THE INFLUENCE OF A MAGNETIC FIELD ON THE INTERACTION
BETWEEN A LAMINAR BOUNDARY LAYER AND A
SUPersonic Mainstream

Thesis submitted for the degree of
Master of Science

by

John Phillip Rayner, B.Sc.

Department of Physics,
Australian National University,
1967.
The work contained in this thesis is my own except where another source is indicated.

[Signature]
ACKNOWLEDGEMENTS

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SUMMARY

A preliminary theoretical and experimental investigation of the interaction between a cool laminar boundary layer and a hot supersonic mainstream in the presence of a magnetic field has been carried out using shock ionized Argon as the test gas. A simple theory of the interaction is modified to include the effect of magnetic body forces on the mainstream by developing an analogy between the forces and the pressure gradients caused by surface curvature. It is shown that the field should increase the pressure rise to separation, while decreasing the associated flow deflection, and that the size of the effect depends on the distance to separation. It is also shown that the field should increase the length of a step-induced separated region, provided the pressure in the region is constant.

A photographic investigation of the self-luminous gas flow over a step was conducted using a free piston shock tube. Since the separation zone itself was too small to observe directly, the experiments concentrated on the length of the separated region. It was found that the separated length increased with time, and that the magnetic field in fact slightly reduced the length. The reasons for the decrease in length are not known, but it
is suggested that it is due to the influence of the field on the flow within the separated region itself.

The field produced no discernible change in the conditions at separation, which suggests that separation distance is very small. This view is supported by previous theories which indicate that the distance is drastically reduced by strong heat transfer effects, so that in the present case the distance is about one boundary layer thickness in length. Therefore a marked increase in the pressure at separation requires an intense magnetic field, situated close to the interaction, whose dimensions are comparable with the boundary layer's thickness.
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LIST OF SYMBOLS

This list consists of those symbols which appear more than once in the text. The symbols commonly accepted in the various areas considered have been retained, which has led to a certain amount of duplication. However, the appropriate meaning for a symbol should be apparent from the context in which it appears. MKS units have been used for electro-magnetic quantities while CGS units have generally been employed in other connections.

Capital Letters.

B Magnetic flux intensity. (Also referred to as the magnetic field).

B_0 Maximum value of the magnetic field.

C_f Skin friction coefficient.

C_{f0} Skin friction coefficient at beginning of interaction.

C_p Specific heat.

C_p Pressure coefficient \( = (p - p_0)/(\frac{1}{2} \rho_0 u_0^2) \)

C_{ps} " " " at separation, \( B = 0 \)

C_s " " " " " \( B \neq 0 \)

C_{pp} " " " plateau \( B = 0 \)

C_p " " " " \( B \neq 0 \)

D_{em} Ambipolar diffusion coefficient.

E Electric field.

\( \sim \) \( \begin{array}{c} G = \frac{C_{pp}}{C_{ps}} \\ M \end{array} \) Magnetic field.

H Hall coefficient \( (= W_e/\nu_e) \).

I Ionization energy per unit mass \( (= eV_i/M_a) \).

J Current density.

K \( = e u^2/\beta \).

L Typical length in dimensional analysis.
Distance from leading edge of model to collar.
Length of shock tube.
L' Reference length.
Le Lewis number \( = (eC_p) \frac{D_{am}}{k_c} \).
M Mass, e.g. \( M_e \) Electron mass.
M Mach number.
M_s Shock Mach number.
M_0 Molecular weight.
N Number density, e.g. \( N_e \) Electron density.
Pr Prandtl number \( = \mu C_p/k \).
Q Electron-atom collision cross-section.
R Reynolds number \( = u x/\mu \).
\( R_L \)\( = e u_\infty x/\mu \omega \)
\( R_w \)\( = e u_\infty x/\mu_\infty \)
\( R_{x_0} \)\( = e u_\infty x_0/\mu_\infty \)
\( R_{\infty} \) Universal gas constant.
\( R_{m} \) Magnetic Reynolds number \( = \mu_0 \sigma u L \)
S Interaction parameter \( = \sigma B^2 x/\rho u \).
S Based on \( x_s \).
T Temperature.
U_S Shock speed.
V Velocity.
V_i Ionization potential.
V_i Diffusion velocity of species i.
W_e Electron cyclotron frequency.

**Lower Case Letters:**

a Factor in fitted curves for specific heat.
b Impact parameter.
c_i Mass fraction of species i.
d Molecular diameter.
e Electronic charge.
g \( = \int (\sigma uB^2/\beta^2) \, dx \).
h Step height.
h Debye length.
h: Enthalpy.
h₀: Enthalpy stagnation.
h₁: Enthalpy of species i.
h₂: Enthalpy of ions.
k: Boltzmann’s constant.
k: Thermal conductivity - Total.
k₁: " " - Atoms.
k₂: " " - Fully ionized gas.
l: Length of separated region - B = 0.
l₁: " " " " - B ≠ 0.
l₀: " " " " - Asymptotic value.
n, s: Intrinsic co-ordinates.
p: Pressure.
Pl: Initial shock tube pressure.
Pl: Upstream pressure - oblique shock waves.
d: Total speed.
d: Energy flux.
r₀: Radius of model.
β = 1 (dS/dx)/2βθ₀: Time.
t₀: Characteristic development time of the interaction.
u, v: Components of velocity.
x, y, z: Rectangular co-ordinates.
xₕ: Distance from beginning of interaction to separation.
xₚ: " " " to plateau B = 0.
xₜ: " " " B ≠ 0.
x₀: " " leading edge to beginning of interaction.

Greek Letters.

α: Degree of ionization.
β = (M² - 1)½
γ: Ratio of specific heats (perfect gas).
δ: Boundary layer thickness.


$\delta_1$ Boundary layer thickness - displacement.

$\delta_2$ " " " " - momentum.

$\delta_0$ " " " " - at beginning of interaction.

$\Delta$ Distance from surface to edge of shear layer.

$\varepsilon = \varepsilon_1/\varepsilon_2$. 

$\varepsilon$ Permittivity of free space.

$\theta_s$ Oblique shock waves - shock angle.

$\theta_w$ " " " " - wall angle.

$\theta$ Deflection of streamlines.

$\theta_s$ " " " " at separation, $B = 0$.

$\theta_{se}$ " " " " $B \neq 0$.

$\theta_p$ " " " " plateau, $B = 0$.

$\theta_{sp}$ " " " " $B \neq 0$.

$\theta_l = eV_i/k$. 

$\lambda = l_s/1$. 

$\lambda$ Debye length / Impact parameter.

$\lambda$ Polhausen shape parameter.

$\mu$ Viscosity - Total.

$\mu_1$ " " - Atomic.

$\mu_2$ " " - Fully ionized gas.

$\mu_e$ Permeability of free space.

$\nu e$ Electron collision frequency.

$= \mu/\varepsilon$. 

$\rho$ Density.

$\sigma$ Electrical conductivity - Total.

$\sigma_1$ " " - Low ionization.

$\sigma_2$ " " - High ionization.

$\tau$ Viscous stress.

$\tau$ Viscous stress tensor.

$\tau$ Skin friction.

$\tau_0$ " " at beginning of interaction.

$\gamma = (1/\beta^2)\sigma B^2 x_s/\varepsilon u^3 v_p$. 

$\psi$ Collision integral.
Subscripts.

- \( a \): Atom.
- \( B \): Magnetic field present.
- \( c \): Electron.
- \( i \): Ion; species.
- \( n \): Normal component.
- \( o \): Conditions at the beginning of the interaction. (Only applies in theory of the interaction itself).
- \( p \): Plateau conditions.
- \( s \): Separation conditions.
- \( s \): Component - Intrinsic co-ordinates.
- \( s \): Shocks: \( M_s \) \( U_s \).
- \( t \): Tangential component.
- \( w \): Conditions at the wall.
- \( x, y, z \): Rectangular components.
- \( I \): Initial shock tube conditions.
- 1: Oblique shock wave - Upstream conditions.
- 2: Oblique shock wave - Downstream conditions.
- \( \infty \): Conditions in freestream - Used in boundary layer theory.
1. INTRODUCTION

The original object of this investigation was to extend the work on magnetic nozzles in shock tubes \(^1,2,3\). In these earlier studies a coil producing a large magnetic field was wrapped around the shock tube and changes in the flow of the ionized gas through the coil were observed. Many of the observations consisted of direct photographs of the flow patterns. However, because the coil itself obscured the flow within it, the intention here was to mount the coil inside a model and to observe the flow over the outer surface.

At this stage interest turned to a consideration of the boundary layer on the model's surface and its possible effect on the flow. Because the mainstream was supersonic it was possible that an interaction existed between the mainstream and the boundary layer of a type usually known as a "shock boundary layer interaction" \(^4\). This interaction has received considerable attention under normal wind tunnel conditions where the model surface temperature is not greatly different from the mainstream stagnation temperature. However, little information is available on the interaction under the extreme heat transfer conditions encountered in the
shock tube. The interaction can be provoked by any agency producing a sudden pressure rise in the flow. The pressure rise induces boundary layer separation and an often extensive region of separated flow is formed. This report is a preliminary investigation of the perturbation effect of a magnetic field on an already established interaction.

In the theoretical section we consider the effect of a field on both the ionized mainstream and the boundary layer on the cold surface. These two regions are then allowed to interact and the effect of the field is assessed. A simple theory of the interaction indicates that the field increases the pressure rise to separation.

Experimental studies were conducted in a free piston shock tube ⁵, in which an increase in area occurred downstream of the diaphragm. Further information on the performance and running times of tubes of this type was obtained and compared with existing theories ⁶.

Interactions were produced by steps of different heights placed on the surface of the model. Interactions observed in the presence of a transverse magnetic field showed that the length of the separated region was slightly reduced. This effect masked any influence the field may have had on the conditions at separation. It is clear that further experiments are required involving
interactions which are not influenced by conditions in
the separated region.

The main features of the interaction are sketched
in fig. (1.1). A supersonic, ionized mainstream flows
over a cold surface. The surface strongly cools the
boundary layer which forms on it. A step placed on the
surface sets up an interaction between the mainstream
and the boundary layer which separates the boundary layer
from the surface to form a shear layer. The shear
layer traps a separated region of slowly circulating
"dead air" beneath it and subsequently reattaches at some
point further downstream. The curving outer edge of the
boundary layer in the separation zone produces a group
of compression waves in the mainstream which coalesce
into an oblique shock wave. A transverse magnetic field
is applied to this situation and its effect upon the
various parts of the interaction is discussed in the
following sections.

Cylindrical models were used in the experiments.
See figure (1.2). This ensured that the induced currents
formed closed current loops within the gas when the
field was present. See figure (1.3). Thus the
complications associated with the electrodes and external
circuit required in a steady, strictly two dimensional
situation, were avoided. Because the currents formed closed
loops, the electric field, $\tilde{E}$, was identically zero.

The radius of the cylindrical model was very large compared with the thickness of the boundary layer, while the step height, $h$, was still fairly small compared with the model radius. See section (8.5.1). Hence it is valid to apply a two-dimensional analysis of the interaction to the experimental configuration.

Part I develops the two-dimensional theory of the interaction in a magnetic field while Part II describes some experimental observations of the effect of the field on the flow.
PART I. THEORY OF THE INTERACTION IN A MAGNETIC FIELD

Outline of Part I

Section Two presents the basic equations of magneto-gas-dynamics (M.G.D.), and the general conditions required for M.G.D. effects in the shock tube are discussed.

In Section Three the results of a number of real gas normal and oblique shock wave calculations are presented. The normal shock results determine the conditions in the shock-heated sample of Argon. The oblique shock wave results relate the experimental flow deflection angles to their associated pressure changes.

Section Four considers the boundary layer equations of a compressible ionized gas in a magnetic field. Estimates of the transport properties of ionized Argon are also given.

From the considerations of Section Four, Section Five develops rough boundary layer profiles showing the variation of different gas properties through the boundary layer. The momentum integral equation in a magnetic field is derived, and the calculated density profile is used in this equation to obtain an estimate of the thickness of the boundary layer.

Section Six discusses the effect of the magnetic field on the mainstream shock relations and also develops
a modified form of the characteristic equations which takes account of the field.

The interaction between the mainstream and the boundary layer is investigated in Section Seven and an outline of a simple theory, due to Hakkinen\textsuperscript{8}, is presented. It is shown how this theory can be modified to allow for the magnetic forces by developing an analogy between them and the effect of a pressure gradient caused by curvature of the surface. Expressions are developed for the change in the pressure rise to separation, and the angle change associated with this pressure rise, due to the magnetic field.

Finally the length of the separated region produced by a step is considered. It is shown that provided the pressure in the separated region is constant, the decrease in the separation angle due to the magnetic field produces an increase in length. The magnetic field also produces a curvature of the shear layer which leads to a further increase in length.
2. GENERAL M.G.D. CONSIDERATIONS

2.1 Basic equations of M.G.D.

For reference purposes, we first write down the basic equations of M.G.D. The equations are essentially the Navier Stokes equations modified by the inclusion of a magnetic body force in the momentum equation, and an electrical dissipation term in the energy equation. The magnetic field is described by Maxwell's equations together with an extended form of Ohm's law. Thus we have the following set of equations:

Continuity:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{\tilde{v}}) = 0 \quad (2.1-1)
\]

Momentum:

\[
\rho \frac{d\mathbf{\tilde{v}}}{dt} = -\nabla p + \nabla \cdot \mathbf{T} + \mathbf{F} \times \mathbf{B} \quad (2.1-2)
\]

Energy:

\[
\rho c \frac{dh}{dt} = \frac{\partial h}{\partial t} + \nabla \cdot (\mathbf{\tilde{v}} \cdot \mathbf{E}) + \nabla \cdot (k \nabla T) + \nabla \cdot (\mathbf{F} \times \mathbf{B}) + \frac{\mathbf{F} \cdot \mathbf{\tilde{F}}}{\sigma} \quad (2.1-3)
\]

Maxwell's equations:

\[
\nabla \times \mathbf{H} = \mathbf{F}, \quad (2.1-4)
\]
\[
\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t, \quad (2.1-5)
\]
\[
\nabla \cdot \mathbf{B} = 0, \quad (2.1-6)
\]
\[
\nabla \cdot \mathbf{E} = 0. \quad (2.1-7)
\]
Ohm's Law:
\[ \tilde{J} = \sigma (\tilde{E} + \nabla \times \tilde{B}) \]  
(2.1-8)
\[ \frac{d}{dt} = \frac{\delta}{\delta t} + \nabla \cdot \tilde{V} \] is the convective derivative
and \( h_0 = h + \tilde{V} \cdot \tilde{V} / 2 \) is the stagnation enthalpy.

In writing down these equations we have assumed that the ionized gas can be regarded as a single, continuous, electrically conducting fluid in which no accumulations of electrical charge occur. To fully specify the system of equations we also require an equation of state and expressions for the various transport properties.

In the energy equation the term \( \nabla \cdot (\tilde{J} \times \tilde{B}) + \tilde{J} \cdot \tilde{J} / \sigma \) can be written as \( \tilde{J} \cdot \tilde{E} \). Hence, in a situation where \( \tilde{E} = 0 \), the energy equation is independent of any term involving the magnetic field.

2.2 Necessary conditions for M.G.D. effects

2.2.1 Interaction Parameter

The necessary conditions for M.G.D. effects have been discussed by previous investigators and verified experimentally in studies of M.G.D. flows in gas driven shock tubes 1,2,3.

Consider the situation sketched in fig. (2.1). An ionized gas moving down a tube with speed \( u \), encounters a transverse magnetic field \( B \). If the electric field,
\( E \), is small, and a current path is provided, the induced current density, \( J \), is given by

\[
J \sim \sigma u B.
\]

This current interacts with the field to produce a retarding force,

\[
F \sim J B / \text{unit length. Thus } F \sim \sigma u B^2, \text{ per unit length, per unit area.}
\]

The integrated effect of this force over a length \( L \), may be compared with the dynamic pressure, to yield the "interaction parameter", \( S \).

\[
i.e. \quad S = \sigma B^2 L / \rho u.
\]

For values of \( S \) near unity, the retarding force is capable of significantly influencing the freestream dynamic pressure.

Values of \( S \) have been calculated for typical flow conditions behind ionizing shock waves in Argon. These values are plotted in fig. (2.2). From this figure it is apparent that \( S \) depends strongly on the initial shock tube pressure, and only weakly on the shock Mach number. The density, \( \rho \), is roughly proportional to the initial pressure, so that \( S \) increases rapidly as the pressure decreases. At constant initial pressure \( \rho \) changes fairly slowly as the shock Mach number increases while \( \sigma \) and \( u \) increase at about the same rate. Thus \( S \) changes slowly with Mach number.
2.2.2 Magnetic Reynolds Number.

Associated with the induced currents is an induced magnetic field, \( \mathbf{B}' \). This field opposes the original field, \( \mathbf{B}_0 \).

If we let \( \mathbf{B}_T = \mathbf{B}_0 + \mathbf{B}' \), equation (2.1-4) becomes

\[
\nabla \times \mathbf{B}_T = \nabla \times (\mathbf{B}_0 + \mathbf{B}') = \mu_0 \mathbf{J}.
\]

In the absence of currents, \( \mathbf{B}_T = \mathbf{B}_0 \), so that \( \nabla \times \mathbf{B}_0 = 0 \). Thus if \( L \) is a typical linear dimension,

\[
\frac{\mathbf{B}'}{L} \approx \mu_0 \mathbf{J} \approx \mu_0 \sigma u \mathbf{B}_0.
\]

\[
\therefore \quad \frac{\mathbf{B}'}{\mathbf{B}_0} \approx R_m;
\]

where \( R_m \) is the magnetic Reynolds number, defined by

\[
R_m = \sigma u \mu_0 L. \quad (2.2.2-1)
\]

\( R_m \) can be interpreted in several different ways, but in this case we are regarding it as a number giving an estimate of the size of the induced field compared with the original magnetic field. Typical values of \( R_m \) behind strong shock waves in Argon are plotted in Fig. (2.3). From these curves it appears that in the present work \( R_m/cm \approx 0.3 \text{ cm}^{-1} \). The field used in the experiments is large over a region \( L \approx 0.5 \text{ cm} \), so that \( R_m \approx 0.15 \). Hence in the following work it is assumed that \( R_m \) is very small, which implies that the induced field is neglected when compared with the original field.
2.3 Hall Effects.

When a plasma moves through a sufficiently strong magnetic field, the electrical conductivity is no longer isotropic. This is because the electrons tend to spiral round the magnetic field lines. A measure of the importance of this effect can be obtained by comparing the frequency of rotation of the electrons about the field lines with their mean collision frequency. If the electrons can make at least one orbit between collisions the effect will be significant. Thus for Hall effects we require

\[ \frac{\omega_e}{\nu_e} \geq 1, \]

where \( \omega_e \) is the electron cyclotron frequency \( (\omega_e = eB/M_e) \) and \( \nu_e \) is the electron collision frequency.

Let us define a "Hall coefficient", \( H_a \), by

\[ H_a = \frac{\omega_e}{\nu_e} = \frac{eB}{M_e} \nu_e. \] (2.3-1)

In this work we require a magnetic field, \( B \), such that \( \sigma B^2 L/\rho u = S \), where \( S \approx 0.5 \) for an appreciable interaction.

A simple expression for the electrical conductivity is given by\(^{10}\)

\[ \sigma = N_e e^2 / M_e \nu. \]
Substituting for \( B \) and \( \nu_e \) in (2.3-1) we find that

\[
H_a^2 = e \sigma u S/N_e^2 e^2 L .
\]  
(2.3-2)

If \( \alpha \) is the degree of ionization, then

\[
\alpha = N_e/(N_e + N_a)
\]

where \( N_a \) is the number density of atoms and \( N_e \) is the electron number density.

Also, if \( \epsilon_2 \) is the density behind the shock, \( = M_a (N_e + N_a) \), and \( \epsilon_1 = M_a \frac{P_1}{76} \times 2.6 \times 10^{-19} \; \text{gm/cc} \), is the initial density, where \( P_1 \) is the initial shock tube pressure in cm. of mercury, then

\[
H_a^2 = \frac{\epsilon_1 \mu_0}{\epsilon_2 \alpha^2 p_1 L^2} \times 6 \times 10^{-2} .
\]  
(2.3-3)

In this expression \( L \) is in centimeters,

\[
\mu_0 = 4 \pi \times 10^{-7} \; \text{henry/metre}
\]

\( e \) = electronic charge \( = 1.6 \times 10^{-19} \; \text{caulomb} \)

and \( M_a = \text{mass of the Argon atom} = 6.63 \times 10^{-26} \; \text{Kg} \).

Thus the Hall coefficient is expressed in terms of the strength of the magnetic interaction, \( S \), and the conditions in the shock tube.

Note also that \( \mu_0 \mu_m = Lu^2 \)

where \( Lu \) is the Lundquist number defined by

\[
Lu = B \sigma L (\mu_0 / e)^{1/2} .
\]  
(2.3-4)

The directions of the normal induced current, \( J_c \), the Hall current, \( J_H \), and the force due to the interaction of the Hall current with the magnetic field are sketched in fig. (2.4).
The modification to the normal current is given by

\[ J_c = \sigma_0 uB \times \frac{1}{1 + Ha^2} \]

while the Hall current is given by

\[ J_h = \sigma_0 uB \frac{Ha}{1 + Ha^2} \]

where \( \sigma_0 \) is the conductivity in the absence of a magnetic field.

Table (2.1) shows the values of \( Ha \) obtained under the conditions used in the experiments. These values show that the Hall effect on the normal current is small enough to be neglected in this analysis. However the Hall current, \( J_h \), can be substantial and may amount to 50% of the normal current. But, because the geometry has been arranged to provide closed loops for the normal current, this current is favoured over the Hall current. Also the forces exerted by the Hall current tend to impart flow motion about the axis of symmetry. Since we are principally interested in forces in the streamwise direction as far as boundary layer separation is concerned, it is not expected that these "Hall forces" will strongly influence the interaction.
Table (2.1)

<table>
<thead>
<tr>
<th>$M_0$</th>
<th>$p_1$ (cm. of Hg)</th>
<th>$e_2 / p_1$</th>
<th>$a$</th>
<th>$S$</th>
<th>$R_m$</th>
<th>$L$ (cm.)</th>
<th>$H_a^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>0.2</td>
<td>7.9</td>
<td>0.14</td>
<td>0.5</td>
<td>0.3</td>
<td>1.0</td>
<td>0.3</td>
</tr>
<tr>
<td>15.6</td>
<td>0.5</td>
<td>7.4</td>
<td>0.12</td>
<td>0.5</td>
<td>0.3</td>
<td>1.0</td>
<td>0.17</td>
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<td>14.3</td>
<td>1.0</td>
<td>6.4</td>
<td>0.09</td>
<td>0.5</td>
<td>0.3</td>
<td>1.0</td>
<td>0.17</td>
</tr>
</tbody>
</table>

2.4 Non-equilibrium effects in the boundary layer.

In his review paper, "Plasma Boundary Layers"¹¹, Fay indicates that non-equilibrium effects exist within the boundary layer when Hall effects are important. He argues that joule heating adds energy to the electrons which must be shared quickly with the heavy particles if equilibrium is to be maintained. If $T_e$ is the electrons temperature, $T_A$ the heavy particle temperature, and $\tau_e$ the mean time for the electrons to lose their thermal energy $kT_e$, then

$$\frac{J^2}{\sigma} \sim N_e k \frac{T_e - T_A}{\tau_e}.$$  

If $\tau_e^{-1}$ is the electron collision frequency and $J \sim \sigma u B$, then

$$\frac{T_e - T_A}{T_A} \left(\frac{W_e \tau_e}{\tau_e}ight)^2 \frac{M^2}{M_i} \frac{T_e}{M_i \tau_e},$$  \hspace{1cm} (2.4-1)

where $M$ is the Mach number $\sim (M_1 u^2 / k T_A)^{1/2}$. 

If the energy is lost by elastic collisions then, because of the large disparity between the electron mass, $M_e$, and the ion mass, $M_i$, $\tau_e \sim \tau_e M_i/M_e$. Inelastic collisions are rather more efficient in their energy exchange so that if these collisions dominate, then $\tau_e < \tau_e M_i/M_e$.

$(W_e \tau_e)$ is the Hall coefficient discussed in the previous section. Hence, when $H_a \gg 1$, equation (2.4-1) shows that non-equilibrium effects may also become important. Thus, because $H_a < 1$ in these experiments, this effect should be small.

Also Camac\textsuperscript{12} in his work on stagnation point heat transfer from ionized Argon, concludes that the boundary layer in his experiments, carried out at $p_1 = 1$ mm. of mercury, is in ionization and thermodynamic equilibrium for freestream temperatures greater than 8000°K. Since the initial pressures and freestream temperatures are considerably larger in these experiments it is reasonable to assume that the boundary layers encountered here are in thermodynamic equilibrium.

2.5 Ionization time behind shock waves in Argon.

This problem has been considered in detail by Petschek and Byron\textsuperscript{13} while experimental results have also been obtained by Camac\textsuperscript{12}. They find that after a strong shock wave has passed through a gas, a finite time
elapses before the shock heated gas reaches equilibrium. This is probably due to the low efficiency of the most likely initial ionization process:

\[ \text{Ar} + \text{Ar} \rightarrow \text{Ar} + \text{Ar}^+ + e. \]

When an appreciable number of electrons have been produced the much more efficient process:

\[ \text{Ar} + e \rightarrow 2e + \text{A}^+ \]

takes over and the system proceeds rapidly towards equilibrium. In these papers the importance of impurities was stressed since they apparently speeded up the process. It is probably during the initial inefficient period that the presence of easily ionized impurities is most important.

Petschek and Byron find that for pure Argon, the ionization time, \( t_{\text{ion}} \), is given by

\[ t_{\text{ion}} = \frac{0.156}{p_1} \exp \left( \frac{87000}{T} \right) \]  \hspace{1cm} (2.5-1)

where \( p_1 \) is the initial shock tube pressure in mm. of mercury and \( T \) is the perfect gas temperature for the particular shock considered. Thus, \( T = T_1 \left( \frac{5}{16} \right) M_S^2 \), where \( M_S \) is the shock Mach number. Figure (2.5) shows the variation of \( t \) with \( M_S \) for different initial pressures calculated on the basis of (2.5-1).

Experiments by both Camac and Petschek confirm equation (2.5-1) when \( p_1 = 1 \) mm.
Table (2.2) compares the experimental shock tube testing time with the ionization time calculated using equation (2.5-1). From the table it is apparent that we may assume that the test gas was in equilibrium during the experiments.

Also, no special precautions were taken as far as the extraction of impurities was concerned, so that ionization times were probably smaller than those quoted.

<table>
<thead>
<tr>
<th>P_l (cm)</th>
<th>M_s</th>
<th>Testing Time (μsec)</th>
<th>t_{ion} (μsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>16.4</td>
<td>12</td>
<td>2.4</td>
</tr>
<tr>
<td>0.5</td>
<td>15.6</td>
<td>18</td>
<td>1.3</td>
</tr>
<tr>
<td>1.0</td>
<td>14.3</td>
<td>26</td>
<td>1.4</td>
</tr>
<tr>
<td>2.0</td>
<td>12.9</td>
<td>35</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Dolder and Hide in their work on M.G.D. nozzles in Shock Tubes found that M.G.D. interactions were observed only when the condition $t_{test} \geq t_{ion}$ was satisfied. Table (2.2) shows that this condition was fulfilled in these experiments.
3. NORMAL AND OBLIQUE SHOCK WAVES

3.1 Oblique shock waves.

One of the principal objectives of this report is to obtain information about changes in pressure along the surface of various models. Since the only basic experimental information gathered was from direct photographic observation of the self-luminous flow over the models, and in particular of shock angles and flow deflection angles, real gas oblique shock wave calculations have been made to relate these angles to the pressure changes.

The basic conservation equations applied to the situation shown in fig. (3.1) are

\[ u_1 t = u_2 t, \quad (3.1-1) \]
\[ \epsilon_1 u_1 n = \epsilon_2 u_2 n, \quad (3.1-2) \]
\[ p_1 + \epsilon_1 u_1 n = p_2 + \epsilon_2 u_2 n, \quad (3.1-3) \]
\[ h_1 + \frac{1}{2} v_1^2 = h_2 + \frac{1}{2} v_2^2 = h_0. \quad (3.1-4) \]

Putting \( \epsilon = \epsilon_1 / \epsilon_2 \) gives

\[ p_2 = p_1 + \epsilon_1 u_1 n \left( 1 - \epsilon \right), \quad (3.1-5) \]
\[ h_2 = h_1 + \frac{1}{2} u_1 n \left( 1 - \epsilon^2 \right), \quad (3.1-6) \]

and

\[ \tan \left( \Theta_s - \Theta_w \right) = \epsilon \tan \Theta_s \quad (3.1-7) \]

where \( u_1 n = v_1 \sin \Theta_s. \)
Most of the flow conditions of interest are covered by tables of the thermodynamic properties of Argon. From these tables plots of $p$ against $e$ for constant enthalpy values were constructed using log-log scales. On such a plot these lines form a family of almost parallel straight lines. The available data extends from pressures of 0.1 to 5.0 atmospheres. In some cases, however, it was necessary to extrapolate the curves to pressures of about 10 atmospheres.

The curves were used in conjunction with the above equations to find the conditions behind various oblique shock waves using an iteration procedure. $p_1, e_1, h_1$ and $u_1n$ are known and a value for $e_2$ is assumed. The value of $e$ using this assumed $e_2$ is substituted in (3.1-6) and (3.1-7) and values for $p_2$ and $h_2$ are obtained. Using these values for $p_2$ and $h_2$ a new value for $e_2$, and hence for $e$, is found from the curves giving the thermodynamic properties. The new value for $e$ is substituted in (3.1-6) and (3.1-7) and the process is continued until the change in $e, (\Delta e)$ between the value substituted and the subsequent value found from the curves is less than some specified amount. This method converged fairly slowly. However, if the assumed value of $e$ was plotted against $\Delta e$, and values of $e$ were chosen which bracketed the true
value, the process converged very rapidly. See figure (3.2). After three trials $\Delta \epsilon/\epsilon$ was usually less than 0.005. Results using the most common shock tube operating conditions as starting points are shown in figs. (3.3) to (3.6).

3.2 Normal shock waves.

If $U_s$ is the speed of a normal shock wave the conservation equations become

$$e_1 U_s = e_2 (U_s - U_2). \quad (3.2-1)$$

$$p_1 + e_1 U_s^2 = p_2 + e_2 (U_s - U_2)^2, \quad (3.2-2)$$

$$h_1 + 2U_s^2 = h_2 + 2(U_s - U_2)^2. \quad (3.2-3)$$

If tables of thermodynamic properties are available the method used for the oblique shock waves can again be used. However, when these calculations were made tables were not available so that the following method was used:\textsuperscript{9,17,18}:

If $\alpha$ is the degree of ionization and $R_o = \bar{R}/M_o$, where \(\bar{R}\) is the gas constant and $M_o$ is the molecular weight, the equation of state becomes

$$p = e (1 + \alpha) R_o T. \quad (3.2-4)$$

The enthalpy, $h$, is given approximately by

$$h = \frac{p}{e} + \frac{R_o T}{\gamma - 1} (1 + \alpha) + \alpha R_o \frac{e V_i}{k}. \quad (3.2-5)$$
The extra equation required to complete the system is the Saha equation,

\[
\frac{\alpha^2}{1 - \alpha^2} p = C T^{5/2} \exp \left( \frac{-eV_1}{kT} \right).
\]  

(3.2-6)

These equations were solved by assuming a value for \(\epsilon (=e_1/e_2)\) from which values of \(p_2\) and \(h_2\) were found. A value for \(\alpha\) was found from \(h = 5/2(p/\epsilon) + \alpha R_0 (eV_1/k)\), and \(T\) was then found from (3.2-4). The values of \(T\) and \(p\) were substituted in the Saha equation giving a second value of \(\alpha\). Usually the two values of \(\alpha\) did not agree. Further values of \(\epsilon\) were chosen until the difference between the two values of \(\alpha\) was less than some specified amount.

If \(p\) is in atmospheres the Saha equation used in these calculations is of the form\(^{19}\):

\[
\log_{10} \left( \frac{\alpha^2 p}{1 - \alpha^2} \right) = 5 \log_{10} T - 7.94 \times 10^4 - 5.47\].

Later when tables became available the most common shock tube operating conditions were rechecked and found to be consistent with the earlier calculations.

These conditions are summarized in Table (3.1).
Table (3.1)

<table>
<thead>
<tr>
<th>$M_s$</th>
<th>$p_1$ (cm of Hg)</th>
<th>$e$ (gm/cc)</th>
<th>$T$ (x10$^5$ K)</th>
<th>$\alpha$</th>
<th>$u$ (x10$^{-3}$ cm/sec)</th>
<th>$h$ (x10$^{-3}$ cal/gm at)</th>
<th>$p_2/p_1$</th>
<th>$e_2/e_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.4</td>
<td>0.2</td>
<td>3.5</td>
<td>1.23</td>
<td>0.153</td>
<td>4.60</td>
<td>3.30</td>
<td>1.05</td>
<td>400</td>
</tr>
<tr>
<td>15.6</td>
<td>0.5</td>
<td>7.9</td>
<td>1.27</td>
<td>0.126</td>
<td>4.32</td>
<td>3.00</td>
<td>2.30</td>
<td>350</td>
</tr>
<tr>
<td>14.3</td>
<td>1.0</td>
<td>13.6</td>
<td>1.25</td>
<td>0.086</td>
<td>3.88</td>
<td>2.51</td>
<td>3.76</td>
<td>286</td>
</tr>
<tr>
<td>12.9</td>
<td>2.0</td>
<td>22.8</td>
<td>1.20</td>
<td>0.048</td>
<td>3.37</td>
<td>2.02</td>
<td>5.90</td>
<td>224</td>
</tr>
</tbody>
</table>

Figs. (3.7) to (3.11) show the variation of the gas properties with initial shock tube pressure ($p_1$) and shock Mach number ($M_s$).
4. M.G.D. BOUNDARY LAYERS

4.1 Outline of earlier work.

Two review papers have been found particularly useful in the preparation of this section\textsuperscript{11,20}. At this stage the theory is also beginning to appear in some text books\textsuperscript{9,21}. No attempt has been made in this section to produce an exhaustive survey of the literature and only those papers which may have some direct bearing on the present work are discussed. Hence we are interested in steady, compressible flow in which the magnetic Reynolds number is small, and the transport properties are variable. Of particular concern is the effect of the magnetic field on separation.

Apart from Hartmann's\textsuperscript{22} early work on channel flow the first paper to deal with a boundary layer in a magnetic field was due to Rossow\textsuperscript{23}. He considered the problem of an incompressible, constant property boundary layer on a flat plate in the presence of a transverse magnetic field. By substituting a stream function represented as a power series in $S$, the interaction parameter, in the momentum boundary layer equation he generated a family of ordinary differential equations. The lowest member of the family was the familiar Blasius equation\textsuperscript{4}.
Numerical integration of the second member of the family showed that

\[ \alpha c R^2 = 0.664 - 1.738 S + \ldots \]  

(4.1-1)

where \( S = \sigma B^2 x / e u_{\infty} \).

Thus he was able to show that the field reduced the skin friction and heat transfer while increasing the boundary layer thickness. Bleviss\(^\text{24}\) has studied the effect of a field on hypersonic couette flow, allowing the gas to be compressible and of variable ionization. He showed that the field produced a large increase in the total drag, i.e. skin friction plus magnetic stress, a large reduction in the skin friction and a small increase in heat transfer.

He also found an odd hysteresis effect for certain temperatures in the boundary layer for which the skin friction and heat transfer became multivalued functions of the applied field.

The same effect was observed by Bush\(^\text{25}\) in his work on hypersonic boundary layers. Bush assumed that the boundary layer was in equilibrium and that \( R_m \ll 1 \). He also assumed that the mainstream was non-conducting and that the field was represented by a power law distribution \( \sim x^{-1/2} \). His analysis showed that both the skin friction and heat transfer were reduced by the field.

Figure (4.1) is a sketch of the variation of shear, \( \tau \),
with external flow velocity \( (\mathbf{U}_\infty) \) for different values of \( S \). As \( \mathbf{U}_\infty \) increases \( \mathcal{C} \) at first also increases. At point \( X \) magnetic effects begin to occur and \( \mathcal{C} \) drops below its non-magnetic value. At \( Y \) the tangent is vertical and any further increase in \( \mathbf{U}_\infty \) causes a discontinuous drop in \( \mathcal{C} \) to the lower branch of the curve. If the velocity is then decreased, \( \mathcal{C} \) traces along the lower branch until \( Z \) is reached, whereupon it jumps discontinuously back to the upper branch.

This hysteresis effect is thought to be connected with the variation of the electrical conductivity with enthalpy through the boundary layer. This variation, for air, is sketched in fig. (4.2). In region "a" the conductivity increases exponentially while electrons are being produced from the reaction \( N + O \rightarrow NO^+ + e^- \). At point "b" production of NO decreases and the extra energy is used to complete the nitrogen dissociation. Thus the temperature increase is small and \( \sigma \) is nearly constant. At "c" oxygen and nitrogen begin to ionize and the conductivity again starts to increase. Bleviss believes that the hysteresis effect is due to the abrupt change in conductivity at "b".

Similar solutions of the boundary layer equations have been obtained by Lykoudis\(^26\) who showed that if the mainstream velocity \( (\mathbf{U}_\infty) \) is given by \( \mathbf{U}_\infty = c x^m \) similar solutions exist if

\[
B \propto x^{m-\frac{3}{2}}. \tag{4.1-2}
\]

For a flat plate \( m = 0 \), so that \( B \propto x^{-\frac{3}{2}} \), which is the form
used by Bush.

Hains and Yoler have carried out a theoretical and experimental investigation of the situation sketched in fig. (4.3). The effect of the field was regarded as a small perturbation and both the viscous and inviscid regions were studied under this assumption. The disturbances produced near the walls propagated inwards to form oblique shock waves downstream of the coil. These waves were observed photographically by Dolder and Hide. The boundary layer was thickened by the field and the heat transfer was slightly reduced near the coil, only to increase again further downstream. The boundary layer equations were solved using both the momentum integral equation and the thermal energy integral equation.

Heiser, while studying the effect of a magnetic field on eddies produced in a mercury bath by a paddle moving through it, observed that the field suppressed the formation of the eddies. He suggested that the field altered the separation characteristics of the boundary layer to prevent the formation of the vortex sheet.

Arguing from the approximate momentum equation

\[ \nu \left( \frac{\partial^2 u}{\partial y^2} \right) = -\frac{u \partial u}{\partial x} - \frac{\sigma u B^2}{c} \]  

he suggested that the usual Polhausen shape parameter
\[ \Lambda = \frac{\sigma^2}{\nu} \frac{du}{dx}, \]

should become
\[ \Lambda^1 = \Lambda \left( 1 + \frac{\sigma B^2}{\rho} \frac{du}{dx} \right). \]

\( \Lambda^1 \) becomes the appropriate parameter to use in the Polhausen method so that separation occurs when \( \Lambda^1 = -12 \).

In his experiments \( \sigma B^2 / (\rho du/dx) \approx -1 \) so that the magnetic field would have a large effect upon the conditions at separation. Finally he suggested that a transverse field might suppress separation in a wide angle diffuser.

More recently Heiser and Bornhorst \(^{29}\) have considered a modified Polhausen approach to M.H.D. boundary layers in which their velocity profile can handle both M.H.D. and freestream pressure gradient effects.

Sherman \(^{30,31}\) has considered the situation sketched in fig. (4.4) in which an incompressible, constant-property fluid encounters the field due to a wire embedded in one wall of a duct. A preliminary paper found the solution for the inviscid flow field \(^{30}\), while a later paper used this solution in the discussion of the related boundary layer problem \(^{31}\). The non-linear equations were solved using a power series expression. He showed that the field produced a large decrease in skin friction and that a moderate field could produce separation. A rapid
increase in displacement thickness occurred, while the change in momentum thickness was fairly small.

A great deal of attention has been given to the problem of the effect of a magnetic field at the stagnation point of a blunt body. Both inviscid and viscous situations have been considered. Bush\textsuperscript{32} has shown that the field increases the stand-off distance of the bow shock and this has been observed experimentally by Ziemer\textsuperscript{33}. Neuringer and McIlroy\textsuperscript{34} and several other workers\textsuperscript{20} investigated the stagnation point itself and showed that the field reduced the skin friction and heat transfer rate. The pressure at the stagnation point only changed slightly and the total drag on the body increased\textsuperscript{20}. A number of the early papers tended to overestimate the reduction in heat transfer because they only considered the viscous flow and neglected the effect of the field on the inviscid region. More recent papers have considered the whole flow field\textsuperscript{20}. At the moment it seems that although the field reduces the heat transfer rate, the effect is not large enough to be of probable engineering interest.

Louis, Gal and Blackburn\textsuperscript{35} have made a detailed investigation of a large M.H.D. generator in which they pay particular attention to its internal
aerodynamics. The flow was supersonic (1 < M < 1.5) and turbulent boundary layers existed on the walls. When a current was drawn from the generator the Lorentz force was sufficient to choke the flow and extensive regions of separated flow formed on the electrodes. They suggested that the behaviour of the boundary layer on the insulated walls of the duct was quite different to that encountered on the electrodes. This can be explained as follows:

In fig. (4.5) let AA' and BB' represent the electrodes and let A'B' be the lower insulated wall. The flow is out of the paper and the field is perpendicular to A'B'. The insulated walls were highly cooled so that the boundary layer on A'B' was very thin and only the outer part of it was conducting.

Because of the current (I) passing through the load (R_L), a certain potential difference \( V_L = IR_L \) exists between AA' and BB'. Now the induced current \( J_\sim \sigma uB \), and as we pass through the boundary layer on A'B', \( \sigma u \) decreases. Thus the induced current also decreases. However because of the fixed potential difference (V_L) we will tend to get eddy currents induced in the boundary layer flowing in the opposite direction. These currents interacting with the field produce a force which tends to accelerate the boundary layer, thus making it more stable. Hence the stabilizing influence of the high heat transfer rate combined with this M.H.D. force should
make it more difficult to separate the boundary layer on this wall. On the electrodes, however, the current \( J \) must be drawn directly through the boundary layer. The joule heating keeps the whole boundary layer hot, and the force, \( (\vec{J} \times \vec{B}) \), is such that it retards the flow in the boundary layer. Thus we can expect extensive regions of separated flow on the electrodes. These regions were observed for \( M > 1.25 \). Calculations were made in which the blockage due to the separated boundary layer was considered. For routine calculations an effective supersonic diffuser efficiency was defined which combined the losses due to shocks, friction and eddy currents.

4.2 The boundary layer equations in a magnetic field.

In the general case several different boundary layers can be defined, each depending on a different dissipation process. Thus we have the usual viscous boundary layer of velocity gradient whose thickness, \( \delta_v \), is proportional to \( R_L^{-\frac{1}{2}} \). Associated with the temperature gradient is a thermal boundary layer whose thickness, \( \delta_T \), is proportional to \( (\Pr R_L)^{-\frac{1}{2}} \). Finally a magnetic boundary layer can be defined for which \( R_m^{-\frac{1}{2}} \) gives a measure of its thickness, \( \delta_B \). The magnetic boundary layer can be regarded as a region over which the magnetic
field changes are due to the presence of electrical currents. If $L$ is a typical length, the present work corresponds to the case where

$$\delta_B > L \gg \delta_v \sim \delta_t .$$

That is, the usual viscous boundary layer is modified by the presence of magnetic body forces.

Applying the usual order of magnitude analysis to the basic M.G.D. equations gives the boundary layer equations of steady, two-dimensional flow.

\[
\frac{\partial}{\partial x} (e u) + \frac{\partial}{\partial y} (e v) = 0 . \tag{4.2-1}
\]

\[
e (u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}) = - \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} (\mu \frac{\partial u}{\partial y}) + (\widetilde{J} \times \widetilde{B})_x . \tag{4.2-2}
\]

\[
\frac{\partial p}{\partial y} = (\widetilde{J} \times \widetilde{B})_y . \tag{4.2-3}
\]

\[
e (u \frac{\partial h_0}{\partial x} + v \frac{\partial h_0}{\partial y}) = \frac{\partial}{\partial y} \left[ \mu \frac{\partial (u^2)}{\partial y} \right] + \frac{\partial}{\partial y} \left( \frac{k \frac{\partial T}{\partial y} }{\alpha} \right) + \nabla \cdot (\widetilde{J} \times \widetilde{B}) + \frac{\widetilde{J} \cdot \widetilde{J}}{\sigma} . \tag{4.2-4}
\]

\[
\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} = 0 . \tag{4.2-5}
\]

Initially a uniform transverse magnetic field, $H_{yo}$, is present. Let $H'_y ( = H'_y - H_{yo})$ and $H'_x$ be the fields due to the induced currents. Equation (4.2-5) shows that over the boundary layer

$$B'_y / B'_x \sim H'_y / H'_x \sim \delta / L \sim \nu / \alpha \sim R_L^{-1} .$$
In these experiments the geometry was such that the induced currents formed closed loops within the plasma. See figure (1.3).

Hence \( \vec{E} = 0 \)

and

\[
\vec{J} = \sigma (u B_y - v B'_x) \vec{k}
\]

\[
= \sigma u B_y \left( 1 + B'_y/B_y - v B'_x/u B_y \right) \vec{k}
\]

\[
= \sigma u B_y \left( 1 + 0 \left( B'_y/B_y \right) \right) \vec{k}
\]

Also \( \vec{J} = \nabla \times \vec{H} = \vec{k} \left( \partial H_y/\partial x - \partial H_x/\partial y \right) \)

\[
= \vec{k} \left( B'_y/L - B'_x/s \right) / \mu_0
\]

\[
= \vec{k} B'_y (1 - L^2/s^2) / \mu_0 L
\]

Equating the two expressions for \( \vec{J} \) gives

\[
B'_y/B_y \left( 1 - L^2/s^2 \right) = \sigma u \mu_0 L \left( 1 + 0 \left( B'_y/B_y \right) \right).
\]

Hence \( B'_y/B_y \approx R_m R_L^{-1} \)

and \( B'_x/B_y \approx R_m R_L^{-2} \).

Thus \( \vec{J} = \sigma u B_y \vec{k} \)

and \( \vec{J} \times \vec{B} = -\sigma u B^2 y_0 \left( 1.1 + 0 \left( R_m R_L^{-2} \right) \vec{J} \right) \),

so that

\[
(\vec{J} \times \vec{B})_x = -\sigma u B^2 y_0
\]

and \( (\vec{J} \times \vec{B})_y = 0 \).
Hence the boundary layer equations become

\[ \frac{\partial}{\partial x}(e u) + \frac{\partial}{\partial y}(e v) = 0, \quad (4.2-1) \]

\[ e \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) - \sigma u B^2, \quad (4.2-6) \]

\[ \frac{\partial p}{\partial y} = 0, \quad (4.2-7) \]

\[ e \left( u \frac{\partial e}{\partial x} + v \frac{\partial e}{\partial y} \right) = \frac{\partial}{\partial y} \left[ \mu \frac{\partial ^2 u}{\partial y^2} \right] + k \frac{\partial e}{\partial y} \quad (4.2-8) \]

where putting \( E = 0 \) has removed the magnetic terms from the energy equation \( (4.2-8) \).

Consider the right hand side of equation \( (4.2-8) \).

The specific heat at constant pressure \( (C_p) \) is defined by

\[ C_p = \left( \frac{\partial h}{\partial T} \right)_p \]

and since we have shown that \( \frac{\partial p}{\partial y} = 0 \) we can put

\[ C_p = \left( \frac{\partial h}{\partial y} \frac{\partial \gamma}{\partial T} \right)_p \]

Also since \( \nu \ll u, \quad u^2/2 = h_0 - h \).

Thus the right hand side of \( (4.2-8) \) becomes

\[ \frac{\partial}{\partial y} \left( \mu \frac{\partial h_0}{\partial y} - \mu \frac{\partial h}{\partial y} + k \frac{\partial h}{\partial y} \right). \]

If the Prandtl number, \( P_r \), is defined by

\[ P_r = \mu C_p / k \]

the energy equation can be written as

\[ e \left( u \frac{\partial h_0}{\partial x} + v \frac{\partial h_0}{\partial y} \right) = \frac{\partial}{\partial y} \left( \mu \frac{\partial h_0}{\partial y} \right) + \frac{\partial}{\partial y} \left[ \mu \frac{\partial h}{\partial y} \left( \frac{1}{P_r} - 1 \right) \right] \quad (4.2-9) \]
This equation should still be valid under conditions of variable ionization, provided the Lewis number, $L_e$, is unity. See section (4.2.1).

If $P_r = 1$

$$h_o = \text{constant} \quad (4.2-10)$$

is still an exact integral of the energy equation, even in the presence of magnetic fields, and gives the enthalpy distribution through the boundary layer on an insulated surface.

However, due to the presence of the term $\sigma u B^2$ in the momentum equation, no simple integral of the form

$$h_o = A + Bu \quad (4.2-11)$$

exists for a cooled, constant temperature wall, when a magnetic field is present.

To assess the validity of the assumption that $P_r = 1$ it is necessary to consider the transport properties of Argon at high temperatures. Table (2.1) shows that we are mainly concerned with gas conditions in the range $1 < p < 6$ atm and $300^\circ K < T < 13000^\circ K$. Our discussion is therefore limited to these ranges.

4.2.1 Lewis Number.

The energy flux of equation (4.2-4) is given more precisely by

$$- \dot{q} = k( \partial T / \partial y) - \Sigma e_i h_i V_i \quad (4.2.1-1)$$

where $k( \partial T / \partial y)$ is the usual Fourier heat conduction term.
and \( \sum e_i c_i h_i V_i \) represents the energy flux due to the enthalpy carried by diffusion. In this expression:

- \( e \) = mass density of the mixture,
- \( c_i \) = mass fraction of species \( i \),
- \( h_i \) = enthalpy per unit mass of species \( i \),
- \( V_i \) = diffusion velocity of species \( i \).

The amount of heat carried by diffusion is determined by the Lewis number, \( L_e \), defined by\(^{12}\),

\[
L_e = \left( e C_p \right) \frac{D_{am}}{k_a} \quad (4.2.1-2)
\]

where, for an ionized gas:

- \( D_{am} \) is the ambipolar diffusion coefficient,
- \( e C_p \) is the specific heat at constant pressure of the mixture per unit volume,
- and \( k_a \) is the atomic thermal conductivity.

If the contribution to the heat flux due to the species concentration gradient is included, differentiation of equation (3.2-5) yields

\[
\varphi h / \varphi y = C_p \left( \varphi T / \varphi y \right) + h' \left( \varphi \alpha / \varphi y \right) \quad (4.2.1-3)
\]

where, from equation (4.3.1-1) we have assumed that \( C_p \) is given approximately by

\[
C_p = \left( \gamma / \gamma - 1 \right) R_o (1 + \alpha),
\]

and \( h' \) is the ionic species enthalpy defined by

\[
h' = \left( \gamma / \gamma - 1 \right) R_o T + \alpha I,
\]

where \( I \) is the ionization energy per unit mass \( (= e V_i / M_a) \).
Camac has shown that for a singly ionized gas
\[ \Sigma e C_i V_i h_i = -h' \left( k_e L_e / C_p \right) (\partial \alpha / \partial y) , \]
and substituting for \( (\partial T / \partial y) \) from equation (4.2.1-3), the expression for the heat flux becomes
\[ \dot{q} = -\frac{k}{\partial_p} \left[ \frac{\partial h}{\partial y} + h' \frac{\partial \alpha}{\partial y} (L'_e - 1) \right] \quad (4.2.1-4) \]
where \( L'_e = L_e k_e / k \).

Hence the boundary layer energy equation becomes
\[ e \left( u \frac{\partial h_o}{\partial x} + v \frac{\partial h_o}{\partial y} \right) = \frac{\partial}{\partial y} \left[ \mu \frac{\partial h_o}{\partial y} \right] + \frac{\partial}{\partial y} \left\{ \frac{k}{C_p} \left[ \frac{\partial h}{\partial y} (1-P_r^*) + h' \frac{\partial \alpha}{\partial y} (L'_e - 1) \right] \right\} \]

Camac shows that for Argon \( L_e \) is given by
\[ L_e = 0.255 \left( \frac{T}{10^4} \right)^{-0.16} \]
Hence \( L_e \sim 0.25 \) and \( L'_e \ll L_e \).

Since \( L_e < 1 \), diffusion effects are less important than convective effects, and the heat flux based on the enthalpy is reduced. Therefore, the assumption that \( L_e = 1 \), overestimates the heat flux and should predict a higher electron density near the wall than actually exists.

When \( L'_e = 1 \), equation (4.2.1-5) reduces to equation (4.2-9).

Since the analysis to be given in section (7.4) depends on the assumption that the electrical conductivity near the wall is small, putting \( L'_e = 1 \) should give a conservative estimate of the validity of this assumption.
4.3 Transport Properties.

4.3.1 Specific Heat at Constant pressure \((C_p)\).

Although \(C_p\) is not a transport property in the same sense as the other properties to be discussed, it is included here because it appears in the boundary layer equations through \(P_r\), becoming a function of temperature for \(T \geq 7000^\circ K\).

Originally an analytical expression for \(C_p\) was used \(^{37}\), given by

\[
C_p = \frac{5}{R_0} \frac{(1+\alpha) + (5T + 2\Theta_1^2)\alpha(1-\alpha^2)}{2} \quad (4.3.1-1)
\]

where \(\Theta_1 = eV_i/k\).

This expression is valid for single ionization. It can be derived by differentiating equation \((3.2-5)\) with respect to \(T\) at constant pressure. This result involves the factor \((\partial \alpha/\partial T)_p\) which can be found by differentiating the Saha Equation at constant pressure.

Later Cambel's thermodynamic tables \(^{15}\) containing values of \(C_p\) in the range of interest became available. Some of his results are plotted in fig. (4.6).

Below \(7000^\circ K\), \(C_p = 0.124 \text{ cal/gm}^\circ K\) is constant. For purposes of interpolation the lines drawn in this figure are fitted curves of the form

\[
C_p = 0.124 + a(T-7)^3 \quad (4.3.1-2)
\]
where $T$ is in 1000's of degrees, and "a" is a constant for any particular pressure. Values of "a" are plotted against $p$ in fig. (4.7). The curves show reasonable agreement with the data over the temperature range of interest.

Above the temperatures shown, $C_p$ decreases again, and then exhibits an oscillatory behaviour with temperature as significant numbers of second and higher ions are produced.

4.3.2 Electrical Conductivity.

The electrical conductivity of a plasma depends on the movement of electrons, and this in turn depends on the collisions the electrons are making with other particles. Two limiting cases can be distinguished.

The first case applies when the degree of ionization is very small, so that the conductivity is governed by short range electron-atom collisions. The conductivity is then given by a formula due to Chapman and Cowling \textsuperscript{38},

$$\sigma_1 = \frac{0.53 e^2 \alpha}{(M_e kT)^2} \frac{1}{Q} \text{ mks units}$$

where $Q$ is the electron-atom collision cross section.

For Argon at $10^4$ °K, $Q \sim 10^{-20} M^2$ \textsuperscript{37}.

The second case is for a fully ionized plasma in which the long-range electron-ion encounters dominate.
Under these conditions the conductivity is given by the Spitzer-Harm formula, \[ \sigma = \frac{16 \pi^2 (kT)^{3/2} \epsilon^2}{1.69 M_e^{1/2} \epsilon \log_e \left[ \frac{12 \pi (\epsilon kT)^{3/2} M_0^{1/2}}{e^3 (\alpha e)^{1/2}} \right] } \] mks units.

This particular form of the expression is due to de Leeuw. The change from case one to case two occurs for a comparatively small value of \( \alpha \) (the degree of ionization) \( \sim 10^{-3} \).

For a partially ionized gas it has been suggested that a mixture of the form

\[ \frac{1}{\sigma} = \frac{1}{\sigma_1} + \frac{1}{\sigma_2} \]  

is appropriate.

If the values of the constants are inserted into \( \sigma_1 \) and \( \sigma_2 \),

\[ \sigma_1 = 3.8 \times 10^{-12} \frac{\alpha}{Q T^2} \text{ mho/m} \]

where \( Q \sim 10^{-20} \text{ m}^2 \), and

\[ \sigma_2 = \frac{6.72 \times 10^{-3} T^{3/2}}{\log_10 \left[ 2.27 \times 10^{-6} (T^3/\alpha e)^{1/2} \right]} \text{ mho/m} \]

In fig. (4.3) \( \sigma \) is plotted as a function of initial shock tube pressure for various shock Mach numbers. Usually it was found that for the conditions covered by this figure that \( \sigma_1 \) made a negligible contribution.
Figure (4.9) shows the variation of electrical conductivity with temperature at a pressure of one atmosphere.

4.3.3 Thermal Conductivity, \( k \)

As with electrical conductivity two limiting cases have been considered for thermal conductivity. For slight ionization the expression for the atomic conductivity can be used, since a small amount of ionization has little effect.\(^{37}\)

Hence \( k_1 = \frac{2 \times 10^{-4} (T/M_0)^{1/2}}{d^2 \Omega_k} \)

where \( k_1 \) is in \( \text{cal/cm/sec}/^\circ\text{K} \),

\( T \) is the temperature in \( ^\circ\text{K} \),

and \( M_0 \) is the molecular weight.

\( d \) is a characteristic molecular diameter and \( \Omega_k \) is the collision integral for thermal conductivity.

\( \Omega_k \) varies slowly with temperature.

For Argon \( M_0 = 40 \ \text{gm} \), and \( d \approx 3.4 \times 10^{-8} \ \text{cm} \), while \( \Omega_k \) varies between 0.67 at 5000\(^\circ\)K and 0.59 at 11000\(^\circ\)K.

Thus \( k_1 \approx 4.14 \times 10^{-8} \frac{T^{1/2}}{d^2} \ \text{cal/cm/sec}/^\circ\text{K} \tag{4.3.3-1} \)

where \( \Omega_k \) has been given the value 0.66.

For the case of a fully ionized gas Spitzer has derived an expression for \( k \) which is similar in nature
to his expression for $\sigma$ and is

$$k_2 = 5.58 \times 10^{-8} \frac{k}{(kT)^{5/2}} \text{ cal/cm/sec/°K,}$$

$$e^{4M_e \frac{k}{2}} \log_e \Lambda$$

or, substituting for the other constants

$$k_2 = 1.08 \times 10^{-12} \frac{T^{5/2}}{\log_e \Lambda} \text{ cal/cm/sec/°K,}$$

where $\Lambda = \frac{h}{\nu_0}$, Debye Shielding Length

Impact Parameter

and is given by $\Lambda = 12\pi (\epsilon_0 k)^{3/2} T^{3/2}$;

$$e^3 N_e^{3/2}$$

the values of the constants being in MKS units.

If $N_e$ is the number density of electrons /cc then

$$\Lambda = 1.23 \times 10^4 \frac{T^{3/2}}{N_e^{1/2}}.$$ 

Values for $\log_e \Lambda$ as a function of temperature are shown in fig. (4.10); the data used being from Cambel's tables 15.

If $N_e$ is not known explicitly then $\Lambda$ can be written as

$$\Lambda = 3.16 \times 10^{-6} \frac{T^{3/2}}{(e)^{1/2}} \text{ cal/cm/sec/°K},$$

where $e$ is the gas density in $Kg/m^3$ and $\alpha$ is the degree of ionization. (De Leeuw's formula for $\sigma$ contains an extra factor of $1/\sqrt{2}$ in $\Lambda$ giving $\Lambda = 2.27 \times 10^{-6} \frac{T^{3/2}}{(e)^{1/2}}$.)

From the curves it is apparent that $\log_e \Lambda$ is very weakly dependent upon pressure in the range of interest, and that it varies slowly with temperature.
Thus $k_2$ is governed almost entirely by the $T^{5/2}$ behaviour. Figure (4.11) shows the variation of thermal conductivity with temperature at one atmosphere pressure. Also shown is $k_1$ and $k_2$ calculated from (4.3.3-1) and (4.3.3-2). Thus for $T < 9000^\circ K$ the conductivity is given by $k_1$ while for $T > 14000^\circ$ it is given by $k_2$. The region between $9000^\circ$ and $14000^\circ$ represents the transition region between low ionization and high ionization conditions for the thermal conductivity of Argon. A more detailed consideration of the thermal conductivity is given in a paper by Camac$^{12}$ in which he discusses the mixture rule used to link the two regions. Because the only dependence on pressure is introduced through $\log_e \Lambda$, the conductivity should be almost independent of pressure over the range of interest.

4.3.4 Viscosity

The viscosity of a gas is largely governed by collisions between heavy particles. Thus the viscosity is basically a function of temperature even when a small electron population is present. The viscosity becomes affected by ionization only when the temperature is so high that sufficient ions are produced for long range ion-ion collisions to dominate.
For low ionization,

\[ \mu_1 = \frac{2.67 \times 10^{-5} (NT)^{1/2}}{d^2 \Omega}, \]

where quantities \( M, d \) and \( \Omega \) are the same as for \( k_1 \).

Hence \( \mu_1 = 2.22 \times 10^{-5} T^{3/2} \text{ gm/cm/sec}. \quad (4.3.4-1) \)

This formula becomes less accurate as room temperature is approached. Table (4.1) compares experimental values of viscosity with those calculated using equation (4.3.4-1).

<table>
<thead>
<tr>
<th>( T^{\circ}K )</th>
<th>( \mu ) (EXPT)</th>
<th>( \mu_1 )</th>
<th>( \frac{\mu_1}{\mu} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>293</td>
<td>2.2 \times 10^{-4}</td>
<td>3.8 \times 10^{-4}</td>
<td>1.7</td>
</tr>
<tr>
<td>1100</td>
<td>5.63</td>
<td>7.36</td>
<td>1.3</td>
</tr>
</tbody>
</table>

An accurate representation is given by Sutherland's formula,

\[ \mu \propto T^{3/2}/(T + C), \]

where \( C \) is a characteristic temperature.

For Argon \( C = 170^{\circ}K \).

If an expression of the form \( \mu \propto T^w \) is to give an accurate variation of viscosity with temperature then \( W = 1.5 - T'/T' + 1 \) where \( T' = T/C^{4.3} \). In the high temperature limit \( T'/T' + 1 \rightarrow 1 \) and \( W \rightarrow \frac{1}{2} \). Though \( \mu_1 \)
overestimates the viscosity near the wall, it probably
gives a reasonable variation over most of the boundary
layer.

For a fully highly ionized gas the viscosity is given
by

$$\mu_2 = \frac{5}{16} \frac{M_i kT}{\pi} \left( \frac{2kT}{e^2} \right)^2 \text{gm/cm/sec.}$$

where $M_i$ is the mass of the ion, $Z$ is the charge ratio
($=1$ for single ionization) and $A$ is a function of $\alpha$:

$$A = \log_e \left(1 + \alpha^2\right) - \frac{\alpha^2}{1 + \alpha^2}.$$ 

Thus $\mu_2 = 2.02 \times 10^{-16} \frac{T^{5/2}}{A} \text{gm/cm/sec.} \ (4.3.4-2)$

In fig. (4.12) $\mu_2$ is plotted as a function of $T$, and values of $\alpha$ used in $A$ are for $p = 1$ atmosphere.

At 12000°K, $\alpha \simeq 0.12$, so that $A \approx \alpha^4$. Hence $A$, and
therefore $\mu$, is a very strong function of $\alpha$; decreasing
rapidly as $\alpha$ is increased. Once $\alpha$ reaches $\sim 0.5$ the variation
of $A$ with $\alpha$ is much less violent, and for very high
temperatures, the $T^{5/2}$ variation becomes apparent.

Also shown in fig. (4.12) is the expression for $\mu_1$. The dashed line joining the two expressions is of reciprocal
form,

$$i.e. \ \frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2}.$$ 

From fig. (4.12) we can conclude that $\mu_1$ is satisfactory
up to $T \sim 11000^\circ$ and that the discrepancy in assuming that $\mu_1$ holds up to our temperature limit of $12500^\circ$ is only small.

For higher pressures, $\alpha$ at a particular temperature is less than at one atmosphere. Hence curves for $\mu_2$ at higher pressures would be further to the right, and $\mu_1$ would hold for higher temperatures.

Hence, under the present conditions it will be assumed that $\mu$ is given by $\mu_1$ over the whole temperature range of interest.

4.3.5 Prandtl number (Pr)

The Prandtl number, $Pr$, is defined by

$$Pr = \frac{\mu C_p}{k}. \quad (4.3.5-1)$$

For temperatures below $6000^\circ C$, $C_p = 0.124 = \text{constant}$ and $Pr = \frac{\mu_1 C_p}{k_1} = 0.68 = \text{constant}.$

Figure (4.13) shows the variation of $Pr$ with temperature for $T > 5000^\circ$ at one atmosphere pressure. Initially the Prandtl number rises due to the increase in $C_p$ while $\mu/k$ remains constant. Above $11000^\circ$, $k$ begins to rise and $\mu$ decreases rapidly so that $Pr$ drops sharply. For a highly ionized monatomic gas
\[
\text{Pr} \rightarrow \frac{2}{3} \left( \frac{M_o}{N_1} \right)^{1/2} = 2.5 \times 10^{-3} \quad \text{(for Argon)}.
\]

For temperatures less than 13000°K, Pr remains of order one, with a maximum value of 1.85 at 12000°K. Hence, in these experiments Pr is reasonably well behaved, and does not exhibit the extreme values characteristic of a very highly ionized gas.

For higher pressures the behaviour of Pr is similar to that shown for one atmosphere. However, because \( C_p \) increases more slowly, the maximum value reached is smaller.
5. PROPERTIES OF THE BOUNDARY LAYER

5.1 Boundary Layer Profiles.

Since the Prandtl number remains of order one, rough profiles of the variation in flow properties through the boundary layer can be obtained by considering the simple integrals of the energy equation which exist when 
P_T \equiv 1.

For simplicity, a linear velocity profile of the form

$$\frac{u}{u_\infty} = \frac{y}{\delta} = y'$$  \hspace{1cm} (5.1-1)

is assumed, where $u_\infty$ is the freestream velocity and $\delta$ is the boundary layer thickness. In the absence of magnetic fields this is a reasonable approximation to the actual profile when the mainstream is supersonic.

For a cooled, constant temperature wall in the absence of magnetic fields a simple integral of the energy equation is given by

$$h_0 = A + B u.$$  

At the wall $y' = u = 0$ and $h_0 = h_0 w$, where $h_0 w$ is the stagnation enthalpy at the wall, given by

$$h_0 w = C_p T_w,$$

and $T_w \approx 300^\circ K$.

In the mainstream, $u = u_\infty$ and $h_0 = h_0 \infty$. Thus

$$h + u^2/2 = h_0 w + (h_0 \infty - h_0 w) \frac{u}{u_\infty}.$$  \hspace{1cm} (5.1-2)

Putting (5.1-1) into (5.1-2) gives

$$h = h_0 w + (h_0 \infty - h_0 w) y' - \left( \frac{u_\infty^2}{2} \right) y'^2.$$  \hspace{1cm} (5.1-3)
In figs. (5.1) to (5.6) various flow properties are plotted as functions of $y'$ for the enthalpy distribution of equation (5.1-3).

The mainstream conditions used are those shown in Table (3.1) for $p_1 = 0.5 \text{ cm.}, M_s = 15.6$.

One of the main features of these curves is that the high heat transfer rate cools the inner 15% of the boundary layer to such an extent that the electrical conductivity and therefore any magnetic forces become very small. Magnetic forces are important only in the outer layer, and even here they are much smaller than for the corresponding insulated wall.

The profiles assume that the gas remains in equilibrium through the boundary layer. In a low density boundary layer the ion-electron recombination rate may be too slow to "keep up with" the high temperature gradients within the layer. Thus some ionization may be frozen into the boundary layer, producing a substantial electron density near the wall.

These results are strictly valid only in the absence of the magnetic force term in equation (4.2-6). However, since the experimental freestream interaction parameter is not large (fig. (2.2)), and the values of the parameter within the boundary layer are not very different from the
freestream value, it is reasonable to employ these results to describe the characteristics of the boundary layer studied here.

5.2 Momentum Integral Equation

A number of useful approximate methods in the theory of boundary layers stem from an application of the momentum integral equation. Whereas equation (4.2-6) expresses local momentum conservation at points within the boundary layer, this equation, when integrated across the boundary layer considers the total momentum conservation within the boundary layer as a whole.4 Thus

\[
\int_0^s \left[ \rho u \frac{du}{dx} \right] dy + \int_0^s \left[ \rho v \frac{dv}{dy} \right] dy = - \int_0^s \left[ \frac{d \rho}{dx} \right] dy + \left[ \rho \frac{u^2}{2} \right]_0^s - \left[ \rho \frac{v^2}{2} \right]_0^s - \int_0^s \sigma u B^2 dy.
\]

Using the continuity equation (4.2-1) to eliminate \( \nu \) gives

\[
\frac{d}{dx} \int_0^s \rho u u_\infty (1 - \frac{u}{u_\infty}) dy - \int_0^s \rho u u_\infty \frac{du}{dx} dy = \int_0^s \frac{d \rho}{dx} dy + \int_0^s \sigma u B^2 dy + \tau \omega.
\]

If the mainstream is described by an equation of the form

\[
\rho_\infty u_\infty \frac{d u_\infty}{dx} = - \frac{d \rho}{dx} - \sigma_\infty u_\infty B^2,
\]

then,

\[
\frac{d}{dx} \rho_\infty u_\infty ^2 \int_0^s \frac{e u}{\rho_\infty u_\infty} (1 - \frac{u}{u_\infty}) dy + \rho_\infty u_\infty \frac{d u_\infty}{dx} \int_0^s (1 - \frac{e u}{\rho_\infty u_\infty}) dy = \frac{\tau \omega}{\sigma_\infty u_\infty} \int_0^s (1 - \frac{\sigma u}{\sigma_\infty u_\infty}) dy.
\]
In the equation, \( \delta_1 = \int_0^\infty (1 - \frac{e\cdot u}{e\cdot u_\infty})dy \) is known as the displacement thickness.

and \( \delta_2 = \int_0^\infty \frac{e\cdot u}{e\cdot u_\infty} (1 - \frac{u}{u_\infty})dy \) as the momentum thickness.

We can also put \( \delta_R = \int_0^\infty (1 - \frac{\sigma\cdot u}{\sigma\cdot u_\infty})dy \).

Thus \( \frac{d}{d\infty} e\cdot u_\infty \delta_2 + e\cdot u_\infty \frac{du}{d\infty} \delta_1 + \sigma\cdot u_\infty B^2 \delta_2 = \tau_\omega \). (5.2-3)

Equation (5.1-3) is known as the momentum integral equation.

### 5.3 Boundary Layer Thickness

For a uniform mainstream and in the absence of magnetic fields, equation (5.2-1) becomes

\[
\frac{d}{d\infty} \int_0^\infty e\cdot u (u_\infty - u)dy = \tau_\omega. \tag{5.3-1}
\]

The variation in density through the boundary layer under the conditions of the experiment was found in section (5.1) for an assumed linear velocity profile. Thus a rough estimate of the boundary layer thickness can be made by using these profiles in equation (5.3-1).

Therefore if we put

\[ u / u_\infty = \frac{y}{\delta} = y' \]

\[ dy = \delta dy' \]

and \( e / e_\infty = R(y') \)

then \( \tau_\omega = \mu_\omega \frac{\partial u}{\partial y}_w = \mu_\omega u_\omega / \delta \),

and equation (5.3-1) becomes

\[
\frac{\mu_\omega}{e_\infty u_\infty} \frac{1}{\delta} = \frac{d}{d\infty} \int_0^1 R(y') y'(1 - y') dy'. \tag{5.3-2}
\]
If we define $A$ by,
$$A = \int_0^1 a(y') dy',$$
where $a(y') = R(y') y'(1 - y')$,
equation (5.3-2) becomes
$$\frac{\delta}{\delta} \frac{d\delta}{dx} = \frac{\mu \omega}{\epsilon_{oo} u_{oo} A}.$$
Hence
$$\frac{\delta}{\delta} = \left( \frac{2}{A} \right)^{1/2} R_L^{1/2}$$
(5.3-3)
where $R_L = \epsilon_{oo} u_{oo} x/\mu_\omega$, is a Reynolds number based on the freestream values of $\epsilon$ and $u_\omega$, and the value at the wall for $\mu$.

The skin friction, $C_f$, is defined by
$$C_f = 2 \frac{\tau_m}{\epsilon_{oo} u_{oo}^2},$$
(5.3-4)
so that $C_f R_L^{1/2} = (2A)^{1/2}$.

(5.3-5)
Also the momentum thickness, $S_2 = A \delta$
giving
$$\frac{S_2}{x} = (2A)^{1/2} R_L^{-1/2}.$$  (5.3-6)

The displacement thickness $S_1 = \delta \int_0^1 (1 - R(y') y') dy'$.

If $B = \int_0^1 b(y') dy'$, where $b(y') = 1 - R(y') y'$,
$$\frac{S_1}{x} = B \left( \frac{2}{A} \right)^{1/2} R_L^{-1/2}.$$  (5.3-7)

Using the density profile shown in fig. (5.2) the functions $a(y')$ and $b(y')$ were plotted against $y'$. The integrals $A$ and $B$ were then evaluated numerically, giving
$$A = 0.23$$
and
$$B = 0.41.$$
Thus $s/x = 2.94 \frac{R_L}{2}$

$C_f R_L^{1/2} = 0.68$

$s_2/x = 0.68 R_L^{-1/2}$

$s_1/x = 1.2 R_L^{-1/2}$

When $M_s = 15.6$, $\nu = 0.5$ cm. of mercury, the mainstream conditions are,

$e_\infty = 7.9 \times 10^{-5}$ gm/cc

$\nu_\infty = 4.32 \times 10^5$ cm/sec

For $T_w = 300^0 K$, $\mu = 2.2 \times 10^{-4}$ c.g.s. units

Thus, $R_L/\text{cm} = e_\infty \nu_\infty/\mu = 1.5 \times 10^5 \text{ cm}^{-1}$.

In the experiments the length along the model is typically $\sim 2$ cm.,

making $R_L \sim 3 \times 10^5$.

Hence, $s \sim 1.1 \times 10^{-2}$ cm.

$C_f \sim 1.2 \times 10^{-3}$

$s_2 \sim 2.5 \times 10^{-3}$ cm.

$s_1 \sim 4.3 \times 10^{-3}$ cm.

If the mainstream value is used for $\mu$, $\mu = 2 \times 10^{-3}$

c.g.s. units, the unit Reynolds number becomes,

$R_\infty/\text{cm} = e_\infty \nu_\infty/\mu \sim 1.5 \times 10^4 \text{ cm}^{-1}$

Over the experimental range of conditions the shape of the density profile only changes very slightly, so that the values of A and B found above may be used in all cases.
6. THE EFFECT OF A MAGNETIC FIELD ON THE MAINSTREAM.

6.1 Shock Relations in a Magnetic Field.

The change in gas conditions across a shock wave can be analysed by assuming that the hydrodynamic and thermodynamic variables change discontinuously across the shock. However, Maxwell's equations must still be satisfied by the magnetic field.

Thus from \( \nabla \times B = \mu_0 J \) we must have across the shock,

\[
J_z = \frac{1}{\mu_0} \frac{\partial B}{\partial x},
\]

where the \( x \)-direction is normal to the shock and the \( z \)-direction is tangential to it.

Since the shock is regarded as a discontinuity

\[
\Delta x \rightarrow 0.
\]

If \( B \) changes discontinuously \( \Delta B > 0 \).

Hence \( \Delta B/\Delta x \rightarrow \infty \).

This implies that an infinite current, \( J_z \), exists which, because the conductivity is finite, is impossible.

Therefore the magnetic field is continuous across the shock.

Because both \( J \) and \( B \) are finite, and the discontinuity is very thin, the body force in any momentum balance across the shock wave is vanishingly small. Hence the usual shock relation applies, namely,

\[
P_1 + e_1 u_1^2 = P_2 + e_2 u_2^2.
\]
However, because the temperature and velocity change discontinuously, and \( \sigma \) is dependent on temperature, the product \( \sigma u \) changes discontinuously. But \( J = \sigma u B \), and \( B \) is constant across the shock.

Hence the induced current changes discontinuously, giving

\[
\frac{J_1}{\sigma_1 u_1} = \frac{J_2}{\sigma_2 u_2}.
\]

Now \( J = \sigma u B = \frac{1}{\mu_0} \frac{\partial B}{\partial x} \),

so that \( u B = \frac{1}{\mu_0 \sigma} \frac{\partial B}{\partial x} \)

If \( \sigma \to \infty \), \( u B \) = constant,

and the jump condition becomes \( u_1 B_1 = u_2 B_2 \).

Thus a discontinuous change in \( B \) occurs and a current sheet forms at the shock. A momentum balance then gives

\[
p_1 + e_1 u_1^2 + B_1^2 / 2 \mu_0 = p_2 + e_2 u_2^2 + B_2^2 / 2 \mu_0.
\]

With oblique shocks the same arguments apply.

Across the shock the non-magnetic jump relations are satisfied, and, although the magnetic field is continuous, the induced current changes discontinuously. "Switch-on" shock waves, in which the tangential field is induced behind the shock when only a normal field exists in front of it, should only occur in the limit \( \sigma \to \infty \).
6.2 Characteristic Equations

In discussing the interaction between the supersonic mainstream and the boundary layer an expression relating deflections of the mainstream to the pressure changes they induce is required. This relation is most easily obtained from the characteristic equations of the flow, in this case modified by the inclusion of magnetic body forces.

Guderley has extended the usual method of characteristics in two dimensions to include the effects of body forces. However, here we will extend the method outlined by Hayes and Probstein.

The equations of motion are written in terms of intrinsic coordinates, s along the streamlines and n, normal to them.

The magnetic field is resolved in these directions. See figure (6.1).

The induced current \( J = \sigma q B_n \), and the induced components of force are \( F_s = -\sigma q B_n^2 \), and \( F_n = \sigma q B_s B_n \), where \( q \) is the total velocity.

The continuity equation is of the form

\[
\frac{\partial}{\partial s} (\varepsilon \rho) + \varepsilon q \frac{\partial \rho}{\partial n} = 0 ,
\]

(6.2-1)
while the momentum equations become

\[ e \varphi \frac{\partial \varphi}{\partial s} + \frac{\partial p}{\partial s} + \sigma \varphi B_n^2 = 0 \]  \quad (6.2-2)

\[ e \varphi^2 \frac{\partial \varphi}{\partial s} + \frac{\partial p}{\partial n} - \sigma \varphi B_s B_n = 0 \]  \quad (6.2-3)

where \( \varphi \) is the deflection of the streamline from some reference direction.

Using \( a^2 = \partial p / \partial \varphi \), and putting \( \varphi / a = M \), (6.2-1) and (6.2-2) give

\[ \frac{M^2 - 1}{e \varphi^2} \frac{\partial p}{\partial s} + \frac{\partial \varphi}{\partial n} - \frac{\sigma}{e \varphi} B_n^2 = 0 \]  \quad (6.2-4)

Now along a characteristic, a relation exists between total differentials. Hence we make use of the expressions

\[ \frac{d \varphi}{ds} = \frac{\partial \varphi}{\partial s} + \frac{\partial \varphi}{\partial n} \frac{dn}{ds} \]  \quad (6.2-5)

and

\[ \frac{d \varphi}{ds} = \frac{\partial \varphi}{\partial s} + \frac{\partial \varphi}{\partial n} \frac{dn}{ds} \]  \quad (6.2-6)

to eliminate \( \partial p / \partial s \), \( \partial \varphi / \partial s \) and \( \partial p / \partial n \).

However, in the final equation a term of the form

\[ \frac{\partial \varphi}{\partial n} \left[ 1 - (M^2 - 1) \left( \frac{dn}{ds} \right)^2 \right] \]

exists, so that one further relation needs to be satisfied.

If we put \( dn / ds = \pm (M^2 - 1)^{-\frac{1}{2}} \), this relation defines the directions in the flow for which we can write an expression in total differentials, i.e. the characteristics.
Hence we find that
\[ dp + \frac{e u^2}{\beta} d\theta = \left( \frac{B_0^2}{\beta^2} \pm \frac{B_n^2}{\beta^2} \right) \sigma u B^2 \, ds, \quad (6.2-7) \]
along \[ \frac{dn}{ds} = \pm \frac{1}{\beta}, \]
where \[ \beta = (M^2 - 1)^{\frac{1}{2}}. \]
These equations can then be written in rectangular coordinates \((x, y)\) by application of the usual transformation.

In the present situation we are interested in flow which is initially moving parallel to the \(x\)-axis in a transverse magnetic field so that equation \((6.2-7)\) becomes
\[ dp + \frac{e u^2}{\beta} d\theta = \frac{\sigma u B^2}{\beta^2} \frac{dy}{dx} \, dx, \quad (6.2-8) \]
along \[ \frac{dy}{dx} = \pm \frac{1}{\beta}. \]

6.3 Simple Waves in a Magnetic Field.

6.3.1 Uniform parallel flow

Let us put \( e u^2/\beta = K \) and \( \int \frac{\sigma u B^2}{\beta^2} \, dx = g(x) \) and assume that, over a limited region, changes in \( K \) and \( \sigma u B^2/\beta^2 \) are small compared with the changes in pressure. Upstream of \( x = 0 \) in fig. (6.2), i.e. for \( x < 0 \), the flow is uniform, while at \( x = 0 \) a magnetic field is established.
Applying the characteristic equation (6.2-8) to the mesh sketched in Fig. (6.2) we see that

\[ p_4 - p_2 + K(\theta_4 - \theta_2) = g(x_4) - g(x_2) \]

and

\[ p_4 - p_3 - K(\theta_4 - \theta_3) = g(x_4) - g(x_3) \]

Since the magnetic effects are uniform in the y direction

\[ g(x_2) = g(x_3) \]

Also, because the flow upstream of \( x = 0 \) is uniform

\[ p_2 = p_3 \text{ and } \theta_2 = \theta_3 \]

Hence

\[ \theta_4 = \theta_2 = \theta_3 \]

\[ p_4 - p_2 = g(x_4) - g(x_2) \]

and

\[ p_4 - p_3 = g(x_4) - g(x_3) \]

Thus, when the field is applied both families of Mach lines remain straight, while an increase in pressure occurs with increasing distance.

6.3.2 Simple Waves

Now consider what happens when this flow is deflected.

In Fig. (6.3) the deflection starts at \( x = 0' \).

Along \( O'Y' \) the pressure is constant (= \( p_1 \)), \( q = q(0') \),
and \( \Theta = \Theta_1 \).

The (+) line 1—4 is the first one to feel the effect of the disturbance. This line is straight, which means that
\[
\Theta_1 = \Theta_2 = \Theta_3 = \ldots.
\]
and that
\[
\eta_2 - \eta_1 = g(x_2) - g(x_1), \quad \eta_3 - \eta_1 = g(x_3) - g(x_1), \ldots
\]
Along the (-) lines,
\[
\eta_5 - \eta_2 - K(\Theta_5 - \Theta_2) = g(x_5) - g(x_2)
\]
and
\[
\eta_6 - \eta_3 - K(\Theta_6 - \Theta_3) = g(x_6) - g(x_3).
\]
For the (+) line
\[
\eta_6 - \eta_5 + K(\Theta_6 - \Theta_5) = g(x_6) - g(x_5).
\]
Solving these equations we find that
\[
\Theta_6 = \Theta_5 ,
\]
\[
\eta_6 - \eta_5 = g(x_6) - g(x_5).
\]
Hence in this region the (+) lines remain straight, while the pressure varies as
\[
dp = d \eta = \sigma u B^2 dx / \beta^2. \tag{6.3.2-1}
\]
Along the (-) lines
\[
dp - \frac{e u^2}{\beta} d \Theta = \frac{\sigma u B^2 dx}{\beta^2} , \tag{6.3.2-2}
\]
Finally, consider any line joining two points "0"
and "3" on different (+) lines. See figure (6.4).

For the (+) lines

\[ p_1 - p_o = g(x_1) - g(x_o) \]
\[ p_3 - p_2 = g(x_3) - g(x_2) \]

Also \( \theta_1 = \theta_0 \) and \( \theta_3 = \theta_2 \).

For the (-) lines

\[ p_3 - p_1 - K(\theta_3 - \theta_1) = g(x_3) - g(x_1) \]
\[ p_2 - p_o - K(\theta_2 - \theta_0) = g(x_2) - g(x_o) \]

Solving these equations for the conditions at "3" gives

\[ p_3 - p_o - K(\theta_3 - \theta_0) = g(x_3) - g(x_o) \]

Thus the flow changes encountered by any line crossing the straight (+) lines are related by

\[ dp = \frac{e u^2}{\beta} \, d\theta + \frac{\sigma u B^2}{\beta^2} \, dx \quad (6.3.2-3) \]
7. INTERACTION BETWEEN A LAMINAR BOUNDARY LAYER AND A SUPersonic MAINSTREAM.

7.1 Physical description of the Interaction.

In this interaction a pressure rise retards the boundary layer flow and increases its thickness, thus deflecting the supersonic mainstream. A pressure rise, proportional to the deflection, is induced in the mainstream and further thickens the boundary layer so that it continues to grow thicker in equilibrium with the increasing pressure. At the wall the pressure rise is balanced by viscous forces which decrease as the thickness increases. Eventually they become too small to support the pressure gradient and separation takes place. The mechanism described is usually referred to as a "free interaction" since it is independent of the agency producing the initial disturbance and is governed purely by the local conditions.

In the mainstream an increase in pressure slows down the gas and increases its density. The decrease in speed tends to expand the streamtubes while the increase in density contracts them. In supersonic flow density changes dominate so that a net contraction occurs. If a body force which retards the flow is present, only the expansion process is aided, so that for a given pressure rise the net contraction is reduced.
Thus in this interaction, a magnetic body force in the mainstream reduces the change in boundary layer thickness associated with a given pressure rise. Hence the viscous forces at a particular point are increased. Therefore a greater pressure rise can be tolerated before separation occurs. Hence we can conclude that if a retarding body force is present in the mainstream only, the pressure rise to separation is increased.

In fig. (7.1) several important features common to all interactions of this type are sketched. As well as giving a general picture of the interaction, fig. (7.1) shows the distribution of pressure, skin friction and heat transfer through the interaction region. Between the start of the interaction at \( x = 0 \) and separation at \( x = x_s \), the pressure rise is roughly linear with distance. At the same time the skin friction decreases rapidly, becoming zero at separation.

Beyond separation the boundary layer breaks away from the wall and a region of slow moving reverse flow is set up underneath it. This is often known as the "dead air" region. The pressure continues to rise until a pressure \( p_s \sim 2p_s \) is reached. At this point the pressure gradient approaches zero and a constant pressure plateau is established.

At the beginning of the interaction the heat transfer
rate drops rapidly from its flat plate value to a very much smaller value in the separated region. Thus, the inner layer of low momentum gas which is built up underneath the main boundary layer effectively shields the surface from heat transfer.

7.2 Outline of earlier work.

A number of different theories have been proposed for the interaction, differing mainly in the method used to describe the boundary layer. Nearly all use an expression similar to equation (6.3.2-3) to describe the mainstream.

The solutions can be roughly classified into three groups. The first group consists of semi-empirical mixing theories based on the mass entrainment rate between the inviscid and viscous flows\(^{47,48}\). The second group, of which the analysis to be discussed is a member\(^8\), divides the boundary layer into an inner thin viscous region and an outer, largely inviscid zone\(^{49}\). An elegant perturbation solution was developed by Lighthill\(^{50}\) for the case where the disturbance was too weak to cause separation. The third group makes use of boundary layer integral relations\(^{51,52,53}\). In several of these solutions it is shown that the change in momentum thickness is much smaller than the change in displacement thickness.

Also because of the violent way in which the velocity
profile varies near separation, one of the energy integrals is often used as well as the momentum integral equation.

Most of these theories predict that

\[ C_{p5} (M^2 - 1)^{\frac{1}{4}} R^{\frac{1}{4}} = D \sim 1 \]

Experimentally \( D \approx 0.93 \), while theoretical values vary from 0.8 to 1.2.

Various attempts have been made to describe the flow beyond separation. This is much harder because of the mathematical difficulty of adequately describing the flow in the dead air region. Honda\(^5^2\) has assumed that the separated boundary layer forms a jet-like flow over the separated region. Lees and Reeves\(^5^1\) assumed that the integral equations were valid over the entire region and used velocity profiles containing regions of reversed flow in them. Hakkinen\(^8\) has also developed a simple expression for the length of the separated region based largely on intuitive notions.

A number of experimental investigations have also been undertaken, so that at this stage the factors governing the separation pressure are fairly well understood under supersonic conditions\(^8,5^4,5^5,5^6\). More recently interest has centered on the hypersonic region and several papers giving pressure and heat transfer distributions have appeared\(^5^7,5^8,5^9\).

Several theories have also attempted to include the
effects of heat transfer$^{49,60,61}$. The general conclusion seems to be that the wall temperature, $T_w$, has very little effect on the separation pressure. However, the streamwise extent of the interaction up to separation is proportional to $T_w$. In our case the wall is very strongly cooled so that the pressure gradients are probably very much larger than for an insulated wall. Early experiments seemed to indicate that the interaction was barely dependent upon heat transfer effects$^{62}$. However a more recent experiment$^{63}$ agrees more closely with the theory.

Two papers have appeared which deal with the interaction on a curved wall$^{64,65}$. The paper by Greber is discussed in some detail in section (7.4).

Finally, Trilling$^{66}$ has considered an unsteady interaction and has suggested that stable vibrations exist for certain gas conditions. These vibrations have been observed by Fiszdon$^{68}$.

7.3 Hakkinen's Theory of the Interaction$^8$.

The basic assumption of Hakkinen's theory is that the boundary layer is made up of an outer zone whose velocity profile is virtually that of the undisturbed layer, joined at $y = \delta_j$ to an inner sub-layer of small momentum. See figure (7.2). The pressure rise induced in the mainstream is due to the apparent lifting of the
undisturbed layer through a height $S$. Hence for the outer layer near $S_i$

$$\tau(y) \approx \tau_0,$$  \hspace{1cm} (7.3-1)

where $\tau$ is the local viscous stress and $\tau_0$ is the skin friction at the beginning of the interaction.

Also $\mu u(y) \approx \tau_0 (y - S).$  \hspace{1cm} (7.3-2)

For the inner layer, the momentum is small, so that we can assume that the viscous stress is balanced by the pressure gradient. Thus

$$\partial \tau/\partial y = dp/dx, \text{ for } y \leq S_i. \hspace{1cm} (7.3-3)$$

Hence $\tau(y) \approx \tau_\infty + y(dp/dx)$  \hspace{1cm} (7.3-4)

and $\mu u(y) \approx \tau_\infty + \frac{1}{2} y(dp/dx).$  \hspace{1cm} (7.3-5)

Matching (7.3-1), (7.3-2), (7.3-4) and (7.3-5) at $y = S_i$ gives

$$S \frac{dp}{dx} = \frac{(\tau_0 - \tau_\infty)^2}{2 \tau_0}. \hspace{1cm} (7.3-6)$$

If the pressure coefficient $(C_p)$ is defined by

$$C_p = (p - p_0) / \frac{1}{2} \rho_\infty u_\infty^2 \hspace{1cm} (7.3-7)$$

and the skin friction coefficient $(C_f)$ by

$$C_f = \tau/\frac{1}{2} \rho_\infty u_\infty^2, \hspace{1cm} (7.3-8)$$

equation (7.3-6) becomes

$$S \frac{dC_p}{dx} = \frac{1}{2} C_f 0 (1 - C_f 0 / C_f 0)^2. \hspace{1cm} (7.3-9)$$

The mainstream is described by the Prandtl-Meyer relation

$$dp = \frac{e u^2}{(M^2-1)^{1/2}} d\theta, \hspace{1cm}$$
where $\Theta$ is the deflection of the streamlines.

Now it is assumed that the pressure rise due to the thickening of the boundary layer is mostly due to the growth of the inner layer.

If $\Theta$ is related to the rate of growth of the boundary layer thickness, $\Theta = \frac{dS}{dx}$.

In this case we assume that we are far enough from the leading edge to neglect the change in $S$, due to the natural growth of the boundary layer. Hence

$$C_p = \frac{2}{\beta} \frac{dS}{dx}, \quad (7.3-10)$$

where $\beta = (\frac{M^2 - 1}{M^2})^{\frac{1}{2}}$.

Hence $S = \frac{\beta}{2} \int C_p \, dx. \quad (7.3-11)$

Combining $(7.3-9)$ with $(7.3-11)$ gives

$$\beta \frac{dC_p}{dx} \int C_p \, dx = C_{f_0} \left(1 - \frac{C_{f_0}}{C_{f_0}}\right)^2. \quad (7.3-12)$$

Integrating by parts gives

$$\frac{dC_p}{dx} \int C_p \, dx = \frac{C_p^2}{2} + \int \frac{d^2C_p}{dx^2} \left[ C_p \, dx \right] \, dx. \quad (7.3-13)$$

Now, $d^2C_p/dx^2$ is small except for a short region near the beginning of the interaction where $C_p$ is also small. Thus the second integral is small compared with $C_p^2$.

Hakkinen, therefore, neglects this integral completely, which is equivalent to assuming that $dC_p/dx$ is constant
through the interaction.

Hence \( C_{p}^2 = \frac{2C_{f0}}{\beta} (1 - \frac{C_{f}C}{C_{f0}})^2 \). \hspace{1cm} (7.3-14)

At separation \( C_{f0} = 0 \), so that the pressure coefficient at separation, \( C_{ps} \), is given by

\[ C_{ps} = \left( \frac{2C_{f0}}{\beta} \right)^{\frac{1}{2}}. \hspace{1cm} (7.3-15) \]

If we take \( \kappa \propto T \) then \( \frac{C_{f0}R^2}{\kappa} = 0.664^4 \)

and \( C_{ps}(M^2 - 1)^{\frac{1}{4}} R^\frac{1}{4} = 1.15 \). \hspace{1cm} (7.3-16)

This result may be compared with the correlation of Chapman\(^{54} \), for which

\[ C_{ps}(M^2 - 1)^{\frac{1}{4}} R^\frac{1}{4} = 0.93. \]

Knowing \( C_{ps} \) we can now estimate the angle through which the flow has turned. Thus

\[ \Theta_s = (d\xi/dx)_s = \beta C_{ps}/2 = (\beta C_{f0}/2)^{\frac{1}{2}}. \hspace{1cm} (7.3-17) \]

Once separation has occurred it becomes difficult to extend the analysis to the plateau. However, experiments\(^{54,64} \) show that the plateau pressure coefficient, \( C_{pp} \), is some multiple of the separation pressure. Thus

\[ C_{pp} = GC_{ps}, \hspace{1cm} (7.3-18) \]

where \( G \) appears to be independent of Reynolds number, while being slightly dependent upon Mach number. Figure (7.3) uses Chapman's data to show this variation. From the figure \( G = 1.9 \) appears to be a reasonable number to choose.
for these experiments. For his experiments at $M = 2$, Greber found that $G = 1.54$ correlated his data for shock waves incident on both flat and curved surfaces and also for separation induced by a corner.

7.4 Theory of the Interaction in a Magnetic Field.

Greber has extended the analysis just given to the case where separation is induced on a curved surface. The equation for the mainstream is

$$\frac{dC_p}{dx} = \frac{2}{\beta} \frac{d\theta}{dx} + \frac{dC_\theta}{dx}$$  \hspace{1cm} (7.4-1)

where $dC_\theta/dx$ is the pressure gradient due to surface curvature alone.

This may be compared with the expression found earlier for the mainstream in a magnetic field. (Equation (6.3.2-3)).

$$dp = \frac{e \bar{u}}{\beta} d\theta + \frac{\sigma u B^2}{\beta^2} dx.$$  

If we again relate $\theta$ to the increase in boundary layer thickness,

$$\frac{dC_p}{dx} = \frac{2}{\beta} \frac{d^2\epsilon}{dx^2} + \frac{2}{\beta^2} \frac{dS}{dx}$$  \hspace{1cm} (7.4-2)

where $S = \int \frac{\sigma \bar{B}^2}{e \bar{u}} dx$ is a variable interaction parameter and $dS/dx = \sigma B^2/e u$.

Comparing equation (7.4-1) and (7.4-2) we see that
we may be able to establish an analogy between the effects of surface curvature and those of the magnetic field.

The theories discussed in section (7.2) indicate that the part of the boundary layer closest to the wall is the critical region in the interaction. These theories rely on the idea that the main boundary layer is displaced from the wall by a low momentum sub-layer. In Hakkinen's theory this is stated explicitly, while in the integral methods an equivalent conclusion is reached when it is shown that the changes in the momentum thickness are much smaller than changes in the displacement thickness.

For the boundary layer on the models in the shock tube, the region near the surface is very strongly cooled. The electrical conductivity is small and the magnetic body forces are much smaller than in the mainstream. See figures (5.1) to (5.6). Hence, for the inner layer we can neglect body forces and write as before

\[ \frac{dp}{dx} = \alpha \frac{v}{y}. \]

In Hakkinen's analysis the outer layer is assumed to be the same shape as the upstream undisturbed profile. That is, it is assumed that the pressure gradients encountered as the boundary layer approaches separation do not substantially influence the velocity profile. If
the magnetic body forces are of the same order, or less than the pressure gradients involved, then it is also reasonable to assume that the outer layer is unaltered in profile when a magnetic field is present.

Upstream of the interaction the boundary layer is subject to the pressure rise induced in the mainstream by the magnetic field. This pressure rise may influence the boundary layer profile before the interaction is reached. Experiments were performed in which the distance upstream of the interaction over which the field was applied was varied by a factor of about five without producing an observable effect on the interaction. See section (9.4). It is therefore concluded that this effect is small.

Thus the outer layer can still be described by equations (7.3-1) and (7.3-2). Using these equations also implies the further assumption that the viscosity is unchanged, being equal to its value at the wall.

Figure (5.1) indicates that this is a reasonable assumption. Although the theory described here is very simple, the many assumptions made have a reasonable physical basis. The theory concentrates almost entirely on the inner layer, but gives values for $C_p^s$ which are in accord with experiments on insulated surfaces. More elaborate
theories indicate that the value of \( C_p \) is unaffected by heat transfer, and this conclusion is supported by experiments to be described in section (9.5'). As the analysis is primarily aimed at predicting \( C_p \), the assumptions are regarded as adequate.

Hence, as a first approximation we can use the same boundary layer relation as before, viz.

\[
\frac{\delta}{\delta} \frac{dC_p}{dx} = \frac{1}{2} C_{r_0} (1 - \frac{C_{r_0} \cdot C_{r_0}}{C_{r_0}})^2. \quad (7.3-9)
\]

Thus we have established that the boundary layer behaves as if body forces were absent, and that the mainstream relation is of the same form as that for a curved surface.

In his analysis Greber assumes that \( dC_p/dx \) is constant, and also takes \( dC_p/dx \) to be constant for the same reasons as for the flat plate. See equation (7.3-13). He also shows that the effect of surface curvature on the boundary layer itself is small compared with its effect on the mainstream. This is similar to the conclusion drawn here that the effect of the field on the boundary layer itself is small compared with its effect on the mainstream.

Hence, if we ensure that \( \sigma B^2/\varepsilon \omega \) is constant, by analogy with the results of Greber's theory we can write down the expression for the pressure coefficient at separation
in the presence of a transverse magnetic field, \( C_{se} \).

Whence

\[
C_{se} = C_{ps} \left[ 1 - \frac{2}{\rho^2} \frac{dS/dx}{(dC_p/dx)} \right]^{-\frac{1}{2}}
\]

(7.4-3)

where \( C_{ps} \) is the pressure coefficient at separation in the absence of a field.

Now \( \frac{dS}{dx} > 0 \), so that \( C_{se} > C_{ps} \).

Hence the presence of the magnetic body force in the mainstream increases the pressure coefficient at separation.

The result found by Greber for a curved plate,

\[
C_{pc} = C_{ps} \left[ 1 - \frac{dC_p/dx}{C_p^2} \right]^{-\frac{1}{2}}
\]

(7.4-4)

has been checked by him for the interaction caused by a shock wave incident upon the boundary layer on a convex surface. Thus, in his experiments \( dC_p/dx < 0 \) and \( C_{pc} < C_{ps} \) by the amount predicted by equation (7.4-4). This result therefore supports the rather drastic assumptions made.

It should be noted however, that for the magnetic field, \( (dS/dx) > 0 \), so that for a strict analogy to be maintained, the magnetic effects should be related to a concave surface.

It should also be noted that the effect considered here can only be regarded as a perturbation of the basic interaction between the thickening boundary layer and
curving mainstream. That is, the magnetic forces must be smaller than the pressure forces due to the curving mainstream. If the magnetic forces become too large the effect described here is swamped and separation is provoked directly by the pressure due to the magnetic forces.

Similarly, with a concave surface, the effect should only occur when the curvature is very small. Once the curvature becomes large the associated pressure rise is sufficient to cause separation. Separation induced in this way has been studied experimentally by Chapman.

If we regard the effect as a small perturbation, then in the expression for $C_{sB}$ we can approximate $dC_p/dx$ by its field free value. That is,

$$\frac{dC_p}{dx}_{B \neq 0} \approx \frac{dC_p}{dx}_{B = 0} \approx \frac{C_{ps}}{x_s}$$

where $x_s$ is the distance from the beginning of the interaction to separation in the absence of a magnetic field.

Hence

$$C_p B \approx C_{ps} (1 + \psi)$$

where

$$\psi = \frac{S_s}{\beta^2 C_{ps}}$$

and $S_s = \sigma B^2 x_s / e u$ is an interaction parameter based on $x_s$.

A comparison of Greber's results for a convex wall with those for a flat plate suggests that changes in $x_s$ due to the convex surface, are about an order of magnitude smaller than changes in $C_{ps}$. Thus, when changes due to the
field are not so small it may be reasonable to put

\[(dC_p/\Delta x)_{B \neq 0} \approx C_{pB}/\Delta x. \]

If this is so, then

\[C_{PB} \approx C_{pB} \left( 1 + \psi + (1 + \psi^2)^{\frac{1}{2}} \right).\]

Beyond separation Greber again finds that the plateau pressure is a constant multiple of the separation pressure. In view of the preceding arguments establishing an analogy between the effects of a magnetic field and a convex surface, it is expected that the same relation between separation pressure and plateau pressure will apply in the present case.

7.5 Angles associated with pressure changes.

In the shock tube experiments it was not possible to obtain pressure distributions. However the change in direction of the flow was observed photographically. Thus we require an expression for the change in the deflection angle at separation due to the magnetic field.

When \(B = 0\) we have shown that

\[\Theta_s = (\beta/2) C_{ps} = (\beta C_{po}/2)^{\frac{1}{2}}. \quad (7.3-17)\]

When \(B \neq 0\) we have for the mainstream

\[\frac{dC_p}{dx} = \frac{2}{\beta} \frac{d\Theta}{dx} + \frac{2}{\beta^2} \frac{dS}{dx}, \quad (7.4-2)\]

while for the boundary layer at separation

\[6 (dC_p/\Delta x) = C_{po}/2. \quad (7.5-1)\]
The analysis given above assumes that \( \frac{dC_p}{dx} \) and \( \frac{dS}{dx} \) are constant. Hence from (7.4-2) \( \frac{d\theta}{dx} = \frac{d^2S}{dx^2} = \) constant.

Since \( S = \int \theta \, dx \), combining (7.4-2) and (7.5-1) gives

\[
\delta \frac{dC_p}{dx} = \frac{2}{\beta} \frac{d\theta}{dx} \int \theta \, dx + \frac{2}{\beta^2} S \frac{dS}{dx} = \frac{C_f}{2}. \tag{7.5-2}
\]

Since \( \frac{d\theta}{dx} = \) constant, \( \frac{d\theta}{dx} \int \theta \, dx = \theta^2/2 \).

Rearranging (7.5-2) gives

\[
\theta^2 = \beta \frac{C_f}{2} \left[ 1 - \frac{2}{\beta^2} \frac{\delta}{(C_f/2) \frac{dS}{dx}} \right]. \tag{7.5-3}
\]

Since \( \delta (\frac{dC_p}{dx}) = C_f/2 \), and \( \theta_s^2 = \beta \frac{C_f}{2} \), (7.5-3) can finally be written

\[
\theta_{SB} = \theta_s \left[ 1 - \frac{2}{\beta^2} \frac{dS/dx}{\frac{dC_p}{dx}} \right]^{1/2}, \tag{7.5-4}
\]

where \( \theta_{SB} \) is the flow deflection angle at separation in the presence of a magnetic field.

The expression inside the bracket is the same as for the pressure coefficient, so that under similar perturbation conditions we can write

\[
\theta_{SB} = \theta_s (1 - \gamma). \tag{7.5-5}
\]

Thus the physical effect we are looking for in the experiments is a decrease in the flow deflection angle of order \( \gamma \).

However, the actual separation point is not visible,
and only the final flow deflection angle related to the pressure coefficient of the plateau is observable. Since the plateau pressure is proportional to the separation pressure, the change in the plateau pressure angle, $\Theta_{BP}$, is given by

$$\Theta_{BP} = G \Theta_{ps} (1 - \gamma)$$

(7.5-6)

$$= \Theta_p (1 - \gamma)$$

(7.5-7)

where $\Theta_p$ is the plateau pressure angle in the absence of a field.

Thus by observing the change in the plateau pressure angle we can find the value of the factor $(1 - \gamma)$.

### 7.6 Expected magnitude of the change in the conditions at separation due to the magnetic field.

Previous theories of the interaction discussed in section (7.2) suggest that the pressure coefficient at separation, $C_{ps}$, is unaffected by heat transfer and that it is governed purely by the mainstream conditions. If we assume that this is so in the present situation then

$$C_{ps} = \frac{0.93}{R_{xo}^{\frac{1}{4}} (M^2 - 1)^{\frac{1}{4}}}$$

(7.6-1)

where $R_{xo}$ is a Reynolds number based on conditions in the mainstream given by

$$R_{xo} = \frac{c_{oo} U_{x0} x_{0}}{\mu_{oo}}$$
and \( x_s \) is the distance from the leading edge to the beginning of the interaction.

Using \( C_{pp} = 1.9 C_{ps} \), values of \( C_{pp} \) and \( \Theta_P \) were calculated for the shock tube conditions summarized in Table (3.1) and the results are shown in Table (7.1).

In section (7.4) it was shown that the change in \( C_p \) and \( \Theta \) due to the magnetic field is governed by a factor \( \psi \) given by

\[
\psi = \frac{1}{\beta^2} \frac{\sigma B^2}{e u} \frac{x_s}{C_{ps}}.
\]

The magnitude of the term \( \sigma B^2/eu \) can be readily found from fig. (2.2) and the measured size of the magnetic field.

In the experiments the maximum value of the field, \( B_0 \), \( \approx 3.0 \, \text{w/m}^2 \), and taking the average value of \( B^2 \) over the field region gives

\[
B^2 \approx 0.7 B_0^2 \approx 5.3 \, (\text{w/m}^2)^2.
\]

The main uncertainty associated with the calculation of \( \psi \) is the difficulty of finding a reliable value for \( x_s \) under the present conditions. Chapman's results at Reynolds numbers and Mach numbers similar to those found in the shock tube indicate that \( x_s \) is typically \( \approx 0.5 \, \text{cm} \) in the absence of heat transfer.

The theories discussed in section (7.2) suggest that \( x_s \)
is strongly dependent upon heat transfer, being probably proportional to \((T_w/T_\infty)\). In the shock tube
\[
T_w/T_\infty \approx 300/12700 \approx 0.0236.
\]

On this assumption,
\[
\chi_s \sim 0.5 \times 0.0236 \sim 1.2 \times 10^{-2} \text{ cm}.
\]

In section (5.3) we showed that the boundary layer thickness \(\sim 10^{-2} \text{ cm}\). Hence the calculated value of \(\chi_s\) suggests that separation is taking place over about one boundary layer thickness. Viewed in this way the value of \(\chi_s\) seems remarkably small since separation usually takes place over tens of boundary layer thicknesses. However, using this value of \(\chi_s\) should give a conservative estimate of the effect of the field.

Combining the various estimates gives the values of \(\gamma\) shown in Table (7.1).

<table>
<thead>
<tr>
<th>(M_s)</th>
<th>(R_\infty) (\times 10^4)</th>
<th>(R_{x_0}) (\times 10^4)</th>
<th>(C_{ps})</th>
<th>(C_{pp})</th>
<th>(\Theta^\circ)</th>
<th>(\frac{\sigma_{oe}(w/m^2)}{e_{10}U_{10}}) cm(^{-1})</th>
<th>(\gamma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.4</td>
<td>0.2</td>
<td>2.45</td>
<td>1.53</td>
<td>0.056</td>
<td>0.106</td>
<td>6.8</td>
<td>36.5</td>
</tr>
<tr>
<td>15.6</td>
<td>0.5</td>
<td>2.37</td>
<td>3.4</td>
<td>0.0466</td>
<td>0.0885</td>
<td>5.5</td>
<td>19</td>
</tr>
<tr>
<td>14.3</td>
<td>1.0</td>
<td>2.14</td>
<td>4.2</td>
<td>0.0472</td>
<td>0.0895</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td>12.9</td>
<td>2.0</td>
<td>1.92</td>
<td>6.3</td>
<td>0.046</td>
<td>0.0875</td>
<td>4.1</td>
<td></td>
</tr>
</tbody>
</table>

(a) \(x_0 = 2 \text{ cm}\).

Hence, when \(P_1 = 0.5 \text{ cm} \) of mercury, the magnetic field should produce about a 7% change in the separation.
conditions while, for \( p_1 = 0.2 \) cm, the change should be about 10%.

Since the plateau pressure angle \( \sim 5^\circ \) the field should produce a change \( \sim 0.5^\circ \). Thus the effect is very small. This is caused primarily by the extremely small value assumed for \( x_3 \), brought about by its strong dependence on heat transfer.

### 7.7 Length of a Step-induced Separated Region

In fig. (1.1) a step of height \( h \) separates the boundary layer and forms a separated region of length \( l \). If the gas in the separated region is completely stagnant a pressure gradient cannot be supported and a pressure plateau is produced. When \( B = 0 \) the mainstream condition, \( \frac{\partial C_p}{\partial p} = \frac{2}{\rho} \frac{\partial \theta}{\partial z} \), shows that if a pressure plateau exists the angle \( \theta \) is constant. Hence the outer edge of the shear layer is straight and the plateau pressure angle, \( \Theta_p \), determines the length, \( l \), through the relation

\[
\frac{h}{l} = \tan \Theta_p \approx \Theta_p.
\]  

(7.7-1)

Because of the small size of the photographs of the flow, it is very difficult to measure \( \Theta_p \) directly. However, \( l \) can be measured fairly accurately and \( h \) is known. Thus if the constant pressure assumption is satisfied, \( \Theta_p \) can be found from (7.7-1). Under these conditions, \( \Theta_p \) is governed by the free-interaction analysis
given in section (7.5). That is,

\[ \Theta_p = \frac{1}{2} G \beta C_{ps} \]

where \( C_{ps} \) is given by (7.6-1).

Hence from (7.7-1) the expression for the free-interaction separated length becomes

\[ l = Kh \beta^{-\frac{1}{2}} R_{x0}^{\frac{1}{4}} \]

where \( K \) is a constant.

If we put \( G = 1.9 \) then

\[ K = \frac{2}{(1.9 \times 0.93)} = 1.13 \]

If a reference length, \( L' \), is defined then

\[ l = Kh \beta^{-\frac{1}{2}} R_{x0}^{\frac{1}{4}} (\frac{x_0}{L'})^{\frac{1}{4}}. \quad (7.7-2) \]

If \( l \) does not follow this variation, pressure gradients may exist within the separated region. If this is so then \( \Theta \) rises above its free interaction value and the observed separated length for a given step height would be less than that expected for a free interaction.

If \( l_B \) is the length of the separated region in a magnetic field and the shear layer remains straight then,

\[ \Theta_{Bp} = h/l_B. \]

Hence, for the same gas conditions and step height, a change in length occurs given by,

\[ \lambda = l_B/l = \Theta_p/\Theta_{Bp} = 1/(1 - \gamma) > 1. \quad (7.7-3) \]

Thus a magnetic field increases the length of the separated
region.

However, when a field is present the mainstream relation becomes

\[ \frac{dC_p}{d\Theta} = \left( \frac{2}{\beta} \right) d\Theta + \left( \frac{2}{\beta^2} \right) dS. \quad (7.4-4) \]

Thus, if it is assumed that the pressure is constant within the separated region, then the presence of the field decreases \( \Theta \). Hence the shear layer is no longer straight and takes up the shape sketched in fig. (7.4).

Since we have assumed that \( \frac{dC_p}{dx} = 0 \) within the separated region, (7.4-2) becomes

\[ \frac{d^2 \Delta}{dx^2} = \left( \frac{-1}{\beta} \right) \left( \frac{dS}{dx} \right), \quad (7.7-5) \]

where \( \Delta \) is the distance from the surface to the edge of the shear layer and \( \Theta = \frac{d\Delta}{dx} \).

If \( \left( \frac{-1}{\beta} \right) \left( \frac{dS}{dx} \right) \) is constant then

\[ \Delta = \delta_0 + \delta_{BP} + \Theta_{BP} (x - x_{BP}) - \left( \frac{dS}{dx} \right) \left( x - x_{BP} \right)^2 / 2 \beta \]

\[ \quad (7.7-6) \]

where \( \delta_0 \) is the undisturbed boundary layer thickness, \( \delta_{BP} \) is the increase in thickness up to the plateau when \( \beta \neq 0 \), and \( x_{BP} \) is the distance from the beginning of the interaction to the plateau when \( \beta \neq 0 \).

At the step, \( \Delta = h \) and \( x = x_{BP} + l_B \).

For a well developed region \( h \gg \delta_0, \delta_{BP} \), and when \( \beta = 0 \) \( h = \Theta_p l \). Hence, evaluating (7.7-6) at the step gives,
\[ \beta \lambda^2 - \mu \lambda + 1 = 0 \quad (7.7-7) \]

where \( \lambda = l_B/l_0 \),
\[ \beta = 1(dS/dx)/2p \theta_p \]

and \( \mu = \theta_B/p \theta_p = (1 - \psi) \).

If \( C_{pp} \) is the plateau pressure coefficient when \( \beta = 0 \) and \( S_1 = \sigma B^2 l/eu \) is an interaction parameter based on \( l \), the length of the separated region when \( \beta = 0 \), \( \beta \) is given by
\[ \beta = S_1/\beta^2 C_{pp} \quad (7.7-8) \]

The solution of (7.7-5) is
\[ \lambda = \frac{\mu}{2\beta} \left[ 1 \pm \left( 1 - \frac{4\beta}{\mu^2} \right)^{1/2} \right]. \quad (7.7-9) \]

If \( \beta \) is very small, \( \beta \) is small and \( \mu \sim 1 \). Whence the two possible solutions become
\[ \lambda \sim (\mu/\beta) (1 + \beta/\mu^2) \text{ or } \lambda \sim (1/\mu) (1 + \beta/\mu^2). \]

In the limit as \( \beta \to 0 \), the first solution \( \to \infty \) and is therefore disregarded.

Hence, when \( \beta \) is small
\[ \lambda \sim 1 + \psi + \beta. \quad (7.7-10) \]

Thus, as well as the increase in length due to the change in plateau pressure angle, \( \sim l/(1 - \psi) \), the curvature of the shear layer produces a further increase in length \( \sim \beta \).

The solution (7.7-9) is strictly only valid when \( 4\beta/\mu^2 \leq 1 \). Table (7.1) shows that when \( p_l = 0.2 \text{ cm}, \psi \sim 0.1 \).
Hence $\mu \approx 0.9$ and $\beta \approx 0.2$, so that $\lambda_{\max} \approx 2.25$.

In the experiments when $\beta = 0$ the separated length, $l$, is typically $\approx 0.5$ cm. Using the values given in Table (7.1) we find that when $p_1 = 0.2$ cm., $\beta \approx 1.15$, and that when $p_1 = 0.5$ cm., $\beta \approx 0.6$.

These values of $\beta$ are larger than the maximum value allowed by the present theory, so that it becomes difficult to visualize the nature of the flow. However, it seems clear that if the assumption that the pressure in the separated region is constant, is valid, then a substantial increase in the separated length should be observed.
PART I: CONCLUSION.

The theory presented in the previous sections suggests that a magnetic field increases the pressure rise to separation and reduces the separation angle. Also, if the pressure in the separated region is constant the field produces a substantial increase in the length of the region due to the decrease in angle and the curvature of the shear layer.

However, the quantitative interpretation of the results of any experiments concerning the effect of the field on the separation conditions will be difficult due to the unknown effect of heat transfer on the distance to separation. Whilst it is clearly important to investigate this effect separately, it was decided to proceed with experiments to determine if the trends indicated above are, in fact, realized in practice.
PART II: SOME EXPERIMENTS ON THE EFFECT OF A MAGNETIC FIELD ON A SHOCK BOUNDARY LAYER INTERACTION AT HIGH TEMPERATURES.

Outline of Part II.

Part II divides roughly into two major sections, the first of which describes the apparatus and methods, while the second presents the experimental results and discusses their significance.

Section (8.1) briefly describes the free piston shock tube while section (8.2) outlines the shock speed measurements. The flow was photographed by the Image Converter Camera described in section (8.3). Values for the useful flow time were obtained from the framing camera sequences taken, and compared in section (8.4) with the times expected from ideal shock tube theory. Section (8.5) describes the models and section (8.6), the generation of the field. Section (8.7) considers some experiments related to the coil current monitor which demonstrate the field's existence during the test time, while section (8.8) presents the measured field distributions.

Section (9.1) discusses the general features of the observed flow patterns, and detailed results showing the growth of the separated region with time are presented in section (9.2). Comparison shots in section (9.3) show...
that the field slightly decreases the steady separated length. Disturbances observed originating from the field itself are discussed in section (9.4). The arguments given in sections (9.5) and (9.6) suggest that pressure gradients exist in the separated region, while section (9.6) also suggests that the decrease in length may be due to the effect of the field on the separated region itself. Finally, section (9.7) offers two explanations for the existence of the pressure gradients.
8. EXPERIMENTAL APPARATUS

8.1 Free Piston Shock Tube

The experiments were conducted in a shock tube in which the high pressures and temperatures required for the driver gas were obtained by a free piston compression process. The compression tube was similar to the one described by Stalker. See figure (8.1). A novel feature of the apparatus was the increase in the shock tube's diameter from ½" i.d. to 1" i.d. immediately downstream of the diaphragm.

Initially a 6" long driver section was located between the shock tube and the compression tube. See figure (8.2). Stalker has developed a theory for this configuration using an unsteady expansion to connect the compression tube's reservoir conditions to those at the upstream side of the nozzle. Using this arrangement it was found that the observed shock speeds were about 20% lower than predicted.

However, in nearly all the experiments the shock tube was coupled directly to the compression tube. See figure (8.3). In this case a steady expansion feeds the nozzle directly. Under these conditions the observed shock speeds were about 5% lower than expected.
The shock tube itself was 5' 2\(\frac{3}{4}\)" long and of 1"
internal diameter. The models were located in the Mach
cone at the end of the shock tube, and two observation
stations, one foot apart, were let into its wall. See
figure (8.4).

8.2 Routine Shock Speed Measurements

In all the experimental work routine measurements
of the shock speed were made. From the observation
stations lengths of fibre optic led the light signals from
the shock heated gas to a pair of 1P42 photodiodes. The
output from the diodes was displayed on the upper beam of
a Tektronix Type 555 Oscilloscope.

On a number of occasions one of the fibre optics
was placed directly opposite the test section so that the
time of arrival of the shock at the model could be
checked. The shock speed measurements obtained with one
optic in this position showed that the shock speed
attenuation at \(p_1 = 0.2\) cm. of mercury, was less than
3%. At higher initial pressures the percentage attenuation
was decreased.

8.3 Image Converter Camera.

All the results concerning the interaction were
obtained by direct photographs of the self-luminosity
patterns in the flow. The camera used was the S.T.L. model 1D Image Converter Camera. It consists essentially of a conventional optical system which produces an image on a photo-sensitive screen. The electrons emitted from the back of the screen pass through accelerating and control grids to a second photo-sensitive screen. The optical image formed on this screen is then photographed by a Polaroid Camera. Different units can be plugged onto the camera in such a way that either streak or framing pictures can be obtained. The characteristics of the units used are summarized in Table (8.1).

<table>
<thead>
<tr>
<th></th>
<th>Standard Framing Unit</th>
<th>Microsecond Framing Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frames</td>
<td>3, (1&quot; x 8&quot;)</td>
<td>3, (1&quot; x 8&quot;)</td>
</tr>
<tr>
<td>Exposure time (μsec)</td>
<td>0.05 0.1 0.2</td>
<td>0.5 1.0 2.0 5.0 10.0</td>
</tr>
<tr>
<td>Time between exposures (μsec)</td>
<td>0.5 1.0 2.0 5.0 10</td>
<td>Variable: 3.5 to 500</td>
</tr>
</tbody>
</table>

Care had to be taken with the stops used to ensure that the photocathode was not overloaded. Overloading distorted the picture and could lead to permanent cathode
The three horizontal lines visible in the photographs are caused by the control grids.

All recording was on Polaroid 3000 A.S.A. film.

Extension bellows mounted on the front of the camera produced a final image magnification of 1.32.

The camera can be triggered by an external pulse or directly from the output of a photo-transistor situated at the end of a fibre optic. The latter method was used in these experiments with the fibre optic pointing directly at the model. A delay generator was situated between the trigger unit and the camera, so that its operation could be delayed for a specified time after the flow started.

8.4 Testing time.

Numerous framing camera sequences of the time development of the flow were made using the Image Convertor Camera. From these sequences the time duration of useful flow could be determined.

Figure (8.5) summarizes the results for the useful testing time expressed in microseconds per foot length of shock tube. The low performance values were obtained from measurements made at the model, 5' 2\(\frac{3}{4}\)" from the diaphragm, with the shock tube coupled directly to the compression tube. The high performance results were obtained, using
the 6" driver section, at a point 4' 1½" from the diaphragm. For comparison, unpublished results of Stalker's for a constant area tube are also plotted. They are derived from measurements made in Argon using a 1" i.d. tube, 5.1 feet from the diaphragm.

The comparison shows that under similar conditions the testing times in the two shock tubes are very similar and that the presence of the driver section has very little effect. Thus the testing time seems to be independent of the method used to generate the flow.

A comparison can also be made between the observed flow duration and that calculated on the basis of ideal shock tube behaviour:

For an ideal shock tube the testing time \( t_{\text{test}} \) at a station distance, \( L \), from the origin is given by:

\[
    t_{\text{test}} = \frac{L}{U_s \left( \frac{e_2}{e_1} - 1 \right)} \quad (8.4-1)
\]

Hence, for Argon

\[
    \tau = \frac{t_{\text{test}}}{\text{foot}} = \frac{9.5 \times 10^2}{M_s \left( \frac{e_2}{e_1} - 1 \right)} \mu\text{sec/foot} \quad (8.4-2)
\]

where \( M_s \) is the shock Mach number and \( \frac{e_2}{e_1} \) is the density ratio across the shock.

The ratio \( \tau_{\text{expt}} / \tau_{\text{theory}} \) is plotted in fig. (8.6) as a function of the initial shock tube pressure, \( p_1 \).

The curves show that the observed times are about...
half the values expected for an ideal shock tube. This is because the boundary layer on the shock tube wall removes gas from the shock-heated region and thus accelerates the contact surface towards the shock front.

Estimates of the length of the shock-heated slug of test gas have been made by equating the increase in mass of the slug to the difference between the rate at which mass enters the region across the shock and the rate at which it leaks past the contact surface via the boundary layer.

The curves show that this effect is only very slightly dependent upon the initial gas pressure until very low pressures are reached. At these low pressures it is possible to achieve a balance between the mass gained and the mass lost, so that the slug remains the same length as it moves down the tube.

The results also show that as the performance of the tube is increased (i.e. the shock Mach number for a particular initial pressure is increased) the value of \( \frac{\tau_{\text{expt}}}{\tau_{\text{theory}}} \) increases.

### 8.5 Models

Numerous models were constructed and tested. Since a certain amount of work was required on the interaction itself in the absence of magnetic fields, a number of simple
models without coils were constructed. When a magnetic field was required it was generated by either one or two coils mounted inside a model.

8.5.1 Models without coils

These models consisted simply of \( \frac{1}{2}'' \) o.d. hollow copper cylinders, several inches long, with the leading edge so machined that the flow passed smoothly over the polished outer surface of the cylinder. See figure (8.7). The rest of the flow passed down the inside of the cylinder and was neglected. Collars of different height were machined so that they could be pushed onto the cylinder to the required distance from the leading edge.

Because of the strong shock waves and possible choking associated with the flow over the collars it was not possible to place the models wholly inside a glass section of the shock tube. Hence all testing was carried out in the small Mach cone at the end of the shock tube. This also avoided the distortion which would be associated with attempts to take photographs through thick, curved glass walls. To obtain a sufficiently long test section for the models the leading edge was located some distance inside the shock tube. The model was so positioned that the interaction region itself was just clear of the end of the tube.

On a straight cylinder at zero angle of attack the
An undisturbed boundary layer is described by the equations used for a flat plate [73]. In section (5.3) it was shown that the boundary layer thickness $\sim 0.1 \text{ mm}$. Since the distance to separation is of the same order, and the separation angle is fairly small, the increase in boundary layer thickness is much less than 0.1 mm.

The radius of the models, $r_0 = 6.35 \text{ mm}$. Hence the increase in thickness to separation is very small compared with the model's radius, and it is therefore valid to describe the separation process on the models by a two-dimensional theory.

The step heights, $h$, of the collars were 0.47, 0.57, 0.87, 1.17 and 1.69 mm.

For the smallest step, $h/r_0 = 0.075$ and therefore the whole separated region should still be basically two-dimensional. When $h = 1.17 \text{ mm}$, $h/r_0 = 0.185$, and the assumption becomes questionable.

### 8.5.2 Models with coils

The main features of the coil models are sketched in fig. (1.2). The early models had only one coil, while most of the later models on which magnetic field effects were observed used two coils. In the experiments to be described double coil models were used in all cases.

The front part of the model consisted of a cylindrical...
brass cavity with a sharp leading edge. The external flow again passed smoothly along the outer surface and the internal flow was trapped by the cavity and ignored. The intention was to photograph the external flow before the reflected shock wave emerged from the cavity. It was found that this condition could be met if about 100 mgm. of steel wool was placed in the cavity. The steel wool provided a heat exchanger for the hot stagnant gas and roughly doubled the time before the reflected internal shock wave re-emerged at the leading edge.

Since the gas in the heat exchanger had been drastically cooled, the pressure of a given volume was reduced. Shock reflection cannot occur until the pressure at the upstream face of the heat exchanger rises to a sufficiently high value, and reflection is therefore delayed by cooling within the heat exchanger.

The main body of the model was machined from a block of nylon or nylatron. The same collars as before were pushed onto these models to the required position.

In all cases the models were sting mounted from the back of the dump tank. For the coil models, the leads from the coils were taken back along the sting and out through a glass seal at the back of the tank.
8.6 Generation of the Magnetic Field.

Because of the small size of the coils and the large fields required, the coil current was supplied by a capacitor discharge triggered from the shock tube. The lower beam of the Tektronix 555 Oscilloscope used for shock timing was delayed with respect to the upper beam, and the "gating signal" associated with the lower beam was used to trigger a 2D21 thyratron. The thyratron triggered a small spark gap controlling the main capacitor discharge. Thus the coil current discharge could be started any time after the shock had passed the first light station. A small non-inductive resistance of 0.1013 ohms was placed in series with the coils, and the signal generated across it was displayed on the lower beam, and thus provided a monitor for the coil current. The circuit is shown in fig. (8.8). The two field coils were wound in opposite senses and connected in parallel. This arrangement produced an enhanced radial component of the magnetic field in the region between the coils. The coil region is drawn to scale in fig. (8.9).

The discharge circuit was arranged to be slightly overdamped, and the coils were so designed that the coil current varied by less than 2% over the duration of the flow\textsuperscript{74}. By adjusting the delay it was possible to ensure
that the field was steady at its maximum value during the observation time. The discharge characteristics are summarized in Table (8.2).

Table (8.2)

<table>
<thead>
<tr>
<th>No. of Turns/coil</th>
<th>Wire gauge</th>
<th>C</th>
<th>V&lt;sub&gt;max&lt;/sub&gt;</th>
<th>Rise</th>
<th>Steady</th>
<th>i&lt;sub&gt;max&lt;/sub&gt;/coil</th>
</tr>
</thead>
<tbody>
<tr>
<td>coils</td>
<td></td>
<td></td>
<td></td>
<td>time</td>
<td>time</td>
<td></td>
</tr>
</tbody>
</table>

| 2 | 80 | 30 S.W.G. | 850 | 900 | 100 | 50 | 650 amps |

* Electrolytic

The field at the centre of a single loop of wire is given by: \( B = \mu_0 I / 2 \pi r \). The effective radius of the present coils is about 4mm., and using the data of Table (8.2) we see that the field at the centre of each coil should be about \( 8 \mu / \text{M}^2 \). Therefore, in the region between the coils on the surface the maximum field should be a few Webers/metre\(^2\).

8.7 Current Monitor.

To check the method of monitoring the current a series of shots were fired in which a search coil was wrapped around the model itself. See figure (8.10). The signal from the search coil was fed through an integrating network and compared with the signal from the current monitor. Figure (8.11) is a tracing taken from an actual shot. The search coil signal is similar in shape to the
current signal, except in the early stages where it lags behind the current. This is probably due to the slow response of the search coil caused by its large diameter. However, the field is certainly present and near its maximum value when the current is a maximum. In this particular shot the downstream fibre optic was positioned directly opposite the model and the signals from light stations recorded. As expected there is a small abrupt change in the search coil trace at the instant when the shock wave reaches the model. The disturbance lasts for about 30 μsec and then stops. Under the conditions of this shot the usual flow duration is about 26 μsec., although a "blob" of hot gas often remains trapped in the corner for a further two or three microseconds. The trace therefore shows that currents are flowing in the shock heated gas and that the distortion of the applied magnetic field is small. This is consistent with the conclusions of section (2.2.2) in which it is shown that the Magnetic Reynolds Number should be small. Usually it was not possible to detect any definite changes in the signal from the current monitor, although on some occasions very small irregularities were observed on the current trace at the expected time of arrival of the shock.
8.8 Distribution of the Magnetic Field.

The distribution of the magnetic field was obtained by a search coil traverse both along and normal to the model's surface. The search coil consisted of 10 turns of 40 S.W.G. wire. Its mean diameter was 2mm, and its thickness was less than 0.5mm.

One particular discharge condition was chosen and the discharge current was displayed on the upper beam of an oscilloscope. The signal from the search coil at different positions along the model was fed through an integrating network to the lower beam. Figure (8.12) shows the variation in the radial magnetic field as a function of distance along the model.

The normal distribution shown in fig. (8.13), was obtained by attaching the search coil to the top of a series of perspex spacers of known thickness placed on the model's surface.

Horizontal lines on the curves shown in fig. (8.12) and fig. (8.13) indicate the effective size of the search coil. Thus each reading represents the average value of the field over the region of the search coil. Figure (8.14) is a composite plot derived from fig. (8.12) and fig. (8.13), which attempts to show the distribution of the field by plotting lines of constant field intensity. The shaded
area roughly corresponds to the region of increased luminosity observed in photographs of the surface in the absence of an obstruction when the maximum field \( \approx 3 \mu T \). See figure (8.15).

In the expression for the interaction parameter, the variation of \( B^2 \) with distance is required. This is plotted in fig. (8.16) and for comparison the fitted curve

\[
(B/B_0)^2 = 1 - 0.135x^{5/2}
\]

where \( x \) is in mm., is also shown.

Figure (8.17) shows the variation of the maximum magnetic field with the voltage observed across the current monitor and with the total coil current itself.
9. EXPERIMENTAL RESULTS.

9.1 General features of the results.

Figure (9.1) is an Image Convertor Camera photograph of the observed self-luminosity pattern. The gas emerges from the end of the shock tube on the right hand side of the photograph and flows over the cylindrical model on which is mounted a collar of height 0.87 mm. The decrease in size of the useful test zone due to the exit Mach cone is visible towards the left hand side of the photograph. The leading edge of the model is located about \( \frac{1}{2} \)" inside the shock tube. The oblique shock wave associated with separation is clearly visible, while the darker, and therefore cooler, separated region occupies the space between the shock and the step. A further disturbance originates from the point where reattachment occurs at the top corner of the step. On some occasions a diffuse line of increased luminosity was observed along the outer edge of the separated region. Also in some cases, particularly when the flow was developing, a small region of increased luminosity was observed close to the face of the step.

Sometimes it was impossible to see the separated region itself, although it was nearly always possible to detect the weak separation shock. Thus the point at which
the line representing the shock intersected the model's surface was used as a marker when determining the position of separation. The distance between this point and the step was one of the few measurements which could be made with any certainty from the photographs. Consequently the results presented here concentrate on this length.

The measurement of the length of the separated region was made directly from the photographs using a set of dividers. An attempt was made to use a travelling microscope, but because of the coarse grain of the film (3000 A.S.A.) required for the camera, and the camera's limited resolution, all detail was lost under the microscope.

In the results the gas conditions are identified by the initial shock tube pressure, $p_1$, in cm. of mercury, while the actual conditions can be found by reference to Table (3.1). $h$ denotes the height of the collar in mm., $L$ is the distance in mm. from the leading edge to the step, and $l$ is the length of the separated region in mm. In all cases $l$ is presented as a function of the time in microseconds after the beginning of the flow. This is because one of the conclusions to emerge from the results is that the separated region takes a finite time to develop. In most of the tests the separated length was
still increasing slowly even at the end of the flow.

In an attempt to find a reasonable value for the final steady length, denoted by $l_0$, a curve of the form

$$ l = l_0 \left[ 1 - \exp \left( -\frac{t}{t_0} \right) \right] $$

was fitted to the experimental points for each set of results, and is shown by the solid line in the figures. This curve was chosen because it appeared to fit the experimental results fairly well. Only when the length was initially increasing very rapidly was there a serious discrepancy due to the slower risetime of the fitted curve. The assumed curve also possessed the correct features that, $l = 0$ when $t = 0$, and that $l \rightarrow l_0$ as $t \rightarrow \infty$. Also from the values of $t_0$ used, an estimate of the characteristic time to establish the separated region could be obtained.

By varying the camera’s delay, the full time development of the separated region could be established. This was done in a few cases, but more usually only sufficient points were obtained to establish the asymptotic length, $l_0$.

The photographs were too small to obtain reliable measurements of the plateau pressure angle, $\theta_p$, directly. However, for the reasons outlined in section (7.7) it was thought that $\theta_p$ could be determined from $\theta_p \approx h/l_0$. Also, because the change in $\theta_p$ due to the field may only
be \( \sim 0.5^\circ \) it was thought that the effect of the field could be deduced from measurable changes in \( l_0 \).  

Prior to each shot a "still photograph" of the model was taken from which the height of the step was measured using a travelling microscope. The step heights shown in Table (9.1) represent the mean values of a number of these measurements.

Figure (9.2) is a photograph taken under exactly the same conditions as fig. (9.1), except that a magnetic field is present. If any change has occurred in the separated length it is too small to observe qualitatively. The only marked change introduced by the field is the increase in luminosity in the free-stream behind the shock. This increase is rather more clearly visible in fig. (9.3) in which separation is taking place outside the influence of the magnetic field.

In all cases the field was produced by the double coil arrangement described in section (8.6). For each set of results the distribution of the magnetic field with respect to the experimental points is plotted on the same figure.

When \( p_1 = 0.2 \text{ cm} \), the separated length was still increasing rapidly at the end of the test time. Hence,
in these experiments, most of the results were taken at $p_1 = 0.5$ cm. This pressure was chosen since it provided a compromise between the low initial pressures required for a strong magnetic interaction, and the higher pressures required for a sufficiently long test time.

Tests were conducted using the four sets of gas conditions listed in Table (3.1) and several different step heights. Results both with and without a magnetic field present were obtained, and curves of the form given by (9.1-1) were fitted to the data. Table (9.1) summarizes the experimental conditions and presents the values of $l_0$ and $t_0$ used for the fitted curves. The detailed experimental results are presented in fig. (9.4) to fig. (9.19).
Table (9.1)

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>$P_1$ (cm. of Hg)</th>
<th>Step Height (h mm.)</th>
<th>$L$ (mm.)</th>
<th>$l_0$ (mm.)</th>
<th>$t_0$ (sec)</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9.4)</td>
<td>0.2</td>
<td>0.57</td>
<td>20</td>
<td>3.58</td>
<td>4.17</td>
<td></td>
</tr>
<tr>
<td>(9.6)</td>
<td>0.5</td>
<td>0.57</td>
<td>20</td>
<td>4.71</td>
<td>8.16</td>
<td></td>
</tr>
<tr>
<td>(9.7)</td>
<td>1.0</td>
<td>0.57</td>
<td>20</td>
<td>3.9</td>
<td>6.06</td>
<td></td>
</tr>
<tr>
<td>(9.8)</td>
<td>2.0</td>
<td>0.57</td>
<td>20</td>
<td>4.0</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td>(9.10)</td>
<td>0.5</td>
<td>1.69</td>
<td>20</td>
<td>8.6</td>
<td>5.1</td>
<td></td>
</tr>
<tr>
<td>(9.12)</td>
<td>0.5</td>
<td>0.57</td>
<td>30</td>
<td>3.0</td>
<td>4.7</td>
<td>Trip Wires$^a$</td>
</tr>
<tr>
<td>(9.13)</td>
<td>0.2</td>
<td>~0.8</td>
<td>20</td>
<td>4.6</td>
<td>4.84</td>
<td></td>
</tr>
</tbody>
</table>
| (9.13) | 0.2             | ~0.8                | 20      | 4.1       | 4.84$^b$  | $B 
eq 0$ |
| (9.15) | 0.5             | 1.17$^b$            | 28      | 6.15      | 6.0       |          |
| (9.15) | 0.5             | 1.17                | 28      | 5.78      | 6.67      | $B 
eq 0$ |
| (9.15) | 0.5             | 1.17                | 28      | 5.59      | 6.0       | $B 
eq 0$. Uses same $t_0$ as $B = 0$ |
| (9.16) | 0.5             | 0.47                | 26.5    | 4.0       | 6.93      |          |
| (9.16) | 0.5             | 0.47                | 26.5    | 4.4       | 14.4      | Values for $l^2 = l_0^2 \left(1 - e^{-t/t_o}\right)$ |
| (9.17) | 0.5             | 0.56                | 26.5    | 4.23      | 6.56      | $B 
eq 0$ |
| (9.18) | 0.5             | 0.47                | 26.5    | 4.02      | 6.93$^b$  | Curves fitted to $l/h$ results |
| (9.18) | 0.5             | 0.56                | 26.5    | 4.14      | 6.06$^b$  | $B 
eq 0$ |
| (9.19) | 0.5             | 0.87                | 28.5    | 5.33      | 4.84$^b$  |          |
| (9.19) | 0.5             | 0.87                | 28.5    | 5.10      | 4.84$^b$  | $B 
eq 0$. Uses same $t_0$ as $B = 0$. |

$^a$ Attempt to produce turbulent interaction.

$^b$ Represents the mean value of several different heights.
9.2 Results for the separated length in the absence of a magnetic field.

9.2.1 \( h = 0.57 \text{ mm} \).

The collar used to obtain these results consisted of a snugly fitting, bent piece of brass strip, pushed onto the model to the required position.

The results obtained at \( p_1 = 0.2 \text{ cm} \) are plotted in fig. (9.4), and show that the separated length is still increasing fairly rapidly at the end of the test time, while the initial rate of increase in length is greater than that suggested by the fitted curve.

Figure (9.5) presents some results obtained at an initial pressure of 0.5 cm. of mercury. In this series an attempt was made to assess the effect of Reynolds number on the separated length by varying the distance, \( L \), from the leading edge to the step. The results for \( L = 10, 30 \) and \( 40 \text{ mm} \) suggest that the length increases with Reynolds number. This conclusion is consistent with the trend predicted by (7.7-2).

The separated length observed when \( L = 20 \text{ mm} \), shown in fig. (9.6), does not appear to be consistent with the other results: being larger than expected and also developing more slowly. Thus this particular set of results is of doubtful validity.

To further test (7.7-2) the results shown in fig. (9.7)
and fig. (9.8) were obtained at initial pressures of 1.0 and 2.0 cm. respectively.

In fig. (9.9) the asymptotic length, $L_0$, inferred from fig. (9.4) to (9.8) is plotted as a function of

$$h \beta^{-2} R_{L}^{1/4} \left( x_0 / L' \right)^{1/4}$$

for a constant step height of 0.57 mm. This figure shows that the separated length seems to be fairly well described by equation (7.7-2). The experimental line has a gradient of 0.215 and does not pass through the origin. The expected line of gradient 1.13 passing through the origin is drawn on the same figure. Thus the experimental gradient is about a factor of five smaller than anticipated. This fact, together with the line's failure to pass through the origin therefore suggests that this simple approach may not be adequate. Hence the gas in the separated region may be in motion and pressure gradients may exist within this region. Consequently tests were undertaken using steps of different heights to further investigate this point.

9.2.2 $h = 1.69$ mm.

Figure (9.10) shows a few points obtained using a much larger step, 1.69 mm. high, located 20 mm. from the leading edge. These results should be treated with a certain degree of reservation since in most cases the actual separation point was obscured by the end of the shock tube. Its position was inferred by producing the lines representing
the separation shock and the model's surface until they met. The distance between this point and the step was regarded as the separated length.

Rather surprisingly the time scale of the interaction is comparable with that observed for the smaller steps, and therefore appears to be insensitive to changes in step height. See figure (9.26). The photographs of large separated regions often showed a thin region of increased luminosity close to the face of the step. See figure (9.11). This suggests that the gas in the separated region is in motion and is retarded close to the step, to form a small stagnation region. Hence a rise in pressure probably exists within the separated region.

9.2.3 Turbulent Flow

Figure (9.9) showed that the observed length was smaller than expected. This may have been brought about by very large shear forces increasing the possible value of \( \Theta_p \). Despite the low free stream Reynolds numbers and high heat transfer rates the possibility existed that these shear forces were present because the undisturbed boundary layer was already turbulent. Consequently a few tests were performed in which trip wires were mounted on the fore part of the model in an attempt to provoke
transition of the assumed laminar boundary layer.

Results are presented in fig. (9.12) in which the 0.57 mm. collar was located 30 mm. from the leading edge. In the first test 14 turns of 40 S.W.G. wire were wound over the first 2 cm. of the model while in the second and third tests 6 turns of 30 S.W.G. wire were wound over the first 1.5 cm. A fairly pronounced decrease in length occurred, indicating that the skin friction had been increased. Figure (9.10) shows some results obtained using the 1.69 mm. collar, 67 mm. from the leading edge, with 10 turns of 30 S.W.G. wire wound onto the model. A marked decrease in the separated length was again observed. The points at 12 μsec are extremely doubtful due to the confused nature of the flow pattern and the absence of a definite separation shock.

From these results we can conclude that the turbulence within the boundary layer has been increased, and gives rise to an increase in the shearing forces. This in turn suggests that the undisturbed boundary layer was probably laminar. However, more precise experiments are required to determine the transition conditions for the present high heat transfer rates before we can be certain of the state of the boundary layer.
9.3 Results obtained in the presence of a magnetic field.

Figure (9.13) compares the results obtained at \( p_1 = 0.2 \) cm. with a field present to those obtained in the absence of a field for a step height \( \approx 0.8 \) mm. The measurement of the step height is rather uncertain in this series only because the "step" consisted of a number of turns of insulation tape wound around the cylindrical model. It is apparent however that the presence of the magnetic field slightly reduces the length of the separated region.

Figure (9.14) represents a large number of scattered results obtained during the time when the coil models were being developed. A number of fairly rough collars of slightly differing heights were used and the actual results are plotted in this figure. One point to note is that for each collar used, the field has decreased the separated length. Since all the heights were of similar magnitude, as measured from the still photographs, fig. (9.15) is an attempted correlation in which the ratio \( 1/h \) has been plotted as a function of time. This figure shows rather more clearly the effect of the field. The mean collar height, \( \bar{H} \), for these results is \( 1.17 \) mm., and this value has been used in Table (9.1) as being typical of this series.

The distribution of the magnetic field is also plotted
in fig. (9.14), and shows that separation is taking place outside the field. However, the decrease in length still occurs, and this suggests the effect depends on the influence of the field on the separated region itself rather than on the conditions at separation.

Some difficulty was experienced with the double coil models due to electrical breakdowns during the test time. These breakdowns were caused by the fairly high dynamic pressure bending the slightly flexible nylatron model, which cracked the thin Araldite coating over the coils. This problem was countered by winding a single layer of \( \frac{3}{4} \)" wide adhesive tape over the coils. This tape was slightly elastic and could be readily replaced every few shots. Comparison shots using the same collars on a polished copper cylinder showed that the presence of the tape produced no discernable change in the separated length. Even if some small change had occurred, the length in the absence of a field which was compared with that observed when the field was present, was obtained on the same model with a similar piece of tape in place.

Figure (9.16) shows the results at \( p_\perp = 0.5 \text{ cm} \) for \( h = 0.47 \text{ mm} \). In this series and also when \( h = 0.87 \text{ mm} \), the collars were machined from a block of nylatron. These collars fitted the models snugly and were of more uniform
height than the other collars made by bending pieces of brass strip. There was some concern that some mass leakage may have occurred from the separated regions formed by these collars. However, the reasonable consistency of the final results indicates that these collars had been adequate. See figure (9.25).

Two points early in the series shown in fig. (9.16) again show that the experimental length is increasing more rapidly than the fitted curve suggests. The second curve drawn in this figure is given by

\[ l^2 = (4.14)^2 \left[ 1 - e^{-t/t_0} \right]. \]

This curve initially increases much more rapidly than the curve usually employed. However, despite its early advantage, it was much more difficult to obtain a reasonable overall fit using the \( l^2 \) curve rather than equation (9.1-1).

Figure (9.17) presents the results obtained when a field was applied to the interaction studied in fig. (9.16). Nominally the same collar was used but microscope measurements showed that the step height was rather larger in this series, \( h = 0.56 \text{ mm} \). These results were taken after the results presented in fig. (9.16), rather than adopting the more acceptable approach of making alternate shots with the field present. The distribution of the field has again been plotted on the same figure and
it is seen that separation is taking place within the field region.

In this series the occurrence of a second disturbance was noted apart from the one due to the step. The disturbance was rather poorly defined but resembled a weak oblique shock wave. The position at which this second shock met the surface has also been plotted and the effect is discussed in greater detail in section (9.4). Figure (9.18) is a replot of the data from fig. (9.16) and (9.17) in terms of \( \frac{1}{h} \), made because of the slightly different collar heights employed in the two series. Again the field reduces the length of the separated region.

Figure (9.19) shows the results obtained for a step height of 0.87 mm. Alternate shots were made with the field present and the length is again slightly decreased. The distribution of the field shows that for most of the results separation is taking place outside the field's region. Exactly the same time constant has been used for the two fitted curves, which indicates that the effect of the field on the development of the separated region must be very small.
9.4 Disturbances due to the Magnetic Field.

Figure (9.20) is a photograph showing the slight increase in luminosity on the upstream side of the shock due to the step, caused by the weak oblique shock wave originating from the magnetic field. In both fig. (9.21) and (9.22) the first frame was taken one \(\mu\text{sec.}\) after the flow started and in both pictures the small, highly luminous separated region is still close to the step. In fig. (9.22) a field is present and the disturbance caused by it is clearly visible on the upstream side of the separated region.

The results obtained in connection with this second shock wave are plotted in fig. (9.23) and (9.24). In these experiments the position of the step was moved with respect to the field. In both series, shock waves were observed which were apparently caused by the field itself. Unlike the shock waves due to the step their position remained fixed with time at a position where \(B \approx 0.8 B_0\) when \(B_0 \approx 3\text{W/m}^2\). A few points were obtained in the absence of the field to give a rough idea of the position of the separation shock, while its expected position derived from fig. (9.16) is shown by the broken line.

The measurements of the position of the separation shock in the presence of the field exhibit a surprisingly
large variation. This may occur because separation is now taking place in a region where the field is rapidly decreasing, which may introduce further disturbances into the flow. However, no major change occurs in the position of separation due to the different lengths the undisturbed boundary layer has to traverse through the field before reaching the interaction zone, which indicates that the undisturbed layer is not strongly affected by the field.

The occurrence of the second shock wave shows that the magnitude of the field is sufficient to induce body forces in the flow which are capable of producing a weak shock wave. Presumably a pressure rise accompanies this shock which could in turn affect the boundary layer. Thus it may be possible to produce a region of separated flow, caused entirely by the effect of the magnetic field on the mainstream.

The results presented so far show that a finite time is required to establish a separated region and that the separation shock moves upstream from the agency causing the disturbance. If the field had been strong enough to produce a separated region then we may have observed the upstream movement of the shock wave. However, the results show that the shock remains in the same position and therefore we can conclude that the field is not strong enough to
produce such a region. A further possibility is that the time scale of an interaction of this type may be longer than usual due to the possible retarding effect of the body forces on the rate at which gas flows into the separated region.

9.5 Correlation of the results in the absence of a magnetic field.

The asymptotic value of the separated length in the absence of a magnetic field is plotted in fig. (9.25) as a function of \( \frac{h \beta^{-\frac{1}{2}} R^4_{L'}}{(x_0/L')^4} \) for the different collar heights used. Also plotted are the results presented previously in fig. (9.9) for a height of 0.57 mm.

Figure (9.25) shows that a line of gradient 0.65 passing through the origin roughly fits the results obtained using different collar heights. This may be contrasted with the line obtained in fig. (9.9) in which only the gas conditions had been varied.

Therefore, the failure of equation (7.7-2) to fully describe the results suggests that the basic assumption made in section (7.7) needs to be re-examined. That is, the results indicate that pressure gradients may exist within the separated region.

If this is so the separated length is smaller than expected and the value of \( \theta_p \) deduced from the ratio \( h/l \) is
larger than that calculated for a free interaction. If the gradient of 0.65 found in fig. (9.25) is compared with the expected gradient of 1.13 (see section (7.7)) then we see that the observed value of $\theta$ is 1.74 times the value for a free interaction. Table (7.1) shows that $\theta_p$ at $p_1 = 0.5$ cm. is typically $\sim 5.5^\circ$, so that the observed angle $\sim 9.5^\circ$. Therefore, under these conditions, the free interaction plateau angle, $\theta_p$, is less than $9.5^\circ$. Since the ratio of freestream temperature to wall temperature, $(T_\infty / T_w)$, $\sim 40$ we can conclude that $\theta_p$ is at most a weak function of $(T_\infty / T_w)$.

Some experiments conducted at $p_1 = 0.5$ cm. using flares rather than collars showed that a definite separated region existed for a flare angle of $15^\circ$. Experiments using smaller angles were inconclusive due to the difficulty of detecting very smaller separated regions. Hakkinen has shown in his zero heat transfer experiments that the plateau pressure coefficient is less than the coefficient for incipient separation by a factor of $1.2$. These experiments using flares therefore show that $\theta_p$ is certainly $< 15^\circ$, and probably $< 12^\circ$, and hence substantiate the result found using the collars.

The characteristic times used for the fitted curves are plotted against collar height in fig. (9.26). Rather
surprisingly the time appears to be almost independent of the height, and therefore of the mass of gas required to fill the separated region. It should also be noted that the magnetic field had no discernable effect on the time, which implies that in both cases the same basic mechanism is governing the development of the separated region.

9.6 Effect of the magnetic field on the length of the separated region.

In fig. (9.27) the asymptotic separated length observed in the presence of a magnetic field is compared with the length observed in the absence of the field for various collar heights at an initial pressure of 0.5 cm., while Table (9.2) presents the same results in tabular form.

Table (9.2)

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$h$ mm.</th>
<th>$l_B$ mm.</th>
<th>$l_0$ mm.</th>
<th>$l_B/l_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>$\sim 0.8$</td>
<td>4.1</td>
<td>4.6</td>
<td>0.891</td>
</tr>
<tr>
<td>0.5</td>
<td>0.56</td>
<td>4.185$^a$</td>
<td>4.3$^b$</td>
<td>0.973</td>
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<tr>
<td>0.5</td>
<td>0.87</td>
<td>5.1</td>
<td>5.33</td>
<td>0.956</td>
</tr>
<tr>
<td>0.5</td>
<td>1.17</td>
<td>5.78</td>
<td>6.15</td>
<td>0.941</td>
</tr>
</tbody>
</table>

$^a$ Mean value from the independent curves of figs. (9.17) & (9.18)

$^b$ Value from $B = 0$ curve in fig. (9.27) for $h = 0.56$ mm.
These results show that for the three collar heights considered, the length of the separated region is slightly reduced by the presence of the magnetic field. In each case the field had the distribution shown in fig. (8.12) with a maximum value, $B_0, \approx 3 \text{ W/m}^2$.

However, the discussion of section (7.7) suggested that the separated length should be increased by the presence of the magnetic field due to the decrease in $\Theta_p$, while the curvature of the shear layer should produce a further substantial increase.

In section (9.5) it was shown that the separated length in the absence of a field was inadequately described by the analysis of section (7.7). Consequently measurements of $l$ cannot be used to infer results concerning small changes in $\Theta_p$. Also the results for $h = 1.17 \text{ mm}$ and $0.87 \text{ mm}$ show that the decrease in length persists even when separation is taking place outside the field. This shows that the observed effect is not directly associated with the influence of the field on the conditions at separation.

The large predicted increase in length due to the shear layer's curvature depended on the assumption that the pressure in the separated region was constant. The discussion of section (9.5) has already suggested that this
was not so, and the fact that the increase was not observed lends support to this argument.

Since we have discounted the effect of the field on separation and also on the edge of the separated region, one possibility that remains is the effect the field may have on conditions within the separated region itself.

In fig. (9.28) the change in the separated length, \((l_0 - l_B)/l_0\), is presented as a function of collar height, from which it is seen that the change is almost linear with height. This suggests that the change in the separated length may be dependent upon an interaction parameter based on the length of the separated region.

Figure (9.28) shows that \((l_0 - l_B)/l_0 = 0.05\) h and from fig. (9.25) we see that \(l_0 \approx 6\) h.

Hence \((l_0 - l_B)/l_0 \approx (0.5/6)l_0\) when \(l_0\) is in cm.

If the change in length is proportional to an interaction parameter then \(\sigma B^2/\varepsilon u \approx 0.5/6 \approx 0.1\) cm.\(^{-1}\).

When \(B_0 = 3\) W/m\(^2\), the mean value of \(B^2\) gives, \(B^2 \approx 6.3(W/m^2)^2\) for a field of 1.0 W/m\(^2\), and a length of 1.0 cm,

\[
\sigma/\varepsilon u = 0.5/6 \times 6.3 \approx 0.013.
\]

This figure can be compared with the free stream value of \(\sigma/\varepsilon u\) found from fig. (2.2) for an initial pressure of
0.5 cm. 

\[ i.e. \left( \sigma /eu \right)_\infty = 0.19 \]

Hence \( \sigma /eu \sim 0.07 \left( \sigma /eu \right)_\infty \)

Therefore, if the assumption that the change in length is proportional to an interaction parameter based on the separated length is correct, then the appropriate gas conditions to use seem to be more typical of a fairly cool separated region rather than of the mainstream. Hence the decrease in length would depend directly on the influence of the field on the separated region.

Hakkinen\(^8\), in his analysis of the length of the separated region produced by an incident shock wave, suggests that the separated length is controlled by the distance required for the shear forces to reduce to zero the momentum of the backflow driven by the reattachment pressure rise. The present experiments indicate that the gas in the separated region is not stagnant and therefore a backflow may exist. If magnetic forces are present in the separated region they would tend to retard the backflow, and therefore assist the viscous forces in reducing the backflow’s momentum. The distance required to destroy this momentum would therefore be reduced by the presence of the magnetic field.
It should be noted that this explanation demands that the gas in the separated region is in motion and therefore provides the necessary momentum and shear forces to balance possible pressure gradients.

9.7 Pressure gradients within the separated region.

Several arguments have indicated that the pressure is not constant within the separated region so that it becomes necessary to suggest possible reasons for the existence of these pressure gradients.

Without understanding fully the mechanism by which the separated region is established, it at least seems likely that a stagnation region with its associated pressure rise is initially formed close to the step. The pressure rise drives a reverse flow upstream and so increases the separated length. Hence, while the region is developing rapidly the trapped gas is in motion. As the length increases the stagnation region, and presumably its attendant pressure rise must decay in some way. This view is supported by the observation that the initially luminosity of the separated region is much higher than that of a well developed region. See figure (9.29). Although the size of the region appears to be fairly steady towards the end of the test time, it may take rather longer for the pressure to become constant. This is because a constant
pressure implies that the temperature, and therefore the heat transfer rate, is constant. Ihrig's work suggests that the characteristic time to establish a constant heat transfer rate is longer than the time required for a constant mass entrainment rate. Also, Rom's shock tube experiments on the heat transfer rate from the separated region formed by a backward-facing step show that, although the separated region is well formed, the heat transfer rate from the region requires a longer time to become steady. Therefore, although the separated region appears to be well developed, changes in pressure within the region may exist due to the non-steady heat transfer rate and the remnants of the early stagnation region.

An alternative explanation stems from Chapman's discussion of a step-induced turbulent separation in which he concludes from his experimental results that a free interaction exists only as far as separation. In a separation of this type the pressure initially increases very rapidly to a value several times larger than that encountered in laminar interactions. Since the separation distance is very small the boundary layer profile has only a short distance in which it can adjust itself to the increasing pressure, and therefore changes shape only near the wall while the increase in thickness is small. Consequently, at separation,
substantial shear forces exist a short distance from the wall and persist within the separated region. The pressure continues to increase slowly beyond separation, and sometimes exhibits a maximum before increasing again close to the step. Chapman therefore suggests that, "a plateau in pressure (characteristic of dead air) does not occur since the eddying motion of the turbulent layer energizes the air". That is, the shear forces of the inner part of the layer are sufficiently active beyond separation to keep the gas in motion and hence provide the necessary balancing forces for the pressure gradients.

Therefore if the undisturbed boundary layer in the present work is turbulent, pressure gradients can exist within the separated region. However, the discussion of section (9.2.3) suggests that the boundary layer is in fact laminar.

For a laminar boundary layer on an insulated surface, separation takes place over a much larger distance than for a turbulent interaction. This allows large scale modifications of the boundary layer profile in which a comparatively thick low speed sub-layer is formed near the wall. Thus the shear forces, which are initially smaller than for a turbulent layer, are substantially reduced over a large part of the boundary layer and are no longer active.
within the separated region. Hence the gas is almost stationary and a pressure plateau exists within the region.

However, in the present instance strong heat transfer effects are present which may substantially increase the shear at the wall in the undisturbed boundary layer. We have also shown that the separation distance is very small, and consequently only a very thin low speed sub-layer is formed while the change in the overall profile is slight. Therefore active shear forces may again exist beyond separation and although the undisturbed layer is laminar, the conditions within the separated region may be more closely related to those encountered in a turbulent interaction. Hence, in these experiments pressure gradients may exist within the separated region.
CONCLUSION

A preliminary theoretical and experimental investigation of the interaction between a laminar boundary layer and a supersonic mainstream in the presence of a magnetic field has been carried out.

Calculations of the conditions behind ionizing shock waves in Argon showed that they were suitable for an M.H.D. investigation. Estimates of the transport properties of Argon under the conditions of the experiment showed that the usual equations of boundary layer theory could be used. These equations were presented and the modifications introduced by the magnetic field were discussed.

Rough profiles showing the variation of different gas properties through the boundary layer showed that magnetic effects should be small close to a cooled wall. Also from these profiles an estimate of the boundary layer thickness was obtained.

The effect of a magnetic field on the mainstream was discussed, and an extension of the Method of Characteristics was made which allowed for the effect of magnetic body forces.

An outline of a simple theory of the interaction due to Hakkinen was given, and it was shown how this theory
could be extended to the present situation by developing an analogy between it and the interaction on a curved wall. An expression was also developed for the angle change associated with separation.

For a step-induced separation the effect of the field on the shape of the shear layer formed once separation had occurred was considered.

These expressions suggest that when the magnetic field is acting only in the mainstream, it should increase the pressure rise to separation, while decreasing the associated angle change. The decrease in angle should increase the length of the separated region in front of a step, while the curvature of the shear layer should add a further contribution.

A photographic investigation of the interaction was carried out using collars on cylinders to provoke separation. The ionized gas flows were generated by a free piston shock tube in which a 1:4 increase in area occurred downstream of the diaphragm. It was found that the testing time was very similar to that measured in a constant area shock tube.

Contrary to the theoretical predictions, it was found that when a magnetic field was applied to the step-induced interaction, the length of the separated region
was slightly decreased.

This decrease persisted when the field was confined to the region downstream of the separation point, indicating that the separation process itself was unaffected by the presence of the magnetic field. This supports the view that heat transfer to the wall reduces the interaction length up to separation to the order of one boundary layer thickness, and in so doing, reduces the magnetic effects to negligible proportions.

Little experimental information exists on shock boundary layer interactions on a cool surface with a high temperature mainstream. The present study suggests that a proper understanding of this phenomenon is a necessary preliminary to an understanding of the M.G.D. effects.

The reasons for the decrease in the length of the separated region are not understood. However, the observed effect suggests that the magnetic field is influencing the flow in the separated region. This point also requires further investigation.

Thus, theoretical indications are that it may be possible to increase the pressure rise to separation in a shock boundary layer interaction by the application of a magnetic field. The experiments suggest that if this is to be done, it must be achieved by an intense magnetic
field, situated close to the interaction, with dimensions comparable with the boundary layer thickness.

If these possibilities are realized then it may be feasible to use a magnetic field to suppress the undesirable shock boundary layer interaction encountered in a hypersonic intake such as the one sketched in fig. (9.30).
APPENDIX I

DISCUSSION OF FURTHER LINES OF INVESTIGATION.

From the present work a number of points have arisen which require further investigation.

First, the major uncertainty associated with a quantitative estimate of the effect of the field on separation is due to the unknown value of the length of the separated region under the present heat transfer conditions, upon which the strength of the magnetic interaction depends. An investigation of the variation in the heat transfer rate through the interaction region would help to fix this quantity more precisely.

Second, the non-steady nature of the interaction means that longer running times in the shock tube must be achieved, particularly at low initial pressures, i.e. for $p_1 < 0.2$ cm. of mercury. Thus a longer tube is required with an increased internal diameter to counteract the adverse effect of the boundary layers on the walls of the tube.

Third, the flow visualization technique must be improved so that much smaller separated regions can be detected. A spark source Schlieren system giving a magnified image of the interaction would be desirable.

Fourth, studies of the effect of the field on incipient separation would be free of the uncertainties
associated with extensive separated regions. An accurate method of detecting incipient separation would then be necessary. Needham\(^7\) has described a method using thin film heat transfer gauges which may be applicable in the present case.

Fifth, further work on the interaction involving magnetic fields only, without any other disturbing influence, e.g. steps, could be pursued.

From a theoretical viewpoint there is a need for a very much more rigorous and complete theory, which would have to take into account a number of factors.

First, the effect of the magnetic field on the velocity profile would need to be considered.

Second, allowance would have to be made for the variation in the transport properties through the boundary layer.

Third, a more complete description of the mainstream would be desirable, with emphasis on the effect of the field on such quantities as the flow Mach number. Also the possible influence of Hall current effects would need to be considered.

Fourth, the effect of Joule heating within the boundary layer must be analysed in more detail. In some cases the currents may be sufficient to produce enough heating to locally increase the degree of ionization and
hence the electrical conductivity.

The rate of recombination and diffusion of ion-electron pairs may help to modify the distribution of currents within the boundary layer.

Fairly recently a number of papers have appeared in which integral techniques have been successfully applied to the interaction\(^5\). Integral forms of the boundary layer equations in a magnetic field can be readily derived. Therefore as a next step it may be possible to apply these equations to the interaction with a field present. Considerable care would have to be exercised regarding the choice of the velocity profile, with due allowance made for the field. To obtain sufficient control it would probably be advisable to employ two of the integral equations, say the zeroth and first moment of the momentum equation.
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Figure:

1. General features of the interaction  
2. Model with coils  
3. Induced currents  
4. Interaction parameter - Sketch  
5. " " - Values  
6. Magnetic Reynolds number  
7. Hall effects  
8. Ionization time  
9. Oblique shock waves - Configuration  
10. " " - Calculation  
11. " " - Shock angles  
12. " " - Pressure ratio  
13. " " - Density ratio  
14. " " - Enthalpy ratio  
15. Normal shock waves - Density ratio  
16. " " - Degree of ionization  
17. " " - Temperature  
18. " " - Velocity  
19. " " - Pressure ratio  
20. Skin friction  
21. Conductivity of air - Sketch  
22. Configuration - Hains and Yoler  
23. " - Sherman  
24. " - M.H.D. generator  
25. Specific heat at constant pressure  
26. As for (4.6) - "a" factor  
27. Electrical conductivity - Behind shock wave  
28. " " - At one atmosphere  
29. Thermal conductivity  
30. $\log_{e} \Lambda$  
31. Viscosity  
32. Prandtl number
(5.1) Boundary layer profiles - Temperature
(5.2) " " " - Density
(5.3) " " " - Degree of ionization
(5.4) " " " - Electrical conductivity
(5.5) " " " - Interaction parameter
(5.6) " " " - Induced currents
(6.1) Intrinsic co-ordinates
(6.2) Characteristics
(6.3) Simple waves
(6.4) Simple flow
(7.1) General features of the interaction
(7.2) Hakkinen profile
(7.3) G factor
(7.4) Curvature of shear layer in a magnetic field
(8.1) Compression tube
(8.2) Driver section
(8.3) Direct coupling
(8.4) Shock Tube
(8.5) Test time in Argon
(8.6) Test time, comparison
(8.7) Model, no coils
(8.8) Coil current, circuit
(8.9) Field coils (detail)
(8.10) Pick-up coil
(8.11) Current monitor experiment
(8.12) Distribution of radial magnetic field - along surface
(8.13) " " " " " - normal to surface
(8.14) Lines of constant radial field
(8.15) I.C.C. photograph: Effect of field on flow in absence of collar
(8.16) Distribution of \((B/B_0)^2\)  
(8.17) Variation of \(B_0\) with current  
(9.1) I.C.C. photograph: Step-induced interaction  
\[ B = 0 \]  
(9.2) As for (9.1), \(B \neq 0\)  
(9.3) As for (9.1), \(B \neq 0\)  
(9.4) Experimental results  
(9.5) " "  
(9.6) " "  
(9.7) " "  
(9.8) " "  
(9.9) Correlation - \(h = 0.57\) mm., \(B = 0\)  
(9.10) Experimental results  
(9.11) I.C.C. photograph: Stagnation region at step  
(9.12) Experimental results  
(9.13) " "  
(9.14) " "  
(9.15) " "  
(9.16) " "  
(9.17) " "  
(9.18) " "  
(9.19) " "  
(9.20) I.C.C. photograph - Disturbance due to magnetic field  
(9.21) I.C.C. photograph - \(B = 0\)  
(9.22) " " - Disturbance due to magnetic field  
(9.23) Experimental results  
(9.24) Experimental results  
(9.25) Separated length, correlation, \(B = 0\)  
(9.26) Characteristic times  
(9.27) Effect of magnetic field on separated length  
(9.28) Fractional change in separated length  
(9.29) I.C.C. photo: Time development of sep. region  
(9.30) Hypersonic intake - Sketch
Supersonic Ionized Flow

Boundary Layer
Cold Wall

Compression Waves

Magnetic Field

Separation Shock

Reattachment
Shear Layer

Fig (1.1) General Features of the Interaction.

Flow
Brass Cavity
Collar
Steel Wool

Coils
Nylatron Cylinder

Fig (1.2) Model with Coils

Fig (1.3) Induced Current
Fig.(2.1) Interaction Parameter

Fig.(2.4) Hall Effects

Fig.(3.1) Oblique Shock Waves

Fig.(3.2) Oblique Shock Calculation
\[ B = 1.0 \text{ W/M}^2 \]
\[ L = 1.0 \text{ cm.} \]

---

**Fig. (2.2) Interaction Parameter, S.**
Fig. (2.3) Magnetic Reynolds Number, \( R_m \).
Fig. (2.5) Ionization Time.
Fig. (3.3) Shock Angles
(Same initial states as in Fig. (3.4).)

Fig. (3.4) Pressure Ratio

<table>
<thead>
<tr>
<th>$p_i$ (cm Hg)</th>
<th>$M_s$</th>
<th>$p_1$ (at.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>16.4</td>
<td>1.05</td>
</tr>
<tr>
<td>0.5</td>
<td>15.6</td>
<td>2.30</td>
</tr>
<tr>
<td>1.0</td>
<td>14.3</td>
<td>3.76</td>
</tr>
<tr>
<td>2.0</td>
<td>12.9</td>
<td>5.90</td>
</tr>
</tbody>
</table>
Fig. (3.5) Density Ratio

<table>
<thead>
<tr>
<th>$p_i$ (cm Hg)</th>
<th>$M_s$ (gm/cm²)</th>
<th>$e_1 \times 10^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>16.4</td>
<td>3.35</td>
</tr>
<tr>
<td>0.5</td>
<td>15.6</td>
<td>7.90</td>
</tr>
<tr>
<td>1.0</td>
<td>14.3</td>
<td>13.6</td>
</tr>
<tr>
<td>2.0</td>
<td>12.9</td>
<td>22.8</td>
</tr>
</tbody>
</table>

Fig. (3.6) Enthalpy Ratio

<table>
<thead>
<tr>
<th>$p_i$ (cm Hg)</th>
<th>$M_s$ (gm/cm²)</th>
<th>$h_1 \times 10^{-3}$ (cal/gm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>As in Fig. (3.5)</td>
<td>3.30</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>3.00</td>
<td>2.51</td>
</tr>
<tr>
<td></td>
<td>2.02</td>
<td></td>
</tr>
</tbody>
</table>
Normal Shock Wave Calculations
(in Argon)

$M_s$ - Shock Mach Number
$p_i$ - Initial Shock Tube Pressure

Fig.
3·7 Density Ratio $\frac{\varepsilon_2}{\varepsilon_1}$
3·8 Degree of Ionization $\alpha_2$
3·9 Temperature behind Shock $T_2$
3·10 Velocity $U_2$
3·11 Pressure Ratio $\frac{p_2}{p_1}$
Fig. (3.8) Degree of Ionization

Fig. (3.9) Temperature
Fig. (3·10) Flow Velocity behind Shock

Flow Velocity $U_2$

$U_2 \times 10^{-5}$ cm/sec.

Pressure $p_i$ (cm Hg):
- 0.1
- 1.0
- 10

Fig. (3·11) Pressure Ratio

Pressure Ratio $\frac{p_2}{p_1}$

$\frac{p_2}{p_1} \times 10^{-2}$

$M_s$ range: 14 to 20
Fig.(4·1) Skin Friction

Fig.(4·2) Conductivity of Air

Fig.(4·3) Hains & Yoler 

Fig.(4·4) Sherman

Fig.(4·5) M.H.D. Generator
Fig. (4.7)
Factor 'a' in Fitted Curve

Fig. (4.6) Specific Heat at Constant Pressure
Fig(4.8) Electrical Conductivity
Fig. (4.9) Electrical Conductivity

$p = 1.0$ atmosphere

$\sigma$ (mho/meter)

$T \times 10^{-3}$ °K

$10^1$

$10^2$

$10^3$

$10^4$
Fig. (4-11) Thermal Conductivity

Fig. (4-10) $\log_e \Lambda$

$p$ at:
1
2
5
Fig. (4.12) Viscosity

$\mu = 1 \text{ at.}$

Fig. (4.13) Prandtl Number

$Pr = 1 \text{ at.}$
Fig. (5.1) to (5.6) Boundary Layer Profiles

$T_\infty = 1.27 \times 10^4 \text{ K}$

$\rho_\infty = 7.9 \times 10^{-5} \text{ gm./cc.}$

$M_s = 15.6 \quad P_1 = 0.5 \text{ cm. Hg.} \quad \text{Linear Velocity Profile}$
\[ \frac{y}{b} \]

\[ \alpha_\infty = 0.126 \]

\[ \sigma_\infty = 6.3 \times 10^3 \text{ mho/m} \]

Fig. (5.3) Degree of Ionization

Fig. (5.4) Electrical Conductivity
Insulated Surface

\[ S_\infty = 0.19 \]
(L = 1 cm.
B = 1 W/m²)

\[ \sigma_\infty U_\infty = 2.8 \times 10^7 \text{ m.k.s. units} \]

(\( R_m = \sigma u \mu_0 L = 0.35 \))
Fig. (6.1) Intrinsic Co-ordinates

Fig. (6.2) Characteristics

Fig. (6.3) Simple Waves

Fig. (6.4) Simple Flow
Fig.(7.1) General Features of the Interaction

- Displacement
- Pressure
- Skin Friction
- Heat Transfer
Fig. (7.2) Hakkinen Profile

Fig. (7.3) Factor, $G = \frac{C_{pp}}{C_{ps}}$

Fig. (7.4) Curvature of Shear Layer in a Magnetic Field
Fig. (8.1) Compression Tube

Fig. (8.2) Driver Section

Fig. (8.3) Direct Coupling
Fig. (8.4) Shock Tube
Fig. (8·5) Test Time in Argon.
Fig. (8.6) Test Time, Comparison.

Fig. (8.7) Model, No Coils.
+ Gate, Lower Beam, 555 C.R.O.

900 V.

850 μF

Field Coils

To Lower Beam

Fig.(8.8) Coil Current Circuit.

Fig.(8.9) Field Coils (Detail)

Flow

2mm

Coils

Brass Cavity

To H.T.

Lead

C.L.

Fig.(8.10) Pick-up Coil

Fig.(8.11) Current Monitor Expt.

20μsec/cm.

Discharge Current

Light Stations

Pick-up Signal
Fig. (8.12) Distribution of Radial Magnetic Field along Surface
Fig. (8.13) Radial Field Distribution Normal to Surface

Fig. (8.14) Lines of Constant Radial Field
FIG. (8.15) I.C.C. PHOTOGRAPH SHOWING THE EFFECT OF THE MAGNETIC FIELD ON THE FLOW OVER THE MODEL IN THE ABSENCE OF A STEP.

Experimental Conditions

\[ p : 0.2 \text{ cm. Hg.} \]

\[ M_s : 16.3 \]

Exposure time : \( 0.2 \mu \text{sec.} \)

Camera delay : \( 7 \mu \text{sec.} \)

Time between frames : \( 2 \mu \text{sec.} \)

Total flow time : \( 12 \mu \text{sec.} \)

\[ B_0 \approx 2 \text{ W/m}^2 \]

Interaction parameter \( \approx 0.5 \)

Legend

C Mach Cone
D Disturbance due to field
F Flow direction
G I.C.C. grid wires
M Magnification
S End of Shock Tube
T Surface of model

Notes: 1) In all I.C.C. photographs the frames number from bottom to top.

2) The prints do not show all the detail visible in the original Polaroid photographs. The tracing accompanying each photograph attempts to supply some of the lost detail.
Fig. (8.16) Distribution of $(B/B_0)^2$

Fig. (8.17) Variation of $B_0$ with Current
FIG. (9.1) I.C.C. PHOTOGRAPH SHOWING THE MAIN FEATURES OF THE INTERACTION.

Experimental Conditions
p : 0.5 cm. Hg.
Mg : 15.5
Exposure time : 0.5 μsec.
Camera delay : 10 μsec.
Time between exposures : 3.5 μsec
Total flow time ~ 18 μsec
Step height : 0.87 mm.
B = 0

Legend
F Flow direction
G I.C.C. grid wires
H Step
M Magnification
S End of Shock Tube
T Surface of model
R Separated region
W Separation Shock Wave
L Increased luminosity outside separated region
N Secondary disturbance originating from near the step
l separated length

Notes: 1) In all I.C.C. photographs flow is from right to left.
2) Separated length increases with time.
3) Decrease in luminosity, implying cooler conditions, as the length increases.
4) Secondary disturbance, N, is more prominent in fig. (9.11).

FIG. (9.2) EFFECT OF THE MAGNETIC FIELD ON THE INTERACTION.

Experimental Conditions
As for fig. (9.1) except
Bo ≈ 3 W/m².

Legend
As for fig. (9.1) except
B Increased luminosity due to magnetic field.

Notes: 1) Lack of change in separated length.
2) Increased luminosity due to magnetic field.
FIG. (9.3) a. EFFECT OF THE MAGNETIC FIELD ON THE INTERACTION.

Experimental Conditions

\[ p = 0.5 \text{ cm. Hg.} \]
\[ M_s = 15.6 \]
Exposure time : 0.5 \( \mu \)sec.
Camera delay : 10 \( \mu \)sec.
Time between exposures : 3.5 \( \mu \)sec.
Total flow time \( \sim 18 \mu \text{sec.} \)
Step height : 1.24 mm.
\[ B_0 \approx 2.5 \text{ W/m}^2. \]

Legend

As for fig. (9.1) except
B Increased luminosity due to magnetic field
P Stagnation zone close to face of step within separated region. See fig. (9.11).

FIG. (9.3) b. Heavier print made from same original as (9.3) a, showing separation shock.

Note: Separation is taking place outside the influence of the magnetic field.
Fig.(9·3)a

Fig.(9·3)b

M: x 2·92

M: x 2·83
Fig.(9.4)

$p_1 = 0.2 \text{ cm. Hg.}$
$h = 0.57 \text{ mm.}$
$L = 20$

Fig.(9.5)

$p_1 = 0.5 \text{ cm. Hg.}$
$h = 0.57 \text{ mm.}$
$L = \text{ mm.}$
$\Delta \ 10$
$+ \ 30$
$\circ \ 40$

Fig.(9.6)

$p_1 = 0.5 \text{ cm. Hg.}$
$h = 0.57 \text{ mm.}$
$L = 20$
Fig. (9.7)

\[ p_1 = 1.0 \text{ cm. Hg.} \]

\[ h = 0.57 \text{ mm.} \]

- \( \bigcirc \) L = 20
- + L = 30

Fig. (9.8)

\[ p_1 = 2.0 \text{ cm. Hg.} \]

\[ h = 0.57 \text{ mm.} \]

- \( \bigcirc \) L = 20
Fig. (9.9) Correlation, $h - 0.57 \text{mm.}, B - 0$. 

Length to Step, Initial Pressure

$L \text{ mm.}, p \text{ cm. Hg.}$
Fig. (9.10)
\[ p_1 = 0.5 \text{ cm Hg.} \]
\[ h = 1.69 \text{ mm.} \]

\[ \bigcirc \text{ L = 20} \]

\[ + \text{ B.L. Trip} \]

Fig. (9.12)
\[ p_1 = 0.5 \text{ cm Hg.} \]
\[ h = 0.57 \text{ mm.} \]

\[ \bigcirc \text{ L = 30} \]

\[ + \text{ B.L. Trip} \]
FIG. (9.11). STAGNATION ZONE WITHIN THE SEPARATED REGION.

Experimental Conditions

- $p = 0.5$ cm. Hg.
- $M_s = 15.6$
- Exposure time: $0.2 \mu$ sec.
- Camera delay: $13 \mu$ sec.
- Time between exposures: $2 \mu$ sec.
- Total flow time: $18 \mu$ sec.
- Step height $1.29$ mm.
- $B = 0$

Legend

- G Grid wires
- H Step
- M Magnification
- S End of Shock Tube
- T Surface of model
- R Separated region
- W Separation shock
- N Secondary disturbance originating from near the step
- P Stagnation zone close to face of step within the separated region

Note: The secondary disturbance appears as a region of increased luminosity originating from the step. It is usually more prominent early in the flow, and is particularly noticeable when a stagnation region exists close to the face of the step. This suggests that it represents the decay of the local stagnation conditions in the mainstream. For a steady well-developed flow we may in fact expect a slight decrease in luminosity originating from the step due to the expansion fan associated with the flow around its top corner. See figure (1.1).
Fig. (9.11)
Fig. (9.13)

- \( p_1 = 0.2 \text{ cm Hg} \)
- \( h = 0.8 \text{ mm} \)
- \( L = 20 \text{ mm} \)
- \( B = 0 \)
- \( B_0 = 3.0 \text{ W/m}^2 \)

Fig. (9.14)

- \( p_1 = 0.5 \text{ cm Hg} \)
- \( L = 28 \text{ mm} \)
- \( h \text{ mm, } B = 0 \text{ B}_0 = 3.0 \text{ W/m}^2 \)
- Symbols correspond to different conditions.
Fig. (9.15)

$\frac{1}{h}$ Plot of Data in Fig. (9.14)
(Same Symbols)

Fig. (9.16)

$p_1 = 0.5 \text{ cm. Hg.}$
$h = 0.47 \text{ mm.}$
$L = 26.5 \text{ mm.}$
$B = 0$
Fig. (9.17)
\[ p_1 = 0.5 \text{ cm. Hg.} \]
\[ h = 0.56 \text{ mm.} \]
\[ L = 26.5 \text{ mm.} \]
\[ B_0 = 3.0 \text{ W/m}^2 \]
\[ \times \text{ Second Disturbance} \]

Fig. (9.18)
\[ p_1 = 0.5 \text{ cm. Hg.} \]
\[ L = 26.5 \text{ mm.} \]
\[ h = 0.47 \text{ cm. Hg.} \]
\[ h = 0.56 \text{ mm.} \]
\[ B_0 = 3.0 \text{ W/m}^2 \]
FIG. (9.20). DISTURBANCE PRODUCED BY THE MAGNETIC FIELD.

Experimental Conditions

- $p : 0.5 \text{ cm. Hg}$.
- $M_s : 15.5$
- Exposure time : $0.5 \mu\text{sec}$.
- Camera delay : $10 \mu\text{sec}$.
- Time between exposures : $3.5 \mu\text{sec}$.
- Total flow time $\approx 18 \mu\text{sec}$.
- Step height : $0.53 \text{ mm}$.
- $B_0 \approx 3 \text{ W/m}^2$

Legend

- G I.C.C. Grid wires
- S End of Shock Tube
- T Surface of model
- H Step
- M Magnification
- W Separation shock
- R Separated region
- N Secondary disturbance from step
- B Disturbance due to magnetic field

Note: Also shown in this figure is a typical "still photograph" taken prior to the actual shot from which measurements of the step height were made.
FIG. (9.21) AND (9.22) DISTURBANCE PRODUCED BY THE MAGNETIC FIELD: COMPARISON WITH OBSERVED FLOW IN THE ABSENCE OF THE FIELD.

**Experimental Conditions** - Fig. (9.21)

- $p$ : 0.5 cm. Hg.
- $M_s$ : 15.6
- Exposure time : 0.5 $\mu$sec.
- Camera delay : 1 $\mu$sec.
- Time between exposures : 3.5 $\mu$sec.
- Total flow time : $\approx 18$ $\mu$sec.
- Step height : 0.53 mm.
- $B = 0$

**Note:** Increase in separated length and decrease in luminosity with time.

**Experimental Conditions** - Fig. (9.22)

As for fig. (9.21) except

- $B_0 \approx 3 W/m^2$

**Legend**

As for figure (9.20)

As for figure (9.21)
Fig. (9.23)
\( p_1 = 0.5 \text{ cm. Hg.} \)
\( h = 0.53 \text{ mm.} \)
\( L = 28 \quad " \)

- \( B_0 = 3.0 \text{ w/m}^2 \)
- Field 'Shock'
- \( B = 0 \)

Fig. (9.24)
\( L = 29.5 \text{ mm.} \)
Rest as in (9.23)
Fig.(9·25) Separated Length, Correlation, B=0.
Fig. (9.26) Characteristic Times

Symbols as in (9.27)

Fig. (9.27) Effect of Magnetic Field on Separated Length

\( p_1 = 0.5 \text{ cm.Hg.} \)

\( L \text{ mm.} \quad B = 0 \quad B_0 = 3.0 \text{ w/m}^2 \)

28
26.5
20
Fig. (9.28) Fractional Change in Separated Length.

\[ \frac{I_0 - I_B}{I_0} \times 10^2 \]

\[ P_1 = 0.5 \text{ cm.Hg.} \]
\[ B_0 = 3.0 \text{ w/m}^2 \]
FIG. (9.29). TIME DEVELOPMENT OF SEPARATED REGION.

Experimental Conditions

\[ p = 0.5 \text{ cm. Hg.} \]
\[ M_s = 15.5 \]
Exposure time: \( 0.5 \mu \text{sec.} \)
Camera delay: \( 5 \mu \text{sec.} \)
Time between exposures: \( 5 \mu \text{sec} \)
Total flow time \( \approx 18 \mu \text{sec.} \)
Step height: 1.26 mm.
\[ B_0 \approx 2.5 \text{ W/m}^2 \]

Legend

G I.C.C. Grid wires
S End of Shock Tube
T Surface of model
H Step
M Magnification
W Separation shock
R Separated region
N Secondary disturbance from step
D Distortion due to high light intensity producing local overloading

Note: 1) Increase in separated length with time. Rate of increase is unaffected by the presence of the field.
2) Decrease in luminosity with time. See also fig. (9.1) and (9.21).
Fig.(9.29) Hypersonic Intake