An Investigation into Passive Modelocking using a Nonlinear Directional Coupler

by

Douglas Body

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This thesis is entirely my own work, except where explicitly indicated.

Douglas Body

Laser Physics Centre
Research School of Physical Sciences and Engineering
Australian National University
Canberra.
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1 Corinthians 13:2
Abstract

This work examines the use of a waveguide device, namely a nonlinear directional coupler to passively modelock a solid state laser.

An examination is undertaken of the available analytical theory and criteria are identified for the optimal conditions for a self-starting, short pulse laser. Experiments were performed using twin core optical fibre as the NLDC element in a bulk optics Nd:YVO$_4$ laser. Pulsing behaviour was not observed in this system, but the important parameters for models were measured. These allowed the analytical models, and a developed numerical model, to be compared and some predictions as to the necessary specifications for the scheme to work can be made.

The more demanding self-starting predictions show that it would be impossible to fabricate a monolithic planar waveguide device that would produce self-starting modelocking using technology available in our laboratories. As a result the investigation of planar waveguide devices was restricted to a study of the properties of waveguide produced by Li ion implantation into Nd:YAG.

It was found that planar waveguides could be produced from a single implant of 6MeV Li ions to a dose of $1 \times 10^{16}$ions/cm$^2$. Absorption measurements show a total waveguide loss of $1.7\pm0.2$dB/cm as implanted reduces to $0.7\pm0.2$dB/cm with annealing. The absorption spectrum showed a number of distinct peaks due to the creation of colour centres. There was a significant change in absorption spectrum; waveguide loss around 700-800nm; and the waveguide mode patterns as the sample was progressively annealed from 300° to 400°C.

Lasing in the ion implanted guide was achieved with an external threshold of $39\pm2.5$mW. The lasing threshold was quite low and suggests that Li ion implantation is a viable technique for fabricating low loss planar active waveguides.
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Symbols Reference List

\( a_1, a_2, a_3 \) coupling terms dependant on waveguide structure and proximity of other guides
\( A(z) \) mode amplitude in waveguide a
\( A_0 \) \( =A(z=0) \), initial mode amplitude in waveguide a
\( A_{\text{beam}} \) cross-section area of beam
\( A_{\text{eff}} \) effective mode area
\( b \) nonlinear loss coefficient
\( b_1, b_2, b_3 \) coupling terms dependant on waveguide structure and proximity of other guides
\( B \) magnetic induction vector
\( B(z) \) mode amplitude in waveguide b
\( B_0 \) \( =B(z=0) \), initial mode amplitude in waveguide b
\( c \) speed of light (m/s)
\( c_1, c_2, c_3 \) dispersion parameters
\( d \) thin film plate thickness
\( D \) group velocity dispersion
\( D(z) \) electric displacement vector
\( D_0 \) normalised dispersion parameter
\( D_1, D_2(z) \) real and imaginary components of \( D(z) \)
\( e_z \) unit vector in z direction
\( E \) electric field vector
\( E_1 \) energy stored in upper level of lasing transition
\( E_2 \) energy stored in lower level of lasing transition
\( E_c \) total circulating energy in the laser cavity
\( E_{\text{circ}} \) circulating energy in the cavity
\( E_p \) pulse energy
\( E_{\text{spont}} \) energy of light captured by the cavity produced by spontaneous emission
\( E_{\text{stim}} \) energy of the light captured by the cavity originally produced by stimulated emission
\( f_0, f_1, f_2 \) parameters in transmission of thin film plate on rotation
\( F(z) \) mode amplitude in waveguide a, \( (=B(z) \exp(-i b_1 z)) \)
\( F_1(z), F_2(z) \) real and imaginary components of \( F(z) \)
\( F_m \) figure of merit for Krausz self-starting criteria
\( g \) gain per pass
\( g_0 \) peak gain
\( g_p \) \( =1/n_3^2 \)
\( H \) magnetic field vector
\( I_i \) incident intensity
\( I_{\text{sat}} \) intensity at which small signal gain halves
\( I_t \) transmitted intensity
\( k \) \( =2 \pi/\lambda \), optical wavenumber (m\(^{-1}\))
\( K \) coupling constant between waveguides
\( K_0 \) number of cavity roundtrips in Herrman theory linear stage
\( K_{ij} \) coupling constant between waveguide i and waveguide j
\( l \) linear cavity loss
- \( L \) device length (m)
- \( L_b \) beatlength (m)
- \( L_{cav} \) length of laser cavity
- \( L_D \) fibre dispersion length (m)
- \( L_f \) fibre length
- \( m_i \) initial number of modes circulating in the cavity
- \( n \) refractive index
- \( n_0 \) refractive index at low light intensity
- \( n_2 \) intensity dependent part of refractive index (W\(^{-1}\))
- \( n_c \) cover/air refractive index
- \( n_f \) film refractive index
- \( n_g \) guide refractive index
- \( n_m \) effective index of mth mode
- \( n_{\text{max}} \) maximum refractive index across waveguide profile
- \( n_s \) substrate refractive index
- \( \Delta n_2 \) = \((n_m^2-n_2^2)\)\(^{1/2}\) (from Pogossian Method)
- \( P \) power necessary to halve the gain
- \( P_e \) steady state intra-cavity power
- \( P_{\text{ext}} \) external pump power
- \( P_{\text{ext}} \) steady state intra-cavity power
- \( P_{\text{ext}} \) external pump power
- \( P_{\text{ext}} \) power necessary to halve the gain
- \( P_{\text{ext}} \) polarisation vector
- \( q_m \) = \((n_m^2-n_2^2)\)\(^{1/2}\) (from Pogossian Method)
- \( r \) reflectivity of thin film surface
- \( r_{\text{ext}} \) reflectivity of an external cavity reflector
- \( r_m \) = \((n_m^2-n_2^2)\)\(^{1/2}\) (from Pogossian Method)
- \( r_{oc} \) reflectivity of the output coupler of the laser
- \( R(z) \) mode amplitude in waveguide a
- \( S_0, S_1, S_2, S_3 \) "Stokes Parameters". Functions of mode powers in 2 coupled waveguides
- \( S_m \) = \((n_m^2-n_i^2)\)\(^{1/2}\) (from Pogossian Method)
- \( S \) self phase modulation slope
- \( t \) guide thickness
- \( t_i \) transmission of ith cavity element
- \( T_0 \) input pulse width (s)
- \( T_{\text{la}} \) response time of the amplifier
- \( T_2 \) polarization relaxation time of the gain medium (s)
- \( T_g \) lifetime of the upper lasing level
- \( T_r \) round-trip time for cavity
- \( u \) field amplitude in fibre core a
- \( U \) \((k^2n_{\text{oo}}^2\beta^2)^{1/2}\)
- \( v \) field amplitude in fibre core b
- \( v_1, v_2, v_3 \) dispersion parameters
- \( v_g \) group velocity
- \( V \) \((k^2n_{\text{oo}}^2-k^2n_2^2)^{1/2}\)
- \( w_i \) width of refractive region in refractive index profile of refractive index n\(_i\)
\( W \) pulse energy

\( W \) \((\beta^2 - k^2n_0^2)^{1/2}\)

\( z \) distance along fibre axis

\( \alpha \) average propagation constant for coupled modes

\( \alpha_c \) fraction of photons emitted into the \( 4\pi \) steradians that are captured by the laser cavity

\( \alpha_i \) \(=(\beta^2-k^2n_i^2)^{1/2} \). Used in step index solution for waveguide modes

\( \alpha_p \) proportionality constant that relates pump power and gain

\( \beta \) mode propagation constant

\( \beta_d \) dispersion parameter=(\(d^2 \beta / d \omega^2\))

\( \gamma \) response of the cavity saturable absorber (W\(^{-1}\))

\( \Gamma \) \(=(\delta^2 + 4K_{ab}K_{ba})^{1/2} \), useful combination of constants

\( \delta \) magnitude of self phase modulation

\( \eta \) phase shift in each round trip of the cavity

\( \eta_j \) \((k^2n_i^2-\beta^2)^{1/2}=i\alpha_j \)

\( \theta(z) \) phase of mode in waveguide a

\( \theta_i \) incident angle of light onto thin film plate

\( \theta_t \) transmitted angle for light onto thin film plate

\( \lambda \) optical wavelength (m)

\( \nu \) optical frequency (Hz)

\( \xi(i) \) propagation constant of mode in guide (i), corrected for presence of the other guide

\( \Xi \) constant of the motion for “stokes parameters” description of 2 coupled waveguides

\( \rho \) difference in mode propagation constants

\( \sigma_{21} \) stimulated emission cross-section

\( \sigma_s \) cross-section of the gain medium

\( \sigma_E \) cross-section for stimulated emission transition, normalised to energy units

\( \zeta \) dimensionless propagation distance for Non-Linear Schroedinger Equation

\( \tau \) dimensionless time for Non-Linear Schroedinger Equation

\( \tau_c \) average lifetime of the most intense peak in the light intensity distribution

\( \tau_f \) fluorescence decay time for upper lasing level

\( \tau_i \) duration of the initial mode beating fluctuation in the free-running laser

\( \tau_{ij} \) lifetime of the level i with respect to transitions to level j

\( \tau_n \) normalised pulsewidth

\( (\Delta u)_{FWHM} \) full width at half maximum of the first beatnote between the longitudinal laser modes

\( \Delta v_0 \) longitudinal mode spacing

\( \Upsilon \) shift in laser oscillation frequency from a Fabry-Perot cavity resonance.

\( \Phi_i \) phase jump at interface i

\( \Phi \) chirp

\( \psi(z) \) phase of mode in waveguide b

\( \Psi \) \( e_x \) or \( h_x \) (solution of waveguide mode equation)

\( \Omega \) gain bandwidth
1 Introduction

1.1 Introduction

A source of short, unchirped optical pulses at a high repetition rate (up to several 10s GHz) from a laser device that is small, simple and robust is in high demand for communications and scientific applications. The work reported in this thesis investigates the use of a waveguide device, namely a non-linear directional coupler, to produce such short pulses, using passive modelocking.

Originally it was intended to construct and characterise laser devices using an all-waveguide geometry such as a doped, twin-core optical fibre to provide both gain and nonlinear coupling in a monolithic fibre device; or by integrating both gain and coupling regions into a planar waveguide. However, it proved too difficult to fabricate the monolithic fibre device, and hence the main experiments were performed instead on a laser having a separate (bulk) laser medium and a resonator containing a fibre-based nonlinear directional coupler. Furthermore, our numerical simulations, supported by the experimental work, suggested that any integrated planar device would only operate successfully if its resonator was physically rather long (several 10s cm). Since devices of this size were beyond the capabilities of the fabrication methods available at the ANU (primarily ion implantation into bulk laser crystals to create surface waveguides), the work reported here concentrates instead on the properties of waveguides produced via ion implantation.

1.2 Passive Modelocking

In any laser, only the optical frequencies for which an integral number of their lasing wavelength fit into the round trip length of the resonator receive net gain. If there is sufficient gain for more than one optical frequency to lase, the round trip condition means that these frequencies will be multiples of $v$, where:

$$v = \frac{c}{2L}$$  \hspace{1cm} (1)
where $L$ is the cavity optical length, and $c$ the speed of light.

In a simple continuous wave (CW) laser, the optical frequencies (called modes) of the cavity have a random phase with respect to each other and amplitudes that are dependent on the spectral properties of the gain medium. As a result, these modes beat randomly together creating fast modulation of the amplitude of the laser output. However, if the relative phase of the modes can be controlled such that they interfere constructively at a certain point in time (space) in the resonator, the laser intensity becomes large at that point and cancels at all other points. The result is that the laser emits a continuous train of narrow pulses of light separated by the round trip time of the resonator and the laser is said to be mode-locked.

There are several methods of affecting the relative phase of the modes using active (eg. electro-optic, acousto-optic) or passive modulators. In the latter case, a device that has a transmission (or alternatively a reflectivity) that increases in response to increasing incident intensity is added to the cavity. Due to the random beating of the oscillating axial modes, the most intense point on the quasi-CW output from the laser experiences marginally less loss than the other points in the cavity intensity distribution (see Figure 1).

![Figure 1](image)

**Figure 1** Effect on an arbitrary, noisy CW laser output of device with intensity dependent transmission leading eventually to one modelocked pulse in the cavity.
After many round trips of the laser resonator, this difference becomes significant and the eventual result is that there is only one intensity peak left in the cavity i.e. a single circulating pulse is created. This operating condition is favoured because it leads to the lowest total loss of power from the laser cavity. The end result is the same as if the individual phases of the oscillating modes had been influenced, but the process itself is not phase dependent. In the frequency domain, the intensity dependent element has acted as a modulator, transferring power between the oscillating modes and thereby locking their phases.

In a real laser cavity, with a real gain medium there are competing processes that work against the formation of a single intensity peak. These include the finite number of frequencies that receive gain, gain narrowing and gain saturation. There are also a number of processes which cause the relative phases of the cavity modes to change with respect to each other and cause the intensity profile to change. These include perturbations (e.g. vibrations), dispersion and self-phase modulation.

The early stages of the pulse formation are particularly critical. At this time, one intensity peak must experience sufficiently different conditions from any other in the cavity that it will eventually be able to dominate. Some passive modulators are unable to meet this condition and require some other mechanism to produce a dominant peak which can be amplified and narrowed by the passive modelocking process. This dominant peak can be provided by, for example, mechanical shocks to the laser cavity; oscillation of a laser mirror; or by adding an active modulator. A laser which will form the modelocked intensity peak from the background noise without such a starting mechanism is said to "Self-Start".

1.3 Problems with Passive Modelocking Schemes

A huge number of passive modelocking schemes are known. Many early experiments used saturable absorber dyes as the mode-locking element within a conventional laser resonator, whilst more recently semiconductor saturable absorber mirrors (SESAMs) have received a lot of attention. However, for many applications, particularly if the source were to be used in an optical fibre communications system, an all fibre device would be desirable because it could be easily spliced to passive fibres in the
network. Applications in communications also demand a high repetition rate (ie. > 1 GHz) pulse train and a system that is robust, and requires no cavity adjustments after manufacture.

Some all fibre systems have been reported, but none of them simultaneously meet these criteria. They include:

- The “Figure of Eight Laser” [1] using Er-doped optical fibre and operating at a wavelength of 1550nm has been reported from several laboratories. Although these devices produce pulses as short as 314fs, the pulse shape is strongly dependent on the pump power. The minimum effective cavity length reported to date seems to contain about 2m[2] of fibre which, when only one pulse exists per cavity round trip, gives an upper limit on the repetition rate of about 100MHz. The design is also prone to producing multiple bursts of pulses and does not appear to operate reliably over long periods of time[3].

- Several all fibre designs that use Er-doped fibre as the gain medium and the nonlinear polarisation rotation in an optical fibre to give the nonlinear reflectivity function needed for passive modelocking have been published[4][5][6]. Pulses as narrow as 700fs have been achieved with this design, but with the available pump power the device was not self-starting unless the total cavity length was greater than 15m. No values of the maximum repetition rate have been published, but a 15m cavity length would restrict operation to ~20MHz.

- Zirngibl et al. [3] report a fibre device which incorporates a semi-conductor saturable absorber as the modelocking element. They reported pulse widths as narrow as 1.2ps. Because there is a limit to the maximum doping level in Er-doped fibre, this means that to get the necessary gain more than 1.5m of fibre was needed in the cavity limiting the maximum pulse repetition rate to ~200MHz.

In spite of the demonstration of these systems for the 1550nm telecommunications window using Er-doped fibre as the active medium, no scheme for self-starting, passively modelocking of an all fibre laser in the 1300nm telecommunications window has been published[7]. Nor has a design for producing a repetition rate greater than 200MHz been demonstrated, although for the case of the systems based on a semiconductor saturable absorber, the limitation in the repetition rate comes from the choice of gain medium (in
that case Erbium doped fibre) rather than something inherent in the process. As a result, other mode-locking configurations are still worthy of consideration.

### 1.4 The non-linear directional coupler as a passive modelocker

Two identical optical waveguides in close proximity are able to interact so that light energy can be exchanged between them. For example, light initially launched into only one of these optical waveguides will be transferred totally to the other waveguide if the coupling region is sufficiently long. This structure is called a directional coupler. This energy exchange, is strongly dependent on the two waveguides being identical. The length of waveguide required for all the light launched into one guide to transfer to the other and back again is known as the coupling length or the beatlength of the device, denoted in this thesis by $L_b$.

If some mechanism is introduced to progressively create a difference between the two guides (for example change the refractive index of one relative to the other), at first the effect will be to increase the coupling length, $L_b$. For larger differences, complete power transfer does not occur (irrespective of length of propagation) and with an even larger difference the coupling between the guides becomes negligible.

Many substances, including glasses and crystals, have a component of the refractive index that is dependent on the local light intensity. ie.

$$n = n_0 + n_2 I$$

Hence, if a pulse with sufficient intensity is launched into one of the guides, it will cause a change in the refractive index of that guide creating a small difference between the waveguides. The greater the light induced "difference" between the guides, the less light that is able to couple over to the other waveguide in a given distance.

This process can be used to make a nonlinear reflection (or transmission) element for modelocking a fibre laser in the following way. The directional coupler is incorporated within the laser cavity so that only one of the waveguides is part of that resonator. Any light transferred to the other waveguide is lost from the resonator. If the laser power increases, the change in the refractive index of the waveguide will reduce coupling to the
"lossy" guide, reducing the cavity loss. Hence the device has a transmission which increases with increasing power - this is the response necessary for a modelocking scheme.

1.5 Thesis Outline

In this work, we have investigated the use of a nonlinear directional coupler to passively modelock a laser system.

Chapter 2 examines some of the physics of the propagation of light in waveguides and looks closely at the behaviour of the nonlinear directional coupler.

In Chapter 3 a number of models of the processes that occur inside a laser cavity are examined and some necessary criteria for self-starting for a general laser system incorporating a directional coupler as the passively modelocking element established.

Chapter 4 describes the experiments and results towards measuring the parameters that the theory indicates are important to obtaining self-starting passive modelocking for a laser using neodymium yttrium orthovanadate (Nd:YVO₄) operating at 1340nm as the gain medium and containing a nonlinear directional coupler in its resonator.

In Chapter 5 the results presented in Chapter 4 are used to refine the criteria for self-starting passive modelocking and these, combined with a numerical simulation of the laser, give indications as to which of the models presented in Chapter 3 will be useful for predicting the self-starting of the passive modelocking for a given laser system. These tools are then used to evaluate a number of possible device configurations. One promising device configuration requires a long, low loss waveguide laser in Nd:YAG.

Chapter 6 describes the process by which such a waveguide could be produced by ion implantation and presents the theory whereby the refractive index profile of guide can be calculated from the measurable parameters - the effective indices of the waveguide modes.

Chapter 7 reports on the experiments to characterise guides produced by high energy Li implantation in Nd:YAG. Several waveguide lasers in Nd:YAG were produced by this method. We particularly concentrate on determining the affect of ion energy, dose and subsequent annealing on the waveguide refractive index profile and loss.

Finally, Chapter 8 summarises and outlines some future directions of this work.
Nonlinear Directional Coupler Theory

2.1 Introduction

Two identical optical waveguides in close proximity form a structure known as a directional coupler which has been described by theory [8][9] and studied experimentally [10][11] for the linear (ie. low light intensity) case since the late 1960s. Early work also investigated different ways of using this structure as an optical switch. For this purpose, the device length was chosen so that light launched into one of the guides coupled completely to the second guide (the so-called cross state configuration). When one of the waveguides was perturbed relative to the other, the coupling was eliminated so that the light then emerged from the first guide (the bar state configuration). Any form of perturbation can, in principle, be used to create this switching action. For example, Campbell et al. [12] studied nonlinear directional couplers in GaAs. By applying terminals to the waveguide they caused switching by changing the refractive index of one of the guides via the electro-optic effect.

The use of the intensity dependence of the refractive index of the waveguide (the Kerr effect) to create the perturbation, and hence induce switching, was first proposed by Jensen [13] in 1982. Such a device could have extremely fast electronic response times (~10^{-14}s). It was, however, not until 1985 that the first experimental confirmation of the theory was reported, first in GaAs/GaAlAs and later in fused silica [14]. Subsequent experiments confirmed the fast response of the device, with switching of 100ps [15], 50ps [16] and 100fs [17] pulses reported. The n_2 component of the refractive index for silica in equation (2) is extremely small, ~ 3.2 \times 10^{-16} cm^2/W [18] and so a very high local intensity is necessary to obtain switching.

The structure, described above, is a far from ideal switch because there will always be some part of the pulse where the intensity is too low to cause switching. Thus only the central, highest intensity part of the pulse will be switched. This property has been experimentally demonstrated with 50ns pulses [19] and later with 100fs pulses [17].
Solutions proposed to overcome this limitation include using femtosecond, flat topped pulses [20] or optical solitons [21] (although to date no experimental confirmation of the latter technique has been reported).

Although this property causes problems for the use of the device for optical switching, it is a useful method for pulse shortening and clean up [22][23] as the more intense parts of a pulse can be retained in one guide while the less intense parts couple to the other. Several theoretical papers have appeared taking advantage of this incomplete switching or pulse compression effect for passive modelocking [24][25][26][27][28][29][30], as well as for "cleaning up" pulses during amplification [31][32].

For the application of a nonlinear directional coupler discussed in this thesis, it will be necessary to accurately determine the nonlinear response of the coupler, particularly at low powers. As will become apparent later, the nonlinear behaviour defines a coefficient, γ, proportional to the slope of the response function near zero power and this determines the mode-locking driving force and through it the final pulse duration, and also the self-starting characteristics of the laser. Although approximate analytic solutions for the response of a nonlinear directional coupler are adequate for many purposes, the predicted values of γ can contain large errors, even changing sign, due to the approximations introduced in such analyses.

In this chapter, therefore, the general equations governing the behaviour of the light in a directional coupler will be derived in some detail. These are evaluated for the two geometries important to this work - a directional coupler made of fibres (ie. two round waveguides in close proximity) which is the geometry used in the experiments described in Chapter 4 with the Nd:YVO₄ laser; and a directional coupler made of two, essentially square waveguides, which has relevance to the work in Chapter 7 on waveguides produced by ion implantation into crystals.

2.2 Directional Coupling in Optical Waveguides

A number of different derivations for coupling between optical waveguides have been published [9][33][34][13][35][36]. Rather than treating the entire coupler as a single structure, the optical modes of the individual waveguides are first calculated separately, then the presence of the “second” waveguide is introduced in the form of a perturbation to
the original modes.

The presence of this perturbation has two main effects:

- the propagation constant of the mode of the original waveguide changes slightly.
- light initially launched into the modes of one waveguide transfers into the modes of the other waveguide, and vice versa.

The full derivation of the equations describing the Nonlinear Directional Coupler are included in Appendix A based on Hardy et al.'s [35] extension of Kogelnik's work [37][38][39]. Hardy et al’s approach was adopted because it makes the fewest assumptions about the waveguides or the modes. Only the most important results are included here.

2.2.1 Coupled Mode Theory for the Directional Coupler

For the case of a directional coupler consisting of two single mode waveguides ("a" and "b"), the propagation equations for the amplitudes of the mode of waveguide a, \(A(z)\) and waveguide b, \(B(z)\) are given by:

\[
\frac{dA}{dz} = i\xi^{(a)}A + iK_{ab}B \\
\frac{dB}{dz} = i\xi^{(b)}B + iK_{ba}A
\]

The assumptions made in deriving (3) are that a linear combination of the modes of the isolated waveguides are a good approximation to the modes of the combined structure and that there is no z dependence in the refractive index profile. Equations (3) have the general solution:

\[
A = e^\frac{iz}{2} \left[ A_0 + \frac{2B_0 K_{ab}}{\Gamma} \sin(\Gamma z) + A_0 \cos(\Gamma z) \right] \\
B = e^\frac{iz}{2} \left[ B_0 \cos(\Gamma z) + \frac{2A_0 K_{ba}}{\Gamma} \sin(\Gamma z) + B_0 \right]
\]

where \(\alpha = \xi^{(b)} + \xi^{(a)}, \rho = \xi^{(a)} - \xi^{(b)}, \Gamma = (\rho^2 + 4K_{ab} K_{ba})^{\frac{1}{2}}, A_0 = A(z=0)\) and \(B_0 = B(z=0)\) (ie. values for A and B at start of coupler).

Qualitatively, \(\xi^{(i)}\) is the propagation constant of the mode, corrected for the presence of the other waveguide; \(\alpha\) is the average propagation constant for the coupled modes; \(\rho\) is
the difference in the mode propagation constants; and $K_{ij}$ describes the coupling between the waveguides (corrected for the difference in propagation constants between them). A qualitative definition for $\Gamma$ is more difficult, however, for very similar waveguides (i.e. $\rho$ small) then the term is dominated by the coupling terms and represents the rate that they exchange energy as a function of propagation distance. For non-identical waveguides, $\rho$ dominates and has an effect in limiting the maximum energy that can couple.

![Diagram](image)

**Figure 2** Maximum coupling fraction for a directional coupler as a function of difference between core refractive indices ($\Delta n$) and wavelength ($\mu m$). ($n_{co}=1.45459$, $n_{cl}=1.44963$, $r=4\mu m$ and core separation = $10\mu m$, planar waveguides).

In the case where $B_0=0$, (4) becomes:

$$A=e^{i\alpha z} \left[ \frac{A_0^2}{i} \frac{\rho}{\Gamma} \sin(\Gamma z) + \frac{A_0}{\Gamma} \cos(\Gamma z) \right]$$

$$B=e^{i\alpha z} \left[ \frac{2A_0K_{ba}}{i\Gamma} \sin(\Gamma z) \right]$$

(5)

which shows that for $\rho=0$, the total power oscillates between the two waveguides.

The exact values of $\alpha$, $\rho$ and $\Gamma$ are dependent on integrals of the modes over the
Figure 3 Maximum coupling fraction for a directional coupler as a function of difference in core radius ($\Delta r$).

$n_{co}=1.45459$, $n_{cl}=1.44963$, $r=4\mu m$, core separation = $10\mu m$, planar waveguides).

entire waveguide geometry. Analytical expressions for the specific cases of a planar, step index waveguide (similar to ion implanted waveguides of Chapter 6) and a step index fibre (relevant to Nd:YVO$_4$ experiments of Chapter 4) are derived and evaluated in Appendix D.

An important trend emerges from evaluation of the various constants in (2) and (3). In eqn (3) $\xi^{(a)}$ is approximately the propagation constant of the waveguide mode and has a value close to $k n_{max}$, where $k=2\pi/\lambda$ and $n_{max}$ is the maximum refractive index of the waveguide. For a waveguide in fused silica, at 1342nm $\xi=8.6 \times 10^6m^{-1}$. From Appendix D, we find that $K\sim2500m^{-1}$ for a directional coupler with 4$\mu$m diameter core and 10$\mu$m core separation. Hence $\xi^{(a)} \gg K$. As a result if the waveguides are not identical, which means that they have different mode propagation constants $\xi^{(a)}$, $\xi^{(b)}$ then $\rho$ becomes rapidly very large ($\gg K$), dominating $\Gamma$ in eqn (4) and the maximum coupling becomes very small. Examples of the sensitivity of the coupling to small perturbations between the two waveguides are shown in Figures 2 and 3 for step index waveguides. The data shows that the coupling is reduced to less than 50% for a change of less than 0.005% in the relative
waveguide refractive index or 4% on the relative core radius.

**Figure 4** Compare $\xi$ and $K$ for TE and TM modes of the same step index fibre guide ($n_{c0}=1.45459$, $n_{cl}=1.44963$, $\lambda=1342\text{nm}$ and $r=4\mu\text{m}$).

These results clearly show that great care must be taken to ensure that the value of $\Gamma$ does not vary either as a function of position along the coupler or with time. Stress and temperature can both change the refractive index profile and/or shape of the two waveguides and could, therefore, have major affect on the coupling between them. For this reason the directional couplers must be stabilised against such external environmental factors. This can be achieved by winding the fibre onto a spool or preferably encasing it in a protective (often rigid) jacket.
Figure 5  % difference between TE and TM mode coupling as a function device length (in beatlengths) (10μm core separation step index fibre)

Another issue that will be of importance for the later experiments, is the effect of polarisation on the coupling between the waveguides. Figure 4 compares the coupling constants for the TE and TM modes for a step index fibre and demonstrates that there is very little difference between the two polarizations for a wide variety of core separations. Figure 5 shows the difference between the coupling fractions of the different polarisations as a function of the length of the fibre. This shows that there is only a small (<5%) difference between the coupling of the different polarisations and should not be a major effect in any of the experiments.

2.3 Nonlinear Waveguides.

If we now allow the refractive index to be intensity dependent, eqns (3) become:

\[
\frac{dA}{dz} = iα_1 A + iα_2 A|A|^2 + iα_3 B \\
\frac{dB}{dz} = iβ_1 B + iβ_2 B|B|^2 + iβ_3 A
\]  (6)
The equations (6) can be studied a number of ways. There are, however, no known analytical solutions that preserve the phase information of the light in the waveguides. The following, however, gives some empirical insight into the effect of nonlinearity.

If we substitute $A = \exp(i \alpha z \cdot D(z))$, $B = \exp(i \beta z \cdot F(z))$ into (6) this gives:

$$\frac{dD}{dz} = i a_2 D |D|^2 + i a_2 e^{i p x} F \quad \frac{dF}{dz} = i b_2 F |F|^2 + i b_2 e^{-i p x} D$$

(7)

where $\rho = b_1 - a_1$. In the case where $\rho = 0$ (i.e. identical guides) and using $D = D_r + i D_i$, $F = F_r + i F_i$ then (7) may be decoupled to give:

$$\frac{dD_r}{dz} = -a_2 D_r (D_r^2 + D_i^2) - a_2 F_i \quad \frac{dD_i}{dz} = a_2 D_r (D_r^2 + D_i^2) + a_2 F_r$$

$$\frac{dF_r}{dz} = -b_2 F_r (F_r^2 + F_i^2) - b_2 D_i \quad \frac{dF_i}{dz} = b_2 F_r (F_r^2 + F_i^2) + b_2 D_r$$

(8)

In the limit of small $D$, $F$ (or alternatively $a_2 = b_2 = 0$) then (8) becomes:

$$\frac{dD_r}{dz} = -a_2 F_i \quad \frac{dF_i}{dz} = b_2 D_r$$

$$\frac{dD_i}{dz} = a_2 F_r \quad \frac{dF_r}{dz} = -b_2 D_i$$

(9)

which has the solutions:

$$D_r(z) = -\sqrt{\frac{a_2 F_i(0) \sin(\sqrt{a_2 b_2} z) + D_r(0) \cos(\sqrt{a_2 b_2} z)}{b_2}}$$

$$F_i(z) = \sqrt{\frac{b_2 D_r(0) \sin(\sqrt{a_2 b_2} z) + F_i(0) \cos(\sqrt{a_2 b_2} z)}{a_2}}$$

$$D_i(z) = D_i(0) \cos(\sqrt{a_2 b_2} z) + \sqrt{\frac{a_2 F_r(0) \sin(\sqrt{a_2 b_2} z)}{b_2}}$$

$$F_r(z) = F_r(0) \cos(\sqrt{a_2 b_2} z) - \sqrt{\frac{b_2 D_i(0) \sin(\sqrt{a_2 b_2} z)}{a_2}}$$

(10)

ie. the $D_r$ and $F_i$ are coupled, as are $D_i$ and $F_r$. 
For high intensity (8) becomes:
\[ \frac{dD}{dz} = i a_2 |D|^2 \quad \frac{dF}{dz} = i b_2 |F|^2 \] (11)

which has the solutions:
\[ D(z) = D(0)e^{i a_2 |D(0)|^2 z} \quad F(z) = F(0)e^{i b_2 |F(0)|^2 z} \] (12)

**Figure 6** Mixing of terms in Directional Coupler.

ie. the behaviour resembles Self Phase Modulation (SPM). As a result the function of \(a_3\), \(b_3\) is to cause oscillation between \(D_r\) and \(F_r\), and \(D_i\) and \(F_i\) while \(a_2\) and \(b_2\) cause phase change within each of the guides. It would appear, that at high intensity the phase within a waveguide is changing so rapidly coupling to its counterpart in the other guide is precluded. This is demonstrated in Figure 6. The interesting physics occurs when these two interactions become similar in magnitude.

This physics is more effectively studied if we make the substitutions:
\[ D = R(z)e^{i \theta(z)} \quad F = Q(z)e^{i \psi(z)} \] (13)

into eqn (7). After some manipulation this yields:
The behaviour of these equations is shown in Figures 7-10. Qualitatively it can be seen that the behaviour is strongly dependent on the phase difference $\theta-\psi$ between the waveguides, rather than the actual phase values. Qualitatively, it can be seen that at low power the two different waveguide modes remain a constant $\pi/2$ out of phase with each other, whereas at high power there is a gradual phase change with propagation distance. Examining eqn 14(c) it is apparent that the dominant behaviour depends on the ratio of $a_2 R^2/a_3$.

\[
\begin{align*}
\frac{dR}{dz} &= a_3 Q \sin(\theta - \delta z - \psi) \\
\frac{dQ}{dz} &= b_3 R \sin(\psi + \delta z - \theta) \\
R \frac{d\theta}{dz} &= a_2 R^3 + a_3 Q \cos(\theta - \delta z - \psi) \\
Q \frac{d\psi}{dz} &= b_3 Q^3 + b_3 R \cos(\psi + \delta z - \theta)
\end{align*}
\] (14)

Figure 7 R(z) (Amplitude in guide a as a function of distance) for different input powers. Graphs are normalised by $P_{\text{crit}}$.
Defining:

\[ P_{\text{crit}} = \frac{4a_3}{a_2} \]  \hspace{1cm} (15)

as a critical power allows the figures to be normalised to this value for generality.

Figure 7 shows the power in the launch guide as a function of propagation distance and power. The normalised (to be 1 at the maximum power) behaviour is shown in Figure 8. This shows that as the launch power increases the beatlength increases. Coupling still remains at 100%. Above \( P_{\text{crit}} \), the coupling reduces rapidly and the beatlength decreases with increasing power.

![Figure 8](image)

\textbf{Figure 8} \hspace{0.5cm} R(z)/R(0) as a function of propagation distance along waveguide.
At the cost of losing the phase information, we can make the following substitutions (known as the Stokes parameters [40]):

\[
\begin{align*}
S_0 &= |D|^2 + |F|^2 \\
S_1 &= |D|^2 - |F|^2 \\
S_2 &= DF^* + D^*F \\
S_3 &= i(D^*F - DF^*)
\end{align*}
\]
Figure 10  Phase difference (ie. $\theta(z) - \psi(z)$) as a function of propagation distance. (for powers normalised to fraction of critical power)

These transform (7) into:

\[
\begin{align*}
\frac{dS_0}{dz} &= 0 \\
\frac{dS_1}{dz} &= (a_3 + b_3)S_3 \\
\frac{dS_2}{dz} &= \frac{S_0 - S_1}{2}b_2S_3 - \frac{S_0 + S_1}{2}a_2S_3 \\
\frac{dS_3}{dz} &= \frac{1}{2}(a_2S_2 - 2b_3) + \frac{S_0 - S_1}{2}(2a_3 - b_2S_2)
\end{align*}
\]

If we allow $a_3 = b_3$, $a_2 = b_2$ (identical guides) then the equations (17) simplifies to:
From these it is possible to see that $S_0$ is a constant of the motion, but there also exists:

$$\Xi = \frac{a_2 S_1^2}{2} + 2a_3 S_2$$  \hspace{1cm} (19)$$

which is also a constant of the motion.

Differentiating (18) and taking advantage of (19) leads to:

$$\frac{d^2 S_1}{dz^2} + (4a_3^2 - a_2 \Xi) S_1 + \frac{a_2^2 S_1^3}{2} = 0$$  \hspace{1cm} (20)$$

which has solutions[13]:

$$S_1(z) = S_1(0) \text{cn} \left[2a_3 z, \left(\frac{a_2 S_1(0)}{4a_3}\right)^2\right]$$  \hspace{1cm} (21)$$

where cn is one of the Jacobi elliptic integrals. Rearranging to give back the equations in their original forms gives:

$$|D(z)|^2 = \frac{S_1 + S_0}{2} = \frac{|D(0)|}{2} \left\{ 1 + \text{cn} \left[2a_3 z, \left(\frac{a_2 |D(0)|}{4a_3}\right)^2\right] \right\}$$  \hspace{1cm} (22)$$

for the case where $F(0)=0$. 
Figure 11 shows the response of the nonlinear directional coupler to different input powers for different device lengths. The output has been normalised to the power launched into core a. The length of the device determines the coupling at low power, according to:

$$\left|D(z)\right|^2\bigg|_{D(0)=0} = \frac{1+\cos[2a_3 z]}{2}$$ (23)

An increase in power changes the refractive index of the waveguide increasing the coupling length, which equivalently changes the coupling for a given length of waveguide. Eventually the critical power is exceeded and 100% coupling is no longer possible. With a further increase in power the coupling length decreases, resulting in the oscillation of the coupling as a function of power. For future reference, it is important to note the differences which exist in the slopes of the device response near zero power for cases where the device lengths correspond to 0.25, 1.25 and 2.25 $L_b$. Clearly the slope increases as the number of beatlengths increases in spite of the fact that the initial coupling is identical. This suggests a stronger mode-locking driving force will be obtained when the device length is several beatlengths long.

![Figure 11](image)

**Figure 11** Response of nonlinear directional coupler to different input powers
2.3.1 Approximate Solutions for Nonlinear guide

While eqn (22) is the exact solution for the behaviour of the light energy in a nonlinear directional coupler it is a somewhat "exotic" function and inconvenient to evaluate. The most important region for our application is that close to zero power, since it is the slope here that provides the nonlinear response to drive the laser modelocking action. In this region several approximations are possible that allow a far more intuitive understanding of what is occurring.

An approximation [41] that holds for small powers (ie. $P/P_{\text{crit}} < 0.1$) allows a "guess" to be made for the parameters and then the error minimised. In this case, we guess that $S_1(z)=A \cos[c_1 z]$ (ie. resembles the 0 power case). Then (20) becomes:

\[ AC\cos[c_1 z](4a_3^2-a_2 \Xi-c_1^2)+\frac{a_2^2}{2}(AC\cos[c_1 z])^3=0 \]  

(24)

and a measure of the error in the approximation is:

\[ E(c_1) = \frac{1}{2\pi} \int_0^{2\pi} \left( AC\cos[c_1 z](4a_3^2-a_2 \Xi-c_1^2)+\frac{a_2^2}{2}(AC\cos[c_1 z])^3 \right)^2 dz \]  

(25)

Next we seek a minimum ie. $\partial E/\partial c_1 = 0$ and solve for $c_1$ to give:

\[ S_1(z)=PC\cos z \left[ \sqrt{\frac{4a_3^2-a_2 \Xi+\frac{3a_2^2 P^2}{8}}{16a_3^2-\frac{a_2^2 P^2}{2}}}-1-\frac{a_2^2 P^2}{32a_3^2} \right] \]  

(26)

for the case when $|D(0)|^2 = P$, $F(0)=0$. Qualitatively, increasing the power increases the
coupling length as expected. For later calculations the slope of the function with power will become important and this approximation will be useful.

![Graph](image)

**Figure 12**  Comparison of actual beatlength (normalised to zero power beatlength) vs incident power and approximation from eqn (26)

The accuracy of the approximation for predicting the beatlength is compared in Figure 12. This is somewhat misleading, however, as the shape of the response changes at the higher powers and is not exactly sinusoidal. However, errors are small for $P/P_{\text{crit}} < 0.1$.

### 2.4 Summary

Two waveguides in close proximity interact and light is able to couple from one to the other. This can be modelled by using Coupled Mode Theory. At low light intensity the behaviour of the light in the guides is given by:

$$\frac{dA}{dz} = i\gamma^{(a)}_A A + iK_{ab} B$$

$$\frac{dB}{dz} = i\gamma^{(b)}_B B + iK_{ba} A$$

(27)
which has the solutions:

\[
A = e^{\frac{iz}{2} \left[ i \frac{A_0 \delta + 2B_0 K_{ab}}{\Gamma} \sin(\Gamma z) + A_0 \cos(\Gamma z) \right]} \quad B = e^{\frac{iz}{2} \left[ B_0 \cos(\Gamma z) + i \frac{2A_0 K_{ba} - B_0 \delta}{\Gamma} \sin(\Gamma z) \right]}
\]

Qualitatively, \( \Gamma \) indicates the "differentness" of the guides and as this gets larger the coupling between the guides decreases.

At high power (i.e. where the guide refractive index is affected by the light intensity) the propagation is given by:

\[
|A(z)|^2 = \frac{|A(0)|^2}{2} \left[ 1 + cn \left( 2a_2 z, \left( \frac{a_2 |A(0)|^2}{4a_3} \right)^2 \right) \right]
\]

where it has been assumed that all the light is launched initially into core A.
3 Self Starting Passively Modelocked Laser Models

3.1 Introduction

Various authors have investigated the theoretical operation of a laser cavity containing a nonlinear directional coupler. To date, work has concentrated on the steady state behaviour and long term stability of such a system. Our interest has been to investigate the self-starting and short term dynamics to establish a set of criteria for the reliable operation of the laser.

In this chapter, the previous work, both theoretical and experimental, will be reviewed. Then, the available analytical theory for the dynamics inside a modelocked cavity will be summarised. From this, the important theoretical criteria for device operation will be derived. This will form the background to the discussion of the experimental results in the subsequent chapters.

3.2 Previous Work: Steady-State Behaviour

The original work by Langridge and Firth[42] used an arrangement of a directional coupler with one pair of ports joined by a length of fibre which provided gain (Fig. 14). Their model included the frequency dependence of the gain as well as pulse broadening terms arising from SPM. They chose the coupler to have ~15% coupling at low power and investigated whether a stable pulse formed in the cavity when initial pulses with envelopes described by either A sech[B t] or A Exp[-x^2] circulated within the resonator. They noted that there were definite parameter combinations (values of A and B) for which a stable pulse formed and for others it did not. The work does not attempt to relate this behaviour to experimentally measurable parameters.
Figure 13  Fibre laser model investigated by Langridge and Firth.

This was independently followed by papers [43][44] by Winful and Walton which modelled the laser as one long directional coupler. One of the waveguides had gain, while the intensity in the other was periodically reset to zero at intervals corresponding to the resonator length. This approach meant that both the gain and nonlinear coupling were handled simultaneously and allows the use of one set of coupled Non-Linear-Schrödinger Equations:

\[
\begin{align*}
\frac{i}{\zeta} \frac{\partial u}{\partial t} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + K \nu + |u|^2 u &= i \frac{1}{2} g_0 L_D \left( u + \tau_2^2 \frac{\partial^2 u}{\partial \tau^2} \right) \\
\frac{i}{\zeta} \frac{\partial v}{\partial t} + \frac{1}{2} \frac{\partial^2 v}{\partial \tau^2} + K \nu + |v|^2 v &= 0
\end{align*}
\]  

(30)

to be solved over the entire propagation distance that characterises one round trip of the laser cavity. In eqn (30) \( u \) and \( v \) are the field amplitudes in the cores; \( L_D = \frac{T_0^2 / \beta_2}{\zeta} \) is the dispersion length (propagation distance for which waveguide dispersion will have a detectable affect on the pulse); \( g_0 \) is the peak gain; \( \tau_2^2 = \frac{T_2}{T_0} \) where \( T_2 \) is the polarization relaxation time of the gain medium and \( T_0 \) is the input pulse width. The dimensionless parameters used are \( \zeta = z / L_D \) and the dimensionless time \( \tau = \frac{t - z / v_g}{T_0} \) where \( v_g \) is the group velocity. \( K \) is the coupling coefficient (see eqns (169), (188), (189) and (190)) between the cores and the condition that the light totally transfers from an input core to the other is \( K L = \pi / 2 \).
In their work, Winful and Walton chose the device and coupling lengths to be $L_D$ (Fig. 14) (although they acknowledge that, while this is optimal for conditions where modelocked pulses have formed, it gives such a large loss at low power that self-starting of the passive modelocking is impossible). Their work used, as an initial condition to the simulation, a Gaussian or hyperbolic secant pulse injected into one core of the "laser". They found that if this seed pulse was sufficiently intense a stable regime could be achieved for the operation of the laser. The steady state pulse had a width reduced from that of the seed and was independent of the input pulse. In a realistic negative group velocity dispersion (GVD) regime the steady-state pulse duration was found to be 58fs for $\beta_2 = -20 \text{ ps}^2/\text{km}$; and 2.15ps for $\beta_2 = 20 \text{ ps}^2/\text{km}$. In this work the gain profile was assumed to have a physically unrealistic, parabolic profile and no account was taken of gain saturation or stimulated Raman scattering.

This early work was followed by that of Essiabre and Vallee [45] who considered the gain and the coupling separately. They simulated a device containing a gain region connected to one port of a four port 50%/50% linear coupler with two of the ports joined by a length of twin core fibre (see Fig. 15). Light enters the device at port 2, 50% remains and traverses the loop in a clockwise direction. The other 50% couples to port 3 and traverses the loop in an anti-clockwise direction. The fibre loop is chosen to be one beatlength long so that at low power all incident light couples to the outer core and is lost through ports 1 and 4. At higher powers, less of the light couples to the outer core and returns via ports 2 and 3 to the main cavity. Their results, in the anomalous dispersion region again, found that the system was (remarkably) stable and they obtained a 63fs pulse using a 500fs seed pulse. Their system, however, required an impractical beatlength of about 40m. Their model had the advantage that it allowed the different effects of the gain
and the nonlinear pulse shaping to be studied with greater independence.

A later paper by Essiambre and Vallee [46] studied a system with separate gain and
coupling regions with feedback provided by a mirror on only one core of the output of the
nonlinear directional coupler (Fig. 15). This configuration is very close to that studied
experimentally in this thesis. They investigated the response of the system with a
Lorentzian and a parabolic gain profile in the anomalous dispersion regime. Their results
again showed that for appropriate choice of powers the device would operate and, in the
case of Lorentzian gain profile, produce almost exactly fundamental soliton shaped pulses.
However, their scheme relied again on an impractical beat length for the fibre of 20-40m.
It was also pointed out in this paper that one of the reasons that the design is so stable is
that small perturbations and noise that would normally build up to levels where it would
disrupt the passive modelocking is constantly being transferred to the second core and is
lost from the resonator. These authors seeded their simulations using Gaussian pulses.

Figure 15  Laser model studied by Essiambre and Vallee [45]

Figure 16  Laser Model used for Essiambre and Vallee.[46]
The work of Kanka [47] extended the model of Winful and Walton focussing on the more realistic Lorentzian gain line shape and including stimulated Raman Scattering. Their simulations used a fibre length such that $K L = \pi/3$ (ie the coupling was initially 50%) in an attempt to consider the problem of self starting (contrasting with the previous simulations that have been run at the unrealistic $K L = \pi/2$ point (i.e. the coupling is initially 100% and the low power cavity has 100% loss obviously precluding any build-up of lasing). Kanka’s model used a ring laser configuration comprising a nonlinear directional coupler incorporating gain, linear output coupling, and a filter to prevent the Stimulated Raman Scattering from shifting the pulse outside the gain window. The results suggest that under these conditions the laser was stable when operating within a specific gain window, below which the pulses died out, and above which they were unstable. The beatlength for the fibre was again unrealisable and of the order of 75m. They reported a steady state with pulse width of 159fs FWHM. As an attempt to investigate self-starting of the device, weak Gaussian pulses (0.1 in normalised units, corresponding to about 3.7W ) were used as the initial conditions and pulses were found to form from them.

The most recently published work by Oh et al. [30] is again an extension of the work of Winful and Walton by including a range of higher order effects. Again a ring geometry was chosen, using a parabolic gain profile and including Stimulated Raman scattering (SRS), as well as soliton self-steepening, gain saturation, and third-order dispersion. They reported regimes for stable modelocking using a 500fs seed pulse of 1.5 soliton units. In the best conditions this resulted in a sech shaped pulse with a FWHM of 160fs circulating in the resonator. They also investigated the operation of a device for which $K L = \pi/4$ and reported little change in the operation or stable regions but some broadening of the pulse. SRS was the strongest extra effect reported, and it resulted in a change of stable pulse duration from 106fs (no SRS) to 160fs (with SRS) as well as producing chirping and an observable shift and reshaping of the pulse spectrum.

A recent paper by Hsu and Yang [48] suggested the directional coupler as a modelocking element for a semiconductor laser. Their model was similar to that of Winful and Walton with both the gain and the coupling lumped together. They included a simple model of gain with saturation and allowed their pulses to build up from an initial Gaussian pulse with 1/e width of 1ns. Their model contained no pulse broadening terms (eg. SPM). In some of their simulations a second, parasitic pulse formed on the main one due to the
recovery of the gain in the media. This is not expected to occur for solid state systems where the lifetimes of the metastable levels are longer than those assumed in these simulations. No values for device lengths were given in the paper.

Although, as indicated above, there have been quite a number of numerical studies of the use of a nonlinear directional coupler as a modelocking element in a laser, experimental work is almost totally absent. It would appear that the only experimental paper published in this area was that of Reinhardt et al. [49] who used a 20m length of twin core optical fibre to increase pulse shortening in a harmonically modelocked laser. They were unable to detect any additional pulse shortening due to the presence of the fibre and, due to the lengths used, had considerably difficulty with changes of phase and coupling in the fibre due to environmental factors.

### 3.3 Dynamic Behaviour

A number of analytical and numerical models have been developed to study the dynamics that occur inside a laser cavity. In general, they have been designed to give predictions on a particular laser parameter (e.g. pulsewidth, self-starting behaviour, etc) and in the following a summary the different models will be presented. The symbol convention used by Haus et al.[50] will be followed.

### 3.4 Predictions of Pulsewidth

Haus et al. [50] produced a model applicable to a wide range of laser cavities which contain optical elements that act as instantaneous saturable absorbers. In their model, the steady state behaviour of the pulse was derived by assuming that the effect of the different cavity elements was additive and summed to zero over one cavity round trip. Under these assumptions one obtains Haus’s so called, master equation:

\[
-\imath \Upsilon - (l + i \eta \gamma + g \left( 1 + \frac{1}{\Omega^2} \frac{d^2}{dt^2} \right) + i D \frac{d^2}{dt^2} + (\gamma - i \delta) |a|^2 \right] a = 0
\]

where \( \Upsilon \) allows for a slight shift of the laser oscillation frequency from one of the Fabry Perot resonances of the cavity; \( l \) is the linear loss; \( \eta \) characterises the phase shift in each
round trip of the cavity; \( g \) is the gain per pass with \( \Omega \) the gain bandwidth. \( D \) describes the group velocity dispersion; \( \gamma \) approximates the response of the saturable absorber at the circulating power (the modelocking driving force); and \( \delta \) gives the magnitude of the self phase modulation.

This equation (31) has the solution:

\[
a = A \text{sech} \left( \frac{t}{\tau} \right) e^{i \Phi_{\text{in}} \text{sech} \left( \frac{t}{\tau} \right)}
\]

(32)

which describes a pulse with variable amplitude \( A \); pulse width \( \tau \); and chirp \( \beta \).

Substituting into (31) leads to:

\[
\Phi = -\frac{3}{2} \chi \pm \sqrt{\left( \frac{3}{2} \chi \right)^2 + 2}
\]

(33)

where:

\[
\chi = \frac{\delta D_n - \gamma}{\delta + \gamma D_n}, \quad D_n = \frac{\Omega^2 D}{g}, \quad \tau_n = \frac{W^2 \Omega^2}{2g}, \quad W = 2A^2 \tau
\]

(34)

where \( W \) is the pulse energy; \( \tau_n \) is a normalised pulsewidth; \( D_n \) is the normalised dispersion parameter and \( \chi \) is a useful collection of constants. The parameter \( g \), represents the gain of the medium and is given by:

\[
g = \frac{g_0}{1 + \frac{2A^2 \tau}{P_{\text{sat}} T_r}}
\]

(35)

where \( 2A^2 \tau \) is the pulse energy; \( T_r \) is the round trip time for the cavity; and \( P_{\text{sat}} \) is the power necessary to halve the gain and is given by:
where $\sigma_{21}$ is the stimulated emission cross-section; $\tau_f$ is the fluorescence decay time of the upper lasing level; and $A_{\text{beam}}$ is the beam cross-sectional area in the gain medium.

The sign of the $\pm$ in eqn (33)a is chosen to give the correct sign in the original equations and is given in Table 1.

<table>
<thead>
<tr>
<th>$\delta D_n - \gamma$</th>
<th>$\delta + \gamma D_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 0$</td>
<td>'−'</td>
</tr>
<tr>
<td>$&lt; 0$</td>
<td>'+'</td>
</tr>
</tbody>
</table>

Table 1 Sign for $\pm$ in eqn (33)a

Brabec et al. [52] extended this model of the laser cavity processes beyond the so-called Weak Pulse Approximation (i.e. that each term in (31) was additive and commutative). Their model allowed the processes to occur sequentially around the cavity and permitted different values for the pulse parameters at different points in the cavity. The limit they found for the pulse width in the cavity in the steady state was

$$\tau = \frac{6.06|D|}{\delta E_p} + \alpha \delta E_p$$

(37)

where $E_p$ is the pulse energy and $\alpha$ is a numerical factor depending on the position within the cavity ranging from 0.1 at the dispersive to 0.25 at the non-dispersive ends of the cavity. Note that, although the amplitude modulation produced by any non-linearity, $\gamma$, does not appear anywhere in (37) the ratio of $\gamma/\delta$ (amplitude nonlinearity to SPM phase nonlinearity) has a strong influence on the pulse quality and stability and through this mechanism has an indirect affect on the limits of the pulse duration.
3.5 Pulse Stability

An extension to Haus's pulsewidth calculations is possible by requiring that only the pulse centre sees net gain. This leads to the pulse stability requirement

\[(1-\Phi^2)-2\Phi D_n>0\]  

(38)

3.6 Self-starting

Krausz et al. [53] derived a criterion for self-starting of passive modelocking which can be summarised by a figure of merit \(F_m\) given by:

\[F_m = \frac{\gamma P \tau_c \ln(m_i)}{T_R}\]  

(39)

where \(\gamma\) is the slope (in \(W^{-1}\)) of the nonlinear loss function at the CW circulating power (mode-locking driving force cf. eqn (31)); \(P\) is the intra-cavity circulating power (in \(W\)); \(m_i\) is the initial number of modes oscillating in the laser; \(T_R\) is the cavity round-trip time; and \(\tau_c\) is the average lifetime of a particular peak in the intensity distribution of the laser. The laser will self start for a parameter set for which \(F_m > 1\).

This parameter quantifies a number of intuitive expectations about the self-starting of the mode locking. ie. Modelocking is more likely if there is:-

- Strong selection of the highest intensity peak (ie. large \(\gamma P\))
- Large intensity modulation at startup (ie. many laser modes oscillation - large \(m_i\))
- Long lifetime for highest peak (ie. large \(\tau_c\))
- Many transits of the nonlinear element (ie. small \(T_c\)). The competing effects to pulse formation that come from the gain medium are essentially time, rather than round-trip number, dependent. Hence, self-starting is more likely if the non linear element is able to act more often in a given time on the laser field.

The parameters of (39) have the advantage that they are apparently readily measurable. In particular, for a Lorentzian line shape:
\[
\tau_c = \frac{1}{\pi (\Delta v)_{\text{FWHM}}} \tag{40}
\]

where \((\Delta v)_{\text{FWHM}}\) is the full width half maximum of the first beatnote between the longitudinal modes of the laser. Other line shapes contribute proportionality factors only a little different from \(\pi\).

It is useful to examine the factors that can influence the width of this beatnote. Several authors [54] [55] have considered broadening in response to frequency pulling effects and spurious reflections inside or outside the resonator.

The major cause of the frequency pulling is the intra-cavity group delay dispersion (GDD). In this case the broadening of the beatnote is given by:

\[
(\Delta v)_{\text{FWHM}} = \frac{\pi |D_d|}{T_r \tau_i} \Delta v_0 \tag{41}
\]

where \(D_d\) is the round trip GDD at the centre frequency of the lasing line, \(\tau_i = T_r / m_i\) is the characteristic duration of the initial mode beating fluctuation in the free-running laser (of the order of 10ps) and \(\Delta v_0\) is the longitudinal mode spacing in the absence of dispersion.

In the early stages of self-starting one peak is trying to dominate over the other fluctuations in light intensity in the cavity. Spurious reflections add weak “copies” of the light field back into the resonator increasing the number of fluctuations over which the one peak must dominate and reducing the chance that this will occur. These stray reflections increase the beatnote width to approximately:

\[
(\Delta v)_{\text{FWHM}} = \frac{r_{\text{ext}}}{2\sqrt{\pi}} \frac{1 - r_{oc}^2}{r_{oc}} \Delta v_0 \tag{42}
\]

where \(r_{\text{ext}}\) is the reflectivity of the reflector external to the cavity; and \(r_{oc}\) is the reflectivity of the output coupler of the laser. Measurement of the beatnote in an attempt to quantify the expected self-starting behaviour will be presented later.

Another method for characterising self starting was developed by Chen et al. [56] and considers the stability against small perturbations of the CW solutions of a “master” equation very similar to eqn (31). First the cw solution of (31) is sought in the form:
(43) \[ a = U_c e^{iP_c x} \]

(U_c and P_c are both real and positive) which leads to a pair of equations:

\[
\frac{g_0}{U_c^2} = 1 - \gamma U_c^2 \\
\frac{1 + \frac{P_c}{P_{sat}}}{P_{sat}^2} \gamma U_c^2
\]

(44) \[ P_c = x - \delta U_c^2 \]

\( (l, \gamma, x, \delta \text{ cf. eqn (31)}) \) Next this solution is perturbed, \( a = (U_c + e)^eP_c \), where \( u = a, e^{i\omega t + \theta} \) and the behaviour of these solutions is examined. Working through the algebra leads to the condition that:

\[
\Theta = \frac{\gamma U_c^2 - g_0 \omega^2}{\Omega^2} - \frac{U_c \epsilon_c}{1 + i\omega T_c} \sqrt{\left( U_c \epsilon_c - \gamma U_c^2 \right)^2 - D^2 \omega^2 - 2D \delta U_c^2 \omega^2}
\]

(45)

where:

\[
\epsilon_c = \frac{g_0 U_c}{1 + \frac{U_c^2}{P_{sat}})^2 P_{sat}} \]

\[
T_c = T_0 \frac{U_c^2}{1 + \frac{U_c}{P_{sat}}}
\]

(46)

g_0 is the gain; and \( T_0 \) is the "decorrelation time" of the gain medium.

The cw solution is unstable (and hence self-starting of the modelocking occurs) when \( \text{Re}[\lambda] > 0 \) for some value of \( \omega \). This method has the advantage that, when coupled with the theory of Haus, we get a measure for both self-starting and pulsewidth. However some of the parameters are difficult to determine accurately for a given laser system.

The final method, studied in depth by Herrmann et al. [57], uses statistical properties of the field of the laser assuming that it has a white Gaussian random noise response. The evolution of a pulse inside the laser is broken down into three stages - an initial linear stage (from threshold to when the laser radiation begins to saturate the gain); a nonlinear intermediate stage (in which the effect of the gain saturation becomes important); and a final cw-stage (in which the modelocked pulse train rapidly forms and is stabilised or the laser subsides into CW emission). This leads to the criterion for self-starting:
\[
\sigma_a \frac{\sqrt{\frac{T_R}{T_{1a} P_{TH}}}}{13.2 b \sqrt{p}} \leq \ln \left[ \frac{T_R}{\tau_c} - 3.15 \right] + \frac{1}{2} \ln \left( \frac{T_R}{\tau_c} - 3.15 \right)
\]  \tag{47}

where \( \tau_c = \frac{2}{\Delta \omega} (2 K_0 \ln 2)^{1/2} \) is the correlation time of the modes; where \( \Delta \omega \) is the filter bandwidth in the cavity, or \( \Delta \omega = \Omega/(2 I)^{1/2} \) if the gain limits the bandwidth; \( T_{1a} \) is the response time of the amplifier; \( b \) is the nonlinear loss coefficient in \( I = I_{\text{linear}} + b I \) where \( I \) is the photon number density (consequently \( b \) has units m \(^2\)s and \( b = \gamma \hbar \nu A_{eg} \)); \( \sigma_a \) is the cross-section of the gain medium; \( p_{TH} = T_R/T_{1a} \) is the threshold pump rate; \( K_0 \) is the number of round trips of the laser cavity in the linear stage and is given by:

\[
K_0 = \sqrt{\frac{2}{p} \ln \left( \frac{\langle I(K_0) \rangle}{I_{\text{noise}}} \right)} = \frac{6.5}{\sqrt{p}} \tag{48}
\]

where \( p \) is the pump rate, \( \langle I(K_0) \rangle \) is the mean intensity of the light inside the laser cavity as a function of the round trip number, \( I_{\text{noise}} \) is the depth of modulation due to the noise.

This derivation also enables an optimum round-trip time to be calculated given by:

\[
T_R^o = \frac{1.61 \times 10^3 b^2 p}{\sigma_a^2 T_{1a} P_{TH}} \tag{49}
\]

Herman rearranges these equations to give a condition where self-starting can always be achieved if the nonlinear coefficient, \( b \), is larger than \( b_{ml} \) with:

\[
b_{ml} = \frac{\sigma_a \sqrt{T_R/T_{1a}}}{13.2 \left( \ln \left[ \frac{T_R}{\tau_c} - 3.15 \right] + \frac{1}{2} \ln \left( \frac{T_R}{\tau_c} - 3.15 \right) \right)} \sqrt{\frac{P}{P_{th}}} \tag{50}
\]

From comparison of values and derivation \( p = g \), and from substitutions, (47), (48), (49) and (50) become:
This has the advantage that it is rigorous and involves few approximations about the pulse shape in the cavity (cf. Ippen and Haus [50]) or phenomenological additions (cf. Krausz [53]).

### 3.7 Self Q-Switching

A likely competing process for passive modelocking process is self Q-switching. "Q" is a quality factor defining the ratio of the energy stored in cavity to the loss from that cavity [58]. In a high loss (low Q) cavity it is possible to have a large amount of energy stored in the laser crystal as excited atoms, since there are comparatively few lasing photons to de-excite them through stimulated emission. However, if the circulating photons build up to a point where they are able to cause an increase in the reflectivity of the nonlinear mirror, then this can lead to a "runaway" effect of increasing numbers of photons in the cavity, causing more stimulated emission causing a further increase in the reflectivity of the nonlinear cavity element. Eventually the inversion is reduced to zero and the runaway process stops. The net result is that the laser output periodically builds up to a giant pulse of light with a pulse length of the order of the cavity round trip time, in which all the stored energy of the gain medium is dumped and then dies away. This is undesirable as it competes with, and tends to dominate, the build up towards CW passive modelocking.

Krausz et al. ([54] referencing [59]) give the criterion for the suppression of Q-switching as:

\[
\frac{\sigma_a T_R^l}{13.2 \gamma h v A_{\text{eff}} g} \leq \ln \left[ \frac{T_R^4 \sqrt{g}}{104 \ln 2} \right] - 3.15 + \frac{1}{2} \ln \left[ \frac{T_R^4 \sqrt{g}}{104 \ln 2} \right] - 3.15
\]

\[
K_0 = \frac{6.5}{\sqrt{g}}
\]

\[
T_R^o = \sqrt{\frac{\gamma^2 h^2 v^2 A_{\text{eff}}^2}{\sigma_a l}}
\]

\[
b_m = \frac{13.2 \sqrt{g} \ln \left[ \frac{T_R^4 \sqrt{g}}{104 \ln 2} \right] - 3.15 + \frac{1}{2} \ln \left[ \frac{T_R^4 \sqrt{g}}{104 \ln 2} \right] - 3.15}{\sigma_a T_R^l}
\]
where $P_e$ is the steady state intra-cavity power (averaged over $T_r$) and $T_g$ is the lifetime of the upper lasing level. Mostly this contains a set of material parameters, but Q-switching could be suppressed by small $P_{sat}$ or large $P_e$ (i.e. low cavity loss).

Note that (52) and (40) (from endnote [53]) are always simultaneously satisfied (as $g_0 \tau(0) \ll T_r$) but that (40) (from endnote [53]) is not a sufficient condition for modelocking. For (39) and (52) to be satisfied simultaneously requires that $T_g \gg \tau_c$ (i.e. the gain relaxation time is very much shorter than the mode lifetime/correlation time).

3.8 Important Parameters for a Self-Starting Laser

In this section we will examine some of the important trends which define appropriate parameters to achieve either good pulse stability or reliable self-starting.

3.8.1 Stability

Setting $D_n$ to zero (i.e. no dispersion) gives reduces eqn (38) to:

$$\gamma P_e < \frac{T_r}{T_g} \left( 1 + \frac{P_e}{P_{sat}} \right)$$

(52)

which is $>0$ (i.e. stable pulses) for $\gamma/\delta > 1/3$.

If $\gamma=0$ then eqn (38) becomes:

$$-1 - \frac{9\gamma^2}{2\delta^2} + \frac{3\gamma}{\delta} \sqrt{\frac{9\gamma^2}{4\delta^2} + 2}$$

(53)

which is never $> 0$ for any value of $D_n$. This is obvious - there is no pulse formation
without the nonlinear mirror!

If $\delta=0$ (no self-phase modulation) then eqn (38) becomes:

$$
\begin{align*}
-2 \left( \frac{3}{2D_n} - \sqrt{2 + \frac{9}{4D_n^2}} \right)^2 + D_n \sqrt{8 + \frac{9}{D_n^2}} & \quad D_n > 0 \\
-2 \left( \frac{3}{2D_n} + \sqrt{2 + \frac{9}{4D_n^2}} \right)^2 - D_n \sqrt{8 + \frac{9}{D_n^2}} & \quad D_n \leq 0
\end{align*}
$$

(55)

which is $>0$ for all $D_n$, however the pulsewidth (eqn (33)b) is undefined.

For the general case things are more complicated. Figure 17, shows the stability criteria for $\delta=1 \, 10^{-4}$ (appropriate to a 10cm length of 4$\mu$m core radius fibre at $\lambda=1.3\mu$m which approximates the conditions used in later work). This indicates that stability is possible with positive dispersion, but only for rather large values of $\gamma$. In contrast, the pulses are stable in the anomalous dispersion ($D_n < 0$) for almost all $\gamma$ shown.

**Figure 17** Stability criteria for a range of $\gamma$ and $D_n$ ($\delta=1 \, 10^{-4}$).
In summary, for pulse stability $\gamma > \delta$ is desirable, as is negative group velocity dispersion.

### 3.8.2 Self Starting

#### 3.8.2.1 Krausz Theory

From eqn (39) it is apparent that self starting is more likely with a large $\gamma$, (largest nonlinearity); large circulating power $P_c$, (implying low resonator losses); large mode correlation time $\tau_c$ (implying low spurious reflections, etc); and for a large number of modes oscillating at startup, $m_i$ (wide bandwidth). Reducing the cavity round trip time, $T_R$ will also help. As a result any laser should be built with a short, high finesse resonator (which implies that the coupling loss would have to be small); operate with high gain at high pump powers; and be carefully prepared to minimise the effects of stray reflections, etc.

#### 3.8.2.2 Chen Theory

Trends here are much more difficult to determine, but looking at some simple cases:--

- No dispersion (i.e. $D=0$) gives

\[ \Theta = -\frac{g\omega^2}{\Omega^2} \]  

or

\[ \Theta = 2\gamma U_c^2 - \frac{g\omega^2}{\Omega^2} \left( \frac{2\varepsilon}{1+i\omega T_c} \right) \]  

Equation (56) shows no solution for real $\omega$ for which $\Theta > 0$, however at $\omega = 0$, (57) becomes:

\[ \Theta = 2\gamma U_c^2 - 2U_c\varepsilon_c \]  

(58)
For the case of fibres, \( P_{\text{sat}} \ll 1 \) and \( \varepsilon_c = g_0 \frac{P_{\text{sat}}}{U_c^3} \) which means RHS of eqn (58) >0 for large \( \gamma \) and \( U_c \).

- Setting \( \gamma = 0 \), gives:

\[
\lambda = -\frac{g\omega^2}{\Omega^2} \frac{U_c \varepsilon_c}{1 + i\omega T_c} \pm \left( \frac{U_c \varepsilon_c}{1 + i\omega T_c} \right)^2 - D^2 \omega^4 - 2D \delta U_c^2 \omega^2
\]  

(59)

which has a maximum of \( \Theta = 0 \), at \( \omega = 0 \) ie. no self-starting as would be anticipated with no nonlinearity.

- Setting \( \delta = 0 \), gives:

\[
\Theta = 0 \quad \Theta = 2\gamma U_c^2 - 2U_c \varepsilon_c
\]  

(60)

which, as discussed above allows \( \Theta > 0 \), for large \( U_c \) (requiring low cavity loss) and large \( \gamma \).

### 3.8.2.3 Hermann Theory

Examination of eqn (51)a and (50)d show that self-starting is more likely for small loss \( l \); large \( \gamma \); large gain \( g \); and short cavity round-trip, \( T_R \). These conclusions are in agreement with the trends from the other models.

### 3.8.3 \( \gamma \) Parameter - Optimising the Mode-Locking Driving Force

An important parameter in any calculation of the pulse width or self-starting behaviour is \( \gamma \), the slope of the intensity dependent loss as a function of power. Recalling eqn (22):

\[
|D(z)|^2 = \frac{S_1 + S_0}{2} = \frac{|D(0)|^2}{2} \left\{ 1 + cn \left[ 2a_2z_c \left( \frac{a_2 |D(0)|^2}{4a_3} \right)^2 \right] \right\}
\]  

(61)

and letting \( a_3 = \pi/L_b \) and \( a_2 = \omega_0 n_2/(c A_{\text{eff}}) = \delta/L_f = S \), where \( L_b \) is the beatlength of the fibre; \( L_f \) is the length of the device; and \( A_{\text{eff}} \) is the effective area of the mode in the fibre gives us
a response of the nonlinear directional coupler as:

\[
P_{\text{out}} = \frac{P_{\text{in}}}{2} \left\{ 1 + \text{cn} \left[ \frac{2\pi z}{L_b} \left( \frac{SL_b P_{\text{in}}}{4\pi} \right)^2 \right] \right\} \tag{62}
\]

So that some general results can be explored, the normalised length is defined as: \( z_c = \frac{L_f}{L_b} \),

\( P_c = \frac{P}{P_{\text{crit}}} \)

where:

\[
P_{\text{crit}} = \frac{4\pi}{SL_b} \tag{63}
\]

so that

\[
P_{\text{out}} = \frac{P_{\text{in}}}{2} \left\{ 1 + \text{cn} \left[ \frac{2\pi z}{L_b} \left( \frac{P_{\text{in}}}{P_{\text{crit}}} \right)^2 \right] \right\} \tag{64}
\]

Note that \( P_{\text{crit}} \) is dependent both the waveguide/material dependant parameters (S) and the fibre beatlength (L_b).
Figure 18  Coupling and slope of Coupling Function (γ) as a function of circulating power (W). This particular example is for a coupler 0.2m long, S=2.799 \times 10^{-3} \text{ W}^{-1}.

Now, γ is the slope of this function, evaluated at the circulating power ie.

\[ \gamma = \frac{d}{dP} \left[ \frac{1}{2} + cn \left( \frac{2\pi z}{L_b} \left( \frac{P}{P_{\text{crit}}} \right)^2 \right) \right] \mid_{P=P_{\text{circ}}} \]  

(65)

Figure 18 shows the coupling fraction and the slope of the coupling function (γ) for the specific case of 0.2m fibre, S=2.799 \times 10^{-3} \text{ W}^{-1} (4\mu \text{m core fibre glass fibre at 1342nm - relevant to later fibre experiments}). Note first that the value of γ increases almost linearly with power. This is a somewhat unusual property which suggests that the mode-locking driving force tends to zero at zero power. As a consequence one can anticipate that large average circulating powers will be essential for mode-locking action to self-start. The figure also shows that if the overall length of the coupler is pre-determined (by for example the need to have a particular pulse repetition rate from the laser) it is best to choose the longest possible beatlength because γ is then largest. This trend holds across the full range of beatlengths and coupling fractions.
It is also apparent that $\gamma$ is sensitive to the power circulating in the resonator, which is itself dependent on the cavity loss. Hence, the optimal coupling will be a function of the cavity linear loss and pump power. A first approximation is $P_{\text{circ}} = P_{\text{in}} / l_{\text{cavity}}$, where $l_{\text{cavity}}$ is the total cavity loss, both linear and nonlinear.
In many circumstances it will not be possible to change the beatlength at will, and hence the question arises as to what the optimum device length would be for a fixed beatlength coupler. Since the maximum value of $\gamma$ is, as noted above, sensitive to circulating power and hence cavity losses, figure 19 plots $\gamma$ for a fixed beatlength and varying device lengths with loss as a parameter. The maxima of the curves indicate the best choice of coupling at low power. Figure 19 also indicates that a greater device length gives a larger response. In other words when the beatlength cannot be varied a stronger mode-locking driving force will be obtained by making the coupler several beat lengths long.

For many of the criteria associated with self-starting of the laser, however, it is not $\gamma$ alone but the ratio $\gamma/T_R$ which is important. Figure 20 plots this ratio for the conditions of Figure 19 and demonstrates that the largest value of $\gamma/T_R$ is obtained when the device is slightly longer than 2 beatlengths long for most values of the cavity loss.

Numerically calculated values for the optimal beatlength for a given cavity loss and pump power are given in Figures 21 and 22. These show the wide variation of the optimal position with loss. As a general trend the optimal fibre length increases with increasing loss and increasing pump power, although this is less reliable a rule for the higher
circulating powers.

As shown above, $P_{\text{crit}}$ is dependent on the fibre length, and the slope increases with the number of beatlengths used. The optimal combination of these (assuming that beatlength is experimentally controllable and realisable) is difficult to calculate analytically and in practice is better done numerically on a case by case basis. An emperical rule of thumb is that the optimal fibre length makes the coupling fraction and the linear loss approximately equal.

3.9 Summary

There are a number of very different analytical models of the processes inside the laser cavity. In general for best stability and highest likelihood of self starting we need negative dispersion, low loss (to give high circulating power), small self-phase modulation.
δ, large gain but small $P_{\text{sat}}$ (helped by small pump volume), long mode correlation time (aided by low dispersion and control of reflections back into cavity) and large gain bandwidth. A long beatlength for the NLDC would be advantageous and while $\gamma$ increases with number of beatlengths used, $\gamma/T_R$ (important to some of the models) does not after about 2 beatlengths.

**Figure 22**  Optimal fibre length (normalised to beatlength) for fibre length ($L_b < L_f < 2 L_b$) for given pump power and cavity loss
4 Nd:YVO₄ Laser Experiments

4.1 Introduction

An obvious way, and our original intention, to achieve many of the desirable parameters for a laser mode-locked using a nonlinear directional coupler would have been to build it using twin core Neodymium doped glass optical fibre. Provided the two cores are similar enough, the twin core fibre structure can act as a directional coupler so that light is able to couple backwards and forwards between them. By doping the cores with Neodymium, the structure can also have the high gain necessary for laser action. Neodymium in glass also has a suitably wide gain bandwidth for short pulse generation. A device fabricated entirely of fibre should be capable of having low resonator loss and a short device length, whilst simultaneously permitting strong beam confinement in both the gain medium (giving a small $P_{\text{sat}}$) and the nonlinear element (giving a large $\gamma$ for a given power).

Unfortunately, several attempts made by Graham Atkins at the Optical Fibre Technology Centre (OFTC) at the University of Sydney to produce a suitable Nd-doped twin core fibre failed, and there was no detectable coupling at any of the wavelengths in any of the samples which were supplied. Measurements of other physical parameters (eg. Core diameter and ellipticity) \[60\] showed a strong variation with length. For example, the core diameter showed up to 1µm variation (in a 7µm diameter core) over several metres. This indicated that there was a large non-uniformity introduced during the drawing process. This variation in parameters means that the cores were not similar enough for coupling to occur.

The inhomogeneities were thought to be due to the use of aluminium co-doping in the fibre manufacture. The co-doping is used to achieve the high neodymium concentration necessary to obtain high gain over short fibre lengths. In spite of several attempts to overcome the problem, no fibre with significant coupling was obtainable and this approach had to be abandoned. Nevertheless some interesting lasing data was obtained using some of
these fibres and details of these experiments are included in Appendix B.

A number of undoped twin core fibre samples where coupling was present were supplied by the OFTC, at the University of Sydney. Although the quality of this fibre was high, it was designed for an application in coarse wavelength division demultiplexing (between 1.32μm/1.55μm) and, therefore, had a rather short beatlength. As outlined in the previous chapter, the fibre beatlength is an important parameter because it affects both the cavity loss and the switching power. The lack of doping in this fibre meant that an external gain medium would have to be used and it would be, therefore, necessary to couple light from the fibre to the gain medium and back again. In general this would be expected to increase the minimum resonator losses compared to the case of a doped fibre.

In this chapter measurements to fully characterise the linear and nonlinear properties of this fibre are described including the power dependence of the switching behaviour, and the tunability of the coupling. The choice and characteristics of the gain medium used with this fibre are described along with measurements of the important lasing parameters. In spite of the fact that the laser that was constructed fulfilled several of the criteria presented earlier for self-starting mode-locking, no pulsations were observed.

4.2 Twin Core Fibre Experiments

4.2.1 Fibre Fabrication and Physical Properties

The fibre used for the experiments was fabricated at the Optical Fibre Technology Centre (OFTC), University of Sydney by Dr. Graham Atkins. Fabrication involved making a conventional preform which was then cut in half and side polished into two "D-shaped" sections (the polishing was done at CSIRO Division of Applied Physics, Lindfield NSW). These sections were then mounted, polished sides together in a specially constructed jig, 320mm long [60]. Approximately 700m of fibre was then drawn at about 1950°C at high tension. The fibre, designated TCF#2C, had a core radius of 4.06±0.03μm (although this is hard to determine precisely as the cores had a graded index); core separation of 18.6±0.5μm and a cutoff wavelength of 910±10nm for the second mode (ie. at wavelengths longer than 910nm the fibre became single moded).
4.2.2 Fibre Absorption

The absorption measured as a function of wavelength for the preform glass can be expected to be the same as that of the resulting fibre. For the TCF #2C fibre used, the preform absorption at wavelengths of interest is shown in Figure 23 giving values around 590±10 dB/km at 1342nm (0.06%/cm). A closer analysis of the data indicates that there are two broad absorption peaks, one at 1244±5nm, the other at 1385±5nm representing overtones of the main OH-absorption. The losses at the other wavelengths are due to the broad wings associated with these peaks. In comparison, standard communications fibre has an absorption at 1340nm of ~0.5dB/cm (0.0001%/cm). The high absorption values point to a mild contamination somewhere in the fabrication process. Nevertheless, fibre absorption losses were sufficiently low to contribute negligibly to the total resonator losses within the laser.

Figure 23  Absorption vs wavelength for twin core fibre (TCF#2C). Fitted curve is Lorentzians with peaks at 1244 and 1385nm.
4.2.3 Fibre Beatlength

There are a number of experimental techniques that have been used to measure the fibre coupling beatlength experimentally, the major ones being the cut-back method; elasto-optic probing [61]; and wavelength scanning [62].

The cut back method, as the name implies, involves measuring the coupling for a known length of fibre; reducing the fibre length by cutting some off, and re-measuring the coupling fraction to establish a trend. However, because the coupling is very sensitive to small differences in the relative properties of the cores, any change due to bending or stress occurring during the cut-back process can change the coupling. This makes cut-back prone to experimental error and rather unreliable. Furthermore, if there is any chance that the beatlength changes along the fibre length the method may never converge. Cutback is also destructive and hence prevents the piece of fibre that has been characterised from being used in the laser.

The Elasto-Optic [61] method is based on stressing a short length of fibre and examining the change in the output as a function of position where the stress is applied. Light is launched into a length of fibre and the output of one of the cores is a recorded as a function of the position where the stress is applied. If the light is almost entirely in one of the waveguides then there will be little modulation of the output due to the applied stress.

![Figure 24 Plot of $W = k (\beta_o^2 - n_{cl}^2)^{1/2}$ for planar waveguide $n_{co} = 1.45459$, $n_{cl} = 1.44963$, $r = 4\mu m$ showing increase of mode size with wavelength.](image)
whereas at a place where the power is about equal in each waveguide the modulation will have a large amplitude. Plotting the depth of modulation with length enables the beatlength of the fibre to be determined. As with the cut-back method, Elasto-Optic method requires the fibre environment to remain constant throughout the measurement. In practice this means that only short lengths of fibre can be used (5-10cm), whilst for high accuracy the this length of fibre should contain several beatlengths. Our fibre had an anticipated beatlength of 4-4.5cm at 1340nm which was rather long for application of this technique.

As a result we adapted another procedure [62] for measuring beatlength - namely to determine the variation of coupling as a function of wavelength.

Equations (188) and (193) do not readily show how the coupling between the cores varies with wavelength. However, eqn (236) shows that the W parameter decreases with increasing wavelength. (This is shown in Figure 24, for the case of a planar waveguide, TE modes where $n_{co}=1.45459$, $n_{cl}=1.44963$, $r=4\mu m$ but is a generally similar for all planar and fibre configurations). A decrease in W indicates that the mode decays outside the fibre more slowly with distance, and hence the mode volume increases with increasing
As a result the coupling between the cores increases (as a greater fraction of one mode "sees" the other) and the coupling length decreases. This variation is plotted in Figure 25 and indicates that for given length of fibre the coupling fraction between the cores also changes with wavelength. The calculated coupling fraction for the case described above and a fibre length of 10cm is shown in Figure 26.

Also not apparent is that while the modes of the individual guides that make up the coupler are not cut off, the coupled modes of the combined structure can be. The lowest order super-modes of the directional coupler comprise a symmetric and anti-symmetric mode. In a similar way to that which occurs for a single waveguide, the symmetric mode propagates for all wavelengths whereas the anti-symmetric mode cannot propagate for wavelengths beyond a specific cut-off wavelength. At wavelengths longer than cut-off, any launched light redistributes itself to 25% in each core and the structure has a large loss of energy (the remaining 50%) into radiation. The cut-off wavelength is given by [63]

\[
\lambda = \frac{2\pi a}{\sqrt{n_{eoo}^2 - n_{cl}^2}} \frac{1}{V_c}
\]
for identical cores, where a is the fibre radius; \( n_{co} \) is the core refractive index; \( n_{cl} \) is the refractive index for the cladding and \( V_c \) is a numerical factor which is dependent on the refractive index profile of the fibre. If the refractive index profile can be approximated by a power law dependence of refractive index on fibre radius, ie.

\[
n^2(r) = n_{co}^2 \left( 1 - 2\Delta \left( \frac{r}{a} \right)^q \right)
\]  

(67)

where \( 2\Delta = 1 - n_{cl}^2/n_{co}^2 \); and \( q \) ranges from 1 (triangular profile refractive index) to \( \infty \) (step index profile), then \( V_c \) is well approximated by

\[
V_c = \sqrt{\frac{2(q+2)}{qa(q) + \ln \left( \frac{2s}{a} \right)}}
\]  

(68)

where \( s \) is the fibre core centre separation and \( a(1)=181/420 \), \( a(2)=73/168 \), \( a(3)=13/32 \) and \( a(\infty)=1/4 \) (equivalent step index profile). Hence, for our fibre, the cutoff wavelength is in the range 2.01\( \mu \text{m} \) (triangular profile) to 3.36\( \mu \text{m} \) (step index).

Hence for any given coupler there will be a long wavelength limit beyond which coupling is no longer observed. It is worth noting that a short wavelength limit also exists where the directional coupler can support a larger number of supermodes. When these higher order modes are excited the coupling behaviour becomes more complex and less predictable using simple analytic expressions. Hence this multi-mode region is not useful for determining the fibre beatlength.

Measurement of the wavelength response in principle allows the beatlength of the fibre for a given wavelength to be accurately determined. The measurement procedure, therefore, involved the determination of the coupling fraction as a function of wavelength for a known length of fibre. The peaks and troughs in the spectrum indicated that the fibre length was an (unknown) integral number of half beatlengths.
In principle the beatlength as a function of wavelength could be uniquely determined from a measurement of a single length of fibre. However, as noted above there are both long and short wavelength limits which restrict the measurement range which limits the accuracy of the beatlengths determined from a single measurement. The

![Graph](image)

**Figure 27** OSA measurement of coupling of a white light source. Fibre length is $107\pm0.5\text{cm}$

wavelength dependence is also a function sensitive to small variations in the refractive index profile. In practice a more accurate measurement was obtained by comparing the results from several measurements using different fibre lengths. The beatlength was then determined by curve fitting to the data.

The measurements were performed using either an Optical Spectrum Analyser (OSA) and a white light source (Figure 27) or with the tunable output from an Optical Parametric Oscillator (OPO) (Figure 28).
Figure 27 shows the intensity as a function of wavelength of the light in one core of a 107±0.5cm length of the twin core optical fibre as measured by the OSA. A quartz tungsten halogen (100W Xenophot) white light source was coupled into a length of single mode communications fibre which has been fusion spliced to a single core of the twin core fibre. A second fusion splice was made from the other end of that core to second length of single core fibre which was then connected to the input of the OSA. The results show a number of interesting features of the coupling behaviour of the fibre:

- At the wavelengths shorter than 1150nm there is some ripple in the light intensity in the core but there is no coupling (the calculated beatlength for this region does not change monotonically, nor does it fit the trend readily apparent for longer wavelengths).
- Coupling is apparent from 1150nm to longer wavelengths, although the exact
behaviour in the region 1360-1420nm is obscured by the large peak due to the overtone of the O-H stretching vibration, most likely due to some trace water contamination of the fibre.

Figure 28 shows the coupling fraction in a 14.85±0.05cm length of fibre as a function of wavelength using the output of an OPO. The OPO was a singly resonant (i.e. resonating the signal) oscillator that used a 10mm long AR coated LBO crystal, synchronously pumped by frequency doubled, compressed pulses from a cw, mode-locked Nd:YLF laser (Coherent Antares). The OPO produced wavelength tunable pulses in the range of 800-900nm (signal) and 1200-1600nm (idler) by temperature tuning of the LBO crystal (using the type 1, non-critically phase matched interaction). Pulses were 2ps in duration and up to 200W peak power. These data obtained using the OPO show that there is good coupling (ie.≈95%) over at least the range 1340-1600nm.

Figure 29 shows the results of a number of measurements of the beatlength with different sample lengths. This shows the good repeatability of the measurement and hence that the uniformity between different fibre samples was good. By curve fitting to the data it was possible to show that the beatlength of the fibre varied according to the relationship:
\[ L_b = 368.188e^{-0.003314\lambda} \]  

(69)

where \( L_0 \) is in cm and \( \lambda \) in nm. This formula gives an error in the beatlength of less than 0.05cm and while different from the curve fit in Figure 28, there is actually less than 15% difference in their predictions in the region 1300-1600nm. Hence, the beatlength was 4.40±0.05cm at 1340nm and 2.15±0.05cm at 1550nm.

Empirically, for coupling to be possible, the two waveguides must remain "similar" for at least as long as the beatlength. The longer the beatlength, or alternatively the weaker the coupling of the waveguides, the more sensitive the structure is to perturbations. For this fibre, it would appear that when the coupling length exceeds about 6.5cm (at the short wavelength end of the measurements) the inherent fluctuations in the core prohibit coupling. There is no coupling in the fibre for wavelengths shorter than 1150nm, and hence no coupling at 1064nm. This indicated than any laser experiments could not be performed using the usual 1064nm transition in Neodymium.

4.2.4 Power Dependence of Coupling

As has already been discussed, coupling in a directional coupler occurs only when the cores are almost identical. The whole purpose of the NLDC in the application described in this thesis is to exploit the sensitivity of the coupling to a mismatch induced, via the Kerr effect, by a high power pulse propagating in the one of the cores. The mismatch turns off the coupling thereby creating the power dependent modulation necessary for achieve passive modelocking.

A number of experiments were performed to confirm that the switching did, in fact, occur at high power in the twin core fibre used in our experiments, and to try to determine if this switching power accurately corresponded to the value of \( P_{\text{crit}} \) predicted by eqn (63). These experiments were performed at 1550nm (restricted by the availability of an appropriate source) for which \( L_0 = 2.15\pm0.05\)cm (see eqn (69)) and \( P_{\text{crit}} = 210kW \).
Figure 30  Experimental arrangement for measuring switching with power at 1550nm

The experimental rig used for these experiments is shown in Figure 30. 60 ps duration pulses from a Coherent Antares Nd:YLF laser operating at 1053.5nm were sent through approximately 30m of single mode optical fibre to provide a broad bandwidth linearly chirped output pulse via SPM, i.e. different parts of the pulse in time had slightly different frequencies. This output pulse was then injected into a Nd:YLF regenerative amplifier. During amplification gain narrowing occurred which meant only a narrow range of the central frequencies in the pulse were amplified. Because of the chirp present on the pulse this was equivalent to amplifying only a short time window in the pulse. This procedure led to narrowing of the pulse from 60ps down to 7ps. Total output energies of up to 2mJ were obtained at 20Hz.

The shortened pulses were then incident on a beam splitter with 95% reflectivity. The transmitted ~5% was focussed into an Optical Parametric Generator formed of a z-cut KTP crystal cut to have gain at ~1.55µm. The OPG created a relatively broad angular emission due to the presence of strong off axis gain. This output beam was collimated and used as the seed for a second KTP crystal which is pumped by the reflected 95% of the 1055nm radiation and forms an OPA with a gain of several million. Filters and mirrors were placed downstream of the OPG/OPA to remove unwanted pump and frequency doubled pump leaving pulses of 1548±1nm. The amplification process is also somewhat
non-linear and the 1550nm pulses are expected to be somewhat shorter than the 7ps incident pulses. The system was able to produce good TEM$_{00}$ mode (important to an efficient fibre launch) pulses with energies up to 400µJ.

Figure 31  Compare experimental results with calculations for switching ($L_S=0.0218$m, $L_F=0.408$m, $\lambda=1.55\times10^{-6}$m, core radius=$5.0\times10^{-6}$m Pulse length=14.02ps 10.07% Jitter)

Figure 31 shows the response of the fibre coupling vs the input energy of the pulse for a 40.8±0.5cm length of twin core fibre (~18.7 beatlengths at this wavelength). It shows the experimental results and a curve fitted to the observed response. The graph also shows the CW response: i.e. the response to a continuous beam with the same peak power as the 7ps pulses, displaying the familiar coupler behaviour. However, this CW response has to be averaged over the pulse envelope to be comparable with the experimental data as indicated by the second trace. Additionally, empirical observations of the behaviour of the OPG show that the pulse energy jitters by about 10% from pulse to pulse and this introduces some scatter which smears any modulation at higher energies (the third trace).

The figure shows that there is reasonable agreement between the experimental results and the predicted behaviour. During the experiments it was found that the fibre
would work well for a period of time and then suddenly the transmitted power would drop, apparently due to a large power pulse (from the statistical jitter) causing damage to the fibre end.

**Figure 32**  Input and Output Energy from twin core fibre samples showing a maximum transmission of ~50%

The maximum fibre transmission (including losses due to launching and collimating lenses) was ~50%. As can be seen from **Figure 32**, the output tended to saturate at under 2.5μJ beyond which increasing the power at the input did not result in any increase in the output.

It is possible that this power is being lost in a number of ways. The most likely causes are Stimulated Raman Scattering (SRS) and Stimulated Brillouin Scattering (SBS). In Stimulated Raman Scattering a photon of the incident field exchanges energy with a vibrational energy level of the material such that a scattered photon is created at a frequency either up (anti-Stokes radiation) or down shifted (Stokes radiation) from the original photon. Stimulated Brillouin Scattering is similar, except that it involves
transitions between the phonon energy states. In either case light is removed from the forward travelling beam to a scattered beam, which can be travelling in an arbitrary direction in the fibre.

For both SBS and SBS the growth of the scattered light is given by [64]

$$\frac{dI_s}{dz} = -G_i I_p I_s$$

$$\frac{dI_p}{dz} = -G_i I_p I_s$$

(70)

where $I_s$ is the intensity of the scattered light; $I_p$ is the intensity of the original light (here designated pump); and $G_i$ is a gain coefficient which depends on the scattering process and the frequency. The process builds up from quantum "noise" at the scattered frequency, is strongly pump power dependent and has a threshold. SRS can occur in either the forwards or backwards directions, and the scattered intensity, $I_s$, can have a frequency shift from that of the pump of as much as 30THz in fused silica, and $G_i = G_r \sim 1 \times 10^{-13}$ m/W. In contrast the Stimulated Brillouin spectrum is extremely narrow with a bandwidth of $\sim$10MHz shifted by $\sim$10GHz and a peak gain $G_i = G_b \sim 6 \times 10^{11}$ m/W for a narrow bandwidth source. SBS always occurs in the backwards direction.

From eqn (70) a threshold criteria can be readily derived given by:

$$\frac{G_i P_{th} L_{th}}{A_{eff}} = N_i$$

(71)

where $A_{eff}$ is the effective area of the fibre mode; $P_{th}$ is the critical power necessary for the onset of the process in a fibre length $L_{th}$; $G_i$ is the gain coefficient appropriate to either Raman or Brillouin Scattering; and $N_i = 16$ for forward Stimulated Raman Scattering; $N_i = 20$ for backward Stimulated Raman Scattering; and $N_i = 21$ for Stimulated Brillouin Scattering. For fused silica, with a 4µm core radius fibre at 1.55µm, this leads to the power length products, $P_{th} L_{th} = 21$ W m for Brillouin Scattering, $P_{th} L_{th} = 8043$ W m for forward Raman Scattering and $P_{th} L_{th} = 10053$ W m for backward Raman Scattering.

A 1µJ pulse in 7 ps leads to a local power of $\sim$143,000W so that it can be seen that for the fibre length used (10-50cm) the experimental powers are well above the relevant thresholds and this could readily explain the power limiting which was observed. It is worth noting that since, in the operating NLDC laser, the lower power, "seed" components have more loss than the main pulse. This means that the stimulated scattering processes
are suppressed at their low power stage and should not be a problem.

To better understand which of these processes were occurring and some of the dynamics inside the fibre, the spectra of the fibre output was taken. In Figure 33 the output spectra of one of the fibre cores is shown as a function of the launch energy. It shows a change in the dominant peak with increasing launch power. The shift of this peak is consistent with SBS.

![Figure 33](image)

**Figure 33** Output spectra from twin core fibre is dependent on launch power

The 0μJ reading came from placing many ND filters in the beam. The other powers were obtained by placing a Glan-Taylor polariser downstream from a Soleil-Babinet compensator. By changing the compensator the polarisation of the input beam was changed from linear to elliptical and a fraction, depending on the angle of the ellipse in the polarisation plane, was rejected. This allowed continuous tuning of the input power without the chance of changing the beam alignment.

In **Figure 34** the FWHM of the spectra are shown as a function of launched power and demonstrate a roughly linear dependence. This is not expected for a spectrum produced purely by SBS and hence the spectrum may also have been affected by Self Phase Modulation.
Figure 34 Width (nm) of output of different cores of 36.6±0.1cm length of twin core fibre with input energy. Shows roughly linear increase in width with increasing pulse energy.

The length scale over which self-phase modulation is likely to be significant is dependent on a number of parameters for a given fibre. These effects are combined in a "nonlinear length", $L_{NL} = (\Delta P_0)^{-1}$ where $\Lambda = n_2 k / A_{eff}$ (where $A_{eff}$ is the effective area of the mode in the fibre)[65]. For our system $\Lambda \sim 1.55 \times 10^{-3}$ W$^{-1}$ m$^{-1}$ and if it is assumed that we get 50% launch of the 7ps pulses this gives $L_{NL} = 18$mm/$P_0$(μJ). So the fibres under consideration which were sufficiently long (30-40cm) for SPM to be of potential importance.

Kean et al. [66] showed that if the effects of group-velocity dispersion in the fibre and any frequency chirp on the input pulses are ignored then the spectral width $\Delta \lambda$ of the SPM pulse is given by:

$$\Delta \lambda = \Delta \lambda_{r} + A \sqrt{\frac{2 \ln 2}{e} \frac{\lambda n_2 L_f P}{c A_{eff} t_p}}$$  \hspace{1cm} (72)
where $\Delta \lambda$ is the bandwidth of the input pulse; $P$ is the peak power of the pulse; and $t_p$ is the pulsewidth (FWHM) in time. From this it is seen that the spectral width grows linearly with pulse peak power, and hence with energy.

The combined affect of SPM and SBS is difficult to predict, but would be consistent with the changing shape and position of the output spectra (SBS) with a broadening provided by SPM.

While SPM is under consideration, then, from [50] the Haus parameter of eqn (31) is

$$\delta = \frac{\omega_0 n_2 L_f}{c A_{\text{eff}}}$$

where $L_f$ is the fibre length, $n_2$ is the Kerr effect nonlinearity and $A_{\text{eff}}$ is the effective mode area, (usually a good approximation is $\pi r^2$ where $r$ is the nominal fibre core radius). For the TCF #2C fibre used $r=4\mu\text{m}$, $n_2 = 3 \times 10^{-20} \text{ m}^2/\text{W}$, giving $\delta(1064\text{nm}) = 3.524 \times 10^3 L_f \text{ W}^{-1}\text{m}^{-1}$ and $\delta(1342\text{nm}) = 2.794 \times 10^3 L_f \text{ W}^{-1}\text{m}^{-1}$.

### 4.2.5 Tuning the Coupling

Both self-starting of the passive modelocking of the laser and the long term stability of the pulses are extremely sensitive to the initial coupling fraction in the NLDC. As was demonstrated in Section 3.8.3, this optimal coupling fraction is dependant on the cavity circulating power and hence the cavity loss. The cavity loss includes the losses of all cavity components only one of which is the twin core fibre. In our laser experiments, we chose to use a cut-back method to adjust the initial coupled fraction. This therefore involved cutting the fibre to a length slightly longer than the expected optimum, measuring the coupling, and then polishing back the fibre length until it was optimal. This method also allowed us to encapsulate the length of twin core fibre inside a glass capillary tube to stabilize it against external environmental factors.

Several experiments were performed on a different method of tuning the coupling, in this case by applying stress to the fibre by bending. Details of these experiments are in Appendix E. Our results show that tuning of the coupling inside the laser cavity would have been possible by this method. However, the method leaves the fibre considerably less
protected against the environment than is possible with the cut-back method and there are problems with long term stability and reproducibility of the coupling which meant that the method was not used for these experiments.

4.2.6 Dispersion

The dispersion parameter, D, is given by

\[ D = \frac{\partial \omega}{\partial \lambda} L_f = \frac{\lambda}{c} \frac{d^2 n}{d\lambda^2} L_f \]  

[67]. Using

\[ n^2(\lambda) = 1 + \sum_{i=1}^{3} \frac{c_i \lambda^2}{\lambda^2 - \nu_i} \]

where \( c_1 = 0.6961663 \), \( c_2 = 0.4079426 \), \( c_3 = 0.8974794 \), \( \nu_1 = 0.004679148 \), \( \nu_2 = 0.01351206 \), \( \nu_3 = 97.934002 \) [68] gives \( D(1064\text{nm}) = 8.237 \times 10^{-27} \text{ L f \ s^2 m^{-1}} \) and \( D(1342\text{nm}) = -3.085 \times 10^{-27} \text{ L f s^2 m^{-1}} \). Note that this gives "normal" dispersion at 1064nm while negative dispersion at 1342nm. The zero dispersion point occurs at \(~1270\text{nm}\) (according to the above numbers, but it is very sensitive) and so the D parameter is likely to be quite small for our fibre.

4.2.7 Fibre Summary

The fibre has no detectable coupling for wavelengths shorter than 1150nm. At longer wavelengths the beatlength as a function of wavelength can be approximated by

\[ L_b = 368.188 e^{-0.003314\lambda} \]

where \( L_b \) is the fibre beatlength in cm and \( \lambda \) is wavelength in nm. The consistency and reproducibility of the fibre measurements show that the fibre has good uniformity along its length. The fibre does show switching behaviour at powers consistent with the theory. At the high powers necessary to take some of the switching measurements a number of stimulated scattering effects appeared to occur limiting the fibre transmission. These
effects are not expected to be a significant in the modelocked laser where such high power are not encountered. Fibre coupling was tuned to the optimal length for the cavity by a cut-back method, but could possibly have been be tuned by appropriate bending.

4.3 Towards a Passively Modelocked Twin Core Fibre Laser

<table>
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<tr>
<th>Material</th>
<th>$\sigma$ (10$^{-21}$ cm$^2$)</th>
<th>$\tau$ Fluorescence Lifetime (µs)</th>
<th>Bandwidth</th>
<th>Lasing Wavelength (µm)</th>
<th>Pump Wavelength (nm)</th>
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<td>1nm</td>
<td>1.3187</td>
<td>808</td>
<td>[73]</td>
</tr>
</tbody>
</table>

Table 2: Comparison of laser hosts for $\sim$1.3µm ((F) indicates a fibre gain media, otherwise bulk crystal)

The fibre was now well characterised and the results showed that the only practical wavelengths that it could be used as a NLDC lay between 1150 and 1600nm, hence it was necessary to identify a gain medium for the subsequent laser experiments within this range. The longer the wavelength used, the shorter the beatlength and the greater the circulating power required to get the laser to mode-lock, hence shorter wavelengths in this range were favoured. Unfortunately no practical fibre based gain medium was available (Erbium doped fibre being impractical both because of its long wavelength and its low gain coefficient which would have meant the fibre would have needed to be impracticably long). Hence the use of a bulk gain medium was contemplated. For some applications, operation in or close to the second telecommunications window (around 1300nm) would be useful. Since this corresponds to a well-known transition in Nd, the possibility of using a Nd doped materials was investigated.
4.3.1 Gain Media for 1.3\(\mu\)m

A number of gain materials have been developed for the second telecommunications window \([69][70][71][72][73][74]\) and their properties are summarised in Table 2. Some important differences between these laser hosts are readily apparent - the glass hosts have broad bandwidths, but small stimulated emission cross-sections \((\sigma)\), while the crystal hosts have large \(\sigma\), but narrow bandwidths. Having both a large bandwidth (giving a large variation in intensity in the cavity at start up and also permitting shorter pulses) and a large cross-section (to enable a large gain) would be ideal.

![Schematic Diagram of All Fibre Laser](image)

**Figure 35** Schematic diagram of all fibre laser, incorporating a non-linear directional coupler.

A schematic of a possible laser where the gain medium is a doped fibre spliced to the characterised undoped twin core is shown in Figure 35. However, fibre lasers in the 1300nm (second telecommunications) window are difficult to handle as the glass is hygroscopic, brittle and difficult to splice to the standard silica glass fibre from which the NLDC was fabricated. Hence we choose instead to use a bulk crystal as the gain medium. The disadvantage of the approach is that the coupling the light between the fibre and gain medium is likely to introduce significant cavity losses, which as shown in Chapter 5, have a very significant limiting effect on the self-starting mode-locking of the laser. However, our experiments on Nd:doped fibre lasers had already shown that these losses could be acceptably low (a few %) and hence this was not expected to be a major problem. A schematic of the cavity including the necessary bulk optical components for coupling between the fibre and gain medium is shown in Figure 36. From the table, Neodymium doped Yttrium Orthovanadate (Nd:YVO\(_4\)) was chosen as the gain medium. It has the largest gain near 1342nm and strong pump absorption meaning that the crystal can be short (a few mm long) helping to minimise the resonator length. Nd:YVO\(_4\) is also easy to handle.
(ie. it is neither hydroscopic nor strongly temperature sensitive) and is ready available.

![Schematic diagram of laser with bulk optics, incorporating a non-linear directional coupler.](image)

**Figure 36** Schematic diagram of laser with bulk optics, incorporating a non-linear directional coupler.

### 4.4 Neodymium Orthovanadate (Nd:YVO₄)

#### 4.4.1 History

Neodymium doped orthovanadate was investigated by O'Connor [75] in 1966 who reported a pulsed laser threshold of ~1J for lasing at 1064nm. Lasing at 1.34μm was achieved by Tucker et al. [76] in 1976. Although the material was seen as promising at the time, difficulties in crystal growth [77] (eg. vanadium loss, vanadium reduction and oxygen defects [78]) prevented this work from progressing further.

Recent advances in the Czochralski method including O₂ annealing and growth with excess vanadium has largely removed these difficulties and the large gain cross-section, highly polarised emission, good mechanical strength and strong absorption at 808nm have made Nd:YVO₄ ideal for a broad range of diode-pumped laser applications with laser devices with CW outputs up several watts commercially available.
4.4.2 Spectroscopy

The energy level diagram for Nd:YVO₄ is shown in Figure 37. The main pump transition used is the ground state \(^{4}I_{9/2}\) to the pump bands \(^{4}F_{5/2}\) which decay to the upper metastable level \(^{4}F_{3/2}\). For Nd:YVO₄ this state has a fluorescence lifetime of 98\(\mu\)s [79], which is shortest of any of the practical Nd hosts, and radiative lifetime 115\(\mu\)s [80]. Lasing is possible between the \(^{4}F_{3/2}\) and a number of levels, the major ones of interest \(^{4}F_{3/2} \rightarrow ^{4}I_{11/2}\) which emits at 1064.1nm and \(^{4}F_{3/2} \rightarrow ^{4}I_{13/2}\) which emits at 1342.5nm [81] and having stimulated emission cross-sections of 15.5 x 10\(^{-19}\) cm\(^2\) and 7.61 x 10\(^{-19}\) cm\(^2\) respectively for the a cut crystal [80]. The relaxation time for the termination level to the ground state has
not been reported for Nd:YVO₄ but would be expected to be similar to the value of 30ns known for Nd:YAG [82]. The finite time to depopulate the lower level of the lasing transition has the effect lowering the saturation intensity of the gain for short pulses because dynamically the system changes from a four level to three level laser for times shorter than this relaxation time.

Orthovanadate is strongly birefringent (n₀=1.958, nₑ=2.168 at 1064nm) and a number of its other spectroscopic properties are polarisation dependent. The stimulated emission cross-section, for example is 15.5 $10^{-19}$ cm² for an a-cut crystal, but only ~7 $10^{-19}$ cm² for the c-cut [83]. Similarly, absorption is polarisation dependent with the absorption coefficient at 808nm increasing from 10.5cm⁻¹ to 40.7cm⁻¹ by going from $\sigma$ to $\pi$ pump light polarisation. Hence, it is important to have the crystal a-cut and pump at the $\pi$-polarisation for optimal laser performance with Nd:YVO₄.

<table>
<thead>
<tr>
<th>Nd:YVO₄ Properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Crystal Structure</td>
<td>Tetragonal</td>
</tr>
<tr>
<td>Crystal Type</td>
<td>Uniaxial</td>
</tr>
<tr>
<td>Density</td>
<td>4.24g/cm³</td>
</tr>
<tr>
<td>Melting Point</td>
<td>1810°C</td>
</tr>
<tr>
<td>Hardness (Moh)</td>
<td>4.5±0.5</td>
</tr>
<tr>
<td>n₀, nₑ</td>
<td>1.958, 2.168 (1064.1nm) 2.009, 2.2405 (550nm)</td>
</tr>
<tr>
<td>dn₀/dT = +8.5 x 10⁻⁶ /°C</td>
<td></td>
</tr>
<tr>
<td>dnₑ/dT =+3.9 x 10⁻⁶ /°C</td>
<td></td>
</tr>
<tr>
<td>Stimulated Emission cross-section (a cut)</td>
<td>15.5 x 10⁻¹⁹ cm² 1064.1nm 7.61 x 10⁻¹⁹ cm² 1342 nm</td>
</tr>
<tr>
<td>Radiative Lifetime</td>
<td>115μs</td>
</tr>
<tr>
<td>Fluorescence Lifetime</td>
<td>100μs $^4F_{3/2}$ (@1 At. wt. %)</td>
</tr>
<tr>
<td>Absorption peak $\lambda$=808nm (1 at. % Nd)</td>
<td>$\sigma$-pol $\alpha$=10.5cm⁻¹ $\pi$-pol $\alpha$=40.7cm⁻¹</td>
</tr>
<tr>
<td>Damage Threshold</td>
<td>&gt;3 GW/cm²</td>
</tr>
</tbody>
</table>

Table 3    Nd:YVO₄ Properties
4.4.3 Physical Properties and Crystal Configuration

The main crystal properties for Nd:YVO₄ are summarised in Table 3. The crystal used for all these experiments was supplied by Spectra Physics Australia (from sources in China), and had a dopant level of 1 atomic weight % Nd, was “a” cut, 6mm diameter, cylindrical, 3mm long, with a 5° wedge to prevent crystal forming an etalon in the cavity.

For later work one crystal face was coated for HR (>99%) at 1340nm and HT (>80%) for the pump at 808nm. Effort was also made to keep the reflectivity at 1064nm low (<20%) to prevent lasing in this unwanted, but higher gain, wavelength.

4.4.4 Absorption

Figure 38 shows the absorption spectra of Nd:YVO₄ using a Coherent 899 ring Ti:Sapphire laser. This confirms the information supplied in datasheets and indicates a strongly peaked, polarisation dependent absorption. The maximum absorption is 25cm⁻¹ for pump polarised || c axis and about 17cm⁻¹ for the orthogonal polarisation. The FWHM

![Figure 38](Nd:YVO₄ absorption showing different absorptions at orthogonal polarisations)
of the absorption peak is difficult to determine due to the presence of the other peaks in the wings, but the main peak has a FWHM of 3.1±0.1nm (although the presence of the other peaks makes this 5.8±0.1nm). The orthogonal polarisation gives 1.4±0.1 nm FWHM of peak.

4.4.5 Fluorescence

Fluorescence measurements were made with a Coherent 899 model ring Ti:Sapphire laser and the results are shown in Figures 39 and 41. A 1m MacPherson spectrometer, slits in the range 150-400μm and a 300lines/mm grating were used, giving a resolution of 0.2-0.6nm. The emission at 1μm is best characterised by a Gaussian λ=1063.9±0.2nm and with FWHM 0.5 ±0.2nm. The 1300nm emission is characterised by two close Lorentzian peaks at 1340.8±0.5nm FWHM 2.1±0.2nm and a much smaller peak (~36% of main) at 1344.5±0.5nm FWHM 1.1±0.2nm.

![Figure 39](Nd:YVO4 fluorescence at ~1064nm.)
4.4.6 Gain and Linear Cavity Loss

Figure 40 Nd:YVO₄ fluorescence at ~1340nm.

The small signal gain, $g$, and the minimum intracavity loss, $l$, are important parameters in the laser models describing self-starting modelocking. We, therefore, performed measurements to determine these parameters in various laser cavities. The technique adopted was to introduce a variable loss into the cavity and measure the resultant threshold of the laser as a function of this loss. The residual cavity loss can be obtained from the resulting data.

We used a microscope cover slip tilted at various angles to the resonator axis to introduce a variable loss. The cover slip itself was quite thin (about 100μm) and acted as a Fabry-Perot etalon and thus it had an intensity transmission [84] given by

$$I_t = \frac{1}{I_i}\frac{1}{1 + F \sin^2 \left( \frac{\delta}{2} \right)}$$

$$F = \left( \frac{2r}{1-r^2} \right)^2 \quad \delta = 2nd\cos[\theta_i] \quad \theta_i = \sin^{-1} \left( \frac{\sin[\theta_i]}{n_f} \right)$$

(77)
where \( r \) is the reflectivity of the glass surface; \( d \) is the plate thickness; \( n_r \) is the refractive index and \( \theta_i \) is the incident angle of the light. \( n_r \) was measured using a Metricon 2010 prism coupler giving \( n_r = 1.52 \), from this \( r \) can be calculated, using

\[
\begin{align*}
    r^2 &= \left( \frac{n_0^2 - n_r^2}{n_0^2 + n_r^2} \right)^2 \\
\end{align*}
\] (78)

Parameter \( d \) can be calculated by measuring the transmission vs angle with the plate outside the cavity using a separate laser.

The gain was calculated by measuring the threshold of a given cavity as a function of plate tilt, this curve was then fitted by:

\[
P_{th} = \frac{a_3}{1 + f_0 \sin^2 \left( \frac{2\pi}{\lambda} f_1 \cos \left( \frac{\sin^{-1} \left( \frac{\sin(\theta - f_2)}{1.52} \right)}{\lambda} \right) \right)}
\] (79)

which is an adaptation of eqn (77). The important parameters from the fit are \( f_1 \) and \( f_2 \). Experimentally \( f_0 \) was measured to be 0.1757 and the induced plate transmission as a function of angle is given by:

\[
t = \frac{0.973}{1 + 0.1757 \sin^2 \left( \frac{2\pi}{\lambda} f_1 \cos \left( \frac{\sin^{-1} \left( \frac{\sin(\theta - f_2)}{1.52} \right)}{\lambda} \right) \right)}
\] (80)

where the factor of 0.973 is the measured maximum transmission of the coverslip due to scattering and other imperfections.

At threshold, the total gain exactly equals the loss. So the product of the transmission of the individual cavity elements, \( t_i \), multiplied by the gain (assumed exponential in pump power, \( P_p \)) gives 1. ie.

\[
e^{\gamma P_p t_1 t_2 t_3 \ldots t_i} = 1
\] (81)

or:
\[ P_p = -\frac{1}{\alpha_p} \ln[t_1t_2t_3...t_M] \] (82)

So by varying one of the losses, \( t_1 \), say (the loss due to the inserted cover slip), and keeping the remaining ones constant, the variation in threshold pump power at each loss gives:

\[ P_p = -\frac{1}{\alpha_p} \ln[t_1] - \frac{1}{\alpha_p} \ln[t_2t_3...t_{M-1}] \] (83)

Hence plotting \( P_p \) vs \( \ln[t_1] \) gives a straight line with a slope \(-1/\alpha_p\) and intercept \(-1/\alpha_p \ln[t_1t_2t_3...t_{M-1}]\) and hence the residual cavity loss can be readily determined.

4.4.6.1 Measuring Gain for Real Laser Cavities

4.4.6.1.1 Lasing at 1064nm Straight Line Cavity

![Figure 41](image)

Figure 41 Experimental Setup for 1064nm laser without fibre in cavity.

Several measurements of the loss for different configurations of the 1.064\(\mu\)m laser cavity were made. In all cases the pump source was a CW Ti:Sapphire laser emitting up to 1W average power and tuned to the 808nm pump band of Nd:YVO\(_4\).

A simple linear cavity (see Figure 41) consisting of the Nd:YVO\(_4\) crystal coated as described in 4.4.3; a nominally 99% reflectivity mirror (from CVI Corporation); and a broadband AR coated lens (Thor Labs C220: \( f=11\)mm, \( NA=0.25 \)), had a round trip loss of 83\% (ie \( t_{av}=0.17 \)) and a threshold of 25mW. The 25mW figure is external to the cavity and does not include losses due to the 5x microscope objective (\( t=0.93\pm0.0025 \)) used to
focus into the Nd:YVO₄ or the crystal end mirror (t>0.90). The design specifications for the crystal coating were chosen to have a low reflectivity at 1.064μm to suppress lasing on this unwanted line later in the experiments and this is the main source of the losses.

4.4.6.1.2 Lasing at 1064nm, Fibre in Straight Line in Cavity

To the "straight" cavity was added a twin core fibre and the necessary launch optics. The fibre was glued inside glass capillary and its ends polished. The flatness over the entire polishing jig surface (~50mm diameter) was better than λ/4, and a better figure than this is expected for the fibre surface. Surface quality was estimated to be 30/10 (Scratch/Dig).

Encapsulating the fibre in a glass capillary tube had the advantage that the fibre environment was constant - it was not subject to variable stress due to handling, bending, temperature or humidity and so the coupling should remain constant across the experiments. This made the handling of the fibre easier and permitted various different
methods to be used for attaching mirrors to the fibre ends. A concern during the encapsulation process was that it would introduce sufficient stress to switch off the coupling, although this did not eventuate. The glue used was Summers Labs UV71. A low viscosity UV curing epoxy was chosen because the glue needed to remain liquid whilst the fibre was inserted into the glue filled capillary. Temperature curing was not favoured as temperature induced stress of the fibre was a concern.

To get lowest loss cavity meant that a mirror coating had to be applied to one end face of the fibre in the capillary tube. Degradation of the UV curing epoxy, and the possible introduction of thermal stresses, however, prevented the assembly being heated above the about 100°C restricting the coating temperature. This limitation ruled out the use of dielectric coatings (which need to be applied using substrate temperatures in the range 200-300°C for good adhesion) and so unprotected silver was used. The expected reflectivity for silver is greater than 98% at the wavelengths of interest [86]. The use of silver as the end mirror coating made alignment of the fibre rather more difficult because there was no transmission at all through the coating.

When the fibre was added to the cavity the calculated linear loss became 90% (ie. \( t=0.1 \)). The fibre has no coupling at this wavelength (see 4.2.3 below) and so the increase in cavity loss must have been due to the launch optics, imperfect reflection from the Ag mirror and unwanted reflections from the uncoated fibre input end face.

The gain \((=1/(\text{total cavity loss}))\) is shown in Figure 42. The figure shows that the gain is reasonably linear for both cases, but the presence of the fibre in the cavity, even without any coupling, reduces the gain significantly. With no fibre the gain is given by:

\[
g = 0.49P_p^{\text{ext}} - 6.1 \tag{84}
\]

where \(P_p^{\text{ext}}\) is the pump power (mW) measured externally (ie. Before 5x microscope objective and crystal face). This gives a gain of 6.15 for a pump power of 25mw. In contrast, with the fibre in the cavity the gain is given by:

\[
g = 0.65P_p^{\text{ext}} - 15 \tag{85}
\]

giving a Gain of 1.25 for the same 25mW pump.
4.4.6.1.3 **Standard 1342nm Cavity**

The layout for the cavity used for most of the laser experiments at 1342nm is shown in Figure 43. The additional mirror which bends the cavity had a reflectivity of >99% at 1342nm, but < 15% at 1064nm and was necessary to suppress lasing on the higher gain 1064nm line. Repeating the above measurements but at 1342nm with no fibre on this three mirror (coated crystal and 2 CVI > 99% mirrors) and two lens (Thor Labs C220, broadband AR coated at for 950-1550nm) cavity, yielded a total round trip loss of 5.00±0.25% (τ=0.95±0.0025). Only one lens is actually necessary to make this cavity stable but the measurement was made including all the optics that would be needed when the fibre was introduced so that the extra loss due to the fibre could be more accurately determined. The external threshold for this resonator was ~3mW.

Next several different lengths of fibre with different couplings were added to the resonator. Initially the lowest loss resonator so-obtained had a total loss of 25±2%, which included 17±1% due to the coupling.

![Experimental Setup for 1340nm laser with fibre in cavity.](image)
The coupling was measurement by (see diagram) magnifying the image of the output from the fibre viewed through mirror 2. The data was recorded using a Find-R-Scope 1800nm video camera - (sensitive from red - 1900nm) - and an image capture system. Much effort was necessary to filter out the stray 1064 and 808nm (pump) radiation.

A coverslip AR coated for 1340 with a reflectivity of, at worst, 1% (the uncertainty was due to measurement error) at the lasing wavelength was added to reduce the losses at the input face of the fibre. As noted above it was not possible to apply a dielectric anti-reflection coating direct to the end of the fibres. The cover slip was attached to the input end of the fibre using silicone index matching oil. The silicone oil was chosen for its low absorption at 1340nm (measured to be less than 0.059cm\(^{-1}\) hence for the anticipated thickness of a few hundred microns the absorption would be negligible), high viscosity, low toxicity and reasonable match to the refractive index of silica \((n_D=1.4950 [87])\). The best results for this cavity (after a slight shortening of the fibre to reduce the coupling) was 19±2% for the total cavity loss including 12.0±0.5% due to coupling.

The results suggest the residual cavity round trip loss was only ~7% which included all the losses associated with coupling into and out from the fibre. It appeared that there were few ways of reducing this short of recoating the multi-element intra-cavity lenses to optimise their performance at 1342nm.
Figure 44 shows the calculated gain for the Nd:YVO$_4$ laser. Again, there is a difference in behaviour for the fibre cavity to the empty cavity - the no-fibre cavity has:

$$g = 0.038P_p^{\text{ext}} + 0.99$$ \hspace{1cm} (86)

while putting the fibre into the cavity gives:

$$g = 0.0168P_p^{\text{ext}} + 0.92$$ \hspace{1cm} (87)

The contrast, at 25mW again, is of a Gain of 1.94 for the no-fibre case against 1.34 for the fibre case.

Orthovanadate is strongly birefringent and the gain is also polarisation dependent. Even short lengths of optical fibre can cause depolarisation and we suspect that a slight reorientation of the polarisation is going on inside the fibre and hence the gain is reduced.
4.5 Cavity Beatnote

A number of the laser models require determination of the "correlation time", $\tau_c$ of the modes in the cavity as an important parameter, particularly in determining the self-starting mode-locking characteristics. From Krausz [53], for a Lorentzian lineshape,

$$\tau_c = \frac{1}{\pi (\Delta \nu)_{FWMH}}$$  \hspace{1cm} (88)

![Diagram of Cavity Beatnote envelope for empty cavity with external pump of 25mW.](image)

where $(\Delta \nu)_{FWMH}$ is the full width half maximum of the first beatnote of the longitudinal modes of the laser. Other line shapes contribute proportionality factors only slightly different from $\pi$. Considering the importance of this parameter to the models, a large amount of effort was put into measuring this, supposedly easily obtained, parameter.

Initially the beatnote of the bent, three mirror, single lens cavity was measured as a function of pump power. The results are shown in Figures 45 and 46. This data was taken using a New Focus InGaAs 1GHz photodetector and a Hewlett-Packard Digital Spectrum Analyser (SA) and are the result of the average of 100 scans. Figure 46 (and subsequent
SA figures) shows the averaged “envelope” of the cavity beatnotes. Individual scans showed a cavity beatnote that was up to an order of magnitude narrower than that of the envelope, but varied in frequency and amplitude between each SA scan.

As such, the envelope for even the "empty" cavity (coated crystal, 3 mirrors, bent cavity and the single lens) contains considerable structure and as many as 7 peaks, each approximately Gaussian in shape. In the example presented, none of the individual envelope peaks is wider than 7000Hz, but the full envelope can be as wide as 80kHz. The area under the envelope increases in a roughly linear fashion with external pump power.

In Figure 46 quite substantial changes in the beatnote structure occur for small changes in the cavity external pump power. There is a general broadening of the beatnote area, (more than simple height increase with power), quite rapidly in the region (5.5 - 20mW external) and little expansion (maybe even a slight contraction above 50mW) and a general clustering around peaks at 830.4MHz and 830.35MHz.
Figure 47 shows the heights of Gaussians fitted to the major peaks of the beatnote spectra. They show a general increase in the beatnote envelope amplitude with increasing pump power. This is to be expected because raising the pump power increases the gain and hence the number of oscillating modes, which in turn increases the strength of the beatnote. Figure 48 shows centre frequencies of the fitted peaks from Figure 47 and demonstrates that the beatnote spectrum expands with increasing pump power. It also shows growing complexity of the signal.

4.5.1 Beatnote Width Factors

The simple cavity measurement yielded a beatnote envelope width of ~80kHz. A narrow beatnote (because it implies a long cavity mode correlation time) is desirable for laser
Figure 48 Fitted beatnote frequencies as a function of external pump power (mW).

self-starting and hence it was important to try to determine the factors affecting the beatnote width with the aim being to then reduce this width. A number of different beatnote broadening sources were examined, these included:

- Cavity Length Jitter
- Dispersion Effects
- Competition between Cavity Modes
- Polarisation Effects
- Loss Effects
- Cavity Reflections.

4.5.1.1 Cavity Length Jitter

The beatnotes come from the beating together of the longitudinal modes of the cavity. These modes are spaced by:
where $L_{\text{opt}}$ is the optical length of the cavity. If there are several regions of length, $L_i$, in the cavity of different refractive index, $n_i$, then the optical length is:

$$L_{\text{opt}} = n_1 L_1 + n_2 L_2 + n_3 L_3 + \ldots$$  \hspace{1cm} (90)

From eqns (89) and (90) it is apparent that if $L_{\text{opt}}$ changes, due to for example vibrations, then this would change the cavity mode spacing and hence the beatnote width. However, to get the measured 80kHz spread in beatnote frequencies would require a jitter in the cavity length of $\sim 20\mu m$. Considering that all mirrors and the crystal were in stable pedestal mounts (with which we have used in the past to construct interferometers) and lenses were on commercial xyz stages (which have been used for long term alignment of single mode fibre systems) a length variation of more than a couple of microns is very unlikely. Hence cavity length changes cannot explain the observed beatnote width.

### 4.5.1.2 Dispersion Effects

From eqns (89) and (90) if any of $n_i$'s change significantly over the spectral width of the laser then this also would cause a broadening of the beatnote. From eqn (41), for our cavity ($\Delta v_0 = 830.35\text{MHz}$, $T_r = 1.2 \times 10^9\text{s}$ and using $\tau_i = 10\text{ps}$) would require a dispersion, $|D| = 368\text{fs}^2$, which is equivalent to a fibre GVD of $\sim 770 \text{ ps/nm km}$ (about 20 times that of standard communications fibre). This again is an unlikely explanation of the data.

### 4.5.1.3 Cavity Mode Competition

If the cavity supports more than one transverse mode then the each longitudinal mode could lase in a different transverse mode. The mode spacing of the different longitudinal/transverse mode combinations would not be regular and could lead to the observed beatnote width.
The higher order transverse cavity modes have a larger beam cross-section within the cavity. Hence, the cavity mode competition experiments consisted of first measuring the output power of the laser for a given pump power and taking the beatnote spectra. An iris was next inserted into the cavity and closed so that the laser output was progressively 75%, 50% and then 25% of the un-apertured cavity output, taking the beatnote spectrum at each successive point.

As is shown in Figures 49-52 there are some trends across the spectra. The un-apertured cavity spectra are the narrowest and often the simplest in structure (i.e. with only one peak). The spectra seem to change more with increasing cavity loss, rather than due to any possible suppression of transverse modes. Hence cavity mode competition cannot explain the main component of the beatnote spectra width.
4.5.1.4 Polarisation Effects

As was noted in Section 4.4.2, the stimulated emission cross-section (and hence the gain) as well as the crystal refractive index are strongly polarisation dependent. A possible cause of the beatnote width was fluctuations in the laser polarisation giving rise to different gain and cavity optical length.

To test this a Glan-Taylor polarising prism was inserted into the "empty" cavity. Since one particular polarisation would see the least cavity loss, the aim was to try to "lock" the laser into that particular polarisation. The behaviour of beatnote of this cavity with power was measured and is shown in Figure 53. As in the preceding examples, the beatnote narrows to a single peak with increasing power. The polarised cavity, however, had a considerably broader beatnote than in the other cases, and a quite large loss, leading to a cavity threshold of ~13mW compared to ~3mW for the "empty" configuration.

A better test of this would be to insert a Wollaston or other polarisation separating prism, take the component at the orthogonal polarisation, put it through a $\lambda/2$ plate and recombine it with the other beam. However, alignment of this system would be very

![Figure 53](image-url) Beatnote spectra as a function of external pump power for cavity with Glan-Taylor prism.
difficult. Polarisation effects on beatnote needs further investigation.

4.5.1.5 Effect of loss on Beatnote

In many of the preceding experiments it appeared that the cavity configurations with the larger loss had the broader beatnote. To examine this more fully, a microscope coverslip was inserted into the laser and the beatnote was measured as a function of the cavity loss. The results of this are shown in Figure 54 and they are not particularly conclusive. In general, for increasing loss the beatnote power seems to be concentrated more in one dominant peak, rather than the sidebands, but there is little variation in beatnote power, width or position across the range of losses considered. This results are quite confusing. All measurements are taken at a common output power of the laser, nominally <1mW through one of the 99% mirrors, but the laser thresholds involved vary from ~4mW -14mW external.
4.5.1.6 Cavity Reflections

A possible cause of the beatnote width is spurious reflections from cavity elements, or elements external to the cavity. Experiments with single core fibre, outlined in Appendix B, show that even a handheld microscope slide angled to reflect the laser light back into the laser cavity could give an increase in the cavity beatnote by a factor of three.

From eqn (42), assuming a $r_{oc}=0.995$ gives a $\sim 3.5\%$ feedback is required to give this width. This is possible from the face of the microscope objective lens used external to the cavity to focus the light for pumping, but unlikely considering the tight focus used. Nevertheless the sensitivity of the beatnote to feedback of this nature suggests that unidentified sources of stray reflections are the most likely cause for the broadened beatnotes.

4.5.1.7 (No) Beatnotes with the Fibre in Cavity

After these preliminary measurements which suggested that the interpretation of beatnotes was not very straightforward, the fibre was inserted into the cavity and no beatnote could be found! This occurs when the only change to the cavity has been the addition of the fibre (ie. there were no major changes in cavity alignment, gain or configuration) and hence this seems to be a fibre induced affect.

Possibly the fibre has the effect of narrowing the gain so that only one mode at a time is able to lase inside the cavity, or it broadens the beatnote to such an extent that the spectrum analyser is unable to pick it from the noise.

4.5.1.8 Beatnote Summary

Despite the apparent simplicity of the measurement, considering its usefulness, the beatnote width parameter proved to be considerably more difficult to measure in practice for the real laser system. This may also explain the scarcity of beatnote width measurements quoted in the relevant literature.

Spurious cavity reflections seem the most likely cause of the broadening of the beatnote for the cavity without the fibre although this has not be conclusively proved. In future calculations, the beatnote width of 80kHz measured for the simple cavity at 1342nm was used. The cavity beatnote was undetectable when the fibre was included in the cavity.
### 4.6 Characterised Laser

In the terms of the Haus model of the laser (eqn (31)) we have built several Nd:YVO₄ lasers with the following characteristics:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Value or Expression</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi )</td>
<td>Shift of laser oscillation frequency from Fabry Perot resonances of Cavity</td>
<td>Unknown</td>
<td>4.4.6</td>
</tr>
<tr>
<td>( I )</td>
<td>Linear Cavity Loss</td>
<td>Fibre in Cavity: 0.9</td>
<td>0.19±0.02 (0.12±0.005 is coupling)</td>
</tr>
<tr>
<td></td>
<td>No Fibre: 0.83</td>
<td>No Fibre: 0.05</td>
<td></td>
</tr>
<tr>
<td>( x )</td>
<td>Phase shift per round trip</td>
<td>Unknown</td>
<td></td>
</tr>
<tr>
<td>( g_0 )</td>
<td>Small signal gain per round trip ( (P_e = \text{external pump power (mW)}) )</td>
<td>Fibre in Cavity: 0.65 ( P_e - 15 )</td>
<td>0.0168 ( P_e + 0.92 )</td>
</tr>
<tr>
<td></td>
<td>No Fibre: 0.49 ( P_e - 6.1 )</td>
<td>No Fibre: 0.038 ( P_e + 0.99 )</td>
<td></td>
</tr>
<tr>
<td>( \Omega )</td>
<td>Gain Bandwidth</td>
<td>0.5±0.1nm (132.47GHz)</td>
<td>2.1±0.2nm (349.55GHz)</td>
</tr>
<tr>
<td>( D )</td>
<td>Group Velocity Dispersion ( (L_f = \text{length of fibre in cavity (m)}) )</td>
<td>8.237 ( 10^{27} L_f ) s⁻² m⁻¹</td>
<td>-3.085 ( 10^{27} L_f ) s⁻² m⁻¹</td>
</tr>
<tr>
<td>( \delta = S L_f )</td>
<td>Self Phase Modulation</td>
<td>3.524 ( 10^3 L_f ) W⁻¹ m⁻¹</td>
<td>2.799 ( 10^3 L_f ) W⁻¹ m⁻¹</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Slope of &quot;saturable absorber&quot; vs circulating Power ( L_b = \text{fibre beat length (m)} ) ( P_e = \text{Circulating laser power (W)} ) ( S = \omega_0 n_2 / (c A_{\text{eff}}) ) ( A_{\text{eff}} = \text{Effective Fibre Mode Area (m²)} \sim \pi (4 \mu m)^2 )</td>
<td>Strictly ( \frac{d}{dP} \left[ 1 + \frac{2\pi l_f}{L_b} \left( \frac{S L_b P}{4\pi} \right)^2 \right] ); but this is not defined analytically. An approximation which seems to be accurate to better than 1% for ( SL_b P / (4\pi) &lt; 0.1 )</td>
<td>4.2</td>
</tr>
<tr>
<td>( (\Delta u)_{\text{FWHM}} )</td>
<td>Cavity Beatnote FWHM</td>
<td>Up to 80kHz</td>
<td>4.5</td>
</tr>
</tbody>
</table>
However, when pumping at the maximum output of our Coherent 899 Ring Ti:Sapphire laser of about 500mW at 810nm with a number of different fibre lengths failed to show any pulsing behaviour. Pulsing could not be induced by any of the methods commonly used to self-start Kerr-lens mode-locked systems (vibrating the mirrors, etc). As a result we concluded that in spite of the indications of some of the analytic models in chapter 3, the self-starting threshold was not exceeded for this laser.

### 4.7 Conclusions

A twin core fibre was characterised and found to have coupling at wavelengths longer than 1150nm with a good uniformity of beatlength along its length. Switching occurs at the power levels expected by the theory.

The coupling in the fibre puts a lower limit on the laser gain medium that can be used and we chose Nd:YVO₄ lasing 1342nm as the host for the experiments. The cavity loss and gain were well characterised. The cavity beatnote was measured for the cavities...
without fibre and it is expected that its width is due to spurious reflections. No beatnote was detected for any cavity containing an optical fibre.

The laser was pumped with up to 500mW at 810nm from a Coherent 899 Ring Ti:Sapphire laser but no pulsing behaviour was observed despite the cavity having many of the identified desirable characteristics including low loss, short cavity round-trip time and non-linear coupling with a close to optimised fibre length.
5 Implications of Nd:YVO$_4$ Experiments

5.1 Introduction

The previous chapter described experimental results on a Nd:YVO$_4$ laser containing a twin core fibre as a mode-locking element. This laser failed to give pulsed output despite meeting many of the criteria introduced in Chapter 3 for self-starting passive modelocking. In particular the laser had:

- low dispersion - operation was within 70nm of the theoretical zero dispersion point
- low cavity loss - 7±2% underlying round-trip loss
- small self-phase modulation - most of cavity length was bulk optics where the local intensities are not sufficient for SPM to be appreciable.
- large gain - Nd:YVO$_4$ had the largest $\sigma$ (stimulated emission cross-section) of any practical laser host at 1300nm
- suitable fibre length - the 7±2% round-trip loss was combined with a fibre of just over 2 beatlengths and almost ideal match of linear loss to coupling with 12±0.5% coupling.

It would appear, therefore, that some of the analytic criteria are not specific enough to guarantee that mode-locking will occur. This Chapter, therefore, examines the self-starting criteria in more detail and compares their predictions with the experimental data from Chapter 4. The analytic criteria are also compared with numerical simulations of the self-starting process. Having established which analytical models are useful, the required specifications for a self-starting passively modelocked laser can be better defined.
Figure 55  Black Region indicates satisfying Krausz's Self-Starting Criteria (ie. $F_m > 1$, eqn (39)). ■ indicates best experimental conditions achieved.

Figure 56  Black region indicates satisfying Chen's self-starting criteria (ie. $\text{Re}[\lambda] > 0$ for some $\omega$, eqn (45)). ■ indicates experimental conditions.

Figure 57  Black region indicates satisfying Herman's self-starting criteria (ie. LHS (51)a < RHS (51)a). ■ is experimental conditions.

Figure 58  Black region indicates satisfying the Krausz/Haus criteria for suppression of self q-switching. (ie. LHS (52) < RHS (52)) ■ indicates experimental conditions.
5.2 Analysis of Nd:YVO₄ Laser.

Figures 55-59 show, in black, the operating regions that satisfy the various analytic criteria summarised in Chapter 3. ie.

- Figure 55 shows Krausz’s self starting Criteria (ie. $F_m > 1$, eqn (39))
- Figure 56 shows Chen’s self starting criteria (ie. $\text{Re}[\Theta] > 0$ for some $\omega$, eqn (45))
- Figure 57 shows Herman’s self-starting criteria (ie. LHS (51)a < RHS (51)a)
- Figure 58 shows the Krausz/Haus criteria for the suppression of self q-switching (ie. LHS (52) < RHS (52))
- Figure 59 shows the region of stable pulses according to Haus's criteria (ie. eqn (38)).

The symbol ■ in each of the figures marks the most favourable combination of cavity parameters achieved in the Nd:YVO₄ experiments detailed in Chapter 4 - 7% cavity loss with a pump power of 500mW. This device failed to show any pulsing behaviour despite passing Krausz’s self-starting criterion - which it should be noted would appear to
be a very optimistic criterion compared to any of the others - and being very close to passing Chen’s. There was no observed self q-switching, which is consistent with Haus’s predictions although the absence of any pulsation behaviour at all probably makes this rather meaningless.

5.3 Computer Simulation

Since the laser failed to produce self-starting, passively modelocked pulsations, in spite of passing Krausz’s criterion and being close to Chen’s, it was decided to develop a computer model of the early stages of the pulse formation in the laser (~first millisecond) to enable us to better understand the processes that are important during this build-up phase.

To date, the published self-starting passively modelocked laser models have been either: a) numerical simulations of pulse propagation within a nonlinear directional coupler with gain [89][90][91][92][93][94][30] or b) analytical models of the starting dynamics [95][96][97][98][99][54]. The previous numerical simulations generally use an input in the form of a single pulse, often weak, but with a smooth shape (usually either Gaussian or Lorentzian) which is propagated using the Non-Linear Schroedinger equation. Stabilization and narrowing of this input pulse has been generally used to identify a set of parameters favourable for passive modelocking. Decay, oscillation or the development of structure on the pulse have been taken to indicate an unfavourable set of parameters. This approach is particularly applicable to the study of the effects of fibre dispersion, Stimulated Raman Scattering and Self Phase Modulation on the laser dynamics, however the models do not adequately represent many of the affects of the gain medium (eg. saturation, gain bandwidth, gain lineshape). Furthermore, by injecting an initial pulse these simulations have assumed that the device self-starts. In many of the simulations a long beatlength (>0.5m) has been used, which our experience has shown to be impossible to produce in real optical fibre with current technology.

The analytical models have made the problem tractable by simplifying the behaviour of the gain medium; approximating the behaviour of the passive modelocking element by a linear term; and assuming that each pulse-shaping element in the cavity perturbs the pulse only very slightly during each roundtrip. However, since it can be
anticipated that, (a) at least in the early stages of pulse formation, large changes occur to
the pulse shape and phase; (b) that the passive modelocking element eventually saturates;
and (c) the bandwidth of the gain medium has an effect on the final pulse shape, width and
stability; means that these models are also inadequate.

Neither the analytic models nor these previous numerical simulations handle
adequately the growth of a pulse from the noise at laser threshold (ie. self starting) or the
frequency dependence and saturation of the gain medium.

As a result, we have developed a computer model specifically aimed at simulating
the laser during the build-up phase. This model examines the effect of the gain medium
and the passive modelocking element on a "window" of intensity as it progresses around
the cavity. A "round-trip" of the laser involves processing the "window" through the non-
linear loss routine, a FFT to transform from intensity to frequency space, then through the
gain medium routine and an Inverse FFT to return to intensity space to begin the cycle
again.

The "window" was a large numerical array. The light field in the cavity is divided
into boxes with the local field represented by a complex number. The resolution used (ie
boxes per unit time) depends on the maximum frequency modelled, which is dependent on
the bandwidth of the gain medium. In practice this yields an array size that makes
calculations prohibitive. Instead, a subsection of the light field is represented and the rest
of the cavity light is assumed to behave similarly.

5.3.1 Gain Medium

The gain acts in frequency space and the routine needs to manage the populations
of the lasing levels and take into account the frequency behaviour of the laser.

The laser levels are simulated by 4 ODEs

\[
\begin{align*}
\frac{dE_1}{dt} &= P_{\text{pump}} - (E_1 - E_2)\sigma(E_{\text{stim}} + E_{\text{spont}}) - \frac{E_1}{\tau_{12} + \tau_{10}} \\
\frac{dE_2}{dt} &= (E_1 - E_2)\sigma(E_{\text{stim}} + E_{\text{spont}}) + \frac{E_1}{\tau_{12}} - \frac{E_2}{\tau_{20}} \\
\frac{dE_{\text{stim}}}{dt} &= (E_1 - E_2)\sigma(E_{\text{stim}} + E_{\text{spont}}) \\
\frac{dE_{\text{spont}}}{dt} &= \alpha \frac{E_1}{\tau_{12}}
\end{align*}
\] (95)
where $E_1$ is the energy stored in the gain medium in the upper level of the lasing transition; $E_2$ is the energy stored in the lower level of the lasing transition; $E_{\text{stim}}$ is the energy of the light captured by the cavity originally produced by stimulated emission; $E_{\text{spont}}$ is the energy of the light captured by the cavity produced by spontaneous emission; $\tau_{ij}$ is the lifetime of the level $i$ with respect to transitions to level $j$; $\alpha$ is the fraction of photons emitted into the 4\pi steradians that are captured by the laser cavity; $\sigma_E$ is the cross-section of the transition for stimulated emission normalised appropriately for the energy units of the equation and is given by:

$$\sigma_E = \frac{\sigma}{h \nu V_{\text{pump}}}$$  \hspace{1cm} (96)

where $V_{\text{pump}}$ is the volume of the gain medium pumped.

These equations "automatically" take care of gain saturation (and this occurs at relatively low fluence in Nd lasers due a "bottle-neck" effect due to the slow (30ns) depopulation of the lower level of the laser transition).

On entry to the gain medium routine, the total energy and phase as a function of frequency of the incoming light is saved for each frequency in the gain bandwidth. The equations (95) are advanced by a total timestep equal to the roundtrip time of the cavity and the produced energy is added back into the cavity proportionally to the lineshape of the gain medium. The stimulated emission contributions are added in phase with the existing energy, the spontaneous component at a random phase. The phase of each box is also incremented so that it would be totally randomised in a time equal to the correlation time of the laser if other effects do not intervene.

### 5.3.2 Non-linear Loss

To each point in the array the combined coupler response function and linear loss were applied ie.

$$E_{\text{circ}}(i) = E_{\text{circ}}(i)(1 - l_{\text{cavity}}) \left[ \frac{1}{2} \left\{ 1 + cn \left[ \frac{2\pi L_f}{L_b} \left( \frac{S L_b E_{\text{circ}}(i)}{4\pi t_{\text{array}}} \right)^2 \right] \right\} \right]$$  \hspace{1cm} (97)
where \( E_{\text{circ}}(i) \) is the energy contained in an array element representing a timewidth of \( t_{\text{array}} \). This, inherently, makes the assumption that the coupler acts instantaneously and that it's response at a given time is not dependent on any preceding history. Since the response time of fused silica is \( \sim 2-4 \text{fs} \) these assumptions are valid. The actual device uses a double pass of the same piece of fibre although this is not specifically taken into account.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity Length</td>
<td>( L_{\text{cav}} )</td>
<td>0.3056m</td>
<td></td>
</tr>
<tr>
<td>Pump Power</td>
<td>( P_{\text{pump}} )</td>
<td>2500W</td>
<td>Physically unrealistic, but here to show behaviour.</td>
</tr>
<tr>
<td>Pump Wavelength</td>
<td>( \lambda )</td>
<td>810nm</td>
<td>Main pump transition for ( \text{Nd:YVO}_4 )</td>
</tr>
<tr>
<td>Gain Cross-section</td>
<td>( \sigma )</td>
<td>( 7.61 \times 10^{-23} \text{ m}^2 )</td>
<td></td>
</tr>
<tr>
<td>Radiative Lifetime</td>
<td></td>
<td>115 ( \mu \text{s} )</td>
<td></td>
</tr>
<tr>
<td>Fluorescence Lifetime</td>
<td></td>
<td>100( \mu \text{s} )</td>
<td></td>
</tr>
<tr>
<td>Dispersion</td>
<td>( D )</td>
<td>( -3.085 \times 10^{-27} \text{ L}_r \text{ W}^{-1} \text{ m}^{-1} )</td>
<td></td>
</tr>
<tr>
<td>Laser Wavelength</td>
<td>( \lambda )</td>
<td>1342nm</td>
<td></td>
</tr>
<tr>
<td>Destination Level Lifetime</td>
<td>( \tau_2 )</td>
<td>30ns</td>
<td></td>
</tr>
<tr>
<td>Gain Bandwidth</td>
<td>( \Omega )</td>
<td>2.1nm</td>
<td></td>
</tr>
<tr>
<td>Fluorescence Capture</td>
<td>( \alpha )</td>
<td>0.008</td>
<td>Spontaneous Photons are emitted into ( 2\pi ) steradians, only 0.8% have correct trajectory to be &quot;captured&quot; by cavity.</td>
</tr>
<tr>
<td>Fraction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-Phase Modulation</td>
<td>( \delta )</td>
<td>( 2.799 \times 10^{-3} \text{ W}^{-1}\text{m}^2 )</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4** Parameters for \( \text{Nd:YVO}_4 \) simulations.

### 5.4 Simulation Results and Bandwidth

Figures 61-63 show sample results of these computer simulation for a strongly pumped \( \text{Nd:YVO}_4 \) laser with parameters given in Table 4. Figure 61 shows the intensity of the 8 highest pulses in the cavity as a function of time. In this simulation the laser self-
starts - an initial peak (actually the second highest of the laser intensity profile at threshold) grows slowly until about the 10\(\mu\)s point, and then rapidly grows to be the dominant peak in the cavity. At the same time, at the end of the growth, there is a rapid change and the other peaks in the cavity are suppressed as the modes lock so that the narrow pulse is formed. Figure 62 shows the intensity profile across the cavity at various times indicating "snapshots" of the intra-cavity intensity.

When the intensity profile in time is Fourier Transformed it forms a peak in frequency space. The height and FWHM of this peak are shown in Figure 63 as a function of simulation time. Since there is only a finite amount of energy in the cavity the peak in frequency space has constant area. Hence as a peak forms in the cavity the frequency width increases and the peak height must decrease. Figure 63 also indicates that a three step process for the pulse formation by passive modelocking (as indicated by Hermann

---

**Figure 60** Intensities of 8 highest peaks as function of simulation time. (Details in Table 4)  
**Figure 61** Representative "snapshots" of intensity profile in cavity window. (Details in Table 4)  
**Figure 62** Bandwidth and peak height of intensity in frequency space. (Simulation Parameters in Table 4).
The simulations have only been shown for a simulation time of 30µs, which is much shorter than the upper state lifetime of Nd:YVO₄. However, simulations for longer timescales (not shown) show that the result above remains stable to the length of the simulation (1ms). In this simulation the pulses become flat-topped with a FWHM of ~3ps. The flat top occurs because at the peak of the pulse the coupling in the NLDC has been made effectively zero. Any further increase in power increases the coupling slightly and leads to a greater loss.

Figures 64-66 show for identical conditions (ie. same parameters for gain, cavity and fibre properties, the only difference being the initial intensity profile at threshold) a simulation that failed to self-start. In this case it is difficult to discern any difference
between the starting

**Figure 66** Peak Intensities for gain FWHM 1nm

**Figure 67** Cavity "snapshots" for gain FWHM 1nm

**Figure 68** Gain information for 1nm FWHM laser simulation
conditions that may have caused the failure, and there were several occasions (at 10\(\mu\)s and again at about 16\(\mu\)s) where a single peak has an intensity up to an order of magnitude greater than the next peak.

An important parameter (that was not apparent from the analytical models) to the self-starting of a passively modelocked laser system is gain bandwidth. Figures 67-69 show the behaviour of a laser with identical parameters to the above, except that the FWHM of the gain is 1nm (approximately half the previous simulations). This fails to self-start and comparison, even with the 2nm gain FWHM case, shows considerably fewer peaks in the intensity “snap-shots”.

![Figure 69] Peak intensities for gain FWHM 4nm

![Figure 70] Cavity “snapshots” for gain FWHM 4nm

![Figure 71] Gain parameters for gain FWHM 4nm
In contrast, Figures 70-71 show the behaviour when the gain FWHM was doubled (to 4nm). The modelocking self-starts and at a slightly earlier time that the 2nm case. Closer inspection (Figure 72) shows that the final pulse width is slightly narrower for the wider gain bandwidth, but again the squaring of the pulse shape hampers the full narrowing.

**5.5 Comparison of Analytical Models and Simulations**
Figure 73  Comparison of Satisfying Krausz's Self Starting Condition (Black Region) and Computer Simulation Results (● self-starts, ■ fails to self-start)

Figure 74  Comparison of Satisfying Chen's Self Starting Condition (Black Region) and Computer Simulation Results (● self-starts, ■ fails to self-start)

Figure 75  Comparison of Satisfying Hermann's Self Starting Condition (Black Region) and Computer Simulation Results (● self-starts, ■ fails to self-start)

Figure 76  Comparison of Satisfying Haus's Suppression of Q-switching Condition (Black Region) and Computer Simulation Results (● self-starts, ■ fails to self-start)

Figure 77  Comparison of Haus's Stability Criteria (Black Region) and Computer Simulation Results (● self-starts, ■ fails to self-start)
Figures 73-77 compare the different criteria of Chapter 3 with the results of the simulations. It would seem that both Krausz and Chen's criteria are optimistic in their predictions for self-starting of the passive modelocking. The equations characterising gain used in our computer simulations allowed self q-switching to occur, but that behaviour was not observed in any simulation where mode-locking was observed, despite a number of them falling in the region (white) where Haus predicts self Q-switching should dominate. Both Herman's self-starting criteria and Haus's pulse stability criteria show good agreement with the simulations in terms of the minimum pump power to achieve self-starting for a given loss, but Herman's criterion predicts that the laser will fail to self-start at high pump powers and that is not observed in the simulations.

For the subsequent work, therefore, we used Haus's stability criteria to investigate the likelihood of self-starting of the passive modelocking for a wide range of cases. The computer simulation can then be used to better understand the dynamics (eg. pulse shape, noise background, long term stability) of a specific set of parameters.
5.6 Implications for further work

5.6.1 Nd:YVO₄ laser at 1341nm

As is shown in Section 5.2, the prospects of getting a self-starting laser from the available materials in Nd:YVO₄ are not good. Figure 67 shows the minimum pump power for a given cavity loss (calculated from Haus’s criteria) for a range of fibre lengths. A theoretical total cavity loss of 5% (modest considering coupling into and out of the fibre) requires an external pump power of about 100W CW, with more than 400m of fibre within the cavity. More power is needed for a shorter fibre. Hence, unless the cavity loss can be reduced to \(<1\%\) then the model indicates that the laser will not work with the available materials for physically realisable powers.

Figure 78 Minimum pump power (W) necessary to achieve Pulse Stability (Haus’s Criteria) for varying cavity loss and fibre lengths. (λ=1341nm, L₉=4.31cm)
5.6.2 Theoretical Nd:YVO$_4$ Laser at 1064nm

Figure 79 Haus's stability criteria for hypothetical Nd:YVO$_4$ laser operating at 1064nm. $L_b=10.8\text{cm}$, fibre length $\sim1$ beatlength single pass.

Figures 80 and 81 show the Haus stability criteria for a hypothetical laser with the 1064nm behaviour of the Nd:YVO$_4$ as measured in Chapter 4, but assuming that the fibre used in our experiments had coupled at 1064nm (giving a beatlength of 10.8cm). This likelihood of self-starting is slightly better due to the larger gain (and hence a greater intracavity circulating power) and longer beatlength of the fibre (giving a lower critical power), but still we are left with the situation that more than 100W CW pump power will be needed to get the device to self-start reliably with losses $>3\%$, while keeping the fibre length reasonable at $\sim11\text{cm}$ (assuming a double pass).

At the current limits of technology bulk optic self-starting passively modelocked Nd:YVO$_4$ lasers operating at either 1342nm or 1064nm are not feasible for the available fibre with the main limitation being the available pump power. The necessary power levels
could be reduced considerably if longer beatlengths and/or greater nonlinearities ($n_2$) were used.

**Figure 80**  Response for different fibre lengths for hypothetical Nd:YVO$_4$ laser operating at 1064nm with parameters as measured in lab (Table in 4.5.1.8) and $L_b=10.8$cm
### 5.6.3 Theoretical Nd:Glass Laser at 1055nm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity Length</td>
<td>$L_{cav}$</td>
<td>0.3056m</td>
</tr>
<tr>
<td>Pump Wavelength</td>
<td>$\lambda_{pump}$</td>
<td>795nm</td>
</tr>
<tr>
<td>Gain Cross-section</td>
<td>$\sigma$</td>
<td>$3.7 \times 10^{-24}$ m$^2$</td>
</tr>
<tr>
<td>Radiative Lifetime</td>
<td>$\tau_2$</td>
<td>30ns</td>
</tr>
<tr>
<td>Laser Wavelength</td>
<td>$\lambda$</td>
<td>1054nm</td>
</tr>
<tr>
<td>Destination Level Lifetime</td>
<td>$\tau_2$</td>
<td>30ns</td>
</tr>
<tr>
<td>Gain Bandwidth</td>
<td>$\Omega$</td>
<td>23nm</td>
</tr>
<tr>
<td>Fluorescence Capture Fraction</td>
<td>$\alpha$</td>
<td>0.008</td>
</tr>
<tr>
<td>Dispersion</td>
<td>$D$</td>
<td>$8.237 \times 10^{-27} \text{ L}_f W^{-1} m^{-1}$</td>
</tr>
<tr>
<td>Beatlength</td>
<td>$L_b$</td>
<td>0.108m</td>
</tr>
<tr>
<td>Self-Phase Modulation</td>
<td>$\delta$</td>
<td>$3.524 \times 10^{-3}$ W$^{-1}$ m$^{-2}$</td>
</tr>
</tbody>
</table>

| Table 5 | Parameters for Nd:Glass simulations

Figures 81 and 82 show the theoretical expressions for a laser made from Nd doped glass fibre (Simulation parameters are in Table 5). The parameters chosen are for a ~1% Nd (atomic weight %) and assumes a glass fibre which has a beatlength of ~10cm. Again we have the problem that for achievable beatlengths the necessary pump powers are large and probably prohibitive.

However, if glass with a greater nonlinearity ($n_2$) could be used in the NLDC the necessary power to achieve passive modelocking could be greatly reduced. Figure 83 shows the decreasing power thresholds for increasing non-linearity. As an example, the figure shows the threshold for laser modelocked with a hypothetical NLDC made from a chalcogenide glass fibre such as described by Asobe et al. [102] (ie. $n_2=2.0 \times 10^{14}$ cm$^2$/W, core diameter=3.0μm)
Figure 81  Haus Stability Criteria for a hypothetical Nd:Glass laser as a function of external pump power (W) and cavity loss ($L_b=10.8\text{cm}$) for our achieved cavity loss of 7% would be $\sim 1\text{W}$. 
Figure 82 Minimum pump power to achieve self-starting passive modelocking (Haus's stability criteria) vs cavity loss and fibre length for hypothetical Nd:Glass laser. Assuming $L_n=10.8\,\text{cm}$. 
Figure 83  Cavity Pump Power necessary to achieve Passive Modelocking (Haus's Stability Criteria) for a hypothetical ~10cm Nd:Glass laser for increasing glass nonlinearity. Best results are for a hypothetical NLDC fabricated from fibre of Asobe et al. [102]

5.6.4 Theoretical Nd:YAG Laser

Table 6 shows the parameters used for the calculations for a hypothetical self-starting passively modelocked Nd:YAG laser operating at 1064nm. Of particular note is that the nonlinear refractive index is more than 2.5 times that of the glass fibre. Figure 73 shows the Haus stability criteria for the device for the range of possible cavity losses and pump powers. Figure 74 shows the minimum pump power, for a given cavity loss and waveguide length, to achieve self starting. The results shown in the figures, suggests that a laser could reasonably be expected to self-start with a pump power of under 10W for a loss of <2%.

The required design needs a beatlength of ~40cm and gives an approximate device length of 23-25cm. The device can be made shorter, but the required pump power also increases. This assumes a waveguide diameter of 6μm, the required threshold decreases if the guide can be made narrower than this.
Figure 84  Stability Criteria (Haus) for hypothetical Nd:YAG laser (shaded region). L_b=40cm, but assuming double pass of ~20cm waveguide length.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ω</td>
<td>Gain Bandwidth</td>
<td>4.5 A (132.47 GHz)</td>
<td>*</td>
</tr>
<tr>
<td>n_2</td>
<td>Nonlinear Refractive index</td>
<td>8.6 \times 10^{-16} \text{cm}^2/\text{W} (average)</td>
<td>†</td>
</tr>
<tr>
<td>σ</td>
<td>Stimulated Emission Cross-section</td>
<td>6.5 \times 10^{-19} \text{cm}^2</td>
<td>*</td>
</tr>
<tr>
<td>I_{sat}</td>
<td>Intensity at which small signal gain halves</td>
<td>1.249 \times 10^7 \text{W/m}^2</td>
<td>*</td>
</tr>
<tr>
<td>τ</td>
<td>Excited State Lifetime</td>
<td>230 \times 10^{-6} \text{s}</td>
<td>*</td>
</tr>
<tr>
<td>D</td>
<td>Group Velocity Dispersion</td>
<td>1.285 \times 10^{-27} L_f \text{s}^2/\text{m}</td>
<td>†</td>
</tr>
<tr>
<td>δ=Γ L_f</td>
<td>Self Phase Modulation</td>
<td>5.079 \times 10^{-13}/A_{eff} L_f \text{W}^{-1}\text{m}^{-1}</td>
<td></td>
</tr>
</tbody>
</table>

References

Table 6  Nd:YAG Parameters at 1064nm.
Figure 85  Necessary external pump power for self-starting passive modelocking (Haus’s stability criteria) for a range of waveguide losses and different beat lengths. All 3 “lasers” considered have approximately the same length (~20cm). Longer beatlengths, but using more than one beatlength works better for low loss.

5.7 Conclusion

This work has demonstrated that many of the analytical models which have been presented in the literature are inadequate to describe the process of self-starting of passive modelocking using a NLDC. Furthermore, numerical simulations focussing on pulse evolution within a resonator containing a nonlinear directional coupler and gain but seeded by a well formed input pulse are also misleading when self-starting is of paramount importance. Our experiments whilst satisfying Krausz’s and being close to Chen’s self-starting criteria did not produce any evidence of mode-locking suggesting these criteria are not realistic. This was confirmed using computer simulations which concentrated on the processes which are expected to be important in the early stages of the self-starting (threshold-50μs). By comparing the results of the analytical models with these
simulations it was found that the best agreement occurred for Haus’s pulse stability criterion.

Haus’s criteria was therefore used to calculate minimum pump powers for a given cavity loss for a number of Nd laser hosts - Nd:YVO₄, Nd:Glass and Nd:YAG. The results indicate that for the available fibre the Nd:YVO₄ and Nd:Glass lasers will not exhibit self-starting passive modelocking for the available laser pump sources. However, a long, low loss (<2% eg. waveguide laser (see Chapter 6)) produced in a Nd:YAG crystal could show self-starting passive modelocking for an achievable but nevertheless rather high pump power.

As a final point it should be noted that the self-starting criteria become much easier to fulfill if the nonlinearity of the NLDC can be increased significantly. In this context it is worth noting that optical fibres based on chalcogenide glasses show third order nonlinearities several orders of magnitude higher than that of silica fibres. If the NLDC were fabricated in such glasses self-starting mode-locking may be achievable at quite modest powers.
6 Ion Implanted Waveguides - Refractive Index Profiles

6.1 Introduction

Although the preferred method of fabricating a laser containing a non-linear directional coupler as the modelocking element was through the use of optical fibre, as described in the previous chapters, we also considered options for fabricating devices in planar waveguides. Of the available planar waveguide fabrication processes the creation of structured waveguides via high energy ion implantation was feasible using the facilities at the ANU.

At the time of the fibre work, the properties of ion implanted waveguides for integrated photonics was also the subject of active research. In the event, the results from our fibre experiments and the numerical studies in the earlier chapters were sufficiently discouraging to suggest that any planar waveguide based device would need to be several 10's cm long before self-starting modelocking could be expected. Since devices of this length could not be fabricated using the available facilities, we did not attempt to fabricate structured devices at all, but rather concentrated on studying the basic properties of ion implanted slab waveguides as a guide to the ultimate feasibility of the technique.

In this chapter a brief introduction to the mechanisms of waveguide formation by ion implantation will be given. This is followed by a summary of theoretical methods used to determine the refractive index profile produced by ion implantation from the measurable parameters. Finally, these methods are tested against a range of "likely" waveguides to determine the best one to use for the experimental results of the next chapter.
6.2 Optical Waveguide Formation by Ion Implantation

Optical waveguides can be produced in a wide variety of materials by bombarding them with high energy (MeV) ions. Much of the previous work has been carried out using the "light" ions - protons and He ions as they have a number of desirable ballistic properties:

- they have the longest possible range in the host
- they deposit most of their energy in a small region towards the end of their range. Hence, the region of lattice damage in crystalline materials which might result in a strong degradation of the lasing properties is minimised.

The mechanism by which an ion beam of given species, energy and dose will affect a given target material is difficult to predict in detail as a rather wide variety of processes can occur. However, for light ions, energy is mainly lost by two distinct processes - electronic excitation and nuclear collisions.

For most of the ion range, when the ion is travelling at high velocity, energy is lost by electronic excitations which couple to the crystal lattice and can lead to the displacement of individual atoms, most often the anions. In addition to the new defects created, the ionisation process will liberate electrons and holes and this allows colour centres to also form at pre-existing defect sites and at impurities. This type of excitation leads to a slight change (usually increase) in the local refractive index.

At the lower energies, near the end of the ion range, there are many more nuclear collisions which displace lattice ions and can lead to cascades of damage leading to the destruction of the original lattice. For example, a typical displacement energy is only 25eV [103], but ions easily have energies as large as 50keV at the end of the implant range, and hence the potential exists to produce 2000 "disturbed" atoms. Although many of these "disturbed" atoms will rapidly return to normal sites, even at 95% recovery, this leaves,

---

1. A colour centre most commonly forms when a negatively charged ion is removed, but an electron is localised in the vicinity. This electron is regarded as bound to what is in effect a positively charged "centre" in the lattice, and as such has a spectrum of energy levels quite different from the normal crystal.

say, 100 defects, per incident ion. The destruction of the lattice often leads to a swelling and a local decrease in the crystal density. The density decrease leads to a reduction in refractive index.

The combined affect of the two processes can produce a region close to the crystal surface with an increased refractive index ("guide") above a "barrier" region of reduced refractive index.

### 6.2.1 History

The technique of creating optical waveguides near the surface of an amorphous or crystalline host by ion implantation is not new, but one for which the advantages are only beginning to be realised as annealing techniques have been shown to reduce the loss of such guides, and the limitations of other, initially more promising, techniques (e.g., ion indiffusion) become apparent.

Ion implantation has the advantage that it can be applied to crystalline and amorphous hosts, and the guide produced can be readily controlled by the accurate application of a known species, energy, and dose and by use of a subsequent annealing cycle. Ion implantation is not temperature specific and so can be used to produce guides in "delicate" materials such as BaTiO₃ and KNbO₃ which have very limited ranges of thermal and phase stability. The procedures for writing waveguide structures where masking of the ion beam is necessary have been pioneered in the semiconductor industry. With the expanding demands for new optical circuitry both for the telecommunications industry, and to fulfill the dream of building an all-optical computer, the demand for planar lightwave circuits will increase. Ion implantation allows the well investigated advantages of existing (and any new) bulk hosts (e.g., good optical quality, wavelength tunability, price) to be combined with the advantages of waveguiding (e.g., beam confinement, low lasing threshold, compatibility with fibre/integrated optics) and has significant relevance to these applications.

The effect of radiation, initially fast neutrons from reactors, on materials has long been studied from the basic science perspective, as well as for its relevance to space science and nuclear power. The first concerted effort (and without the controversy over dose measurement that hampered earlier efforts) to look at the induced refractive index
changes (amongst a host of other physical properties) was performed by Primak[104]. His results showed an increase in refractive index of vitreous silica with neutron dose, up until about $50 \times 10^{18}$ neutrons/cm$^2$ after which there is a slow decrease with dose. This raised the possibility of creating a waveguiding layer in amorphous silica.

Fused silica has a simple chemical structure and a low refractive index, so any changes produced by particle bombardment tend to increase the refractive index of the material by compaction of the lattice or the addition of polarisable ions.

The first experiments aimed at producing a refractive index change appropriate to waveguiding were performed by Schineller et al.[105] who used 1.5MeV protons at a dose of about $10^{17}$ protons/cm$^2$. They also proposed the use of several implant energies to produce thick guides or two different energies to produce vertical directional couplers. The idea to use a mask so that complicated waveguide structures, such as directional couplers, could be produced was also suggested. This was implemented using a PMMA mask by Goell et al. [106] using Lithium ions at a range of energies from 32-200keV and doses from $2.4-10 \times 10^{14}$ ions/cm$^2$ to give a wide guide. Guiding was achieved for 632.8nm radiation from a He-Ne laser.

Silica has subsequently been studied with implantations with H$, He^+$, Li$, B^+$, Na$^+$, Ar$^+$ and Bi$^+$ [107] and at a wide range of energies.

The use of ion implantation to induce refractive index changes for guiding was, however, slow to develop because in most optical hosts the structure produced by the ion bombardment had a refractive index less than that of the original material (eg. soda lime glass due to Na loss; quartz, calcite, LiTaO$_3$, LiNbO$_3$, Al$_2$O$_3$ lose surface oxygen) [108]. Guides were formed by Destefanis et.al. [109] in LiNbO$_3$ by implanting a low index "barrier" region beneath the surface and using the relatively unchanged region between the barrier and the surface as a waveguide. No loss measurements were made. Wei et.al. [110] implanted lithium niobate samples with Ar$^+$ and Ne$^+$ specifically for the effect on the refractive index, but did not succeed in making waveguides.

Waveguides have also been produced in semiconductors, for example GaAs[111] and GaP[112] using protons and ZnTe[113] using protons and B$^+$ in the early 70s.

The first guides in Nd:YAG were produced by Arutyunyan et.al. [114] who implanted with $^4$He ions in the energy range of 2.1-2.7MeV. Measurements of the propagation constants of the waveguide modes were made and their properties with dose
examined. Calculations of the refractive index profile were performed, although the distinction between guided and leaky modes was not made in their analysis in which they used WKB method. Not much was published subsequently until Chandler et.al. produced a planar waveguide laser in 1989[115].

Chandler et.al. produced their guide in Nd:YAG by using a number of different energy implants of He$^+$ ions, with the substrate temperature held at 77K and with ion energies up to 2.8MeV and a total dose of $7 \times 10^{16}$ ions/cm$^2$. This gave a guide with a loss of $\sim 4$dB/cm unannealed and 1.5dB/cm annealed. The laser was pumped at 590nm with an absorbed energy threshold of $\sim 50$mW.

Multiple energy implants in Chandler’s experiments were necessary because at the maximum He$^+$ energy available (2.8MeV) the refractive index increase in the electronic excitation region was not sufficient to produce guiding (it was found that the change in refractive index saturated at about a +0.15% increase and hence required an ion energy of $> 3.5$MeV to achieve guiding) [116]. Curiously, for low ion dose they reported a slight increase of the refractive index in the nuclear stopping region. Implanting at 5 different energies (0.4, 0.8, 1.2, 1.6 and 2.0 MeV) at low dose gave 5 nuclear stopping region induced increases in the refractive index totalling $\sim 0.3\%$. This averaged to $\sim 0.25\%$ over the implant depth which was $\sim 4\mu m$ and allowed guiding.

In response to this work a large number of crystals have been experimented with [117] and to date lasers have been produced in Nd:YAG [115], Nd:YAP[118], Nd:MgO:LiNbO$_3$[119], Nd:GGG [120][121], Nd:Bi$_4$Ge$_3$O$_{12}$ [117], Yb:YAG[122] and Tm:LG/Glass [123][124] by helium ion implantation. Guiding in Nd:YAP and Nd:MgO:LiNbO$_3$ was achieved by several high energy implants giving a broad $>1\mu m$ barrier to a $\sim 3.5\mu m$ guide. This leads to pseudo guided modes as they are technically leaky but because the tunnelling fraction is low due to the thick barrier, low losses can be achieved. Implants into Bi$_4$Ge$_3$O$_{12}$, proved unusual since the unannealed implant gave an increase in the refractive index in both the electronic excitation and nuclear damage regions [125].

For lasers, the best figures to date indicate thresholds of $\sim 10$mW (absorbed power) [124] for Nd:YAG planar guides and 0.5mW [124] for a channel guide formed by masking the substrate prior to implanting. Lateral confinement in the channel was produced by the slight increase in refractive index provided by the ions.
Waveguide formation by implantation of other ions has not been as systematically studied as He$^+$ implants, but a number of guides have been produced with N$^{126}$ implants in SiO$_2$, H$^+$ implants to give photorefractive guides in BaTiO$_3$[127] and Ti implants into LiNbO$_3$ [128]. Mostly the choice of ion has been dependent on the available sources and either a light ion or one that was "chemically similar" to a host ion has been used.

In analysing the waveguides created by ion implantation a method is required to determine the refractive index profile created in the material. This turns out to be a far from trivial task especially when the index changes are rather small and the waveguide quite asymmetrical. This chapter therefore considers the various different methods for determining the index profile based on the measurement of the propagation constants of both guided and leaky modes of the ion implanted waveguide. The chapter starts with a discussion of the methods of predicting the propagation constants of these modes for waveguides with a known index profile before tackling the inverse problem: determining the index profile from measurements of the modes. It will be shown using trial profiles that many of the analytic techniques fail to accurately determine the index profiles and as a result a numerical fitting method was generally adopted to analyse the experimental data.

The experiments on the ion implanted waveguides themselves are presented in the following chapter which describes the use of Li ions to produce planar waveguides and lasers in Nd:YAG. This includes studying the effects of the implantation on the spectroscopy, waveguide loss and waveguide parameters as well the affect of annealing. Lithium ions were chosen as our implanter (National Electrostatics Corporation, 1.7MV tandem accelerator) required a solid ion source preventing the generation of He ions.
6.3 Experimental Determination of Refractive Index Profile - Prism Coupling or Dark Modes

Since the refractive index change produced by ion implantation occurs in a region \(\sim 10\mu m\) deep in the surface layer of a crystal, a major challenge is the determination of the refractive index profile in the implanted region as the profile itself cannot be measured directly. What can be measured are the propagation constants of the modes of the waveguide. This is commonly done by exciting these modes using a prism coupler as seems to have been first described by Tien et.al. [129] in 1969.
The principle of operation is shown in Figure 86. A prism with a refractive index substantially higher than that of the waveguide (i.e. > 110% $n_{\text{guide}}$) is placed almost in contact with the surface containing the waveguide. The laser beam enters the prism and strikes the base of the prism at an angle, $\theta$. Normally all the incident power would totally internally reflected. However, at the angles, $\theta_m$, the evanescent field from the reflection at the prism surface has the same phase velocity and polarisation as a mode (either guided or leaky) of the waveguide. The light is then able to tunnel from the prism into the guide exciting the mode and this registers as a decrease in the reflected beam intensity. The propagation constant/phase velocity of the mode can then be calculated according to $\beta = k n_{\text{prism}} \sin \theta$. Propagation constants are also often represented by their "effective index" given by $n_{\text{eff}} = n_{\text{prism}} \sin \theta$.

An experimental trace of the reflection as a function of angle (as measured in units of $n_{\text{eff}}$) is shown in Figure 87. This was measured for a Nd:YAG sample ($n_{\text{substrate}} = 1.8155$) with a 1064nm laser used as the light source. A dip in the reflectivity indicates coupling to a waveguide mode.
In principle, knowing the propagation constants or effective mode indices of all the waveguide modes, guided and leaky, enables the refractive index profile to be calculated. In practice this inverse problem is difficult. It relies on prior knowledge to provide a reasonable guess of the waveguide shape, and even then will yield a number of quite different profiles for which the propagation constants differ by less than the measurement accuracy.

\[
\begin{align*}
[\nabla^2 + k^2 n(x)^2 - \beta^2] \Psi &= 0
\end{align*}
\]  

(98)

where \( k = 2 \pi/\lambda \); \( \beta \) is the propagation constant of the mode and \( \Psi \) is either \( e_y \) (the electric field in the \( y \) direction) or \( h_y \) (the magnetic field in \( y \) direction). The details of this
derivation are in Appendix C.

If a given solution, \( \Psi \), to (98) has a propagation constant \( \beta \) which is real then the mode propagates in the refractive index profile given by \( n(x) \) without loss and is called a "guided mode". However there exist solutions to (98) for which \( \beta \) is complex. These modes have an inherent loss and are "leaky" or "unbound" modes of the guide. In some situations these leaky modes can have quite low inherent losses, for example when the waveguides are rather short, as is the case in our experiments. The real part of their propagation constants are measurable and they can be used to determine the structure of the guide providing more information than obtained from the guided modes alone.

The experimental trace in Figure 87 shows only one guided mode (at \( n_{\text{eff}} = 1.8162 \)) whilst at least 7 leaky modes are discernable.

### 6.5 Solutions for the ion implanted waveguide profile

It is expected that an ion implanted guide will have two distinct regions - a guiding region with refractive index elevated (if only slightly i.e. \( n_{\text{guide}} - n_{\text{substrate}} < 0.005 \)) above that of the substrate, and a barrier region (corresponding to the major energy loss region) where the refractive index is less than that of the substrate.

These two features mean that the expected refractive index profile will have only a very small number of guided modes; a large number of leaky modes with low loss (loss dependent on the barrier width); and a refractive index profile which is not monotonically decreasing but contains at least one turning point. The refractive index profile, except for the air/guide interface, is expected to be smoothly varying with depth. Measurements also suggest that the refractive index profiles "seen" by the different polarisations are different and so each polarisation should be considered separately.

These expectations put some strong limitations on the available analytical and numerical methods that can be used to analyse the mode structure.

In this section the analytical solutions for Step Index [130] and the Morse and Feshbach [133] refractive index profiles will be presented. The similarity between these...
profiles and those expected for an ion implanted waveguide and the fact that they have exact analytic solutions makes them useful in this context. The Matrix Method \cite{144,134} presented next is the method eventually implemented for solving the "forward" problem (ie. given a refractive index profile, find the modes) in the numerical optimisation routines.

Following this, the methods of Ulrich \cite{136}, Pogossian \cite{137} and several variations on the WKB method \cite{144,142} for "inverse" problem (ie. from the modes, calculate the refractive index profile) will be discussed.

6.5.1 Analytical Methods

6.5.1.1 Exact solutions

6.5.1.1.1 Step Index Solutions

One exact method for determining the propagation constants of the modes is to divide the refractive index profile into a number of smaller regions of constant refractive index, \( n_i \) and width \( w_i \). In this situation, (98) becomes:

\[
(V^2 + k^2 n_i^2 - \beta^2) \Psi = 0
\]  

(99)

for which the solutions are:

\[
\Psi_j = A_j e^{x\sqrt{\beta^2 - k^2 n_i^2}} + B_j e^{-x\sqrt{\beta^2 - k^2 n_i^2}}
\]  

(100)

Continuity of the fields must occur at the boundaries. For the case of TE modes \( e_y \) is continuous at the boundaries between regions of different \( n_i \)'s, whilst for TM it is \( h_y \) that is continuous. This leads to an equation of the form:

\[
A_{j+1} + B_{j+1} = A_j e^{\alpha_j w_j} + B_j e^{-\alpha_j w_j}
\]  

(101)

where \( \alpha_j = (\beta^2 - k^2 n_i^2)^{1/2} \). For TE modes continuity of \( h_x \) leads to:

\[
\alpha_j A_{j+1} - \alpha_j B_{j+1} = \alpha_j A_j e^{\alpha_j w_j} - \alpha_j B_j e^{-\alpha_j w_j}
\]  

(102)

whereas for TM modes continuity of \( e_z \) leads to:
This gives a set of simultaneous linear equations. The modes of the waveguide are then the non trivial solutions (ie. all $A_i's$ and $B_i's$ not equal to zero) of this set of simultaneous equations. Combining (101) and (102) or (103) into a single large matrix, ie.

\[
\begin{bmatrix}
1 & 1 & -e^{a_{j-1}} & 0 & 0 & \cdots \\
0 & 0 & 1 & 1 & -e^{a_{j-1}} & -e^{-a_{j-1}} & \cdots \\
0 & 0 & a_{j-1} & 0 & 0 & \cdots \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & a_{j-1} & 0 & 0 & \cdots \\
\end{bmatrix}
\begin{bmatrix}
A_0 \\
B_0 \\
A_1 \\
B_1 \\
A_2 \\
B_2 \\
\vdots \\
A_j \\
B_j \\
\end{bmatrix}
\]

(104)

where $g_j = 1$ for TE modes and $g_j = 1/n_j^2$ for TM. The eigenvalues of this matrix represent the modes of the waveguide.

Alternatively (101) and (102) can be represented in the form:

\[
\begin{bmatrix}
A_{j-1} \\
B_{j-1}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
1 + \frac{\alpha_j g_j}{\alpha_{j-1} g_{j-1}} & e^{a_j} \\
1 - \frac{\alpha_j g_j}{\alpha_{j-1} g_{j-1}} & e^{-a_j}
\end{bmatrix} \begin{bmatrix}
1 - \frac{\alpha_j g_j}{\alpha_{j-1} g_{j-1}} & e^{a_j} \\
1 + \frac{\alpha_j g_j}{\alpha_{j-1} g_{j-1}} & e^{-a_j}
\end{bmatrix} \begin{bmatrix}
A_j \\
B_j
\end{bmatrix}
\]

(105)

In this way a matrix, $T$, for the structure can be formed according to:

\[
T = \prod_{j=1}^{N} m_j
\]

(106)

A mode exists at a value of $\alpha$ (and hence $\beta$) for which one of the elements of $T$, usually $T_{21}$ is zero.

This method has been used by Smith et al. [130] and has the advantage that by exploiting the Cauchy Integral Theorem that if there are no zeros of $f(z)$ on the chosen
contour and there are no poles within the closed contour then \( S_0 \), given by

\[
S_m = \frac{1}{2\pi i} \oint_C \frac{f'(z)}{f(z)} \, dz
\]  

(107)

gives the number of roots of \( f(z) \) inside the contour. Also, a polynomial \( p(z) \) defined by:

\[
p(z) = \sum_{i=1}^{S_0} c_i z^i
\]  

(108)

can be formed that has the same roots as \( f(z) \) where:

\[
c_k = \frac{1}{k - S_0} \sum_{j=1}^{S_0 - k} S_j c_{k+j}
\]  

(109)

The authors suggest also that the roots be found in terms of \( \alpha_N \) where now \( \alpha_j = (\alpha_N^2 - (k^2 n_j^2 - k^2 n_N^2))^{1/2} \). In this space all the guided modes are then found on the real axis (in the complex plane) in the region from \( 0 < \alpha_N < k (n_{\text{max}}^2 - n_0^2)^{1/2} \) where \( n_{\text{max}} \) is the maximum refractive index in the structure.

The leaky modes are better modelled by using the representation:

\[
m_j = \frac{1}{2} \left[ \begin{array}{c}
1 + \frac{\eta_j g_j}{\eta_{j-1} g_{j-1}} e^{-i\eta_j} \\
1 - \frac{\eta_j g_j}{\eta_{j-1} g_{j-1}} e^{i\eta_j}
\end{array} \right] \left[ \begin{array}{c}
1 - \frac{\eta_j g_j}{\eta_{j-1} g_{j-1}} e^{i\eta_j} \\
1 + \frac{\eta_j g_j}{\eta_{j-1} g_{j-1}} e^{-i\eta_j}
\end{array} \right] \]  

(110)

where \( \eta_j = -i \alpha_j = (k^2 n_j^2 - \beta^2)^{1/2} = (k^2 (n_j^2 - n_0^2) + \eta_0^2)^{1/2} \). Where the solutions are again along the real axis in the \( \eta_N \) plane.
The simplest example of a waveguide, the three layer step index guide is shown in Figure 89. In our cases, $n_0$ will usually be air ($n=1.0$) and $n_2$ will be the refractive index of the substrate/bulk material. The modes of such a waveguide are the solutions of:

$$\tanh(a_1 w_1) + \frac{\alpha_0 \alpha_1 g_0 g_1 + \alpha_1 \alpha_2 g_1 g_2}{\alpha_1 g_1^2 + \alpha_0 \alpha_2 g_0 g_2} = 0$$

(111)

[131] whilst for the four layer waveguide (Figure 90) the mode equation becomes:

$$\tanh(a_1 w_1) + \frac{\alpha_1 g_1 (\alpha_0 \alpha_2 g_0 g_3 + \alpha_2 \alpha_3 g_2 g_3) + \alpha_1 g_1 (\alpha_0 \alpha_2 g_0 g_2 + \alpha_2 g_3^2) \tanh(a_3 w_3)}{\alpha_3 g_3 (\alpha_1 g_1^2 + \alpha_0 \alpha_2 g_0 g_2) + (\alpha_1^2 + \alpha_2^2 + \alpha_0^2 \alpha_3^2) \tanh(a_3 w_3)} = 0$$

(112)

[132]. A comparison of Figures 88 and 90 shows that the four layer guide is a crude approximation to the expected ion implanted refractive index profile.

With more regions the problem rapidly becomes intractable and despite the apparent simplicity of the equations above they form a problem that is already ill conditioned numerically.

As an aside, when $\alpha_3 w_3 \to \infty$, $\tanh[\alpha_3 w_3] \to 1$ and (112) becomes:
\[
\tanh[\alpha_3 d] = \frac{\alpha_1 (\alpha_0 \alpha_2 + \alpha_3) + \alpha_3 (\alpha_0 \alpha_2 + \alpha_3)}{\alpha_3 (\alpha_1 + \alpha_0 \alpha_2) + (\alpha_1 \alpha_2 + \alpha_0 \alpha_3)} \\
= -\frac{(\alpha_1 \alpha_0 + \alpha_1 \alpha_3)(\alpha_2 + \alpha_3)}{(\alpha_1 + \alpha_0 \alpha_3)(\alpha_2 + \alpha_3)} \\
= -\frac{(\alpha_1 \alpha_0 + \alpha_1 \alpha_3)}{(\alpha_1 + \alpha_0 \alpha_3)} \\
\]

(113)

(i.e. Despite their apparent differences (112) → (111) for \( \alpha_3 w_3 \to \infty \)).

**Figure 91**  Waveguide in isolation and Waveguide with prism coupler

The effect of the prism used to measure the modes, on the modes themselves needs to be minimised. The comparison of the three layer guide in isolation (substrate, guide, air) and the quasi-four layer guide (substrate, guide, air, prism) with the inclusion of the high index prism used to measure the waveguide modes is shown in Figure 91. As just noted, when \( \tanh[\alpha_3 d] \to 1 \), (112) → (111). So, for the case of a waveguide in YAG where \( n_{YAG} = 1.83 \) and \( \lambda = 1.064 \mu m \) gives \( \alpha_3 = 9.05 \times 10^6 m^{-1} \). Hence, the required air gaps are 0.16\( \mu m \) (\( \tanh[x] = 0.9 \)), 0.20\( \mu m \) (\( \tanh[x] = 0.95 \)) and 0.29\( \mu m \) (\( \tanh[x] = 0.99 \)) respectively. As can be seen, for most achievable air gaps the effect of the prism is negligible.
6.5.1.1.2 Morse and Feshbach Index Profile

A smooth potential, which is a reasonable match to the expected waveguide profile (and is shown in Figure 92) comes from Morse and Feshbach [133] in which the refractive index takes the form:

$$n^2(x) = n_0^2 + n_1^2 \cosh^2 a \left[ \tanh \left( \frac{x - ad - r}{d} \right) + \tanh a \right]^2$$  \hspace{1cm} (114)

This defines a profile with an asymmetric dip whose width is dependent on d, with a minimum index value of $n_0$ at $x=r$; a left side index value of $(n_0^2 + n_1^2 \ e^{-2\alpha})^{1/2}$ and the right side index value of $(n_0^2 + n_1^2 \ e^{2\alpha})^{1/2}$. After much straight forward, if tedious, algebra solutions to the wave equation can be found in the form:

$$\Psi = e^{-z} \text{sech}^2 \frac{z}{2} \left( h + \frac{1}{2} \sqrt{\frac{1}{4} - A^2 \cosh^2 a} h + \frac{1}{2} \sqrt{\frac{1}{4} - A^2 \cosh^2 a} \right)$$  \hspace{1cm} (115)

where $F(a,b;c;x)$ is the Hypergeometric function, $z = (x - ad - r)/d$, $A = k n_1 d$, $B = (\beta^2 d^2 - k^2 n_0^2 d^2)^{1/2}$ and $g$ and $h$ are the solutions of

$$g^2 + h^2 - B^2 + A^2 \cosh 2a = 0 \quad 2gh + A^2 \sinh 2a = 0$$  \hspace{1cm} (116)
Continuity of $\Psi$ and $d\Psi/dx$ at the surface of the guide (ie. at $x=0$) gives the eigenvalue equation

$$\sqrt{\beta^2-k^2n_s^2} = -(g+h \tanh z) - \frac{\alpha \beta}{\gamma} F\left(\alpha+1,\beta+1,\gamma+1;\frac{e^{-z}}{e^z+e^{-z}}\right) \left[ \frac{e^{-z}}{e^z+e^{-z}} \right] \left[ e^{-z} \right] \left( e^z+e^{-z} \right)^2$$

where

$$\alpha = h + \frac{1}{2} \sqrt{\frac{1}{4} - A^2 \cosh^2 a}$$

$$\beta = h + \frac{1}{2} \sqrt{\frac{1}{4} - A^2 \cosh^2 a}$$

$$\gamma = 1 + g + h$$

This has exact solutions, although difficult to calculate.

### 6.5.2 Approximate solutions

The need to obtain a range of solutions for not just the bound modes but also for the "leaky" modes, combined with the fact that the refractive index profile is discontinuous at the guide/air interface, is asymmetric, and leads to an eigenvalue problem with boundaries at $\pm \infty$, means that there are very few methods for predicting the mode indices that are sufficiently accurate and numerically or analytically tractable.

#### 6.5.2.1 Matrix Method

One method, first published by Chandler and Lama [134] and then refined by Mathey et.al. [144] treats the prism, the air gap and the refractive index structure itself as a big multilayer dielectric stack. Hence, the set of electromagnetic fields $e_j(z)$ and $h_j(z)$ is related to the electromagnetic fields in the next region $e_{j+1}(z)$ and $h_{j+1}(z)$ by the relation

$$(119)$$

$$\begin{bmatrix} e(z) \\ h(z) \end{bmatrix} = \begin{bmatrix} \cos(kw_j) & -i \sin(kw_j) \\ ig_j & \cos(kw_j) \end{bmatrix} \begin{bmatrix} e(z+w_j) \\ h(z+w_j) \end{bmatrix} = N_j \begin{bmatrix} e(z+w_j) \\ h(z+w_j) \end{bmatrix}$$
where $g_j = k_j / (\mu_o c k)$ for TE modes, $g_j = n_j^2 k_j / (\mu_o c k)$ for TM modes, $k_j^2 = k_0^2 (n_j^2 - n_m^2)$. The entire structure is hence represented by:

$$\prod_{j=1}^{N} = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}$$

(120)

and the reflection of the structure is given by:

$$r = \frac{g_0 n_{11} + g_0 g_{n+1} n_{12} - n_{21} - g_{n+1} n_{22}}{g_0 n_{11} + g_0 g_{n+1} n_{12} + n_{21} + g_{n+1} n_{22}}$$

(121)

and the intensity reflection coefficient is given by:

$$R = |r|^2$$

(122)

The modes of the waveguide occur when $R = 0$. Hence, finding the waveguide modes becomes a case of looking for the solutions of:

$$g_0 n_{11} + g_0 g_{n+1} n_{12} - n_{21} - g_{n+1} n_{22} = 0$$

(123)

This method has the advantage that it is able to calculate the bound and leaky modes using the same numerical routine. It also gives a "realistic" response for the reflectivity, in that the dark modes get broader at higher orders (cf. Figure 87). As will be demonstrated later, this method proved important in solving the inverse problem of obtaining the index profile from the measured mode indices.

### 6.5.3 The Inverse Problem

In general the aim was not to find the dark modes of a given refractive index profile, but the inverse problem - to try to reconstruct the refractive index profile from the experimentally determined dark mode effective indices. One approach to solving this problem is calculate numerically the mode indices for a "guessed" profile defined by as few variables as practical (for example, a step index with a variable width and height) and adjust these variables until the best fit between predicted and measured mode indices is obtained. However, there are several other methods which have been presented for directly solving this inverse problem in specific cases. In what follows these direct methods are
described and compared with the numerical optimization method outlined above.

6.5.3.1 Ulrich Method

The oldest and simplest of the methods of solving inverse problem was provided by Ulrich et. al. [136]. A mode of a guide of width $w_l$ and constant refractive index $n_1$ (i.e. a three layer step index guide) satisfies the equation:

$$kw_l\sqrt{n_1^2-n_m^2} = \Psi_m(n_1,n_m)$$  \(124\)

where:

$$\Psi_m(n_1,n_m) = m\pi + \Phi_0(n_1,n_m) + \Phi_2(n_1,n_m)$$  \(125\)

and:

$$\Phi_j(n_1,n_m) = \arctan \left[ \frac{\sqrt{n_m^2-n_{j+1}^2}}{\sqrt{n_m^2-n_{j-1}^2}} \right]$$  \(126\)

where $n_0 = n_c$ is the refractive index of the covering layer, usually air; $n_2 = n_s$ is the refractive index of the substrate; and $r_s = 1$ for TE modes and $r_s = (n_1/n_0)^2$ for TM modes. So if two modes, $N_\mu$ and $N_v$, are known then (124) can be rearranged to give:

$$n_1^2 = F(n_1^2)$$  \(127\)

where:

$$F(n_1^2) = \frac{N_\mu^2\Psi_\mu^2 - N_v^2\Psi_v^2}{\Psi_\mu^2 - \Psi_v^2}$$  \(128\)

The equation (127) has $n_1$ on both sides, but as written it forms an iterative scheme for solving for $n_1$ from two modes for which the mode numbers $\mu$ and $v$ are known. When $n_1$ is known, it is comparatively simple to calculate $w_l$ from (124). If more than two modes are known, then it is usual to consider all the known modes in pairs and calculate a set of $n_1$'s and $w_l$'s. The mean and standard deviation of this set of solutions gives an indicator of the accuracy of the fit (and its inherent assumptions).
### 6.5.3.2 Pogossian Method

Another method for the inverse problem comes from the work of Pogossian et al. [137] If the three layer guide (air, guide, substrate) is sufficiently large to allow the use of ray optics then it is readily possible to derive the condition for a mode to be guided as [138]

\[
2q_m k w_1 - 2\tan^{-1}\left(\frac{r_m}{q_m}\right) - 2\tan^{-1}\left(\frac{p_m}{q_m}\right) = 2\pi m
\]  

(129)

where \(q_m^2 = n_f^2 - n_m^2\), \(r_m^2 = n_m^2 - n_c^2\), \(p_m^2 = n_m^2 - n_s^2\), where \(n_f\) is the index of the film; \(n_c\) is the refractive index of the cover/air; and \(n_s\) is the refractive index for the substrate; and \(w_1\) is the thickness of the guide. Note that after taking the tan of both sides and expanding, this equation can be converted to:

\[
\tan U w_1 = \frac{U W_1 + U W_2}{U^2 - W_1 W_2}
\]

(130)

which is the result of using the Snyder and Love [139] theory and solving the differential equations (it is much easier to go from (129) to (130) than the other way). Now, by recognising that the Arctans give back angles and constructing triangles with sides of \(r_m\), \(q_m\) and \((r_m^2 + q_m^2)^{1/2} = n_f^2 - n_c^2 = c_1\) so that \(\tan^{-1} r_m/q_m = \pi/2 + \sin^{-1}(q_m/c_1)\) allows (130) to be transformed to:

\[
q_m k w_1 = \pi (m + 1) - \sin^{-1}\left(\frac{q_m}{c_1}\right) - \sin^{-1}\left(\frac{q_m}{c_2}\right)
\]

(131)

ie. this has the form of:

\[
y(x). x = f(y)
\]

(132)

This special type of equation can be approximated by a Lagrange series expansion [140] so that

\[
y(x) = \frac{f(0)}{x - f^{(1)}(0)} + \frac{f^2(0)f^{(2)}(0)}{2[x - f^{(1)}(0)]^3} + \frac{f^3(0)f^{(3)}(0)}{6[x - f^{(1)}(0)]^4} + \frac{f^4(0)[f^{(2)}(0)]^2}{2[x - f^{(1)}(0)]^5} + \ldots
\]

(133)

Expanding \(f(q_m)\), where:
about $q_m=0$ and noting that:

\[
x = kw_1 \\
J_0 = \pi (m+1) \\
J_1 = \left( \frac{1}{c_1} + \frac{1}{c_2} \right) c_2 - n_f^2 - n_s^2 \\
J_2 = 0 \\
J_3 = \left( \frac{1}{c_1^3} + \frac{1}{c_2^3} \right)
\]

which gives:

\[
q_m = \frac{\pi (m+1)^3 \left( \frac{1 + 1}{c_1^3 c_2^3} \right)}{6H^4} + ...
\]

where

\[
H = kw_1 + \frac{1}{c_1} + \frac{1}{c_2}
\]

now remembering that $q_m^2 = n_f^2 - n_s^2$ allows (136) to be rearranged to give:

\[
n_m^2 = n_f^2 - \frac{\pi^2 (m+1)^2}{H^2} \left[ \pi^2 \left( \frac{1 + 1}{c_1^3 c_2^3} \right)^2 \left( \frac{(m+1)^4}{3H^5} \right)^2 + \frac{\pi^6 \left( \frac{1 + 1}{c_1^3 c_2^3} \right)^2}{36H^8} \right] (m+1)^6
\]

so that if the experimental modes ($n_m$) are plotted against $(m+1)^2$, and the coefficients to the fit determined by least squares, then it is possible to calculate $n_f$, $n_s$ and $w_1$ ($n_c$ is assumed to be known).

For a three layer step index guide structure, the equivalent equation to (138) for the TM modes of the structure is readily determined to be:
\[ n_m^2 = n_j^2 - \frac{\pi^2}{K^2(m+1)^2} \]

\[ n_m^2 = n_j^2 - \frac{\pi^2}{3K^2(m+1)^2} \]

\[ K = k_n + \frac{n_n^2}{c_1^2} + \frac{n_n^2}{c_2^2} \]

If the more general 4 layer step index guide is considered, the situation changes.

From the differential equations for the 4 layer guide it is possible to derive the condition:

\[ \frac{U(W_4 W_2 - W_1 W_2) + U(W_1 W_2 - W_1 W_2) \tanh[W_3 W_2]}{W_3(U^2 - W_1 W_2) + (U^2 W_2 - W_1 W_2) \tanh[W_3 W_2]} = 0 \]

which is equivalent to the equation:

\[ q_m k_n W_1 - \tan^{-1}\left( \frac{p_m}{q_m} \right) - \tan^{-1}\left( \frac{r_m s_m + s_m^2 \tanh[s_m W_2]}{q_m s_m + q_m r_m \tanh[s_m W_2]} \right) = 2\pi m \]

where

\[ q_m^2 = n_j^2 - n_m^2 \quad p_m^2 = n_m^2 - n_c^2 \quad r_m^2 = n_m^2 - n_s^2 \quad s_m^2 = n_m^2 - n_1^2 \]

This is readily converted to:

\[ q_m k_n W_1 = \pi(m+1) - \tan^{-1}\left( \frac{q_m}{\sqrt{c_1^2 - q_m^2}} \right) - \tan^{-1}\left( \frac{q_m \sqrt{c_3^2 q_m^2 + c_2^2 q_m^2 \tanh[c_3^2 - q_m^2 W_2 k]}}{\sqrt{(c_3^2 - q_m^2)(c_3^2 - q_m^2) + (c_3^2 - q_m^2 \tanh[c_3^2 - q_m^2 W_2 k])}} \right) \]

and noting that:
\[ f(0) = \pi (m+1) \]
\[ f^{(1)}(0) = -\frac{A}{c_1} \frac{1}{B} \]
\[ f^{(2)}(0) = 0 \]
\[ f^{(3)}(0) = \frac{1}{c_1^3} + 2 \frac{A^3}{B^3} \]
\[ 3A \left( \frac{-c_2^2 + c_3^2}{c_2 c_3} - c_3 d \text{sech}^2[c_3 w_2 k] - 2 \text{tanh}[c_3 w_2 k] \right) \]
\[ \frac{1}{B^2} \left( \frac{3 + 3c_2 w_2 k \text{sech}^2[c_3 w_2 k]}{c_3} + \frac{3 \text{tanh}[c_3 w_2 k]}{c_2} \right) \]
\[ A = c_3 + c_2 \text{tanh}[c_3 w_2 k] \]
\[ B = c_2 c_3 + c_3 ^2 \text{tanh}[c_3 w_2 k] \]
gives:
\[ n_m^2 = n_f^2 - \frac{\pi^2}{H^2 (m+1)^2} - \frac{f^{(3)}(0) \pi^4}{3H^5 (m+1)^4} - \frac{(f^{(3)}(0))^2 \pi^6}{36H^8 (m+1)^6} \]  
(146)

Also, once it has been recognised that the equation for the prism coupler modes has the form:
\[ n_m^2 = n_f^2 + b_2 (m+1)^2 + b_4 (m+1)^4 + b_6 (m+1)^6 + .. \]  
(147)

then by using each equation to eliminate higher terms in the next it is possible to derive equations for \( n_f \) from the measured mode indices, \( n_m \), in a given experiment by:
\[ n_f^2 = \frac{4n_2^2}{3} - \frac{n_1^2}{3} \]
\[ = \frac{3n_0^2}{2} - \frac{3n_1^2}{5} + \frac{n_2^2}{10} \]
\[ = \frac{8n_0^2}{5} - \frac{4n_1^2}{35} + \frac{8n_2^2}{35} - \frac{n_3^2}{35} \]
\[ = \ldots \]  
(148)

A number of tests of waveguide profiles close to those expected to be created by ion implantation were made to test the method. A lot of difficulty was realised in
calculating the profiles from the given parameters. In practice to solve for the three unknowns in (147) is a very ill-posed problem. In practice the data is fitted by a quadratic, and from this $H$ and $f^{(3)}(0)$ are determined. A value for $w_2$ is guessed and from that $n_f$ and $w_1$ are calculated. The modes for this profile are then calculated and the quality of the fit is determined. A one variable optimisation routine chooses the best fit.

The method for TM modes is similar, (144) becomes:

$$q_m kw_1 = \pi(m + 1) - \tan^{-1}\left(\frac{q_m n_c^2}{n_f \sqrt{c_2^2 - q_m^2}}\right)$$

$$- \tan^{-1}\left(\frac{q_m n_1^2}{n_f \sqrt{c_3^2 - q_m^2} + n_1^2 \sqrt{c_1^2 - q_m^2}}\right)\left[\frac{c_3^2 - q_m^2 w_2 k}{c_3^2 - q_m^2 w_2 k}\right]$$

$$= f(q_m)$$

(149)

and noting that:

$$f_{tm}^{(0)} = \pi(m + 1)$$

$$f_{tm}^{(1)}(0) = -\frac{n_c^2}{n_f c_1} - \frac{C}{D}$$

$$f_{tm}^{(2)}(0) = 0$$

$$f_{tm}^{(3)}(0) = -\frac{3n_c^2}{c_3 n_f^2} + \frac{2n_c^6}{c_3 n_f^6} + \frac{2C^3}{D^3} + \frac{n_1^2 n_f^2 + c_1 d_k n_f n_1^2 c_3^4 Sech^2[c_3 w_2 k]}{D} + \frac{n_1^4 Tanh[c_3 w_2 k]}{c_3} + \frac{n_1^4 Tanh[c_3 w_2 k]}{c_1}$$

(150)

$$+ \frac{3 c_3 c_1 n_f^2 n_1^2 + c_3 d_k n_f n_s^2 Sech^2[c_3 w_2 k] + 2n_f n_s^2 Tanh[c_3 w_2 k]}{D^2}$$

$$C = c_1 n_1^2 n_f^2 + c_1 n_1^4 Tanh[c_3 w_2 k]$$

$$D = c_1 c_3 n_f^2 n_1^2 + c_3 n_f^2 n_s^2 Tanh[c_3 w_2 k]$$

gives:

$$n_m^2 = n_f^2 \frac{\pi^2}{K^2} (m + 1)^2 - \frac{f_{tm}^{(3)}(0) \pi^4}{3K^5} (m + 1)^4 - \frac{(f_{tm}^{(3)}(0))^2 \pi^6}{36K^8} (m + 1)^6 + ...$$

(151)
The disadvantage of the Pogossian method is that it cannot be performed for a waveguide for which the eigenvalue equation is not known. Another disadvantage of the method is that it relies on an expansion about \( q_m = 0 \). This is equivalent to an expansion around \( B = 0 \) for the potential in (114) and while we have an eigenvalue expression (117) for this guide it is not readily obvious how it depends on \( B \).

6.5.3.3 WKB Method

It has been realised since the very early days of optics that there were similarities between the Schrödinger equation:

\[
\left(-\frac{\hbar^2}{8\pi^2m} \frac{\partial^2}{\partial z^2} + V(z)\right)\Psi(z) = E\Psi(z)
\]

and the equations for the modes:

\[
\left(\frac{1}{k^2} \frac{\partial^2}{\partial z^2} - n^2(z)\right)\Psi(z) = -n_m^2\Psi(z)
\]

and hence the well established Wentzel-Kramers-Brillouin (WKB) approximation from quantum mechanics can be applied [141][134]. In its formulation in the optical regime, the modes of a refractive index profile, \( n(z) \) are the solutions of the equation:

\[
I = k \int_0^{z(m)} \sqrt{n^2(z) - n_m^2} \, dz = m\pi + \Phi_a + \Phi_b + \rho_m
\]

\[
n(z(m)) = n_m
\]

where \( \Phi_a \) is the phase jump at the "top" interface, usually considered to be a step between the guide and the air and is given by:

\[
\Phi_a(n_m) = \tan^{-1}\left[r_a \left(\frac{n_m^2 - n_c^2}{n(0)^2 - n_m^2}\right)\right]
\]

[142] where \( r_a = 1 \) for TE modes and \( r_a = (n(0)^2/n_c^2) \) for TM modes. The refractive index change at the "bottom" "turning point", \( \Phi_b \), is generally accepted to be [142] \( \pi/4 \) if the variation of the refractive index is sufficiently slow.

Despite its apparent simplicity (154) still only has analytical solutions for a very
limited number of profiles (namely step, linear, parabolic, exponential, Gaussian, hyperbolic secant\cite{143}, Fermi \cite{144}). The method has the disadvantage that the function may have only two points on the profile where the refractive index has the same value. The anticipated refractive index profile for an implanted guide has three (at the air interface, going down into the refractive index dip, and coming up the other side). Furthermore the method is not well suited to the calculation of leaky modes. Its value is in the ability to calculate from the refractive indices themselves, without requiring a guess for the profile shape (see below).

The importance of the WKB method for the study of our profile lies in some approximations to the integral \eqref{154} which enable the refractive index profiles to be built up from the effective refractive indices of the modes themselves and a few assumptions (eg. that the refractive index profile is slowly varying, monotonically decreasing).

The method of \cite{142} is to fit the measured modes as a function of the mode number, \( m \), to a polynomial, ie. \( N(m) \). The surface refractive index is given by \( N(-0.75) \) and a number of regions to calculate is chosen, for example 100 (giving \( m_0 \) through to \( m_{100} \)) and from there a series of refractive indices are calculated (ie. \( N(m_i) \)). If we assume that the refractive index profile consists of a series of steps of refractive index \( N_i=(N(m_i)+N(m_{i+1}))/2 \) then the integral in \eqref{154} can be replaced by the sum

\[
I=k\left[\sqrt{N_1^2-n_1^2}(z_1-z_0)+\sqrt{N_2^2-n_2^2}(z_2-z_1)+...+\sqrt{N_i^2-n_i^2}(z_i-z_{i-1})\right]
\]

(156)

Using (156) and \( z_0 = 0 \) enables us to calculate the rest of the function giving

\[
z_i=\frac{\rho_i}{k}\sum_{j=1}^{i-1}z_j\left(\sqrt{\tilde{N}_j^2-n_i^2}-\sqrt{\tilde{N}_{j+1}^2-n_i^2}\right)
\]

(157)

where:

\[
\rho_i=m_i\pi+\frac{\pi}{4}+\Phi_0(N(m_i))
\]

(158)

Recovering the refractive index profile becomes a case of applying (157) iteratively.
Alternatively, from Mathey et al. [144] we approximate $I$ in (154) by:

$$I = \sum_{i=1}^{m} \int_{z_i}^{z_{i+1}} \left[ n_i + \frac{n_{i-1} - n_i}{i - z_i} \right]^2 \left[ n_m - n_i \right]^{2} \, dz$$

(159)

hence:

$$z_m = z_{m-1} + \frac{2n_m - n_{m-1}}{n_m^2}$$

$$= \frac{n_m - 1}{n_m} \left[ \left( \frac{n_{m-1}}{n_m} \right)^2 - 1 \right] - \ln \left[ \frac{n_{m-1}}{n_m} + \left( \frac{n_{m-1}}{n_m} \right)^2 - 1 \right]$$

$$[\frac{\lambda}{4\pi} (2m\pi + 2(\phi_\alpha(n_m) + \phi_\beta))]$$

(160)

and

$$z_1 = \frac{n_0 - n_1}{n_1}$$

$$= \frac{n_0}{n_1} \left[ \left( \frac{n_0}{n_1} \right)^2 - 1 \right] - \ln \left[ \frac{n_0}{n_1} + \left( \frac{n_0}{n_1} \right)^2 - 1 \right]$$

(161)

This leaves one free parameter, $n_0$, which is chosen to give the smoothest profile. In practice this involves minimising the areas of triangles with vertices $(n_i z_i), (n_{i+1}, z_{i+1})$, and $(n_{i+2}, z_{i+2})$.

### 6.6 Testing the methods

In the following, a number of the methods will be tested on examples expected to be close to the refractive index of the experimentally produced ion implanted waveguides.
The various symbols are defined with reference to Figures 93 and 95.

The cases considered are:

a) three layer step index profile guides (8μm guide width (w), guide refractive index (nₐ) = 1.83 (ie. YAG like):

\[ n_c \rightarrow n_1 \rightarrow w \rightarrow n_s \]

![Figure 93](image)

**Figure 93** Three layer step index guide for comparison of methods

\[ n_c \rightarrow n_1 \rightarrow w_1 \rightarrow n_2 \rightarrow w_2 \rightarrow n_s \]

![Figure 94](image)

**Figure 94** Four layer step index guide for comparisons

i) \( n_s = 1.5 \) (10+ guided modes, see Table 7)

ii) \( n_s = 1.8 \) (5 guided modes)

iii) \( n_s = 1.828 \) (1 guided mode)

b) double step profile guides (8μm guide width (w₁), guide refractive index \( n_1 = 1.83 \) with a “barrier” region of refractive index \( n_2 \) and width \( w_2 \)

i) \( n_2 = 1.4, n_s = 1.5 \), width of \( n_2 \) region = \( w_2 = 1 \) μm, 0.1μm and 0.01μm (10+ guided modes, see Table 8)

ii) \( n_2 = 1.75, n_s = 1.8, w_2 = 1 \) μm, 0.1μm and 0.01μm (5 guide modes, 3 leaky modes)

---

2 All the test cases presume air above the guide (ie. \( n_c = 1 \)) and are for \( \lambda = 1.064 \)μm (ie. Nd:YAG laser).
(b iii is expected to be close to the real profile. Others are included as transitions to this difficult case). The modes were calculated using Eqns (111) and (112) which give exact

| $n_s=1.8$, $n_r=1.828$, $w_2=1\mu m$, $0.1\mu m$ and $0.01\mu m$ (1 guided mode, 4 leaky modes) |

| $n_s=1.8$ | 1.82896062 | 1.82893762 |
| $n_s=1.828$ | 1.82926214 | 1.82925075 |

| $n_s=1.8$, $n_r=1.83$, $w=8\mu m$, $\lambda=1.064\mu m$ |
| $n_s=1.5$ | 1.8286959 | 1.82893762 |
| $n_s=1.8$ | 1.82896062 | 1.82893762 |

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<th>TM Modes</th>
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<tr>
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<td>1.77867322</td>
</tr>
<tr>
<td>1.75635707</td>
<td>1.76134235</td>
</tr>
<tr>
<td>1.73637783</td>
<td>1.74132993</td>
</tr>
</tbody>
</table>

| $n_s=1.8$, $n_r=1.83$, $w=8\mu m$, $\lambda=1.064\mu m$ |
| $n_s=1.5$ | 1.8286959 | 1.82893762 |
| $n_s=1.8$ | 1.82896062 | 1.82893762 |

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<td>1.82575764</td>
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<td>1.81320307</td>
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<td>1.80418726</td>
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<td>1.73637783</td>
<td>1.74132993</td>
</tr>
</tbody>
</table>

| $n_s=1.8$, $n_r=1.83$, $w=8\mu m$, $\lambda=1.064\mu m$ |
| $n_s=1.5$ | 1.8286959 | 1.82893762 |
| $n_s=1.8$ | 1.82896062 | 1.82893762 |

<table>
<thead>
<tr>
<th>TE Modes</th>
<th>TM Modes</th>
</tr>
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<tbody>
<tr>
<td>1.8286959</td>
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<td>1.82547458</td>
<td>1.82575764</td>
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<td>1.81980367</td>
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<td>1.80154999</td>
<td>1.80418726</td>
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<td>1.79339814</td>
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<td>1.77385837</td>
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<td>1.75635707</td>
<td>1.76134235</td>
</tr>
<tr>
<td>1.73637783</td>
<td>1.74132993</td>
</tr>
</tbody>
</table>

| $n_s=1.8$, $n_r=1.83$, $w=8\mu m$, $\lambda=1.064\mu m$ |
| $n_s=1.5$ | 1.8286959 | 1.82893762 |
| $n_s=1.8$ | 1.82896062 | 1.82893762 |

<table>
<thead>
<tr>
<th>TE Modes</th>
<th>TM Modes</th>
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<td>1.82048738</td>
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<tr>
<td>1.73637783</td>
<td>1.74132993</td>
</tr>
</tbody>
</table>

**Table 7** Propagation Constants for the TE and TM modes of a 3 layer step index waveguides solutions for their respective profiles.
Table 8  
TE and TM modes for 4 layer step index guides

As can be seen from Tables 7 and 8 there is very little difference between the modes for these vastly different refractive index profiles and this makes any sort of fitting and calculation routine difficult to implement.

Based on these cases a comparison of the various methods was attempted. The “8 dec places” columns in a number of the tables mean that the calculation method was given the mode values to the full 8 decimal places they were originally calculated to. However, the prism coupler we use to take the measurements only gives results to the 4th decimal place and so the results were calculated again with methods given only the data to “experimental accuracy” to see if this affected the results significantly.
6.6.1 Three Layer Step Index Guide

Table 9 shows a comparison of the results obtained with "Ulrich Method" (ie. calculated using a routine based on (127)). The correct results (ie. values that were used to calculate the original mode profiles) are \( n_i=1.83 \) and \( w_i=8\mu m \). Common across all the

<table>
<thead>
<tr>
<th>( n_i=1.83, \lambda=1.064\mu m, )</th>
<th>Ulrich (8 dec places)</th>
<th>Ulrich (4 dec places)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_i=8\mu m )</td>
<td>( n_i )</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>TE</td>
<td>( n_i=1.5 )</td>
<td>1.829999998</td>
</tr>
<tr>
<td></td>
<td>( n_i=1.8 )</td>
<td>1.82946535</td>
</tr>
<tr>
<td></td>
<td>( n_i=1.828 )</td>
<td>1.83112954</td>
</tr>
<tr>
<td>TM</td>
<td>( n_i=1.5 )</td>
<td>1.83000002</td>
</tr>
<tr>
<td></td>
<td>( n_i=1.8 )</td>
<td>1.829999999</td>
</tr>
<tr>
<td></td>
<td>( n_i=1.828 )</td>
<td>1.8308233</td>
</tr>
</tbody>
</table>

Table 9 Comparison of results for Ulrich method for three layer step index guide. Exact results are \( n_i=1.83 \) and \( w_i=8\mu m \).

calculations is that as the substrate refractive index \( (n_s) \) approaches the guide index \( (n_i) \), more of the nine modes, which the method was given for the calculations, are leaky.

For its simplicity, this method gives excellent results when it is given the problem it was designed to solve (ie. from list of guided modes, calculate an appropriate \( n_i \) and \( w_i \)) but (understandably) the accuracy decreases as more leaky modes are used.

<table>
<thead>
<tr>
<th>( n_i=1, \lambda=1.064\mu m, n_i=1.83, )</th>
<th>Pogossian 8 dec places</th>
<th>Pogossian 4 dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_i=8\mu m )</td>
<td>( n_i )</td>
<td>( n_i )</td>
</tr>
<tr>
<td>TE</td>
<td>( n_i=1.5 )</td>
<td>7.98514512</td>
</tr>
<tr>
<td></td>
<td>( n_i=1.8 )</td>
<td>8.82833146</td>
</tr>
<tr>
<td></td>
<td>( n_i=1.828 )</td>
<td>8.85700250</td>
</tr>
<tr>
<td>TM</td>
<td>( n_i=1.5 )</td>
<td>7.97656942</td>
</tr>
<tr>
<td></td>
<td>( n_i=1.8 )</td>
<td>8.89774541</td>
</tr>
<tr>
<td></td>
<td>( n_i=1.828 )</td>
<td>8.78416717</td>
</tr>
</tbody>
</table>

Table 10 Results of the method of Pogossian for the 3 layer step index guide. Exact results are \( w_i=8.0\mu m, n_i=1.83 \).
Implementing the method of Pogossian et al. involves plotting the square of the measured modes, i.e. $n_m^2$ vs the mode number plus one squared, i.e. $(m+1)^2$ and obtaining the coefficients for the least squares fit. The zeroth order term then gives $n_1^2$ (guide index). The linear and quadratic terms then give 2 nonlinear equations from which $n_s$ and $w_j$ can be calculated from (137) and (138) for the TE modes and (139) and (140) for the TM modes. The exact results for the comparison with results are given in Table 10. The exact values are $n_1=1.83$ and $w_1=8.0\mu m$. The calculations for this method are based on the assumption that the modes are pure guided and hence it is not surprising that the results are worse the less accurate this assumption. The method shows no major differences in accuracy if the modes are TE or TM and while, in general, gets the $n_s$ very close (worst case scenario above gets $n_s$ to 0.06%), the results for the other parameters are considerably less accurate.

Table 11 shows the results of using WKB method as implemented by Chiang et al. (i.e. Based on eqns (157) and (158)). A 3rd order (4 terms) polynomial is fitted to the refractive indices as a function of their mode number. The interval from the term at the surface to the $9^{th}$ mode value was divided into 200 equal steps and the depth for each of these was calculated. The 200 steps were chosen as it was found that any extra steps did not produce variations above the $9^{th}$ decimal place. In Table 11 the calculated peak refractive index (i.e. at the surface) was used as $n_s$ and the depth at which the known substrate refractive index was predicted to occur is denote $w_j$. The * in the table indicate regions in which the fitted curve of $n_m$ was not a monotonically decreasing function of $m$ and the method failed.

<table>
<thead>
<tr>
<th>$n_s=1.83$, $\lambda=1.064\mu m$, $w_1=8\mu m$</th>
<th>Chiang 8 dec places</th>
<th>Chiang 4 dec places</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_s=1.5$</td>
<td>1.82992961</td>
<td>8.03285566</td>
</tr>
<tr>
<td>$n_s=1.8$</td>
<td>1.83011244</td>
<td>8.43219024</td>
</tr>
<tr>
<td>$n_s=1.828$</td>
<td>1.82896441</td>
<td>*</td>
</tr>
<tr>
<td>$n_s=1.5$</td>
<td>1.82998583</td>
<td>7.98961393</td>
</tr>
<tr>
<td>$n_s=1.8$</td>
<td>1.83014217</td>
<td>8.44128532</td>
</tr>
<tr>
<td>$n_s=1.828$</td>
<td>1.82931700</td>
<td>*</td>
</tr>
</tbody>
</table>

Table 11: Comparison of method of Chiang with results for three layer step index guide. (Exact results are $w_1=8\mu m$ and $n_s=1.83$).
Table 12 shows the results for using the WKB method as implemented by Mathey et al. [144] (ie. eqns (160) and (161)). As with the other methods that rely on guided modes for the calculations, the method tends to give worse results when the leaky modes are included. Surprisingly the predictions for the surface refractive index get closer to the exact result for the higher $n_s$ values (implies more leaky modes used in the calculations).

### 6.6.2 Four Region Step Index Guide

<table>
<thead>
<tr>
<th>$n_s$</th>
<th>8 dec Pogossian</th>
<th>4 dec Pogossian</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n=1.4$, $n=1.5$</td>
<td>$1.8300007$, $1.7990196$, $1.4631576$, $2.59309928$, $1.8300533$, $7.8646526$, $1.7633858$, $0.1834951$</td>
<td>$0.14095645$, $1.75704337$, $1.8299710$, $0.0141910$, $1.3479484$, $2.4538465$</td>
</tr>
<tr>
<td>$n=1.75$, $n=1.8$</td>
<td>$1.8300013$, $7.6934491$, $1.6613623$, $1.10923934$, $1.8300494$, $7.8870605$, $1.8040961$, $0.1327964$</td>
<td>$0.14095645$, $1.75704337$, $1.8299710$, $0.0141910$, $1.3479484$, $2.4538465$</td>
</tr>
<tr>
<td>$n=1.8$, $n=1.828$</td>
<td>$1.8300024$, $7.9591698$, $1.6946274$, $7.2927102$, $1.8299771$, $7.9615992$, $1.6943473$, $7.3469102$</td>
<td>$0.14095645$, $1.75704337$, $1.8299710$, $0.0141910$, $1.3479484$, $2.4538465$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$n_s$</th>
<th>8 dec Pogossian</th>
<th>4 dec Pogossian</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n=1.4$, $n=1.5$</td>
<td>$1.8300013$, $7.6934491$, $1.6613623$, $1.10923934$, $1.8300494$, $7.8870605$, $1.8040961$, $0.1327964$</td>
<td>$0.14095645$, $1.75704337$, $1.8299710$, $0.0141910$, $1.3479484$, $2.4538465$</td>
</tr>
<tr>
<td>$n=1.75$, $n=1.8$</td>
<td>$1.8300013$, $7.6934491$, $1.6613623$, $1.10923934$, $1.8300494$, $7.8870605$, $1.8040961$, $0.1327964$</td>
<td>$0.14095645$, $1.75704337$, $1.8299710$, $0.0141910$, $1.3479484$, $2.4538465$</td>
</tr>
<tr>
<td>$n=1.8$, $n=1.828$</td>
<td>$1.8300024$, $7.9591698$, $1.6946274$, $7.2927102$, $1.8299771$, $7.9615992$, $1.6943473$, $7.3469102$</td>
<td>$0.14095645$, $1.75704337$, $1.8299710$, $0.0141910$, $1.3479484$, $2.4538465$</td>
</tr>
</tbody>
</table>

Table 13 shows the results of the inverse method of Pogossian for the 4 layer step index refractive index profiles. The calculations use all 9 modes. The expressions in (146) and (151) are badly conditioned and so it is better to use an iterative minimisation routine, than attempt to solve the equations. This is done by first using a least squares fit to
the modes to determine \( n_1 \), \( w_1 \), \( n_2 \) and \( w_2 \) are determined by minimising the error term using \( w_2 \) as the variable - ie \( w_2 \) is guessed and then from the curve fit \( w_1 \) and \( n_2 \) are calculated, the modes for this profile are then determined and this gives a function to minimise.

\[
\begin{align*}
n_1 &= 1.83, \quad w_1 = 8\, \mu m, \quad \lambda = 1.064\, \mu m \\
8\, \text{dec Chiang} & \quad 4\, \text{dec Chiang} \\
n_1 & \quad w_1 & \quad n_1 & \quad w_1 \\
1\, \mu m & 1.82999305 & 8.01075403 & 1.83000937 & 8.01605033 \\
0.1\, \mu m & 1.82999079 & 8.01766096 & 1.83003310 & 8.01475297 \\
0.01\, \mu m & 1.829798956 & 8.03042126 & 1.83004439 & 8.03262664 \\
n_1 &= 1.75, \quad n_2 = 1.8 \\
1\, \mu m & 1.82995616 & 8.34463320 & 1.82981706 & 8.34540326 \\
0.1\, \mu m & 1.83066500 & 8.06987316 & 1.83062150 & 8.0679026 \\
n_1 &= 1.8, \quad n_2 = 1.828 \\
1\, \mu m & 1.82994798 & 8.59559224 & 1.83009147 & 8.53218756 \\
0.1\, \mu m & 1.82956661 & * & 1.82951273 & * 
\end{align*}
\]

**Table 15** Results from Chiang method for the four layer step index guide. Exact results should be \( n_1 = 1.83 \) and \( w_1 = 8.0 \).

**Table 15** shows the results for the Chiang inverse WKB method when applied to the 4 layer profile. Again, because the profile does not actually conform to the monotonically decreasing requirements of the method it cannot give the correct answer. Nevertheless it does quite well. As in the three guide case, the * indicates that the fit to the

\[
\begin{align*}
n_1 &= 1.83, \quad w_1 = 8\, \mu m, \quad \lambda = 1.064\, \mu m \\
\text{Mathey 8 dec} & \quad \text{Mathey 4 dec} \\
n_1 & \quad w_1 & \quad n_1 & \quad w_1 \\
w_1 = 1\, \mu m & 1.83098317 & 8.00764401 & 1.83075079 & 8.01332692 \\
w_1 = 0.1\, \mu m & 1.83098970 & 8.01469002 & 1.83098970 & 8.014752141 \\
w_1 = 0.01\, \mu m & 1.83097956 & 8.02752141 & 1.83073379 & 8.00935685 \\
w_1 = 1\, \mu m & 1.83094251 & 8.21134504 & 1.83070770 & 8.22323157 \\
w_1 = 0.1\, \mu m & 1.83093197 & 8.03877638 & 1.83102943 & 8.03434168 \\
w_1 = 0.01\, \mu m & 1.83093197 & 8.03877638 & 1.83102943 & 8.03434168 \\
w_1 = 1\, \mu m & 1.83088914 & 7.94842949 & 1.83086567 & 7.90405890 \\
w_1 = 0.1\, \mu m & 1.83029340 & 8.68557759 & 1.83027188 & 8.68876304 \\
w_1 = 0.01\, \mu m & 1.83002355 & 10.29741948 & 1.82965781 & 9.35445380 
\end{align*}
\]

**Table 14** Comparison of results for Mathey method for 4 layer step index guide. Exact values are \( n_1 = 1.83 \) and \( w_1 = 8.0 \)
refractive index curve has a turning point, usually for m<0 and hence the method fails. No information as to \( w_2 \) is available from this method.

Table 14 shows the results of using the Mathey etal. method for determining the refractive index profile of a 4 layer step index guide. Again, we are asking the method to make predictions in a region which it is not suited but since it is simple to implement and not computational intensive it was worthwhile in order to make the comparisons. In general, the method gives better results for the guide refractive index (\( n_1 \)) parameter but worse results for the guide width (\( w_1 \)) as the barrier width (\( w_2 \)) gets smaller.

6.6.3 Numerical Methods

A general routine to calculate the modes of a given refractive index profile numerically, using the Matrix Method of Section 6.5.2.1, was written. A number of different numerical methods were investigated to calculate the refractive index profile by starting with a "guess" to the important parameters, calculating the modes of the "guessed" guide and then minimising the difference. A number of different minimisation routines are evaluated in Appendix F.

In general the numerical methods did better than the analytical methods when given an initial set of parameters that was "close" to the exact solution. For a refractive index profile that is more complicated than the step index profiles numerical minimisation is the only way to get a solution to the problem.

6.7 Summary

For its computational simplicity, Ulrich's method seems the best for getting a "ball park" figure for the guide refractive index and width. The more complicated analytical methods do not seem to provide significantly better results, despite their extra complexity and computational effort.

Numerical methods were, in general, more accurate than the analytical methods if given a set of initial parameters that was close to the exact solution. Using Ulrich's method to provide the initial "guess" to a numerical routine gives the best of both approaches. This method was therefore adopted in analysing the profiles generated by ion implantation and discussed in the following chapter.
7 Ion Implanted Waveguide Experiments

7.1 Introduction

The numerical simulations in Chapter 5 indicated that a waveguide laser in Neodymium doped Yttrium Aluminium Garnet (Nd:YAG) could be designed to have the right combination of parameters to obtain self-starting passive modelocking using a nonlinear directional coupler. The results, however, indicated that the NLDC would need to be at least 20cm long to achieve this. The only fabrication technique available at the ANU capable of producing waveguides in Nd:YAG was high energy ion implantation, and unfortunately devices of this type and length were too big to be fabricated using the available equipment.

Hence, the work described in this chapter focussed on the properties of waveguides produced by ion implantation in much smaller Nd:YAG samples - a topic itself of considerable interest. In particular, the work concentrated on understanding the refractive index profile produced by ion implantation; the effect of the implantation on the lasing properties; and on the mechanisms that cause loss in the waveguides. The aim was to produce a low loss, single mode waveguide with gain at 1064nm as a preliminary step before fabrication of the larger mode-locked device could be contemplated.

7.2 Experiments

7.2.1 Preliminary Implants

Little work had been done to produce ion implanted waveguides in crystalline materials at the ANU prior to that described in this PhD. Hence some preliminary implants were performed into industrial sapphire which has a similar density and hardness to YAG, but was cheaply available. Almost all the previous reports on the creation of waveguides via ion implantation had used He ions. However, the 1.7MV NEC Tandem (5SDH) ion
implanter in the Department of Electronic Materials Engineering at the Research School of Physical Sciences and Engineering (RSPhysSE) at the ANU requires a solid source (for sputtering) and so Lithium was chosen for the implants as it was the next heaviest ion. A light ion is desirable because of its range and ballistic properties (as outlined in Section 6.2). The ANU implanter can deliver energies up to 10MeV, depending on the ion charge.

<table>
<thead>
<tr>
<th>Dose</th>
<th>Wavelength (nm)</th>
<th>Ulrich Method</th>
<th>Numerical Gaussian Fit</th>
<th>Error per mode (x 10^-5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 \times 10^{16}</td>
<td>514.5</td>
<td>TE</td>
<td>n_x (uncert.)</td>
<td>t_x (uncert. (\mu m))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TM</td>
<td>1.767 (0.002)</td>
<td>3.77 (0.11)</td>
</tr>
<tr>
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<td>632.8</td>
<td>TE</td>
<td>1.762 (0.002)</td>
<td>3.65 (0.43)</td>
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<tr>
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<td></td>
<td>TM</td>
<td>1.765 (0.003)</td>
<td>3.69 (0.45)</td>
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<td>810</td>
<td>TE</td>
<td>1.755 (0.004)</td>
<td>3.73 (0.40)</td>
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<td></td>
<td>TM</td>
<td>1.766 (0.003)</td>
<td>4.10 (0.08)</td>
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<tr>
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<td>1064</td>
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<td>4.42 (0.05)</td>
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<td></td>
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<td>1.7560 (0.0001)</td>
<td>4.99 (0.02)</td>
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<tr>
<td>3 \times 10^{16}</td>
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<td>TE</td>
<td>1.76 (0.01)</td>
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</tr>
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<td>3.72 (0.33)</td>
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<td>TM</td>
<td>1.765 (0.003)</td>
<td>3.78 (0.32)</td>
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<td>810</td>
<td>TE</td>
<td>1.761 (0.002)</td>
<td>4.11 (0.03)</td>
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<td>TM</td>
<td>1.768 (0.002)</td>
<td>4.11 (0.04)</td>
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<td>TE</td>
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<td>4.18 (0.25)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TM</td>
<td>1.7565 (0.0004)</td>
<td>4.48 (0.06)</td>
</tr>
</tbody>
</table>

Table 16 Refractive index profiles for 3MeV Li implanted Sapphire (higher doses including 1064.1nm data)

Implants were performed at 1 \times 10^{15}, 3 \times 10^{15}, 1 \times 10^{16} and 3 \times 10^{16} ions/cm² (as this gives an approximately linear distribution on a logarithmic scale). In each case the waveguide modes were measured using the Metricon Prism Coupler at 514.5nm (Argon Ion laser); 632.8nm (Helium Neon laser); and 810 nm (Gallium Arsenide diode laser) light, for both TE and TM polarisations. Further modes were taken for 1064.1nm (Nd:YAG laser) light for implant doses greater than 1 \times 10^{16} ions/cm². A three layer step index guide model was fitted to the measured mode data using Ulrich's method (cf. 6.5.3.1) and the calculated waveguide width (w_i) and refractive index (n_i) are shown in Tables 16 and 17.
The three layer step index calculations make the assumption that all the modes used for the calculation are guided even though in this case almost all of them are not. Computationally, this translates into an assumption that the mode effective indices are real. If a mode is leaky then it has an imaginary component to its effective index - the larger the loss the larger the imaginary component. However, the barrier region in these guides is expected to be relatively wide and hence while technically leaky their loss is quite low and hence the imaginary component of the mode indices is small. Hence, using Ulrich’s method and ignoring the presence of the imaginary component still yields reasonable results (cf. [145] and below).

Ulrich’s method needs only 2 modes to calculate the guide thickness and refractive index. When there were more than 2 modes measured, all the 2 mode pairs are considered, taking into account the mode number, and a guide thickness and refractive index calculated for each one. This generates a distribution of the results for which a mean and standard deviation can be calculated (shown as “uncertainty” in the table). A low uncertainty in the results indicates that the three layer step index profile is a good model of the true refractive index profile.

Table 17  Summary of calculated refractive index profiles for 3MeV Li implanted into Sapphire.
Sapphire is birefringent, but the prism coupler also allows for the refractive index of the substrate to be determined for both TE and TM polarisations and these are included in the table and used for subsequent calculations.

![Figure 95 TRIM results for ion range and ionisation for 3MeV Li into Sapphire.](image)

The step index refractive index profile suggests the waveguide depth was 3.5-4μm. Figure 95 shows the results for a TRIM[146] simulation for 3MeV Li ions into sapphire (Al₂O₃, density 3.98g/cm³[147]) and indicates that the Li ion range is 3.95±0.13μm. Hence, the guiding occurs in a region above that where the Lithium ions accumulate and where severe lattice damage occurs. This should give a lower loss than if the waveguide mode was centred on the contaminated region. From the TRIM simulations it is apparent that ionisation, thought to produce a refractive index increase, climbs slightly in the first 2μm and then decreases rapidly to 4μm: the refractive index profile of the waveguide would be expected to follow this trend.

The data show a decrease in the refractive index at the shorter wavelengths so that, with the exception of 1 10¹⁵ ions/cm² at 514.5nm TM, there are no guided modes produced at 514.5nm or 632.8nm for any of the doses tried. The data also shows that apparently the different polarisations "see" different waveguides - in general the index changes being
It is important to realise that although there were a number of instances of increased refractive index, the only actual guided modes (mode for which the effective index of any mode was greater than that of the substrate) were the TM modes at 632.8nm and 810nm for the guides implanted at $3 \times 10^{16}$ ions/cm$^2$.

Following on from the work of Chandler et al.[148] a numerical procedure (similar to that outlined in Appendix F) was used to optimise parameters for a Gaussian shaped barrier model. The profile shape is shown in Figure 96 and mathematically the profile is given by:

$$n = n_1 + (n_2 - n_1) e^{-\frac{(x-t)^2}{w_0^2}} \quad x \leq t$$

$$= n_s + (n_2 - n_s) e^{-\frac{(x-t)^2}{w_0^2}} \quad x > t$$

where $n_1$ is the surface refractive index; $n_2$ is the refractive index at the bottom of the
barrier; \( t \) is the depth of the barrier; while \( w_1 \) and \( w_2 \) are the widths "into" and "out from" the barrier (allowing the structure to be asymmetric). In Figure 96, \( n_1=1.831, n_2=1.83 \), \( t=4\mu m \) and \( w_1=w_2=0.6\mu m \).

The results of the fitting routines are shown in Tables 16 and 17 and in Figures 97 and 98. These results are in general agreement with the trends from the Ulrich Method.

*Figure 97*  Comparison of guide refractive index \( n_1 \) vs dose (ions/cm\(^2\)) at 632.8nm calculations:

- Guide thickness is, in general, in the range of 3.5-4\( \mu m \) (although there is a range from 2.9-5.03\( \mu m \)). It is, on average, inside the 3.95±0.13\( \mu m \) predicted by the TRIM simulations.

- In 11 out of the 28 measurements the implantation process produces an increase in the refractive index above that of the substrate. For all but the 3 \( 10^{16} \) ions/cm\(^2\) case this occurs only for the TM polarisation (it only occurs for both TE and TM for 810nm and 1064nm at the highest dose).

- As a general trend the \( n_1 \) refractive index increases slightly with dose for both TE and TM polarisations, but the other parameters show no consistent
behaviour.

A number of absorption measurements were taken on the samples using a Shimadzu UV - 3101PC UV-NIR Scanning Spectrophotometer with a resolution of 2nm. A transmission measurement of the implanted region taken through the layer (rather than along the guide) was compared to that of an un-implanted sample across the range from 200-1200nm. Only the region 200-600nm was found to contain any features of interest. The difference in the spectra is presumed to be due to the implanted layer which, from the TRIM simulations and waveguide mode measurements, is expected to be less than 10μm thick. The loss of this layer in dB/cm was calculated assuming the implanted region was 10μm thick. The variation of this loss with ion dose and wavelength is shown in Figure 99.

**Figure 98**  Fitted guide refractive index ($n_g$) vs dose for modes at 810nm.
Figure 99 Comparison of calculated guide loss (dB/cm) vs Wavelength and dose for 3MeV Li ions into sapphire.

The figure shows that the loss generally decreases from a number of peaks in the UV to longer wavelength. The most prominent peak in the UV is located at 205±1nm and has a calculated loss of nearly 10000dB/cm for the highest dose implant. Other peaks are located at 259±3nm, 303±2nm, 357±2nm and 454±1nm. There is evidence in the scans for the 1 and 3 $10^{15}$ ions/cm² dose measurements of an additional peak at ~225nm which becomes obscured at the higher implant doses. Figure 100 shows the dependence of the height of the peaks on the ion dose. The log-log graph shows a roughly linear increase with dose. Clearly ion implantation in sapphire results in the production of large numbers of colour centres.
Figure 100 Peak Heights for Lorentzians fitted to absorption spectra for 3MeV Li ions into Sapphire vs dose.

7.2.2 Li Implants into Nd:YAG

Next, two 5mm x 5mm samples of Nd:YAG were prepared from a Nd:YAG laser rod. The rod contained 1 atomic weight % of Neodymium and all subsequent YAG samples were also prepared from this rod. The lengthy preparation and the cost of the material needed for a sample large enough for mode and loss measurements, limited the total number of experimental samples to a short series each attempting to converge on the desired low loss, single mode guide.
Refractive indices measured and used for calculations:
514.5nm 1.8383
632.8nm 1.8306
810.0nm 1.8230
1064.1nm 1.8155

Table 18 3MeV Li at a dose of $1 \times 10^{16}$ ions/cm$^2$ into Nd:YAG. Calculated refractive index profiles.

7.2.2.1 3MeV Li Ions, $1 \times 10^{16}$ ions/cm$^2$ into Nd:YAG

The first sample was implanted with 3 MeV Li ions to a dose of $1 \times 10^{16}$ ions/cm$^2$. The refractive index profiles calculated from experimentally determined mode indices are shown in Table 18. The Ulrich method fits show that, for this energy and dose, there was no increase in refractive index above that of the substrate and so no waveguide layer, although the uncertainties in some of the results are large enough that a slight increase above the substrate value cannot be totally ruled out (in fact, only for TE polarisations for 810nm and 1064.1nm is guiding impossible). The only guided mode identified was a TM mode at 632.8nm.

The Gaussian fit data is a little more interesting. With the exception of the 810nm TE modes, all the other data show an increase in the refractive index in the ion transition region. With the exception of 514.5nm TE modes, all show a decrease in refractive index, below that of the substrate in the “damage” region, some as much as 3%.
As is shown in Figure 101, the ion range for 3MeV Li into YAG (YAG has a different density to sapphire and, although close, it could not be assumed that the results from sapphire were directly transferable) is calculated to be 4.19±0.20μm. However, the data in Table 18 indicate that low refractive index “damage” region is shallower than this, in some cases by as much as 0.4μm. This is consistent with the expected model for a refractive index change induced by ion implantation, in as much as the mechanism leading to a reduction in the refractive index is due to the energy lost as the ions suddenly slow down towards the end of their range, rather than, for example, a chemical effect produced in the region where the implanted ions accumulate.

### 3MeV Li Ions, 3 \times 10^{16} ions/cm² in Nd:YAG

In the light of these results a second 5mm x 5mm sample was implanted at 3 \times 10^{16} ions/cm². The calculated refractive index profiles are shown in Table 19. In contrast to the lighter dose implant, there are guided TM modes at both 514.5nm and 632.8nm.

The Ulrich method results suggest that there is only one situation which shows a definite increase in refractive index in the surface region (the 514.5nm TM mode) although the uncertainty in the results does not conclusively rule out this possibility for the other
cases. The main change the higher dose has brought seems to come in the change in the TE mode refractive indices.

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Ulrich Method</th>
<th>No. of guided modes</th>
<th>Numerical Gaussian Fit</th>
<th>Error per mode (x 10^-4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n (uncert)</td>
<td>t (μm) (uncert)</td>
<td>n1</td>
<td>n2</td>
</tr>
<tr>
<td>514.5 TE</td>
<td>1.838 (0.003)</td>
<td>4.17 (0.09)</td>
<td>0</td>
<td>1.83867</td>
</tr>
<tr>
<td>514.5 TM</td>
<td>1.8398 (0.0005)</td>
<td>4.19 (0.04)</td>
<td>1</td>
<td>1.84134</td>
</tr>
<tr>
<td>532.8 TE</td>
<td>1.830 (0.001)</td>
<td>3.93 (0.05)</td>
<td>0</td>
<td>1.83083</td>
</tr>
<tr>
<td>532.8 TM</td>
<td>1.83 (0.02)</td>
<td>4.02 (0.47)</td>
<td>1</td>
<td>1.83297</td>
</tr>
<tr>
<td>810.0 TE</td>
<td>1.821 (0.007)</td>
<td>3.90 (0.19)</td>
<td>0</td>
<td>1.82373</td>
</tr>
<tr>
<td>810.0 TM</td>
<td>1.82 (0.02)</td>
<td>4.03 (0.60)</td>
<td>0</td>
<td>1.82522</td>
</tr>
<tr>
<td>1064.1 TE</td>
<td>1.815 (0.006)</td>
<td>3.91 (0.08)</td>
<td>0</td>
<td>1.81674</td>
</tr>
<tr>
<td>1064.1 TM</td>
<td>1.813 (0.006)</td>
<td>4.00 (0.18)</td>
<td>0</td>
<td>1.81645</td>
</tr>
</tbody>
</table>

Table 19 Calculated refractive index profiles for 3 MeV Li at a dose of 3 \(10^{16}\) ions/cm\(^2\) into Nd:YAG.

The Gaussian fit results again suggests the refractive index has increased in the

![Figure 102 Absorption spectra for 3 \(10^{16}\) ions/cm\(^2\).](image-url)
surface ion transition region in all cases, and, with the exception of 514.5nm TE modes and 1064.1nm TM modes, that a decrease in the refractive index occurs in the ion stopping region. In general, the changes in the refractive index between 1 \(10^{16}\) ions/cm\(^2\) to 3 \(10^{16}\) ions/cm\(^2\) does not seem to show a discernable pattern.

The measured as-implanted absorption spectrum for this sample is shown in Figure 102 and the spectral dependence of the absorption coefficients in Figure 103. The neodymium absorption lines greatly interfere with this measurement and this results in a lack of smoothness in these data compared with that obtained for sapphire in Figure 99. In contrast to the sapphire implants and the later work, the absorption profiles show remarkably little structure and can be fitted with two broad Lorentzian peaks centred at 252nm and 291nm. As with the sapphire implants, the higher dose means that one of these absorption peaks dominates, whereas the lower dose more of the structure is apparent on the spectrum. The loss coefficients are based on an assumed Li range of 10\(\mu\)m as before.

The loss at the wavelengths of interest (ie >800nm), is not discernable above the "noise", however for wavelengths < 500nm the absorption is too large for waveguiding unless measures can be taken (such as annealing) to reduce it.

![Figure 103](image)

**Figure 103** Calculated loss (dB/cm) vs ion dose and wavelength
Attempts were made to have both the 5mm x 5mm samples re-polished so that light could be end-coupled into the guides. The aim was to launch 810nm pump light into the guide and look for lasing. Since the waveguides are located very close to the surface of the crystal, the corners need to be polished "square" to within about a micron of the surface to make end coupling possible. These corners were very easy to damage particularly during the prism coupler measurements.

Re-polishing was, however, not possible because the implantation process caused swelling of the crystal by about 10-20nm\(^3\). The polishing procedure required two crystals to be butted against each other forming an optical contact to provide a continuous end surface which allows the corners to be polished square without chipping. This proved impossible for post implanted crystals because of the swelling. As a result, laser tests on these waveguides were abandoned.

7.2.3 First End-Fire Guide

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Ulrich Method</th>
<th>No. of Guided Modes</th>
<th>Numerical Gaussian Fit</th>
<th>Error per mode (x 10(^4))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(n_0) (uncert.)</td>
<td>(t) ((\mu)m) (uncert.)</td>
<td>(n_1)</td>
<td>(n_2)</td>
</tr>
<tr>
<td>632.8</td>
<td>TE</td>
<td>1.8299 (0.0007)</td>
<td>3.99 (0.03)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>TM</td>
<td>1.831 (0.007)</td>
<td>4.04 (0.11)</td>
<td>1</td>
</tr>
<tr>
<td>810</td>
<td>TE</td>
<td>1.82 (0.01)</td>
<td>3.99 (0.10)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>TM</td>
<td>1.82 (0.01)</td>
<td>4.04 (0.17)</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 20 Calculated refractive index profiles for 3MeV Li ions into Nd:YAG at a dose of 2 \(10^{16}\) ions/cm\(^2\).

Learning from these earlier experiences, a 8mm x 16mm x 4mm Nd:YAG sample was prepared with “square” corners and this was implanted over a 3mm wide stripe parallel to the 16mm long axis with a dose of 2 \(10^{16}\) ions/cm\(^2\) at an energy of 3MeV. The swelling due to implanting of 6MeV Li ions.

---

\(^3\) It was only recognised at the time that there had been swelling because optical contacting was not possible. Measurements on the sample were difficult as most of the 5mm x 5mm face had been implanted and so rose in a dome shape, for which an accurate zero point was difficult to find. Subsequent samples were implanted in a stripe and a new measurement probe had been purchased enabling a swelling of 15\(\pm\)10nm to be measured for the swelling due to implanting of 6MeV Li ions.
refractive index profile deduced for this sample is shown in Table 20 and displays only one guided mode, the TM mode at 632.8nm.

The Ulrich Method calculations indicated that the refractive index had probably increased in surface region. The Gaussian profile results also showed an increase in the refractive index in this region and, with the exception of 632.8 TE modes, a decrease in the "barrier" region. The guide width, t, was also shorter than the ion range. As was apparent in Table 19, the TM modes "see" a slightly wider guide than the TE modes (in addition to a raised refractive index).

Light at 632.8nm was successfully end fire coupled into this waveguide. To achieve this the chip was placed on a tilt stage and a 10x microscope objective used to focus the beam into the waveguide.

To try to reduce the guide loss the sample was annealed at 400°C in an Argon atmosphere for 1 hour (temperature increasing at 1°/minute and decreasing at the same rate to allow any thermal stress within the crystal to be slowly released). These conditions were chosen after a survey of recommendations from the available literature. However, what has been published was neither systematic nor well explained and was mostly used with waveguides produced by He implantation. Influential data came from Chandler et al. [149] who observed a 90% reduction in loss in a He implanted guide with annealing and no substantial effect on the guide refractive index profile for anneal temperatures below 800°C.

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Ulrich Method</th>
<th>No. of Guided Modes</th>
<th>Numerical Gaussian Fit</th>
<th>Error per mode ($\times 10^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_1$ (uncert)</td>
<td>$t$ ($\mu$m) (uncert)</td>
<td>$n_1$</td>
<td>$n_2$</td>
</tr>
<tr>
<td>514.5</td>
<td>TE 1.8380 (0.0002) 4.20 (0.01) 0</td>
<td>1.84015</td>
<td>1.82149</td>
<td>3.99154</td>
</tr>
<tr>
<td></td>
<td>TM 1.839 (0.001) 4.24 (0.05) 1</td>
<td>1.84035</td>
<td>1.83112</td>
<td>4.17968</td>
</tr>
<tr>
<td>632.8</td>
<td>TE 1.830 (0.002) 3.97 (0.03) 0</td>
<td>1.83053</td>
<td>1.82799</td>
<td>3.86091</td>
</tr>
<tr>
<td></td>
<td>TM 1.8307 (0.0007) 4.02 (0.06) 1</td>
<td>1.83215</td>
<td>1.80419</td>
<td>3.93795</td>
</tr>
<tr>
<td>810</td>
<td>TE 1.822 (0.001) 3.96 (0.03) 0</td>
<td>1.82301</td>
<td>1.81141</td>
<td>3.96026</td>
</tr>
<tr>
<td></td>
<td>TM 1.822 (0.008) 4.01 (0.02) 0</td>
<td>1.82458</td>
<td>1.79109</td>
<td>4.0911</td>
</tr>
<tr>
<td>1064.1</td>
<td>TE 1.81 (0.01) 3.99 (0.15) 0</td>
<td>1.81629</td>
<td>1.80176</td>
<td>3.93883</td>
</tr>
<tr>
<td></td>
<td>TM 1.815 (0.007) 4.02 (0.13) 0</td>
<td>1.81625</td>
<td>1.81127</td>
<td>3.87445</td>
</tr>
</tbody>
</table>

Table 21 Refractive index profile for 3MeV Li into Nd:YAG to a dose of 2 \(10^{16}\) ions/cm\(^2\) post 400°C anneal
The affect of annealing on the refractive index profile is shown in Table 21. The calculated refractive indices for the Ulrich Method show, if anything, a slight increase after annealing, but the guide width slightly decreases. For the Gaussian fit there is a slight decrease in the guide refractive index and a slight increase in the waveguide width!

Despite the fact that the guided modes could still be measured using the prism coupler, light could no longer be end-fire coupled into the waveguide at 632.8nm, despite several attempts. We can only deduce that annealing had in fact reduced the strength of the waveguide but the changes were too subtle to be identified via the procedures used to analyse the index profile. Since the annealing conditions were far less extreme than those described in the literature, we proceeded cautiously in our later investigations of the effect of annealing which seems to have a stronger effect for our Li implants than has been reported using He.

7.2.4 H implants into Nd:YAG

Guiding of the modes at 810 and 1064nm was not achieved for any of the 3MeV Li implants, but is essential for the production of a laser at 1064nm. Hence, either a deeper guide or a greater change in refractive index, or both, was needed. The implanter provides higher energy ions in 3MeV steps in proportion to the degree of ionisation of the implanting atom (eg. doubly ionized Li allows 6 MeV ions to be produced). However, the implantation time is greatly increased as one goes to the higher ionisation states because of reduction of the ion current.

As can be seen from Figure 104, H implants have the advantage that they have a considerably greater range for a given energy due to their small mass. We were not able to perform implants at energies greater than 1MeV because above this energy X-rays dangerous to the operators would be produced during the implanting process. The ionisation profile with depth is significantly different from the Li ion profiles and the affect this has on the guide refractive index was not known a priori.
Table 22 shows the refractive index profiles deduced for the H implant in Nd:YAG. The lighter ion does give considerably greater range (as expected), but seems to cause a decrease in refractive index in the surface region. In the Li implanted waveguides it is the TM modes that see the greatest ion induced refractive index increase. With H implants, the TM modes have the largest decrease in refractive index. The data showed that H implants would not give the desired waveguide and hence we resorted to the use of higher energy Li implants instead.

Figure 104 Comparison of TRIM predicted range and ionisation produced for a 1MeV H implant into YAG. Mean TRIM calculated ion range is 9.26±0.30μm. (Trimhyag.wmf from htrim.dgr)
7.2.5 6 MeV Li Ion Implants into Nd:YAG

A second 4mm x 8mm x 16mm Nd:YAG sample, with edges polished square to within 1μm of the surface, was implanted with 6 MeV Li ions to a dose of $1 \times 10^{16}$ ions/cm² in a 3mm wide stripe along the 16mm axis. The refractive index parameters deduced for this sample are shown in Table 23. An increase in the refractive index is apparent at all wavelengths and guided modes were observed for all except the 810nm TM mode.

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Ulrich Method</th>
<th>No. of Guided Modes</th>
<th>Numerical Gaussian Fit</th>
<th>Error per mode (x 10^-4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>514.5</td>
<td>TE 1.839 (0.001) 7.45 (0.50)</td>
<td>1</td>
<td>1.838486</td>
<td>1.831705</td>
</tr>
<tr>
<td>TM 1.839 (0.003) 7.47 (0.60)</td>
<td>1</td>
<td>1.839797</td>
<td>1.828480</td>
<td>8.248332</td>
</tr>
<tr>
<td>532.8</td>
<td>TE 1.832 (0.009) 8.12 (0.07)</td>
<td>1</td>
<td>1.831173</td>
<td>1.825572</td>
</tr>
<tr>
<td>TM 1.833 (0.004) 7.90 (0.28)</td>
<td>1</td>
<td>1.831514</td>
<td>1.829506</td>
<td>7.929241</td>
</tr>
<tr>
<td>810</td>
<td>TE 1.825 (0.005) 8.64 (0.92)</td>
<td>1</td>
<td>1.822945</td>
<td>1.814410</td>
</tr>
<tr>
<td>TM 1.825 (0.005) 8.33 (0.32)</td>
<td>0</td>
<td>1.823248</td>
<td>1.816973</td>
<td>8.079545</td>
</tr>
<tr>
<td>1064.1</td>
<td>TE 1.818 (0.001) 9.55 (0.10)</td>
<td>1</td>
<td>1.816434</td>
<td>1.784804</td>
</tr>
<tr>
<td>TM 1.818 (0.001) 9.53 (0.01)</td>
<td>1</td>
<td>1.817869</td>
<td>1.802437</td>
<td>9.483672</td>
</tr>
</tbody>
</table>

Table 23 Calculated refractive index profiles for 6 MeV Li into Nd:YAG to a dose of $1 \times 10^{16}$ ions/cm².
Dielectric mirrors were pushed against the endfaces of the implanted waveguide and held in place using a drop of Dimethyl Sulphoxide (DMSO, CH₃SOCH₃, mp:18.4°C, bp:189°C [150]) which was chosen because of its high surface tension, relatively high refractive index (n=1.477) and low toxicity. In this configuration, lasing at 1064nm was observed when the waveguide was pumped at 808nm by a CW Ti:Sapphire Laser. The external threshold was ~200mW, and lasing was sustainable for several minutes until the mirrors generally shifted due to air currents. Launching and imaging of the laser end faces were achieved using 10x microscope objectives and a telescope arrangement upstream of the launching lens allowed the pump beam waist and divergence to be tuned for optimum coupling. As shown in Figure 105, a cylindrical lens was added to give a highly elliptical beam (beam size approximately 7mm (vertical) x 1mm (horizontal)) just prior to the 10x objective. This allowed the beam to be tightly focussed into the waveguide in the vertical plane, but produced an approximately collimated beam in the horizontal direction.

Repeatedly contacting the mirrors onto the end faces of the chip, rapidly led to
chipping of the waveguide corners and it was eventually not possible to achieve lasing because the both poor pump power launch efficiency and increased cavity loss due to scattering.

7.2.5.1 **Fluorescence Spectra of the Laser Transition**

![Figure 106](image.png)

Figure 106  Fluorescence spectra around Nd lasing wavelength for different ion implantation dose and energy.

It had been reported by Field et al. [151] that the formation of guides using He implantation into Nd:YAG produced changes in the fluorescence spectra at 1064nm and reduced the stimulated emission cross-section for the lasing transition in this region.

Figure 106 shows the fluorescence spectra for an unimplanted Nd:YAG sample and after implantation with 6MeV Li atoms to a dose of $1 \times 10^{16}$ ions/cm². These show that there is very little change in the spectra after implantation. There is a slight broadening of the
spectra discernable, particularly in the trough at ~1063nm (and expanded in the figure). It is not expected to have any significant effect on the lasing properties of the crystal. Hence our data does not show the strong effects reported by Field et al.

7.2.5.2 The Effect of Annealing

It was decided to systematically anneal the sample implanted with 6MeV ions at a dose of $1 \times 10^{16}$ ions/cm$^2$ and measure the effect it had on the guided modes; the waveguide loss (as measured using the spectrometer); and the waveguide loss at 808nm (monitored by measuring the decay of 1064nm fluorescence).

7.2.5.2.1 Guided Mode Measurements

The waveguide modes (both guided and leaky) were measured at 4 wavelengths
(514.5nm (air cooled Ar ion laser), 632.8nm (HeNe), 810nm (GaAs Diode laser) and 1064.1nm (Nd:YAG laser)) for both TE and TM polarisations using the prism coupler, as described elsewhere. The resulting effective mode indices are plotted in Figures 108-114. The solid line in the figures indicates the value of the substrate refractive index. Any mode with an effective index above this line is guided.

The mode indices tend to decrease with increasing annealing temperature. This causes the sample to have no guided modes at 632.8nm TE above 300°C and no 632.8nm TM modes above 475°C. The exception is the 810nm modes, for which there are no guided modes until ~300°C (when they “appear”) and these are then annealed away at higher temperatures. In general the TM modes remain guided to higher temperatures than their corresponding TE modes. A number of modes are also seen to “disappear” eg. 3rd mode 632.8nm TE at ~350°C, 2nd mode 632.8nm TM ~275°C, 4th mode 1064nm TM at ~300°C.

The behaviour of guide refractive index and width as a function of polarisation, wavelength and annealing temperature shows some interesting behaviour:-

- Figure 116 shows the waveguide refractive index ($n_g$) calculated using the Ulrich Method as a function of annealing temperature and wavelength for TE polarisation (The graph shows the "height" of the step over the substrate refractive index). Both
the 514.5nm and 1064.1nm TE indices are above the substrate value across the full annealing temperature range but a gradual decrease is evident with increasing annealing temperature. The 632.8nm index value rises slightly above the substrate refractive index until ~350°C where it dips suddenly and decreases steadily from there on. In contrast, the guide at 810nm starts off with a refractive index below that of the substrate and at ~350°C undergoes a sudden increase, above the substrate value and then steadily decreases from there on. Both the sudden jumps and the fact that there is no gradual behaviour at the different wavelengths are surprising.

• Figure 117 shows the behaviour of the guide refractive index (n₁) from Ulrich’s Method for TM polarisation data. Again we see the refractive index values at 514.5nm and 1064.1nm decrease steadily with increasing annealing temperature. Again the refractive index at 632.8nm starts off above the substrate value and then undergoes a sudden drop, this time at ~300°C and the 810nm refractive index
begins below that of the substrate and undergoes a sudden "jump" to a level above the substrate value.

That there are "jumps" in the calculated refractive index values is not really surprising, considering the relative crudeness of the model and its sensitivity to the appearance or disappearance of a particular mode. That the "jumps" occur at approximately the same annealing temperature and are observed for both the TE and TM polarisations suggests, however, that the changes are more likely to be real.

Figures 118 and 119 show the calculated guide thickness for the various wavelengths and polarisations. The calculated guide thicknesses also undergo a sudden transition to a narrower width at ~325°C. What is also of interest is that there are only 3 guide widths "used" by all the wavelengths and polarisations. Again, that the thicknesses
"jump" is not surprising and could well be an artefact of the simplicity of the calculation technique. The fact that the "jump" occurs at approximately the same annealing temperature and that the same thickness keep recurring seems to make the observation more significant.

7.2.5.2.2 Absorption

Figure 108, with expanded view of the 200-500nm region in Figure 109, shows the absorption of the implanted region after different annealing temperatures as a function of wavelength.

Figure 108: Absorption Spectra for 6MeV Li implanted 1 \(10^{16}\) ions/cm\(^2\) vs annealing temperature.

Figure 110 shows the calculated absorption of the waveguide as function of wavelength in the as-implanted state. It shows that there is a strong absorption in the UV, peaking at about 250nm which trails off at the longer wavelengths and is lost in the measurement noise at about 700nm. To scale the figure for an average loss in dB/cm involves a free parameter, namely the depth of crystal that has been "changed" by the
Figure 109 Absorption Spectra (Nd lines included) as a function of annealing temperature and wavelength for 6MeV Li implanted Nd:YAG to $1 \times 10^{16}$ ions/cm$^2$. implantation process. A comparison of the measurements taken with the spectrometer and the decay of fluorescence measurements (see below) enable the effective depth parameter to be determined as ~25μm. This is surprising since both the TRIM calculations and the waveguide mode calculations suggest a waveguide depth/Li ion range of ~10μm.
The measurement noise comes from the baseline drift of the spectrophotometer - the empty cavity spectra is taken before and after the implanted YAG transmission measurements, the smaller is divided by the larger to give a "loss" figure and then scaled.

![Graph showing absorption spectra](image)

**Figure 121** Calculated absorption spectra for 6MeV Li implanted to $1 \times 10^{16}$ ions/cm$^2$ as implanted.

with the other measurements. This provides a noise floor for the calculated loss of about 10dB/cm.

The absorption spectra show a number of interesting features. If an extrapolation to longer wavelengths is valid (i.e., that the loss at 1064nm is due mostly to the tails of the absorptions in the visible) then the, as implanted loss at 1064nm (lasing wavelength) is < 5dB/cm and closer to 2dB/cm.
Figure 122  Calculated waveguide loss (dB/cm) vs wavelength and anneal temperature.

The implantation process gives not just a single absorption peak, but the absorption spectrum would seem to be the result of a combination of absorptions centres of Lorentzian profile with centres at 252nm, 300nm, 360nm and 485nm. Several of the peaks change wavelength after annealing. The 360nm peak migrates to ~350nm before disappearing. The 300nm peaks migrates to 285nm by the time the annealing temperature has reached 800°C. The other 2 peaks show no major changes in wavelength. There are substantial decreases in loss, between 300 and 400 degrees, 400 and 500 degrees and then between 500 and 600 degrees, above which there is very little change in the absorption with further annealing.

As Figures 123 and 124 show, the 300-400deg transition sees the disappearance of the 485nm absorption and consequently the longer absorption tails which extend into the IR. As a result a decrease in the absorption at the laser and pump wavelengths could be expected. The transition from 400-500°C sees the disappearance of the 360nm line and 500-600 sees the decay of the 250nm absorption leaving the major feature as a Lorentzian absorption peak located at 285nm. The rising "tails" in Figure 122 at the UV end of the
spectrum starting from the 400°C measurements may indicate another absorption further into the UV of magnitude \(~500\text{dB/cm}\) (guessing from the direction of the "tails" at 300°C (ie. down) and at 400°C (ie. up)) or simply reflect an artefact of the absorption edge in YAG (as separate to neodymium) at \(~240\text{nm}\).

It is expected that the disappearance of various lines indicates that annealing 

![Graph](image.png)

**Figure 123**  Fitted peak heights (above noise floor) for absorption spectra vs annealing temperature for 6MeV Li implanted Nd:YAG.

removes various colour centres formed during the implantation process. The disappearance of the 485nm feature and the sudden changes in the refractive index values for many of the modes at the same temperature appears significant.

7.2.5.2.3 **Loss at 700-800nm From Fluorescence Measurements**

The technique of measuring the waveguide loss based on measurements of decay of fluorescence involved the following. Light was launched into the waveguide at a wavelength close to one of the pump bands of Nd and, by the use of interference filters, an image of the decay of the 1064nm fluorescence as a function of distance along the guide was captured using
a CCD camera and frame grabber. The strength of the fluorescence reflects the intensity at the "pump" (which lay in the 700-800nm range) as a function of propagation distance. By tuning the pump wavelength progressively further from the pump absorption band, the observed decay rates can be extrapolated to determine the inherent waveguide loss. Alternatively, since there are large regions of the chip which have not been implanted, a reference image indicating the rate of decay of the pump due to absorption by the Nd pump bands can be recorded, and used as a baseline for the waveguide measurements. This allowed the net loss (ie. due to effects of the waveguide alone) of the waveguide to be determined.

A typical measurement is shown in Figure 124. It shows a spread of values, reflecting inconsistency between successive sets of data, with a general trend towards lower loss at longer wavelengths. Figures 125 and 126 show frequency histograms for different losses in specific wavelength ranges obtained from the fluorescence measurements after annealing at 350°C and 400°C. These show the same trends observed in other measurements: namely a sudden change occurs between 300 and 400°C - the waveguide at 400°C having considerably
Figure 125  Histogram of measured waveguide losses after 350°C annealing.

lower losses, as indicated by the cluster of results closer to zero.

Figure 127 shows the behaviour of the mean, median and histogram peak losses as functions of the annealing temperature. The high result at 550°C does not fit with any of the other measurements and is expected to be in error. The major feature seems to be, rather than a gradual decrease in the waveguide loss (as is seen in the spectrometer results for the shorter wavelengths), a plateau at ~ 1.7dB/cm up to 350°C and a similar plateau at 0.7dB/cm at higher temperatures. This is consistent with the sudden changes in the refractive index profiles observed at similar temperatures. It demonstrates that there is little advantage in annealing much above 400 degrees for these implant conditions.
7.2.5.2.4 Annealing Behaviour Summary

In summary, whilst annealing in an inert atmosphere results in a gradual reduction in absorption at shorter wavelengths, particularly in the UV, with increasing temperature, the waveguide losses at ~800nm and the refractive index structures show step-like changes at ~350°C with little variation on either side. This temperature also corresponds to a region where the 485nm absorption peak is observed to anneal away, suggesting this colour centre has a strong role in determining the properties of the Li ion induced waveguides in Nd:YAG.

7.2.6 5mm x 8mm Planar Laser

A final 5mm x 8mm x 4mm Nd:YAG laser sample was prepared in the CSIRO's polishing laboratory in Sydney (parallelism of the laser mirror faces 0.5 arc minutes,
Figure 127  Variation of Loss measures with annealing temperature. Shows sudden change from ~1.7dB/cm to 0.7dB/cm in annealing from 300°C - 400°C

scratch/dig 20/10, surface flatness of top surface λ/10 and edge sharpness, 0.5μm chipping). This was implanted with 6MeV Li ions to a dose of 3×10^{16} ions/cm^2 in a strip ~2 mm wide along the 5mm axis of the 5mm x 8mm face.

The end faces were subsequently coated using alternate layers of the dielectric SiO\(_2\) (n=1.452 at 632.8nm [152]) and TiO\(_2\) (n=2.35 at 632.8nm [152]) (ie. a high (H) and a low index (L) in a H (HL)\(_7\) 0.5L configuration). This gave a high reflection at 1064nm (>99% on the test slides coated at the same time - the reflection could not be measured for the actual sample) and a reasonable transmission (>85%) at the pump wavelength (808nm). The 0.5L layer goes towards smoothing out some of the "ripples" in the reflectivity and makes it easier to obtain good transmission at 808nm (which lies in the wings of the 1064nm main reflection peak). In the coating process, however, some of the dielectric formed a thin coating on the top and bottom surfaces of the chip. This was not nearly thick enough to cause any change in the guiding properties, but it prevented using the spectrometer to measure guide absorption and the measurement of the guided modes by prism coupling (which in any case was to be avoided to reduce the likelihood of damaging
the corners of the sample).

To enable the dielectric coating to adhere well the crystal was heated to ~200°C during the coating process. This was undertaken in high vacuum (2 \(10^{-4}\) Torr) and is equivalent to an initial 200°C anneal.

The laser crystal underwent subsequent annealing in an Ar atmosphere successively at 300°C, 400°C and 500°C. The annealing, however, caused the dielectric coating on one of the faces to peel and come off. Hence, this face had to be recoated after each annealing. The peeling only seemed to occur for the one mirror face - the other coped with the annealing as well as a prolonged batch of cleaning to remove the peeling layer from the other face and clean away any surface contaminants (oil, dust, grease, polishing compound) - no reason was found for the behaviour.
7.2.6.1 Laser Threshold

<table>
<thead>
<tr>
<th>Anneal Temperature (°C)</th>
<th>% Reflectivity</th>
<th>External Threshold (mW)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>808nm</td>
<td>1064nm</td>
</tr>
<tr>
<td>200</td>
<td>3.3</td>
<td>99.6</td>
</tr>
<tr>
<td>300</td>
<td>4.2</td>
<td>99.2</td>
</tr>
<tr>
<td>400</td>
<td>10.6</td>
<td>99.1</td>
</tr>
<tr>
<td>500</td>
<td>1.0</td>
<td>99.2</td>
</tr>
</tbody>
</table>

**Table 24** Laser External Threshold (mW) vs annealing temperature for Chip Laser implanted with 6MeV Li at a dose of $3 \times 10^{16}$ ions/cm².

The coated laser crystal was end pumped with a Coherent Mira laser operated in a CW mode and using the optical system described in Figure 105 to give focussing in the vertical direction and collimation in the horizontal direction. **Table 24** shows a comparison of the external (ie. before the launching objective and crystal face) pump power needed to get the chip laser to threshold. Also indicated in the table is the reflectivity for the pump and lasing wavelengths as indicated by the microscope slides used as test samples.

![Figure 128](image_url)  
**Figure 128** Slope Efficiency for Nd:YAG chip laser after annealing at 300°C. Laser has a slope efficiency of 9.4%.
There would appear to be a minimum threshold after the sample was annealed at 300°C although the values were difficult to determine precisely due to the sensitivity of the lasing thresholds to alignment of the pump beam.

Lasing was achieved using either TE or TM polarised pump beams although the TM polarization was found to give a slightly lower (~10%) threshold.

Figure 129  Slope efficiency of 200°C annealed Nd:YAG chip laser. Line is $y=0.0701 x -1347.3$ ($r^2=0.92$). Indicates a slope efficiency of ~7%.

Figure 130  Intensity profile for 6MeV Li implanted Nd:YAG (Dose = $3 \times 10^{16}$ ions/cm²) lasing mode. Intensity cross-section taken at line in figure.
7.2.6.2 Slope Efficiency

The slope efficiency of the laser was measured in two situations - for the initial laser and after the annealing to 300°C. The results are shown in Figures 129 and 128. The laser has an increase in slope efficiency of 2.4% from ~7% to 9.4% after annealing.

Figure 131  Lasing in a leaky mode 1

Figure 132  Lasing in a leaky mode 2

Figure 133  Lasing in a Leaky Mode 3
7.2.6.3 Lasing Modes

Near threshold the chip lased in a single waveguide mode and Figure 130 shows that the beam had a Gaussian cross-section at the waveguide output.

Apart from the one guided mode, a number of "leaky" modes were also observed to lase (these are shown in Figure 132-135 but required successive increases in pump power to reach threshold as shown in Figure 135). The different modes were excited by tracking the input beam down the crystal face by up to several hundred microns. The fact that so many modes could be excited and then propagate through 16mm of YAG when the guide
technically supported only one guided mode, validates the inclusion of the leaky modes in
the earlier calculations.

7.2.6.4 Waveguide Loss

As with previously implanted chip, the waveguide loss was measured using the
fluorescence at 1064nm to investigate the loss at 770-850nm. The loss results are shown in
Figure 136. The waveguide loss decreases after the 300°C anneal, but surprisingly when
compared with the 1 $10^{16}$ ions/cm$^2$ data, the losses increase with further annealing. There
is however a good correlation of the loss and waveguide threshold data.

![Graph showing waveguide loss via mode number](Image)

**Figure 135** External threshold (mW) vs mode number for 200°C anneal chip laser.
7.3 Summary

Industrial sapphire samples were implanted with 3MeV Li ions at a range of doses from \(1 \times 10^{15} \) to \(3 \times 10^{16} \) ions/cm\(^2\). These measurements established that Li implants gave an increase in refractive index proportional to ion dose and that colour centres with absorptions at wavelengths < 500nm were produced. 3MeV Li implants into Nd:YAG confirmed these effects also occurred in this material, but the waveguides produced did not have any guided modes for the laser pump wavelength at ~808nm or the lasing wavelength at 1064nm. An increase in the ion implantation induced refractive index change or the waveguide thickness was necessary to allow guided modes at these wavelengths.

1 MeV H implants into Nd:YAG were then performed as this would have a greater range in Nd:YAG than Li. However measurements showed that H gave a decrease in the refractive index in the guide region.

6 MeV Li implants were then performed into Nd:YAG samples carefully prepared to have high quality square edges at doses of 1 and \(3 \times 10^{16} \) ions/cm\(^2\). Both samples showed
waveguiding at 810 and 1064nm and were made to lase.

The 6MeV Li sample, implanted to a dose of $3 \times 10^{16}$ ions/cm$^2$ had mirror coatings applied directly to the crystal faces and lased with an external threshold of $\sim 50$ mW. Annealing this sample at 300°C caused a decrease in this threshold to $\sim 39$ mW but subsequent annealing produced an increase in the threshold that was paralleled in an increase in the waveguide loss in the 800nm region of the spectrum.

An extensive study was made of the waveguide modes, absorption and waveguide loss as the $1 \times 10^{16}$ ions/cm$^2$ sample was progressively annealed to 800°C. While the absorption measurements show a fairly continuous reduction with increasing absorption temperature, both the refractive index profile calculated from the waveguide modes and the guide loss measurements show sudden changes as the waveguide is annealed from 300-400°C. This occurs simultaneously with the disappearance of the colour centre at 485nm. Further work to determine relationship between the behaviour of the colour centres formed by different implant energies and doses; and the waveguide loss and refractive index is required.
8 Summary, Conclusions and Future Directions

8.1 Summary

This work examined the use of a waveguide device, namely a nonlinear directional coupler to passively modelock a solid-state laser. Ideally the laser would have been fabricated in such a way that coupling, gain and feedback were all integrated within a monolithic structured waveguide thereby minimising the optical losses as well as the pump power needed to produce self-starting passive mode-locking. In practice it proved too difficult to fabricate this monolithic device and hence the experimental work focussed on the use of a bulk gain medium combined with a passive, fibre-based nonlinear directional coupler. Additionally the possibility of creating the monolithic device in a planar waveguide created by ion implantation into the surface of a laser crystal was examined. The laser constructed using a bulk gain medium and a separate nonlinear directional coupler failed to produce self-starting passive mode-locking in spite of the fact that it satisfied several of the threshold criteria from the literature. As a result considerable attention was paid to the predictions relating to self-starting mode-locking in this type of laser.

Examination of the available theory identified a number of important parameters for the self-starting modelocking and its long term stability. These included:-

- High cavity circulating power. This suggests the laser should have large gain, high pump power and a low cavity loss. In practice this favours the use of a monolithic all-waveguide or all-fibre design.
- High local intensity. This is not only essential to obtain low power switching in the nonlinear directional coupler but also leads to a small saturation power, \( P_{\text{sat}} \), in the gain medium, which was important in some aspects of the theory.
- Low Dispersion, preferably anomalous. This could become significant in an all-fibre device especially if the fibre is rather long. In conditions where the
light intensity in the nonlinear directional coupler is high self-phase modulation, SPM, can also become important and a small amount of anomalous dispersion could counteract this and prevent pulse break-up. It should be noted, however, that if the nonlinearity introduced by the coupler is sufficiently strong, mode-locking will occur for all reasonable values of dispersion.

- Long Mode Correlation time. This should be achievable by ensuring the laser resonator is mechanically stable and if care is taken to suppress all back reflections into and from within the cavity.

- Large Gain Bandwidth. This leads to both a narrow pulsewidth and greater intensity modulations in the cavity light field during the critical build-up phase and hence a greater chance of self-starting.

- Optimal Fibre Length. The strength of the mode-locking driving force, $\gamma$, is strongly dependent on the length of the nonlinear directional coupler. The optimal length depends on the circulating power and number of fibre beatlengths used. In general, longer fibre lengths and longer beatlengths give a more favourable $\gamma$, although the $\gamma/T_R$ parameter, important in many of the self-starting criteria, means that the returns for going greater than 2 beatlengths are small.

Not all of these conditions could be well met with a laser using separate gain and coupling sections. Nevertheless a laser was built using Nd:YVO$_4$ as the gain material for 1342nm with parameters as close as possible to those outlined above.

This laser failed to give any mode-locking behaviour in spite of the fact that its resonator losses were quite low (\(\sim 19\pm 2\%\), including \(12\pm 0.5\%\) due to the coupler). Measurements of the mode-correlation time by monitoring the beatnote between the longitudinal modes of the laser were performed but proved rather inconclusive. No beatnote, at all could be detected with the NLDC within the cavity, despite it being readily measurable, (if wider and containing considerably more structure that expected) without the fibre. As a result the mode-correlation time was unknown. A number of mechanisms were explored to explain the beatnote structure of the resonator without the coupler. Higher order cavity modes, polarisation rotation and sources of cavity loss were eliminated as causes. We suspect that reflections into, and from within the cavity were the cause of
the observed broadening of the signal.

The twin core fibre used as the NLDC was extensively characterised. Its absorption and beatlength as a function of wavelength were both measured, as was its switching power. Options for tuning the beatlength to obtain the optimum coupling conditions were explored.

In the light of the results and the relatively favourable parameters measured in the experiments, the implications of not obtaining mode-locking were examined against the predictions of the available analytical models. It was found that several of the models were overly optimistic and some slightly pessimistic: i.e. most predicted that self-starting should have occurred in the conditions achieved in the experiments. A numerical simulation focussing on the laser behaviour during the critical build-up phase was developed and its predictions compared with the analytical models. It showed good agreement with the more conservative of those models confirming that the required experimental conditions were far more stringent than those that were actually achieved.

The conservative analytic models and numerical results demonstrated that self-starting mode-locking would only occur if the resonator loss could be markedly reduced and the pump power significantly increased (by orders of magnitude in each case). Alternatively a large increase in mode-locking driving force was needed, and this could be obtained by increasing the beatlength of the NLDC to greater than 20cm or by increasing the nonlinearity of the material from which the NLDC was fabricated. None of these options was practical.

The more demanding self-starting predictions showed that it would be impossible to fabricate a monolithic planar waveguide device that would produce self-starting mode-locking using technology available in our laboratories. As a result the investigation of planar waveguide devices was restricted to a study of the properties of waveguides produced by Li ion implantation into Nd:YAG.

To this end, a number of preliminary implants were performed in sapphire, followed by implants in Neodymium doped Yttrium Aluminium Garnet (YAG). The problem of determining the refractive index profiles in these waveguides was extensively analysed, and a numerical method adopted based on measurement of the effective indices of both guided and leaky modes as the only one capable of giving meaningful results. It was found that planar waveguides could be produced from a single implant of 6 MeV Li
ions to a dose of $10^{16}$ ions/cm$^2$. Absorption measurements show a total waveguide loss of $1.7\pm0.2$ dB/cm as implanted which reduces to $0.7\pm0.2$ dB/cm with annealing. The absorption spectrum showed a number of distinct peaks due to the creation of colour centres. There was a significant change in absorption spectrum; waveguide loss around 700-800 nm (measured using fluorescence techniques); and the waveguide mode patterns, as the sample was progressively annealed from 300°C to 400°C.

Lasing in the ion implanted planar guide was achieved with an external threshold of $39\pm2.5$ mW. The lasing occurred in the single guided mode which existed in the waveguide at 1064 nm, but was also possible in at least 4 other “leaky” modes. This threshold is quite low and suggests that Li ion implantation is a viable technique for fabricating low loss planar active waveguides.

### 8.2 Conclusions and Future Directions

A waveguide passively modelocked laser based on a Non Linear Directional Coupler (NLDC) is achievable but will require reduced cavity loss <1%; a longer beatlength fibre >20 cm; and preferably much high material nonlinearities to obtain self-starting at low pump power. The use of a glass host for the active laser ion is also preferable since generally this leads to an increased bandwidth (albeit as the expense of reduced gain), and this favours self-starting. It would appear possible to use a chalcogenide glass as the host material to obtain much more favourable conditions for self-starting; whether this is technically feasible or a useful goal remains an open question.

One puzzling aspect of the results was the absence of any cavity beatnotes from the laser containing the NLDC, despite the fact that the beatnote was readily detected in other circumstances. This would seem to indicate that the fibre has an affect on the dynamics inside the cavity that is not currently understood, but may become important in implementing an all fibre passively modelocked laser using the NLDC.

From the comparison of the computer simulations with real experimental parameters and the available analytic theory, Haus’s pulse stability criteria would appear to be the most useful for predicting the self-starting behaviour of this class of device.

Planar waveguides were produced by a single Li implant into Nd:YAG. Thresholds of 39 mW external were obtained. This could be decreased by confinement in two
dimensions by implanting through a mask or strip loading to obtain lateral confinement using a high refractive index layer. Strip loading using a highly nonlinear material could also be investigated as a means of increasing the effective nonlinearity of the structured waveguide as a means of increasing the mode-locking driving force. The refractive index changes created by ion implantation would appear to depend on the production of colour centres at 252, 300, 360 and 485nm. Future work to determine whether these act together or a specific absorption determines the refractive index changes could provide a route to reducing the waveguide loss allowing the NLDC design to be implemented on a single, large Nd:YAG chip.
A. Appendix A: Coupled Mode Theory for the Nonlinear Directional Coupler

A.1 Linear Guides

Taking Maxwell's Equations for source free, time dependent fields

\[
\nabla \times E = -\frac{\partial B}{\partial t}
\]

\[
\nabla \times H = \frac{\partial D}{\partial t}
\]  

(163)

where \( \nabla = (\partial / \partial x, \partial / \partial y, \partial / \partial z) \) and \( E, H, D, B \) are the time dependent vectors of the electric and magnetic field, the electric displacement and the magnetic induction respectively.

Assuming \( E(t) = E e^{i \omega t}, D = \varepsilon E \) and \( B = \mu H \) gives

\[
\nabla \times E = -i\omega \mu H \quad \nabla \times H = i\omega E
\]  

(164)

Now for purposes of further analysis, we break the fields up into transverse \((x,y)\) and longitudinal components \((z)\) then (164) becomes

\[
\nabla_t \times E_t = -i\omega \mu H_z
\]

\[
\nabla_t \times H_t = i\omega E_z
\]

\[
\nabla_z \times E_z + e_z \times \frac{\partial E_t}{\partial z} = -i\omega \mu H_t
\]

\[
\nabla_z \times H_z + e_z \times \frac{\partial H_t}{\partial z} = i\omega E_t
\]  

(165)

where \( \nabla_t = (\partial / \partial x, \partial / \partial y, 0) \) and \( e_z \) is the unit vector in the \( z \) direction.
Now, looking for solutions of the form \( E(x,y,z) = E_v \exp(-i \beta_v z) \) (where we anticipate a number of solutions, marked by a mode number \( v \)) changes (165) to

\[
\begin{align*}
\nabla_x \times E_v & = -i \omega \mu H_{tv} \\
\nabla_x \times H_v & = i \omega \varepsilon E_v \\
\n\nabla_x \times E_v - i \beta_v e_z \times E_v & = -i \omega \mu H_{tv} \\
\n\nabla_x \times H_v - i \beta_v e_z \times H_v & = i \omega \varepsilon E_v
\end{align*}
\]

(166)

We postulate that the overall field \( E_t \) for any given two waveguide structure can be represented by an expansion over the guided modes of the waveguides \( E_{tl}(a) \) or \( E_{tl}(b) \). ie.

\[
\begin{align*}
E_t(x,y,z) & = \sum_{v=1}^{\infty} a_v(z)E_{rv}^{(a)}(x,y) \\
& = \sum_{v=1}^{\infty} b_v(z)E_{rv}^{(b)}(x,y) \\
H_t(x,y,z) & = \sum_{v=1}^{\infty} a_v(z)H_{rv}^{(a)}(x,y) \\
& = \sum_{v=1}^{\infty} b_v(z)H_{rv}^{(b)}(x,y)
\end{align*}
\]

(167)

where the summation, as in [35] includes integrations over the guided modes as well as the leaky or radiation modes. Next expand the guided mode of each guide over the complete set of the other waveguide modes ie.

\[
\begin{align*}
E_{tl}^{(b)} & = \sum_{v=1}^{\infty} C_v E_{nv}^{(a)}(x,y) \\
E_{tl}^{(a)} & = \sum_{v=1}^{\infty} D_v E_{nv}^{(b)}(x,y)
\end{align*}
\]

(168)

where we define

\[
\begin{align*}
C_v & = 2\pi \int_{-\infty}^{\infty} E_{tl}^{(b)} \times H_{tv}^{(a)} \, dx \, dy \\
D_v & = 2\pi \int_{-\infty}^{\infty} E_{tl}^{(a)} \times H_{tv}^{(b)} \, dx \, dy
\end{align*}
\]

(169)

Now assuming that the field of the waveguide structure can be described by

\[
E_t(x,y,z) = A(z)E_t^{(a)}(x,y) + B(z)E_t^{(b)}(x,y) + R_t(x,y,z)
\]

(170)

we substitute (167)-(168) into (170) to give
\[ E_{i}(x,y,z) = \left[ a_{i}(z) - A(z) - C_{i}B(z) \right] E_{n}^{(a)}(x,y) + \sum_{\nu=2}^{\infty} \left[ a_{\nu}(z) - C_{\nu}B(z) \right] E_{n}^{(b)}(x,y) \]

\[ = \left[ b_{i}(z) - B(z) - D_{i}A(z) \right] E_{n}^{(b)}(x,y) + \sum_{\nu=2}^{\infty} \left[ b_{\nu}(z) - D_{\nu}A(z) \right] E_{n}^{(b)}(x,y) \]

(171)

Now, if we require that \( E_{i}(x,y,z) \) be orthogonal to the guided modes for all \( z \), it follows that

\[ a_{i}(z) = A(z) + C_{i}B(z) \quad b_{i}(z) = B(z) + D_{i}A(z) \]

(172)

In the presence of sources \( P(x,y,z) \), exciting various waveguide modes Maxwell's Equations from (164) become

\[ \nabla \times \mathbf{E} = -i\omega \varepsilon \mathbf{H} \quad \nabla \times \mathbf{H} = i\omega \mu \mathbf{E} + i\omega \mathbf{P} \]

(173)

From this it is possible to derive, for any two arbitrary electromagnetic fields

\[ \nabla \cdot \left( \mathbf{E}_{1} \times \mathbf{H}_{2}^{*} + \mathbf{E}_{2}^{*} \times \mathbf{H}_{1} \right) = -i\omega \mathbf{P}_{1} \cdot \mathbf{E}_{2}^{*} + i\omega \mathbf{P}_{2} \cdot \mathbf{E}_{1} \]

(174)

Setting \( \mathbf{P}_{2} = 0 \), integrating over the guide cross-section and using the divergence theorem gives

\[ z \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial}{\partial z} \left( \mathbf{E}_{1} \times \mathbf{H}_{2}^{*} + \mathbf{E}_{2}^{*} \times \mathbf{H}_{1} \right) dxdy = -i\omega \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{P}_{1} \cdot \mathbf{E}_{2}^{*} dxdy \]

(175)

Now, if we expand the transverse components of the perturbed field over the complete set of unperturbed modes of the waveguide ie.

\[ E_{n}(x,y,z) = \sum_{\nu=1}^{\infty} p_{\nu}(z) E_{n}(x,y) \]

(176)

and we are looking at interaction with the mode designated \( \mu \) in guide 2. ie. \( E_{2}(x,y,z) = E_{\mu}(x,y) \exp(-i\beta_{\mu}z) \) then substituting (176) into (175) and making use of the orthogonality relationship for the modes ie.
\begin{equation}
2z \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_v \times H_\mu dxdy = \delta_{\nu\mu}
\tag{177}
\end{equation}

gives

\begin{equation}
\frac{dp_\mu}{dz} + i\beta_\mu p_\mu = -i\omega \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P.E_\mu^* dxdy
\tag{178}
\end{equation}

For the purposes of this derivation the polarizations, P, are the result of deviations of the refractive index/dielectric constant from the nominal distribution for which the modes were calculated and as such can be represented by

\begin{equation}
P_\mu = \Delta \varepsilon E_\mu
\tag{179}
\end{equation}

This is straightforward for P_t, i.e.

\begin{equation}
P_t = \Delta \varepsilon E_t = \Delta \varepsilon \sum_{\mu=1}^{\infty} p_\mu E_{\mu t}(x,y)
\tag{180}
\end{equation}

but for P_z we need a more roundabout route. From (173)b

\begin{equation}
\nabla \times H_t = i\omega \varepsilon E_t + i\omega \Delta \varepsilon E_x
= i\omega (\varepsilon + \Delta \varepsilon) E_z
\tag{181}
\end{equation}

Hence

\begin{equation}
P_z = \Delta \varepsilon E_{zt} = \frac{\Delta \varepsilon}{i\omega (\varepsilon + \Delta \varepsilon)} (\nabla \times H_{t})
= \frac{\Delta \varepsilon}{i\omega (\varepsilon + \Delta \varepsilon)} (\nabla \times \sum_{v=1}^{\infty} a_v H_{tv})
= \frac{\Delta \varepsilon}{i\omega (\varepsilon + \Delta \varepsilon)} \sum_{v=1}^{\infty} a_v (\nabla \times H_{tv})
= \frac{\Delta \varepsilon}{i\omega (\varepsilon + \Delta \varepsilon)} \sum_{v=1}^{\infty} a_v E_{zv}
\tag{182}
\end{equation}

So substituting (173) and (182) into (178) gives
\[
\frac{dp_{\mu}}{dz} + i \beta_{\mu} p_{\mu} = -i \omega \sum_{v=1}^{\infty} a_v \kappa_{\nu \mu} \tag{183}
\]

where

\[
\kappa_{\nu \mu} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Delta E_{\nu \mu} E_{\nu v} - \frac{\varepsilon \Delta e}{\varepsilon + \Delta e} E_{\nu \mu} E_{\nu v} dxdy \tag{184}
\]

Substituting (172) into (183) gives

\[
\frac{dA}{dz} + C_1 \frac{dB}{dz} = i [ \bar{\beta}^{(a)} + \kappa_{11}^{(a)} ] (A + C_1 B) + i \sum_{v=2}^{\infty} a_v(z) \kappa_{v1}^{(a)} \tag{185}
\]

Adding and subtracting \( \sum_{v=2}^{\infty} C_v \kappa_{v1}^{(a)} B(z) \) from RHS of (185) gives

\[
\frac{dA}{dz} + C_1 \frac{dB}{dz} = i [ \bar{\beta}^{(a)} + \kappa_{11}^{(a)} ] A + i [ C_1 \bar{\beta}^{(a)} + \kappa_{ab} ] B + i \sum_{v=2}^{\infty} \kappa_{v1}^{(a)} [ a_v(z) - C_v B(z) ] \tag{186}
\]

where use has been made of

\[
\kappa_{ab} = \sum_{v=1}^{\infty} C_v \kappa_{v1}^{(a)} \tag{187}
\]

and, finally, rearranging the order of the summation in (187) gives

\[
\kappa_{ab} = \omega \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Delta E^{(a)} \sum_{v=1}^{\infty} C_v E_{\nu v}^{(a)} E_{\nu v}^{* (a)} dxdy + \omega \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\varepsilon \Delta e^{(a)}}{\varepsilon^{(a)} + \Delta e^{(a)}} \sum_{v=1}^{\infty} C_v E_{\nu v}^{(a)} E_{\nu v}^{* (a)} dxdy \tag{188}
\]

Rearranging (186) and the equivalent for B gives

\[
\frac{dA}{dz} = i \xi^{(a)} A + i K_{ab} B \quad \frac{dB}{dz} = i \xi^{(b)} B + i K_{ba} A \tag{189}
\]
where

\[ \xi^{(a)} = \beta^{(a)} + \frac{\kappa_{aa} - C_1 \kappa_{ba} + C_1 D_1 (\beta^{(a)} - \beta^{(b)})}{1 - C_1 D_1} \]

\[ \xi^{(b)} = \beta^{(b)} + \frac{\kappa_{bb} - D_1 \kappa_{ab} + C_1 D_1 (\beta^{(b)} - \beta^{(a)})}{1 - C_1 D_1} \]

\[ K_{ab} = \kappa_{ab} + C_1 (\beta^{(a)} - \beta^{(b)} - \kappa_{bb}) \]

\[ K_{ba} = \kappa_{ba} + D_1 (\beta^{(b)} - \beta^{(a)} - \kappa_{aa}) \]

### A.2 Nonlinear Guides.

If we now allow for the guides to be nonlinear (i.e., the refractive index has a component that changes with light intensity \( n = n_0 + n_2 I \)) then the expression for \( \Delta e^{(a)} \) (from eqns (179) and (188)) has two components, one for the presence of the other guide and one for the nonlinear refractive index change induced by the light intensity profile across the entire guide. i.e.

\[ \Delta e^{(a)} = \epsilon (n_a^2 - n_i^2) + n_2 \epsilon^2 c |AE^{(a)} + BE^{(b)}|^2 \]

\[ = \epsilon (n_a^2 - n_i^2) + n_2 \epsilon^2 c [A |E^{(a)}|^2 + AB \langle E^{(a)} E^{(b)} \rangle + A \langle B^{(a)} E^{(b)} \rangle + B |E^{(b)}|^2] \quad (191) \]

The advantage of the above derivation is that, as long as the approximation of eqn (179) is valid, it can be used to model a variety of waveguide shapes and properties. This could be done on a waveguide by waveguide basis, but often the guide itself remains constant and it is the launched powers that we wish to vary. In this case it is better to move some of the terms from (191) into the equations and simply compute the integrals across the guided modes.

The expressions are tractable (just) for the TE modes of a planar step index guide. Equation (189a) then becomes

\[ \frac{dA}{dz} = \frac{A}{1 - C^2} [\beta (1 - C^2) + (I_1 - CI_2)] \]

\[ + |A|^2 (I_2 - CI_3) + AB \langle I_3 - CI_5 \rangle + A \langle B (I_3 - CI_2) \rangle + B |I_2 (I_2 - CI_3) |^2 ] \]

\[ + iB (I_6 - CI_5) \]

\[ + |A|^2 (I_7 - CI_2) + AB \langle I_2 - CI_5 \rangle + A \langle B (I_2 - CI_3) \rangle + B |I_2 (I_2 - CI_3) |^2 ] \]

\[ + iB (I_6 - CI_5) \quad (192) \]
where

\[
I_6 = \omega \int \int (n_a^2 - n^2) E^a_y E^b_y \, dx \, dy
\]

\[
I_7 = \omega^2 n_c E^a_0 E^b_0 \, dx \, dy
\]

\[
I_5 = \omega^2 n_c E^a_2 E^b_2 \, dx \, dy
\]

\[
I_3 = \omega^2 n_c E^a_3 E^b_3 \, dx \, dy
\]

\[
I_1 = \omega \int \int (n_a^2 - n^2) E^a_1 \, dx \, dy
\]

\[
I_2 = \omega^2 n_c E^a_4 \, dx \, dy
\]

(193)

where \(E^a_y\) are the modes of the guide \(a\), \(n_a\) is the refractive index profile with just guide \(a\) in, \(n\) is the total refractive index profile including both guides and \(C\) is given by (169). Qualitatively \(I_1\) indicates the effect of the second guide, \(b\), on the modes of guide \(a\), \(I_6\) indicates the overlap (and hence the coupling) of the modes of guide \(a\) to the modes of guide \(b\) and \(I_2, I_3, I_5\) and \(I_7\) indicate the effect of various combinations of the intensities of the different modes on the local refractive index.

Choosing the largest term in each bracket in (192) allows a considerably simplification to be obtained, leading to

\[
\frac{dA}{dz} = iA \left[ \beta(1 - C^2) + (I_1 - CI_6) + |A|^2 (I_2 - CI_7) \right] + iB[I_6 - CI_1]
\]

(194)

\[
= ia_1 A + ia_2 |A|^2 + ia_3 B
\]

and similarly for \(B\)
\[
\frac{dB}{dz} = i b_1 B + i b_2 B|B|^2 + i b_3 A
\] (195)

Qualitatively, \(a_1\) is the effect of the other guide on the mode, \(a_2\) is the effect of the intensity dependent refractive index on the mode and \(a_3\) is the coupling of the mode to the mode of the other guide.

Note that the expression for \(I_2\) (which is the important term in \(a_2\) and \(b_2\)) can be related to the expression derived by, for example Agrawal[153], for the self phase modulation of light in a fibre. Here, expanding the expression for \(I_2\) (including the normalisation of the modes) gives

\[
I_2 = \frac{\omega \varepsilon n_2 c}{A_{\text{eff}}} \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_y^4 \, dx \, dy \right]^{\frac{1}{2}} = \frac{\omega \varepsilon n_2 c}{A_{\text{eff}}} \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \beta^2 E_y^2 \, dx \, dy \right]^{\frac{1}{2}} 
\]

(196)

\[
A_{\text{eff}} = \frac{\beta^2}{k^2} \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_y^2 \, dx \, dy \right]^{\frac{1}{2}} \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_y^4 \, dx \, dy \right]^{\frac{1}{2}} 
\]

with the only difference between this and Agrawal’s result being the factor of \(\beta^2/k^2 = n_c^2\)

Evaluating the expressions for the TE modes of the planar guide leads to
and the comparative values for these expressions are shown in Fig 137 for a silica waveguide similar to the fibre used for the later experiments (ie. \( n_{co} = 1.455, n_{cl} = 1.45, \lambda = 1.342\mu m \) and \( r = 4\mu m \)). The expressions show a decrease in all the terms with separation of the guides except \( I_2 \), (which is not surprising as this integral quantifies the effect of the guide on itself).
Figure 137  Comparison of the expressions of eqn 7 For step index fibre ($n_c=1.455$, $n_o=1.45$, $\lambda=1.342\mu m$, $a=4\mu m$)
B. Appendix B: Relevant Neodymium Doped Fibre Experiments

B.1 Relevant Fibre Experiments

The original concept for the passively modelocked device had called for an all fibre design based on a twin-core Neodymium-doped glass fibre. Several attempts were made by Graham Atkins at the OFTC at University of Sydney to produce this, but there was no detectable coupling (at any of the wavelengths tried) in any of the samples.

Before attempts to produce fibre with gain and coupling were abandoned, a number of experiments were performed particularly looking at the effects of fibre in a laser cavity on the cavity beatnote and lasing spectra. These are included here, as they have relevance to issues which will need to be addressed in a final fibre laser device.

B.2 Fibre Laser Description

The lasers under consideration, consisted of 10cm lengths of nominally 1% Nd doped glass fibre (UNSW code No. 9341S). These were glued into glass capillary (using Summers Labs UV71 UV curable epoxy) and the ends polished back so that the end of the fibre was flush and parallel with the end of the capillary tube. This enabled dielectric mirrors (~98% reflectivity at 1.055μm) to be pressed against the ends by a spring arrangement or stuck on using UV curing epoxy - in either case the mirrors were self-aligning. Alternatively, the light from the fibre was collimated using a short focal length lens and feedback provided by an external mirror. This is referred to as the external cavity geometry in what follows.
Figure 138 shows the external threshold versus cavity loss for a 12cm long fibre laser with one mirror glued on and the other in an external cavity. The slope of the line fitted to this data indicates that the cavity has a total round-trip loss of ~35%. When the external cavity section was replaced by a mirror glued onto the fibre end, the external threshold dropped to ~1.2mW indicating that a substantial fraction of this loss is due to the external cavity and its associated optics. From these calculations the small signal gain is
given by:

\[ g = 0.28 P^\text{ext}_p + 0.52 \]  \hspace{1cm} (198)

where \( P^\text{ext}_p \) is the pump power to the cavity measured externally (i.e., ignoring the losses due to the end mirror, focussing optics and fibre coupling into account). The low threshold values obtained using this laser are worth noting since they indicate that such a laser could have been driven very far (500 times) above threshold using the available pump power. This would have provided conditions more favourable to self-starting that were achieved using the laser described in the body of this thesis.

**B.3 Fluorescence**

![Fluorescence spectra of Nd:Glass fibre](image)

Figure 140  Fluorescence spectra of Nd:Glass fibre showing emission at 1057±2nm (FWHM=16.5nm) and 896±1nm (FWHM 41nm)

The 10cm long samples were end pumped at 797nm using either a single mode diode laser or the Coherent Laser Group Mira laser configured to provide CW output. Figure 140 shows the fluorescence spectrum of the optical fibre, when pumped at 797nm.
It has two broad peaks, one at 1057±2nm that is Lorentzian in shape with a FWHM of 16.5±0.1nm. A second, much more highly structured peak is at 896±1nm and this has a FWHM of approximately 41±0.1nm. The 1057nm transition is much broader than the equivalent transition in Nd:YVO₄ again providing more favourable conditions for self-starting.

### B.4 Fibre Laser Spectra

![Graph showing fibre laser spectra](image)

**Figure 141** Output of fibre laser when pumped at ~200mA (140mW) showing “comb” of output wavelengths at ~1nm spacing. Figure shows the FWHM of the individual peaks.
Figure 142  Comparison of individual peak FWHM (nm) and spacing (nm) vs laser pump power (mA) for fibre laser. Mode spacing shows little variation with power, but FWHM shows a trend upwards.

The output from a 10cm long fibre laser shows a “comb” of narrow peaks, 1-2Å wide and spaced at approximately 1nm intervals. The spectrum for a diode pump power of 200mA (~140mW) is shown in Figure 141. The number of peaks grows with the laser power - from 2 at 55mA (~40mW) to 10 at 200mA (~182mW) as the gain bandwidth broadens. As is indicated in Figure 142, there is a trend for the peak spacing to decrease and the peak width to increase with increasing pump power and so this effect would seem to be power dependant.

The regular nature of this spectrum led us to suspect that we had formed a small etalon in the cavity, possibly enhanced by the presence of gain. The electric field inside a structure consisting of two identical mirrors of reflectivity $r$; a medium between the mirrors with gain $g$; and a round-trip phase shift, $\delta=2 \pi/\lambda 2 n_r d$, where $d$ is the etalon thickness and $n_r$ is the refractive index; is given by:

$$E_s [1 + r^2 g e^{-\delta} + r^4 g^2 e^{-2\delta} + r^6 g^3 e^{-3\delta} + ...]$$

(199)
where $E_s$ is the incident electric field. This forms a geometric progression with $a=1$ and ratio, $R=r^2g e^{i\delta}$. The sum to $n$ terms of such a series is:

$$S_n = \frac{a}{1-R} \frac{R^{n+1}}{1-R}$$

(200)

and hence:

$$E_t = E_s \left[ \frac{1}{1-r^2ge^{-i\delta}} \frac{\left(r^2ge^{-i\delta}\right)^{n+1}}{1-r^2ge^{-i\delta}} \right]$$

(201)

and so for intensity:

$$I_t = \frac{1}{1+(r^2g)^2-2r^2g\cos\delta} \left[ 1+(r^2g)^2-2(r^2g)^{(n+1)}\cos(n+1)\delta \right]$$

(202)

In practice, $r^2g \leq 1$ and so (202) reduces to:

$$I_t = \frac{1-Cos(n+1)\delta}{1-Cos\delta}$$

(203)

for the case of $r^2g = 1$ or:

$$I_t = \frac{1}{1+r^4g^2-2r^2g\cos\delta}$$

(204)

for $r^2g < 1$

The behaviour for a number of different values of $r^2g$ is shown in Figure 144. This figure shows the narrowing of the response with increasing $r^2g$, but also that, for even quite small values there is an appreciable change in the transmission of the structure. This is more clearly shown in Figure 143 where the modulation depth given by:

$$I_t = \frac{1+2r^2g}{1+r^4g^2+2r^2g}$$

(205)
is plotted as a function of $r^2g$. This shows that even quite small values, for example $r^2g=0.003$, gives a modulation depth in the transmission of $>1\%$.

From Figure 144 we also see that the transmission is a strongly peaked function with peaks at a spacing of $\delta=\pi$. If $n_f=1.45$ (ie. that of the glass) then this requires $d\approx 300\mu m$ to give the approximate peak spacing of 1nm. This is an difficult distance to explain. The fibre was $\sim 10cm$ long; the mirrors were attached using UV cured epoxy, and would need to be considerably closer to the fibre ends than 0.3mm for the laser to work (the strong divergence of the beam from the fibre would result in the reflection losses from the end mirrors becoming very high).
Figure 144  Comparison of the normalised transmission for a range of values for $r^2g$.

Figure 145  Single core fibre laser output pumped at 100mW showing etalon like structure.
These experiments were first carried out using some doped twin core fibre (for which it appeared there was no coupling, but these experiments made us question that belief). However, similar experiments using a single core, Nd doped fibre, with one mirror in an external cavity (ie. not glued to the end of the fibre) produced the results in Figure 145. In this instance the peaks were a little broader ~0.3nm FWHM and the spacing was ~2nm (implying d ~200μm for the etalon). The spacing of the modulation did not change when the end mirror was removed and reconfigured in an external cavity. This did, however, cause a change in the pattern of peaks (ie. the glued mirror produced single peaks, the external cavity gave the 3 peak pattern of the figure).

B.5 Beatnote Behaviour

The beatnote spectra of a number of fibre laser cavities were recorded. As with the simple cavity using a Nd:YVO₄ crystal as the gain medium for 1340nm which produced the beatnotes described in Chapter 4, the fibre laser beatnote traces represent the envelope

![Figure 146](image)

**Figure 146** Fibre in cavity beatnote for external pump of 245mW pump showing one central beatnote envelope.
of the peaks (the individual scans produced beatnotes that were considerably narrower, but their position and intensity varied dramatically between scans). The measurements, therefore, do not represent the instantaneous cavity behaviour but a statistical average over time.

As can be seen from Figure 146 the contrast is quite marked with the bulk element cavity. The beatnotes are generally a single, broad peaks with FWHM of the order of 100-200kHz, and approximately Gaussian in shape. The figure shows the running average of 100 scans.

![Figure 147](image)

**Figure 147** Comparison of beatnote width (Hz) vs laser pump power (mW) for a range of single core fibre samples.

The behaviour of the beatnote as a function of power is shown in Figure 147. As can be seen, the beatnotes show a slight increase in width as the external pump power is increased but this tends to saturate, often at quite low external pump powers, even as low as 100-200mW. The behaviour as a function of power is well fitted by curves of the form:

\[ W_{bn} = W_\infty e^{-cx} \]  

(206)

where \( W_{bn} \) is the beatnote width and \( W_\infty \) is the width at "infinite" power. This behaviour is
in contrast to the bulk case where the beatnotes showed a tendency to narrow with increase pump power.

**Figure 148** Comparison of cavity beatnote widths (Hz) vs core area for a range of single core fibres.

It was thought that the beatnote behaviour may be related to self-phase-modulation in the fibre core. To test this a 10m sample of fibre was drawn from a preform while continuously varying the pull speed giving fibre with identical glass composition (and hence core refractive index, dopant concentration, etc) but different core areas. This was then sectioned into five pieces and a laser, 10±0.2cm long was made from each. The beatnote as a function of power was measured for each fibre (**Figure 147**). Next the cores were imaged and their beam areas measured using a Spiricon Beam Analyser. As is shown in Figure 148 there is a trend towards broader beatnotes as the core area increases using either the average beatnote width, or the $W_\infty$ measurements.

We postulated that the width of the beatnote might be related to the maximum
induced SPM phase shift, $\theta_{\text{max}}$, for a given length, $z$ which is given by:

$$
\theta_{\text{max}} = \frac{z}{L_{\text{NL}}} = \alpha P_0 \alpha = \frac{n_2 \Omega_0 P_0 z}{c A_{\text{eff}}} 
$$

(207)

[154] where $L_{\text{NL}}$ is the characteristic length for SPM and $A_{\text{eff}}$ is the effective core area of the fibre. That is SPM, effects should decrease with increasing core area, which is not the case for this laser.

**B.6 Feedback into laser cavity**

Several different mirrors, to provide external feedback into the laser resonator, were placed on a translation stage and aligned to retro-reflect the output from a 9.8±0.1cm single core fibre laser. This laser had mirrors glued to both ends of the capillary and an external lens (NA 0.4 $f=6.24\text{mm}$) to collimate its output. Sample results are shown in **Figure 149**. They show that the beatnote is strongly affected by the light feeding back into the laser -

![Figure 149](image_url)
the beatnote width, height and frequency (despite the fact that the laser length remains constant) are all affected.

Figure 150 Comparison of beatnote behaviour with different reflecting surfaces.
The response of the beatnote width is shown in Figure 150 for the mirrors in Figure 151. The greatest affect comes with the 1064nm mirror, followed by the 800nm reflector and then the microscope slide. This seems to indicate that it is the feedback of the laser wavelength rather than the pump or other wavelengths that causes the beatnote broadening. The behaviour did not require careful alignment of the reflected light and it was possible to hold a microscope slide by hand and align it accurately enough to give a variation of a factor of three in the beatnote width.

The position of the external mirror also strongly affected the beatnote width. The beatnote width peaked when the mirror was 4.5±0.5cm and 12.5±0.5cm from the laser output and troughs occurred at ~0 and 9.5±0.5cm. The effect of the feedback became negligible when the external mirror was more than 20cm from the output. The laser cavity had a physical length of 9.8±0.1cm, giving an optical length of 14.2±0.5cm. The peaks/troughs in the width are separated by 9.5±0.5cm (ie. very close to the physical laser length, but not the optical length). The nulling of the effect past 20cm could indicate the coherence length of the laser.

It must be admitted that the beatnote behaviour is not well understood.
B.7 Summary

It was deduced that the beatnote width in fibre lasers is:
- considerably broader than for bulk lasers
- increases with increasing power (tending to saturate) and increasing core area.
- extremely sensitive to feedback from external sources into the laser at the lasing wavelength. Depending on the distance of the feedback source to the laser, this can cause a considerable broadening of the beatnote and also shift the cavity beatnote centre frequency.
C. Appendix C: Waveguide Mode Equations

C.1 Introduction

In this chapter the eigenvalue equation for the waveguide modes (Eqn (98)) will be derived from Maxwell's equations. This derivation is based on Snyder and Love [155], but explains the steps in greater detail.

C.2 The Vector Wave Equations

In a region of free space free from sources of electric or magnetic fields Maxwell's equations satisfy

\[ \nabla \times E = -\frac{\partial B}{\partial t} \quad \nabla \times H = \frac{\partial D}{\partial t} \] (208)

and using

\[ B = \mu_0 H \quad D = \varepsilon_0 E = \varepsilon_0 n^2 E \]

\[ H(x,y,z,t) = H(x,y,z)e^{-i\omega t} \quad E(x,y,z,t) = E(x,y,z)e^{-i\omega t} \] (209)

gives

\[ \nabla \times E = i \left[ \frac{\mu_0}{\varepsilon_0} k H \right] \quad \nabla \times H = -i \left[ \frac{\varepsilon_0}{\mu_0} kn^2 E \right] \] (210)

Now, if we assume that

\[ E = (e_1 + e_z z)e^{i\beta z} \quad H = (h_1 + h_z z)e^{i\beta z} \] (211)
then qualitatively this is presuming that the modes propagate at a constant velocity, \( \beta \), and this velocity will be important later. \( \beta \) is often referred to as the propagation constant for the mode.

If we substitute (211) into (210) and compare longitudinal and transverse components we get

\[
e_t = \frac{1}{\sqrt{\varepsilon_0 k n^2}} z \left[ \beta h_t + i \nabla h_z \right]
\]

\[
h_t = \frac{1}{\sqrt{\mu_0 k n^2}} z \left[ \beta e_t + i \nabla e_z \right]
\]

or rearranging

\[
e_t = \frac{i}{k^2 n^2 - \beta^2} \left[ \beta \nabla e_z - \frac{\mu_0 k n^2 \times \nabla h_z}{\varepsilon_0} \right]
\]

\[
h_t = \frac{i}{k^2 n^2 - \beta^2} \left[ \beta \nabla h_z - \frac{\varepsilon_0 k n^2 \times \nabla e_z}{\mu_0} \right]
\]

Taking \( \nabla \times (210)(a) \), using

\[
\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}
\]

and recalling that

\[
\nabla \cdot \mathbf{D} = \nabla \cdot (\varepsilon \mathbf{E}) = 0
\]

\[
= \nabla \cdot (n^2 \varepsilon_0 \mathbf{E}) = 0
\]

\[
= n^2 \nabla \cdot \mathbf{E} + \mathbf{E} \cdot \nabla n^2 = 0
\]

Hence
\[ \nabla E = -\frac{E \nabla n^2}{n^2} = -E \nabla \ln(n^2) \]  

(216)

and so

\[ \nabla \times (\nabla \times E) = -\nabla [E \nabla \ln(n^2)] - \nabla^2 E = k^2 n^2 E \]

(217)

or rearranging

\[ (\nabla^2 + k^2 n^2)E = -\nabla [E \nabla \ln(n^2)] \]

(218)

Next, assuming that the refractive index profile is constant in \( z \) allows us to look for solutions of the form \( E = e(x,y) e^{i \beta z} \) and defining \( \nabla = \partial/\partial z + \nabla_t \) changes (218) to

\[
\begin{align*}
[\nabla_t^2 + k^2 n^2 (x,y) - \beta^2] e_t &= -\nabla [e_r \nabla_t \ln(n^2)] \\
[\nabla_t^2 + k^2 n^2 (x,y) - \beta^2] e_z &= -i \beta e_r \nabla \ln(n^2)
\end{align*}
\]

(219)

Similarly we can derive the equations of the magnetic fields to be

\[
\begin{align*}
[\nabla_t^2 + k^2 n^2 (x,y) - \beta^2] h_t &= (\nabla_t \times h_r) \times \nabla \ln(n^2) \\
[\nabla_t^2 + k^2 n^2 (x,y) - \beta^2] h_z &= (\nabla_t h_r - i \beta h_r) \cdot \nabla \ln(n^2)
\end{align*}
\]

(220)

The equations (219) and (220) along with the continuity of \( H \), the tangential components of \( E \) and the normal component of \( D \) (or equivalently \( n^2 E \)) at any interfaces give the full set of equations necessary to solve for a given refractive index profile.

**C.2.1 Planar Guide Equations**

On a planar waveguide (i.e., one in which \( n \) is a function of \( x \) only and very accurately describes our ion implanted guides), (219) and (220) become
\[
\begin{align*}
\frac{d^2 e_x}{dx^2} + \frac{d}{dx} \left[ e_x \frac{d \ln n^2}{dx} \right] + (n^2 k^2 - \beta^2) e_x &= 0 \\
\frac{d^2 e_y}{dx^2} + (n^2 k^2 - \beta^2) e_y &= 0 \\
\frac{d^2 e_z}{dx^2} + i \beta \left[ e_x \frac{d \ln n^2}{dx} \right] + (n^2 k^2 - \beta^2) e_z &= 0 \\
\frac{d^2 h_x}{dx^2} + (n^2 k^2 - \beta^2) h_x &= 0 \\
\frac{d^2 h_y}{dx^2} \left[ \frac{d \ln n^2}{dx} \right] \frac{dh_y}{dx} + (n^2 k^2 - \beta^2) h_y &= 0 \\
\frac{d^2 h_z}{dx^2} + e_x \frac{d \ln n^2}{dx} \left( i \beta \frac{dh_z}{dx} \right) + (n^2 k^2 - \beta^2) h_z &= 0
\end{align*}
\]

and (213) becomes

\[
\begin{align*}
e_x &= \frac{i}{k^2 n^2 - \beta^2} \left\{ \beta \frac{\partial e_z}{\partial x} + \left[ -\frac{\mu_0 k}{e_0} \frac{\partial h_z}{\partial y} \right] \right\} \\
h_z &= \frac{i}{k^2 n^2 - \beta^2} \left\{ \beta \frac{\partial h_z}{\partial x} - \left[ -\frac{\mu_0 k}{e_0} \frac{\partial e_z}{\partial y} \right] \right\} \\
e_y &= \frac{i}{k^2 n^2 - \beta^2} \left\{ \beta \frac{\partial e_z}{\partial y} - \left[ -\frac{\mu_0 k}{e_0} \frac{\partial h_z}{\partial x} \right] \right\} \\
h_y &= \frac{i}{k^2 n^2 - \beta^2} \left\{ \beta \frac{\partial h_z}{\partial y} + \left[ -\frac{\mu_0 k}{e_0} \frac{\partial e_z}{\partial x} \right] \right\}
\end{align*}
\]

### C.2.2 Step Index Profile

Now (221) has exact solutions for the case of constant \( n \). In this case

\[
\frac{d \ln(n^2)}{dx} = 0
\]
In the regions where \( n^2(x) k^2 > \beta^2 \) (or alternatively \( n(x) > n_{\text{eff}} \)) then (224)(c) and (f) have the solutions

\[
e_z = A \sin(x \sqrt{k^2 n^2 - \beta^2}) + B \cos(x \sqrt{k^2 n^2 - \beta^2})
\]
\[
h_z = C \sin(x \sqrt{k^2 n^2 - \beta^2}) + D \cos(x \sqrt{k^2 n^2 - \beta^2})
\]

where \( A, B, C \) and \( D \) are constants to be determined from the boundary conditions.

In the region where \( n^2(x) k^2 < \beta^2 \) then the solutions are

\[
e_z = P e^{-x \sqrt{\beta^2 - k^2 n^2}} + Q e^{x \sqrt{\beta^2 - k^2 n^2}}
\]
\[
h_z = R e^{-x \sqrt{\beta^2 - k^2 n^2}} + S e^{x \sqrt{\beta^2 - k^2 n^2}}
\]

where, again, \( P, Q, R \) and \( S \) are constants determined from the boundary conditions.

Equations (225) and (226) can also be more simply represented by solutions of the form

\[
e_z = A e^{-x \sqrt{\beta^2 - k^2 n^2}} + B e^{x \sqrt{\beta^2 - k^2 n^2}}
\]
\[
h_z = C e^{-x \sqrt{\beta^2 - k^2 n^2}} + D e^{x \sqrt{\beta^2 - k^2 n^2}}
\]

where the value of \( (\beta^2 - k^2 n^2)^{1/2} \) is allowed to be complex.
C.2.3 Weak Guidance Approximation

However, this vectorial picture, while being exact, has very few analytical solutions, but several approximations allow some important simplifications.

In the limit of weak guidance (ie. when there only small differences between refractive indices across the guide) or mathematically expressed as

$$-\nabla_i [e_r \nabla_i \ln(n^2)] = 0 \quad (228)$$

and so

$$[\nabla_i^2 + k^2 n^2 - \beta^2] e_i = 0 \quad (229)$$

Similarly $\nabla_i \cdot e_i = (\nabla_i e_x, \nabla_i e_y)$ and so (221) is replaced by

$$[\nabla^2 + k^2 n^2 - \beta^2] \Psi = 0 \quad (230)$$

where $\nabla^2$ is now the scalar Laplacian and $\Psi$ represents either $e_y$ or $h_y$. Note that $\beta$ is the propagation constant for the scalar wave equation, as distinct from the exact propagation constant $\beta$ which is the solution for the exact, vector wave equation.

In the case where the refractive index is isotropic in both the y and z directions (which is used as an approximation in most planar guides) then it is possible to considerably simplify and separate (212) and (213) into two "sets" of solutions - Transverse Electric (TE) for which $e_x = e_z = h_y = 0$ and

$$h_x = -\frac{\beta}{k} \sqrt{\frac{\epsilon_0 e_y}{\mu_0}} \quad h_z = -i \frac{\epsilon_0}{k} \frac{de_y}{\mu_0 dx} \quad (231)$$

or Transverse Magnetic (TM) for which $h_x = h_z = e_y = 0$ and

$$e_x = \frac{\beta}{kn^2} \sqrt{\frac{\mu_0 h_y}{\epsilon_0}} \quad e_z = \frac{i}{kn^2} \frac{\mu_0 dh_y}{\epsilon_0 dx} \quad (232)$$
D. Appendix D: Linear Waveguide Parameters

D.1 Introduction

In this appendix the case of a step index planar waveguide (a good approximation for the ion implanted waveguides) and a step index fibre (useful for the Nd:YVO₄ work) will be considered. Expressions for the parameters in eqn (3) will be calculated (where possible) and these serve as the basis for an understanding of the behaviour waveguide and fibre behaviour.

Many of the results here are not new, but while easy, in principle, to calculate are rarely published. This enables a number of intuitive results to be quantified.

D.2 Linear Planar Waveguide

The waveguide and its relevant parameters are shown in Figure 152. This case is useful for the ion implanted waveguides that will be considered in Chapters 6 and 7.

Figure 152: Planar Waveguide, Step Index, Directional Coupler
In the figure, we have allowed the two waveguides to differ slightly (i.e. different refractive indices and widths) on a common cladding refractive index.

We will, however, first consider the simpler case where \( n_2 = n_3 = n_{co} \), \( r_1 = r_2 = r \), then \( C_1 = D_1 = C \), \( \beta^{(a)} = \beta^{(b)} = \beta \), \( \kappa_{na} = \kappa_{nb} = \kappa_1 \), \( \kappa_{ab} = \kappa_{ba} = \kappa_2 \), then

\[
\xi^{(a)} = \xi^{(b)} = \xi = \beta + \frac{\kappa_1 - \kappa_2}{1 - C^2} K_{ab} = K_{ba} = K = \kappa_2 - \kappa_1 C
\]

and \( \alpha = 2\xi \), \( \rho = 0 \), \( \Gamma = K \) and (4) becomes

\[
A(z) = e^{i\gamma z}[iB_0 \sin(Kz) + A_0 \cos(Kz)]
\]
\[
B(z) = e^{i\gamma z}[B_0 \cos(Kz) + iA_0 \sin(Kz)]
\]

For the step-index profile guides above, the lowest order TE mode (i.e. only \( E_y = 0 \)) is the even mode, given by

\[
E_y = \frac{\cos(Ux)}{\cos(Ur)}
\]

\[
\begin{align*}
&= e^{wx} & -r \leq x \leq r \\
&= e^{-wr} & x < -r \\
&= e^{-wx} & x > r
\end{align*}
\]

where

\[
U = \sqrt{k^2 n_{co}^2 - \beta^2}
\]
\[
W = \sqrt{\beta^2 - k^2 n_{cl}^2}
\]
\[
V = k_1 \sqrt{n_{co}^2 - n_{cl}^2}
\]

The modes in eqn (235) are normalised to unity at the core/cladding boundary. The allowed modes of the waveguide are given by the characteristic equation for the even modes which is

\[
W = U \tan(U r)
\]

For the case of a single mode waveguide the expressions become simpler and the
coefficients in eqn (233) become

\[ N = \frac{\beta}{U^2 W \omega \mu} \left[ V^2 (1 + r W) \right] \]

\[ \beta e^{(2r-s)W} \left[ s - 2r + \frac{4W}{V^2} + e^{-2rW} \right] \]

\[ C = \frac{N \omega \mu}{N \omega \mu} \]

\[ \kappa_1 = \frac{\omega \varepsilon_0 V^2}{2 W k^2 N} e^{-2rW} (e^{4rW} - 1) \]

\[ \kappa_2 = \frac{\omega \varepsilon_0 e^{(2r-s)W} (W + UTan[Ur])}{k^2 N} \]  \[ (238) \]

**Figure 153** Comparison of values for \( C, \kappa_1 \) and \( \kappa_2 \) (from eqn (238)) for TE\(_0\) mode of step index guide with \( n_c = 1.455, n_d = 1.45, \lambda = 1342\text{nm} \) and \( r = 4\mu\text{m} \).

where \( N \) is a normalisation factor so that the total area under the curves is unity. While the expressions themselves are not difficult to calculate they are included here as, to the author's knowledge, they are not readily available in the literature.

Examining the expression for \( \kappa_1 \) it is intuitive that the coupling between the guides should decrease with increasing separation of the guides (ie. \( s \) increasing) and decrease as the guide width gets smaller (\( r \) decreasing).
As can be seen from Figures 153 and 154 the propagation constant, $\beta$, dominates the other terms in (233), except for the case of waveguides extremely close together.

For non-identical guides the expressions for the TE modes (238) become much nastier (even for the case of two single mode guides) giving
\[N_1 = \frac{\beta_1}{U_1^2 W_1 \omega \mu} \left[ V_1^2 (1 + r_1 W_1) \right] \quad N_2 = \frac{\beta_2}{U_2^2 W_2 \omega \mu} \left[ V_2^2 (1 + r_2 W_2) \right]\]

\[
C_1 = \frac{\beta_1}{\omega \mu \sqrt{N_1 N_2}} \begin{bmatrix}
    e^{(r_1 r_2 - s) W_1} & \left( -\frac{1}{W_1 - W_2} + \frac{W_1 + U_2 \tan(U_2 r_2)}{U_2^2 + W_2^2} \right) + \\
    e^{(r_1 r_2 - s) W_2} & \left( -\frac{1}{W_1 - W_2} + \frac{W_2 + U_1 \tan(U_1 r_1)}{U_1^2 + W_1^2} \right) + \\
    e^{(r_1 r_2 - s) W_1} & \left( -\frac{W_1 - U_2 \tan(U_2 r_2)}{W_1 + W_2} + \frac{W_2 + U_1 \tan(U_1 r_1)}{U_1^2 + W_1^2} \right) + \\
    e^{(r_1 r_2 - s) W_2} & \left( -\frac{W_1 - U_2 \tan(U_2 r_2)}{W_1 + W_2} + \frac{W_2 + U_1 \tan(U_1 r_1)}{U_1^2 + W_1^2} \right) + \\
\end{bmatrix}
\]

\[
\kappa_{11} = \frac{\omega \mu V_1^2}{2 W_1 k^2 N_1} e^{-2s W_1 (e^{4r_1 W_1} - 1)}
\]

\[
\kappa_{22} = \frac{\omega \mu V_2^2}{2 W_2 k^2 N_2} e^{-2s W_2 (e^{4r_2 W_2} - 1)}
\]

\[
\kappa_{12} = \frac{\omega \mu V_2^2}{k^2 \sqrt{N_1 N_2 (U_2^2 + W_1^2)}} \begin{bmatrix}
    e^{(r_1 r_2 - s) W_1} (W_1 - U_2 \tan(U_2 r_2)) - \\
    e^{(r_1 r_2 - s) W_2} (W_1 + U_2 \tan(U_2 r_2)) + \\
    e^{(r_2 r_1 - s) W_1} (W_2 - U_1 \tan(U_1 r_1)) - \\
    e^{(r_2 r_1 - s) W_2} (W_2 + U_1 \tan(U_1 r_1)) + \\
\end{bmatrix}
\]

\[
\kappa_{21} = \frac{\omega \mu V_1^2}{k^2 \sqrt{N_1 N_2 (U_1^2 + W_2^2)}} \begin{bmatrix}
    e^{(r_1 r_2 - s) W_1} (W_1 - U_2 \tan(U_2 r_2)) - \\
    e^{(r_1 r_2 - s) W_2} (W_1 + U_2 \tan(U_2 r_2)) + \\
    e^{(r_2 r_1 - s) W_1} (W_2 - U_1 \tan(U_1 r_1)) - \\
    e^{(r_2 r_1 - s) W_2} (W_2 + U_1 \tan(U_1 r_1)) + \\
\end{bmatrix}
\]

Hence, the degeneracy of \(\kappa_{11}\) and \(\kappa_{22}\), and \(\kappa_{12}\) and \(\kappa_{21}\) that made the identical guide case elegant is not likely here.

For the other polarisation, TM case, again assuming identical cores we get solutions of the form
\[ N = \frac{\kappa n_{co}^2}{c \mu \beta n_{cl}^2 U^2 W} \left[ V^2 + r W \left( \frac{n_{cl}^2 U^2 + n_{co}^2 W^2}{n_{co}^2 n_{cl}^2} \right) \right] \]

\[ C = \frac{\kappa n_{co}^2 e^{(2r_i-s)W}}{c \mu \beta n_{cl}^2 N} \left( \frac{n_{co}^2 + n_{cl}^2}{V^2} \right) \frac{n_{co}^2}{n_{cl}^2} \left( 1 + \frac{n_{co}^2}{n_{cl}^2} \right) \left( 1 - \frac{n_{co}^2}{n_{cl}^2} e^{-2rW} \right) \]

\[ \kappa_1 = \frac{\omega \varepsilon_0 n_{co}^2 V^2 e^{-2sW}}{2k^2 n_{cl}^2 N} \left( 1 + \frac{k^2 n_{co}^2 W}{\beta^2 (k^2 n_{co}^2 + V^2)} \right) \left( e^{4rW} - 1 \right) \]

\[ \kappa_2 = \frac{\omega \varepsilon_0 n_{co}^2 e^{(2r_i-s)W}}{k^2 n_{cl}^2} \left( \frac{k^2 W}{\beta^2 (k^2 n_{co}^2 + V^2)} \right) - \frac{U^2 n_{cl}^2}{n_{co}^2} - \frac{W}{1 + \frac{n_{co}^2}{n_{cl}^2}} e^{-2rW} \left( 1 - \frac{n_{co}^2}{n_{cl}^2} + \frac{n_{co}^2 W}{n_{cl}^2} - \frac{n_{cl}^2}{n_{co}^2} \right) \]

and their relative strengths are shown in Figures 155 and 156.

As with the TM modes, the presence of the other guide makes a very small correction to the propagation constant, \( \beta \), of the mode, with the major effect being the coupling of the field, which drops off exponentially with distance.

Also of interest is the comparison of the values for the TE and TM modes of the same waveguide (as shown in Figure 157). This shows that for chosen case (\( n_{co} = 1.45459 \), \( n_{cl} = 1.44963 \), \( \lambda = 1342 \text{nm} \), \( r = 4 \mu \text{m} \)) the TM \( \xi \) is slightly higher than that of the TE, but is less affected by the presence of the other guide. Similarly the coupling constant, \( K \), of the TM modes is slightly less than that of the TE modes.

Figure 158 shows the % difference between the different polarisations for the linear planar guide. It shows that a planar guide is quite susceptible to input polarisation. Using an unpolarised source would give a strong smearing of response, and device orientation will be important for a polarised source.
Figure 155  Comparison of normalization constant $C$ and coupling constants $\kappa_1$ and $\kappa_2$ for TM$_0$ modes for a step index guide $n_{co}=1.455$, $n_{cl}=1.45$, $\lambda=1.342\mu m$ and $r=4\mu m$.

Figure 156  Compare the $\xi$ ($m^{-1}$) and $K$ ($m^{-1}$) for TM$_0$ mode of a step index guide with $n_{co}=1.455$, $n_{cl}=1.45$, $\lambda=1.342\mu m$ and $r=4\mu m$. 
Figure 157  Compare $\xi$ and $K$ for TE and TM modes of the same guide ($n_{co}$=1.45459, $n_{cl}$=1.44963, $\lambda$=1342nm and $r$=4$\mu$m).

Figure 158  % difference between coupling at TE and TM polarisations as a function of device length (10$\mu$m core separation)
D.3 Linear Fibre Waveguide

The equations for fibres are, in general, considerably more complicated than those for the planar case (without much increase in understanding!). But, for completeness, the case of a step index fibre the modes have the form [157]

\[
\begin{align*}
E_r &= -\frac{i\beta}{U^2} \left[ \frac{-\mu l}{\beta} \frac{\mu l}{r} A U J'_i(Ur) + \frac{-\mu l}{\beta} B J_i(Ur) \right] e^{i(\omega t + l\phi - \beta z)} \\
E_\phi &= -\frac{i\beta}{U^2} \left[ \frac{il}{r} A J_i(Ur) - \frac{\omega l}{\beta} B U J'_i(Ur) \right] e^{i(\omega t + l\phi - \beta z)} \\
E_z &= A J_i(Ur) e^{i(\omega t + l\phi - \beta z)} \\
H_r &= -\frac{i\beta}{U^2} \left[ B U J'_i(Ur) - \frac{i\omega e_0 n_c^2 l}{\beta} A J_i(Ur) \right] e^{i(\omega t + l\phi - \beta z)} \\
H_\phi &= -\frac{i\beta}{U^2} \left[ \frac{il}{r} B J_i(Ur) + \frac{i\omega e_0 n_c^2 U}{\beta} A J_i(Ur) \right] e^{i(\omega t + l\phi - \beta z)} \\
H_z &= B J_i(Ur) e^{i(\omega t + l\phi - \beta z)}
\end{align*}
\]

in the core and

\[
\begin{align*}
E_r &= \frac{i\beta}{W^2} \left[ C W K'_i(Wr) + \frac{i\omega e_0 n_c^2 l}{\beta} D K_i(Wr) \right] e^{i(\omega t + l\phi - \beta z)} \\
E_\phi &= \frac{i\beta}{W^2} \left[ \frac{il}{r} C K_i(Wr) - \frac{\omega l}{\beta} D W K'_i(Wr) \right] e^{i(\omega t + l\phi - \beta z)} \\
E_z &= C K_i(Wr) e^{i(\omega t + l\phi - \beta z)} \\
H_r &= \frac{i\beta}{W^2} \left[ D W K'_i(Wr) - \frac{i\omega e_0 n_c^2 l}{\beta} C K_i(Wr) \right] e^{i(\omega t + l\phi - \beta z)} \\
H_\phi &= \frac{i\beta}{W^2} \left[ \frac{il}{r} D K_i(Wr) + \frac{i\omega e_0 n_c^2 W}{\beta} C K_i(Wr) \right] e^{i(\omega t + l\phi - \beta z)} \\
H_z &= D K_i(Wr) e^{i(\omega t + l\phi - \beta z)}
\end{align*}
\]

in the cladding. \(J_i\) are Bessel function of the First Kind, order \(l\) and \(K_i\) are modified Bessel functions of the First Kind.

The characteristic equation for the guided modes satisfies
\[
\left( \frac{J'_1(Ua)}{UaJ'_1(Ua)} + \frac{K'_1(Wa)}{WaK'_1(Wa)} \right) \left( \frac{n_{c2}^2 J'_1(Ua)}{UaJ'_1(Ua)} + \frac{n_{c2}^2 K'_1(Wa)}{WaK'_1(Wa)} \right) = \left( \frac{i\beta}{k} \right)^2 \left( \frac{1}{Ua} \right)^2 + \left( \frac{1}{Wa} \right)^2 \] (243)

and

\[
B = \frac{i\beta \left( \frac{1}{U^2 a^2} + \frac{1}{W^2 a^2} \right)}{\omega \mu \left( \frac{J'_1(Ua)}{UaJ'_1(Ua)} + \frac{K'_1(Wa)}{WaK'_1(Wa)} \right)} A
\]

(244)

where \( A \) is chosen to satisfy normalisation conditions and \( a \) is the guide radius.

The equations in this form do not readily calculate the important coupling parameters as they are in spherical polar coordinates. However, for the case that \( n_{c2} - n_{c1} \ll 1 \) the continuity condition for tangential components of \( \mathbf{H} \) at the core cladding interface becomes identical to that for \( \mathbf{E} \). This leads to a considerable simplification of the field expressions and also the use of Cartesian coordinates (which greatly simplifies the relevant integrations). The fields are then

\[
\begin{align*}
E_x &= 0 \\
E_y &= AJ'_1(Ur)e^{i(\omega t - k_2 z)} \\
E_z &= \frac{UA}{2\beta} \left[ J_{l+1}'(Ur)e^{i(l+1)\phi} + J_{l-1}'(Ur)e^{i(l-1)\phi} \right] e^{i(\omega t - k_2 z)} \\
H_x &= -\frac{\beta}{\omega \mu} AJ'_1(Ur)e^{i(l\phi + \omega t - k_2 z)} \\
H_y &= 0 \\
H_z &= -\frac{iUA}{2\omega \mu} \left[ J_{l+1}'(Ur)e^{i(l+1)\phi} - J_{l-1}'(Ur)e^{i(l-1)\phi} \right] e^{i(\omega t - k_2 z)}
\end{align*}
\] (245)

in the core and
\[ E_x = 0 \]
\[ E_y = B K_1(Wr) e^{i(\omega t + \phi - \beta z)} \]
\[ E_z = \frac{\beta}{2} K_{l+1}(Wr) e^{i(l+1)\phi} + K_{l-1}(Wr) e^{i(l-1)\phi} \]
\[ H_x = -\frac{\beta}{\omega \mu} B K_1(Wr) e^{i(\phi + \omega t - \beta z)} \]
\[ H_y = 0 \]
\[ H_z = \frac{i\beta}{2\omega \mu} K_{l+1}(Wr) e^{i(l+1)\phi} - K_{l-1}(Wr) e^{i(l-1)\phi} \]

in the cladding. Where \( B = A J_1(Ua)/K_0(Wa) \), where \( a \) is the radius of the cores.

For the case when \( l=0 \), (which fortunately corresponds to the lowest order mode), the fields simplify greatly as the solutions become radially symmetric. In this state there exist two solutions, known as TE for the first bracket in (243) equalling 0 and TM for the second bracket in (243).

The TE case the solutions have the form

\[ E_\phi = \frac{-J_1(Ur)}{J_1(Ua)} \]
\[ H_r = \frac{\varepsilon_0 \beta J_1(Ur)}{\mu_0 k J_1(Ua)} \]
\[ H_z = i \frac{\varepsilon_0 U J_0(Ur)}{\mu_0 k J_1(Ua)} \]

in the core and

\[ E_\phi = \frac{-K_1(Wr)}{K_1(Wa)} \]
\[ H_r = \frac{\varepsilon_0 \beta K_1(Wr)}{\mu_0 k K_1(Wa)} \]
\[ H_z = i \frac{\varepsilon_0 W K_0(Wr)}{\mu_0 k K_1(Wa)} \]

in the cladding. Unfortunately, these expressions do not give analytical results for the integrals (169) or (204), but the numerical evaluations are shown in Figure 159 and Figure 160.
The TM modes of the step index fibre are given by

\[
E_r = \frac{J_1(Ur)}{J_1(Ua)} \\
E_z = i \frac{U J_0(Ur)}{\beta J_1(Ua)} \\
H_\phi = \sqrt{\frac{\varepsilon_0 k n_{co}^2 J_1(Ur)}{\mu_0 \beta J_1(Ua)}}
\]

(249)

in the core and

\[
E_r = \frac{n_{co}^2 K_1(Wr)}{n_{cl}^2 K_1(Wa)} \\
E_z = -i \frac{n_{co}^2 W K_0(Wr)}{n_{cl}^2 \beta K_1(Wa)} \\
H_\phi = \sqrt{\frac{\varepsilon_0 k n_{co}^2 K_1(Wr)}{\mu_0 \beta K_1(Wa)}}
\]

(250)

in the cladding. Again, analytical expressions for the integrals are not obtainable, but the coupling relationships are shown in Figures 161 and 162.
In general the behaviour of the mode parameters in the guides is not significantly different from that of the planar waveguide modes. However, comparing Figures 158 and 163 shows that the step index fibre has considerably less polarisation dependence to the coupling.

Figure 160  Compare Values for $\xi$ (m$^{-1}$) and $K$ (m$^{-1}$) for the TE$_{0}$ mode of a step index fibre ($n_{e}=1.455$, $n_{cl}=1.45$, $\lambda=1.342\mu$m and $a=4\mu$m)

Figure 161  $\xi$ (m$^{-1}$) and $K$ (m$^{-1}$) for TM$_{0}$ for step index fibre ($n_{e}=1.455$, $n_{cl}=1.45$, $\lambda=1.342\mu$m and $a=4\mu$m).
Figure 162  C and $k_1$ and $k_2$ for TM$_0$ for step index fibre ($n_{co}=1.455$, $n_{cl}=1.45$, $\lambda=1.342\,\mu\text{m}$ and $a=4\,\mu\text{m}$)

Figure 163  % difference between coupling at TE and TM polarisations for step index fibre. (10\,\mu\text{m} core separation).
E. Appendix E: Bend Tuning the Fibre Coupling

E.1 Theory

Rewriting eqn (7) as

\[ \frac{dD}{dz} = ia_2|D|^2D + ia_3Fe^{i\Delta\beta z} \]
\[ \frac{dF}{dz} = ia_2|F|^2F + ia_3De^{i\Delta\beta z} \]

(251)

it becomes apparent that any mismatch in the propagation constants of the modes has a strong affect on the coupling. This is taken advantage of in optical switching, but it is this sensitivity that also makes fabrication of twin core fibre with long coupling lengths or any coupling at short wavelength an extremely difficult problem.

A need for a method whereby the propagation constant mismatch, and hence the coupling, may be tuned has led to several different techniques. [158][159] Vallee et al. [158] notes that by bending the fibre in the plane parallel to the plane of the fibre cores, gives both a different propagation length in the fibre cores but also, through stress or stretching, a change in the propagation constants in those cores. Arkwright et al. [159] displaced one end of a self-supporting length of optical fibre. This lead to a cantilever bend in the fibre. In this situation, the induced difference in the propagation constants between the modes of initially identical cores is given by

\[ \Delta\beta = \frac{3\beta v_1 d}{L^3} (L-z) \]

(252)

where L is the length of the cantilevered section and \( v_1 \) is the displacement of the fibre end. This method has the advantage that the experimental set up is simple, but only a small section of the fibre (for which \( \Delta\beta \) is small) is producing the main effect.
In the case of low power (ie. so that the first terms in the LHS of (251)a and (251)b can be neglected) and using

\[ \Delta \beta = \Delta \beta_0 + \frac{3\beta v d}{L^3} (L - z) \]

\[ = \beta_1 - \beta_2 L \]

where \( \beta_1 = \Delta \beta_0 + 3 \beta v d/L^2 \) and \( \beta_2 = 3 \beta v d/L^3 \) eqn (251) readily transforms to

\[ \frac{d^2 D}{dz^2} - i(\beta_1 - 2\beta_2 z) \frac{dD}{dz} + a_2^2 D = 0 \]

which, despite its simple form, has solutions only in the form of Gaussian Hypergeometric \( F_1 \) functions. Hence, numerical methods must be used to investigate the equations.

Computer simulations show that a \( \Delta \beta \approx 60 \text{m}^{-1} \) is sufficient to restrict an identical core fibre (ie. with 100% coupling initially) to less than 10% coupling, which for light at 1.064\( \mu \text{m} \) is about \( 10^{-5} \beta \).
Figure 164 Fibre coupling as a function of end displacement for a ~30cm length of twin core fibre at 1.064μm (100mW CW)

E.2 Experiments

Figure 164 shows some of the results for launching a CW, 100mW 1064nm Nd:YAG beam into ~30cm of twin core fibre, glued to a piece of springsteel. The end was deflected in the horizontal direction by a vertical post on a translation stage and it is this displacement that forms the x-axis scale. This fibre does not have any coupling at 1064nm in its unstressed state, but the figure clearly shows that almost any coupling fraction can be achieved by a suitable bending of the fibre. The two traces are for the same fibre, but different days and give an indication of how sensitive the measurement is to the fibre environment and of the likely reproducibility.
Quite a bit of energy was put into trying to fit Figure 164 to fibre parameters, but no combination could be found that gave anything like the behaviour predicted by (251) and (252) particularly in the region from -20 to +20mm deflection. (The line in the plot simply joins the points!).

This discrepancy is not surprising as eqn (251) assumes identical guides with different mode propagation constants, whereas in these experiments we have used fibre for which there is no initial coupling, indicating non-identical guides.

However, the experiments demonstrated that the coupling would be tunable and, if a system for ensuring stability and reproducibility can be found, then this could be a practical method for tuning the fibre coupling which could be used inside the operating laser.
F. Appendix F: Numerical Methods for the Ion Implanted Refractive Index Profile

F.1 Introduction

A common method for finding the refractive index profile from an experimentally determined set of mode indices is to choose an appropriate model for profile with a number of free parameters. A guess is made at the values of the free parameters and the modes of the profile are calculated. The difference between these modes and the experimentally determined values is calculated and this value is minimised numerically.

In this appendix the efficiency and effectiveness of a number of different numerical minimisation methods at reconstructing a refractive index profile from its modes will be compared. The quality of the initial "guess" for parameters is important and the affect this has on the convergence of the different routines to the correct parameters will be explored.

F.1.1 Three Layer Step Index Guide

A general routine to calculate the modes of a given refractive index profile numerically, using the Matrix Method of Section 6.5.2.1, was written. It was given the appropriate model (ie. 3 region step index profile with two free parameters - n₁ and w₁ ) and a "guess" for the parameters (see below) and then a number of different methods were tried to minimise the difference between the "guess" and the known modes. The results of this are summarised in Table 25.
Results of different minimisation routines. \( e \) is the calculated error \((x 10^{-3})\). Exact result is \( n_1=1.83 \) and \( w_1=8\mu m \).

The function to be minimised, in general uses the first 9 modes for the configuration. A function evaluation involves calculating the modes lying in the region from the \( n_1 \) for the configuration to \((n_{m_9} - (n_{m_8} - n_{m_9})) \) where \( n_{m_i} \) is the \( i \)th mode and this gives a range of refractive indices to search which will not result in finding the next mode down when the parameters are close, but allows the 9th mode to be included in the calculations for the situation where the calculated mode indices are lower than the real ones. The numerical methods are from "Numerical Recipes" [161] otherwise Numerical Algorithms Group routine E04JAF (denoted N in the following) has been used and this is apparently quasi-Newton based. Powell's Method (P) and the Downhill Simplex method (DS) have the advantage that they do not require the use of derivative information from the function. The quasi-Newton (QN) and Conjugate Gradient methods (CG) do need derivatives, but these are calculated numerically and seem to be accurate enough for the method, without increasing the number of necessary evaluations significantly.
To understand some of the issues involved the results of the minimisation the function is plotted in Figure 154 (And expanded in Figure 155). The minima occurs at (1.83,8) (ie. Inside the grey box in Figure 154). As is apparent, there is little to mark this region until the readings are very close. This gives just about any minimisation routine challenges, even for this 2 variable problem, as there are numerous local minima as well as a large region of fairly flat response.

Figures 168 and 157 show the progress of the different minimisation routines tested. In this example, two of the routines (DS and P) failed to converge to the solution. The implementation of CG routine used tends to choose a “line” in the parameter space and
minimise along that line, in the process it makes some quite big excursions away from its best minima in an attempt to get the next line minimisation smaller.

In Table 25, the different minimisation routines are tested on the different three layer step index guide modes (see section 6.6) used to test the analytical methods. They are arranged according to the different modes used and different starting parameters. An i% perturbation means that the initial parameters are all the \( n_i \)'s and \( w_i \)'s perturbed up by that amount. e.g. for the first situation \( n_1 = 1.8483 \) and \( w_1 = 8.08 \mu m \) are given to the program. An i% "special" perturbation recognises that from inspection it is possible to get close to the refractive index of the guide (ie. \( n_i \)) simply by giving the \( n \) for the lowest order mode as the starting estimate for \( n_i \). Hence for a 1% special with \( n_s = 1.5 \) then the starting point is \( n_1 = 1.8289 \) and \( w_1 = 8.08 \mu m \).

The \( e \) column gives the differences between the modes of the "minimum" and the exact modes (added in quadrature) \( (x 10^3) \) for the cases when the given method fails. Failure is readily discerned as the error term is large and the number of iterations is usually small. The sample for \( n_s = 1.828 \) and using the 4 decimal places of the modes as supplied by the prism coupler shows no significant differences compared with that achieved with the 8 decimal place results. It is also apparent that the quasi-Newton and Conjugate gradient methods do not gain any great benefit from the "special" cases.

On the basis of these results, the NAG routine would seem to be the best method, followed by Downhill Simplex and distantly Powell's method. The quasi-Newton method and Conjugate Gradient lose on both counts that they have high numbers of function evaluations and still seem to converge less readily to the minima.

**F.1.2 Four Layer Step Index Guide**

Table 26 and Table 27 show the results of applying the numerical methods used in Table 25 to the problem of the 4 region guide in the case of \( n_1 = 1.83 \), \( n_2 = 1.4 \) and \( n_s = 1.5 \) for which the analytical methods were more accurate (Table 26) and the "harder" case of \( n_1 = 1.83 \), \( n_2 = 1.8 \) and \( n_s = 1.828 \) (Table 27) (ie. analytical methods were less accurate).

As with the case for the 3 layer step index guide, in general the downhill-Simplex method and NAG routine outperform the others for getting "close" to the correct value with reliability. In general the downhill-Simplex method does this with far less function
evaluations than the NAG routine, but is less robust. The results given in the table are meant to summarise the behaviour. That is, for each case considered, they show an "easy" example for which most of the routines converged and a "hard" example for which most did not.

<table>
<thead>
<tr>
<th>λ=1.064μm</th>
<th>Start Point</th>
<th>Method</th>
<th>w₁</th>
<th>w₂</th>
<th>Mean error per mode 10⁻⁶</th>
<th>iters</th>
<th>Max % err</th>
<th>Max err var</th>
</tr>
</thead>
<tbody>
<tr>
<td>n₁=1.83, w₁=8μm, n₂=1.4, w₂=1μm</td>
<td>1% Increase n₁=1.8483 w₁=8.08μm n₂=1.414 w₂=1.01μm</td>
<td>QN 1.82998451</td>
<td>8.01738825</td>
<td>1.27319765</td>
<td>1.07228013</td>
<td>3.34</td>
<td>1796</td>
<td>9.1 n₂</td>
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<tr>
<td></td>
<td>P 1.84343947</td>
<td>8.11724213</td>
<td>1.42738619</td>
<td>2.16138769</td>
<td>1818.89</td>
<td>14608</td>
<td>116.1 w₂</td>
<td></td>
</tr>
<tr>
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<td>DS 1.84845843</td>
<td>7.07566046</td>
<td>1.41461681</td>
<td>1.69379441</td>
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<td>69.4 w₂</td>
<td></td>
</tr>
<tr>
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<td>1.0722337</td>
<td>3.33</td>
<td>387</td>
<td>10.8 n₂</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CG 1.82998992</td>
<td>8.00064824</td>
<td>1.40009986</td>
<td>1.08935262</td>
<td>3.70</td>
<td>325</td>
<td>8.9 w₂</td>
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<tr>
<td></td>
<td>2% Increase n₁=1.866 w₁=8.16μm n₂=1.428 w₂=1.02μm</td>
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<td>8.16450342</td>
<td>1.07002732</td>
<td>1.01208065</td>
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<td>524</td>
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<td>8289</td>
<td>116.7 w₂</td>
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<tr>
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<td>1.42822298</td>
<td>1.92605766</td>
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<td>1.17913683</td>
<td>3.70</td>
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</tr>
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<td></td>
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<td>8.01579846</td>
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<td>1.07017756</td>
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</tr>
<tr>
<td></td>
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<td>8.00064954</td>
<td>1.40009214</td>
<td>1.08935166</td>
<td>3.71</td>
<td>372</td>
<td>8.9 w₂</td>
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</tr>
<tr>
<td></td>
<td>16% Special n₁=1.8288 w₁=9.28μm n₂=1.624 w₂=1.16μm</td>
<td>QN 1.80236979</td>
<td>9.27949279</td>
<td>1.62367438</td>
<td>1.160607</td>
<td>6644.44</td>
<td>147</td>
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<tr>
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<td>9.44416413</td>
<td>1.62552451</td>
<td>2.86732224</td>
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<td>DS 1.82881406</td>
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<td>1.7928125</td>
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</tr>
<tr>
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<td>16.1 w₂</td>
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</table>

Table 26 Comparison of different numerical methods. Exact results would be n₁=1.83, w₁=8μm, n₂=1.4 and w₂=1μm.

In general the major errors occur in the w₂ (barrier width) term with the n₂ (barrier refractive index) being the next most common. This can be understood in the context of the relative size of the effect on the modes of these two terms (as was alluded to in connection with Table 8). It is also interesting to note that there are some examples for which the error per mode is quite large, while the maximum error in any particular term < 10%.

As was apparent in the three layer step index guide results, the optimisation routines are able to get closer to the results for the n₁=1.5 case than for the n₁=1.828. This
Table 27  Comparison of results for different numerical methods for the "easy" guide. Exact results would be $n_1=1.83$, $w_1=8.0\mu m$, $n_2=1.4$ and $w_2=0.1\mu m$ is because the minimisation function is less well behaved for the $n_s=1.828$ case. This is unfortunate, considering that the real implanted case is much more likely to be closer to the second example!

<table>
<thead>
<tr>
<th>$\lambda=1.064\mu m$</th>
<th>Initial Position</th>
<th>Method</th>
<th>$n_1$</th>
<th>$w_1$</th>
<th>$n_2$</th>
<th>$w_2$</th>
<th>Mean error per mode $10^6$</th>
<th>iters</th>
<th>max err %</th>
<th>max err var</th>
</tr>
</thead>
<tbody>
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<td>QN</td>
<td>1.83</td>
<td>7.996</td>
<td>1.44</td>
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<td>7.996</td>
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<td>1.590</td>
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<td>8.080</td>
<td>1.413</td>
<td>2.419</td>
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<td>921</td>
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<td>100.0</td>
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<td>545</td>
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</table>

is because the minimisation function is less well behaved for the $n_s=1.828$ case. This is unfortunate, considering that the real implanted case is much more likely to be closer to the second example!
<table>
<thead>
<tr>
<th>$\lambda = 1.06$</th>
<th>Initial Point</th>
<th>Method</th>
<th>$n_1$</th>
<th>$w_1$</th>
<th>$n_2$</th>
<th>$w_2$</th>
<th>error per mode $10^6$</th>
<th>iters</th>
<th>max err %</th>
<th>max err var.</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=1.83 w=8μm</td>
<td>1% Increase</td>
<td>QN</td>
<td>1.82831144</td>
<td>8.09087621</td>
<td>1.82182473</td>
<td>0.00115492</td>
<td>495.55</td>
<td>492</td>
<td>99.9</td>
<td>w₂</td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>1.82971139</td>
<td>8.5444187</td>
<td>1.36035072</td>
<td>1.37308169</td>
<td>67.89</td>
<td>6413</td>
<td>37.3</td>
<td>w₂</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DS</td>
<td>1.84836147</td>
<td>7.20460938</td>
<td>1.81824214</td>
<td>1.3794824</td>
<td>445.56</td>
<td>58</td>
<td>37.9</td>
<td>w₂</td>
<td></td>
</tr>
<tr>
<td>n=1.8 w=1μm</td>
<td>N</td>
<td>1.82993893</td>
<td>7.95913617</td>
<td>2.95404517</td>
<td>0.22578275</td>
<td>146.67</td>
<td>1613</td>
<td>77.4</td>
<td>w₂</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CG</td>
<td>1.82829283</td>
<td>8.08354763</td>
<td>1.82172428</td>
<td>0.00303428</td>
<td>523.33</td>
<td>133</td>
<td>99.7</td>
<td>w₂</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1% Special</td>
<td>QN</td>
<td>1.82840413</td>
<td>8.08358345</td>
<td>1.82650606</td>
<td>0.00345372</td>
<td>493.33</td>
<td>327</td>
<td>99.7</td>
<td>w₂</td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>1.82840430</td>
<td>8.0796420</td>
<td>1.72640873</td>
<td>0.00003698</td>
<td>493.33</td>
<td>17153</td>
<td>99.9</td>
<td>w₂</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DS</td>
<td>1.82988503</td>
<td>7.93172554</td>
<td>1.80876075</td>
<td>1.0036502</td>
<td>28.56</td>
<td>394</td>
<td>0.9</td>
<td>w₂</td>
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<tr>
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<td>N</td>
<td>1.82840418</td>
<td>8.07055593</td>
<td>1.82309715</td>
<td>0.00000000</td>
<td>493.33</td>
<td>445</td>
<td>100.0</td>
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<tr>
<td></td>
<td>CG</td>
<td>1.82840424</td>
<td>8.08482453</td>
<td>1.82230911</td>
<td>0.00099602</td>
<td>493.33</td>
<td>299</td>
<td>100.0</td>
<td>w₂</td>
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</tr>
</tbody>
</table>

| Table 28 | Comparison of the results of different numerical methods for the "harder" case. Exact result would be $n_1=1.83$, $w_1=8\mu m$, $n_2=1.8$ and $w_2=1\mu m$. |

Table 28 and Table 29 give the results for the case which is expected to be closest to the real situation ie. $n_1=1.83$, $n_2=1.8$, $n_4=1.828$ - one guided mode, several leaky modes.
<table>
<thead>
<tr>
<th>λ=1.064μm</th>
<th>Initial Position</th>
<th>Metho d</th>
<th>n₁</th>
<th>w₁</th>
<th>n₂</th>
<th>w₂</th>
<th>error per mode</th>
<th>iters</th>
<th>max err</th>
<th>max err var</th>
</tr>
</thead>
<tbody>
<tr>
<td>n₁=1.83</td>
<td>w₁=8μm</td>
<td>n₂=1.8</td>
<td>w₂=0.1μm</td>
<td>1% Increase n₁=1.8483 w₁=8μm n₂=1.818 w₂=0.101μm</td>
<td>ON 1.82999519 8.15997726 1.82172649 0.09957558 411.11 400 2.0 w₂</td>
<td>P 1.82999515 8.04351471 1.58742730 0.01249713 9.30 30173 60.0 w₂</td>
<td>DS 1.82868228 7.77752096 1.83664818 0.00000245 188.89 382 100.0 w₂</td>
<td>N 1.82807264 8.15381247 1.83577644 0.09467148 2188.89 312 5.3 w₂</td>
<td>CG 1.8284926 8.15998192 1.83588405 0.10197859 2166.67 100 2.0 w₂/n₂</td>
<td></td>
</tr>
<tr>
<td>2% Special n₁=1.8291 w₁=8.08μm n₂=1.818 w₂=0.101μm</td>
<td>ON 1.82805888 8.15997726 1.82172649 0.09957558 411.11 400 2.0 w₂</td>
<td>P 1.82999515 8.04351471 1.58742730 0.01249713 9.30 30173 60.0 w₂</td>
<td>DS 1.82868228 7.77752096 1.83664818 0.00000245 188.89 382 100.0 w₂</td>
<td>N 1.82807264 8.15381247 1.83577644 0.09467148 2188.89 312 5.3 w₂</td>
<td>CG 1.8284926 8.15998192 1.83588405 0.10197859 2166.67 100 2.0 w₂/n₂</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1% Special n₁=1.8291 w₁=8.16μm n₂=1.836 w₂=0.102μm</td>
<td>ON 1.82999519 8.00071792 1.80021685 0.09741238 9.32 490 2.6 w₂</td>
<td>P 1.82999515 8.02977139 1.5720849 0.03998834 9.30 30173 60.0 w₂</td>
<td>DS 1.82999466 7.93068275 1.81735796 0.23874864 9.48 272 138.0 w₂</td>
<td>N 1.82999520 8.03979873 1.73267922 0.02998159 9.30 1611 70.0 w₂</td>
<td>CG 1.82985777 8.02682526 1.79139949 0.0683288 30.00 34451 31.7 w₂</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2% Increase n₁=1.8666 w₁=8.16μm n₂=1.836 w₂=0.102μm</td>
<td>ON 1.83039999 8.15918497 1.83372224 0.10370887 1911.11 244 3.7 w₂</td>
<td>P 1.83085880 7.72777905 1.22583800 0.00020315 454.44 3153 99.8 w₂</td>
<td>DS 1.86692831 7.18777405 1.83623427 1.15672755 8733.33 31 115.7 w₂</td>
<td>N 1.82999521 8.02738000 1.7650480 0.04479019 9.30 1611 55.3 w₂</td>
<td>CG 1.83478366 8.15890712 1.83916327 0.10248883 1155.56 200 2.5 w₂</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1% Increase n₁=1.8483 w₁=8μm n₂=1.818 w₂=0.101μm</td>
<td>ON 1.82999511 8.01002946 1.79335612 0.07953634 9.31 676 20.5 w₂</td>
<td>P 1.82999515 8.04820884 0.15501757 0.00310727 9.30 3940 96.9 w₂</td>
<td>DS 1.84847133 6.94945333 1.8144964 1.5145068 4011.11 64 151.5 w₂</td>
<td>N 1.82999494 8.03032830 1.75513553 0.03887944 9.30 1611 83.9 w₂</td>
<td>CG 1.83006330 8.02671465 1.76294000 0.05007932 11.44 35533 49.9 w₂</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 29** Comparison of numerical methods for "hard" example. Exact result would be n₁=1.83, w₁=8μm, n₂=1.8 and w₂=0.1μm

Table 28 is shorter as all the methods gave such poor results for even the supposedly simple cases tried, that no other attempts were made. As with the other examples for 4 layer guides the universally worst calculated parameter is w₂. Table 29 is longer and indicates that the routines, in general, had less trouble with this set of parameters. Again w₂ is consistently calculated incorrectly and badly so.
Table 30 shows the results of combining several methods to solve the 4 layer guide problem. ie. The end point of one method is used as the starting point for the next. In general, the improvement achieved was not worth the effort involved - often the initial points were themselves local minima and the routines could do little to improve them or find their way out to the global minima.

F.2 Conclusions

Table 30 summarises the results for the 5 numerical methods considered in this Appendix, comparing their success (as determined by an average error per mode) and the time taken to do that (measured by the iteration count) and a combined figure (ie. error x iteration count) to give some measure of efficiency. This comparison quite obviously shows that the
NAG routine is the most efficient method, followed by quasi-Newton and then the Downhill Simplex.

Table 31  Summary of method performance across the given 4 layer step index guide examples.


7. An all optical scheme was proposed by D.M. Pataca et al. (D.M. Pataca, M.L. Rocha, R. Kashyap, K. Smith, Bright and dark pulse generation in an optically modelocked fibre laser at 1.3µm, Electronics Letters 31, p35 (1995)). This device has no bulk optics, AOM or EOM but does require a source of optical pulses from a diode laser at 1564nm which it uses to produce the pulses at 1300nm.


10. N.S. Kapany, J.J. Burke, K. Frame, "Radiation Characteristics of Circular Dielectric Waveguides", Applied Optics 4, p1534 (1965) showed the presence of coupling in fibers and that it had a variation with wavelength that conformed to theory.


39. This work goes further than the derivation of [38] in that it allows for the modes to have dissimilar $\beta$s and has complex constants.

40. Used heaps, reference for this and much of the following derivation is A. Ackiewicz, Novel effects in non-linear coupling, Optical and Quantum Electronics 20, p329, 1988.


Similarly work was performed earlier to this based on an earlier Haus papers (eg. H.A. Haus, "Theory of mode locking with a fast saturable absorber", Journal of Applied Physics 46, p3049 (1975).) by Martinez et al. (O.E. Martinez, R.L. Fork, J.P. Gordon, "Theory of passively mode-locked lasers for the case of a nonlinear complex-propagation coefficient", Journal of the Optical Society of America 2B, p753 (1985).) but apart from being earlier do not add any simplifications or useful results.


\[ F_m = \frac{\kappa}{g \beta \sigma \tau_p} \tag{255} \]

where \( \kappa \) is the slope of the nonlinear reflectivity, \( g \) is the small signal gain, \( \beta \) is a numerical factor \( \sim 1 \) which is different for different pulse shapes, \( \sigma \) is the stimulated emission cross-section and \( \tau_p \) is the duration of the pulse. This is, however, considerably more difficult to measure experimentally.

Kraus et al. derive a parallel condition for modelocking

\[ \kappa > \alpha \frac{g_0}{2P_s} \frac{T_0}{T_g} \tag{256} \]

where \( \alpha \) is a pulse shape dependent factor \( \sim 1 \), \( P_s \) is the saturation power of the gain medium and is given by \( P_s = h \nu A_g (\sigma T_g) \) where \( A_g \) is the average beam cross section in the gain medium, \( T_g \) is the lifetime of the upper laser level and \( \tau \) is the duration of the fluctuation in the background level that may grow to be a modelocked pulse.


71. Castech Specification Sheets


76. A.W. Tucker, M. Birnbaum, C.L. Fincher, L.G. DeShazer, "Continuous-wave operation of Nd:YVO_{4} at 1.06 and 1.34\mu m", Journal of Applied Physics 47, p232 (1976).


80. Data Sheet from ITI Electro-Optics Corporation


83. Castech Data Sheet, Fujian Castech Crystals, Inc., PO Box 143, Fuzhou, Fujian, 350002, P.R. China.


85. ibid. p375.

86. Optics Guide 5, Melles Griot, pg 4-29 (1990)


Similarly work was performed earlier to this based on an earlier Haus papers (eg.


Guiding was achieved with 300keV protons at a doses of less than $2 \times 10^{15}$ ions/cm² although the refractive index change is produced by the compensation of $p$ and $n$ type materials giving a smaller free-carrier concentration that the substrate and hence a smaller plasma contribution to the refractive index.


117. Townsend et.al. have published a number of review articles and lists of results for He and/or H ions into a variety of crystal hosts. Most useful table to date is in P.D. Townsend, "Ion implanted waveguides and waveguide lasers", Nuclear Instruments and Methods in Physics Research B65, p243 (1992).


The theory was developed in a subsequent pair of articles:


R.E. Smith, S.N. Houde-Walter, G.W. Forbes, "Numerical determination of planar waveguide modes using the analyticity of the dispersion relation", Optics Letters 16,
268


131. This readily converts to the more familiar

\[ \tan(Uh) = \frac{g_0g_1UW_1 + g_1g_2UW_2}{g_1^2U^2 - g_0g_2W_1W_2} \]

of the Snyder & Love theory where \( U = (k^2 n_1^2 - \beta^2)^{1/2} = i \alpha_1 \), \( W_1 = (\beta^2 - k^2 n_0^2)^{1/2} = \alpha_0 \), \( W_2 = (\beta^2 - k^2 n_2^2)^{1/2} = \alpha_2 \) and \( h = l_1 \) is the guide width.

132. Again this readily is transformed to

\[ \tan(Uh) = \frac{Ug_1(W_1W_2g_2g_3 + W_2W_2g_2g_3) + Ug_1(W_1W_2g_2g_2 + W_2g_2^2) \tanh(W_2d)}{W_3g_3(U^2g_1^2 - W_1W_2g_2g_3) + (U^2W_2g_1^2g_2 - W_1W_2g_2^2g_3) \tanh(W_2d)} = 0 \]

133. Methods of Theoretical Physics, Philip M. Morse and Herman Feshbach, McGraw-Hill Book Company p1651 (1953).


152. Measured from deposited thin films. These are slightly different from synthetic fused silica and rutile numbers.


156. These parameters are chosen to be similar to the fibre used in later experiments.


160. According to Stephen Hawking’s Editor (Stephen Hawking, *A Brief History of Time*, Bantam Books, (1996).) the readership halves for every included equation. Hence if you have read this far it makes you $1 \text{ in } 2^{254} \sim 2.89 \times 10^{76}$.