Essays on Sovereign Debt Crisis

Michinao Okachi

A thesis submitted for the degree of
Doctor of Philosophy of
the Australian National University
May, 2017

Research School of Economics
College of Business & Economics
To my family
Declaration

I hereby declare that this thesis is my own work and contains no material which has been accepted for the award of a degree or diploma at any university or institution. To the best of my knowledge, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

..............................................................

Michinao Okachi
Research School of Economics
College of Business & Economics
The Australian National University
Canberra ACT 0200
Australia
Acknowledgments

First and foremost, I would like to express my sincere gratitude to my principal supervisor, Dr. Chung Tran, for giving me exceptional guidance and advice. Under his generous supervision, I was able to tackle the challenging topic of sovereign debt crisis, and his sharp intellect and abundant knowledge of actual economic issues has led to the completion of this PhD dissertation. He also supported me to have many presentation opportunities in not only in Australia but also New Zealand and Japan.

Likewise, I show my profound gratitude to another of my supervisors, Dr. Tatsuyoshi Okimoto, for providing advice based on deep understanding of the economy. His insightful suggestions and comments inspired me to understand the interaction between models and actual economic phenomena.

Many thanks also to Dr. Richard Dennis who was my previous supervisor at the Australian National University (ANU). Unfortunately, he left the ANU during my PhD programme, but he kindly provided me the opportunity to study at the ANU under his supervision. I also thank to my past supervisors, Dr. Kazuo Ueda and Dr. Naoyuki Yoshino. Because of their guidance in my academic career and support for my study, I was able to reach the PhD programme at the ANU.

I am also appreciative of the comments, advice and discussion on policy issues through my study at ANU. In particular, I am grateful to Dr. Kentaro Asai, Dr. Ippei Fujiwara, Dr. Cody Yu-Ling Hsiao, Dr. Timothy Kam, Dr. Cagri Kumru, and Dr. Kazuki Onji.

I have also benefited from comments received at conferences and presentations: the Brown-Bag Seminar Series of the Research School of Economics at the ANU, the Brown-Bag seminar of the AJRC (the Australia–Japan Research Centre) at the ANU, the Kansai Public Economics Study Group at Kwansei Gakuin University, the Brown-Bag seminar of the Faculty of Economics at the Kyoto University, the NZ Macroeco-
nomics Dynamics Workshop 2016 at Victoria University, the Workshop of the School of Economics at Nagoya University, the Workshop of the Faculty of Economics at Hitotsubashi University, and the Summer Workshop on Economic Theory 2016 of the Section of Bubble and Banking Crises at Otaru University of Commerce.

My thanks for the friendship of my PhD schoolmates. In particular, I thank Minhee Chae, Kimlong Chheng, Jamie Cross, Syed Abul Hassan, Chenghan Hou, Wee Chain Koh, Sanghyeok Lee, Anpeng Li, Qingyin Ma, Arm Nakornthab, Tomohito Okabe, Amir-Reze Rahmani, Sen Xue, Jilu Zhang and Beili Zhu.

I am grateful to have ANU Scholarship Tuition Fee Exemption Sponsorship and ANU University Research scholarship. Because of these two secure scholarships, I could focus on my research without any financial worries. I also appreciate Karin Hosking to edit this dissertation.

Last but not least, I thank my parents, brother and sister. They have always encouraged me to study abroad with their warm hearts. In particular, advice from my father, who also studied economics abroad and obtained his PhD, has been my motivation to overcome difficult times.
Abstract

This thesis consists of three chapters that aim to develop economic models to explain sovereign debt crises. Chapter 2 provides the dynamic general equilibrium model of endogenous sovereign default, incorporating financial intermediaries. By a government’s decision to default, government bonds become non-performing and financial intermediaries eliminate them from their net worth. While other literature on endogenous default models assumes that the default state is exogenously given, only depending on TFP or endowment, the model in Chapter 2 creates a mechanism by which the default state is contingent on the amount of debt outstanding in the non-default state. Through this feature, the model quantifies the financial amplification effect on the economy and shows the phenomenon of "Too-Big-to-Default". The model explains the important features of the Argentinean default in 2001, capturing the default frequency, the debt-to-GDP ratio and moments of main variables.

Chapter 3 proposes a new sovereign debt crisis model which is applicable to an advanced country, assuming the government’s incapability to serve its debts rather than willingness of repayment. The fiscal limit is defined as the sum of discounted future primary surplus under the tax rate to maximize tax revenue. When the debt outstanding exceeds the fiscal limit, the government falls into debt crisis. The economic contraction in the crisis results from the exogenous tax rate and decreased imported inputs. The model replicates the high debt-to-GDP ratio, which the endogenous model cannot assume, and captures movements of important variables of the Spanish debt crisis in around 2012.

Chapter 4 introduce foreign bonds based on the model in Chapter 3, for the analysis of several countries such as Greece and Ireland in which a majority of bonds is held by foreign agents. In the model, the government issues bonds for foreign investors, and that leads the outflow of domestic output. Instead of government bonds, households adopt capital for their inter-temporal utility maximization. Also, the fiscal limit
is drawn from the exogenous distribution. The simulation result for the Greek economy generally explains the contraction of its economy by the crisis in around 2012 although the effect of imported inputs is overestimated and that of domestic inputs is underestimated.

**Keywords:** Dynamic Stochastic General Equilibrium, Financial Intermediaries, Non-Performing Bonds, Fiscal Limit, Laffer Curve, Developing and Advanced Countries, Domestic and Foreign Debts
# Contents

1 Introduction .................................................. 1
   1.1 Motivation ................................................. 1
   1.2 Key Research Questions and Findings .................. 4
   1.3 Organization ............................................ 7

2 Sovereign Default and Financial Intermediaries ......... 8
   2.1 Introduction ............................................. 8
   2.2 Empirical Evidence of Relation between Sovereign Debt Crises and Financial Intermediaries .............. 11
   2.3 Model .................................................. 14
      2.3.1 Households ......................................... 15
      2.3.2 Goods Producing Firms ............................ 16
      2.3.3 Financial Intermediaries ......................... 17
      2.3.4 The Sovereign Government ....................... 21
      2.3.5 Foreign Investors and Government Bond Pricing 24
      2.3.6 Competitive Equilibrium ......................... 25
      2.3.7 Timing ........................................... 27
   2.4 Quantitative Analysis .................................. 28
      2.4.1 Calibration and Functional Forms ............... 29
      2.4.2 Cyclical Co-movements of the Model ........... 33
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>Simulation Results</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>2.5.1 Baseline Results</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>2.5.2 Alternative Scenarios</td>
<td>41</td>
</tr>
<tr>
<td>2.6</td>
<td>Conclusion</td>
<td>46</td>
</tr>
<tr>
<td>3</td>
<td>A General Equilibrium Model of Sovereign Debt Crisis in an Advanced Country</td>
<td>48</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>48</td>
</tr>
<tr>
<td>3.2</td>
<td>Model</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>3.2.1 Government</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>3.2.2 Households</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>3.2.3 Final Goods Firms</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td>3.2.4 Intermediate Goods Firms</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>3.2.5 Crisis Scheme</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>3.2.6 Equilibrium</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>3.2.7 Fiscal Limit</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>3.2.8 Timing</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>3.3 Functional Form and Calibration</td>
<td>70</td>
</tr>
<tr>
<td>3.4</td>
<td>Quantitative Results</td>
<td>73</td>
</tr>
<tr>
<td></td>
<td>3.4.1 Fiscal Limit</td>
<td>73</td>
</tr>
<tr>
<td></td>
<td>3.4.2 Decision Rules</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td>3.4.3 Simulation results</td>
<td>81</td>
</tr>
<tr>
<td>3.5</td>
<td>Conclusions</td>
<td>91</td>
</tr>
<tr>
<td>4</td>
<td>A General Equilibrium Model of Sovereign Debt Crisis in an Advanced Country - The Case of Foreign Debt -</td>
<td>93</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>93</td>
</tr>
<tr>
<td>4.2</td>
<td>Model</td>
<td>95</td>
</tr>
</tbody>
</table>
4.2.1 Government .................................................. 96
4.2.2 Households .................................................. 98
4.2.3 Final Goods Firms .......................................... 99
4.2.4 Intermediate Goods Firms ................................ 100
4.2.5 Foreign Investors .......................................... 101
4.2.6 Crisis Scheme ............................................. 102
4.2.7 Competitive Equilibrium .................................. 103
4.3 Functional Form and Calibration ............................ 104
4.4 Quantitative Results ......................................... 108
    4.4.1 Decision Rule ............................................. 108
    4.4.2 Simulation Results ...................................... 110
4.5 Conclusion .................................................. 116

5 Concluding Remarks ........................................ 118

A Chapter 1 Appendix ........................................... 126
    A.1 Data in Figure 1.1 ........................................ 126

B Chapter 2 Appendix ......................................... 127
    B.1 Diagrams .................................................. 127
    B.2 Computational Algorithm ................................. 128
    B.3 The Assumption of Exogenous Proportion of Domestic Bond Holding .......................... 129
    B.4 Financial Intermediaries’ Optimization Problem .............................................. 131
    B.5 Debt Restructuring ........................................ 133
    B.6 Sensitivity Analysis ...................................... 135
    B.7 Data in Figure 2.4 ........................................ 137

C Chapter 3 Appendix ........................................... 138
    C.1 Diagrams .................................................. 138
# List of Tables

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Balance Sheet</td>
<td>19</td>
</tr>
<tr>
<td>2.2</td>
<td>Calibration</td>
<td>32</td>
</tr>
<tr>
<td>2.3</td>
<td>Simulation Results</td>
<td>39</td>
</tr>
<tr>
<td>2.4</td>
<td>Simulation Results: Alternative Scenarios</td>
<td>42</td>
</tr>
<tr>
<td>2.5</td>
<td>Simulation Results: Various Scenarios</td>
<td>43</td>
</tr>
<tr>
<td>3.1</td>
<td>Calibration</td>
<td>72</td>
</tr>
<tr>
<td>3.2</td>
<td>Simulation Results</td>
<td>83</td>
</tr>
<tr>
<td>3.3</td>
<td>Alternative Simulation Results</td>
<td>90</td>
</tr>
<tr>
<td>4.1</td>
<td>Calibration</td>
<td>106</td>
</tr>
<tr>
<td>4.2</td>
<td>Simulation Results</td>
<td>112</td>
</tr>
<tr>
<td>4.3</td>
<td>Alternative Simulation Results</td>
<td>115</td>
</tr>
<tr>
<td>B.1</td>
<td>Sensitivity Analysis</td>
<td>135</td>
</tr>
<tr>
<td>C.1</td>
<td>Sensitivity Analysis</td>
<td>144</td>
</tr>
<tr>
<td>D.1</td>
<td>Sensitivity Analysis</td>
<td>149</td>
</tr>
</tbody>
</table>
List of Figures

1.1 Public Debt .................................................. 3

2.1 Interest Rate and Lending .................................... 34
2.2 Value Function ............................................... 35
2.3 Default Set and Government Bond Prices ................. 37
2.4 Default Effects on Main Variables ....................... 40
2.5 High Maximum Debt Amount .............................. 45

3.1 Proportion of government bond held by domestic agents . 53
3.2 Laffer Curve and GDP ...................................... 74
3.3 Fiscal Surplus ............................................... 75
3.4 Fiscal Limit .................................................. 76
3.5 Crisis Probability (Fiscal Limit) ......................... 76
3.6 Bond Related Variables .................................... 78
3.7 Decision Rules of Main Variables ....................... 80
3.8 Simulation Result 1 ......................................... 85
3.9 Simulation Results 2 ....................................... 87
3.10 Simulation Results 2 Cont. ............................... 88

4.1 Fiscal Limit .................................................. 107
4.2 Decision Rule of Government Bond .................... 108
4.3 Decision Rule ............................................... 109
Chapter 1

Introduction

1.1 Motivation

Sovereign debt crises have huge impacts on the economy. The Argentinean government declared default and suspended all debt repayments in December 2001. The amount of privately held debt was more than 100 billion US dollars\(^1\), and this default caused an enormous economic contraction. Real GDP dropped by about 18.9\% in 2002Q1 from the average in 1998, the unemployment rate exceeded 22\% and the poverty ratio reached 14\%\(^2\). Depositors were limited in how much cash they could withdraw, and riots broke out in many large cities. The recent European debt crisis also drove a severe recession in several eurozone countries, especially Portugal, Ireland, Italy, Greece and Spain (PIIGS). The Greek case was the hardest among these countries, and even though five years has passed since the Greek government virtually defaulted in 2012, GDP is still stagnant and the debt-to-GDP ratio remains high.

If the occurrence of sovereign debt crises were exceptional, the analysis of sovereign debt crisis would not be so meaningful. However, according to Das, Papaioannou and

\(^1\)Before the Greek default, Russia’s default in 1918 was regarded as the largest in history. According to Zettelmeyer et al. (2013), the estimated present value of that default is equivalent to about 100 billion euros.

\(^2\)I adopt the definition of poverty rate as people who are living on less than 1.9 dollar per day. The data source is the World Bank.
Trebesch (2012), sovereign debt restructuring were implemented more than 600 times in 95 countries from 1950 to 2010. Although 447 debt exchanges were through the Paris Club, 186 cases were conducted with private creditors. In addition to this, public debt overhang is one of the big economic concerns today. Figure [1.1] (a) depicts the transition and projection of gross debt-to-GDP ratios in several selected countries and the averages of advanced and emerging economies. The ratio in Japan is the worst among OECD countries, exceeding 230% of GDP. After the collapse of the bubble economy in the early 1990s, the Japanese government sustained aggregate demand through its expenditure. Recently, social security costs have been a huge burden for the government due to the aging society. The debt-to-GDP ratio in the U.S. increased in the 2008 financial crisis. Since then, political turmoil related to the debt ceiling problem has occurred several times. Argentina defaulted in 2001 with massive debts and most of its bonds were issued in dollars, so the devaluation of the peso by abandoning the dollar-pegged exchange rate triggered a dramatic increase in its debt-to-GDP ratio. In 2005 and 2010, the government and most creditors agreed to debt restructuring with high haircut rates. In Greece, the sovereign debt crisis has caused severe recession since 2010 and the government virtually defaulted in 2012. International organizations sustained the government’s budget by providing bailouts in exchange for austerity. However, the ratio remains high due to weak economic recovery and political turmoil. The average debt-to-GDP ratios in both advanced and emerging economies are expected to remain high, reaching around 100% and 50% respectively. Although the ratio in emerging economies is not as high as in advanced economies, it does not indicate that emerging economies are safer than advanced economies, because of lower credit provided to these governments by investors.

The main cause of high debts is social security costs, especially in advanced countries. As the number of people who need financial assistance from a government increases, the expenditure on social security costs increases. The elderly tend to rely on
social security such as pensions, subsidies for medical treatment and livelihood protection. Thus, if the proportion of elderly people over working people (or taxpayers) increases, it will be a burden for the government. Figure 1.1 (b) is the projection of the dependency ratio estimated by the OECD. The Japanese dependency ratio is expected to keep increasing at a rapid pace, reaching 80% in 2050. The increased pace of the dependency ratio in the EU is not as fast as in Japan but it will exceed 50% in 2050. The ratio in the U.S. will also increase until about 2030 but will be more constant after that. In China, although the current dependency ratio is not high, the ratio is anticipated to increase with high speed because of the one child policy which was in place from 1979 to 2015. The average ratio of OECD countries will keep increasing and approach 50% in 2050. Considering these situations, it is expected that social security costs will put long-term pressure on government expenditure. Besides, the growth rates in developed countries are relatively low, so these governments are not able to count on increases in tax revenue from economic growth. Hence, public debts are a large problem and we cannot simply deny the possibility of public debt-related crises in relatively large economic countries in future. It is important to have models
to understand the mechanism of crises, quantify their effects on the economy and have policy implications to minimize social welfare loss.

However, the current models to analyze the sovereign debt crises are far from meeting these goals. The biggest reason is the intricacy of sovereign debt crises. The crises are caused by many factors and affect the economy in many ways. For example, a vast literature emphasizes the amplification mechanism through the financial sector. Also, sovereign default models usually assume a government’s willingness rather than the ability to pay debt. However, it is hard to affirm that sovereign defaults are decisions of governments’ willingness in many cases. In addition, the current account is closely related to fiscal surplus and debt sustainability. In fact, the current account deficits in Greece, Spain and Portugal were quite high at the time of the European debt crisis, and also the Argentinean default in 2001. The exchange rate system is also a matter of debt crisis. The Argentinean peso was pegged to the U.S. dollar, and that reduced Argentinean competitiveness. However, the devaluation of the peso caused a sudden increase in nominal debt value because most bonds are issued in dollars. Not only these factors but also many other factors affect sovereign debt crises, such as market sentiment, political uncertainty and economic size. It is extremely difficult to include many factors in the model. Thus, this thesis focuses on the interaction between sovereign default and financial intermediaries, and provides a new model which is applicable to advanced economies in order to capture the characteristics of these countries by assuming the government’s incapability of repayment of debts rather than its willingness.

1.2 Key Research Questions and Findings

This thesis consists of three self-contained chapters. The principal objective is to provide economic models to analyze sovereign debt crises. In Chapter 2, the model focuses on the relation between sovereign default and financial contraction. Chapters 3
and 4 comprise models for the analysis of sovereign debt crises in advanced countries.

Chapter 2 proposes a dynamic general equilibrium model of endogenous sovereign default with financial intermediaries. The overall framework of the model is as follows. The benevolent government maximizes households’ utility by two instruments: issuance of new government bonds and the decision to default or repay its debts. In the case of default, the government loses its access to the bond market, its bonds become non-performing, and financial intermediaries have to eliminate a certain proportion of these non-performing bonds from their net worth. This balance sheet effect causes contraction in lending and output. The characteristic of the model is that the state of default is contingent on the state of non-default through the amount of debt outstanding, while prior research assumes that the default state is exogenously given, depending only on TFP or endowment. The model can quantify the magnitude of economic contraction through the financial sector by changing the haircut rate or the proportion of government bonds held by domestic agents. In addition to this, the model’s features explain the phenomenon of "Too-Big-to-Default", meaning the government does not choose to default under the accumulation of huge debts because default effects on the economy would be too severe. The model is calibrated by the Argentinean economy around its default in 2001, and its compatibility with the actual default is generally sound. The simulation result captures important features such as default frequency, the debt-to-GDP ratio and moments of main variables.

Chapter 3 provides a new sovereign debt crisis model which is applicable to an advanced country. The canonical endogenous default model, assuming the government’s willingness to default, is not regarded as an appropriate tool for the analysis of sovereign debt crises in advanced countries because the model needs to assume that the government can estimate the default’s effect on the economy precisely before the decision of default and its effects have to be relatively small and short. In addition, the model cannot replicate the high debt-to-GDP ratio as accumulated by many advanced
countries. Thus, I propose a new model for the analysis of sovereign debt crisis in an advanced country, which assumes the government’s capability of debt repayment. The mechanism of the model is as follows. The government issues new bonds to households in order to finance the deficiency in its budget, but if government debts exceed a fiscal limit, the government falls into crisis. The fiscal limit is defined as the summation of discounted future primary surplus when the government sets its tax rate to maximize tax revenue. Due to the crisis, the government is obligated to set the exogenous tax rate during the crisis as an austerity measure and the amount of imports of intermediate goods decreases due to higher interest rates for working capital in the crisis state. The model assumes domestic bonds for the analysis of the Spanish economy. It replicates characteristics of advanced countries such as high enough debt-to-GDP ratio, reasonable crisis frequency and also moments of main variables.

Chapter 4 extends the model of sovereign debt crisis in an advanced country for the case of foreign bonds. Although the majority of government bond holders are domestic agents in most advanced countries, foreign nations hold the majority of government bonds in some countries such as Greece and Ireland which suffered in the European debt crisis. Thus, in this model, I assume foreign investors purchase and price government bonds instead of domestic households. The aggregate resource constraint is affected by the net outflow of domestic output. Also, I introduce capital for the maximization of inter-temporal utility because households do not have access to the bond market. In addition to this, I assume the fiscal limit is drawn from the exogenous distribution because the government has a lower incentive to serve debts. I calibrate the Greek economy around its virtual default in 2012. Although the model underestimates the domestic contraction and overestimates the effects of restriction of imported inputs due to the assumption of financial autarky of working capital acquisition, the model fits the important moments of the economy.
1.3 Organization

This thesis is organized as follows. Chapters 2 proposes a dynamic general equilibrium model of endogenous sovereign default incorporating financial intermediaries, evaluating the Argentinean economy around its default in 2001. Chapter 3 provides a sovereign debt crisis model which is applicable to an advanced country instead of the endogenous sovereign default model, and examines its compatibility with the debt crisis in Spain. Chapter 4 extends the previous chapter’s model for a foreign debt country’s case, and analyzes the virtual Greek default. Chapter 5 concludes with a brief discussion of results and future research.
Chapter 2

Sovereign Default and Financial Intermediaries

2.1 Introduction

Sovereign defaults entail financial crises. Before the 21st century, most sovereign defaults happened in less financially developed countries. However, recent sovereign defaults such as those in Argentina in 2001 and Greece in 2012 alert us that default can occur in financially established countries. Sovereign defaults in these countries cause financial contractions and amplify economic downturns. Today, some countries with relatively large financial systems accumulate a large proportion of public debts, so we cannot deny the possibility of the occurrence of another Argentina or Greece in the future. Thus, I believe it is important to comprehend the mechanism of interaction between sovereign default and financial intermediaries.

I propose incorporating financial intermediaries into dynamic stochastic general equilibrium model of endogenous sovereign default introduced by Eaton and Gersovitz (1981). The economy is divided into states of non-default and default, and a government has two policies in the non-default state: the issuance of new government bonds
in order to smooth households’ consumption and the decision of default or repayment. This government’s decision is conducted by comparing value functions of default and non-default\(^1\). If the government chooses default, the economy receives two penalties. The first penalty is the government’s non-access to the bond market, which is a common assumption of the endogenous sovereign default model. The government loses the trust of lenders, so it cannot issue new bonds during the time the default state. The second penalty is financial contraction. Government bonds become non-performing and financial intermediaries have to eliminate the non-performing bonds from their net worth. Then, the size of financial intermediaries’ balance sheet shrinks, and that leads to a decline in working capital lending to firms. Eventually, output and consumption drop. The low level of net worth continues until the government regains the non-default state. Aside from financial intermediaries and the government, there are three other agents in my model: households, goods producing firms and foreign investors. Households possess goods producing firms and financial intermediaries, provide their labor force to the firms, deposit to financial intermediaries and consume goods. The firms borrow working capital from financial intermediaries, acquire labor force and produce goods. Foreign investors undertake the rest of government bonds that domestic financial intermediaries do not hold and price the government bonds depending on the probability of default.

The contributions of this chapter can be summarized in two points. First, while prior research assumes that the default state depends only on TFP or endowment regardless of the amount of bonds in the non-default period, the state of default in my model is contingent on the state of non-default. That is, because government bonds in the non-default state become non-performing, the amount of non-performing bonds

\(^1\)Private financial contracts are guaranteed by law. If private agents break contracts, penalties are forcibly imposed by courts. However, this mechanism does not function on the sovereign bond market because no organization can force it to repay its borrowing to creditors. Instead, it is considered that the government will lose access to the bond market and the economy will be punished or penalised due to the government’s decision to default. In theory, the endogenous sovereign default model usually assumes exclusion from bond markets and direct punishments such as autarky or low endowment.
that financial intermediaries have to eliminate from their net worth depends on the amount of government debts at the time of the government’s decision to default. This feature allows us to quantify the financial amplification effects on the economy by changing the haircut rate or proportion of government bonds held by domestic agents. In addition to this, the model can describe the phenomenon as "Too-Big-to-Default". If the government accumulates too much debt, the amount of non-performing bonds that financial intermediaries have to eliminate will be large and the government will not be able to choose default by itself\(^2\). Second, my model fits the actual default well. I calibrate the model for the Argentinean economy, which defaulted in December 2001, and the simulation result captures important features such as moments of variables, the default frequency and the premium of government bonds.

–Related Literature–


However, Borensztein and Panizza (2008) point out that exclusion from the financial market and direct sanctions are not enough to grasp the mechanism of default effects. They emphasize that the domestic financial system amplifies the default effect on the economy because financial intermediaries hold a large proportion of sovereign bonds. Several papers incorporate the financial sector into the endogenous sovereign default. Sosa Padilla (2014) studies the effect of sovereign default via the financial system. Perez (2015) quantifies financial contraction through the effects of balance sheets

\(^2\)This feature implies that governments, especially in developed countries, which accumulate massive debts cannot choose default by themselves due to the huge impact on the economy. Bi (2012) mentions that the endogenous sovereign default model is not applicable for developed countries. Instead she uses the fiscal limit to analyze sovereign defaults in these countries. Refer to the next chapter about fiscal limits.
and liquidity. Engler and Steffen (2014) assume heterogeneous banks and introduce
dysfunction in the interbank market as the default punishment. The main difference
of my model from theirs is that the default state is contingent on the amount of debts
in the non-default state while their papers assume the default state depends only on
exogenous shocks. Theoretically, Gennaioli et al. (2014a) present a framework of sov-
ereign default, internalizing its effect on domestic banks. They show that if banks hold
more government bonds, a government’s incentive to choose default is smaller because
damages to the banks’ balance sheets and contractions in lending are larger.

Some papers that extend the framework of the endogenous sovereign default mech-
anism. Yue (2010) studies endogenous default and economic recovery by incorporating
ex post debt renegotiation. D’Erasmo (2011) includes the government’s reputation to
explain the high debt-to-GDP ratio. Hatchondo and Martinez (2009) introduce long-
term bonds within the sovereign default model. Gu (2015) assumes a two countries’
model and analyzes the default effect on trades. Kushwah (2014) explores default
effects on financial flows in a monetary union.

This chapter is organized as follows: Section 2.2 shows empirical evidence of the
relation between sovereign default and the financial sector. Section 2.3 presents a model
to analyze the effect of sovereign default incorporating financial intermediaries. Section
2.4 sets functional forms and parameters and explains the cyclical co-movements of the
model. Then, it delivers the quantitative results. Section 2.5 concludes.

2.2 Empirical Evidence of Relation between Sov-
ereign Debt Crises and Financial Intermediaries

As the Greek default shows, a default in an economically big nation has a huge im-
pact on the economy. One of the biggest reasons would be that developed countries
usually have large financial systems. Financial intermediaries hold a large proportion of government bonds as safe assets and the interest rates of government bonds are the benchmark of the nation’s other interest rates such as housing loans and corporate bonds. Thus, if a debt crisis entails a financial crisis, the effect on the economy would be larger than the debt crisis without a financial crisis. In the next subsection, I will explore prior studies about these two relations. There is a vast literature of studies of the relation between financial and sovereign crises. Gennaioli et al. (2014a) investigate 110 default episodes from 1980 to 2005 and report that 30 banking crises occurred before defaults, 44 banking crises entailed sovereign defaults almost at the same time as default and 36 defaults happened without banking crises. Borensztein and Panizza (2008) point out that sovereign default is more likely to arouse a banking crisis, considering 149 countries’ data between 1975 and 2000. They report that while the unconditional probability of a banking crisis is 2.9%, the probability of the crisis conditional on default jumps up to 14.1%\(^3\).

How, and how much, does sovereign default affect financial contraction? Gennaioli et al. (2014b) find that banks which hold a large proportion of sovereign bonds become financially fragile and reduce loans during sovereign crises. They analyze the role of government bonds for 20,000 banks in 191 countries during times of default including Russia in 1998, Argentina in 2001 and Greece in 2012. They report that 10% increase in government bond holdings causes 3.6% cumulative decline of lending during the first two years of default due to deleveraging, preference for high risk sovereign bonds and deficiency of liquidity. Acharya et al. (2014) study four channels of the lending contraction in the European debt crisis. The first channel is balance sheet contraction. The high sovereign credit risk damages banks’ balance sheets, so these banks reduce lending in order to satisfy their balance sheet constraints. The second channel is banks’ high incentives to hold risky government bonds to obtain high returns. The

\(^3\)They also report that the unconditional probability of a sovereign default is 2.2% and the probability of crisis conditional on a banking crisis is 4.5%. 

12
third channel is financial repression. Governments obligate banks to possess their bonds. The final channel is the low value of government guarantees, which leads to high financing costs. The authors point out that the first channel, i.e. balance sheets, was the main cause of lending contraction and the second channel also affected it.

Some papers study the effects of sovereign premiums on the economy. Corsetti et al. (2012) report that high sovereign risk increases borrowing costs to the private sector and has an amplification effect on cyclical shocks. Albertazzi et al. (2014) investigate the behavior of Italian banks during the European sovereign crisis and estimate that 100 basis points of sovereign spread shock causes 70 basis points increase in the lending interest rate and 0.7% point decrease in the growth rate of loans.

There are several papers that study the effects of financial contraction on the economy in sovereign debt crises. Acharya et al. (2014) found that firms depending on banks in GIIPS (Greece, Ireland, Italy, Portugal and Spain) countries reduced their net debts and held more cash during the European debt crisis. In addition to this, investment, sales growth and employment growth in financially constrained firms are lower than in less financially constrained firms. Neumeyer and Perri (2005) show that country risk in emerging economies is one of the main factors of output volatility. They estimate that if Argentina could eliminate its country risk, the output volatility would be 27% lower than the actual volatility.

From these evidences, the financial sector plays an important role in sovereign debt crises. These crises can cause banking crises, and instability in the financial sector amplifies an economic downturn. However, most defaults have happened in relatively less financially developed countries, so the analyses were restricted to these countries. This suggests that if sovereign default happens in financially developed economies, the effects of sovereign default entailing a banking crisis could be larger than in these empirical results.
2.3 Model

The framework of the model is a dynamic stochastic general equilibrium of a small open economy, consisting of four domestic agents and one foreign agent: households, goods producing firms, financial intermediaries, the sovereign government and foreign investors. The first three agents are a continuum of unit measure. Households provide labor for goods producing firms and receive wages from them. Goods producing firms produce output goods from labor and sell them to households. Financial intermediaries receive deposits from households, hold government bonds and lend working capital to goods producing firms. The government is benevolent, maximizing the households’ utility by two instruments: the decision to default or not and issuance of government bonds. Foreign investors also hold government bonds. The first graph of Appendix B.1 depicts the overall picture of the economy in the non-default state.

Default is in full for the government, so it does not need to repay its debts to lenders at all. However, two penalties are imposed on the economy by the decision to default\footnote{Any damages to the economy will be penalties for the benevolent government.}. The first penalty is the government’s non-access to the bond market. It cannot issue any bonds and smooth households’ consumption during the time of default. The second penalty is financial contraction. Government bonds become non-performing and financial intermediaries have to eliminate a part of these non-performing bonds from their net worth. This decline in net worth causes contractions in finance and output. The government in the default state has an opportunity to recover to the non-default state with an exogenous stochastic probability at the end of every period. In the case of recovery, the amounts of both debts and non-performing bonds are zero at the beginning of the next period.
2.3.1 Households

Households are infinitely lived and derive utility from consumption and disutility from labor. Their total income consists of wages, interest revenue from deposits, dividends from goods producing firms and financial intermediaries and government transfers. In terms of deposits, households lend them to financial intermediaries and receive the principal and interest within the period \( t \), so the terms of deposits consist only of their interest revenue. Thus, households cannot maximize inter-temporal households’ utility, but the benevolent government maximizes their inter-temporal utility by adjusting the transfers\(^5\).

The households’ utility maximization problem is

\[
\max_{c_t, L_t} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \tag{2.1}
\]

subject to

\[
c_t = w_t L_t + r^d d_t + \Pi_t + T_t \tag{2.2}
\]

where \( \beta \in (0, 1) \) is the discount factor of the household, \( c_t \) is consumption, \( w_t \) is the wage rate, \( L_t \) is the labor supply, \( r^d \) is the interest rate on deposit, \( d_t \) is deposit, \( \Pi_t \) is dividends from goods producing firms and financial intermediaries, \( T_t \) is government transfers. The utility function \( u : \mathbb{R}^2_+ \to \mathbb{R} \) is continuous, twice differential and satisfies

\[
\frac{\partial u}{\partial c} > 0, \quad \frac{\partial^2 u}{\partial c^2} < 0, \quad \frac{\partial u}{\partial L} < 0, \quad \frac{\partial^2 u}{\partial L^2} < 0 \quad \text{and} \quad \frac{\partial^2 u}{\partial c \partial L} - \left( \frac{\partial u}{\partial c} \frac{\partial u}{\partial L} \right)^2 > 0.
\]

The first-order condition of the households’ optimization problem is

\[
u_{L_t} + u_{c_t} w_t = 0 \tag{2.3}
\]

The equation represents the equality of the wage rate to the marginal rate of substitution between labor supply and consumption.

\(^5\)This assumption is common in the endogenous sovereign default model (see Arellano (2008), Mendoza and Yue (2012) and Perez (2015)).
2.3.2 Goods Producing Firms

Goods producing firms maximize profits by producing output from labor and capital. I assume the production technology follows the Cobb-Douglas function

\[ y_t = e^{A_t}(L^d_t)^{1-a}K^a \] (2.4)

where \( y_t \) is output, \( A_t \) is total factor productivity, \( L^d_t \) is labor input, \( K \) is the fixed capital stock and \( \alpha \) is the capital share of output. I assume the capital stock is exogenous in order to reduce the number of state variables for simplicity.

\( A_t \) follows the AR(1) process

\[ A_t = \rho A_{t-1} + \varepsilon_t \] (2.5)

where \( \rho \) is the persistence of temporary TFP shock and \( \varepsilon_t \sim N(0,\sigma^2) \).

Firms need to borrow working capital \( k^d_t \) from financial intermediaries in order to pay wages to households before the commencement of production. They secure a fraction \( \gamma \) of working capital to wages in each period.

\[ k^d_t = \gamma w_t L^d_t \] (2.6)

More generally, this equation is held inequality since firms can borrow working capital exceeding the necessary amount to finance wages. However, the excess amount of working capital directly leads to a decrease in their profits, so the working capital requirement always binds with equality. This working capital is repaid with its interest.

---

\footnote{As Mendoza and Yue (2012) mention, capital accumulation is usually omitted in the sovereign default model because empirical evidence justifies this assumption. For example, Meza and Quintin (2007) point out that the effect of capital on the economy is limited during crises from their analyses in Mexico and Argentina in 1994 and Indonesia, South Korea and Thailand in 1997. However, there are several papers that include capital such as Gordon and Guerron-Quintana (2016) and Roldán-Pena (2012).}
$r_t^k$ within the period $t$. The profit maximization problem is

$$\max_{L_t^d, k_t^d} \Pi_t^{GP} = y_t - w_t L_t^d - r_t^k k_t^d$$

(2.7)

The first-order condition of goods producing firms with respect to the labor input $L_t^d$ is

$$(1 - \alpha)e^{A_t}(L_t^d)^{-\alpha}K^\alpha = (1 + r_t^k \gamma)w_t$$

(2.8)

### 2.3.3 Financial Intermediaries

The market for financial intermediaries is monopolistic competitive\(^7\). They possess net worth $n$, receive deposits $d_t$ from households, lend working capital $k_t^s$ to goods producing firms and hold government bonds $a_{t+1}$. I assume net worth $n$ is constant in the non-default state because financial intermediaries do not keep their retained earnings but allot all profits (including losses) to households which are the owners of the financial intermediaries\(^8\). However, when the government decides to default and government bonds become non-performing, the net worth declines depending on the amount of non-performing bonds. The characteristics of government bonds are one period maturity, zero coupon and non-contingent. I assume the government forces financial intermediaries to hold a certain proportion of total bonds $v$. The explanation of this assumption is in Appendix B.3. The amount of government bonds held by financial intermediaries $a_{t+1}$ is determined as follows

$$a_{t+1} = -v b_{t+1}$$

(2.9)

\(^7\)De Bandt and Davis (2000) estimate the competitiveness of the banking sectors of some EU countries by using H-statistics. They classify that large banks in Germany and France are monopolistic competition and large and small banks in Italy are likely to be monopolistic competition. Similarly, Yildirim and Philippatos (2007) apply this estimation to the banking market in Latin American countries and classify them also as monopolistic competition.

\(^8\)In addition to this, although the method of finance is indifferent in the Modigliani-Miller theorem, it is theoretically and empirically shown that issuing new equity is quite costly (See Stein (1998), Diamond and Rajan (1999), Cornett and Tehranian (1994)). Thus, it would be a legitimate assumption that financial intermediaries do not acquire additional capital within the analysis of sovereign default.
where \( b_{t+1} \) represents newly issued government bonds at time \( t \). The value of government bonds \( b \) is non-positive since bonds are a liability for the government. Foreign investors hold the rest of government bonds \(-(1 - v)b_{t+1}\) and set the price of government bonds to \( q_t \in \left[ 0, \frac{1}{1+r^d} \right] \). Similarly, if the government is in the non-default state at period \( t - 1 \), the amount of government bonds held by financial intermediaries at the beginning period \( t \) is

\[
a_t = -vb_t
\]  

(2.10)

In the case that the government regains the non-default state from the default state at the end of period \( t - 1 \), \( b_t \) is zero and so is \( a_t \).

If the government chooses default at time \( t \), the government bonds \( b_t \) become non-performing, so financial intermediaries have to eliminate the non-performing bonds from their net worth. However, not all non-performing bonds inflict damage upon financial intermediaries' net worth; an international financial organization disburses a certain fraction of the bonds in order to sustain the financial system of the economy. I assume the proportion of non-performing bonds that financial intermediaries have to eliminate, which is the haircut rate \( h \) for them, is determined exogenously and the financial organization accepts the burden of the rest of the non-performing bonds \((1 - h)a_t\) at the time of default\(^9\). Thus, the amount of non-performing bonds that financial intermediaries have to eliminate \( z_{nd,t} \) is formulated as

\[
z_{nd,t} = ha_t
\]  

(2.11)

The subscript of non-performing bonds is \( nd \) because the government state is non-default at the beginning of period \( t \). The financial intermediaries' net worth in the default state \( n_t^d \) is

\[
n_t^d = n - z_{nd,t}
\]  

(2.12)

\(^9\) Another interpretation of \((1 - h)a_t\) is that financial intermediaries contract the insurance in the case of government default.
The financial organization takes the burden of the remaining domestic non-performing bonds \((1 - h) a_t\). If the government is in the default state at the beginning of period \(t\), there is \(z_{d,t}\) amount of non-performing bonds that is carried over from the previous period \(t - 1\). Then, the non-performing bonds at the beginning of period \(t\) in the case of the default state is

\[
n^d_t = n - z_{d,t}
\]  

(2.13)

This net worth lowered by non-performing bonds continues until the government regains the non-default state. The decrease in net worth affects the contraction of finance in three ways. First, financial intermediaries are more likely to face the constraint of a capital adequacy ratio. The proportion of net worth over risk-weighted assets has to be larger than the minimum capital adequacy ratio. Thus, if it binds the capital adequacy ratio, the decline in lending will be larger than in the non-binding case. Second, because of the assumption that the lending cost increases as the amount of lending deviates from a lending criterion and the lending criterion decreases as non-performing bonds increase, financial intermediaries have to pay additional costs on lending. Finally, financial intermediaries have to acquire more deposits from households due to the lower level of net worth in order to finance adequate lending of working capital.

The balance sheets of financial intermediaries in both non-default and default cases are in Table 2.1. In the non-default case, assets comprise from lending of working capital \(k^a_t\) and government bonds, and liabilities are deposits and net worth. In the default state, financial intermediaries do not hold government bonds, and their net worth is the default state value \(n^d_t\).

<table>
<thead>
<tr>
<th>Table 2.1: Balance Sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Non-Default Case</td>
</tr>
<tr>
<td>(k^a_t)</td>
</tr>
<tr>
<td>(q\mu a_{t+1})</td>
</tr>
<tr>
<td>(k^d_t)</td>
</tr>
<tr>
<td>(n^d_t)</td>
</tr>
</tbody>
</table>
Financial intermediaries face different profit maximization problems in the non-default and default states. First, their profit maximization problem in the former state is

\[
\max_{k^s_t, d_t} \Pi^{FI}_t = r^s_t k^s_t + a_t - q_t a_{t+1} - r^d_t d_t - \Psi_{nd}(k^s_t) \tag{2.14}
\]

where \( \Pi^{FI}_t \) is profits of financial intermediaries and \( \Psi_{nd} : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) is the lending cost function which is twice continuously differentiable. The interest rate on deposit \( d_t \) is the risk-free interest rate \( r^d \) because of the zero probability of default risk of financial intermediaries, and the loan market decides the interest rate on working capital \( r^k_t \).

Assets and liabilities in the balance sheet have to be equal as:

\[
k^s_t + q_t a_{t+1} = d_t + n \tag{2.15}
\]

They also need to satisfy the capital adequacy ratio constraint. The ratio is defined as net worth over the risk-weighted assets.

\[
\chi \leq \frac{n_t}{k^s_t + \xi q_t a_{t+1}} \tag{2.16}
\]

where \( \chi \) is the minimum capital adequacy requirement and \( \xi \) is the risk-weight on government bonds.

When the government is in the default state, financial intermediaries solve the different maximization problem under different constraints from the non-default state. The financial intermediaries’ profit maximization problem in the default state at time \( t \) is

\[
\max_{k^s_t, d_t} \Pi^{FI}_t = r^s_t k^s_t - r^d_t d_t - \Psi_d(k^s_t, z_{s,t}) \tag{2.17}
\]

where \( \Psi_d : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+ \) is the lending cost function in the default state which is twice continuously differentiable with respect to the first argument.
The balance sheet equation and capital adequacy constraint are as follows:

\[ k_t^a = d_t + n_t^d \]  \hspace{1cm} (2.18)

\[ \chi \leq \frac{n_t^d}{k_t^a} \]  \hspace{1cm} (2.19)

The minimum capital adequacy ratio \( \chi \) does not change from the non-default case.

### 2.3.4 The Sovereign Government

The government is assumed to be benevolent, and its purpose is to maximize households’ lifetime utility by using two policies in the non-default state: the decision of repayment or default \( \delta_{s,t} \) and the issuance of new bonds \( -b_{t+1} \). If the government is in the default state, it has no policy to make. I assume that the government cannot save for recession during good times, so the value of government bonds is non-positive. In addition, the maximum amount of bonds that the government can issue is exogenously given as \( b_{\text{max}} \). Thus, the range of issuance of government bonds is defined as \( -b_{t+1} \in [0, b_{\text{max}}] \). After the realization of TFP, the government decides on repayment or default by comparing non-default and default value functions. The government’s optimal default decision can be formulated as

\[ V(A_t, b_t, z_{nd,t}, m_{nd,t}) = \max \{ V^{nd}(A_t, b_t, m_{nd,t}), V^d(A_t, z_{nd,t}, m_{nd,t}) \} \]  \hspace{1cm} (2.20)

where \( V^{nd} \) and \( V^d \) represent the government’s values in the non-default and default states respectively. The selection of \( V^{nd} \) or \( V^d \) denotes the government’s policy of repayment or default \( \delta_s \in \{ \delta_{nd}, \delta_d \} \). There are two cases of the default value \( V^d \), depending on whether the government is in the non-default or default state at the beginning of period \( t \). The government makes a decision on repayment or default \( \delta_s \) in the state of non-default at the beginning of time \( t \), so \( V^d \) is the function of \( m_{nd,t} \) in
the equation (2.20).

First of all, the value function of government in the non-default state $V^{nd}$ is defined as

$$V^{nd}(A_t, b_t, m_{nd,t}) = \max_{\{b_{t+1}\}} \left[u(c_t(A_t, b_t, m_{nd,t}), L_t(A_t, b_t, m_{nd,t}), \delta_{nd,t}) + \beta E_t[V(A_{t+1}, b_{t+1}, m_{nd,t+1})|A_t, b_t, m_{nd,t}, \delta_{nd,t}]\right]$$

(2.21)

The government chooses the optimal amount of transfer to households by issuing new bonds $-b_{t+1}$ in order to maximize households’ lifetime utility. The government can choose the policy $\delta_s$ every period, so the expected value in the next period is the general function of $V$. The government faces the aggregate resource constraint of the economy.

$$c_t = y_t - (1-v)(q_t b_{t+1} - b_t) - \Psi_{nd}(k_t^s)$$

(2.22)

The first term of the right-hand-side is output, the second term is net payments of government bonds for foreign investors and the final term is the lending cost of financial intermediaries. Next, the government transfer is defined as

$$T_t = -q_t b_{t+1} + b_t$$

(2.23)

The government controls the amount of transfers to households by issuance of new government bonds $-b_{t+1}$. By combining the above two equations, the aggregate resource constraint is

$$c_t = y_t + a_t - q_t a_{t+1} - \Psi_{nd}(k_t^s) + T_t$$

(2.24)

If the government chooses default in the case that the government is the non-default
state at the beginning of period $t$, the default value function is

$$V^d(A_t, z_{nd,t}, m_{nd,t}) = u(c_t(A_t, z_{nd,t}, m_{nd,t})|\delta_{d,t}, L_t(A_t, z_{nd,t}, m_{nd,t})|\delta_{d,t})$$

$$+ \beta(1 - \vartheta)E_t\{V^d(A_{t+1}, z_{d,t+1}, m_{d,t+1})|(A_t, z_{nd,t}, m_{nd,t}, \delta_{d,t})\}$$

$$+ \beta \vartheta E_t\{V(A_{t+1}, 0, 0, m_{nd,t+1})|(A_t, m_{nd,t}, \delta_{d,t})\}$$

(2.25)

There exists the amount of $-b_t$ government bonds before the default decision by government, and financial intermediaries have to incur the non-performing bonds following the haircut rule (2.11). If the default state continues to the next period $t + 1$ with the probability $(1 - \vartheta)$, non-performing bonds $z_{nd,t}$ are carried over from time $t$ to the next period $t + 1$, $z_{d,t+1}$. If the government regains the non-default state with the probability $\vartheta$ at the end of period $t$, the amount of government debts and non-performing bonds will be zero at the beginning of next period $t + 1$. The government constraint, which is equal to the aggregate constraint, in the default state is

$$c_t = y_t - \Psi_d(k^s_t, z_{s,t})$$

(2.26)

The points of difference from the non-default state are no transactions of government bonds and transfers and the formulation of a lending cost function.

The default set $\Gamma^A$ is the set of TFP for the government bonds outstanding such that the value of default is larger than the non-default value under the condition of non-default state at the beginning of period $t$:

$$\Gamma^A(b_t, m_{nd,t}) = \{A_t \in A : V^{nd}(A_t, b_t, m_{nd,t}) \leq V^d(A_t, z_{nd,t}(b_t), m_{nd,t})\}$$

(2.27)

The amount of non-performing bonds $z_{nd,t}$ depends on the current bonds outstanding $b_t$ because non-performing bonds are determined from the financial intermediaries’ government bond holding rule (2.10) and the haircut rule (2.11).
The default probability $\pi_t^e$ is defined as the conditional cumulative probability density over the transition of TFP from time $t$ to $t+1$ within the range of the default set $\Gamma^A_t$.

$$\pi_t^e(A_t, b_{t+1}, m_{nd,t+1}) = \int_{\Gamma^A_t(b_{t+1}, m_{nd,t+1})} f(A_{t+1}, A_t) dA_{t+1} \quad (2.28)$$

If the government is in the default state at the beginning of period $t$ (i.e. $m_s = m_d$), the default value function is,

$$V^d(A_t, z_{d,t}, m_{d,t}) = u(c_t(A_t, z_{d,t}, m_{d,t}), L_t(A_t, z_{d,t}, m_{d,t}))$$

$$+ \beta(1 - \delta)E_t[V^d(A_{t+1}, z_{d,t+1}, m_{d,t+1})|A_t, z_{d,t}, m_{d,t})]$$

$$+ \beta \delta E_t[V(A_{t+1}, 0, 0, m_{nd,t+1})|A_t, m_{d,t})] \quad (2.29)$$

The amount of non-performing bonds $z_{d,t}$ is different from the amount if the government chooses default during period $t$ $z_{nd,t}$. However, the constraint is identical to that in the previous case.

### 2.3.5 Foreign Investors and Government Bond Pricing

Foreign investors also undertake the rest of the government bonds that are not held by financial intermediaries (i.e. $-b_{t+1} - a_{t+1}$) and price the government bonds $q_t$. Foreign investors’ expected profits are

$$\Pi_t^F = -q_t(-b_{t+1} - a_{t+1}) + \frac{1}{1 + r^d} \frac{-\pi_t^e(A_t, b_{t+1}, m_{nd,t+1})}{1 + r^d} (-b_{t+1} - a_{t+1})$$

$$+ \frac{\pi_t^e(A_t, b_{t+1}, m_{nd,t+1})}{1 + r^d} (-b_{t+1} - a_{t+1})(1 - h) \quad (2.30)$$

The first term is the expenditure for the newly issued government bonds, and the second and third terms are the expected return in the cases of non-default and default respectively. Unlike in other endogenous sovereign default models, even though the government chooses default, foreign investors can receive $(1 - h)$ times the amount of
bond holdings from the international financial organization. Besides, foreign investors are risk neutral and behave perfectly competitively, so the price of government bonds is derived from the first-order condition with respect to $-b_{t+1} - a_{t+1}$ as

$$q_t(A_t, b_{t+1}, m_{nd,t+1}) = \frac{1}{1 + r^d} \left\{ (1 - \pi^e(A_t, b_{t+1}, m_{nd,t+1})) + \pi^e(A_t, b_{t+1}, m_{nd,t+1})(1 - h) \right\}$$

(2.31)

The price of government bonds reflects the bailout from the financial organization, and that prevents drastic movement in its price. Domestic financial intermediaries also purchase the government bonds with the same price $q_t$ since the government bonds are indifferent between domestic and foreign agents.

### 2.3.6 Competitive Equilibrium

I summarize the state vector as $S \in \{A, b, z_d, m_s\}$, where $m_s \in \{m_{nd}, m_d\}$ indicates the government’s state of non-default or default at the beginning of time $t$.

**Definition**

*The recursive equilibrium of this economy is defined as policy functions of the private sector \( \{c(S), L(S), L^d(S), k^d(S), k^s(S), d(S)\} \), value functions \( \{V(A, b, z_{nd}, m_{nd,t}), V^{nd}(A, b, m_{nd}), V^d(A, z_{nd}, m_{nd})\} \) for government, government policies \( \{b'(A, b, m_{nd}), \delta_s(A, b, m_{nd})\} \), prices \( \{w(S), r^k(S)\} \) and the government bond pricing $q(A, b', m'_{nd})$ such that:

1. Given the government policies and the price of government bond, the households policies solve their utility maximization problem.
2. Given the productivity, goods producing firms policies solve their profit maximization problem.
3. For $(m_s, \delta_s) = (m_{nd}, \delta_{nd})$:
   Given the government policy of newly issuance of government bonds, the amount of current bond and the price of government bond, $a(b)$ and $a'(b')$ satisfy financial intermediaries’ government bond holding rules at time $t$ (2.10) and at time

25
t + 1 \((2.9)\), and the financial intermediaries policies solve the non-default state’s profit maximization problem of financial intermediaries in \((2.14) – (2.16)\).

For \((m_s, \delta_s) = (m_{nd}, \delta_d)\):

Given the amount of current bond, \(a(b)\), \(z_{nd,t}(b)\) and \(n^d_t(b)\) satisfy the rules \((2.10)\), \((2.11)\) and \((2.12)\), and the financial intermediaries policies solve the default state’s profit maximization problem of financial intermediaries in \((2.17) – (2.19)\).

For \(m_s = m_d\):

Given the non-performing bonds carried over from previous period \(z_d\), \(n^d_t(b)\) satisfies the rule \((2.13)\), and the financial intermediaries policies solve the default state’s profit maximization problem of financial intermediaries in \((2.17) – (2.19)\).

(4) The working capital and labor markets clear:

\[
k^s(S) = k^d(S) \tag{2.32}
\]

\[
L(S) = L^d(S) \tag{2.33}
\]

(5) For \(m_s = m_{nd}\):

Given the amount of current bond and the price of government bond, the government policies solve its optimization problem and the government transfer satisfies equation \((2.23)\).

(6) The plan of consumption satisfies the aggregate resource constraint \((2.24)\) in the normal state and \((2.26)\) in the default state.

(7) For \((m_s, \delta_s) = (m_{nd}, \delta_{nd})\):

Given the default set \(\Gamma^A(b, m_{nd})\) and the default probability \(\pi^c(A, b', m'_{nd})\), the price of government bonds satisfies the foreign investors’ government bond pricing equation \((2.31)\).
Condition (1) and (2) require households’ and goods producing firms’ optimization problems respectively. Condition (3) requires decision rules of financial intermediaries’ government bonds holding and non-performing bonds and financial intermediaries’ optimization problems divided into three cases: the non-default state at the beginning of time $t$ and the government’s choice of repayment, the non-default state at the beginning of time $t$ and the government’s choice of default and the default state at the beginning of period $t$. Condition (4) requires the market clearing of working capital and labor. Condition (5) requires the government’s optimization problem in the non-default state. Condition (6) requires the aggregate constraint in each state of the economy. Condition (7) requires the determination of the price of government bonds.

### 2.3.7 Timing

The overview of time flow around time $t$ is depicted in the second graph of Appendix B.1. I divide timelines into the non-default and default states.

---

**The Timeline of the Non-Default State**

At the beginning of period $t$, the government owes the amount of $-b_t$ debts (if it regains the non-default state from the default state, $-b_t$ is zero). First of all, TFP is revealed. Then, the government makes the decision to either repay or default. If it chooses the former, foreign investors and domestic financial intermediaries receive the full amount of repayments and the government issues new bonds $-b_{t+1}$ priced by $q_t$. Afterwards, financial intermediaries borrow deposits from households and lend working capital to goods producing firms. The market determines the interest rate for working capital. Households provide labor force to goods producing firms and receive wages. The wage rate is determined by the labor market. Then, the firms produce output goods and repay working capital with interest to financial intermediaries. Households purchase output goods, receive government transfers and deposit with interest and
finally consume. Time $t$ ends here and goes to next time $t + 1$. If the government had chosen default after the realization of TFP, the timeline goes to the default state.

The Timeline of the Default State—

There are two initial points in the default state timeline: continuing default state and government’s default decision during time $t$. If the default state continues from the previous period, there are $z_{d,t}$ amount of non-performing bonds, which are carried over from time $t - 1$. Then, the TFP is revealed. If the government in the non-default state chooses default after the TFP is revealed, the timeline shifts to the default state with non-performing bonds $z_{nd,t}$, which is equivalent to $(-b_t \cdot v \cdot h)$. Subsequently, each agent transacts goods and households consume similarly to in the non-default state case. Finally, nature decides to regain the non-default state or not with exogenous probability $\vartheta$. If the government does not recover the non-default state, the non-performing bonds $z_{s,t}$ are carried over to the next period $t + 1$.

2.4 Quantitative Analysis

This section provides the quantitative analysis based on the calibration for the Argentinean economy. The biggest reason for my selection of the Argentinean default for the calibration is that the country had a relatively well-established financial system, among the countries which experienced default. The Argentinean case was one of the biggest defaults in history before the Greek default, and a banking crisis had also happened\textsuperscript{10}.

In my model, the financial system is essential to generate the amplification mechanism of default, so Argentina was considered appropriate for the analysis of the financial amplification effect by default. Of course, the default in Greece was huge and a banking crisis also broke out. However, as I explain in the next chapter, the endogenous sovereign default model cannot be applicable to the analysis of a country which accu-

\textsuperscript{10}See Gennaioli et al. (2014a).
mulates massive public debts, such as 170% of debt-to-GDP ratio in Greece\textsuperscript{11}. Another reason for my choice of Argentina is that this country’s default is often discussed in analyzing sovereign default, so it is easy to compare with prior research\textsuperscript{12}.

After explaining the calibration and functional forms, I report the cyclical co-movements of the model, focusing on the explanation of financial variables, values functions and government bond pricing. Finally, I show the simulation results of baseline and alternative scenarios. I also provide results of changing the parameters related to non-performing bonds and the amount of maximum debt.

2.4.1 Calibration and Functional Forms

Following Greenwood et al. (1988), I specify the utility function as

\[
u(c_t, L_t) = \frac{(c_t - \frac{1}{\omega}L_t^\omega)^{1-\eta}}{1-\eta}
\]

(2.34)

where \( \eta \) is the degree of risk aversion and \( \omega \) is the elasticity of labor supply. I assume this function to eliminate the wealth effect on the labor supply, so the labor supply depends only on the wage rate. I set \( \eta \) and \( \omega \) to 2 and 1.455 respectively from the standard value of the RBC model. Following Mendoza and Yue (2012), I set the household’s discount factor \( \beta \) to 0.88\textsuperscript{13}, and the persistence of transitory TFP shock \( \rho \) and the standard deviation of the shock \( \sigma \) to 0.95 and 0.017 respectively. I set the capital share of output \( \alpha \) and the risk free interest rate on deposit \( r^d \) to 0.36 and 0.02 respectively, which are common values used in the RBC model. Under the Basel Accord, banks are required to hold no less than 8% of net worth proportional to risk-

\textsuperscript{11}Data source is OECD.
\textsuperscript{12}For example, canonical papers that calibrate the Argentinean economy are Aguiar and Gopinath (2006), Arellano (2008) and Mendoza and Yue (2012).
\textsuperscript{13}This value is lower than the standard RBC model. This is because if it takes a higher value, the government will not choose default in order to relieve the burden of its debts in the first place. The one way to interpret this low value is that the government can be regarded as less patient for political reasons. Other papers also set low values on the discount factor. For example, Aguiar and Gopinath (2006) and Perez (2015) set to 0.8 and 0.9 respectively.
weighted assets. However, because the Argentinean banking system is more volatile than those of other advanced countries, the Central Bank of Argentina sets its original minimum capital adequacy ratio to 11.5% ($\chi = 0.115$). Next, I set the risk weight on government bonds $\xi$ to 0 because the value is zero for domestic bond holders under the current Basel regulation. I assume the financial intermediaries’ lending cost function in the non-default and default states as the following quadratic forms,

$$
\Psi_{nd}(k^s_t) = \phi_{nd,1} k^s_t + \frac{\phi_2}{2} \left\{ \frac{k^s_t - \left( \frac{n}{\chi_{Arg}} - \xi q_t a_{t+1} \right)}{k^s} \right\}^2
$$

$$
\Psi_d(k^s_t, z_{s,t}) = \phi_{d,1} k^s_t + \frac{\phi_2}{2} \left\{ \frac{k^s_t - \left( \frac{n^d(z_{s,t})}{\chi_{Arg}} - \phi_3 \right)}{k^s} \right\}^2
$$

where $\phi_{nd,1}$, $\phi_{d,1}$, $\phi_2$ and $\phi_3$ are parameters taking positive values. The parameters $\phi_{nd,1}$, $\phi_{d,1}$ are the base premium on lending of working capital in the non-default and default state respectively, $\phi_2/k^s$ is the elasticity of lending normalized by the steady state lending of working capital and $\phi_3$ is the extra penalty on lending in the default state. The first term in each function represents the simple linear relation of lending and its cost, and the second term indicates the quadratically increasing cost as the amount of lending deviates from the lending criterion. I set the lending criteria to $\left( \frac{n}{\chi_{Arg}} - \xi q_t a_{t+1} \right)$ in the non-default case and $\left( \frac{n^d(z_{s,t})}{\chi_{Arg}} - \phi_3 \right)$ in the default case. As the amount of lending moves from these criteria, financial intermediaries have to pay additional lending costs. The criterion in the non-default state is net worth over the Argentinean capital adequacy ratio minus risk-weighted government bonds, and the criterion in the default case is net worth lowered by non-performing bonds over the Argentinean capital adequacy ratio minus the extra penalty $\phi_3$.\footnote{Gerali et al. (2010) assume a similar function that increases costs quadratically as the capital to assets ratio deviates from the exogenous fixed value. As they point out the quadratic cost function is ad hoc and is not established from the microfoundation, but they show the reconciliation of their model with the euro area economy. Several papers also assume the similar à la Rotemberg (1982) quadratic cost function. For example, Iacoviello (2015) adopts adjustment cost functions on deposits and loans, and Dib (2010) also applies the adjustment cost on capital and deposit interest rates.} I explain the economic
interpretation of the lending cost function by deriving the first-order conditions in Appendix B.4. I set the parameter \(\phi_{nd,1}\) to 0.02, which is considered to be a reasonable risk premium on lending in the non-default state. I set the capital adequacy ratio of financial intermediaries \(\chi_{Arg}\) to 14.6% from the data in December 1996 before the Argentinean economic downturn started\(^{15}\). Next, according to Dias and Richmond (2008), it takes 5.7 years on average to regain partial market access after default\(^{16}\). Thus, I set the probability of recovery to the non-default state \(\vartheta\) to 0.044 per quarter\(^{17}\). In terms of the proportion of domestic agents’ government bond holdings, Sturzenegger and Zettelmeyer (2006) estimate that domestic agents hold 60% of total defaulted debt, but according to the data from Shapiro and Pham (2006), about 41.5% of government debts were held by domestic agents in 2001. Thus, I set the proportion of domestic bond holdings \(v\) to 0.5 as a benchmark and examine different values in the section on alternative scenarios. Next, it is extremely difficult to calibrate parameters for the haircut rate \(h\) because the Argentinean government implemented the debt swaps or reductions not only one time but five times and the rate varied in each restructuring, and moreover the reduction rates for each kind of bonds were also different. According to Sturzenegger and Zettelmeyer (2006), the average haircut rates were 71–75% at the main debt restructuring in 2005. However, the government granted GDP-indexed warrants equivalent to 2 cents on one dollar, but the value increased significantly due to the high recovery of the Argentinean economy after the default. Thus, I set the haircut rate \(h\) to 0.6 as a benchmark and also examine other values in the alternative scenarios section. A detailed explanation of the debt restructuring in Argentina is in Appendix B.5.

\(^{15}\)See International Monetary Fund (1998).

\(^{16}\)Dias and Richmond (2007) define the partial market access as "the first year in which there are positive net bond and bank transfers to the public or private sector" and full market access as "the first year of positive net bond and bank transfers to the private or public sector greater than 1.0% of GDP."

\(^{17}\)Some other sovereign default models such as Mendoza and Yue (2012), Perez (2015) set \(\vartheta\) to 0.083. They refer to the median duration to regain partial market access.
Table 2.2: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\eta): Degree of risk aversion</td>
<td>2</td>
<td>Standard Value</td>
</tr>
<tr>
<td>(\omega): Elasticity of labor supply</td>
<td>1.455</td>
<td>Standard Value</td>
</tr>
<tr>
<td>(\beta): Discount factor</td>
<td>0.88</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>(\rho): Persistence of transitory TFP shock</td>
<td>0.95</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>(\sigma): Standard deviation of TFP shock</td>
<td>0.017</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>(\alpha): Capital share of output</td>
<td>0.36</td>
<td>Standard Value</td>
</tr>
<tr>
<td>(r^d): Risk–free interest rate</td>
<td>0.02</td>
<td>Standard Value</td>
</tr>
<tr>
<td>(\chi): Minimum capital adequacy requirement</td>
<td>0.115</td>
<td>Regulation in Argentina</td>
</tr>
<tr>
<td>(\xi): Risk weight on government bonds</td>
<td>0.0</td>
<td>Basel Regulation</td>
</tr>
<tr>
<td>(\phi_{nd,1}): Base premium on lending in non-default</td>
<td>0.02</td>
<td>Standard Value</td>
</tr>
<tr>
<td>(\chi_{Argentina}): Argentina’s capital adequacy ratio</td>
<td>0.146</td>
<td>Average of Argentinean Banks</td>
</tr>
<tr>
<td>(\vartheta): Probability of the recovery to non-default</td>
<td>0.044</td>
<td>Dias and Richmond (2008)</td>
</tr>
<tr>
<td>(v): Proportion of domestic bond holdings</td>
<td>0.5</td>
<td>Sturzenegger and Zettelmeyer (2006) and Data</td>
</tr>
<tr>
<td>(h): Haircut rate</td>
<td>0.6</td>
<td>Sturzenegger and Zettelmeyer (2006)</td>
</tr>
</tbody>
</table>

\(calibrated\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K): Fixed capital stock</td>
<td>5.4</td>
<td>(E[K/y] \approx 2.7)</td>
</tr>
<tr>
<td>(\gamma): Ratio of working capital to wages</td>
<td>2.7</td>
<td>(E[wL/y] \approx 0.57)</td>
</tr>
<tr>
<td>(n): Net worth in the non-default case</td>
<td>0.5</td>
<td>(E[r^\eta] \approx 0.025)</td>
</tr>
<tr>
<td>(\phi_2/k^\eta): Lending elasticity to its interest rate</td>
<td>0.07</td>
<td>(r^\eta</td>
</tr>
<tr>
<td>(b_{max}): Maximum debt</td>
<td>-0.7</td>
<td>Decline of net worth by default (\approx 0.35)</td>
</tr>
<tr>
<td>(\phi_{d,1}): Base premium on lending in default</td>
<td>0.035</td>
<td>(E[r^\eta_d] \approx 0.11)</td>
</tr>
<tr>
<td>(\phi_3): Extra financial penalty by default</td>
<td>0.8</td>
<td>(r^\eta</td>
</tr>
</tbody>
</table>

Then, I jointly calibrate the fixed capital stock \(K\), the ratio of working capital to wages \(\gamma\), net worth in the non-default case \(n\), and lending elasticity to the interest rate \(\phi_2/k^\eta\) by targeting moments of the expected capital-to-GDP ratio, labor income-to-GDP ratio, lending interest rates under neutral TFP, and deviation of lending interest rates under 5% lower and higher than the TFP level. According to the estimation by Coremberg et al. (2007), the capital-to-GDP ratio from 1980 to 2001 in Argentina was about 2.7. Labor income share in GDP was about 57% in 2000 (Frankema (2010)). The expected lending interest rate is set to be slightly higher than the risk-free interest rate at 2.5% and to change \(\pm 1\%\) points under the condition of \(\pm 5\%\) variation of TFP from the neutral level. From these targeted moments, I obtain \(K = 5.4\), \(\gamma = 2.7\),
\[ n = 0.5^{18} \text{ and } \phi_2/k^*_t = 0.07. \]

In addition to this, I separately calibrate parameters related to the default state: the maximum debt amount \( b_{\text{max}} \), the base premium on lending in the default state \( \phi_{d,1} \) and the additional financial penalty due to the government’s decision to default \( \phi_3 \) by targeting net worth decline by default, the average working capital interest rate in the default state and interest rate deviation by the debt outstanding. Net worth of financial intermediaries shifted relatively constant before the default, but decreased about 35% after default due to the devaluation of Argentinean peso\(^{19}\). Next, the average interest rate in the default state was about 11% after the Argentinean default in 2002, and the rate changes \( \pm 3\% \) points if the amount of bonds changes to \( \pm 0.35 \). From these three targeted values, I obtain \( b_{\text{max}} = -0.7, \phi_{d,1} = 0.035 \) and \( \phi_3 = 0.8 \). Table 2.2 summarizes values of all parameters.

### 2.4.2 Cyclical Co-movements of the Model

The decrease of net worth is one of the main channels to amplify the effect of sovereign default in my model. If the government chooses to default, its bonds become non-performing and financial intermediaries have to eliminate a part of the non-performing bonds from their net worth. Because the lending of working capital and its interest rates depend on net worth, the financial contraction is contingent on the amount of non-performing bonds in the default state. The left panel of Figure 2.1 shows the relation between non-performing bonds and the lending interest rate under different levels of TFP and government states. As TFP and non-performing bonds are higher, the interest rate in the default state increases. Obviously, the interest rate in

\(^{18}\) The value of net worth in my model is higher than in the data. I assume all firms need working capital to acquire labor force, but firms which have enough net worth do not need borrow money from financial intermediaries in the actual economy. Taking their net worth into account, the amount of net worth in the economy will be larger than only financial intermediaries’ net worth.

\(^{19}\) I normalize the net worth based on the real effective exchange rate. I obtained the net worth data from the Central Bank of Argentina and real effective exchange rates from the Federal Reserve Bank of St. Louis.
Note: High, middle and low represent the TFP values of -10%, 0% and 10% deviation from the steady state level respectively, and D and ND indicate default and non-default states respectively.

the non-default state does not depend on non-performing bonds. The interest rate in the default state is higher than in the non-default state under the same TFP level. For example, in the neutral TFP case, while the interest rate in the case of the non-default state is about 2.5%, that of the default state is about 8% at zero non-performing bonds and 14% at the maximum amount of non-performing bonds. The right graph shows the relation between non-performing bonds and the lending of working capital under the same conditions as the previous graph. Lending in the default state increases as the level of TFP is higher and non-performing bonds are lower. In the neutral TFP case, the amount of lending in the default state under maximum and zero non-performing bonds is about 40% and 20% lower than that of lending in the non-default state respectively.

Next, Figure [2.2] depicts the value functions of default and non-default states in four different TFP cases. The horizontal axis of the default value function represents the amount of debts at the time of default, so the scales correspond to the non-performing bonds following equations (2.10) and (2.11). Other endogenous sovereign default models assume that the default value function depends only on TFP or endowment and
non-performing bonds do not affect the default value function. Thus, default can happen in most TFP or endowment cases if the government accumulates massive debts. However, in my model, not only the non-default value function but also the default value is decreasing the amount of debts. In the cases of steady state level and -5% lower than steady state TFP, the non-default value function is always higher than the default value function regardless of the amount of government bonds, so the government does not choose default at all, and the price of government bonds is equal to the value at zero default probability. However, as TFP takes lowers values, the distance between default and non-default values is closer, and the default value exceeds the non-default
value in the case of -15% TFP if debts exceed the threshold. The pace of decline in the non-default value becomes steep because the price of government bonds decreases due to the possibility of occurrence of default. If the price of government bonds declines, the government cannot transfer enough to households by the issuance of its bonds. Thus, the amount of government transfer to households defined in equation (2.22) will decline and could be negative. Then, the amount of consumption declines and so does households’ utility.

The left panel of Figure 2.3 represents the default set depending on TFP and government debts. Default only happens under the condition of low TFP and high debt\textsuperscript{20}. The important feature is that the default area does not exist if the TFP is larger than about -10% because the non-default value function is always higher than the default value function. The right panel of Figure 2.3 shows government bond prices depending on the TFP and government debts. In the case of low TFP and high government debt, the probability of default is high so foreign investors set a low price on government bonds following the bond pricing equation (2.31). Even though the probability of default is almost certain, the price does not fall below 0.4 because the international financial organization guarantees the repayment of \( -(1 - h)b_{t+1} \) debts to financial intermediaries and foreign investors, and foreign investors know the financial organization’s behavior at the time of their pricing of the government bonds. The price of government bonds shows quite drastic movement because as Arellano (2008) points out, the region of the government debts that the investors account for in the default premium (i.e. the default probability is between zero and one) is limited in the endogenous sovereign default model.

\textsuperscript{20}Aguiar and Amador (2013) point out that although default usually happens in bad times, more than one-third of defaults occurred when income was above the trend.
2.5 Simulation Results

I simulate the model to obtain statistical properties to analyze sovereign debt default. I generate stochastic TFP processes 2000 times for 500 periods and trace the movement of variables. The government starts from the non-default state, and the initial values of TFP and government bonds are neutral and zero respectively, but I discard the first 100 periods in order to eliminate the effect of these initial conditions.

2.5.1 Baseline Results

First of all, I explain the data source and period. The average debt-to-GDP ratio is taken from the historical public debt database of the IMF Fiscal Affairs Department for 1993–2000. In terms of default frequency, Argentina had experienced six defaults since 1820, so the default frequency per quarter is about 0.77%. The data source for the average bond spread is J.P. Morgan’s EMBI+ spread on foreign currency dominated Argentinean bonds from 1998\(^{21}\) to 2016 excluding the time of debt crisis from 2002 to 2005. I set this time span for the crisis because the government defaulted in December 2001 and implemented its first debt restructuring after the default in 2005. The GDP

\(^{21}\)I start from 1998 due to the limitation of data.
data is from OECD, consumption and the interest rate on lending data are from IMF’s International Financial Statistics and lending data is from the Central Bank of Argentina. Detailed explanations are in Appendix B.7. In my model, I remove the data’s trend effect of output, consumption and lending from the original data by using the Hodrick-Prescott (HP) filter\(^{22}\). Then, I calculate the correlation coefficient and effects of sovereign default for the sample period from 1999Q1 to 2005Q1.

Table 2.3 summarizes the main results of the actual Argentinean economy’s data and baseline simulation. The average debt-to-GDP ratio in simulation (37.06\%) obtains a similar value to the data (33.13\%), although other papers usually report lower values on the debt-to-GDP ratio\(^{23}\). Next, the frequencies of default in the data and the simulation are 0.77\% and 0.71\% respectively. Although the default frequency in the data is taken from the long history and it is controversial to judge that the actual default frequency is 0.77\%, it is an intuitively acceptable value that default happens one time in about 35 years derived from 0.71\% default probability per quarter. The simulation result for the average bond spread reports a lower value than the data. This is because while the bond spread in the data takes a constantly high value\(^{24}\) as the country risk, in simulation it is zero under the zero default probability state and the bond spread jumps up only in the case that investors take into account the possibility of default.

The coefficient correlations between output and consumption in the data and the simulation are 0.960 and 0.990 respectively, so their movements around the default period are quite similar in both the data and the simulation. The relation between output and lending is not strong in the data. This suggests that other factors also affected the contraction of output\(^{25}\). However, in my simulation, output depends on

\(^{22}\) The smoothing parameter is 1600.

\(^{23}\) For example, Arellano (2008), Mendoza and Yue (2012) and Perez (2015) report the debt-to-GDP ratio at about 6\%, 23\% and 22\% respectively.

\(^{24}\) The average bond spread is 1.76\% from 2006 to 2016.

\(^{25}\) For example, Mendoza and Yue (2012) assume that final goods producing firms cannot import intermediate goods from foreign countries in the default state.
Table 2.3: Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Debt/GDP</td>
<td>37.06%</td>
<td>33.13%</td>
</tr>
<tr>
<td>Default Frequency</td>
<td>0.77%</td>
<td>0.71%</td>
</tr>
<tr>
<td>Average Bond Spread</td>
<td>1.92%</td>
<td>0.19%</td>
</tr>
<tr>
<td>The correlation with Output</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Consumption</td>
<td>0.960</td>
<td>0.990</td>
</tr>
<tr>
<td>- Lending</td>
<td>0.662</td>
<td>0.988</td>
</tr>
<tr>
<td>- Interest rate on lending</td>
<td>-0.740</td>
<td>-0.932</td>
</tr>
<tr>
<td>The Default Effect on Variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Output</td>
<td>-10.56%</td>
<td>-16.15%</td>
</tr>
<tr>
<td>- Consumption</td>
<td>-17.31%</td>
<td>-16.34%</td>
</tr>
<tr>
<td>- Lending</td>
<td>-27.94%</td>
<td>-26.44%</td>
</tr>
<tr>
<td>- Interest rate on lending</td>
<td>17.50%</td>
<td>9.76%</td>
</tr>
</tbody>
</table>

Note 1: The values of output, consumption and lending show the deviation from trend.
Note 2: The default effect on variables indicates values that show the most extreme effect around the default period.
Note 3: The average bond spread is on a quarterly basis.

lending and TFP, so the relation between output and lending is very high. Output and interest rate on lending in both data and simulation are negatively related. Finally, I summarize the default effects on the main four variables in the last section of the table. The baseline simulation results of contractions of consumption and lending are similar to the data, but the baseline results overestimate output and underestimate the interest rate on lending.

Figure 2.4 shows the transition of main variables around the default period. In the horizontal axis, year zero represents the time of default and the values of scale are years before and after the default. Figure 2.4 (a) describes the interest rate on lending. The interest rate in the data shifted between 2 and 4% until one year before the default, but the rate sharply increased due to increased uncertainty around debt sustainability. The interest rate hit a maximum value of 17.5% half a year after the default, and then the rate gradually fell and recovered to the pre-default level one and half years after default. In my simulation, the interest rate is almost constant at around 2% before the
default, but the interest rate jumps up to about 10% at the time of default and in the next quarter. Then, the rate begins to decrease but it takes time to converge to the pre-default level.

Figure 2.4 (b) shows the amount of lending. The lending in the data had started to decline about half a year before the default and dropped sharply at the time of default. Then, the rate stayed at a low level for about two years before beginning to recover quickly. The lending in my simulation keeps shifting above the trend just before the
default and hit the bottom at the time of default, reaching about 26% of decline. Then, it gradually recovers and becomes positive in two years.

Figure 2.4(c) depicts the GDP movement. GDP in the data fell below the trend half a year before the default and reached 10.56% below trend at the time of default. The rate stayed around -10% for three quarters and then started to increase and regained a positive value three years after the default. GDP in the simulation drops 16% from the trend at the time of default but begins to recover soon and becomes positive in two years. GDP and lending are closely related in my simulation.

Finally, Figure 2.4(d) reports the transition of consumption. The consumption in the data and the simulation show similar movement. In the data, consumption started to decline three quarters before default and hit the bottom half a year after default. Then, it gradually recovered and regained a positive value about two years after default. In my simulation, consumption is composed of output, profits from goods producing firms and financial intermediaries and government transfers, so I can separately derive their contributions to consumption. The biggest contributor to consumption is output, and it determines most of the fluctuation in consumption. However, transfers also affect its movement, especially before the default, and pushing down consumption to almost zero percent. This is because as the amount of debt becomes higher, foreign investors take into account the possibility of default and the price of government bonds declines. Thus, the amount of transfers contributes to consumption negatively before the default.

2.5.2 Alternative Scenarios

In this section, I examine two alternative scenarios: the financial intermediaries’ burden of non-performing bonds and maximum amount of issuance of government bonds.

\[26\] The reason why consumption jumped up and lending dropped two years after the default in the actual Argentinean economy is high inflation. Because of the increase in demand for natural gas, the GDP deflator increased 22.7% in 2004Q1 from the previous quarter. Although I subtract the effect of inflation from nominal consumption and lending, it is hard to obtain the true real values.
I report changing haircut rates \((h)\) and the proportion of government bonds held domestically \((v)\) as the first alternative scenario because these variables are related to the non-performing bonds which are the main cause of the financial amplification mechanism. The second half is the case that the government can issue more bonds than the baseline case. This feature creates the phenomenon as "Too-Big-to-Default", meaning the government cannot choose default by itself when it accumulates too much debt.

**Non-Performing Bonds**

Table 2.4: Simulation Results: Alternative Scenarios

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Haircut Rate (h = 0.48)</th>
<th>Haircut Rate (h = 0.72)</th>
<th>Domestic Bond Holding (v = 0.45)</th>
<th>Domestic Bond Holding (v = 0.55)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Debt/GDP</td>
<td>33.13%</td>
<td>29.32%</td>
<td>34.85%</td>
<td>28.83%</td>
<td>34.87%</td>
</tr>
<tr>
<td>Default Frequency</td>
<td>0.71%</td>
<td>1.29%</td>
<td>0.32%</td>
<td>1.05%</td>
<td>0.27%</td>
</tr>
<tr>
<td>Average Bond Spread</td>
<td>0.19%</td>
<td>0.28%</td>
<td>0.06%</td>
<td>0.35%</td>
<td>0.05%</td>
</tr>
<tr>
<td>The correlation with Output</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Consumption</td>
<td>0.990</td>
<td>0.994</td>
<td>0.989</td>
<td>0.994</td>
<td>0.991</td>
</tr>
<tr>
<td>- Lending</td>
<td>0.988</td>
<td>0993</td>
<td>0.983</td>
<td>0.994</td>
<td>0.987</td>
</tr>
<tr>
<td>- Interest rate on lending</td>
<td>-0.932</td>
<td>-0.954</td>
<td>-0.906</td>
<td>-0.957</td>
<td>-0.915</td>
</tr>
<tr>
<td>The Default Effect on Variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Output</td>
<td>-16.15%</td>
<td>-14.55%</td>
<td>-18.94%</td>
<td>-14.33%</td>
<td>-18.96%</td>
</tr>
<tr>
<td>- Consumption</td>
<td>-16.34%</td>
<td>-14.76%</td>
<td>-19.59%</td>
<td>-14.47%</td>
<td>-19.55%</td>
</tr>
<tr>
<td>- Lending</td>
<td>-26.44%</td>
<td>-23.79%</td>
<td>-30.92%</td>
<td>-23.40%</td>
<td>-30.90%</td>
</tr>
<tr>
<td>- Interest rate on lending</td>
<td>9.76%</td>
<td>9.29%</td>
<td>10.97%</td>
<td>9.19%</td>
<td>10.71%</td>
</tr>
</tbody>
</table>

Note: The haircut rate and financial intermediaries’ bonds holding in the baseline case are 0.6 and 0.4 respectively.

Table 2.4 shows the simulation results under different values for the haircut rate and the proportion of government bonds held by domestic financial intermediaries. First of all, when the haircut rate is 20% smaller than the baseline case of 60%, declines in output, consumption and lending are about 2.7%, 2.6% and 3.5% smaller than the baseline because financial intermediaries are burdened with less non-performing bonds. This weak effect of default on the economy leads to a higher incentive for the government to choose default, so the frequency of default changes from 0.67% to
1.29%. Also, the average debt-to-GDP ratio declines from 33.2% to 29.3% since the
government will choose default with a relatively lower level of debt, and the average
bond spread increases from 0.16% to 0.28% since the risk of default appears more
frequent as the government chooses default more. The case of the higher haircut rate
shows opposite effects on the economy. The domestic financial intermediaries need to
eliminate more non-performing bonds, so the damage on the economy is more severe
than in the baseline case. Thus, the government is less willing to choose default, and
that leads to higher average debt-to-GDP and lower bond spread.

Next, the change in the proportion of financial intermediaries’ government bond
holdings influences the economy similar to the change in the haircut rate. In the case
of lower domestic bond holdings, financial intermediaries are burdened with a smaller
amount of non-performing bonds. Thus, the damage on the economy is milder and the
government will choose default more. Also, the average debt-to-GDP ratio decreases
and bond spread increases.

Table 2.5: Simulation Results: Various Scenarios

<table>
<thead>
<tr>
<th>( v )</th>
<th>0.36</th>
<th>0.48</th>
<th>0.6</th>
<th>0.72</th>
<th>0.84</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>(1.51, -9.72)</td>
<td>(1.07, -11.27)</td>
<td>(0.87, -12.24)</td>
<td>(0.85, -12.71)</td>
<td>(0.80, -13.26)</td>
</tr>
<tr>
<td>0.40</td>
<td>(1.57, -11.00)</td>
<td>(1.28, -11.96)</td>
<td>(1.19, -12.97)</td>
<td>(0.98, -14.05)</td>
<td>(0.60, -14.66)</td>
</tr>
<tr>
<td>0.45</td>
<td>(1.70, -11.53)</td>
<td>(1.43, -12.88)</td>
<td>(1.05, -14.33)</td>
<td>(0.74, -15.99)</td>
<td>(0.49, -17.67)</td>
</tr>
<tr>
<td>0.5</td>
<td>(1.57, -12.64)</td>
<td>(1.29, -14.55)</td>
<td>(0.67, -17.20)</td>
<td>(0.32, -18.94)</td>
<td>(nan, nan)</td>
</tr>
<tr>
<td>0.55</td>
<td>(1.25, -13.93)</td>
<td>(0.85, -16.52)</td>
<td>(0.27, -18.96)</td>
<td>(nan, nan)</td>
<td>(nan, nan)</td>
</tr>
<tr>
<td>0.60</td>
<td>(0.85, -15.32)</td>
<td>(0.27, -18.46)</td>
<td>(nan, nan)</td>
<td>(nan, nan)</td>
<td>(nan, nan)</td>
</tr>
<tr>
<td>0.65</td>
<td>(0.37, -16.90)</td>
<td>(nan, nan)</td>
<td>(nan, nan)</td>
<td>(nan, nan)</td>
<td>(nan, nan)</td>
</tr>
</tbody>
</table>

Note 1: The first and second elements in each set of parentheses represent the default frequency
and the decline in output from the trend respectively.

Note 2: "nan" represents no occurrence of default.

Table 2.5 summarizes the various combinations of both parameters, the proportion
of financial intermediaries’ government bond holdings \( v \) and the haircut rate \( h \), re-
porting on the frequency of default (the first element) and output decline (the second
element). The frequency of default is monotonic in inverse relation with the hair-
cut rate, but its relation with the proportion of domestic government holdings is not monotonic. The reason for the former monotonic relation is quite straightforward because the financial intermediaries have to eliminate more non-performing bonds as the haircut rate increases. However, as equation (2.22) shows, the lower proportion of government bond holdings by domestic agents means a higher outflow of goods by issuance of government bonds. Thus, the government becomes more conservative to accumulating debts, so these two effects lead to a nonlinear relation between the frequency of default and the proportion of domestic government bond holdings. If both values are high, the government does not choose default at all because the damage on the economy is too strong to choose default. Next, the default effect on output sees a monotonic decrease as the values of these two parameters increase. This result is consistent with our intuition because as the amount of non-performing bonds that financial intermediaries have to eliminate increases, the contractions of lending and output are larger due to the government’s decision to default.

**Too-Big-to-Default**

Next, I examine the case that the government issues its bonds twice as high as the baseline case, 0.7. Figure 2.5 (a) shows the default and non-default value functions under the condition of 10% lower than neutral TFP. The non-default value is higher than the default value in the case of a low debt amount (from -0.38 to 0) and the default value exceeds the non-default value in the middle level of issuance of debt (from -1.27 to 0.38). However, the non-default value exceeds the default value again around the maximum debt region (from -1.4 to -1.27). This is because while the default value function is quadratically decreasing, the non-default value is not a monotonously decreasing function. The former effect comes from the characteristics of the lending cost function (2.36). The amount of non-performing bonds that the government has to
Figure 2.5: High Maximum Debt Amount
(a) Value Functions (-10% TFP)  (b) Default Set

(c) Price of Government Bond
eliminate increases quadratically as the government accumulates debts, so the damage on the economy from the default is larger if the government issues more bonds. The latter effect comes from the increase in the price of government bonds. When the default value function decreases and the value becomes closer to the non-default value function, or the default value falls below the non-default value, the government is less likely to choose default and the price of government bonds increases. Thus, the government can transfer more to households, and the decrease in households’ utility is mitigated.

Figure 2.5 (b) and 2.5 (c) show this phenomenon. The non-default area expands and the price of government bonds increases around the high debt amount. This phenomenon can be expressed as "Too-Big-to-Default". If the government accumulates massive debts, it cannot choose default by itself because of the huge effect on the economy. This would be a part of the reason why defaults do not happen frequently in developed countries, and this result implies that the endogenous default model might be inappropriate to use to analyze a country where a government accumulates massive debts.

2.6 Conclusion

This chapter proposes a dynamic general equilibrium model of endogenous sovereign default incorporating financial intermediaries. The government selects default or repayment by comparing values of default and non-default. If the government chooses default, its bonds become non-performing and the net worth of financial intermediaries is reduced by eliminating these non-performing bonds. This leads to contraction in lending and output. I calibrate the Argentinean economy around its default in 2001 in order to explore the mechanism of financial amplification by sovereign default.

My main contributions can be summarized two points. First, while other papers
simplify the government value of default depending only on TFP or endowments, the
default value in my model is contingent outstanding government debts in the non-
default state. Thus, as the government issues more bonds, financial intermediaries
have to eliminate more non-performing bonds from their net worth in the default state.
This feature allows us to examine the financial amplification effect on the economy by
changing some important parameters such as the haircut rate and the proportion of
government bonds held by domestic agents. Besides, the model explains the phenom-
enon of "Too-Big-to-Default". It shows that the government will not be able to choose
default by itself when it accumulates too much debt. Second, the compatibility of my
model to the default in Argentina in 2001 is relatively sound, capturing moments of
important variables and frequency of default.
Chapter 3

A General Equilibrium Model of Sovereign Debt Crisis in an Advanced Country

3.1 Introduction

The recent European sovereign debt crisis awakened the idea that the occurrence of debt crises was not restricted to developing countries. After the revelation of the Greek government’s manipulation of its financial deficit in 2009, investors became skeptical toward the fiscal sustainability of European countries, especially Portugal, Italy, Ireland, Greece and Spain. The credit default swap (CDS) rate in these countries increased significantly around the time that the Greek government virtually defaulted in 2012. Although the IMF, ECB and European Commission (the so-called Troika) and the European Stability Mechanism (ESM) attempted to stabilize the crisis, mainly by injecting bailouts to governments and banking sectors in these countries, the crisis caused serious contractions, decreasing real GDP by about 26.5% in Greece, 8.9% in Spain,
8.7% in Italy, 7.9% in Ireland and 7.9% in Portugal\(^1\). The huge impact of sovereign debt crisis on the economy became apparent.

According to a recent report in the IMF Fiscal Monitor, average debt in advanced countries is projected to remain high, shifting to over 70% of the GDP ratio in the near future. The main cause of this high debt projection is that governments which have generous social welfare systems cannot reduce their social security expenditure easily due to the aging society. Although debt crises are triggered not only by the debt-to-GDP ratio but also by many other factors, the ratio is one of the indisputably significant causes of crises. As long as the debt level is expected to be high, it is important to have tools to analyze sovereign debt crises and quantify their effects on the economy.

When a government faces a sovereign debt problem, it had been usually chosen default or restructuring as a method to reduce the burden of huge debts. The most authoritative model to analyze sovereign debt default is the endogenous sovereign default which was introduced by Eaton and Gersovitz (1981). The model assumes a government’s willingness rather than the ability to pay debt. In other words, if the value of default is larger than that of non-default, the government chooses to default. However, it is hardly considered that this mechanism can be applicable to advanced economies or at least to the debt crisis in Europe for the following reasons. First of all, the European debt crisis case does not satisfy the two presuppositions of the endogenous default model: that the government can estimate the effect of the default precisely and that its effect on the economy has to be small and short. Otherwise, the government cannot compare values of default and non-default and would not choose default in any economic situation. These presuppositions might be regarded as legitimate assumptions for most defaults which have happened in developing countries because, judging from

\(^1\)These values are the percentage decline of annual GDP from the maximum year before the financial crisis in 2008 to the minimum year after the crisis.
data on recent default episodes\textsuperscript{2}, only six out of forty-one default episodes experienced more than 5\% of GDP decline and three countries recorded more than 10\% decline. Besides, as Borensztein and Panizza (2008) report, most of the default effects on the economy are short. Thus, if situations or economic characteristics are similar to these majority prior defaults, the government would estimate the effect of default relatively well. However, these two presuppositions are not applicable to at least the Greek case. First of all, its economic contraction was the worst among recent sovereign defaults in the world, reducing 26.5\% of GDP, and its effect is still continuing five years after the default in 2012. It is hardly assumed that the government estimated this catastrophic effect accurately before the default and chose default by itself. Not only the government, but also IMF, failed to estimate its effect well. IMF’s Independent Evaluation Office (IEO) concluded that "The IMF’s pre-crisis surveillance mostly identified the right issues but did not foresee the magnitude of the risks that would later become paramount".

Secondly, as Bi (2012) mentions, the endogenous default model cannot assume high amounts of debts\textsuperscript{3}. The first reason for this is that it is necessary to assume quite drastic and sticky TFP or endowment. The government issues bonds if a low level TFP or endowment is realized since its purpose in issuing government bonds is to smooth inter-temporal households' utility. Thus, it is difficult to accumulate a massive amount of debts like many developed countries under the conventional parameter settings of TFP or endowment unless the model assumes that the extremely low TFP or endowment continues for a long time. The second reason is that the model has to assume the government’s reluctance to choose to default. If the government chooses default with a

\textsuperscript{2}Most data is taken from 1983 to 2006 and Greece in 2012. GDP is on a purchasing-power-parity basis. The economic contraction is measured as the GDP drop from the maximum year between three years and one year before default to the minimum year between the time of default and three years after the default. The three countries which have experienced more than 10\% GDP decline are Indonesia in 1998, Argentina in 2001 and Greece in 2012.

\textsuperscript{3}For example, Arellano (2008) and Mendoza and Yue (2012) replicate the mean debt-to-GDP ratio to 5.95\% and 23\% respectively, which is much smaller than the actual Argentinean debt-to-GDP level of about 37\%. 

50
small amount of debt, the average debt-to-GDP ratio will also be low even though the model assumes the drastic and sticky TFP or endowment. Thus, the government needs to accumulate its debts instead of selecting default, and price makers of government bonds trust that the government will not choose default with a relatively small amount of debts. However, this point contradicts the first reason that the effect of default should be small and short because the government is reluctant to choose default but the default effects on the economy have to be small enough to choose default. Therefore, it is hard to coincide with high debt-to-GDP ratio and an appropriate level of default frequency. Of course, as I have mentioned, debt crises occur not only from high debt amounts but for many other reasons. No one denies that high debt is one of the main causes of crises although the criterion of high debt itself varies among countries. Hence, we should not underestimate the problem of the endogenous default model, that it is incapable of replication of the high debt amount.

In addition to these reasons, another problem of the endogenous default model is that it is not useful for policy implications. The function of government is only the issuance of bonds as I have explained, so the model excludes tax revenue and government spending\textsuperscript{4}. Therefore, I cannot address the issue to study any fiscal policies in the first place.

Finally, European countries except for Greece avoided default by all means\textsuperscript{5}. These governments implemented mixed policies to mitigate the debt crisis through austerity and financial management (financial stability, smooth redemption of debt and lowering the interest rate). For example, the Spanish government implemented an austerity plan involving about 65 billion euro, including an increase in the value added tax (VAT) and decrease of its transfer in 2012. Also, the ESM approved 100 billion euro of rescue

\textsuperscript{4}There are several exceptions. For example, Arellano and Bai (2016) assume that a government faces aggregate constraint and fiscal constraint.

\textsuperscript{5}The reasons why most eurozone governments avoided defaults were not only the strong effects of default on their countries’ economies but also the prevention of breakdown of the eurozone economic system.
loans, and the ECB announced that it would purchase short-term debt unlimitedly.

Therefore, considering these factors, the endogenous sovereign default model is not regarded as an appropriate tool for the analysis of the European debt crisis. Instead I propose a new sovereign debt crisis model which is applicable to an advanced country. The overall framework of the model is as follows. The government issues new bonds to households in order to finance the deficiency of its budget. As the amount of government debts increases, it increases the tax rate. However, if government debts exceed a fiscal limit, the government falls into crisis. Following Bi (2012), I define the fiscal limit as the summation of discounted future primary surplus when the government sets its tax rate on output to maximize its tax revenue. The price of government bonds depends on the probability that the debt amount exceeds the fiscal limit in the next period. In order to prevent drastic movement in prices, the government guarantees the exogenous amount of repayment reduced by the cutback rate at the time of the crisis. Then, it rolls over the bonds until it recovers to its normal state. Also, the government is obligated to set the exogenous tax rate during the crisis as an austerity measure. In addition to this, following Mendoza and Yue (2012), I assume that the amount of imports of intermediate goods decreases due to the higher interest rate of working capital in the crisis state.

Next, it is important to point out the distinction between domestic and foreign government bonds. According to Gros (2013), while domestic government bond holders put pressure on the government to serve debts, the government does not have a high incentive to repay debt by increasing tax or reducing expenditure in the case of foreign debt. Usually, a high risk premium would be an incentive to serve debts even in the case of foreign debts, but monetary union loosened the discipline of fiscal management in the Euro countries. In addition to this, he also points out that the higher debt repayment burden leads to welfare loss in the case of foreign debts, while domestic debts are internal redistributed. In the European case, more than half of total government
bonds are held by domestic agents in Italy and Spain, but foreigners hold the majority of Greek and Irish government bonds as Figure 3.1 shows. According to Reinhart and Rogoff (2011), the proportion of total central government debt held by domestic agents has been quite high in developed countries. Thus, I assume domestic government bonds in this chapter and calibrate the Spanish economy. However, I also provide a model for the foreign government bond case and analyze the Greek crisis in the next chapter.

Figure 3.1: Proportion of government bond held by domestic agents

Source: The central bank in each country.

The results of my analysis can be summarized in three points. First, fundamental economic conditions provide the mechanism for a debt crisis in an advanced country from the fiscal limit and the issuance of government bonds. In the case of recession or expansionary government policy, the fiscal limit is lower and the government needs to issue more debt, and that leads to higher risk of debt crisis. Second, the compatibility of the simulation of the data to sovereign debt crisis in Spain is generally sound, matching the average debt-to-GDP ratio, the frequency of crisis and moments of main variables. The average debt-to-GDP ratio is high enough for an advanced country, and the high debt is compatible with the frequency of crisis. The third point is that unlike endogenous default models, my model is flexible enough to quantify many varieties
of government policies. For instance, the 5% reduction of government consumption reduces the average debt-to-GDP from 47% to 9% and crisis frequency from 0.75% to 0.10% but sacrifices the level of GDP by 0.8% in the normal state.

–Related Literature–


In terms of the fiscal capability, Ghosh et al. (2012) analyze the debt limit that the government cannot roll over. Bi (2012) models a partial default when the government debt outstanding exceeds a fiscal limit derived from the Laffer curve. However,
their research includes only the risk of default and does not assume the default state explicitly. Leeper and Walker (2011), Bi and Leeper (2012) also study the fiscal limit in advanced countries from the Laffer curve. Bocola (2016) studies the pass-through of sovereign risk incorporating financial intermediaries, assuming the exogenous probability of sovereign default. Davig et al. (2010, 2011) also assume the fiscal limit from the exogenous logistic distribution function.

This chapter is organized as follows: Section 3.2 presents the model to analyze the sovereign debt crisis. Section 3.3 sets functional forms and calibrates parameters. Section 3.4 delivers the quantitative results; consisting of fiscal limits, decision rules and simulations. Section 3.5 concludes.

### 3.2 Model

The framework of the model is a dynamic stochastic general equilibrium of the small open economy, consisting of four domestic agents: households, final goods firms, intermediate goods firms and a government, and one foreign agent. Households and both types of firms are a continuum of unit measure. The government imposes taxes for output production, pays transfers to households, consumes final goods and issues new bonds in order to finance the deficiency in its budget. The tax rate is determined by a rule depending on the amount of debt. The government falls into the crisis if its debts exceed a fiscal limit, which is defined as the discounted summation of future maximum primary surplus. Then, the government repays its debts to households reduced by an exogenous cutback rate. International financial organizations intervene to balance the government’s budget constraints in the state of crisis. In this state, the amount of importation of intermediate goods declines from the amount in the normal state because of the high interest rate for working capital. As long as the crisis state continues, the
same amount of debts is carried over to the next period. Finally, the government has a chance to go back to the normal state with exogenous stochastic probability after a certain mandatory crisis duration. The first graph of Appendix C.1 depicts the overall picture of the economy in the normal state.

The economic state can be categorized into three situations. The first situation is that the government is in the normal state at the beginning of period $t$ and does not fall into the crisis during the period $t$. The second case is the government is in the normal state at the beginning of period $t$ and falls into the crisis during the period $t$. The last state is that the crisis state continues from the previous period. For simplicity, $\xi_t$ denotes the state of the economy at the beginning of period $t$, taking $\xi_{n,t}$ in the normal state and $\xi_{c,t}$ in the crisis state. Similarly, $\chi_t$ represents whether the government falls into the crisis or not during the period $t$, taking $\chi_{n,t}$ and $\chi_{c,t}$ as normal and crisis states respectively. Thus, the first economic situation can be expressed as $(\xi, \chi) = (\xi_n, \chi_n)$, and so as $(\xi, \chi) = (\xi_n, \chi_c)$ and $\xi = \xi_c$ in the second and third situations respectively.

### 3.2.1 Government

In the normal state, the government’s expenditure consists of the reimbursement of government bonds issued in the previous period $B_{t-1}$, government consumption $G_t$ and transfers to households $Z_t$, and its revenue consists of tax revenue $T_t$ and newly issued government bond $B_t$ priced by households as $q_t$. The government issues its bonds only for domestic households to finance the deficiency in its budget. Characteristics of government bonds are one period maturity, zero coupon and non-contingent. In the crisis state, the government issues bonds not to balance its budget but to roll over the bonds.

First of all, I assume that government consumption $G_t$ is determined exogenously. Following Gali et al (2007), government consumption measured as the deviation from
the steady state criteria $G$ standardized by the steady state GDP\textsuperscript{6} is

$$g_t \equiv (G_t - G)/GDP$$

(3.1)

$g_t$ follows the AR(1) process as:

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$$

(3.2)

where $\varepsilon_{g,t} \sim N(0, \sigma^2_g)$. Next, following Bi (2012), the government transfer depends on TFP as:

$$Z_t = Z + \zeta^z(e^A_t - e^A)$$

(3.3)

where $Z$ and $A$ are the criteria of transfer and TFP respectively and the parameter $\zeta^z(\leq 0)$ is the elasticity of TFP to transfer. The parameter $\zeta^z$ takes a non-positive value because the government transfer is considered to be countercyclical toward the business cycle as a function of social insurance for households. TFP also follows the AR(1) process as:

$$A_t = \rho_A A_{t-1} + \varepsilon_{A,t}$$

(3.4)

where $\varepsilon_A \sim N(0, \sigma^2_A)$. The government imposes tax only on final goods $Y_t$, so the total tax revenue is defined as:

$$T_t = \tau_t Y_t$$

(3.5)

where $\tau_t$ is the tax rate on output.

If the government does not fall into crisis (i.e. $\chi_t = \chi_{n,t}$), following Bi (2012), the government sets the simple tax rule in the normal state as:

$$\tau_t = \tau + \kappa(B_{t-1} - B)$$

(3.6)

\textsuperscript{6}In my model, firms import intermediate goods from foreign countries, so GDP is defined as output of final goods minus the amount of imports.
where $\tau$ and $B$ are criteria of the tax rate and government bonds outstanding respectively and $\kappa(\geq 0)$ is the elasticity parameter of government bonds to the tax rate. The tax rate depends only on previous issuance of government bonds $B_{t-1}$, and the government increases its tax rate as the amount of bonds increases in order to reduce its high level of debts. Then, the government repays the full amount of debt issued in the previous period $B_{t-1}$ to households and issues new bonds $B_t$. Thus, the amount of issuance of government bonds $B_t$ is:

$$ q_t B_t = B_{t-1} + G_t + Z_t - T_t $$

(3.7)

However, if the government falls into the crisis (i.e. $\chi_t = \chi_{cr}$), it repays the amount of remitted bonds $(1 - \delta)B_{t-1}$ where $\delta$ is the exogenous cutback rate, and issues the same amount of bonds.

$$ B = (1 - \delta)B_{t-1} $$

(3.8)

I assume the price of government bonds is unity in the state of crisis$^8$, so government bonds do not affect the budget constraint. Thus, the budget constraint in the case of $(\xi_t, \chi_t) = (\xi_{n,t}, \chi_{c,t})$ is $T_t + \Phi^I_{t} = G_t + Z_t$ where $\Phi^I_t$ is the international financial organizations’ intervention to satisfy the budget constraint. I assume that government consumption $G_t$ and transfers $Z_t$ in the crisis state also follow the same process as in the normal state. However, the organizations obligate the government to set an exogenous fixed tax rate $\tau^d$ as an austerity policy. Thus, the tax rate in the crisis state is

$$ \tau_t = \tau^d $$

(3.9)

$^7$There is a possibility that the government cannot prepare for this amount of repayment in the crisis, so I assume international financial organizations guarantee the exogenous amount of repayment at the time of crisis.

$^8$The economic background of this assumption is that financial international organizations provide perfect warrant of the repayment of debts, so the price of government bonds is indifferent to the domestic currency.
The government’s revenue $T_t$ and expenditure $G_t + Z_t$ are determined independently in the crisis state and the government cannot issue its bonds to satisfy the budget constraint, so the international financial organizations need to satisfy the budget constraint by providing the bailout $\Phi^{I/O}_t$. As long as the crisis state continues, government bonds are rolled over to the next period.

When the government regains the normal state from the crisis at the end of period $t - 1$, the debt amount that the government has to repay is set to the exogenously given amount $B^r$. Normally, $B^r$ should be equal to the rolling-over value, but if the cutback rate is low, the government starts from a high debt amount when it regains the non-default state. Thus, I set a different debt value when the government regains the non-default state. I assume the international financial organizations pay the difference between the rolling-over value and $B^r$.  

3.2.2 Households

Households are infinitely lived and derive utility from consumption $C_t$ and disutility from labor measured by hours worked $L_t$. They provide labor force to both final and intermediate goods firms, and receive wages $w_t L_t$ and profits $\Upsilon_t$ from them. The government transfers final goods $Z_t$ and transacts its bonds with households. Finally, they consume final goods.

The households’ utility maximization problem is

$$\max_{C_t, L_t, B^H_t} E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right]$$

(3.10)

In the case $(\xi_t, \chi_t) = (\xi_{n,t}, \chi_{n,t})$,

$$s.t. \quad C_t + qtB^H_t(\xi_{n,t}, \chi_{n,t}) \leq w_t L_t + B_{t-1} + \Upsilon_t + Z_t$$

(3.11)

---

9 The bailout can be negative if the government’s revenue is higher than expenditure.

10 I examine the case that $B^r$ is equal to the rolling-over value in the alternative scenario section.
In the case \((\xi_t, \chi_t) = (\xi_{n,t}, \chi_{c,t})\),

\[
\text{s.t.} \quad C_t + B_t^H(\xi_{n,t}, \chi_{c,t}) \leq w_t L_t + (1 - \delta) B_{t-1} + \Upsilon_t + Z_t \tag{3.12}
\]

where \(\beta \in (0, 1)\) is the discount factor of household, \(B_t^H\) is the households’ demand for government bonds and \(w_t\) is the wage rate. The utility function \(u : \mathbb{R}_+^2 \to \mathbb{R}\) is continuous, twice differential and satisfies \(\frac{\partial u}{\partial C} > 0\), \(\frac{\partial^2 u}{\partial C^2} < 0\), \(\frac{\partial u}{\partial L} < 0\), \(\frac{\partial^2 u}{\partial L^2} < 0\) and \(\frac{\partial^2 u}{\partial C^2} \frac{\partial^2 u}{\partial L^2} - (\frac{\partial u}{\partial C} \frac{\partial u}{\partial L})^2 > 0\).

The first-order conditions of the households’ optimization problem are

\[
w_t = \frac{u_L(C_t, L_t)}{u_C(C_t, L_t)} \tag{3.13}
\]

\[
q_t = \beta \mathbb{E}_t u_C(C_{t+1}, L_{t+1}) \frac{1}{u_C(C_t, L_t)} \left\{ (1 - P_{c,t+1}) + P_{c,t+1}(1 - \delta) \right\} \tag{3.14}
\]

where \(\mathbb{E}_t[P_{c,t+1}]\) is the probability that the government falls into the crisis in the next period. These two equations hold in the case of \((\xi_t, \chi_t) = (\xi_{n,t}, \chi_{n,t})\), but the only first equation holds in the case of \((\xi_t, \chi_t) = (\xi_{n,t}, \chi_{c,t})\) because the government falls into the crisis and the price of government bonds is exogenously determined as unity.

When the government is in the default state at the beginning of period \(t\) (i.e. \(\xi_t = \xi_{c,t}\)), the constraint is

\[
C_t \leq w_t L_t + \Upsilon_t + Z_t \tag{3.15}
\]

Because the newly issued and repaid government bonds are of equal value, these bonds do not affect the households’ constraint. Thus, the equation \((3.13)\) is also the only first-order condition of the households’ optimization problem in this state.
3.2.3 Final Goods Firms

Final goods firms purchase intermediate goods $M_t$ from both domestic intermediate goods firms and foreign agents and acquire labor force $L_t^f$ from households. They borrow working capital $\kappa_t$ from foreign agents for the importation of a certain fraction of intermediate goods. Then, final goods firms produce final goods from the intermediate goods and labor force and sell them to households and the government.

The technology of production follows the Cobb-Douglas production function:

$$Y_t = e^{A_t} M_t^{\alpha_M} (L_t^f)^{\alpha_L}$$ (3.16)

where $M_t^{\alpha_M}$ is intermediate inputs which are composed of domestic intermediate inputs $m_t^d$ and imported intermediate inputs $m_t^s$, and parameters $0 < \alpha_M, \alpha_L < 1$ are the shares of intermediate goods and labor inputs respectively, satisfying $\alpha_M + \alpha_L < 1$. I abstract capital input for simplicity.\(^{11}\)

The technology of the combination of domestic intermediate goods and imported intermediate goods follows the CES aggregator:

$$M_t = \left[ \lambda \left( m_t^d \right)^{\frac{\psi - 1}{\psi}} + (1 - \lambda) \left( m_t^s \right)^{\frac{\psi - 1}{\psi}} \right]^{\frac{\psi}{\psi - 1}}$$ (3.17)

where $\lambda \in [0, 1]$ is the share of domestic intermediate goods in total intermediate goods composition and $\psi(> 0)$ is the elasticity of substitution across domestic and imported intermediate goods. These intermediate goods are defined as the summation of the continuum of differentiated domestic and imported intermediate goods $m_{j,t}^d, m_{j,t}^s$.

---

\(^{11}\)Refer to the previous chapter for the justification of abstraction of the capital input. I introduce capital in the next chapter.
respectively by using the following Dixit-Stiglitz aggregator:

\[ m_i^d = \left[ \int_0^1 (m_{j,t}^d d) \right]^{\frac{\nu-1}{\nu}} \quad \text{and} \quad m_i^* = \left[ \int_0^\theta (m_{j,t}^* d) + \int_0^\theta (m_{j,t}^* d) \right]^{\frac{\nu-1}{\nu}} \]

(3.18)

where \( \nu(>1) \) is the substitution elasticity between differentiated domestic and imported intermediate goods, setting the same value on this substitution elasticity of these two intermediate goods. Final goods firms have to borrow working capital with its interest rate \( r_t^* \) in advance in order to import a certain fraction \( \theta \in [0,1] \) of intermediate goods from foreign agents. Thus, the working capital constraint for these imported intermediate goods is

\[ \frac{\kappa_t}{1 + r_t^*} \geq \int_0^\theta p_{j,t}^* m_{j,t}^* dj \]

(3.19)

where \( p_{j,t}^* \) is the price of imported intermediate input \( m_{j,t}^* \). The firms do not borrow working capital exceeding the minimum requirement, so the constraint is held with equality.

I adopt the two-stage budgeting method to solve the optimization problem of final goods producers: the profit maximization problem in the first stage and the cost minimization problem of imported intermediate goods in the next stage. First of all, the firms maximize the following problem in the first stage:

\[ \max_{m_t^*, m_t^d, L_t^f} \pi_t^f = Y_t(1 - \tau_t) - p_t^* m_t^* - p_t^m m_t^d - w_t L_t^f \]

(3.20)

subject to the production function (3.16) and the combining technology of domestic and imported intermediate goods (3.17). The choice variables are inputs of production \( m_t^*, m_t^d \) and \( L_t^f \). The government imposes tax on outputs with the rate \( \tau_t \), so after tax sales of final output are \( Y_t(1 - \tau_t) \). The first-order conditions with respect to each
control variable are

$$e^{At} \alpha_M M_t^{\alpha_M - \frac{\alpha_M}{\alpha_L}} (1 - \lambda)(m_t^*)^\frac{1}{\alpha_L} \left(L_t^f\right)^{\alpha_L} (1 - \tau_t) = p_t^*$$

(3.21)

$$e^{At} \alpha_M M_t^{\alpha_M - \frac{\alpha_M}{\alpha_L}} \lambda (m_t^d)^\frac{1}{\alpha_L} \left(L_t^f\right)^{\alpha_L} (1 - \tau_t) = p_t^m$$

(3.22)

$$e^{At} \alpha_L M_t^{\alpha_M} \left(L_t^f\right)^{\alpha_L-1} (1 - \tau_t) = w_t$$

(3.23)

The next stage is the cost minimization problem with respect to imported intermediate goods. By substituting the working capital constraint (3.19) into the cost function, the problem is as follows:

$$\min_{m_{j,t}} \int_\theta^1 p_{j,t}^* m_{j,t}^* dj + (1 + r_t^*) \int_0^\theta p_{j,t}^* m_{j,t}^* dj$$

(3.24)

subject to the Dixit-Stiglitz aggregator of imported intermediate goods (3.18). The solution to the problem is derived as:

$$m_{j,t}^* = \left(\frac{p_{j,t}^*}{p_t^*}\right)^{-v} m_t^* \text{, for } j \in [\theta, 1]$$

(3.25)

$$m_{j,t}^* = \left(\frac{(1 + r_t^*)p_{j,t}^*}{p_t^*}\right)^{-v} m_t^* \text{, for } j \in [0, \theta]$$

(3.26)

$$p_t^* = \left(\int_\theta^1 (p_{j,t}^*)^{-v} dj + \int_0^\theta ((1 + r_t^*)p_{j,t}^*)^{-v} dj\right)^\frac{1}{-v}$$

(3.27)

where $p_t^*$ is the shadow price of combining imported intermediate goods with the minimum cost, which can be interpreted as the aggregate price of imported intermediate goods. Following Mendoza and Yue (2012), imported intermediate goods are traded in
the world market at the price of numeraire. Thus, their price is simplified as:

\[ p_t^* = [(1 - \theta) + \theta(1 + r_t^*)^{1-\nu}]^{\frac{1}{1-\nu}} \] (3.28)

In the normal state, the interest rate on working capital \( r_t^* \) is same as the world risk-free interest rate \( r^f \) because final goods firms borrow it through the world market and I abstract the possibility of firms’ default. However, interest rates for working capital will surge in the crisis because of the increase in the country’s specific risk premium. The interest rate in the crisis state is assumed to be the return on government bonds with the same marginal utility with respect to current and next period consumption under the condition of definite occurrence of crisis\(^{12}\). Therefore, the interest rate on working capital is

\[ r_t^* = \begin{cases} r^f & \text{if } (\xi, \chi) = (\xi_n, \chi_n) \\ 1/(\beta \cdot (1 - \delta)) - 1 & \text{otherwise} \end{cases} \] (3.29)

Finally, GDP can be defined as outputs minus the amount of imports as:

\[ GDP_t = Y_t - p_t^* m_t^* \] (3.30)

\[ 3.2.4 \text{ Intermediate Goods Firms} \]

Intermediate goods firms produce intermediate goods from labor \( L_t^m \) and sell them to final goods firms. The technology of intermediate goods is

\[ m_t^d = A^I (L_t^m)^\gamma \] (3.31)

\(^{12}\)Mendoza and Yue (2012) assume that firms in the default state cannot import intermediate goods which are required as working capital for \( j \in [0, \theta] \) because the interest rate on working capital is infinite due to exclusion from world credit markets. Unlike in their model, I do not assume the government debt crisis involves full default, so it is legitimate not to assume financial autarky of the economy during the crisis state.
where $A^I$ is an invariant TFP coefficient and $\gamma \in [0,1]$ is the share of labor inputs for intermediate goods production. Then, the profit maximization problem of intermediate goods firms is

$$\max_{L^m_t} \pi_t^m = p_t^m A^I (L^m_t)\gamma - w_t L^m_t$$

(3.32)

Then, the first-order condition with respect to $L^m_t$ is

$$w_t = \gamma p_t^m A^I (L^m_t)^{\gamma-1}$$

(3.33)

### 3.2.5 Crisis Scheme

In this section, I restrict the government state to the normal state $\xi_{n,t}$ at the beginning of period $t$ because the transition in economic state only matters in crisis scheme. The state vector is summarized as $S^n_t \in \{A_t, g_t, B_{t-1}, \xi_{n,t}\}$.

If the expected issuance of government bond $B(S^n_t)$ is lower than or equal to the fiscal limit $B^{\text{max}}(A_t, g_t)$, which I will define in the section 3.2.7, the government stays in the normal state during the period $t$. However, if the newly issued government bond is higher than the fiscal limit, the government falls into the crisis state. Thus, the transition of government state $\chi(S^n_t, B^{\text{max}}_t)$ can be defined as:

$$\chi(S^n_t, B^{\text{max}}_t) = \begin{cases} 
\chi_{n,t} & \text{if } B(S^n_t) \leq B^{\text{max}}(A_t, g_t) \\
\chi_{c,t} & \text{otherwise}
\end{cases}$$

(3.34)

Next, in order to obtain the price of government bonds (equation (3.14)), it is necessary to derive the probability of crisis in the next period. First of all, I define the crisis set in order to acquire the probability of crisis. The crisis set $\Gamma(\xi_{n,t}, B_{t-1}, B^{\text{max}}_t)$ is for the set of TFP and government consumption such that the newly issued government
bond is larger than the fiscal limit.

\[ \Gamma(\xi_{n,t}, B_{t-1}, B_{t-1}^{\text{max}}) = \{ A_t \in A, g_t \in G : B(\xi_t, B_{t-1}) > B_{t-1}^{\text{max}}(A_t, g_t) \} \] (3.35)

The probability of crisis in the next period \( \mathbb{E}_t[P_c(S^n_{t+1}, B_{t+1}^{\text{max}}|S^n_t, \chi_{n,t})] \) is defined from the transition of TFP \( f^A(A_{t+1}, A_t) \) and government consumption \( f^g(g_{t+1}, g_t) \) from time \( t \) to \( t+1 \) and the default set \( \Gamma(\xi_{n+1}, B_{t+1}^{\text{max}}) \). The probability of crisis in the next period can be expressed by the current variables \( (A_t, g_t, B_t, \xi_{n,t}, \chi_{n,t}) \) and exogenous variable \( B_{t+1}^{\text{max}} \) as \( P_c(A_t, g_t, B_t, \xi_{n,t}, \chi_{n,t}, B_{t+1}^{\text{max}}) \). Then, the probability of crisis in the next period is

\[
P_c(A_t, g_t, B_t, \xi_{n,t}, \chi_{n,t}, B_{t+1}^{\text{max}}) = \mathbb{E}_t[P_c(S^n_{t+1}, B_{t+1}^{\text{max}}|S^n_t, \chi_{n,t})] = \int \int_{\Gamma(\xi_{n+1}, B_t, B_{t+1}^{\text{max}})} f^A(A_{t+1}, A_t)f^g(g_{t+1}, g_t)dA_{t+1}dg_{t+1}
\] (3.36)

### 3.2.6 Equilibrium

The new state vector is summarized as \( S_t \in \{ A_t, g_t, B_{t-1}, \xi_t \} \). \( A_t \) and \( g_t \) are the transitions of TFP and government consumption measured by the deviation from the steady state criteria divided by steady state GDP respectively. These two state variables follow the AR(1) process defined in equations (3.1) and (3.4). \( B_{t-1} \) is the government bond carried over from the previous period, and \( \xi_t \) is the government state at the beginning of period \( t \).

**Definition**

*A competitive equilibrium is defined as policy functions for private agents \( \{C(S), L(S), B^H(S), m^*(S), m^d(S), L^f(S), L^m(S)\} \), prices \( \{w(S), p^m(S)\} \), the government plan of new issuance of government bonds \( B(A, g, B_{-1}, \xi_n, \chi_n) \), the law of motion of transition*\(^{13}\)

\(^{13}\)The government can fall into another crisis immediately after regaining the normal state from the crisis state, but the government bond is priced only in the case of \( (\xi_t, \chi_t) = (\xi_{n,t}, \chi_{n,t}) \).
of the economic state $\chi(S^n, B^{max})$ and the price of government bond $\{q(A, g, B, \xi_n, \chi_n, B^{max'})\}$ such that:

1. For $(\xi, \chi) = (\xi_n, \chi_n)$:
   Given the price of government bond, the government plan of new issuance of government bonds satisfies equation (3.7).

2. Households’ policies satisfies condition (3.13) in any government state and for $(\xi, \chi) = (\xi_n, \chi_n)$: given the crisis set and the probability of crisis, their policies satisfies the equation (3.14) and the market clearing condition of government bonds.

3. Both final and intermediate goods firms solve their profit maximization problems respectively.

4. The market for domestic intermediate good clears.

5. The market for labor clears:

   $$L(S) = L^f(S) + L^m(S)$$  

(3.37)

6. The market for final goods clears as

   for $(\xi, \chi) = (\xi_n, \chi_n)$:

   $$Y - p^*m^* = C + G$$  

(3.38)

   for $(\xi, \chi) = (\xi_n, \chi_c)$ and $\xi = \xi_c$:

   $$Y - T - p^*m^* = C - Z$$  

(3.39)

7. For $\xi_t = \xi_{nt}$:

   The law of motion of transition of the economic state satisfies the equation (3.34).
In terms of condition (6), because the international financial organizations cover the fiscal surplus or deficits by the amount of $\Phi_t^{IO}$ in the crisis state, the aggregate constraint is $Y_t - p_t^s m_t^s + \Phi_t^{IO} = C_t + G_t$. By substituting the government budget equation $T_t + \Phi_t^{IO} = G_t + Z_t$, the aggregate resource constraint is derived as in equation (3.39).

### 3.2.7 Fiscal Limit

I define the fiscal limit $B_{\text{max}}$ as the maximum amount of debt that the government is able to serve. Instead of the tax rule defined in equation (3.6), the government sets its tax rate on output to maximize tax revenue. The government can increase tax revenue by increasing the rate when the tax rate is low enough, but the revenue will decline if the tax rate exceeds a threshold of the maximum rate because the rate of revenue loss from output decline is larger than the rate of revenue increase by raising the tax rate. The revenue maximization problem depends on $A_t$ and $g_t$, so the government’s tax maximization problem is defined as:

$$\max_{\tau_t} T_{t}^{\text{max}}(A_t, g_t) = \tau_t Y_t(A_t, g_t)$$

subject to (3.13), (3.16), (3.17), (3.21)-(3.23), (3.28), (3.31), (3.33), (3.37), (3.38), where $T_{t}^{\text{max}}$ is the maximum tax revenue. As a matter of course, the government does not need to satisfy its budget constraint (3.7) and the government state is restricted to the normal state.

Government consumption and government transfers depend on $A_t$ and $g_t$ respectively, so the maximum primary surplus at time $t$ is

$$T_{t}^{\text{max}}(A_t, g_t) - G_t(g_t) - Z_t(A_t)$$
Then, the fiscal limit is defined as the summation of discounted maximum future primary surplus\textsuperscript{14}:

\[ B^{\text{max}}(A_t, g_t) = E_0 \sum_{t=0}^{\infty} \beta^t u^{\max}_C(C_t, L_t) (T^{\max}(A_t, g_t) - G_t(g_t) - Z_t(A_t)) \tag{3.42} \]

where \( u^{\max}_C \) denotes the marginal utility of consumption under the condition of the tax rate set to maximize the government’s tax revenue. If the amount of debt exceeds this fiscal limit, the government is unable to repay its debts no matter how it changes the tax rate.

\subsection*{3.2.8 Timing}

The overview of time flow around time \( t \) is depicted in the second graph in Appendix C.1. I divide timelines into normal and crisis states.

\textless The Timeline of the Normal State \textgreater

At the beginning of period \( t \), the government has debts \( B_{t-1} \) (if it regains the normal state from the crisis state, the amount of debt \( B_{t-1} \) is equivalent to \( B^* \)). First, TFP and government consumption are revealed, and the fiscal limit \( B^{\text{max}} \) is obtained. Then, the government estimates the maturity-based government bond issuance \( B_t \), and if this value is lower than the fiscal limit, the government stays in the normal state. Otherwise, the economy falls into the crisis state. Afterwards, firms produce their goods, markets determine prices, and all agents complete their transactions including the government’s repayment of its previous debt \( B_{t-1} \) and issuance of new bonds \( q_t B_t \). Then, households consume. Time \( t \) ends here and goes to next time \( t + 1 \). If the maturity-based government bond \( B_t \) is higher than the fiscal limit, the timeline goes to the crisis state.

\textsuperscript{14}For simplicity, the primary surplus is derived in order to obtain the fiscal limit. If the model adopts the fiscal surplus instead, which includes transactions of government bonds, the fiscal limit is different from that in the primary surplus case.
There are two initial points in the timeline of the crisis state. If the government is in the crisis state at the beginning of period $t$, it has debts $B_{t-1}^d$. Then, the TFP and the amount of government consumption are revealed. If the government falls into the crisis from the normal state, the timeline shifts to the crisis state with the debt obligation $(1 - \delta)B_{t-1}$. Then, firms produce their goods, markets determine prices, and all agents complete their transactions including the government’s repayment of the previous debt $B_{t-1}^c$ in the case of $\xi_t = \xi_{c,t}$ and $(1 - \delta)B_{t-1}$ in the case of $(\xi_t, \chi_t) = (\xi_{n,t}, \chi_{c,t})$, and issuance of new bonds $B_{t}^c$. Then, households consume. Finally, nature decides to regain the normal state with exogenous probability $\vartheta$ after a certain mandatory crisis period that the government must be in the crisis state. If nature does not choose the normal state or the mandatory duration of the crisis state has not finished yet, the crisis state continues to the period $t + 1$ with debt $B_{t}^c$.

### 3.3 Functional Form and Calibration

The model is calibrated to the Spanish data from 1995 to 2014. The reason why I adopt the Spanish economy is that Spain is one of the European countries which fell into a debt crisis around 2012, and the majority of government bonds are held domestically.

First of all, the discount factor $\beta$ has to be relatively small. Otherwise, the government can take into account far future fiscal surpluses and the fiscal limit will be too high. I set the baseline value of the discount factor to 0.91, which is lower than the standard RBC model\(^ {15}\), in order to target the probability of crisis at about 50% under 110% of debt-to-GDP ratio in the case of low TFP (-5% from the steady state) and high government consumption (+5% from the steady state) from the Spanish average.

\(^{15}\)A low discount factor value is often taken in the literature of sovereign default. For example, the value is 0.8 in Aguiar and Gopinath (2006), 0.88 in Mendoza and Yue (2012), 0.9 in Perez (2015), 0.95 in Bi (2012) and 0.953 in Arellano (2008).
debt-to-GDP ratio after the crisis. Next, I specify the households’ utility function as:

\[ u(c_t, L_t) = \log C_t - \omega \frac{L_t^{1+\eta}}{1+\eta} \]  

(3.43)

where \( \eta > 0 \). Following Gertler and Karadi (2011), I set the inverse Frisch elasticity of labor supply \( \eta \) to 0.276. The parameter of relative utility weight of hours worked \( \omega \) is set to 2.23 by targeting one-third of the working time in a day. Using the HP filter, the persistence and standard deviation of government consumption shock are 0.74 and 0.0175 respectively, and those of TFP shock are 0.82 and 0.0337 respectively\(^{16}\).

I set the intermediate input share in final goods production \( \alpha_M \) to 0.515 to match the total intermediate consumption over gross output in the early 2000s\(^{17}\) from the OECD’s STAN Input-Output data. I set the labor input share in final goods production \( \alpha_L \) to 0.339 because the proportion of labor share is assumed to be 70% of summation of labor and capital inputs in most literature. I set the substitution elasticity between domestic and foreign intermediate goods \( \psi \) to 0.4 in order to target about 10% decline in imported intermediate goods by crisis from the actual Spanish data. Following Feenstra et al. (2014), I set the Dixit-Stiglitz curvature parameter \( \nu \) to 3. The Armington weight of domestic inputs \( \lambda \) is set to 0.81 from the share of domestic intermediate goods to GDP in the OECD’s STAN Input-Output data. I set the upper bound of imported inputs with working capital \( \theta \) and the invariant TFP coefficient for intermediate goods production \( A^I \) to 0.7 and 0.31 from Mendoza and Yue (2012). The world risk-free interest rate \( r^f \) is set to 0.01, which is the standard value of the RBC model. The labor share in the production of intermediate goods \( \gamma \) is also set to 0.7.

I set the criteria of government consumption \( G \), transfer \( Z \), government debt \( B \) and tax rate \( \tau \) to 0.0187, 0.0147, 0.066 and 0.307 respectively so as to match the ratios of the steady state level of GDP to government consumption 0.190, government

---

\(^{16}\)I adopt TFP as GDP per person employed.

\(^{17}\)This data does not specify the exact period.
Table 3.1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ): Discount factor of households</td>
<td>0.91</td>
<td>Fiscal Limit</td>
</tr>
<tr>
<td>( \eta ): Inverse Frisch elasticity of labor supply</td>
<td>0.276</td>
<td>Gertler and Karadi (2011)</td>
</tr>
<tr>
<td>( \omega ): Relative utility weight of hours worked</td>
<td>2.23</td>
<td>1/3 working time</td>
</tr>
<tr>
<td>( \rho_g ): Persistence of gov’t consumption shock</td>
<td>0.74</td>
<td>OECD</td>
</tr>
<tr>
<td>( \sigma_g ): Standard deviation of gov’t consumption shock</td>
<td>0.0175</td>
<td>OECD</td>
</tr>
<tr>
<td>( \rho_A ): Persistence of TFP shock</td>
<td>0.82</td>
<td>OECD</td>
</tr>
<tr>
<td>( \sigma_A ): Standard deviation of TFP shock</td>
<td>0.0337</td>
<td>OECD</td>
</tr>
<tr>
<td>( \alpha_M ): Intermediate input share in final goods production</td>
<td>0.515</td>
<td>OECD</td>
</tr>
<tr>
<td>( \alpha_L ): Labor input share in final goods production</td>
<td>0.339</td>
<td>Labor Share (70%)</td>
</tr>
<tr>
<td>( \psi ): Substitution elasticity across intermediate goods</td>
<td>0.4</td>
<td>10% decline by crisis</td>
</tr>
<tr>
<td>( \nu ): Substitution elasticity within intermediate goods</td>
<td>3</td>
<td>Feenstra et al. (2014)</td>
</tr>
<tr>
<td>( \lambda ): Armington weight of domestic inputs</td>
<td>0.81</td>
<td>OECD</td>
</tr>
<tr>
<td>( \theta ): Imported goods with working capital</td>
<td>0.7</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>( A_I^f ): Invariant TFP coefficient for int. goods production</td>
<td>0.31</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>( r^f ): World risk-free interest rate</td>
<td>0.01</td>
<td>Standard Value</td>
</tr>
<tr>
<td>( \gamma ): Labor input share in int. goods production</td>
<td>0.7</td>
<td>Standard Value</td>
</tr>
<tr>
<td>( G ): Government consumption criterion</td>
<td>0.0187</td>
<td>OECD ((G/GDP = 0.190))</td>
</tr>
<tr>
<td>( Z ): Government transfer criterion</td>
<td>0.0147</td>
<td>OECD ((Z/GDP = 0.149))</td>
</tr>
<tr>
<td>( B ): Government debt criterion</td>
<td>0.066</td>
<td>OECD ((B/GDP = 0.67))</td>
</tr>
<tr>
<td>( \tau ): Tax rate criterion</td>
<td>0.307</td>
<td>OECD ((R_n/GDP = 0.379))</td>
</tr>
<tr>
<td>( \tau_d ): Tax rate criterion in the crisis state</td>
<td>0.321</td>
<td>OECD ((R_d/GDP_d = 0.40))</td>
</tr>
<tr>
<td>( \kappa ): Elasticity of tax</td>
<td>0.37</td>
<td>Bi (2012)</td>
</tr>
<tr>
<td>( \zeta^z ): Elasticity of transfers to productivity</td>
<td>0</td>
<td>data</td>
</tr>
<tr>
<td>( \delta ): Cutback rate</td>
<td>0.10</td>
<td>Households’ financial assets</td>
</tr>
</tbody>
</table>

transfers 0.149, government debt 0.67 and tax revenue 0.379\(^\text{18}\). I obtained the data on government transfers, government debt and tax revenue from OECD data between 1995 and 2014, but government consumption consists of many components, so I set the above value in order to obtain a balanced budget at 45% of debt-to-GDP ratio. In this situation, tax revenue is equal to government expenditure including government bond repayments under the condition that the price of government bonds is equivalent to the discount factor \( \beta \) in the steady state. I set the exogenous tax rate in the crisis state to 0.321 from 40% of the tax revenue to GDP so as to be about 5% higher than the normal rate.

\(^{18}\)The tax is not imposed on GDP but on outputs, so I subtract the amount of imported intermediate goods following equation (3.30).
Following Bi (2012), I set the elasticity of the tax rate $\kappa$ to 0.37. In terms of the elasticity of transfers to productivity $\zeta^2$, transfers over GDP had been constant and were not elastic to TFP variation before the global financial crisis in 2008, so I set this rate to 0. Finally, government bonds are the only asset for households in my model, so I set the proportion of government bonds’ cutback rate $\delta$ by targeting the households’s asset data. According to the OECD’s data, the average household’s financial assets per capita discounted by inflation from 2010 to 2012 was about 10% lower than the maximum value before the financial crisis, so I set $\delta$ to 0.1 as the baseline. However, I also examine other values in the alternative scenario section.

3.4 Quantitative Results

The quantitative results consist of three layers: the fiscal limit, decision rules and simulations. In the section of the fiscal limit, I report the Laffer curve and GDP mapped by the tax rate, and fiscal surplus movement at first. Then, the fiscal limit is derived from the summation of discounted maximum future fiscal surplus as defined in equation (3.42). After that, I show the criterion of probability of crisis depending on the state variables. Next, I show the decision rules of new issuance of government bonds, probability of crisis and price of government bonds and also the decision rules of the main endogenous variables. Finally, I report simulation results both in the long-run and around the crisis period, and also results of alternative scenarios in order to examine the effects of different government policies.

3.4.1 Fiscal Limit

The above two graphs in Figure 3.2 show the Laffer curve under different values of state variables, $A_t$ and $g_t$. The left graph is the Laffer curves in three different states of $A_t$ fixing $g_t$ at the steady state, and the right graph is the curves in three different
states of $g_t$ fixing $A_t$ at the steady state. If both variables are at the steady state, the tax revenue over GDP will be maximized to be 47.0% at 53.6% of the tax rate. Also, the maximum tax revenues over GDP are 49.8% and 44.4% by setting the tax rate to 53.9% and 53.6% in the high and low TFP cases respectively. Similarly, the maximum tax revenues are 49.0% and 45.2% over GDP when the tax rates are 54.8% and 52.7% in the high and low government consumption cases respectively. The tax revenue in the high government consumption case will be higher than the baseline because the amount of outputs will be higher. Corresponding to the Laffer curves, the two graphs below in Figure 3.2 show the values of GDP mapping from the tax rate. The pace of GDP decline is accelerating as the tax rate increases. For example, the decline in GDP
from changing the tax rate from 70% to 80% is about 47% larger than that from 20% to 30% under the steady state of \( g_t \) and \( A_t \).

Figure 3.3: Fiscal Surplus

(a) with \( g \) at steady state

(b) with \( A \) at steady state

Note: Time shows quarterly basis

Next, given the Laffer curve, I consider the fiscal surplus to obtain the fiscal limit. I simulated 2000 times for 200 periods, depending on the initial states of \( A_t \) and \( g_t \). A detailed explanation of the computational algorithm is in Appendix C.2. Figures 3.3 shows the transition of fiscal surplus under the tax rate to maximize the tax revenue in three different initial states of \( A_t \) and \( g_t \), fixing another variable at the steady state. The fiscal surplus under the steady states of \( A_t \) and \( g_t \) is about 13.1% over steady state GDP. The left graph reports that the fiscal surplus in the high \( A_t \) case is about 2.8% points higher than the steady state and that in low \( A_t \) is about 2.6% points lower than the steady state in the first period. However, as time passes, \( A_t \) is converging to the steady state and so is the fiscal surplus. The right graph shows the different levels of \( g_t \) under a steady state of \( A_t \). In the case of low and high \( g_t \), the fiscal surplus is 2.9% higher and 3.0% lower than the steady state case in the initial period respectively, and it also converges to the steady state level as time goes on.

Figure 3.4 describes the average of total trials of fiscal limits, which is defined as the summation of discounted fiscal surplus derived from Figure 3.3. This figure shows
Figure 3.4: Fiscal Limit

Note: The fiscal limit is the average of total trials.

the combination of debt-to-GDP ratio and $A_t$ under three different levels of $g_t$, and
the government falls into the crisis when the amount of debt exceeds these colored
areas. For example, if $g_t$ is the middle level in the initial period, the fiscal limit is
about 133% of GDP under the middle $A_t$ level. As the value of $A_t$ changes, the fiscal
limit shifts in direct proportion to TFP, taking 146% and 119% over GDP in the case
of 5% points higher and lower than the neutral case respectively. Also, the fiscal limit
changes -11.6% and +12.1% from the middle level of $g_t$ by changing its value to high
and low respectively.

Figure 3.5: Crisis Probability (Fiscal Limit)

(a) with $g$ at steady state (b) with $A$ at steady state
Next, Figure 3.5 is the probability of crisis under three different values of $A_t$ and $g_t$. While Figure 3.4 is the average of all trials, Figure 3.5 is the cumulative density functions of the fiscal limit depending on the stochastic transitions of $A_t$ and $g_t$. If both $A_t$ and $g_t$ are at the steady state (the black curve), the probability of crisis is less than 1% in 114% debt-to-GDP level and jumps up, reaching 50% and 99% in 134% and 149% of debt-to-GDP ratio respectively. If $A_t$ is low and $g_t$ is in steady state, the probabilities of crisis in 1%, 50% and 99% are 104%, 119% and 134% of debt-to-GDP ratio respectively. Similarly, if $A_t$ is high, the probabilities of crisis in 1%, 50% and 99% are 126%, 146% and 164% debt-to-GDP ratio respectively. The case of different $g_t$ reports similar results to $A_t$ in the right graph.

### 3.4.2 Decision Rules

In this section, I explain the relation between the newly issued government bonds $B_t$, the probability of crisis $E_t[P_{c,t+1}]$ and the price of government bonds $q_t$. Figure 3.6 maps the debt-to-GDP ratio to these three variables, fixing TFP and government consumption at the steady state.$^{19}$

Graph (a) is the decision rule of newly issued government bonds under the condition that the government does not fall into crisis (i.e. the maturity base of current issuance of government bonds $B_t$ is smaller than the fiscal limit $B_{\text{max}}$). Both horizontal and vertical axes are standardized by steady state GDP. The red and pink curves represent the maturity base ($B_t$) and discount by its price ($q_tB_t$) respectively, and the black dotted line is the 45 degree line. If the red curve is above the line, the government has to repay a larger amount of debt in the next period than the current period, and if the pink curve is in the same condition, the government needs to acquire more than the amount of repayments. The decision rules of government bonds show that the maturity and discount base curves are slightly higher and lower than the 45 degree line.

---

$^{19}$Different TFP and government consumption cases are in Appendix C.3
Figure 3.6: Bond Related Variables

(a) Government Bond

(b) Probability of Crisis

(c) Price of Government Bond

Note: Debt-to-GDP ratios in the horizontal axis represent the amount of repayment of government bond \( B_{t-1} \), except for the red curve of maturity base debt \( B_t \) respectively under the condition of below about 100% of debt-to-GDP ratio. However, as the debt amount increases, exceeding 100% of debt-to-GDP ratio, the government has to repay a larger amount of debt in the next period than the current period due to the decrease in the price of government bonds. For instance, while the reimbursement of debt in the next period (\( B_t \)) is 105% if the previous debt-to-GDP ratio is 100%, the amounts of debt are 129% and 152% in the cases of previous debt at 120% and 140% respectively.

The middle graph (b) shows the probability of crisis. The brown curve marked by diamond shapes and the red curve are mapped from the previous debt (\( B_{t-1} \)) and the maturity-based debt (\( B_t \)) respectively. The solid black and dotted green and light blue curves show the probability of crisis based on the fiscal limit under the steady
state, low and high levels of TFP respectively, which are obtained from Figure 3.5. As I have explained in section 2.7, the criterion of debt amount whether the government falls into crisis or not depends on maturity-based government bonds. The graph shows that as the previous debt increases, the maturity-based debt is larger following the decision rules of government bonds. Then, the probability of crisis is derived as the cumulative probability that maturity-based debt exceeds the fiscal limit. For example, the probability of crisis is 50% when the maturity-based debt is about 132% of debt-to-GDP ratio, which corresponds to 124% of previous debt.

The bottom graph (c) depicts the price of government bonds following the households’ bond pricing equation (3.14). If the debt-to-GDP ratio is in the safe zone (less than about 100% of debt-to-GDP ratio), the bond price is relatively constant at about 0.91, which is the value of the discount factor. Then, the price of government bonds declines when the debt-to-GDP ratio is in the risky zone (between about 100% and 150% of debt-to-GDP ratio) due to the increased probability of crisis. The price keeps decreasing even after the risky zone because the marginal utility of consumption in the current period increases by the increase in tax rate, but is constant in the next period because of the fixed tax rate in the crisis state.

Therefore, all three graphs in Figure 3.6 are related to each other. The high amount of debt induces the increase of the probability of crisis, the high crisis probability decreases the price of government bonds, and the low government price leads to high debt amount.

Next, Figure 3.7 describes the decision rules of the main endogenous variables: tax rate, imported intermediate goods, output, tax revenue, GDP, consumption, domestic intermediate goods, wages and hours worked. These graphs report the cases of steady state, low TFP and high government consumption in both crisis and normal states. Obviously, the debt amount is irrelevant to any variables in the case of crisis, so the values are parallel to the horizontal axis. Also, government consumption in the crisis
state does not affect these variables because of the intervention of international financial organizations.

First of all, the tax rate in the normal state is the function of only the debt amount regardless of other state variables, so its rate increases linearly as debts rise. Second, the crisis effects on imported intermediate goods are relatively large. When the debt-to-GDP ratio is 100%, imported goods in the crisis state are 7% lower than the normal state under the steady state of $A_t$ and $g_t$. This difference between crisis and normal states is one of the main channels of economic contraction due to the crisis. Output
is also a decreasing function in the amount of debt because the government increases its tax rate as the amount of debt increases as graph (a) shows. Next, tax revenue in the normal state is increasing as debt-to-GDP increases but the pace of increase is sluggish due to lower output levels. The tax revenue exceeds the crisis case when the debt amount is higher than about 110% of debt-to-GDP ratio. GDP and consumption show similar movement to output. Households’ consumption in the high government consumption case is lower than the steady state level because of the crowding out effect. Besides, the decline in consumption in the crisis state is larger than in the GDP case because the aggregate resource constraint is different from the normal state and the tax revenue negatively affects consumption. Next, domestic intermediate goods and wages are decreasing in the debt-to-GDP ratio and TFP, but the proportion of decline of domestic intermediate goods is limited in the low TFP case. Wages decline under high government consumption due to lower levels of household consumption. Finally, hours worked increase in the case of low levels of TFP and the crisis state. According to equation (3.13), wages are determined by the marginal utility in hours worked over consumption, and the latter effect is larger than the former effects under low TFP. In addition to this, low productivity increases cost per unit of production, so more labor is necessary to maintain the given amount of output\textsuperscript{20}. Hours worked increase under the crisis state because the prices of imported and domestic intermediate goods decline more than wages decrease, so final goods firms sustain production by increasing labor inputs and that leads to an increase in hours worked in the crisis state.

### 3.4.3 Simulation results

I generate stochastic TFP processes 2000 times for 500 periods and trace the movement of variables. The government starts from the non-default state, and the initial values of TFP and government bonds outstanding are neutral and 30% of debt-to-GDP ratio. \textsuperscript{20}This result is consistent with other canonical papers such as Gali (1999) and Smets and Wouters (2007).
respectively, but I discard the first 100 periods in order to eliminate the effect of these initial conditions.

In terms of the crisis duration, other literature using endogenous sovereign default models assumes that the government has a chance to regain the normal state soon after the crisis. Otherwise, it will be an intricate model due to the assumption of a mandatory crisis period that a government has to stay in the crisis, and the government would not choose default in the first place under this assumption. Besides, it might be an appropriate assumption for developing countries that the government can soon recover its normal state, as Borenstein and Panizza (2009) report, the default duration is usually short. However, the European debt crisis, at least, was different from those in developing countries because governments’ fiscal and economic situations had stagnated for many years after the debt problems occurred. Thus, in order to make the model more flexible and suitable to the actual economy, I set the duration of the mandatory crisis state to eight quarters from the time the Greek government defaulted in March 2012 until it returned to the bond market in April 2014. I refer to the Greek bond market because, as De Grauwe and Ji (2013) point out, negative self-fulfilling market sentiments significantly affected the surge of spreads in eurozone countries. I also report the zero mandatory crisis period as endogenous default models assume. After the mandatory crisis period, the government can regain the normal state with exogenous probability. Dias and Richmond (2007) report that the average duration of exclusion from international capital markets by default is 5.7 years, so I set the probability to regain the normal state at 0.044. If the government regains the normal state, it can start from the half of the steady state GDP level as the benchmark. However, I examine different values in the section on alternative scenarios.
Table 3.2: Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Debt-to-GDP</td>
<td>67.6%</td>
<td>47.0%</td>
</tr>
<tr>
<td>Default/Crisis Frequency</td>
<td>0.60%</td>
<td>0.75%</td>
</tr>
<tr>
<td>Average Bond Spread</td>
<td>0.71%</td>
<td>5.78%</td>
</tr>
<tr>
<td>The correlation with GDP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Output</td>
<td>99.9%</td>
<td>92.4%</td>
</tr>
<tr>
<td>- Consumption</td>
<td>98.6%</td>
<td>74.9%</td>
</tr>
<tr>
<td>- Imported Intermediate Goods</td>
<td>83.8%</td>
<td>72.1%</td>
</tr>
<tr>
<td>- Domestic Intermediate Goods</td>
<td>98.8%</td>
<td>61.6%</td>
</tr>
<tr>
<td>- Hours Worked</td>
<td>-77.4%</td>
<td>12.8%</td>
</tr>
<tr>
<td>- Tax Rate</td>
<td>-12.2%</td>
<td>-17.0%</td>
</tr>
<tr>
<td>- Tax Revenue</td>
<td>98.1%</td>
<td>70.9%</td>
</tr>
</tbody>
</table>

Note 1: The Spanish government has experienced default 13 times since 1476. Source: Reinhart and Rogoff (2008).

Note 2: Bond spread in data is 10-year government bond premium toward the yields of German bonds, while that in simulation represents the annual rate of reciprocal of government bond price minus that of bond price without the possibility of crisis.

Baseline Results

Table 3.2 reports the average debt-to-GDP ratio, frequency of default or crisis, average bond spread and the correlations of GDP with main variables in both data of the actual Spanish economy and the simulation results under the baseline parameter settings. These values are taken from 1995 to 2014 including the crisis period, except for the bond spread which is taken from 1995 to 2007.

First of all, although the average debt-to-GDP ratio in data (67.6%) is relatively higher than the simulation (47.0%), the model replicates a high enough value to represent advanced economies, unlike the endogenous default models. The reason for the simulation’s lower debt than the data is that once the probability of crisis takes a large enough value and high government consumption or low TFP continue for several quarters, the government bond is on a divergent path as its decision rule (Figure 3.6 (a)) shows, and the government falls into crisis within a relatively short period. Next, the default or debt crisis frequencies in the data (0.60% per quarter) and simulation
(0.75\% per quarter) show similar values. In the endogenous default model, the default frequency is trade-off with debt-to-GDP ratio because a high amount of debt indicates a government’s reluctance to choose default. For example, Arellano (2008) obtains about 6\% of debt-to-GDP ratio in order to target 3\% of default frequency, but Aguiar and Gopinath (2006) report about 23\% debt-to-GDP ratio and 0.02\% of default frequency using a similar model. However, my model is compatible with the high debt-to-GDP ratio and high enough crisis frequency. The average bond spread of baseline simulation is higher than that of the data. In terms of data, government bonds in Euro countries had been treated as the same or almost the same as the most secure bonds in the region (i.e. German bonds) before the debt crisis, so the bonds’ risk premium had been almost zero for a long time\textsuperscript{21}. In terms of the simulation, the risk premium on an annual basis takes a high value in the risky debt region because the price of government bonds takes a low value due to the 10\% of cutback rate.

The correlations between GDP and other main variables\textsuperscript{22} are in the second half of the Table 3.2. The relations between GDP and output, consumption and imported and domestic intermediate goods are high in both the data and the simulation. There is no quarterly or annual data on imported intermediate goods, so I adopt the amount of total imports as an alternative variable. Hours worked shows a negative correlation with GDP in the data because of the downward trend though time, and the correlation with GDP in the simulation is low because hours worked is procyclical with debt and government consumption but countercyclical with TFP and economic state of crisis. The tax rate is in weak negative relation with GDP in both data and simulation. Finally, in terms of tax revenue, while the tax revenue in the data is highly related to GDP, the simulation result reports a weaker relation than the data because the high tax rate pushes down GDP. I report figures for these variables in order to grasp the

\textsuperscript{21}This low interest rate was the prime reason for the housing bubble in Spain.

\textsuperscript{22}There is no annual or quarterly data for imported and domestic intermediate goods, so I refer to total imports as the former variable and intermediate consumption as the latter variable.
economic movements around the crisis period in Figure 3.9 and 3.10.

Figure 3.8: Simulation Result 1

(a) Debt-to-GDP ratio  
(b) Probability of Crisis  
(c) Bond Spread

Note 1: Year zero is set to 2012Q1.
Note 2: The left vertical axis in graph (b) represents the basis point of five-year government bond CDS.

Figure 3.8 depicts the transitions in debt-to-GDP ratio from three years before to three years after the crisis and the probability of crisis and bond spread from three years before to one year after the crisis. I set year zero to the first quarter of 2012 at the time that the debt crisis was most serious due to the Greek government’s implementation of a huge magnitude of debt restructuring in March. First of all, the debt-to-GDP ratio in the simulation increases and the pace accelerates as the crisis is approaching. Then, the government falls into the crisis with over 120% debt-to-GDP ratio and regains the normal state probabilistically after the mandatory crisis period of two years. The graph shows the average of all trials, so the decline in debt is moderate after the mandatory crisis period, but the movement is quite drastic in each trial. The reason why the simulation result is always higher than the data is that I collect cases going into the crisis state. Even though a government is accumulating debts, it can lower the debts if high TFP or low government consumption continues, but the model does not include these cases. Besides, GDP hit the bottom five or six quarters after the first quarter in 2012, and the debt-to-GDP ratio was unusually low from about 5 years before the crisis because of the housing bubble.

Graph (b) reports the probability of crisis. I adopt the credit default swap (CDS)
of five-year government bonds as the alternative variable of probability of crisis in the data. The simulation result shows the probability of crisis at less than ten percent until one and a half years before the crisis, but then the rate surges, reaching over 80% just before the crisis. In the data, after the 2008 global financial crisis settled down, the Spanish CDS was still low. However, fears around debt sustainability in some eurozone countries after the extent of Greek debt manipulation was revealed in 2009 increased the rate of CDS, exceeding 400 basis points in 2012. Graph (c) describes the bond spread. Similar to the previous graph, the bond spread in the simulation begins to increase rapidly one and a half years before the crisis, reaching more than 70% just before the crisis, and the data also shows similar movement with CDS.

Next, Figures 3.9 and 3.10 show transitions in the main endogenous variables around the crisis. The red dotted curve shows the actual movement of the Spanish economy, and the black curve is the simulation result before and at the time of crisis, and the blue and green solid curves are the recovery after the crisis, showing mandatory crisis durations for two and zero years respectively. The upper left graph in Figure 3.9 shows the transition of tax rates. Both the simulation and data keep gradually increasing until the crisis. While the data continues increasing after the crisis, the increase in simulation stops at the time of the crisis and begins to decrease after the mandatory crisis period. Next, in fact, the amount of imports had been recovering from the global financial crisis until one year before the crisis, but it sank about 14% from the maximum point before the crisis. In simulation, the amount of imports declines about 10% due to the crisis. Thus, although the time it takes for the crisis effects to appear in imports is different in the data and the simulation, the magnitude is similar. The output result also depicts similar declines in both data and simulation, decreasing slightly more than 4% from three years before the crisis. The important feature of the simulation result is that the output hits the bottom one quarter before the crisis for two reasons. First, as the next graph of TFP shows, the sudden drop in TFP before
the crisis contributes to an increase in the probability of crisis and decrease in the price of government bonds, and that leads to increased issuance of government bonds and the crisis. Second, the economic contraction related to the crisis state itself is not so harsh in my model. As the graphs of tax rates and imported intermediate goods show, the tax rate in the crisis state does not change much from the normal state and the effects of imported intermediate goods on production is at most about 10%. Mendoza
and Yue (2012) assume autarky as the punishment of default, so the external effect on the economy is strong, but my model assumes only a surge in interest rates for working capital. Next, TFP decreases around the crisis in both data and simulation, but the decline in data is larger than in the simulation. Government consumption, which is another state variable, increases one period before the crisis and contributes to the occurrence of the crisis in the next period. The data also shows the high level of government consumption before the crisis. In terms of the simulation result for tax revenue, the government increases it by raising the tax rate before the crisis. Then, revenue decreases due to the lower levels of output around the crisis but increases again because of the gradual recovery of output. The data shows constant increases in tax revenue. The movement of the simulation result for GDP in the upper left graph in Figure 3.10 is similar to that for output, but there are differences: GDP hit the bottom
at the time of the crisis due to the decrease in the amount of imported intermediate goods, and crisis effects on the economy in the data are slightly larger than in the baseline simulation result. Also, the transition in households’ consumption resembles output and GDP. However, the financial organizations’ intervention negatively affects consumption, so the drop in consumption is larger than GDP. Domestic intermediate goods had been constant in the data before the crisis, but it dropped sharply at the time of the crisis. In the simulation, however, the decline of domestic intermediate goods is only 2% at the time of crisis. Hours worked in the data are almost constant around the crisis, but increases due to the crisis in the simulation. This is mainly because final goods firms supplement the shortage of intermediate goods with labor inputs as I have shown in the decision rules in Figures 3.7.

Overall, it is possible to say that the model generally accommodates features of the complicated movements of the actual debt crisis in Spain, capturing the magnitude of crisis shock and stickiness. The biggest single difference is that my model underestimates the effects of domestic inputs such as hours worked and domestic intermediate goods. That implies that not only the restriction of imports of intermediate goods but also domestic factors would amplify the effects of a crisis.\(^\text{23}\)

**Alternative Scenario**

In this section, I examine the effect of mainly government-related policies on the economy by changing the values of six parameters. Table 3.3 shows the results of average debt-to-GDP ratio, crisis frequency, average bond spread and changes of GDP in each state of the economy from the baseline scenario and its decline from the normal state to the crisis state.

Section (1) of Table 3.3 shows results that the cutback rate changes 10% points from the 10% baseline cutback rate. The average debt-to-GDP ratio and crisis frequency

\(^{23}\text{For example, Acharya et al. (2014) emphasize the effect of financial contraction in the European debt crisis.}\)
Table 3.3: Alternative Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Average D/GDP</th>
<th>Average Freq.</th>
<th>Average Spread</th>
<th>Normal GDP</th>
<th>Crisis GDP</th>
<th>Decline GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>61.1%</td>
<td>0.66%</td>
<td>1.18%</td>
<td>-</td>
<td>-</td>
<td>-6.19%</td>
</tr>
<tr>
<td>Baseline</td>
<td>47.0%</td>
<td>0.75%</td>
<td>5.78%</td>
<td>-</td>
<td>-</td>
<td>-4.78%</td>
</tr>
<tr>
<td>(1) Cutback rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0:</td>
<td>48.3%</td>
<td>0.74%</td>
<td>0.0%</td>
<td>0.17%</td>
<td>1.98%</td>
<td>-3.07%</td>
</tr>
<tr>
<td>0.2:</td>
<td>45.1%</td>
<td>0.74%</td>
<td>10.99%</td>
<td>-0.05%</td>
<td>-2.40%</td>
<td>-7.02%</td>
</tr>
<tr>
<td>(2) Government Bond Criterion (B/GDP: 0.67)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.52:</td>
<td>24.6%</td>
<td>0.35%</td>
<td>3.00%</td>
<td>-0.73%</td>
<td>-0.20%</td>
<td>-4.27%</td>
</tr>
<tr>
<td>0.82:</td>
<td>71.8%</td>
<td>1.23%</td>
<td>9.12%</td>
<td>0.67%</td>
<td>0.06%</td>
<td>-5.36%</td>
</tr>
<tr>
<td>(3) Tax revenue criterion (R_n/GDP: 0.38)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5%:</td>
<td>100.7%</td>
<td>2.09%</td>
<td>18.45%</td>
<td>2.08%</td>
<td>0.12%</td>
<td>-6.61%</td>
</tr>
<tr>
<td>+5%:</td>
<td>1.1%</td>
<td>0.01%</td>
<td>0.17%</td>
<td>-1.73%</td>
<td>0.51%</td>
<td>-2.61%</td>
</tr>
<tr>
<td>(4) Government consumption criterion (G/GDP: 0.219)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5%:</td>
<td>9.2%</td>
<td>0.10%</td>
<td>1.01%</td>
<td>-0.79%</td>
<td>-0.32%</td>
<td>-4.33%</td>
</tr>
<tr>
<td>+5%:</td>
<td>86.4%</td>
<td>1.71%</td>
<td>13.94%</td>
<td>0.86%</td>
<td>0.11%</td>
<td>-5.49%</td>
</tr>
<tr>
<td>(5) Government transfer criterion (Z/GDP: 0.143)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5%:</td>
<td>14.6%</td>
<td>0.18%</td>
<td>1.65%</td>
<td>-0.17%</td>
<td>0.55%</td>
<td>-4.10%</td>
</tr>
<tr>
<td>+5%:</td>
<td>79.5%</td>
<td>1.51%</td>
<td>11.83%</td>
<td>0.24%</td>
<td>-0.52%</td>
<td>-5.50%</td>
</tr>
<tr>
<td>(6) Recovery (B^* : 50%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-cutback:</td>
<td>110.1%</td>
<td>2.43%</td>
<td>41.9%</td>
<td>-0.50%</td>
<td>0.55%</td>
<td>-3.78%</td>
</tr>
<tr>
<td>0%</td>
<td>16.1%</td>
<td>0.23%</td>
<td>1.91%</td>
<td>0.02%</td>
<td>0.05%</td>
<td>-4.76%</td>
</tr>
</tbody>
</table>

Note: *Normal" and "Crisis" in the section on GDP represent GDP decline from the baseline and the decline shows GDP decrease from the maximum value 3 years before the crisis.

are almost the same as the baseline result because the cut back rate does not affect the economy under the zero crisis probability time and the probability of crisis jumps up just before one and a half years before the crisis. However, the bond spread takes a high value as the cutback rate increases because of the low repayment at the time of the crisis. Thus, the interest rate for working capital also increases, so the amount of imports of intermediate goods declines and that leads to lower GDP in the crisis state. Both sections (2) and (3) relate to the tax rate rule following equation (3.6). The former section reports the case that the government bond criteria changes 15% points from the baseline level of 67%, and the latter section shows the case that the tax revenue criteria changes 5% from the baseline. The low value of government bonds
and high value of tax revenue criteria represent the government’s austerity attitude, imposing a high tax rate on outputs, and the fiscal limit is also higher than the baseline case. Thus, the average debt, crisis frequency and average spread are lower than in the baseline case. However, this safe economy sacrifices GDP in the normal state, decreasing 0.73% and 1.73% from the benchmark respectively. GDP does not change much in the crisis because the tax rate is exogenously given. Sections (4) and (5) report results of 5% change of government consumption and transfer criteria from the baseline respectively. The low values mean the government is taking an austerity policy similar to the previous revenue side. Thus, low government consumption and transfer economy lead to low debt-to-GDP ratios, crisis frequencies and bond spreads, and the GDP levels are also lower than the baseline in the normal case. Transfers in the crisis state negatively affect GDP due to different aggregate resource constraints from the normal state. Finally, I examine two cases in section (6) in which the government starts with the amount of debt carried over during the crisis period $B^c_t$ and zero debt amount instead of the baseline value of 50% debt-to-GDP ratio when the government regains the normal state from the crisis state. In the former case, the debt amount is too high when the government regains the normal state, so the government falls into the crisis again very quickly and the average debt-to-GDP ratio and average bond spread also take high values. In the second case, if the government debt is zero when it regains the normal state, these three statistics decline and the economy is more stable than in the baseline case.

### 3.5 Conclusions

This chapter proposes a sovereign debt crisis model which is applicable to an advanced country. The model features the government’s capability of repayment of its debts rather than its willingness to repay as is assumed in the endogenous sovereign default
model. Concretely, the government falls into debt crisis if its debt exceeds a fiscal limit, and the fiscal limit is defined as the summation of discounted future primary surplus when the government sets its tax rate on outputs to maximize tax revenue. The contraction of the economy by the crisis is attributed to austerity measures and a decline in intermediate goods importation due to high interest rates. Besides, a low value of TFP amplifies its effect. The model’s compatibility to the Spanish debt crisis is generally sound, capturing important features such as high debt-to-GDP ratio, crisis frequency and moments of main variables. Also, the model is sufficiently flexible to examine a variety of government policies quantitatively.

This chapter merely provides a basic framework to analyze sovereign debt crises in advanced countries. The crises are caused by many factors and cause many effects on the economy. It would be expected to deepen the analyses based on the framework.
Chapter 4

A General Equilibrium Model of Sovereign Debt Crisis in an Advanced Country - The Case of Foreign Debt -

4.1 Introduction

The Greek virtual default in 2012 had a significant impact on the country’s economy. Real GDP declined 26.5% in 2013 from the level before the financial crisis and the unemployment rate reached 27.5%. Although five years has passed since the default, the recovery of the economy is quite sluggish and the debt-to-GDP ratio remains high. I proposed a dynamic stochastic general equilibrium (DSGE) model for the analysis of sovereign debt crisis in advanced economies in the previous chapter. However, the model is for economies with domestic bonds, so it is inappropriate to apply the model for the analysis of the Greek crisis because the majority of its government bonds are held by foreign countries. Thus, in this chapter I introduce a model of sovereign debt
crisis in an advanced country for the case of foreign debt.

The modelling of foreign debts accompanies several modifications from the previous chapter. First of all, while domestic households purchase and price the government bonds in the previous chapter, foreign investors play this role in the model. Thus, the difference between the repayment and acquisition of new bonds is the net outflow of output in this case. Second, the model introduces capital for households’ inter-temporal utility maximization problem. In the previous chapter, households use government bonds for their inter-temporal utility maximization and I abstract capital for simplicity because the effect of capital on the economy is small around financial crises. However, the data suggests that capital cannot be ignored in this case due to the huge magnitude of the Greek crisis. Finally, following Bi and Traum (2014), the model assumes that the fiscal limit is drawn from the exogenous distribution depending on the debt-to-GDP ratio. In the case of foreign debt, the government has less incentive to serve debts by taking austerity measures to repay the debt than in the case of domestic debts.

The contraction of the economy by the crisis is caused by several paths. I assume the country falls into financial autarky in the state of crisis, so it is restricted from importing intermediate goods which require working capital. Also, the international financial organizations force the government to impose an exogenous tax rate in return for taking over a certain fraction of government debt. Finally, the lower capital return decreases capital stock and that leads to a lower level of output.

I calibrate the model for the Greek economy in order to analyze its crisis around 2012, and the simulation results capture drastic movement in its economy such as the high debt-to-GDP ratio, crisis frequency and economic contraction. The main difference is that my model overestimates the decrease in imported intermediate goods and underestimates the decrease in domestic intermediate goods.

This chapter is organized as follows. Section 4.2 presents the model to analyze the

\[ \text{See Mendoza and Yue (2012).} \]
sovereign debt crisis for the case of foreign debts, focusing on the modification from the previous chapter. Section 4.3 sets functional forms and calibrates parameters. Section 4.4 delivers the quantitative results: consisting of decision rules and simulation. Finally, section 4.5 concludes.

4.2 Model

The overall framework of the model is similar to the previous chapter. The model is a dynamic stochastic general equilibrium of a small open economy, consisting of four domestic agents: households, final goods firms, intermediate goods firms and a government, and two foreign agents: foreign investors and foreign firms.

The government imposes taxes for final output production, issues new bonds for foreign agents, pays transfers to households and consumes final goods. Households provide labor force to both final and intermediate goods firms and lend capital for final goods firms. They receive wages, the return of capital with interest and profits from both types of firms, and receive government transfers. Intermediate goods firms produce intermediate goods from households’ labor force. Final goods firms produce output goods from imported and domestic intermediate goods, labor and capital. They need to borrow working capital from foreign firms in order to import a certain fraction of intermediate goods. In this foreign debt model, I assume the exogenous fiscal limit depending on the previous debt outstanding because of less incentive to service debts. If the debt amount exceeds the fiscal limit, the government falls into crisis. The transition of normal and crisis states follows the previous chapter. I depict the overall flow of the transaction in Appendix D.1.
4.2.1 Government

The government’s expenditure in the normal state consists of government consumption $G_t$, transfers to households $Z_t$ and the repayment of government bonds $B_{t-1}$. Its revenue consists of tax revenue $T_t$ and new issuance of government bond $B_t$ priced by foreign investors as $q_t$. Government bonds are characterized one period maturity, zero coupon and non-contingent. Government bonds in the crisis state are rolled over until it regains the normal state.

Government consumption is determined exogenously following to the process:

$$g_t = (G_t - G)/GDP$$ (4.1)

where $g_t$ is government consumption measured as the deviation from the steady state criteria $G$ standardized by the steady state GDP, which follows the AR(1) process:

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$$ (4.2)

where $\varepsilon_{g,t} \sim N(0, \sigma^2_g)$.

The amount of government transfer is decided following to the rule:

$$Z_t = Z + \zeta^z(e^{A_t} - e^A)$$ (4.3)

where $Z$ and $A$ are the steady state criteria of transfer and TFP respectively and $\zeta^z(\leq 0)$ is the elasticity of TFP to transfer. Similar to $g_t$, TFP follows the AR(1) process as:

$$A_t = \rho_A A_{t-1} + \varepsilon_{A,t}$$ (4.4)

where $\varepsilon_A \sim N(0, \sigma^2_A)$. The government’s tax revenue is determined as follows:

$$T_t = \tau_t Y_t$$ (4.5)
where \( \tau_t \) is the tax rate on output and \( Y_t \) is final output goods. For simplicity, I assume the government imposes tax only on \( Y_t \).

In the normal state, the government set its tax rate depending on the previous government bonds \( B_{t-1} \) as:

\[
\tau_t = \tau + \kappa(B_{t-1} - B)
\]

(4.6)

where \( \tau \) and \( B \) are criteria of the tax rate and outstanding of government bonds respectively and \( \kappa(\geq 0) \) is the elasticity of government bonds to the tax rate.

The issuance of new government bonds in the normal state is determined to balance its budget:

\[
q_t B_t = B_{t-1} + G_t + Z_t - T_t
\]

(4.7)

The government in the crisis state repays the remitted amount of debts to foreign investors and issues the same amount of bonds. The remitted amount of bonds is determined as \( B_t = (1 - \delta)B_{t-1} \), where \( \delta \) is the exogenous cutback rate. In this case, because the government budget constraint does not hold, the international financial organizations provide the amount of output \( \Phi_t^{IO} \) to satisfy the government constraint. Thus, the government budget constraint in the case is \( T_t + \Phi_t^{IO} = G_t + Z_t \). Instead of tax rule defined in the equation (4.6) in the normal state, the government are required to set an exogenous fixed tax rate from the international financial organizations as:

\[
\tau_t = \tau^d
\]

(4.8)

The amount of bonds in the crisis state is rolled over until the government regains the normal state. When the government recovers the normal state, the debt amount is exogenously given to \( B^r \). The difference between the amount of debt government needs to repay and \( B^r \) is covered by the international financial organizations.
4.2.2 Households

Households are infinitely lived and derive utility from consumption $C_t$ and disutility from labor measured by hours worked $L_t$. They maximize the following utility maximization problem:

$$\max_{C_t, L_t, K_t} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right]$$

subject to

$$C_t + I_t = w_t L_t + r_{k,t} K_t + \Upsilon_t + Z_t$$

$$K_t = I_t + (1 - \mu)K_{t-1}$$

where $\beta \in (0, 1)$ is the discount factor of the household, and the utility function $u : \mathbb{R}_+^2 \to \mathbb{R}$ is continuous, twice differential and satisfies $\frac{\partial u}{\partial C} > 0$, $\frac{\partial^2 u}{\partial C^2} < 0$, $\frac{\partial u}{\partial L} < 0$, $\frac{\partial^2 u}{\partial L^2} < 0$ and $\frac{\partial^2 u}{\partial C \partial L} - \left( \frac{\partial u}{\partial C} \frac{\partial u}{\partial L} \right)^2 > 0$. The first equation is the households’ budget constraint. The right-hand side of the equation consists of wages $w_t L_t$ and the return from capital holdings $r_{k,t} K_t$, profits $\Upsilon_t$ and government transfers. The left-hand side is consumption and investment $I_t$. The second equation is the transition of capital. Households allot investment $I_t$ from their income and create new capital by combining the depreciated previous capital $(1 - \mu) K_{t-1}$, where $\mu$ is the depreciation rate of capital.

The first-order conditions of the households’ optimization problem are

$$w_t = \frac{u_L(C_t, L_t)}{u_C(C_t, L_t)}$$

$$r_{k,t} = \beta \mathbb{E}_t \frac{u_C(C_{t+1}, L_{t+1})}{u_C(C_t, L_t)} (1 + r_{k,t} - \mu)$$

The households’ optimization problem does not change due to the economic states of normal or crisis.
4.2.3 Final Goods Firms

Final goods firms produce output goods from intermediate goods $M_t$, labor force $L_t$ and physical capital $K_t$ following the Cobb-Douglas production function:

$$ Y_t = e^{A_t} M_t^{\alpha_M} (L_t)^{\alpha_L} K_t^{1-\alpha_M-\alpha_L} \quad (4.13) $$

They purchase intermediate goods from both domestic intermediate goods firms $m_d^t$ and foreign firms $m^*_t$, and obtain labor force $L_t^f$ and borrow physical capital $K_t$ with its interest rate $r_{k,t}$ from households. The combination of technology of domestic and imported intermediate goods follows the CES aggregator:

$$ M_t = \left[ \lambda^{\frac{\psi}{\psi-1}} (m_d^t)^{\frac{\psi-1}{\psi}} + (1-\lambda)^{\frac{\psi}{\psi-1}} (m^*_t)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \quad (4.14) $$

where $\lambda$ is the Armington weight of domestic inputs and $\psi$ is the parameter of substitution elasticity. Then, the maximization problem of this firm is

$$ \max_{m_t, m_t^*, L_t^f, K_t} \pi_t^f = Y_t (1-\tau_t) - p_t^* m_t^* - p_t^m m_t^d - w_t L_t^f - r_{k,t} K_t \quad (4.15) $$

Both types of intermediate goods are a summation of the continuum of differentiated domestic and imported intermediate goods $m_{d,t}^d, m_{d,t}^*$ following the Dixit-Stiglitz aggregator:

$$ m_t^d = \left[ \int_0^1 (m_{d,t}^d)^{\frac{\psi-1}{\psi}} dj \right]^{\frac{\psi}{\psi-1}} \quad \text{and} \quad m_t^* = \left[ \int_0^1 (m_{d,t}^*)^{\frac{\psi-1}{\psi}} dj + \int_0^\theta (m_{d,t}^*)^{\frac{\psi-1}{\psi}} dj \right]^{\frac{\psi}{\psi-1}} \quad (4.16) $$

Final goods firms have to satisfy the working capital constraint for the importation of a certain fraction $\theta \in [0,1]$ of intermediate goods:

$$ \frac{K_t}{1+r_t^*} \geq \int_0^\theta p_{j,t} m_{j,t}^* \, dj \quad (4.17) $$
where $\kappa_t$ is working capital, $r_t^*$ is the interest rate of working capital and $p_{j,t}^*$ is the price of imported intermediate inputs. From the cost minimization problem with respect to imported intermediate goods and the assumption that the price of each unit of imported intermediate goods $p_{j,t}^*$ is the numeraire, the aggregate price of imported intermediate goods is

$$p_t^* = [(1 - \theta) + \theta(1 + r_t^*)^{1-\nu}]^{\frac{1}{1-\nu}} \quad (4.18)$$

In the previous chapter, the interest rate on working capital $r_t^*$ is set to be equal to the return of government bonds with the same marginal utility with respect to current and next period consumption under the condition of definite occurrence of crisis. However, following Mendoza and Yue (2012), I assume that firms in the crisis state cannot import intermediate goods which are required for the working capital for $j \in [0, \theta]$ in this model because the interest rate on working capital is infinity (i.e. $r_t^* \to \infty$) due to exclusion from world credit markets. Then, the aggregate price of imported intermediate goods in the crisis state is

$$p_t^* = [(1 - \theta)]^{\frac{1}{1-\nu}} \quad (4.19)$$

**4.2.4 Intermediate Goods Firms**

Intermediate goods firms produce intermediate goods from labor force $L_t^m$ following to the technology:

$$m_t^d = (L_t^m)^{\gamma} \quad (4.20)$$

where $\gamma \in [0, 1]$ is the labor inputs share for the production of intermediate goods. The profit maximization problem of these firms is

$$\max_{L_t^m} \pi_t^m = p_t^m (L_t^m)^{\gamma} - w_t L_t^m \quad (4.21)$$
The first-order condition with respect to $L_t^m$ is

$$w_t = \gamma p_t^m (L_t^m)^{-1} \tag{4.22}$$

### 4.2.5 Foreign Investors

Foreign investors are risk neutral and behave perfectly competitively. In the normal state, they can receive the full amount of bonds issued in the previous period $B_{t-1}$ and purchase new government bonds $B_t^F$ with the price $q_t$. However, if the government falls into crisis, they receive a discounted amount of repayment $(1 - \delta)B_{t-1}$ from the government, where $\delta$ is the cutback rate of the repayment. It is uncertain that they will be repaid the full amount for the bonds at time $t$, so their profit maximization problem is

$$\max_{B_t^F} \mathbb{E}_0 \sum_{t=0}^{\infty} \left( \frac{1}{1 + r^f} \right)^t \left[ \{(1 - P_{c,t}) + P_{c,t}(1 - \delta)\} B_{t-1} - q_t B_t^F \right] \tag{4.23}$$

where $r^f$ is the risk-free interest rate and $P_{c,t}$ is the probability that the government is in a crisis state during the period $t$. By taking the first-order condition with respect to $B_t^F$, the price of government bonds is derived as:

$$q_t = \frac{1}{1 + r^f} \left\{ (1 - P_{c,t+1}) + P_{c,t+1}(1 - \delta) \right\} \tag{4.24}$$

In the case of crisis, foreign investors keep rolling over the discounted amount of repayment $(1 - \delta)B_{t-1}$ until the government regains the normal state. When the government recovers the normal state, the bond amount is exogenously given as $B^r$. 

101
4.2.6 Crisis Scheme

Instead of the endogenous fiscal limit in the previous chapter, I assume that the fiscal limit is drawn from an exogenous distribution depending on the previous debt amount. If government bonds are held by domestic agents, the government has a higher incentive to serve its debts by taking austerity measures, but the government does not have such an incentive to serve its debts in the case of foreign debt. Of course, the lower rate of the government’s repayment of debts leads to higher interest rates, so the government is not totally irresponsible of debt repayments. However, government bonds of eurozone countries are treated as having the same secured level as German bonds, so the government’s fiscal control would be superficial. Therefore, instead of the endogenous fiscal limit which is defined as the government’s maximum capability to serve its debt, I assume an exogenous fiscal limit in the case of foreign debts.

The government falls into crisis if the debt amount $B_{t-1}$ exceeds the fiscal limit $B_t^*$. I assume that the fiscal limit $B_t^*$ is drawn from an exogenous standard logistic distribution. The transition of the crisis follows this process:

$$
\chi_t = \begin{cases} 
\chi_{c,t} & \text{if } B_{t-1} > B_t^* \\
\chi_{n,t} & \text{otherwise} 
\end{cases}
$$

where $\chi_t = \chi_{c,t}$ and $\chi_t = \chi_{n,t}$ indicate the crisis and normal states respectively. Following Bi and Traum (2014), I specify the probability that the government falls into crisis in the next period as:

$$
\mathbb{E}_t[P_{c,t+1}] \equiv P(\chi_{t+1} = \chi_{c,t+1}) = \frac{\exp(\eta_1 + \eta_2 B_t)}{1 + \exp(\eta_1 + \eta_2 B_t)}
$$

where $\eta_1$ and $\eta_2$ are parameters defining the shape of the cumulative distribution function of crisis probability.
4.2.7 Competitive Equilibrium

The state vector is summarized as $S_t \in \{A_t, g_t, K_{t-1}, B_{t-1}, \xi_t\}$. $A_t$ and $g_t$ are the transitions of TFP and government consumption measured by the deviation from the steady state criteria divided by steady state GDP respectively. $K_{t-1}$ and $B_{t-1}$ are capital and the government bond carried over from the previous period respectively. $\xi_t$ is the government state at the beginning of period $t$.

Definition

A competitive equilibrium is defined as policy functions for private agents $\{C(S), K(S), L(S), B^F(S), m^d(S), m^* (S), L^f(S), L^m(S)\}$, prices $\{w(S), p^m(S), r_k(S)\}$, the government plan of new issuance of government bonds $B_t(A, g, K_{-1}, B_{-1}, \xi_n, \chi_n)$, the economic state $\chi(B_{-1}, B^*, \xi_n)$ and the price of government bond $q(B, \xi_n, \chi_n)$ such that:

1. For $(\xi, \chi) = (\xi_n, \chi_n)$:
   
   Given the price of government bond, the government plan of new issuance of government bonds satisfies equation (4.7) and the market clearing condition of government bonds.

2. Households, final and intermediate goods firms solve their problems respectively.

3. The markets for domestic intermediate good and capital clear.

4. The market for labor clears:

   $$L_t = L^f_t + L^m_t$$  \hspace{1cm} (4.27)

5. The market for final goods clears as

   for $(\xi_t, \chi_t) = (\xi_{nt}, \chi_{nt})$:

   $$Y_t - p^*_{mt} m^*_t - (B_{t-1} - q_t B_t) = C_t + I_t + G_t$$  \hspace{1cm} (4.28)

103
for \((\xi_t, x_t) = (\xi_{n,t}, x_{c,t})\) and \(\xi_t = \xi_{c,t}\):

\[ Y_t - T_t - p^*_tm^*_t = C_t + I_t - Z_t \]  \hspace{1cm} (4.29)

(6) For \(\xi_t = \xi_{n,t}\):

The law of motion of transition of the economic state satisfies the equation \(\text{(4.25)}\).

(7) For \((\xi_t, \chi_t) = (\xi_{n,t}, \chi_{n,t})\):

Given the crisis set and the probability of crisis, the price of government bonds satisfies the foreign investor’s no-arbitrage condition \(\text{(4.24)}\).

—Explanation—

In terms of condition (5), the government in the normal state acquires \(q_tB_t\) and repays \(B_{t-1}\) amount of goods from foreign investors, so net outflow of resources to foreign countries is \(B_{t-1} - q_tB_t\). In the state of crisis, the net outflow of goods is zero and international financial organizations intervene to balance the government’s budget \(\Phi^\text{IO}_t\). Thus, the aggregate constraint of the economy is \(Y_t - \bar{p}^*_tm^*_t + \Phi^\text{IO}_t = C_t + I_t + G_t\). By substituting the government budget equation \(T_t + \Phi^\text{IO}_t = G_t + Z_t\), the aggregate resource constraint in the crisis state can be rewritten as is in the equation \(\text{(4.29)}\).

### 4.3 Functional Form and Calibration

The model is calibrated to the Greek data in the normal state from 1995 to 2007 and in the crisis state from 2012 to 2014. I specify the households’ utility function as:

\[ u(c_t, L_t) = \log c_t - \omega \frac{L_t^{1+\eta}}{1+\eta} \]  \hspace{1cm} (4.30)

I adopt the same values on several parameters as in the previous chapter such as the inverse Frisch elasticity of labor supply \(\eta\), the Dixit-Stiglitz curvature parameter \(\nu\), the
upper bound of imported inputs with working capital $\theta$, the risk-free interest rate $r^*$, the labor share in the production of intermediate goods $\gamma$ and the elasticity of tax rate $\kappa$.

Although I set the value of the discount factor quite low in the previous chapter, the value of this model is set to 0.99, which is standard for an RBC model. Next I set the substitution elasticity between domestic and foreign intermediate goods $\psi$ to 0.7 to capture the higher amount of decline of domestic intermediate goods and GDP decline. The depreciation rate of capital $\mu$ is 0.06 in order to target the return rate of capital minus the depreciation rate; this is slightly higher than the risk-free interest rate. From the Greek data, the persistence and standard deviation of government expenditure shocks are 0.65 and 0.014 respectively and those of TFP shocks are 0.64 and 0.015 respectively. The intermediate input share in final goods production $\alpha_M$ is set to 0.42 from the OECD’s STAN Input-Output data. I obtain the labor input share in the production of output goods $\alpha_L$ to 0.41 from 70% of the rest of intermediate goods. The Armington weight of domestic inputs $\lambda$ is calculated from the OECD’s STAN Input-Output data so that the proportion of imported intermediate goods to GDP is 0.12. Next, in the normal period, the criteria of government consumption $\overline{C}$, government transfer $\overline{Z}$, government debt $\overline{B}$ and tax rate $\tau$ are set to be 0.083, 0.051, 0.327 and 0.343 respectively from the ratios of GDP to government consumption 0.240, government transfers 0.147, government debt 1.07 and tax revenue 0.39. I set the exogenous tax rate in the crisis period $\tau^d$ to 0.409 based on data that the government’s tax revenue over GDP $\overline{R}_d/\overline{GDP}_d$ reached about 48% in 2013 due to austerity measures. I set the elasticity of transfers to productivity $\zeta^z$ to $-0.137$ from the OLS estimation. Next, as in the previous chapter, I set the value of the cutback rate from the decline in household assets. In fact, the haircut rate of Greek debt restructuring was about 60% (see Zettelmeyer et al. 2013). However, if investors expect this haircut rate, the price of government bond will be much lower and the bond spread will be much higher.
than the actual economy. For example, if the probability of crisis is 30% under a 60%
cutback rate, the price of government bond is about 0.812 and the annual rate of risk
premium exceeds 120%. In order to avoid drastic movement in these two variables, I
refer to the decline in households’ assets. According to the OECD data, the average
household’s financial assets per capita discounted by inflation from 2010 to 2012 were
about 25% lower than the maximum value before the financial crisis, so I set $\delta$ to 0.25
as the baseline value.

Table 4.1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$: Inverse Frisch elasticity of labor supply</td>
<td>0.276</td>
<td>Gertler and Karadi (2011)</td>
</tr>
<tr>
<td>$\nu$: Substitution elasticity within intermediate goods</td>
<td>3</td>
<td>Feenstra et al. (2014)</td>
</tr>
<tr>
<td>$\theta$: imported goods with working capital</td>
<td>0.7</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>$r^*$: Risk-free interest rate</td>
<td>0.01</td>
<td>Standard Value</td>
</tr>
<tr>
<td>$\gamma$: Labor input share in int. goods production</td>
<td>0.7</td>
<td>Standard Value</td>
</tr>
<tr>
<td>$\kappa$: Elasticity of tax</td>
<td>0.37</td>
<td>Bi (2012)</td>
</tr>
<tr>
<td>$\beta$: Discount factor of households</td>
<td>0.99</td>
<td>Standard Value</td>
</tr>
<tr>
<td>$\psi$: Substitution elasticity across intermediate goods</td>
<td>0.7</td>
<td>Decline of int. goods and GDP</td>
</tr>
<tr>
<td>$\mu$: Depreciation of capital</td>
<td>0.06</td>
<td>Interest rate of $K$ ($r_{kt}=0.075$)</td>
</tr>
<tr>
<td>$\rho_g$: Persistence of gov’t expenditure shock</td>
<td>0.65</td>
<td>OECD</td>
</tr>
<tr>
<td>$\sigma_g$: Standard deviation of gov’t expenditure shock</td>
<td>0.014</td>
<td>OECD</td>
</tr>
<tr>
<td>$\rho_A$: Persistence of TFP shock</td>
<td>0.64</td>
<td>OECD</td>
</tr>
<tr>
<td>$\sigma_A$: Standard deviation of TFP shock</td>
<td>0.015</td>
<td>OECD</td>
</tr>
<tr>
<td>$\alpha_M$: Intermediate input share in final goods output</td>
<td>0.42</td>
<td>OECD</td>
</tr>
<tr>
<td>$\alpha_L$: Labor input share in output of final goods</td>
<td>0.41</td>
<td>Labor Share (70%)</td>
</tr>
<tr>
<td>$\lambda$: Armington weight of domestic inputs</td>
<td>0.6</td>
<td>OECD ($m^*/GDP = 0.120$)</td>
</tr>
<tr>
<td>$\overline{G}$: Government expenditure criterion</td>
<td>0.083</td>
<td>OECD ($\overline{G}/GDP = 0.240$)</td>
</tr>
<tr>
<td>$\overline{Z}$: Government transfer criterion</td>
<td>0.051</td>
<td>OECD ($\overline{Z}/GDP = 0.147$)</td>
</tr>
<tr>
<td>$\overline{B}$: Government debt criterion</td>
<td>0.327</td>
<td>OECD ($\overline{B}/GDP = 1.07$)</td>
</tr>
<tr>
<td>$\tau$: Tax rate criterion</td>
<td>0.343</td>
<td>OECD ($\overline{R}/GDP = 0.39$)</td>
</tr>
<tr>
<td>$\overline{\tau}_d$: Tax rate criterion in the default state</td>
<td>0.409</td>
<td>OECD ($\overline{R}_d/GDP_d = 0.48$)</td>
</tr>
<tr>
<td>$\zeta^*$: Elasticity of transfers to productivity</td>
<td>-0.137</td>
<td>Estimation</td>
</tr>
<tr>
<td>$\delta$: Cutback rate</td>
<td>0.25</td>
<td>Households’ financial assets</td>
</tr>
<tr>
<td>$(b_l, p_l)$: Fiscal limit shape, low probability</td>
<td>(1.4, 0.3)</td>
<td>Bi and Traum (2014)</td>
</tr>
<tr>
<td>$(b_h, p_h)$: Fiscal limit shape, high probability</td>
<td>(2.0, 0.999)</td>
<td>Bi and Traum (2014)</td>
</tr>
</tbody>
</table>

Finally, following Bi and Traum (2014), I set the parameters of probability of crisis
\( \eta_1 \) and \( \eta_2 \) as

\[
\eta_1 = \ln \left( \frac{p_l}{1 - p_l} \right) - \eta_2 b_l, \quad \eta_2 = \frac{1}{b_l - b_h} \ln \left( \frac{p_l (1 - p_h)}{p_h (1 - p_l)} \right)
\]

where \( b_l \) and \( b_h \) are the fiscal limit criteria of debt-to-GDP ratios, taking \( b_l < b_h \). \( p_l \) and \( p_h \) are the probability of falling into debt crisis corresponding to \( b_l \) and \( b_h \). I set the gap between \( b_h \) and \( b_l \) to 60% GDP, and \( p_h \) and \( p_l \) are 0.999 and 0.3 respectively from Bi and Traum (2014). I set the lower and higher criteria of debt-to-GDP ratios \( b_l \) and \( b_h \) to 140% and 200% because debt-to-GDP ratio was about 170% one quarter before the crisis. Figure 4.1 shows the probability of crisis and price of government bonds following equations (4.26) and (4.24) respectively. The black curve shows the baseline fiscal limit, setting 30% crisis probability at 140% of debt-to-GDP ratio. The dotted blue and red curves report the cases that the criteria shift \( \pm 10\% \) of debt-to-GDP ratio from the baseline. I examine the cases of these two different values in the section on sensitivity analysis.

Figure 4.1: Fiscal Limit

(a) Probability of Crisis  
(b) Price of Government Bonds

Note: Debt-to-GDP ratios in the horizontal axis represent previous debt outstanding.
4.4 Quantitative Results

4.4.1 Decision Rule

Figure 4.2 depicts the decision rule for government bonds. The red and pink curves show newly issued government bonds at maturity base ($B_t$) and discounted base ($q_t B_t$) respectively. There are two main differences in the decision rule between the previous chapter and this chapter. First, because of the high value of the discount factor, the price of government bonds is high under the region of zero crisis probability. Thus, the maturity base newly issued government bonds take close values to the discounted base. Second, the gap between maturity base and discounted base is wider as the debt-to-GDP ratio increases, exceeding the threshold of 130% of debt-to-GDP ratio because the cutback rate is higher than in the previous chapter. As Figure 4.1 (b) shows the price of government bonds declines sharply, but the government cannot reduce the discount base of issuance of government bonds in order to satisfy the equation $q_t B_t = B_{t-1} + G_t + Z_t - T_t$. Thus, the government needs to repay a higher level of government bonds in the next period.

Figure 4.2: Decision Rule of Government Bond

Figure 4.3 describes the decision rules of the main endogenous variables. The black
Figure 4.3: Decision Rule

The blue and green curves correspond to high government consumption and low TFP under the normal government state, and the pink dotted line is low TFP under the crisis state.

First of all, the tax rate in the crisis state is always higher than the normal state under 200% of debt-to-GDP ratio because of the high exogenous tax rate in the crisis state. Next, imported intermediate goods decline dramatically due to the crisis. In the case of 150% of debt-to-GDP ratio, the amount drops more than 50% under the
steady state case. These two effects, the high tax rate and low amount of imported intermediate goods, are the main causes of output decline in the crisis. The amount of output declines about 15% from the normal to the crisis state in the case of 150% of debt-to-GDP ratio. Tax revenue in the crisis state is higher than in the normal state under about 190% of debt-to-GDP ratio in the steady state because the high tax rate outweighs the low output. The GDP difference between the crisis and normal states does not change so much from output because although the amount of imports is low in the crisis state, their price is higher than in the normal state. Next, the movement of consumption is highly non-linear in the normal state. Because the government bonds are held by foreign investors, the outflow of output goods reduces by the amount of $B_{t-1} - q_t B_t$. Thus, as the price of government bond decreases and the maturity base repayment increases, the amount of outflow is larger and domestic consumption shrinks. In the crisis state, the consumption level is the almost same regardless of TFP because the low tax revenue sustains the amount of consumption. Next, wages are a the function of consumption and labor as equation (4.11) shows, so wages declines as consumption declines. The decision rules of domestic intermediate goods and labor measured in hours worked are straightforward. Although the low TFP does not affect hours worked much, these two variables increase in high government consumption and decreases in a crisis.

4.4.2 Simulation Results

I generate stochastic TFP processes 2000 times for 500 periods, truncating the first 100 periods in order to invalidate the effects of initial conditions\(^2\). After the two-year mandatory crisis period, the government has an opportunity to regain the normal state with 0.083\% probability every period, which is widely used in the endogenous default model.

\(^2\)I set the initial condition of the economy as the normal state of the government, steady state of TFP, government consumption and capital stock, and 30\% of debt-to-GDP ratio.
Baseline Results

Table 4.2 shows the results of the baseline simulation, focusing on the average debt-to-GDP ratio, crisis frequency (default frequency in data), average bond spread and the correlations of GDP with important variables. Unlike the endogenous default model, the model replicates the high debt-to-GDP ratio (97.2%), which is close to the actual Greek ratio (106.7%). The probability of crisis reports are also similar to the data value, showing 0.55% per quarter in the baseline simulation and 0.53% in the data. The average bond spread is higher than the data mainly due to the 25% cutback rate of government bonds. The second half of Table 4.2 shows correlations between GDP and the main variables. Output, consumption and imported and domestic intermediate goods show quite high relations with GDP in both the data and the simulation. In terms of hours worked, similar to the Spanish case, the GDP relation with hours worked is negative due to the downward trend in working time. However, labor input is highly related to GDP in the simulation. Next, while the tax rate is almost irrelevant with to GDP in the data, the simulation result shows a negative relation between tax rate and GDP because high tax rates decrease output. Tax revenue indicates a high relation to GDP in the data, but the simulation shows quite a low relation because the effects of output and tax rate offset each other.

Figure 4.4: Simulation Result 1

(a) Debt-to-GDP ratio    (b) Probability of Crisis    (c) Bond Spread

Note: The left vertical axis in graph (b) represents the basis point of five-year government bond CDS.
Table 4.2: Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Debt/GDP</td>
<td>106.7%</td>
<td>97.2%</td>
</tr>
<tr>
<td>Default/Crisis Frequency</td>
<td>0.53%</td>
<td>0.55%</td>
</tr>
<tr>
<td>Average Bond Spread</td>
<td>2.27%</td>
<td>9.44%</td>
</tr>
<tr>
<td>The correlation with GDP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Output</td>
<td>99.6%</td>
<td>99.8%</td>
</tr>
<tr>
<td>-Consumption</td>
<td>98.5%</td>
<td>94.6%</td>
</tr>
<tr>
<td>-Imported Intermediate Goods</td>
<td>92.8%</td>
<td>98.9%</td>
</tr>
<tr>
<td>-Domestic Intermediate Goods</td>
<td>99.3%</td>
<td>94.4%</td>
</tr>
<tr>
<td>-Hours Worked</td>
<td>-69.7%</td>
<td>79.2%</td>
</tr>
<tr>
<td>-Tax Rate</td>
<td>7.55%</td>
<td>-87.0%</td>
</tr>
<tr>
<td>-Tax Revenue</td>
<td>96.4%</td>
<td>5.1%</td>
</tr>
</tbody>
</table>

Note 1: The average debt-to-GDP ratio is the period of the non-default state.

Note 2: The Greek government had experienced default four times since 1829.

Note 3: Bond spread in data is 10-year government bond premium toward the yields of German bonds, while that in simulation represents the annual rate of reciprocal of government bond price minus the world risk-free interest rate.

Next, Figure 4.4 depicts the transition of debt-to-GDP ratio, probability of crisis and bond spread from four years to one quarter before the Greek government’s virtual default in the first quarter of 2012. The debt-to-GDP ratio in data is always higher than in the simulation before the crisis, but both ratios are increasing with rapid pace as the crisis approaches. The simulation result for the probability of crisis was lower than 10% one year before the crisis, but the rate jumps up significantly within one year. The data also shows a similar result in that the CDS begins to increase about one year before the crisis. The bond spread in the simulation corresponds to the probability of default, so the rate surges from one year before the crisis. The bond spread in the data also begins to increase about two years before the crisis, with fluctuation.

Figures 4.5 and 4.6 show the transitions of the main endogenous variables around the crisis. The red dotted curve is the data for the actual Greek economy, and the black curve shows the simulation results before and at the time of crisis, and the blue and green curves are the simulation result of recovery after the crisis, representing
mandatory crisis periods of two and four years respectively. The simulation result generally captures the important features of the actual Greek crisis such as GDP (figure h) and tax revenue (figure g). I set the two-year mandatory crisis duration as the baseline, but the result of the four-year simulation is more compatible with the data because the level of actual GDP has been stagnant since the crisis. Output (figure c) is decreasing in debt-to-GDP during the crisis period because the return from capital
holdings is low during the crisis period, so the stock of capital is diminishing due to depreciation. Government consumption (figure e) is different between the data and the simulation. The Greek government needed to take drastic fiscal austerity measures, and government consumption decreased by about 25% after the crisis from the level of three years before the crisis. In my simulation, I calibrated the parameters of exogenous government consumption from the normal state, then proceeded to the crisis state.
Thus, the government consumption effect is underestimated in the simulation. This difference suggests that government behavior in the crisis state should be transformed from the normal state. On the other hand, following Mendoza and Yue (2012), I assume financial autarky from the world credit market in the crisis state, but the simulation result (figure b) overestimates the effect of the decline of imported intermediate goods. The decline in domestic intermediate goods in the data (figure j) is more severe than that in the simulation.

**Alternative Scenario**

### Table 4.3: Alternative Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Crisis</th>
<th>Average</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D/GDP</td>
<td>Freq.</td>
<td>Spread</td>
<td>Non-def</td>
</tr>
<tr>
<td>Data</td>
<td>106.7%</td>
<td>0.53%</td>
<td>2.27%</td>
<td>-</td>
</tr>
<tr>
<td>Baseline</td>
<td>97.2%</td>
<td>0.55%</td>
<td>9.44%</td>
<td>-</td>
</tr>
<tr>
<td>(1) Cutback rate ($\delta$:0.25)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>99.3%</td>
<td>0.33%</td>
<td>8.86%</td>
<td>0.18%</td>
</tr>
<tr>
<td>0.35</td>
<td>95.4%</td>
<td>0.73%</td>
<td>10.05%</td>
<td>1.08%</td>
</tr>
<tr>
<td>(2) Government Bond Criterion ($B/GDP$: 1.07)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.92</td>
<td>84.1%</td>
<td>0.01%</td>
<td>8.11%</td>
<td>-1.43%</td>
</tr>
<tr>
<td>1.22</td>
<td>95.6%</td>
<td>1.64%</td>
<td>11.33%</td>
<td>2.77%</td>
</tr>
<tr>
<td>(3) Tax revenue criterion ($R_n/GDP$: 0.39)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5%</td>
<td>96.1%</td>
<td>2.05%</td>
<td>11.97%</td>
<td>3.36%</td>
</tr>
<tr>
<td>+3%</td>
<td>82.34%</td>
<td>0.01%</td>
<td>8.10%</td>
<td>-0.74%</td>
</tr>
<tr>
<td>(4) Government consumption criterion ($G/GDP$: 0.240)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5%</td>
<td>82.6%</td>
<td>0.01%</td>
<td>8.10%</td>
<td>-0.14%</td>
</tr>
<tr>
<td>+5%</td>
<td>97.0%</td>
<td>1.63%</td>
<td>11.27%</td>
<td>1.71%</td>
</tr>
<tr>
<td>(5) Government transfer criterion ($Z/GDP$: 0.147)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5%</td>
<td>89.7%</td>
<td>0.07%</td>
<td>8.36%</td>
<td>0.87%</td>
</tr>
<tr>
<td>+5%</td>
<td>97.9%</td>
<td>1.26%</td>
<td>10.68%</td>
<td>0.61%</td>
</tr>
<tr>
<td>(6) Recovery ($B^r$: 30%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-cutback</td>
<td>108.7%</td>
<td>1.17%</td>
<td>11.31%</td>
<td>-2.28%</td>
</tr>
<tr>
<td>0%</td>
<td>95.2%</td>
<td>0.53%</td>
<td>9.37%</td>
<td>0.68%</td>
</tr>
</tbody>
</table>

Note: "Normal" and "Crisis" in the section on GDP represent GDP decline from the baseline and the decline shows GDP decrease from the maximum value 3 years before the crisis.
Table 4.3 shows the crisis effects on the economy by changing six parameters mainly related to government policies. First, section (1) shows the cases of different cutback rates. A low cutback rate reduces the probability of crisis because the amount of government bonds that the government has to issue will be lower than the baseline under the risky debt region. Corresponding to the crisis frequency, the average bond spread also declines from the baseline level. Sections (2) and (3) report changes in revenue criteria. The criteria of low government bond and high tax revenue induce high tax rates, and that leads to low debt-to-GDP ratio, crisis frequency and bond spread. However, high tax rates sacrifice the level of GDP. Next, sections (4) and (5) show the results of different criteria of government expenditure. The austerity policy, and the low government consumption and transfers, also lead to low debt-to-GDP ratio, crisis frequency and bond spread. Finally, section (6) reports different cases of the government’s debt when it regains the normal state. If the government regains the normal state with the amount of debt outstanding remitted by the cutback rate at the time of crisis, the amount of debts is too high and the economy becomes less stable as the high debt-to-GDP ratio and crisis frequency shows. However, even though the government regains the normal state with zero debt, the economy does not become especially stable because the government rapidly increases its debts due to the low tax rate.

4.5 Conclusion

I provide the DSGE model to analyze sovereign debt crises in advanced countries in which government bonds are held by foreigners. Based on the previous chapter, the model requires three modifications due to the introduction of foreign debts. First, the difference between the government’s repayment and the issuance of new bonds is the net outflow of domestic output. Second, instead of government bonds, capital
is introduced for inter-temporal utility maximization. The diminishing capital stock during the crisis decreases output. Finally, the fiscal limit is drawn from the exogenous distribution depending on the debt outstanding.

I calibrate the Greek economy for the analysis of the country’s debt crisis around 2012. The compatibility of the model to the actual Greek crisis is generally sound, capturing the high debt-to-GDP ratio, crisis frequency and economic contraction. However, the model overestimates the decline in imported intermediate goods and underestimates the path of domestic contraction. This suggests that it would be important to consider how the domestic path amplifies the effects of debt crises in the future research.
Chapter 5

Concluding Remarks

This thesis consists of three chapters which focus on the development of a dynamic stochastic general equilibrium model to analyze sovereign debt crises. I summarize the main points of the economic model and important findings in this concluding chapter. Then, I discuss opportunities for future research.

In Chapter 2, I incorporate financial intermediaries into the framework of the dynamic general equilibrium model of endogenous sovereign default. The main feature of the model is the contingency of the default state from the non-default state because government bonds become non-performing by the decision of default and financial intermediaries have to eliminate a certain proportion of these non-performing bonds from their net worth. This characteristic make it possible to quantify the financial contraction effect on the economy. In addition to this, the model captures the phenomenon of "Too-Big-to-Default". A government which accumulates a large amount of debts may choose not to default due to the high damage to the economy. The simulation result captures important features of the Argentinean default in 2001 such as debt-to-GDP ratio, frequency of crisis and contraction of the economy.

The endogenous default model is not applicable to advanced countries because the government is assumed to be able to estimate the effect of default precisely and its effect
has to be small and short, and also the model replicates only relatively low amounts of debt outstanding. Thus, in Chapter 3, I propose a new model of sovereign debt crisis in an advanced country. The model assumes the government’s incapability to repay its debts rather than its willingness. If government debt outstanding exceeds an endogenous fiscal limit, the government falls into crisis. The model defines the fiscal limit as the summation of discounted future primary surplus under the tax rate to maximize the government’s tax revenue. In the crisis state, the government is forced to impose an exogenous tax rate and the amount of imported intermediate goods declines because of the increase of interest rate of working capital for the purchase of a certain fraction of these imports. The simulation result of the model can replicate high debt-to-GDP, and the model’s compatibility with the debt crisis in Spain in 2012 is generally sound.

In Chapter 4, I introduce foreign bonds based on the model in Chapter 3 for the analysis of a country where the majority of bonds are held by foreign countries. In the model, because of the foreign investors’ purchase of government bonds, they price the government bonds and the net outflow of domestic output occurs. I introduce capital, instead of government bonds, for the maximization of inter-temporal utility. Also, the fiscal limit is drawn from the exogenous distribution due to the government’s lower incentive to serve its debts. I calibrate the Greek debt crisis in around 2012. The model overestimates external effects and underestimates the domestic contraction effects of the amplification of the crisis, but it explains the overall movement around the crisis.

Two directions for future research are proposed. The first direction is to extend the current debt crisis models to be adoptable for the analysis of actual debt crises. Sovereign debt crises are caused by many factors and affect the economy in many ways, so it is extremely intricate to capture the mechanism. However, I would like to shed light on debt crises from many angles. The next direction is policy implications.
According to Reinhart and Rogoff (2013), there are five ways to decrease the debt-to-GDP ratio: economic growth, austerity measures, default or restructuring, inflation surprise and financial repression. I would like to investigate which policy is achievable and minimizes social welfare loss under different countries’ economic environments.

I research the sovereign debt crisis not for academic achievement or curiosity but to contribute to people and society. Today, many countries accumulate massive public debts, and some of them are projected to be unsustainable. I am apprehensive of the consequences of these high debts, because sovereign debt crisis could ultimately be a matter of lives. I hope from the bottom of my heart that research into sovereign debt crises proceeds further and finds ways to minimize negative effects on the economy.
Bibliography


Appendix A

Chapter 1 Appendix

A.1 Data in Figure 1.1

- Public Debt-to-GDP Ratio: Government debts are measured in gross. The data source from 1990 to 2005 is the historical public debt database (September 2012 version), IMF Fiscal Affairs Department. The data source from 2006 to 2021 is IMF Fiscal Monitor (October 2016 version).

- Dependency Ratio: This ratio is defined as (population aged 65 years or over) / (population aged 20 to 64 years) in the graph. The data source is OECD.
Appendix B

Chapter 2 Appendix

B.1 Diagrams

Figure B.1: The Sketch of My Model
Figure B.2: Timing

Note: Parentheses and brackets represent government debts and non-performing bonds respectively.

B.2 Computational Algorithm

I solve the model using the discrete state space (DSS) with a two-loop algorithm. The outside loop is the price of government bonds consistent with each state space, and the inside loop is the iteration of government value functions in both non-default and default states. In order to obtain precise results, I apply linear interpolation to the government bonds.

<Flow of Programming>

1. Discretize the state space of TFP \( (A) \) and total government bonds \((b)\). I use Tauchen’s method (1986) to gain the state space of TFP, following equation (2.5). I set the center point to zero and the width to three standard deviations and take 25 discrete grid points. Also, I uniformly generate 51 discrete grid points of government bonds within the range of \([b_{\text{max}}, 0]\).

2. Initialize the government value functions \(V^{nd}_0(A, b)\) and \(V^d_0(A, b)\) and the govern-
ment bond price \( q_0(A, b') \). The initial guess of government price is \( q_0 = 1/r^d \).

3. The government’s value function iteration.

(a) Solve the government optimal debts \( b' \) by using the global search procedure in each non-default state matrix of \( A \) and \( b \) so as to be consistent with the competitive equilibrium of private agents.

(b) Obtain allocations, prices and utilities in each state matrix of \( A \) and \( b^1 \).

(c) Calculate updated value functions \( V^{nd}_1(A, b) \) and \( V^d_1(A, b) \).

(d) If the difference between current and updated government value functions in each non-default and default case is small enough (i.e. \( \sup \{|V^{nd}_1(A, b) - V^{nd}_0(A, b)|, |V^d_1(A, b) - V^d_0(A, b)|\} < \varepsilon \)), stop the iteration. Otherwise, go back to procedure (a) with the updated government value functions.

4. Calculate the default set \( \Gamma^A(b) \), the probability of default \( \pi^e(A, b') \) and the government bond price \( q_1(A, b') \).

5. If the difference between current and updated government bond prices is a small enough value (i.e. \( \sup \||q_1(A, b') - q_0(A, b')|| < \varepsilon \)), stop the iteration. Otherwise, go back to procedure 3 with the updated government bond prices.

### B.3 The Assumption of Exogenous Proportion of Domestic Bond Holding

I assume the proportion of domestic financial intermediaries’ government bond holdings over debt outstanding is exogenously determined for the following three reasons.\(^1\)

\(^1\)In the default state, \( b \) represents non-performing bonds. That is, the value of each grid point of \( b \) is multiplied by the proportion of domestic agent’s bond holding (\( v \)) and the haircut rate (\( h \)).
First, governments often intervene in domestic financial intermediaries’ portfolio allocations in sovereign debt crises. Becker and Ivashina (2014) investigate the relationship between governments and banks during the European sovereign debt crisis and point out that the governments urged their domestic banks to hold newly issued government bonds through the intervention of banks’ management, especially in countries which accumulated massive public debts and exposed high sovereign default risk such as in Greece and Italy. Similar to the European debt crisis, the government’s intervention toward banks was also observed in Argentina before the government officially defaulted in December 2001. Domestic financial intermediaries were practically enforced to conclude the reduction of interest payments and extension of bonds’ maturities with the (unreliable) guarantee of future tax income\(^2\), although the Argentinean government stressed it was "voluntary" debt restructuring. Thus, financial intermediaries could not freely choose the amount of bond holdings around the default period.

The second reason is that the proportion of domestic debt has been a narrow value range of narrow over a long period. Reinhart and Rogoff (2011) provide data that the ratio of domestic agents to total central government debt has been shifting in a range between about 80% and 95% in advanced countries and 40 to 60% in emerging countries since the 1960s.

The final reason is the limitation of obtaining the inner solution within the framework of an endogenous sovereign default model. As Arellano (2008) points out, the risky borrowing region (the default probability is between 0% and 100%) is quite narrow, so the price of government bonds would take corner solutions in most economic states. In other words, if financial intermediaries can choose the amount of government bonds \(a_{t+1} \in [0, -b_{t+1}]\) freely, they will hold the maximum amount of bonds \(a_{t+1} = -b_{t+1}\) or \(a_{t+1} = 0\) in most cases and the region to take \(a_{t+1} \in (0, -b_{t+1})\) will be limited. Even though I introduce additional assumptions on financial intermediaries’ portfolios such

\(^2\)See International Monetary Fund (2004).
as the Value-at-Risk constraint or portfolio theories of risk diversification, they would contribute little to obtaining the inner solutions.

### B.4 Financial Intermediaries’ Optimization Problem

In this appendix, I derive the financial intermediaries’ first-order conditions including lending cost functions in both non-default and default states, and explain the economic interpretations. The financial intermediaries’ maximization problem in the non-default state \((2.14)\) substituting the lending cost function \((2.35)\) is as follows.

\[
\max_{k_t^s, d_t} \Pi_{t}^{FI} = r_t^k k_t^s + a_t - q_t a_{t+1} - r^d d_t - \left[ \phi_1 k_t^s + \frac{\phi_2}{2} \left( k_t^s - \left( \frac{n}{\chi A_{Arg}} - \xi q_t a_{t+1} \right) \right) \right]^2
\]

(B.1)

The constraints are the equality of balance sheet \((2.15)\) and the capital adequacy requirement \((2.16)\). The first-order conditions of financial intermediaries’ optimization problems in the non-default state are:

\[
r_t^k = r^d + \phi_1 + \frac{\phi_2}{k_t^s} \left( k_t^s - \left( \frac{n}{\chi A_{Arg}} - \xi q_t a_{t+1} \right) \right) + \lambda_t^{nd} n(k_t^s + \xi q_t a_{t+1})^{-2}
\]

(B.2)

\[
0 \leq \left( \frac{n}{k_t^s + \xi q_t a_{t+1}} - \chi \right) \perp \lambda_t^{nd} \geq 0
\]

(B.3)

where \(\lambda_t^{nd}\) is the Lagrange multiplier associated with the constraint \((2.16)\).

Similarly, their maximization problem in the default state \((2.17)\) with lending cost function \((2.36)\) is
max \Pi^{FI}_{k_t^d, d_t} = r^s_t k_t^s - r^d_t d_t - \left[ \phi_1 k_t^s + \frac{\phi_2}{2} \left( \frac{n_t^d}{\chi_{Arg}} - \phi_3 \right) \left( k_t^s \right)^2 \right]

(B.4)

The first-order conditions of financial intermediaries’ optimization problems in the default state are:

\[ r^k_t = r^d_t + \phi_1 + \frac{\phi_2}{k_t^s} \left( k_t^s - \left( \frac{n_t^d}{\chi_{Arg}} - \phi_3 \right) \right) + \lambda_t^d n_t^d (k_t^s)^{-2} \]  

(B.5)

\[ 0 \leq \left( \frac{n_t^d}{k_t^s} - \chi \right) \perp \lambda_t^d \geq 0 \]

(B.6)

where \( \lambda_t^d \) represents the Lagrange multipliers associated with the capital adequacy constraint in the default state \((2.19)\).

Equations \((B.2)\) and \((B.5)\) indicate the decision rules of interest rates in the non-default and default states respectively. The interest rates in both cases are determined by four terms: the risk-free interest rate (the first term), base premium on lending (the second term), deviation from the criteria (the third term) and the additional premium in the case that the capital adequacy ratio hit the constraint (the fourth term). With respect to the third term, the interest rate is an increase function on lending, so it will be costly for goods producing firms to acquire working capital. The interest rate in the default state is higher than in the non-default state under the same TFP level because the lending criteria in the default state are lower than in the non-default state. Also, if the risk weight on government bonds is strictly larger than zero in the non-default state, the interest rate increases as the expenditure for newly issued government bonds \(q_t a_{t+1}\) increases. With respect to the fourth term, the Lagrange multipliers take strictly positive values if the capital adequacy ratio hits the minimum requirement constraint \((\chi = 0.115)\), so the interest rate on lending is higher than in the unconstrained case. Equations \((B.3)\) and \((B.6)\) are complementary conditions
regarding to the capital adequacy constraint in each the non-default and default states.

B.5 Debt Restructuring

The Argentinean debt restructuring was extremely intricate. There were five main restructures around the default in December 2001. Moreover, government bonds consisted of 152 series, eight governing laws and six different currencies at the time of default. Thus, each bond had different haircut rates applied, different currencies affected the real value of bonds and debt restructuring lawsuits proceeded under different laws when bond holders did not agree on haircut rates and sued the Argentinean government.

The first debt restructuring was implemented in June 2001. The government extended the maturities of its bonds equivalent to 29 billion dollars in order to reduce short-term repayments. However, that caused higher debt obligation in the medium and longer terms. In November 2001, the government needed to reduce repayments again due to low tax revenue. This debt restructuring was originally designed in two phases: phase 1 for domestic agents and phase 2 for foreign lenders, but phase 2 was not implemented because the government chose default soon after phase 1 took place. The government reduced its interest payments and extended maturities again in the phase 1 debt restructuring. This exchange was backed by a financial transaction tax, and Standard & Poor's graded the restructuring as "selective default". After the default in December 2001, the government exchanged dollar-based domestic bonds into peso-based due to the abandonment of the fixed exchange rate.

In 2005, the government implemented full-scale debt restructuring, achieving about 71% to 75% haircut rates\(^3\) toward approximately 76% defaulted bonds. Edwards (2015) states that this haircut rate was "excessively high". In order to compensate for the high rate, the government granted GDP-indexed warrants, which were contingent on

\(^3\)See Sturzenegger and Zettelmeyer (2006).
GDP level and growth. The value of this warrant was equivalent to only about 2 cents in a dollar in February 2005. However, the value increased significantly because of the rapid recovery of the Argentinean economy. Thus, the real haircut rate would be considered to be lower than the original haircut rate due to the benefit from this warrant.

The next debt restructuring was held in 2010 for holdouts. Although the deal in 2010 was less attractive than in 2005 because of missing the opportunity to obtain repayments of GDP warrants, the majority of creditors agreed to exchange their defaulted bonds. The participation rate reached 91% of the defaulted bonds.

After that, the Argentinean government neglected holdouts. However, some holdouts filed a lawsuit against the government under the U.S. jurisdiction. In June 2014, because the U.S. Supreme Court declined to hear the case, a lower court determined that the Argentinean government must pay full principal plus interest to the plaintiffs. In addition to this, the court prohibited proceeding to payments of restructured bonds unless the government had completed paying for them. Because the government did not obey the court’s decision and rejected these payments, it fell into (technical) default after the one month grace period\(^4\).

Because not only Argentina but also many other defaulted countries such as Greece had faced difficulties in dealing with debt restructuring, the UNCTAD (UN General Assembly, 2015) set nine principles for debt restructuring: sovereignty, good faith, transparency, impartiality, equitable treatment, sovereign immunity, legitimacy, sustainability and majority restructuring. Based on these principles, international organizations are expected to build frameworks to settle their debt restructuring problems.

\(^4\)Unlike other typical defaults, the Argentinean government had the resources and willingness to disburse payments, so this case can be categorized as "technical default".
B.6 Sensitivity Analysis

I have already reported different values for several parameters such as the haircut rate \((h)\), the proportion of domestic bond holdings \((v)\), and the maximum debt amount in the alternative scenario section. In this appendix, I examine the sensitivity analysis by changing the values of several parameters: the discount factor of households \((\beta)\), the probability of recovering the non-default state \((\vartheta)\), the average capital adequacy ratio \((\chi_{Arg})\) and the extra penalty on lending by default \((\phi_3)\). The main results are in Table B.1. The first and second rows show the data and baseline results respectively.

Table B.1: Sensitivity Analysis

<table>
<thead>
<tr>
<th></th>
<th>Average Debt/GDP</th>
<th>Default Frequency</th>
<th>Average Bond Spread</th>
<th>Decline Y</th>
<th>Decline C</th>
<th>Decline L</th>
<th>Decline Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>37.06%</td>
<td>0.77%</td>
<td>1.92%</td>
<td>-10.56%</td>
<td>-17.31%</td>
<td>-27.94%</td>
<td>17.50%</td>
</tr>
<tr>
<td>Baseline</td>
<td>33.13%</td>
<td>0.71%</td>
<td>0.19%</td>
<td>-16.15%</td>
<td>-16.34%</td>
<td>-26.44%</td>
<td>9.76%</td>
</tr>
<tr>
<td>(1) Discount factor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta = 0.84)</td>
<td>32.15%</td>
<td>1.29%</td>
<td>0.32%</td>
<td>-15.70%</td>
<td>-16.47%</td>
<td>-26.50%</td>
<td>10.62%</td>
</tr>
<tr>
<td>(\beta = 0.92)</td>
<td>34.29%</td>
<td>0.30%</td>
<td>0.04%</td>
<td>-17.85%</td>
<td>-18.10%</td>
<td>-28.62%</td>
<td>9.93%</td>
</tr>
<tr>
<td>(2) Probability of recovering non-default</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\vartheta = 0.030)</td>
<td>34.49%</td>
<td>0.50%</td>
<td>0.08%</td>
<td>-17.46%</td>
<td>-18.02%</td>
<td>-28.94%</td>
<td>10.62%</td>
</tr>
<tr>
<td>(\vartheta = 0.083)</td>
<td>28.34%</td>
<td>1.59%</td>
<td>0.39%</td>
<td>-14.99%</td>
<td>-15.54%</td>
<td>-25.02%</td>
<td>9.89%</td>
</tr>
<tr>
<td>(3) The average capital adequacy ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\chi_{Arg} = 0.13)</td>
<td>15.73%</td>
<td>2.20%</td>
<td>0.86%</td>
<td>-10.21%</td>
<td>-10.11%</td>
<td>-17.64%</td>
<td>7.33%</td>
</tr>
<tr>
<td>(\chi_{Arg} = 0.15)</td>
<td>34.86%</td>
<td>0.13%</td>
<td>0.03%</td>
<td>-19.00%</td>
<td>-19.64%</td>
<td>-30.14%</td>
<td>10.52%</td>
</tr>
<tr>
<td>(4) Extra penalty on lending in the default state</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\phi_3 = 1.08)</td>
<td>31.49%</td>
<td>0.93%</td>
<td>0.26%</td>
<td>-14.98%</td>
<td>-15.06%</td>
<td>-24.34%</td>
<td>9.15%</td>
</tr>
<tr>
<td>(\phi_3 = 1.32)</td>
<td>34.45%</td>
<td>0.43%</td>
<td>0.08%</td>
<td>-17.90%</td>
<td>-18.35%</td>
<td>-29.24%</td>
<td>10.49%</td>
</tr>
</tbody>
</table>

Note 1: Y, C and L represent output, consumption and lending respectively.
Note 2: Parentheses in each section represent baseline values.

Section (1) reports that the discount factor changes ±0.04 points from the baseline value of 0.88. Changing this parameter affects the behavior of government in two ways. In the case of \(\beta = 0.92\) (i.e. the government is more farsighted), the government is more reluctant to compensate for current low consumption by the issuance of bonds and is less likely to choose default to reduce the burden of its debt. These two effects
offset each other and average debt-to-GDP increases only about 1% point from the baseline case. The default frequency and average bond spread decline from the baseline case because the latter effect of avoidance of default is larger than the former effect. However, the economic impact on default will be larger than the baseline because the government defaults with more debts and the amount of non-performing bonds is larger. This phenomenon is implied for advanced economies. It would be considered that the discount factor in these countries is higher than for developing economies due to low interest rates, so this is one reason that these governments are less likely to choose default. The effects of default on advanced economies are larger than the developing economies.

In the second section (2), I examine the cases of lower ($\vartheta = 0.030$) and higher ($\vartheta = 0.083$) probability of recovering the non-default state than the baseline case ($\vartheta = 0.044$). The higher value at 0.083 is the median duration to regain partial market access according to the analysis by Dias and Richmond (2008). When $\vartheta$ is 0.083, the government knows it can recover the non-default state from the default state earlier than the baseline case. Thus, the default value increases and the government chooses default more frequently. Besides, the government chooses default with lower level of debts, so financial intermediaries eliminate lower levels of non-performing bonds. Thus, the default effects on the economy are lower than in the baseline case.

Section (3) shows the changes in the average capital adequacy ratio $\chi_{Arg}$. If this value lowers from 14.6% to 13.0%, financial intermediaries increase the amount of lending in both non-default and default states, but the effect on the latter state is larger than in the former state. Thus, the default effects on main variables such as output, consumption and lending are smaller than the baseline. However, these effects increase the default frequency from 0.71% to 2.20% per quarter due to the lower effect of default. The average debt-to-GDP is about half of the baseline case because the government chooses default with lower debts. When the parameter increases to 0.15,
the effect on the economy is opposite to the case of lower value. I also examine the cases that the minimum capital adequacy ratio $\chi$ and risk weight on government bonds $\xi$ are higher than the baseline case, but the capital adequacy constraints in the both non-default state (2.16) and default state (2.19) hit the upper bound only in the case of lower TFP. Thus, the results do not change much around the default period.

Section (4) reports results for different values of extra penalty on lending in the default state $\phi_3$. The low value on this parameter mitigates the effect of financial contraction by default, so the decrease in lending by default becomes smaller than the baseline. Thus, declines in output and consumption are also lower, the interest rate decreases and the default frequency and bond spread increase from the baseline case.

### B.7 Data in Figure 2.4

- **GDP**: The data is quarterly gross domestic product of the expenditure approach, fixed PPPs, seasonally adjusted. The source is OECD.

- **Lending and Consumption**: I calculate seasonally adjusted real lending and consumption from nominal raw data by subtracting inflation and seasonal effects. The method of seasonal adjustment is X-12-ARIMA, and the inflation data is the seasonally adjusted quarterly GDP deflator. The definition of lending is financial institutions’ credit to the private sector measured in peso. The definition of nominal consumption is quarterly household consumption expenditure, nominal annualized rate and measured in peso. The data source of lending is the Central Bank of Argentina and both nominal consumption and inflation are International Financial Statistics.

- **Interest rate on lending**: The data source is International Financial Statistics.
Appendix C

Chapter 3 Appendix

C.1 Diagrams

Figure C.1: The Sketch of My Model
C.2 Computational Algorithm

I obtain the fiscal limit first, and derive the discrete state space of the model by using the fiscal limit as one of the state variables.

<Fiscal Limit>

I adopt the Markov Chain Monte Carlo (MCMC) simulation in order to calculate the distribution of fiscal limit.

– Flow of Programming –

1. Discretize the state space of the tax rate ($\tau$), TFP ($A$) and government consumption in deviation from steady state normalized by GDP ($g$). I set the tax rate between 0.15 and 0.85\(^1\) and uniformly take 300 grid points. I use Tauchen’s method (1986) to gain the state space of TFP and government consumption, following equations (3.4) and (3.2). In both variables, I take 31 grid points uniformly, setting the center points to zero.

\(^1\)This is because if the tax rate is close to one, the first-order conditions of final goods firms will not hold due to negative profits after tax. Also, if the tax rate is too low, the fiscal limit will be negative because of low tax revenue. Therefore, I restrict the tax rate to between 0.15 and 0.85.
2. Calculate values of endogenous variables \((C_t, L_t, w_t, M_t, m_t^*, m_t^d, L_t^f, L_t^m, Z_t, T_t, G_t, Y_t, p_t^m, p_t^s)\) for each tax rate and state variable given the interest rate of working capital \(r_t^*\), which is equivalent to \(r_f\) by using equations (3.1)(3.3)(3.5)(3.13)(3.16)(3.17)(3.21)-(3.23)(3.28)(3.31)(3.33)(3.37)(3.38).

3. Find the tax rate that maximizes the tax revenue in each state variable.

4. Draw the future shocks of these two variables \((A_{t+i}, g_{t+i})\) given the initial state \((A_t, g_t)\) and calculate fiscal surplus for \(i = 1, \ldots, 200\). Then, aggregate the discounted future fiscal surplus following the definition of fiscal limit (3.42).

5. Repeat procedure 4 for 2000 times for all initial states of \((A_t, g_t)\)

6. Take the average of the all simulations for the fiscal limit and calculate the cumulative density function of crisis probability.

<Discrete State Space>

I separately calculate the normal and crisis states. In terms of the crisis state, I solve the endogenous variables of current and expected values in the next period for each grid point of \((A_t, g_t)\).

In the normal state, following Coleman (1991) and Davig (2004), I find a fixed point in the decision rule for government bonds by using the monotone map method as \(B_t = f^b(A_t, g_t, B_{t-1})\). From the budget constraint of government (3.7), the price of government bonds is

\[
q_t = \frac{B_{t-1} + G_t + Z_t - T_t}{f^b(A_t, g_t, B_{t-1})}
\]

Also from the households’ optimization problem (3.14) and the specification of utility
function (3.43), the price of government bonds is

\[
q_t = \beta \mathbb{E}_t \frac{C(A_t, g_t, B_{t-1})}{C(A_{t+1}, g_{t+1}, f^h(A_t, g_t, B_{t-1}), B_{t+1}^{\max})} \cdot \left[ (1 - P_c(A_{t+1}, g_{t+1}, f^h(A_t, g_t, B_{t-1}), B_{t+1}^{\max})) + P_c(A_{t+1}, g_{t+1}, f^h(A_t, g_t, B_{t-1}), B_{t+1}^{\max})(1 - \delta) \right]
\]  

(C.2)

Then, from the above two equations (C.1) and (C.2), I obtain the following equation:

\[
\frac{B_{t-1} + G_t + Z_t - T_t}{f^b(A_t, g_t, B_{t-1})} = \beta \mathbb{E}_t \frac{C(A_t, g_t, B_{t-1})}{C(A_{t+1}, g_{t+1}, f^h(A_t, g_t, B_{t-1}), B_{t+1}^{\max})} \cdot \left[ (1 - P_c(A_{t+1}, g_{t+1}, f^h(A_t, g_t, B_{t-1}), B_{t+1}^{\max})) + P_c(A_{t+1}, g_{t+1}, f^h(A_t, g_t, B_{t-1}), B_{t+1}^{\max})(1 - \delta) \right]
\]

(C.3)

The decision rule for the newly issued government bonds, \( B_t = f^h(A_t, g_t, B_{t-1}) \) is derived as follows,

---Flow of Programming---

1. Discretize the state space of TFP (\( A_t \)), government consumption (\( g_t \)) and government bonds issued at the previous period (\( B_{t-1} \)). The number of grid points of TFP and government consumption is the same as the computation of the fiscal limit. I set the government bonds (\( B_{t-1} \)) between 0 and 2.5 times the steady state level of GDP, taking 100 grid points. Make an initial guess for the issuance of government bonds \( f^b_0(A_t, g_t, B_{t-1}) \).

2. Evaluate the probability of crisis \( \mathbb{E}_t P_{c,t+1} \) outside grid points of conjectured government bond issuance \( f^b_{t-1}(A_t, g_t, B_{t-1}) \) by using the piecewise linear interpolation method, and compute endogenous variables of current and expected values in the next period for each grid point.

3. Update the guess of government bond issuance \( f^b_t(A_t, g_t, B_{t-1}) \) from the old decision rule \( f^b_{t-1}(A_t, g_t, B_{t-1}) \) by calculating the numerator of the left hand side and the right hand side of the equation (C.3).
4. Check convergence of decision rule $f^h(A_t, g_t, B_{t-1})$. If the difference between the updated and old decision rules is small enough (i.e. $\sup||f^h_t(A_t, g_t, B_{t-1}) - f^h_{t-1}(A_t, g_t, B_{t-1})|| < \epsilon$), $f^h(A_t, g_t, B_{t-1})$ is the decision rule. Otherwise, go back to procedure 2.

C.3 Additional Explanation of Decision Rule

In the section 4.2, I explain decision rules of issuance of government bonds, probability of crisis and price of government bonds only in the case of steady state of TFP and government consumption. This appendix explores the decision rule on three variables in different cases of TFP and government consumption in Figure 3.6.

The left side of the three graphs shows three different levels of TFP fixing government consumption at the steady state, and the right graphs are the cases of different government consumption. Decision rules of government bonds indicate that government bonds are on divergent paths under the situation of low TFP and high government consumption. On the other hand, the government can lower its debt in the case of high TFP and low government consumption, but the decision rule is also on a divergent path as the price of its bonds declines.

The middle two graphs show the probability of crisis based on the previous debt amount. The black dotted curve corresponds to the fiscal limit in the steady state. The probability of crisis begins to increase in a relatively low debt-to-GDP ratio under low TFP and high government consumption cases. The debt-to-GDP ratios in low TFP and high government consumption cases are 13% and 10% lower than the steady state to reach the 50% of probability of crisis respectively. Finally, the characteristics of price of government bonds are two points. First, the prices in high TFP and low government consumption cases are higher than in the steady state case because the expected next marginal utility of consumption over the current marginal utility of
consumption in these states is higher. Second, because the probability of crisis begins to increase in the relatively low debt-to-GDP ratio (around 100%) in low TFP and high government consumption states, the price of government bonds also begins to decrease at a relatively lower level of debt.
C.4 Sensitivity Analysis

In addition to the alternative scenario related to government policies in section 3.4.3, this appendix reports changes in several parameters. The effects on main variables in the table’s columns show changes from baseline values.

Table C.1: Sensitivity Analysis

<table>
<thead>
<tr>
<th></th>
<th>Average D/GDP</th>
<th>Crisis Freq.</th>
<th>Average Spread</th>
<th>Effects on Main Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>67.6%</td>
<td>0.60%</td>
<td>0.71%</td>
<td>-</td>
</tr>
<tr>
<td>Base</td>
<td>47.0%</td>
<td>0.75%</td>
<td>5.78%</td>
<td>-</td>
</tr>
</tbody>
</table>

(1) Discount factor ($\beta$: 0.91)

<table>
<thead>
<tr>
<th>($\beta$)</th>
<th>Average D/GDP</th>
<th>Crisis Freq.</th>
<th>Average Spread</th>
<th>Import G.</th>
<th>Dome. G.</th>
<th>Tax Rev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.89</td>
<td>66.9%</td>
<td>1.61%</td>
<td>16.14%</td>
<td>(0.2, -0.2)</td>
<td>(0.4, -1.0)</td>
<td>(-0.0, -0.3)</td>
</tr>
<tr>
<td>0.93</td>
<td>23.1%</td>
<td>0.20%</td>
<td>1.32%</td>
<td>(-0.8, 0.1)</td>
<td>(-1.0, 0.9)</td>
<td>(-0.8, 0.3)</td>
</tr>
</tbody>
</table>

(2) Imported intermediate goods with working capital ($\theta$: 0.7)

<table>
<thead>
<tr>
<th>($\theta$)</th>
<th>Average D/GDP</th>
<th>Crisis Freq.</th>
<th>Average Spread</th>
<th>Import G.</th>
<th>Dome. G.</th>
<th>Tax Rev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>47.5%</td>
<td>0.76%</td>
<td>5.82%</td>
<td>(0.5, 2.3)</td>
<td>(0.5, 5.3)</td>
<td>(0.4, 1.7)</td>
</tr>
<tr>
<td>1.0</td>
<td>47.7%</td>
<td>0.76%</td>
<td>5.81%</td>
<td>(-0.3, -2.8)</td>
<td>(-0.4, -5.9)</td>
<td>(-0.3, -2.0)</td>
</tr>
</tbody>
</table>

(3) Armington weight of domestic inputs ($\lambda$: 0.81)

<table>
<thead>
<tr>
<th>($\lambda$)</th>
<th>Average D/GDP</th>
<th>Crisis Freq.</th>
<th>Average Spread</th>
<th>Import G.</th>
<th>Dome. G.</th>
<th>Tax Rev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.76</td>
<td>44.0%</td>
<td>0.71%</td>
<td>6.35%</td>
<td>(-3.6, -4.3)</td>
<td>(6.0, 5.0)</td>
<td>(-3.9, -4.7)</td>
</tr>
<tr>
<td>0.86</td>
<td>50.0%</td>
<td>0.78%</td>
<td>4.86%</td>
<td>(4.9, 5.4)</td>
<td>(-7.4, -6.5)</td>
<td>(4.6, 5.1)</td>
</tr>
</tbody>
</table>

(4) Elasticity of transfers to productivity ($R_d/GDP_d$: 0.40)

<table>
<thead>
<tr>
<th>($R_d/GDP_d$)</th>
<th>Average D/GDP</th>
<th>Crisis Freq.</th>
<th>Average Spread</th>
<th>Import G.</th>
<th>Dome. G.</th>
<th>Tax Rev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.42</td>
<td>47.9%</td>
<td>0.76%</td>
<td>5.92%</td>
<td>(-0.1, -1.0)</td>
<td>(-0.1, -2.9)</td>
<td>(-0.3, -1.2)</td>
</tr>
</tbody>
</table>

Note: The first and second elements in the parentheses show the percentage difference from the baseline in the normal and crisis states respectively.

Figure C.4: Crisis Probability (Fiscal Limit) under Different Discount Factors
First, in section (1) of Table C.1, I examine the case that the discount factor changes two points from the baseline scenario of 0.91. When the discount factor is 0.89, the fiscal limit and price of government bonds are lower than in the baseline scenario (the lower and higher fiscal limits are depicted in Figure C.4). Thus, the crisis frequency and average bond spread are higher than the baseline, and that leads to a high average debt-to-GDP ratio. The effects on main variables are not so strong because the discount factor does not affect these variables directly. Section (2) shows cases of changing the value of the upper bound proportion of imported intermediate goods which require working capital. If this proportion increases, final goods firms have to disburse more interest payments on imported intermediate goods especially in the crisis state. Thus, the amount of imports of intermediate goods from foreign countries decreases and eventually GDP decreases. Section (3) reports the result of different values of Armington weight of domestic inputs, λ. A high value of λ means that the fixed amount of production is less costly because the price of domestic intermediate goods is lower than for imported intermediate goods. Thus, the amount of imports declines, but other variables such as GDP, domestic intermediate goods and tax revenue increase. However, the debt-to-GDP ratio, crisis frequency and bond spread do not change much because the steady state level of GDP increases, and that increases the base values of government consumption and transfers. Section (4) shows the effect of high tax rates in the crisis state. The 5% increase of tax rate reduces GDP, the amount of imports and domestic intermediate goods production by 1.0%, 2.9% and 1.2% respectively in the crisis state, but tax revenue increases 3.9% from the baseline value.
C.5 Data

- Public Debt-to-GDP: The measurement is annual gross rate. The data source is OECD. The data used in the baseline simulation in figure 3.9 is quarterly debt-to-GDP ratio from Eurostat.

- GDP: The data is seasonally adjusted quarterly real gross domestic product of the expenditure approach. The source is OECD.

- Government Transfer: The data is annual social benefits and transfers in kind. The source is OECD.

- Households’ Consumption: The measurement is quarterly, real and seasonally adjusted. The source is OECD.

- Imported Intermediate Goods: The period is early 2000s. The source is OECD’s STAN Input-Output data.

- Imports of goods and services: The measurement is quarterly, real and seasonally adjusted. The source is OECD.

- Domestic Intermediate Goods: The measurement is annual and real. The source is ECB.

- Hours Worked: The data is total employment of the average annual hours actual worked per worker. The source is OECD.

- Tax Revenue: The measurement is annual and real. The source is OECD.

- CDS: The data is senior 5-year government bonds. The source is Thomson Reuters.

- Bond Spread: The data is quarterly interest rate on 10-year Spanish government bonds minus that of German government bonds. The source is OECD.
• TFP: The data is seasonally adjusted quarterly GDP per person employed. The source is OECD.
Appendix D

Chapter 4 Appendix

D.1 Diagrams

Figure D.1: The Sketch of My Model

D.2 Computational Algorithm

Similar to chapter 3, I separately calculate the normal and crisis states. In the

148
(K_{t-1}, A_t, g_t). In the normal state, the computational algorithm is as follows:

1. Discretize the state space of TFP ($A_t$), government consumption ($g_t$), capital stock ($K_{t-1}$) and government bonds issued at the previous period ($B_{t-1}$). I adopt Tauchen’s method (1986) for the state space of TFP and government consumption and uniformly take 25 points, following equations (4.4) and (4.2). Capital stock is discretized between 0.8 and 1.5 times of the steady state level of capital, taking 40 grid points, and the government bonds is also discretized between 0 and 2.5 times the steady state level of GDP, taking 40 grid points. Make an initial guess for the issuance of government bonds $f^b_0(A_t, g_t, K_{t-1}, B_{t-1})$.

2. Evaluate the probability of crisis in the next period $E_t[P_c; t+1]$ following the equation (4.26) and compute endogenous variables for each grid point.

3. Update the guess of government bond issuance $f^b_i(A_t, g_t, K_{t-1}, B_{t-1})$ from the old decision rule $f^b_{i-1}(A_t, g_t, K_{t-1}, B_{t-1})$ by calculating the government new bond issuance rule (4.7).

4. Check convergence of decision rule $f^b(A_t, g_t, K_{t-1}, B_{t-1})$. If the difference between the updated and old decision rules is small enough (i.e. $\sup ||f^b_i(A_t, g_t, K_{t-1}, B_{t-1}) - f^b_{i-1}(A_t, g_t, K_{t-1}, B_{t-1})|| < \epsilon$), $f^b(A_t, g_t, K_{t-1}, B_{t-1})$ is the decision rule. Otherwise, go back to procedure 2.

D.3 Sensitivity Analysis

Table D.1 reports effects on the economy by changing several important parameters.

Section (1) is the case that changes the fiscal limit by ±10% points of the debt-to-GDP ratio from the baseline value of 140%. In the case of a low fiscal limit, the average debt-to-GDP ratio is lower, the frequency of crisis is higher and the average bond spread is higher than the baseline case. Because the government falls into the
Table D.1: Sensitivity Analysis

<table>
<thead>
<tr>
<th>D/GDP</th>
<th>Crisis D/GDP</th>
<th>Average D/GDP</th>
<th>Effect on Main Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>95.9%</td>
<td>0.53%</td>
<td>2.27%</td>
</tr>
<tr>
<td>Base</td>
<td>97.2%</td>
<td>0.55%</td>
<td>9.44%</td>
</tr>
</tbody>
</table>

(1) Fiscal limit shape, low probability ($b_l$: 1.4)

- 1.3: 89.4% 1.24% 10.75% (1.2, 0.2) (1.6, 0.1) (-1.7, 0.2)
- 1.5: 100.7% 0.11% 8.42% (-1.2, -0.2) (-3.3, -0.2) (-1.3, 0.1) (1.7, -0.0)

(2) Armington weight of domestic inputs ($\lambda$: 0.6)

- 0.5 86.5% 0.10% 8.36% (-7.1, -12.7) (18.7, 9.5) (-13.3, -16.2) (-8.1, -10.8)
- 0.7 100.5% 1.10% 10.65% (6.6, 13.1) (-22.0, -16.0) (12.3, 15.7) (7.6, 10.4)

(3) Substitution elasticity across intermediate goods ($\psi$: 0.7)

- 0.5 91.4% 0.14% 8.55% (-0.7, -5.9) (21.2, 23.6) (-10.9, -21.2) (-1.0, -3.4)
- 0.9 96.8% 1.03% 10.16% (1.1, 5.4) (-17.7, -23.4) (9.6, 16.5) (0.1, 3.1)

Note: The first and second elements in the parentheses show the percentage difference from the baseline in the normal and crisis states respectively.

crisis with a lower level of debt, GDP and both imported and domestic intermediate goods are higher than the baseline. That leads to a lower tax rate and less tax revenue.

Section (2) reports the case of Armington weight of domestic inputs $\lambda$, shifting 0.1 points from the baseline scenario. The low value of this parameter leads to low average debt-to-GDP ratio, low frequency of crisis and low average spread. The level of GDP declines because final goods firms need to rely on costly imported intermediate goods.

Section (3) shows the result of changing the elasticity of substitution across domestic and foreign intermediate goods $\psi$. As the previous section, the low value is more secure than the baseline case, but that leads to a slightly lower level of GDP.

D.4 Data

- Government Debt to GDP: The measurement is annual gross rate. The source from 1995 to 2014 is the historical public debt database (September 2012 version) of IMF Fiscal Affairs Department. The data in the baseline simulation is quarterly debt-to-GDP ratio from Eurostat.
- GDP: The data is seasonally adjusted quarterly real gross domestic product of the expenditure approach. The source is OECD.

- Government Consumption: The data is annual final consumption expenditure of general government. The source is OECD.

- Government Transfers: The data is annual social benefits and transfers in kind. The source is OECD.

- Households’ Consumption: The measurement is quarterly, real and seasonally adjusted. The source is OECD.

- Imported Intermediate Goods: The period is early 2000s. The source is OECD’s STAN Input-Output data.

- Imports of goods and services: The measurement is quarterly, real and seasonally adjusted. The source is OECD.

- Domestic Intermediate Goods: The measurement is annual and real. The source is ECB.

- Hours Worked: The data is total employment of the average annual hours actual worked per worker. The source is OECD.

- Capital: The data is private capital stock measured in constant 2011 international dollars. The source is IMF Fiscal Affairs Department.

- Tax Revenue: The measurement is annual and real. The source is OECD.

- CDS: The data is senior 5-year government bond. The source is Thomson Reuters.

- Bond Spread: The data is the quarterly interest rate on 10-year Greek government bonds minus that of German government bonds. The source is OECD.
• TFP: The data is seasonally adjusted quarterly GDP per person employed. The source is OECD.