MODIFIED DECISION FEEDBACK EQUALIZATION TECHNIQUES FOR DATA COMMUNICATIONS

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Doctor of Philosophy
of the Australian National University

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DECLARATION

The contents of this thesis are the result of original research and have not been submitted to any other university for the purpose of obtaining a postgraduate degree.

Much of the work in this dissertation has been published or has been submitted for publication in IEEE research journals or conference proceedings. The following papers have been submitted to IEEE journals.


☆ S. Hasnie, R.A. Kennedy and I. Fijalkow, "Modified MMSE design for decision feedback equalizers," to be submitted to IEEE Transactions on Communications, April 1999


The following papers have been published in Conference Proceedings.


The following paper is in preparation.

S. Hasnie and R. Kennedy, "Soft quantization for decision feedback equalization," to be submitted to *IEEE Transactions on Communications*, April 1999

The research presented in this dissertation is done under the supervision of Dr. Rodney A. Kennedy. Some research projects were done in collaboration with the international researchers Raúl Casas and Prof. Rick Johnson¹, and Dr. Inbar Fijalkow². The majority of the work is, however, my own.

Shazia Hasnie
June, 2000

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ACRONYM EXPLANATIONS

[Text continues without specific content extractible in this view]
ABSTRACT

The conventional minimum mean square error (MMSE) design approach applied to decision feedback equalization (DFE) is based on the correct past decision assumption. This assumption seems necessary in order to obtain a closed form solution. In this dissertation we argue that the MMSE-DFE design based on the correct past decision assumption gives a false sense of optimality. It inherently negates the non-linearity of the DFE structure. We critically examine the philosophy behind this assumption and the limitations it can put on the system performance.

We analyse the adverse effects of the nonlinear recursive character of the DFE which makes its presence felt primarily in the phenomenon of error propagation. A hybrid MMSE design criterion is developed which is based on reducing the damaging effect of large taps in the feedback path by using a convex combination of two MMSE designs (linear equalizer and MMSE-DFE) so as to reduce the symbol error rate and to make the design more robust against error propagation.

We show via a simulation study that the actual MMSE solution for a DFE is in fact far from the MMSE solution based on the correct past decision assumption, especially lower signal to noise ratios. We study DFE design without making the correct past decision assumption. This results in a new design called the Modified MMSE-DFE design, which accounts for the possibility of feedback errors.

We generalize the fractionally spaced DFE (FS-DFE) structure setting by making both the feedforward filter and feedback filter fractionally spaced and by replacing the scalar decision quantizer with a vector decision quantizer. We draw two significant findings regarding the design and performance of this new struc-
Abstract

ture. Firstly, when optimized under the correct past decision assumption the new structure is functionally equivalent to an appropriately configured conventional FS-DFE. We demonstrate that, by taking due regard to the likelihood of decision errors (and error propagation), the new FS-DFE structure, when optimized using a maximum \emph{a posteriori} criterion, outperforms the conventional FS-DFE.

We derive an optimal nonlinear quantizer to be used as a decision function in a decision feedback structure where the criterion is the suppression of error propagation. We show that on channels where noise is impulsive a soft saturation nonlinearity outperforms the standard nearest neighbor quantizer in a DFE in terms of its ability to suppress error propagation.
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<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>AWGN</td>
<td>additive white Gaussian noise</td>
</tr>
<tr>
<td>BER</td>
<td>bit error rate</td>
</tr>
<tr>
<td>BS</td>
<td>baud spaced</td>
</tr>
<tr>
<td>SCORE</td>
<td>spectral coherence restoral algorithm</td>
</tr>
<tr>
<td>CDMA</td>
<td>code division multiple access</td>
</tr>
<tr>
<td>CI</td>
<td>channel interference</td>
</tr>
<tr>
<td>CMA</td>
<td>constant modulus algorithm</td>
</tr>
<tr>
<td>COFDM</td>
<td>coded orthogonal frequency division multiplexing</td>
</tr>
<tr>
<td>CPD</td>
<td>correct past decision</td>
</tr>
<tr>
<td>DAB</td>
<td>digital audio broadcast</td>
</tr>
<tr>
<td>dB</td>
<td>decibel</td>
</tr>
<tr>
<td>DDFSE</td>
<td>delayed decision feedback sequence estimation</td>
</tr>
<tr>
<td>DFE</td>
<td>decision feedback equalizer</td>
</tr>
<tr>
<td>DFNPI</td>
<td>decision feedback noise prediction with interleaving</td>
</tr>
<tr>
<td>DSL</td>
<td>digital subscriber lines</td>
</tr>
<tr>
<td>DSP</td>
<td>digital signal processing</td>
</tr>
<tr>
<td>FDM</td>
<td>frequency division multiplexing</td>
</tr>
<tr>
<td>FIR</td>
<td>finite impulse response</td>
</tr>
<tr>
<td>FS</td>
<td>fractionally spaced</td>
</tr>
<tr>
<td>FSK</td>
<td>frequency shift keying</td>
</tr>
<tr>
<td>HDTV</td>
<td>high definition television</td>
</tr>
<tr>
<td>Acronyms</td>
<td>Full Form</td>
</tr>
<tr>
<td>------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>ISDN</td>
<td>integrated services digital network</td>
</tr>
<tr>
<td>ISI</td>
<td>intersymbol interference</td>
</tr>
<tr>
<td>kHz</td>
<td>kilohertz</td>
</tr>
<tr>
<td>LAN</td>
<td>local area network</td>
</tr>
<tr>
<td>LE</td>
<td>linear equalizer</td>
</tr>
<tr>
<td>LMS</td>
<td>least mean square</td>
</tr>
<tr>
<td>LTE</td>
<td>linear transversal equalizer</td>
</tr>
<tr>
<td>LTI</td>
<td>linear time invariant</td>
</tr>
<tr>
<td>MAI</td>
<td>multiple access interference</td>
</tr>
<tr>
<td>Mbps</td>
<td>mega bits per second</td>
</tr>
<tr>
<td>MAP</td>
<td>maximum a posteriori</td>
</tr>
<tr>
<td>MCM</td>
<td>multicarrier modulation</td>
</tr>
<tr>
<td>MF</td>
<td>matched filter</td>
</tr>
<tr>
<td>ML</td>
<td>maximum likelihood</td>
</tr>
<tr>
<td>MLSD</td>
<td>maximum likelihood sequence detection</td>
</tr>
<tr>
<td>MMSE</td>
<td>minimum mean square error</td>
</tr>
<tr>
<td>µs</td>
<td>microsecond</td>
</tr>
<tr>
<td>MSE</td>
<td>mean square error</td>
</tr>
<tr>
<td>OFDM</td>
<td>orthogonal frequency division multiplexing</td>
</tr>
<tr>
<td>PAM</td>
<td>pulse amplitude modulation</td>
</tr>
<tr>
<td>PCS</td>
<td>personal communication systems</td>
</tr>
<tr>
<td>QAM</td>
<td>quadrature amplitude modulation</td>
</tr>
<tr>
<td>QPSK</td>
<td>quaternary phase shift keying</td>
</tr>
<tr>
<td>RF</td>
<td>radio frequency</td>
</tr>
<tr>
<td>RLS</td>
<td>recursive least squares</td>
</tr>
<tr>
<td>SER</td>
<td>symbol error rate</td>
</tr>
<tr>
<td>SIR</td>
<td>signal-to-interference ratio</td>
</tr>
<tr>
<td>SNR</td>
<td>signal-to-noise ratio</td>
</tr>
<tr>
<td>TH</td>
<td>Tomlinson Harashima coding</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>---------------------------</td>
</tr>
<tr>
<td>VA</td>
<td>Viterbi algorithm</td>
</tr>
<tr>
<td>WAN</td>
<td>wide area network</td>
</tr>
<tr>
<td>WMF</td>
<td>whitening matched filter</td>
</tr>
<tr>
<td>ZF</td>
<td>zero forcing</td>
</tr>
</tbody>
</table>
## Notation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_k$</td>
<td>transmitted symbol at time $k$</td>
</tr>
<tr>
<td>$\hat{a}_k$</td>
<td>estimate of the transmitted symbol at time $k$</td>
</tr>
<tr>
<td>${a_m}$</td>
<td>transmitted symbol stream</td>
</tr>
<tr>
<td>$\mathbf{A}$</td>
<td>finite alphabet of transmit symbols</td>
</tr>
<tr>
<td>$A_k$</td>
<td>vector of transmit symbols</td>
</tr>
<tr>
<td>$\hat{A}_k$</td>
<td>vector of previous estimates of transmit symbols</td>
</tr>
<tr>
<td>$c(t)$</td>
<td>continuous time representation of the physical channel</td>
</tr>
<tr>
<td>$C$</td>
<td>actual equivalent subchannel</td>
</tr>
<tr>
<td>$\mathcal{C}$</td>
<td>concatenation of $\mathbf{F}$ and $\mathbf{D}$ weights</td>
</tr>
<tr>
<td>$\mathbf{D}$</td>
<td>vector of feedback filter coefficients</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>cursor delay</td>
</tr>
<tr>
<td>$e_k$</td>
<td>$a_k - \hat{a}_k$ at time $k$</td>
</tr>
<tr>
<td>$\tilde{e}_k$</td>
<td>soft error</td>
</tr>
<tr>
<td>$\widehat{e}_k$</td>
<td>hard or final decision error</td>
</tr>
<tr>
<td>$E_k$</td>
<td>state error vector</td>
</tr>
<tr>
<td>$E$</td>
<td>error set</td>
</tr>
<tr>
<td>$\epsilon(\sigma_e^2)$</td>
<td>base probability</td>
</tr>
<tr>
<td>$\text{erfc}(\cdot)$</td>
<td>complementary error function</td>
</tr>
<tr>
<td>$f_s$</td>
<td>sampling rate</td>
</tr>
<tr>
<td>$F(z)$</td>
<td>transfer function of linear filter</td>
</tr>
<tr>
<td>$\mathbf{F}$</td>
<td>vector of feedforward filter coefficients</td>
</tr>
</tbody>
</table>
Notation

**F₀** feedforward filter for a joint MMSE design

**F₁** feedforward filter for BSE or FSE only design

**F₂** feedforward filter for hybrid MMSE-DFE design

*g(t)* impulse response of the transmit filter

**G** lower triangular matrix from Cholesky factorization

*h(t)* equivalent channel

*h₀* leading channel tap

**H(e^{jwt})** discrete Fourier transform of *h(t)*

**H(z)** equivalent channel transfer function

**H** equivalent channel vector

**H** channel convolution matrix

**I** identity matrix

**J** performance index for the MSE criterion

**J** Jordan matrix

**κ** average number of secondary errors per primary error

**l₂** space of square summable sequences

**λ** ratio of noise to signal variance \((σ_μ^2/σ_ν^2)\)

**M** masking or indexing matrix for **D**

**M** number of elements in the alphabet of transmit symbol

*n(t)* white Gaussian noise

**N** set of *N* parallel channel paths

**N₀** noise spectral density

**N_f** number of feedforward filter taps

**N_d** number of feedback filter taps

**N_k** zero mean vector Gaussian process

*p(t)* convolution of the transmit filter *g(t)* and the physical channel *c(t)*

*p(−t)* matched filter

*pₙ(.)* multivariate Gaussian density of a zero mean vector Gaussian process

*p(.)* probability density of a continuous random variable
\( P_{e(\text{Actual})} \) probability of error when actual decisions are fed back

\( P_e \) probability of error

\( P_{e(CPD)} \) probability of error under the correct past decision assumption

\( P_R \) projection matrix onto the one dimensional subspace spanned by \( R \)

\( P_R^{\perp} \) projection onto the orthogonal complement of the space spanned by \( R \)

\( \text{Pr}(\cdot) \) probability of a discrete random variable

\( P_{a_k} \) nonlinear, memoryless mapping

\( Q(\cdot) \) vector quantizer

\( Q_A(\cdot) \) nearest neighbor quantizer

\( r(t) \) received pulse

\( r_k \) residual ISI

\( R \) vector of non-zero channel cursors

\( |R(jw)|^2 \) Fourier transform of the matched filter output

\( \mathbb{R} \) set of real numbers

\( \rho \) correction vector on \( \mathbf{D}_{\text{cpcd}} \)

\( s(t) \) transmitted signal

\( S_r(e^{jwT}) \) folded spectrum of the received pulse

\( S(\cdot) \) soft decision quantizer

\( \sigma_a^2 \) variance of the transmit symbol

\( \sigma_n^2 \) noise variance

\( \Sigma \) positive definite covariance matrix

\( T_\Delta \) vector of the cursor delay index

\( \mathbf{T} \) vector of overall channel-feedforward filter convolution

\( T \) symbol period

\( u_k \) signal at the feedforward filter output at time \( k \)

\( v(t) \) filtered noise

\( w \) sampling frequency in radians per second

\( \mathbf{W} \) vector of weights

\( \omega \) hybrid MMSE-DFE design parameter
Notation

$y_k$  equivalent channel output at time $k$
$z(t)$  input signal to the quantizer
$z_k$  quantizer input at time $k$
$*$  convolution operator
$*$  complex conjugate
$/$  transpose
Modified Decision Feedback Equalization Techniques for Data Communications

Chapter 1

Introduction
This chapter sets the stage for the problems and issues dealt with in this dissertation. The generic digital communication model is described. When data symbols are transferred through the communication system from transmitter to receiver at high speed, the symbols get smeared in time, making it harder to detect them correctly. Channel equalization is the technique used to remedy the situation. There are various channel equalization techniques, one of which—decision feedback equalization—is the topic of this thesis. These channel equalization techniques are outlined in this chapter, although a detailed discussion on decision feedback equalizers is delayed until chapter 2. Furthermore, this chapter introduces the thesis or idea which is the focal point of this whole dissertation. The thesis is the validity and consequences of the correct past decision assumption in the MMSE-DFE design. This chapter also describes how the research is organized in this thesis. Finally the major contributions of the thesis are listed in a point summary.
1.1 Communications System Model

All communication systems have the basic function of information transfer. The three basic components of any communication system are the transmitter, the channel and the receiver. Furthermore, for an analog input information signal (such as voice or video), electric transducers are used at both the source and destination to convert the information into an electric signal and vice versa, respectively. At the information source this electrical signal is sampled, source encoded, channel encoded and then mapped into sequences of symbols such as a stream of bits. In digital communications the symbol stream is inherently discrete in time and amplitude while the physical system including the channel is continuous in time and amplitude.

The transmitter processes the symbol stream, \( \{a_m\} \), time indexed by \( m \), to produce a transmitted signal, given by

\[
s(t) = \sum_{m=-\infty}^{\infty} a_m g(t - mT),
\]

(1.1)
1.1 Communications System Model

suited to the characteristics of the transmission channel (typically its center frequency and bandwidth). In equation (1.1), \( t \) is time, \( T \) is the symbol period and \( g(t) \) is the impulse response of the transmit filter or the pulse shape filter. The transmit filter produces a continuous-time signal for transmission over the continuous-time channel. Transmission almost always involves modulation and may also include channel coding [1].

The physical connection between the transmitter and the receiver, called the channel, has many forms: e.g., microwave radio, twisted pair copper wire, coaxial cable or optical fibre. In all cases it is assumed that the channel can be modeled as a linear system.

Broadly speaking we encounter two types of channels in high speed data communications through band limited channels: (i) channels whose characteristics are time-invariant and usually unknown, for example a dial up telephone network; and (ii) channels, whose frequency response characteristics are typically time-varying and unknown, such as radio channels and underwater acoustic channels. For these channels, it is not possible to design optimum fixed demodulation filters [2]. In this thesis we consider the problem of channel equalization in the presence of known, time invariant channel distortion, and noise, assumed to be additive white and Gaussian (AWGN). Thus this thesis focuses on linear time invariant (LTI) channels. While time variation is a feature of many important practical channels, it is a fact that the performance of a DFE is not practically well understood even on known, time invariant channels.

One objective of the thesis is the design and analysis of the DFE on a structural level. It implicitly involves determining optimal parameter settings when the channel is known\(^1\).

1.1.1 The Equivalent Channel

The equivalent channel used in this thesis is given in Figure 1.2.

\(^1\) While this may seem to be an already solved problem we show that classical designs are made under strong assumptions whose validity may be questionable.
1.1 Communications System Model

Figure 1.2: The equivalent channel. The received signal is \( r(t) = c(t) \ast s(t) + n(t) \) where \( n(t) \) is the random Gaussian noise process, \( c(t) \) is the physical channel impulse response, and \( s(t) \) is the input signal.

The receiver operates on the output signal from the channel, which is also the input to the receiver. It is given by

\[
r(t) = \sum_{m=-\infty}^{\infty} a_m p(t - mT) + n(t)
\]

where \( n(t) \) is white Gaussian noise and \( p(t) = c(t) \ast g(t) \) is the convolution of the channel \( c(t) \) (Figure 1.1) with the pulse shaping filter \( g(t) \). The operation ‘\( \ast \)’ denotes convolution. This continuous time received signal \( r(t) \) is filtered by a receive filter with impulse response \( q(t) \) to limit the noise power. The received filter output is denoted \( y(t) \). The received filter is commonly a matched filter (i.e., \( q(t) = p(-t) \)) or a whitening matched filter (WMF), but other filters can be conceived, such as a root Nyquist filter (so that the filtered, sampled noise remains white). When the receiver comprises just a matched filter \( p(-t) \), \( y(t) \) can be written as

\[
y(t) = \sum_{m=-\infty}^{\infty} a_m h(t - mT) + v(t),
\]

where \( v(t) \) is the filtered noise (colored noise) and \( h(t) = p(t) \ast p(-t) \). In general, the receiver operations include synchronization, channel equalization, detection and decoding.

This thesis concentrates on the receiver block (shown in Figure 1.1) and, in contrast to equation (1.3), the use of non-linear processing to perform equalization.
1.1.2 Wireless Channel Interface Techniques

In this section, we will discuss wireless\(^2\) channel interface techniques for future broadband communications. The emergence of equalized quaternary phase shift keying using a decision feedback equalizer as a wireless channel interface technique has made this discussion relevant here. The research effort on this technique is mainly by Ariyavisitakul \textit{et al.} \cite{3-6}. The major considerations in choosing a wireless channel interface technique are the following:

\textbf{Flexible data rates:} With the increasing popularity of multimedia communications the need will be to support bandwidth-on-demand services. The future wireless links must be able offer a high peak data rate and a means for traffic multiplexing.

\textbf{Low power consumption:} Mobile terminals are battery powered. Therefore, power consumption is a major system design issue. Future broadband services will require increased transmitted power (RF radiation). Advanced digital signal processing (DSP) techniques help to a certain extent to overcome this problem but can themselves impose a battery power overhead. Therefore the solution lies in a reduced complexity wireless channel interface technique.

\textbf{High frequency reuse:} The total bandwidth available for broadband communications will be limited due to the already congested spectrum. Therefore, frequency reuse is desirable as high frequency reuse means low required bandwidth. High frequency reuse can be achieved by a number of techniques (see reference \cite{7}). In tandem with these techniques, ISI cancellation and smart antenna array processing techniques will enable the receiver to operate at low signal to interference ratios (SIR).

\(^2\) Communications in which the transmission is not carried over wire or cable but is broadcast by radio waves, as in cellular, satellite, microwave, etc.
**Robustness:** The wireless channel interface technique should allow for differing channel environments and high mobility.

**Low transmission overhead:** Transmission overhead includes guard bands, guard time, coding redundancy, and all the bandwidth dedicated to receiver training and synchronization [6].

For example, future broadband communications will demand data rates in the order of 1 to 2 Mbps for wireless wide area networks (WANs) where the delay spread is up to 40 $\mu$s, and data rates of up to 20 Mbps for wireless local area networks (LANs) where the delay spread is up to 4 $\mu$s. Research has already been conducted on the possible wireless channel interface techniques for these kinds of applications. Promising candidates for future broadband communications include:

**Broadband Code Division Multiple Access (B-CDMA)** [8]: CDMA techniques use spread spectrum modulation\(^3\). In order to support a high peak data rate or information bandwidth within a limited transmission bandwidth, a broadband CDMA must rely on multiple access interference (MAI) cancellation techniques. Interference cancellation algorithms are very complicated and challenging to implement. A CDMA system also requires fast and accurate “closed loop”\(^4\) power control, which does not lend itself well to short packet data transmission [8].

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\(^3\) Spread spectrum modulation is a wireless communication technique which uses a transmission bandwidth many times greater than the information bandwidth [9] of any user.

\(^4\) Closed loop power control means that when the base station determines that any user’s received signal on the reverse link has too high or too low a power level, a one-bit command is sent by the base station to the user over the forward link to command it to lower or raise its relative power by a fixed amount [9].
Orthogonal Frequency Division Multiplexing (OFDM) [10]: OFDM\(^5\) is an example of a multicarrier modulation (MCM)\(^6\) approach. The principal advantage of OFDM scheme is its near optimum transmission performance in multipath channels without channel equalization. However, OFDM is more sensitive to carrier frequency offset compared to single carrier modulation schemes [13]. A well known disadvantage of OFDM is its large peak to average signal power ratio. This calls for the use of highly linear transmit amplifiers which are not power efficient [6].

QPSK using a DFE [6]: A widely assumed limitation of this approach is that the required length of the equalizer grows linearly with the bit rate, leading to fundamental tracking problems and excessive complexity. A DFE is also notorious for error propagation. These and other limitations are addressed in a series of research efforts on reduced complexity broadband equalization and a joint coding and DFE scheme for remedying the error propagation problem [3-6]. Thus the solution to error propagation remains a very vital issue for future broadband communications.

A Study on the Performance of QPSK using a DFE

Figure 1.3 shows a simulation of a FS-DFE equalized QPSK constellation. Even at a high noise level the signal is well equalized using the FS-DFE. Channel equalization is a practical and feasible option for future high rate mobile communication [6]. The DFE's low power consumption (because of its low complexity) is one of its major attractions for the mobile terminal. Thus it becomes imperative

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5 Cimini in [11] proposed an OFDM scheme in which blocks of bits called packets are transmitted in parallel. Each packet is transmitted at a low baud rate. The idea is to spread out the effects of fading over many bits rather than a few adjacent bits being completely destroyed by fading. The entire bandwidth is divided into many subchannels. The spectra of the sub-channels do not overlap and there is a guard band between adjacent channels to isolate them at the receiver end. The spectra of individual subchannels are zero at other subcarrier frequencies in order to combat interchannel interference.

6 MCM is a form of frequency division multiplexing (FDM), where the bits in a packet are transmitted in parallel, each at a low bit rate [10]. The European digital audio broadcast (DAB) standard uses MCM, which allows digital broadcasting to be added to analog AM and FM broadcasting [12].
Figure 1.3: The above figure shows the equalized quaternary phase shift keying (QPSK) constellation using a FS-DFE. The system parameters are: a feedforward filter with 4 fractionally spaced taps, a feedback filter with 4 $T$-spaced taps, and a channel with taps $\mathbf{H} = [0.4, 1.0, -0.75, 0.6, 0.45]^T$. (a) Equalized constellation at 18dB SNR and (b) equalized constellation at 6dB SNR. Even at this very high noise power, the equalization performance is good.

that DFEs are studied more profoundly and the philosophies behind the design and analysis are revisited.

1.2 Intersymbol Interference Mitigation

A linear channel may introduce, as a function of frequency

- Amplitude distortion
- Phase distortion

into the received signal. There is additive noise also. Any amplitude or phase distortion introduced into the data signal by the linear channel results in intersymbol interference (ISI). Typically a received sample is a linear function of several data symbols. The energy from one symbol spills over to the adjacent symbol periods. The received symbols increasingly interfere with each other at high symbol rates, thus placing an upper limit on the transmission data rate for reliable communications. Tolerance of the detector to noise may also be reduced and, when the
distortion is sufficiently severe, it can prevent the transmitted symbols from being correctly detected from the received signal. When the root mean square (rms) value of the delay spread is higher than approximately 0.3 of the symbol period, the channel impulse response becomes frequency selective [12]. A frequency selective channel may have nulls or fades in the passband of the channel. Thus to increase the data rate, some form of intersymbol interference (ISI) mitigation technique has to be adopted.

Two upper bounds on the figure of merit of a receiver in the presence of ISI are provided by the matched filter (MF) figure of merit and the maximum likelihood sequence detector (MLSD) figure of merit. In the presence of ISI, the receiver will generally not achieve a matched filter figure of merit because the matched filter receiver does not take ISI into account. However, the matched filter figure of merit represents a very useful benchmark, since the difference between an actual receiver's figure of merit and the matched filter figure of merit is a measure of the effectiveness of our methods in countering the effects of ISI. The other upper bound on the figure of merit is the MLSD figure of merit. Most equalization techniques are suboptimal. An MLSD is an example of an optimal detector. No receiver based on equalization or any other technique for compensating ISI can generally outperform an MLSD (at least asymptotically at high SNR). Moreover, any gap between the MLSD and the MF figures of merit is due to the ISI and not due to shortcomings in the receiver design. This difference is a fundamental measure of the severity of the ISI. Surprisingly, in many cases this difference is zero, meaning that within a multiplicative constant the error probability of the MLSD is essentially the same as the MF bound even in the presence of ISI [2].

While probability of error results allow us to predict the impact of ISI when channel coding is not used, channel capacity results allow us to assess the effect of ISI in the presence of channel coding. For strictly bandlimited channels,

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7 The figure of merit of a receiver is defined as the ratio of the receiver output SNR to the channel output SNR.
8 A measure of the maximum amount of data that a channel can transfer. Mathematically it is the maximum average mutual information between the input and the output of the channel in any given signaling interval.
ISI always reduces the capacity of a channel which is surprising as it does not necessarily increase the probability of error [2, page 488].

A channel capacity, reached with a particular coding technique developed for the ideal, zero ISI Gaussian channel combined with a DFE designed under the correct past decision assumption, can also be reached asymptotically in any band-limited Gaussian channel using the same coding technique [14]. Thus the “dB gap” between a coded modulation system and capacity can be as close on channels with ISI as on ideal zero ISI channels, at least at high SNR.

There are several ISI mitigation techniques available [12], namely:

**Equalization:** If the modulation bandwidth exceeds the coherence bandwidth\(^9\) of the radio channel, ISI occurs and the transmit pulses are spread in time. An equalizer within a receiver compensates for the average range of expected channel amplitude and delay characteristics.

**Spatial Diversity:** This is the most common diversity technique. Multiple antennas are placed strategically in space. All these antennas are connected to a common receiving system. While one antenna sees a signal null, one of the other antennas may see a signal peak, so spatial diversity provides redundancy. The receiver may select the antenna with the best signal at any time or somehow combine all antennas’ outputs.

**Multicarrier/Multilevel Modulation:** Multicarrier modulation (MCM), such as coded orthogonal frequency division multiplexing (COFDM), relies on sending several low data rate streams in parallel which combine to form the desired high data rate transmission. MCM is being rediscovered for wireless communications as a way to avoid the demands of equalization [12].

Using multilevel modulation to transmit a fixed data rate decreases the

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\(^9\) The SNR gap to capacity is the difference between the normalized SNR required to achieve a certain probability of error and the normalized SNR that the channel capacity theorem says is required for a vanishingly small probability of error [2].  
\(^{10}\) The coherence bandwidth is the range of frequencies over which two frequency components are strongly correlated in amplitude [15].
symbol rate and thus the ratio of delay spread to the symbol period. However, increasing the modulation level requires increased transmitter power to achieve the same bit error rate (BER). In practice, most systems use four level modulation, with some using eight-level modulation\textsuperscript{11} [12].

Out of the three ISI mitigation techniques discussed above, channel equalization is the topic of this thesis and is dealt with in the following sections in greater detail.

1.3 Channel Equalization

The classical receiver front-end processing is to sample the output of a matched filter at the symbol rate. The output of this matched filter normally contains ISI [2]. As discussed in Section 1.2, the ISI can be removed by means of channel equalization. Thus an equalizer can be a discrete time filter that compensates for the amplitude and phase distortions introduced by the channel [12]. Unfortunately any equalization of amplitude distortion also results in noise enhancement and therefore there is a tradeoff between minimizing ISI and noise at the input to the detector [2]. The residual ISI can be set to zero or minimized in conjunction with the noise. Typically at low SNR, allowing a low value of residual ISI that takes into account the noise gives much better performance than zero residual ISI, because of noise enhancement. At high SNR, the two ISI settings give similar performance, meaning that noise enhancement is not an issue [16].

There are three main groups of simple and practical equalization structures:

1. Linear Equalizers
2. Decision Feedback Equalizers
3. Transmitter Precoding

\textsuperscript{11} Generally FSK is not used in situations where ISI is an important phenomenon, since it is a nonlinear modulation technique and is therefore difficult to equalize properly.
The description of maximum likelihood sequence detection (MLSD) is deliberately omitted as it is rarely used in practise to equalize ISI channels. Firstly, the linear equalizer and the DFE are considerably less complex to implement than MLSD. Secondly, in high performance data communication systems, error control coding is almost always used and surprisingly, it has been shown theoretically that coding can be used to approach channel capacity in the presence of ISI without the need to use MLSD for equalization [2, page 445].

The linear equalizer will be described in Section 1.4. Decision feedback equalization will also be described in Section 1.5. However, being the topic of this thesis, will be dealt with more thoroughly in Chapter 2.

### 1.4 Linear Equalizers

A linear transversal equalizer (LTE) is the simplest form of equalizer. Figure 1.4 shows an equivalent channel and a discrete time transversal filter, $n_k$ is the additive white noise filtered by the receive filter then sampled. For simplicity and generality of results it is still assumed to be white. The equivalent channel $h(t)$, is taken as monic\(^{12}\) and causal with transfer function $H(z)$. Since the transversal filter has feedforward taps only, it is a non-recursive finite impulse response (FIR)

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\(^{12}\) A particularly important class of causal or anti-causal sequences have a unity-valued sample at time zero ($h_0 = 1$). These sequences are said to be monic.
1.4 Linear Equalizers

filter.

As shown in Figure 1.2 a whitening matched filter can be used as the optimal front-end receiver filter. Typically if the pulse shape at the output of the matched filter satisfies the Nyquist criterion for zero ISI, then the matched filter is the optimal receive filter, in the sense that it maximizes the SNR. For a received pulse \( p(t) \), the pulse at the output of the matched filter has Fourier transform \( |P(jw)|^2 \). The Nyquist criterion for zero ISI thus becomes, at the output of the matched filter,

\[
S_p(e^{jwT}) = \frac{1}{T} \sum_{m=-\infty}^{\infty} |P(j(w + m \frac{2\pi}{T}))|^2 = \text{constant}
\]

(1.4)

where the quantity \( S_p(e^{jwT}) \) is called the folded spectrum of the received pulse \( p(t) \) [2].

Since \( H(z) \) is a monic and causal filter, the equalizer must have all its poles inside or on the unit circle, hence \( F^{-1}(z) \) is a causal filter, though not necessarily minimum phase or stable, because of the possibility of poles on the unit circle. When \( H(z) \) has zeros on the unit circle, the zero-forcing linear equalizer is not useful since the noise at the slicer input would have to have infinite variance [2, page 451]. The transfer function of the zero forcing linear equalizer is the inverse of the folded spectrum and is given by

\[
\frac{1}{H(z)} = \frac{1}{F(z)F^*(z^{-1})}.
\]

(1.5)

The number of taps of the equalizer depends on the severity of the ISI introduced by the channel. When the ISI is severe, spreading the information bearing symbol energy over a several adjacent symbols, the number of equalizer taps required can be quite large.

High data rates (over voice band channels, cable channels, mobile radio channels etc.) lead to the search for more effective equalization techniques. High data rates place signal energy within the heavily attenuated portion of the transmission spectrum. This results in intersymbol interference.
In the zero-forcing linear equalizer, the tap weights are adjusted to invert the channel independent of the noise. For example this can pose a serious problem, for fading channels possessing nulls in the amplitude characteristics. Zero forcing linear equalizers put a large gain in the fade region and thus cannot be expected to perform well for such channels as this results in noise enhancement [17]. This property makes the zero forcing linear traversal equalizer (LTE) unsuitable for radio mobile channels and other high data rate channels. The solution is the minimum mean square error (MMSE) linear equalizer or the DFE. The MMSE linear equalizer provides a moderated gain at the fading frequencies. The mean square error (MSE) criterion reduces noise enhancement by allowing residual ISI at the slicer input and attempting to minimize the sum of ISI power and noise power.

The MMSE that a linear equalizer can achieve is given as [15, page 311]

$$\frac{T}{2\pi} \int_{-rac{\pi}{T}}^{\frac{\pi}{T}} \frac{N_0}{|H(e^{jwT})|^2 + N_0} dw$$  \hspace{1cm} (1.6)

where $w = 2\pi f$ is the sampling frequency in radians, $H(e^{jwT})$ is the discrete time Fourier transform of the equivalent channel $h(t)$, known also as the frequency response, and $N_0$ denotes the noise spectral density.

In the presence of pure phase distortion and additive white Gaussian noise, a linear equalizer followed by a threshold-level detector is the optimum detection device. However, by the late 1960s, it was recognized that over highly dispersive channels even the best linear equalizers fall short of the matched filter performance bound\(^\text{13}\), obtained by considering the reception of an isolated transmitted pulse [18]. Thus in the presence of amplitude distortion the linear equalizer does not give a high enough tolerance to noise and a strategy based on nonlinear equalization becomes inevitable [16]. The simplest nonlinear equalizer is the DFE and this is treated next.

\(^{13}\) The probability of error for a matched filter in the isolated data symbol case is called the matched-filter bound, because it represents a lower bound on the probability of error of a detector that has to deal with intersymbol interference [2].
1.5 Decision Feedback Equalizers

The topic of decision feedback equalization is introduced here briefly in order to flesh out the ideas on which this dissertation is based. A more detailed discussion on DFEs will be provided in Chapter 2. The points mentioned in this section will facilitate the explanations given in the chapter summary of Section 1.7.

The topic of this thesis is decision feedback equalization for dispersive channels when high data rates are used. DFE is an attractive choice for channel equalization because it has low complexity, and is simple to implement. The DFE is not without its vices however, mainly because it has a non-linear recursive feedback structure. In order to obtain a closed form design solution for the DFE filter parameters, the design analysis uses the correct past decision assumption. This assumption regards decisions fed back into the feedback filter as correct (as in an ideal world). In reality this is not the case and some decisions fed back into the feedback filter are erroneous. When these erroneous symbols are used to cancel intersymbol interference (ISI), they trigger further errors and a phenomenon known as error propagation.

Researchers have proposed a number of techniques to combat error propagation in DFEs. Notable techniques include Tomlinson-Harashima precoding [19] and decision feedback noise prediction with interleaving (DFNPI) [20]. These techniques are discussed in more detail in Section 1.6 and in Chapter 2, respectively. The point to be noted here is that these techniques are often either impractical or inadequate for future high data rate mobile communications. Therefore, the need remains to study the effects of DFE error propagation on the structural level. By accounting for inevitable errors and error propagation, better designs and better performance can be achieved. Thus this dissertation is based on the following thesis:

The MMSE-DFE design analysis under the correct past decision as-
sumption is not optimal. The theoretical DFE performance derived under the correct past decision assumption is not the same as the DFE performance in practice. When the correct past decision assumption is not relied on, one can re-examine the filter weight design or even investigate structural variation to the standard DFE to give better performance in the actual operating environment.

We argue that unless these design issues are solved at the grass root level the theoretical performance and the practical performance of the DFE will remain worlds apart.

1.6 Transmitter Precoding

Transmitter precoding, also called Tomlinson-Harashima coding (TH), can be used to avoid error propagation triggered by the noise induced errors in the DFE. A brief discussion of TH is included in this chapter because it is used in practice as one of the techniques to control error propagation and for channel equalization. Notable research efforts on transmitter precoding include [21, 22], with Tomlinson in 1971 [19] as the first to introduce the concept of TH precoding.

The idea of precoding is to move the cancellation of the postcursor\textsuperscript{14} ISI to the transmitter where the past transmitted symbols are known without the possibility of error. However, this means that the postcursor ISI impulse response must be known precisely at the transmitter. Uns suited to broadcast applications, when there are many different channels. This is the major disadvantage of transmitter precoding. In practice, the postcursor ISI impulse response is estimated at the receiver end through adaptive filtering techniques. This ISI impulse response is send back to the transmitter in order to be cancelled using transmitter precoding techniques. This technique is feasible on time-invariant or slowly varying channels but is not feasible on fast fading channels such as mobile radio.

\textsuperscript{14} Postcursor ISI is defined as the interference from the past data symbols.
The basis of transmitter precoding is the observation that the linear equalizer can be put at the transmitter end, i.e., before the front end receiver called the whitening matched filter (WMF), without compromising the requirement that the Nyquist criterion for zero ISI be satisfied at the slicer input. TH precoding is in some ways similar to DFE. In TH precoding an effect comparable to DFE operation (i.e., the cancellation of ISI before a decision is made on the current symbol) is achieved by a “precoding” operation in the transmitter using modulo arithmetic (see Figure 1.5) [14]. There are two benefits:

1. The receiver now comprises the WMF followed directly by a slicer. The noise at the slicer input is filtered by the WMF. Thus even though the receiver uses linear equalization, it does not suffer the noise enhancement of standard linear equalization.

2. The error probability may actually be slightly lower than that of the DFE, because the postcursor ISI cancellation is done in the transmitter and so there is no possibility of error propagation.

Simply doing the equalization in the transmitter as shown in Figure 1.5 is not practical, however, because it increases the average and peak power of the transmitted signal.

TH has recently gained importance because it does provide a performance improvement on channels with ISI when incorporated with channel coding. The
DFE on the other hand is fundamentally incompatible with channel coding because delay-free decisions are required to cancel postcursor ISI. In the absence of channel coding, transmitter precoding does not offer a substantial advantage over the DFE.

In [14] it is shown that TH precoding can be further extended to obtain a more general transmitter equalization technique called trellis precoding, which has the desirable attribute of potentially operating very close to channel capacity.

1.7 Thesis Layout

This section presents the complete layout of the thesis by providing a chapter by chapter account.

Chapter 2: Review of Decision Feedback Equalization Techniques
Chapter 2 gives a detailed introduction to the design and structure of the DFE. The DFE concepts such as MMSE-DFE filter optimization under the correct past decision assumption and error propagation (which form the basis of some of the studies in the thesis) are explained. A thorough bibliography is included which covers every major paper on DFEs written in the last thirty years. DFE development is monitored on a decade to decade basis starting from the 1970s, then moving on to the 1980s and finally more recent results in the literature from the 1990s.

Chapter 3: Conventional MMSE-DFE Design
This chapter addresses the conventional MMSE-DFE structure and pinpoints the fact that the conventional MMSE-DFE based on the correct past decision assumption may not give the optimal value of the actual mean square error (MSE). A new design strategy called a “hybrid MMSE design” is given which is based on a convex combination of the MMSE linear equalizer design and the MMSE-DFE design. Some of the linear equalizer’s degrees of freedom are used to combat large
Chapter 4: Actual MMSE bound for DFE
This chapter continues from Chapter 3 on the problem of conventional MMSE-DFE design. It deals mainly with the optimization of filter parameters to reach a true/actual MMSE solution for the DFE. The MMSE-DFE analysis is undertaken without making the correct past decision assumption. Simulation results are presented which show that the actual value of MMSE is in fact different from the MMSE value given by the conventional MMSE-DFE design using the correct past decision assumption. Thus it gives a comparison of where the existing MMSE solution is and where the true solution lies. It shows that the MSE under the correct past decision (CPD) assumption may be used to bound the actual MSE of DFEs.

Chapter 5: Vector Quantizer Design for FS-DFE
The process of optimizing the MMSE-DFE design continues from earlier chapters. Issues related to the key assumption of correct past decisions are discussed in the context of quantizer optimization. The motivation behind the work done in this chapter is the finding that under the correct past decision assumption, a fractionally spaced feedback filter and a vector quantizer as the decision device boils down to the much simpler conventional FS-DFE with a scalar quantizer. However, without the CPD assumption, the vector quantizer design based on the maximum \textit{a posteriori} (MAP) criterion is shown to give better SER performance.

Chapter 6: Optimal Quantizer Design for the DFE
This chapter continues with the topic of optimal quantizer design for DFE. A soft decision function is used in lieu of the nearest neighbor quantizer. This
arrangement is shown to outperform the standard hard decision quantizer design (in terms of error propagation).

Chapter 7: Conclusions
This chapter wraps up the thesis. First it presents the big picture by describing the existing and upcoming wireless personal communications systems and the technical problems faced by these systems. Second it gives an account of how this thesis helps to address some of those problems. Third, this chapter gives an overview of possible future research directions with which to extend this thesis.

1.8 Contributions
The following list is a point summary of the major technical contributions made in this thesis.

☆ A Hybrid MMSE design technique which combines the performance advantages of both LE and DFE without a SER degradation with respect to the MMSE-DFE’s SER performance.

☆ A Modified MMSE-DFE design is presented which does not rely on making the correct past decision assumption. A simulation study is given which shows the extent of the conventional MMSE-DFE design’s suboptimality compared to the actual MMSE-DFE design.

☆ A new FS-DFE structure using a vector quantizer is investigated and shown to be equivalent to the baud spaced DFE (BS-DFE) structure if the design is made under correct past decision assumption. A new quantizer based on the MAP criterion is introduced which takes into account the error propagation in DFE. It is shown to exhibit superior performance.

☆ A soft decision quantizer used in lieu of the nearest neighbor quantizer is shown to give improved error propagation characteristics in comparison to
the standard (nearest neighbor) hard decision quantizer.

CHAPTER 2

REVIEW OF DECISION FEEDBACK EQUALIZATION TECHNIQUES
CHAPTER 2

REVIEW OF DECISION FEEDBACK EQUALIZATION TECHNIQUES

This chapter provides an in depth review of decision feedback equalization. The idea is to present necessary background material and also to develop the reader’s intuition about the operation of a DFE and the problems which arise. The important papers in the 30 year history of decision feedback equalization are highlighted. The main breakthroughs in the theory and practice of decision feedback equalization over this period are mentioned to facilitate further research.
2.1 Decision Feedback Equalizer

Figure 2.1: The equivalent channel model and a decision feedback equalizer for it. In this case, both the feedforward and feedback filters in the DFE have 3 tap weights and both are symbol spaced. The decision device is typically a nearest neighbour scalar quantizer.

The decision feedback equalizer (DFE) consists of a feedforward filter, a feedback filter, a summer and a decision device as shown in Figure 2.1. The input to the feedforward filter is the equivalent channel output $y_k$. The feedforward filter is identical to the linear transversal equalizer (LTE) (discussed in Section 1.4), but in the DFE context it is used to remove precursor\(^1\) intersymbol interference (ISI). The input to the feedback filter is the sequence of decisions made by the decision device on $z_k$. The feedback filter is used to cancel the postcursor ISI. Figure 2.2 shows an example of a discrete time channel impulse response and Figure 2.3 shows the corresponding cancellation of precursor and postcursor ISI.

The basic idea behind decision feedback equalization is that once an information symbol has been detected, the ISI that it induces on future symbols can be

\(^1\) Precursor ISI is defined as the interference from future data symbols.
eliminated and subtracted out before subsequent symbols are detected.

It is very desirable to reduce the DFE complexity for future wireless mobile terminals. The most complex part of a DFE, in terms of implementation, is the linear feedforward transversal filter that forms the first part of the equalizer. One reason for this is that the remainder of the equalizer, comprising the threshold-level detector and the feedback transversal filter, does not in fact require the use of multipliers. This is because the detected data symbols take on only a finite number of values (such as ±1 for a binary signal and ±1, ±3 for a 4-PAM signal) and there are only a finite number of feedback filter taps. Consequently (in the non-adaptive case), the feedback transversal filter can be implemented with a look-up table.

A useful reduction in the complexity of a decision feedback equalizer can be achieved by reducing the number of feedforward taps [16], as long as it does not require a great increase in the number of taps in the feedback filter to achieve equalization. The number of feedforward filter taps must be minimized, since it dictates the complexity and training requirements. Three factors contribute to the reduced number of feedforward taps chosen. Firstly, the length of the feedforward filter need not grow linearly with the channel dispersion. In an approximate
2.1 Decision Feedback Equalizer

Figure 2.3: (a) Feedforward filter taps. (b) Feedback filter taps. (c) The equalization done by the feedforward filter acting upon the precursor ISI. The feedforward filter minimizes phase, reducing the precursor (d) The overall effective impulse response of the equalized signal after the postcursor ISI has been equalized as well by the feedback filter.
2.1 Decision Feedback Equalizer

Figure 2.4: A 16-QAM constellation at 12dB input SNR equalized with a [5 5] FS-DFE.

sense, when the length of the feedback filter is sufficient, the feedforward filter is used to capture multipath energy. As the delay spread increases, a fixed-length filter may not be able to span the entire impulse response, but the receiver can take advantage of the increased number of resolved paths to achieve better multipath diversity. As a result, we usually find that a DFE with a fixed-length feedforward filter can give relatively stable performance over a broad range of delay spread [6]. The other two factors contributing to a low number of required feedforward taps are the timing recovery technique and the tap-selectable structure\(^2\) proposed in [3]. Both timing recovery and tap selection are performed prior to equalizer training and are based on the use of channel estimates.

Unfortunately, with severe amplitude distortion, more taps may be required in both the feedforward and feedback filters of the DFE. At high transmission rates this may lead to an unacceptable increase in the cost of the equipment [16].

When a multilevel data signal is transmitted and the amplitude distortion in

\(^2\) A technique for optimization of the spacings between different feedforward taps so that, when the channel consists of sparsely distributed multipath, the taps can also be sparsely assigned to wherever the energy is concentrated [6].
the received data signal is not too severe, a DFE gives a relatively good tolerance to additive white Gaussian noise (AWGN), which is approximately 2 to 3 dB [16, page 2] below that attainable by the corresponding MLSD. However, when a binary data signal is transmitted and there is severe amplitude distortion in the received data signal, the tolerance of a DFE to noise may be considerably below that of MLSD [16]. However the complexity of the ML algorithm is roughly $M^4$ times that of DFE, where $M$ is the constellation size and $l$ is the length of the channel impulse response [23].

The feedforward filter in the DFE can be implemented in either one of two forms:

- Baud spaced\(^3\)
- Fractionally spaced

For baud spaced equalizers, the channel is sampled at the rate $\frac{1}{f}$ and the equalizer is sensitive to distortion near the frequency $f = 0.5T$ caused by delay distortion or a poor choice of sample timing phase. This problem can be solved by using fractionally spaced equalizers, where the taps are separated by a fraction of a symbol period. The fractionally spaced tapped delay line is shown in Figure 2.5. More taps are now needed to span the length of the significant ISI, but the resultant structure tends to be less sensitive to timing errors. Figure 2.6 compares the performance of the baud spaced (BS) DFE to the fractionally spaced (FS) DFE on a specific channel. It shows a typical comparison whereby the FS-DFE outperforms the BS-DFE [12].

When fractionally spaced or symbol spaced equalizers are implemented with fixed point arithmetic and a limited word length, the inaccuracies in tap settings cause the performance to degrade.

A DFE designed with a fractionally spaced feedback filter has not been considered previously in the literature, and this is the subject of Chapter 5.

\(^3\) The baud rate is just another term for symbol rate named after the French telegraph engineer Baudot [2].
2.2 Optimization of DFE Filter Coefficients

The most meaningful measure of performance for a digital communication system is the average probability of error. However this criterion is a highly non-linear function of equalizer coefficients and does not admit a closed form solution to any optimization [24]. Therefore, the probability of error as a performance criterion to optimize the tap weight coefficients of the equalizer is difficult to use. As a consequence the two basic methods of determining the optimum parameter values for the linear filters in the DFE (as with the linear equalizer) are the zero-forcing (ZF) design criterion and the minimum mean square error (MMSE) criterion.

Zero Forcing Criterion: In this case, the tap gains are computed to force the overall sampled impulse response after equalization to have minimum ISI. For an infinite tap equalizer, the sampled frequency response of the equalizer should be the inverse of the frequency response of the channel. The drawback of the zero-forcing criterion is that it introduces significant noise enhancement [12].
2.2 Optimization of DFE Filter Coefficients

Figure 2.6: The figure shows improvement in SER performance of the FS-DFE over the BS-DFE versus SNR for 256-QAM modulation, and the channel $H = [0.4, 1.0, -0.75, 0.6, 0.45]^T$. 4 feedforward taps and 5 feedback taps are used by both equalizers.
2.2 Optimization of DFE Filter Coefficients

**Minimum Mean Square Criterion:** To minimise noise enhancement additionally, the minimum mean square error (MMSE) taps are computed to minimize the mean square error between the received and the transmitted symbols. The tap gains are computed to minimize both channel distortion and additive noise, and so the frequency response of the equalizer depends on the noise statistics. Most equalizers for fading radio channels are designed to be MMSE equalizers [12]. Based on classical equalization theory the most common cost function is the mean square error between the desired signal and the input to the decision device [15].

### 2.2.1 Mathematical Derivation for Optimum DFE Filter Coefficients

In this section we will present the solution to the problem of the joint optimization of feedforward and feedback filter taps under the MMSE criterion assuming CPDs. This mathematical formulation is the basis of a generalization in Chapter 4. Further, the form of the MMSE solution will give important insight into some of the potential deficiencies of the MMSE criterion.

The data symbol sequence denoted by \( \{ a_k \} \), consisting of independent symbols from the alphabet \( A \), are passed through a linear non-ideal, band limited channel. This channel will be represented by a convolution matrix given by

\[
H = \begin{bmatrix}
    h_0 & 0 & \ldots & 0 \\
    h_1 & h_0 & \ldots & 0 \\
    \vdots & h_1 & \ddots & 0 \\
    \vdots & \vdots & \ddots & h_0 \\
    h_{N_h} & \vdots & \ddots & h_1 \\
    0 & h_{N_h} & \ldots & \vdots \\
    0 & 0 & \ddots & \vdots \\
    0 & \ldots & 0 & h_{N_h}
\end{bmatrix}
\tag{2.1}
\]

where \( h_0, \ldots, h_{N_h} \) are the tap weights of the channel filter. This channel matrix has a Toeplitz structure. The channel output samples are further corrupted by
2.2 Optimization of DFE Filter Coefficients

AWGN which is independent of the data. Thus the received signal plus noise \( \{ n_k \} \) is processed by:

1. a baud spaced feedforward filter with \( N_f \) taps, \( f_i \), represented by

\[
\sum_{i=0}^{N_f-1} f_i z^{-i}
\]

or equivalently in vector notation

\[
F = [f_0, f_1, \ldots, f_{N_f-1}]'.
\]

(2.2)

2. a baud spaced feedback filter with \( N_d \) taps, \( d_i \), represented by

\[
\sum_{i=1}^{N_d} d_i z^{-i}
\]

or equivalently in vector notation

\[
D = [d_1, \ldots, d_{N_d}]'.
\]

(2.3)

The received symbol sequence, denoted by \( \{ z_k \} \) (which is also the equalized signal), is further processed by the nearest neighbor quantizer denoted by \( Q_A(z) \) to get the output symbol denoted by \( \hat{a}_k \). The objective is to minimize the mean square deviation of point \( z_k \) away from some nominal target \( a_{k-\Delta} \), written mathematically as \( E\{(z_k - a_{k-\Delta})^2\} \). This is the mean square error (MSE) where \( \Delta \) is some arbitrary delay. The performance index for the MSE criterion is denoted by \( J \) and is given as

\[
J = E\{(z_k - a_{k-\Delta})^2\}
\]

(2.4)

which is a function of the receiver parameters \( F, D \) and \( \Delta \).

Considering the definitions above, Figure 2.1, and using the correct past decision assumption, it can be shown that the input to the nearest neighbor quantizer \( Q_A(z) \) is

\[
z_k = (HF)'A_k + F'N_k - D'A_k
\]

(2.5)
2.2 Optimization of DFE Filter Coefficients

where $\mathcal{H}F$ is the convolution of the channel filter and $F$, which has the length $N_t \triangleq N_f + N_h$,

$$A_k \triangleq [a_k, a_{k-1}, \ldots, a_{k-N_t+1}]'.$$

(2.6)

$N_k$ is the Gaussian noise vector

$$N_k \triangleq [n_k, n_{k-1}, \ldots, n_{k-N_t+1}]'.$$

(2.7)

with zero mean and independent of the input sequence $\{a_k\}$, and $D'A_k$ represents the contribution of the feedback filter. Here $D$ is a zero padded version of $D$ so that it is compatible with the dimension of $A_k$ (i.e., $N_t$). Further, we have implicitly applied the correct past decision assumption in $D'A_k$.

The relationship between $D$ and $D$ can be expressed through

$$-D'A_k = -A_k' D = -A_k' M'D$$

where

$$M = (0_{N_d \times \Delta} I_{N_d \times N_d} 0_{N_d \times N_t - \Delta - N_d}).$$

(2.8)

The $M$ matrix is designed to mask all the input symbols except those that are to be multiplied with the limited DFE taps given as $D$. Thus it plays the role of a masking or indexing factor.

Let $T_\Delta \triangleq [0, 0, \ldots, 0, 1, 0, \ldots, 0]'$ where 1 is in the $(\Delta + 1)^{th}$ position. Then

$$a_{k-\Delta} = T_\Delta A_k$$

and, therefore, the error is

$$z_k - a_{k-\Delta} = \{(\mathcal{H}F)' - D'M - T_\Delta \}A_k + F'N_k.$$

(2.9)

Equation (2.9) is written as

$$z_k - a_{k-\Delta} = \begin{bmatrix} A_k' \mathcal{H} + N_k' & -A_k' M' \end{bmatrix} \begin{bmatrix} F \\ D \end{bmatrix} - A_k' T_\Delta.$$

(2.10)
Let

\[ \xi_k = \begin{bmatrix} A_k' \mathcal{H} + N_k' & -A_k'M' \end{bmatrix} \]  

so that the mean square error is given by

\[ J = E\{(z_k - a_k - \Delta)^2\} \]

\[ = E\{F' \xi_k \xi_k' F \} + E\{T \xi_k \xi_k' T\Delta - 2E\{F' \xi_k \xi_k' \} + E\{\xi_k \xi_k' F' \} \} \]

Let \( C = \begin{bmatrix} F' & D' \end{bmatrix} \). Then the minimum mean square error (MMSE) is obtained when

\[ \frac{\partial J}{\partial C} = 2E\{\xi_k \xi_k' \} C - 2E\left\{ \begin{bmatrix} \mathcal{H}'A_k + N_k' & -A_kM' \end{bmatrix} \right\} T\Delta = 0 \]

where

\[ E\{\xi_k \xi_k' \} = E\left\{ \begin{bmatrix} \mathcal{H}'A_k + N_k' \end{bmatrix} \right\} E\left\{ \begin{bmatrix} A_k' \mathcal{H} + N_k' & -A_k'M' \end{bmatrix} \right\} \]

\[ = E\left\{ \begin{bmatrix} \mathcal{H}'A_kA_k' \mathcal{H} + \mathcal{H}'A_k'N_k + N_kA_k' \mathcal{H} + N_kN_k' - (\mathcal{H}'A_kA_k'M' + N_kA_k'M') & MA_kA_k'M' \end{bmatrix} \right\} \]

This can be simplified by applying the independence of the data

\[ E\{A_kA_k'\} = \sigma_s^2 I, \]

where \( \sigma_s^2 \) is the symbol variance, the independence of the noise,

\[ E\{N_kN_k'\} = \sigma_n^2 I \]

where \( \sigma_n^2 \) is the noise variance, and the independence of data and noise, which implies \( E\{A_k'N_k\} = 0 \).
The optimum joint filter $C_{opt}$ is given as

$$C_{opt} = \begin{bmatrix} F_{opt} \\ D_{opt} \end{bmatrix} = \begin{bmatrix} \mathcal{H}'\mathcal{H} + \lambda I & -\mathcal{H}'\mathcal{M}' \\ -M\mathcal{H} & MM' \end{bmatrix}^{-1} \begin{bmatrix} \mathcal{H}' \\ -I \end{bmatrix} T_\Delta \tag{2.17}$$

where

$$\lambda = \frac{\sigma_n^2}{\sigma_w^2}. \tag{2.18}$$

The point here is that the MMSE has a closed form solution. (This solution has a simple generalization to the fractionally spaced case also.) This closed form only comes about because of the assumption of correct past decisions. In Chapter 4 we address how the solution changes when such an assumption is relaxed.

### 2.3 Adaptive Equalization

Adaptive equalization is very important for time varying channels. However, this thesis does not examine or implement these algorithms because its focus is on understanding the DFE in terms of its design procedures and philosophies. This understanding is incomplete even when the channel is perfectly known, and therefore a discussion on time varying channels and adaptive algorithms becomes rather irrelevant.

In most communication scenarios, the channel dynamics are unknown a priori, or, in cases such as the mobile radio channel, the channel characteristics are time-varying. Therefore, the equalizer’s coefficients must be adapted so that they can converge to a solution that satisfactorily reduces the SER and can also track the channel as it varies in time. Adaptation is either during a training sequence to achieve channel identification or via blind adaptation.

#### 2.3.1 Adaptive Equalization Algorithms

In this section we present algorithms for automatically adjusting the equalizer coefficients to optimize a specified performance index and to adaptively compensate for time variations in the channel characteristics.
2.3 Adaptive Equalization

The optimal tap weight setting involves measuring the instantaneous channel impulse response and the noise variance, forming a covariance matrix, and then solving a large system of linear equations. These extensive calculations tend not to be feasible in real time. Therefore, the simple LMS algorithm is often applied to incrementally adapt the weights towards this optimal solution. However, it typically requires many symbols to converge, prompting much overhead per equalized packet. Designers implement this estimator by applying the steepest decent algorithm to minimize the mean square error.

The recursive least squares (RLS) algorithm allows the taps to converge to their values quickly, thus introducing minimal overhead. Unfortunately it is also complex, requiring many computationally-intensive multiplies per sample. The complexity of the RLS algorithm is proportional to $N^2$ whereas the LMS complexity is proportional to $N$, where $N$ is the total number of taps used in the DFE.

2.3.2 Blind Equalization Algorithms

In some cases, radio modems need training when the channel between the transmitter and the receiver changes. Often, the training sequence is not repeated at that time, so that the equalizer has to train on unknown symbols, in a technique referred to as blind equalization. Falconer [25] introduced a method that uses the square of the error signal to adjust equalizer tap weights.

A more recent class of adaptive algorithms is able to exploit the characteristics of the transmitted signal and thus does not require training sequences. These algorithms are able to acquire equalization by adaptively attempting to restore known properties of the transmitted signal, and are called blind algorithms because they provide equalizer convergence without burdening the transmitter with training overhead. These techniques include algorithms such as the constant modulus algorithm (CMA) and the spectral coherence restoral algorithm (SCORE). CMA is used for constant envelope modulation, and guides the equalizer weights...
to achieve, as close as possible, a constant modulus received signal. Although blind equalization is robust and does not require much overhead per packet, it is an order of magnitude slower than LMS to converge.

One of the applications of blind adaptation is in broadcast communication systems, where adaptation must be done without the aid of the transmitter. Again, due to the nonlinear feedback nature of DFEs, blind adaptation may be prone to false convergence [26].

### 2.4 Error Propagation

![Figure 2.7](image)

**Figure 2.7:** An illustration of the bursty nature of errors made by a decision feedback equalizer in the presence of error propagation. The figure shows a sample of error sequences versus time for 16-QAM at a moderate noise level. Soft errors (before quantization) are shown by 'x', and hard errors (the difference between detected and actual symbols) are shown by 'o'. Any non-zero hard error implies a symbol error. Generally an initial error is caused by noise (called the primary error). That subsequent symbol errors follow a primary error and are grouped into bursts (called secondary errors) illustrates the phenomenon of error propagation.
2.4 Error Propagation

![Figure 2.8](image)

Figure 2.8: This figure zooms in on a long error burst for a 64-QAM constellation and a moderate level of noise. Under such severe error propagation, the performance of a decision feedback equalizer is very poor.

In this section error propagation, the major source of performance degradation in DFEs, is discussed. Subsequent chapters in the thesis will refer to the concepts described in this section, therefore it was important to flesh out these concepts here.

The feedback filter is designed to cancel the residual ISI at the output of the feedforward filter. In fact, when the quantizer makes an incorrect decision, the nominal ISI correction of the feedback filter becomes flawed for future decisions and actually increases the risk of further errors. The result is, a single error causes a reduction in the margin against noise for a number of future decisions [2], i.e., the soft decision error \((a_k - \hat{a}_k)\) induces a hard decision error \((a_k - \hat{a}_k)\). The hard decision error acts as a “primary error” causing a burst of errors called “secondary errors”. This phenomenon is known as error propagation and results in an error rate greater than would be predicted on the basis of SNR calculation alone [2].

Error propagation is major challenge in the DFE approach. In practical wire-
less systems, error correction coding is a critical component for achieving reliable communications with high frequency reuse. Most coding schemes are designed to correct random, isolated errors. A DFE on the other hand tends to produce errors that are bursty in nature (see Figure 2.7), due to the fact that the DFE relies on delay-free hard decisions before decoding in order to cancel ISI. The resulting error propagation severely limits the achievable coding gain of any coding technique. In order to remedy the situation, a number of techniques have been proposed in the past, including: (i) transmitter precoding [19], (ii) the use of interleaving to introduce the delay necessary for reliable feedback [20, 27], (iii) the use of tentative soft decisions for feedback [28, 29] and (iv) a soft-decision feedback approach [5]. Transmitter precoding requires a priori channel information which is not available in many wireless systems. The interleaving approach requires the DFE to be implemented in a noise-predictive form, with independent adaptations of feedforward and feedback filters. From a performance perspective the noise-predictive structure is not recommended for a low complexity DFE with a short feedforward filter. More details on this scheme are given in Section 2.5.

There are two main methods for studying the error propagation in DFEs:

- Markov Chain Analysis
- Stability Analysis

In Chapter 6, we use nonlinear stability analysis using passivity theory to study error propagation. A complete literature review on these techniques is given in Section 2.7.

2.5 DFE in Coded Modulation Systems

Although coded modulation systems are not the topic of this thesis, they are an area of continuing research effort by many investigators. Hence a bri
introduction of the topic is desirable in the context of this chapter as it introduces the decision feedback equalization.

Coded modulation and adaptive equalization have proved to be extremely effective in overcoming the two major impediments to high speed data transmission on bandlimited channels, i.e., noise and ISI. There has been a considerable amount of research over the last decade, and it has produced effective coded modulation schemes [30, 31]. These coding schemes notably improve a system’s noise performance. On ISI channels however, coded modulation schemes have to be combined with effective equalization techniques to achieve reliable high data rate transmission.

Furthermore, it appears that, in coded systems, the DFE may be of central importance [20]. Long before coded modulation schemes were discovered, it was shown by Price [32] that on strictly bandlimited channels with sufficiently high SNR, the additional SNR necessary to reliably operate an uncoded PAM system at the channel capacity rate, using an ideal ZF-DFE, is independent of the channel spectrum. Indeed for many channels found in practice, e.g., the voice band telephone channel, including the Nyquist channel4, the dB gap between uncoded QAM with an ideal DFE and the channel capacity is approximately fixed at 9 dB (for $P_e \approx 10^{-6}$) [20]. Therefore, it appears that on many distorted channels capacity can be approached by using a powerful coded modulation scheme designed for Nyquist channels, combined with the equalization power of the DFE [20]. However this coding/equalization combination may not be realized in a straightforward manner, since, in general, the DFE requires delay-free decisions for feedback and in coded systems such decisions are not sufficiently reliable.

Most coding techniques are designed to correct random independent errors. A DFE on the other hand produces errors that are bursty in nature, due to the fact that the DFE relies on delay-free hard decisions (before decoding) to cancel ISI in

4 When the channel has a flat amplitude response over the transmission band, Nyquist filtering equally split between the transmitter and receiver will produce a discrete equivalent channel free of ISI with an additive independent Gaussian noise sequence. This discrete equivalent channel is referred to as the Nyquist channel.
subsequent data symbols. Decision errors tend to trigger strings of improper ISI cancellation. The resulting error propagation severely limits the achievable coding gain of any coding technique [6]. A number of schemes have been proposed in the past to remedy the error propagation problem, notably Tomlinson-Harashima (TH) precoding (a detailed discussion of which was given in Section 1.6) and a technique called decision feedback noise prediction with interleaving (DFNPI) by Eyuboglu [20]. The DFNPI technique allows the combination of DFE with coded modulation schemes. A noise predictive DFE is combined with an interleaver/deinterleaver pair such that the feedback decisions during an error event are distributed over several decoding instances; the result is that the effect of error propagation is reduced. When sufficient delay in the interleavers can be tolerated, this technique can attain the conventional DFE’s performance. In [27] the same concept is applied to the MMSE-DFE so that the channels with spectral nulls can be taken into account as well.

2.6 DFE Applications

DFEs have applications in many areas of digital communications. Examples are high-speed data transmission over: digital radio, coaxial cable, subscriber loop, high performance optical fibre systems [33] and more recently in HDTV and other multimedia applications.

The behaviour of DFEs is quite distinct in two types of operating scenarios, depending on the SNR.

- In the first case, the channel to be equalized has strong ISI but low noise power or high SNR, as is the case in a high data rate microwave link. Here, the job of the DFE is to provide a very low symbol error rate (SER). Noise induced primary errors occur infrequently and may result in short bursts of errors. But these error bursts do not effect the symbol error rate significantly.
In the second scenario, the DFE operates in a low SNR environment in conjunction with a coded system. The DFE is used to reduce the ISI enough for the decoder to operate correctly and achieve very low BERs. This equalization scheme has been proposed for high definition television (HDTV) standards.

### 2.7 A Chronology of DFE Research

#### 2.7.1 Prehistory of the DFE

In the middle and late 1960s researchers were focusing on linear equalizers, both adaptive and non-adaptive, for equalization of linear dispersive channels. For the channels of that time, generally those exhibiting small distortion, linear systems performed well. This was the time when the interest in non-linear receiver structures was initially academic.

In late 1960s and early 1970s there was a shift from the ideal channel models of the past toward models which more closely resembled real communication links. Within the digital field, the most popular research area was channels with memory [34]. Thus the research took the direction of nonlinear techniques and researchers started to show that optimum receivers (which optimize various statistical criteria) for the dispersive channel are nonlinear [35]. In terms of the number of published papers, research on nonlinear systems was one of the most popular topics. There are several factors which contributed to the move from linear to nonlinear systems. The exhaustion of the linear model was becoming apparent, and some interesting results applying to nonlinear structures motivated further study. In practice an interest arose in partial response (correlative) signaling schemes, in which a large amount of ISI was intentionally introduced for spectrum control [34]. Furthermore, a trend toward higher symbol repetition rates led to more severe ISI. For these badly distorted pulses the linear receiver is often several dBs poorer than a optimum nonlinear receiver.
Statistically optimum nonlinear receivers are almost always very computationally complex when there is a large amount of ISI (i.e., a long overall channel impulse response). This suggested that one should seek a statistically sub-optimum receiver that is practical and has a performance that is significantly better than that of any linear equalizer and in some circumstances comparable to an optimum receivers’ performance. A variety of techniques were proposed, but aside from certain noteworthy but isolated papers, the solution which has stood the test of time was provided by Austin in his classic paper [36] in 1967. The solution was the decision feedback equalizer. Thus in the early 1970s the transition began from linear to non-linear (a performance improvement) and from statistically optimum nonlinear (MLSD) to suboptimal nonlinear (DFE) receivers (an implementation improvement). Following is a brief review of some of the major results in decision feedback equalization theory in the last three decades. It shows how the DFE structure has maintained its position as a favored equalization scheme for many applications.

2.7.2 DFE Research in the 1970s

Main DFE Researchers of the decade:

* Austin, Salz, Monsen, George, Price, Falconer, Foschini, Duttweiler, Messerchmitt, Ungerboeck, Cantoni, Butler

Key Results:

- adaptive DFE scheme
- explicit formula for MMSE-DFE
- geometric theory of equalization and ISI
- derivation of an upper bound on the probability of error
- fractionally spaced equalization
- stability concept for decision feedback inverses

Monsen made the first attempt to obtain an optimum DFE parameter setting by solving the MMSE-DFE optimization problem in 1971 [37]. Under the correct past decision assumption, DFE filters were optimized for a minimum mean
square error criterion in a slowly fading dispersive channel environment and shown to be superior in performance than their linear equalizer counterparts. A steepest descent adaptive algorithm was also developed in [37]. It requires neither knowledge of the channel nor the source statistics. The receiver, in addition to mitigating noise and ISI, also acts to eliminate timing jitter and Doppler shift. The results show that channel dispersion provides a beneficial diversity effect that improves the performance over the non dispersive case. Thus an adaptive DFE’s performance is shown to be better in the presence of multipath than without multipath because of the diversity gain potential [37].

Work had already been started on adaptive DFE schemes. One most notable work was in 1971 by George, Bowen, and Storey [38]. They did a comparative study and showed that the DFE was an attractive option due to its moderate complexity and good performance compared to the optimum schemes (which were only theoretically possible) and the transversal linear equalizers in use at the time. They showed that both fixed and adaptive decision feedback equalizers are superior in performance to linear equalizers. They used PAM modulation over a noisy dispersive linear channel in the performance analysis of all the above receiver types.

For the PAM data transmission system, an explicit formula for the MMSE-DFE was derived by Salz in 1973 [17] under the correct past decision assumption. This work augments and completes the earlier work done on the same problem by Monsen [37] which considered only binary signaling. Furthermore, it was shown that for most practical applications signaling at a rate higher than the Nyquist rate for zero ISI (while keeping the information rate fixed by decreasing the number of levels) increases the mean-square error.

Following Monsen and Salz, Falconer and Foschini (1973) optimized DFE filters via a matrix Wiener-Hopf analysis [39]. All previous theoretical studies of DFEs had assumed a baseband linear PAM channel model. In this paper the application of DFEs to QAM modulation was considered and it was shown
that the treatment of decision feedback equalization for two or higher dimensional signals is a nontrivial generalization of the baseband signal case.

Messerschmitt gave a geometric theory of equalization and intersymbol interference for the PAM communication system in [40] (1973). It is shown that the geometric theory approach gives a unified mathematical framework for linear and decision feedback equalization. An equivalence between the structure of ISI and a wide-sense stationary discrete random process was demonstrated. On the basis of this equivalence a further equivalence was demonstrated between zero-forcing (decision feedback) equalization and MSE prediction of a random process.

Results from prediction theory are used to develop computational methods for determining the tap weights of the IIR equalizers for both rational and non rational channel power spectra. Furthermore, the theory of reproducing kernel Hilbert space is used to develop a theory of equalization for nonstationary channels and noise.

Salazar in [41] (1973) used a maximum SNR criterion to optimize DFE filter coefficients for both baseband PAM and QAM modulation. Ignoring error propagation in DFEs, it was shown that the DFE requires fewer taps for acceptable system performance than the linear equalizer. The problem of postcursor size is shown to diminish in importance when a hybrid equalization procedure - a combination of LE and DFE - is used. The price is a lower SNR for this hybrid design procedure. The idea behind this hybrid design was that some of the linear equalizer's degrees of freedom will be used to combat some postcursor ISI although decision feedback is still used.

It should be noted that, in [41], the design is derived ignoring error propagation. In Chapter 3 of this thesis, we have developed a somewhat similar hybrid MMSE scheme, but the fundamental difference lies in the fact that we have taken error propagation inherent in the feedback path of the DFE into account. The design given in Chapter 3 was developed without the knowledge of [41] by Salazar.

Reference [42] by Duttweiler, Mazo, and Messerschmitt in 1974 was the
second of the two initial papers which successfully addressed the topic of **error propagation in the DFE** (the first being [43]). Austin pointed out in [36] that error propagation can be modeled as a discrete Markov chain process. Duttweiler *et al.* noted in [42] the limitation of Markov chain process for the calculation of error probabilities of the DFEs and pointed out that it is feasible for only very short feedback loops (3 to 4 past decisions).

Duttweiler *et al.* derived an **upper bound on the probability of error** of the decision feedback equalizer which applies to any arbitrary equalized channel even when the number of past decisions entering into the current decision is large or possibly infinite. Successive noise samples are assumed to be independent which is true when the equalizer is designed under the zero forcing criterion and the channel noise is stationary and Gaussian [42]. The question of a **tight lower bound on the error probability** is left open for further research and it was tackled much later by Kennedy [44] in 1987.

In August 1976 Ungerboeck [45] presented a novel equalizer structure called a **fractionally spaced equalizer** as a cure for clock-phase recovery in adaptive equalizers. The adaptive equalizer’s performance critically depends on the symbol-clock phase recovery in the receiver by means of an open loop clock phase recovery scheme such that aliasing signal components lead to addition and not the subtraction (spectral nulls) in the spectral roll-off regions.

Ungerboeck investigated the possibility of eliminating the need for clock phase recovery by proposing an equalizer with a tap spacing smaller than the symbol period $T$. He showed that the new equalizer scheme leads to satisfactory performance without the overhead of any clock-phase recovery hardware. He called his equalizer the fractionally spaced equalizer as the feedforward filter tap spacing is only a fraction of the symbol period $T$. It was also shown that for the same length of equalizer, a fractionally spaced equalizer converges to a minimum MSE value at practically the same speed as a $T$-spaced equalizer.

The feedback part of the DFE can be viewed as an approximate inverse of
the convolution of channel impulse response and feedforward filter. However its presence in the system makes the stability consideration difficult. Cantoni and Butler in [46] (1976) formulated a stability concept for this sort of inversion by deriving sets of necessary and sufficient conditions for stability.

A stable system is defined as one which will necessarily recover from any error condition in finite time. It is pointed out that unless certain necessary stability conditions are satisfied, there exist input sequences whose application would ensure that the system never recovered from error. That makes the system potentially unstable. However, for data communications, even unstable systems which are ensured to recover from error in finite time given random input sequences are likely to be highly suitable. So the main point of concern is not the stability of the system but the error recovery time for the random input sequences which are used in data communications. Cantoni and Butler in [46] showed that even an unstable system driven by a random input will recover from error with probability 1. An upper bound on the mean recovery time from the error condition and a lower bound for the probability of recovery from error are obtained [46].

In the late 1970s maximum-likelihood sequence detection implemented by the Viterbi algorithm (VA) emerged as an ultimate solution to the detection problem in a dispersive channel environment. However, the complexity of such a receiver grows exponentially with the length of the channel. Various approaches have been adopted by the researchers to reduce the complexity of MLSD through the years.

Most of the earlier work concentrated on ignoring the tails of the overall channel impulse response model and thus working with one with a shorter length (i.e., direct truncation). Another approach used by the researchers at the time (Forney, Qureshi, Newhall et al. and the references in [47]), is to use a linear equalizer as the prefilter in order to shape the channel impulse response, and then to employ Viterbi decoding to this partially equalized channel. The idea of using
A decision feedback equalizer implemented with the Viterbi algorithm was first proposed by Qureshi [48]. Later on in 1977, Lee and Hill presented the idea with more detail in [47]. They call the structure a DFE-MLSE. A DFE is proposed as a prefilter to a MLSD using the VA for channels having long impulse responses. The total impulse response of the equalized channel is truncated to a shorter length. With this receiver a choice can be made between complexity and the performance by choosing the length of the impulse response. Results given for a single pole channel show that the proposed scheme can reduce the complexity of the receiver while retaining much of its performance advantage.

In addition to the research on hybrid optimum-suboptimum receivers as explained above, some work was done on adaptive DFE techniques for multipath channels. In [49] an adaptive equalization technique was developed by Monsen (1977) for use on tropospheric scatter channels at data rates up to 12.6 Mbps. A multichannel DFE provides the functions of diversity combining, elimination of ISI effects and coherent recombining of the fading multipath components. “These functions are accomplished in a time-varying environment with rms Doppler spreads up to 10 Hz and specular Doppler components such as air plane reflections up to 150 Hz” [49]. In this paper Monsen develops a theoretical approach for the calculation of the average bit error rate in a fading multipath channel. Measured and calculated bit error rate performances are shown to be tightly tied together [49].

In [50] and [51], which is a two part paper by Falconer and Magee, QAM systems for 12 kbps and 14.4 kbps transmission were studied. It was shown that a Viterbi detector is less affected by linear distortion than the decision feedback receiver but is more sensitive to the channel’s phase jitter. Therefore a DFE is a reasonable compromise on voice band channels where the phase jitter may be substantial.

In 1977, [52] was one of the first papers in which the timing phase jitter problem was discussed in the DFE context. Salz studied the the effects of
sampling phase on the baud spaced MMSE-DFE. It was known that DFEs are less sensitive to the effects of timing phase than linear equalizers but no analytical reasoning was available. It was shown in [52] that this phenomenon is primarily due to the manner in which the aliased signal components add to the Nyquist band component. In systems with excess bandwidth, the aliased components can add destructively, resulting in deep nulls at the band edges. This adverse effect is better compensated by DFEs. It should be noted that the earlier work by Ungerboeck showed that the performance of fractionally spaced equalizers is independent of phase jitter.

A good summary of equalization techniques in the 1970's, especially DFEs, is given by Belfiore and Park [53]. It can be regarded as a good tutorial paper on the topic. In short the 1970s was the decade when decision feedback equalization matured as a major equalization technique for dispersive channels. There were still source of impaired performance in DFEs, such as error propagation. The problem of error propagation and the study of error recovery properties are crucial and attempts were made to understand these concepts in 1980s. Along with these problems, researchers continued to apply the DFE in different communications scenarios such as coded modulation systems, etc. The discussion on these topics comes in the next section.

2.7.3 DFE Research in the 1980s

Main DFE Researchers of the decade:
*Clark, Kennedy, O'Reilly, Eyuboglu, Qureshi, Heegard, Duel-Hallen.*

Key Results:
- hybrid DFE-MLSE structures
- characterization of error propagation
- error recovery properties of DFEs
- DFEs with coded modulation

In the mid 1980s, the long pending problem of characterizing the error propagation in DFEs was tackled by many researchers, prominent among those
being O’Reilly and Duarte [33, 54] and Kennedy, Anderson, and Bitmead [44, 55, 56]. The following paragraphs give an account of these works.

Since [42] by Duttweiler et al. in 1974, no major results appeared on the characterization of error propagation in DFEs until 1985 when O’Reilly and Duarte addressed the problem in more detail in [33]. The limitation of the previous work by Duttweiler et al., as pointed out in [33], was that the error probability bounds took into account only the postcursor ISI. Furthermore, binary signaling over an AWGN channel was assumed and little account of the precise form of the received signal was given. In [33] the work was extended and new bounds on error propagation statistics for DFEs were obtained by using state transition diagrams. They employed state reduction procedures to reduce the complexity. The bounds were applied to full-response and partial-response systems of practical interest. For a long ISI duration, the complexity of the state transition diagrams is high and further simplification becomes necessary as discussed in [54] (1987).

In the late 1980s Kennedy, Anderson, and Bitmead did some major work [44, 55–57] on the characterization of error recovery properties. They introduced several classes of channels with common properties. Some classes are of value in practice, though others are more interesting theoretically only.

It is shown in [55] (1987) that it is possible to define the kinds of channels for which the error performance of DFEs is much better than that given by Duttweiler, Mazo, and Messerschmitt in [42] or O’Reilly and Duarte in [33, 54]. The analysis in [55] is restricted to exponential impulse response channels in a high SNR environment assuming binary signaling. An upper bound on the error recovery time of DFEs is given and is shown that exponential impulse response channels form a favorable class of channels with regard to DFE error recovery time.

Furthermore, in [56] the error recovery properties of DFEs over noiseless channels are analyzed and the way channel parameters affect the stochastic
dynamics of DFEs is presented. It is found that the nonlinear decision device in DFE receivers has the effect of partitioning the channel parameter space into a finite number of sets. Some important non-adaptive properties of DFEs can be classified by examining these partitions: namely: (i) the error recovery time statistics, (ii) the input data sequence which results in arbitrarily long recovery times, and (iii) the identification of channels for which it is inappropriate to use DFEs. The discussion in [56] is extended to include noisy channels in [44]. A subclass of such channels is found to realize the worst-case error probability performance of DFEs. The question of the tightness of the DFE error probability bound, as raised by Duttweiler [42] in 1974, is answered in [44] (1987).

In [57] a nontrivial class of channels with severe ISI is defined for which the error recovery time is rapid and finite regardless of the specific statistical model of the input sequence and the initial conditions. These channels form a class of channels for which the DFE may be used effectively in practice. A previous result given in [56] is revisited to show that the channel’s minimum phase property and strict passivity leads to satisfactory recovery from error in DFEs.

In addition to work on error recovery properties, some efforts were made by different researchers on the problem of reducing the complexity of the optimal detectors. Various techniques were investigated in this regard, e.g., [58, 59]. Most of the earlier work concentrated on preprocessing techniques to reduce the channel impulse response to a shorter length. Using DFEs to achieve this end was proposed by Lee and Hill and was called the DFE-MLSE structure in [47] (as discussed earlier). A similar idea was used in [58] for QAM modulation much later in 1987.

In [59] Clark, Lee and Marshall studied the incorporation of the DFE structure with an MLSD as the decision device. The Viterbi algorithm and various reduced complexity versions such as the Reduced State Viterbi Algorithm [23] are used as possible variations of the basic DFE-MLSD structure. The aim is to study these generalizations through simulations to investigate the improvement.
2.7 A Chronology of DFE Research

in tolerance to AWGN for different levels of ISI. It is found that an improvement in noise tolerance can be achieved at the expense of an increase in complexity. Reference [59] was one of the first major papers in the 1980s to investigate the DFE-MLSD type of scheme. So the trend which started with Qureshi in [48] continued.

Some non-linear equalization (DFE) techniques were presented in [16] by Clark. They are suited to high data rates (of the order of Mbps), a binary signaling scheme and a channel with severe amplitude distortion of the received data signal. A conventional design requires a large number of filter taps in such a situation, and this increases the implementation cost and complexity. Furthermore, the equalizer settings are no longer optimum and can lead to poor performance. Improved performance can be achieved by modifying the tap weights of the equalizer without any alteration in the basic structure of the equalizer. It is shown in the paper that by replacing the linear feedforward filter by a simple two-tap filter, performance can be either improved or at least there will be no degradation in performance whereas the cost of implementing a long feedforward filter can be significantly reduced.

Duel-Hallen and Heegard proposed a detection algorithm which provides a direct tradeoff between complexity and performance over linear dispersive channels [60,61] in 1985. They call the algorithm delayed decision-feedback sequence estimation (DDFSE). The complexity of a DDFSE algorithm can be varied by varying a parameter from zero to infinity. At zero, the DFE is realised; and at higher values a VA detector is realized, with increased complexity but improved performance. Later Eyuboglu and Qureshi used a similar kind of idea in [23].

On ISI channels, the coded modulation schemes (which reduce the effect of noise) have to be combined with equalization techniques to achieve reliable high speed transmission. But the combination may not be straightforward, as the DFE requires delay free decisions. In coded systems such delay free decisions
are not sufficiently reliable. In [20] a new technique was proposed by Eyuboglu called decision-feedback noise prediction with interleaving or DFNPI. It combines periodic interleaving with a noise-predictor in the DFE feedback loop so that the delayed decision obtained by the decoder can be made reliable enough to be fed back into the DFE. This work set the ball rolling for the future research in combined coding and equalization schemes.

2.7.4 DFE Research in the 1990s

Main DFE researchers of the decade:

Key Results:
- DFE receivers for CDMA systems
- New upper/lower bounds on the mean error recovery time
- DFEs in coded modulation systems
- Analysis tools for the FIR MMSE-DFE
- Channel-estimate based adaptive DFEs

The work in [27] by Zhou and Proakis extends the work done in [20] on DFEs in coded modulation systems. It is shown with the help of simulations that a MMSE-DFE adapted by the RLS algorithm in a coded modulation system can successfully combat severe ISI while maintaining a coding gain over the corresponding uncoded system. The performance of such systems is studied on both: voice band channels and linear dispersive channels with spectral nulls.

On the digital subscriber lines (DSL) used for ISDN, one of the major performance limiting factors is crosstalk. The MMSE-DFE performance is evaluated in [62] by Abdulrahman and Falconer for channel interference (CI), ISI and AWGN. It is found that both baud spaced and fractionally spaced DFEs have the ability to substantially suppress CI (FS-DFE being better in performance).

A DFE detector for a synchronous CDMA system was proposed in [63] by Duel-Hallen. The complexity is linear in the number of users and it requires one decision per user. Under the CPD assumption, the feedforward and feedback
filters are optimized to eliminate multiuser interference. The performance advantage of using decision feedback becomes profound in the case of weak users. For stronger signals the decision feedback is not involved and the best receiver design is just a decorrelator.

An adaptive fractionally spaced DFE receiver for a CDMA system is proposed by Abdulrahman, Sheikh and Falconer in [64] as an interference cancellation scheme. Two configurations of the system are used: one in which the DFE receiver knows the desired user’s spreading code and the other configuration in which it assumes no knowledge of any spreading codes. The analysis shows the advantage that this system has over other systems in terms of channel capacity.

Error recovery properties are revisited again in [65] by Beaulieu. New upper and lower bounds on the mean recovery times of DFEs are derived. “The method used to study error recovery times is based on writing difference equations for conditional, state dependent, mean recovery times leading to simpler expressions for the solutions” [65].

Optimal symbol-by-symbol detection can be obtained by a MAP probability criterion based on Bayes’s rule. In [66] by Chen, McLaughlin, Mulgrew and Grant this Bayesian equalization scheme was implemented with decision feedback equalization to gain a performance improvement over the conventional adaptive DFEs and adaptive MLSD for stationary and multipath fading channels, respectively. This equalization scheme is referred to as a Bayesian DFE. In terms of computational complexity, the adaptive Bayesian DFE is slightly more complex than the conventional DFE but is much simpler than the adaptive MLSD.

The Unbiased MMSE-DFE structure for coded modulation systems and ISI channels was defined and shown to be an excellent choice by Cioffi, Dudevoir, Eyuboglu and Forney in [67, 68]. It is shown that removing bias from the MMSE-DFE improves the probability of error performance but reduces the SNR [67]. The feedforward filter is realized as a mean square whitened matched filter. The feedback filter is implemented in the transmitter in order to
reduce error propagation in a coded system by means of precoding (Tomlinson-Harashima or trellis precoding). The relationship between channel capacity and the SNR value of the unbiased MMSE-DFE design is established for ISI channels. It is found to be equal to an ideal ISI free channel and independent of other channel characteristics.

Also in [67, 69], it is shown that it is probably not worthwhile using an MLSD to combat ISI in combined equalization and coding systems as long as precoding can be used. Furthermore, it is shown that optimising the feedforward and feedback filters has more influence on the system’s performance than optimizing the transmit spectrum, as long as it is not too far from optimum and the coding gain is large enough. These results are extended in [70] by Dhahir and Cioffi to a finite length MMSE-DFE by means of the Cholesky factorization and displacement structure theory.

Lee and Cioffi in [71] and Dhahir and Cioffi in [72] present non-recursive adaptive DFE algorithms for the computation of optimum DFE filter settings in an unknown channel environment. This is referred to as the channel-estimate based DFE.

In the channel-estimate based DFE, which is gaining popularity for DFE adaptation, the channel is estimated, the estimates are stored in the receiver and then the DFE coefficients are calculated indirectly or non-recursively. However, the non-recursive computation of equalizer settings can impose a signal processing overload compared to direct adaptation using the classical gradient search algorithms (i.e., LMS, RLS etc.). The advantage remains in the fact that the non-recursive algorithms avoid the eigenvalue spread problem that slows the convergence of LMS based adaptive algorithms, and thus provide better tracking performance than the LMS based methods.

In [72] a new computationally efficient algorithm for non-recursive adaptation was introduced for calculating the optimal filter settings of the FIR MMSE-DFE structure. It employs the analytical tools of Cholesky factorization and
displacement structure theory, introduced in [70] by the same authors. The work is further extended in [69] by including decision delay optimization, an aspect which was overlooked for the sake of simplicity in [72]. In [73] the same techniques are used for ISI, co-channel interference (CI) and colored noise cases. In [71] a similar problem was solved by Lee and Cioffi. However, no matrix operations are used and instead the discrete Fourier transform and inverse discrete Fourier transform are used for optimizing the filter settings.

2.8 Synopsis

The decision feedback equalization continues to draw the attention of the researchers. The popularity of DFEs in practice is because of their good performance (lying between that of linear equalizers and optimal detectors) and their structural simplicity (which challenges the linear equalizer). Through the years decision feedback equalization has gone through many developments, both in terms of analysis techniques and structural modifications. The DFE’s applications have ranged from single dimension signal constellations, to multidimensional constellations and then in the 1990s to coded modulation schemes; from steepest descent and recursive least squares adaptive algorithms to channel-estimate based algorithms; and from single user detectors to multiuser detectors.

We note from the account given on the DFEs in this chapter that there is a common thread in the design and analysis of DFEs and its generalizations, namely the correct past decision assumption. There is a dearth of results treating its true nonlinear form and the reality of error propagation. This sets the stage for the content of this thesis, where the correct past decision assumption is critically examined.

A considerable amount of basic DFE research is still required to analyze its performance completely. This will be the work of theoreticians. In practice the
DFE receiver continues to be an effective choice.
When applied to decision feedback equalization (DFE), the conventional minimum mean square error (MMSE) design approach is shown to be suboptimal in performance. In the first part of the chapter we argue that the MMSE-DFE design based on the correct past decision assumption gives a false sense of optimality on the basis that it negates the non-linearity of the DFE structure. We emphasize the philosophy behind the assumption and the limitations it can put on the system performance. In the second part of the chapter we analyse the adverse effects of the nonlinear recursive character of the DFE which makes its presence felt in the error propagation phenomenon, and a hybrid MMSE design criterion is developed. Our hybrid MMSE design approach is based on reducing the damaging effect of large taps in the feedback path by using the convex combination of two MMSE designs (the linear equalizer and the MMSE-DFE) so as to reduce the symbol error rate and to make the design more robust against error propagation.
3.1 Introduction

3.1.1 Correct Past Decision Assumption Revisited

There are two principal criteria which dominate communication system design: the probability of error (PE) criterion (of bits, symbols or sequences of symbols) and the minimum mean square error (MMSE) criterion. These criteria are often used to select the parameter values within the various signal processing structures. The first criterion is a highly nonlinear function of filter coefficients and often proves difficult to optimize. As a consequence the MMSE criterion is more commonly encountered.

The most salient features that contribute to the MMSE criterion’s attractiveness are:

- It is a quadratic function of the filter coefficients, generically leading to a unique solution.
- It permits a closed form solution using linear algebra.
- It shows negligible performance degradation when compared with the PE criterion for high signal to noise ratios, at least for well studied systems.

This work flags the idea that the MMSE equalization criterion cannot be used as effectively on nonlinear structures as it can on linear structures. In this work we focus on the decision feedback equalizer (DFE) (see Figure 3.1), as typical of the non-linear structures one can encounter. We demonstrate that the conventional MMSE approach may not actually give an optimal solution to the problem of minimizing the actual (measured) MSE [36].

There are further deficiencies in what is actually presented in the literature. A review of MMSE-DFE literature shows that it is based on the assumption that there are no decision errors being made at the decision device, thus treating all decisions made on past symbols as being correct. Thus linearity is induced by making the correct past decision assumption. As a consequence:
• it unnaturally regards the DFE as a linear device

• the feedback path is eliminated (by converting it to an equivalent forward linear path).

Having made the assumption that past decisions are correct, authors fail to establish (beyond limited simulation evidence) that the result of their MMSE design is actually compatible with their original assumption.

The mean square value of the error is given as \( E\left\{ (z_k - a_k - \Delta)^2 \right\} \) (see Figure 3.1). Under the correct past decision assumption, the signal \( z_k \) is an explicit function of the data \( \{a_k\} \), as will be revealed later, in contrast to the actual MMSE case where \( z_k \) is also a function of \( \{\bar{a}_k\} \) (which is an implicit function of the data \( \{a_k\} \)). This fundamental aspect can make the detailed studies into MMSE design, such as the one in [70], questionable. In short, they are generally establishing an optimal set of parameter values for a problem which does not make direct contact with practical systems.

The contribution of this chapter is that we demonstrate that the conventional MMSE approach, which relies on the correct past decision assumption to determine the optimal parameter values, can lead to inferior performance (although usually acceptable performance).

### 3.1.2 Reformulation of the MMSE Criterion

The ability to perfectly cancel postcursor channel intersymbol interference (ISI) without enhancing noise has made the decision feedback equalizer (DFE) a popular choice for a channel equalization device. In this chapter we reformulate the conventional MMSE criterion for designing DFEs. It is shown in the DFE literature that it is possible to combat precursor ISI by introducing a feedforward ISI filter before the feedback filter [36]. The main drawback of the DFE feedback structure is that the residual precursor ISI and noise may lead to decision errors and consequently error propagation (as explained in Chapter 2) due to the nonlinear feedback nature of the DFEs. The dynamics and effects of error prop-
propagation are not fully understood, and only a limited amount of work has been
done on the characterization of the error propagation phenomenon [33]. The con-
ventional MMSE procedure for jointly designing a feedforward filter/DFE com-
bination minimizes mean square error at the input to the decision device under
the key assumption of correct past decisions [37].

In this chapter we take into account the fact that decision errors are inevitable
and that error propagation may be significant. In the uncoded system, we expect
an increase in symbol error rate (SER) due to error propagation. In coded systems
the error propagation can affect the channel capacity and the performance of the
decoder.

![Figure 3.1: Channel plus a T-spaced decision feedback equalizer. The channel is given by
H. F is the feedforward filter, D is the feedback filter, Δ is the cursor delay, and Qₐ(·) is the
nearest neighbour quantizer]

Our first step is to interpret the way in which a conventional MMSE-DFE
design for a white source and white noise works. A MMSE-DFE design using a
sufficiently long DFE lets the feedforward filter (either a T-spaced or fractionally
spaced linear filter) convolutionally reduce the overall channel-feedforward filter
precursor ISI, while the feedback filter additively cancels the remaining postcursor
ISI. When only a short length feedback filter is used, it cancels a window of
postcursor ISI, so in that case the feedforward filter reduces the remaining ISI
from the precursor and also from the end of the tail of the postcursor.

Using this interpretation, we then show through examples that a MMSE-
DFE design can yield an overall channel-feedforward filter response with large taps after the cursor. Since the DFE cancels ISI from the postcursor by using these large taps in the feedback path, error propagation will be more severe and can reduce performance significantly.

In an attempt to ameliorate this problem, we modify the MMSE-DFE design: our strategy is to shape the postcursor to reduce the effect of error propagation. In our strategy, which we call a “hybrid MMSE design procedure”, the feedforward filter is used not only to combat precursor ISI but also to reduce postcursor ISI. This should reduce the number of secondary errors\(^1\), consequently improving the SER performance over the conventional MMSE-DFE design. Thus, some of the linear equalizer’s degrees of freedom will be used to combat postcursor ISI, despite decision feedback being used. The idea is to reduce the possibility that a large postcursor tail will be produced by the feedforward filter, whose sole job would otherwise be to reduce precursor ISI only. In other words, the design allows the feedforward filter to act on the postcursor to control error propagation. As expected there is a gain in mean square error when error propagation is taken into account [41]. Now we move onto the analysis and the formal development of this hybrid MMSE design.

### 3.2 Joint MMSE Feedforward Filter-DFE Design

MMSE-DFE optimization under the MMSE criterion has been dealt with in Section 2.2.1. The design presented in this chapter requires the decoupled optimization of the \(F\) and \(D\) filters. Thus we revisit the MMSE-DFE optimization.

---

\(^1\) The first decision error, also called the primary error, when fed back into the nonlinear feedback path of the DFE induces further errors called the secondary errors. The concept is dealt with in more detail in Section 2.4.
3.2.1 Mathematical Model

We transmit a symbol sequence \( \{a_k\} \) from an \( M \)-ary alphabet

\[
A \triangleq \{ \pm 1, \pm 3, \pm 5, \ldots, \pm (M - 1) \}
\]

over a real linear dispersive channel with additive noise.

The communication system consists of either a \( T \)-spaced or fractionally-spaced channel \( H = [h_0, \cdots, h_{N_h}]' \), a \( T \)-spaced or fractionally-spaced equalizer \( F = [f_0, \cdots, f_{N_f-1}]' \), with an overall channel-feedforward filter combination

\[
HF = T = [t_0, \cdots, t_{N_t}]'
\]

(where \( H \) is either the \( T \)-spaced or fractionally-spaced convolution matrix \([74]\) as given in equation (2.1)) and a decision feedback filter is given by \( D = [d_1, \cdots, d_{N_d}]' \).

The objective of the MMSE-DFE design is to minimize the average mean square deviation of the signal \( z_k \) (which is the input to the quantizer, as shown in Figure 3.1) away from the delayed source symbol \( a_{k-\Delta} \), where \( \Delta \) is some arbitrary delay, under the assumption that there are no past decision errors:

**Correct Past Decision Assumption:** \( \hat{a}_k = a_k \ \forall k \).

The effect of the assumption is to regard the transmitted symbols \( a_k \) as being fed back through \( D \) rather than the detected symbols \( \hat{a}_k \), yielding the mean square error

\[
\sigma_k^2(F, D, \Delta) = E\{(z_k - a_{k-\Delta})^2\} = E\{(A'_kHF - \hat{A}'_kD + N'_kF - a_{k-\Delta})^2\}
\]

(3.1)

\[
\hat{A}_k = [a_{k-\Delta-1}, \cdots, a_{k-\Delta-N_d}]'
\]

is a regressor of past decisions (equal to past inputs by the key assumption) being operated on by the DFE, and \( N_k = [n_k, \cdots, n_{k-N_f+1}]' \) is a regressor of
the additive white noise with autocorrelation $\sigma_n^2 I$ where $\sigma_n^2$ is the noise variance. The MMSE solution corresponds to finding the $F$ and $D$ that minimize equation (3.1), given $H$ and $\Delta$.

In deriving the solution to the joint MMSE parameter optimization, the following calculations make it evident that it is possible to largely decouple the $D$ optimization from the optimization of $F$. Hence we proceed to find the best solution $D_{min}$ for an arbitrary $F$ and $\Delta$ by setting the gradient of $\sigma_e^2$ with respect to $D$ equal to zero, as

$$
\nabla_D \sigma_e^2(F, D, \Delta) = E\{-2\hat{A}_k A_k' H F + 2\hat{A}_k A_k' D - 2\hat{A}_k N_k' F + 2a_k - \Delta \hat{A}_k\} = 0
$$

(3.2)

Using the property that the input sequence is white and uncorrelated with the noise (which is also white), the expression simplifies to

$$
\nabla_D \sigma_e^2(F, D, \Delta) = -2\sigma_e^2 M H F + 2\sigma_a^2 D
$$

= 0

(3.3)

where $M = (0_{N_d \times \Delta} I_{N_d \times N_d} 0_{N_d \times N_t - \Delta - N_d})$ as defined in equation (2.8). Thus, the best $D$ (as a function of $F$) can be written as

$$
D_{min}(F) = M H F
$$

(3.4)

which is in the middle portion of the overall channel-feedforward filter combination, i.e., $H F$ is the channel feedforward filter convolution and $M$ serves to retain the middle $N_d$ terms with offset $\Delta$. Now that we have an expression for the best feedback filter setting, we must find the best feedforward filter ($F$) setting. We proceed in the same manner, and again, using the whiteness of the source and the noise, we obtain

$$
\nabla_F \sigma_e^2(F, D, \Delta) = 2\sigma_e^2 H' H F - 2\sigma_e^2 H' M' D_{min} - 2\sigma_a^2 H' T_\Delta + 2\sigma_a^2 F
$$

(3.5)

where $T_\Delta = (0, \ldots, 0, 1, 0, \ldots, 0)^\top_{\Delta}$ has a 1 at the $(\Delta + 1)^{th}$ position. Substituting
\[\textbf{D} = \textbf{D}_{\text{min}}(\textbf{F}), \text{ and using equation } (3.4), \text{ we get} \]

\[
\nabla_{\textbf{F}} \sigma^2(\varepsilon|\textbf{F}, \textbf{D}, \Delta) = 2\sigma_n^2 \mathcal{H}'(\textbf{I} - \textbf{M}'\textbf{M})\mathcal{H}\textbf{F} - 2\sigma_n^2 \mathcal{H}'\textbf{T}_\Delta + 2\sigma_n^2 \textbf{F} \quad (3.6)
\]

yielding

\[
\textbf{F}_{\text{min}} = (\mathcal{H}'\textbf{P}\mathcal{H} + \lambda\textbf{I})^{-1}\mathcal{H}'\textbf{T}_\Delta \quad (3.7)
\]

where \(\lambda = \sigma_n^2/\sigma_n^2\), and we define \(\textbf{P} = (\textbf{I} - \textbf{M}'\textbf{M})\) as the \(N_t \times N_t\) diagonal matrix having the form

\[
\begin{bmatrix}
1 \\
& \ddots \\
& & 1 \\
& & & 0 \\
& & & & \ddots \\
& & & & & 0 \\
& & & & & & 1 \\
& & & & & & & \ddots \\
& & & & & & & & 1
\end{bmatrix}
\begin{bmatrix}
\Delta \\
\textbf{N}_d \\
\textbf{N}_d - \Delta - \textbf{N}_d
\end{bmatrix}
\quad (3.8)
\]

The interpretation is that joint equalization is done by trying to reduce the ISI outside the DFE feedback filter \textit{window} (the taps of the overall channel-feedforward filter that are equalized by the DFE feedback taps) as much as possible with the feedforward filter, while controlling the noise gain. The DFE then takes care of the taps inside its window of width \(N_d\). The role of the \(\textbf{P}\) matrix is to mask the contribution to the MSE from the ISI of the channel-feedforward filter combination taps inside the DFE window.

There still remains the problem of determining the best delay \(\Delta_{\text{min}}\). The simplest way to find the best delay is just to directly compute the MSE for each design resulting from each possible delay and select the minimum.
Thus, the problem of minimizing equation (3.1) for a white source and white noise translates to

\[
\{F_{\text{min}}, \Delta_{\text{min}}\} = \arg\min_{F, \Delta} \left\{ \|P(HF - T_{\Delta})\|_2^2 + \lambda \|F\|_2^2 \right\}
\]

\[
D_{\text{min}} = MHF_{\text{min}}.
\]

(3.9)

Notice that the joint optimization has been reduced to an essentially decoupled problem of successive optimizations.

### 3.2.2 Error Mechanism Model of DFEs

In this section we model the error propagation mechanism of the DFE. It is verified that noise induced errors, called primary errors, lead to subsequent errors, termed secondary errors, and consequently error propagation. Figure 2.8 in Chapter 1 shows this phenomenon.

Let the probability of error \( P_e \) at high SNRs be given by

\[
P_e = P_{e(CPD)}(1 + \bar{\kappa}) \quad \bar{\kappa} > 0
\]

(3.10)

where \( P_{e(CPD)} \) is the probability of error under the correct past decision assumption (called the base probability) and \( \bar{\kappa} \) can be interpreted as the average number of secondary errors per primary error. Assuming the transmission of \( M \)-PAM over a real channel, the base probability \( e(\sigma^2_e) \) is given as

\[
e(\sigma^2_e) = \frac{M - 1}{M} \text{erfc} \left( \sqrt{\frac{3T^2_{\Delta}\sigma^2_e}{2(M^2 - 1)\sigma^2_e}} \right)
\]

(3.11)

where \( \text{erfc}(.) \) is the complementary error function [24] and \( \sigma^2_e \) denotes MSE. In the MMSE-DFE literature, the correct past decision assumption is equivalent to explicitly assuming that \( \bar{\kappa} = 0 \); thus the probability of error is based on the distribution of primary errors alone. Our argument is that assuming \( \bar{\kappa} \) to be zero does not lead to the optimal MMSE-DFE coefficients. The quantity \( \bar{\kappa} \) can be calculated or estimated through various techniques [42]– [57], or through simulation, as is done in this chapter.
To illustrate the relevance of the above ideas, we experiment with the MMSE design using the truncated, $T$-spaced microwave channel model $g_{201}$ from the Applied Signal Technology Inc.\textsuperscript{2} database (shown in Figure 3.2) [74], a 16-tap baud spaced equalizer (BSE), and a DFE matching the postcursor of the channel-BSE combination, at a 30 dB SNR. It is evident from Figure 3.2 that the postcursor taps which end up in the feedback path are quite large. In fact they dwarf the cursor of the channel-BSE response (at delay 19), flagging that the error propagation will be severe.

For the $T$-spaced case, we have a heuristic explanation of how postcursor taps may turn out to be large. The feedforward filter, which reduces precursor ISI, also approximately inverts the effect of the maximum phase roots of the channel (with a suitable decision delay). In general, these roots generate the channel’s precursor. As these roots approach the unit circle, larger feedforward filter taps are required for the inversion. Large feedforward filter taps may then yield an overall channel-feedforward filter response with large postcursor taps.

![Truncated response of the $T$-spaced channel model $g_{201}$.](image)

Figure 3.2: Truncated response of the $T$-spaced channel model $g_{201}$.

We simulate the transmission of 4-PAM at a 30 dB SNR to estimate the distribution of secondary errors. The result is shown in Figure 3.3. Notice that the

\textsuperscript{2} http://www.appsig.com
burst of secondary errors can be very long, yielding an average of approximately 30 errors (in the example shown). Thus, the symbol error rate increases from a rate of $3.5 \times 10^{-6}$ (primary errors only) to $1.1 \times 10^{-5}$ (primary plus secondary errors).

![Secondary Error Distribution](image)

Figure 3.3: Distribution of secondary errors $P(\kappa)$.

### 3.3 Strategy for Hybrid MMSE Design

As seen in the previous section, a MMSE-DFE design may yield an overall channel-feedforward filter response with large postcursor taps, which may in turn lead to high error propagation. Our objective is to modify this existing or conventional MMSE-DFE design to reduce the damaging effect of the potentially large postcursor by applying constraints to the design given in Section 3.2.1. Basically, in the joint MMSE design, the job of the feedforward filter is to cancel precursor ISI. However, in the new design, called the “hybrid MMSE design”, the feedforward filter is also used to shape or reduce postcursor ISI.

In the hybrid MMSE design we construct a convex function that mixes a joint feedforward/feedback filter MMSE-DFE design (with the feedforward filter operating on the precursor only), with a feedforward filter MMSE design (operating
3.3 Strategy for Hybrid MMSE Design

on the ISI of the overall response). Let the feedforward filter for a MMSE-DFE design be

\[
F_0 = \arg \min_{F} \left\{ \|P(\mathcal{H}F - T_\Delta)\|^2 + \lambda \|F\|^2 \right\}
\]  

(3.12)

for some fixed delay \( \Delta \). Now, let the feedforward filter for a feedforward filter (BS or FS) only design be

\[
F_1 = \arg \min_{F} \left\{ \|\mathcal{H}F - T_\Delta\|^2 + \lambda \|F\|^2 \right\}. 
\]

(3.13)

Create the convex combination,

\[
F_\omega = (1 - \omega)F_0 + \omega F_1, \quad \omega \in [0, 1]. 
\]

(3.14)

Thus \( \omega = 0 \) corresponds to the conventional MMSE-DFE design and \( \omega = 1 \) corresponds to the MMSE linear equalizer (LE) feedforward filter design.

The motivation behind constructing the convex function of the MMSE-DFE and MMSE LE design is to make use of the advantages of both the designs. The MMSE-DFE design yields error propagation but less noise enhancement whereas the MMSE-LE design leads to noise enhancement but no error propagation due to the absence of the nonlinear feedback path. Thus in order to adjust the MMSE-DFE coefficients for reduced error propagation, it makes sense to move towards the MMSE-LE design. The degree to which the receiver moves towards the MMSE-LE design is governed by the scalar parameter \( \omega \).

The intention is that the combination of both designs, \( F_0 \) reducing precursor ISI and \( F_1 \) reducing total ISI, will yield an intermediate design \( F_{\omega_{\text{opt}}} \) with a better SER performance than either extreme. Naturally \( F_{\omega_{\text{opt}}} \) is never worse than the best performance of \( F_0 \) and \( F_1 \).

3.3.1 Numerical Study

We conduct a numerical study to illustrate the above concept. We use the following parameters for the experiment.
3.3 Strategy for Hybrid MMSE Design

- a $T$-spaced channel given as $H = [0.01, 0.07, 1.0, 1.1, 0.5]'$ (shown in Figure 3.6)

- 6 $T$-spaced tap feedforward filters $F_0$ and $F_1$

- a DFE matching the entire postcursor of the overall channel-feedforward filter combination

- the design parameters are an 8-PAM constellation, a 23 dB SNR$^3$, a delay of $\Delta = 5$, and $\omega \in [0, 1]$;

- for the above design parameters the feedforward filter coefficients are

  $$F = [-0.0118, -0.0520, 1.0557, -0.0315, 0.0698, -0.0209]'$$

  and the feedback filter coefficients are

  $$D = [1.1084, 0.5615, 0.0401, 0.0119, -0.0105]'$$

  determined by the optimum value $\omega_{opt} = 0.08$

- we adopt symbol error rate as a measure of performance

**Case Study 1: SER Performance**

We plot the SER versus the parameter $\omega$ as shown in Figure 3.4 and verify that in fact there is an intermediate value $\omega_{opt} \neq 0$ which offers improvement in SER performance over both the MMSE-DFE and MMSE-LE designs' SER performance.

The fact that there is actually an intermediate value of $\omega$ which gives the best SER performance is further highlighted by the following table. The same parameters as detailed above are used to generate the SERs at various values of $\omega$. At $\omega = 0.12$ the hybrid MMSE design does better than the MMSE-DFE design ($\omega = 0$) and the MMSE-LE design ($\omega = 1$). Similar tables can be generated at

$^3$ SNR in dB here is defined by $\text{SNR} = 10 \log_{10}(\sigma_0^2/\sigma_n^2)$. 

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different SNR levels to show the improvement in SER performance obtained by the hybrid MMSE design.

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>0</th>
<th>0.12</th>
<th>0.24</th>
<th>0.36</th>
<th>0.48</th>
<th>0.60</th>
<th>0.72</th>
<th>0.84</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>SER</td>
<td>0.0004</td>
<td>0.0003</td>
<td>0.0006</td>
<td>0.0012</td>
<td>0.0029</td>
<td>0.0071</td>
<td>0.0141</td>
<td>0.0271</td>
<td>0.0520</td>
</tr>
</tbody>
</table>

Figure 3.4: The above figure shows the SER as a function of the parameter $\omega$. The design is done for 8-PAM at a 23 dB SNR. The channel is $H = [0.01, 0.07, 1.0, 1.1, 0.5]^T$. The dashed line shows the theoretical SER at each $\omega$, determined by equation (3.11). The solid line shows the simulated SER (which takes into account error propagation). '*' shows the $\omega_{\text{opt}}$ value, which is approximately 0.08 in this example.
Figure 3.5: The above figure shows the average number of secondary errors per primary error ($\bar{K}$) as a function of the parameter $\omega$. The design is done for 8-PAM at a 23 dB SNR and the channel is $H = [0.01, 0.07, 1.0, 1.1, 0.5]'$. The average number of errors in each burst decreases as the $\omega$-design moves from the MMSE-DFE design to the MMSE-LE design. For example, at $\omega = 0.2$, there are on average approximately 6 secondary errors for every primary error.
Case Study 2: Robustness Test

Our second observation is the significant reduction in the average number of secondary errors. This makes the hybrid MMSE design more robust compared to the MMSE-DFE design, as shown in Figure 3.5.

We notice from Figure 3.7 that the hybrid MMSE design procedure diminishes the size of the large postcursor relative to the information bearing cursor (at $\Delta = 5$) in the above experiments. The $T$-spaced equivalent channel is shown in Figure 3.6. The large postcursor phenomenon in the MMSE-DFE design was shown in Figure 3.2. Thus the hybrid MMSE design distributes the postcursor energy more evenly. It is now shown in Figure 3.7 that no one postcursor is large enough to reverse the polarity of the signal in the case of a decision error.

Waterfall Curves

Waterfall curves are generated to compare the SER performance of the MMSE-DFE with the hybrid MMSE design for different values of SNR. Figure 3.8 plots the SER of the conventional MMSE-DFE design (i.e., $\omega = 0$) and the hybrid MMSE design with the optimum choices of $\omega$ at each different value of SNR (for example $\omega = \omega_{opt} = 0.08$ at 23dB SNR as shown in the previous example). The channel is $H = [0.01, 0.07, 1.0, 1.1, 0.5]'$. The idea is to show that the new design is robust to a range of SNRs like the MMSE-DFE design. Furthermore, the new
Figure 3.7: Overall impulse response of the channel and feedforward filter convolution for (a) the MMSE-LE design, (b) the MMSE-DFE design, (c) the hybrid MMSE design for $\omega = 0.08$ and (d) the hybrid MMSE design for $\omega = 0.5$. 
design's SER performance is not degraded at the optimum value of \( \omega \) (by definition). Figure 3.9 shows that even when \( \omega = 0.16 \), which is much further from the \( \omega = 0 \), hybrid MMSE-DFE design performance is as good as the MMSE-DFE up to 22.5 dB. Above 22.5 dB the performance degrades very slightly, and it is reasonable to ignore this in light of the improved secondary error performance that the hybrid MMSE design presents.

![Figure 3.8: SER vs. SNR for two choices of design for 8-PAM transmission. 'x' and 'o' show simulated SERs of the \( \omega = 0 \) (MMSE-DFE) and \( \omega = 0.08 \) (hybrid MMSE-DFE) designs respectively. '+' and 'x' show the theoretical SERs of the \( \omega = 0 \) (MMSE-DFE) and \( \omega = 0.08 \) (hybrid MMSE-DFE) designs respectively. The formula used to calculate the theoretical values is given by equation (3.11).](image-url)
Figure 3.9: SER vs. SNR for two choices of design for 8-PAM transmission. ‘*’ and ‘o’ show simulated SERs of the $\omega = 0$ (MMSE-DFE) and $\omega = 0.16$ (hybrid MMSE-DFE) designs respectively. ‘+’ and ‘x’ show the theoretical SERs of the $\omega = 0$ (MMSE-DFE) and $\omega = 0.16$ (hybrid MMSE-DFE) designs respectively. The formula used to calculate the theoretical values is given by equation (3.11).
3.4 Discussion

The main message of this chapter is that care should be taken when making the correct past decision assumption. Adopting a strategy to take into account secondary errors by appropriately combining the conventional MMSE-DFE and linear equalizer designs gives SER performance comparable to that of the conventional MMSE-DFE design. The advantage is that it provides robustness in terms of error propagation, as the average number of secondary errors is reduced. Experiments have shown that as the size of the constellation increases, so does the effect of error propagation. It is here that the hybrid MMSE design can provide an improvement in performance.

The fact that this procedure does not greatly increase complexity, due to its use of simple design tools, provides motivation for further work. As a first idea, it would be interesting to develop an analytical method for determining the optimum combination value \( \omega_{\text{opt}} \). Another extension to the work would be to make the design adaptive, which is clearly feasible since the separate designs (i.e., \( F_0 \) and \( F_1 \)) themselves can be implemented as adaptive or even blind adaptive schemes.
In this chapter we continue with the idea that the conventional MMSE-DFE design may not be optimal and may not correspond to a practically meaningful measure, as introduced in Chapter 3. First we show numerically through a simulation study that for the low signal to noise ratio case the true or actual MMSE solution for the DFE is not close in parameter space to the MMSE solution calculated using the correct past decision assumption. This simulation study motivates the need for a new design methodology that achieves the minimum actual MSE. The second part of the chapter is devoted to the analysis of the actual MMSE, deriving bounds on performance and developing a novel design strategy. We call this new design the “Modified MMSE-DFE design”. The Modified MMSE-DFE design analysis is done without using the correct past decision assumption. The chapter concludes by highlighting that a similar issue exists for optimizing the coefficients of a DFE to minimize the probability of error (without the correct past decision assumption).
4.1 Introduction

In Chapter 3 we flagged the suboptimality of the MMSE-DFE design. It was shown that the correct past decision (CPD) assumption has the underlying effect of ignoring the true nature of the nonlinear feedback path in the DFE and hence the error propagation. In Chapter 3 the concept was developed without providing a detailed study of how suboptimal the conventional MMSE-DFE design really is compared to the actual MMSE-DFE design. We define the actual MMSE-DFE design as the one which is compatible to the situation where the actual decisions are fed back in the feedback path instead of the transmitted symbols. The actual MSE is what would normally be measured in the DFE during its normal (non-training) operation.

In this chapter, we first undertake a simulation study and show that the actual value of the DFE's MSE is quite different from the MSE value calculated using correct past decision assumption for the DFE. Thus this numerical study reveals that the conventional MMSE-DFE design solution differs significantly from the actual MMSE-DFE solution. MSE contours for a DFE operating in the CPD mode and in the actual mode are given for a range of signal to noise ratios. As expected the conventional MMSE-DFE solution and the actual MMSE-DFE solution are quite close for the low noise case. As the noise level is increased and higher order constellations are used the two solutions diverge. The study reinforces the argument that the conventional MMSE-DFE design using a CPD assumption, is an unnatural design and any performance advantage it shows over alternative designs (which are based on feeding back actual decisions) is meaningless. Thus this simulation study sets the stage for the analysis and design done later in the chapter.

In the second part of the chapter the MMSE-DFE analysis and design is undertaken without making the assumption of correct past decisions. A Modified MMSE-DFE design is given which does not require the correct past decision assumption. This Modified MMSE-DFE design is an approximation to the actual
4.2 Motivation for Improved MMSE-DFE Design

4.2.1 Preamble

The criterion used for evaluation is the measured or actual MSE. Due to the nonlinear and recursive nature of the actual MSE, there is no closed form solution to the problem of determining the actual coefficients. The strategy for evaluation is based on locally exhaustively searching the feedforward and feedback filter parameter space and estimating the actual MSE using simulation. In the simulation study a low order system is used as this lends itself naturally to a graphical interpretation. The idea is to show how MSE values evaluated under the correct past decision assumption readily differ from actual MSE values even for the most simple conceivable system. The low order system comprises a two tap channel, given as \( H = [1, 0.9]' \), a one tap feedforward filter \( F = [f_0] \) and a one tap feedback filter \( D = [d_1] \). The decision device is a nearest neighbor scalar quantizer. The decision delay is chosen to be zero.

4.2.2 Simulation Study – BPSK, 3,6,9 (dB) SNR Results

In this section we will highlight the results of our simulation study for BPSK modulation at 3, 6, and 9 dB SNR levels. The simulation study is done for two cases:

**Actual Case or Mode:** When the DFE is operated with the actual decision at the output of the quantizer being fed back into the feedback filter.

**CPD Case or Mode:** When the DFE is run with the transmit data being fed into the feedback filter.
Motivation for Improved MMSE-DFE Design

(a) Contours of the DFE’s actual MSE plotted as a function of the feedback tap $d_1$ and feedforward tap $f_0$. MMSE is 0.4680.

(b) Contours of the DFE’s MSE under in the CPD mode, as a function of the feedback tap $d_1$ and feedforward tap $f_0$. MMSE is 0.3372.

Figure 4.1: Performance of a DFE evaluated for the Actual case and the CPD case. The channel is $\mathbf{H} = [1, 0.9]^T$, the SNR is 3 dB and the modulation is BPSK. The actual MSE shown in (a) is the mean square error obtained when running the DFE with actual decisions and gives a true indication of performance. Plot (b) shows the mean square error when running the DFE with correct decisions and gives an artificial indication of performance.
4.2 Motivation for Improved MMSE-DFE Design

Figure 4.2: The above figure shows the comparison of actual MSE versus MSE under CPD as given in Figure 4.1(a) and (b) respectively. The minimum of the MSE under CPD is the conventional MMSE design and is denoted by $MSE_{CPD}$. The minimum of the actual MSE is labeled $MSE_{Actual}$. The channel is $H = [1, 0.9]'$, the SNR is 3 dB and the modulation is BPSK. Notice that the two minima are not close.
First we run our system for 3dB SNR for two-level data; the results are shown in Figure 4.1. Figure 4.1(a) shows the MMSE-DFE design for 3 dB SNR when the actual decisions are fed back instead of the transmit symbol as in the correct past decision assumption, shown in Figure 4.1(b). The comparison of the two cases for 3 dB SNR can be seen in Figure 4.2.

(a) Contours of the DFE’s actual MSE plotted as a function of the feedback tap \( d_1 \) and feedforward tap \( f_0 \). MMSE is 0.2780.

(b) Contours of the DFE’s MSE under correct past decision, as a function of the feedback tap \( d_1 \) and feedforward tap \( f_0 \). MMSE is 0.2015.

Figure 4.3: Performance of a DFE evaluated for the Actual case and the CPD case. The channel is \( H = [1, 0.9] \)'', the SNR is 6 dB and the modulation is BPSK. The actual MSE shown in (a) is the mean square error obtained when feeding the DFE with actual decisions and gives a true indication of performance. Plot (b) shows the mean square error when running the DFE using correct decisions and gives an artificial indication of performance.

The results show that there is a discrepancy between the DFE operating under the condition of Actual mode and the DFE operating under the CPD mode. The result is not surprising as the conventional MSE-DFE under CPD ignores the decision errors (or primary errors) and subsequently error propagation. The actual decision feedback takes into account all the errors induced in the nonlinear feedback path.
4.2 Motivation for Improved MMSE-DFE Design

Figure 4.4: Comparison of actual MSE versus MSE under CPD as shown in Figure 4.3(a) and (b) respectively. The channel is $\mathbf{H} = [1, 0.9]'$, the SNR is 6 dB and the modulation is BPSK. The minimum of the MSE under CPD is the conventional MMSE design and is denoted by $MSE_{CPD}$. The minimum of the actual MSE is denoted $MSE_{Actual}$. The MSE values of the two cases are closer at 6 dB SNR than at 3 dB SNR (Figure 4.2). The difference is due to the reduction in noise induced errors for higher SNR. Hence $MSE_{CPD}$ moves closer to $MSE_{Actual}$. 

\[ \text{Figure 4.4: Comparison of actual MSE versus MSE under CPD as shown in Figure 4.3(a) and (b) respectively. The channel is } \mathbf{H} = [1, 0.9]'. \text{ The SNR is 6 dB and the modulation is BPSK. The minimum of the MSE under CPD is the conventional MMSE design and is denoted by } MSE_{CPD}. \text{ The minimum of the actual MSE is denoted } MSE_{Actual}. \text{ The MSE values of the two cases are closer at 6 dB SNR than at 3 dB SNR (Figure 4.2). The difference is due to the reduction in noise induced errors for higher SNR. Hence } MSE_{CPD} \text{ moves closer to } MSE_{Actual}. \]
4.2 Motivation for Improved MMSE-DFE Design

As the signal to noise (SNR) ratio goes up, or in other words noise level comes down, the MMSE values obtained while running the DFE in the Actual mode and in the CPD mode come closer. We witness this effect in the 6 dB SNR case shown in Figure 4.3. Again Figure 4.3(a) shows the MSE contours for the mode where the actual decisions are fed back and Figure 4.3(b) for the CPD mode where the transmit data symbols are fed back. Their comparison is given in Figure 4.4.

Figure 4.5: Performance of a DFE evaluated for the Actual case and the CPD case. The channel is $H = [1.0, 0.9]'$, the SNR is 9 dB and the modulation is BPSK. The actual MSE shown in (a) is the mean square error obtained when feeding the DFE with actual decisions and gives a true indication of performance. Plot (b) shows the mean square error when running the DFE using correct decisions and gives an artificial indication of performance.

The effect is more profound for 9 dB case shown in Figure 4.5. It can be seen from Figure 4.5 that the MSE values given by the systems operating in two modes are more similar than in the lower SNR cases. This is because the noise induced errors are less frequent and hence the CPD assumption becomes more valid. As a consequence the actual MSE converges to the ideal conventional MSE.

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Motivation for Improved MMSE-DFE Design

Figure 4.6: Comparison of actual MSE versus MSE under CPD as shown in Figure 4.5(a) and (b) respectively. The minimum of the MSE under CPD is the conventional MMSE design and is denoted by $MSE_{CPD}$. The minimum of the actual MSE is denoted by $MSE_{Actual}$. The SNR is 9 dB and the modulation is BPSK. The minimum MSE values for both cases are almost the same. The above figure indicates the fact that the conventional MMSE-DFE design performance is relatively close to actual DFE performance at 9 (dB) SNR.
4.2 Motivation for Improved MMSE-DFE Design

Thus the real world DFE performance is what is shown in Figure 4.1(a), Figure 4.3(a) and Figure 4.5(a) for 3, 6, and 9 dB SNRs respectively. It flags the fact that the conventional MMSE-DFE design performance given by the minima of Figure 4.1(b), Figure 4.3(b) and Figure 4.5(b) for 3, 6, and 9 dB SNRs respectively is not well connected with the real world performance of the DFE and is an acceptable design strategy only when the SNR is sufficiently high.

![Contours of the DFE’s actual MSE plotted as a function of the feedback tap d1 and feedforward tap f0. MMSE is 2.3708.](image)

(a) Contours of the DFE’s actual MSE plotted as a function of the feedback tap \(d_1\) and feedforward tap \(f_0\). MMSE is 2.3708.

![Contours of the DFE’s MSE under correct past decision, as a function of the feedback tap \(d_1\) and feedforward tap \(f_0\). MMSE is 1.6836.](image)

(b) Contours of the DFE’s MSE under correct past decision, as a function of the feedback tap \(d_1\) and feedforward tap \(f_0\). MMSE is 1.6836.

Figure 4.7: Performance of a DFE evaluated for the Actual case and the CPD case. The channel is \(H = [1, 0.9]'\), the SNR is 3 dB and the constellation is 4PAM. The actual MSE shown in (a) is the mean square error obtained when feeding the DFE with actual decisions and gives a true indication of performance. Plot (b) shows the mean square error when running the DFE using correct decisions and gives an artificial indication of performance.

### 4.2.3 Simulation Study – 4PAM, 3 dB

In this section, in analogy with Section 4.2.2, we show the difference in MSE values calculated for 4PAM data by running the two systems in the Actual mode and in the CPD mode. In Figure 4.7(a) we show the actual MSE value for the 4PAM constellation when the rest of the system parameters are the same as
4.2 Motivation for Improved MMSE-DFE Design

Figure 4.8: Performance of a DFE evaluated for the Actual case and the CPD case at 3 dB SNR. Notice from the above figure that the two minima are very widely apart from each other. The above figure gives an indication of how far the conventional MMSE-CPD design can be from actual performance at higher constellation data.
4.3 Actual MMSE-DFE bounds

before. Notice from Figure 4.8 that the difference between the two minima is more significant than in the BPSK case for the same SNR (Figure 4.2). The interpretation of this observation is that as the number of signal points increase in the transmit signal constellation so does the probability of error and the error propagation in the DFE receiver. Hence the DFE performance given by the MMSE design under the correct past decision assumption is more unrealistic than in the BPSK case.

4.2.4 Synopsis

With the low order system under study we can infer that there is value in finding a design strategy to minimize the actual MSE (since the filter weights obtained from the conventional MMSE design are very different from the optimal actual MSE values). This motivates the desirability for systematic analytical design techniques to determine the actual MMSE-DFE parameter settings\(^1\).

4.3 Actual MMSE-DFE bounds

It is analytically difficult to explicitly determine the actual MSE, the filter weights that achieve the minimum actual MSE and the value of the (actual) MSE obtained at that minimum setting. However, it may well be possible to analytically bound such quantities and this motivates the work in this section.

In this section we will give a bound on the actual MMSE-DFE. The study is relevant in the sense that it analytically shows a relationship between the conventional MMSE-DFE design and the actual MMSE-DFE design.

The output of the feedforward filter is

\[
  u_k = (\mathcal{H}F)'A_k + F'N_k. \tag{4.1}
\]

where \( F \) is the feedforward filter of length \( N_f \) as defined in equation (2.2), \( \mathcal{H} \) is

\(^{1}\) While an exhaustive search, as we have done in Section 4.2, constitutes a design procedure, albeit a crude one, we do not propose it as a practically useful design procedure.
4.3 Actual MMSE-DFE bounds

the channel convolution matrix, given in equation (2.1), \( A_k \) is defined in equation (2.6) and \( N_k \) is defined in equation (2.7). These are reproduced here for convenience as

\[
A_k \triangleq [a_k, a_{k-1}, \ldots, a_{k-(N_t-1)}]',
\]

where \( N_t \) is the length of the convolution \( \mathcal{H}F \), and \( N_k \) is zero mean Gaussian noise vector given as

\[
N_k \triangleq [n_k, n_{k-1}, \ldots, n_{k-(N_j-1)}]'.
\]

The feedback signal (without assuming CPD) is

\[
x_k = D'A_{k-\Delta-1} + D'N_{k-\Delta-1}.
\]

(4.2)

where \( D \) is the feedback filter of length \( N_d \) as defined in equation (2.3) and (with only a mild abuse of notation)

\[
A_{k-\Delta-1} \triangleq [a_{k-\Delta-1}, \ldots, a_{k-\Delta-N_d}]'.
\]

\( \Delta \) is the cursor delay, and

\[
E_{k-\Delta-1} \triangleq [e_{k-\Delta-1}, \ldots, e_{k-\Delta-N_d}]' \in \mathbb{R}^{N_d}
\]

where \( e_k \triangleq \hat{a}_k - a_k \). Assuming BPSK modulation, then \( \mathbb{E} \triangleq \{0, \pm 2\} \). The equalized signal at the input of the quantizer is given as

\[
z_k = u_k - x_k
\]

\[
= (\mathcal{H}F)'A_k + F'N_k - D'A_{k-\Delta-1} - D'E_{k-\Delta-1}.
\]

(4.3)

The mean square deviation of the point \( z_k \) from the nominal target \( a_{k-\Delta} \) is given as

\[
\text{MSE} = \mathbb{E}\{(z_k - a_{k-\Delta})^2\}.
\]

(4.4)

From equation (4.3), we can write the error as

\[
z_k - a_{k-\Delta} = (\mathcal{H}F)'A_k + F'N_k - D'A_{k-\Delta-1} - D'E_{k-\Delta-1} - A_{k'}T_{\Delta}.
\]

(4.5)
where $\mathbf{T}_\Delta = (0, \cdots, 0, 1, 0, \cdots, 0)_{N_t-\Delta-1}^\Delta$ has a 1 at the $(\Delta + 1)^{th}$ position. If we compare the above equation with a similar equation written for $\text{MSE}$ under the CPD assumption, we find that there is one additional term in the above equation, namely $\mathbf{D}'\mathbf{E}_{k-\Delta-1}$. The actual $\text{MSE}$, equation (4.4), can be written

$$MSE = E\{ (e_{k}^{\text{cpd}} - \mathbf{D}'\mathbf{E}_{k-\Delta-1})^2 \}. \quad (4.6)$$

where $e_{k}^{\text{cpd}}$ is the error (between $z_k$ and $a_{k-\Delta}$) under the CPD assumption. That is, $E\{ (e_{k}^{\text{cpd}})^2 \} = \text{MSE}_{CPD}$. Thus

$$MSE_{Actual} = MSE_{CPD} - 2E\{ e_{k}^{\text{cpd}} e_{k-\Delta-1}' \} \mathbf{D} + \mathbf{D}'E\{ \mathbf{E}_{k-\Delta-1} \mathbf{E}_{k-\Delta-1}' \} \mathbf{D}$$

$$= MSE_{CPD} - 2E\{ e_{k}^{\text{cpd}} e_{k-\Delta-1}' \} \mathbf{D} + \| \mathbf{D} \|^2 4P_eN_d. \quad (4.7)$$

where $P_e = Pr(a_k \neq a_k)$ is the probability of error, and we have assumed the independence of the errors leading to $E\{ \mathbf{E}_{k-\Delta-1} \mathbf{E}_{k-\Delta-1}' \} = 4P_e\mathbf{I}$ (an $N_d \times N_d$ diagonal matrix).

Applying Schwartz's inequality to the second term of equation (4.7) we get

$$|E\{ e_{k}^{\text{cpd}} e_{k-\Delta-1}' \} \mathbf{D}| \leq 2E\{ (e_{k}^{\text{cpd}})^2 \} E\{ \| \mathbf{E}_{k-\Delta-1} \|^2 \} \| \mathbf{D} \|^2$$

$$\leq MSE_{CPD} 4P_eN_d \| \mathbf{D} \|^2. \quad (4.8)$$

or

$$|E\{ e_{k}^{\text{cpd}} e_{k-\Delta-1}' \} \mathbf{D}| \leq 2 \| \mathbf{D} \| \sqrt{P_e} \sqrt{N_d} \sqrt{MSE_{CPD}}. \quad (4.9)$$

Thus from equation (4.7) the actual MMSE is upper bounded as follows

$$MSE_{Actual} \leq MSE_{CPD} + 4\| \mathbf{D} \| \sqrt{P_e} \sqrt{N_d} \sqrt{MSE_{CPD}} + 4P_eN_d \| \mathbf{D} \|^2. \quad (4.10)$$

This can be written as is given by

$$MSE_{Actual} \leq MSE_{Actual}^{upperbound} \quad (4.11)$$

where

$$MSE_{Actual}^{upperbound} \equiv \left( \sqrt{MSE_{CPD}} + 2\sqrt{P_e} \sqrt{N_d}\| \mathbf{D} \| \right)^2. \quad (4.12)$$
The above equation relates $MSE_{CPD}$ to $MSE_{Actual}$. It can be interpreted easily that as the probability of error goes down, $P_e \to 0$, the bound becomes tighter and the $MSE_{Actual}$ merges with the $MSE_{CPD}$.

In Section 4.7 optimum feedback filter weights are derived for the actual MSE case. Later on in Section 4.7, the equation (4.30) relates $D$ given in equation (4.12) to $D_{cpd}$ with a small correction vector $\rho$ given as

$$D = D_{cpd} + \rho$$  \hspace{1cm} (4.13)

### 4.4 Actual MMSE-DFE Approximation

Section 4.3 showed that $MSE_{Actual}$ is upper bounded by $MSE_{CPD}$ plus a correction factor depending on the weights of the feedback filter $D$. This section provides an approximation on the actual MMSE-DFE design under some simplifying assumptions given later. The important fact is that the analysis is done without making the correct past decision assumption. From equation (4.5) we can equivalently write

$$z_k - a_{k-\Delta} = (HF)'A_k + F'N_k - D'\hat{A}_{k-\Delta-1} - A_k'T_\Delta.$$  \hspace{1cm} (4.14)

Then

$$MSE_{Actual} = (HF)'E\{A_kA_k'\}(HF) + F'E\{N_kN_k'\}F + D'E\{\hat{A}_{k-\Delta-1}\hat{A}_{k-\Delta-1}'\}D$$
$$+ T'\Delta E\{A_kA_k'\}T_\Delta + 2D'E\{\hat{A}_{k-\Delta-1}A_k'\}T_\Delta - 2(HF)'E\{A_kA_k'\}(HF)$$
$$- 2D'E\{\hat{A}_{k-\Delta-1}N_k'\}F - 2D'E\{\hat{A}_{k-\Delta-1}A_k'\}(HF)$$  \hspace{1cm} (4.15)

where $\hat{A}_{k-\Delta-1}$ is, strictly speaking, a function of $F$ and $D$, i.e., $\hat{A}_{k-\Delta-1} = \hat{A}_{k-\Delta-1}(F, D)$. Taking the gradient of $MSE_{Actual}$ with respect to $D$ we get

$$\nabla_D(MSE_{Actual}) = 2E\{\hat{A}_{k-\Delta-1}\hat{A}_{k-\Delta-1}'\}D + 2E\{\hat{A}_{k-\Delta-1}A_k'\}T_\Delta$$
$$- 2E\{\hat{A}_{k-\Delta-1}N_k'\}F - 2E\{\hat{A}_{k-\Delta-1}A_k'\}(HF).$$  \hspace{1cm} (4.16)
Note that the dependence of $\hat{A}_{k-\Delta-1}$ on $D$ is ignored and in this sense equation (4.16) is an approximation. Similarly taking the gradient with respect to $F$ we get

$$\nabla_F(MSE_{Actual}) = 2\mathcal{H}'E\{A_kA_k'\}(HF) + 2E\{N_kN_k'\}F - 2\mathcal{H}'E\{A_kA_k'\}T_\Delta - 2\mathcal{H}'E\{A_k\hat{A}_{k-\Delta-1}'\}D - 2E\{N_k\hat{A}_{k-\Delta-1}'\}D.$$  

(4.17)

Note that the dependence of $\hat{A}_{k-\Delta-1}$ on $F$ is also ignored and in this sense is an approximation. From equation (4.16) and equation (4.17), we equate

$$\nabla_D(MSE_{Actual}) = 0$$

and

$$\nabla_F(MSE_{Actual}) = 0$$

to get

$$\begin{bmatrix}
\mathcal{H}'E\{A_kA_k'\} + E\{N_kN_k'\} & -\mathcal{H}'E\{A_k\hat{A}_{k-\Delta-1}'\} - E\{N_k\hat{A}_{k-\Delta-1}'\} \\
-E\{\hat{A}_{k-\Delta-1}N_k'\} - E\{\hat{A}_{k-\Delta-1}A_k'\} & E\{\hat{A}_{k-\Delta-1}\hat{A}_{k-\Delta-1}'\}
\end{bmatrix}
\begin{bmatrix}
F \\
D
\end{bmatrix}
= \begin{bmatrix}
\mathcal{H}'E\{A_kA_k'\} \\
E\{\hat{A}_{k-\Delta-1}A_k'\}
\end{bmatrix}T_\Delta.$$  

(4.18)

This is a linear equation which is readily solved for $F$ and $D$. However, not all components in this equation can be determined in closed form. In the next section we make further approximations to yield a closed form, approximate solution.

### 4.4.1 Assumptions

We make the following assumptions.

1. The input signal, noise and detected signal $\hat{a}_{k-\Delta-1}$ are white with zero mean, so that

- $E\{A_kA_k'\} = E\{a_k^2\}I = \sigma_a^2 I$
4.4 Actual MMSE-DFE Approximation

- \( E\{N_kN'_k\} = E\{n_k^2\}I = \sigma_n^2I \)
- \( E\{\hat{A}_{k-\Delta-1}\hat{A}'_{k-\Delta-1}\} = E\{\hat{a}_{k-\Delta-1}^2\}I = \sigma_s^2I \) where \( \sigma_s^2 = E\{a_k^2\} \).

2. The signal \( \hat{a}_{k-\Delta-1} \) and noise are independent, so that \( E\{\hat{A}_{k-\Delta-1}N_k'\} = 0 \).

3. \( E\{A_k\hat{A}'_{k-\Delta-1}\} = \eta\sigma_s^2J \) where the interpretation of \( J \) is that it is the \((\Delta + 1)\)th power of a Jordan matrix which has unit entries on its first sub-diagonal and zero everywhere else. It is used as a shift/masking matrix. Finally,

\[ \eta = E\{a_k\hat{a}_{k-\Delta-1}\}/\sigma_s^2. \]

Applying all the above simplifying assumptions, we rewrite equation (4.18) as

\[
\begin{bmatrix}
\mathcal{H}'\mathcal{H} + \lambda I & -\eta\mathcal{H}'J \\
-\eta J'\mathcal{H} & I
\end{bmatrix}
\begin{bmatrix}
F \\
D
\end{bmatrix}
= \begin{bmatrix}
\mathcal{H}' \\
0
\end{bmatrix}T_\Delta \tag{4.19}
\]

where \( \lambda = \sigma_n^2/\sigma_s^2 \). Thus the solution of equation (4.19) is

\[ F^{opt} = \{\mathcal{H}(I - |\eta|^2JJ')\mathcal{H}' + \lambda I\}^{-1}\mathcal{H}'T_\Delta \] \( \tag{4.20} \)

\[ D^{opt} = \eta J'\mathcal{H}F^{opt}. \] \( \tag{4.21} \)

These values of \( D^{opt} \) and \( F^{opt} \) are the approximations to the actual MMSE-DFE design values conditioned on assumptions 1 to 3. Nonetheless, the correct past decision assumption is not made in deriving the filter weights. The design presented in this section is called the “Modified MMSE-DFE design”.

It should be noted that \( \eta = 1 \) will correspond to optimum filter weights under the CPD design. Therefore, equation (4.20) and equation (4.21) for \( \eta = 1 \) correspond to optimum feedforward and feedback filter weights, derived under the correct past decision assumption. These optimum filter weights are given in equation (3.7) and equation (3.9), respectively, in Chapter 3.

A simulation study is needed so as to see the actual MMSE-DFE performance. To achieve the end we present three examples dealing with three different channels. The modulation is BPSK in all three examples. The \( \eta \) used to calculate the
optimum filter weights for the Modified MMSE-DFE design in this BPSK case is $\eta = 1 - 2P_e$. For calculating the optimum filter weights for the conventional MMSE-DFE design, $\eta = 1$.

**Example I**

Channel is $H = [1, 0.9]'$

SNR = 3dB

$N_f$ and $N_d = 1$

$F^{opt}(CPD) = [0.6661]$ at $\eta = 1$

$D^{opt}(CPD) = [0.5995]$ at $\eta = 1$

Actual MSE(0.6661,0.5995)=0.51065

$F^{opt}(Actual) = [0.5756]$ at $\eta = 0.842$

$D^{opt}(Actual) = [0.4361]$ at $\eta = 0.842$

Actual MSE(5756,0.4361)=0.4647

**Example II**

Channel is $H = [1, 0.9, -0.7]'$

SNR = 6dB

$N_f = 2$ and $N_d = 1$

$F^{opt}(CPD) = [0.9755]$ at $\eta = 1$

$D^{opt}(CPD) = [0.8779, -0.6828]$ at $\eta = 1$

Actual MSE($F^{opt}(CPD), D^{opt}(CPD))=0.522$

$F^{opt}(Actual) = [0.7153, -0.0540]$ at $\eta = 0.842$

$D^{opt}(Actual) = [0.4964, -0.4624]$ at $\eta = 0.842$

Actual MSE($F^{opt}(Actual), D^{opt}(Actual))=0.4525$

**Example III**

Channel is $H = [1, 0.86, 0.43, -0.06, 0.027]'$

SNR = 3dB

$N_f = 4$ and $N_d = 4$
4.5 Probability of Error as an Evaluation Criterion

The calculation of the probability of error is of prime importance in communications engineering. In this section we present a brief simulation study analogous to Section 4.2 when the MSE criterion is replaced by a $P_e$ criterion. We evaluate how the probability of error differs for the two modes of operation, i.e., the Actual mode and the CPD mode. In Figure 4.9, Figure 4.11 and Figure 4.13 we consider a 4PAM system for 3, 6, and 9 dB SNRs, respectively. In Figure 4.15, BPSK modulation is used for a 3dB SNR. The relevant feature is the difference in the contours and their respective minima for the two modes of DFE operation described in Section 4.2.2.

As equation (4.12) shows, when the probability of error is low the performance of the DFE operating in the Actual mode can be very close to the DFE performance operating in the CPD mode. The effect is more profound at higher SNRs as shown in Figure 4.14.

4.6 Conclusions

In general the conventional MMSE parameter settings need not be close to the parameter settings that give actual MSE. This was illustrated by a simple example. The conventional MMSE-DFE parameter settings under the correct past decision assumption give the MSE values corresponding to the actual MSE
4.6 Conclusions

Figure 4.9: The probability of error performance of a DFE evaluated for the Actual case and the CPD case. The channel is $H = [1, 0.9]^T$, the SNR is 3 dB and the signal constellation is 4PAM. The actual $P_e$ is the probability of error obtained when feeding the DFE with actual decisions and gives a true indication of performance as shown in (a). The $P_e$ under CPD is the probability of error when running the DFE using correct decisions as shown in (b). It gives an artificial indication of performance.

(a) Contours of the DFE’s actual $P_e$ plotted as a function of the feedback tap $d_1$ and feedforward tap $f_0$. The minimum $P_e$ is 0.4750.

(b) Contours of the DFE’s $P_e$ under correct past decision, as a function of the feedback tap $d_1$ and feedforward tap $f_0$. The minimum $P_e$ is 0.3966.
Figure 4.10: The above figure shows the comparison of actual $P_e$ versus $P_e$ under CPD as given in Figure 4.9(a) and (b) respectively. The minimum of the $P_e$ under CPD is the conventional $P_e$ and is denoted $P_e(CPD)$. The minimum of the actual $P_e$ is denoted by $P_e(Actual)$. 
(a) Contours of the DFE's actual $P_e$ plotted as a function of the feedback tap $d_1$ and feedforward tap $f_0$. The minimum $P_e$ is 0.4017.

(b) Contours of the DFE's $P_e$ under correct past decision, as a function of the feedback tap $d_1$ and feedforward tap $f_0$. The minimum $P_e$ is 0.3966.

Figure 4.11: Probability of error performance of a DFE evaluated for the Actual case and the CPD case. The channel is $H = [1, 0.9]'$, the SNR is 6 dB and the signal constellation is 4PAM. The actual $P_e$ is the probability of error obtained when feeding the DFE with actual decisions and gives a true indication of performance as shown in (a). Plot (b) shows the probability of error when running the DFE using correct decisions. It gives an artificial indication of performance.
Figure 4.12: The above figure shows the comparison of actual $P_e$ versus $P_e$ under CPD as given in Figure 4.11(a) and (b). The minimum of the $P_e$ under CPD is the conventional $P_e$ and is denoted by $P_e(CPD)$. The minimum of the actual $P_e$ is denoted $P_e(Actual)$. The SNR is 6 dB and the modulation is 4PAM.
4.6 Conclusions

(a) Contours of the DFE’s actual $P_e$ plotted as a function of the feedback tap $d_1$ and feedforward tap $f_0$. The minimum $P_e$ is 0.3009.

(b) Contours of the DFE’s $P_e$ under correct past decision, as a function of the feedback tap $d_1$ and feedforward tap $f_0$. The minimum $P_e$ is 0.1564.

Figure 4.13: Probability of error performance of a DFE evaluated for the Actual case and the CPD case. The channel is $H = [1, 0.9]'$, the SNR is 9 dB and the signal constellation is 4PAM. The actual $P_e$ as shown in (a) is the probability of error obtained when feeding the DFE with actual decisions and gives a true indication of performance. Plot (b) shows the probability of error when running the DFE using correct decisions. It gives an artificial indication of performance.
Figure 4.14: The above figure shows the comparison of actual $P_e$ versus $P_e$ under CPD as given in Figure 4.13(a) and (b) respectively. The minimum of the $P_e$ under CPD is the conventional $P_e$ and is denoted $P_e^{(CPD)}$. The minimum of the actual $P_e$ is denoted by $P_e^{(Actual)}$. The SNR is 9 dB and the modulation is 4PAM.
4.6 Conclusions

(a) The actual probability of error versus the ratio of feedback filter and feedforward filter weights for BPSK modulation at 3dB SNR. The actual probability of error is calculated when running the DFE in the Actual mode as described in Section 4.2.

(b) The probability of error obtained when operating the DFE in CPD mode (described in Section 4.2) versus the ratio of feedback filter and feedforward filter weights for BPSK modulation at 3dB SNR.

Figure 4.15: The above figures show that the minimum value of $P_e$ under the Actual mode operation does not tally with the minimum value of $P_e$ under the CPD mode operation. There is an obvious shift in the two minima.
values (obtained when running the DFE with actual decisions) only at sufficiently high SNRs. Particularly for systems that are expected to work at low signal to noise ratios, new design methodologies need to be developed in an attempt to achieve the actual MMSE and the actual minimum \( P_e \). The results presented in this chapter can be seen as a first step towards such methodologies.

4.7 Appendix

This appendix takes some of the results in Section 4.3 further. Here we attempt to solve for optimum \( D \) filter weights for Actual MMSE-DFE.

The expression for \( MSE_{CPD} \) is

\[
MSE_{CPD} = E\{((H')'A_k + F'N_k - D'A_{k-\Delta-1} + A_k'T)\}^2
\]

\[
= \sigma_n^2(F'H'F + \sigma_n^2D'D + \sigma_n^2F'F - 2\sigma_n^2T'H'F - 2\sigma_n^2D'M'H'F)). \tag{4.22}
\]

where \( E\{A_kA_{k-\Delta-1}'\} = M \) and \( M \) is defined in equation (2.8). We take the gradient of equation (4.22) with respect to \( F \) to get

\[
\nabla_F(MSE_{CPD}) = 2\sigma_n^2F'H'F + 2\sigma_n^2F - 2\sigma_n^2T'H'F - 2\sigma_n^2M'H'D. \tag{4.23}
\]

To obtain the optimum weights for \( F \), we equate the above gradient to zero as

\[
\nabla_F(MSE_{CPD}) = 0 \Rightarrow F_{min}.
\]

Thus,

\[
F_{min} = (H'H + \lambda I)^{-1}(H'M'D + H'T)\]. \tag{4.24}

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Putting this $F_{\text{min}}$ in to equation (4.22) we get

$$MSE_{CPD}(F_{\text{min}}, D) = \sigma^2_a (D'M\mathcal{H} + T'_\Delta \mathcal{H})(\mathcal{H}'H + \lambda I)^{-1}\mathcal{H}'\mathcal{H}(\mathcal{H}'H + \lambda I)^{-1}$$

\[ \begin{align*}
& (\mathcal{H}'M'D + \mathcal{H}'T_\Delta) + \sigma^2_a (I + D'D) \\
& + \sigma^2_a (D'M\mathcal{H} + T'_\Delta \mathcal{H})(\mathcal{H}'H + \lambda I)^{-2}(\mathcal{H}'M'D + \mathcal{H}'T_\Delta) \\
& - 2\sigma^2_a T'_\Delta \mathcal{H}(\mathcal{H}'H + \lambda I)^{-1}(\mathcal{H}'M'D + \mathcal{H}'T_\Delta) \\
& - 2\sigma^2_a D'M\mathcal{H}(\mathcal{H}'H + \lambda I)^{-1}(\mathcal{H}'M'D + \mathcal{H}'T_\Delta) \\
\end{align*} \] (4.25)

Taking the gradient of $MSE_{CPD}$ with respect to $D$, we get

$$\nabla_D \{MSE_{CPD}(F_{\text{min}}, D)\} = 2\sigma^2_a M\mathcal{H}(\mathcal{H}'H + \lambda I)^{-1}\mathcal{H}'\mathcal{H}(\mathcal{H}'H + \lambda I)^{-1}$$

\[ \begin{align*}
& (\mathcal{H}'M'D + \mathcal{H}'T_\Delta) + 2\sigma^2_a D \\
& + 2\sigma^2_a M\mathcal{H}(\mathcal{H}'H + \lambda I)^{-2}(\mathcal{H}'M'D + \mathcal{H}'T_\Delta) \\
& - 2\sigma^2_a T'_\Delta \mathcal{H}(\mathcal{H}'H + \lambda I)^{-1}\mathcal{H}'M' \\
& - 2\sigma^2_a M\mathcal{H}(\mathcal{H}'H + \lambda I)^{-1}(2\mathcal{H}'M'D + \mathcal{H}'T_\Delta). \\
\end{align*} \] (4.26)

Now we take the second derivative as it is used in deriving the optimum feedback filter weights in the Actual MMSE-DFE case (equation (4.31)).

$$\nabla^2_D \{MSE_{CPD}(F_{\text{min}}, D)\} = 2\sigma^2_a M\mathcal{H}(\mathcal{H}'H + \lambda I)^{-1}\mathcal{H}'\mathcal{H}(\mathcal{H}'H + \lambda I)^{-1}\mathcal{H}'M'$$

\[ \begin{align*}
& + 2\sigma^2_a I \\
& + 2\sigma^2_a M\mathcal{H}(\mathcal{H}'H + \lambda I)^{-2}\mathcal{H}'M' \\
& - 4\sigma^2_a M\mathcal{H}(\mathcal{H}'H + \lambda I)^{-1}\mathcal{H}'M' \\
\end{align*} \] (4.27)

In order to derive the $D_{\text{min}}$ weights for the Actual MMSE-DFE we proceed as follows. From equation (4.12) we get

$$MSE_{\text{Actual}}^{\text{upperbound}}(F_{\text{min}}, D) = MSE_{CPD}(F_{\text{min}}, D)$$

\[ \begin{align*}
& + 4\|D\| \sqrt{P_c} \sqrt{N_d} \sqrt{MSE_{CPD}(F_{\text{min}}, D)} \\
& + 4P_c N_d \|D\|^2. \end{align*} \] (4.28)
Taking the gradient of this equation with respect to $D$ we get

$$
\nabla_D MSE_{upperbound}^{Actual}(F_{\text{min}}, D) = \left(1 + 2 \frac{\sqrt{D} \sqrt{P_e \sqrt{N_d}}}{\sqrt{MSE_{CPD}(F_{\text{min}}, D)}}\right) \nabla_D MSE_{CPD}(F_{\text{min}}, D)
+ 2 \frac{D}{\sqrt{D} \sqrt{P_e \sqrt{N_d}}} \sqrt{MSE_{CPD}(F_{\text{min}}, D)} \\
+ 8 P_e N_d D.
$$

(4.29)

In the light of the above equation we make the following assumption.

$$
D = D_{\text{cpd}} + \rho
$$

(4.30)

where $\rho$ is a vector of correction terms on $D_{\text{cpd}}$ such that $\|D_{\text{cpd}}\| \gg \|\rho\|$. Writing equation (4.29) in terms of $D_{\text{cpd}}$ and $\rho$, and expanding $\nabla_D MSE_{CPD}(F_{\text{min}}, D + \rho)$ in a two terms Taylor’s series around $D_{\text{cpd}}$ we get

$$
\nabla_D MSE_{upperbound}^{Actual}(F_{\text{min}}, D_{\text{cpd}} + \rho) \approx \left(1 + 2 \frac{\|D_{\text{cpd}}\| \sqrt{P_e \sqrt{N_d}}}{\sqrt{MSE_{CPD}(F_{\text{min}}, D_{\text{cpd}})}}\right) \\
\nabla_D^2 MSE_{CPD}(F_{\text{min}}, D_{\text{cpd}}) \rho \\
+ 2 \frac{D_{\text{cpd}} + \rho}{\|D_{\text{cpd}} + \rho\|} \sqrt{P_e \sqrt{N_d}} \\
\sqrt{MSE_{CPD}(F_{\text{min}}, D_{\text{cpd}})} + 8 P_e N_d (D_{\text{cpd}} + \rho).
$$

(4.31)

Approximating $\frac{D_{\text{cpd}} + \rho}{\|D_{\text{cpd}} + \rho\|}$, by

$$
\frac{D_{\text{cpd}} + \rho}{\|D_{\text{cpd}} + \rho\|} = \frac{D_{\text{cpd}}}{\|D_{\text{cpd}}\|} + \frac{\rho}{\|D_{\text{cpd}}\|} - \frac{D'_{\text{cpd}} \rho}{\|D_{\text{cpd}}\|^2} D_{\text{cpd}}
$$

(4.32)

can rearrange equation (4.31) for values of $\rho$ as

$$
\rho \approx -\frac{(2 \sqrt{P_e \sqrt{N_d}} \sqrt{MSE_{CPD}(F_{\text{min}}, D_{\text{cpd}})} + 8 P_e N_d) D_{\text{cpd}}}{\nabla_D^2 MSE_{CPD}(F_{\text{min}}, D_{\text{cpd}}) + 8 P_e N_d}
$$

(4.33)

This equation provides the correction vector $\rho$ with which to obtain the actual $D$ weights from the $D_{\text{cpd}}$ weights.
We propose a generalization of the conventional fractionally spaced decision feedback equalization (FS-DFE) structure which replaces the symbol spaced feedback filter with a fractionally spaced feedback filter and replaces the scalar decision quantizer with a vector decision quantizer. We draw two significant findings regarding the design and performance of the new structure. Firstly, when optimized under the commonly used correct past decision assumption the new structure is functionally equivalent to an appropriately configured conventional FS-DFE. However, we show that this equivalence, like many other optimal designs in the DFE literature, is a misleading manifestation of the correct past decision assumption. By taking due regard to the likelihood of decision errors (and error propagation) we demonstrate that the new structure when optimized using a maximum a posteriori criterion outperforms the conventional FS-DFE.
5.1 Introduction

Equalization is the process of diminishing the effects of intersymbol interference and noise in a received signal to improve the estimate of the transmitted data. A quick review of equalization reveals that it can be broken up into various classes depending on the criterion for optimality, the structure employed, sampling rates and the computational complexity [18]. Receiver structures which are linear have been extensively studied and correspond to using a linear system inverse concept to effect equalization. The performance of linear equalizers falls well short of what is actually achievable because no account of the typical discrete nature of the input is used. The key to the improvements seen in nonlinear equalization methods lies in recognizing that equalization is more to do with recovery of discrete valued symbols than system inversion. Good equalization strategies therefore exploit the known discrete levels of the data symbols.

While the performance of nonlinear equalization strategies can obtain performance which is optimal against measures such as mean square error, maximum likelihood sequence or maximum a posteriori symbol detection, their implementation complexity is often too great to warrant practical use. However, decision feedback equalization is one nonlinear but suboptimal technique that does combine the good signal recovery attributes of the well designed nonlinear equalization strategies and a simplicity that challenges linear equalization.

The previous two chapters have dealt with the optimization of the DFE filters. In this chapter we will deal with the optimization of the decision device or the quantizer. This chapter is thematically linked with the previous chapters: it also deals with a generalization of the conventional DFE and it involves a critical re-examination of the correct past decision assumption.

Conventional fractionally spaced decision feedback equalization (FS-DFE) uses: a fractionally spaced forward filter, a symbol spaced spaced feedback filter, and a nearest neighbor scalar decision quantizer. Generally the fractionally spaced samples are taken at integral multiples of the baud rate. Receivers based
on fractionally spaced samples are more tolerant of sampler phase error and thus allow channel estimation without any explicit need for matched filtering [45].

It is anomalous in the conventional FS-DFE that the feedback filter operates at the baud rate whereas the forward processing, including the channel, is fractionally spaced. There exists no compelling argument in the literature why fractionally spaced feedback filtering would be ineffective or unnecessary. Here we rigorously examine this structural design issue and attempt to glean some insights. We consider a modified structure which uses a fractionally spaced feedback filter and replaces the scalar decision quantizer with a vector decision quantizer. The decision rule used in this vector quantizer is based on the maximum a posteriori criterion.

A secondary issue we address is the effect of employing the common simplifying design assumption that the equalizer does not make any errors. This is a strong assumption and is clearly not supported by operational evidence. A decision feedback equalizer makes errors and exhibits error propagation [57]. This assumption, for example, is employed to simplify the design process of determining the coefficient values of the FS-DFE filters under the minimum mean square error (MMSE) criterion [18]. In this work we look at the parallel question of how a vector decision quantizer design is affected by this assumption.

5.2 System Definition and Notation

5.2.1 Parallel Representation

Consider a real baseband model of a communication channel. The data symbols are denoted by $a_k$ where $k$ is a time index corresponding to multiples of the symbol period $T$. The channel model is the result of sampling at the receiver at a rate $f_s = N/T$ where $N \geq 2$ is an integer representing the number of samples per symbol period.

A discrete time channel model generated by fractionally spaced sampling can
be represented equivalently by either a higher rate serial representation or a lower rate parallel one (polyphase representation). The polyphase representation is used in this chapter and Figure 5.1 shows an example where the channel has a parallel representation clocked at the symbol rate. The oversampling parameter $N$ also reflects the degree of parallelism.

### 5.2.2 Channel Model

The input data symbols $a_k$ are drawn from a finite alphabet of symbols represented by

$$\mathbb{A} \overset{\Delta}{=} \{ \pm 1, \pm 3, \ldots, \pm (M - 1) \}$$

with cardinality $|\mathbb{A}| = M$ (positive and even). The symbols are assumed to be drawn independently from a uniform distribution on $\mathbb{A}$, i.e., $\Pr(a_k) = 1/M$.\(^\dagger\)

Note that the nearest neighbor quantizer for the alphabet $\mathbb{A}$ is given by the scalar nonlinearity

$$Q_{\mathbb{A}}(z) \overset{\Delta}{=} \arg \min_{a \in \mathbb{A}} \left\{ (z - a)^2 \right\}$$

\(^\dagger\) We use the notation $\Pr(\cdot)$ to denote probability (of a discrete random variable) and the notation $p(\cdot)$ to denote probability density (of a continuous random variable).
5.2 System Definition and Notation

where \( z \in \mathbb{R} \).

The fractionally spaced channel model is assumed to have a finite impulse response and be decomposable into a set of \( N \) symbol spaced subchannels (with \( L \) parameterizing the length of the subchannels). Each subchannel is given by

\[
[h_0^{(n)}, \ldots, h_L^{(n)}] \in \mathbb{R}^{L+1}, \quad n \in \mathbb{N}
\]

and it is indexed by \( n \), lying in the set \( \mathbb{N} = \{1, 2, \ldots, N\} \).

Define the \( nth \) subchannel, which excludes the leading tap \( h_0^{(n)} \), as

\[
H^{(n)} \triangleq [h_1^{(n)}, \ldots, h_L^{(n)}] \in \mathbb{R}^L, \quad n \in \mathbb{N}
\]

and a state vector of symbols

\[
\mathbf{A}_k \triangleq [a_{k-1}, \ldots, a_{k-L}] \in \mathbb{A}^L
\]

which is common to all subchannels.

Then the \( nth \) subchannel output is given by

\[
y_k^{(n)} = h_0^{(n)} a_k + \mathbf{A}_k' H^{(n)} + n_k^{(n)}, \quad n \in \mathbb{N}
\]

where \( n_k^{(n)} \) is assumed to be a zero mean, Gaussian process but not necessarily white. An explicit form of the noise distribution will be given later as it depends on the context. Note that the inner product term in equation (5.5) represents intersymbol interference.

5.2.3 Receiver Structures—Preamble

Figure 5.2 shows a novel fractionally spaced receiver which generalizes the FS-DFE. This structure can be contrasted with the conventional FS-DFE shown in Figure 5.3. Firstly, the new structure, shown in Figure 5.2, uses \( N \) feedback linear filters \( D^{(1)}, \ldots, D^{(N)} \) and a vector quantizer \( Q(\cdot): \mathbb{R}^N \rightarrow \mathbb{A} \) in contrast to the conventional FS-DFE, shown Figure 5.3, which uses a single feedback filter \( D_{\text{conv}} \) and the nearest neighbor scalar quantizer \( Q_h(\cdot) \) given in equation (5.2). Both the conventional FS-DFE and the new generalized FS-DFE use \( N \) forward
5.2 System Definition and Notation

linear filters; denoted $F_{\text{conv}}^{(1)}, \ldots, F_{\text{conv}}^{(N)}$ in the former case and $F^{(1)}, \ldots, F^{(N)}$ in the latter.

![Diagram](image)

Figure 5.2: Generalized fractionally spaced decision feedback equalizer incorporating fractionally spaced forward filters, fractionally spaced feedback filters and a vector quantizer.

![Diagram](image)

Figure 5.3: Conventional fractionally spaced decision feedback equalizer incorporating fractionally spaced forward filters, a symbol spaced feedback filter and a scalar nearest neighbor quantizer.

Initially to simplify the presentation, we will consider a simplification of Figure 5.2 shown in Figure 5.4. The receiver in Figure 5.4 excludes consideration of the fractionally spaced forward filters by setting

$$F^{(n)} = 1, \forall n \in \mathbb{N}. \tag{5.6}$$

Later we will return to an analysis of the system in Figure 5.2 and demonstrate
that it is a mild generalization of the analysis of the simpler system once channel and noise transformations are applied.

\[ y^{(1)}_k = y^{(N)}_k = F^{(1)}(N) = F^{(N)} = I \]

\[ z^{(l)}_k \]

\[ z^{(1)}_k \]

\[ z^{(N)}_k \]

\[ Q(.) \]

\[ D^{(1)} \]

\[ D^{(N)} \]

\[ \hat{a}_k \]

Figure 5.4: Simplified fractionally spaced decision feedback equalizer incorporating fractionally spaced feedback filters and a vector quantizer, where \( F^{(n)} = 1, \forall n \in \mathbb{N} \).

### 5.2.4 Simplified Novel FS-DFE Structure

Towards the goal of equalization for each subchannel, shown in Figure 5.4, an estimate of the intersymbol interference is subtracted, viz.,

\[ z^{(n)}_k = y^{(n)}_k - \hat{A}^{(n)}_k D^{(n)} \]  

(5.7)

where \( \hat{A}^{(n)}_k \) is an estimate of the state vector defined in equation (5.4) and is common to all subchannels, and

\[ D^{(n)} \triangleq [d^{(n)}_1, \ldots, d^{(n)}_L]^{'} \in \mathbb{R}^L \]  

(5.8)

is the \( n \)th feedback filter, in Figure 5.2. The mechanism for generating decisions \( \hat{a}_k \) of the symbols, which are the constituent elements of \( \hat{A}^{(n)}_k \), will be described shortly.

We assume that the feedback filters \( D^{(n)} \) defined in equation (5.8) are tuned to the corresponding postcursor subchannel coefficients of equation (5.3) viz.,

\[ D^{(n)} = H^{(n)}, \forall n \in \mathbb{N}. \]  

(5.9)
This setting ensures the best possible fit to the ISI term in equation (5.5) and means that any mismatch can only be due to an error in estimating the state, i.e., \( \hat{A}_k \neq A_k \).

The signals in equation (5.7) are combined to form the vector

\[
Z_k = \left[ z_k^{(1)}, \ldots, z_k^{(N)} \right]' \in \mathbb{R}^N
\]

which is the input to the vector quantizer \( Q(\cdot) \). As shown in Figure 5.2, its output is the symbol decision

\[
\hat{a}_k = Q(Z_k) \in A
\]

obtained with no decoding delay. The vector quantizer serves to partition the input space \( Z_k \in \mathbb{R}^N \) into \( M \) regions. Each is identified by a symbol decision corresponding to one of the \( M \) data symbols in the alphabet \( A \) of equation (5.1).

Various generalizations can be made to this formulation, such as imperfect tuning where equation (5.9) does not hold exactly and more general quantizers than equation (5.11) which may incorporate decision delay in the spirit of [75]. The most important generalization, from Figure 5.4 to Figure 5.2, is studied later in this chapter. These variations do not change the essential aspects of the work considered here which focuses on why parallel decision feedback gives performance advantages and how to design the vector quantizer \( Q(\cdot) \).

## 5.3 Problem Formulation

The essential problem to be solved is determining the explicit form of the optimal vector quantizer in the receiver of Figure 5.4. It is a function of the channel (and tuned feedback filter parameters) expressed through the quantizer input vector \( Z_k \), which is the subject of Section 5.3.1, and the vector noise process \( N_k \), which is the subject of Section 5.3.2.
5.3 Problem Formulation

5.3.1 Vector Quantizer Input

Given tuned parameters, as in equation (5.9), the nth quantizer input can be written as

$$z_k^{(n)} = h_0^{(n)} a_k + E_k^{(n)} + n_k^{(n)}$$  (5.12)

where the state error vector is defined by

$$E_k = [e_{k-1}, \ldots, e_{k-L}]' \in \mathbb{E}^L.$$  (5.13)

and $e_k \triangleq a_k - \hat{a}_k$ takes one of $2^M - 1$ discrete values from the set

$$E \triangleq \{0, \pm 2, \pm 4, \ldots, \pm 2(M - 1)\}.$$  

The vector form of equation (5.12) is given by

$$Z_k = R a_k + H E_k + N_k$$  (5.14)

where

$$R = [h_0^{(1)}, \ldots, h_0^{(N)}]' \in \mathbb{R}^{N \times 1} \setminus \{0\}$$  (5.15)

is a non-zero vector of subchannel cursors,

$$H = [H^{(1)}, \ldots, H^{(N)}]' \in \mathbb{R}^{L \times N}.$$  (5.16)

is a channel matrix composed of all subchannel vectors, and $N_k = [n_k^{(1)}, \ldots, n_k^{(N)}]'$ is a zero mean vector Gaussian process. Next we consider two cases based on the assumptions made on the vector noise process $N_k$.

5.3.2 Noise Modeling

In general the density function of the zero mean noise process $N_k$ can be written as a multivariate Gaussian density [76]

$$p_{N}(x) \triangleq \frac{1}{(2\pi)^{N/2}|\Sigma|^{1/2}} e^{-x'\Sigma^{-1}x/2}$$  (5.17)
where $\Sigma$ is the noise’s positive definite covariance matrix and $|\Sigma|$ is its determinant. The exact form of $\Sigma$ depends on the context of the problem as we will describe later.

A special case of the density given in equation (5.17) occurs when $\Sigma = \sigma^2 I_{N \times N}$ where $\sigma^2$ is the variance, meaning that $N_k$ is a zero mean vector white Gaussian process, uncorrelated from channel to channel. Then equation (5.17) simplifies to

$$p_N(x) \triangleq \frac{1}{(2\pi)^{N/2} \sigma^N} \prod_{n=1}^{N} e^{-x_n^2/2\sigma^2},$$

(5.18)

where $x_n$ is the nth component of $x$.

The decision rule used in the vector quantizer is the maximum a posteriori (MAP) criterion. It attempts to maximize

$$\Pr(a_k | Z_k)$$

(5.19)

over all possible values of $a_k \in A$ [77]. This maximizes the probability of making a correct symbol decision, i.e., $a_k = a_k$, given the measurements. We can rewrite equation (5.19) using Bayes’s Rule as

$$\Pr(a_k | Z_k) = \frac{p(Z_k | a_k) \Pr(a_k)}{p(Z_k)}.$$  

(5.20)

Being able to evaluate equation (5.20) plays an important role in determining the form of the vector quantizer (equation (5.21)). In the following sections we will consider two different means to determine equation (5.11). The first, considered in Section 5.4, relies on the simplifying assumption of correct past decisions. The second, in Section 5.5, makes no such assumption.

5.3.3 Vector Quantizer Design

The problem is to determine explicitly the memoryless nonlinear vector quantizer $Q(\cdot)$ in our generalized fractionally spaced decision feedback equalization structure, Figure 5.4 (and eventually Figure 5.2) that maximizes the probability of a correct decision given the measurements $Z_k$. 

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Mathematically the design problem is as follows: Given $Z_k$, determine

$$
\hat{a}_k = Q(Z_k) \triangleq \arg\max_{a_k \in A} \{ \Pr(a_k | Z_k) \}. \tag{5.21}
$$

The degree to which we obtain a solution to this design problem and the degree to which the performance is optimal, as we will see, depends crucially on whether a simplifying assumption is made regarding the existence of past errors.

### 5.4 Correct Past Decision Vector Quantizer Design

A common simplifying assumption used in DFE design is that there are no past decision errors—here, because the state is finite dimensional, this is tantamount to taking $E_k = 0$ in equation (5.14), so that we obtain

$$
Z_k = R a_k + N_k. \tag{5.22}
$$

We now attempt to use this simplifying assumption in designing the vector quantizer equation (5.21). The appropriate decision rule relies on a modification of the criterion equation (5.19), and its Bayes equivalent, equation (5.20), by including a conditioning on $E_k = 0$, viz.,

$$
\hat{a}_k = \arg\max_{a_k \in A} \left\{ \Pr(a_k | Z_k, E_k = 0) \right\}
= \arg\max_{a_k \in A} \left\{ \frac{p(Z_k | a_k, E_k = 0) \Pr(a_k | E_k = 0)}{p(Z_k | E_k = 0)} \right\}.
$$

We can simplify this expression by noting that the conditioning in $\Pr(a_k | E_k = 0)$ can be dropped (by independence and causality). Then since the data is assumed to have a uniform distribution, this is a constant. Also $p(Z_k | E_k = 0)$ is invariant to the parameter being maximized. Therefore, the decision rule becomes

$$
\hat{a}_k = \arg\max_{a_k \in A} \left\{ p(Z_k | a_k, E_k = 0) \right\}
= \arg\max_{a_k \in A} \left\{ p(N_k = Z_k - R a_k) \right\}
= \arg\max_{a_k \in A} \left\{ p_N(Z_k - R a_k) \right\}. \tag{5.23}
$$
Two design examples of a correct past decision vector quantizer will be presented: one based on assuming white Gaussian noise which is uncorrelated between the subchannels (Section 5.4.1), and the other (Section 5.4.2) which arises when fractionally space sampling induces correlation and linear forward filters, as shown in Figure 5.2) which further correlate the noise, are employed.

5.4.1 Quantizer Design Example I

In the simplest case we take the completely uncorrelated noise model given by equation (5.18). This design, which has no fractionally spaced forward filters and only has fractionally spaced feedback filters, is useful for channels which exhibit little precursor energy. In this chapter this simplest case is also useful as a means to illustrate the vector quantizer design in some detail without the complications of noise coloration before or after any forward linear filtering. In the following subsection we will give a generalization which makes no simplifications, and show through appropriate transformations that is mathematically equivalent to the development here.

In this case by examining \( p(Z_k | a_k, E_k = 0) \) component by component some simplifications becomes evident. Under the conditioning, the density of \( z_k^{(n)} \), from equation (5.12), can be written as the scalar Gaussian density

\[
p(z_k^{(n)} | a_k, E_k = 0) = p_n(z_k^{(n)} - h_0^{(n)}a_k) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(z_k^{(n)} - h_0^{(n)}a_k)^2 / \sigma^2}.
\]

Since we assumed that the noise is white and uncorrelated from subchannel to subchannel, then

\[
p(Z_k | a_k, E_k = 0) = \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(z_k^{(n)} - h_0^{(n)}a_k)^2 / \sigma^2} = \frac{1}{(2\pi)^{\frac{N}{2}}\sigma^N} \exp \left(-\frac{1}{2\sigma^2} \sum_{n=1}^{N} (z_k^{(n)} - h_0^{(n)}a_k)^2 \right).
\]

The maximization of equation (5.25), which is the objective of the decision rule in equation (5.23), is equivalent to maximizing the exponent in equation
(5.25), which is the same problem as the Euclidean metric minimization

\[ \hat{a}_k = \arg \min_{a_k \in \mathcal{A}} \left\{ \| Z_k - R \hat{a}_k \|^2 \right\} \]  

(5.26)
on \mathbb{R}^N$. We now show this is equivalent to a Euclidean metric minimization on $\mathbb{R}$ with the aid of Figure 5.5. This implies a considerable simplification.

![Euclidean metric interpretation of the optimal MAP vector quantizer assuming correct past decisions.](image)

Figure 5.5: Euclidean metric interpretation of the optimal MAP vector quantizer assuming correct past decisions. The solution to the original Euclidean distance minimization problem in $\mathbb{R}^N$ is equivalent to the solution of a scalar Euclidean distance minimization problem in $\mathbb{R}$.

Given $R$ from equation (5.15), let $P_R \triangleq RW'$ define the projection matrix onto the one dimensional subspace spanned by $R$ where

\[ W \triangleq R/\| R \|^2. \]  

(5.27)

Then

\[ Z_k = P_R Z_k + (I - P_R) Z_k \]

\[ = (W'Z_k) R + P_R^\perp Z_k \]  

(5.28)

where $P_R^\perp \triangleq I - P_R$ is a projection onto the orthogonal complement of the space spanned by $R$ (refer to Figure 5.5). Then the argument of equation (5.26)
becomes
\[
\|Z_k - R a_k\|^2 = \|(W'Z_k - a_k)R + P_R^\perp Z_k\|^2 \\
= (W'Z_k - a_k)^2 \|R\|^2 + \|P_R^\perp Z_k\|^2,
\]
where we have used the orthogonality of $P_R^\perp$ and $P_R$. Note that $R$ and $P_R^\perp Z_k$ are independent of $a_k$. Hence, the minimization in equation (5.26) is equivalent to the scalar minimization
\[
\tilde{a}_k = \arg \min_{a_k \in A} \left\{ (W'Z_k - a_k)^2 \right\}
\]
(5.29)
This is nothing less than
\[
\tilde{a}_k = Q_A(W'Z_k).
\]
(5.30)
from equation (5.2). We defer interpretation of this result until after the next two subsections.

5.4.2 Quantizer Design Example II

Many generalizations of the system considered in Section 5.4.1 are possible. Here two generalizations are made: a correlated noise model case as given in equation (5.17) and the use of feedforward filters in the receiver as shown in Figure 5.2.

We will show that the design example presented in this section is essentially equivalent to that presented in Section 5.4.1 by applying various transformations to the channel and noise models.

In order to show the equivalence of the two designs, i.e., the white noise model without feedforward filters and the colored noise model with feedforward filters, we proceed as follows. Let $C(n)$ denote the actual subchannels and $F(n)$ denote the corresponding feedforward filters; then in the light of linearity, the symbols are acted upon by the effective channel
\[
H(n) = C(n) * F(n).
\]
We assume that the feedback filter parameter settings are tuned, such that equation (5.8) holds (where $H^{(n)}$ is now the effective channel, incorporating the feedforward filters). Also $R$ is given by equation (5.15) with a similar reinterpretation of the channel. In what follows we say nothing about how to optimize the parameter values in $F^{(n)}$ and $D^{(n)}$. This is because the vector quantization optimization is valid for arbitrary filter settings except that we do require tuned feedback filters using equation (5.9). This implies that the following derivations are quite general.

The significant difference between this case and the one in Section 5.4.1 is that the noise is correlated now. The sources of the noise correlation are manifold. There can be correlation between $N$ subchannels as well as within any physical subchannel $n$—this arises naturally from sampling to obtain the discrete time model. Further correlation occurs because the noise in filtered through the feedforward filters, as shown in Figure 5.2. Let $\Sigma > 0$ in equation (5.17) model the totality of such correlations from sampling and filtering. Then from equation (5.23), the decision rule is

$$\hat{a}_k = \arg \max_{a_k \in A} \left\{ p_N(Z_k - R a_k) \right\}$$

$$= \arg \max_{a_k \in A} \left\{ \frac{1}{(2\pi)^{N/2} |\Sigma|^{1/2}} e^{-[Z_k - \Sigma^{-1} R a_k]/2} \right\}$$

$$= \arg \min_{a_k \in A} \left\{ [Z_k - \Sigma^{-1} R a_k] \Sigma^{1/2} [Z_k - \Sigma^{-1} R a_k] \right\}$$

$$= \arg \min_{a_k \in A} \left\{ \|Z_k - \Sigma^{-1} R a_k\|^2 \right\}$$

where $\Sigma^{-1}$ is positive definite ($\Sigma^{-1} > 0$) and so has a Cholesky factorization $\Sigma^{-1} = GG'$ where $G$ is lower triangular. Let $X = Z_k - \Sigma^{-1} R a_k$ and consider the

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2 Usually the receiver’s analog front end filter $q(t)$ has a bandwidth which matches that of the transmitted signal. In turn the transmitted signal tends to have a small excess bandwidth so that neither the symbol spaced nor fractionally spaced noise samples can be white—usually the noise is strongly correlated. This issue has been avoided in previous chapters.
metric of interest

\[ \|X\|_{\Sigma^{-1}}^2 = X'GG'X \]
\[ = \|G'X\|^2 \]  \hspace{1cm} (5.32)

Thus, an interpretation of \( G \) is that it transforms the coordinate system such that the metric to be minimized is now Euclidean. This permits us to transform the problem of equation (5.31) into one directly analogous to equation (5.26). Thus equation (5.31) simplifies to

\[ \hat{a}_k = \arg\min_{a_k \in A} \left\{ \|\tilde{Z}_k - \tilde{R}a_k\|^2 \right\} \]  \hspace{1cm} (5.33)

where \( \tilde{Z}_k = G'Z_k \) and \( \tilde{R}_k = G'R_k \). Now equation (5.33) and hence equation (5.31) are mathematically equivalent to equation (5.26). Therefore, the simplification of the minimization problem applies, as in equation (5.29), and we can reduce the norm minimization problem in equation (5.31) and equation (5.33) in \( \mathbb{R}^N \) to the scalar minimization problem

\[ \hat{a}_k = \arg\min_{a_k \in A} \left\{ \left( \frac{\tilde{R}'\tilde{Z}_k - a_k}{\|\tilde{R}\|} \right)^2 \right\} \]
\[ = \arg\min_{a_k \in A} \left\{ \left( \frac{R'\Sigma^{-1}Z_k - a_k}{R'\Sigma^{-1}R} \right)^2 \right\}. \] \hspace{1cm} (5.34)

Utilizing the nearest neighbor quantizer of equation (5.2) we can write

\[ \hat{a}_k = Q_A(\hat{W}'Z_k) \] \hspace{1cm} (5.35)

where

\[ \hat{W} = \Sigma^{-1}R/\|R\|_{\Sigma^{-1}} \] \hspace{1cm} (5.36)

is a vector of weights.

Equation (5.35) has a simple geometrical interpretation. A general vector quantizer partitions the \( Z_k \)-space into \( M \), possibly irregular, decision regions. For equation (5.35), these decision regions are bounded by a set of parallel hyperplanes perpendicular to \( \hat{W} \), i.e., \( \hat{W}'Z_k = 0 \), \( \hat{W}'Z_k = \pm 2 \), etc.
Figure 5.6 shows the receiver structure with the optimal vector quantizer.

We now move onto interpreting the significance of the simplifications found in this section.

### 5.4.3 Equivalence to Conventional FS-DFE with Scalar Quantizer

We can conclude that under the correct past decision assumption the optimal MAP vector quantizer of equation (5.10) reduces to linearly combining the components of the input vector $Z_k$ with weights $\tilde{W}$, followed by a nearest neighbor quantizer for the alphabet $\mathcal{A}$. Since all processing, apart from the quantizer, is linear this means that we can replace the fractionally spaced feedback processing structure with a scalar symbol spaced structure and a scalar nearest neighbor quantizer.

Thus the system shown in Figure 5.6 boils down to the conventional fractionally spaced decision feedback equalization structure shown in Figure 5.3. Comparing the two systems we can draw the following equivalences:

$$D_{\text{conv}} = \sum_{n=1}^{N} \tilde{w}^{(n)} D^{(n)}$$

$$F_{\text{conv}}^{(n)} = \tilde{w}^{(n)} F^{(n)}, \quad n \in \mathbb{N}$$

$$F_{\text{conv}}^{(n)} n_k^{(n)} = \tilde{w}^{(n)} F^{(n)} n_k^{(n)}, \quad n \in \mathbb{N}$$

This means that there seems to be no advantage in using the new fractionally spaced feedback structure and vector quantizer because when optimized it reduces to the conventional structure. However, this equivalence, like many other supposedly optimal designs in the DFE literature, is based on the assuming correct past decisions. This is misleading because it cloaks the potential performance benefits of our new structure (as will be revealed in the following section).
5.5 Maximum A Posteriori Vector Quantizer Design

5.5.1 Decision Rule

In this section we drop the assumption that past decisions are correct. Note that the error vectors $E_k$ of equation (5.13) are mutually exclusive, and so we can employ Bayes’s rule to rewrite equation (5.19) as

$$
\Pr(a_k | Z_k) = \sum_{E_k} \Pr(a_k | Z_k, E_k) \Pr(E_k | Z_k)
$$

$$
= \sum_{E_k} \frac{p(Z_k | a_k, E_k) \Pr(a_k | E_k)}{p(Z_k | E_k)} \Pr(E_k | Z_k)
$$

$$
= \sum_{E_k} \frac{p(Z_k | a_k, E_k) \Pr(a_k)}{p(Z_k)} \Pr(E_k)
$$

(5.37)

where the summations are taken over all possible values of the state error vector. From equation (5.37) and equation (5.21), and eliminating terms invariant to the
maximization, the decision rule can be written as

\[ \hat{a}_k = \arg \max_{a_k \in \mathcal{A}} \left\{ \sum_{E_k} p(Z_k | a_k, E_k) \Pr(E_k) \right\} \]

\[ = \arg \max_{a_k \in \mathcal{A}} \left\{ \sum_{E_k} \Pr(E_k) \times p_N (Z_k - R a_k - E_k' H) \right\}. \quad (5.38) \]

Note that equation (5.38) is an extension of equation (5.23) except we now make provision for the possibility of decision errors and weight them according to their likelihood of occurring. However, unlike the results in Section 5.4, this maximization does not reduce to a Euclidean distance minimization. Instead we have a weighted sum of exponents of Euclidean distances which needs to be minimized, viz.,

\[ \hat{a}_k = \arg \min_{a_k \in \mathcal{A}} \left\{ \sum_{E_k} \Pr(E_k) \times \frac{1}{(2\pi)^{N/2} \sigma^N} \times \exp \left( -\frac{1}{2\sigma^2} \|Z_k - R a_k - E_k' H\|^2 \right) \right\}. \]

\[ \quad (5.39) \]

5.5.2 Iterative Design Procedure

There is a difficulty in implementing equation (5.39); the weighting \( \Pr(E_k) \) is a function of the decisions, \( \hat{a}_k \), which in turn depends on the equation (5.39). In other words, the decision rule equation (5.39) only implicitly defines how the decisions are made.

The difficulty in obtaining a closed form expression for \( \Pr(E_k) \) motivates a numerical solution which we call the iterative design procedure. The vector quantizer equation (5.39) is tuned through the steps which determine progressive estimates of the density \( \Pr(E_k) \).

**Step (1)**

\[ \Pr(E_k) = \begin{cases} 
1 & \text{if } E_k = 0 \\
0 & \text{otherwise}
\end{cases} \]

Call this the zeroth iteration of the estimate of the density \( \Pr(E_k) \). Note that this simply implements the optimal vector quantizer assuming correct past decisions (equation (5.35)).
5.5 Maximum A Posteriori Vector Quantizer Design

**Step (2)** Simulate a communication system with equation (5.39) using the estimates of the density \( \Pr(E_k) \) (obtained from the previous step).

**Step (3)** Use Monte Carlo techniques to determine revised estimates of \( \Pr(E_k) \).

If these estimates differ significantly from those used from the previous iteration return to Step (2) and use the revised estimates.

Under this iterative design procedure, the convergence of \( \Pr(E_k) \) to some steady state value typically occurs within three to four iterations.

![Decision regions in \( \mathbb{Z}_k \)-space for a \( N = 2 \) input BPSK vector quantizer](image)

Figure 5.7: Decision regions in \( \mathbb{Z}_k \)-space for a \( N = 2 \) input BPSK vector quantizer for a channel model with \( \mathbf{H}^{(1)} = [0.55, 1.00]' \), \( \mathbf{H}^{(2)} = [0.10, 0.90]' \) at 10dB SNR. The dashed line is the decision boundary for the optimal vector quantizer assuming correct past decisions. The solid line is the decision boundary for the MAP vector quantizer obtained from the iterative design procedure. The ‘o’ (\( a_k = -1 \)) and ‘+’ (\( a_k = +1 \)) mark the various centers for the Gaussian terms in equation (5.38).
5.5 Maximum A Posteriori Vector Quantizer Design

Figure 5.8: Simulated BER vs SNR for a channel model with $H^{(1)} = [0.55, 1.00]'$ and $H^{(2)} = [0.10, 0.90]'$. The points ‘*’ are for the optimal vector quantizer assuming correct past decisions. The points ‘x’ are for the MAP vector quantizer obtained from the iterative design procedure.
5.5 Maximum A Posteriori Vector Quantizer Design

5.5.3 Design Example

Figure 5.7 shows an example of the results of using the iterative design procedure. The parameters chosen in the simulation are: $M = 2$ (BPSK), $N = 2$ (i.e., an oversampling factor of 2), $H^{(1)} = [0.55, 1.00]$, $H^{(2)} = [0.10, 0.90]$ and a 10dB SNR. Depicted are the decision boundaries in $Z_k$-space for the initial vector quantizer (the dashed line) and the final vector quantizer (the solid line). As discussed earlier the dashed line case is equivalent to Step (1) which is the design assuming correct past decisions. Clearly there is a significant difference between the two decision boundaries and hence we can expect different performances.

The corresponding BER results are shown in Figure 5.8. The MAP vector quantizer obtained from the iterative design procedure shows an improvement of up to 1.9dB over the optimal vector quantizer assuming correct past decisions\(^3\).

Finally, we give a simple explanation of the superior operation of the new receiver over the one where correct past decisions are assumed. We refer back to Figure 5.7. The conventional receiver would output $+1$ shown by ‘+’, whereas the new receiver would output a $-1$ decision shown by ‘o’. The conventional receiver’s decision is based on the premise that noise accounts for the difference between any point ‘+’ and any other point in the decision region. The new FS-DFE equalizer with its vector quantizer exploits the explanation that the error in the previously decoded decision plus noise accounts for the difference between any point ‘o’ and any other point in the decision region. While this explanation contains a degree of oversimplification it does explain the essence of the idea—the actual decision is made by weighting all possible decision and noise scenarios and selecting the most likely one.

\(^3\) While only significant improvements are seen at lower SNRs, there are practical systems which operate well into this regime relying on powerful error correcting codes to improve the overall error rate. One such example is the current ATSC Digital Television standard which recommends the use of a DFE and needs to operate down to a symbol error (8-level data) rate of 0.2 at the so-called threshold of visibility.
5.6 Summary and Conclusions

A generalized decision feedback equalizer structure which uses fractionally spaced feedback filters and a vector quantizer has been developed and studied. Two MAP designs for the vector quantizer were considered. Under the simplifying assumption of correct past decisions the structure was shown to be functionally equivalent to a conventional fractionally spaced decision feedback equalizer. Under a more general MAP criterion, which takes into account the possibility of decision errors a different structure was obtained with superior performance.

These results highlight the ever-present danger of using the common correct past decision assumption to tune decision feedback based receiver structures—in no sense do these designs reflect optimal performance. Superior performance can be obtained by taking into account the possibility of decision errors in the design process.
We continue with the problem of quantizer optimization in a DFE structure from Chapter 5. In this chapter we derive an optimal nonlinear quantizer to be used as the decision function in a decision feedback structure. The criterion used to determine the quantizer function is the degree to which the propagation of decision errors is suppressed. We show that a soft saturation nonlinearity outperforms the standard nearest neighbor quantizer in a decision feedback equalizer structure in terms of error propagation suppression. This implies that on channels where noise is impulsive a soft saturation quantizer decision feedback structure can perform better than a conventional decision feedback structure on a broad class of channels. The theoretical results are supported by a set of simulations.
6.1 Introduction

Decision feedback is an equalizer design philosophy which greatly simplifies practical implementation while maintaining reasonable performance relative to more optimal structures [18]. The philosophy expounds that estimates of past data symbols can be used as if correct in determining current and future symbol estimates [56]. This is a rather strong assumption. However, the alternative is to use a symbol sequence detection scheme, such as the Viterbi algorithm, which typically has high complexity and thus is impractical [18]. The simplicity of decision feedback structures is further emphasized by the recognition that, in terms of implementation, the feedback filters are simpler than transversal filter structures (in the form of a linear equalizer).

The decision feedback equalizer is usually synonymous with the notion of decision feedback [36]. However, as discussed in Section 2.7.3, recently, decision feedback has become prominent as a device in generating simplified reduced state sequence estimation schemes which attempt to approximate the optimal Viterbi algorithm. These schemes implicitly utilize multiple past symbol estimates (decisions) to reduce the complexity and dimensionality of the trellis. Thus it can be said that decision feedback is an effective tool in producing implementable and practical receiver structures in telecommunications. The ideas in this chapter, which are formulated in terms of the simpler decision feedback equalizer, should have ready applicability to the more recent reduced state Viterbi schemes.

The symbol error rate or the bit error rate are central measures to evaluate the performance of any communications receiver structure. For structures which utilize decision feedback, symbol errors are made in two ways: (i) primary errors which can be thought of as being generated by channel noise; and (ii) secondary errors due to error propagation triggered by a primary error (see Section 2.4). The distinction need not be so explicit but it is convenient for the work done in this dissertation. Primary errors tend to be straightforward to understand and are unavoidable in any receiver structure. Error propagation is well recognized
but less well understood \[42, 56, 57\]. This difficulty in understanding arises since decision feedback is inherently nonlinear and recursive, and thereby difficult to analyse.

The analysis of decision feedback equalizers has concentrated on the study of primary errors. To make such an analysis authors make the assumption that past decisions are correct. The details are not important but this has the rather idealistic effect of permitting linear systems analysis tools to be used. Necessarily this analysis glosses over the effect of error propagation and researchers often resort to simulations to characterize it.

The study of error propagation has concentrated on two approaches: (i) finite state machine analysis \[42, 56\]; and (ii) nonlinear stability analysis using passivity theory\(^1\) \[57\]. This chapter uses the latter approach to devise a new structure that has improved (diminished) error propagation.

This chapter considers the question about what is the best way to generate (preliminary) decisions given that they are fed back. It is convenient to refer to the conventional nearest neighbor decision device as a \textit{hard} decision device. A decision device which admits the possibility that more levels than the symbol alphabet are possible is referred to as a soft decision device.

\section*{6.2 Problem Formulation}

\subsection*{6.2.1 Channel and Equalizer Model}

A communication channel, shown in Figure 6.1, subject to intersymbol interference (ISI), is represented by the following input-output equivalent baseband

\(^1\)Passivity theory has its origins within circuit theory \[78\]. A passive circuit dissipates all energy in a finite time and hence is stable. The application of the passivity theory in the DFE leads to conditions on the channel for the DFE to have a finite error recovery time for all initial conditions and for all input sequences \[26\].
Figure 6.1: Variant on decision feedback equalizer. \( H(z) = 1 + \sum_{i=1}^{\infty} h_i z^{-i}, h_0 = 1, \) and \( H(z) - 1 = \sum_{i=1}^{\infty} h_i z^{-i}. \) Note that the feedforward filter structure is omitted as it is not relevant for the analysis done in this chapter.

Discrete time representation,

\[
y_k = a_k + \sum_{i=1}^{\infty} h_i a_{k-i} + n_k
\]  

(6.1)

where \( k \) denotes time, \( a_k \) is the discrete data belonging to some alphabet \( A, \) \( \{h_i\} \) are the channel parameters, \( n_k \) is additive noise (not necessarily Gaussian) and \( y_k \) in the noisy channel output. Note that in equation (6.1), the leading channel coefficient (which could be represented by \( h_0 \)) is defined to be unity without loss of generality.

With regards to correcting for the intersymbol interference found on this channel, consider the simple variant on the standard DFE in Figure 6.1. Ordinarily a single quantizer

\[
\mathcal{Q}_A(\cdot): \mathbb{R} \mapsto A
\]

is employed which maps a continuous value to an alphabet symbol in a nearest neighbor fashion, e.g., if the data were binary then this quantizer would be the standard signum function. For the standard DFE this type of nearest neighbor quantizer is found inside the feedback loop.

The problem we investigate concerns the optimum choice, in the appropriate sense, of a suitable memoryless nonlinearity (in lieu of the nearest neighbor
6.2 Problem Formulation

quantizer), which we call the soft decision function or quantizer.

\[ S(\cdot): \mathbb{R} \rightarrow \mathbb{R} \]

As shown in Figure 6.1, it is placed inside the feedback loop. Naturally, we ultimately need to generate estimates of the transmitted data and thereby force the estimates at the output of the equalizer to take on one of the alphabet values. This justifies the use of the terminating nearest neighbor quantizer \( Q_A(\cdot) \) seen in Figure 6.1. We refer to the output of the first nonlinearity as the soft decision and denote it by the symbol \( \tilde{a}_k \). The hard decisions are denoted by \( \hat{a}_k \) and the two decisions are related via

\[ \hat{a}_k = Q_A(\tilde{a}_k). \quad (6.2) \]

From equation (6.1) the defining recursive equation for the new nonlinear equalizer structure is given by

\[ \tilde{a}_k = S(a_k + r_k) \quad (6.3) \]

where

\[ r_k = \sum_{i=1}^{\infty} h_i (a_{k-i} - \tilde{a}_{k-i}) \quad (6.4) \]

is the residual intersymbol interference now. The noise term is omitted here for the sake of simplification.

Finally we present some important but straightforward properties that the soft decision quantizer \( S(\cdot) \) should satisfy if the nonlinear equalizer is to behave as expected. That is, if the ISI is zero or sufficiently close to zero then the soft decisions should be such that the related hard decisions be correct.

**Property 1:** \( S(a) = a \) for all \( a \in A \).

**Property 2:** \( S(\cdot) \) is continuous at \( a \) for all \( a \in A \).
6.2 Problem Formulation

6.2.2 Error Model

This chapter focuses on means to mitigate the detrimental effects of error propagation in a nonlinear feedback structure by optimally selecting the soft decision function. Towards this goal we define two error signals: the soft error,

\[ \tilde{e}_k \triangleq a_k - \tilde{a}_k, \]  

and the hard or final decision error

\[ \hat{e}_k \triangleq a_k - \tilde{a}_k. \]

An equation that captures the soft decision feedback recursion but expressed in terms of the soft error signals, \( \tilde{e}_k \), is therefore given by

\[ \tilde{e}_k = a_k - S(a_k + r_k) \]

where from equation (6.4),

\[ r_k = \sum_{i=1}^{\infty} h_i \tilde{e}_{k-i}. \]

Figure 6.2: Relationship between nonlinearities for fixed \( a_k \): (a) \( S(\cdot) \) (b) \( P_{a_k}(\cdot) \)

We can then express the new error signal, \( \tilde{e}_k \), as a memoryless nonlinear function of the signals \( a_k \) and \( r_k \). In what follows it is convenient to write

\[ w_k \triangleq -\tilde{e}_k = \begin{align*} & = S(a_k + r_k) - a_k. \end{align*} \]  

(6.8)
6.2 Problem Formulation

to permit the error system to be represented in a conventional negative feedback control form. A crucial but simple task is to relate geometrically the nonlinear, memoryless mapping which takes $r_k$ to $w_k$ (and is parametrized by $a_k$)

$$\mathcal{P}_{a_k}: r_k \mapsto w_k$$

(6.9)
in terms of the memoryless nonlinearity $S(\cdot)$. The nature of $\mathcal{P}_{a_k}(\cdot)$ can be directly inferred from equation (6.8). For a frozen $a_k$, $\mathcal{P}_{a_k}(\cdot)$ is identical to $S(\cdot)$ apart from a shift in the input and an equal shift in magnitude and sign at the output. For example, if $S(\cdot)$ is given by the nonlinearity in Figure 6.2(a) then $\mathcal{P}_{a_k}(\cdot)$ is shown in Figure 6.2(b). To characterize $\mathcal{P}_{a_k}(\cdot)$ completely it remains to take into account the finite variation of $a_k$ over $\Lambda$. This equivalent nonlinear map may be represented by the superposition of the transposed nonlinear maps for various $a_k$.

Since only one input $a_k$ is operative at any time instant $k$ then the mapping is truly a function (one-to-one). However, it is convenient to represent the mapping from $r_k$ to $w_k$ (independently of $a_k$) as a multivalued nonlinearity because as will be seen later the effect of the variation in $a_k$ is minor.

![Figure 6.3: Error system for the soft decision errors $\tilde{e}_k$.](image)

Bringing the above results together we consider Figure 6.3. The top block, which is linear and strictly proper, is a representation of equation (6.7), i.e., takes $\tilde{e}_k$ and generates the residual ISI $r_k$. The bottom block is the nonlinearity $\mathcal{P}_{a_k}(\cdot)$ which maps $r_k$ (and $a_k$) to $w_k$ (equivalent to $-\tilde{e}_k$).
6.3 Stability and Error Propagation

6.3.1 Decision Feedback Philosophy

Our beginning point is the recognition that the error propagation phenomenon is a major detrimental influence on the performance of decision feedback structures. Here it is important to clarify this statement because means to overcome potential problems (i.e., error propagation) will be suggested and be shown to improve performance, yet maintain the spirit of the decision feedback philosophy—characterized by simplicity of practical implementation and superior performance than the linear equalizer.

Decision feedback is a key notion that permits a considerable simplification and practical implementation of nonlinear equalizers. It asserts that under the assumption of correct past decisions an optimal decision rule based on a reduced order problem can be derived. When the assumption fails, it can be argued that the performance is adversely influenced well beyond the detrimental effect of an isolated, noise generated error. This argument says that if an error is made, the recursion is such that future errors are more likely to be generated. Indeed due to the linear feedback filter the influence of the error may be present for a considerable number of symbol periods during which further errors are more likely.

6.3.2 Stability Concept

The idea of reformulating the error recovery problem as a stability problem originated with Cantoni et al. [46]. Our analysis here takes up this concept since it is natural to investigate the use of stability ideas in proving that, under certain conditions, a decision feedback structure has a finite recovery time (for all initial conditions and input sequences). The ideas we need have their origins within circuit theory. Our main result uses Passivity Theory [78] to give an easily checked frequency domain condition that guarantees a finite recovery time.

The problem of curtailing error propagation naturally leads to the study of
error recovery time. It is defined as the time required for an initial error state to return to zero. This could mean that the hard decision error becomes identically zero or the soft decision error becomes negligible. We define it in terms of the hard decision error, as follows.

**Error Recovery:** Given an arbitrary initial error state, the decision feedback structure has *recovered from error by time* $K$ if

$$\hat{e}_k = 0, \quad \forall k \geq K.$$  \hfill (6.10)

The relationship of this definition of error recovery to stability is given next.

**Stability:** A decision feedback structure is *stable* if the error recovery time is finite for all possible initial error states and for all possible input sequences.

This can be regarded as a stability concept in the sense that the zero error condition can be regarded as the natural (desirable) condition and an initial error state acts like a transitory perturbation away from zero. Qualitatively the faster the structure can return to the zero error condition, the better the performance will be in terms of lower transmission errors.

### 6.3.3 Conditions for Stability

In this subsection we determine the conditions which are equivalent to the previous stability notion. These results form the basis of this chapter’s key results which parallel those presented in [57].

The first result relates to the hard decision errors $\hat{e}_k$. It follows directly from the observation that $\hat{e}_k$ takes on only discrete values from the finite alphabet $\mathcal{A}$:

**Theorem 1 (Hard Decision Errors)**

$$\hat{e} \in l_2 \Rightarrow \hat{e}_k = 0, \quad \forall k \geq K \quad K < \infty.$$
6.3 Stability and Error Propagation

Here $l_2$ denotes the space of square summable sequences. (The choice of $l_2$ for this argument is not crucial, indeed $l_p$ is also fine for any $1 \geq p < \infty$. However, separate arguments relying on passivity ideas for which energy is a central concept heavily favor the use of $l_2$.)

Our analysis makes use of the soft decision errors, $\tilde{e}_k$, and therefore we need a second key result that relies on equation (6.5) and equation (6.6):

**Theorem 2 (Soft to Hard Decision Relationship)**

$\tilde{e} \in l_2 \Rightarrow \hat{e} \in l_2$.

**Proof:** Using the properties of a soft decision quantizer (i.e., that they leave the alphabet symbols unaffected and locally continuous at those points):

$\tilde{e} \in l_2 \Rightarrow \tilde{e}_k \rightarrow 0 \text{ as } k \rightarrow \infty$

$\Rightarrow \tilde{a}_k \rightarrow a_k \text{ as } k \rightarrow \infty$

$\Rightarrow \exists K(\epsilon) \text{ s.t. } |\tilde{a}_k - a_k| < \epsilon \quad \forall k \geq K(\epsilon)$

$\Rightarrow Q_A(\tilde{a}_k) = a_k \quad \forall k \geq K(\epsilon)$

whenever

$$\epsilon < \min_{a^{(1)} \neq a^{(2)}} \left\{ \frac{1}{2} |a^{(1)} - a^{(2)}| \right\}, \quad a^{(1)}, a^{(2)} \in \mathbb{A}$$

because $Q_A(\cdot)$ is a nearest neighbor rule. However, $Q_A(\tilde{a}_k) - a_k = \tilde{a}_k - a_k = \hat{e}_k$, whereby we infer

$$\hat{e}_k = 0 \quad \forall k \geq K(\epsilon) \Rightarrow \hat{e} \in l_2.$$ 

6.3.4 Sector Bound Constraint

The error system in Figure 6.3 comprises of two blocks back to back. The use of passivity pivots around having the back to back systems passive, i.e., they
6.3 Stability and Error Propagation

dissipate energy. Concentrating on the lower block; the passivity of the lower block (which is influenced by the receiver structure design) can be guaranteed by having it sector bounded in an input-output sense. Figure 6.4 shows what it means for a nonlinear system to be \([0, \alpha]\) sector bounded; the curve defining the input-output relationship is bounded above by a line of slope \(\alpha > 0\) (and bounded below by 0). An interpretation of this condition is that the instantaneous input output gain is non-negative and no more than \(\alpha\).

We can use equation (6.8) to determine conditions on \(S(\cdot)\) such that \(P_{a_k}(\cdot)\) in equation (6.9) is \([0, \alpha]\) sector bounded. Just as we noted that \(P_{a_k}(\cdot)\) could be determined from \(S(\cdot)\) via a simple translation (whose magnitude was determined by \(a_k\)); conversely, we can say that \(S(\cdot)\) can be determined via translation from \(P_{a_k}(\cdot)\). The only interesting or nontrivial aspect of this reverse relationship is that there is ambiguity due to \(a_k\) which can take any of the discrete values in \(A\). The result is shown in Figure 6.5 which shows the intersection of the translations of Figure 6.4 for the alphabet

\[A = \{-3, -1, +1, +3\}.\]

Other alphabets are analogously determined.

The interpretation of Figure 6.5 is straightforward. Any (soft) decision func-

Figure 6.4: Sector \((0, \alpha)\) bounded nonlinearity.
tion, $S(\cdot)$, which lives in the shaded region is guaranteed to have a corresponding $P_{ah}(\cdot)$ which is sector bounded to $[0, \alpha]$ as in Figure 6.4.

### 6.4 Main Results

For the error system in Figure 6.3 we have the following key result (the proof is a generalization of the one given in [56]. As it is somewhat technical in nature, it has been simplified for this presentation).

**Theorem 3 (Stability Condition)** Suppose the lower block in Figure 6.3 is $[0, \alpha]$ sector bounded, then the overall system is stable in the sense of error propagation if

$$\frac{1}{\alpha} + \sum_{i=1}^{\infty} h_i \cos(i \theta) > 0, \quad \forall \theta \in [0, 2\pi]. \quad (6.11)$$

The condition is a positive real condition, which leads to the deduction that $\tilde{e}_k \in l_2$, which in turn implies, via Theorem 2, that $\tilde{e}_k \in l_2$ (stability).

As an illustration, if the ISI is limited in extent to two terms then the stability condition determines conditions on the sizes of $h_1$ and $h_2$. Figure 6.6 shows the region in $h_1$ and $h_2$ space corresponding to equation (6.11) as the shaded rounded triangle its axis intercepts are at magnitude $\alpha^{-1}$.

The objective in terms of designing a quantizer $S(\cdot)$ is to maximize the stability region described by equation (6.11). As a designer we have control over $S(\cdot)$ and this will influence $\alpha$. From Figure 6.5 the equivalent objective is to minimize $\alpha$ and it is clear that this is achieved when $\alpha = 1$ (the curve passes through points $(a, a)$ for all $a \in A$. The constraint in this case narrows to a 45 degree line for inputs from $-(M - 1)$ to $(M - 1)$ for the usual $M$-ary quantization levels. Outside this range the quantization characteristic is not uniquely determined but still constrained. Note that a saturation characteristic satisfies this condition and we select this as a representative soft quantization function for our simulations.
Figure 6.5: Constraint on the Quantizer characteristic for a sector $(0, \alpha)$ bounded nonlinear error mapping.

Figure 6.6: Stability condition on a channel with two ISI weights.
### 6.5 Simulations

The model used in the simulations was geared towards demonstrating the extent of error propagation. A simple impulsive noise model was adopted consisting of a Gaussian noise spike (variance 10) every 100 data samples. Depending on the amplitude of the noise at each of these instants a hard decision error (primary error) may or may not have been made. The data record length was $10^6$ samples and the symbols were binary. Table 6.1 gives results for 3 indicative channels. The conventional (hard) decision feedback equalizer (DFE) and the soft decision feedback equalizer that uses a saturation type soft quantizer (soft DFE) are compared.

The first channel in Table 6.1 represents a channel such that both types of equalizer are stable in the error propagation sense. For this channel there is no discernible difference in the error propagation. Although the ISI for this channel is mild, it is sufficiently large to close the eye pattern. The second channel is one for which only the soft DFE is stable and the improvement in bit error rate

<table>
<thead>
<tr>
<th>Channel Model</th>
<th>DFE</th>
<th>Soft DFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.0 + 0.3z^{-1} - 0.3z^{-2}$</td>
<td>0.00374</td>
<td>0.00374</td>
</tr>
<tr>
<td>$1.0 + 0.6z^{-1} - 0.6z^{-2}$</td>
<td>0.01474</td>
<td>0.00874</td>
</tr>
<tr>
<td>$1.0 + 0.6z^{-1} + 0.6z^{-2}$</td>
<td>0.01128</td>
<td>0.00626</td>
</tr>
</tbody>
</table>

Table 6.1: Bit error rate performance comparison

In comparison with the soft saturation characteristic we can consider the hard nearest neighbor quantization function. With this selection it can be readily deduced that $\alpha = 2$. Thus a tentative conclusion using Theorem 3 is that the stability region of the soft quantizer is a two times scaling of the stability region of the hard quantizer.
is evident. The third channel is one for which both channels are stable but the stability margin of the soft DFE is much greater. Given that this explains its performance advantage, we can qualitatively extend the result and argue that the degree of stability relates inversely to the extent of error propagation.

6.6 Conclusions

Using passivity theory to examine the stability of a decision feedback structure we determined the optimum (in the sense of maximizing a stability region) soft decision quantizer. The new structure was shown to have suppressed error propagation relative to the classical nearest neighbor quantizer. The results and techniques should have wider applicability to advanced receiver structures that utilize decision feedback principles.


The model used in the simulation is the equivalent circuit of an FIF. A simple numerical model was used to simulate Tr and Tr measured at 310 every 100 days. Considering the variability of the model, some of these simulations may converge to different steady states or may not converge at all. The simulation results show that 100 points were used for the analysis. Table 3 gives results for a hypothetical example. The simulations used Quasi-Ballistic Resonant (QBR) and the self-driven resonant ballast resonator that uses a set of values from a generator and a DPF as explained.

For the simulation, the total capacitance of each type of ballast resonator is shown in the error propagation case. For the configuration, both the ballast and the resonator are simulated with the same number of points. Although the DPF is not shown in this case, it is an important component which only the soft DPF is stable and the hard DPF is not.
This chapter is divided into three main sections. The first section describes how the communications systems of the near future will look. It flags the current trends in and the anticipated future demands on telecommunications services. It pinpoints the technical hindrances to fulfil those demands. Furthermore, it gives possible solutions to these technical challenges so that the ultimate telecommunications goal of global connectivity can be reached. The second section describes how the DFE addresses some of the challenges faced by future personal communication systems and how the DFE research fits into this big picture. The last section describes possible extensions of the research done in this dissertation.
7.1 Future Communication Systems

In the late 1990s, personal communications systems\(^1\) (PCS) represent the next major step forward in the story of wireless communications that began one and a half decades ago with the arrival of cellular and cordless telephones. Mobility is at the heart of future telecommunications. The promise of personal communications to make all kinds of information available anywhere, anytime at low cost to a large mobile population [79] has a widespread appeal. People want to transmit and receive information wherever they are and whenever they choose, and in whatever format they like including: sound, text, graphics, and streaming video, etc. [79]. Thus the next generation of mobile radio systems should provide a wide variety of integrated multimedia services and an increased system capacity. But practical systems that can deliver it remain several years in the future.

The three main technical challenges which play a major role in PCS are (i) mobility, (ii) the wireless channel interface, and (iii) power consumption. The impediments in the way of information transfer over radio channels are:

- the limited spectrum that must be shared efficiently among many users;
- transmission impairments, such as fading, interference, and multipath propagation, that can change abruptly with time, location, and frequency bands;
- interrupted connections associated with handoff procedures; and
- the limited power available to portable and mobile terminals.

The physical layer confronts directly the technical challenges of personal communications systems. To meet these challenges, personal communications systems must deploy a myriad of techniques including modulation, source coding, channel coding, interleaving, diversity reception, and channel equalization. Thus the

\(^1\) The term personal communications means wireless access to information services. The term originated in 1989 in the United Kingdom to refer to spectrum assignments around 1800 MHz. Systems specifically designated as personal communication systems by spectrum regulators in Europe, the United States, and Japan operate in frequency band between 1700 MHz and 2000 MHz [79, page 30].
worldwide research activities in the field of mobile communications is focused on meeting the technical challenges of the current and future personal communications systems.

Currently there are two distinct yet complementary system approaches for addressing the demand for mobility. They are Centralized Wireless Networks\(^2\) and Decentralized Wireless Networks\(^3\) [6]. These wireless networks provide collection of services, each capable of performing some of the functions of a complete personal communications system. Examples of these services include: wireless telephony and wireless data networks, using the technologies of cellular networks, satellite communications, wireless LAN and wireless local loop etc. [79]. Compared to today’s cellular systems, the next-generation mobile and fixed wireless systems will have a much higher bit rate capability, e.g., 10 Mbps to provide 1 Mbps multimedia services to multiple users within each coverage area. We require new technology to merge today’s separate systems into the integrated information delivery system of the twenty-first century [79, page 1].

Such future PCS which will provide multimedia capabilities to portable or mobile terminals will require a high speed but low complexity wireless technique to combat fading and multipath delay spread, while minimizing the power requirement of the wireless modem. Broadband\(^4\) wireless techniques, such as multicarrier modulation (MCM) and adaptive equalization, will be required to overcome the effects of multipath delay spread over mobile radio channels [3]. Power consumption becomes an increasingly important consideration in the design of new wireless systems. The DFE fits these criteria well [5]. Depending on the nature of the application the future broadband wireless channel interface will either be

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\(^2\) Centralized wireless networks are wide-area cellular networks with increased bit rate capabilities. This system approach permits high mobility and global communications through handoff and roaming.  
\(^3\) Decentralized wireless LAN can offer data rates of the order of 20 Mbps as the coverage area is restricted, facilitating reduced signal attenuation and multipath delay spread.  
\(^4\) The term broadband is used for a channel with more bandwidth than a standard voice grade channel. Broadband channels are used for high-speed voice and data communications, radio and television broadcasting, some local area networks (LANs), and many other communications applications. The term usually implies a bit rate of more than 2Mbps.
7.2 Future DFE Research Directions

When wireless communications mature and the data rates go higher than those available today, sophisticated techniques for ISI mitigation will become inevitable. Thus in future high data rate communication system the need to equalize the channel distortion will become even more profound. Some form of ISI mitigation needs to be adapted to increase the data rates to support future multimedia traffic. System designers typically use a combination of the ISI-mitigation techniques namely equalization, multicarrier modulation, multilevel modulation, directional antennas and spread spectrum [12, page 124]. With channel equalization, diversity, and channel coding are the other techniques which can be used independently or in tandem to further improve the received signal quality [15, page 229]. This dissertation deals with equalization only and it focuses entirely on decision feedback equalization.

Decision feedback equalization is the key equalization technique for wireless broadband modems. To provide multimedia capabilities to wireless PCS terminals high data rates will be required but without the overhead of complexity in the wireless equipment both at the mobile terminal and at the base station. The high power consumption of channel equalization algorithms is a major design challenge. Most of the DSP performance is needed in the receiver circuits where channel equalization is performed. In many DSP-based modem applications, such as broadband modems for high-speed Internet access to the home or gigabit ethernet transceivers, channel equalization requires such high signal processing power that power consumption and clock speed become major design challenges [80]. Often the DFE, because of its reduced complexity, is one of only a few equalization techniques that can be contemplated.

Although minimizing the power requirement of wireless modem is crucial,
fading and multipath delay spread are also vital issues to be tackled. While it is generally thought to be extremely difficult to implement a DFE on channels with severe ISI, a low complexity DFE technique employing a reduced number of taps for feedforward filter and a training-free feedback filter can lead to low training overhead and a practical hardware realization (see reference [6]).

Major candidates for future wireless channel interface techniques are: broadband CDMA, OFDM or an equalized single carrier system using QPSK and a DFE [6]. The major disadvantage of OFDM is that it requires a large peak-to-average signal power ratio and hence the use of highly linear transmit amplifiers which are not power efficient [5].

Future broadband modems for high-speed Internet access in portable PCS and HDTV mounted on vehicles, etc., need adaptive equalization techniques, which are a hybrid of adaptation during a training sequence and blind adaptation. Again DFEs are popular in such modems. Equalization and coded modulation will be used in tandem to meet the SNR performance and channel equalization needs of future wireless multimedia traffic. However, the incorporation of DFE into coded systems imposes a major design problem. The DFE needs delay free decisions while codecs in coded modulation system introduce delay. Furthermore the problem of error propagation, which at present is of concern only to the scientific community, will also become a matter of concern from a practical implementation point of view. Transmitter precoding will not win in the long run as it requires precise knowledge of the channel at the transmitter. MLSD will often be inappropriate, at least in the mobile terminal, because of its high complexity. It may be able to survive in the base station where the complexity is not such a constraint as in the mobile terminal.
7.3 Possible Thesis Extensions

There are two major issues addressed in this dissertation. Both avoid the standard design assumption of correct past decisions. The first issue is the optimization of feedforward and feedback filter parameters and the second issue is the optimization of the decision device or quantizer in the DFE structure. Chapter 3 and Chapter 4 deal with the optimization of DFE filters and Chapter 5 and Chapter 6 deal with the optimization of the quantizer. Based on the material covered in this thesis, a number of extensions are possible. A natural extension to the research presented in the thesis is the joint optimization of both the DFE filter parameters and the quantizer.

The material presented in Chapter 3 shows that the conventional MMSE-DFE structure optimized under the correct past decision assumption may not be optimal. A hybrid MMSE design procedure is presented which controls the error propagation in the DFE to a certain extent. The research presented in Chapter 3 can be extended in a number of ways. As pointed out in Section 3.4, it would be interesting to implement the design as an adaptive or even as a blind adaptive scheme. More interesting channels, i.e., fast fading channels, can be considered to determine the performance of the design. The hybrid design presented is robust against error propagation and therefore it would be worthwhile to implement it for complex constellations, e.g., high order QAM constellations or QPSK.

Chapter 3 flagged the fact that conventional MMSE-DFE design might not be optimal as it is based on the assumption of correct past decision. In Chapter 4 we continued on this idea and showed, by simulation, that the actual MMSE-DFE solution is very different from conventional MMSE-DFE solution for the low signal to noise ratio case. The optimal MMSE-DFE analysis was done without making the correct past decision assumption. We called this new design the Modified MMSE-DFE design. It is based on calculating the covariance of the transmitted symbol and its estimate. The study done could be further extended by making
Further simulation studies of the Modified MMSE-DFE design. The important point to note here is that the Modified MMSE-DFE design, unlike the conventional MMSE-DFE design, is close to the real world situation and hence reflects the actual performance of the DFE.

Chapter 5 introduces the topic of quantizer optimization. A generalization of the FS-DFE structure is presented such that the feedback filter is also fractionally spaced and the conventional scalar quantizer is replaced by a vector quantizer. We find that when this structure is optimized under the correct past decision assumption it is equivalent to the conventional, simpler structure of $T$-spaced feedback filter and scalar quantizer. Next we optimized the vector quantizer in the generalized FS-DFE structure without making the correct past decision assumption. We again find that an improvement in performance is possible in the proposed FS-DFE structure when the decision errors are taken into account. A further generalization of the FS-DFE structure is possible if we implement the forward FS filter as a $N \times P$ linear network, where $N$ is the oversampling factor and $P$ is the number of feedback filters.

Chapter 6 continues with the topic of quantizer optimization. The optimality of the nearest neighbor quantizer used in the classical DFE structure is questioned again. An optimum soft memoryless decision quantizer is determined using passivity theory. This new quantizer is shown to outperform the nearest neighbor quantizer in terms of its ability to suppress error propagation. An extension to general nonlinear quantizers with memory is possible using the framework given in the chapter.
BIBLIOGRAPHY


