AN ANALYSIS OF MONETARY POLICY RULES

By

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A Thesis submitted for the degree of Doctor of Philosophy of The Australian National University

September 2000

DECLARATION

This thesis represents my own original work except where otherwise acknowledged in the text.

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May 2000

ACKNOWLEDGEMENTS

First I would like to acknowledge and express my gratitude for the comments and support I have received from my committee members: Adrian Pagan, Graeme Wells and Warwick McKibbin. Graeme Wells and Adrian Pagan have read over and commented on every chapter in this thesis. From when I began this thesis through to the very end, Adrian Pagan has been a great source of advice and encouragement and I am especially grateful to him.

While working on this thesis my research has benefited from the comments and suggestions people have made on various chapters. For their time and comments my thanks go to John Pitchford, Laurence Ball, Louis Maccini, Charlie Bean, and Peter Stemp. I would also like to thank participants at the Australian Macroeconomics Workshop (April, 1999) and the PhD conference in Business and Economics (November, 1999).

The decision to undertake a P.hD was not a small one, but at no time have I regretted it. The process of completing this work has been made easier by the fellowship of other students in the program, in particular: Andrew Wait, Vlad Pavlov, and Steven Kennedy. I also benefited from visiting the Economics Department of the Reserve Bank of Australia over the summer of 1998-1999. For their resources and financial support go my thanks. Thanks also to the Economics Department of Johns Hopkins University whom I visited early in 2000. Finally I would like to thank my parents, Lee and Owen, my sisters, Katherine and Elizabeth, and my girlfriend, Junko Inoue, for their support and encouragement.

ABSTRACT

This thesis investigates the performance of optimal simple monetary policy rules in dynamic macroeconomic models. In keeping with much modern macroeconomics the majority of the models considered contain forward-looking rational expectations. These expectation terms complicate policy analysis because they introduce time-inconsistency into optimal policy making. To overcome this problem this thesis develops techniques to examine both pre-commitment and discretionary rules.

Three interrelated issues are focused on in this thesis. The first issue is how to construct simple policy rules whose performance closely approximates that of the optimal state contingent rule. This thesis argues that rather than comparing a range of simple rules and selecting the best one analysis should begin with the optimal state contingent rule. State variables can then be excluded from the optimal state contingent rule if their policy relevance is 'small'. The key to finding a simple rule that performs well is finding a small set of variables that is approximately sufficient for the information in the full system.

The second issue investigated is whether simple rules that have been found to perform well in closed economy models also perform well in open economy models. For the United States it is widely held that a Taylor type rule closely approximates the optimal state contingent rule. In open economy models, however, exchange rate movements play an important role and consequently rules such as the Taylor rule that overlook the exchange rate may perform poorly. Results obtained in this thesis support the notion that rules developed in closed economy models do not perform well in open economy models. These results question the usefulness of studies that apply closed economy policy rules to open economy models.

The final issue is whether information about the policy regime implemented by the monetary authority can be extracted from the policy rule it employs. A central bank's policy objectives and the intensity with which it goes about meeting these objectives are unobserved. However, policy reaction functions that relate the monetary authority's policy decisions to the state of the economy can be estimated. How policy makers respond to the economic state depends on its objectives and preferences. This

aspect of the thesis establishes conditions under which information about policy preferences and objectives can be uncovered from estimated policy rules.

TABLE OF CONENTS

Declaration ii

Acknowledgement iii

Abstract iv

Chapter 1: Introduction **1**

Chapter 2: Solving for Optimal Simple Rules in Rational Expectations Models **5**

2.1 Introduction 5

2.2 Existing approaches 7

2.2.1 Optimal pre-commitment Rules 8

2.2.2 Optimal simple pre-commitment rules 9

2.2.3 Optimal discretionary rules 10

2.3 Optimal simple discretionary rules 11

2.3.1 A matrix decomposition solution method 13

2.3.2 An undetermined coefficients solution method 16

2.4 An application 18

2.5 Conclusions 21

Appendix A: Taylor (1979) 23

Appendix B: A simple analytic example 25

Chapter 3: Instability under Nominal GDP Targeting: The role of Expectations Models **32**

3.1 Introduction 32

3.2 Growth targeting 35

3.2.1 Accelerationist Phillips Curve 36

3.2.2 Adaptive expectations 39

3.2.3 Rational expectations 41

3.2.4 Mixed expectations 43

3.2.5 Expectations and stability 47

3.3 Level targeting 47

3.4 Exact versus inexact targeting 52
3.5 Interest rate stabilization 55
3.6 Conclusions 57

Chapter 4: Conditionally Optimal Rules in a Simple Closed Economy Model under Discretion and Commitment **60**

4.1 Introduction 60
4.2 A simple closed economy model 63
4.3 Analysis of results: efficiency frontiers and impulse responses 64
4.3.1 Model A 65
4.3.2 Model B 68
4.3.3 Model C 70
4.3.4 Efficiency frontiers 72
4.4 Conditionally optimal rules 74
4.4.1 Model A 75
4.4.2 Model B 76
4.4.3 Model C 76
4.5 Conclusions 78
Appendix A: Conditionally optimal rules 80
Appendix B: Sensitivity analysis 86

Chapter 5: Optimal and Conditionally Optimal Targeting Rules for Small Open Economies **88**

5.1 Introduction 88
5.2 A small open economy framework 91
5.2.1 Domestic economy 91
5.2.2 Foreign economy 92
5.3 Optimal policy rules 96
5.3.1 Model A 96
5.3.2 Model B 101
5.4 Conditionally optimal rules 106
5.5 The role of monetary conditions indicators 111

5.6 Conclusions 114

Appendix A: Some sensitivity analysis for section 5.3 117

Chapter 6: Exploring the Role of the Terms-of-Trade in Australian Monetary Policy 119

6.1 Introduction 119

6.2 Model outline 121

6.3 Study design and results 124

6.3.1 Inflation targeting 125

6.3.2 Price level targeting 129

6.4 Dynamic homogeneity and the shape of the efficiency frontier 133

6.5 Conclusions 135

Chapter 7: Steps Toward Identifying Central Bank Policy Preferences 138

7.1 Introduction 138

7.2 A general macroeconomic setting 140

7.3 Identifying the policy preference matrix 141

7.3.1 Step one 143

7.3.2 Step two 144

7.3.3 Step three 145

7.3.4 In addition... 147

7.4 Some examples 149

7.4.1 Example one 149

7.4.2 Example two 150

7.4.3 Example three 152

7.5 Conclusions 153

Chapter 8: Discretionary Inflation Bias with Costly Inflation 155

8.1 Introduction 155

8.2 Rules versus discretion with costly inflation 156

8.3 Discretionary inflation bias 157
8.4 Conclusions 160

References **162**

Chapter 1

INTRODUCTION

The introduction of rational expectations into mainstream macroeconomics in the 1970s has had major implications for monetary policy. On one level theory predicted that policy decisions would be ineffective because agents would have already accounted for them, and that only unanticipated policy would effect the economy. By their nature these unanticipated policy actions would be random serving only to create greater volatility in the economy. On a separate level the potential for time-inconsistency in policy decision making was believed to rule out control theory where the application was an economic model in which expectations were formed rationally.

In the early 1980s it was realized that the rational expectations hypothesis by itself was insufficient to generate policy ineffectiveness. What was also needed was the natural rate hypothesis whereby price changes affected the supply side of the economy only to the extent to which they were unanticipated. If this natural rate hypothesis did not hold in each period, as sticky price theories and over-lapping wage contracting models suggested, then a role for policy could be motivated. At the same time control theory found ways to adapt to rational expectations. Time-inconsistency occurred because of an inability to pre-commit. Imposing pre-commitment as an optimization constraint led to optimal pre-commitment rules. Alternatively, one could accept the planner's inability to pre-commit and solve for the optimal time-consistent policy. Moreover, it was found that not all classes of rational expectation model led to time-inconsistency. Models where the expectations were of contemporaneous variables and not future variables were found not to create time-inconsistency (Chow, 1980).

Even though the control theory techniques have been established and the prominence of the policy ineffectiveness hypothesis has declined, the 1990s have not seen a great deal of research into optimal monetary policy in rational expectations models. There are some notable exceptions of which almost all assume pre-commitment before solving for the optimal policy rule. A much larger proportion of the monetary policy literature has shied away from analyzing optimal policy choosing instead to examine and contrast various simple rules.

Against this backdrop this thesis at its widest interpretation examines monetary policy in rational expectations models. More specifically a large part of this thesis looks at the efficiency of optimal simple policy rules in dynamic open economy macroeconomic models. Simple rules have merit because they are easy to construct, easy to evaluate, are highly transparent, and aid communication of policy decisions. Moreover, one simple rule – the Taylor rule – has been widely analyzed in closed economy settings and has been advocated as a stabilization tool. One focus of this thesis is on extending existing analysis of Taylor type rules to open economy models, thereby examining whether its performance in open economy models matches that it enjoys in closed economy frameworks.

We begin in Chapter 2 by developing the numerical techniques required to solve for optimal simple rules in dynamic rational expectations models. The focus of this Chapter differs from standard treatments of optimal control in rational expectations models in that it emphasizes optimal simple rules rather than fully optimal rules. In particular, Chapter 2 offers a solution method for optimal simple rules in the absence of pre-commitment that is new to the literature. The techniques developed in Chapter 2 are tested on existing models in the literature, and applied widely throughout the remainder of this thesis.

Chapter 3 looks at the stabilizing properties of a specific policy rule: nominal GDP targeting. While it might not be the optimal policy rule to apply in any given model it is often felt that nominal GDP targeting is a robust policy rule, one that is likely to perform reasonably well across a range of plausible models. Chapter 3 formulates an economic framework that encompasses many popular theoretical models and uses analytical and numerical techniques to establish that where inflation expectations contain some forward-looking element nominal GDP targeting is unlikely to create instability in the economy. These analytic results confirm what numerical studies on more general models have typically found, and demonstrate the sensitivity of Ball's (1999) result showing nominal GDP targeting to be unstable.

Following our exploration into the stability properties of nominal GDP targeting, Chapters 4, 5, and 6 turn to the issue of how well optimal simple policy rules perform relative to the optimal state contingent rule. Chapter 4 builds a closed economy model, and contrasts optimal state contingent policy rules with the Taylor rule and the Henderson-McKibbin rule. Both pre-commitment and discretionary equilibria are solved for and compared. A version of the open economy Buiter-Miller model is constructed in Chapter 5 and using this model consumer price inflation targeting is contrasted with non-tradables inflation targeting. Only pre-commitment solutions are solved for, but optimal state contingent rules, optimal simple rules, and optimal Taylor type rules are considered. This analysis finds that optimal Taylor type rules do not perform well in open economy models, but that Taylor type rules extended with the inclusion of the real exchange rate perform much better.

Building on a theme, Chapter 6 turns to an estimated model of the Australian economy. In Australia it is a stylized fact that the terms-of-trade and the real exchange rate are highly correlated with increases in the terms-of-trade leading to appreciations of the real exchange rate. With the terms-of-trade driving the real exchange rate Chapter 6 explores which of the terms-of-trade and the real exchange rate contains more useful information for policy makers. Simulation results reveal that monetary policy is more effective when it responds directly to the real exchange rate than it does when it responds directly to the terms-of-trade. Moreover, Taylor type rules are notable in this model for their poor performance. In this Australian model Taylor types rules are able to mount very little leverage over inflation and the output gap, and this compromises their ability to stabilize these variables.

Methodologically, Chapters 4, 5, and 6 argue that the most nature benchmark against which to assess the performance of any simple policy rule is the optimal state contingent rule – the policy rule that optimally exploits all available information. The fact that a Taylor type rule performs better than a nominal GDP targeting rule in some given model is of secondary interest if both rules perform poorly relative to the optimal rule. Once the performance of the optimal policy rule is established, optimal simple policy rules that perform well can be constructed by individually removing state variables from the optimal rule and seeing how the exclusion of that variable affects the rule's performance.

Chapter 7 turns away from evaluating policy rules and instead looks at how information obtained from estimated policy reaction functions can be used to cast light of the objectives of policy makers. Consistent with the first part of the thesis the framework used is one where rational expectations are present, and where central banks optimize to set monetary policy. Chapter 7 establishes a number of identification conditions that must hold if the rational expectations model is to be identified, and subsequently if the parameters in the policy objective function are to be identified.

The idea behind Chapter 7 rests on the fact that the feedback coefficients in the optimal policy rule are nonlinear combinations of the parameters in the economic model and those in the objective function. If the economic model can be identified and estimated, then these estimated parameters together with estimates of the policy feedback coefficients can possibly be used to extract information about the objective function parameters. A maintained assumption throughout this process is that the policy maker is optimizing some function.

The final Chapter in this thesis continues with the theme of comparing pre-commitment and discretionary monetary policy. Chapter 8 takes the canonical 'rules vs discretion' model and builds in an economic cost to anticipated inflation. This cost might be motivated by inflation tax considerations, indexing problems, or the menu cost/shoe leather cost literature. Chapter 8 shows that such an inflation cost has ambiguous implications for the magnitude of the discretionary inflation bias: the bias may go up or down.

Chapter 2

SOLVING FOR OPTIMAL SIMPLE RULES IN RATIONAL EXPECTATIONS MODELS

2.1) Introduction

This chapter shows how to solve for optimal simple policy rules in dynamic rational expectations models. Such models naturally contain jump variables, variables that are not predetermined. Kydland and Prescott (1977) and Calvo (1978) were the first to bring to popular attention the implications these non-predetermined variables have for control theory. These implications, forcefully brought home by Barro and Gordon (1983), were that dynamic programming did not generate an optimal policy program, and that optimal programs were unlikely to be implemented due to time-inconsistency. This literature recommended that policy makers adhere to simple rules.

Following Barro and Gordon (1983) numerous studies were performed where central banks and governments were assumed to follow simple rules. These studies, however, found it hard to resist evaluating the economic consequences of these simple policy rules, or to argue that one simple rule outperformed another according to some criterion. Of course with a welfare criteria in place advocating one simple rule over another implicitly amounts to an inefficient form of numerical optimization. Which begs the question of how policy makers should choose the simple rule to implement.

An alternate strand of the literature sought to develop techniques for applying control theory methods to rational expectations models. Initially this literature identified rational expectations models in which time-inconsistency did not arise. In this vein Taylor (1979) developed a model of overlapping wage contracts where wage expectations were formed rationally and solved for an optimal monetary policy rule using standard dynamic programming. This was possible because the expectations in Taylor's model were contemporaneous. Chow (1980) further argued the case for control theory by formally showing that time-inconsistency would not arise if the expectations were not of future variables. Preston and Pagan (1982) followed up this point, establishing conditions for stabilizability in models with expectations of contemporaneous variables.

In models with forward-looking rational expectations this strand of the literature did not pass up on optimization, but rather chose to formally build the requirement of time-consistency into the optimization problem. In this vein Cohen and Michel (1984) solved a one state variable problem for the optimal time-consistent rule, and subsequently Currie and Levine (1985), Oudiz and Sachs (1985), Miller and Salmon (1985), and Backus and Driffill (1986) largely developed the theory of optimal control in rational expectations models. In this literature one can assume the existence of some pre-commitment mechanism and then solve for an optimal pre-commitment policy rule, or alternatively solve for an optimal time-consistent rule.

While these optimization techniques have been available for some time, and there is an extensive literature on the properties of various monetary policy rules, relatively few studies have used control theory to develop policy rules. This is despite the fact that optimal rules are the natural benchmark against which to compare simple rules. However, Taylor (1979), McKibbin and Sachs (1988, 1991), Svensson (1994, 1998), Fair and Howrey (1996), Fuhrer (1997), and Rudebusch (1999) are notable for applying control theory to address monetary policy questions.

Against this background this chapter has two aims. The first is to bring to wider attention the central methods in the literature for constructing optimal monetary policy rules in dynamic rational expectations models. The second aim is to address a gap in this literature. The solution methods currently available can solve for optimal and optimal simple pre-commitment rules, but only optimal discretionary rules, not optimal simple discretionary rules. This chapter presents methods for solving for optimal simple discretionary rules.

We begin in Section 2.2 by briefly surveying the solution techniques in the literature. The emphasis in this Section is on closed-loop solution methods in discrete time models. Having discussed the available solution methods, Section 2.3 turns to the outstanding problem of solving for optimal simple discretionary rules. Two methods are presented. The first, in Section 2.3.1, relies on matrix decomposition methods for solving the rational expectations model, the second, in Section 2.3.2, employs the method of undetermined coefficients. In Section 2.4 an example is presented and the methods in Section 2.3 applied to it. Section 2.5 concludes. Appendix A applies the techniques described in this chapter to a second example, Taylor (1979), while Appendix B uses an analytic example to compare the methods in Oudiz and Sachs (1985) with those developed here.

2.2) Existing Approaches

This Section examines the key papers in the literature, with an emphasis on closed-loop solution methods for optimal simple rules in discrete time models. The key references in this literature are: Oudiz and Sachs (1985); Backus and Driffill (1986); and the recent paper by Soderlind (1999). In principle we would like to be able to solve for four types of policy rule: optimal pre-commitment rules; optimal simple pre-commitment rules; optimal discretionary rules; and optimal simple discretionary rules. The latter two rules are time-consistent, the former two are only time-consistent in the presence of some pre-commitment mechanism. Simple rules are sub-optimal in that they exclude information from the policy rule that could improve welfare, as measured by some objective function. Each of these key papers solves for optimal pre-commitment rules and optimal discretionary rules. In addition, Oudiz and Sachs (1985), and Soderlind (1999) present methods for solving for optimal simple pre-commitment rules. What is absent from this literature is a method of solving for optimal simple discretionary rules.

The solution methods in Oudiz and Sachs (1985), Backus and Driffill (1986), and Soderlind (1999) are very similar. Soderlind (1999) follows Oudiz and Sachs' (1985) Lagrangean approach to solve for optimal pre-commitment rules; Backus and Driffill (1986) use dynamic programming to achieve the same result. Both Soderlind (1999) and Backus and Driffill (1986) draw heavily on Oudiz and Sachs (1985) to solve for optimal discretionary rules.[1]

All of the solution methods contained in the references above are couched in terms of the following linear economic structure

$$\begin{bmatrix} \mathbf{y}_{1t+1} \\ E_t\mathbf{y}_{2t+1} \end{bmatrix} = \boldsymbol{\Pi}_1 \begin{bmatrix} \mathbf{y}_{1t} \\ \mathbf{y}_{2t} \end{bmatrix} + \boldsymbol{\Pi}_2 \mathbf{x}_t + \begin{bmatrix} \mathbf{v}_{1t+1} \\ \mathbf{0} \end{bmatrix}, \qquad \mathbf{v}_{1t} \sim \text{iid}[\mathbf{0}, \boldsymbol{\Sigma}]. \tag{1}$$

where $\mathbf{y}_{1t}$ is an (m×1) vector of predetermined variables, $\mathbf{y}_{2t}$ an ((n-m)×1) vector of jump, or free, variables, and $\mathbf{x}_t$ a (p×1) vector of policy instruments. Model coefficients are contained in the (n×n) and (n×p) matrices $\boldsymbol{\Pi}_1$ and $\boldsymbol{\Pi}_2$ respectively. The expectation of $\mathbf{y}_{2t}$ conditional upon information up to and including time t is denoted $E_t\mathbf{y}_{2t}$. Period t information, denoted I_t, is defined as $I_t = \{\mathbf{y}_{1t}, \mathbf{y}_{2t}, I_{t-1}\}$.

Throughout this paper the policy objective function is taken to be

$$\text{Loss}[0,\infty] = \text{tr}[\mathbf{W}\boldsymbol{\Omega}], \tag{2}$$

where $\mathbf{W}$ is a symmetric, positive semi-definite, time-invariant matrix of known policy weights, $\boldsymbol{\Omega}$ is the variance-covariance matrix of the vector containing the predetermined variables, the jump variables, and the policy instruments, and 'tr' is the trace operator.

2.2.1) Optimal Pre-Commitment Rules

Because the focus of this paper is not on solving for optimal pre-commitment rules, the treatment given here, which follows Soderlind (1999), but with different notation, is brief. Form the Lagrangian

$$L = E_0 \sum_{t=0}^{\infty} [\mathbf{y}_t^T \mathbf{V} \mathbf{y}_t + 2\mathbf{y}_t^T \mathbf{U} \mathbf{x}_t + \mathbf{x}_t^T \mathbf{R} \mathbf{x}_t] + 2\boldsymbol{\rho}_{t+1}^T [\boldsymbol{\Pi}_1 \mathbf{y}_t + \boldsymbol{\Pi}_2 \mathbf{x}_t + \xi_t - \mathbf{y}_{t+1}], \tag{3}$$

[1] See Currie and Levine (1985) for a treatment of optimal pre-commitment rules in continuous time models.

where $\mathbf{y}_t = [\mathbf{y}_{1t}{}^T\ \mathbf{y}_{2t}{}^T]^T$, $\xi_{t+1} = [\nu_{1t+1}{}^T\ (\mathbf{y}_{2t+1}\text{-E}_t\mathbf{y}_{2t+1})^T]^T$, and ρ_{t+1} is a vector of Lagrange multipliers, and the dynamic constraints come from (1). The matrices **V**, **U**, and **R** are components of the weighting matrix **W** defined in (2). Differentiating (3) with respect to ρ_{t+1}, $\mathbf{y}_t$ and $\mathbf{x}_t$ and collecting the first order conditions in matrix form gives

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Pi}_1^T \\ \mathbf{0} & \mathbf{0} & -\mathbf{\Pi}_2^T \end{bmatrix} \begin{bmatrix} \mathbf{y}_{t+1} \\ \mathbf{x}_{t+1} \\ \mathrm{E}_t\rho_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{\Pi}_1 & \mathbf{\Pi}_2 & \mathbf{0} \\ -\mathbf{V} & -\mathbf{U} & \mathbf{I} \\ \mathbf{U}^T & \mathbf{R} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{y}_t \\ \mathbf{x}_t \\ \rho_t \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}. \tag{4}$$

For the purposes of this paper it suffices to stop here. The important message from (4) is that the optimal pre-commitment policy rule depends not only on the state vector, but also the vector of Lagrange multipliers. Rules that exclude these Lagrange multipliers, such as simple rules, cannot be fully optimal and therefore will not be certainty equivalent.

2.2.2) Optimal Simple Pre-Commitment Rules

Consider the class of macroeconomic models in (1). Take the policy reaction function to be

$$\mathbf{x}_t = \varphi \begin{bmatrix} \mathbf{y}_{1t} \\ \mathbf{y}_{2t} \end{bmatrix}. \tag{5}$$

This policy rule sets the vector of instruments as a function of the vector of state variables. Together (1) and (5) imply

$$\begin{bmatrix} \mathbf{y}_{1t+1} \\ \mathrm{E}_t\mathbf{y}_{2t+1} \end{bmatrix} = \mathbf{\Pi} \begin{bmatrix} \mathbf{y}_{1t} \\ \mathbf{y}_{2t} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_{1t+1} \\ \mathbf{0} \end{bmatrix}, \tag{6}$$

where $\mathbf{\Pi} = \mathbf{\Pi}_1 + \mathbf{\Pi}_2\varphi$. Taking the period t conditional expectation of (6), then applying a spectral decomposition to $\mathbf{\Pi}$ gives $\mathbf{\Pi} = \mathbf{M}^{-1}\mathbf{\Lambda}\mathbf{M}$, where Λ is a matrix with the eigenvalues of $\mathbf{\Pi}$ along its leading diagonal, and zeros elsewhere. Partition Λ as

$\Lambda = \begin{bmatrix} \lambda_1 & \mathbf{0} \\ \mathbf{0} & \lambda_2 \end{bmatrix}$, and $\mathbf{M}$ conformably, $\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix}$, having reordered Λ such that the eigenvalues enter in ascending magnitude. All eigenvalues with modulus less than one are contained in λ_1 and all those with modulus greater than one are contained in λ_2. The number of eigenvalues with modulus less than one and greater than one define the dimensions of λ_1 and λ_2 respectively. Provided a φ exists such that the number of eigenvalues in λ_2 equals the number of jump variables, the system is stablizable and has a unique rational expectations equilibrium. If the eigenvalues in λ_2 are distinct, $\mathbf{M}_{22}$ has full rank and can be inverted. To place the system on its saddle-arm requires

$$\mathbf{y}_{2t} = -\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\mathbf{y}_{1t}. \tag{7}$$

If $\mathbf{M}_{22}$ is singular, then a spectral decomposition of Π cannot be used and a Jordan canonical form (Blanchard and Kahn, 1980) or a Schur decomposition (Klein, 2000) might be used instead.

With the jump variables evolving according to (7) and the transition of the predetermined variables given in (6), the evolution of the entire system is determined as an implicit function of the policy parameters φ. The objective function (2) can be evaluated and then minimized with respect to φ. Minimization occurs subject to the constraint that the rank of λ_2 is 'n-m': that the system has a unique rational expectations solution. Some elements of φ can be fixed and the remaining unconstrained parameters optimized over.

2.2.3) Optimal Discretionary Rules

To solve for optimal discretionary rules, Oudiz and Sachs (1985) use dynamic programming methods. They summarize their solution procedure as follows:

> *'The time-consistent, non-cooperative equilibrium is found as a limit to a finite time (T period) problem, for T large. The solution is derived in two steps. The finite-horizon problem is solved for the last period, T,*

and then it is solved for period t given the solution for period t+1. We find the limit of the rule for period 0 as $T \to \infty$.'

Keep in mind that the optimization and time-recursion take place subject to the restriction that the system's rational expectations solution be unique. First take the Lagrangean function under pre-commitment, equation (3), and augment it with a constraint linking the jump variables to the predetermined variables:

$$\mathbf{y}_{2t} = \mathbf{N}_t \mathbf{y}_{1t}. \tag{8}$$

Minimization of the Lagrangean function continues as previously, generating a first order condition linking the instrument vector to the predetermined variables:

$$\mathbf{x}_t = \mathbf{S}_t \mathbf{y}_{1t}. \tag{9}$$

Equations (8) and (9) are then solved simultaneously, (with $\mathbf{N}_t$ an implicit function of $\mathbf{S}_t$, and $\mathbf{S}_t$ an implicit function of $\mathbf{N}_t$), subject to the restriction that $\mathbf{N}_t$ places the system on its stable manifold. The initial $\mathbf{N}_T$ is chosen as the solution to the terminal period problem. Further details are provided in Backus and Driffill (1986), McKibbin and Sachs (1991), and Soderlind (1999).

2.3) Optimal Simple Discretionary Rules

Having briefly described existing techniques for solving for optimal rules in dynamic rational expectations models we now turn to the outstanding problem of solving for optimal simple discretionary rules. The approach we take to discretion is conceptually different to the Lagrangean approach discussed in Section 2.2.3. Under pre-commitment a policy rule is proposed whose structure cannot be altered as time passes. Knowing this agents form their expectations using the proposed rule and the feedback coefficients in the proposed rule are then optimized over. With discretion we look for a time-consistent rule: a rule that the monetary authority has no incentive to alter as time passes. The idea of discretion is implemented by drawing a distinction between the rule the monetary authority proposes to follow today and that it proposes to follow in the future. Distinguishing between today's rule and the future rule captures the notion that today's monetary authority cannot tie the hands of future policy makers.

Consider dynamic specifications of the form

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2 E_t \mathbf{y}_{t+1} + \mathbf{A}_3\mathbf{x}_t + \mathbf{v}_t, \qquad \mathbf{v}_t \sim \text{iid}[\mathbf{0},\Sigma] \tag{10}$$

where $\mathbf{y}_t$ is an n×1 vector of jump and predetermined variables, $\mathbf{x}_t$ is an p×1 vector of policy instruments, and $\mathbf{v}_t$ is an n×1 vector of stochastic innovations. $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$, and $\mathbf{A}_3$ are matrices of policy invariant coefficients. The variance-covariance matrix Σ may be singular. Finally, E_t is the mathematical expectation operator conditional upon period t information, I_t. As previously defined $I_t = \{\mathbf{y}_t, I_{t-1}\}$. The model's structure and parameters are assumed known.

Equation (10) is more general than may first appear. Systems with lags of the instrument vector or more general lead and lag structures in $\mathbf{y}_t$ can all be manipulated into the form (10). Moreover, by redefining variables, expanding the state vector, and exploiting the law of iterated expectations, expectations of future variables conditional on period t-s (s > 0) information are also possible (Binder and Pesaran, 1995).

Provided $\mathbf{A}_2$ has full rank (10) and (1) are largely equivalent. The two specifications differ slightly in so much as (10) permits $\mathbf{x}_t$ to affect $\mathbf{y}_t$ contemporaneously, while in (1) a one period control lag is assumed. For the mechanics of the approach described below this difference is unimportant, a one period control lag could easily be accommodated.

The information available to the monetary authority when it sets policy is contained in the state vector, $\mathbf{y}_{t-1}$, and the vector of innovations, $\mathbf{v}_t$. Accordingly it is these variables that form the basis of the policy rule. Excluding $\mathbf{v}_t$ from the rule imposes the restriction that the rule is formed using period t-1 information. With simple rules some state variables and/or innovation terms are omitted from the policy rule.

Alternatively, at the cost of elements in $\mathbf{y}_{t-1}$ and $\mathbf{v}_t$ expectations of future variables may be built into the reaction function.[2]

Recall that we draw a distinction between the policy rule proposed for today and that proposed for the future. Accordingly we posit the rule

$$\mathbf{x}_t = \varphi_1 \mathbf{y}_{t-1} + \varphi_2 \mathrm{E}_t \mathbf{y}_{t+1} + \varphi_3 \mathbf{v}_t \tag{11}$$

for today, but a different rule

$$\mathbf{x}_{t+j} = \psi_1 \mathbf{y}_{t+j-1} + \psi_2 \mathrm{E}_{t+j} \mathbf{y}_{t+j+1} + \psi_3 \mathbf{v}_{t+j}, \qquad \forall\, j > 0. \tag{12}$$

for the future.[3] An important special case of (11) and (12) is that where every state variable and stochastic innovation is included. In this case expected future variables contain no additional information and φ_2 and ψ_2 would naturally be restricted to equal null matrices. The special case identified relates to the optimal discretionary rule, illustrating that the methods presented below can be applied to solve for optimal as well as optimal simple discretionary rules. Two solution methods are presented; the first uses matrix decomposition methods to solve the rational expectations model while the second employs method of undetermined coefficients and solves a matrix quadratic. Uhlig (1999) discusses how these two rational expectations solution methods relate to each other.

2.3.1) A Matrix Decomposition Solution Method

To solve the system, while capturing the essence of time-consistency, we begin with the future period. Defining $\mathbf{z}_t = [\mathbf{y}_t^T\ \mathbf{x}_t^T]^T$, advancing the time subscript on (10), and combining (10) with (12) gives the system

[2] Note that additional information can only be obtained from expectations of the future if some elements in $\mathbf{y}_{t-1}$ or $\mathbf{v}_t$ are directly excluded from the rule. In which case placing expected future variables in the policy rule amounts to an indirect way of accessing this information.

[3] Assuming the same policy rule for all future periods is without loss of generality. What is important is that policy decisions made today do not constrain future policy makers. Thus indexing the ψ's by time would not alter the solution. Another way of thinking about this is that the optimization problem facing today's central bank has already been solved by future central banks. We are looking for a stationary solution so all future central banks implement the same policy rule. However, today's

$$\mathbf{A}^*\mathbf{z}_{t+1} = \mathbf{B}^*\mathbf{z}_t + \mathbf{F}^*\mathrm{E}_{t+1}\mathbf{z}_{t+2} + \mathbf{G}^*\mathbf{u}_{t+1}, \qquad \mathbf{u}_t \sim \mathrm{iid}[\mathbf{0},\boldsymbol{\Phi}] \tag{13}$$

where $\mathbf{A}^* = \begin{bmatrix} \mathbf{A}_0 & -\mathbf{A}_3 \\ \mathbf{0}_{p\times n} & \mathbf{I}_p \end{bmatrix}$, $\mathbf{B}^* = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0}_{n\times n} \\ \psi_3 & \mathbf{0}_{p\times p} \end{bmatrix}$, $\mathbf{F}^* = \begin{bmatrix} \mathbf{A}_2 & \mathbf{0}_{n\times n} \\ \psi_2 & \mathbf{0}_{p\times p} \end{bmatrix}$, and $\mathbf{G}^* = \begin{bmatrix} \mathbf{I}_n & \mathbf{0}_{n\times n} \\ \psi_3 & \mathbf{0}_{p\times p} \end{bmatrix}$. The variance-covariance matrix of $\mathbf{u}_t = [\mathbf{v}_t^T\ \mathbf{0}^T]^T$, $\boldsymbol{\Phi}$, is directly and clearly related to the variance-covariance matrix of $\mathbf{v}_t$, Σ. Provided $\mathbf{F}^*$ has full rank (13), with an expanded state vector to define $\mathbf{u}_{t+2}$, can be re-expressed as

$$\begin{bmatrix} \mathbf{u}_{t+2} \\ \mathbf{z}_{t+1} \\ \mathrm{E}_{t+1}\mathbf{z}_{t+2} \end{bmatrix} = \mathbf{K}\begin{bmatrix} \mathbf{u}_{t+1} \\ \mathbf{z}_t \\ \mathbf{z}_{t+1} \end{bmatrix} + \Theta[\mathbf{u}_{t+2}], \tag{14}$$

where $\mathbf{K} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{F}^* \end{bmatrix}^{-1}\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{G}^* & \mathbf{B}^* & \mathbf{A}^* \end{bmatrix}$ and $\Theta = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{F}^* \end{bmatrix}^{-1}\begin{bmatrix} \mathbf{I} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$. The case where $\mathbf{F}^*$ is singular is discussed below. Equation (14) has a form analogous to (6), and is amenable to a variety of solution methods. Following our discussion of the pre-commitment case in Section 2.2, take the period t+1 conditional expectation of (14) and apply a spectral decomposition to K. This decomposition gives $\mathbf{K} = \mathbf{M}^{-1}\Lambda\mathbf{M}$, where Λ contains the eigenvalues of K along its leading diagonal and otherwise equals the null matrix. As earlier, we now reorder the eigenvalues in Λ so that they are in ascending magnitude and partition Λ into $\Lambda = \begin{bmatrix} \lambda_1 & \mathbf{0} \\ \mathbf{0} & \lambda_2 \end{bmatrix}$, where the dimensions of λ_1 and λ_2 are such that any eigenvalues in λ_2 have modulus greater than one. **M** is also reordered in accordance with Λ and it is then partitioned as $\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix}$, conformably with Λ. Provided the number of eigenvalues in λ_2 equals the number of jump variables in $\mathbf{z}_t$ and the eigenvalues in λ_2 are distinct, $\mathbf{M}_{22}$ is non-singular and the unique stable solution is given by

central bank does not know what the future period's solution is and must take a guess at it. This guess is summarized in (12).

$$\mathbf{z}_{t+1} = -\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\begin{bmatrix}\mathbf{u}_{t+1} \\ \mathbf{z}_t\end{bmatrix} \stackrel{\text{def}}{=} \boldsymbol{\theta}_1^*\mathbf{z}_t + \boldsymbol{\theta}_2^*\mathbf{u}_{t+1}. \tag{15}$$

Where $\mathbf{F}^*$ is singular (14) is not a valid representation of the system. Instead the model must be left in the form

$$\begin{bmatrix}\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{F}^*\end{bmatrix}\begin{bmatrix}\mathbf{u}_{t+2} \\ \mathbf{z}_{t+1} \\ \mathrm{E}_{t+1}\mathbf{z}_{t+2}\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{G}^* & \mathbf{B}^* & \mathbf{A}^*\end{bmatrix}\begin{bmatrix}\mathbf{u}_{t+1} \\ \mathbf{z}_t \\ \mathbf{z}_{t+1}\end{bmatrix} + \begin{bmatrix}\mathbf{I} \\ \mathbf{0} \\ \mathbf{0}\end{bmatrix}[\mathbf{u}_{t+2}],$$

in which case it can then be solved using one of the methods described in Anderson and Moore (1985), King and Watson (1998) or Klein (2000). These methods all return a solution in the form of (15). Note that the solution (15) depends on the feedback parameters in the future policy reaction function, (12).

Exploiting the definition of $\mathbf{z}_t$, combining (10) and (11), and inserting the period t conditional expectation of (15) into the resulting system gives

$$\mathbf{z}_t = [\mathbf{A} - \mathbf{F}\boldsymbol{\theta}_1^*]^{-1}[\mathbf{B}\mathbf{z}_{t-1} + \mathbf{G}\mathbf{u}_t] \stackrel{\text{def}}{=} \boldsymbol{\theta}_1\mathbf{z}_{t-1} + \boldsymbol{\theta}_2\mathbf{u}_{t+1}, \tag{16}$$

provided of course that $[\mathbf{A}\text{-}\mathbf{F}\boldsymbol{\theta}_1^*]$ is nonsingular. The process driving $\mathbf{z}_t$ in (16) depends on the future policy rule through $\boldsymbol{\theta}_1^*$ and on today's policy rule through $\mathbf{F}$, $\mathbf{B}$, and $\mathbf{G}$. From (16) the unconditional variance-covariance matrix of $\mathbf{z}_t$, $\boldsymbol{\Omega}$, is easily obtained by solving for the fixed point of

$$\boldsymbol{\Omega} = \boldsymbol{\theta}_1\boldsymbol{\Omega}\boldsymbol{\theta}_1^{\mathrm{T}} + \boldsymbol{\theta}_2\boldsymbol{\Phi}\boldsymbol{\theta}_2^{\mathrm{T}}. \tag{17}$$

As long as the spectral radius of $\boldsymbol{\theta}_1$ is less than one – a result that holds if the system is stablizable - $\boldsymbol{\Omega}$ can be solved from (17) using standard fixed-point solution methods. Once $\boldsymbol{\Omega}$ is obtained the objective function (2) is easily evaluated and can then be maximized with respect to φ_1, φ_2, and φ_3 holding constant ψ_1, ψ_2, and ψ_3. Now recognizing that the future monetary authority faces the same optimization problem as the current monetary authority we can use the newly optimized values of φ_1, φ_2, and

φ_3 as a better guess at the rule applied by the future central bank. That is we set $\psi_1 = \varphi_1$, $\psi_2 = \varphi_2$, and $\psi_3 = \varphi_3$, and with this new guess at the future policy reaction function re-solve the future period rational expectations model to obtain updated values of θ_1^* and θ_2^*. A new solution for (15) is obtained, the current central bank's objective function is evaluated and again minimized with respect to φ_1, φ_2, and φ_3 while again holding ψ_1, ψ_2, and ψ_3, constant. This iterative procedure is continued until a fixed point is obtained whereby the newly optimized feedback parameters in today's policy rule equal those proposed in the future policy rule: $\varphi_1 = \psi_1$, $\varphi_2 = \psi_2$, and $\varphi_3 = \psi_3$. Note that coefficients in the policy rule can be arbitrarily restricted and hence the solution obtained is that for an optimal simple discretionary rule.[4]

2.3.2) An Undetermined Coefficients Solution Method

Section 2.3.1 provided a solution method for optimal simple discretionary rules that relied on matrix decomposition methods to solve the underlying rational expectations model. Matrix decomposition solution methods are popular because they naturally impose the uniqueness and stability conditions arising from transversality conditions. But matrix decomposition solution methods are not always the most convenient technique to apply to a specific problem. Another popular solution method is the method of undetermined coefficients, and in this Section we show how the method of undetermined coefficients can be brought to bear in solving for optimal simple discretionary rules.

From (13) the solution to the system is postulated to be

$$\mathbf{z}_{t+1} = \boldsymbol{\theta}_1^* \mathbf{z}_t + \boldsymbol{\theta}_2^* \mathbf{u}_{t+1}, \tag{18}$$

where $\boldsymbol{\theta}_1^*$ and $\boldsymbol{\theta}_2^*$ are parameter matrices whose values have yet to be determined. Advancing (18) and taking conditional expectations as necessary and substituting back into (13) and equating coefficients gives the restrictions

[4] However, for some simple rule structures convergence may not be obtained. In some cases no stable rule of the postulated form exists. More generally the convergence properties of the algorithm described are unknown.

$$[\mathbf{A}^* - \mathbf{C}^*]\boldsymbol{\theta}_1^* = \mathbf{B}^* + [\mathbf{D}^* + \mathbf{F}^*]\boldsymbol{\theta}_1^{*2} \tag{19}$$

$$\mathbf{A}^*\boldsymbol{\theta}_2^* = \mathbf{F}^*\boldsymbol{\theta}_1^*\boldsymbol{\theta}_2^* + \mathbf{G}^*. \tag{20}$$

From (20), $\boldsymbol{\theta}_2^* = [\mathbf{A}^*\text{-}\mathbf{F}^*\boldsymbol{\theta}_1^*]^{-1}\mathbf{G}^*$. Equation (19) is a matrix quadratic in $\boldsymbol{\theta}_1^*$ and thus permits multiple solutions. One solution, satisfying McCallum's (1983) continuity restriction that $\boldsymbol{\theta}_1^* = \mathbf{0}$ when $\mathbf{B}^* = \mathbf{0}$, designed to rule out sunspots, is

$$\boldsymbol{\theta}_1^* = [(\mathbf{A}^* - \mathbf{C}^*) - (\mathbf{D}^* + \mathbf{F}^*)\boldsymbol{\theta}_1^*]^{-1}\mathbf{B}^*. \tag{21}$$

Provided $[(\mathbf{A}^*\text{-}\mathbf{C}^*)\text{-}(\mathbf{D}^*+\mathbf{F}^*)\boldsymbol{\theta}_1^*]$ is non-singular, and (21) is a contraction mapping, standard fixed-point techniques[5] can be employed to solve for $\boldsymbol{\theta}_1^*$. In the solution $\boldsymbol{\theta}_1^*$ is implicitly a function of the future policy feedback parameters ψ_1, ψ_2, and ψ_3. When setting policy today policy makers optimize over φ_1, φ_2, and φ_3 recognizing that the decisions they make do not influence ψ_1, ψ_2, or ψ_3. From the viewpoint of today's policy maker the structure of the rule that will be implemented in the future is established. So in period t the policy maker can assume that the economy evolves according to (18), with the coefficient matrices in (18) given by the solutions to (20) and (21).

Note that $\boldsymbol{\theta}^*_1$ and $\boldsymbol{\theta}^*_2$ in (18) depend upon ψ_1, ψ_2, and ψ_3 but not φ_1, φ_2, or φ_3. The current policy decision cannot influence *how expectations are formed*, but can alter *the expectation formed*, by changing the state $\mathbf{z}_t$. With $\mathbf{z}_{t+1}$ formed by (18) the transition equation for $\mathbf{z}_t$ becomes

$$\mathbf{z}_t = [(\mathbf{A} - \mathbf{C}) - (\mathbf{D} + \mathbf{F})\boldsymbol{\theta}_1^*]^{-1}\mathbf{B}\mathbf{z}_{t-1} + [\mathbf{A} - \mathbf{F}\boldsymbol{\theta}_1^*]^{-1}\mathbf{G}\mathbf{u}_t \stackrel{\text{def}}{=} \boldsymbol{\theta}_1\mathbf{z}_{t-1} + \boldsymbol{\theta}_2\mathbf{u}_t. \tag{22}$$

Equation (22) expresses $\mathbf{z}_t$ as a VAR(1) process. As earlier, $\boldsymbol{\Omega}$ can now be solved numerically using (17) provided the spectral radius of $[(\mathbf{A}\text{-}\mathbf{C})\text{-}(\mathbf{D}\text{+}\mathbf{F})\boldsymbol{\theta}^*_1]^{-1}\mathbf{B}$ is less than one. With the system cast in the form of a VAR(1) process optimization follows directly that described in Section 2.3.1. Guesses at the feedback parameters in the future rule are made. Conditional on this guess of the future rule optimal feedback

[5] This is Binder and Pesaran's (1995) 'brute force' method.

parameters for today's rule are found, and these are used to revise the guess at the future rule. This iterative procedure stops once a fixed point is obtained with $\psi_1 = \varphi_1$, $\psi_2 = \varphi_2$, and $\psi_3 = \varphi_3$ occurring naturally as the outcome of the optimization.

2.4) An Application

Our example is taken from Clarida, Gali and Gertler's (1999) (CGG) *Journal of Economic Literature* paper. CGG's analysis is theoretical, but we take their New Keynesian model and parameterize it for simulation purposes. Their model has two key equations: those for the output gap and inflation. Both demand and supply shocks are persistent, modeled as simple auto-regressive processes. The model also has a policy reaction function determined optimally, so the combined system has five equations. Using standard notation, the system is[6]

$$y_t = E_t y_{t+1} - \gamma[i_t - E_t \pi_{t+1}] + g_t \qquad \gamma > 0$$

$$\pi_t = E_t \pi_{t+1} + \lambda y_t + u_t \qquad \lambda > 0$$

with the demand and supply shocks modeled respectively as

$$g_t = \mu g_{t-1} + \varepsilon_t \qquad 0 \le \mu < 1$$

$$u_t = \rho u_{t-1} + v_t . \qquad 0 \le \rho < 1$$

In matrix notation the CGG model can be written

$$\begin{bmatrix} 1 & 0 & \gamma \\ -\lambda & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} E_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \varphi_1 & \varphi_2 & 0 \end{bmatrix} \begin{bmatrix} g_t \\ u_t \\ 0 \end{bmatrix}, \qquad (23)$$

with the error vector driven by

[6] One important parameterization that is worth noting is that the coefficient on the expected inflation term in the Phillips curve would more generally equal the discount factor. Here the discount factor is

$$\begin{bmatrix} g_t \\ u_t \\ 0 \end{bmatrix} = \begin{bmatrix} \mu & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{t-1} \\ u_{t-1} \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ v_t \\ 0 \end{bmatrix}.$$

As indicated in (23) the monetary authority sets the level of the nominal interest rate as a linear function of the two observed structural disturbances g_t and u_t in order to minimize Loss$[0,\infty] = \alpha Var[y_t] + (1-\alpha)Var[\pi_t]$. CGG's model assumes that period t expectations are formed, and period t policy decisions made, with all agents knowing the structure of the economy and aware of all variables dated period t or earlier. Thus policy makers know the demand and supply shocks before they set policy.

Because demand innovations move output and inflation pro-cyclically, it is always optimal for policy makers to eliminate the influence of these demand innovations from the system – regardless of policy preferences. Consequently the coefficient applied to the demand disturbance in the optimal policy reaction function is invariant to the weight placed on output in the policy objective function, α. In what follows we solve the system using the method of undetermined coefficients, following Section 2.3.2.

Table 2.1 presents the optimal policy reaction functions for a range of values of α assuming the monetary authority can pre-commit to a course of action.[7] As expected the coefficient applied to the demand disturbance (g_t) is invariant to α. Also observe that in the extreme cases where α = 0, 1 all volatility in inflation and output (respectively) can be completely eliminated. This is a consequence of agents and policy makers knowing the disturbances before they make their decisions.

set to one to make the Phillips curve consistent with the loss function used, which weights the present and the future equally.

[7] For simulation purposes we set: $\gamma = 0.8$; $\lambda = 0.4$; $\mu = 0.5$; $\rho = 0.5$; and $\sigma_\varepsilon = \sigma_v = 1$. The correlation between the supply and demand disturbances is set to zero.

Table 2.1 – Clarida-Gali-Gertler (1999) model under pre-commitment					
	Feedback coefficients		Std. Deviations %		
1-α	g_t	u_t	y_t	π_t	i_t
0	1.25	1.00	0.00	2.31	1.85
0.2	1.25	1.08	0.40	1.99	1.91
0.4	1.25	1.17	0.86	1.62	1.98
0.6	1.25	1.28	1.41	1.18	2.06
0.8	1.25	1.40	2.08	0.65	2.17
1	1.25	1.56	2.89	0.00	2.31

Table 2.2 derives optimal discretionary rules for the CGG model. Unsurprisingly, in the cases where α = 0, 1 the optimal pre-commitment and discretionary solutions coincide. In these two special cases the problem collapses to that where there is one instrument matched against a single policy goal. With one instrument and one goal the system is controllable, ruling out time-inconsistent behavior.

Table 2.2 – Clarida-Gali-Gertler (1999) model under discretion					
	Feedback coefficients		Std. Deviations %		
1-α	g_t	u_t	y_t	π_t	i_t
0	1.25	1.00	0.00	2.31	1.85
0.2	1.25	1.04	0.21	2.14	1.88
0.4	1.25	1.10	0.51	1.90	1.92
0.6	1.25	1.18	0.94	1.56	1.99
0.8	1.25	1.32	1.62	1.01	2.10
1	1.25	1.56	2.89	0.00	2.31

Comparing Tables 2.1 and 2.2 reveals a property derived analytically in CGG: for a given value of α the variance of output is higher and the variance of inflation is lower under pre-commitment than discretion. CGG point to this property as a benefit accruing to pre-commitment, even in the absence of a discretionary bias.

Figure 2.1: Efficiency Frontiers for Clarida *et al* (1999)

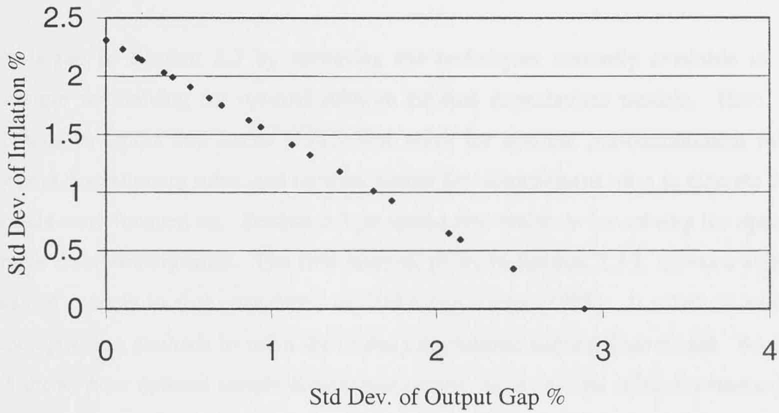

Figure 2.1 plots the efficiency frontiers from the CGG model under pre-commitment and discretion. These frontiers map out a straight line and it is hard to distinguish between them. Clearly it is not the case that the efficiency frontier for the pre-commitment case dominates that for discretion. So while for a given value of α pre-commitment generates a lower variance of inflation and a higher variance of output than discretion does, by choosing α suitably, discretion can match the variances of both inflation and output produced by pre-commitment. Intuitively it is clear that to match both the variance of output and inflation the α under discretion must be less than that under pre-commitment. This is an example where appointing a Rogoff (1985) optimally conservative central banker would eliminate the time-inconsistency.

2.5) Conclusions

In dynamic economies where agents' form rational forward-looking expectations optimal policy rules are typically time-inconsistent. Control theory has addressed this problem by developing techniques to solve for optimal pre-commitment rules and optimal time-consistent (discretionary) rules. In the case of pre-commitment the literature contains methods that solve for optimal rules and for optimal simple rules. But for discretion solution methods are currently only available to solve for optimal discretionary rules. This paper expands on this literature by presenting algorithms that solve for optimal simple discretionary rules.

We began in Section 2.2 by surveying the techniques currently available in the literature for solving for optimal rules in rational expectations models. Here, the methods in Oudiz and Sachs (1985) that solve for optimal pre-commitment rules, optimal discretionary rules, and optimal simple pre-commitment rules in discrete time models were focused on. Section 2.3 presented two methods for solving for optimal simple discretionary rules. The first method, given in Section 2.3.1, solves a similar class of models to that considered in Oudiz and Sachs (1985). It relies on matrix decomposition methods to solve the underlying rational expectations model. Section 3.2 shows how optimal simple discretionary rules can be solved using the method of undetermined coefficients. As natural byproducts Sections 2.3.1 and 2.3.2 also show how to solve for optimal simple pre-commitment rules as well as optimal discretionary rules. Having presented the solution algorithms, Section 2.4 took a recent paper by Clarida, Gali, and Gertler (1999) and applied the methods in Section 2.3 to their model.

The techniques derived in this paper are widely applicable. Much of the literature on monetary policy considers the properties and relative merits of simple rules. Relatively few papers analyze rules constructed using optimization and those that do invariably impose pre-commitment as a constraint. Optimal discretionary rules are rarely considered, and optimal simple discretionary rules have been completely neglected. Yet only by comparing pre-commitment rules with discretionary rules can the advantages of pre-commitment be assessed. Moreover, it is as interesting to compare optimal rules with optimal simple rules under discretion as it is for pre-commitment.

Appendix A: Taylor (1979)

This example comes from Taylor's (1979) *Econometrica* paper. Taylor's model has five equations, two of which are estimated behavioral equations. These two estimated equations are for output and inflation respectively. Their structures are as follows

$$y_t = a_1 d_t + a_2 y_{t-1} + a_3 y_{t-2} + a_4 d_{t-1} + a_5 E_{t-1}\pi_t + a_6 \varepsilon_{t-1} + \eta_t,$$

$$\pi_t = b_1 \pi_{t-1} + b_2 \varepsilon_{t-1} + b_3 E_{t-1} y_t + \varepsilon_t.$$

Real money balances are denoted d_t, η_t and ε_t are demand and supply innovations respectively and the remaining notation is standard. Two identities ($y_{t-1} = y_{t-1}$ and $\varepsilon_{t-1} = \varepsilon_{t-1}$) are used to manipulate the system into companion form. The fifth equation is a policy reaction function linking the policy instrument, d_t, to the observed state.

The objective function Taylor uses is a weighted average of the unconditional variances of output and inflation with α representing the weight placed on output stabilization and $1-\alpha$ that on inflation stabilization. Applying the pre-commitment algorithm of Section 2.3.2 to this model produces the following optimal closed-loop policy reaction functions.[8]

Table 2.3 can be compared to Table II in Taylor (1979); some differences are evident.[9] In light of Chow (1980), Preston and Pagan (1982) it is unsurprising that the results in Table 2.3 also pertain to the case of policy discretion. Quite why these results differ from those in Taylor is unclear. Notably, the three coefficients that both algorithms agree on are those stemming from the demand side of the model. From a control perspective it is always optimal to fully offset shocks propagated through the demand side of the model because such shocks move output and inflation pro-cyclically. Observe also that the differences between those here and Taylor's are increasing in α.

[8] The simulations use Taylor's estimated parameters: $a_1 = 0.578$; $a_2 = 1.167$; $a_3 = -0.324$; $a_4 = -0.484$; $a_5 = -0.447$; $a_6 = 0.38$; $b_1 = 1$; $b_2 = -0.67$; $b_3 = 0.018$. Further $\sigma_\eta = 0.7916\%$ per quarter, $\sigma_\varepsilon = 0.3661\%$ per quarter, and the correlation between the demand and supply shocks is 0.012.

Table 2.3 – Taylor (1979) model under pre-commitment and discretion							
	Feedback coefficients					**Std. Deviations %**	
α	y_{t-1}	y_{t-2}	π_{t-1}	ε_{t-1}	d_{t-1}	$sd(y_t)$	$sd(\pi_t)$
0.01	-2.02	0.56	-15.09	9.46	0.84	2.16	1.64
0.1	-2.02	0.56	-4.32	2.24	0.84	1.36	2.05
0.2	-2.02	0.56	-2.65	1.12	0.84	1.20	2.30
0.5	-2.02	0.56	-0.96	-0.02	0.84	1.02	2.92
0.7	-2.02	0.56	-0.36	-0.42	0.84	0.94	3.46
0.9	-2.02	0.56	0.19	-0.79	0.84	0.87	4.64
α	**Some Representative Results from Taylor (1979)**						
0.01	-2.02	0.56	-15.11	9.49	0.84	2.14	1.64
0.5	-2.02	0.56	-0.86	-0.09	0.84	1.01	2.88
0.9	-2.02	0.56	0.29	-0.86	0.84	0.85	3.96

[9] In an effort to understand why these differences are occurring I have also solved the model using standard dynamic programming, following Taylor (1979). The results obtained were the same as those generated from my algorithm. The reason for these differences in results remains unresolved.

Appendix B: A Simple Analytic Example

Consider the following simple model. Output, y_t, is described by the process

$$y_t = \beta E_t y_{t+1} + \gamma r_t + u_t. \qquad 0 \leq \beta \leq 1, \gamma < 0 \qquad \text{(B1)}$$

The policy instrument is the real interest rate, r_t. Demand shocks, u_t, follow the AR(1) process

$$u_t = \rho u_{t-1} + \varepsilon_t. \qquad 0 \leq \rho < 1$$

All variables have had their averages removed. The objective function is a weighted average of the unconditional variances of output and the policy instrument:

$$\text{Loss}[0, \infty] = \alpha \text{Var}(y_t) + (1-\alpha)\text{Var}(r_t). \qquad 0 \leq \alpha \leq 1 \qquad \text{(B2)}$$

B1) The Optimal Rule under Pre-Commitment using the Method of Undetermined Coefficients

The state variable for the system is the demand shock, u_t, so the policy reaction function takes the form

$$r_t = \varphi u_t \qquad \text{(B3)}$$

Substituting (B3) into (B1) gives

$$y_t = \beta E_t y_{t+1} + (1 + \gamma\varphi) u_t. \qquad \text{(B4)}$$

To solve this rational expectations model we posit the solution

$$y_t = \theta u_t \qquad \text{(B5)}$$

and proceed to solve for θ. Leading (B5) by one period and taking the conditional expectation results in

$$E_t y_{t+1} = \theta\rho u_t . \tag{B6}$$

Substituting (B6) and (B5) into (B4) and equating coefficients gives $\theta = \frac{(1+\gamma\varphi)}{(1-\rho\beta)}$, and therefore from (B5) the solution for output is

$$y_t = \frac{(1+\gamma\varphi)}{(1-\rho\beta)} u_t \tag{B7}$$

The objective function is now written as $\text{Loss}[0,\infty] = \alpha\left(\frac{1+\gamma\varphi}{1-\rho\beta}\right)^2 \sigma_u^2 + (1-\alpha)\varphi^2\sigma_u^2$.

Minimizing this loss function with respect to φ produces $\varphi = \frac{-\alpha\gamma}{(1-\alpha)(1-\rho\beta)^2 + \alpha\gamma^2}$,

which is positive under the parameter assumptions above: policy tightens in response to positive demand shocks. In the special case where $\rho = 0$, β does not appear in the solution and the forward-looking component of (B1) is non-consequential.

B2) The Optimal Rule under Pre-Commitment using Oudiz and Sachs (1985)

Recall that the model under consideration is

$$y_t = \beta E_t y_{t+1} + \gamma r_t + u_t \qquad 0 \le \beta \le 1, \gamma < 0 \tag{B8}$$

$$u_t = \rho u_{t-1} + \varepsilon_t . \qquad 0 \le \rho < 1$$

Where the problem is to set the real interest rate, r_t, as a function of u_t to minimize (B2). This system can be expressed in matrix form:

$$\begin{bmatrix} u_{t+1} \\ E_t y_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ -\beta^{-1}(1+\gamma\varphi) & \beta^{-1} \end{bmatrix} \begin{bmatrix} u_t \\ y_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}. \tag{B9}$$

φ is the feedback parameter to be optimally selected in the simple rule $r_t = \varphi u_t$. The eigenvalues of the coefficient matrix are clearly ρ and β^{-1}, thus our assumptions regarding these two parameters ($0 < \rho, \beta < 1$) ensure that our system has a saddle-point independent of policy. That the system is stablizable with a unique rational expectations solution follows. We assume that these eigenvalues are distinct and employ a spectral decomposition to solve the system.[10] Specifically:

$$\begin{bmatrix} u_{t+1} \\ E_t y_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{1+\gamma\varphi}{1-\rho\beta} & 1 \end{bmatrix} \begin{bmatrix} \rho & 0 \\ 0 & \beta^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\dfrac{1+\gamma\varphi}{1-\rho\beta} & 1 \end{bmatrix} \begin{bmatrix} u_t \\ y_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}.$$

Restricting the system to lie on its saddle-arm implies $y_t = \dfrac{(1+\gamma\varphi)}{(1-\rho\beta)} u_t$, which gives the same transition equation for output as that derived using the method of undetermined coefficients, equation (B7). Therefore, the optimal value of φ will also be the same, given the same objective function.

B3) The Optimal Rule under Discretion using the Method of Undetermined Coefficients

To solve the system we assume that current policy is set using the rule (B3) while future policy follows $r_{t+j} = \psi u_{t+j}$, $\forall\, j \geq 1$. We are looking for a stationary solution, so the equilibrium will have $\psi = \varphi$, but the important point is that policy makers today must optimize over their choice of φ allowing for the fact that *the process by which agents form their expectations will be a function of ψ and not φ.*

Consider the period t+1 problem. The transition equation is

$$y_{t+1} = \beta E_{t+1} y_{t+2} + (1+\gamma\psi) u_{t+1}. \tag{B10}$$

Positing the solution

[10] That the eigenvectors have been arbitrarily normalized and not normalized to length one, as is common, in no way alters the solution. The normalization applied was chosen for convenience.

$$y_{t+1} = \theta u_{t+1} \tag{B11}$$

and using the method of undetermined coefficients produces the solution for θ:

$$\theta = \frac{1+\gamma\psi}{1-\rho\beta}. \tag{B12}$$

Using (B12) and (B11) in (B10) gives $y_t = \left(1+\gamma\varphi+\rho\beta\frac{1+\gamma\psi}{1-\rho\beta}\right)u_t$. The loss function is therefore given by $\text{Loss}[0,\infty] = \alpha\left(1+\gamma\varphi+\rho\beta\frac{1+\gamma\psi}{1-\rho\beta}\right)^2\sigma_u^2 + (1-\alpha)\varphi^2\sigma_u^2$, which when differentiated with respect to φ - *holding ψ constant* - gives the first order condition

$$\frac{d\text{Loss}[0,\infty]}{d\varphi} = 2\sigma_u^2\alpha\gamma\left(1+\gamma\varphi+\rho\beta\frac{1+\gamma\psi}{1-\rho\beta}\right) + 2(1-\alpha)\varphi\sigma_u^2. \tag{B13}$$

Now, recognizing that the objectives, incentives, and constraints facing future policy makers are the same as those faced today, we set $\psi = \varphi$ and find the zero of (B13):

$$\varphi = \frac{-\alpha\gamma}{(1-\alpha)(1-\rho\beta)+\alpha\gamma^2}. \tag{B14}$$

Under our parameter assumptions the feedback coefficient under discretion is positive with slightly smaller magnitude than that under commitment, implying a less activist stance. The feedback coefficients under pre-commitment and discretion are the same only in the special case where $\rho = 0$, or $\alpha = 0, 1$. The latter two cases correspond to situations where the number of instruments equals the number of goals.

B4) The Optimal Rule under Discretion using Oudiz and Sachs (1985)

To translate the example above to one with a finite time horizon the policy loss function is taken to be $\text{Loss}[0,T]=\sum_{i=0}^{T}\alpha[y_i]^2+(1-\alpha)[r_i]^2$. The dynamic constraints are:

$$y_t=\beta E_t y_{t+1}+\gamma r_t+u_t, \qquad 0\le\beta\le 1, \gamma<0 \qquad \text{(B15)}$$

$$u_t=\rho u_{t-1}+\varepsilon_t. \qquad 0\le\rho<1 \qquad \text{(B16)}$$

Consider the period T problem. Expected variables can be written as a function of the state and therefore

$$E_T y_{T+1}=H_{T+1}E_T u_{T+1}=H_{T+1}\rho u_T, \qquad \text{(B17)}$$

where H_{T+1} has yet to be determined. Importantly, H_{T+1} is independent of the current policy. Substituting (B17) into (B15) gives

$$y_T=(1+\rho\beta H_{T+1})u_T+\gamma r_T. \qquad \text{(B18)}$$

Equation (B18) allows a standard dynamic programming solution and we therefore proceed to minimize the period T loss function, Loss[T,T], subject to (B18). The first order condition gives the policy reaction function

$$r_T=-\alpha\gamma[(1-\alpha)+\alpha\gamma^2]^{-1}[1+\rho\beta H_{T+1}]u_T. \qquad \text{(B19)}$$

Next substitute (B19) back into (B18) producing

$$y_T=(1-\alpha)[(1-\alpha)+\alpha\gamma^2]^{-1}[1+\rho\beta H_{T+1}]u_T=H_T u_T.$$

We look for a time-invariant solution and therefore seek $H_0 = H$ generated from the time-recursion as $T \to -\infty$ of

$$H_T = (1-\alpha)[(1-\alpha)+\alpha\gamma^2]^{-1}[1+\rho\beta H_{T+1}] \tag{B20}$$

The fixed-point, H, of (B20) is

$$H = \frac{1-\alpha}{(1-\alpha)+\alpha\gamma^2-\alpha\rho\beta}. \tag{B21}$$

Finally substituting (B21) into (B19) yields

$$r_t = \frac{-\alpha\gamma}{[(1-\alpha)(1-\rho\beta)+\alpha\gamma^2]}u_t. \tag{B22}$$

The solution given by (B22) is identical to that derived using the method of undetermined coefficients, equation (B14). In general numerical methods are needed to solve for the fixed-point of (B20). In the example given any sensible value for H_{T+1} is permissible as a starting value in the time recursion used to solve for H. This is because the system has a saddle-point independently of policy and (B20) is a contraction mapping. More generally, the starting value for H_{T+1} must place the system on its stable-manifold. To make this clearer, assume that the solution to the period T problem gives the feedback rule

$$r_T = b_T u_T. \tag{B23}$$

where b_T is a non-linear function of the system parameters, determined by the period T optimization.[11] We now allow for the fact the y_t is a jump variable and impose the restriction that the system has a saddle-point. To do this, substitute (B23) into (B15) and combine (B15) with (B16) in matrix form

$$\begin{bmatrix} u_{T+1} \\ E_T y_{T+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ -\beta^{-1}(1+\gamma b_T) & \beta^{-1} \end{bmatrix} \begin{bmatrix} u_T \\ y_T \end{bmatrix} + \begin{bmatrix} \varepsilon_{T+1} \\ 0 \end{bmatrix}. \tag{B24}$$

[11] This optimization requires imposing a terminal condition. Here we set $E_T y_{T+1} = 0$. Optimizing subject to this terminal condition produces the solution $b_T = -\alpha\gamma/[(1-\alpha)+\alpha\gamma^2]$. This value of b_T would be used to initialize the time-recursion.

Note that equation (B24) has the same form as (B9), and that under the parameter assumptions of the model the system is saddle-point stable independent of policy. As with the system (B9) for the system to be on its sable-arm requires

$$y_T = \frac{(1+\gamma b_T)}{(1-\rho\beta)} u_T. \tag{B25}$$

With output in the last period given by (B25),

$$H_T = \frac{(1+\gamma b_T)}{(1-\rho\beta)}. \tag{B26}$$

Clearly H_T depends on b_T. Now consider the period just prior to the terminal period. From (B25) $E_{T-1}y_T = \frac{(1+\gamma b_T)}{(1-\rho\beta)}\rho u_{T-1}$, which implies

$$y_{T-1} = \left[\frac{1+\rho\beta\gamma b_T}{1-\rho\beta}\right] u_{T-1} + \gamma r_{T-1}. \tag{B27}$$

The next step is to minimize the loss function Loss[T-1,T] subject to (B27) as a constraint. This minimization gives the policy reaction function $r_{T-1} = b_{T-1}u_{T-1}$ where

$$b_{T-1} = -\gamma(1-\alpha+\alpha\gamma^2)^{-1} H_T. \tag{B28}$$

Finally, use (B28) to find the time-invariant steady-state policy feedback parameter. The solution to the problem is found as the joint backward time-recursion of (B26) and (B28) until convergence. The solution is

$$b = \frac{-\alpha\gamma}{(1-\alpha)(1-\rho\beta)+\alpha\gamma^2}. \tag{B29}$$

Equation (B29) is of course identical to (B14). Consequently, Oudiz and Sach's (1985) method produces the same solution as the techniques developed in this paper.

Chapter 3

INSTABILITY UNDER NOMINAL GDP TARGETING: THE ROLE OF EXPECTATIONS

3.1) Introduction

Over recent years there has been increasing interest in monetary policy, with much of this interest focussing upon the properties of monetary policy rules. Whether they employ activist or non-activist methods the common basis shared by these rules is that they aim to provide the economy with a nominal anchor. Three rules that have received particular, and recent, attention are Bryant, Hooper and Mann (1993) (BHM) rules (of which Taylor rules - Taylor, 1993 - are a special case), Nominal GDP targeting rules (McCallum, 1989a), and inflation forecast targeting rules (Svensson, 1997a).

The BHM rule first came to popular attention when Taylor (1993) showed that his variant of it could accurately describe actual U.S monetary policy decisions over recent years.[1] It also has the virtue of being simple and easy to compute. Subsequent studies have suggested that monetary policy in a range of other countries can be well approximated by BHM type rules (Clarida, Gali, and Gertler, 1998). One of the criticisms leveled at BHM rules is that in their proposed form they are not operational. They assume unrealistically that policy makers know the current levels of output, potential output, and inflation when they set policy. Publication lags along with subsequent data revisions (Orphanides, 1997) suggest that these informational requirements are too stringent.

An operational BHM rule could, therefore, be more usefully based on lagged values of these variables or on current expected values formed using period t-1 information. Rudebusch and Svensson (1998), on the other hand, argue that central banks do have information in addition to that held by private agents. While central banks may not know the current values of output and inflation, assuming that they do may usefully capture the presence of this additional information.

Proponents of nominal GDP targeting, McCallum (1989a, 1989b), Hall and Mankiw (1994) for example, have emphasised its operationality - it depends only on variables known to policy makers – and its robustness. Because policy makers do not know the correct specification of the economy they require a rule that performs adequately over a range of models rather than optimally over just one. They argue that nominal GDP targeting provides such a rule. Finally, Svensson (1997a, 1997b) has argued that a central bank's inflation forecasts provide the ideal intermediate target and that inflation forecast targeting better captures the forward-looking nature of actual policy making.

In an interesting and important paper, Ball (1999) uses a simple macro-economic model to analyse the properties of each of these three policy rules. First Ball (1999) finds that a BHM rule is optimal regardless of the preferences of policy makers and, moreover, that inflation forecast targeting rules can always be expressed as a BHM rule. Thus Ball (1999) unites BHM rules and inflation targeting rules and shows them to have many desirable properties. Ironically, Ball (1999) further shows that nominal GDP targeting is not robust to his specification. Instead nominal GDP targeting is 'disastrous' leading to instability in inflation and output. Svensson (1997b) replicates Ball's instability result and suggests that it is the stylised fact that policy affects real output before inflation that Ball builds into his model that is at the heart of the instability result.

In reply to Ball (1999) and Svensson (1997b), McCallum (1997) argues that the Ball-Svensson instability result is a special case, and, furthermore, not a very interesting case. McCallum (1997) shows that the stability properties of the system come down to how the Phillips curve, or supply-side of the economy, is specified. By considering a range of five different supply-side formulations – all of which result in stable systems – McCallum (1997) concludes that the Ball-Svensson instability result is fragile.

[1] This does not imply, however, that the US federal Reserve actually follows a Taylor rule when

This dependence of the properties of nominal GDP targeting on the supply side of the economy parallels an earlier debate between Bean (1983) and West (1986). In an early analytical paper examining the properties of nominal GDP targeting, Bean (1983) uses a rational expectations model, with inelastic labour supply, and the policy objective of stabilising real output, to show that nominal GDP targeting is the optimal policy. When labour supply is elastic, Bean (1983) shows that a nominal GDP rule responds optimally to demand shocks but is sub-optimal in the face of supply shocks. In response to Bean (1983), West (1986) shows that Bean's results depend crucially upon how the supply side of the economy is specified. By modifying the supply side of the model West (1986) generates results precisely the opposite to those derived by Bean (1983).

In this paper we examine further the nature and robustness of the Ball-Svensson instability result. Our examination focuses on the important role expectations play in the short-run aggregate supply curve. Ball (1999) uses an accelerationist Phillips curve that essentially takes agents' expectation of inflation to equal last period's inflation rate. We show that if inflation expectations are formed using any of a number of other processes, then economic stability generally prevails. We further show that this is true for nominal GDP growth targeting and nominal GDP level targeting.

The structure of this paper is as follows. We begin in Section 3.2.1 by presenting and discussing the Ball-Svensson instability result, and discussing the time-series properties of the unstable system. We use Sections 3.2.2, 3.2.3, and 3.2.4 to generalise on the accelerationist Phillips curve and show that stability is restored under adaptive expectations, fully forward-looking expectations, and partly forward-looking expectations respectively. Section 3.2.5 provides a discussion of these results.

Section 3.3 turns to nominal GDP level targeting. We begin by outlining the stability of the system using the baseline accelerationist Philips curve. Next we examine level targeting when private agents form adaptive expectations. We find for both of these supply-side specifications that exact level targeting generates identical stability

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making its decisions.

properties to growth targeting, and we present a result showing why this must be the case.

Inexact targeting is briefly discussed in Section 3.4. Here we show that when inexact targeting rather than exact targeting is applied to Ball's (1999) system then the model is stable for all plausible parameter values. Sections 3.5 discusses interest rate smoothing while Section 3.6 concludes.

3.2) Growth targeting

Consider the simple two equation model of inflation and excess demand

$$y_t = \lambda y_{t-1} - \gamma r_{t-1} + v_t, \qquad \gamma > 0, 0 \leq \lambda \leq 1; \tag{1}$$

$$\pi_t = \pi_t^e + \alpha y_{t-1} + u_t, \qquad \alpha > 0 \tag{2}$$

where y_t is the output gap measured as a percent, π_t is the difference between inflation and its target rate, π_t^e is a measure of expected π_t, and r_t is the difference between the real interest rate and its equilibrium level. The stochastic errors, u_t and v_t, are assumed to be independent iid$[0,\sigma^2]$ processes.

The instrument available to the monetary authority is assumed to be r_t. Policy makers actually set the nominal interest rate, but by setting it equal to their desired real interest rate plus expected inflation they effectively set r_t itself. Setting to one side inflation expectations, monetary policy impacts on real demand through the IS curve, equation (1), after one period with aggregate demand then having a flow on effect to inflation after a further period. With these policy lags the model is best viewed as an annual one.

Policy makers are assumed to target nominal GDP growth. That is they set the current level of the real interest rate such that next period's expected nominal GDP growth equals zero.[2] In mathematical terms r_{t-1} is set so that

[2] A target rate of zero is chosen for simplicity. Any other rate could be chosen without affecting the stability properties we derive.

$$E_{t-1}(\pi_t + y_t - y_{t-1}) = 0 , \tag{3}$$

where E_{t-1} is the mathematical expectations operator conditional upon all information dated period t-1 and earlier. The monetary authority, therefore, is assumed to form its expectations rationally even if private agents do not. Moreover, we are assuming that the monetary authority fully knows the state of the system when it sets its policy instrument, ie it sets r_{t-1} with an information set that includes t-1 information.

Next we examine the stability of output and inflation under a range of assumptions about how private agents form their inflation expectations – assuming that monetary policy is constrained by equation (3).

3.2.1) Accelerationist Phillips Curve

Result one (Ball-Svensson): Assume that the system is given by equations (1), (2), and (3) and that $\pi_t^e = \pi_{t-1}$, ie that the short-run Phillips curve is of the accelerationist variety. Then the system is unstable with inflation and output non-stationary processes.

Setting $\pi_t^e = \pi_{t-1}$ in (2) and substituting (1) and (2) into (3) produces the following state-contingent policy reaction function:[3]

$$r_{t-1} = \frac{1}{\gamma}[\pi_{t-1} + (\alpha + \lambda - 1)y_{t-1}]. \tag{4}$$

Substituting (4) back into (1) and ignoring the stochastic error terms gives us the VAR(1) model

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1-\alpha & -1 \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \end{bmatrix}. \tag{5}$$

[3] Interestingly, we see that the implied policy reaction function from the moment condition (3) is in the form of a BHM rule.

Stability depends upon the eigenvalues of the coefficient matrix. These eigenvalues can be found by solving for the roots of the quadratic

$$\mu^2 - (2-\alpha)\mu + 1 = 0. \tag{6}$$

These roots are

$$\mu_{1,2} = \frac{(2-\alpha) \pm \sqrt{\alpha(\alpha-4)}}{2}.$$

For $\alpha > 4$, one of either μ_1, μ_2 lie outside the unit circle. When $\alpha < 4$ the modulus of μ_1 and μ_2 equal

$$\sqrt{\left(\frac{2-\alpha}{2}\right)^2 + \alpha\frac{(4-\alpha)}{4}},$$

which after canceling terms can be seen to equal 1. Thus when the roots are imaginary they lie on the unit circle. Finally, when $\alpha = 4$ both roots equal –1. Consequently, for all values of $\alpha > 0$ the system is either unstable or has roots on the unit circle. In either case y_t and π_t are non-stationary. This is the Ball-Svensson instability result.

Note that the roots of the system depends only on α and not on λ or γ. The specification of the IS curve in no way affects the stability of the system (see also McCallum, 1997).

The cause of this instability can best be understood as follows. In period t when policy makers are setting r_t they do so on the basis that $E_t\pi_{t+1}$ and E_ty_t are predetermined. Thus the policy objective is met by policy makers fixing r_t to set E_ty_{t+1} appropriately. Policy makers behave this way every period so r_{t+1} is set taking $E_{t+1}\pi_{t+2}$ and $E_{t+1}y_{t+1}$ as predetermined. Consequently the policy objective is met with policy makers only considering the interest rate's effect on the output gap and ignoring the subsequent effect the output gap has on inflation. But the output gap permanently alters the following period's inflation rate. Because of this follow on effect, the interest rate setting in period t permanently effects both $E_{t+1}\pi_{t+2}$ and $E_{t+1}y_{t+1}$, but this effect is ignored by policy makers when they set r_t. Since the channel policy makers are ignoring has permanent implications for inflation in subsequent periods policy makers find themselves having to offset the permanent effect of earlier policy decisions. This sets up a permanent cycle in the interest rate, which transmits itself into the output gap and inflation.

Ball-Svensson instability is an example of instrument instability. One proposed solution to instrument instability is to take a more medium term or forward-looking approach to policy (Holbrook, 1972). In this spirit, Svensson (1997b) shows that if monetary policy is constrained by a restriction such as $E_{t-1}(\pi_{t+1} + y_t - y_{t-1}) = 0$, then instability would not appear. The intuition here is that monetary policy affects both π_{t+1} and $y_t - y_{t-1}$ with the same lag, and hence policy makers consider policy's effect on both $E_t\pi_{t+2}$ and E_ty_{t+1} when setting r_t. Policy makers no longer find themselves having to fully offset earlier policy decisions. Yet constraining $E_{t-1}(\pi_{t+1} + y_t - y_{t-1})$ is unsatisfactory because it has no natural interpretation or justification.

The policy constraint (3) can be obtained from the first order condition of the minimisation problem

$$\text{Loss}[0,\infty] = E_{t-1}\sum_{i=0}^{\infty}\varphi^{i}(\pi_{t+i} + y_{t+i} - y_{t-1+i})^{2}, \qquad 0 < \varphi \leq 1$$

subject to equations (1), (2).

Accordingly the Ball-Svensson instability result depends upon policy makers being constrained by a loss function that has perfect substitutability between inflation and output growth. The levels of inflation and output themselves do not matter. But if the policy loss only depends upon the level of nominal GDP growth why are we then concerned that this loss function leads to infinite variances in output and inflation? Afterall, output and inflation do not separately enter the loss function. Alternatively, if policy makers are concerned with the variances in inflation and output, why do we impose on them a loss function that states that only the sum of inflation and output growth matter?

To better analyse the statistical nature of the instability present in this system we represent the system (5) as

$$y_t = (2-\alpha)y_{t-1} - y_{t-2} - u_t + v_t - v_{t-1} \tag{7}$$

and

$$\pi_t = (2-\alpha)\pi_{t-1} - \pi_{t-2} + \alpha v_{t-1} + u_t - (1-\alpha)u_{t-1}. \tag{8}$$

Equations (7) and (8) have identical autoregressive structures. Therefore any unit root(s) present must lie at the same point(s) in the spectrum for both y_t and π_t. We note that in the special case when $\alpha = 0$ both y_t and π_t contain double unit roots; the system is unstable but does not contain a cycle. When $\alpha = 2$ y_t and π_t are integrated with roots of $L = \pm i$, suggesting a cyclical pattern in which a full cycle is completed every four years. Similarly, for $\alpha = 4$ y_t and π_t are integrated, each containing the two roots $L = -1, -1$. The root $L = -1$ indicates the presence of a cycle with a two year duration. We note that as α declines from 4 toward 0 the length of the cycle increases.

In the literature on seasonality, roots of $L = -1, \pm i$ represent integration at seasonal frequencies (Hylleberg, Engle, Granger, and Yoo, 1990). But as we discussed earlier our model is best interpreted in annual terms, and hence the roots described above do not represent seasonality. Instead they indicate the presence of something akin to a business cycle. By targeting nominal GDP in this model, policy makers, instead of controlling inflation by eliminating the demand cycle, by their very actions introduce a pronounced cycle into the economy.

3.2.2) Adaptive expectations

Consider now an alternative form of backward-looking expectations – adaptive expectations. Under adaptive expectations we write

$$\pi_t^e = \pi_{t-1} + \delta(\pi_{t-1}^e - \pi_{t-1}), \quad 0 \le \delta \le 1. \tag{9}$$

When $\delta = 0$ adaptive expectations simplifies to produce an accelerationist Phillips curve. Alternatively, $\delta = 1$ implies that inflation expectations equal some fixed constant.

Result two: Assume that the system can be represented by equations (1), (2), and (3), and that inflation expectations are formed adaptively by (9). Then the system is stable provided $0 < \delta < 1$ and $0 < \alpha < 2$. As α increases above 2 stability requires smaller values for δ. The model is unstable for $\alpha \ge 4$.

Introducing the lag operator ($z_{t-1} = Lz_t$), (9) can be written as

$$\pi_t^e = \frac{(1-\delta)L}{(1-\delta L)}\pi_t. \tag{10}$$

Substituting both (10) and (2) into (3) yields

$$E_{t-1}\left[c + (1-L)^{-1}(u_t - \delta u_{t-1} + \alpha y_{t-1} - \alpha\delta y_{t-2}) + y_t - y_{t-1}\right] = 0,$$

where c is an arbitrary constant. Multiplying through by the difference operator (1-L) and recognizing that $E_{t-1}y_t = y_t + \varepsilon_t$, where ε_t is an innovation orthogonal to the monetary authority's information set, produces the AR(2) process

$$y_t + (\alpha - 2)y_{t-1} + (1 - \alpha\delta)y_{t-2} = \varepsilon_t + \delta u_{t-1}. \tag{11}$$

Equation (11) will be stable provided the following three conditions are met (see Harvey, 1981, or Sargent, 1987):

a) $1 + (\alpha\text{-}2) + (1\text{-}\alpha\delta) > 0$;

b) $1 - (\alpha\text{-}2) + (1\text{-}\alpha\delta) > 0$;

c) $1 - (1\text{-}\alpha\delta) > 0$.

Because we have assumed that $\alpha > 0$, condition c) is satisfied provided $\delta > 0$. Similarly, condition a) is met if we further restrict $\delta < 1$. Re-writing condition b) we obtain $\alpha(1+\delta) < 4$. Consequently, provided $0 < \delta < 1$ the system is stable if $0 < \alpha < 2$. As α increases above 2 the system requires increasingly smaller values of δ if stability is to be obtained. For $\alpha \geq 4$ the system is unstable. However, if Svensson (1997b, page 20) is correct in his conjecture that $\alpha < 1$, then the system is stable under adaptive expectations except in the extreme cases where $\delta = 0, 1$.

3.2.3) Rational expectations

Now let us assume that inflation expectations are formed by $\pi_t^e = E_{t-1}\pi_{t+1}$. Thus it is next period's inflation rate that is relevant for the Phillips curve, and we assume that private agents form their expectations rationally using all information dated period t-1 or earlier.

Roberts (1995) shows that a Phillips curve with inflation expectations of this form can be justified by a number of models including the costly price adjustment model of Rotemberg (1982), and the staggered contracts time-dependent pricing model of Calvo (1983). In these models it is the firm's inability to set prices costlessly and instantaneously that directs firms to anticipate future price movements when setting prices today.

Result three: Assume that the system can be represented by (1), (2), and (3) and that $\pi_t^e = E_{t-1}\pi_{t+1}$. Then the system is stable provided $0 < \alpha < 4$.

Using (1), (2) and (3) along with our assumption that $\pi_t^e = E_{t-1}\pi_{t+1}$ we obtain the optimal state contingent policy reaction function[4]

[4] We should note that the expectation $E_{t-1}\pi_{t+1}$ is not predetermined at time t-1, and that this reduces the control lag to inflation to that of output: one period. Svensson (1997b) argues this situation is conducive to stability. That it is the expectations channel and not the reduced control lag that is behind *Result three* can be seen by the fact that $E_{t-2}\pi_{t+1}$ and $E_{t-1}\pi_{t+1}$ differ only in that with the latter agents know u_{t-1} and v_{t-1}. But $E_{t-2}\pi_{t+1}$ is predetermined at t-1, and these innovation terms do not affect the system's stability. Thus the stability properties of the system are the same regardless of whether expectations are formed using t-1 or t-2 information.

$$r_{t-1} = \frac{1}{\gamma}[E_{t-1}\pi_{t+1} + (\alpha + \lambda - 1)y_{t-1}]. \tag{12}$$

Substituting (12) into (1) results in the reduced form system

$$y_t = -E_{t-1}\pi_{t+1} - (\alpha - 1)y_{t-1} + v_t \tag{13}$$

$$\pi_t = E_{t-1}\pi_{t+1} + \alpha y_{t-1} + u_t. \tag{14}$$

The state variables of (13), (14) are y_{t-1}, v_t and u_t. Thus we posit the solution

$$y_t = \theta_{11}y_{t-1} + \theta_{12}u_t + \theta_{13}v_t \tag{15}$$

$$\pi_t = \theta_{21}y_{t-1} + \theta_{22}u_t + \theta_{23}v_t. \tag{16}$$

From (15) and (16)

$$E_{t-1}\pi_{t+1} = \theta_{21}\theta_{11}y_{t-1}. \tag{17}$$

Note that stability of the system depends only upon the magnitude of θ_{11}. Substituting (15), (16), and (17) into (13) and (14) and equating coefficients gives the following set of restrictions on the undetermined coefficients:

i) $\theta_{11} = -\theta_{21}\theta_{11} - (\alpha - 1)$;

ii) $\theta_{12} = 0$;

iii) $\theta_{13} = 1$;

iv) $\theta_{21} = \theta_{21}\theta_{11} + \alpha$;

v) $\theta_{22} = 1$;

vi) $\theta_{23} = 0$.

We can solve for θ_{11} from (i) and (iv) by recognizing that $\theta_{11} + \theta_{21} = 1$. Following this strategy we substitute $\theta_{21} = 1 - \theta_{11}$ into (i) resulting in the quadratic

$$\theta_{11}^2 - 2\theta_{11} + 1 - \alpha = 0. \tag{18}$$

Solving (18) for θ_{11} yields

$$\theta_{11} = 1 \pm \sqrt{\alpha} \ .$$

From (13) and (14), however, we know that when $\alpha = 1$, y_{t-1} cannot affect y_t. Then McCallum's (1983) minimum state variable criteria dictates that we take the negative root, which ensures that our model is stable for all $0 < \alpha < 4$. Inflation and the output gap are non-stationary when $\alpha \geq 4$, but $\alpha < 4$ is the relevant case. Notice, however, that for $1 < \alpha < 4$, θ_{11} is negative implying that output is negatively correlated with its own first lag over this parameter range. Under Svensson's (1997b) conjecture that $\alpha < 1$ this correlation is positive.

3.2.4) Mixed Expectations

In our final example we consider the generalisation whereby inflation expectations contain forward-looking and backward-looking components:

$$\pi_t^e = \beta E_{t-1}\pi_{t+1} + (1-\beta)\pi_{t-1}, \qquad 0 \leq \beta \leq 1. \tag{19}$$

This specification for inflation expectations arises naturally from the analyses in Sections 3.2.1 and 3.2.3. With instability present when $\beta = 0$ but absent when $\beta = 1$ it is natural to examine β values between 0 and 1 to uncover the cross-over point between stability and instability. Moreover, equation (19) could arise if some agents were forward-looking and others backward-looking (heterogenous expectations) or possibly through contracting behaviour. When $\beta = 0$ equation (19) reduces to the accelerationist Phillips curve we studied in Section 3.2.1. Alternatively, when $\beta = 1$ we have the fully forward-looking case studied in Section 3.2.3.

Result four: Assume that the system can be represented by (1), (2), and (3), and that inflation expectations are formed by (19). Then for $0 < \beta \leq 1$ the system is stable provided $0 < \alpha < 4$.

By substituting (19) into (2) and then (1) and (2) into (3) we can easily derive the optimal policy reaction function. We then substitute this policy reaction function back into (1) to produce the reduced form system below.

$$y_t = -\beta E_{t-1}\pi_{t+1} - (1-\beta)\pi_{t-1} - (\alpha - 1)y_{t-1} + v_t \tag{20}$$

$$\pi_t = \beta E_{t-1}\pi_{t+1} + (1-\beta)\pi_{t-1} + \alpha y_{t-1} + u_t . \tag{21}$$

The state variables for this system are: π_{t-1}, y_{t-1}, u_t and v_t. Accordingly we posit the solution

$$y_t = \theta_{11}y_{t-1} + \theta_{12}\pi_{t-1} + \theta_{13}u_t + \theta_{14}v_t \tag{22}$$

$$\pi_t = \theta_{21}y_{t-1} + \theta_{22}\pi_{t-1} + \theta_{23}u_t + \theta_{24}v_t . \tag{23}$$

Using (22) and (23) along with (20) and (21) we equate coefficients to derive the following restrictions upon θ_{11}, θ_{12}, θ_{21} and θ_{22} (we ignore the remaining coefficients because they do not affect the stability of the system).

i) $\theta_{11} = -\beta\theta_{21}\theta_{11} - \beta\theta_{22}\theta_{21} + 1 - \alpha$;

ii) $\theta_{12} = -\beta\theta_{21}\theta_{12} - \beta\theta_{22}^2 - 1 + \beta$;

iii) $\theta_{21} = \beta\theta_{21}\theta_{11} + \beta\theta_{22}\theta_{21} + \alpha$;

iv) $\theta_{22} = \beta\theta_{21}\theta_{12} + \beta\theta_{22}^2 + 1 - \beta$.

From ii) and iv) we observe that $\theta_{12} = -\theta_{22}$. Similarly, i) and ii) imply the relationship $\theta_{11} + \theta_{21} = 1$. Using these relationships we can reduce our system down to two non-linear simultaneous equations in θ_{21}, θ_{22}.

$$\beta\theta_{22}^2 - (1 + \beta\theta_{21})\theta_{22} + 1 - \beta = 0 \tag{24}$$

$$\beta\theta_{21}^2 - (\beta + \beta\theta_{22} - 1)\theta_{21} - \alpha = 0 \tag{25}$$

From (24) the solutions for θ_{22} are

$$\theta_{22} = \frac{\beta\theta_{21}+1}{2\beta} \pm \frac{\sqrt{(1+\beta\theta_{21})^2 - 4(1-\beta)\beta}}{2\beta}.$$

However, from Section 3.2.3 we know that when $\beta = 1$, θ_{22} must equal zero because π_{t-1} is no longer a member of the minimum set of state variables. Therefore, the appropriate root is the negative one and the solution for θ_{22} is

$$\theta_{22} = \frac{\beta\theta_{21}+1}{2\beta} - \frac{\sqrt{(1+\beta\theta_{21})^2 - 4(1-\beta)\beta}}{2\beta}. \tag{26}$$

Similarly, the roots of (25) are

$$\theta_{21} = \frac{\beta+\beta\theta_{22}-1}{2\beta} \pm \frac{\sqrt{(\beta+\beta\theta_{22}-1)^2 + 4\alpha\beta}}{2\beta}.$$

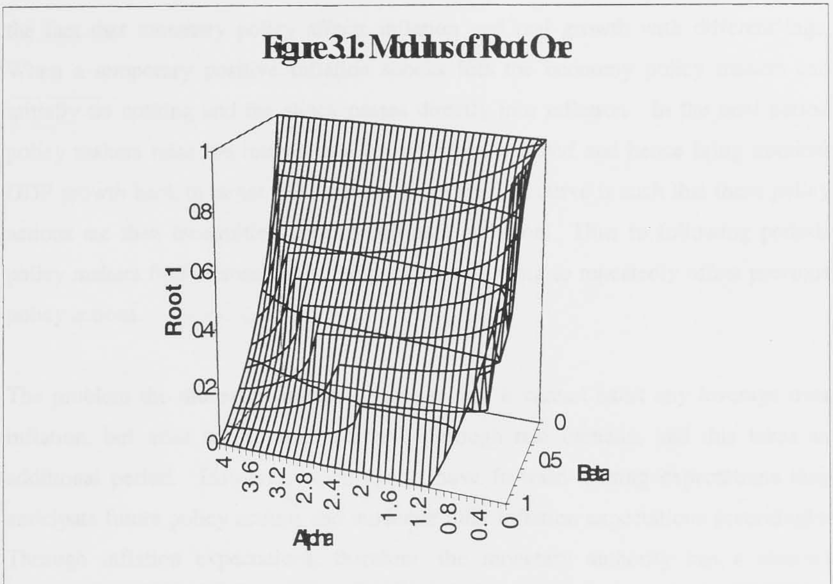
But from Section 3.2.3 we know that when $\beta = 1$, $\theta_{22} = 0$ and $\theta_{21} = \sqrt{\alpha}$. Therefore, the appropriate root is the positive one and the solution for θ_{21} is

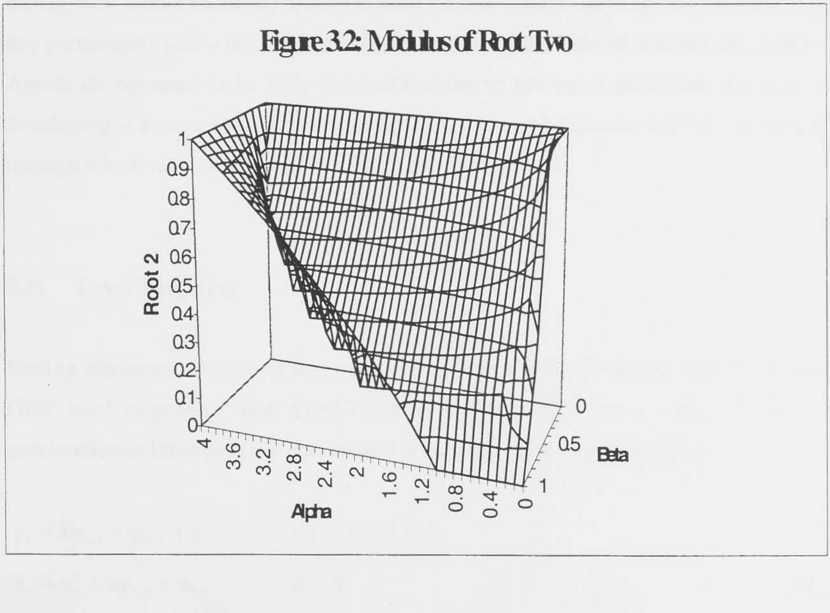
$$\theta_{21} = \frac{\beta+\beta\theta_{22}-1}{2\beta} + \frac{\sqrt{(\beta+\beta\theta_{22}-1)^2 + 4\alpha\beta}}{2\beta}. \tag{27}$$

Clearly (26) and (27) are not closed form solutions for θ_{21} and θ_{22}. However, given values for α and β we can numerically solve for θ_{21} and θ_{22}. Once we know θ_{21} and θ_{22} we can easily determine the stability of the system by checking the roots of

$$\mu^2 - (1-\theta_{21}+\theta_{22})\mu + \theta_{22} = 0.$$

Performing these numerical simulations reveals that for $0 < \alpha < 4$, $0 < \beta \leq 1$, the system is stable. Figures 3.1 and 3.2 graph the modulus of the roots of the system over these parameter ranges. These figures show that when $\beta = 0$ the model is unstable (the Ball-Svensson result) and that for all other values of β the model is stable provided $0 < \alpha < 4$. Thus for plausible values of α the system will be stable provided private agents' inflation expectations place some positive weight on future inflation outcomes.





3.2.5) Expectations and Stability

We described earlier how the cause of the Ball-Svensson instability was generated by the fact that monetary policy affects inflation and real growth with different lags. When a temporary positive inflation shocks hits the economy policy makers can initially do nothing and the shock passes directly into inflation. In the next period policy makers raise the interest rate to lower real demand and hence bring nominal GDP growth back to target. The nature of the Phillips curve is such that these policy actions are then transmitted permanently into inflation. Thus in following periods policy makers find themselves in the position of having to repeatedly offset previous policy actions.

The problem the monetary authority faces is that it cannot exert any leverage over inflation, but must act upon it indirectly through real demand, and this takes an additional period. However, when agents have forward-looking expectations they anticipate future policy actions and moderate their inflation expectations accordingly. Through inflation expectations, therefore, the monetary authority has a channel through which it can exert some influence over current inflation. By changing the timing with which monetary policy affects inflation, inflation expectations eliminate any permanent cycle – any cycle that exists will be in the form of damped oscillations. Agents do not need to be fully forward-looking to prevent a permanent cycle from developing – even a small amount of forward-looking behaviour provides a channel through which monetary policy can affect current inflation.

3.3) Level Targeting

Having discussed the case of nominal GDP growth targeting we now turn to nominal GDP level targeting. Ball (1999) has shown that GDP level targeting with an accelerationist Phillips curve also creates instability. Consider the system

$$y_t = \lambda y_{t-1} - \gamma r_{t-1} + v_t, \qquad \gamma > 0, 0 \leq \lambda \leq 1; \tag{28}$$

$$\pi_t = \pi_t^e + \alpha y_{t-1} + u_t, \qquad \alpha > 0 \tag{29}$$

$$p_t = p_{t-1} + \pi_t, \tag{30}$$

where p_t is the (logged) current price level. The monetary authority is assumed to choose the interest rate in period t-1 such that

$$E_{t-1}(p_t + y_t) = z \qquad\qquad 0 < z < \infty. \qquad (31)$$

Result five (Ball): Assume that the system can be represented by (28), (29), (30) with $\pi_t^e = \pi_{t-1}$, and that policy is constrained by (31). Then the system will be unstable with properties identical to those of *Result one*.

Deriving the optimal policy reaction function, substituting it back into (28), and ignoring the constant and innovation terms, which do not affect stability, results in the VAR(1) model

$$\begin{bmatrix} y_t \\ \pi_t \\ p_t \end{bmatrix} = \begin{bmatrix} -\alpha & -1 & -1 \\ \alpha & 1 & 0 \\ \alpha & 1 & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ p_{t-1} \end{bmatrix}.$$

Clearly the coefficient matrix is singular and can have at most two non-zero eigenvalues. Using a co-factor expansion the eigenvalues of the system can be found by solving for μ in

$$\mu[\mu^2 - (2-\alpha)\mu + 1] = 0.$$

As expected one eigenvalue is zero while the remaining two are determined by the quadratic expression in the square brackets. However, this quadratic is identical to equation (6) used to determine stability under nominal GDP growth targeting with an accelerationist Phillips curve. Therefore the stability properties of the model under level targeting are the same as those under growth targeting: the model is unstable.

Result six: Assume that the system can be represented by (28), (29), (30), and (31) with inflation expectations formed adaptively through (9). Then the system has stability properties identical to those of *Result two*.

Equations (28), (29), and (30) become

$$y_t = \lambda y_{t-1} - \gamma r_{t-1} + v_t, \qquad \gamma > 0, 0 \le \lambda \le 1;$$

$$\pi_t = \frac{(1-\delta)L}{(1-\delta L)}\pi_t + \alpha y_{t-1} + u_t, \qquad \alpha > 0, 0 \le \delta \le 1 \tag{32}$$

$$p_t = p_{t-1} + \pi_t. \tag{33}$$

Canceling terms (32) can be re-written as

$$\pi_t = c + \frac{1}{1-L}[(1-\delta L)u_t + \alpha y_{t-1} - \alpha\delta y_{t-2}], \tag{34}$$

where again c is an arbitrary constant. Similarly, (33) can be expressed as

$$p_t = a + \frac{1}{1-L}[\pi_t]. \tag{35}$$

By using (34) and (35) in (31) we produce

$$E_{t-1}\left[a + \frac{1}{(1-L)}\left[c + \frac{1}{(1-L)}((1-\delta L)u_t + \alpha y_{t-1} - \alpha\delta y_{t-2})\right] + y_t\right] = z. \tag{36}$$

Multiplying (36) through by $(1\text{-}L)^2$ and recognising that $y_t = E_{t-1}y_t + \varepsilon_t$ allows us to express (36) as the AR(2) process

$$y_t + (\alpha - 2)y_{t-1} + (1-\alpha\delta)y_{t-2} = \varepsilon_t + \delta u_{t-1}. \tag{37}$$

Equation (37), however, is simply a re-statement of the AR(2) process we derived under nominal GDP growth targeting with adaptive expectations (equation 11). Clearly the characteristic equations of (37) and (11) are the same and therefore the systems have the same roots, implying that their stability properties are identical. As with the accelerationist Phillips curve, here again with adaptive expectations we have the result that the system's stability properties are identical under either nominal GDP growth or nominal GDP level targeting.

That level targeting and growth targeting produce identical stability properties under an accelerationist Phillips curve and adaptive expectations is no coincidence. The stability properties will be the same regardless of how private agents form their expectations. We now show this result.

Result seven: Assume that the monetary authority knows the system and forms its expectations rationally using current period information. Then the stability properties of the system under nominal GDP level targeting are identical to those under nominal GDP growth targeting.

Define the current price level as $p_t = p_{t-1} + \pi_t$. Now general forms of exact nominal GDP growth targeting and exact nominal GDP level targeting can be represented by the following constraints respectively.

$$E_{t-1}(\pi_t + y_t - y_{t-1}) = q_t \qquad \text{(growth targeting)} \tag{38}$$

$$E_{t-1}(p_t + y_t) = z_t, \qquad z_t > 0 \ \forall\, t, \qquad \text{(level targeting)} \tag{39}$$

We assume that q_t and z_t are independent of y_t and π_t and hence note that setting (38) and (39) to target q_t and z_t respectively is arbitrary and does not affect the stability of the system. By substituting $p_t = p_{t-1} + \pi_t$ into (39) we obtain

$$E_{t-1}(p_{t-1} + \pi_t + y_t) = z_t \tag{40}$$

However, since the monetary authority targets nominal GDP in each period, it must be the case that we also have

$$E_{t-2}(p_{t-1} + y_{t-1}) = z_{t-1}$$

and hence that

$$E_{t-2}p_{t-1} = -E_{t-2}y_{t-1} + z_{t-1}. \tag{41}$$

Adding and subtracting $E_{t-2}p_{t-1}$ from the LHS of (40) and exploiting (41) gives us

$$E_{t-1}(p_{t-1} - E_{t-2}p_{t-1} + \pi_t + y_t - E_{t-2}y_{t-1}) = z_t - z_{t-1}. \tag{42}$$

Now, provided the monetary authority forms its expectations rationally we can define $p_{t-1} = E_{t-2}p_{t-1} + \eta_{t-1}$ and $y_{t-1} = E_{t-2}y_{t-1} + \varepsilon_{t-1}$, where ε_{t-1} and η_{t-1} are forecast errors orthogonal to the monetary authority's period t-2 information set, but known to policy makers in period t-1. Substituting these definitions into (42) and canceling terms yields

$$E_{t-1}(\pi_t + y_t - y_{t-1}) = z_t - z_{t-1} - \varepsilon_{t-1} - \eta_{t-1}. \tag{43}$$

But (43) is identical to (38) where $q_t = (1\text{-}L)z_t - \varepsilon_{t-1} - \eta_{t-1}$, thus the stability properties of the two targeting rules are identical. This result obtains because once we know the nominal GDP level forecast error, we can offset this error the next period by essentially choosing a time varying growth rate target. However, the result shows that whether we offset these forecast errors or not does not alter the system's stability properties.

Note that this result has been derived quite independently of the underlying behavioral economic model. The model itself is not important provided the monetary authority knows what it is. Moreover, the result has been derived without reference to how private agents form their expectations. All that matters is that the monetary authority forms its expectations rationally.

To give an example of the usefulness of *Result seven* let us consider the simulation study of Hall and Mankiw (1994). In their study of how nominal GDP targeting would have affected the US economy, Hall and Mankiw (1994) calibrated a small stylised model of the US economy. This model's supply side was a Phillips curve with adaptive expectations and was in the form of (9) with parameter values (in our notation) of $\delta = 0.9$ and $\alpha = 0.05$. When applying exact nominal GDP targeting to their model they found first that their model was stable and second that nominal GDP growth targeting and nominal GDP level targeting produced identical output and inflation variances. Commenting on these results Hall and Mankiw (1994) simply observed without explanation that 'with perfect achievement of the target, the level and growth rate policies are the same.'

With $\delta = 0.9$ and $\alpha = 0.05$ the conditions of *Result two* and *Result six* are satisfied. *Result two* tells us that with these parameter values the system will be stable. *Result six*, or the more general *Result seven*, tells us that not only will the system be stable but that the model stability properties will be identical under the two targeting rules.

3.4) Exact versus Inexact Targeting

To this point our entire analysis has been couched in terms of exact targeting, rather than the less stringent inexact targeting. Exact targeting requires that the monetary authority meet its targeted objective up to a random error each and every period. In the Ball model studied above the monetary authority is able to achieve this because r_{t-1} has leverage over y_t even though y_{t-1} and π_t are predetermined. Inexact targeting on the other hand requires only that the monetary authority act to move nominal GDP growth (say) back towards the target rate period by period – systematic misses are allowed (see Bryant, Hooper and Mann, 1993).

Again consider the model

$$y_t = \lambda y_{t-1} - \gamma r_{t-1} + v_t, \qquad \gamma > 0, 0 \leq \lambda \leq 1; \tag{44}$$

$$\pi_t = \pi_{t-1} + \alpha y_{t-1} + u_t, \qquad \alpha > 0, \tag{45}$$

but now append to it the policy reaction function

$$r_{t-1} = \rho E_{t-1}(\pi_t + y_t - y_{t-1}), \quad 0 < \rho < \infty. \tag{46}$$

This policy reaction function is the natural analogue to the exact targeting case because in the limit as $\rho \to \infty$ exact targeting results. Consequently we must expect instability to occur in this limiting situation. We further eliminate the no adjustment case, $\rho = 0$, because it is not an example of nominal GDP targeting. However, it is trivial to show that with the policy instrument held fixed the system is unstable with π_t following a random walk.

Result eight: Assume that the system can be represented by (44), (45), and (46). Then the system is stable provided $\lambda < 1$ and $\alpha < 4$. If $\alpha > 4$, then the system will still be stable provided ρ is 'small'.

Substituting (46) into (44) gives

$$y_t = \lambda y_{t-1} - \gamma\rho E_{t-1} y_t - \gamma\rho E_{t-1}\pi_t + \gamma\rho y_{t-1} + v_t . \tag{47}$$

Taking conditional expectations of (45) and (47) produces

$$E_{t-1}\pi_t = \pi_{t-1} + \alpha y_{t-1}, \tag{48}$$

and

$$E_{t-1} y_t = \frac{(\lambda + \gamma\rho)}{(1+\gamma\rho)} y_{t-1} - \frac{\gamma\rho}{(1+\gamma\rho)} E_{t-1}\pi_t . \tag{49}$$

Inserting (48) and (49) in (47) and (45) allows us to derive the reduced form system

$$y_t = \frac{(\lambda + \gamma\rho(1-\alpha))}{(1+\gamma\rho)} y_{t-1} - \frac{\gamma\rho}{1+\gamma\rho}\pi_{t-1} + v_t , \tag{50}$$

$$\pi_t = \pi_{t-1} + \alpha y_{t-1} + u_t . \tag{51}$$

Using (51) to solve for π_t, lagging, substituting into (50), and collecting terms produces the AR(2) process for y_t (where again error terms have been ignored)

$$y_t = \frac{1 + \lambda + \gamma\rho(2-\alpha)}{1+\gamma\rho} y_{t-1} - \frac{\lambda + \gamma\rho}{1+\gamma\rho} y_{t-2} .$$

For our system to be stable we now require three conditions to hold:

i) $$\frac{1 + \lambda + \gamma\rho(2-\alpha)}{1+\gamma\rho} - \frac{\lambda + \gamma\rho}{1+\gamma\rho} < 1, \tag{52}$$

ii) $$-\left(\frac{1+\lambda+\gamma\rho(2-\alpha)}{1+\gamma\rho}\right)-\frac{\lambda+\gamma\rho}{1+\gamma\rho}<1, \tag{53}$$

iii) $$\left(\frac{\lambda+\gamma\rho}{1+\gamma\rho}\right)<1. \tag{54}$$

We consider each of these conditions in turn. First, by cancelling terms, (52) reduces to the requirement that $-\alpha\gamma\rho < 0$. The parameter restrictions we have placed on the system mean that this condition is satisfied. Second, provided $|\rho| < \infty$, (54) amounts to the requirement that $\lambda < 1$. We had earlier assumed that $\lambda \leq 1$ but by ruling out $\lambda = 1$ condition (54) is met. Notice, here, that under exact targeting the stability of the system did not depend on the demand side of the model – any value of λ was admissible. Now under inexact targeting the demand side of the economy is relevant, but only a very mild restriction is placed on it.

Finally, (53) implies the restriction

$$\rho<\frac{2(1+\lambda)}{\gamma(\alpha-4)}, \text{ when } \alpha>4$$

$$\rho>\frac{2(1+\lambda)}{\gamma(\alpha-4)}, \text{ when } \alpha<4.$$

In the unlikely case that $\alpha > 4$, stability requires that ρ be small, and instability is associated with large values of ρ, or, equivalently, more aggressive policy behaviour. If we take the more likely case where $\alpha < 4$, then the system is stable under our assumption that $0 < \rho < \infty$.

Consequently, under a very mild restriction upon the demand side of the economy and the plausible restriction that $\alpha < 4$, the system is stable under inexact targeting. Clearly, once we get away from the extreme case of exact targeting, nominal GDP growth targeting does not cause instability even if private agents form their inflation expectation using a simple naive backward-looking process.

3.5) Interest Rate Stabilization

Along with their other duties, central banks also often have the role of overseeing or ensuring the soundness and stability of the financial sector. One way of modelling this responsibility is to constrain the movements of financial variables, such as interest rates. This process leads to interest rate smoothing. A further reason explaining why central banks smooth interest rates is that the costs of reversing a policy decision are such that it pays to alter policy stance gradually rather than in large movements (Lowe and Ellis, 1997).

In the Ball-Svensson framework (Section 3.2.1) the instability in π_t and y_t is translated into interest rates through the policy reaction function. In this framework the standard loss function, which penalises separately deviations of π_t and y_t from their target values, results in a stable system. Consequently, we might expect that a loss function that imposes stability upon the policy instrument, r_t, combined with a nominal GDP growth target, would also result in a stable system.

To capture the effects of interest rate stabilization on the economy we assume that the central bank minimises the loss function

$$\mathrm{Loss}[0,\infty] = \frac{1}{2}\mathrm{E}_{t-1}\sum_{i=0}^{\infty}\varphi^{i}[(\pi_{t+i} + y_{t+i} - y_{t-1+i})^{2} + Q(r_{t+i})^{2}], \tag{55}$$

where Q is the relative weight placed upon financial stability and $0 < \varphi \leq 1$ is the discount factor, subject to the transition equations

$$\begin{bmatrix} \pi_{t+1} \\ y_{t+1} \\ y_t \end{bmatrix} = \begin{bmatrix} 1 & \alpha & 0 \\ 0 & \lambda & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \pi_t \\ y_t \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ -\gamma \\ 0 \end{bmatrix}[r_t] + \begin{bmatrix} u_{t+1} \\ v_{t+1} \\ 0 \end{bmatrix} = A\begin{bmatrix} \pi_t \\ y_t \\ y_{t-1} \end{bmatrix} + B[r_t] + \begin{bmatrix} u_{t+1} \\ v_{t+1} \\ 0 \end{bmatrix}. \tag{56}$$

Denoting the vector $[\pi_t\ y_t\ y_{t-1}]^T$ by $\mathbf{z}_t$ the loss function can be written recursively as

$$L(\mathbf{z}_t) = \max_r[\mathbf{z}_t^T\mathbf{R}\mathbf{z}_t + r_tQr_t + \varphi\mathrm{E}_{t-1}L(\mathbf{z}_{t+1})], \tag{57}$$

where

$$\mathbf{R} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}.$$

The system (56), (57) is an example of the 'stochastic linear optimal regulator problem' (Sargent, 1987), and is known to have a solution of the following form:

$$L(\mathbf{z}_t) = \mathbf{z}_t^T \mathbf{P} \mathbf{z}_t + d;$$

$$d = \frac{\varphi}{1-\varphi} \mathrm{tr}[\mathbf{P}\mathbf{\Omega}];$$

$$r_t = -\varphi(Q + \varphi \mathbf{B}^T \mathbf{P} \mathbf{B})^{-1} \mathbf{B}^T \mathbf{P} \mathbf{A} \mathbf{z}_t; \tag{58}$$

$$\mathbf{P} = \mathbf{R} + \varphi \mathbf{A}^T \mathbf{P} \mathbf{A} - \varphi^2 \mathbf{A}^T \mathbf{P} \mathbf{B} (Q + \varphi \mathbf{B}^T \mathbf{P} \mathbf{B})^{-1} \mathbf{B}^T \mathbf{P} \mathbf{A}, \tag{59}$$

where $\mathbf{\Omega}$ is the (singular in our case) variance-covariance matrix of the innovation vector $[u_t\ v_t\ 0]^T$. Following Rudebusch and Svensson (1998) and Svensson (1998), in the limit as $\varphi \rightarrow 1$ the loss function (55) converges to the unconditional mean of the period loss function. For our problem we define $\varphi = 1$ and reinterpret the intertemporal loss function as

$$\mathrm{Loss}[0, \infty] = \mathrm{Var}[\pi_t + y_t - y_{t-1}] + Q\mathrm{Var}[r_t].$$

Consistent with $\varphi = 1$, (58) and (59) become

$$r_t = -(Q + \mathbf{B}^T \mathbf{P} \mathbf{B})^{-1} \mathbf{B}^T \mathbf{P} \mathbf{A} \mathbf{z}_t = -\mathbf{F} \mathbf{z}_t, \tag{60}$$

$$\mathbf{P} = \mathbf{R} + \mathbf{A}^T \mathbf{P} \mathbf{A} - \mathbf{A}^T \mathbf{P} \mathbf{B} (Q + \mathbf{B}^T \mathbf{P} \mathbf{B})^{-1} \mathbf{B}^T \mathbf{P} \mathbf{A}, \tag{61}$$

respectively. Substituting (60) into (56) results in the reduced form system

$$\mathbf{z}_{t+1} = [\mathbf{A} - \mathbf{B}\mathbf{F}] \mathbf{z}_t,$$

where the stochastic error terms have been ignored. The system's stability properties are then determined by the eigenvalues of $[\mathbf{A} - \mathbf{BF}]$.

The parameter Q indexes the degree of interest rate stabilization. We know from Section 3.2.1 that when Q = 0 our system is unstable. Similarly, in the limit as $Q \to \infty$ the loss function restricts the policy instrument to a constant. This limiting case is therefore identical to the inexact targeting case (Section 3.4) where $\rho = 0$. As we mentioned in Section 3.4, this perverse case leads to instability, but is not an example of nominal GDP targeting. Nevertheless, in the limiting cases where $Q = 0, \infty$ the system is unstable. The remainder of this Section considers the stability of the system for $Q \in (0, \infty)$.

To address this issue we proceed numerically as follows. We allow the parameters α, γ, λ, and Q to vary independently over the closed intervals [0.1, 4], [0.1, 2], [0, 1], and [0.2, 5] in increments 0.1, 0.1, 0.1, and 0.2 respectively. For each of these parameter values we construct the matrices **A**, **B**, and Q. With **R** known we then iterate over (61) until convergence to establish the **P** matrix.[5] The vector **F** is now easily constructed and the stability of the system can be checked by examining the eigenvalues of $[\mathbf{A} - \mathbf{BF}]$. With 40 different values of α, 20 different values for γ, 11 different values for λ, and 25 different values for Q, this process requires solving for 220,000 different **F** vectors. With three eigenvalues associated with each $[\mathbf{A} - \mathbf{BF}]$ matrix we generate 660,000 eigenvalues in total. Each of these eigenvalues were examined and found to be less than one in magnitude. Thus for all 220,000 specifications we consider, nominal GDP growth targeting combined with some degree of interest rate smoothing did not produce instability.

3.6) Conclusions

In this paper we have explored the Ball-Svensson result that nominal GDP targeting can cause economic instability. Following McCallum (1997) we have shown that the stability properties of the system depend on how the supply side of the economy is

[5] To iterate over **P** we express the matrix Riccati difference equation in the following iterative form $\mathbf{P}_{j+1} = \mathbf{R}+\mathbf{A}^T\mathbf{P}_j\mathbf{A}-\mathbf{A}^T\mathbf{P}_j\mathbf{B}[Q+\mathbf{B}^T\mathbf{P}_j\mathbf{B}]^{-1}\mathbf{B}^T\mathbf{P}_j\mathbf{A}$. We begin the iteration with $\mathbf{P}_0 = \mathbf{R}$.

specified. This sensitivity to the supply side of the model parallels the earlier debate between Bean (1983) and West (1986) on conditions under which nominal GDP targeting is optimal.

We began by outlining the Ball-Svensson instability result, relating it to the timing lags with which monetary policy affects output and inflation, and discussed the statistical nature of the instability. Concentrating on the role of inflation expectations in the Phillips curve, we then extended Ball's model to allow for adaptive expectations, rational forward-looking expectations, and partly rational forward-looking and partly backward-looking expectations. For each of these expectations processes we showed that, for plausible parameter values, exact nominal GDP growth targeting does not lead to instability.

Turning from nominal GDP growth targeting to level targeting we showed that while exact level targeting may generate instability with an accelerationist Phillips curve, other expectation formulations result in stability. Specifically, for plausible parameter values, we showed that adaptive expectations and forward-looking expectations lead to stable models under exact nominal GDP level targeting. Moreover, we derived conditions under which a model's stability properties when level targeting would be the same as those under growth targeting, and showed that Ball's model met these conditions. Lastly, we used our results to explain some of the simulations results found in Hall and Mankiw (1994).

In Section 3.4 we explored inexact targeting and found that even if an accelerationist Phillips is an appropriate specification of the short-run aggregate supply curve, that inexact targeting does not generally lead to instability.

Finally, in Section 3.5 we presented simulation evidence indicating that nominal GDP growth targeting together with the simultaneous objective of smoothing interest rates was likely to generate a stable system. This stability arises because the optimal policy reaction function has the interest rate as a linear combination of output and inflation. With the policy objective function restricting the variance of the interest rate to be finite the variances of output and inflation are similarly constrained. Thus ruling out Ball-Svensson type instability.

The over-riding conclusion of this paper is that while the Ball-Svensson instability result is interesting in itself, it appears fragile and does not carry over to more general specifications of the Phillips curve, or policy loss function. The analysis in McCallum (1997) supports this conclusion. We should point out however that the fact that nominal GDP targeting is unlikely to result in instability does not imply that it should be applied in practice. We have not argued that nominal GDP targeting is optimal in any sense, but rather that it is unlikely to be 'disastrous.'

Chapter 4

OPTIMAL INFLATION TARGETING IN A SIMPLE CLOSED ECONOMY MODEL UNDER COMMITMENT AND DISCRETION

4.1) Introduction

Viewed as an application of control theory monetary policy has an easy solution. The monetary authority is given control over one or more instruments and is assigned an objective, or loss function, by the government. With instrument independence, but not goal independence, the monetary authority then minimizes this loss function, subject to the (dynamic) constraints represented by the structure of the economy. From this minimization the monetary authority uncovers the implicit instrument rule it should optimally follow.

But in practice policy makers face several important complications. The macro-models, which constrain the optimization, are often large making the optimization process complicated, and the resulting policy reaction function large and potentially unwieldy. Moreover, imprecisely estimated relationships and uncertainty over the form of economic relationship to be estimated, cast doubt on the appropriateness and robustness of any optimal rule. Finally, the presence of forward-looking expectations in the dynamic constraints introduces the possibility of time-inconsistency, a divergence between the optimal discretionary and optimal commitment rules, potentially leading to an inflation bias (see Kydland and Prescott, 1977 or Barro and Gordon, 1983).

For these reasons, among others, a blossoming literature on monetary policy has turned away from optimal control, exploring instead the relative properties and merits of various simple instrument rules. A short list of such simple instrument rules would include: price level targeting; inflation targeting; inexact nominal GDP level targeting; inexact nominal GDP growth targeting; the Henderson and McKibbin rule (Henderson and McKibbin, 1993); and the popular Taylor rule (Taylor, 1993). One advantage simple rules have over optimal rules is that they are not motivated from the standpoint of any given model, suggesting that they may perform well across a range of models. Furthermore, they are simple to compute, highly transparent, and can potentially be calibrated to loosely replicate historical outcomes.

But in the presence of forward-looking agents simple rules do not overcome the problem of time-inconsistency (Fair and Howrey, 1996). If a simple rule can be calibrated to match historical outcomes, then it can also be calibrated to attain some other goal. In practice, policy simulations with simple rules have simply assumed the existence of some pre-commitment technology.

Despite the weaknesses of optimal policy rules raised above it remains the case that, in the context of any given model, optimal policy rules still provide a valuable benchmark against which the performance of simple rules can be compared. Because analyses of simple rules assume commitment to these rules, comparisons between simple rules and optimal rules should probably take the optimal commitment rule as the baseline, rather than the optimal discretionary rule.

But there are further advantages to analyzing optimal policy rules. By comparing the optimal discretionary rule to the optimal commitment rule the practical effects of not having a commitment technology can be measured. This measurement can take place not just in terms of the variances of variables but also in terms of the feedback coefficients applied to variables in the policy rule. Additionally, one would hope that the variables entering the optimal policy rule would include in a non-trivial way those variables whose movements policy makers use to justify their policy interventions. These 'significant', or important, variables should arguably form the basis of any well performing simple rule.

With the prospect that some, or many, variables may enter the optimal rule in a trivial, or negligible, way, the possibility presents itself that these variables could be excluded from the optimal rule without significantly impairing the rule's performance.[1] By excluding some state variables one may gain some of the simplicity and transparency advantages of simple instrument rules while retaining the good properties of fully optimal rules. Such conditionally optimal rules, as we might call them, may be of particular use to institutions who wish to analyze or predict central bank behavior but who do not have available, or cannot afford, the entire data-set available to the monetary authority.

With these issues in mind, this paper explores two avenues. The first avenue is an analysis and comparison of optimal discretionary policy with optimal policy assuming commitment. Results from these two optimal policies are contrasted with those produced by a Taylor rule and two variants of the Henderson and McKibbin rule. The second avenue is an examination of the feasibility and performance of conditionally optimal rules. This examination takes place under both discretion and commitment.

The paper proceeds as follows. In Section 4.2 we outline a simple descriptive closed economy macroeconomic model of the type previously used by Ball (1999) and Svensson (1997a, 1997b). Using this macroeconomic model Section 4.3 uses both analytical methods and simulations to construct and compare the performance of a Taylor rule and two Henderson and McKibbin rules with optimal commitment and optimal discretionary rules. Variations on the model of Section 4.2 are used to check the robustness of the results, and impulse response functions are presented to illustrate the properties of each model. Section 4.3 also plots and discusses the efficiency frontiers for each model specification.

Section 4.4 introduces the conditionally optimal rule to be analyzed, and compares its structure to a simple inflation targeting rule. Following this introduction, Section 4.4 addresses whether the conditionally optimal rule can stabilize the economy, and when it can asks how effective it is at doing so. As with Section 4.3, the analysis in Section 4.4 takes place under both discretion and commitment. Section 4.5 concludes and discusses further applications of the techniques used in the paper. Appendix A contains proofs and technical details, while Appendix B performs sensitivity analysis to investigate how robust the results are to the exact parameter values employed.

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[1] Cecchetti (1997) argues that '...if the solution to the complex problem can be approximated by a

4.2) A Simple Closed Economy Model

To introduce the ideas and techniques used throughout this paper consider the following simple closed economy macroeconomic model:

$$y_t = \lambda y_{t-1} - \gamma[i_t - \eta E_{t-1}\pi_{t+1} - (1-\eta)\pi_{t-1}] + v_t, \qquad 0 \leq \lambda < 1,\ 0 \leq \eta \leq 1,\ \gamma > 0 \tag{1}$$

$$\pi_t = \delta E_{t-1}\pi_{t+1} + (1-\delta)\pi_{t-1} + \alpha y_{t-1} + u_t. \qquad \alpha > 0,\ 0 \leq \delta \leq 1 \tag{2}$$

All variables have been de-meaned and represent: the output gap, y_t; the nominal interest rate, i_t; and inflation, π_t, respectively. Demand and supply innovations are represented by v_t and u_t, and are assumed to follow independent iid[0, σ^2] processes. Finally E_{t-1} represents the mathematical expectations operator conditional upon period t-1 information. Equation (1) is a dynamic IS relationship expressed in real interest rate/output gap space, while equation (2) represents a short-run aggregate supply curve, or expectations augmented Phillips curve. The nominal interest rate, i_t, is assumed to be the monetary policy instrument. Real interest rate changes effect excess demand during the first period with the flow on effect of demand through to inflation coming after an additional period. In the special case where $\delta = \eta = 0$, this model has been studied previously by Svensson (1997a, 1997b), Ball (1999), and McCallum (1997).[2]

In our numerical work we take λ to equal 0.9, γ to equal 0.8, and α to equal 0.4 – implying a sacrifice ratio[3] of 2.5 ($1/\alpha$) when $\delta = \eta = 0$. The standard deviation of each innovation is set to one. The timing of events is as follows. At the end of period t-1 period t-1 variables are realized. Then, during period t the monetary authority sets the value of the nominal interest rate, i_t. Subsequently, period t shocks, u_t and v_t, occur, and y_t and π_t are realized. With this timing the monetary authority makes its period t policy decision based on period t-1 information and the *ex ante* distributions of the innovations.

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simple rule, there may be substantial virtue in adopting the approximate solution.'

[2] This model differs slightly in the timing with which the real interest rate impacts on demand from that used by the authors referenced above. Here we have the impact occurring contemporaneously while they impose a one period lag.

[3] Here we define the sacrifice ratio in terms of the transition between two steady states. A sacrifice ratio of 2.5 means that if the inflation rate is one percentage point lower in the second steady state than the first then the cumulative output loss between the two steady states will be 2.5%.

We test the robustness of our results by considering three variants of this model. These variants are indexed by their values for δ and η, and are denoted: Model A; Model B; and Model C. Model A is backward-looking with $\delta = \eta = 0$, Model B has rational financial markets, $\eta = 1$, but other markets form naive backward-looking expectations, $\delta = 0$, and Model C is fully forward-looking with $\delta = \eta = 1$.

4.3) Analysis of Results: Efficiency Frontiers and Impulse Responses

The loss function faced by the monetary authority when deriving its optimal policy rule is[4]

$$\mathrm{Loss}[0,\infty] = (1-\beta)\mathrm{Var}(y_t) + \beta\mathrm{Var}(\pi_t)\,, \qquad 0 \le \beta \le 1 \qquad (3)$$

where $\mathrm{Var}(y_t)$ and $\mathrm{Var}(\pi_t)$ are the unconditional variances of the output gap and inflation.[5] Rudebusch and Svensson (1998) and Svensson (1998) show that this loss function can be motivated from the standard intertemporal quadratic loss function as the limiting case where future losses are valued equally with current losses.

Two of the model specifications described above – Models B and C – contain forward-looking variables. For these models the optimal discretionary rule and the optimal commitment rule may differ. Rather than just presenting results for just the discretionary case or just the commitment case, this paper presents results for both cases. This is done for two reasons. First, it is interesting in itself to see how the optimal discretionary and the optimal commitment rules differ in the absence of a discretionary inflation bias.

Second, we wish to compare how simple rules – such as the Taylor rule or a simple inflation targeting rule – compare to optimal rules. If the monetary authority

[4] Svensson (1997a) argues that this loss function is consistent with the motivation behind inflation targeting.

[5] Because all variables in the system have been demeaned this loss function implicitly targets a value for π_t of zero. Because of the demeaning, however, this is without loss of generality. Further, although it is the inflation rate that is targeting by policy makers, the price level is still determined by the system. Each of the model specifications considered are sticky price models and as a result today's price level

announces that it will follow a Taylor rule and agents believe this, then the monetary authority will then face an incentive to depart from the Taylor rule. Because of this incentive, simulations performed using a Taylor rule assume that a commitment mechanism exists holding the central bank to its rule. For this reason simulation results for the Taylor rule should most appropriately be compared to those from the optimal commitment rule. But in general central banks do not commit themselves to predetermined courses of action. As a consequence, the optimal discretionary rule also provides a useful benchmark for comparison.

4.3.1) Model A

Model A contains two state variables (excluding the innovations, u_t and v_t, which are not part of the period t-1 information set): y_{t-1} and π_{t-1}. Accordingly, the optimal policy reaction function may be represented as a linear combination of these two variables, with parameters φ_1, φ_2 respectively. Table 4.1 contains the optimal policy rules as well as the variances of inflation, output and the nominal interest rate for both 'strict inflation targeting' (SIT, $\beta = 1$) and 'flexible inflation targeting' (FIT, $\beta = 0.5$). Comparative results for the Taylor rule[6] and two Henderson and McKibbin[7] rules are also included.

There is no unambiguously superior rule among those in Table 4.1. However, the Taylor rule and the second of the two Henderson and McKibbin rules is strictly dominated by FIT. Interestingly, Table 4.1 illustrates that as we move from FIT to SIT the optimal policy rule places increased rather than reduced weight on output. By placing a larger weight on output SIT is able to dampen future inflationary pressures, but at the cost of greater output fluctuations. Further we see that the Taylor rule places too little weight on output.[8] Also note that the optimal rules apply a coefficient greater than one to inflation. This ensures that monetary policy 'leans against the wind', and does not accommodate inflation increases.

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is anchored by last period's price level. Under these assumptions the price level will follow a random walk.

[6] The Taylor rule considered in this paper is based on the observed variables: y_{t-1}; and π_{t-1} with coefficients of 0.5 and 1.5 respectively.

[7] The two Henderson and McKibbin rules are used here. The first applies the feedback coefficient 1.5 to both y_{t-1} and π_{t-1}, the second the feedback coefficient 2. Feedback coefficients less than one generate instability.

[8] This result has been derived analytically by Ball (1999).

Table 4.1					
Regime	**Var(y_t)**	**Var(π_t)**	**Var(i_t)**	**φ_1 (y_{t-1})**	**φ_2 (π_{t-1})**
FIT	2.42	3.12	7.7	1.535	2.025
SIT	8.25	2.16	26.47	2.375	4.125
Taylor	3.41	4.89	8.96	0.5	1.5
H-M	1.61	5.25	8.37	1.5	1.5
H-M	2.55	3.55	10.33	2	2

Among the simple rules the Henderson and McKibbin rule with a feedback coefficient of 2 dominates the Taylor rule and comes closest to matching the performance of FIT. The Henderson and McKibbin rule with 1.5 as its feedback coefficient performs best in terms of the output gap's variance, but worst in terms of inflation's variance.

To better illustrate the properties of the model, Figures 4.1 and 4.2 plot impulse response functions for independent, temporary, 1% demand and supply shocks respectively. Here we are interpreting v_t, the innovation to the dynamic IS curve, as the demand shock, and u_t, the innovation in the Phillips curve, as the supply shock. These responses are drawn for FIT ($\beta = 0.5$).

From Figure 4.1, in period 10 when the demand shock hits, output increases by the full 1% with no inflation or interest rate response. This increase in demand pressure flows through to inflation in subsequent periods causing inflation to rise and eliciting a tightening in monetary policy. Interest rates rise by more than the rate of inflation and the higher real interest rate reduces demand pressure. Excess capacity results, lowering inflation. Subsequently, the output gap, inflation, and the nominal interest rate all converge geometrically back to baseline. The shock passes through the system after about 10 periods.

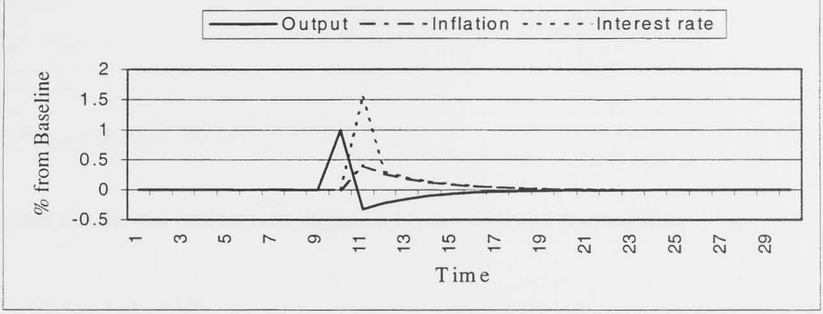


Figure 4.1: Model A - 1% Demand Shock

The supply shock impulse responses are plotted in Figure 4.2. In response to the supply shock inflation increases directly by the full 1%. Higher inflation generates a policy tightening so the nominal interest rate rises the following period, and rises by more than inflation does to generate a real interest rate rise. In turn the higher real interest rate dampens demand pressure and the excess capacity puts downward pressure on the inflation rate, returning inflation to baseline. As the supply shock passes through the system inflation gradually declines and as it does so the output gap and interest rate also return to baseline. Again after 10 periods the shock has almost completely passed through the system

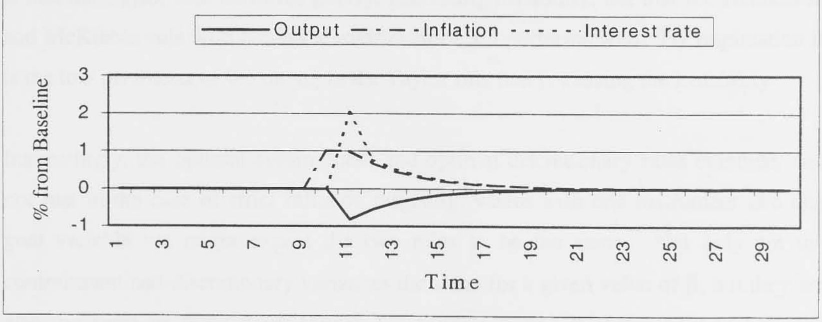


Figure 4.2: Model A - 1% Supply Shock

Finally, we should note that in Model A we arrive at identical policy conclusions regardless of whether we consider the real or nominal interest rates as the policy instrument. The reason for this is that with backward-looking expectations, inflation expectations are a state variable of the system. Hence we can write the optimal rule:

$$i_t = \varphi_1 y_{t-1} + \varphi_2 \pi_{t-1}$$

as

$$r_t + \pi_{t-1} = \varphi_1 y_{t-1} + \varphi_2 \pi_{t-1}, \qquad (4)$$

where r_t is the real interest rate. Equation (4) can easily be rearranged as

$$r_t = \varphi_1 y_{t-1} + (\varphi_2 - 1)\pi_{t-1}.$$

Thus the shift to treating the real rather than the nominal interest rate as the policy instrument only alters the coefficient on lagged inflation in the optimal policy rule.[9]

4.3.2) Model B

In Model B agents operating in financial markets are forward-looking while the remainder are backward-looking. Both y_{t-1} and π_{t-1} remain state variables and form the basis for the policy rule. Table 4.2 presents simulation results for SIT, FIT, the Taylor rule and the two Henderson and McKibbin rules, along with the respective feedback coefficients for each policy rule. The first thing we observe from Table 4.2 is that the Taylor rule performs poorly, generating instability, but that the Henderson and McKibbin rule with feedback coefficient of 1.5 performs well. By implication it is the low parameter of 0.5 on y_{t-1} in the Taylor rule that is causing the instability.

Interestingly, the optimal commitment and optimal discretionary rules coincide, and not just in the case of strict inflation targeting, where with one instrument and one goal variable we might expect the two rules to be the same. Not only are the commitment and discretionary variances the same for a given value of β, but they are also the same as those from Model A. Models A and B differ only in how the dynamic IS curve is specified, thus these results underscore the point that it is

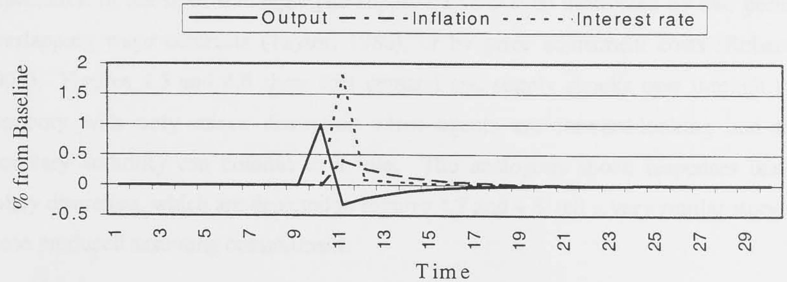
[9] When inflation expectations contain some forward-looking element both coefficients in the optimal policy rule change when we transform the policy rule from the nominal to real interest rate. When $\delta = 1$, π_{t-1} is no longer a state variable for the system.

parameters in the supply side of the system that determine the shape of the efficiency frontier.

Table 4.2						
	Regime	**Var(y_t)**	**Var(π_t)**	**Var(i_t)**	φ_1 (y_{t-1})	φ_2 (π_{t-1})
Discretion	**FIT**	2.42	3.12	6.23	1.804	1.697
	SIT	8.25	2.16	24.58	2.375	3.125
Commitment	**FIT**	2.42	3.12	6.23	1.804	1.697
	SIT	8.25	2.16	24.58	2.375	3.125
	Taylor	∞	∞	∞	0.5	1.5
	H-M	2.29	3.55	5.44	1.5	1.5
	H-M	3.18	2.65	8.23	2	2

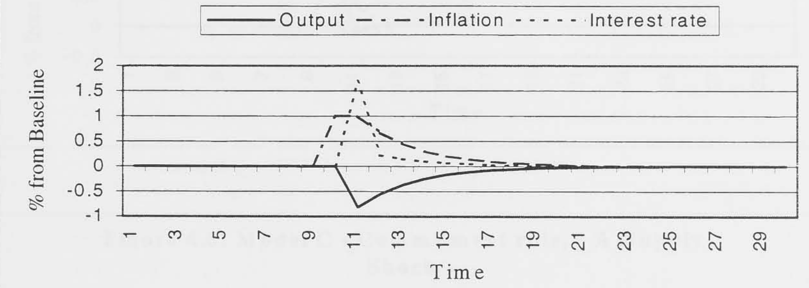
Figures 4.3 and 4.4 plot the dynamic responses of Model B to demand and supply innovations in the case of both policy commitment and policy discretion. With private agents forward looking the decisions of the monetary authority are anticipated, helping policy makers stabilize the economy. With a 1% demand shock (Figure 4.3) output rises by 1% and this positive output gap places upward pressure on inflation. Anticipating higher inflation policy makers tighten policy and agents anticipating this tightening moderate their inflation expectations. The policy tightening leads to an excess of capacity that places downward pressure on inflation. As inflation declines, policy begins to ease and output starts to rise, returning the economy to baseline.

Figure 4.3: Model B - 1% Demand Shock



The 1% supply shock, shown in Figure 4.4, increases inflation by 1%. However, once agents identify the shock the monetary authority lifts the interest rate, contributing to a fall in the output gap, which reduces inflationary pressures. Subsequently, policy eases and inflation and the output return to baseline.

Figure 4.4: Model B - 1% Supply Shock



In many ways the simulation results for Model B are similar to those of Model A. The key difference is that with forward-looking financial markets inflation expectations return to the inflation target faster than lagged inflation does, and this allows the monetary authority to shorten the duration of policy activism.

4.3.3) Model C

In this model all agents are forward-looking and rational, which, as we might expect, greatly helps policy makers achieve their objectives. The forward-looking expectation in the short-run aggregate supply curve can be motivated by two period overlapping wage contracts (Taylor, 1980), or by price adjustment costs (Roberts, 1995). Figures 4.5 and 4.6 show that demand and supply shocks pass through the economy with only minor disruption when agents are forward-looking and the monetary authority can commit to a rule. The analogous shock responses under policy discretion, which are depicted in Figures 4.7 and 4.8, tell a very similar story to those produced assuming commitment.

Figure 4.5: Model C - Commitment rule, 1% Demand Shock

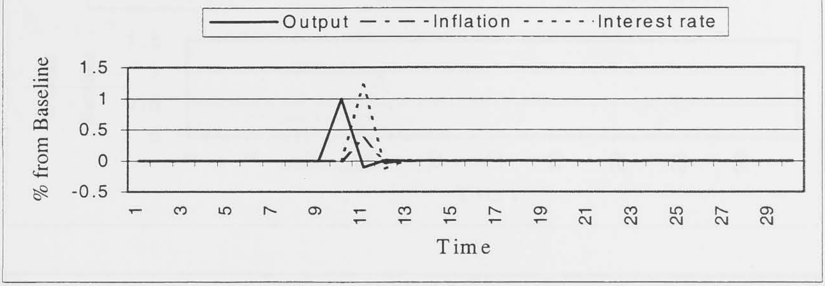


Figure 4.6: Model C - Commitment rule, 1% Supply Shock

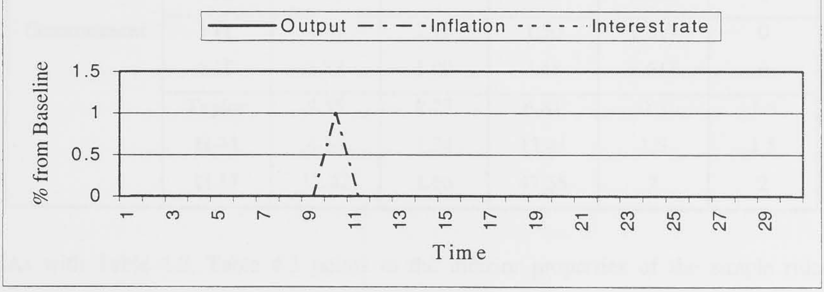


Figure 4.7: Model C - Discretionary rule, 1% Demand Shock

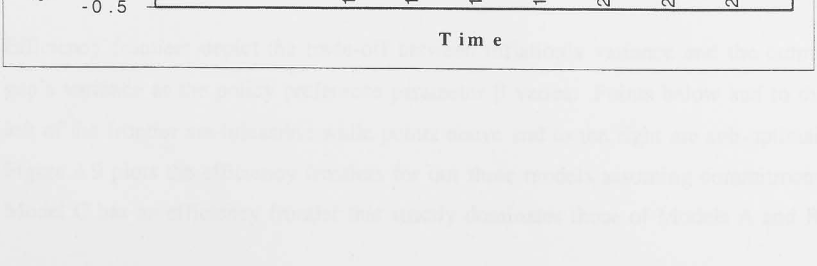


Figure 4.8: Model C - Discretionary rule, 1% Supply Shock

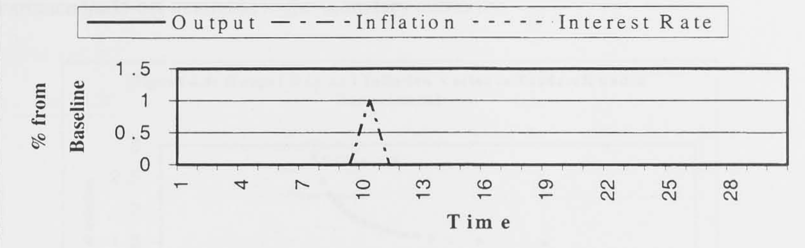


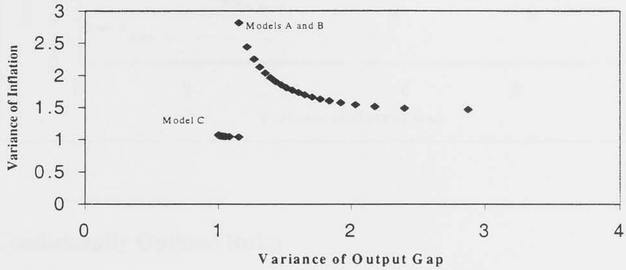
Table 4.3						
	Regime	**Var(y_t)**	**Var(π_t)**	**Var(i_t)**	φ_1 (y_{t-1})	φ_2 (π_{t-1})
Discretion	**FIT**	1.01	1.13	1.52	1.226	0
	SIT	1.62	1.10	4.93	1.745	0
Commitment	**FIT**	1.01	1.13	1.50	1.217	0
	SIT	1.33	1.09	3.48	1.617	0
	Taylor	4.55	1.73	6.81	0.5	1.5
	H-M	4.56	1.34	13.41	1.5	1.5
	H-M	12.42	1.66	47.55	2	2

As with Table 4.2, Table 4.3 points to the inferior properties of the simple rules relative to either FIT or SIT, under either commitment or discretion. Again - as expected - neither FIT nor SIT dominate the other, although with commitment FIT seems to gain a large reduction in output's variance with only a small increase in inflation's variance. This result seems to imply that the variance 'sacrifice ratio' in the economy is large.

4.3.4) Efficiency Frontiers

Efficiency frontiers depict the trade-off between inflation's variance and the output gap's variance as the policy preference parameter β varies. Points below and to the left of the frontier are infeasible while points above and to the right are sub-optimal. Figure 4.9 plots the efficiency frontiers for our three models assuming commitment. Model C has an efficiency frontier that strictly dominates those of Models A and B, due, of course, to the fact that all agents in Model C are forward-looking and rational. The forward-looking expectations help stabilize the economy and improve the variance trade-off available to the monetary authority.

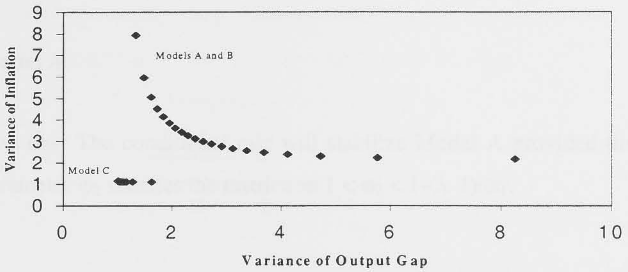
Figure 4.9: Output Gap and Inflation Variance Trade-off under Commitment



As noted earlier the efficiency frontiers for Models A and B coincide. Models A and B differ only in how inflation expectations are formed in the Fisher equation. Consequently, Models A and B differ only in their specifications of the dynamic IS curve. The demand side of the economy does not change what monetary policy can optimally achieve, but it does change the policy rule needed to achieve it. This point can be understood more readily once one appreciates that it is the existence of the supply innovation, u_t, that produces the variance trade-off to begin with (Clarida, Gali, and Gertler, 1999).

Figure 4.10 plots the corresponding efficiency frontiers for the three models under discretion. Again the efficiency frontier for Model C strictly dominates those for Models A and B. As with commitment, under discretion the efficiency frontiers for Models A and B coincide.

Figure 4.10: Output Gap and Inflation Variance Trade-off under Discretion



4.4) Conditionally Optimal Rules

As outlined in the introduction, conditionally optimal rules are rules that optimize the central bank's loss function conditional upon a policy reaction function that excludes some state variables. For a conditionally optimal rule to perform well only variables that have 'small' coefficients in the optimal rule should be excluded. Conditionally optimal rules may be useful because they merge the properties of simple rules with those of optimal rules. Any advantages of conditionally optimal rules are likely to become clearer in larger scale macroeconomic models. It is in these large models that the optimal policy rule becomes truly complicated, and where many variables in the rule may have negligible coefficients. With this point in mind, this Section is intended only to provide a simple illustration of conditionally optimal rules.

The model of Section 4.2, together with the assumption that expectations are formed using period t-1 information, means that the optimal policy rule is a linear combination of the two state variables[10] y_{t-1} and π_{t-1}. Naturally, then, the conditionally optimal rule we consider is of the form:[11]

$$i_t = \varphi_2 \pi_{t-1}.$$

[10] In Model C, which has all agents forward-looking and rational, π_{t-1} is not a state variable for the system.

[11] The other possible conditional rule: $i_t = \varphi_1 y_{t-1}$ is not formally considered because it has the nominal policy instrument responding to a real variable and is therefore likely to lead to indeterminacy (Edey, 1989).

This conditional rule differs from a simple inflation targeting rule only in that the parameter φ_2 is chosen to optimize a loss function and not simply imposed.

4.4.1) Model A

Proposition one: The conditional rule will stabilize Model A provided the feedback parameter φ_2 satisfies the restriction $1 < \varphi_2 < 1-(\lambda-1)/\alpha\gamma$.

Proof:

See Appendix A. ∎

Proposition one is very intuitive. If φ_2 is less than 1, then the nominal interest rate rises less than one-for-one with inflation expectations, so the real interest rate declines. A decline in the real interest rate in turn stimulates future inflation. The process repeats itself ending in instability. Alternatively, if φ_2 is too large then inflation stimulates a real interest rate rise that it too contractionary, generating unstable oscillations.

Minimizing the loss function, (3), subject to the restrictions imposed by the structure of Model A and the conditional rule produces results for SIT and FIT as summarized in Table 4.4.

Table 4.4

Regime	Var(y_t)	Var(π_t)	Var(i_t)	φ_1 (y_{t-1})	φ_2 (π_{t-1})
FIT	10.46	43.88	55.08	0	1.120
SIT	11.88	43.25	56.08	0	1.139
Taylor	3.41	4.89	8.96	0.5	1.5
H-M	1.61	5.25	8.37	1.5	1.5
H-M	2.55	3.55	10.33	2	2

Table 4.4 clearly illustrates that while Model A can be stabilized using a policy rule based solely of lagged inflation such a rule is not very effective, producing large variances in both output and inflation. The simpler rules, which use the optimal

information set, but with sub-optimal parameter values, easily dominate both FIT and SIT derived as conditionally optimal rules. The underlying reason for the poor performance of the conditionally optimal rule in Table 4.4 is the fact that in the optimal rule for Model A the coefficient on y_{t-1} is far from zero.

4.4.2) Model B

Model B has forward-looking inflation expectations in financial markets but backward-looking inflation expectations elsewhere. However, we know from Section 4.3 that for Model B the optimal commitment and optimal discretionary rules coincide.

Proposition two: For Model B the conditional rule will only stabilize the economy in the event that $\alpha\gamma < (1-\lambda)/2$ or $\alpha\gamma > (3+\lambda)/2$.

Proof:

See Appendix A. ∎

The numerical model from which our simulations are drawn assumes $\alpha = 0.4$, $\gamma = 0.8$ and $\lambda = 0.9$. Consequently, for our system neither the condition that $\alpha\gamma < (1-\lambda)/2$ nor $\alpha\gamma > (3+\lambda)/2$ hold so the system cannot be stabilized by a conditionally optimal rule based on lagged inflation.

4.4.3) Model C

In Model C the only period t-1 observable state variable is y_{t-1}, and Table 4.3 clearly shows that the economy can be stabilized with policy based solely on this variable. However, with forward-looking agents the possibility presents itself that the economy can be stabilized using extraneous information, information that is not directly informative of the state of the economy; π_{t-1} is such a variable. Agents will base their expectations on π_{t-1} if they believe policy makers are using this variable to set policy. A correlation between π_{t-1} and both π_t and y_t is then formed that is available for policy makers to exploit. Once agent's inflation expectations depend on π_{t-1}, π_{t-1} effectively becomes a state variable containing information exploitable by policy makers. Note that this policy channel arises as a simple consequence of self-fulfilling behavior.

Proposition three: Assuming commitment, for Model C policy makers can stabilize the economy using the conditional rule provided $0 < \alpha\gamma < 4$.

Proof:

See Appendix A. ∎

Table 4.5 presents simulation results based on the conditionally optimal rule.

Table 4.5						
	Regime	**Var(y_t)**	**Var(π_t)**	**Var(i_t)**	φ_1 (y_{t-1})	φ_2 (π_{t-1})
Discretion	**FIT**	6.74	4.79	3.31	0	0.831
	SIT	6.24	4.15	3.29	0	0.890
Commitment	**FIT**	5.88	2.99	4.29	0	1.199
	SIT	7.39	2.73	7.58	0	1.666
	Taylor	4.55	1.73	6.81	0.5	1.5
	H-M	4.56	1.34	13.41	1.5	1.5
	H-M	12.42	1.66	47.55	2	2

As with Model B, the simple rules (excluding the Henderson and McKibbin rule with feedback parameter 2) dominate the conditionally optimal rule for both FIT and SIT. What is most interesting about Table 4.5 is that under discretion the variances of inflation and the output gap are simultaneously reduced as $\beta \rightarrow 1$. Appointing a central bank governor who cares only about inflation – an 'inflation nutter' is optimal. The inability of the central bank to commit to policy announcements, combined with the fact that lagged inflation is only a useful variable for policy makers if agents believe the rule, leads to an optimal rule where policymakers 'accommodate' inflation increases. As the weight on inflation in the loss function is increased the degree of accommodation is reduced, producing a joint reduction in the variances of inflation and the output gap.

4.5) Conclusions

In some respects the conclusions drawn here are unsurprising. The more agents look to the future the easier it is for policy makers to achieve their policy objectives, the more stable the system is, and the quicker the effects of supply and demand shocks dissipate. However, there are also a number of results that could not so obviously be anticipated:

i) In a dynamic setting with discretionary policy appointing an 'inflation nutter' can be optimal.

ii) In a relative sense, the simple rules perform best where there is no forward-looking expectations. With forward-looking agents SIT and FIT dominate all three simple rules.

iii) Sometimes - depending on parameter values and how much information is discarded - policy makers can stabilize the economy using conditional rules.

iv) When all agents are forward-looking and rational, the monetary authority can stabilize the economy using variables that, if not used by policy makers, would not be informative about the state of the economy.

v) The three simple rules, which apply sub-optimal coefficients to an optimal state variable set, tend to out-perform the conditionally optimal rule we consider, which applies optimal coefficients to a sub-optimal state variable set.

vi) Simple inflation targeting rules perform poorly, particularly in the context of models without forward-looking agents.

While the model of this paper is descriptive and highly simplified it does demonstrate that a conditionally optimal inflation targeting rule can stabilize the economy. The poor performance of the conditionally optimal rule in this paper stems from the fact that with only two state variables in the system a huge amount of information is lost when the output gap is omitted from the policy rule. Equivalently, in the optimal rule the output gap does not have a coefficient close to zero. To better understand whether conditionally optimal rules can make a useful contribution to monetary policy, a larger, more comprehensive, model that accounts for a wider range of shocks might be used. In light of the fact that currently all explicit inflation targeting countries have large tradable goods sectors, future work on conditionally optimal rules could usefully be placed in the context of a small open economy.

Appendix A: Conditionally Optimal Rules

Proof of proposition one: The conditional rule will stabilize Model A provided the feedback parameter φ_2 satisfies the restriction $1 < \varphi_2 < 1+(1-\lambda)/\alpha\gamma$.

Take the system

$$y_t = \lambda y_{t-1} - \gamma[i_t - \pi_{t-1}] + v_t \qquad 0 \le \lambda < 1, \gamma > 0 \qquad \text{(A1)}$$

$$\pi_t = \pi_{t-1} + \alpha y_{t-1} + u_t \qquad \alpha > 0. \qquad \text{(A2)}$$

and add to it the policy reaction function:

$$i_t = \varphi_2 \pi_{t-1}. \qquad \text{(A3)}$$

Eliminating the nominal interest rate from the system, then exploiting the lag operator gives the IS curve

$$y_t = \frac{\gamma(1-\varphi_2)}{1-\lambda L}\pi_{t-1} + \frac{1}{1-\lambda L}v_t. \qquad \text{(A4)}$$

Lagging A4, and substituting into A2, gives the inflation equation $\pi_t = (1+\lambda)\pi_{t-1} + (\alpha\gamma(1-\varphi_2) - \lambda)\pi_{t-2} + \alpha v_{t-1} + u_t - \lambda u_{t-1}$, which will be stable provided

i) $1 + \lambda + \alpha\gamma(1 - \varphi_2) - \lambda < 1$;

ii) $-1 - \lambda + \alpha\gamma(1 - \varphi_2) - \lambda < 1$;

iii) $-\alpha\gamma(1 - \varphi_2) + \lambda < 1$.

Now (i) requires $\varphi_2 > 1$. With $\alpha, \gamma > 0$ and $0 \le \lambda < 1$ condition (ii) always holds when (i) does. Finally, condition (iii) requires $\varphi_2 < 1+(1-\lambda)/\alpha\gamma$. Ruling out $\lambda = 1$, $(1-\lambda)/\alpha\gamma$ is always positive, so there exists a range of values for φ_2, $1 < \varphi_2 < 1+(1-\lambda)/\alpha\gamma$, for which the system is stable. ∎

Proof of proposition two: For Model B the conditional rule will only stabilize the economy in the event that $\alpha\gamma < (1-\lambda)/2$ or $\alpha\gamma > (3+\lambda)/2$.

The model is

$$y_t = \lambda y_{t-1} - \gamma[i_t - E_{t-1}\pi_{t+1}] + v_t \qquad 0 \le \lambda < 1, \gamma > 0 \qquad \text{(A5)}$$

$$\pi_t = \pi_{t-1} + \alpha y_{t-1} + u_t \qquad \alpha > 0. \qquad \text{(A6)}$$

$$i_t = \varphi_2 \pi_{t-1}. \qquad \text{(A7)}$$

Combining A7 with A5 yields

$$y_t = \lambda y_{t-1} - \gamma\varphi_2\pi_{t-1} + \gamma E_{t-1}\pi_{t+1} + v_t \qquad \text{(A8)}$$

To solve this rational expectations system we propose the solution

$$y_t = \theta_{11} y_{t-1} + \theta_{12}\pi_{t-1} + \theta_{13} v_t + \theta_{14} u_t \qquad \text{(A9)}$$

$$\pi_t = \theta_{21} y_{t-1} + \theta_{22}\pi_{t-1} + \theta_{23} v_t + \theta_{24} u_t. \qquad \text{(A10)}$$

Now substituting A9 and A10 into A8 and A6 and equating coefficients gives

$$\theta_{11} = \lambda + \gamma(\theta_{21}\theta_{11} + \theta_{22}\theta_{21}) \qquad \text{(A11)}$$

$$\theta_{12} = -\gamma\varphi_2 + \gamma(\theta_{21}\theta_{12} + \theta_{22}^2) \qquad \text{(A12)}$$

$$\theta_{21} = \alpha \qquad \text{(A13)}$$

$$\theta_{22} = 1. \qquad \text{(A14)}$$

Exploiting A14 and A13 we get[12] $\theta_{11} = \dfrac{\lambda + \alpha\gamma}{1 - \alpha\gamma}$ and $\theta_{12} = \dfrac{\gamma(1-\varphi_2)}{1-\alpha\gamma}$. The system can be expressed as the process $y_t = (1+\theta_{11})y_{t-1} - (\theta_{11} - \alpha\theta_{12})y_{t-2} + \Delta v_t + u_{t-1}$, so stability requires the three conditions:

i) $1 + \alpha\theta_{12} < 1$;

ii) $\alpha\theta_{12} - 2\theta_{11} - 1 < 1$;

iii) $\alpha\theta_{12} - \theta_{11} > -1$.

First consider the case where $\alpha\gamma < 1$. For condition (i) to hold then requires $\varphi_2 > 1$. Condition (ii) holds provided $\varphi_2 > (\alpha\gamma - 2(1+\lambda))/\alpha\gamma$, which is implied by $\varphi_2 > 1$. Finally condition (iii) holds when $\varphi_2 < (1-\lambda-\alpha\gamma)/\alpha\gamma$. For a stabilizing policy rule to exist therefore requires $(1-\lambda-\alpha\gamma)/\alpha\gamma > 1$, which holds when $\alpha\gamma < (1-\lambda)/2$. So in the unlikely case that $\alpha\gamma < (1-\lambda)/2$ a conditionally optimal rule based on π_{t-1} exists that stabilizes the economy.

Alternatively consider the case where $\alpha\gamma > 1$. Condition (i) requires $\varphi_2 < 1$. Similarly, condition (ii) requires $\varphi_2 < (\alpha\gamma - 2(1+\lambda))/\alpha\gamma$, which implies $\varphi_2 < 1$. Finally condition (iii) holds when $\varphi_2 > (1-\alpha\gamma-\lambda)/\alpha\gamma$. Accordingly, stability can be achieved provided $(\alpha\gamma - 2(1+\lambda))/\alpha\gamma > (1-\alpha\gamma-\lambda)/\alpha\gamma$. But this restriction only holds when $\alpha\gamma > (\lambda+3)/2$. Therefore a conditionally optimal rule based on π_{t-1} that stabilizes the economy only exists when $\alpha\gamma < (1-\lambda)/2$ or $\alpha\gamma > (3+\lambda)/2$. ∎

Proof of proposition three: Assuming commitment, for Model C policy makers can stabilize the economy using the conditional rule provided $0 < \alpha\gamma < 4$.

Consider the system

$$y_t = \lambda y_{t-1} - \gamma[i_t - E_{t-1}\pi_{t+1}] + v_t, \qquad 0 \le \lambda < 1, \gamma > 0 \qquad \text{(A15)}$$

$$\pi_t = E_{t-1}\pi_{t+1} + \alpha y_{t-1} + u_t, \qquad \alpha > 0 \qquad \text{(A16)}$$

$$i_t = \varphi_2 \pi_{t-1}. \qquad \text{(A17)}$$

We eliminate the nominal interest rate from the system by substituting A17 into A15 giving

$$y_t = \lambda y_{t-1} - \gamma\varphi_2\pi_{t-1} + \gamma E_{t-1}\pi_{t+1} + v_t. \qquad \text{(A18)}$$

[12] The case where $\alpha\gamma = 1$ leads directly to instability.

The postulated solution is

$$y_t = \theta_{11} y_{t-1} + \theta_{12}\pi_{t-1} + \theta_{13} v_t + \theta_{14} u_t, \tag{A19}$$

$$\pi_t = \theta_{21} y_{t-1} + \theta_{22}\pi_{t-1} + \theta_{23} v_t + \theta_{24} u_t. \tag{A20}$$

Substituting A20 and A19 into A18 and A16 and equating coefficients produces

$$\theta_{11} = \lambda + \gamma(\theta_{21}\theta_{11} + \theta_{22}\theta_{21}) \tag{A21}$$

$$\theta_{12} = -\gamma\varphi_2 + \gamma(\theta_{21}\theta_{12} + \theta_{22}^2) \tag{A22}$$

$$\theta_{21} = \alpha + \theta_{21}\theta_{11} + \theta_{22}\theta_{21} \tag{A23}$$

$$\theta_{22} = \theta_{21}\theta_{12} + \theta_{22}^2 \tag{A24}$$

From A23 and A21 we deduce $\theta_{11} = \lambda + \gamma(\theta_{21} - \alpha)$, and similarly from A24 and A22, $\theta_{12} = -\gamma\varphi_2 + \gamma\theta_{22}$. Substituting these relationships back into A23 and A24 produces the two quadratic equations $\theta_{22}^2 - (1 - \gamma\theta_{21})\theta_{22} - \gamma\varphi_2\theta_{21} = 0$, $\gamma\theta_{21}^2 + (\lambda - \alpha\gamma - 1 + \theta_{22})\theta_{21} + \alpha = 0$, which can be solved as

$$\theta_{22} = \frac{(1-\gamma\theta_{21})}{2} \pm \frac{\sqrt{(\theta_{21}\gamma - 1)^2 + 4\varphi_2\theta_{21}\gamma}}{2} \tag{A25}$$

and

$$\theta_{21} = \frac{(1+\alpha\gamma - \lambda - \theta_{22})}{2\gamma} \pm \frac{\sqrt{(\lambda + \theta_{22} - \alpha\gamma - 1)^2 - 4\alpha\gamma}}{2\gamma}. \tag{A26}$$

Clearly A25, A26, together with A21 and A22 do not represent a unique solution. However, we know that when $\varphi_2 = 0$, then $\theta_{22} = \theta_{12} = 0$, suggesting that we take the positive root of A25. Furthermore, when $\alpha = \lambda = 0$ we expect $\theta_{11} = \theta_{21} = 0$ implying that the positive root of A26 is the appropriate one. Ignoring the innovation terms our system can be written as

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \end{bmatrix}. \qquad \text{(A27)}$$

For our system to be stable we require the eigenvalues of the coefficient matrix in A27 to both have a modulus less than one. The two eigenvalues are:

$$\mu = \frac{(\theta_{11} + \theta_{22})}{2} \pm \frac{\sqrt{(\theta_{11} + \theta_{22})^2 - 4(\theta_{12}\theta_{21} + \theta_{11}\theta_{22})}}{2}. \qquad \text{(A28)}$$

Now consider $\varphi_2 = 0$. Under this restriction we know that $\theta_{22} = \theta_{12} = 0$ and, further from A28, that $\mu = 0, \theta_{11}$. Thus we only need to show that $|\theta_{11}| < 1$ to prove that the system can be stabilized when $\varphi_2 = 0$. Under these restrictions

$$\theta_{11} = \frac{(1 + \lambda - \alpha\gamma)}{2} + \frac{\sqrt{(\lambda - \alpha\gamma - 1)^2 - 4\alpha\gamma}}{2}.$$

First consider the case where $(\lambda\text{-}\alpha\gamma\text{-}1)^2 < 4\alpha\gamma$, which is where θ_{11} is complex. The modulus of θ_{11} is

$$\sqrt{\left(\frac{(\lambda - \alpha\gamma + 1)}{2}\right)^2 + \left(\frac{\sqrt{4\alpha\gamma - (\lambda - \alpha\gamma - 1)^2}}{2}\right)^2}. \qquad \text{(A29)}$$

After expanding, canceling terms, and simplifying, A29 reduces to $\sqrt{\lambda}$. However we have restricted $0 \leq \lambda < 1$, and hence θ_{11} always has a modulus less than one. The condition $(\lambda\text{-}\alpha\gamma\text{-}1)^2 < 4\alpha\gamma$ requires $1+\alpha\gamma-2\sqrt{\alpha\gamma} < \lambda < 1+\alpha\gamma+2\sqrt{\alpha\gamma}$. But with $\alpha, \gamma > 0$ and $\lambda < 1$ the upper constraint is non-binding. Hence provided λ lies in the parameter range $1+\alpha\gamma-2\sqrt{\alpha\gamma} < \lambda < 1$ the system will be stable. For this condition to have any chance of holding we require $0 < \alpha\gamma < 4$.

Next consider the case where $(\lambda\text{-}\alpha\gamma\text{-}1)^2 = 4\alpha\gamma$, i.e. when $\lambda = 1+\alpha\gamma-2\sqrt{\alpha\gamma}$, then θ_{11} will have a modulus less than one provided $0 < \alpha\gamma < 4$. Finally consider the case where $(\lambda\text{-}\alpha\gamma\text{-}1)^2 > 4\alpha\gamma$, that is where $\lambda < 1+ \alpha\gamma-2\sqrt{\alpha\gamma}$. Now when $0 < \alpha\gamma < 4$, $\theta_{11} < 1$.

Furthermore, the function $1-\sqrt{\alpha\gamma}$ is everywhere below θ_{11} over the interval $0 < \alpha\gamma < 4$. To show this we subtract $1-\sqrt{\alpha\gamma}$ from θ_{11} giving

$$\frac{\sqrt{(\lambda-\alpha\gamma-1)^2-4\alpha\gamma}}{2}+\frac{\lambda-(1+\alpha\gamma-2\sqrt{\alpha\gamma})}{2}. \tag{A30}$$

But we know that $\lambda -(1+\alpha\gamma-2\sqrt{\alpha\gamma})$, which we denote by φ, is negative. Adding and subtracting $2\sqrt{\alpha\gamma}$ in the squared part of A30's first term produces $\frac{\sqrt{\varphi^2-4\varphi\sqrt{\alpha\gamma}}}{2}+\frac{\varphi}{2}$. That $1-\sqrt{\alpha\gamma}$ lies below θ_{11} for $0 < \alpha\gamma < 4$ now follows directly from the fact that $\varphi < 0$. The function $1-\sqrt{\alpha\gamma}$, however, lies strictly between -1 and 1 when $0 < \alpha\gamma < 4$, which completes the proof. ∎

Comment: Our method of proving *Result three* has been to show that when $0 < \alpha\gamma < 4$, setting $\varphi_2 = 0$ produces a stable system. If $\varphi_2 = 0$ is the only parameter value producing a stable system, then $\varphi_2 = 0$ is obviously the optimal policy. More generally a range of values for φ_2 - about 0 - will stabilize the economy and the policy maker will choose that value that minimizes the policy loss function. If, however, the above condition, $0 < \alpha\gamma < 4$, does not hold, then this proof does not rule out that the system can be stabilized. It does, however, say that the range of values for φ_2 that might stabilize the economy under these circumstances does not include $\varphi_2 = 0$.

Appendix B: Sensitivity Analysis

The results of the main text were generated for specific parameter values. Of course in practice we are never sure of what the exact parameter values in any given economy are. In the absence of analytical solutions for the unconditional variances of y_t, π_t, and i_t, the robustness of the results can be checked using sensitivity analysis. In essence sensitivity analysis amounts to changing the parameters in the model and seeing how the variances respond.

Some observations about the robustness of the simulation results can be made, however, without resorting to further simulations. In particular, it is always optimal for the monetary authority to offset demand pressures coming through the output gap, regardless of the policy preference parameter, β. Demand pressure represents future inflationary pressure. Thus by stamping down on demand shocks the monetary authority can simultaneously reduce the variances of both the output gap and inflation, which is always optimal. Consequently, changes to the parameters in the demand side of the economy do not shift the efficiency frontier. Instead, such parameter changes, alter the policy rule required to reach any point on the efficiency frontier. The variances of y_t and π_t (but not i_t) are therefore invariant to the parameters λ and γ (excluding $\gamma = 0$). The same, however, cannot be said for parameter changes in the Phillips curve, because they do move the efficiency frontier.

Consider the macroeconomic model given by:

$$y_t = \lambda y_{t-1} + (1-\lambda)E_{t-1}y_{t+1} - \gamma[i_t - E_{t-1}\pi_{t+1}] + v_t \tag{B1}$$

$$\pi_t = E_{t-1}\pi_{t+1} + \alpha y_{t-1} + u_t. \tag{B2}$$

In the case where $\lambda = 1$ the dynamic IS curve (B1) is the same as that used for Model C. Alternatively, when $\lambda = 0$, it is the same as that used by Clarida, Gali and Gertler (1999) and McCallum (1997), and can be supported by utility maximizing agents. The Phillips curve, equation (B2), is also that used in Model C, and it can be derived from a firm optimizing in the face of price adjustment costs (Roberts, 1995). Under policy commitment this system will produce variances for y_t and π_t identical to those of Model C – independently of the values for λ and γ (excluding $\gamma = 0$). Similarly, replacing $E_{t-1}\pi_{t+1}$ in equation (B2) with π_{t-1} and setting $\alpha = 0.4$ and assuming commitment, the system above will produce variances for y_t and π_t identical to those of Models A and B.

By earlier considering two processes for inflation expectations (fully backward and fully forward/rational) the robustness of the results to that aspect of the Phillips curve has already been considered. It just remains to examine how/whether the results change as α varies. Figure B4.1 plots how the value of the minimized loss function changes as α varies between 0 and 1. Models B and C are considered with $\beta = 0.5$ under the assumption of policy commitment. The parameters λ an γ are kept at 0.9 and 0.8 respectively.

Figure B4.1 - Sensitivity of loss function to Alpha, assuming Commitment

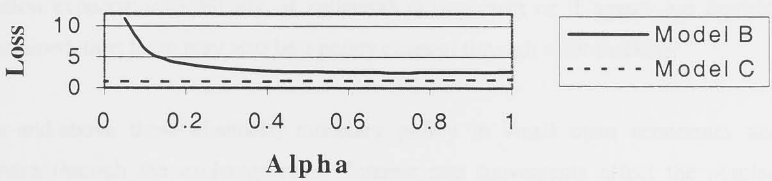


Figure B4.1 illustrates that for Model C the results are very insensitive to the value of α, with the evaluated loss remaining pretty constant for all values of α between 0 and 1. For Model B the evaluated loss is relatively insensitive to values of α greater than 0.2.

Chapter 5

OPTIMAL AND CONDITIONALLY OPTIMAL TARGETING RULES FOR SMALL OPEN ECONOMIES

5.1) Introduction

In closed economy macroeconomic models there are three key monetary policy transmission mechanisms. The first of these is the demand pressure channel: output above capacity places upward pressure on inflation as workers demand higher wages in compensation for the increased working hours. The second channel is through inflation expectations. If people expect higher inflation in the future then inflation will rise now as workers negotiate wage contracts that insure them against future expected price rises. Monetary policy credibility plays an important role in anchoring inflation expectations. Finally, if collateral is important or if agents are liquidity constrained, then there may also be a policy channel through asset markets.

Over-and-above these channels, monetary policy in small open economies also operates through the exchange rate. Interest rate movements affect the nominal exchange rate, and the exchange rate in turn influences tradable goods prices, which form a component of consumer prices. In theory, the quickest acting of these channels should be the expectations channel. In practice the expectations channel appears less important for lowering inflation than it does for maintaining a low rate of inflation. That is the inflation expectations channel provides a useful long-run anchor for inflation but is less effective as a short-run anchor.

It is a stylized fact however, that monetary policy's effect on inflation through the exchange rate channel is swifter acting than its effect through aggregate demand.[1] As a direct consequence, the policy control lag in a small open economy is shorter than that of a closed economy. A corollary of this shorter control lag is that in a small open economy the effects of policy decisions can be more closely associated with the

[1] See Ball (1998) and Reserve Bank of New Zealand (1996).

decision maker. This may have contributed to the fact that the recent shift toward inflation targeting has been concentrated among central banks in small open economies. It is these central banks that can be more effectively held accountable for their actions.

When setting monetary policy to meet its objectives, the central bank must take all of these channels into account. The task of satisfying these objectives can be viewed as an exercise in control theory. The monetary authority sets its instrument (typically a short-term nominal interest rate) to minimize the stipulated objective function subject to dynamic constraints imposed by the structure of the economy.

Yet a large part of the literature on monetary policy concerns itself with analyzing the relative merits and advantages of different simple instrument rules[2] at meeting various policy objectives.[3] The aim of this literature is to find a simple transparent instrument rule that performs well across a range of plausible models, but which is not necessarily optimal for any given model. A rule that is optimal in some given model may perform poorly – perhaps even generating instability – in the context of another economic framework. In the face of uncertainty over which model best reflects reality, robustness - not optimality - is the over-riding criteria.

Nevertheless, the techniques of control theory can still make a useful contribution to the analysis of monetary policy and policy rules. In the context of any given model the optimal rule provides a benchmark, or lower bound, against which the performance of other simple rules can be compared. Moreover, as policy makers form stronger views about the structure and interactions of their economy the set of models over which a rule needs to be robust declines and the importance of optimality increases. And it remains the case that if simple rules are to be useful they must incorporate or be based around information that features significantly in the optimal rule.

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[2] Such simple rules include inexact nominal GDP targeting (McCallum, 1989a, and Bryant, Hooper and Mann, 1993), the Taylor rule (Taylor, 1993), the Henderson and McKibbin rule (Henderson and McKibbin, 1993). Further examples include partial adjustment rules involving deviations of inflation from target or deviations of the price level from target (see for example Edey, 1997).

[3] Examples include targeting linear combinations of output's variance, inflation's variance, and the price level's variance, along with associated special cases (such as inflation targeting and price level targeting).

It is on this latter point that this paper makes its main contribution. In the context of a small open economy we ask the question which variables are important in the optimal policy rule (given, of course, some pre-specified objective). Alternatively, we ask the question how much is lost if some variables are excluded from the optimal policy rule. These rules, which are based on a restricted set of state variables, are called conditionally optimal rules, and they will perform well provided the information retained in the conditionally optimal rule adequately summarize the current state of the system. It is possible that some simple rules (Taylor rule, Henderson and McKibbin rule, etc) can be reinterpreted as conditionally optimal rules. That is some simple rules may be sub-optimal in the broader context, but optimal conditional upon a given information set. Knowing which variables are important for the performance of the optimal rule should help in the design of simple instrument rules.

Rudebusch and Svensson (1998) and Chapter 4 have previously examined conditionally optimal rules in the context of closed economy models. In these studies conditionally optimal rules performed badly, generally being dominated by simple rules such as the Taylor rule. The poor performance of conditionally optimal rules in these closed economy environments was due to the fact that the models had few state variables to begin with. Too much information was lost when some state variables were discarded. Essentially, the optimal policy rules in those models had no state variables with coefficients close to zero.

This paper is structured as follows. Section 5.2 introduces the model, which in many ways can be viewed as a discrete time version of the Buiter and Miller (1981) model. Buiter and Miller (1981) is in turn an extension of Dornbusch's (1976) seminal sticky price exchange rate model. Section 5.3 considers the stabilizing properties of optimal inflation targeting rules for two model specifications. The first where all agents are backward-looking; the second where all agents are to some extent forward-looking and rational. Impulse response functions are presented to better analyze the effects shocks have on the system.

Conditionally optimal rules are discussed and analyzed in Section 5.4, where it is found that rules based on only two or three particular state variables can well approximate the optimal rule. Monetary Conditions Indicators (MCIs), indicators that supposedly summarize the stance of monetary policy in small open economies, are introduced in Section 5.5 and discussed as a tool for aiding policy makers. Finally, Section 5.6 offers concluding comments and directions for future research. Appendix A analyses the sensitivity of the results of Section 5.3 to different parameterizations of the model.

5.2) A Small Open Economy Framework

This Section introduces a small open economy model. The model considered has its origins in Dornbusch (1976) and is developed further in Buiter and Miller (1981). Its structure is as follows:

5.2.1) Domestic Economy[4]

$$y_t = \lambda y_{t-1} - \gamma[i_t - \pi_t^{e,c}] + \mu q_{t-1} + \theta y_{t-1}^f + v_t \qquad \gamma, \mu, \theta > 0,\ 0 \le \lambda < 1 \qquad (1)$$

$$\pi_t^{e,c} = \delta E_{t-1}\pi_{t+1}^c + (1-\delta)\pi_{t-1}^c \qquad 0 \le \delta \le 1 \qquad (2)$$

$$\pi_t = \rho E_{t-1}\pi_{t+1} + (1-\rho)\pi_{t-1} + \alpha y_{t-1} + u_t \qquad \alpha > 0,\ 0 \le \rho \le 1 \qquad (3)$$

$$\pi_t^c = \kappa\pi_t^i + (1-\kappa)\pi_t \qquad 0 \le \kappa \le 1 \qquad (4)$$

$$p_t^i = e_t + p_t^f \qquad (5)$$

$$e_t = \delta E_{t-1}e_{t+1} + (1-\delta)e_{t-1} + i_t^f - i_t + \varepsilon_t \qquad (6)$$

$$q_t = e_t + p_t^f - p_t^c \qquad (7)$$

Where:

y_t	= domestic output gap
π_t	= nontradables inflation rate
π_t^c	= consumer price inflation rate
$\pi_t^{e,c}$	= expected consumer price inflation rate
π_t^i	= import price inflation (in domestic dollars)
p_t^i	= import price level
p_t^c	= consumer price level
p_t^f	= foreign price level

i_t	= domestic nominal interest rate
i_t^f	= foreign nominal interest rate
q_t	= real exchange rate
y_t^f	= foreign output gap
e_t	= effective nominal exchange rate
v_t	= domestic demand shock
u_t	= domestic supply shock
ε_t	= portfolio preference shock.

5.2.2) Foreign Economy

$$y_t^f = \lambda^f y_{t-1}^f - \gamma^f [i_t^f - \pi_t^{e,f}] + v_t^f \qquad \gamma^f > 0, 0 \le \lambda^f < 1 \qquad (8)$$

$$\pi_t^{e,f} = \tau E_{t-1} \pi_{t+1}^f + (1-\tau)\pi_{t-1}^f \qquad 0 \le \tau \le 1 \qquad (9)$$

$$\pi_t^f = \omega E_{t-1} \pi_{t+1}^f + (1-\omega)\pi_{t-1} + \alpha^f y_{t-1}^f + u_t^f \qquad \alpha^f > 0, 0 \le \omega \le 1 \qquad (10)$$

$$i_t^f = \varphi_1^f y_{t-1}^f + \varphi_2^f \pi_{t-1}^f \qquad (11)$$

Where:

π_t^f	= foreign inflation rate
$\pi_t^{e,f}$	= foreign expected inflation rate
v_t^f	= foreign demand shock
u_t^f	= foreign supply shock.

Aside from the interest rates and the inflation rates, all variables are in logs, and all price levels are linked to inflation rates using the standard identity.

Equation (1) is a dynamic IS curve expressed in interest rate/output gap space, but where the curve is conditioned upon the real exchange rate, q_{t-1}, and the foreign output gap, y^f_{t-1}. Expectations of consumer price inflation are formed using equation (2). Nontradables inflation is driven by a Phillips curve, equation (3). Consumer price inflation, equation (4), is a weighted average of tradable goods price inflation and nontradables inflation, where the weight on tradable goods inflation is the direct

[4] Throughout this paper the domestic economy will be associated with the small open economy and the foreign economy with the large economy.

exchange rate pass-through coefficient.[5] Equation (5) defines import prices in term of foreign prices and the nominal exchange rate. The nominal exchange rate, e_t, represents the number of domestic dollars it takes to purchase one foreign dollar. An increase in e_t represents a nominal exchange rate depreciation.

We model the nominal exchange rate using (6), which is simply an uncovered interest parity (UIP) condition, but with mixed expectations as per equation (2). This UIP condition simply states that the expected change in the exchange rate fully offsets the foreign-domestic nominal interest rate differential. The real exchange rate is defined by equation (7) in terms of consumer prices rather than the nontradable output price. However, these two measures of the real exchange rate are related. If we denote q_t^d as the real exchange rate in terms of domestic goods prices, then $q_t = (1-\kappa)q_t^d$. Consequently, either real exchange rate measure can be used in practice. Throughout we define our real exchange rate in terms of consumer prices and to keep things consistent we also define our real interest rate in terms of consumer price inflation. The inflation expectation present in the Phillips curve relates to nontradable goods prices.[6]

The foreign sector of the system we model is of the same genre used by Svensson (1997a) and Ball (1999). In addition we have an explicit policy reaction function, equation (11).

To reduce the dimensions of our system we exploit the real exchange rate definition and write the UIP equation in real terms.

$$q_t = q_{t+1}^e + [i_t^f - \pi_{t+1}^{e,f}] - [i_t - \pi_{t+1}^{e,c}] + \varepsilon_t. \tag{12}$$

Substituting (7) into (5) gives

$$p_t^i = q_t + p_t^c, \tag{13}$$

[5] This direct exchange rate pass-through coefficient is a partial elasticity. In the long-run a 1% permanent depreciation of the nominal exchange rate will cause both domestic prices and consumer prices to rise by 1%. Thus the long-run exchange rate pass-through coefficient is one, not κ.

[6] Svensson (1998) in his study of open economy inflation targeting takes the alternative approach of defining the real exchange using domestic goods prices and then also using expected domestic goods inflation when defining the real interest rate.

which when differenced produces

$$\pi_t^i = \Delta q_t + \pi_t^c . \tag{14}$$

Import price inflation, from equation (14), can now be substituted into the definition of consumer price inflation, equation (4), resulting in

$$\pi_t^c = \frac{\kappa}{1-\kappa}\Delta q_t + \pi_t . \tag{15}$$

This simplified model, which excludes the levels of all nominal variables, now contains just seven endogenous variables, of which only five are stochastic endogenous. The system is:

$$y_t = \lambda y_{t-1} - \gamma[i_t - \delta E_{t-1}\pi_{t+1}^c - (1-\delta)\pi_{t-1}^c] + \mu q_{t-1} + \theta y_{t-1}^f + v_t \tag{16}$$

$$\pi_t = \rho E_{t-1}\pi_{t+1} + (1-\rho)\pi_{t-1} + \alpha y_{t-1} + u_t \tag{17}$$

$$\pi_t^c = \frac{\kappa}{1-\kappa}\Delta q_t + \pi_t \tag{18}$$

$$q_t = \delta E_{t-1}q_{t+1} + (1-\delta)q_{t-1} + [i_t^f - \tau E_{t-1}\pi_{t+1}^f - (1-\tau)\pi_{t-1}^f] - [i_t - \delta E_{t-1}\pi_{t+1}^c - (1-\delta)\pi_{t-1}^c] + \varepsilon_t \tag{19}$$

$$y_t^f = \lambda^f y_{t-1}^f - \gamma^f[i_t^f - \tau E_{t-1}\pi_{t+1}^f - (1-\tau)\pi_{t-1}^f] + v_t^f \tag{20}$$

$$\pi_t^f = \omega E_{t-1}\pi_{t+1}^f + (1-\omega)\pi_{t-1}^f + \alpha^f y_{t-1}^f + u_t^f \tag{21}$$

$$i_t^f = \varphi_1^f y_{t-1}^f + \varphi_2^f \pi_{t-1}^f \tag{22}$$

Events occur in this model as follows. At the end of period t-1 all t-1 variables are realized. Then, during period t, policy makers set the level of their instrument and expectations are formed. After policy is set and expectations are formed shocks occur and period t variables are realized. With this timing of events and lag structure the model is best thought of as an annual one with policy set and expectations formed on the basis of period t-1 information and the *ex ante* distributions of the shocks.

The two model specifications considered in this paper, which vary in how expectations are formed, have not been estimated or formally calibrated using any economy's data. An advantage of this is that the results produced are not specific to any one country. Following Svensson (1998) the aim has been to set values for α, γ, δ, etc that are not obviously at odds with those one might expect to find if the model were estimated. In Appendix A, sensitivity analysis is performed to examine the robustness of the results to different parameter settings.

Considering the foreign economy first we set $\lambda^f = 0.9$ and $\gamma^f = 0.8$ in the dynamic IS curve. In the Phillips curve we set $\alpha^f = 0.4$. The parameters φ^f_1 and φ^f_2 are chosen using an optimization procedure. The foreign central bank sets φ^f_1 and φ^f_2 to minimize the loss function:[7]

$$\text{Loss}^f[0,\infty] = (1-\beta^f)\text{Var}(y^f_t) + \beta^f \text{Var}(\pi^f_t) \tag{23}$$

with $\beta^f = 0.5$, that is with equal weight placed on the unconditional variances of output and inflation.

The parameters in the domestic economy are set in symmetry with those of the foreign economy[8]. That is $\alpha = 0.4$ in the Phillips curve, $\lambda = 0.9$, and $\gamma = 0.8$ in the dynamic IS curve. To complete the IS curve specification we set $\mu = 0.4$, and $\theta = 0.1$. This value for μ implies that the IS curve has a Monetary Conditions Ratio (MCR) of 2. A typical estimate of the MCR for a small open economy[9] is between 1.5 and 3.5. Our ratio is comfortably within this range. The direct exchange rate pass-through coefficient is set equal to 0.3, $\kappa = 0.3$. We complete the stochastic specification of the system by setting $\sigma_u = \sigma_v = \sigma_\varepsilon = \sigma_u{}^f = \sigma_v{}^f = 1$, and all covariance terms to zero. With only five stochastic endogenous variables (and two identities) the covariance matrix for the random disturbances is singular with rank five.

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[7] This loss function, as well as that presented later for the domestic monetary authority, is in terms of the variances of variables, which assumes that variable averages are being targeted. Ball (1999), Fair and Howrey (1996), and Rudebusch and Svensson (1998) also take this approach, which leaves the question of what the optimal levels to target is unanswered (see Pagan, 1997).

[8] An advantage of assuming symmetry here is that it makes clear that the differing results between the domestic and foreign sectors are due to the small open economy nature of the domestic economy and not due to other forms of asymmetry brought about by coefficient differences.

[9] See Duguay (1994), Gerlach and Smets (1996), and Eika, Ericsson, and Nymoen (1996) for typical MCR estimates for small open economies.

Two model specifications are considered in this paper, each specification differing in how expectations are formed. The first model – Model A – has all expectations backward-looking. That is $\delta = \rho = \tau = \omega = 0$ and all agents simply take last period's value of a variable as their expectation of next period's value. The second model – Model B – introduces rational forward-looking behavior. Agents operating in financial markets are taken to be fully forward-looking and rational ($\delta = \tau = 1$) while agents operating in factor markets form mixed expectations ($\rho = \omega = 0.5$).

5.3) Optimal Policy Rules

Consider the loss, or objective, function

$$\text{Loss}[0,\infty] = (1-\beta)\text{Var}(y_t) + \beta\text{Var}(\pi_t^*) \qquad (24)$$

where because policy makers can target either consumer price inflation or nontradables inflation π_t^* may represent either π_t or π_t^c. The parameter β describes the preferences of the monetary authority and it dictates the propensity for policy makers to either 'lean against the wind' or accommodate supply shocks. When $\beta = 1$ the policy regime is referred to as Strict Inflation Targeting (SIT); when $\beta = 0.5$ the regime is called Flexible Inflation Targeting (FIT).[10] For a given value of β the domestic monetary authority minimizes equation (24) subject to the dynamic constraints provided by the structure of the economy – either Model A or Model B. The method used to solve for these optimal policy rules is described in Chapter 2.

5.3.1) Model A

Recall that in Model A all agents are assumed to be naïve and backward-looking; the parameters δ, ρ, τ, and ω are all set to zero. The observed state variables upon which the monetary authority bases its feedback policy rule are: y_{t-1}; π_{t-1}; π^c_{t-1}; q_{t-1}; y^f_{t-1}; and π^f_{t-1}. Recall also that the foreign policy reaction function is set by minimizing equation (23) with $\beta^f = 0.5$. Table 5.1 presents summary simulation results in the form of unconditional variances produced by optimal policy rules under a selection of policy regimes. The respective optimal policy rules themselves are presented in Table 5.2.

Table 5.1: Unconditional Variances for Model A								
Regime	Var(y_t)	Var(π_t)	Var(π_t^c)	Var(q_t)	Var(i_t)	Var(y_t^f)	Var(π_t^f)	Var(i_t^f)
FIT(π_t)	2.42	3.12	4.51	9.15	12.04	2.42	3.12	7.70
SIT(π_t)	8.25	2.16	7.12	22.56	63.90	2.42	3.12	7.70
FIT(π_t^c)	2.52	3.21	4.15	8.55	8.02	2.42	3.12	7.70
SIT(π_t^c)	8.80	3.29	3.02	18.31	11.67	2.42	3.12	7.70

Table 5.1 shows how the variances of the eight variables in the system (including the domestic nominal interest rate) are affected by the particular policy reaction function used by policy makers. Unsurprisingly, SIT - based on either nontradables inflation or consumer price inflation - leads to greater variances for output and the real exchange rate than FIT. For each of the values of β considered targeting π_t^c rather than π_t leads to a lower variance for the real exchange rate and the domestic nominal interest rate. Intuitively this result occurs because dampening that variance of consumer price inflation requires dampening the variance of tradable goods inflation. A volatile nominal interest rate generates a volatile nominal exchange rate, which given the sluggish price adjustment present in the model translates into increased variances for the real exchange rate and tradable goods prices.

Table 5.2: Optimal Policy Rules for Model A						
Regime	y_{t-1}	π_{t-1}	π^c_{t-1}	q_{t-1}	y^f_{t-1}	π^f_{t-1}
FIT(π_t)	1.535	1.025	1.000	0.500	0.125	0.000
SIT(π_t)	2.375	3.125	1.000	0.500	0.125	0.000
FIT(π_t^c)	1.307	0.883	1.000	0.433	0.347	0.173
SIT(π_t^c)	1.206	1.565	1.000	0.293	0.822	0.530

In Table 5.2 we present the policy reaction functions associated with each of the policy regimes considered in Table 5.1. Rows one and two reveal that when domestic

[10] This terminology follows Svensson (1998). There is, however, no obvious reason why Flexible Inflation Targeting could not alternatively be called Flexible Output Targeting.

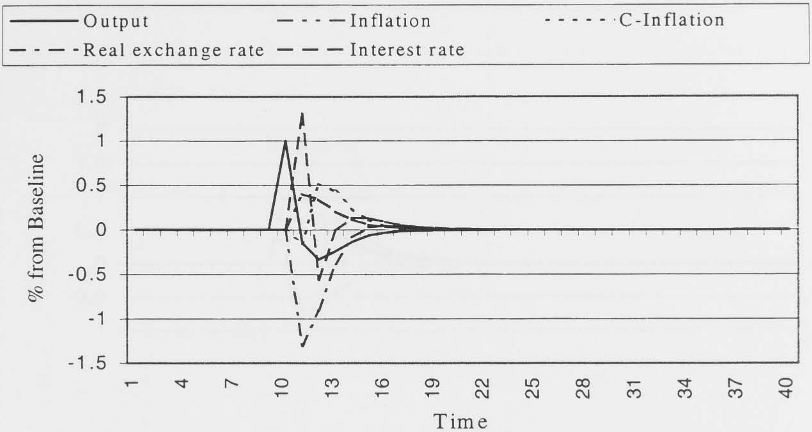
inflation is targeted monetary policy responds similarly to foreign variables and the real exchange rate regardless of the policy preference parameter β. The reason for this is that nontradables inflation is affected by foreign variables and the real exchange rate only through their influence on excess demand. By setting these coefficients to offset foreign demand and supply shocks policy makers can then concentrate on minimizing the effects of domestic shocks on output and domestic inflation. Clearly, none of the rules in Table 5.2 resembles the Taylor rule. In fact the Taylor rule is sub-optimal and inefficient for two reasons: first it is based on a sub-optimal set of state variables; and second it applies inappropriate coefficients to lagged output and inflation.

To better understand the dynamic properties of Model A, and to appreciate how monetary policy responds to shocks, Figures 5.1 through 5.3 present impulse response functions for the system. In these impulse response functions the foreign monetary authority is assumed to optimize policy over the two foreign state variables (with $\beta^f = 0.5$) while the domestic monetary authority minimizes a weighted average ($\beta = 0.5$) of the variances of output and consumer price inflation.[11]

In response to a domestic demand shock Figure 5.1 shows that output rises immediately by 1%, the full size of the shock. In the following period the increased output gap flows through to higher nontradables inflation and monetary policy tightens, raising the nominal interest rate. The higher nominal interest rate brings about a rise in the real interest rate which generates an appreciation of the real exchange rate and a reduction in tradable goods prices, which lowers consumer price inflation. The policy tightening also dampens aggregate demand leading to an excess supply of capacity, which in turn places downward pressure on nontradables inflation. As nontradable inflationary pressures begin to ease policy loosens, the interest rate falls and the real exchange rate starts to appreciate. After 10 periods or so the economy has returned to baseline.

[11] These impulse response functions trace the effect of temporary 1% supply, demand, and portfolio preference shocks. However, because each of the disturbance terms has unit variance a 1% shock is the same as a one standard deviation shock.

Figure 5.1: Model A - 1% Demand Shock



The effects of a 1% domestic supply shock are shown in Figure 5.2. Both domestic inflation and consumer price inflation rise by 1% contemporaneously with the shock. Subsequently monetary policy tightens raising the nominal interest rate and the inflationary pressures subside. The higher interest rate, however, leads to a fall in output and an appreciation of the real exchange rate. This excess capacity puts further downward pressure on inflation and allows the interest rate to begin to fall. As the interest rate falls output recovers and the real exchange rate begins to depreciate, returning the economy to baseline.

Figure 5.2: Model A - 1% Supply Shock

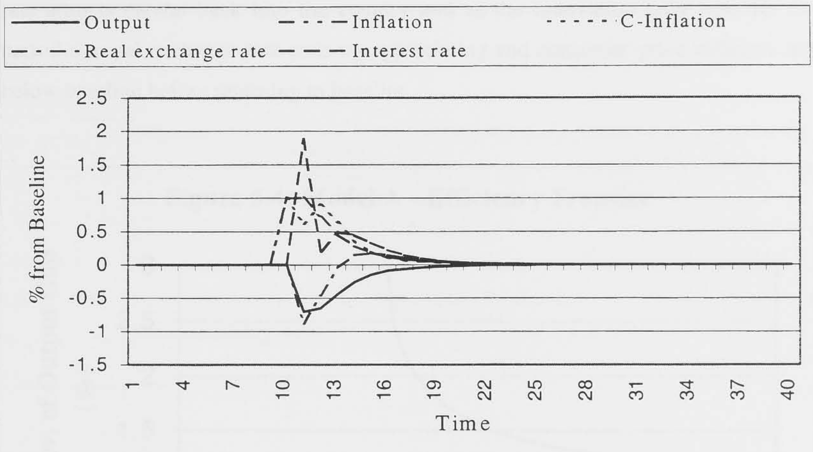
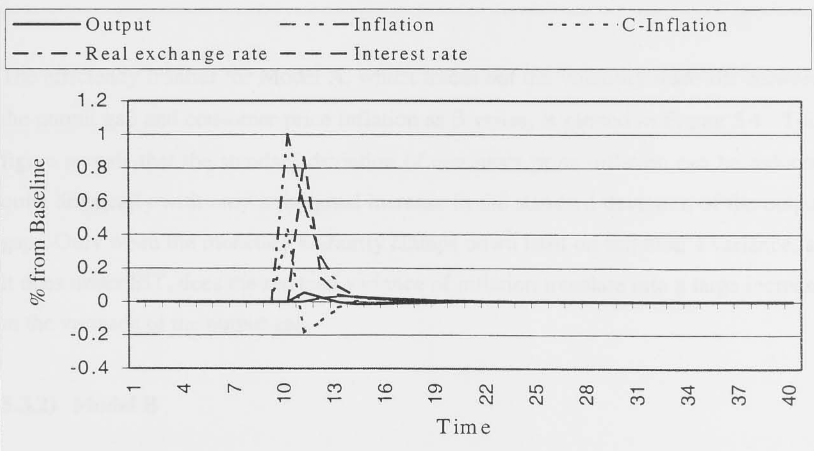
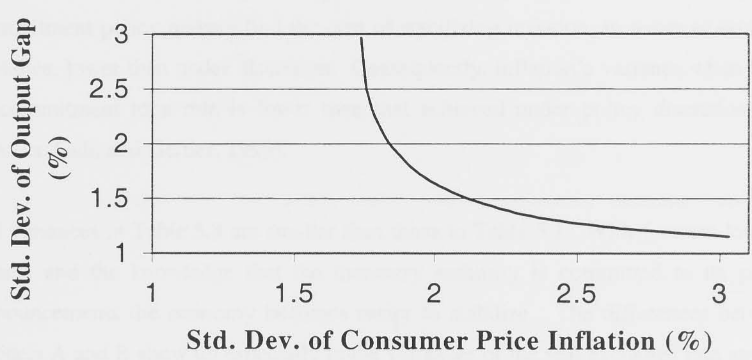


Figure 5.3: Model A - 1% Portfolio Preference Shock



Finally, Figure 5.3 reveals the effects of a portfolio preference shock. This portfolio preference shock comes in the form of a preference for domestic agents to hold their assets in foreign currency. Its immediate effect is to depreciate the real exchange rate by 1%, which contemporaneously results in a rise in consumer price inflation as tradable goods prices rise. Monetary policy tightens in response to the higher consumer price inflation, partly offsetting the stimulus the depreciating real exchange rate has on real output. Real output still rises in the short-term. The higher interest rate attracts capital back into the country and as the innovation only lasts for one period the real exchange rate ends up appreciating and consumer price inflation falls below baseline before returning to baseline.

Figure 5.4: Model A - Efficiency Frontier



The efficiency frontier for Model A, which traces out the volatility trade-off between the output gap and consumer price inflation as β varies, is plotted in Figure 5.4. This figure reveals that the standard deviation of consumer price inflation can be reduced quite drastically with only a marginal increase in the standard deviation of the output gap. Only when the monetary authority clamps down hard on inflation's variance, as it does under SIT, does the reduced variance of inflation translate into a large increase in the variance of the output gap.

5.3.2) Model B

Unlike Model A, which assumes all agents are backward-looking, Model B assumes that financial markets form rational forward-looking expectations ($\delta = \tau = 1$). Furthermore, other agents are assumed to form mixed expectations, their expectations are partly forward-looking and partly backward-looking ($\rho = \omega = 0.5$). Because financial market expectations are now forward-looking π^c_{t-1} is now longer a state variable in the system. The set of observable state variables is: y_{t-1}; π_{t-1}; q_{t-1}; y^f_{t-1}; and π^f_{t-1}.

With forward-looking agents in the model the optimal discretionary policy rule and the optimal commitment policy rule no longer coincide.[12] The results of this Section are generated assuming policy commitment.[13] Policy commitment can be achieved by either an optimal contracting arrangement between the government and the governor (Walsh, 1995) or through reputation effects (Barro and Gordon, 1983). Under policy commitment policy makers find the cost of stabilizing inflation, in terms of output's variance, lower than under discretion. Consequently, inflation's variance when there is commitment to a rule is lower than that achieved under policy discretion (see Clarida, Gali, and Gertler, 1999).

All variances in Table 5.3 are smaller than those in Table 5.1. With forward-looking agents and the knowledge that the monetary authority is committed to its policy announcements the economy becomes easier to stabilize. The differences between Models A and B show up especially in the variances of the real exchange rate and the domestic nominal interest rate, indicating the less activist role monetary policy plays in stabilizing the economy when agents are forward-looking.

Table 5.3: Unconditional Variances for Model B

Regime	Var(y_t)	Var(π_t)	Var(π_t^c)	Var(q_t)	Var(i_t)	Var(y_t^f)	Var(π_t^f)	Var(i_t^f)
FIT(π_t)	1.27	1.57	3.17	3.52	7.84	1.27	1.57	3.01
SIT(π_t)	2.80	1.33	4.18	6.38	26.21	1.27	1.57	3.01
FIT(π_t^c)	1.29	1.67	2.97	3.07	5.15	1.27	1.57	3.01
SIT(π_t^c)	2.19	1.65	2.74	3.42	5.76	1.27	1.57	3.01

Table 5.4 presents the policy reaction functions associated with each of the targeting regimes shown in Table 5.3.

[12] The policy objective function used throughout this study directs the monetary authority to target potential output – not some rate of output above potential. As a consequence the difference between the optimal commitment and discretionary rules does not manifest itself in the form of an inflation bias. However, the time inconsistency does alter the trade-off between the variances of output and inflation facing the monetary authority, which changes the slope of the efficiency frontier.

[13] Assuming commitment has the advantage that the simulation results presented can be easily be compared with studies exploring the performance of simple rules, which implicitly assume the existence of some pre-commitment technology.

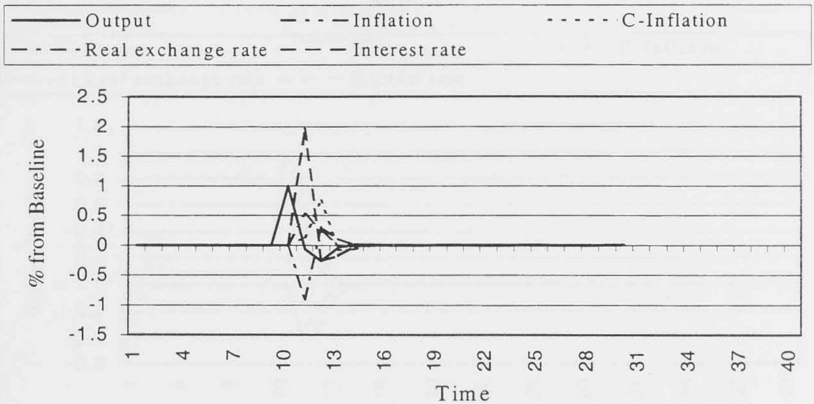
Table 5.4: Optimal Policy Rules for Model B					
Regime	y_{t-1}	π_{t-1}	q_{t-1}	y^f_{t-1}	π^f_{t-1}
FIT(π_t)	2.185	0.851	0.716	-0.432	-0.221
SIT(π_t)	2.796	1.683	0.710	-0.431	-0.220
FIT(π_t^c)	1.954	0.770	0.595	-0.257	-0.145
SIT(π_t^c)	1.707	0.898	0.348	0.062	-0.025

Of special interest is the fact that with forward-looking agents monetary policy now optimally responds negatively to the foreign output gap and foreign inflation (excluding SIT(π^c_t) where policy tightens in response to y^f_{t-1}). In Table 5.2 the feedback coefficients applied to y^f_{t-1} and π^f_{t-1} were positive across all policy regimes considered. The intuition behind these negative feedback coefficients is unclear, but may well involve the interaction between domestic and foreign monetary policies. The foreign monetary authority responds positively (raises interest rates) in response to both y^f_{t-1} and π^f_{t-1}. The higher foreign interest rate causes capital to flow from the domestic to the foreign economy inducing the exchange rate to depreciate. Consequently, movements in the foreign output gap and foreign inflation always occur in conjunction with movements to the real exchange rate. Table 5.4 shows that domestic monetary policy responds more aggressively to the real exchange rate when agents are forward-looking, which may in part explain the negative feedback coefficients in columns six and seven of Table 5.4.

Observe also that as monetary policy places greater weight on stabilizing inflation's variance, the feedback coefficient applied to y_{t-1} rises under nontradables inflation targeting, but falls under consumer price inflation targeting. This feature is also present in Table 5.2. Under nontradables inflation targeting a more aggressive response to the domestic output gap serves to dampen future inflationary pressures. When consumer price inflation is being targeted such a strong response to the domestic output gap creates large swings in the domestic interest rate and the exchange rate. Volatility in the exchange rate adds directly to volatility in tradables inflation, raising the variance of consumer price inflation. Because variability of consumer price inflation is to be avoided the response of policy makers is to lower to feedback coefficient applied to y_{t-1} in the optimal policy reaction function.

Impulse response functions showing the dynamic responses of Model B to demand, supply, and portfolio preference innovations are shown in Figures 5.5 through 5.7. These impulses relate to the policy regime FIT(π^c_t).

Figure 5.5: Model B - Commitment, 1% Demand Shock



In response to a domestic demand shock Figure 5.5 shows that the output gap immediately opens, which in the following period stimulates inflation, bringing about a policy tightening (nominal interest rate rise) and an exchange rate appreciation. The high interest rate and the noncompetitive real exchange rate slow aggregate demand and create an excess of capacity, which places downward pressure on nontradables inflation. As nontradable inflationary pressures ease policy begins to loosen: the nominal interest rate falls and the exchange rate is allowed to depreciate. In the short-term the depreciation of the exchange rate causes tradables inflation to rise, which also raises consumer price inflation. In the long-run consumer price inflation falls as nontradables inflation evaporates. The shock passes through the system in about five years, which is about half the time of the equivalent shock in Model A.

For a domestic supply innovation, Figure 5.6 indicates that the *impact* of the shock is the same in both Model A and Model B: both nontradables and consumer price inflation rise by one percentage point. Subsequently, monetary policy tightens, the nominal interest rate rises, the real exchange rate appreciates slightly, inflationary pressures ease, and a negative output gap opens. As inflationary pressures dissipate monetary policy eases and the economy returns to baseline.

Figure 5.6: Model B - Commitment, 1% Supply Shock

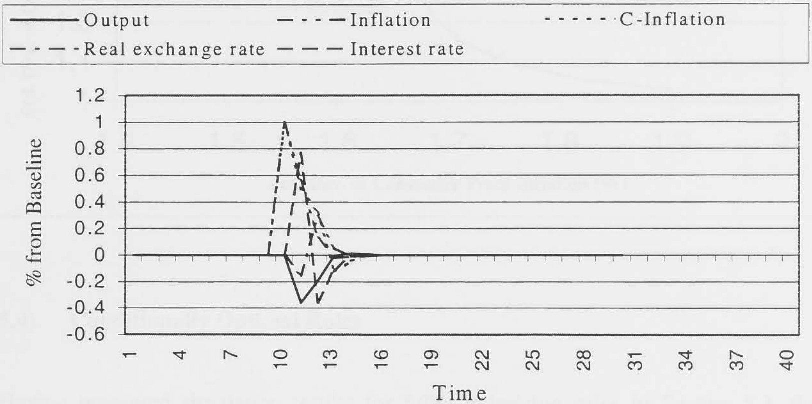
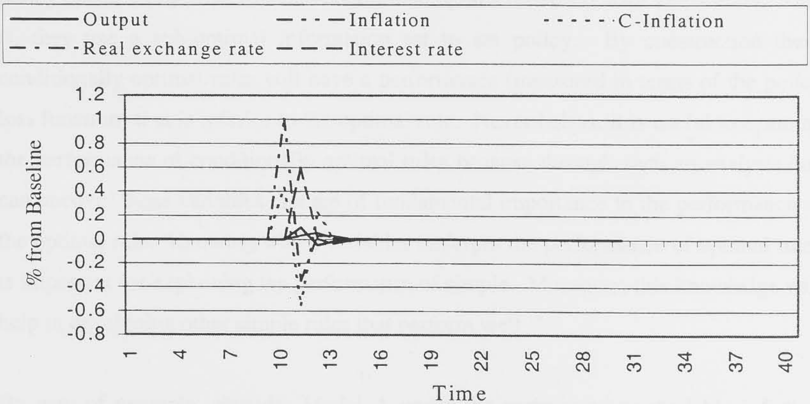
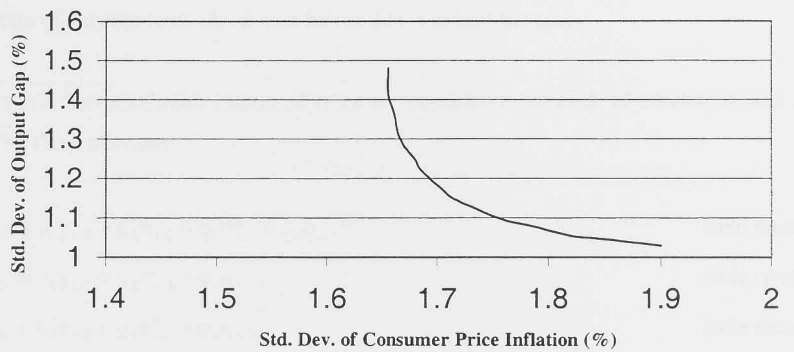


Figure 5.7: Model B - Commitment, 1% Portfolio Preference Shock



The efficiency frontier for Model B is plotted in Figure 5.8. Given the unconditional variances shown in Table 5.3 it is no surprise that the efficiency frontier for Model B lies closer to the origin than that for Model A.

Figure 5.8: Model B - Efficiency Frontier



5.4) Conditionally Optimal Rules

Having presented simulation results for fully optimizing rules in Section 5.3, this Section turns to the relative efficiency and stabilizing properties of conditionally optimal rules. Conditionally optimal rules are rules that endeavor to optimize the policy maker's objective function conditional upon a restricted state variable set. That is, they use a sub-optimal information set to set policy. By construction these conditionally optimal rules will have a performance (measured in terms of the policy loss function) that is inferior to the optimal rule. Nevertheless, it is useful to examine the performance of conditionally optimal rules because through such an analysis one can uncover those variables that are of fundamental importance to the performance of the optimal rule. Knowing which variables underpin the performance of optimal rules is important for explaining the performance of simple. Moreover, this knowledge will help in developing other simple rules that perform well.[14]

By way of example, consider Model A under the regimes of nontradable inflation targeting (Table 5.1). The coefficients on the two foreign variables: y^f_{t-1} and π^f_{t-1} are each either zero or very close to zero. Removing these two foreign variables from the

[14] Cecchetti (1997) makes the observation that '...if the solution to the complex problem can be approximated by a simple rule, there may be substantial virtue in adopting the approximate solution'.

set of state variables forming the policy rule is therefore unlikely to drastically undermine the performance of the optimal rule. Effectively one might expect the four variables: y_{t-1}; π_{t-1}; π^c_{t-1}; and q_{t-1} to come close to forming a sufficient statistic for the state of the economy. As shown below this is indeed the case.

In this Section three conditional rules are considered for each of Models A and B. The three rules are:

$$i_t = \varphi_1 y_{t-1} + \varphi_2 \pi_{t-1} + \varphi_3 \pi^c_{t-1} + \varphi_4 q_{t-1}; \qquad \text{(rule one)}$$

$$i_t = \varphi_1 y_{t-1} + \varphi_2 \pi_{t-1} + \varphi_4 q_{t-1}; \qquad \text{(rule two)}$$

$$i_t = \varphi_1 y_{t-1} + \varphi_3 \pi^c_{t-1} + \varphi_4 q_{t-1}. \qquad \text{(rule three)}$$

Rules two and three are nested inside rule one and each of the three rules exclude the two foreign variables y^f_{t-1} and π^f_{t-1}. In addition, for Model A, we also consider the two rules:[15]

$$i_t = \varphi_1 y_{t-1} + \varphi_3 \pi^c_{t-1}; \qquad \text{(rule four)}$$

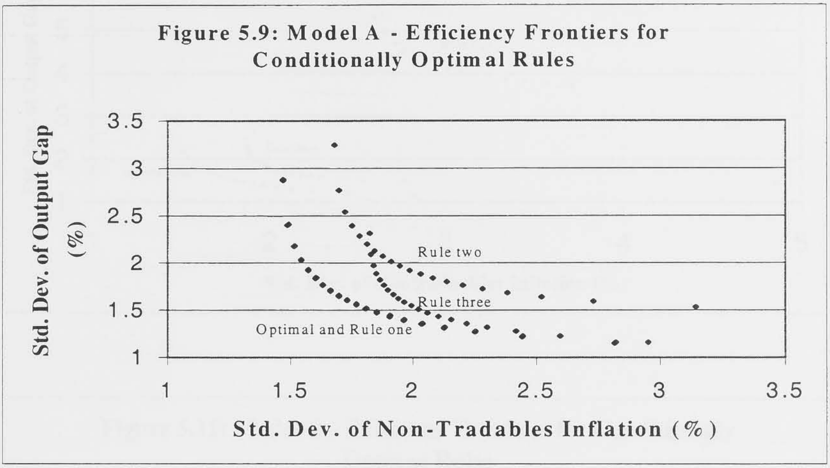
$$i_t = \varphi_1 y_{t-1} + \varphi_2 \pi_{t-1}. \qquad \text{(rule five)}$$

Rules four and five represent the class of Bryant, Hooper, and Mann (1993) rules. In the case where $\varphi_1 = 0.5$ and $\varphi_3 = 1.5$ rule four can be thought of as a form of Taylor rule (similarly when $\varphi_1 = 0.5$ and $\varphi_2 = 1.5$ in rule five). Alternatively, when $\varphi_1 = \varphi_3$ (in rule four) and $\varphi_1 = \varphi_2$ (in rule five) rules four and five can be thought of as examples of Henderson and McKibbin (1993) rules. Rules two and three generalize on the Bryant, Hooper, and Mann class of rules by including the level of the real exchange rate. Intuitively, adding the real exchange rate is an obvious and potentially important extension for a small open economy model.

The results of this Section can best be illustrated in the form of efficiency frontiers. Figure 5.9 plots the efficiency frontiers for the optimal rule, rule one, rule two, and rule three for Model A, assuming it is nontradables inflation that is included in the

[15] These two rules were not considered for Model B because convergence problems prevented results from being constructed with any useful accuracy.

policy objective function.[16] Analogous results for Model B are presented in Figure 5.11, while Figure 5.10 compares rule four and rule five with the optimal rule for Model A.



Beginning with Figure 5.9, the efficiency frontiers for the optimal rule and for rule one basically lie on top of each other. From a practical standpoint very little is lost by excluding the two foreign variables y^f_{t-1} and π^f_{t-1} from the rule. Rule three is superior to rule two except in the extreme case of strict inflation targeting. Thus, conditional upon the output gap and the real exchange rate, consumer price inflation contains more useful information than nontradables inflation regarding the state of the economy. This point is further underscored in Figure 5.10 where rule four (which contains y_{t-1} and π^c_{t-1}) is vastly superior to rule five (which uses y_{t-1} and π_{t-1}). From Figures 5.9 and 5.10, together with Table 5.2, it appears that in Model A the three most important state variables are the output gap, consumer price inflation, and the real exchange rate.

[16] Results for the case where it is consumer price inflation that is included in the policy objective function follow closely those for the nontradable inflation targeting case.

Figure 5.10: Model A - Efficiency Frontiers for Rules four and five

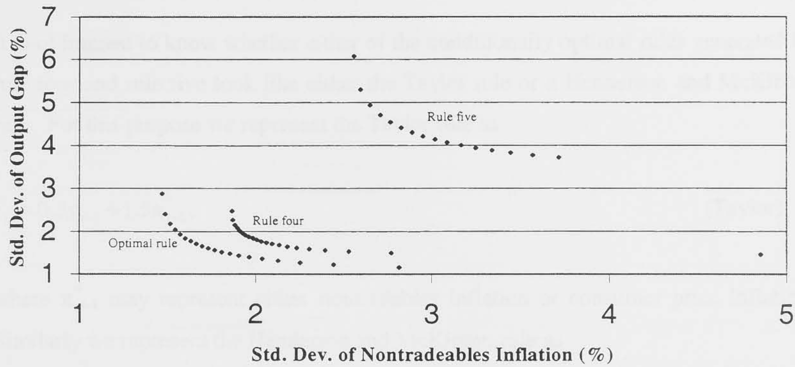


Figure 5.11: Model B - Efficiency Frontiers for Conditionally Optimal Rules

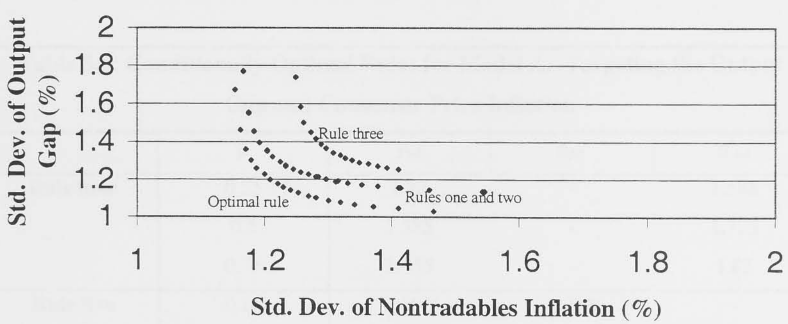


Figure 5.11 shows that in Model B information is lost when the foreign variables (y^f_{t-1} and π^f_{t-1}) are excluded. Note that for Model B lagged consumer price inflation is not a state variable for the system: it provides no additional information about the state of the economy. When the two foreign variables are excluded, however, lagged consumer price inflation does become informative, but only marginally so. Rules one and two, which only differ in that rule two excludes π^c_{t-1} perform nearly identically. The fact that rule three performs worse than rule two points to the superior

information present in nontradables inflation over consumer price inflation, which is the opposite of the result found for Model A.

It is of interest to know whether either of the conditionally optimal rules generated by rule four and rule five look like either the Taylor rule or a Henderson and McKibbin rule. For this purpose we represent the Taylor rule as

$$i_t = 0.5y_{t-1} + 1.5\pi^*_{t-1}, \qquad \text{(Taylor)}$$

where π^*_{t-1} may represent either nontradables inflation or consumer price inflation. Similarly we represent the Henderson and McKibbin rule as

$$i_t = \sigma[y_{t-1} + \pi^*_{t-1}]. \qquad \text{(HM)}$$

Tables 5.5 and 5.6 show optimal conditional feedback rules for a number of policy regimes generated from Model A.

Table 5.5: Conditionally Optimal Rules for Model A. Targeting the Output Gap and Consumer Price Inflation

	β	y_{t-1}	π_{t-1}	π^c_{t-1}
Rule four	0.25	1.72	-	1.588
	0.5	1.585	-	1.723
	0.75	1.455	-	1.83
Rule five	0.25	1.892	1.208	-
	0.5	1.721	1.271	-
	0.75	1.499	1.311	-

The feedback coefficient on the lagged output gap is much greater than 0.5 for all of the policy regimes considered in Table 5.5. The Taylor rule's coefficient of 1.5 on the lagged inflation term is, however, broadly of the correct magnitude. If we were to set σ = 1.6 (roughly), then a Henderson and McKibbin rule would approximate reasonably well the simulation results for rule four.

Table 5.6: Conditionally Optimal Rules for Model A. Targeting the Output Gap and Nontradables Inflation				
	β	y_{t-1}	π_{t-1}	π^c_{t-1}
Rule four	0.25	1.804	-	1.609
	0.5	1.744	-	1.779
	0.75	1.697	-	1.935
Rule five	0.25	2.14	1.24	-
	0.5	2.273	1.37	-
	0.75	2.477	1.541	-

Table 5.6 tells a similar story. The feedback coefficient on the output gap for the Taylor rule needs to be raised considerably if it is to be interpreted as a conditionally optimal rule. But with σ = 1.7 (or so) a Henderson and McKibbin rule is not dissimilar to the conditionally optimal rule generated by rule four.[17]

5.5) The Role of Monetary Conditions Indicators

Monetary Conditions Indicators (MCI) are meant to be summary indicators of the current stance of monetary policy in small open economies. They are formed as a linear combination of the interest rate and the exchange rate, and are used by some central banks – notably the Bank of Canada and the Reserve Bank of New Zealand – as operating targets. This Section explores the role Monetary Conditions Indicators have in setting monetary policy and asks the question of whether using an MCI can lead to improved policy outcomes. Our findings are summarized in two results.

Proposition one: Using a Monetary Conditions Indicator as an operational target does not hamper the monetary authority in meeting its policy objectives. Nor does it allow the monetary authority to achieve superior outcomes.

[17] While it is interesting that a Henderson and McKibbin rule based on the lagged output gap and lagged consumer price inflation can be viewed (loosely) as a conditionally optimal rule for Model A, it is not surprising that the Taylor rule cannot. The Taylor rule imposes two restrictions upon the feedback coefficients in the conditional rule while the Henderson and McKibbin rule only imposes one.

Proof:

Denoting the vector of known state variables by $\mathbf{y}_{t-1}$ the optimal policy reaction function has the form:

$$i_t = \psi^* \mathbf{y}_{t-1}, \tag{25}$$

where ψ^* represents the unique parameter vector that minimizes the monetary authority's objective function. Now define the MCI as[18]

$$MCI_t = E_{t-1}[i_t - \chi q_{t-1}]. \tag{26}$$

This MCI[19] is an operating target that is set as a linear function of the state vector $\mathbf{y}_{t-1}$, giving

$$MCI_t = \mathbf{f}\mathbf{y}_{t-1}. \tag{27}$$

Because the nominal interest rate is set to keep the operating target on track (26) can be combined with (27) to produce the implied policy reaction function

$$i_t = \mathbf{f}\mathbf{y}_{t-1} + \chi q_{t-1}. \tag{28}$$

Introducing the selection vector $\mathbf{s}$ = [0, 0, .., 1, 0, .. ,0], where the 1 corresponds q_{t-1}, (28) can be written as

$$i_t = [\mathbf{f} + \chi \mathbf{s}]\mathbf{y}_{t-1} \tag{29}$$

Thus for the Henderson and McKibbin rule we have a degree of freedom with which to fit the rule to the data.

[18] By defining our MCI as we have, and recognizing that with q_{t-1} predetermined when it is set, the MCI can be manipulated as one would a policy instrument. Viewed in this way the result of this proof is unsurprising.

[19] As we have defined it, our MCI is a combination of the *nominal* interest and the *real* exchange rate. This differs from the MCIs used by policy makers, which are typically based on nominal variables.

With the vector **f** and the parameter χ free to be chosen by the policy maker it is always possible to choose a vector **f**, **f***, such that $\mathbf{f}^* + \chi\mathbf{s} = \psi^*$. Accordingly, when using an MCI as an operating target one can always re-create the optimal policy rule.[20]

∎

Proposition two: When monetary policy is based on a restricted set of state variables, using an MCI as an operational target may produce better outcomes than the conditionally optimal rule.

Proof:

The conditionally optimal rule is based on a subset of the complete set of state variables. Let us denote the variables included in the rule by the vector $\mathbf{z}_t$, where $\mathbf{z}_t = \mathbf{K}\mathbf{y}_t$, and **K** is a conformable selection matrix. We write the conditionally optimal rule as

$$i_t = \psi^{**}\mathbf{z}_{t-1}, \tag{30}$$

where ψ^{**} is the unique parameter vector that optimizes the policy maker's loss function. As before our MCI is defined as the linear combination

$$MCI_t = E_{t-1}[i_t - \chi q_{t-1}]. \tag{31}$$

Similarly, the optimal rule will express the operational target as a linear combination of the included state variables

$$MCI_t = \mathbf{f}\mathbf{z}_{t-1}. \tag{32}$$

Now substituting (32) into (31) and re-arranging produces the relationship

$$i_t = \mathbf{f}\mathbf{z}_{t-1} + \chi q_{t-1}. \tag{33}$$

[20] Svensson (1998) also finds this result in his model, which uses a different definition of the MCI.

Remembering that the policy maker can choose the vector **f** and the parameter χ two possibilities present themselves. First, if q_{t-1} is an element of $\mathbf{z}_{t-1}$, then a vector **f**, $\mathbf{f}^*$, can always be found, for any choice of χ, such that the conditional MCI rule (32) replicates the conditionally optimal rule. The conditional MCI rule cannot improve upon the optimal conditional rule. Second, if q_{t-1} is not an element of the vector $\mathbf{z}_{t-1}$, then by setting $\mathbf{f} = \psi^{**}$ in equation (33) with $\chi = 0$ the monetary authority can replicate the conditional optimal rule. However, with χ a free parameter and q_{t-1} representing additional information, the policy maker can always find a value for χ such that the conditionally optimal MCI rule outperforms the conditionally optimal instrument rule.[21]

∎

5.6) Conclusions

The first part of this paper considered the stabilizing properties of optimal rules under a number of targeting regimes. These optimal rules were expressed in the form of feedback relationships, which allowed us to visually assess the respective contributions various state variables made to the optimal policy rule. When agents form rational forward-looking inflation expectations we found that the system was easier to stabilize with the variances of all variables lower than those generated under extrapolative expectations.

In the second part of the paper the stabilizing properties of conditionally optimal rules were examined. Overall foreign variables – specifically the lagged foreign output gap and lagged foreign inflation – were found to be relatively unimportant; omitting these two variables from the rule did not seriously detract from its performance. The process by which inflation expectations were formed was found to alter the relative performance of consumer price inflation and nontradables inflation in the optimal rule. When financial market expectations were formed in a simple backward manner, consumer price inflation was found to be more informative than nontradables

[21] The key reason why using an MCI produces superior outcomes is that it exploits a broader information set. Because the real exchange rate is likely to feature as an important variable in any small open economy policy reaction function, it is unlikely that using an MCI as an operational target will produce superior economic outcomes in practice.

inflation. When financial market expectations were formed rationally the opposite was the case.

Of the three principle conditional rules considered in this paper the best performing was rule one, which only excluded the two foreign variables. Given that this conditional rule encompasses the other conditional rules considered this result is unsurprising. More interesting is the fact that rule two (based on y_{t-1}, π_{t-1}, and q_{t-1}) and rule three (based on y_{t-1}, π^c_{t-1}, and q_{t-1}) perform well relative to the fully optimal rule. Indeed, in Model A, where agents form backward-looking expectations, the conditional rule containing only y_{t-1} and π^c_{t-1} performs surprisingly well. None of these conditionally optimal rules, however, correspond to the Taylor rule, but for values of the policy preference parameter, β, between about 0.25 and 0.75 the conditionally optimal rule using y_{t-1} and π^c_{t-1} resembles a Henderson and McKibbin rule.

This paper also examined the role of Monetary Conditions Indicators as a tool for setting monetary policy in small open economies with relatively large tradable goods sectors. For plausible contexts in which the central bank accounts for the exchange rate when setting the nominal interest rate no advantages to using an MCI were found. This result confirms the finding of Svensson (1998).

Throughout our analysis we have assumed that the model structure, parameter values, and *ex ante* distributions of disturbances are known with certainty. The only uncertainty present in the model relates to the realization of the structural disturbances. But clearly while our assumption of model certainty is a common and convenient one it is far from realistic. In practice neither policy makers nor private agents know the true structure of the economy, and this uncertainty is likely to affect their decisions. Uncertainty about the structure of the economy can be allowed for with optimizing agents through the use of risk-sensitive optimal control (Whittle, 1990) or robust optimal control (Zhou, Doyle and Glover, 1996). Applying risk-sensitive optimal control to our analysis may well be a fruitful avenue for further research.

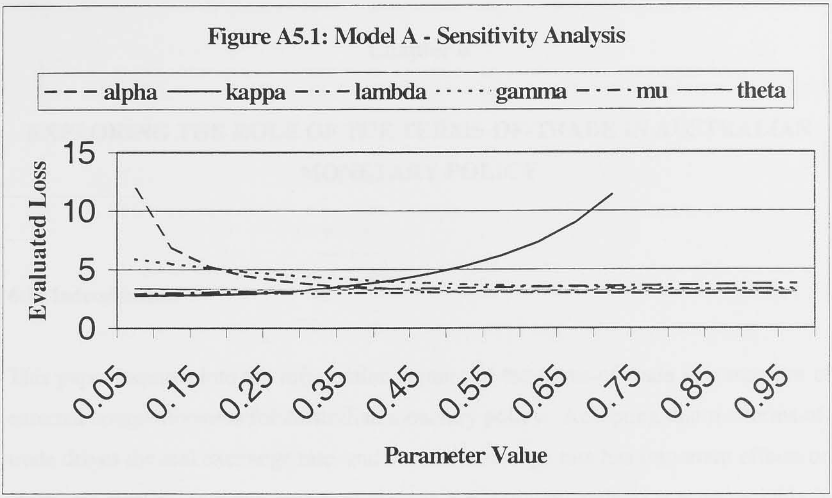
Finally, one weakness of this paper is that the model supporting it has not been estimated. In one sense this is an advantage because it does not tie the analysis down to a specific country's economy, but is it also a disadvantage in that it no policy recommendations can be made. Exploring conditionally optimal rules within the context of an estimated model for a small open economy is an obvious direction for further analysis.

Appendix A: Some Sensitivity Analysis for Section 5.3.

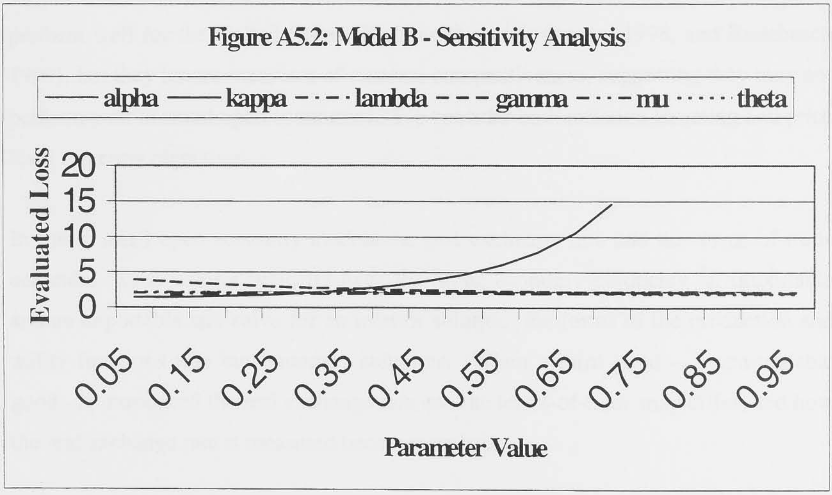
This appendix investigates the robustness of the results presented in Section 5.3 to different parameterizations of the model. The system has fourteen parameters: too many to be systematically explored, especially if interactions between parameters are also considered. The two specifications considered in the text of the paper go some way toward illustrating how the process by which expectations are formed affects the results. So the sensitivity of our results to changes in the parameters ρ, δ, τ, and ω is not considered here. The parameters in the foreign sector of the model (λ^f, α^f, and γ^f) are also not considered because they are likely to only have a second order effect on the variances of domestic variables. This leaves the six domestic parameters: α; κ; λ; γ; μ; and θ.

To analyze the sensitivity of the model to changes in these six parameters, we evaluate the minimized policy loss function as these six parameters vary between 0.05 and 0.95. Both Model A and Model B of Section 5.3 are considered and the policy regime maintained throughout is FIT(π^c_t) ($\beta = 0.5$) for the domestic monetary authority and FIT(π_t) ($\beta^f = 0.5$) for the foreign monetary authority. Results are summarized in figures A5.1 and A5.2.

Figure A5.1 shows that Model A is most sensitive to α and κ. These two parameters are the inverse of the sacrifice ratio and the direct exchange rate pass-through coefficient respectively. That our results are sensitive to the values of these two parameters is perhaps not surprising given that both are of direct importance to the policy transmission mechanism. The results are also sensitive to γ. However, the minimized loss function is relatively flat locally about the values of α (0.4) and γ (0.8) chosen. For κ our results are quite robust for values less than that chosen (0.3), but sensitive to values above 0.3.



For Model B the results are similar: our results are most sensitive to the parameters α and κ, while the parameter γ does not appear as important.



Chapter 6

EXPLORING THE ROLE OF THE TERMS-OF-TRADE IN AUSTRALIAN MONETARY POLICY

6.1) Introduction

This paper inquires into the information content of the terms-of-trade as a measure of external competitiveness for Australian monetary policy. Accepting that the terms-of-trade drives the real exchange rate, and the real exchange rate has important effects on aggregate demand, we ask how important is the terms-of-trade as a state variable in the optimal monetary policy rule? This inquiry is not a theoretical one, but is based on a well known, data based, model of the Australian economy: the de Brouwer-O'Regan model. A second objective of the paper is to investigate the usefulness of Taylor type rules in a small open economy. Taylor type rules have been shown to perform well for the United States (Rudebusch and Svensson, 1998, and Rudebusch 1999), but they ignore measures of external competitiveness, suggesting they may not perform well in small open economies. We consider both inflation targeting and price level targeting objectives.

In many small open economy models the real exchange rate and the terms of trade coincide. Such models typically have the home economy producing an importable and an exportable and solve for an interior solution; the forms of the production and utility functions rule out boundary solutions. When a third good – a non-tradable good - is introduced the real exchange rate and the terms-of-trade may differ, and how the real exchange rate is measured becomes an issue.

A country's terms-of-trade is defined as the ratio of its export price relative to its import price, measured in terms of that country's dollars. The terms-of-trade therefore measures how many foreign goods can be imported for an additional home good exported. The real exchange rate is typically defined as the ratio of the foreign country price level to the home country price level, once both price levels are measured in terms of the same currency. As such the real exchange rate measures how many home goods must be given up in exchange for a single foreign good. With an importable, an exportable, and a non-traded good there are three prices and two relevant relative prices. Choosing the import price as the numeraire the relative prices of interest are the terms-of-trade and the non-traded price relative to the import price (Pitchford, 1993).

Measuring the prices of non-traded goods is difficult. As a consequence standard practice is to employ a real exchange rate measured using the ratio of national price levels. Provided all relative prices are constant whether one uses non-traded goods prices or aggregate price levels is immaterial. When relative prices are allowed to vary Dwyer and Lowe (1993) show how terms-of-trade changes affect the real exchange rate. The relationship between the real exchange rate and the terms-of-trade depends on whether the nominal exchange rate is fixed or floating, and on whether the terms-of-trade change arises through an import or an export price change. Gruen and Dwyer (1995) use a small theoretical model to show that with a fixed exchange rate terms-of-trade increases are inflationary, and that with a floating exchange rate the effect of terms-of-trade rises on inflation is ambiguous.

Empirically the Australian evidence is that the terms-of-trade is a stationary variable (the terms-of-trade decline around a deterministic trend, Gruen and Kortian, 1996), and that there is a strong relationship between the terms-of-trade and the real exchange rate. Increases in the terms-of-trade cause the real exchange rate to appreciate (Blundell-Wignell and Gregory, 1990). These stylized facts are built into models of the Australian economy, particularly that of de Brouwer and O'Regan (1997).

Using closed-loop control methods this study expresses the optimal policy reaction function as a linear combination of the system's state variables. Then using exact methods, rather than stochastic simulations, unconditional variances for each variable in the system are obtained. These techniques allow us to integrate points on the efficiency frontier, policy reaction functions, with parameters in the policy objective function. Moreover, they allow us to easily optimize over a restricted set of state variables to assess the relative contributions the real exchange rate and the terms-of-trade have to the location of the efficiency frontier.

The structure of the paper is as follows. In the following Section the basic economic model underlying the paper is discussed. The model itself is a simplified version of the de Brouwer-O'Regan model, revised and estimated using data up to June 1998. Section 6.3 describes the policy rules considered in the study and summarizes the results. Both inflation targeting and price level targeting are examined. Section 6.4 examines how dynamic homogeneity affects the slope of the efficiency frontier. Section 6.5 concludes.

6.2) Model Outline

The model analyzed in this paper is a slightly simplified version of the de Brouwer-O'Regan model. For the most part the simplifications made are those necessary to convert the model to a VAR(2) process. Expressing the model as a VAR(2) process is useful because it facilitates solving for the optimal policy rule.

The model is:

$$y_t = 0.75y_{t-1} - 0.1q_{t-1} + 0.05T_{t-1} - 0.22[i_{t-1} - 4\pi_{t-1}] + \rho_t \tag{1}$$

$$\pi_t = -.16p_{t-1} + .11c_{t-1} + .05p^i_{t-1} + .15\Delta c_{t-1} + .16y_{t-2} - .18\Delta c_{t-2} + .02\pi^i_{t-2} + e_t \tag{2}$$

$$q_t = 1.09\Delta T_t + 0.63q_{t-1} + 0.25T_{t-1} + 0.66[i_{t-1} - 4\pi_{t-1}] + v_t \tag{3}$$

$$c_t = 0.9c_{t-1} + 0.1p_{t-1} + 0.2y_{t-1} + 0.42\pi_{t-1} + 0.58\pi_{t-2} + \varepsilon_t \tag{4}$$

$$p^i_t = .58\pi_t - .58q_t + .66p^i_{t-1} + .19\pi_{t-1} + .34p_{t-1} + .05q_{t-1} - .21\pi^i_{t-1} + .19q_{t-2} + \eta_t \tag{5}$$

$$T_t = 1.68T_{t-1} - 0.81T_{t-2} + \omega_t \tag{6}$$

Where: y_t = domestic output

q_t = real exchange rate

T_t = terms-of-trade

π_t = quarterly consumer price inflation

i_t = annualized nominal short term interest rate

p_t = level of consumer prices

c_t = unit labor costs

p^i_t = level of import prices (in domestic dollars)

π^i_t = quarterly import price inflation.

All variables are represented as deviations from steady-state. Each stochastic innovation is assumed to be a zero mean white noise process, uncorrelated with the other shocks.[1] Equation (1) is a dynamic open economy IS curve. The output gap depends on its lag, the real exchange rate, the terms-of-trade, and an *ex post* measure of the real interest rate. Consumer price inflation is modeled using a mark-up pricing equation, (2). The terms-of-trade and the *ex post* real interest rate drive the real exchange rate, equation (3), shocks to which are propagated by the lagged real exchange rate. The unit labor costs equation, equation (4), restricts unit labor costs to equal the price level in the long-run, with the change in unit labor costs equaling the consumers' price inflation rate. Equation (5) describes the evolution of import prices. Import prices rise proportionally with foreign price level increases and depreciations in the nominal exchange rate in the long-run. The stochastic process underlying the terms-of-trade is summarized in equation (6). Note that while static homogeneity holds for each of consumer prices, import prices, and unit labor costs, dynamic homogeneity only holds for unit labor costs.[2]

More generally the de Brouwer-O'Regan model contains foreign variables, such as foreign output, foreign prices, and the foreign interest rate. This paper does not consider foreign variables as a source of shocks and excludes these variables from the system. The main reason for this exclusion is that each foreign variable in the de Brouwer-O'Regan model is modeled autoregressively without interaction. Consequently, foreign demand can rise above potential without affecting foreign prices, and foreign interest rates can rise without reducing foreign demand. Without these interactions foreign shocks are unlikely to propagate into the domestic economy realistically, and hence they are omitted.

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[1] The variances of ρ_t, e_t, v_t, ε_t, η_t, and ω_t are 0.2855, 0.0454, 8.172, 0.07427, 0.7704, and 3.0276 respectively. At time of writing these variances come from the most recently estimated equations, using data extending to 1998q2. The exact sample sizes and periods differ across equations.

[2] For this reason this paper does not consider the policy implications of changes to the central banks inflation target, preferring to keep the inflation target constant.

Aside from the coefficients on the real exchange rate and the terms-of-trade in equation (1), and the long-run of the unit labor costs equation, all coefficients are estimated. The two non-estimated coefficients in equation (1) have their values taken from Shuetrim and Thompson (1999). The long-run of the unit labor costs equation, equation (4), is chosen to impose the theoretical property of static homogeneity, but with an adjustment speed slow enough to only marginally affect the short-run dynamics. In addition there are several identities linking levels and growth rates.

In this model the instrument of monetary policy is the nominal interest rate. Because prices adjust slowly in the model, increases in the nominal interest rate raise the real interest rate, which in turn causes the real exchange rate to appreciate. The higher real interest rate and real exchange rate dampen aggregate demand, which lowers output and the mark-up in the price equation. Price changes eventually pass-through fully into unit labor costs. The change in the real exchange rate also has a direct impact on prices through its effect on traded goods prices.

The model has several key features. The first is that monetary policy is not all that effective in the model, having a relatively weak influence on aggregate demand. The real interest rate does have a large influence on the real exchange rate, but the real exchange rate too has a very modest affect on aggregate demand. Consequently, monetary policy has only weak influence on firms' mark-ups, and gains most of its kick through the direct exchange rate channel into prices.

The second key feature is that dynamic homogeneity does not hold in the model. It follows that the model's steady-state has real variables affected by nominal variables, and inflation is not super-neutral. The absence of dynamic homogeneity has an impact on the slope of the policy efficiency frontiers derived later. In particular, dynamic homogeneity appears necessary if standard rectangular efficiency frontiers are to be obtained.

The final key feature of the model is the reliance of the real exchange rate on the terms-of-trade. Real interest rate differentials are important for the real exchange rate, but more important is the terms-of-trade. Movements in these two driving factors are propagated through time by the lagged real exchange rate. From equation (3), increases in the terms-of-trade are found to generate numerically large appreciations in the real exchange rate.

To complete the model specification we summarize the policy makers' preferences through an objective function

$$\text{Loss}[0,\infty] = \alpha \text{Var}[y_t] + \beta \text{Var}[\pi_t^a] + \gamma \text{Var}[p_t]. \qquad \alpha, \beta, \gamma \geq 0$$

The terms $\text{Var}[y_t]$, $\text{Var}[\pi^a_t]$ and $\text{Var}[p_t]$ each represent unconditional variances.[3] This objective function can be motivated from an intertemporal objective function under the assumption of zero discounting (see Rudebusch and Svensson, 1998, and Svensson, 1998). Under inflation targeting we set $\gamma = 0$ and restrict $\alpha + \beta = 1$; under price level targeting we set $\beta = 0$ and restrict $\alpha + \gamma = 1$.

6.3) Study Design and Results

In this Section we consider optimal simple inflation and price level targeting rules, and their effects on the economy as summarized by unconditional standard deviations. Of interest is how these standard deviations change, particularly those of output, inflation, and the price level, as the real exchange rate and the terms-of-trade are alternately included and excluded from the policy rule. Both inflation targeting and price level targeting regimes are considered.

Because dynamic homogeneity does not hold in the model it is worthwhile discussing the nature of the policy regime. In models where the Classical dichotomy holds in the long-run the issue of what price level or rate of inflation to target is of second-order importance. In such models the nominal steady-state does not impose a real cost, so the natural question to address is that of how monetary policy can minimize the cost of bringing the economy to its steady-state. In models like that used in this paper where the Classical dichotomy does not hold in the long-run, the issue of what price level or inflation rate to target is important. For every nominal steady-state there is an

[3] In all simulations it is the annualized inflation rate ($\pi^a_t = 4\times\pi_t$) that enters the policy loss function.

associated real steady-state. The question of what inflation rate should optimally be targeted is an interesting one, and one that could potentially be explored in the de Brouwer-O'Regan model. However we do not investigate that question in this paper. Instead we assume that the monetary authority has chosen the inflation rate or price level it wants to target. Associated with this nominal target is some real steady-state. The monetary authority is then assumed to respond to shocks by returning the economy to this steady-state in a way that minimizes the costs of doing so. These costs are defined by the quadratic policy objective function above. Implicitly we are assuming that the unconditional means of inflation and the price level can be chosen independently of α, β, and γ.

For purposes of exposition it is convenient to present the results in two subsections: inflation targeting and price level targeting.

6.3.1) Inflation Targeting

For inflation targeting the policy loss function is taken to be

$$\text{Loss}[0,\infty] = \alpha \text{Var}[y_t] + (1-\alpha)\text{Var}[\pi_t^a] \qquad 0 \le \alpha \le 1.$$

The policy preference parameter α regulates how the policy maker trades-off the variances of output and inflation in response to supply shocks. It also indexes how quickly policy returns inflation to its target rate following shocks.

Four simple policy rules are considered. The monetary authority is assumed not to know the values of period t variables when setting the nominal interest rate in period t, so only lagged variables enter the policy rule. The four policy rules considered are:

$$i_t = \varphi_1 y_{t-1} + \varphi_2 \pi_{t-1} + \varphi_3 q_{t-1} + \varphi_4 T_{t-1} \qquad (I_1)$$

$$i_t = \varphi_1 y_{t-1} + \varphi_2 \pi_{t-1} + \varphi_3 q_{t-1} \qquad (I_2)$$

$$i_t = \varphi_1 y_{t-1} + \varphi_2 \pi_{t-1} + \varphi_4 T_{t-1} \qquad (I_3)$$

$$i_t = \varphi_1 y_{t-1} + \varphi_2 \pi_{t-1} \qquad (I_4)$$

Rules I_2, I_3, and I_4 are nested within rule I_1. Rule I_4 represents the class of Taylor type rules. If a Taylor type rule were appropriate for Australia, then rules I_1, I_2, and I_3 would offer little improvement over rule I_4 in terms of the variances of output and inflation. Similarly, if the real exchange rate offers more information for policy makers than the terms-of-trade, then the efficiency frontier associated with rule I_2 will lie closer to the origin than that for rule I_3. In this Section we are interested in how the Taylor type rule performs relative to the fully optimal rule, and also in how rules I_2 and I_3 compare to I_1.

Table 6.1 presents the unconditional standard deviations for the key variables in the system, for each of the four policy rules, under two targeting regimes: flexible inflation targeting (FIT, $\alpha = 0.5$) and strict inflation targeting (SIT, $\alpha = 0$). These two regimes are presented to allow comparisons with previous studies (Svensson, 1998, and Chapter 5). For inflation, it is the unconditional standard deviation of annualized inflation that is shown.

Table 6.1

		Unconditional Std. Deviations					
	Regime	y	π^a	p	q	u	i
Rule I_1	FIT	0.87	1.70	∞	6.85	∞	3.64
	SIT	0.94	1.69	∞	6.70	∞	4.05
Rule I_2	FIT	0.98	1.74	∞	6.80	∞	3.90
	SIT	1.05	1.73	∞	6.61	∞	4.31
Rule I_3	FIT	1.30	2.03	∞	7.53	∞	4.38
	SIT	1.35	2.02	∞	7.37	∞	4.74
Rule I_4	FIT	1.40	2.17	∞	8.18	∞	4.23
	SIT	1.40	2.17	∞	8.16	∞	4.31

Comparing first rule I_1 with rule I_2 we note that excluding the terms-of-trade from the policy reaction function raises the variance of output slightly while the variance of inflation remains essentially unchanged. In contrast, comparing rule I_3 with rule I_1 we observe that excluding the real exchange rate from the policy reaction function significantly raises both the variance of output and the variance of inflation. In fact the performance of rule I_3 appears closer to that of rule I_4 (the Taylor type rule) than it does to rule I_1, implying that the terms-of-trade does not contain much information over-and-above that in output and inflation. Also of interest is the fact that the variance of inflation does not change much as the policy regime, α, changes, for each of the four rules.

The policy reaction functions associated with each rule and regime in Table 6.1 are given in Table 6.2. For each policy rule as we move from flexible inflation targeting (FIT) to strict inflation targeting (SIT) the coefficient applied to (quarterly) inflation rises while that applied to output falls. This result is consistent with the simulation studies of Svensson (1998), Chapters 4 and 5, and with the analytical work in Ball (1999). Note that output features significantly in the policy reaction function even when it is absent from the policy objective function. This result occurs because higher output today represents higher inflation tomorrow. By responding to current demand pressures policy makers can help stabilize inflation.

Table 6.2

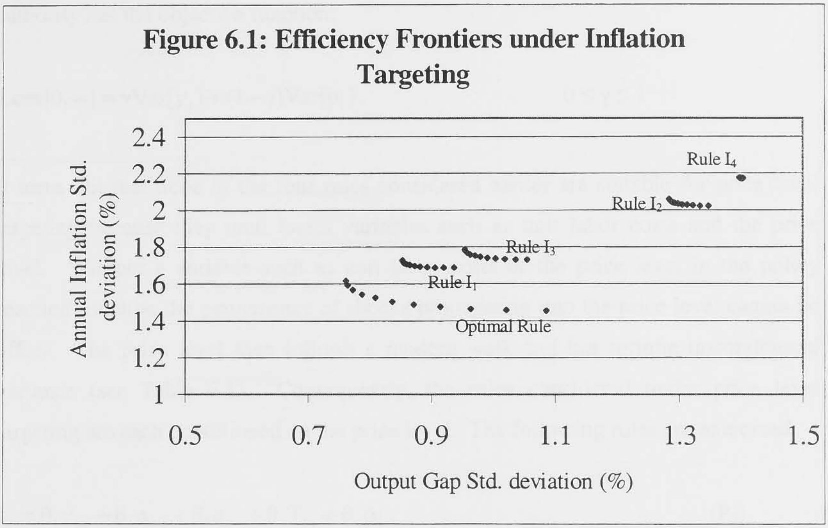
		Policy Reaction Function Coefficients			
	Regime	y_{t-1}	π_{t-1}	q_{t-1}	T_{t-1}
Rule	FIT	1.644	0.823	-0.581	0.232
I_1	SIT	1.709	0.970	-0.636	0.224
Rule	FIT	2.334	-0.418	-0.399	-
I_2	SIT	2.358	0.098	-0.466	-
Rule	FIT	3.091	-0.747	-	-0.214
I_3	SIT	3.227	-0.552	-	-0.271
Rule	FIT	2.817	0.886	-	-
I_4	SIT	2.795	1.168	-	-

A typical property of optimal policy rules is that the real interest rate must rise in response to higher inflation if the system is to have a unique stable equilibria. In terms of the simulation results, this typically means that the feedback coefficient applied to inflation in the policy rule should be greater than 4 (π_t is quarterly inflation while i_t is an annual interest rate). The policy rules in Table 6.2 to not reflect this theoretical property, and may seem unusual as a consequence. It is the fact that dynamic homogeneity does not hold in the de Brouwer-O'Regan model that accounts for this unusual result. One consequence of dynamic homogeneity not holding is that the uncontrolled model is stable with respect to inflation. In the uncontrolled economy inflation does not have a unit root and thus the role for policy makers in the economy is reduced.

Also of interest in Table 6.2 is the coefficient applied to the real exchange rate in each policy reaction function. Chapter 5 shows that the real exchange rate coefficient in the policy reaction should be closely related to (minus) the ratio of the real exchange rate and the real interest rate coefficients in the IS curve. From equation (1) this ratio of coefficients is 0.45, which after allowing for the minus sign is broadly consistent with the results in Table 6.2.[4] Observe also that with the exception of the Taylor type rule the coefficients on both output and inflation rise as increased weight is placed on inflation in the objective function. Increased weight on inflation stabilization involves responding to current inflation and future inflation, as indicated by demand pressures.

To drive home the results of Table 6.1, Figure 6.1 depicts the efficiency frontiers for each of the four policy rules, along with that for the fully optimal rule. This figure clearly reveals the superior performance of rule I_2 over rule I_3 emphasizing the usefulness of the real exchange rate as a state variable for policy makers over that of the terms-of-trade. Intuitively, this result arises because the real exchange rate itself is influenced by monetary policy while the terms-of-trade is not. Having the real exchange rate in the policy reaction functions allows policy makers to partly mitigate the economic consequences of real exchange rate shocks.

[4] The coefficient applied to the real exchange rate in the policy rule would equal –0.45 if all state variables entered the rule and a 100% weight were placed on output in the policy objective function.



The performance of each rule is clearly and unambiguously ranked in Figure 6.1, where the efficiency frontiers of rules I_1 - I_4 are plotted alongside that for the optimal rule, which is presented as a benchmark. Of particular interest is that rule I_3 is far superior to rule I_2. This reflects the superior information present in the real exchange rate over-and-above that in the terms-of-trade. Another interesting feature of Figure 6.1 is the weak influence monetary policy has over the variance of inflation. For all five rules (including the optimal rule) moving along the frontier brings much greater change in the variance of the output gap than it does the variance of inflation. A plausible explanation of this feature is the lack of dynamic homogeneity in the pricing sector of the model. The absence of dynamic homogeneity means that lagged inflation, which is included in all rules considered, is not very informative about future inflation. Consequently the variables contained in rules I_1 - I_4 have very little effect on inflation, contributing to the relatively flat efficiency frontiers.

6.3.2) Price Level Targeting

To complement our analysis of inflation targeting, this subsection explores whether the standing of the terms-of-trade as a state variable for monetary policy is raised under price level targeting. By price level targeting we mean that the monetary authority has the objective function:

$$\text{Loss}[0,\infty] = \gamma \text{Var}[y_t] + (1-\gamma)\text{Var}[p_t]. \qquad 0 \le \gamma \le 1$$

It turns out that none of the four rules considered earlier are suitable for price level targeting because they omit levels variables such as unit labor costs and the price level. Without a variable such as unit labor costs or the price level in the policy reaction function the permanence of shocks propagating into the price level cannot be offset. The price level then follows a random walk and has infinite unconditional variance (see Table 6.1). Consequently, the rules considered under price level targeting are each conditioned on the price level. The following rules are examined:

$$i_t = \theta_1 y_{t-1} + \theta_2 \pi_{t-1} + \theta_3 q_{t-1} + \theta_4 T_{t-1} + \theta_5 p_{t-1} \qquad (P_1)$$

$$i_t = \theta_1 y_{t-1} + \theta_2 \pi_{t-1} + \theta_3 q_{t-1} + \theta_5 p_{t-1} \qquad (P_2)$$

$$i_t = \theta_1 y_{t-1} + \theta_2 \pi_{t-1} + \theta_4 T_{t-1} + \theta_5 p_{t-1} \qquad (P_3)$$

$$i_t = \theta_1 y_{t-1} + \theta_2 \pi_{t-1} + \theta_5 p_{t-1} \qquad (P_4)$$

$$i_t = \theta_1 y_{t-1} + \theta_5 p_{t-1} \qquad (P_5)$$

Capturing our interest is the performance of rules P_2 and P_3 relative to rules P_1 and P_4. Note that Taylor type rules are inappropriate for price level targeting because they cannot stabilize the price level. The performances of these five rules under flexible price targeting (FPT, $\gamma = 0.5$) and strict price targeting (SPT, $\gamma = 0$) are summarized in Table 6.3.

Table 6.3							
		Unconditional Std. Deviations					
	Regime	y	π^a	p	q	u	i
Rule	FPT	1.32	1.89	1.38	6.82	1.68	5.29
P_1	SPT	2.41	2.58	1.09	7.77	1.62	10.06
Rule	FPT	1.44	1.93	1.43	6.74	1.75	5.53
P_2	SPT	2.54	2.63	1.13	7.73	1.74	10.22
Rule	FPT	1.84	2.33	1.82	7.75	2.23	6.60
P_3	SPT	3.13	3.26	1.48	9.47	2.12	12.64
Rule	FPT	1.96	2.55	2.09	8.66	2.56	6.67
P_4	SPT	3.34	3.69	1.77	11.02	2.37	13.68
Rule	FPT	1.97	2.56	2.09	8.70	2.57	6.78
P_5	SPT	2.93	3.33	1.81	10.11	2.36	11.08

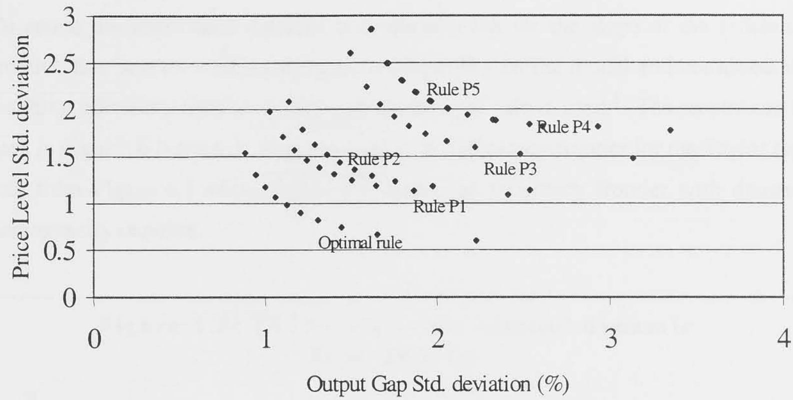
Aside from the variances of the price level and unit labor costs, the variances of all variables are greater under price level targeting than inflation targeting; the cost of not allowing bygones be bygones. Rule P_2, which contains the real exchange rate, produces lower variances for all variables than Rule P_3, suggesting that the real exchange rate is more informative for monetary policy than the terms-of-trade. In the steady-state of the system unit labor costs equal consumer prices. Yet for all regimes considered the volatility in unit labor costs is higher than that for consumer prices. By targeting consumer prices the central bank prevents the price level from changing to clear the labor market. Instead the nominal wage rate moves to clear the labor market, making unit labor costs more volatile than consumer prices.

Table 6.4 complements Table 6.3 and shows the policy reaction functions associated with each of the policy regimes presented above.

Table 6.4						
		Policy Reaction Function Coefficients				
	Regime	y_{t-1}	π_{t-1}	q_{t-1}	T_{t-1}	p_{t-1}
Rule	FPT	2.116	1.866	-0.703	0.273	1.358
P_1	SPT	2.568	2.092	-1.025	0.337	4.173
Rule	FPT	2.887	0.662	-0.478	-	1.302
P_2	SPT	3.509	0.543	-0.734	-	4.416
Rule	FPT	3.909	-0.711	-	-0.275	1.331
P_3	SPT	5.441	-3.188	-	-0.532	4.433
Rule	FPT	3.659	0.458	-	-	1.352
P_4	SPT	5.519	-3.084	-	-	4.883
Rule	FPT	3.817	-	-	-	1.446
P_5	SPT	4.427	-	-	-	2.950

Table 6.4 indicates that the feedback coefficients are typically very large, implying an aggressive policy adjustment in response to state variables, which helps explain the large standard deviations in Table 6.3. The magnitude of these feedback coefficients typically rises as the policy regime switches from flexible price targeting to strict price targeting. Finally, Figure 6.2 plots the efficiency frontiers for each of the policy rules in price standard deviation/output standard deviation space. An efficiency frontier for the fully optimal rule is also provided as a benchmark. Aside from the fully optimal rule and Rule P_1, that closest to the origin is Rule P_2 – that containing the real exchange rate. Rule P_2 clearly dominates Rule P_3, which contains the terms-of-trade in place of the real exchange rate, but is otherwise identical in structure. Thus under price level targeting, as with inflation targeting, the real exchange rate appears to be a more important variable than the terms-of-trade for the monetary authority to base policy on – despite the fact that the terms-of-trade is a key driver of real exchange rate movements.

Figure 6.2: Efficiency Frontiers under Price Level Targeting



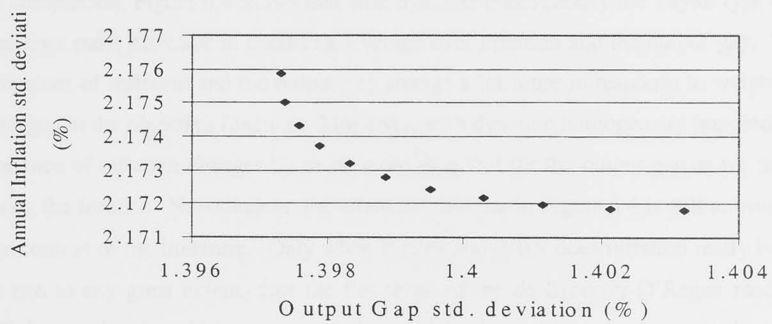
6.4) Dynamic Homogeneity and the Shape of the Efficiency Frontier

An intriguing feature of Figure 6.1 is how flat the efficiency frontiers are. This flatness translates into the inflation rate being relatively impervious to the parameters in the policy objective function. Debelle and Stevens (1995) also found similar flat efficiency frontiers in their policy control simulations, using a model simpler, but not dissimilar, to the de Brouwer-O'Regan model. This flatness is unusual in the context of the literature, which typically finds the efficiency frontier to be roughly a rectangular hyperbola.

One explanation for the frontiers obtained in the de Brouwer-O'Regan model is that they are a consequence of dynamic homogeneity not holding. With the inflation's dynamic coefficients summing to much less than one, inflation has an automatic stabilizer. In fact inflation has finite variance even in the case where only the variance in the output gap is targeted. Normally with such a policy regime inflation would follow a unit root and have infinite variance. Moreover, without the lags of inflation present for dynamic homogeneity to hold, past inflation contains very little power for predicting future inflation. The output gap's impact on inflation through the mark-up term is also small and together these features make Taylor type rules an ineffective stabilization tool.

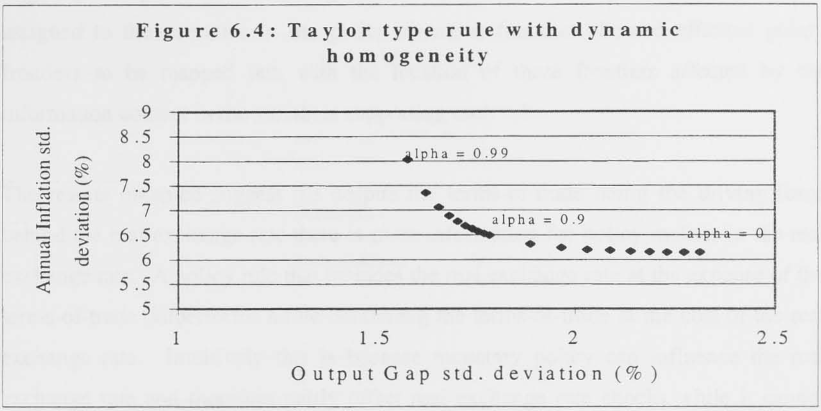
To assess the importance dynamic homogeneity has for the slope of the efficiency frontier, this Section imposes dynamic homogeneity on the model and constructs the resulting efficiency frontier assuming a Taylor type rule is used.[5] The results can be seen in Figures 6.3 and 6.4. Figure 6.3 takes the efficiency frontier for the Taylor type rule from Figure 6.1 while Figure 6.4 shows the efficiency frontier with dynamic homogeneity imposed.

Figure 6.3: Taylor type rule without dynamic homogeneity



Note the scale of Figure 6.3 is such that changes to the policy objective function's parameterization lead to very small changes in the variances of inflation and the output gap. This illustrates the ineffectiveness of the Taylor type rule as a stabilization tool in this model. Along the frontier the variance of the output gap changes by more that that for inflation.

[5] There is no unique way of introducing dynamic homogeneity into the model so there is a certain amount of arbitrariness in this process. Nevertheless, it is likely that imposing dynamic homogeneity is more important for the frontier than the lag structure assumed. For this reason the results obtained are illustrative if not conclusive.



In comparison, Figure 6.4 shows that with dynamic homogeneity the Taylor type rule becomes more effective in obtaining leverage over inflation and the output gap. The variances of inflation and the output gap change a lot more in response to weighting changes in the objective function. Moreover, with dynamic homogeneity imposed the variance of inflation changes by much more than that for the output gap as we move along the frontier. Nevertheless, the efficiency frontier in Figure 6.4 is still unusual in the context of the literature. Only when α rises above 0.9 does inflation really begin to rise to any great extent, thus the flat shape of the de Brouwer-O'Regan model's efficiency frontier is not completely explained by the absence of dynamic homogeneity. It may well be the case that it is the presence of the error correction terms, which introduces feedback from levels into growth rates, that is at the heart of the explanation behind the frontier's shape.

6.5) Conclusions

This paper takes the de Brouwer-O'Regan model of the Australia economy and uses it to explore the relative information content in the terms-of-trade and the real exchange rate. In Australia these two variables have a strong statistical association with the terms-of-trade the dominant force behind real exchange rate movements in the model. Using control theory, optimal simple rules that alternately included and excluded the real exchange rate and the terms-of-trade were constructed, motivated on inflation targeting and price level targeting policy objectives. Altering the relative weights assigned to the variables in the policy objective function allowed efficient policy frontiers to be mapped out, with the location of these frontiers affected by the information content in the variables supporting each rule.

The results obtained suggest the despite the terms-of-trade being the driving force behind the real exchange rate there is more information for policy makers in the real exchange rate. A policy rule that includes the real exchange rate at the expense of the terms-of-trade outperforms a rule containing the terms-of-trade at the cost of the real exchange rate. Intuitively this is because monetary policy can influence the real exchange rate and therefore partly offset real exchange rate shocks while it cannot influence the terms-of-trade.

Regarding the performance of Taylor type rules, the simulations show that Taylor type rules perform poorly in the de Brouwer-O'Regan model, a result that is in stark contrast to findings from closed economy models. Intuitively this is because in open economy models inflation and the output gap are influenced by a greater range of variables than just lags of the output gap and lags of inflation. Consequently, lags of the output gap and inflation are poor predictors of where future inflation and excess demand are heading, and rules based on just these variables will naturally tend to perform poorly.

In Section 6.4 we explored how the shape of policy efficiency frontiers was affected by whether dynamic homogeneity held in the model. The standard de Brouwer-O'Regan model does not possess dynamic homogeneity and its efficiency frontiers are unusually shaped. Imposing dynamic homogeneity did remove some of this flatness, making the frontiers more of a standard rectangular hyperbola shape, but this was not the whole answer. It is possible that it is the error correction terms that the de Brouwer-O'Regan model has that many monetary policy models do not have that provides the final piece of the answer.

Future work analyzing Australian monetary policy might concentrate on actual outcomes and ask the question how close to optimal is the Reserve Bank's behavior. Alternatively, one could go a step further and attempt to estimate the policy preference parameters assuming that they were behaving optimally. Chapter 7 explores how these parameters might be identified and estimated.

Chapter 7

STEPS TOWARD IDENTIFYING CENTRAL BANK POLICY PREFERENCES

7.1) Introduction

Modern analyses of central bank behavior begin with a policy objective function and construct policy rules by optimizing the objective function subject to a system of constraints. Descriptions of actual central bank behavior can also be obtained by estimating policy reaction functions directly. For the United States, Clarida, Gali, and Gertler (1998), Fuhrer (1997), and Judd and Rudebusch (1998) have all estimated reaction functions for the Federal Reserve. Taylor (1993) also developed a rule describing Federal Reserve policy decisions, popularly known as the Taylor rule. Clearly these estimated policy reaction functions and those developed through optimization are not unrelated. Optimal policy rules set the policy instrument as a linear function of the state vector. The feedback coefficients in these optimal rules are nonlinear functions of the parameters in the model constraining the optimization, as well as the parameters in the policy objective function. In principle it is these nonlinear parameter combinations that applied studies estimate.

A better understanding of monetary policy decisions can be had if the monetary authority's preferences can be disentangled and extracted from estimated policy rules. With these preferences in hand we would know which variables enter the policy objective function; which aspects of the economy the central bank is concerned about; and how senior central bank appointments affect the policy regime in operation. Because they relate directly to the policy regime in place, policy preferences, not estimated policy rules, are more informative of the objectives and incentives underpinning policy decisions.

Given a plausible economic model, and provided the estimated policy rule is the outcome of a constrained optimization process, it should be possible to find objective function parameters such that the optimal rule closely resembles the estimated rule. Of course, if these implied policy objective function parameters are to be informative it is important that the model constraining central bank behavior realistically capture the relationships at work in the economy. The objective of this chapter is to present conditions under which a policy regime in operation can be uncovered from the data.[1]

One of the most common objective functions employed in the monetary policy rules literature defines loss in terms of a linear combination of the unconditional variances of a vector of economic variables.[2] To formalize this, let $\mathbf{z}_t$ be a vector of economic variables, including the policy instrument(s). We assume that $\mathbf{z}_t$ is weakly stationary with unconditional mean vector $\mathbf{z}^*_t$. Each element in $\mathbf{z}_t$ has its counterpart in $\mathbf{z}^*_t$. Without loss of generality $\mathbf{z}^*_t$ is taken to equal the null vector.[3] Further, it is assumed that policy makers target the unconditional mean of $\mathbf{z}_t$, and therefore that $\mathbf{z}^*_t$ is also the target vector. With this notation every variable in $\mathbf{z}_t$ has a nominal target value. Of course, for many of these variables zero weight may be applied to their deviations from target in the objective function.

Denote the unconditional variance-covariance matrix of $\mathbf{z}_t$ by $\mathbf{\Omega}$. Let $\mathbf{W}$ be a symmetric, positive semi-definite, matrix of policy weights; $\mathbf{\Omega}$ and $\mathbf{W}$ share the same dimensions. The infinite horizon policy objective function is: $\text{Loss}[0,\infty] = \text{tr}[\mathbf{W\Omega}]$, where 'tr' is the trace operator. In many applications $\mathbf{W}$ is a diagonal matrix. Given this objective function, a policy regime is defined by the matrix of policy weights (preferences), $\mathbf{W}$, and the vector of targets, $\mathbf{z}^*_t$. It is the elements in this $\mathbf{W}$ matrix that we seek to identify.

The structure of the chapter is as follows. Section 7.2 develops the general economic structure within which subsequent analysis takes place. Using this general economic framework Section 7.3 systematically examines the conditions under which policy preferences can be identified. To illustrate how the identification conditions are applied in practice Section 7.4 considers several popular models and examines

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[1] Soderlind (1999) estimates the parameters in an objective function using a model of the United States. He does not consider identification however.
[2] See, for example, Ball (1997), Svensson (1999), Svensson (1998), Fair and Howrey (1996), Rudebusch and Svensson (1998), Clarida, Gali, and Gertler (1999), and Fuhrer (1997), among others.

whether their structure permits identification of the policy regime. Section 7.5 concludes.

7.2) A General Macroeconomic Setting

Consider the following macroeconomic specification:

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2 E_{t-1}\mathbf{y}_{t+1} + \mathbf{A}_3\mathbf{x}_t + \mathbf{v}_t, \qquad \mathbf{v}_t \sim \text{iid}[\mathbf{0}, \boldsymbol{\Sigma}] \tag{1}$$

where $\mathbf{y}_t$ is an n×1 vector of dependent variables, $\mathbf{x}_t$ a p×1 vector of policy instruments, $\mathbf{v}_t$ an n×1 vector of stochastic innovations, and E_{t-1} is the mathematical expectations operator conditional upon information set I_{t-1}, where $I_t = \{\mathbf{y}_t, \mathbf{x}_t, I_{t-1}\}$. Matrices $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$, and $\mathbf{A}_3$ contain structural parameters with dimensions conformable with $\mathbf{y}_t$ and $\mathbf{x}_t$ as needed. An alternative specification would have the expectations in (1) formed using period t rather than period t-1 information. More will be said about this alternative specification later, particularly in Section 7.4. Specification (1) is more general than may first appear. Models with complicated lag and lead structures can be manipulated into this form (Binder and Pesaran, 1995). Variables that are predetermined and time changes in policy instruments may be included in $\mathbf{y}_t$.

Assumption one: The instrument vector, $\mathbf{x}_t$, is set as a linear function of the state vector, $\mathbf{y}_{t-1}$.

Policy therefore follows the rule:

$$\mathbf{x}_t = \boldsymbol{\Psi}\mathbf{y}_{t-1}, \tag{2}$$

where the p×n matrix $\boldsymbol{\Psi}$ contains the policy feedback coefficients. Where necessary, lags of the instrument vector enter into this rule through $\mathbf{y}_{t-1}$. It is desirable to allow

[3] Normalizing $\mathbf{z}^*_t$ to equal zero is without loss of generality when policy decisions are constrained by a system of linear equality constraints. This normalization is not appropriate if some of the constraints are inequality constraints, such as a constraint preventing the nominal interest rate from going negative.

some elements of Ψ to equal zero or be otherwise restricted, thereby accommodating simple rules. Let the unrestricted elements of Ψ not set to zero be represented by the (b×1) vector φ.

Defining $\mathbf{z}_t = [\mathbf{y}_t^T\ \mathbf{x}_t^T]^T$ and combining (1) and (2) produces the system:

$$\mathbf{B}_0\mathbf{z}_t = \mathbf{B}_1\mathbf{z}_{t-1} + \mathbf{B}_2 E_{t-1}\mathbf{z}_{t+1} + \mathbf{u}_t. \qquad \mathbf{u}_t \sim \text{iid}[\mathbf{0}, \Xi] \tag{3}$$

Clearly $\mathbf{z}_t$ has dimensions (n+p)×1, and hence $\mathbf{\Omega}$ and $\mathbf{W}$ are (n+p)×(n+p) matrices. The central bank's behavior is formalized as follows:

Assumption two: The monetary authority operates under the regime: $\mathbf{W}$, $\mathbf{z}^*_t = \mathbf{0}$ $\forall$ t, and selects the unique $\varphi \in Q \subset \Re^b$, Q convex, that minimizes Loss[0,∞] = tr[$\mathbf{W\Omega}$], subject to (3).

7.3) Identifying the Policy Preference Matrix

This Section is central to the chapter. It provides necessary and sufficient conditions for identification of the policy preference matrix $\mathbf{W}$. Before turning to the details of these identification conditions, which are presented in a sequence of propositions, it is useful to underline from the outset what is known and what is to be determined.

Substituting (2) into (1) gives:

$$\mathbf{A}_0\mathbf{y}_t = (\mathbf{A}_1 + \mathbf{A}_3\mathbf{\Psi})\mathbf{y}_{t-1} + \mathbf{A}_2 E_{t-1}\mathbf{y}_{t+1} + \mathbf{v}_t. \tag{4}$$

The solution to (4) takes the form (see McCallum, 1983, or Uhlig, 1999):

$$\mathbf{y}_t = \mathbf{\Pi}_1\mathbf{y}_{t-1} + \mathbf{\Pi}_2\mathbf{v}_t, \tag{5}$$

where $\mathbf{\Pi}_2 = \mathbf{A}_0^{-1}$ and $\mathbf{\Pi}_1$ satisfies:

$$\mathbf{A}_0\mathbf{\Pi}_1 = \mathbf{A}_1 + \mathbf{A}_3\mathbf{\Psi} + \mathbf{A}_2\mathbf{\Pi}_1^2 . \qquad (6)$$

It is assumed that the reduced form parameters $\mathbf{\Pi}_1$, and $\mathbf{\Pi}_2$ and the feedback coefficients in the policy rule, $\mathbf{\Psi}$, are known. The identification strategy that follows does not impose restrictions on the error variance-covariance matrix and as a consequence information in $\mathbf{\Pi}_2$ is not employed for identification. Clearly this assumption could be relaxed and the information in $\mathbf{\Pi}_2$ brought into play. Nevertheless, using just $\mathbf{\Pi}_1$ and $\mathbf{\Psi}$, this Section establishes conditions for identifying $\mathbf{W}$.

Identification problems arise on several levels: first because the system is simultaneous; second because rational expectations terms are present; and third because the system is subject to control. As a consequence the identification strategy proposed below is a recursive one.

At its most simplistic the identification problem is one of imposing enough structure on the system so that estimates of the structural parameters can be backed out from the reduced form.[4] Elements in the feedback matrix $\mathbf{\Psi}$ are nonlinear functions of the structural parameters, $\mathbf{A_0}$, $\mathbf{A_1}$, $\mathbf{A_2}$, and $\mathbf{A_3}$, and the policy preferences, $\mathbf{W}$. The variables in the 'p' policy reaction function(s), entering nontrivially are predetermined state variables so identifying $\mathbf{\Psi}$ is not an issue.

For ease of exposition, define:

$$\mathbf{C} \overset{\text{def}}{=} \mathbf{A}_1 + \mathbf{A}_3\mathbf{\Psi} .$$

This $\mathbf{C}$ matrix is a commingling of the parameters applied to the state vector in equation (4). Now, variance-covariance matrix restrictions aside, (6) implies:

[4] Identification of simultaneous systems is analyzed thoroughly in Fisher (1966). Pesaran (1988) examines identification in rational expectations models.

$$[\mathbf{A}_0 \quad -\mathbf{C} \quad -\mathbf{A}_2]\begin{bmatrix}\boldsymbol{\Pi}_1 \\ \mathbf{I} \\ \boldsymbol{\Pi}_1^2\end{bmatrix} \overset{\text{def}}{=} [\mathbf{H}][\boldsymbol{\Gamma}] = [\mathbf{0}], \tag{7}$$

where, in matrix form

$$[\mathbf{C}] = [\mathbf{A}_1 \quad \mathbf{A}_3]\begin{bmatrix}\mathbf{I} \\ \boldsymbol{\Psi}\end{bmatrix} \overset{\text{def}}{=} [\mathbf{A}][\boldsymbol{\Lambda}]. \tag{8}$$

7.3.1) Step One

The first step in the recursive identification strategy involves identifying **H** in equation (7). Necessary and sufficient conditions for identifying **H** are summarized in:

Proposition one: Let the parameters in the i'th row of **H**, $\mathbf{h}_i$ (1×3n), be subject to 'r_i' linear inhomogeneous restrictions, $\mathbf{h}_i\mathbf{R}_i = \mathbf{r}_i$, where $\mathbf{R}_i$ has dimensions (3n×r_i), then a necessary condition for identifying **H** is $r_i \geq 2n$, $\forall$ i $\in$ [1,…,n]. A sufficient condition for identifying **H** is rank[$\boldsymbol{\Gamma}$ $\mathbf{R}_i$] = 3n, $\forall$ i $\in$ [1,…,n].

Proof:

The row vector $\mathbf{h}_i$ is subject to the following linear inhomogeneous restrictions:

$$[\mathbf{h}_i][\mathbf{R}_i] = [\mathbf{r}_i]. \tag{9}$$

Combining (7) with (9) produces:

$$[\mathbf{h}_i][\boldsymbol{\Gamma} \quad \mathbf{R}_i] = [\mathbf{0} \quad \mathbf{r}_i].$$

The dimensions of $\mathbf{h}_i$, $\boldsymbol{\Gamma}$, and $\mathbf{R}_i$ are 1×3n, 3n×n, and 3n×r_i respectively. Accordingly, $\mathbf{h}_i$ contains 3n parameters jointly subject to n+r_i restrictions. The restrictions in (9) include the normalization restriction arising when a dependent variable is chosen. Consequently, identifying $\mathbf{h}_i$ necessarily requires $r_i \geq 2n$. For these linear inhomogeneous restrictions to be sufficient requires $[\Gamma\ \mathbf{R}_i]$ to be such that rank$[\Gamma\ \mathbf{R}_i]$ = 3n (see Fisher, 1966).

∎

Proposition one is very intuitive. In a standard simultaneous equations system without rational expectations identifying an equation requires at least as many restrictions be imposed as there are endogenous variables ($r_i \geq n$). With the rational expectations term present each equation has an additional 'n' parameters to identify, but the number of reduced form parameters available is unchanged. It directly follows that 'n' additional restrictions must be imposed to achieve identification.

7.3.2) Step Two

While *proposition one* provides conditions under which **H** is identified, and identification of **H** implies identification of **C**, it does not separately identify $\mathbf{A}_1$ and $\mathbf{A}_3$. This leads to:

Proposition two: Let the i'th row of **A**, $\mathbf{a}_i$ $(1\times(n+p))$, be subject to 'q_i' linear inhomogeneous restrictions, $\mathbf{a}_i\mathbf{Q}_i = \mathbf{q}_i$, where $\mathbf{Q}_i$ has dimensions $((n+p)\times q_i)$, then a necessary condition for identifying **A** is $q_i \geq p$, $\forall\ i \in [1,\ldots,n]$. A sufficient condition for identifying **A** is that rank$[\Lambda\ \mathbf{Q}_i] = n+p$, $\forall\ i \in [1,\ldots,n]$.

Proof:

From equation (8):

$$[\mathrm{A}][\Lambda]=[\mathrm{C}]. \tag{10}$$

Assume further that $\mathbf{a}_i$ is subject to 'q_i' restrictions of the form:

$$[\mathbf{a}_i][\mathbf{Q}_i]=[\mathbf{q}_i]. \tag{11}$$

Combining (10) and (11) gives:

$$[\mathbf{a}_i][\Lambda \quad \mathbf{Q}_i]=[\mathrm{C}_i \quad \mathbf{q}_i].$$

The dimensions of $\mathbf{a}_i$, Λ, and $\mathbf{Q}_i$ are $1\times(n+p)$, $(n+p)\times n$, and $(n+p)\times q_i$ respectively. Therefore $\mathbf{a}_i$ contains n+p parameters that are collectively subject to $n+q_i$ linear restrictions. Consequently, a necessary condition for identification is $q_i \geq p$. A sufficient condition for identification of $\mathbf{a}_i$ is $\text{rank}[\Lambda\ \mathbf{Q}_i] = n+p$.

∎

In terms of the recursive identification strategy the role of *proposition two* is to disentangle the elements in $\mathbf{A}_3$ from those in $\mathbf{A}_1$. The elements in these two matrices are mingled because monetary policy is set conditional upon the state vector. In words *propositions two* states that a necessary condition for identification is that a restriction on the elements in $\mathbf{A}_1$ and $\mathbf{A}_3$ be imposed for each control variable in the system. Clearly if an equation contains all state variables and an instrument is set as a linear function of all of the state variables, then the coefficient in $\mathbf{A}_3$ associated with that instrument in that equation cannot be identified.

7.3.3) Step Three

Thus far in the identification strategy information contained in the policy feedback matrix Ψ has not been used. As long as the rank conditions of *proposition one* and *two* hold, then information in the reduced form coefficients, Π_1 is sufficient to identify all the coefficients in the structural model. In this final identification step we introduce Ψ. If Ψ is determined optimally, then its elements will be nonlinear functions of the structural parameters ($\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$, and $\mathbf{A}_3$), and also $\mathbf{W}$. Provided the structural parameters are identified a crucial ingredient in Ψ is known. This third and final step establishes necessary and sufficient conditions under which knowledge of Ψ and the structural parameters can be used to identify $\mathbf{W}$. These conditions are summarized in:

Proposition three: Let the column vector $\mathbf{w} = \text{vech}(\mathbf{W})$ be subject to 's' linear inhomogeneous restrictions, $\mathbf{S}^T\mathbf{w} = \mathbf{s}$, then a necessary condition for global identification of $\mathbf{W}$ is: $s \geq \frac{(n+p)(n+p+1)}{2} - b$. A sufficient condition for global identification of $\mathbf{W}$ is: $\text{rank}[\mathbf{J}(\mathbf{w}^{**})^T\ \mathbf{S}]^T = k = [(n+p)(n+p+1)/2]$, $\forall\ \mathbf{w}^{**}$

$\in P \subset \Re^{k}_{+}$, where $\mathbf{J}(\mathbf{w}^{**})$ is the Jacobian of the transform f: P → Q defined below.

Proof:

In its most general form **W** is a square, symmetric, matrix containing $(n+p)^2$ parameters. Symmetry reduces the number of independent parameters in **W** to $[(n+p)(n+p+1)]/2$. In what follows let $k = [(n+p)(n+p+1)/2]$.

An outcome of the policy optimization is a continuously differentiable function f: P → Q relating the policy preferences to the coefficients in the policy rule:[5]

$$\varphi = f(\mathbf{w})\,. \tag{12}$$

Recall that φ is a (b×1) vector containing the elements of Ψ that are unrestricted. **w** is also subject to 's' linear inhomogeneous restrictions of the form:

$$\mathbf{S}^{T}\mathbf{w} = \mathbf{s}\,. \tag{13}$$

The policy objective function is only defined up to a scalar allowing one element of **w** to be normalized upon. This normalizing restriction is subsumed into (13). Remaining restrictions on **w** are most likely to take the form of exclusion restrictions, particularly on the covariance elements of $\mathbf{\Omega}$. The non-linearity of (12) complicates identification. From the mean value theorem there exists a $\mathbf{w}^{**}$ between $\mathbf{w}^{*}$ and **w**, each elements of P, such that:

$$\varphi^{*} = \varphi + \mathbf{J}(\mathbf{w}^{**})(\mathbf{w}^{*} - \mathbf{w})\,, \tag{14}$$

where $\mathbf{J}(\mathbf{w}^{**})$ is an b×k Jacobian matrix. Combining (13) and (14) allows the restrictions on **w** to be represented as:

[5] The structural parameters have been subsumed into the functional form.

$$\begin{bmatrix} \mathbf{J}(\mathbf{w}^{**}) \\ \mathbf{S}^T \end{bmatrix} \mathbf{w} = \begin{bmatrix} \boldsymbol{\varphi} - \boldsymbol{\varphi}^* + \mathbf{J}(\mathbf{w}^{**})\mathbf{w}^* \\ \mathbf{s} \end{bmatrix}. \tag{15}$$

From (15) a sufficient condition for global identification of $\mathbf{w}$ is rank$[\mathbf{J}(\mathbf{w}^{**})^T \ \mathbf{S}]^T = k$, $\forall \ \mathbf{w}^{**} \in P$ (see Rothenberg, 1971). Notice, however, that $\forall$ n, p > 0, k > np ≥ b. Therefore, rank$[\mathbf{J}(\mathbf{w}^{**})]$ is at most 'b', which implies that a necessary condition for global identification is s ≥ k–b, or after substituting for k, $s \geq \frac{(n+p)(n+p+1)}{2} - b$.

∎

The necessary condition of *proposition three* has a clear interpretation: the 'b' coefficients in $\boldsymbol{\varphi}$ can be used to identify at most 'b' elements in $\mathbf{W}$. An interesting aspect of *proposition three* is that the Jacobian matrix $\mathbf{J}(\mathbf{w}^{**})$ itself need not have full rank for all $\mathbf{w}^{**} \in P$. A singularity in the Jacobian matrix means that there is no information in the functional relationship between $\mathbf{w}$ and $\boldsymbol{\varphi}$ to tie down one or more parameters in $\mathbf{w}$. However, provided this lack of information in $\mathbf{J}(\mathbf{w}^{**})$ can be offset by additional outside information in the form of additional columns in $\mathbf{S}$ identification is still possible.

7.3.4) In Addition...

Of course equation (2) implies that the relationship between the policy instruments and the predetermined variables is a deterministic one. Rarely would this be the case. In practice the information set used by agents to form their expectations, and that an econometrician uses when estimating policy reaction functions, may only be a subset of that available to the monetary authority when it sets policy. This can arise if the monetary authority uses a more recent information set than other agents. Where this is the case deviations between the actual path of $\mathbf{x}_t$ and that predicted by (2) are to be expected. These deviations are accommodated by adding a p×1 innovation vector $\mathbf{e}_t$, uncorrelated with $\mathbf{v}_t$ and $\mathbf{y}_{t-1}$, to equation (2) giving:

$$\mathbf{x}_t = \boldsymbol{\Psi}\mathbf{y}_{t-1} + \mathbf{e}_t. \tag{16}$$

Intuitively, adding this innovation vector facilitates identification of $\mathbf{A_1}$ and $\mathbf{A_3}$ because it automatically imbues $\mathbf{x}_t$ with volatility absent from $\mathbf{y}_{t-1}$. The addition of the policy innovation term leads to:

Proposition four: Given (1) and (16) $\mathbf{A_1}$ and $\mathbf{A_3}$ are identified provided the rank condition of *proposition one* holds.

Proof:

Substituting (16) into (1) produces:

$$\mathbf{A}_0\mathbf{y}_t = (\mathbf{A}_1 + \mathbf{A}_3\mathbf{\Psi})\mathbf{y}_{t-1} + \mathbf{A}_2\mathrm{E}_{t-1}\mathbf{y}_{t+1} + \mathbf{A}_3\mathbf{e}_t + \mathbf{v}_t. \qquad (17)$$

Equation (16) is identified because it contains only predetermined variables and therefore both $\mathbf{\Psi}$ and $\mathbf{e}_t$ are assumed to be known. The rational expectations solution to (17) takes the form:

$$\mathbf{y}_t = \mathbf{\Pi}_1\mathbf{y}_{t-1} + \mathbf{\Pi}_2\mathbf{v}_t + \mathbf{\Pi}_3\mathbf{e}_t, \qquad (18)$$

where $\mathbf{\Pi}_2 = \mathbf{A}_0^{-1}$, $\mathbf{\Pi}_1$ satisfies (6), and $\mathbf{\Pi}_3 = \mathbf{A}_0^{-1}\mathbf{A}_3$. The reduced form parameter matrices $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_3$ are known and the solution to (17) asserts that once $\mathbf{A}_0$ is identified so too is $\mathbf{A}_3$. *Proposition one* presents a sufficient condition for $\mathbf{A}_0$ to be identified.

∎

Finally, we may wonder how these identification conditions would be affected if the expectations in (1) were formed using period t rather than period t-1 information. It is not difficult to show that provided variance-covariance matrix restrictions are not used for identification, and provided the policy rule continues to depend only on $\mathbf{y}_{t-1}$ and does not contain $\mathbf{v}_t$, *propositions one - four* remain unaffected. The intuition behind this result is that the solution for $\mathbf{\Pi}_1$ is unaltered by the change to period t information. The solution for $\mathbf{\Pi}_2$ does change, but it is not required for identification provided $\mathbf{v}_t$ does not enter the policy rule.

7.4) Some Examples

This Section takes some popular models from the literature and examines whether they satisfy the conditions necessary for identification of policy regimes. The aims of this Section are twofold. First the Section aims to illustrate how *propositions one*, *two*, and *three* are applied in practice. Second, the Section aims to investigate the suitability of various models as vehicles for identifying policy regimes.

All models considered contain the variables: y_t, π_t, and i_t, representing the output gap, inflation and the nominal interest rate respectively, and as a consequence the policy objective function used throughout this Section is taken to be:

$$\text{Loss}[0,\infty] = \alpha \text{Var}[\pi_t] + (1-\alpha)\text{Var}[y_t] + \sigma \text{Var}[i_t].$$

Accordingly, the systems examined require that we identify just two policy preference parameters. As such, for each system **S** has four independent columns implying s = 4.

7.4.1) Example One

Consider the following system:

$$y_t = \beta y_{t-1} - \gamma[i_t - E_{t-1}\pi_{t+1}] + g_t, \tag{19}$$

$$\pi_t = \pi_{t-1} + \lambda y_{t-1} + u_t, \tag{20}$$

$$i_t = \varphi_y y_{t-1} + \varphi_\pi \pi_{t-1}. \tag{21}$$

Equation (19) is a dynamic IS curve, (20) an accelerationist Phillips curve, and (21) the policy reaction function. The stochastic terms - g_t and u_t - are assumed to be finite variance white noise processes. Observe first that equations (20) and (21) are identified in so much as for them the rank conditions of *propositions one* and *two* hold. For subsequent identification of other coefficients the coefficients in equations (20) and (21) are assumed known. Next note that with two feedback parameters in (21) and two independent policy preference coefficients the necessary condition of *proposition three* is satisfied. It just remains to be seen whether *propositions one* and *two* hold when applied to (19).

In terms of equation (4) an unconstrained representation of (19) takes the form:

$$\eta_1 y_t = \eta_2 \pi_t + c_1 y_{t-1} + c_2 \pi_{t-1} + \rho_1 E_{t-1} y_{t+1} + \rho_2 E_{t-1} \pi_{t+1} + g_t . \tag{22}$$

The system (19) - (21) has n = 2 and p = 1. Therefore, the order condition of *proposition one* requires that (19) place at least four restrictions ($r_i \geq 2n$) on the structure of (22). Relative to (22), equation (19) imposes: $\eta_1 = 1$; $\eta_2 = 0$; $\rho_1 = 0$; and $\rho_2 = c_2/\varphi_\pi$. Thus *proposition one*'s order condition for identification is satisfied.

Proposition two requires the number of restrictions on the elements of $\mathbf{A_1}$ and $\mathbf{A_3}$ associated with the IS curve be greater than or equal to the number of policy instruments. It is useful to rewrite (22) as:

$$\eta_1 y_t = \eta_2 \pi_t + \lambda_1 y_{t-1} + \lambda_2 \pi_{t-1} + \rho_1 E_{t-1} y_{t+1} + \rho_2 E_{t-1} \pi_{t+1} + \gamma i_t + g_t . \tag{23}$$

In light of (23), the restrictions on the IS curve's structural parameters take the form:

$$[\lambda_1 \quad \lambda_2 \quad -\gamma] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ \varphi_y & \varphi_\pi & 0 \end{bmatrix} = [c_1 \quad c_2 \quad 0].$$

Provided π_{t-1} enters the policy reaction function non-trivially ($\varphi_\pi \neq 0$) the rank condition of *proposition two* is satisfied.[6] In this system the policy regime can be identified.

7.4.2) Example Two

The second example is adapted from McCallum (1997) and consists of the following equations for the output gap and inflation:

[6] Notice that φ_y can equal zero, but not φ_π, and the system is still identified. Thus the optimal simple inflation rule $i_t = \varphi_\pi \pi_{t-1}$ can still be examined. A policy rule where the interest rate responds only to the output gap might usually be expected to lead to nominal indeterminacy. In this model it produces an unidentified system. Moreover, with only one parameter in the policy rule the order condition for *proposition three* is not met so the two policy preference parameters cannot be identified.

$$y_t = E_{t-1}y_{t+1} - \gamma[i_t - E_{t-1}\pi_{t+1}] + g_t \tag{24}$$

$$\pi_t = E_{t-1}\pi_{t+1} + \lambda y_t + u_t . \tag{25}$$

To add some persistence both g_t and u_t are assumed to follow AR(1) processes:

$$\begin{bmatrix} g_t \\ u_t \end{bmatrix} = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \begin{bmatrix} g_{t-1} \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} \bar{g}_t \\ \bar{u}_t \end{bmatrix} .$$

McCallum's system is completed by the addition of the policy reaction function:

$$i_t = \varphi_g g_{t-1} + \varphi_u u_{t-1} . \tag{26}$$

Analogous to example one the policy reaction function (26) is identified and contains two feedback coefficients. With only two independent parameters in the policy objective function *proposition three*'s order condition is meet. Unfortunately neither (24) nor (25) are identified. In unrestricted form equation (25) has the form:

$$\eta_1\pi_t = \eta_2 y_t + \eta_3 g_t + \eta_4 u_t + \lambda_1\pi_{t-1} + \lambda_2 y_{t-1} + \lambda_3 g_{t-1} + \lambda_4 u_{t-1}$$

$$+ \rho_1 E_{t-1}\pi_{t+1} + \rho_2 E_{t-1} y_{t+1} + \rho_3 E_{t-1} g_{t+1} + \rho_4 E_{t-1} u_{t+1} + \bar{u}_t . \tag{27}$$

Comparing equation (25) with (27) the independent restrictions imposed on (27) are: $\eta_1=1$; $\eta_3=0$; $\eta_4=0$; $\lambda_1=0$; $\lambda_2=0$; $\lambda_3 = 0$; $\rho_1=1$; $\rho_2=0$; $\rho_3=0$; and $\rho_4=0$. These restrictions number ten while the order condition of *proposition one* requires only eight restrictions. In terms of this necessary condition equation (25) is over-identified. Appearances are deceiving, however. For while equation (25) satisfies *proposition one*'s order condition it fails the rank condition. To see this, observe that the restrictions listed exclude π_{t-1}, y_{t-1}, and g_{t-1} from the system. At the same time the rational expectations solution to the system expresses π_t and y_t in terms of just u_{t-1}, also excluding[7] π_{t-1} y_{t-1}, and g_{t-1}. Thus three of the columns in the [Γ $\mathbf{R}_i$] matrix

[7] That all elements in Π_1 associated with π_{t-1} and y_{t-1} equal zero is clear because these two variables do not appear in the system's structure and hence only enter the system's state vector trivially. That the elements associated with g_{t-1} also equal zero (or for one equation μ_1) is a consequence of the system being subject to control.

associated with (25) depend linearly on the others and $[\Gamma\ \mathbf{R}_i]$ has rank = 11 < 12 (3n, where n = 4). In a similar vein it can be shown that equation (24) is also unidentified.

7.4.3) Example Three

Our final example comes from Clarida, Gali and Gertler (1999) and is of the same genre to those just analyzed. It differs, however, in that the persistence in the system is endogenous, determined by lagged dependent variables, and not exogenous, driven by autocorrelated shocks. In fact the shocks g_t and u_t are assumed to be finite variance white noise processes.

$$y_t = \beta y_{t-1} + (1-\beta)E_{t-1}y_{t+1} - \gamma[i_t - E_{t-1}\pi_{t+1}] + g_t \tag{28}$$

$$\pi_t = \delta\pi_{t-1} + (1-\delta)E_{t-1}\pi_{t+1} + \lambda y_t + u_t \tag{29}$$

$$i_t = \varphi_y y_{t-1} + \varphi_\pi \pi_{t-1}$$

Like the previous two examples the order condition of *proposition three* is satisfied because the policy reaction function contains two feedback coefficients. Now consider equation (28). In the structure of equation (4), at its most general, equation (28) becomes:

$$\eta_1 y_t = \eta_2\pi_t + c_1 y_{t-1} + c_2\pi_{t-1} + \rho_1 E_{t-1}y_{t+1} + \rho_2 E_{t-1}\pi_{t+1} + g_t\,. \tag{30}$$

Relative to (28) equation (30) imposes the four restrictions: $\eta_1 = 1$; $\eta_2 = 0$; $c_2 = -\rho_2\varphi_\pi$; and $c_1 = 1-\rho_1-\rho_2\varphi_y$, which with n = 2 means that the order condition of *proposition one* is met. Provided neither β nor δ equal zero the rank condition is also satisfied.[8] Moreover, if φ_π is non-zero the rank condition of *proposition two* is also satisfied implying that equation (28) is identified. If φ_π does equal zero, then the order condition of *proposition three* does not hold and the policy preference parameters cannot be identified.

Now consider the Phillips curve, equation (29). The unrestricted Phillips curve is:

$$\eta_2\pi_t = \eta_1 y_t + c_1 y_{t-1} + c_2\pi_{t-1} + \rho_1 E_{t-1} y_{t+1} + \rho_2 E_{t-1}\pi_{t+1} + u_t \tag{31}$$

Relative to (31) equation (29) imposes the restrictions: $\eta_2 = 1$; $c_1 = 0$; $c_2 + \rho_2 = 1$; and $\rho_1 = 0$. These four restrictions satisfy *proposition one*'s necessary condition for identification. Like the IS curve (28), provided neither β nor δ equal zero the rank condition of *proposition one* is also met. As a consequence for this system the policy regime is identifiable.

What these three examples illustrate is that provided we consider only the order condition for identifying the policy preferences (*proposition three*) the major obstacle faced when identifying **W** is that of identifying the structure of the economy. Identifying the economy's structure is essential, however, because it constrains the optimization process leading to the policy rule.

7.5) Conclusions

The aim of this chapter was simple. We wanted to lay the foundations for estimating central bank policy preferences by establishing conditions under which these preference parameters could be identified. It was demonstrated that optimizing central banks apply policy rules whose feedback coefficients are nonlinear functions of its policy preferences. Before these policy preferences can be backed out from these feedback coefficients several identification conditions need to hold. As a consequence this chapter proposes a recursive identification strategy consisting of three steps. The first two steps, summarized in *propositions one* and *two*, identify the parameters in the structural model constraining the central bank's optimization. Only once the structural model is identified can enough structure be placed on the policy reaction function to disentangle the policy preference coefficients. *Proposition three* provides necessary and sufficient conditions for the policy preference coefficients to be identified.

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[8] When $\beta = 0$, for example, y_{t-1} is not a state variable in the system and φ_y appropriately equals zero. Consequently, the column of Π_1 associated with y_{t-1} equals zero, leading to the rank condition of *proposition one* failing.

To illustrate how the conditions developed in *propositions one*, *two* and *three* are applied in practice three examples were provided and their identification properties examined. The identification conditions developed in this chapter are important because only by identifying and estimating the policy regime in operation can we tell what the objectives of the monetary authority truly are. In particular, this chapter serves to emphasize that it is not necessarily possible to say anything meaningful about a policy regime purely on the basis of an estimated policy rule. Future work will seek to identify and estimate actual policy regimes in operation, and to document how these policy regimes have changed over time.

Chapter 8

DISCRETIONARY MONETARY POLICY WITH COSTLY INFLATION

8.1) Introduction

Following Kydland and Prescott (1977) it is commonly recognized that through time inconsistency discretionary monetary policy can lead to a positive inflation bias. Several authors have addressed this issue proposing methods to either reduce or eliminate this inflation bias. Rogoff (1985) suggests appointing an inflation averse central bank governor while Walsh (1995) shows that an optimal contract for the central bank governor can solve what is essentially a principal agent problem. Other approaches rely on the central bank governor either being concerned with their reputation as an 'inflation fighter' or making the governor deposit a nominal bond that is released only at the conclusion of their contract.

More recently Pearce and Sobue (1997) demonstrate that any inflation bias present is lowered when the central bank is uncertain about the strength with which its policy instrument affects inflation. Uncertainty leads policy makers to err on the side of caution, resulting in cautious policy, and a lower inflation bias.

This paper explores the implications for discretionary monetary policy of costly inflation. We consider a Phillips curve where a sub-optimally high inflation rate reduces welfare by permanently lower real output. Section 9.2 motivates the case for a non-vertical Phillips curve and describes the economic model used to analyze discretionary policy. Section 9.3 examines how costly inflation effects the magnitude of the discretionary inflation bias. Section 9.4 concludes.

8.2) Rules versus Discretion with Costly Inflation

Many authors have suggested that there may be a negative relationship between inflation and output or, alternatively, between inflation and output growth. Feldstein (1982) argues that inflation can interact with a nominal based tax system raising the cost of capital and lowering investment and output. De Gregorio (1993) moots that firms have to hold money balances to purchase capital. The cost of holding money increases as inflation rises, thus high inflation retards investment, which lowers output.

Another common argument is that inflation distorts price signals inducing agents to waste resources searching for bargains or otherwise avoiding the effects of inflation (menu costs, shoe leather costs, etc). Similarly, high inflation is generally thought to come hand-in-hand with high inflation variability, or inflation uncertainty. In the face of this uncertainty, Fischer (1993) argues, firms may delay investing until the uncertainty is resolved, and this delay lowers output.[1]

It is standard in the monetary policy literature (see Walsh, 1998) for the central bank to choose inflation to minimize the loss function:[2]

$$\text{Loss} = E_{t-1}[\frac{1}{2}(\pi_t - \pi^*)^2 + \frac{\lambda}{2}(y_t - ky_t^*)^2], \quad k > 1, \lambda > 0, \tag{1}$$

subject to

$$y_t = y_t^* + \alpha(\pi_t - E_{t-1}\pi_t) + u_t, \qquad \alpha > 0. \tag{2}$$

where π_t is the inflation rate, π^* is the inflation target, y_t represents (logged) real output, y_t^* represents (logged) potential real output, and $u_t \sim iid[0,\sigma^2]$ is a supply shock. Finally E_{t-1} is the mathematical expectations operator conditional upon period

[1] Further arguments regarding the costs of inflation can be found in De Gregorio (1993) and Stockman (1991). Alternatively, the Tobin effect suggests that inflation may not always be detrimental for output, Tobin (1965).

[2] We use this loss function because it is standard in the literature and because we want to explore the consequences of costly inflation for the inflation bias in the context of the core literature. An

t-1 information. We assume that period t-1 information includes all variables dated period t-1 or earlier as well as the structure of the economy.

At this juncture it is useful to ask what is optimal about the inflation target rate π^*, or, more fundamentally, why inflation is in the loss function at all? The point to be made is that in the economy described by equation (2) only unanticipated inflation matters. Anticipated inflation has no effect on the economy – even in the short run – making it unclear why the loss function contains inflation. Moreover, with neutral inflation any rate of anticipated inflation is as good as any other; any rate of inflation is optimal.

In light of these arguments, and the literature on costly inflation discussed above, we respecify the (inverted) Phillips curve as

$$y_t = y_t^* + \alpha(\pi_t - E_{t-1}\pi_t) - f(\pi_t) + u_t, \qquad \alpha > 0 \tag{3}$$

where $f(\pi_t)$, $f' = \beta \geq 0$, represents the output cost associated with anticipated inflation. We assume that $f(\pi_t)$ equals zero at π^*. It is in this sense that π^* is an optimal inflation rate - it eliminates any effect of anticipated inflation on output. A linear Taylor series approximation to $f(\pi_t)$ about π^* gives

$$f(\pi_t) = f(\pi^*) + \beta(\pi_t - \pi^*). \tag{4}$$

Imposing $f(\pi^*) = 0$ and substituting (4) into (3) generates the Phillips curve

$$y_t = y_t^* + \alpha(\pi_t - E_{t-1}\pi_t) - \beta(\pi_t - \pi^*) + u_t. \tag{5}$$

8.3) Discretionary Inflation Bias

Substituting (5) into (1) and maximising, while holding inflation expectations constant, yields the inflation equation

alternative approach might have used a utility based loss function following Cubitt (1993) or Ireland

$$\pi_t = \pi^* + \frac{(k-1)(\alpha-\beta)\lambda y_t^*}{(1-(\alpha-\beta)\lambda\beta)}. \tag{6}$$

The second term on the RHS of (6) represents the inflation bias. If we had used the Phillips curve (2) in place of (5), then the inflation bias would be $(k\text{-}1)\alpha\lambda y_t^*$. There are several points we can make about the magnitude and sign of the inflation bias when anticipated inflation is costly.

1) When $\beta = 0$ the bias collapses down to $(k\text{-}1)\alpha\lambda y_t^*$: the costless inflation case.
2) Depending on α and β the inflation bias can be either positive or negative. When $\beta = 0$ the inflation bias is strictly non-negative.
3) When $\beta = \alpha$ the output cost of anticipated inflation completely offsets any benefits to surprise inflation and the inflation bias equals zero.
4) In the limit as $\beta \to \infty$, the inflation bias $\to 0$ from below.
5) If $\beta > \alpha$, then the inflation bias with costly inflation must be smaller that the inflation bias without the output cost.

The inflation bias under costless inflation equals that for costly inflation when $\frac{(\alpha-\beta)}{(1-(\alpha-\beta)\lambda\beta)} = \alpha$. This occurs when $\beta = 0$, $(\alpha^2\lambda\text{-}1)/\alpha\lambda$, which we denote β_1 and β_2 respectively. The first of these roots, β_1, produces the familiar result that, if $\beta = 0$, equation (5) collapses to equation (2). With the second root, β_2, note that the denominator $\alpha\lambda$ is always positive, implying that β_2 will be positive or negative depending on whether $\alpha^2\lambda\text{-}1$ is greater or less than zero. Given our theoretical assumption that β is non-negative the only economically interesting case is that where $\alpha^2\lambda\text{-}1 \geq 0$.

To examine how the inflation bias changes with β we differentiate it with respect to β, holding all else constant. Evaluating this derivative at $\beta = 0$ gives

$$\frac{\partial(\text{bias})}{\partial\beta}\Big|_{\beta=0} = (k-1)\lambda)\lambda^2\lambda - 1)y_t^* \tag{7}$$

(1997).

When $\alpha^2\lambda$-1 > 0, the slope of the bias equation is positive at $\beta = 0$, and conversely negative if $\alpha^2\lambda$-1 < 0.

Similarly, the derivative of the bias with respect to β, evaluated at β_2, equals

$$\frac{\partial(\text{bias})}{\partial\beta}\bigg|_{\beta=\beta_2} = \frac{(1-k)(\alpha^2\lambda-1)y_t^*}{(1-(\alpha-\beta)\lambda\beta)^2} \tag{8}$$

Clearly the denominator of (8) is positive. Consequently, because k > 1, we have the result that at $\beta = \beta_2$ the derivative is positive if $\alpha^2\lambda$-1 < 0 and negative if $\alpha^2\lambda$-1 > 0

Therefore, we can make two further observations about the inflation bias

6) If $\alpha^2\lambda$-1 < 0, then so too is β_2, and the only economically interesting point where the biases are equal is at $\beta = 0$. The slope of the bias function is negative at $\beta = 0$ indicating that the bias with costly inflation is less that that where inflation is costless for all $\beta > 0$.

7) If $\alpha^2\lambda$-1 > 0, then there are two economically interesting points at which the biases are equal. For $\beta \in [\beta_1, \beta_2]$ the bias with costly inflation is higher than that for costless inflation. As β increases above β_2 the opposite result prevails.[3]

Of interest here is the fact that when $\alpha^2\lambda$-1 > 0, for all $\beta \in [0, (\alpha^2\lambda-1)/\alpha\lambda]$ the inflation bias actually increases despite the fact that this inflation bias has a permanent output cost. The inflation bias rises when α and λ are large and β is small. This is the case where the monetary authority places a large weight on output stabilization and gains a large benefit from surprise inflation. If the permanent cost of inflation is small then the monetary authority is prepared to incur this cost, but to offset the lost output due to the inflation cost the monetary authority must create a greater inflation surprise than it otherwise would. Consequently, the inflation bias is greater in this case than it would be if inflation were costless.

[3] Here we further assume that $\alpha^2\lambda < 4$ to rule out discontinuous jumps in the bias as β increases.

Figure 9.1 graphs the inflation bias (as a percent of potential output) under hypothesized values for α, and k, for two values of λ as β changes. The solid line has $\alpha^2\lambda-1 < 0$ while the dashed line has $\alpha^2\lambda-1 > 0$.

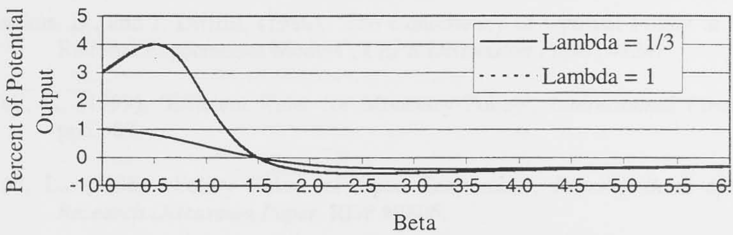
To examine whether anticipated costs of inflation are likely to have any material impact of the size of the inflation bias we parameterize the $f(\pi_t)$ function using estimates from Fischer (1993). Table 4 of Fischer (1993) presents an estimate for β of 0.046 using panel estimation.[4] Taking $\beta = 0.046$, $\lambda = 0.5$ (inflation receives twice the weight of output in the loss function), $k = 1.02$, and $\alpha = 2.5$ gives an inflation bias of 0.025% of potential when inflation is not costly and a bias that is approximately 0.026% of potential when inflation is costly. In this example the presence of an output cost of inflation actually raises the inflation bias by about 4%. While small this increase in inflation is nevertheless noteworthy, given the plausibility of the underlying parameters.

8.4) Conclusions

In this paper we have extended the standard 'rules versus discretion' model to include the case where anticipated inflation has a permanent output cost. When inflation is costly we have shown that the inflation bias associated with discretionary policy is generally smaller than that when inflation is costless. Interestingly, if the marginal cost of inflation is large enough the inflation bias can even be negative. Finally, we have also shown that for some parameter values the presence of costly inflation actually increases the size of the bias. Interestingly, using an estimate of the cost of anticipated inflation we have presented an example where the inflation bias actually increases.

[4] Details of the sample period and countries included to generate this estimate are available in Fischer (1993).

Figure 9.1: Inflation Bias.
Alpha = 1.5, k =1.02



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