure 7.1. For example, to construct the series relating to “high volatility”, when the volatility in a period exceeds the average level of volatility over the sample it is recorded as 1, but if it is lower then it is denoted 0. This is repeated for each successive hedge period, to construct a simple distribution. Similarly, to construct the series for “low volatility”, when the standard deviation of returns in a period is more than 15% smaller than the average standard deviation, it is recorded as 1, but if it does not meet this criteria, it is recorded as 0. These three series are constructed independently of each other.

Second, the periods in which a hedge using a particular futures contract is superior to one using any other contract (on the basis of expected utility where \( \lambda = 3 \)) are identified for each contract in turn. A distribution of 1s and 0s is constructed to indicate whether the futures contract provides the best hedge in that period (1), or not (0). Five such series are created, one for each futures contract.

Finally, correlations between the series based on relative volatility and the series based on futures contract performance are calculated, to determine the strength of association between
the periods in which volatility behaves in a particular way and the periods in which a specific contract gives the best hedge. The entire procedure is repeated using four different hedge ratio estimation methods. Key results are summarised in Table 7.4, which indicates the futures hedge with the highest positive correlation to the volatility classification, and what that correlation is, for each of the three volatility scenarios examined. For example, when constant hedge ratios are calculated using OLS regressions, and when portfolios of 85% Australian and 15% Templeton foreign equity are examined, periods in which volatility is low (below 15% under the average level) have the highest positive correlation (0.61) with periods in which the AOI futures contract hedges outperform all other hedges.

Table 7.4: The futures contract that has the highest positive correlation between that contract being the best and a particular volatility condition

<table>
<thead>
<tr>
<th>Hedge ratio estimation method</th>
<th>Australian equity %</th>
<th>Period risk is more than 15% below average risk</th>
<th>Period risk exceeds average risk</th>
<th>Period risk is more than 15% above average risk</th>
<th>Period risk is more than 15% below average risk</th>
<th>Period risk exceeds average risk</th>
<th>Period risk is more than 15% above average risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant OLS</td>
<td>85</td>
<td>AOIF</td>
<td>FTSE</td>
<td>BOV</td>
<td>AOIF</td>
<td>FTSE</td>
<td>BOV</td>
</tr>
<tr>
<td>70</td>
<td>AOIF</td>
<td>FTSE</td>
<td>BOV</td>
<td>AOIF</td>
<td>FTSE</td>
<td>BOV</td>
<td>55</td>
</tr>
<tr>
<td>55</td>
<td>AOIF</td>
<td>FTSE</td>
<td>BOV</td>
<td>AOIF</td>
<td>FTSE</td>
<td>BOV</td>
<td>55</td>
</tr>
<tr>
<td>Constant GARCH</td>
<td>85</td>
<td>AOIF</td>
<td>FTSE, BOV</td>
<td>SP</td>
<td>AOIF</td>
<td>NIKK</td>
<td>BOV</td>
</tr>
<tr>
<td>55</td>
<td>AOIF</td>
<td>FTSE, BOV</td>
<td>SP</td>
<td>AOIF</td>
<td>NIKK</td>
<td>BOV</td>
<td>55</td>
</tr>
<tr>
<td>Varying OLS</td>
<td>85</td>
<td>AOIF</td>
<td>FTSE</td>
<td>SP, BOV</td>
<td>AOIF</td>
<td>FTSE, BOV</td>
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<td>55</td>
<td>AOIF</td>
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<td>FTSE</td>
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<td>55</td>
<td>AOIF</td>
<td>FTSE</td>
<td>SP, BOV</td>
<td>AOIF</td>
<td>FTSE, BOV</td>
<td>SP</td>
<td></td>
</tr>
</tbody>
</table>

“TWF” indicates foreign equity portfolios based on the asset allocation of Templeton World Fund. “MSCI” indicates the foreign equity portfolios represented by the MSCI World Excluding Australia Index. “AOIF” indicates the Australian All Ordinaries Share Price Index futures contract. “SP” indicates the U.S. S&P 500 Share Price Index futures contract. “FTSE” indicates the U.K. FTSE 100 Index futures contract. “NIKK” indicates the Japanese Nikkei 225 Index futures (SIMEX) contract. “BOV” indicates the Brazilian Bovespa Index futures contract. Correlations are listed underneath the associated futures contract. The “average” level of volatility associated with each unhedged equity portfolio is calculated as the standard deviation of returns on that portfolio over the full period from 02/01/90 to 31/07/01.
Generally, Table 7.4 demonstrates a clear link between periods of low volatility and periods where the AOI futures hedge is superior to hedges using other futures contracts. This is not sensitive to foreign equity portfolio construction or to the division between Australian and foreign equity. In contrast, periods where volatility exceeds the average level by more than 15% are always more strongly positively correlated with periods where a foreign futures hedge outperforms the domestic futures hedge, and this holds even when the volatility condition is relaxed to simply “above average”. This supports the suggestion in Section 7.3.1 based on Table 7.1, that in periods of relatively low volatility of returns on the unhedged equity portfolio, Australian futures hedges perform best but in periods of high volatility foreign futures hedges are superior on the basis of expected utility. However, in periods of relatively low volatility, a manager may elect not to hedge or may not require a full hedge because equity risk is lower than usual. The strengthening of the preference for AOI futures hedges relative to foreign futures hedges when the risk aversion parameter is increased from three to ten reflects changing risk preferences, not changing volatility.

The AOI futures hedges are identified in Tables 7.1, 7.2 and 7.3 as always having the highest frequency of first rankings based on expected utility, which coincides exactly with the uniform preference for AOI futures hedges in periods of below-average risk in Table 7.4. In contrast, neither the average rank criteria in Table 7.1 nor the examination of periods of high risk in Table 7.4 indicate that any single futures contract provides superior hedges for every portfolio. The fact that the futures hedges identified in Table 7.4 as having the highest correlation with high volatility are not uniformly identical to those identified in Table 7.1 as having the best average rank does not indicate any conflict between these sets of results. It simply reflects the frequency with which the state of relatively high volatility arises. For instance, using constant OLS hedge ratios, Table 7.1 shows that the Bovespa hedge of the portfolio of 70% Australian and 30% Templeton foreign equity is superior to other futures hedges using the average rank criteria, but Table 7.4 shows that in periods of high volatility the S&P 500 futures hedge generally performs the best. However, in that case, only three hedge periods of fifteen are classed as high-risk for the purposes of Table 7.4, while the full fifteen periods are utilised in Table 7.1. The distributions of volatilities in various hedge periods are provided in Figures 7.2 through to 7.4, which show the volatility of the returns on each of the unhedged equity portfolios containing Templeton foreign equity in each hedge period relative to the general level of volatility, as measured by the volatility over the full period between 02/01/1990 and 31/07/01. The results are similar for portfolios containing MSCI foreign equity. The hedge period volatility is indicated by dots, and the general level is the straight line. For each portfolio, there is a sufficient mixture of hedge periods with
relatively "high" and "low" volatilities to demonstrate that the results in Tables 7.2 to 7.4 are based on a general sample rather than exclusively in high-risk or low-risk periods.

**Figure 7.2: Volatility of returns on portfolios of 85% Australian and 15% Templeton foreign equity, in various hedge periods, relative to volatility over the full period from 02/01/90 to 31/07/01**

The volatility in each hedge period, listed along the horizontal axis in chronological order, is compared with the average volatility of portfolio returns over the entire sample period from 1992 to 2001, indicated by the horizontal line. The hedge periods are: 02/01/92 to 29/01/92; 01/04/92 to 30/04/92; 01/07/92 to 28/07/92; 01/10/92 to 28/10/92; 03/01/95 to 30/01/95; 03/04/95 to 02/05/95; 03/07/95 to 28/07/95; 02/10/95 to 27/10/95; 02/01/98 to 29/01/98; 01/04/98 to 30/04/98; 01/07/98 to 28/07/98; 01/10/98 to 28/10/98; 02/01/01 to 30/01/01; 02/04/01 to 01/05/01; 02/07/01 to 27/07/01.

**Figure 7.3: Volatility of returns on portfolios of 70% Australian and 30% Templeton foreign equity, in various hedge periods, relative to volatility over the full period from 02/01/90 to 31/07/01**

The volatility in each hedge period, listed along the horizontal axis in chronological order, is compared with the average volatility of portfolio returns over the entire sample period from 1992 to 2001, indicated by the horizontal line. The hedge periods are defined in Figure 7.2.
Figure 7.4: Volatility of returns on portfolios of 55% Australian and 45% Templeton foreign equity, in various hedge periods, relative to volatility over the full period from 02/01/90 to 31/07/01

The volatility in each hedge period, listed along the horizontal axis in chronological order, is compared with the average volatility of portfolio returns over the entire sample period from 1992 to 2001, indicated by the horizontal line. The hedge periods are defined in Figure 7.2.

In the next chapter, this analysis is extended to include both Australian and foreign futures contracts in the hedge strategy. In the absence of perfect foresight, it is not possible to identify for certain which future hedge periods will exhibit relatively high or low volatility, and hence whether domestic or foreign futures hedges are more appropriate. The volatility of portfolio returns may be driven by the volatility of domestic or foreign equity. The use of domestic and foreign futures contracts simultaneously in Chapter Eight may provide a better approach in general because it caters for both sources of equity risk.

7.33 Section Summary

In summary, when the diversified equity portfolio is treated as a single spot asset and performance is measured using expected utility, AOI futures contracts generally provide superior hedges in periods of relatively low volatility of unhedged returns and foreign futures contracts provide superior hedges in periods of relatively high volatility. In the periods examined, the AOI futures hedge generally performs well relative to other hedges. From the perspective of an Australian managed fund, a preference for AOI futures hedges is reasonable given that Australian equity constitutes a large proportion of the entire portfolio. In the context of hedging all equity risk using one futures contract, the relatively good performance of AOI futures contracts is convenient for Australian managers, because the domestic contract can be used to effectively hedge the equity risk of diversified equity portfolios.

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7.4 Hedging Australian and foreign equity separately using AOI futures contracts

To examine portfolio effects, results in the previous section are compared with the results in this section, where the Australian equity component of the portfolio is hedged using Australian AOI futures contracts, and the foreign equity component is hedged separately using the AOI futures contracts. Hedging the foreign equity component of the portfolio is addressed in detail in Chapter Six, and the discussion of hedging the Australian equity component of the portfolio in this section is brief because the issue has been dealt with extensively in prior literature, such as Thomas and Brooks (2001).

Australian equity is represented using the Australian All Ordinaries Share Price Index 32, and the hedging vehicle employed is the futures contract written on that index. When the effectiveness of constant and dynamic hedge ratios estimated using OLS price changes, error correction 33, and bivariate GARCH(1,1) models are compared in each of fifteen periods, an ANOVA test fails to detect any statistically significant difference between the expected utility associated with any of the hedge ratio estimation methods 34, a result not sensitive when the risk aversion parameter is varied from 3 to the extremes of 0.1 and 10. This finding is consistent with that of Thomas and Brooks (2001), who also examine hedging the All Ordinaries Index using its associated futures contract. Thus, on the basis of expected utility, there is no statistically significant difference between the benefits of each method of hedge ratio estimation when hedging the AOI using the AOI futures contract. In the remainder of this chapter, the method used to estimate the hedge ratio applied to the Australian equity component of the portfolio subject to “separate” hedges of domestic and foreign risk is the same as that applied to the foreign equity component.

7.5 Portfolio effects: Hedging using Australian AOI futures contracts

This section examines whether there is any advantage in calculating hedge ratios separately for Australian and foreign equity, relative to treating the entire portfolio as a single spot series, when using a single futures contract. This is of interest because “portfolio effects”, or interactions between different components of the portfolio, are ignored when Australian and foreign equity are hedged separately. For instance, the returns on Australian equity may interact with the returns on the foreign equity component such that a natural hedge is created and fewer futures are required to complete the hedge. In contrast, when the entire portfolio is

32 See Section 5.2.2 in Chapter 5 for comments on the introduction of new ASX equity indices in 2000.
33 Phillips Perron tests fail to reject the presence of cointegration between the AOI and its associated futures contract at the 1% significance level over the sample period.
34 The ANOVA F-statistic for the difference between the means of the expected utilities of the three constant and three dynamic methods of hedge ratio estimation is 0.0491 (p-value 1.00).
cross-hedged as a single asset, the interactions between the returns on the Australian and foreign components are already captured in the total return.

7.5.1 Illustrating portfolio effects

To illustrate, consider the case where there are two assets, namely Australian and foreign equity, and one futures contract, the Australian All Ordinaries SPI futures contract. Assume that the correlation matrix is as follows, where the variables are as defined in Section 7.2, and where \(-1 \leq x \leq 1\) and \(-1 \leq y \leq 1\).

<table>
<thead>
<tr>
<th></th>
<th>(R_{UD})</th>
<th>(R_{UF})</th>
<th>(R_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_{UD})</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(R_{UF})</td>
<td>(x)</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>(R_f)</td>
<td>0.96*</td>
<td>(y)</td>
<td>1</td>
</tr>
</tbody>
</table>

The association between the three variables can be complex. For instance, it can be shown empirically that a negative correlation between returns on Australian and foreign equity \((x < 0)\) does not imply negative correlation between returns on foreign equity and the AOI futures contract. For example, in the hedge period commencing 03/07/95, using portfolios containing Templeton foreign equity, the correlation between returns on Australian and foreign equity was \(-0.11\), but the correlation between returns on foreign equity and the AOI futures contract was \(0.04\), despite the high correlation of \(0.96\) between the AOI and its futures contract. Thus, the interrelationships between the three return series are not as obvious as in the one-asset, one-futures case.

In seven of fifteen hedge periods between 1992 and 2001, the correlation between Australian and foreign equity is negative. Given a negative correlation between Australian and foreign equity, some natural hedging will occur when the two assets are combined into a portfolio. The extent of natural hedging will depend on the relative proportion of each asset, and on the strength of the negative association. When a hedge ratio is estimated using the entire portfolio return as the dependent variable (the “combined” method), the relationship between the Australian and foreign components is taken into account through the single return series, and the relationship between the futures contract and the portfolio as a whole is captured. In contrast, when Australian and foreign equity are hedged separately using the same contract, any natural offset in returns is ignored, and the overall futures position may be inappropriate. For instance, if the correlation between Australian and foreign equity is strongly negative, there may be a reduced demand for futures contracts using the combined hedge relative to when the Australian and foreign components are hedged separately. An additional complication may arise in cases where the association between the AOI futures and foreign

* This correlation is calculated over the full sample period, from 05/01/89 to 26/07/01, as shown in Chapter 5 in Table 5.7.
equity is negative, but the correlation between the Australian equity and futures is positive, because there may be some canceling out of the separate futures positions, which would prove expensive given transaction costs and management costs.

Hence, hedging the Australian and foreign equity components separately, the relationships between Australian equity and AOI futures contracts and between foreign equity and the AOI futures contract are accounted for. However, the potentially offsetting or reinforcing connection between Australian and foreign equity is ignored. By estimating hedge ratios using the entire portfolio return, the latter relationship is already incorporated into a single return series, and there is only one correlation between spot and futures assets that needs to be accounted for in the hedge ratio.

The interrelationships between the components of the hedged portfolio become increasingly complex as the number of futures contracts increases. The analysis conducted in this chapter is limited to the simple case of hedging the Australian and foreign equity components using a single futures contract, namely the Australian AOI futures contract. This examination of portfolio effects is extended in Chapter Eight to include pairs of futures contracts.

7.5.2 Empirical results
Table 7.5 indicates the percentage of hedge periods in which a single hedge of the entire equity portfolio using AOI futures contracts is superior to separate hedges of the Australian and Templeton foreign equity components using AOI futures, where performance is measured by expected utility with various risk. In each hedging scenario, the preference for combined hedges is based on results in the fifteen individual hedge periods. For example, using varying OLS hedge ratios to hedge a portfolio consisting of 85% Australian equity and 15% Templeton foreign equity, combined hedges are preferred in 60% of hedge periods, where performance is measured by expected utility and A equals three. Table 7.6 provides analogous results for portfolios containing MSCI foreign equity.

Table 7.5 shows that when foreign equity is represented using the Templeton portfolio, hedges of the entire portfolio generally outperform separate hedges of Australian and foreign equity in the majority of hedge periods, regardless of the risk aversion parameter. Specifically, combined hedges are preferred in the majority of hedge periods for all equity portfolios hedged using constant OLS, dynamic OLS and dynamic GARCH ratios, and for all constant GARCH hedges except those on portfolios containing 70% Australian equity, 30% Templeton foreign equity. This preference is not generally sensitive to the risk aversion parameter A.
Table 7.5: Percentage of hedge periods where the combined hedge using AOI futures contracts is superior to separate hedges of Australian and foreign equity using AOI futures contracts, where foreign equity is represented using the Templeton portfolio and hedge performance is measured using expected utility, for a variety of hedge periods between 1992 and 2001

<table>
<thead>
<tr>
<th>Hedge ratio estimation method</th>
<th>Australian equity</th>
<th>E(U), λ=0.1</th>
<th>E(U), λ=3</th>
<th>E(U), λ=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant OLS</td>
<td>85</td>
<td>60</td>
<td>60</td>
<td>60</td>
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<td></td>
<td>70</td>
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<td>60</td>
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<tr>
<td>Constant GARCH</td>
<td>85</td>
<td>67</td>
<td>67</td>
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<td>70</td>
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<td>53</td>
</tr>
<tr>
<td>Varying OLS</td>
<td>85</td>
<td>60</td>
<td>60</td>
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<td>70</td>
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<tr>
<td>Varying GARCH</td>
<td>85</td>
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<td>67</td>
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</table>

Expected utility is calculated using equation (6.5). Australian equity is represented using the Australian All Ordinaries Index and foreign equity is represented using the Templeton World Fund foreign portfolio. “Australian equity %” indicates the percentage of the equity portfolio made up by Australian shares. For example, where “Australian equity %” is 85, foreign equity constitutes 15% of the portfolio.

Summarising the results in Table 7.5, combined hedges of the entire portfolio are almost always preferred to separate hedges in the majority of hedge periods when foreign equity is represented using the Templeton portfolio, and this result is generally insensitive to the relative proportions of Australian and foreign equity, the hedge ratio estimation method, and the expected utility risk aversion parameter. However, while this preference is evident in the majority of periods, the percentage is never 100%, indicating that there are individual hedge periods where the fund benefits from separate hedges of the Australian and foreign equity components. The strongest results favouring combined hedges are associated with varying GARCH hedge ratios, but the percentage figures do not alter dramatically between hedge ratio estimation methods.

In contrast, Table 7.6 shows that when foreign equity is represented using the MSCI index, results are more varied as to whether combined hedges are preferred over separate hedges, depending in part on the hedge ratio estimation method. When constant OLS ratios are employed, separate hedges are preferred in the majority of hedge periods, regardless of equity portfolio construction, or risk aversion. Similarly, when implementing varying OLS hedge ratios, separate hedges are preferred for all equity portfolio constructions except for portfolios containing 70% Australian equity and 55% Australian equity. However, combined hedges are generally superior when using constant or dynamic GARCH ratios.
Table 7.6: Percentage of hedge periods where the combined hedge using AOI futures contracts is superior to separate hedges of Australian and foreign equity using AOI futures contracts, where foreign equity is represented using the MSCI portfolio and hedge performance is measured using expected utility, for a variety of hedge periods between 1992 and 2001

<table>
<thead>
<tr>
<th>Hedge ratio estimation method</th>
<th>Australian equity %</th>
<th>E(U), λ=0.1</th>
<th>E(U), λ=3</th>
<th>E(U), λ=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant OLS</td>
<td>85</td>
<td>47</td>
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<tr>
<td>Varying OLS</td>
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</table>

Expected utility is calculated using equation (6.5). Australian equity is represented using the Australian All Ordinaries Index and foreign equity is represented using the MSCI World Excluding Australia Index. “Australian equity %” indicates the percentage of the equity portfolio made up by Australian shares. For example, where “Australian equity %” is 85, foreign equity constitutes 15% of the portfolio.

Comparing Tables 7.5 and 7.6, it is evident that the benefits of accounting for portfolio effects by using combined hedging as opposed to separate hedging are sensitive to the foreign equity portfolio construction. This indicates that although the Australian equity forms the bulk of every portfolio, varying from 55% to 85% of the total portfolio, the foreign equity has an impact on the benefits of combined versus separate hedges. Recall that the Templeton and MSCI foreign equity portfolios emphasise different investment allocations in foreign markets. Hence, managers may benefit from accounting for portfolio effects when hedging equity risk associated with internationally diversified equity portfolios, depending on foreign equity portfolio construction, the relative proportion of Australian and foreign equity, and the hedge ratio estimation method. The results in this section relate exclusively to hedges using the Australian AOI futures contract. In the following section, this analysis is extended to include a wider range of futures contracts. These findings are important because the choice of hedging strategy is critical both to obtaining an effective hedge and to the level of transaction costs incurred. Over-hedging or under-hedging due to a failure to account for complex interrelationships between portfolio components could be costly.

7.5.3 Portfolio effects and correlation

Correlation between investments is described by Ahmed (2001, p 1187) as “one of the most important determinants of portfolio risk”. Table 7.7 illustrates the connection between the preference for combined or separate hedges and the relationship between returns on different components of the hedge portfolio. The relative performance of combined and separate
hedges is compared using expected utility, where the risk aversion parameter equals three. For example, when dynamic hedge ratios are calculated using OLS regressions, and the equity portfolio contains 70% Australian and 30% Templeton foreign equity, the correlation between cases where combined hedges are preferred to separate hedges and cases where there is a positive relationship between returns on the Australian and foreign equity components is -0.07. In Table 7.7, the correlations are stated in terms of cases where combined hedges are preferred to separate hedges. These correlations are equivalent in magnitude but opposite in sign to correlations when separate hedges are preferred to combined hedges. For example, using constant OLS hedge ratios on a portfolio of 85% Australian and 15% Templeton foreign equity, the correlation between cases where combined hedges are preferred over separate hedges and cases where there is a positive association between returns on Australian and Templeton foreign equity is -0.22. Thus, the correlation between cases where separate hedges are preferred and cases where there is a positive relationship between returns on Australian and Templeton foreign equity is 0.22.

The first two columns of Table 7.7 show that under constant and time varying hedge ratios calculated using OLS regressions, the correlation between cases where combined hedges are superior to separate hedges and cases where there is a positive relationship between returns on Australian and foreign equity is invariably negative. This is true for both foreign equity portfolio constructions. This is consistent with the hypothesis that combined hedges that take into account natural hedges between Australian and foreign equity are best when a negative relationship exists between the returns on those components. Such a strategy will reduce costs associated with excess futures positions. However, under time-varying GARCH hedge ratios, the correlation results in Table 7.7 are less consistent with this theory, with some positive and some negative figures. Apparently, when GARCH hedge ratios are used, correlation between the Australian and foreign equity components is not the dominant factor driving the preference for combined hedges.

Turning to the examination of the relationship between returns on foreign equity and Australian futures contracts, consider the last two columns of Table 7.7. Generally, when OLS hedge ratios are applied, a negative correlation exists between periods where combined hedges are preferred to separate hedges and periods where there is a positive relationship between returns on foreign equity and Australian futures contracts, regardless of foreign portfolio construction. For GARCH ratios, the results are mixed. As with the earlier results relating hedge strategy to the relationship between Australian and foreign equity, these results are generally consistent and correlations between different factors appear to explain the results. In contrast, the GARCH results are not generally consistent, suggesting that some further factor is influencing the results. The additional complexity of GARCH hedge ratio
estimation models raises the issue of whether their implementation is practical, or whether the benefits of such models outweigh the considerable difficulties associated with their application. Further, GARCH models are more difficult to explain to investors if required.

| Table 7.7: Correlations between the preference for combined hedges over separate hedges and various relationships between returns on different components of the hedged portfolios |
|---|---|---|---|---|
| Hedge ratio estimation method | Australian equity % | Correlation between "Combined preferred over separate", and "positive relationship between returns on AOI and TWF" | Correlation between "Combined preferred over separate", and "positive relationship between returns on AOI and MSCI" | Correlation between "Combined preferred over separate", and "positive relationship between returns on TWF and AOIF" | Correlation between "Combined preferred over separate", and "positive relationship between returns on MSCI and AOIF" |
| Constant OLS | 85 | -0.22 | -0.20 | -0.11 | -0.07 |
| | 70 | -0.22 | -0.20 | -0.11 | -0.07 |
| | 55 | -0.22 | -0.20 | -0.11 | -0.07 |
| Constant GARCH | 85 | -0.09 | 0.19 | 0.00 | -0.47 |
| | 70 | 0.22 | -0.05 | 0.11 | -0.22 |
| | 55 | -0.07 | 0.04 | -0.22 | -0.04 |
| Varying OLS | 85 | -0.22 | -0.20 | -0.11 | -0.07 |
| | 70 | -0.07 | -0.49 | 0.05 | -0.05 |
| | 55 | -0.22 | -0.22 | -0.11 | 0.22 |
| Varying GARCH | 85 | 0.60 | -0.07 | 0.72 | 0.07 |
| | 70 | 0.20 | 0.05 | 0.27 | -0.05 |
| | 55 | 0.19 | 0.19 | 0.29 | 0.09 |

"TWF" indicates the Templeton foreign equity portfolio. "MSCI" indicates the MSCI World excluding Australia Index portfolio. "AOIF" indicates the Australian All Ordinaries Share Price Index futures contract. "Combined" and "separate" hedges are as defined in Sections 7.2 and 7.5. "Australian equity %" indicates the percentage of the equity portfolio made up by Australian shares. The relative performance of combined and separate hedges is compared using expected utility, where the risk aversion parameter equals three.

In sum, correlation between different components of the hedged portfolio are consistent with some explanations of the preference for combined hedges over separate hedges, in the context of OLS hedge ratios, but the factors driving the results appear to be more complex for GARCH hedges. This is an issue for future research. Further, correlation is a simple linear measure of association, and may not accurately reflect complex relationships between returns on components of diversified portfolios. More advanced descriptions of co-dependence such as copulas may prove more accurate, but this too is a matter for further research.

7.6 Conclusion

This chapter extends the previous chapter by examining the hedging of equity risk given diversified portfolios containing Australian and foreign equity, where foreign equity is represented using the Templeton and MSCI portfolios. A key issue is whether there is any
benefit in accounting for "portfolio effects" by hedging the entire equity portfolio as a single spot series, or whether separate hedges of the Australian and foreign equity components provide an equally effective strategy. The results are examined for sensitivity to the asset allocation between Australian and foreign equity, the construction of foreign equity portfolios, the hedge ratio estimation method, the measure of hedge performance, and the time period.

For hedges of the entire equity portfolio using a single futures contract, Australian futures hedges provide the highest expected utility in periods of relatively low volatility of returns on the unhedged equity portfolio, but foreign futures hedges are superior in periods of high volatility. Risk management strategies are arguably most important in periods of high volatility. In periods of relatively low volatility, a manager may elect not to hedge or may not require a full hedge because equity risk is lower than usual. Managers may improve hedge performance by thoughtful selection of the hedging instrument, depending on the expected level of equity portfolio volatility. The volatility of portfolio returns may be attributable to volatility in returns on domestic Australian equity or foreign equity or both, and the interaction between different components of the portfolio is addressed in more detail in the following chapter.

An initial examination of the impact of portfolio effects is conducted in the context of hedging equity risk using the AOI futures contracts as the exclusive hedging vehicle. Specifically, the effectiveness of hedging the entire equity portfolio using the AOI futures is compared to the effectiveness of ignoring portfolio effects and hedging the Australian and foreign components separately using AOI futures contracts. A general preference for combined hedges in the majority of hedge periods is evident using the expected utility measure of performance, for portfolios containing Templeton foreign equity. The relatively good performance of AOI futures contracts is convenient for Australian managers, because the domestic contract can be used to hedge the equity risk of diversified equity portfolios with reasonable effectiveness. In contrast, results are varied for portfolios containing MSCI foreign equity. Hence, whether fund managers may benefit from accounting for portfolio effects when hedging equity risk associated with internationally diversified equity portfolios appears to be case-specific, depending on the foreign equity component. Correlation between different components of the hedged portfolio are consistent with some explanations of the preference for combined hedges over separate hedges, in the context of OLS hedge ratios, but the factors driving the results appear to be more complex for GARCH hedges. These issues are important because adopting excessive or inadequate positions in futures contracts through an inappropriate choice of hedging strategy is costly.
The analysis of combined and separate hedging strategies in this chapter is limited to the simple case of hedging the Australian and foreign equity components using a single futures contract, namely the Australian AOI futures contract. This examination of portfolio effects is extended in the following chapter to include multiple futures contracts.
CHAPTER EIGHT

HEDGING INTERNATIONALLY DIVERSIFIED EQUITY PORTFOLIOS USING PAIRS OF FUTURES CONTRACTS: PORTFOLIO EFFECTS

8.1 Introduction

This chapter extends the work in the previous chapter by identifying which futures hedging strategy provides the best hedge from the perspective of an Australian investor holding a combination of Australian and foreign equity, and who wants to hedge both domestic and foreign equity risks simultaneously. Chapter Seven examined the choice of a single futures contract to hedge diversified equity portfolios, and provides preliminary analysis of portfolio effects in the context of hedging using Australian AOI futures contracts. The present chapter extends the analysis to consider a wider range of hedging strategies using pairs of futures contracts.

A key issue is whether accounting for “portfolio effects”, or the interactions between different components of the portfolio, improves hedge effectiveness. This complex problem involves both direct hedging and cross-hedging, in that a futures contract used to directly hedge one risk will also interact with other elements of the portfolio, which affects the return distribution. The impact of portfolio effects on the performance of hedged portfolios is examined by comparing the relative effectiveness of hedging the domestic and foreign components of a diversified equity portfolio separately using distinct bivariate models, and hedging the whole portfolio as a single spot asset using a single trivariate model. Hedging separately may not be optimal given that portfolio effects may change the level of hedging required. For instance, a natural hedge may occur if the returns on the domestic and foreign components of the portfolio have a negative association, reducing the required level of futures hedging. The relative performance of combined and separate hedges based on simple OLS regressions and more complex GARCH techniques is also examined.

Prior research on the impact of portfolio interactions when hedging is scarce generally focuses on foreign exchange risk (e.g., Gagnon et al (1998)). The application of a multivariate GARCH model to hedge risks in the context of fund portfolios has not been attempted. This chapter contributes to the literature by examining the issue in the context of...
Australian and foreign equity risk, from an Australian perspective. What differentiates the method in this chapter from that of researchers such as Kroner and Sultan (1993) and Park and Switzer (1995a, 1995b) is the consideration of a trivariate GARCH model of hedge ratio estimation to account for portfolio effects, in addition to traditional models that ignore portfolio effects. Further, prior evidence on the superiority of bivariate GARCH hedge ratios over simple OLS techniques is mixed and appears to depend on the particular data, as discussed in Chapter Three, so the analysis of bivariate hedge ratio estimation models in this chapter is of interest even in the absence of the additional trivariate models.

This chapter is composed of five sections. First, the method and data are described. Second, the hedge ratios are briefly examined, and their connection with results in previous chapters is explained. In the third section, a comparison is drawn between the methods of combined and separate hedging for given methods of hedge ratio estimation. Fourth, the performance of hedges using simple and complex methods of hedge ratio estimation are compared, and the impact of volatility and correlation on the findings is examined. Finally, conclusions are drawn.

8.2 Research Method and Data

Two broad approaches to hedging Australian and foreign equity are adopted. First, Australian and foreign equity may be hedged separately, and the expected returns on the hedged portfolio components, given in equations (8.1) and (8.2) respectively, are then combined for performance measurement.

\[ E(R_{HD,t}) = E(R_{UD,t}) - h_D E(R_{JD,t}) \]  
\[ E(R_{HF,t}) = E(R_{UF,t}) - h_F E(R_{JF,t}) \]  

where \( E(R_{HD,t}) \) = hedged return on domestic Australian equity component at time t  
\( E(R_{HF,t}) \) = hedged return on foreign equity component at time t  
\( E(R_{JD,t}) \) = expected return on domestic Australian equity futures contract at time t  
\( E(R_{JF,t}) \) = expected return on foreign equity futures contract at time t  
\( E(R_{UD,t}) \) = return on the unhedged domestic Australian equity portfolio at time t  
\( E(R_{UF,t}) \) = return on the unhedged foreign equity portfolio at time t  
\( h_D \) = hedge ratio for the domestic Australian equity futures hedge  
\( h_F \) = hedge ratio for the foreign equity futures hedge
The returns on the hedged portfolio are:

\[ E(R_{HP,t}) = w_D \{E(R_{UP,t}) - h_D \cdot E(R_{fD,t})\} + w_F \{E(R_{UF,t}) - h_F \cdot E(R_{fF,t})\} \]  \hspace{1cm} (8.3)

Rearranging equation (8.3):

\[ E(R_{HP,t}) = E(R_{UP,t}) - (w_D h_D \cdot E(R_{fD,t}) + w_F h_F \cdot E(R_{fF,t})) \]  \hspace{1cm} (8.4)

Alternatively, the returns on Australian and foreign equity may be combined to form a single equity portfolio return series, which is cross-hedged using two futures contracts. The expected return on the hedged portfolio is:

\[ E(R_{HP,t}) = E(R_{UP,t}) - (h_D E(R_{fD,t}) + h_F E(R_{fF,t})) \]  \hspace{1cm} (8.5)

In this chapter, Australian All Ordinaries futures contracts are always used to hedge Australian equity risk, because direct hedging is preferable to cross-hedging when a futures contract is written on the particular underlying asset under consideration, a point demonstrated empirically by Eaker and Grant (1987). However, no futures contract is written directly on internationally diversified foreign equity portfolios, so cross-hedging is employed to manage foreign equity risk. Cross-hedging the foreign equity component of the portfolio using single futures contracts was examined in detail in Chapter Six, and the hedging of Australian equity using Australian AOI futures contracts was discussed in Chapter Seven. In this chapter, the benefits from hedging using pairs of futures contracts are compared, using the “separate” and “combined” hedges, represented in equations (8.4) and (8.5) respectively.

Sensitivity analysis is conducted to examine the impact of hedge ratio estimation method, by comparing hedge ratios from price-changes OLS regressions and GARCH(1,1) models. As in previous chapters, hedge effectiveness is measured by expected quadratic utility, \( E(U) \), given in equation (6.5). The sensitivity of the results to the risk aversion parameter is also examined. The sensitivity of results to equity portfolio construction is assessed by comparing results using the Templeton and MSCI foreign equity portfolios, constructed such that the ratios of investment in Australian to foreign equity are 85:15, 70:30 and 55:45, for reasons detailed in Chapter Five. All data employed in this chapter are described in Chapter Five.

To estimate hedge ratios, two techniques are used, namely OLS regressions and GARCH(1,1) models. In the context of separate hedges of Australian and foreign equity components, the bivariate price changes OLS regression model described in equation (6.1) is used. To estimate the hedge ratio on the Australian equity component, \( \Delta S_t \) is the continuously compounded return on the All Ordinaries Index, and \( \Delta F_t \) is the continuously compounded return on the All Ordinaries Share Price Index futures contract. To estimate the
hedge ratio on the foreign equity component, \( \Delta S_t \), represents the returns on either the Templeton or MSCI foreign equity portfolio, and \( \Delta F_t \) is the return on the futures contract written on one of the S&P 500, FTSE 100, Nikkei 225, or Bovespa index.

To estimate hedge ratios for the “combined” hedge of the entire equity portfolio using OLS regressions, the following model is applied:

\[
\Delta P_t = \alpha_p + h_A \Delta AOIF_t + h_{FF} \Delta FF_t + \epsilon_{P,t} \tag{8.6}
\]

where 
- \( \Delta P_t \) = continuously compounded return on the entire equity portfolio
- \( \Delta AOIF_t \) = continuously compounded return on the AOI futures contract
- \( \Delta FF_t \) = continuously compounded return on the foreign equity futures contract
- \( h_A \) = hedge ratio associated with the Australian futures contract
- \( h_{FF} \) = hedge ratio associated with the foreign futures contract
- \( \epsilon_{P,t} \) = error term

However, returns on the Australian and foreign equity components exhibit ARCH effects, as shown in Table 5.5. Thus, it is reasonable to consider GARCH hedge ratio estimation models that adjust for heteroskedasticity and past information. To obtain the conditional hedge ratio, the conditional covariance matrix is estimated using the BEKK parameterization of Engle and Kroner (1995), given by the system of equations (6.2), (6.3), and (6.4). For separate hedges of Australian and foreign equity, a bivariate GARCH(1,1) model is employed (i.e., \( n=2 \)). For the combined hedge of the entire portfolio, a trivariate model is used (i.e., \( n=3 \)). The GARCH models are estimated using RATS software, and applying the BFGS algorithm.

### 8.3 Hedge Ratios

#### 8.3.1 Separate Hedges

In this Chapter, “separate” hedges are those where the Australian equity component is hedged using the Australian AOI futures contract, and the foreign equity component is hedged separately using futures contracts written on one of the S&P 500, FTSE 100, Nikkei 225, or Bovespa indices. Separate hedges using price-changes OLS regressions are estimated using equation (6.1), and separate hedges using bivariate GARCH(1,1) models are estimated using equations (6.2), (6.3) and (6.4), where \( n=2 \). Hedging the foreign equity component of the portfolio was addressed in detail in Chapter Six, and hedging the Australian equity component was analysed in Chapter Seven, so the discussion here is brief.
Separate hedges of Australian and foreign equity do not take into account potential interactions between all components of the portfolio. In contrast, “combined” hedges in the following section are calculated by treating the whole equity portfolio as a single spot asset, capturing the co-movement between Australian and foreign equity in the single return series, and by estimating the hedge ratios in a single model.

8.3.2 Combined Hedges

This section examines “combined” hedges, or those in which the entire equity portfolio is hedged using the Australian AOI futures contract together with a foreign equity futures contract. The performance of hedges based on price change OLS regressions is compared with the performance of hedges using a more complex trivariate GARCH(1,1) model. Recall that the Australian futures contracts are always considered as part of hedge due to the large proportion of Australian equity in all the equity portfolios under consideration and due to the empirically demonstrated preference for direct hedges over cross-hedges (Eaker and Grant (1987)). A foreign futures contract is included to hedge the component of risk introduced by the foreign equity in the portfolio.

Price changes OLS regression hedge ratios are calculated using equation (8.6), and the trivariate GARCH(1,1) hedge ratios are calculated using the system of equations given by (6.2), (6.3) and (6.4), where \( n = 3 \). To illustrate, Tables 8.1 and 8.2 show the hedge ratios for portfolios containing 70% Australian and 30% Templeton foreign equity, where hedge ratios are estimated using OLS and GARCH methods respectively. Hedge ratios for the Australian and foreign futures contracts are listed in pairs. For instance, for OLS hedges involving AOI futures contracts and FTSE 100 futures contracts, Table 8.1 shows that the hedge ratio for the former is 0.6696 and for the latter is 0.1550, in the estimation period commencing on 05/01/89. Generally, the OLS hedge ratios are more stable across time relative to the GARCH hedge ratios.

Table 8.1 shows that when hedge ratios are estimated using OLS regressions, general levels of hedge ratios emerge for each futures contract in each scenario. For instance, for hedges involving both the AOI futures contracts and S&P 500 futures contracts (AOIF/SP), the AOI futures hedge ratio is generally around 0.6 and the S&P 500 futures hedge ratio is generally around 0.2. For hedges involving Nikkei futures contracts (AOIF/NIKK) and Bovespa futures contracts (AOIF/BOV), the foreign futures hedge ratios are small relative to the ratios for hedges involving S&P 500 futures contracts (AOIF/SP) and FTSE 100 futures contracts (AOIF/FTSE). Table 8.1 also shows that the behaviour of the AOI futures hedge ratio depends on the foreign futures contract it is paired with. For example, the largest AOI
futures hedge ratio is approximately 0.68 or 0.69 when matched with the S&P 500 futures contracts or FTSE 100 futures contracts respectively, but increases to approximately 0.80 when paired with the Nikkei 225 futures contracts or Bovespa futures contracts. The AOI futures hedge ratios are more variable when forming a joint hedge with Nikkei or Bovespa futures contracts.

Table 8.2 shows that under GARCH(1,1) hedge ratio estimation, the range of AOI futures hedge ratios for any given futures pair is noticeably larger than when hedge ratios are estimated using OLS regressions. Compare, for instance, the ratios for AOI/FTSE hedges in Tables 8.1 and 8.2. In addition, for any given futures pair, the range of foreign equity futures hedge ratios over time is also larger under the GARCH method than under the OLS regression technique. Indeed, some Nikkei and Bovespa futures hedge ratios in Table 8.2 are negative. Negative hedge ratios are caused by negative correlation between the spot and futures returns in the time period used to estimate the hedge ratio, as discussed and illustrated in Chapter Six, Section 6.3. In general, for each hedging strategy, the OLS regression hedge ratios for each contract are more stable over time than the GARCH(1,1) hedge ratios.

Table 8.1: Hedge ratios for portfolios containing 70% Australian and 30% Templeton foreign equity, calculated using price-changes OLS regressions

<table>
<thead>
<tr>
<th>Estimation period Start date</th>
<th>Estimation period End date</th>
<th>AOI/SP AOI/FTSE AOI/NIKK AOI/BOV</th>
</tr>
</thead>
<tbody>
<tr>
<td>05/01/89</td>
<td>26/12/91</td>
<td>0.6198 0.2315 0.6696 0.1550</td>
</tr>
<tr>
<td>06/07/89</td>
<td>25/06/92</td>
<td>0.6185 0.2417 0.6751 0.1478</td>
</tr>
<tr>
<td>09/01/92</td>
<td>29/12/94</td>
<td>0.5849 0.2369 0.5795 0.1286</td>
</tr>
<tr>
<td>09/07/92</td>
<td>29/06/95</td>
<td>0.5942 0.2274 0.5832 0.1381</td>
</tr>
<tr>
<td>05/01/95</td>
<td>25/12/97</td>
<td>0.6505 0.1882 0.6571 0.1513</td>
</tr>
<tr>
<td>06/07/95</td>
<td>25/06/98</td>
<td>0.6515 0.1903 0.6577 0.1569</td>
</tr>
<tr>
<td>08/01/98</td>
<td>28/12/00</td>
<td>0.6766 0.2174 0.6796 0.2319</td>
</tr>
<tr>
<td>09/04/98</td>
<td>29/03/01</td>
<td>0.6818 0.2240 0.6866 0.2416</td>
</tr>
<tr>
<td>09/07/98</td>
<td>28/06/01</td>
<td>0.6849 0.2263 0.6935 0.2427</td>
</tr>
</tbody>
</table>

"AOI" indicates the Australian All Ordinaries Share Price Index futures contract. "SP" indicates the U.S. S&P 500 Share Price Index futures contract. "FTSE" indicates the U.K. FTSE 100 Index futures contract. "NIKK" indicates the Japanese Nikkei 225 Index futures (SIMEX) contract. "BOV" indicates the Brazilian Bovespa Index futures contract. "AOI/SP" indicates a hedge involving AOI futures contracts and S&P 500 futures contracts simultaneously, and similarly for the other futures combinations. Hedge ratios are calculated using the price changes OLS regression in equation (8.6).
Table 8.2: Hedge ratios for portfolios containing 70% Australian and 30% Templeton foreign equity, calculated using trivariate GARCH(1,1) models

| Estimation period Start date | Estimation period End date | AOIF/SP S&P AOIF futures AOIF FTSE futures AOIF/NIKK futures AOIF/BOV futures |
|-------------------------------|-----------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| 05/01/89                      | 26/12/91                    | 0.6540 0.3896           | 0.6358 0.3349           | 0.6545 0.2234           | 0.8043 0.0961           |
| 06/07/89                      | 25/06/92                    | 0.5820 0.1765           | 0.6579 0.3636           | 0.6720 0.2119           | 0.5671 0.1114           |
| 09/01/92                      | 29/12/94                    | 0.5514 0.3647           | 0.5272 0.2058           | 0.6071 0.1071           | 0.6461 0.0490           |
| 08/06/95                      | 29/06/95                    | 0.6418 0.3716           | 0.5989 0.1864           | 0.6486 0.2188           | 0.6790 -0.0072          |
| 05/01/95                      | 25/12/97                    | 0.7569 0.3700           | 0.7035 0.1519           | 0.7757 0.1001           | 0.7860 0.1781           |
| 06/07/95                      | 25/06/98                    | 0.7252 0.4808           | 0.4357 0.1788           | 0.4530 0.0122           | 0.5592 0.0681           |
| 09/01/98                      | 28/12/00                    | 0.8514 0.2075           | 0.8430 0.5844           | 0.8349 -0.0286          | 0.7173 0.0774           |
| 09/04/98                      | 29/03/01                    | 0.7144 0.3179           | 0.8594 0.1979           | 0.7409 0.3874           | 0.8233 0.1475           |
| 09/07/98                      | 28/06/01                    | 0.9386 0.4555           | 0.8404 0.4301           | 0.8475 0.2344           | 0.8884 0.1506           |

"AOIF" indicates the Australian All Ordinaries Share Price Index futures contract. "SP" indicates the U.S. S&P 500 Share Price Index futures contract. "FTSE" indicates the U.K. FTSE 100 Index futures contract. "NIKK" indicates the Japanese Nikkei 225 Index futures (SIMEX) contract. "BOV" indicates the Brazilian Bovespa Index futures contract. "AOIF/SP" indicates a hedge involving AOI futures contracts and S&P 500 futures contracts simultaneously, and similarly for the other futures combinations. Hedge ratios are calculated using the trivariate version of the GARCH(1,1) model in equations (6.2), (6.3) and (6.4), where n=3. Negative hedge ratios are discussed in detail in Chapter 6, Section 6.3.

Recall that Tables 8.1 and 8.2 refer to portfolios consisting of 70% Australian and 30% Templeton foreign equity. The observations made concerning these results also hold for portfolios containing different proportions of Australian and Templeton foreign equity. Further, results using portfolios containing MSCI foreign equity are similar, and are therefore not reported.

The performance of combined hedge ratios are compared with the performance of separate hedge ratios in the following section.

8.4 Comparison of combined and separate hedges for a given hedge ratio estimation method

This section examines whether there is any advantage in calculating hedge ratios separately for Australian and foreign equity, relative to treating the entire portfolio as a single spot asset for a given hedge ratio estimation method. This is of interest because "portfolio effects", or interactions between different components of the portfolio, are ignored when Australian and foreign equity are hedged separately. For instance, the returns on Australian equity may interact with the returns on the foreign equity component such that a natural hedge is created and fewer futures are required to complete the hedge. In contrast, when the entire portfolio is cross-hedged as a single asset, the interactions between the returns on the Australian and foreign components are already captured in the total return. Separate and combined hedges using Australian futures contracts exclusively were examined in the previous chapter as a
preliminary analysis in the simple case of one futures contract. This section extends the analysis to the more complex scenario using two futures contract simultaneously, specifically where Australian futures contracts hedge the Australian equity component of the portfolio, and one of a range of foreign futures contracts hedge the foreign equity component. This illustrates the use of multiple futures contracts to hedge risk, an issue taken up in greater depth in the following chapter where portfolios of futures contracts are used to hedge equity risk.

8.4.1 Comparing combined and separate hedges

The key issue in this section is whether, for a given hedge ratio estimation method, separate or combined hedges are preferred. The following section then examines whether any hedge ratio estimation method is preferred. Recall that “combined” hedges, or those in which the entire equity portfolio is hedged using a pair of futures contracts, are compared with “separate” hedges, or those where the Australian equity risk is hedged using Australian futures and foreign equity risk is hedged using one of a variety of futures. The expected utilities associated with both scenarios are calculated. The percentage of hedge periods in which the expected utility of the combined hedge exceeds the expected utility of the separate hedge scenario is recorded in Tables 8.3 and 8.4 for portfolios containing Templeton foreign equity and MSCI foreign equity respectively. Nine hedge periods between 1992 and 2001 are employed in this analysis. To illustrate the interpretation of the results in these tables, take the case in Table 8.3 where hedge ratios are calculated using OLS regressions, the risk aversion parameter \( \lambda = 3 \), and the equity portfolio contains 85% Australian equity. Using AOI futures contracts and FTSE futures contracts, a combined hedge using the trivariate regression equation (8.6) is preferred in 44% of hedge periods. By deduction, separate hedges using two distinct bivariate regressions are preferred in 56% of hedge periods, because combined and separate hedges are the only options available.

Consider first the results in Table 8.3, which relate to portfolios containing Templeton foreign equity. When hedge ratios are calculated using price changes OLS regressions, separate hedges of Australian and foreign equity are preferred in the majority of hedge periods when Australian AOI futures contracts are used in conjunction with either S&P 500 futures contracts or FTSE 100 futures contracts, for all asset allocations and risk aversions. A similar result occurs when AOI futures contracts are used with Bovespa futures, except for portfolios containing 85% Australian equity. In contrast, when OLS hedge ratios are calculated, and AOI futures contracts and Nikkei 225 futures contracts are used together, combined hedges are superior in the majority of hedge periods for most portfolio construction scenarios. Results using OLS hedge ratios appear to be generally insensitive to
the risk aversion parameter, and generally insensitive to the allocation between Australian and foreign equity for hedges involving S&P 500 futures contracts and FTSE 100 futures contracts. Hedges involving Nikkei 225 futures contracts and Bovespa futures contracts are more sensitive to the allocation between Australian and foreign equity.

Table 8.3: Percent of hedge periods in which expected utility associated with combined hedges is higher than expected utility associated with separate hedges of Australian and Templeton foreign equity, for hedge periods between 1992 and 2001

<table>
<thead>
<tr>
<th>Hedge ratio estimation method</th>
<th>Australian Equity %</th>
<th>Risk Aversion, λ</th>
<th>AOI futures and S&amp;P 500 futures</th>
<th>AOI futures and FTSE 100 futures</th>
<th>AOI futures and Nikkei 225 futures</th>
<th>AOI futures and Bovespa futures</th>
</tr>
</thead>
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<tr>
<td>OLS</td>
<td>85</td>
<td>0.1</td>
<td>33</td>
<td>44</td>
<td>44</td>
<td>56</td>
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<td>44</td>
<td>56</td>
<td>56</td>
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<tr>
<td>GARCH</td>
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<td>44</td>
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<td>56</td>
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<td>10</td>
<td>44</td>
<td>44</td>
<td>56</td>
<td>56</td>
<td>44</td>
</tr>
</tbody>
</table>

Expected utility \( E(U) \) is calculated using equation (6.5), where \( \lambda \) is the risk aversion parameter. Combined hedges are those which account for the Australian and foreign components in a single model, while separate hedges address the domestic equity risk and foreign equity risk in two distinct models. Separate OLS hedges are calculated using equation (6.1). The combined OLS model is given in equation (8.6). Separate and combined GARCH hedges employ variations on the model in equations (6.2), (6.3) and (6.4), where \( n=2 \) and \( n=3 \) respectively.

From Table 8.3, in comparison with results based on OLS hedge ratios, results based on GARCH hedge ratios are less clear with regard to a preference between combined and separate hedges. The strongest result is associated with scenarios combining AOI futures contracts and FTSE 100 futures contracts, where separate hedges are preferred in the majority of hedge periods for all but one portfolio construction. Results involving the other foreign futures contracts appear to be portfolio-specific, in that separate hedges are preferred in the majority of hedge periods for some portfolio constructions, but combined hedges are preferred in the majority of hedge periods for other portfolios. The results based on GARCH hedge ratios appear to be more sensitive to \( \lambda \) than results based on OLS hedge ratios.

The results for portfolios containing MSCI foreign equity in Table 8.4 are similar to those in Table 8.3 for portfolios containing Templeton foreign equity. Table 8.4 shows that when
hedge ratios are calculated using price changes OLS regressions, separate hedges of Australian and foreign equity are preferred in the majority of hedge periods when Australian AOI futures contracts are used with either S&P 500 futures contracts or FTSE futures contracts, for all asset allocations and risk aversions. Similarly, for OLS hedges involving AOI futures contracts combined with either Nikkei 225 futures contracts or Bovespa futures contracts, separate hedges are preferred in the majority of hedge periods. Generally, the results in Table 8.4 based on OLS regression hedge ratios are not sensitive to the risk aversion parameter.

Table 8.4: Percent of hedge periods in which expected utility associated with combined hedges is higher than expected utility associated with separate hedges of Australian and MSCI foreign equity, for hedge periods between 1992 and 2001

<table>
<thead>
<tr>
<th>Hedge ratio estimation method</th>
<th>Australian equity %</th>
<th>Risk Aversion, ( \lambda )</th>
<th>AOI futures and S&amp;P 500 futures</th>
<th>AOI futures and FTSE 100 futures</th>
<th>AOI futures and Nikkei 225 futures</th>
<th>AOI futures and Bovespa futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>85</td>
<td>0.1</td>
<td>22</td>
<td>44</td>
<td>44</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>44</td>
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<td>70</td>
<td>0.1</td>
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<td></td>
<td>10</td>
<td>44</td>
<td>44</td>
<td>56</td>
<td>44</td>
</tr>
<tr>
<td>GARCH</td>
<td>85</td>
<td>0.1</td>
<td>56</td>
<td>56</td>
<td>44</td>
<td>44</td>
</tr>
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<td>3</td>
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<td>33</td>
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<td>10</td>
<td>56</td>
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<td>70</td>
<td>0.1</td>
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<td>55</td>
<td>0.1</td>
<td>33</td>
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<td>56</td>
<td>44</td>
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</tbody>
</table>

Expected utility \( E(U) \) is calculated using equation (6.5), where \( \lambda \) is the risk aversion parameter. Combined hedges are those which account for the Australian and foreign components in a single model, while separate hedges address the domestic equity risk and foreign equity risk in two distinct models. Separate OLS hedges are calculated using equation (6.1). The combined OLS model is given in equation (8.6). Separate and combined GARCH hedges employ variations on the model in equations (6.2), (6.3) and (6.4), where \( n=2 \) and \( n=3 \) respectively.

An examination of the GARCH hedge results in Table 8.4 shows that for scenarios combining AOI futures contracts and Bovespa futures contracts, separate hedges are preferred in the majority of hedge periods for all but one portfolio construction. In contrast, hedges combining the AOI futures contracts and Nikkei futures contracts show preference for combined hedges in the majority of hedge periods, for portfolios with less than 85% Australian equity. The preference for separate or combined hedges when the other foreign futures contracts are employed appears to be portfolio-specific. A lack of preference for
combined hedges in some hedge periods may be attributed to the low correlation between Australian futures contracts and each of the foreign futures contracts, and between Australian equity and each of the foreign futures contracts, as shown in Chapter Five in Table 5.7. There may not be a sufficiently large interaction between different hedged portfolio components that is not already captured in the separate hedges to make the combined hedges preferable in some periods.

As in Table 8.3, Table 8.4 confirms that the results based on GARCH hedge ratios appear to be more sensitive to \( \lambda \) than results based on OLS hedge ratios.

To assess the impact of the hedge effectiveness measure on these results, performance is reassessed according to the expected return on the hedged portfolio \( E(R_{HP}) \) standardised by risk, where risk is measured by the standard deviation of returns \( \sigma_{HP} \). Thus, the measure is \( E(R_{HP})/\sigma_{HP} \). Although a generally accepted aim of hedging is to minimise risk, managers also consider expected returns when making investment decisions for reasons discussed in detail in Chapter Two. This alternative measure of performance gives an intuitive perspective on the risk-return benefits associated with various futures hedges, in that managers seek to maximise returns per unit of risk, even while attempting to minimise risk. This simple measure is adopted in preference to the more commonly used Sharpe measure because the latter requires the identification of an appropriate risk-free rate of interest, which is not obvious in the context of globally diversified portfolios. Further, the use of the Sharpe measure and the choice of a risk-free rate of interest becomes more problematic as the complexity of the hedging scenarios increases, such as the separate hedging of Australian and foreign equity. In the interests of consistency throughout this chapter, and for the purposes of comparing simple and complex hedging strategies in different chapters, the expected return per unit risk measure offers a more appropriate alternative than the Sharpe measure.

Table 8.5 shows the percent of hedge periods in which combined hedges outperform separate hedges of diversified equity portfolios, where hedge performance is measured using the expected hedged return per unit of risk. Results associated with price changes OLS regression hedge ratios uniformly favour separate over combined hedges in the majority of hedge periods. This finding is true for all combinations of futures contracts, both foreign equity portfolio constructions, and each division between Australian and foreign equity. Table 8.5 confirms the general preference for separate hedges that is evident in Tables 8.3 and 8.4 when OLS hedge ratios are applied. The mixed results under GARCH hedge ratios evident in Table 8.5 also coincides with the earlier results in Tables 8.3 and 8.4. The
preference between combined hedges of the entire equity portfolio and separate hedges of the Australian and foreign equity components does not appear to be very sensitive to the performance measure.

Table 8.5: Percent of hedge periods in which combined hedges outperform separate hedges of Australian and foreign equity, for hedge periods between 1992 and 2001, where hedge effectiveness is measured using the expected hedged return per unit of risk

<table>
<thead>
<tr>
<th>Hedge ratio estimation method</th>
<th>Equity portfolio</th>
<th>Australian equity %</th>
<th>AOI futures and S&amp;P 500 futures</th>
<th>AOI futures and FTSE 100 futures</th>
<th>AOI futures and Nikkei 225 futures</th>
<th>AOI futures and Bovespa futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>AOI, TWF</td>
<td>85</td>
<td>33</td>
<td>44</td>
<td>44</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70</td>
<td>44</td>
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<td>44</td>
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<td></td>
<td></td>
<td>55</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
</tr>
<tr>
<td>AOI, MSCI</td>
<td>85</td>
<td>22</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>70</td>
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<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
</tr>
<tr>
<td>GARCH</td>
<td>AOI, TWF</td>
<td>85</td>
<td>56</td>
<td>67</td>
<td>44</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70</td>
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<td>44</td>
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<td>55</td>
<td>56</td>
<td>44</td>
<td>44</td>
<td>33</td>
</tr>
<tr>
<td>AOI, MSCI</td>
<td>85</td>
<td>56</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>70</td>
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<td>67</td>
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<td></td>
<td>55</td>
<td>33</td>
<td>56</td>
<td>56</td>
<td>56</td>
<td>44</td>
</tr>
</tbody>
</table>

Risk is measured by standard deviation of returns. Combined hedges are those which account for the Australian and foreign components in a single model, while separate hedges address the domestic equity risk and foreign equity risk in two distinct models. Separate OLS hedges are calculated using equation (6.1). The combined OLS model is given in equation (8.6). Separate and combined GARCH hedged employ variations on the model in equations (6.2), (6.3) and (6.4), where n=2 and n=3 respectively. Performance is measured as: \( E(R_{HP})/\sigma_{HP} \).

In summary, for a given hedge ratio estimation method, the relative benefits of using the AOI futures contract and a foreign futures contract to hedge the entire equity portfolio as opposed to separately hedging Australian and foreign equity depends on the foreign futures contract. When hedge ratios are estimated using price changes OLS regressions, separate hedges generally outperform combined hedges in the majority of hedge periods for all foreign futures contracts, except when Nikkei futures are applied to portfolios containing Templeton foreign equity. Thus, a manager motivated by expected utility usually benefits from separate hedges of Australian and foreign equity when using OLS regression hedge ratios. This suggests that explicitly accounting for portfolio effects through the hedge ratio estimation model is generally unnecessary when using the OLS technique. However, when hedge ratios are estimated using the GARCH(1,1) method, preferences for separate or combined hedges appear to be portfolio-specific and not readily generalisable. Thus, the benefits of accounting for portfolio effects through the hedge ratio estimation method are ambiguous under the GARCH model.

Whether to account for complex co-movements between the returns on different components of multi-asset portfolios is an important issue for managers because if accounting for
portfolio effects generally improves the accuracy of a hedge, then the complexity of the hedge ratio estimation method must increase as the number of assets in the portfolio increases. There may be a tradeoff between improvements in accuracy and computational complexity. Further, the finding that accounting for portfolio effects does not generally improve hedge performance under the OLS hedge ratio estimation method for portfolios containing domestic and foreign equity and a maximum of two futures contracts may not discount the benefits of a "combined" approach to hedge ratio estimation because simple OLS methods may be outperformed by more complex GARCH methods. Given that preferences for separate or combined hedges depend on whether hedge ratios are estimated using OLS regressions or a GARCH method, the following section examines which hedge ratio estimation method is preferred in different scenarios.

8.5 Comparison of hedge ratio estimation methods

Previous sections examine the performance of combined and separate hedges in the context of a given hedge ratio estimation method. This section examines whether complex methods of hedge ratio estimation outperform simple methods, and specifically the relative performance of hedges calculated using bivariate and trivariate GARCH(1,1) models and bivariate and trivariate price changes OLS regressions.

8.5.1 Overview of results

The performance of a variety of combined and separate hedging strategies involving two futures contracts are compared in Table 8.6, which indicates the hedge ratio estimation method associated with the highest frequency of first rankings, where ranking is based on expected utility. As in previous sections, hedge effectiveness is measured out of sample in all cases, such that the estimation time period and subsequent hedge period are distinct and non-overlapping. In Table 8.6, the classification of a method as "Cmb" indicates that hedge ratios are calculated using the entire equity portfolio as the dependent variable, referred to in this chapter as a "combined" hedge. Similarly, "Sep" indicates that hedge ratios are calculated separately for distinct or "separate" hedges of the Australian and foreign equity components of the portfolio.
Table 8.6: Best performing hedge ratio estimation method, applied to two futures contracts to hedge equity portfolios, where performance is assessed by the highest frequency of first ranking over hedge periods between 1992 and 2001, where ranking is based on expected utility

<table>
<thead>
<tr>
<th>Equity Portfolio</th>
<th>Aust Equity aversion</th>
<th>AOIF and S&amp;P 500 futures hedge</th>
<th>AOIF and FTSE 100 futures hedge</th>
<th>AOIF and Nikkei 225 futures hedge</th>
<th>AOIF and Bovespa futures hedge</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOI, TWF</td>
<td>85</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb), OLS (Cmb)</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb), OLS (Cmb)</td>
<td>GARCH (Cmb)</td>
<td></td>
</tr>
<tr>
<td>AOI, MSCI</td>
<td>55</td>
<td>GARCH (Cmb), OLS (Cmb)</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>GARCH (Cmb), OLS (Cmb)</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
<td></td>
</tr>
</tbody>
</table>

"AOI, TWF" indicates equity portfolios containing Australian and Templeton foreign equity, and "AOI, MSCI" indicates equity portfolios containing Australian and MSCI foreign equity. "Cmb" indicates that hedge ratios are calculated using the entire equity portfolio as the dependent variable, referred to in this chapter as a "combined" hedge. "Sep" indicates that hedge ratios are calculated separately for distinct or "separate" hedges of the Australian and foreign equity components. Combined hedges are those which account for the Australian and foreign components in a single model, while separate hedges address the domestic equity risk and foreign equity risk in two distinct models. Separate OLS hedges are calculated using equation (6.1). The combined OLS model is given in equation (8.6). Separate and combined GARCH hedges employ variations on the model in equations (6.2), (6.3) and (6.4), where \( n = 2 \) and \( n = 3 \) respectively. Rankings are based on expected utility, which is calculated according to equation (6.5), where \( \lambda \) is the risk aversion parameter. The frequency of first rankings is calculated over nine different hedge periods between 1992 and 2001. Where two hedge ratio estimation methods attract the same number of first rankings over the time periods considered, both are listed in the Table.

To illustrate, "OLS (Cmb)" refers to hedge ratios calculated using trivariate price changes OLS regressions, described in equation (8.6). "OLS (Sep)" refers to hedge ratios estimated using two separate bivariate price changes OLS regression equations, one for the association between Australian equity and Australian futures contracts, and the other for the association...
between foreign equity and foreign futures contracts, using variations on equation (6.1) as described above in Section 8.3. Where two hedge ratio estimation strategies attract the same number of first rankings over the time periods considered, both are listed in Table 8.6. For instance, when hedging portfolios containing 85% Australian equity and 15% Templeton foreign equity using Australian and FTSE 100 futures contracts, and assuming a risk aversion parameter of 0.1, trivariate GARCH(1,1) hedge ratios and trivariate OLS regression hedge ratios both perform equally best relative to other methods.

Table 8.6 shows that hedges applied using the trivariate GARCH(1,1) model, the combined GARCH approach, are either superior or jointly superior to other models for hedges involving the Australian and Bovespa futures contracts and the Australian and Nikkei 225 futures contracts, and this is robust to foreign equity portfolio construction, asset allocation between Australian and foreign equity, and the level of risk aversion. Similarly, the trivariate GARCH(1,1) model performs best for combinations of Australian and FTSE 100 futures contracts and Australian and S&P 500 futures contracts, although these results are more sensitive to equity portfolio construction and risk aversion. The preference for the trivariate GARCH(1,1) model over other methods of hedge ratio estimation are consistent with the findings of Gagnon et al (1998) who conducted a similar study in the context of hedging foreign exchange risk.

To assess the sensitivity of these findings to the performance measure, performance is re-evaluated using the expected return per unit of risk measure, and the results are reported in Table 8.7. The findings in Table 8.7 confirm the general preference for hedge ratios calculated using the trivariate GARCH model for hedges combining the Australian futures contracts with futures contracts written on each of the FTSE 100, the Nikkei 225, and the Bovespa index. While the combined GARCH hedge approach also performs well for half of the equity portfolio scenarios hedged using the Australian and S&P 500 futures contracts, separate bivariate OLS regression hedge ratios also perform well. Further, when using the Nikkei 225 futures contract with the Australian futures contract, there is little difference between the performance of separate bivariate and combined trivariate GARCH hedge ratios. Thus, like the results in Table 8.6 based on expected utility, the results in Table 8.7 based on the expected return per unit of risk measure reveal the trivariate GARCH model as performing well, depending on the combination of futures contracts and portfolio construction, but do not offer unqualified support for the use of combined over separate hedge ratio estimation methods.
Table 8.7: Best performing hedge ratio estimation method, applied to two futures contracts to hedge equity portfolios, where performance is assessed by the highest frequency of first ranking over hedge periods between 1992 and 2001, where ranking is based on expected return per unit risk

<table>
<thead>
<tr>
<th>Equity portfolio</th>
<th>Aust. Equity %</th>
<th>AOI and S&amp;P 500 futures hedge</th>
<th>AOI and FTSE 100 futures hedge</th>
<th>AOI and Nikkei 225 futures hedge</th>
<th>AOI and Bovespa futures hedge</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOI, TWF</td>
<td>85</td>
<td>OLS (Sep)</td>
<td>OLS (Cmb), GARCH (Cmb)</td>
<td>GARCH (Cmb), GARCH (Sep)</td>
<td>GARCH (Cmb)</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>OLS (Sep)</td>
<td>OLS (Cmb), GARCH (Cmb)</td>
<td>GARCH (Cmb), GARCH (Sep)</td>
<td>GARCH (Cmb)</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>GARCH (Cmb)</td>
<td>OLS (Sep)</td>
<td>GARCH (Cmb), GARCH (Sep)</td>
<td>OLS (Cmb)</td>
</tr>
<tr>
<td>AOI, MSCI</td>
<td>85</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb), GARCH (Sep)</td>
<td>GARCH (Cmb), OLS (Sep)</td>
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<td>70</td>
<td>OLS (Sep)</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb), GARCH (Sep)</td>
<td>GARCH (Cmb)</td>
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<tr>
<td></td>
<td>55</td>
<td>OLS (Sep), GARCH (Cmb), OLS (Sep)</td>
<td>GARCH (Cmb), GARCH (Sep)</td>
<td>GARCH (Cmb), GARCH (Sep)</td>
<td>GARCH (Cmb), OLS (Sep)</td>
</tr>
</tbody>
</table>

“AOI, TWF” indicates equity portfolios containing Australian and Templeton foreign equity, and “AOI, MSCI” indicates equity portfolios containing Australian and MSCI foreign equity. “Cmb” indicates that hedge ratios are calculated using the entire equity portfolio as the dependent variable, referred to in this chapter as a “combined” hedge. “Sep” indicates that hedge ratios are calculated separately for distinct or “separate” hedges of the Australian and foreign equity components. Combined hedges are those which account for the Australian and foreign components in a single model, while separate hedges address the domestic equity risk and foreign equity risk in two distinct models. Separate OLS hedges are calculated using equation (6.1). The combined OLS model is given in equation (8.6). Separate and combined GARCH hedges employ variations on the model in equations (6.2), (6.3) and (6.4), where n=2 and n=3 respectively. Rankings are based on expected utility, which is calculated according to equation (6.5), where λ is the risk aversion parameter. The frequency of first rankings is calculated over nine different hedge periods between 1992 and 2001. Where two hedge ratio estimation methods attract the same number of first rankings over the time periods considered, both are listed in the Table.

To summarise, when portfolio performance is measured using expected utility, hedge ratios estimated using the trivariate GARCH(1,1) model generally perform as well as or better than other methods of hedge ratio estimation, suggesting that accounting for portfolio effects when calculating hedge ratios generally improves hedged portfolio performance. However, the benefits of accounting for portfolio effects using combined hedges appear to be futures contract specific and sensitive to the equity portfolio construction. Specifically, when other models are preferred to the trivariate GARCH model, there is no clear preference for combined hedges, particularly for hedges using both the Australian and S&P 500 futures contracts simultaneously. When performance is measured using expected return per unit of risk, the trivariate GARCH model generally performs well, depending on the combination of futures contracts and portfolio construction.

A comparison of the findings in Sections 8.5 and 8.6 highlights an important issue. Consider the results based on expected utility in Table 8.6 with the earlier results in Tables 8.3 and 8.4. From Table 8.6, combined hedges using the trivariate GARCH(1,1) method of hedge ratio estimation generally perform well when all models are considered together. However, when separate and combined hedges are compared in the context of GARCH hedges only in Tables 8.3 and 8.4, there is no clear general preference for combined hedges. For example,
Table 8.6 shows that when AOI and S&P 500 futures contracts hedge portfolios containing 85% Australian equity and 15% Templeton foreign equity, and assuming $\lambda = 10$, the combined GARCH model is preferred to other models. However, Table 8.3 shows that for the same portfolio construction and contracts, combined GARCH hedges are preferred to separate GARCH hedges in only 44% of hedge periods. This seemingly contradictory result occurs because in hedge periods where the expected utility associated with the separate GARCH hedge exceeds that of the combined GARCH hedge, neither GARCH hedge is the best overall method. In contrast, when the expected utility of the combined GARCH hedge exceeds that of the separate GARCH hedge in a particular hedge period, the combined GARCH model is the best model for that period. In this way, combined hedges using the trivariate GARCH model attract the greatest number of first rankings across all hedge periods between 1992 and 2001, even though that model is not preferred to the separate GARCH model in all periods. In short, when the combined GARCH model is preferred to the separate GARCH model, it is also the best of all the models, but when performance based on the separate GARCH model is superior to that based on the combined GARCH model, an OLS model is preferred to both. From the perspective of a manager, combined hedges using trivariate GARCH hedge ratios generally perform well relative to all other models, but if a manager desires separate calculations for each part of the portfolio then the simple OLS regressions appear to be generally preferable to the more complex GARCH method.

8.5.2 Best hedge strategy in each hedge period

While the summary results provided above indicate the general findings, an analysis of factors potentially causing the results requires the reporting of more detailed information for individual hedge periods. Results in Section 8.5.1 analyse the best hedge ratio estimation method for each different futures combination. This presupposes that the particular futures combination will be used and the issue is the selection of a hedge ratio estimation method. In contrast, Tables 8.8 and 8.9 below show the best hedge strategies in each hedge period for portfolios containing Templeton and MSCI foreign equity respectively, when both the hedge ratio estimation method and futures contract combinations are varied. Performance is measured using expected utility where $\lambda = 3$.

Tables 8.8 and 8.9 indicate two key pieces of information for the best hedge of each equity portfolio in each hedge period, namely the futures contract combination and the hedge ratio estimation method. For instance, Table 8.8 shows that the best hedge of the portfolio consisting of 85% Australian and 15% Templeton foreign equity in the hedge period commencing 02/01/92 combined AOI futures contracts and Nikkei futures contracts, and
used the trivariate GARCH(1,1) hedge ratio estimation model (the “combined” approach). Recall that the best hedge is that which attracts the highest expected utility, where \( \lambda = 3 \).

As summarised previously, Tables 8.8 and 8.9 show that generally “combined” GARCH hedge ratio estimation models perform well relative to other models. The GARCH model is preferred in most hedge periods, regardless of relative weightings of Australian and foreign equity, and regardless of the foreign equity portfolio construction. The second last rows of Tables 8.8 and 8.9 reveal the number of hedge periods in which a GARCH model is preferred to an OLS model. Generally, as the proportion of foreign equity increases, the number of periods where the GARCH method is preferred decreases slightly, while still accounting for most periods.

Table 8.8: The hedge associated with the highest expected utility, where foreign equity is represented by the Templeton portfolio and the risk aversion parameter is three

<table>
<thead>
<tr>
<th>Hedge period Start date</th>
<th>Hedge period End date</th>
<th>85 AOI, 15 TWF</th>
<th>70 AOI, 30 TWF</th>
<th>55 AOI, 45 TWF</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP1 02/01/92</td>
<td>29/01/92</td>
<td>AOIF, NIKK</td>
<td>AOIF, NIKK</td>
<td>AOIF, NIKK</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
</tr>
<tr>
<td>HP2 01/07/92</td>
<td>28/07/92</td>
<td>AOIF, FTSE</td>
<td>AOIF, FTSE</td>
<td>AOIF, FTSE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
</tr>
<tr>
<td>HP3 03/01/95</td>
<td>30/01/95</td>
<td>AOIF, BOV</td>
<td>AOIF, BOV</td>
<td>AOIF, NIKK</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
</tr>
<tr>
<td>HP4 03/07/95</td>
<td>28/07/95</td>
<td>AOIF, SP</td>
<td>AOIF, SP</td>
<td>AOIF, SP</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OLS (Sep)</td>
<td>OLS (Sep)</td>
<td>OLS (Sep)</td>
</tr>
<tr>
<td>HP5 02/01/98</td>
<td>29/01/98</td>
<td>AOIF, BOV</td>
<td>AOIF, BOV</td>
<td>AOIF, NIKK</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
</tr>
<tr>
<td>HP6 01/07/98</td>
<td>28/07/98</td>
<td>AOIF, FTSE</td>
<td>AOIF, FTSE</td>
<td>AOIF, FTSE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
<td>OLS (Sep)</td>
</tr>
<tr>
<td>HP7 02/01/01</td>
<td>30/01/01</td>
<td>AOIF, NIKK</td>
<td>AOIF, NIKK</td>
<td>AOIF, NIKK</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GARCH (Sep)</td>
<td>GARCH (Sep)</td>
<td>GARCH (Sep)</td>
</tr>
<tr>
<td>HP8 02/04/01</td>
<td>01/05/01</td>
<td>AOIF, BOV</td>
<td>AOIF, BOV</td>
<td>AOIF, BOV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GARCH (Sep)</td>
<td>GARCH (Sep)</td>
<td>OLS (Sep)</td>
</tr>
<tr>
<td>HP9 02/07/01</td>
<td>27/07/01</td>
<td>AOIF, NIKK</td>
<td>AOIF, NIKK</td>
<td>AOIF, BOV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
<td>GARCH (Cmb)</td>
</tr>
</tbody>
</table>

Freq(GARCH preferred to OLS) 8 8 6
Freq(Cmb preferred to Sep) 6 6 5

Expected utility is calculated using equation (6.5), where \( \lambda = 3 \). “85 AOI, 15 TWF” indicates an equity portfolio consisting of 85% Australian and 15% Templeton foreign equity, and similarly for the other asset allocations. “AOIF” indicates the Australian All Ordinaries Share Price Index futures contract. “SP” indicates the U.S. S&P 500 Share Price Index futures contract. “FTSE” indicates the U.K. FTSE 100 Index futures contract. “NIKK” indicates the Japanese Nikkei 225 Index futures (SIMEX) contract. “BOV” indicates the Brazilian Bovespa Index futures contract. “AOIF, SP” indicates a hedge involving AOI futures contracts and S&P 500 futures contracts simultaneously, and similarly for the other futures combinations. “HP1” indicates the first hedge period, “HP2” indicates the second hedge period, and so on.

“Cmb” indicates that hedge ratios are calculated using the entire equity portfolio as the dependent variable, referred to in this chapter as a “combined” hedge. “Sep” indicates that hedge ratios are calculated separately for distinct or “separate” hedges of the Australian and foreign equity components. Combined hedges are those which account for the Australian and foreign components in a single model, while separate hedges address the domestic equity risk and foreign equity risk in two distinct models. Separate OLS hedges are calculated using equation (6.1). The combined OLS model is given in equation (8.6). Separate and combined GARCH hedges employ variations on the model in equations (6.2), (6.3) and (6.4), where \( n=2 \) and \( n=3 \) respectively.
Table 8.9: The hedge associated with the highest expected utility, where foreign equity is represented by the MSCI portfolio and the risk aversion parameter is three

<table>
<thead>
<tr>
<th>Hedge period</th>
<th>Hedge period Start date</th>
<th>EOC</th>
<th>85 AOI, 15 MSCI</th>
<th>70 AOI, 30 MSCI</th>
<th>55 AOI, 45 MSCI</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP1</td>
<td>02/01/92</td>
<td></td>
<td>AOIF, NIKK GARCH (Cmb)</td>
<td>AOIF, NIKK GARCH (Cmb)</td>
<td>AOIF, NIKK GARCH (Cmb)</td>
</tr>
<tr>
<td>HP2</td>
<td>01/07/92</td>
<td></td>
<td>AOIF, FTSE GARCH (Cmb)</td>
<td>AOIF, NIKK GARCH (Cmb)</td>
<td>AOIF, FTSE GARCH (Cmb)</td>
</tr>
<tr>
<td>HP3</td>
<td>03/01/95</td>
<td></td>
<td>AOIF, NIKK GARCH (Cmb)</td>
<td>AOIF, BOV GARCH (Cmb)</td>
<td>AOIF, BOV GARCH (Cmb)</td>
</tr>
<tr>
<td>HP4</td>
<td>03/07/95</td>
<td></td>
<td>AOIF, SP GARCH (Cmb)</td>
<td>AOIF, SP OLS (Sep)</td>
<td>AOIF, SP OLS (Sep)</td>
</tr>
<tr>
<td>HP5</td>
<td>02/01/98</td>
<td></td>
<td>AOIF, BOV GARCH (Cmb)</td>
<td>AOIF, BOV GARCH (Cmb)</td>
<td>AOIF, SP GARCH (Cmb)</td>
</tr>
<tr>
<td>HP6</td>
<td>01/07/98</td>
<td></td>
<td>AOIF, FTSE GARCH (Cmb)</td>
<td>AOIF, FTSE GARCH (Cmb)</td>
<td>AOIF, FTSE GARCH (Cmb)</td>
</tr>
<tr>
<td>HP7</td>
<td>02/01/01</td>
<td></td>
<td>AOIF, NIKK GARCH (Sep)</td>
<td>AOIF, NIKK GARCH (Sep)</td>
<td>AOIF, NIKK GARCH (Sep)</td>
</tr>
<tr>
<td>HP8</td>
<td>02/04/01</td>
<td></td>
<td>AOIF, BOV GARCH (Sep)</td>
<td>AOIF, BOV OLS (Sep)</td>
<td>AOIF, BOV OLS (Sep)</td>
</tr>
<tr>
<td>HP9</td>
<td>02/07/01</td>
<td></td>
<td>AOIF, NIKK GARCH (Cmb)</td>
<td>AOIF, BOV GARCH (Cmb)</td>
<td>AOIF, NIKK GARCH (Cmb)</td>
</tr>
</tbody>
</table>

Freq(GARCH preferred to OLS) 9 7 7
Freq(Cmb preferred to Sep) 7 6 5

Expected utility is calculated using equation (6.5), where \(\lambda = 3\). "85 AOI, 15 MSCI" indicates an equity portfolio consisting of 85% Australian and 15% MSCI foreign equity, and similarly for the other asset allocations. "AOIF" indicates the Australian All Ordinaries Share Price Index futures contract. "SP" indicates the U.S. S&P 500 Share Price Index futures contract. "FTSE" indicates the U.K. FTSE 100 Index futures contract. "NIKK" indicates the Japanese Nikkei 225 Index futures (SIMEX) contract. "BOV" indicates the Brazilian Bovespa Index futures contract. "AOIF, SP" indicates a hedge involving AOIF futures contracts and S&P 500 futures contracts simultaneously, and similarly for the other futures combinations. "HP1" indicates the first hedge period, "HP2" indicates the second hedge period, and so on.

"Cmb" indicates that hedge ratios are calculated using the entire equity portfolio as the dependent variable, referred to in this chapter as a "combined" hedge. "Sep" indicates that hedge ratios are calculated separately for distinct or "separate" hedges of the Australian and foreign equity components. Combined hedges are those which account for the Australian and foreign components in a single model, while separate hedges address the domestic equity risk and foreign equity risk in two distinct models. Separate OLS hedges are calculated using equation (6.1). The combined OLS model is given in equation (8.6). Separate and combined GARCH hedges employ variations on the model in equations (6.2), (6.3) and (6.4), where \(n = 2\) and \(n = 3\) respectively.

Similarly, combined models of hedge ratio estimation are preferred to separate models in the majority of hedge periods, a result robust to the division between Australian and foreign equity and to the foreign equity portfolio construction. The final row of each Table shows the number of hedge periods where combined models are superior to separate models on the basis of expected utility. As the proportion of foreign equity increases, the number of periods where combined models are preferred decreases, although combined models are still preferred in the majority of hedge periods.

By comparing the results within a particular column, it is clear that the favoured futures contract combinations vary over time. However, the preferred contract combinations are
similar across different portfolio constructions, and also across TWF and MSCI. This is expected as conditions change in different equity markets over time.

The results indicate that the U.S. S&P 500 futures hedge is rarely preferred. This finding is particularly important because the foreign equity portfolios of Australian managed funds generally contain relatively large proportions of U.S. equity, and managers may assume that U.S. futures contracts will provide the most effective hedge of the foreign portfolio. Hence, fund managers should exercise caution when selecting the most appropriate futures contracts to hedge their portfolios, because the proportion of investment in each asset does not necessarily indicate the best futures contract when this is considered in isolation from other factors that influence hedge performance.

Two factors that play a key role in the estimation of hedge ratios are the correlations between returns on hedged portfolio components, and the volatility of returns on those components. Specifically, the hedge ratio is the ratio of the correlation between the spot and futures contract returns to the volatility of the futures contract returns. For many-asset, many-futures portfolios, the analysis becomes more complex due to the interrelations between a larger number of portfolio components. Whether the behaviour of the correlation and volatility of returns provides simple explanations for the patterns of hedge performance reported in this section is examined in the final two sections of this chapter.

8.5.3  Correlation and hedge performance

Correlation is a linear measure of association between two variables, in this case returns on financial instruments. By definition, correlation ranges between \(-1\) and \(+1\), where the former indicates the strongest negative association and the latter indicates the strongest positive association. Two aspects of correlation are important in the following analysis: the sign, which shows the direction of the relationship between the return series, and the magnitude, which indicates the strength of the relationship between the return series. The sign and strength of correlations between returns on equity and futures contracts are already accounted for explicitly in the hedge ratios estimated in previous sections. However, two sets of potentially relevant correlations that may not be explicitly accounted for in the hedge ratio are those between returns on the Australian and foreign equity components that constitute the unhedged portfolio, and those between returns on Australian and foreign futures contracts. This section examines the impact of the sign and strength of these groups of correlations on the preference for a particular hedge ratio estimation method in each hedge period.
Figure 8.1 presents correlations between domestic and foreign equity, and between AOI futures contracts and foreign futures contracts, in each hedge period. Recall that, as discussed in Chapter 5, all returns and correlation calculations are denominated in Australian dollars, to eliminate foreign exchange risk. Nine discrete sets of results are presented, in chronological order from left to right, where “HP1” indicates the first hedge period and “HP9” is the ninth hedge period. The dates corresponding with these hedge periods match those in Tables 8.8 and 8.9.

Figure 8.1 shows that the signs and magnitudes of the correlations between different hedged portfolio components vary between time periods. For instance, in the fifth and seventh hedge periods correlations are relatively strong, but in the ninth period they are relatively weak. The correlations tend to be more strongly positive than strongly negative. The correlations between domestic and foreign equity almost always have the same sign and similar magnitude when foreign equity is represented using Templeton and MSCI portfolios. The relationship between domestic and foreign futures contracts fluctuates over time, as expected given changing market conditions.

When correlations are positive, the actions of the returns on the two variables reinforce each other, and when correlations are negative, the actions of the two variables offset each other. Hedge ratios estimation methods explicitly account for interactions between returns on equity and futures contracts but do not explicitly account for interactions between other components of multi-asset hedged portfolios. However, hedge performance may be affected by these additional interactions. As such, the findings in Tables 8.8 and 8.9 on the performance of different hedge strategies are compared with the behaviour of the correlations in Figure 8.1 to determine whether there is any obvious link between the two sets of results.

Focusing on the preference for separate or combined hedge ratio estimation, there is no obvious connection between either the sign or magnitude of the correlation between domestic and foreign equity and the performance of combined hedges relative to separate hedges. For instance, in the first and second hedge periods, the correlations between AOI and TWF are negative and positive respectively, but in both periods combined method is preferred. Also, the fifth and seventh hedge periods both show strong correlations between AOI and the foreign equity, but the combined method is preferred in the fifth period and the separate method is preferred in the seventh. Similarly, there is no obvious relationship connecting the correlations between domestic and foreign futures returns and the preference for combined or separate hedge ratio estimation. Further, when the choice of futures contract...
combination is examined in the light of Figure 8.1, there is no obvious connection between the magnitude of the correlations between domestic and foreign futures contracts and the preferred futures combination. Generally, there is a positive association between the preferred foreign futures contract and the AOIF. In short, the correlations between components of the hedge portfolio that are not taken into account explicitly in the hedge ratio estimation method do not appear to explain the performance of different hedging strategies.

Figure 8.1: Correlations between different components of hedged portfolios, in nine separate hedge periods between 1992 and 2001

"AOI" is Australian equity, as represented by the Australian All Ordinaries Index. "TWF" is the foreign equity portfolio based on Templeton World Fund asset allocations. "MSCI" is the foreign equity portfolio represented by the MSCI World Excluding Australia Index. "AOIF" indicates the Australian All Ordinaries Share Price Index futures contract. "SP" indicates the U.S. S&P 500 Share Price Index futures contract. "FTSE" indicates the U.K. FTSE 100 Index futures contract. "NIKK" indicates the Japanese Nikkei 225 Index futures (SIMEX) contract. "BOV" indicates the Brazilian Bovespa Index futures contract. When calculating correlations, all returns and calculations are denominated in Australian dollars. "HP1" indicates the first hedge period, "HP2" indicates the second hedge period, and so on. The dates defining each hedge period are listed in Table 8.9, with their corresponding acronyms.

Despite the popularity of correlation as a measure of co-movement, it is subject to a number of important criticisms. For instance, Embrechts et al (2000) argue that correlation does not provide full information on the dependence structure of risks, as it fails to account for the non-linear dependence relationships present between many real-world risk factors. Further, a correlation of zero does not necessarily indicate independence of risks. Perfectly positively dependent risks do not necessarily have a correlation of 1, and negatively dependent risks don’t necessarily have a correlation of -1. Hence, although a linear measure of association such as correlation may provide a useful starting point for hedge ratio estimation, it may not
fully capture the complex relationships between returns on spot and futures assets. This may provide an explanation for the results in this section.

8.5.4 Volatility and hedge performance

Hedging is arguably of greatest benefit in periods of high volatility in the spot market. Indeed, Chang et al (2000) present empirical evidence that the demand for hedging using stock index futures contracts by institutional hedgers increases in periods of relatively high volatility. In this section, the relationship between the volatility of the equity portfolios and the performance of hedge ratio estimation methods is examined.

Figures 8.2 and 8.3 depict the volatility, as measured by standard deviation of returns, of equity portfolios and their domestic and foreign components, where foreign equity is represented by Templeton and MSCI portfolios respectively. As in Figure 8.1, distinct sets of results are presented in chronological order of hedge periods (HP1 to HP9), where the dates are as defined in Table 8.8. Returns on the equity portfolios display the highest volatility in the fifth hedge period, regardless of the foreign equity portfolio construction, possibly because the estimation period includes the Asian currency crisis of 1997. This is true even

Figure 8.2: Volatility of returns on equity components and portfolios in nine hedge periods between 1992 and 2001, where volatility is measured as standard deviation of returns, and foreign equity is represented using the Templeton portfolio

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"AOI" is Australian equity, as represented by the Australian All Ordinaries Index. "TWF" is the foreign equity portfolio based on Templeton World Fund asset allocations. "85AOI, 15TWF" indicates an equity portfolio consisting of 85% Australian and 15% Templeton foreign equity, and similarly for the other asset allocations. All returns and calculations are denominated in Australian dollars. Volatility is measured as the standard deviation of returns in a given hedge period. "HP1" indicates the first hedge period, "HP2" indicates the second hedge period, and so on. The dates defining each hedge period are listed in Table 8.9, with their corresponding acronyms.
“AOI” is Australian equity, as represented by the Australian All Ordinaries Index. “MSCI” is the foreign equity portfolio represented by the MSCI World Excluding Australia Index. “85AOI, 15MSCI” indicates an equity portfolio consisting of 85% Australian and 15% MSCI foreign equity, and similarly for the other asset allocations. All returns and calculations are denominated in Australian dollars. Volatility is measured as the standard deviation of returns in a given hedge period. “HP1” indicates the first hedge period, “HP2” indicates the second hedge period, and so on. The dates defining each hedge period are listed in Table 8.9, with their corresponding acronyms.

Though the highest volatility of the individual foreign equity components occurs in the eighth period. In the fifth period, the high volatility of returns on the individual domestic and foreign equity components and the correlation between those series result in high portfolio volatility. In the eighth period, the high volatility of the foreign equity component is offset to some extent by the lower volatility of the Australian equity component, resulting in lower portfolio volatility relative to those in hedge period five.

The results in Tables 8.8 and 8.9 are analysed in conjunction with the results in Figures 8.2 and 8.3 to determine the relationship between the volatility of equity portfolio returns and hedge performance. The findings for portfolios containing Templeton foreign equity and MSCI foreign equity are similar. Specifically, there does not appear to be a simple relationship between the relative volatility of domestic and foreign equity components, and whether combined or separate hedge ratio estimation methods are preferred.

Similarly, there is no clear relationship between the relative amount of volatility of either the domestic or foreign equity component and the preference for separate or combined hedges.
Combined hedges are preferred in periods of low and high volatility. Finally, no obvious connection exists between equity portfolio volatility and whether OLS method or the GARCH method is preferred in a given hedge period. In short, the volatility of the diversified equity portfolio does not offer a simple explanation for the relative performance of different hedge strategies, which are likely driven by more than one factor. This is an area for future research.

8.6 Conclusion

This chapter extends the previous chapter by examining the hedging of equity risk given diversified portfolios containing Australian and foreign equity, using pairs of futures contracts. A key issue is whether there is any benefit in accounting for “portfolio effects”, which are the complex interactions between different components of portfolios containing multiple assets, when hedging such portfolios. This is investigated by comparing the overall performance of separate traditional hedges of Australian and foreign equity with the performance of “combined” hedges of the entire equity portfolio using two futures contracts, where the interactions between domestic and foreign equity are captured in a single return series before hedge ratios are estimated. The results are examined for sensitivity to the asset allocation between Australian and foreign equity, the construction of foreign equity portfolios, the hedge ratio estimation method, the measure of hedge performance, and the time period. This study can be distinguished from the wealth of prior empirical research comparing traditional hedge ratio models, in that the latter ignore portfolio effects, are generally limited to foreign (non-Australian) data, and are not conducted in the context of diversified fund portfolios. Further, this study can be differentiated from the work of researchers such as Gagnon et al (1998) on portfolio effects, given differences in data and the focus on different risks, and because the current study relies on out-of-sample measures of performance whereas prior work relies on in-sample analysis which unrealistically assumes perfect foresight by the practitioner.

An initial examination of the impact of portfolio effects in the context of hedging equity risk using the AOI futures contracts as the exclusive hedging vehicle is done in Chapter Seven, where it was found that accounting for portfolio effects (through “combined” hedges) is generally beneficial on the basis of expected utility. Extending that analysis to include a wider range of futures contracts, and as a preliminary investigation into the use of multiple futures to hedge risk in Chapter Nine, domestic and foreign futures contracts are used simultaneously as hedging vehicles. When hedge ratios are estimated using price changes OLS regressions, separate hedges generally outperform combined hedges in the majority of
hedge periods for all foreign futures contracts, except when Nikkei futures are applied to portfolios containing Templeton foreign equity. In contrast, when hedge ratios are estimated using GARCH(1,1) methods, preferences for separate or combined hedges appear to be portfolio-specific and not readily generalisable. However, when the performance of all hedge ratio estimation methods is compared for each futures contract combination, hedge ratios estimated using the trivariate GARCH(1,1) model generally perform as well as or better than other methods of hedge ratio estimation, suggesting that accounting for portfolio effects when calculating hedge ratios improves hedged portfolio performance.

When both the estimation method and the futures contract combination are allowed to vary, and analysis is conducted to determine the overall best hedge in each period, the best futures contract combinations change over time, but hedge ratios calculated using the combined method attract the highest expected utility in the majority of hedge periods. These results are robust to the relative proportions of domestic Australian and foreign equity in each equity portfolio, and to the construction of the foreign equity component. This suggests that accounting for portfolio effects through the hedge ratio estimation method is generally beneficial in terms of expected utility. Generally, managers may improve the performance of their hedges through the selection of combined methods of hedge ratio estimation, and in particular the trivariate GARCH(1,1) model. However, the trivariate GARCH model has considerably greater computational complexity than the other models considered, a factor that may deter practitioners.

Simple relationships linking the relative performance of combined and separate hedges with the correlation between domestic and foreign equity and between different futures contracts are not evident. This may be because correlation is a simple linear measure of co-dependence that fails to capture any non-linearity in the relationship between spot and futures returns. Similarly, periods of high volatility of equity portfolio returns are not clearly linked with a preference for combined or separate hedges. Connections between these factors and hedge performance are hypothesised because these factors are not accounted for explicitly in separate hedge ratio estimation methods. The findings suggests that the causes of hedge performance are complex, and highlights an avenue for further research. If managers could identify factors driving hedge performance, and develop ways to predict those factors, they could tailor the hedge ratio estimation method more accurately to the particular hedge period. Regardless of the factors driving hedge performance, this chapter shows that combined hedges perform better generally than separate hedges, indicating that accounting for portfolio effects is important and beneficial in the majority of hedge periods.

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This chapter has examined the hedging of diversified equity portfolios using pairs of futures contracts, which is the most simple case involving more than one hedging vehicle. In the following chapter, the hedging of diversified portfolios using single and multiple futures contracts is examined, to determine the relative benefits of hedging using increasingly large portfolios of futures contracts.
CHAPTER NINE

HEDGING INTERNATIONALLY DIVERSIFIED EQUITY PORTFOLIOS USING MULTIPLE FUTURES CONTRACTS: SINGLE VERSUS MULTIPLE FUTURES CONTRACTS

9.1 Introduction

This chapter extends previous chapters by examining the performance of portfolios of futures contracts used to hedge risk associated with diversified equity portfolios consisting of Australian and foreign investments. A key issue is the performance of simple hedges using single futures contracts relative to complex hedges using multiple futures contracts simultaneously. Transaction costs and ease of portfolio management may dictate that the number of futures contracts used to hedge an equity portfolio be limited. However, although such costs may cast doubt on the practicality of hedges using portfolios of futures contracts, some prior empirical research, such as work by Eaker and Grant (1987), DeMaskey (1997) and Miller (1985), indicates that hedges using multiple futures contracts out-perform hedges using single futures contracts. Using multiple futures may improve portfolio protection through greater “accuracy” or matching of spot and futures contracts. It may be technically possible to hedge all parts of a complex portfolio, but it is likely to be less expensive and more convenient to hedge using only one or a small number of instruments. A key contribution of this chapter is to address this issue in the context of hedging equity risk, in contrast to prior studies which examine foreign exchange risk or commodity risk.

Hedging investments with portfolios of futures contracts has been investigated empirically in the context of foreign exchange risk (Eaker and Grant (1987), Mun and Morgan (1997), Benet (1990), DeMaskey (1997)), commodity risk (Grant and Eaker (1989)), and the risk of fixed-rate mortgaged-backed securities (Koutmos and Pericli (2000)). An issue central to these studies is whether superior hedge effectiveness is obtained using a single type of futures contract or a portfolio of futures. There is little consensus in prior literature as to whether multiple futures hedges outperform single futures hedges. Empirical evidence favouring the use of single futures over portfolios of futures is provided by Grant and Eaker (1989) in the context of commodities, and Koutmos and Pericli (2000), in the context of interest rate risk. In contrast, empirical evidence supporting the use of portfolios of futures to hedge single risks is presented by Eaker and Grant (1987) and DeMaskey (1997) in the
context of foreign exchange risk, and Miller (1985) in the context of commodities. Miller (1985) and DeMaskey (1997) similarly find that multiple hedges are superior to single hedges in the contexts of commodities and European and Asian currency risks respectively. The lack of consensus is exemplified by the study of Mun and Morgan (1997) who, measuring performance using the Sharpe index, find that a portfolio of futures provides superior protection relative to single futures hedge for Indonesia, Singapore, and Thailand, while a single currency futures hedge is superior in the cases of Korea and Malaysia. In short, the results are country-specific, and not readily generalisable. This chapter contributes to the literature by considering these issues from an Australian perspective.

In the context of Australian equity risk, hedges using single futures contracts are examined in Chapters Six and Seven, and hedges involving pairs of futures contracts are investigated in Chapter Eight. This chapter extends these results, addressing three key issues. First, the relative benefit from the use of portfolios of futures contracts to hedge equity risk as opposed to individual contracts is examined. Second, the impact of the number of futures contracts included in futures portfolios on hedge ratio estimation model fit and hedge performance is investigated. Third, the performance of different combinations of futures contracts when hedging Australian and foreign equity risk using portfolios of futures contracts is examined.

This chapter is comprised of four sections. In the first, the research method and data are described. In the second section, results using hedge ratios based on multivariate price changes OLS regressions are analysed, with reference to changes in model fit as the number of futures contracts included in the model increases. The impact of multicollinearity on models involving portfolios of futures contracts is addressed. Third, results comparing hedges using multiple and single futures contracts are analysed. Finally, the results are summarised and conclusions are drawn.

9.2 Research Method and Data

All data are as described in detail in Chapter Five, and the construction of equity portfolios is also described in Chapter Five. Recall that the diversified equity portfolios consist of Australian and foreign components, where foreign equity is represented by the Templeton and MSCI foreign equity portfolios. The portfolios are constructed using different proportions of Australian and foreign equity, where the proportions are based on Australian managed fund practice as reported by Mercer Investment Consulting and discussed in detail in Chapter Five. Specifically, three portfolios are constructed such that the ratios of investment in Australian to foreign equity are 85:15, 70:30 and 55:45. Together, these
proportions represent the range of asset allocations for Australian capital stable, balanced and growth funds. The choice of the specific futures contracts included in this analysis is justified in detail in Section 4.3 of Chapter Four.

The methods used to estimate hedge ratios have been introduced and discussed at length in previous chapters, so their treatment in this section is brief. As in Chapter Eight, combined and separate hedge strategies are analysed. Hedge ratios for multiple futures contracts are estimated using multivariate OLS regressions. The model for combined hedges, where the return on the entire unhedged portfolio as the dependent variable, is:

\[ ALLEQ = \alpha + \beta_1 AOIF + \beta_2 SP + \beta_3 FTSE + \beta_4 NIKK + \beta_5 BOV + \varepsilon \]  

(9.1)

where \( ALLEQ \) = continuously compounded return on the entire unhedged portfolio

\( AOIF \) = continuously compounded return on AOI futures contract

\( SP \) = continuously compounded return on S&P 500 futures contract

\( FTSE \) = continuously compounded return on FTSE 100 futures contract

\( NIKK \) = continuously compounded return on Nikkei 225 futures contract

\( BOV \) = continuously compounded return on Bovespa futures contract

This model is an extension of the bivariate price-changes OLS regression model, given that the change in the natural logarithm of a price is equivalent to the continuously compounded return. Such OLS regression models are commonly employed to estimate hedge ratios in studies of hedging using multiple futures contracts by researchers including Eaker and Grant (1987), Grant and Eaker (1989), Benet (1990), Mun and Morgan (1997), DeMaskey (1997), Koutmos and Pericli (2000), and Clare et al (2000).

When separate hedges are used, the Australian equity is hedged using Australian AOI futures contracts as in Chapter Seven by applying regression model (9.2), and foreign equity is hedged using regression model (9.3):

\[ AUSEQ = \alpha + \beta_1 AOIF + \varepsilon \]  

(9.2)

\[ FOREQ = \alpha + \beta_1 SP + \beta_2 FTSE + \beta_3 NIKK + \beta_4 BOV + \varepsilon \]  

(9.3)

where \( AUSEQ \) is the continuously compounded return on the Australian component of the portfolio and \( FOREQ \) is the continuously compounded return on the foreign equity component of the portfolio.

For hedges involving fewer than the full complement of futures contracts, the models (9.1) and (9.3) are simply adjusted by removing the contracts not required in a particular test. For example, for a combined hedge using futures contracts written on the AOI, S&P 500 and
FTSE 100 indices, the restriction is made $\beta_2 = \beta_3 = 0$. As in all previous chapters, all returns are denominated in Australian dollars.

When hedging using more than one futures contract, multicollinearity may result. Strong correlation between the returns on the equity portfolio, the dependent variable, and returns on the futures contracts, the independent variables, is desirable because hedge effectiveness is likely to improve as the strength of association between these series increases. However, strong positive correlation between changes in the prices of different futures contracts is undesirable, because the independent variables "rob" each other of explanatory power.

Table 9.1 shows the correlations between changes in the natural logarithms of the prices of the futures contracts over the full sample period. As alluded to briefly in Chapter Five, the correlation between the futures contracts is generally low, with the exception of the association between the S&P 500 futures contract and the FTSE 100 futures contract. Whether this particular relationship is sufficient to cause problematic multicollinearity is a question of degree. Australian futures contracts form part of every hedge strategy examined, and the correlations between Australian and foreign futures returns are low, ranging between 0.26 and 0.32. With the exception of the relationship between the S&P 500 and FTSE 100 futures contracts, correlations between returns on foreign futures contracts are generally low, ranging between 0.13 and 0.37. Of all the foreign contracts, the Brazilian Bovespa generally has the lowest correlation with other futures contracts, which is expected given that it is the only contract written on the equity index of a developing market. In short, the relationships between the various futures contracts are diverse enough to form portfolios of futures contracts with a reasonable expectation that multicollinearity will not seriously affect results.

More detailed analysis is undertaken in Section 9.3 to investigate this issue.

Table 9.1: Correlations between the changes in the natural logarithms of the prices of various futures contracts, for the period 05/01/1989 to 26/07/2001

<table>
<thead>
<tr>
<th></th>
<th>AOI Futures</th>
<th>S&amp;P 500 futures</th>
<th>FTSE 100 futures</th>
<th>Nikkei futures</th>
<th>Bovespa futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOI Futures</td>
<td>1.00</td>
<td>0.29</td>
<td>0.32</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>S&amp;P 500 futures</td>
<td>0.29</td>
<td>1.00</td>
<td>0.37</td>
<td>0.37</td>
<td>0.13</td>
</tr>
<tr>
<td>FTSE 100 futures</td>
<td>0.32</td>
<td>0.63</td>
<td>1.00</td>
<td>0.37</td>
<td>1.00</td>
</tr>
<tr>
<td>Nikkei futures</td>
<td>0.26</td>
<td>0.37</td>
<td>0.37</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Bovespa futures</td>
<td>0.26</td>
<td>0.32</td>
<td>0.26</td>
<td>0.13</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Correlation is calculated as the covariance between the asset returns divided by the product of the standard deviations of each asset return series. All returns used in the calculations are denominated in Australian dollars. Correlations involving the Bovespa futures contracts are calculated over the period 26/07/1990 to 26/07/2001.

The performance of hedged portfolios is assessed using expected utility, $E(U)$, which is given in equation (6.5). Further, the sensitivity to the performance measure is determined by comparing the results based on expected utility with those based on the expected return of
the hedged portfolio standardised by the risk of that portfolio, where risk is measured by
standard deviation.

9.3 Hedge Ratios and Model Fit

As a prelude to the examination of the performance of different hedging strategies in the
following section, this section analyses the behaviour of the price-changes OLS regression
hedge ratios. General model fit is illustrated with reference to R-squared statistics, and the
specific combination of futures contracts included in each variation of the model is analysed
using the statistical significance of each beta coefficient.

9.3.1 General Model Fit

The R-squared statistic is both a goodness-of-fit measure in the context of OLS regressions
and Ederington’s (1979) measure of hedge effectiveness. Figures 9.1 and 9.2 depict the R-
squared statistics for “combined” hedges involving a variety of futures contracts, where the
dependent variable is the return on a portfolio consisting of 70% Australian equity and 30%
foreign equity, and where foreign equity is represented using the Templeton and MSCI
portfolios respectively. These results are based on variations on the model in equation (9.1).
Similarly, Figures 9.3 and 9.4 show R-squared statistics for the foreign component of
“separate” hedges of Australian and foreign equity, based on the model in equation (9.3).
The different futures combinations are all indicated, and the R-squared statistics for each of
nine estimation periods between 1992 and 2001 are shown. In each of these figures, “A”,
“S”, “F”, “N”, and “B” indicate futures contracts written on the AOI, S&P 500, FTSE 100,
Nikkei 225, and Bovespa indices respectively. Where more than one futures contract is used
in the hedge, each symbol is separated by a comma. For example, “A,S,F,N” indicates a
hedge simultaneously employing futures contracts written on the AOI, S&P 500, FTSE 100
and Nikkei 225 indices.

Figures 9.1 and 9.2 indicate that the general levels of R-squared statistics are high, around
0.9, suggesting good model fit when the dependent variable is the return on the entire
unhedged portfolio. This does not alter greatly as the number of futures contracts increases.
Further, there is little difference between the goodness of fit of the hedge ratio estimation
model as the combinations of futures contracts are altered. The combinations of AOI futures
contracts and Bovespa futures contracts, and AOI futures contracts and Nikkei futures
contracts generally attract the lowest R-squared statistics. The earlier and later hedge periods

35 Consistent with previous chapters, the estimation periods are: 05/01/89 to 26/12/91, 06/07/89 to 25/06/92,
09/01/92 to 29/12/94, 09/07/92 to 29/06/95, 05/01/95 to 25/12/97, 06/07/95 to 25/06/98, 08/01/98 to 28/12/00,
09/04/98 to 29/03/01, and 09/07/98 to 28/06/01.
are generally associated with lower R-squared statistics relative to the middle periods. This suggests that model fit is generally better in the middle time periods, although the improvement for some contract combinations is marginal. These findings are consistent for both the MSCI and TWF foreign equity representations.

In contrast, in “separate” hedges of domestic and foreign equity, Australian equity is hedged using AOM futures contracts as in equation (9.2), and the foreign equity component is hedged using foreign futures contracts as in equation (9.3). The R-squared statistics on the Australian component in each estimation period range between 0.9558 and 0.8607, indicating good model fit, as expected in direct hedging. Regarding foreign equity hedges, Figures 9.3 and 9.4 depict R-squared statistics for hedges of the foreign component of the equity portfolio. These Figures show that for both Templeton and MSCI foreign equity portfolio hedges, the R-squared statistic varies considerably. For example, when foreign equity is hedged separately using S&P 500 and Nikkei futures contracts, the R-squared statistics for the foreign equity hedge vary within the approximate range of 0.6 to 0.9 as the estimation period changes. Recall that the results in Figures 9.3 and 9.4 relate exclusively to the foreign hedge, and that Figures 9.1 and 9.2 relate to combined hedges involving both Australian and foreign equity simultaneously. A comparison of Figures 9.1 and 9.3, and Figures 9.2 and 9.4 suggests that the inclusion of the Australian equity component in the dependent variable of the “combined” hedge stabilises the results over time for combined hedges relative to separate hedges. This is expected because Australian equity constitutes the majority of the entire portfolio, and the R-squared statistics for Australian hedges are relatively stable and high. In contrast, the separate foreign results are more varied, depending on the combination of futures contracts.
Figure 9.1: R-squared statistics for OLS regressions where the dependent variable is the return on the portfolio of 70% Australian equity, 30% TWF foreign equity, and independent variables are returns on various futures contracts.

Futures contracts
“A”, “S”, “F”, “N”, and “B” indicate the AOI futures, S&P 500 futures, FTSE 100 futures, Nikkei 225 futures, and Bovespa futures respectively. Hedge ratios are estimated using variations of the OLS regression model in equation (9.1). Nine estimation periods between 1992 and 2001 are used to estimate hedge ratios. The end date of each estimation period, provided in the legend, occurs one business day prior to the commencement of the hedge period. The full estimation periods are: 05/01/89 to 26/12/91, 06/07/89 to 25/06/92, 09/01/92 to 29/12/94, 09/07/92 to 29/06/95, 05/01/95 to 25/12/97, 06/07/95 to 25/06/98, 08/01/98 to 28/12/00, 09/04/98 to 29/03/01, and 09/07/98 to 28/06/01.

Figure 9.2: R-squared statistics for OLS regressions where the dependent variable is the return on the portfolio of 70% Australian equity, 30% MSCI foreign equity, and independent variables are returns on various futures contracts.

Futures contracts
“A”, “S”, “F”, “N”, and “B” indicate the AOI futures, S&P 500 futures, FTSE 100 futures, Nikkei 225 futures, and Bovespa futures respectively. Hedge ratios are estimated using variations of the OLS regression model in equation (9.1). The end date of each estimation period, provided in the legend, occurs one business day prior to the commencement of the hedge period. The full estimation periods are as defined in Figure 9.1.
Figure 9.3 shows that when using a single futures contract to hedge the Templeton foreign equity portfolio, the model using the S&P 500 futures contract generally attracts the highest R-squared, while those for models using the Nikkei and Bovespa futures are relatively low. When two futures contracts are used, the model combining the S&P 500 and FTSE 100 futures contracts generally has the best fit. The interaction between these particular contracts is investigated in the following subsection, as their returns also have the highest correlation with each other. When using three futures contracts, the models containing the S&P 500 futures contracts fit well, but the model combining the FTSE, Nikkei and Bovespa futures contracts has the lowest R-squared. When employing hedges of four futures contracts, the result is very similar to that for three futures contracts. Figure 9.4 shows similar results for MSCI foreign portfolios. For both foreign equity portfolio constructions, there is considerable variation in R-squared statistics between estimation periods when each futures combination is considered in isolation.

In short, for combined hedges where more than one futures contract is used, model fit as measured by the R-squared statistic, is generally good and does not appear to alter greatly as the number of futures contracts used to hedge the equity portfolio increases. For separate hedges, the hedged Australian component has consistently high R-squared statistics, indicating good model fit, but the foreign hedged component exhibits greater variation in model fit which is sensitive to both the estimation period and combinations of futures contracts.

While this analysis indicates performance, in the sense that the R-squared statistic is also Ederington’s (1979) measure of hedge effectiveness, that measure focuses on risk to the exclusion of expected returns. As observed in Chapter Three, managers are motivated by both risk and return according to the theoretical framework on which this work is based. As such, this section is intended as an indication of model fit, while hedged portfolio performance is assessed in Section 9.4 using expected utility.
Figure 9.3: R-squared statistics for OLS regressions where the dependent variable is the returns on the TWF foreign equity portfolio, and the independent variables are returns on various futures contracts

![Graph](image)

Futures contracts

"A", "S", "F", "N", and "B" indicate the AOI futures, S&P 500 futures, FTSE 100 futures, Nikkei 225 futures, and Bovespa futures respectively. Hedge ratios are estimated using variations of the OLS regression model in equation (9.3). Nine estimation periods between 1992 and 2001 are used to estimate hedge ratios. The end date of each estimation period, provided in the legend, occurs one business day prior to the commencement of the hedge period. The full estimation periods are: 05/01/89 to 26/12/91, 06/07/89 to 25/06/92, 09/01/92 to 29/12/94, 09/07/92 to 29/06/95, 05/01/95 to 25/12/97, 06/07/95 to 25/06/98, 08/01/98 to 28/12/00, 09/04/98 to 29/03/01, and 09/07/98 to 28/06/01.

Figure 9.4: R-squared statistics for OLS regressions where the dependent variable is the returns on the MSCI foreign equity portfolio, and the independent variables are returns on various futures contracts

![Graph](image)

Futures contracts

"A", "S", "F", "N", and "B" indicate the AOI futures, S&P 500 futures, FTSE 100 futures, Nikkei 225 futures, and Bovespa futures respectively. Hedge ratios are estimated using variations of the OLS regression model in equation (9.3). The end date of each estimation period, provided in the legend, occurs one business day prior to the commencement of the hedge period. The full estimation periods are: 05/01/89 to 26/12/91, 06/07/89 to 25/06/92, 09/01/92 to 29/12/94, 09/07/92 to 29/06/95, 05/01/95 to 25/12/97, 06/07/95 to 25/06/98, 08/01/98 to 28/12/00, 09/04/98 to 29/03/01, and 09/07/98 to 28/06/01.
9.3.2 Multicollinearity

Moving from an examination of general model fit in the previous section, this section analyses the impact of multicollinearity between the returns on futures contracts on the performance of the hedge ratio estimation model. This section has two parts: a general discussion of the p-values associated with the hedge ratios in each regression, and specific identification of which futures hedge ratios are significant and which are not.

Overview of statistical significance of hedge ratios

One symptom of multicollinearity is the presence of unusually high standard errors of individual slope estimators, which makes the beta coefficients, or hedge ratios, appear to be statistically insignificant. Hence, the p-values associated with each hedge ratio are examined, and the information summarised in Figures 9.5 and 9.6, which indicate the percentage of hedge ratio estimation periods in which all the futures variables included in the OLS regression are statistically significant at the 10% significance level, where foreign equity is represented using the Templeton and MSCI portfolios respectively. These graphs refer to “combined” hedges. Analogous results for “separate” hedges are provided in Figure 9.7.

In the legend for Figures 9.5 and 9.6, “85A,15TWF” indicates an equity portfolio where 85% is Australian equity and 15% is Templeton foreign equity, and similarly for the other portfolios. To illustrate the interpretation of these figures, consider Figure 9.5. For portfolios containing 85% Australian and 15% foreign equity that are hedged using a portfolio of AOI futures contracts and FTSE 100 futures contracts, both $\beta_1$ and $\beta_3$ from regression (9.1) are significant at the 10% level in all estimation periods. In contrast, when that equity portfolio is hedged using a portfolio of five futures contracts, one or more of $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$, or $\beta_5$ are not statistically significant at the 10% level in just under 80% of estimation periods.

Figure 9.5 shows that in general, as the number of futures contracts included in the regression increases, the percentage of estimation periods in which all futures hedge ratios are significant decreases. For instance, when portfolios of two futures contracts (A,S and A,F, and generally A,N) are used to hedge various equity portfolios, each futures hedge ratio is significant in all nine estimation periods between 1992 and 2001. In contrast, for portfolios of five futures contracts, all futures hedge ratios are significant in less than 60% of estimation periods depending on the equity portfolio. However, for hedges using two futures contracts, and for all but one case using three futures contracts, the number of estimation periods where all hedge ratios are statistically significant exceeds the 50% level. The particular futures contracts associated with non significant hedge ratios in each hedging scenario are identified in detail later.
The results in Figure 9.5 are influenced by the allocation between Australian and foreign equity in the equity portfolio, the dependent variable. As the proportion of investment in foreign equity increases, the percentage of estimation periods in which all futures hedge ratios are significant at the 10% level remains equal or increases. This pattern is consistent for each of the futures contract combinations. Further, this relationship becomes more pronounced as the number of futures contracts included in the hedge increases. For instance, for portfolios containing 30% and 45% foreign equity, the results are the same for all two-futures hedges, the same for three of three-futures hedges, and always different for all four-futures and five-futures hedges with portfolios containing 45% foreign equity associated with more periods where all hedge ratios are significant.

Figure 9.6 shows analogous results for spot portfolios containing MSCI foreign equity. Like the results involving Templeton foreign equity, there is a negative relationship between the number of futures contracts included in the OLS regression, and the percentage of estimation periods in which all futures hedge ratios are significant at the 10% level. This indicates that in each estimation period, at least one hedge ratio is not significant. The strongest result occurs for portfolios of two futures contracts, where all hedge ratios are significant in 100% of estimation periods for all futures combinations except the AOI and Bovespa futures contracts, and the weakest result is associated with portfolios consisting of all five futures contracts. However, while evident, the result is less consistent when hedging MSCI portfolios, and extreme results occur, such as two cases (A,S,F,B and A,S,F,N,B) where the number of periods where all hedge ratios are significant is zero.

Similarly, the relationship between the allocation between Australian and foreign equity in the spot portfolio and the percentage of estimation periods in which the hedge ratios on all futures contracts are significant is less consistent when foreign equity is represented using the MSCI portfolio rather than the Templeton portfolio. The positive relationship between the proportion of foreign equity and the percentage of periods is evident for all contract combinations except the A,F,B and A,S,N,B. The range between the results for portfolios of 85% and 55% Australian equity are also noticeably larger for portfolios containing MSCI foreign equity. For example, when portfolios of Australian and MSCI foreign equity are hedged using portfolios of futures contracts written on the AOI, S&P 500, FTSE 100, and Nikkei 225 indices (A,S,F,N), the percentage of periods in which all hedge ratios are significant ranges from 33% to 100% for equity portfolios containing MSCI compared to 33% to 78% for Templeton portfolios. Combined hedges of portfolios containing MSCI foreign equity have more cases where larger numbers of futures combinations have all hedge
ratios significant in 100% of estimation periods, particularly for 30% and 45% foreign equity.

The positive correlation between returns on the S&P 500 and FTSE 100 futures contracts flagged in Section 9.2 does not appear to cause problematic multicollinearity. Figures 9.5 and 9.6 show that for portfolios containing 30% and 45% foreign equity, all hedge ratios are significant at the 10% level in all estimation periods for futures portfolios containing AOI, S&P 500 and FTSE 100 futures contracts. Results are weaker for portfolios containing 15% foreign equity, especially in the case of MSCI foreign equity, where at least one ratio is not significant in more than 50% of hedge periods. Later in this section, the specific futures contracts associated with non significant hedge ratios are identified.

Regarding the relative benefits of particular combinations of futures contracts, the best model fit based on the significance of all hedge ratios favours hedges using the A,S and A,F futures combinations. This is robust to estimation period, foreign equity construction, and the division between Australian and foreign equity. When the performance of different single futures hedges of foreign equity was examined in Chapter Six in terms of expected utility, the FTSE 100 and Nikkei 225 futures contracts generally provide the best hedges of the foreign equity portfolios, depending on the hedge ratio estimation method. In Chapter Seven, hedges of Australian equity were conducted using AOI futures contracts, because direct hedging is superior to cross-hedging when the former is available (Eaker and Grant (1987)). It follows intuitively in the case of two-futures hedges in the current chapter that using a combination of AOI and FTSE futures contracts provides among the best model fits when hedging both Australian and foreign equity together. That the combination of AOI and S&P 500 futures contracts is also identified as having good model fit in this section, while the results in Chapters Six and Seven suggest that the AOI and Nikkei futures contracts would provide a relatively good hedge combination is attributable to the differences in performance measurement. In this section, in-sample performance is examined through R-squared statistics and significance of hedge ratios, but in previous chapters and later in the current chapter, out-of-sample hedge performance is measured using expected utility.
Figure 9.5: Percentage of estimation periods in which the hedge ratios of all futures contracts in the OLS regression are statistically significant at the 10% level, where equity portfolios consist of Australian and Templeton foreign equity.

Futures contracts in OLS regression

“A”, “S”, “F”, “N”, and “B” indicate the AOI futures, S&P 500 futures, FTSE 100 futures, Nikkei 225 futures, and Bovespa futures respectively. In the legend, “85A,15TWF” indicates an equity portfolio where 85% is Australian equity and 15% is TWF foreign equity, and similarly for the other portfolios. Hedge ratios are calculated in nine estimation periods: 05/01/89 to 26/12/91, 06/07/89 to 25/06/92, 09/01/92 to 29/12/94, 09/07/92 to 29/06/95, 05/01/95 to 25/12/97, 06/07/95 to 25/06/98, 08/01/98 to 28/12/00, 09/04/98 to 29/03/01, and 09/07/98 to 28/06/01. Hedge ratios are estimated using variations on the OLS regression (9.1).

Figure 9.6: Percentage of estimation periods in which the hedge ratios of all futures contracts in the OLS regression are statistically significant at the 10% level, where equity portfolios consist of Australian and MSCI foreign equity.

Futures contracts in OLS regression

“A”, “S”, “F”, “N”, and “B” indicate the AOI futures, S&P 500 futures, FTSE 100 futures, Nikkei 225 futures, and Bovespa futures respectively. In the legend, “85A,15MSCI” indicates an equity portfolio where 85% is Australian equity and 15% is MSCI foreign equity, and similarly for the other portfolios. Hedge ratios are calculated in nine estimation periods: 05/01/89 to 26/12/91, 06/07/89 to 25/06/92, 09/01/92 to 29/12/94, 09/07/92 to 29/06/95, 05/01/95 to 25/12/97, 06/07/95 to 25/06/98, 08/01/98 to 28/12/00, 09/04/98 to 29/03/01, and 09/07/98 to 28/06/01. Hedge ratios are estimated using variations on the OLS regression (9.1).
Results discussed previously in this section relate to combined hedges of the entire equity portfolio using single OLS regressions. Figure 9.7 shows the results when Australian and foreign equity are hedged separately using two distinct regressions, namely equation (9.3) for the foreign component, and equation (9.2) for the Australian component. For instance, when hedging Australian equity with the AOI futures contract, and Templeton foreign equity with futures contracts written on the S&P 500, FTSE 100 and Nikkei 225 indices, (the A,S,F,N strategy), all hedge ratios in both regressions are significant in approximately 80% of estimation periods. Generally, the AOI futures hedge ratios are always significant at the 10% level, so non significance of any hedge ratios in a given period is attributable exclusively to the foreign futures hedges in regression (9.3). However, both Australian and foreign hedge results are included in Figure 9.7 for comparison with the combined hedges in Figures 9.5 and 9.6.

Figure 9.7 shows that all hedge ratios associated with all futures in the two-contract hedges are always significant at the 10% level, regardless of foreign equity portfolio construction. Four of six three-futures scenarios are also all significant. The Templeton hedge ratios are all significant in more than 50% of estimation periods for all futures combinations. The weakest results in terms of significance are from the strategy using all five futures contracts. In comparison, the MSCI hedge ratios are all significant in more than 50% of periods for all but four futures combinations, each involving more than two contracts.

Comparing Figures 9.5 and 9.6 with Figure 9.7, separate and combined hedge strategies both result in the statistical significance of all hedge ratios in portfolios of two futures contracts, and fewer cases where all hedge ratios are significant in portfolios of larger number of futures contracts. Generally, separate hedges result in hedge ratios that are all significant in 100% of estimation periods for a larger number of futures contract combinations relative to combined hedges. This may be due to the fact that for separate hedges, one fewer contract is included in regression (9.3), as the Australian component is hedged separately using regression (9.2). Thus, from a statistical point of view, for separate hedges, there is less scope for independent variables to rob each other of explanatory power because there is one fewer contract in the foreign component regression and the Australian component is treated in a separate regression. However, this statistical observation on model fit does not alter the fact that when both Australian and foreign futures hedges are applied simultaneously, they will both impact on overall out-of-sample hedge performance, which is a separate issue dealt with in Section 9.4 in terms of expected utility. Specifically, the comments in this paragraph do not diminish the importance of the Australian futures contract, which can be expected to
play a dominant role in hedge performance given the large proportion of Australian equity relative to foreign equity in the unhedged portfolio.

Thus far, the focus is on in-sample measures of model fit including Ederington's (1979) R-squared measure and the p-values of each hedge ratio. Comparing these sets of results for combined hedges, it is evident that while R-squared statistics generally remain high regardless of the number of futures contracts, the percentage of hedge periods for which all hedge ratios are statistically significant tends to decrease as the number of futures contracts included in the hedge strategy increases. For separate hedges, the Australian component generally exhibits high R-squared statistics and consistently significant hedge ratios, as expected in direct hedging. The foreign hedge component shows greater variability of R-squared statistics, depending on the particular futures contract combination. While the general trend for the percentage of hedge periods in which all hedge ratios are significant to decrease as the number of futures contracts increases is also evident for the separate foreign hedge, exact results are influenced by the futures contract combinations.

Figure 9.7: Percentage of estimation periods in which the hedge ratios of all futures included in the OLS regression are statistically significant at the 10% level, where Australian and foreign equity are hedged separately

Futures contracts in OLS regression

“A”, “S”, “F”, “N”, and “B” indicate the AOI futures, S&P 500 futures, FTSE 100 futures, Nikkei 225 futures, and Bovespa futures respectively. In the legend, “AOI,TWF” indicates equity portfolios consisting of Australian and Templeton foreign equity. “AOI,MSCI” indicates equity portfolios consisting of Australian and MSCI foreign equity. Hedge ratios are calculated in nine estimation periods: 05/01/89 to 26/12/91, 06/07/89 to 25/06/92, 09/01/92 to 29/12/94, 09/07/92 to 29/06/95, 05/01/95 to 25/12/97, 06/07/95 to 25/06/98, 08/01/98 to 28/12/00, 09/04/98 to 29/03/01, and 09/07/98 to 28/06/01. Hedge ratios for the Australian and foreign equity components are calculated using regression (9.2) and variations on regression (9.3) respectively.
A more formal examination of the impact of various restrictions on beta coefficients in equations (9.1) and (9.3) is conducted using the Wald $\chi^2$ statistic. This statistic is used to test the joint hypothesis that all betas assumed to equal zero to restrict the full model using all futures contracts in fact do equal zero. Table 9.2 indicates the Wald statistics and associated p-values for variations of the model (9.1), where the equity portfolios consist of 70% Australian and 30% Templeton foreign equity. Similarly, Table 9.3 shows the Wald statistics and associated p-values for variations of the model (9.3), where the equity portfolios consist of Templeton foreign equity. In these tables, the first row in the heading indicates which beta coefficients are restricted to zero for each estimation model, and the second row indicates which futures contracts feature in each hedge. For instance, in the second column of Table 9.2, the beta coefficients for FTSE 100, Nikkei 225 and Bovespa futures contracts in equation (9.1) are restricted to zero, so that particular model estimates hedge ratios for AOI futures contracts and S&amp;P 500 futures contracts only. The associated p-values are indicated below each Wald statistic.

In the majority of cases, the null hypothesis that the indicated foreign futures betas are zero is rejected at the 1% significance level, regardless of whether separate or combined hedges are used, and irrespective of foreign equity portfolio construction. This indicates that in most estimation periods, not all of the betas that are simultaneously restricted to equal zero in many of the hedge ratio estimation models in fact equal zero given the data. There are a small number of exceptions. For combined hedges in Table 9.2, in twelve cases out of one hundred and thirty-five, the null hypothesis is not rejected at the 10% level. At random, non-rejection is expected to occur in about thirteen cases of one-hundred and thirty-five at the 10% level. Similarly, for separate hedges in Table 9.3, there are only three cases of non-rejection out of one hundred and twenty-six, at the 10% significance level. The results for portfolios containing MSCI foreign equity are very similar.

In short, these results suggest that may be less appropriate to restrict the hedge ratio estimation models to include only a small number of futures contracts, by assuming that some foreign futures betas are zero. This is because not all of the hedge ratios on the excluded contracts are zero. However, the Wald statistic does not indicate which excluded contracts would have non-zero hedge ratios, or the magnitude or significance of any non-zero hedge ratios. Because the Wald statistic only provides an overview of the impact of restrictions on the full models, the following section examines in detail the behaviour of each individual contract.
Table 9.2: A comparison of Wald χ² statistics and associated p-values for different restrictions on the hedge ratio estimation model in equation (9.1), where the equity portfolios consist of 70% Australian and 30% Templeton foreign equity

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The p-value is listed below each Wald χ² statistic. Hedge ratios are estimated using variations of equation (9.1), which is

\[ \text{ALLEQ} = \alpha + \beta_1 \text{AOIF} + \beta_2 \text{SP} + \beta_3 \text{FTSE} + \beta_4 \text{NIKK} + \beta_5 \text{BOV} + \epsilon, \]

where \( \text{ALLEQ} \) is the continuously compounded return on the entire unhedged portfolio, and \( \text{AOIF}, \text{SP}, \text{FTSE}, \text{NIKK}, \) and \( \text{BOV} \) are continuously compounded returns on the AOI futures contract, S&P 500 futures contract, FTSE 100 futures contract, Nikkei 225 futures contract and Bovespa futures contract respectively. In the table heading, "A", "S", "F", "N", and "B" represent the AOI futures contract, S&P 500 futures contract, FTSE 100 futures contract, Nikkei 225 futures contract and Bovespa futures contract respectively.
Table 9.3: A comparison of Wald $\chi^2$ statistics and associated p-values for different restrictions on the hedge ratio estimation model in equation (9.3), where the equity portfolios consist of Templeton foreign equity

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<td>22.56</td>
<td>31.36</td>
<td>26.82</td>
<td>30.70</td>
<td>74.61</td>
<td>8.13</td>
<td>14.50</td>
<td>13.14</td>
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<tr>
<td>09/07/1998</td>
<td>42.04</td>
<td>52.00</td>
<td>168.79</td>
<td>166.01</td>
<td>16.83</td>
<td>29.19</td>
<td>31.33</td>
<td>34.58</td>
<td>33.41</td>
<td>117.06</td>
<td>7.46</td>
<td>8.30</td>
<td>20.03</td>
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<td></td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The $p$-value is listed below each Wald $\chi^2$ statistic. Hedge ratios are estimated using variations of equation (9.3), which is

$$
FOREQ = \alpha + \beta_1SP + \beta_2FTSE + \beta_3NIKK + \beta_4BOV + \varepsilon,
$$

where $FOREQ$ is the continuously compounded return on the foreign unhedged component, and $SP$, $FTSE$, $NIKK$, and $BOV$ are continuously compounded returns on the S&P 500 futures contract, FTSE 100 futures contract, Nikkei 225 futures contract and Bovespa futures contract respectively. In the table heading, "S", "F", "N", and "B" represent the S&P 500 futures contract, FTSE 100 futures contract, Nikkei 225 futures contract and Bovespa futures contract respectively.
Statistical significance of futures hedge ratios

Given this overview of the impact of increasing the number of futures contracts on the significance of the beta coefficients in the OLS regressions, it is important to examine which particular futures contracts are associated with hedge ratios that are insignificant. Table 9.4 shows two key pieces of information for each of the fifteen futures portfolios, namely which (if any) contract’s hedge ratio is not significant, and the number of hedge ratio estimation periods in which that situation occurs, where the total number of periods is nine. The latter is indicated in brackets following the contract to which it relates. A dash (-) indicates that all beta coefficients for all futures contracts are significant at the 10% level for all nine hedge periods for that particular portfolio of futures contracts.

To illustrate, consider the tenth row of Table 9.4, which presents the results for hedges using the AOI, Nikkei and Bovespa futures contracts together. For combined hedges where the dependent variable is returns on the portfolio consisting of 85% Australian and 15% Templeton foreign equity, the Nikkei futures hedge ratio is insignificant in two of nine estimation periods, and the Bovespa futures hedge ratio is insignificant in three periods.

Consider the results for combined hedges. As the asset allocation between Australian and foreign equity changes, the particular futures contracts for which hedge ratios are not significant at the 10% level are more varied for the portfolio containing 15% foreign equity. For portfolios containing 30% and 45% Templeton foreign equity, insignificant hedge ratios are generally associated with Nikkei and Bovespa futures contracts, but for portfolios containing 15% Templeton foreign equity, the FTSE futures hedge ratios are also not significant in some cases. For portfolios containing 30% or 45% MSCI foreign equity, the only hedge ratios that are not significant are associated exclusively with the Bovespa futures contract, but for portfolios containing 15% MSCI foreign equity, the S&P 500 and FTSE 100 futures contracts also feature. Thus, the proportion of Australian to foreign equity impacts on the number of different futures contracts that have non-significant hedge ratios when regression (9.2) is implemented. Specifically, the lower the proportion of foreign equity, the greater the number of different contracts that have insignificant hedge ratios and the more frequently each is insignificant, and this is true for both Templeton and MSCI foreign equity portfolio constructions. For instance, the last row in Table 9.4 shows that for portfolios containing 15% Templeton foreign equity, FTSE and Nikkei futures hedge ratios are not significant in six of nine estimation periods and the Bovespa futures hedge ratios are not significant in four periods. When the proportion of foreign equity is increased to 30%, FTSE futures hedge ratios are not significant in two periods, and Nikkei and Bovespa futures hedge
ratios are not significant in three periods. When foreign equity is increased to 45% of the portfolio, Nikkei and Bovespa beta coefficients are not significant in only two periods each.

Remaining focused on combined equity portfolios, changes in the frequency of estimation periods in which hedge ratios for each futures contract are not significant at the 10% level become evident as the number of futures contracts included in the hedge strategy increases. When portfolios containing Templeton foreign equity are hedged using two futures contracts, the number of estimation periods in which a hedge ratio on any contract is not significant ranges from zero to three. When three futures contracts are used, it ranges from zero to four, and for hedge strategies using four or five contracts, it ranges from one to six. Similarly, for hedges of portfolios containing MSCI foreign equity, with two futures it ranges from 0 to 2, with 3 and 4 futures it ranges from 0 to 8, and for 5 futures it ranges from 1 to 8. In short, as the number of futures contracts used to hedge the portfolio increases, the number of estimation periods in which one or more futures hedge ratio is not significant at the 10% level increases. Observations about hedges of portfolios containing Templeton and MSCI foreign equity are generally similar, although foreign equity portfolio construction causes some differences in the detailed behaviour of the hedges. Hedges of portfolios containing Templeton and MSCI foreign equity components are expected to exhibit some differences due to the variations in asset allocations between global regions, as described in detail in Chapter Five.

The beta coefficient on the AOI futures contract is always significant at the 10% level. This is expected as Australian equity makes up the majority of each portfolio. The S&P 500 futures hedge ratios are generally significant in all nine hedge periods, although this futures contract did not perform well in previous chapters. However, recall that this section examines the statistical significance of each hedge ratio, while the performance of various hedged portfolios is examined later in terms of other measures including expected utility. The preference for particular futures contracts depends on the construction of the equity portfolio at any given time. For example, if the proportion of Asian equity in the foreign equity portfolio increases, the significance of the Nikkei hedge ratio may be expected to increase.

When Australian and foreign equity are hedged separately, any hedge ratios that are not significant are generally associated with Bovespa futures contracts, a finding true for portfolios containing Templeton and MSCI foreign equity. The frequency of insignificance is very low for separate hedges involving Templeton foreign equity, reaching a maximum of two periods of a total of nine regardless of the number of futures contracts used in the hedge
The frequency of non-significant hedge ratios is higher for separate hedges involving MSCI foreign equity, reaching a maximum of five periods.

Separate hedges appear to have better model specification relative to combined hedges. Specifically, separate hedges are associated with equal or lower numbers of different futures contracts for which non-significant ratios are detected, and equal or lower frequencies of non-significance across time, relative to combined hedges. These findings are true regardless of foreign equity portfolio construction, and regardless of the number of futures contracts included in the hedge strategy. This may be because the hedge ratios for separate hedges are estimated using two regressions which use fewer independent variables, as opposed to one regression for combined hedges. However, separate hedges ignore portfolio effects, and while the model fit based on significance of independent variables is equal to or better than that of combined hedges, this preference may not emerge when the hedge ratios are implemented and performance is measured by the expected utility of the hedged portfolio.

In short, increasing the number of futures contracts decreases the percentage of hedge periods in which all hedge ratios are statistically significant. Hedges involving smaller numbers of futures contracts generally have better model specification relative to hedges involving larger numbers of contracts, in terms of the significance of hedge ratios. In terms of R-squared statistics for combined hedges, the models do not appear to be highly sensitive to the number of futures contracts, although the R-squared statistics for separate hedges of the foreign equity component vary considerably depending on the particular futures contract combination. While model specification, examined in this section and in the previous section, gives an indication of which hedge strategies will perform best, it is an in-sample assessment which assumes perfect foresight on the part of the manager. More realistic out-of-sample hedge performance is measured in the following section using expected utility, consistent with all previous chapters. Specifically, the out-of-sample expected utility performance of hedges using several futures contracts simultaneously is examined in Section 9.4.
Table 9.4: Futures contracts for which the beta coefficient (hedge ratio) is not statistically significant at the 10% level, and the number of estimation periods in which that occurs, for a variety of equity portfolios and futures portfolios

<table>
<thead>
<tr>
<th>Futures portfolio</th>
<th>Combined Hedges</th>
<th>Separate Hedge</th>
<th>Combined Hedges</th>
<th>Separate Hedge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>85A, 15TWF</td>
<td>70A, 30TWF</td>
<td>55A, 45TWF</td>
<td>TWF</td>
</tr>
<tr>
<td>A,S</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A,F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A,N</td>
<td>N(2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A,B</td>
<td>B(3)</td>
<td>B(2)</td>
<td>B(2)</td>
<td></td>
</tr>
<tr>
<td>A,S,F</td>
<td>F(4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A,S,N</td>
<td>N(3)</td>
<td>N(3)</td>
<td>N(1)</td>
<td></td>
</tr>
<tr>
<td>A,S,B</td>
<td>D(4)</td>
<td>B(3)</td>
<td>B(2)</td>
<td>B(2)</td>
</tr>
<tr>
<td>A,F,N</td>
<td>F(2), N(3)</td>
<td>N(2)</td>
<td>N(1)</td>
<td></td>
</tr>
<tr>
<td>A,F,B</td>
<td>B(3)</td>
<td>B(2)</td>
<td>B(2)</td>
<td>B(2)</td>
</tr>
<tr>
<td>A,N,B</td>
<td>N(2), B(3)</td>
<td>B(2)</td>
<td>B(2)</td>
<td>B(2)</td>
</tr>
<tr>
<td>A,S,F,N</td>
<td>F(4), N(6)</td>
<td>F(2), N(3)</td>
<td>N(2)</td>
<td>N(1)</td>
</tr>
<tr>
<td>A,S,F,B</td>
<td>F(5), B(4)</td>
<td>B(3)</td>
<td>B(2)</td>
<td>B(2)</td>
</tr>
<tr>
<td>A,S,N,B</td>
<td>N(3), B(4)</td>
<td>N(3), B(3)</td>
<td>N(1), B(2)</td>
<td>B(2)</td>
</tr>
<tr>
<td>A,F,N,B</td>
<td>F(2), N(3), B(3)</td>
<td>N(2), B(2)</td>
<td>N(1), B(2)</td>
<td>(2)</td>
</tr>
<tr>
<td>A,S,F,N,B</td>
<td>F(6), N(6), B(4)</td>
<td>F(2), N(3), B(3)</td>
<td>N(2), B(2)</td>
<td>N(1), B(2)</td>
</tr>
</tbody>
</table>

"A", "S", "F", "N", and "B" indicate the AOI futures, S&P 500 futures, FTSE 100 futures, Nikkei 225 futures, and Bovespa futures respectively. "85A, 15TWF" indicates an equity portfolio where 85% is Australian equity and 15% is TWF foreign equity, and similarly for the other asset allocations. "85A, 15MSCI" indicates an equity portfolio where 85% is Australian equity and 15% is MSCI foreign equity, and similarly for the other asset allocations. "TWF" and "MSCI" represent the Templeton and MSCI foreign components of "separate" hedges respectively. All other portfolios are associated with "combined" hedges as they contain both Australian and foreign equity. The number in the brackets indicates how many estimation periods the beta coefficient on that contract is insignificant. The total number of periods is nine. The dash (-) indicates that all beta coefficients on all futures contracts are significant at the 10% level for all nine hedge periods for that particular portfolio of futures contracts.
9.4 Comparison of hedges using multiple futures contracts

This section examines the relative performance of hedges using single futures contracts and hedges using portfolios of futures contracts, where hedge ratios are calculated using price-changes OLS regressions. Performance is measured using expected utility as defined in equation (6.5). Specific issues of interest are whether hedges involving multiple futures contracts outperform hedges using single futures contracts, whether there is a "best" number of futures contracts to use, and whether any particular combination of futures contracts produces superior hedge effectiveness.

Table 9.5 shows the futures contract portfolio associated with the highest expected utility of a hedged diversified equity portfolio, where the futures contract portfolios consist of one, two, three, four or five futures contracts, over nine hedge periods between 1992 and 2001. There are fifteen futures portfolios in total. For example, when combined hedges of equity portfolios containing 85% Australian and 15% Templeton foreign equity are undertaken with each futures combination, the hedged portfolio that shows the highest expected utility in the hedge period commencing 02/01/92 is that using a portfolio of futures written on the AOI and Nikkei 225 indices.

Strong patterns emerge in Table 9.5 when the results in each hedge period are considered in isolation. Specifically, in any given hedge period, one contract or one combination of contracts is preferred regardless of the allocation between Australian and foreign equity, foreign equity portfolio construction. In six of nine hedge periods, the preference for one particular futures portfolio is robust to the choice between separate or combined hedge methods. Although different futures contract combinations are preferred in different periods, hedges using single futures contracts perform well relative to hedges using multiple contracts. For example, in five of nine hedge periods, single futures contracts are preferred to portfolios of two, three, four or five contracts, regardless of equity portfolio construction and whether separate or combined hedge methods are used. Further, the largest futures portfolios preferred under separate or combined methods consists of three contracts. Portfolios of four or five futures contracts are never preferred to smaller portfolios. This is true, even in the absence of transaction costs. If this analysis was extended to incorporate transaction costs, this result would be strengthened, because the larger the number of futures contracts, the greater the cost of trading and management. Thus, managers are justified in using only small numbers of contracts to hedge diversified portfolios, on the basis of expected utility. Recall that these expected utility results are based on out-of-sample performance measurement that
Table 9.5: The futures contract portfolio associated with the highest expected utility of a hedged diversified equity portfolio, where the futures contract portfolios consist of one, two, three, four or five futures contracts, over nine hedge periods between 1992 and 2001.

<table>
<thead>
<tr>
<th>Equity portfolio</th>
<th>Australian equity %</th>
<th>Hedge method</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOI, TWF</td>
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<td>Combined</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A,N, A,F</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A,B</td>
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<td>B</td>
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<td>F</td>
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<td></td>
<td></td>
<td>B</td>
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<tr>
<td></td>
<td></td>
<td>A,N,B</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>Combined</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A,N, A,F</td>
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<tr>
<td></td>
<td></td>
<td>A,B</td>
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<td></td>
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<td>S</td>
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<td>F</td>
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<td>A,N,B</td>
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<td></td>
<td>55</td>
<td>Combined</td>
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<td>A,B</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>A,N,B</td>
</tr>
<tr>
<td>AOI, MSCI</td>
<td>85</td>
<td>Combined</td>
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<tr>
<td></td>
<td></td>
<td>A,N, A,F</td>
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<td>B</td>
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<td>A,N,B</td>
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</table>

"A", "S", "F", "N", and "B" indicate the AOI futures, S&P 500 futures, FTSE 100 futures, Nikkei 225 futures, and Bovespa futures respectively. "AOI, TWF" and "AOI, MSCI" represent portfolios containing Australian and Templeton foreign equity, and Australian and MSCI foreign equity respectively. Hedge ratios for combined hedges are estimated using regression (9.1): 

\[ ALLEQ = \alpha + \beta_1 AOIF + \beta_2 SP + \beta_3 FTSE + \beta_4 NIKK + \beta_5 BOV + \varepsilon \]

and hedge ratios for separate hedges are estimated using regressions (9.2) and (9.3):

\[ AUSEQ = \alpha + \beta_1 AOIF + \varepsilon \]

and \[ FOREQ = \alpha + \beta_1 SP + \beta_2 FTSE + \beta_3 NIKK + \beta_4 BOV + \varepsilon \], where the variables are as defined in Section 9.2. Expected utility is calculated using equation (6.5) where \( \lambda \) is assumed to equal three.
incorporates aspects of both risk and expected return, in contrast to analysis in earlier sections based on Ederington’s (1979) R-squared which is in-sample and ignores expected returns.

In hedge periods where single futures contracts are the preferred hedging vehicle, the AOI futures contract is not the best choice. This requires comment, given the dominance of Australian equity in every diversified equity portfolio. A possible explanation is that even though the majority of the equity portfolio consists of Australian stock, the foreign equity component makes the greatest contribution to volatility in some periods, and hence the best hedge of overall risk is a foreign contract which moves most closely with foreign equity. The particular futures contract, or combination of futures contracts, that is preferred depends on the hedge period, reflecting the impact of changing market conditions over time. In Chapter Seven, results indicate that for hedges of the entire equity portfolio using a single futures contract, Australian futures hedges provide the highest expected utility in periods of relatively low volatility of returns on the unhedged equity portfolio, but foreign futures hedges are superior in periods of high volatility. When the results in the present chapter and those in Chapter Seven are taken together, they imply that when volatility is high, single contract hedges using foreign futures contracts are preferred to hedges using either the domestic contract only or multiple contracts simultaneously.

Empirical support for this implication is provided in the example of portfolios containing 70% Australian and 30% foreign equity. Figure 9.8 shows the volatility of portfolios containing 70% Australian and 30% foreign equity, in each of the nine hedge periods, where volatility is measured by standard deviation of returns. The three periods of highest volatility in Figure 9.8 (HP5, HP8 and HP6) coincide with the uniform preference for hedges using a single foreign futures contract as shown in Table 9.5. Further, in the period of lowest volatility (HP2), hedges using more than one futures contract are preferred, and the domestic contract is included in the contract combination. Hence, the results in Chapter Seven regarding the link between volatility and the preference for domestic or foreign futures contracts in single contract hedges is consistent with the results in Table 9.5. The preference for single futures contract hedges in periods of high volatility is especially important, because hedges are most valuable in highly volatile markets.
Figure 9.8: Volatility of diversified equity portfolios containing 70% Australian and 30% foreign equity, in various hedge periods between 1992 and 2001, where volatility is measured as standard deviation of returns

"70A,30TWF" indicates an equity portfolio where 70% is Australian equity and 30% is TWF foreign equity, and "70A,30MSCI" indicates an equity portfolio where 70% is Australian equity and 30% is MSCI foreign equity. The hedge periods are listed in chronological order, with the most recent period identified as "HP 9". The hedge periods are: 02/01/92 to 29/01/92, 01/07/92 to 28/07/92, 03/01/95 to 30/01/95, 03/07/95 to 28/07/95, 02/01/98 to 29/01/98, 01/07/98 to 28/07/98, 02/01/01 to 30/01/01, 02/04/01 to 01/05/01, 02/07/01 to 27/07/01.

Returning to the discussion of the results in Table 9.5, the use of separate or combined hedge ratio estimation methods does not appear to alter the results greatly. In six of nine periods, the same futures portfolio is preferred under both the separate and combined methods. In periods where a different preference is found, it consists of adding an extra contract under the separate method. For example, in the hedge period commencing 01/07/92, the best futures hedge of portfolios of Australian and Templeton foreign equity consisted of futures contracts written on the AOI and FTSE indices under the combined method. When the separate method is used, futures portfolios of AOI, FTSE and Nikkei futures contracts are preferred. Generally, separate hedges require the same or larger numbers of futures contracts than combined hedges, which would translate into an equally costly or more costly hedge.

The results in Table 9.5 are based on expected utility. To assess the sensitivity of results to the measure of hedged portfolio performance, Table 9.6 exhibits results based on the expected return per unit of risk, which is a useful measure given the previous discussion on volatility. The general patterns are very similar, particularly the preference for a specific futures portfolio in each hedge period regardless of equity portfolio construction. Further, in six of nine hedge periods, the preferred futures portfolio for all equity portfolio constructions is the same under the expected utility measure and the expected return per unit risk measure.
The preferred futures portfolio is different between the performance measures in only one period; that commencing on 03/07/95. However, under the expected return per unit risk measure, a single futures contract is still generally preferred to multiple contracts in that period. The results in Table 9.6 generally confirm those in Table 9.5, indicating that the results are generally robust to the performance measure.

The results in this chapter show that in the context of hedging equity risk, the preference for hedges using single or multiple futures contracts depends on the hedge period. However, hedges using small numbers of futures contracts are always preferred to hedges using larger numbers of futures contracts. Specifically, hedges using four or five futures contracts are never preferred to hedges using smaller numbers of futures contracts, as demonstrated in Table 9.5. Further, in many hedge periods, hedges using single futures contracts are preferred over those employing multiple futures contracts. As highlighted in the introduction to this chapter, there is little consensus in prior literature as to whether single futures hedges outperform hedges using multiple futures contracts, and this issue has not been considered in the context of diversified managed fund portfolios. The results presented in the current section are consistent with the research of Grant and Eaker (1989) and Koutmos and Pericli (2000), which supports the finding that single futures hedges perform better than multiple futures hedges in the contexts of commodities and interest rate risk respectively. The contribution of this chapter is to provide evidence on this issue in the previously unexamined context of domestic and foreign equity risk associated with fund portfolios.

9.5 Conclusion

This chapter contributes to the literature by examining the use of single and multiple futures contracts in the previously unresearched context of hedging the equity risk of diversified equity portfolios from the perspective of Australian investors. Hedges using single futures contracts are examined in Chapters Six and Seven, and hedges involving pairs of futures contracts are investigated in Chapter Eight. The present chapter extends these results, examining whether single or multiple futures contracts result in superior hedges, the impact of increasing the number of futures contracts on hedge ratio estimation model fit and hedge performance, and the performance of different combinations of futures contracts when hedging both Australian and foreign equity risk simultaneously.
### Table 9.6: The futures contract portfolio associated with the highest expected return per unit risk of a hedged diversified equity portfolio, where the futures contract portfolios consist of one, two, three, four or five futures contracts, over nine hedge periods between 1992 and 2001

<table>
<thead>
<tr>
<th>Hedge period start</th>
<th>02/01/92</th>
<th>01/07/92</th>
<th>03/01/95</th>
<th>03/07/95</th>
<th>02/01/98</th>
<th>01/07/98</th>
<th>02/01/01</th>
<th>02/04/01</th>
<th>02/07/01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedge period end</td>
<td>29/01/92</td>
<td>28/07/92</td>
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<th>Australian equity %</th>
<th>Hedge method</th>
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<td>85</td>
<td>Combined</td>
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<td>AOI, MSCI</td>
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<tr>
<td>55</td>
<td>Combined</td>
<td>A,N</td>
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"A", "S", "N", and "B" indicate the AOI futures, S&P 500 futures, FTSE 100 futures, Nikkei 225 futures, and Bovespa futures respectively. "AOI,TWF" and "AOI,MSCI" represent portfolios containing Australian and Templeton foreign equity, and Australian and MSCI foreign equity respectively. Hedge ratios for combined hedges are estimated using regression (9.1), and hedge ratios for separate hedges are estimated using regressions (9.2) and (9.3). Expected return per unit risk is measured as the expected return on the hedged portfolio divided by the standard deviation of returns on the hedged portfolio.
Model specification is examined using in-sample measures of performance including the R-squared measure, Wald $\chi^2$ statistics, and the p-values of each hedge ratio. These are in-sample indications of model performance, in contrast to out-of-sample measures of the performance of various hedged portfolios such as expected utility and return per unit of risk, which are examined later. For combined hedges, it is evident that while R-squared statistics generally remain high regardless of the number of futures contracts, the percentage of hedge periods for which all hedge ratios are statistically significant tends to decrease as the number of futures contracts included in the hedge strategy increases. For separate hedges, the Australian component generally exhibits high R-squared statistics and consistently significant hedge ratios, as expected in direct hedging. The foreign hedge component shows greater variability of R-squared statistics, depending on the particular futures contract combination. While the general trend for the percentage of hedge periods in which all hedge ratios are significant to decrease as the number of futures contracts increases is also evident for the separate foreign hedge, exact results are influenced by the futures contract combinations.

An examination of the impact of volatility on the preference for hedges using single or multiple futures contracts reveals that when volatility is high, single contract hedges using foreign futures contracts are preferred to hedges using either the domestic contract only or multiple contracts simultaneously. The preference for single futures contract hedges in periods of high volatility is especially important, because hedges are most valuable in highly volatile markets.

Hedge performance is measured out-of-sample using expected utility, consistent with the procedure in earlier chapters. Although the preference for single or multiple futures contracts depends on the hedge period, hedges using small numbers of futures contracts are always preferred to hedges using larger numbers of futures contracts, and in many hedge periods, hedges using single contracts are preferred over those employing multiple contracts. This result is generally robust to foreign equity portfolio construction, the allocation between domestic and foreign equity, and the measure of hedge performance. Further, this finding is expected to be strengthened if transaction costs and ease of portfolio management were taken into account explicitly. Thus, although some prior research suggests that using multiple futures contracts as opposed to single contracts may improve hedge performance through superior matching of returns on spot and futures portfolios, this chapter demonstrates that in the context of hedging equity risk associated with diversified managed fund portfolios, this is not generally the case. It may be technically possible to hedge all parts of a complex portfolio, but it is likely to be less expensive and more convenient to hedge using only one or a small number of instruments. Thus, these results indicate that fund managers wishing to hedge equity risk associated with internationally diversified equity portfolios may hedge effectively using one or a small number
of futures contracts, and do not have to engage in more expensive and time-consuming management of a large number of contracts. Further, in periods of relatively high volatility when hedging is most valuable, single foreign futures contracts generally provide better hedges than multiple futures contracts. In practical terms, these results indicate a way in which managers may reduce the transaction costs and management costs of their equity futures hedge without loss of hedge effectiveness.
CHAPTER TEN

CONCLUSIONS

10.1 Introduction

In this chapter, the key empirical findings in this thesis are summarised, and conclusions are drawn. The importance and contribution of this research to the literature and to practitioners is reinforced, the limitations of the research are discussed, and avenues for further research are identified.

This thesis examines the hedging of equity risk associated with diversified portfolios using futures contracts. Australian investors benefit from holding diversified portfolios, and equity is an important asset class, which is generally considered to be more risky than other investment classes. Methods of risk management based on simple diversification and complex dynamic asset allocation have serious disadvantages, and derivatives such as futures contracts provide alternative risk management vehicles. Investors implementing superior risk management strategies have a competitive advantage over those using less effective methods. In particular, the application of a superior risk management strategy should improve hedge performance relative to that of less effective hedging strategies. Although conducted in the context of Australian managed fund portfolios, the findings in this thesis are relevant to all Australian managers and investors who use futures contracts to hedge equity risk, whether the hedge is applied to the whole portfolio or only part of it.

Prior literature on hedging using futures contracts is generally limited to comparisons of hedge ratio estimation techniques in the context of simple hedges involving single assets and single futures contracts. However, managed funds hold diversified portfolios that are subject to multiple risks. Further, a variety of futures contracts may be used as hedging tools, and multiple derivatives may be used simultaneously. This thesis extends the prior literature through four related empirical studies that investigate increasingly complex aspects of hedging risk using futures contracts, presented in Chapters Six, Seven, Eight and Nine. Detailed conclusions from each chapter are presented in the following section.
10.2 Conclusions

10.2.1 Chapter Six: Single asset, single futures contract

Chapter Six presents empirical evidence on the hedging of diversified foreign equity portfolios using futures contracts. Specifically, the choice of a futures contract to hedge the foreign component of equity portfolios held by Australian investors is investigated to determine which, if any, single hedging instrument is superior, and whether Australian futures are effective risk management tools against foreign equity risk. The use of a superior hedging instrument would more effectively prevent losses on the underlying portfolio. The use of a single contract is appealing because transaction costs are lower than if a portfolio of futures is used, and relatively less information and less management time are required.

Much prior literature examines the direct hedging of an underlying asset using a derivative contract written on that asset, or the cross hedging of individual assets using individual derivatives. In contrast, this chapter examines the cross-hedging of equity portfolios using an individual futures contract not written on those portfolios. The results are examined for sensitivity to time periods, the construction of the equity portfolios, and the risk aversion parameter adopted in the calculation of hedge effectiveness.

A comparison of the unhedged position with all hedges using each futures contract indicates that a hedged position outperforms the unhedged position on the basis of expected utility. This result is robust to foreign equity portfolio construction, hedge ratio estimation technique, frequency of re-estimation of the hedge ratio, and time period. However, because the unhedged position is generally not associated with the lowest expected utility in each hedge period, it remains superior to poorly performing hedged positions. This reveals the importance of examining which futures contracts, if any, produce superior hedge protection relative to other futures because all hedges do not perform equally, and while some are superior to an unhedged position, others are relatively inferior. Thus, the examination of hedging techniques, including the relative merits of different futures contracts, in this chapter is important because hedging may improve expected utility.

In a comparison of futures contracts, the FTSE 100 and Nikkei 225 futures contracts generally provide the best hedges of the foreign equity portfolios, depending on the hedge ratio estimation method. Although Australian futures are not generally the most effective tools for hedging foreign equity risk, the performance of Australian futures relative to foreign futures improves as the complexity of hedge ratio estimation method increases. S&P 500 futures do not generally
provide good hedges of the foreign equity portfolios in terms of expected utility, indicating that simple linear correlation does not fully explain hedge effectiveness.

Another issue examined is the relative effectiveness of complex and simple hedge ratio estimation techniques. Although addressed in prior literature, the performance of complex GARCH hedges relative to simple OLS and naïve models remains unresolved, and is revisited in the context of hedging foreign equity portfolios. Time varying GARCH(1,1) ratios generally provide the highest expected utility for individual futures contracts, although this result depends on the equity portfolio.

Overall, the results highlight the importance of an awareness of the interaction between the choices facing Australian fund managers when hedging foreign equity risk, particularly the hedge ratio estimation technique, the futures contract, and the construction of the equity portfolio.

10.2.2 Chapter Seven: Multiple assets, single futures contracts
While Chapter Six examines the hedging of the foreign equity component of diversified equity portfolios, Chapter Seven extends the analysis to examine the hedging of diversified equity portfolios consisting of both Australian and foreign equity, where portfolio effects complicate the hedging process. A key issue is whether there is any benefit in accounting for “portfolio effects” by hedging the entire equity portfolio as a single spot series, or whether separate hedges of the Australian and foreign equity components provide an equally effective strategy. Portfolio effects are the complex interactions between different components of portfolios containing multiple assets. The results are examined for sensitivity to the asset allocation between Australian and foreign equity, the construction of foreign equity portfolios, the hedge ratio estimation method, the measure of hedge performance, and the time period.

For hedges of the entire equity portfolio using a single futures contract, Australian futures hedges provide the highest expected utility in periods of relatively low volatility of returns on the unhedged equity portfolio, but foreign futures hedges are superior in periods of high volatility. Risk management strategies are arguably most important in periods of high volatility. In periods of relatively low volatility, a manager may elect not to hedge or may not require a full hedge because equity risk is lower than usual. Managers may improve hedge performance by thoughtful selection of the hedging instrument, depending on the expected level of equity portfolio volatility. The volatility of portfolio returns may be attributable to volatility in returns on domestic Australian equity or foreign equity or both, and the interaction between different components of the portfolio is addressed in more detail in the following chapter.
An examination of the impact of portfolio effects is conducted in the context of hedging equity risk using the AOI futures contracts as the exclusive hedging vehicle. Specifically, the effectiveness of hedging the entire equity portfolio using the AOI futures is compared to the effectiveness of ignoring portfolio effects and hedging the Australian and foreign components separately using AOI futures contracts. A general preference for combined hedges in the majority of hedge periods is evident using the expected utility measure of performance, for portfolios containing Templeton foreign equity. The relatively good performance of AOI futures contracts is convenient for Australian managers, because the domestic contract can be used to hedge the equity risk of diversified equity portfolios with reasonable effectiveness. In contrast, results are varied for portfolios containing MSCI foreign equity. Hence, whether fund managers may benefit from accounting for portfolio effects when hedging equity risk associated with internationally diversified equity portfolios appears to be case-specific, depending on the foreign equity component. Correlation between different components of the hedged portfolio are consistent with some explanations of the preference for combined hedges over separate hedges, in the context of OLS hedge ratios, but the factors driving the results appear to be more complex for GARCH hedges. These issues are important because adopting excessive or inadequate positions in futures contracts through an inappropriate choice of hedging strategy is costly.

10.2.3 Chapter Eight: Multiple assets, pairs of futures contracts

Chapter Eight examines the hedging of equity risk given diversified portfolios containing Australian and foreign equity using pairs of futures contracts, to determine whether there is any benefit in accounting for “portfolio effects”. This is investigated by comparing the overall performance of separate traditional hedges of Australian and foreign equity with the performance of “combined” hedges of the entire equity portfolio using two futures contracts, where the interactions between domestic and foreign equity are captured in a single return series before hedge ratios are estimated. This study can be distinguished from prior empirical research comparing traditional hedge ratio models, in that the latter ignore portfolio effects, are generally limited to foreign (non-Australian) data, and are not conducted in the context of diversified fund portfolios. Further, this study can be differentiated from the work of researchers such as Gagnon et al (1998) on portfolio effects, given differences in data and the focus on different risks, and because the current study relies on out-of-sample measures of performance whereas prior work relies on in-sample analysis which unrealistically assumes perfect foresight by the practitioner.

Extending the analysis in Chapter Seven, and as a preliminary investigation into the use of multiple futures to hedge risk in Chapter Nine, domestic and foreign futures contracts are used simultaneously as hedging vehicles. When the performance of all hedge ratio estimation
methods is compared for each futures contract combination, hedge ratios estimated using the trivariate GARCH(1,1) model generally perform as well as or better than other methods of hedge ratio estimation, suggesting that accounting for portfolio effects when calculating hedge ratios improves hedged portfolio performance. However, the best futures contract combinations change over time. These results are robust to the relative proportions of domestic Australian and foreign equity in each equity portfolio, and to the construction of the foreign equity component. Hence, managers may improve hedge performance through the use of the trivariate GARCH(1,1) model that incorporates portfolio effects, although it has considerably greater computational complexity than the other models, a factor that may deter practitioners.

Simple relationships linking the relative performance of combined and separate hedges with the correlation between domestic and foreign equity and between different futures contracts are not evident. This may be because correlation is a simple linear measure of co-dependence that fails to capture any non-linearity in the relationship between spot and futures returns. Similarly, periods of high volatility of equity portfolio returns are not clearly linked with a preference for combined or separate hedges. Connections between these factors and hedge performance are hypothesised because these factors are not accounted for explicitly in separate hedge ratio estimation methods. The findings suggest that the causes of hedge performance are complex, and highlights an avenue for further research. Regardless of the factors driving hedge performance, this chapter shows that combined hedges perform better generally than separate hedges, indicating that accounting for portfolio effects is important.

10.2.4 Chapter Nine: Multiple assets, multiple futures contracts

Chapter Nine contributes to the literature by examining the use of single and multiple futures contracts in the previously unresearched context of hedging the equity risk of diversified equity portfolios from the perspective of Australian investors. The present chapter extends earlier results, examining whether single or multiple futures contracts result in superior hedges, the impact of increasing the number of futures contracts on hedge ratio estimation model fit and hedge performance, and the performance of different combinations of futures contracts when hedging both Australian and foreign equity risk simultaneously.

Model specification is examined using in-sample measures of performance including the R-squared measure, Wald $\chi^2$ statistics, and the p-values of each hedge ratio. These are in-sample indications of model performance, in contrast to out-of-sample measures of the performance of various hedged portfolios such as expected utility and return per unit of risk, which are examined later. For combined hedges, it is evident that while R-squared statistics generally remain high regardless of the number of futures contracts, the percentage of hedge periods for
which all hedge ratios are statistically significant tends to decrease as the number of futures contracts included in the hedge strategy increases. For separate hedges, the Australian component generally exhibits high R-squared statistics and consistently significant hedge ratios, as expected in direct hedging. The foreign hedge component shows greater variability of R-squared statistics, depending on the particular futures contract combination. While the general trend for the percentage of hedge periods in which all hedge ratios are significant to decrease as the number of futures contracts increases is also evident for the separate foreign hedge, exact results are influenced by the futures contract combinations.

An examination of the impact of volatility on the preference for hedges using single or multiple futures contracts reveals that when volatility is high, single contract hedges using foreign futures contracts are preferred to hedges using either the domestic contract only or multiple contracts simultaneously. The preference for single futures contract hedges in periods of high volatility is especially important, because hedges are most valuable in highly volatile markets.

Hedge performance is measured out-of-sample using expected utility, consistent with the procedure in earlier chapters. Although the preference for single or multiple futures contracts depends on the hedge period, hedges using small numbers of futures contracts are always preferred to hedges using larger numbers of futures contracts, and in many hedge periods, hedges using single contracts are preferred over those employing multiple contracts. This result is generally robust to foreign equity portfolio construction, the allocation between domestic and foreign equity, and the measure of hedge performance. Further, this finding is expected to be strengthened if transaction costs and ease of portfolio management were taken into account explicitly. It may be technically possible to hedge all parts of a complex portfolio, but it is likely to be less expensive and more convenient to hedge using only one or a small number of instruments. These results indicate that fund managers wishing to hedge equity risk associated with internationally diversified equity portfolios may hedge effectively using one or a small number of futures contracts, and do not have to engage in more expensive and time-consuming management of a large number of contracts. Further, in periods of relatively high volatility when hedging is most valuable, single foreign futures contracts generally provide better hedges than multiple futures contracts. In practical terms, these results indicate a way in which managers may reduce the transaction costs and management costs of their equity futures hedge without loss of hedge effectiveness.

10.2.5 Conclusion

In conclusion, this thesis offers a variety of methods for improving the performance of hedges of diversified equity portfolios using Australian and foreign futures contracts. Extensive
sensitivity analysis with regard to equity portfolio construction, hedge ratio estimation method, hedge effectiveness measures, futures contracts and time periods enhances the generalisability of the findings. The results are of direct relevance to investors exposed to equity risks.

10.3 Suggestions for further research

The research in this thesis suggests several avenues for further research. For instance, managers and investors are likely to be interested in the factors driving hedge performance in different periods. While preliminary investigations of the relationship between hedge performance and variables such as correlation and volatility are conducted in this thesis, a more detailed study could examine a wider range of factors. Other factors of interest include transaction costs, liquidity and thin trading, returns, local market size, knowledge of the local market, and local market regulations. Indeed, Eaker and Grant (1987, p 101) acknowledge that "the underlying structural factors which make a currency or commodity a desirable cross-hedge are not well developed". If performance is driven by simple, identifiable factors, future research may focus on improving hedge performance using this information. If hedge performance is more complex, further research may continue to explore the causes of hedge effectiveness. The results are of interest to hedgers because the utilisation of a superior hedge strategy or instrument translates into a competitive advantage.

Further, as discussed briefly in Chapters Seven and Eight, correlation is a linear measure of association which fails to account for any non-linear dependence between returns on spot assets and futures contracts. For instance, Aczel (1996, p 436) observes that correlation analysis relies on the assumption that the variables are normally distributed random variables. In a multivariable case, Embrechts et al (2000, p 71) note that correlation is unproblematic only when the risks "have jointly a multivariate normal distribution", and correlation is inappropriate when risks are heavy-tailed. More complex measures of co-dependence such as copulas could be investigated as the basis for alternative hedge ratio estimation methods, which take into account the joint distribution of risks rather than simple correlation. Such a model would provide complete information about these risks, and we could work out the dependence structure using copulas (Embrechts et al (2000)).

This thesis restricts analysis to foreign and domestic equity risk. Research on hedging multiple assets using multiple futures contracts simultaneously could be extended to include different risks associated with diversified fund portfolios. For instance, the hedging of interest rate risk and equity risk simultaneously could be examined. Further, the examination of multiple risks simultaneously could be conducted in the context of production and price risk, where the limited
existing research indicates that many traditional results fail to hold when both types of risk are considered simultaneously, as observed by Viaene and Zilcha (1998).

The analysis in this thesis is conducted within the traditional mean-variance framework. An alternative study could examine the issues addressed in this work within a different framework such as stochastic dominance. Other assumptions such as the selection of hedge ratio effectiveness measures are also open to further investigation.

In summary, the analysis in this thesis suggests many areas for further research.
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