An Investigation of Wrinkling Behaviour in Woven Thermoplastic Composite Material

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June 2017

A thesis submitted for the degree of Master of Philosophy
of The Australian National University

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Declaration

This thesis is an account of research undertaken between August 2015 and June 2017 at The Research School of Engineering, College of Engineering and Computer Science, Australian National University, Canberra, Australia.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.
Publications


4. L. Chen, S. Kalyanasundaram, "A Novel Wrinkling Indicator Using the Abrupt Change in Strain Behaviour of Woven Thermoplastic Composite Material”, in preparation for submission

Acknowledgements

I would like to thank A/Prof. Shankar Kalyanasundaram, my supervisor in chief, for the firstly believing in my abilities to be a part of this great research and secondly for his excellent advise and continued support throughout.

This project would not have been successful without the advice, support and knowledge provided by Nima Akhavan Zanjani, Jae Nam, Davood Rahiminejad and Jiaai Liang. I would like to thank them for helping me with the equipment, software, for providing the material properties and UMAT scripts for simulations, for their patience with my questions from the beginning and for their discussions, support and help in overcoming problems.

I can never thank my father (Yingsheng Chen) and mother (Xiaopeng Wu) enough for their emotional and financial support, without which I would never achieve so much in my life.

Most of all, I would like to thank to my husband Pu Fu, for his support, encouragement, and love during my studies, especially in those difficult moments, became the true motivation for me to carry on.
Abstract

This work is designed to develop a viable universal indicator to predict the onset of wrinkling for woven thermoplastic fiber-reinforced composite materials. A range of experiments on Yoshida test are carried out to investigate the local strain behaviour at the onset of wrinkling. An approach of using the abrupt change in the slope of the evolution of the strain path to predict the onset of wrinkling is proposed in this study. This approach is the first of its kind in predicting the onset of wrinkling. Different metrics in the quantification of abrupt change in the slope of strain path are examined and compared. A wrinkling indicator of using strain increment ratio to predict the onset of wrinkling is the fundamental contribution of the present work. The validity of this indicator is explored in a series of the dome forming experiments involving composite materials and steel. The results reveal that this proposed indicator can accurately predict the onset and the propagation of wrinkling in Yoshida test and dome forming operations. This indicator is also applicable to other class of materials. Analytical approach of using energy approach at a small wrinkling affected region (effective region) is also developed to predict the onset of wrinkling in Yoshida tests. This approach provides a direct relationship between the critical wrinkling stress and material properties, boundary conditions and geometrical parameters.
# Table of Contents

Chapter 1  Introduction .............................................................................................................. 1

1.1 Motivation .......................................................................................................................... 1

1.2 Research Objectives ........................................................................................................ 2

1.3 Thesis Structure ................................................................................................................ 3

Chapter 2  Literature Review ................................................................................................... 5

2.1 Introduction ...................................................................................................................... 5

2.2 Buckling and Wrinkling .................................................................................................... 5

2.3 The Necessity of Developing a Wrinkling Indicator ......................................................... 6

2.4 Existing Wrinkling Indicators .......................................................................................... 7

2.4.1 Wrinkling Limit Diagram ............................................................................................ 7

2.4.2 Energy Method ........................................................................................................... 13

2.5 Summary .......................................................................................................................... 23

Chapter 3  Materials and Methodology ..................................................................................... 25

3.1 Introduction ...................................................................................................................... 25

3.2 Woven Thermoplastic Composites .................................................................................. 25

3.2.1 Curv® ......................................................................................................................... 27

3.2.2 Twintex® .................................................................................................................... 30

3.3 Experimental Methodology ............................................................................................... 33
3.4 Finite Element Analysis ................................................................. 38
  3.4.1 Explicit Formulation ................................................................. 38
  3.4.2 Material Modeling ................................................................. 40
  3.4.3 Yoshida Test ................................................................. 41
  3.5 Summary ........................................................................ 44

Chapter 4 Experimental Results ......................................................... 45
  4.1 Introduction ........................................................................ 45
  4.2 Out-of-plane Displacement .................................................... 45
  4.3 Evolution of Strain at the Central Region (node 1) ....................... 48
    4.3.1 Strain Path ................................................................. 49
    4.3.2 Strain Ratio ................................................................. 52
    4.3.3 Strain Increment Ratio .................................................... 54
  4.4 Evolution of Strain Increment Ratio at Four Points of Interest .......... 58
    4.4.1 Strain increment ratio ....................................................... 59
  4.5 Summary ........................................................................ 63

Chapter 5 Development and Validation of Wrinkling Indicator ............. 65
  5.1 Introduction ........................................................................ 65
  5.2 The Development of Wrinkling Indicator for the Implementation in Simulations ................................................................. 65
  5.3 Examination of the Wrinkling Indicator in Yoshida test ................. 73
Chapter 5  Examination of the Wrinkling Indicator in Dome Forming

5.4  Examination of the Wrinkling Indicator in Dome Forming .........................77

5.4.1  Dome Forming for Curv® Specimens ..................................................80

5.4.2  Dome Forming for Twintex® Specimens .............................................83

5.4.3  Dome Forming for Steel Specimens .......................................................85

5.5  Summary .................................................................................................87

Chapter 6  Energy Approach at Effective Region

6.1  Introduction ..............................................................................................89

6.2  The Significance of Using Energy Method in Wrinkling Prediction ..........90

6.3  Effective Region in Yoshida test .............................................................91

6.3.1  Size, Shape and Location of the Effective Region .................................92

6.3.2  Boundary Condition of the Effective Region .........................................97

6.4  Critical Stress for Effective Region ........................................................99

6.4.1  Expression for Critical Stress Estimation ............................................99

6.4.2  Effective Engineering Constants along Non-Principal Coordinates ......103

6.5  Validation of Analytical Critical Stress ................................................104

6.5.1  Critical Stress from Experimental Results .........................................105

6.5.2  Comparison between Analytical, Experimental and FEA Results ......105

6.6  Summary .................................................................................................107

Chapter 7  Conclusion and Recommendations

7.1  Thesis Contribution to knowledge ..........................................................109
7.2 Recommendations for future work ................................................................. 110

Chapter 8 Bibliography ........................................................................................ 111
List of Figures

Figure 2.1 Buckling of a Tubular Beam Column [11] ................................................................. 6

Figure 2.2 Wrinkling of a Tubular Part [12] ........................................................................... 6

Figure 2.3 An example of Wrinkling Limit Diagram (WLD) ................................................... 8

Figure 3.1 Commonly used weaving styles: plain weave (1/1T), twill weaves (2/1T, 2/2T) [57] .............................................................................................................................. 25

Figure 3.2 Single layer laminates from different weave styles showing increasing warpage with weave style unbalance. [57] ........................................................................... 26

Figure 3.3 (a) Twill-weave fabric Curv®; (b) representative unit cell of Curv® [74] ... 28

Figure 3.4 Co-extrusion, cold drawing of a tape [57] ................................................................. 29

Figure 3.5 Coextrusion technology with additional stretching to produce high-strength tapes [57, 77] ...................................................................................................................... 29

Figure 3.6 Principle sketch of hot compaction in the example of unidirectional arranged fibres [72] .......................................................................................................................... 30

Figure 3.7 (a) Twill-weave fabric Twintex® with (b) representative unit cell [74] ....... 31

Figure 3.8 Twintex® commingled glass/polypropylene yarns [69] ................................. 31

Figure 3.9 Schematic diagram of a cross section of a (a) pre-consolidated and (b) consolidated Twintex® bundle [82] .................................................................................................. 32
Figure 3.10 Schematic diagram of commingling process with the use of Air-Jet Texturing Machine [83] .......................... 32

Figure 3.11 A schematic of a compression mould [82] ................................................................. 33

Figure 3.12 Experimental set up used in the Yoshida test ......................................................... 34

Figure 3.13 Camera configuration [95] with dimensions shown for (a) 175*150mm measuring volume for ARAMIS® 5M system [94]; (b) 175*140mm measuring volume for ARAMIS® 1.3M system [92] ........................................................................... 36

Figure 3.14 An example of high contrast stochastic pattern ....................................................... 37

Figure 3.15 Schematic illustration of the Yoshida test ............................................................... 38

Figure 3.16 Presentation of integration point in shell formation in LS DYNA [100] .... 40

Figure 3.17 Model Geometries and Mesh used in the Finite Element Simulation ......... 42

Figure 3.18 Boundary Condition used in Simulation ............................................................... 43

Figure 4.1 Position of Points of Interests Marked in the Out-of-Plane Displacement Contour ................................................................................................................................. 46

Figure 4.2 Evolution of out-of-plane displacement at points of interest of (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°] samples ........................................................................................................................................ 47

Figure 4.3 Strain path at the central point of (a) Curv® [0°/90°]; (b) Curv® [-45°/45°]; (c) Twintex® [0°/90°]; (d) Twintex® [-45°/45°] samples .................................................................................. 51
Figure 4.4 Strain ratio at the central point of (a) Curv® [0°/90°]; (b) Curv® [45°/-45°];
(c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°] samples........................................53

Figure 4.5 Strain increment ratio at the central point of (a) Curv® [0°/90°]; (b) Curv®
[45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°] samples. ..................55

Figure 4.6 Method for calculating (a) strain ratio; (b) strain increment ratio ............56

Figure 4.7 Strain increment ratio at point of interest on the concave surface of (a)
Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-
45°] samples. ........................................................................................................59

Figure 4.8 Strain increment ratio at point of interest on the convex surface of (a) Curv®
[0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°]
samples. ...................................................................................................................62

Figure 5.1 The evolution of SIR and the Critical SIR on (a) the face subjected to
additional compression after wrinkling; (b) the opposite face at the centre point for
Curv® [0°/90°] samples..............................................................................................68

Figure 5.2 Flow Chart of Wrinkling Indicator Defined in this Study.........................70

Figure 5.3 (a) Predicted Results from wrinkling indicator; (b) experiment results of the
out-of-plane displacement at three states of interest for Curv® [0°/90°] samples .74

Figure 5.4 (a) Predicted Results from wrinkling indicator; (b) experiment results of the
out-of-plane displacement at three states of interest for Curv® [45°/-45°] samples
......................................................................................................................................75
Figure 5.5 (a) Predicted Results from wrinkling indicator; (b) experiment results of the out-of-plane displacement at three states of interest for Twintex® [45°/-45°] samples..........................................................75

Figure 5.6 (a) Predicted Results from wrinkling indicator; (b) experiment results of the out-of-plane displacement at three states of interest for Twintex® [0°/90°] samples ..........................................................76

Figure 5.7 Model Geometries of Dome Forming used in the Finite Element Simulation ........................................................................................................................................78

Figure 5.8 Wrinkling Indicator of two states of interest for (a) Curv®; (b)Twintex®; (c) Steel samples..........................................................79

Figure 5.9 Comparison between Predicted Results and Experimental Results of Full Circular Curv® Specimens after Dome Forming (a) predicted result; (b) top view of the formed part; (c) side view; (d) detailed view of flange region with fibre orientation of [0°/90°]; (e) detailed view of flange region with fibre orientation of [45°/-45°] of the formed part at Forming Depth of 40 mm. ........................................81

Figure 5.10 Comparison between Predicted Results and Experimental Results of Full Circular Curv® Specimens after Dome Forming (a) predicted result; (b) top view of the formed part; (c) side view; (d) detailed view of flange region with fibre orientation of [0°/90°]; (e) detailed view of flange region with fibre orientation of [45°/-45°] of the formed part at Forming Depth of 50mm. ...........................................82

Figure 5.11 The Experimental Results of Full Circular Twintex® Specimen after Dome Forming at Forming Depth of (a) 30mm; (b) 35mm; (c) 50mm.........................84
Figure 5.12 Comparison between Predicted Results and Experimental Results of Full Circular Steel Specimen after Dome Forming at Forming Depth of 30 mm (a) predicted result; (b) top view of the formed part; (c) side view of the formed part. ...............................................................................................................................85

Figure 5.13 Comparison between Predicted Results and Experimental Results of Full Circular Steel Specimen after Dome Forming at Forming Depth of 50 mm (a) predicted result; (b) top view of the formed part; (c) side view of the formed part. ...............................................................................................................................86

Figure 6.1 Example contours of minor principal stress (unit: MPa) at elongation of (a) 0.3mm; (b) 0.67mm; (c) 0.92mm for Curv® [0°/90°] samples. ..........................................................92

Figure 6.2 Effective region for Curv® [0°/90°] samples at elongation of (a) 0.3mm; (b) 0.5mm; (c) 0.68mm; (d) 0.92; (e) 1.2mm; (f) 1.5 mm.................................................................93

Figure 6.3 Effective region and the approximated rectangular region for (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°] samples at elongation of 0.3mm. ...............................................................................................................................95

Figure 6.4 Buckling results (load multiplier) for (a) rounded corner rectangle; (b) rectangle effective region.................................................................96

Figure 6.5 Buckling results (load multiplier) for rectangular effective region (a) with a tensile loading; (b) without tensile loading.........................................................98

Figure 6.6 Boundary conditions of effective region. .................................................................99
Figure 6.7 Plate buckling coefficient at different width to length ratio for the effective region. ............................................................................................................................................. 102

Figure 6.8 Comparison between Experimental, FEA Compressive Stress and Analytical Critical Stress for (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°] ............................................................................................................................................. 106

Figure 8.1 Strain path at point of interest on one face of the sample. (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°] .................. A

Figure 8.2 Strain path at point of interest on one face of the sample. (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°] .............. B
List of Tables

Table 3.1 Specifications of the ARAMIS® 5M and 1.3M system [92, 93] ..................35

Table 3.2 Details of Experiment Parameters........................................................................37

Table 4.1 Elongation Period during the Onset of Wrinkling ...........................................48

Table 4.2 Results of strain ratio and strain increment ratio for stage at elongation of 0.67
and 0.92mm defined in Figure 4.6 ......................................................................................57

Table 5.1 The value calculated in the wrinkling indicator for one element..................69

Table 5.2 The Summarized Wrinkling Criteria for Concave and Convex surface .........73

Table 5.3 The differences between the out-of-plane displacement of the central node
and the displacement of the node locating at the wing region for different samples
........................................................................................................................................74

Table 6.1 The dimension of effective region for different samples .........................96

Table 6.2 Mechanical data for Curv® and Twintex® Along Different Orientation Angle
[106, 107]............................................................................................................................103

Table 6.3 Critical Stress in Analytical Approach for Curv® [0°/90°], Curv® [-45°/45°],
Twintex® [0°/90°] and Twintex® [45°/-45°].....................................................................104
## Nomenclature

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>WLC</td>
<td>Wrinkling Limit Curve</td>
</tr>
<tr>
<td>WLD</td>
<td>Wrinkling Limit Diagram</td>
</tr>
<tr>
<td>FLD</td>
<td>Forming Limit Diagram</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
</tr>
<tr>
<td>PP</td>
<td>polypropylene</td>
</tr>
<tr>
<td>BHF</td>
<td>Blank Holder Force</td>
</tr>
<tr>
<td>PEEK</td>
<td>Poly-Ether-Ether-Ketone</td>
</tr>
<tr>
<td>PPS</td>
<td>Polyphenyle Sulphi</td>
</tr>
<tr>
<td>SRPP</td>
<td>Self-Reinforced Polypropylene</td>
</tr>
<tr>
<td>GRPP</td>
<td>Glass-fibre Reinforced Polypropylene</td>
</tr>
<tr>
<td>CFRP</td>
<td>Carbon Fibre Reinforced Plastic</td>
</tr>
<tr>
<td>SR</td>
<td>Strain Ratio</td>
</tr>
<tr>
<td>SIR</td>
<td>Strain Increment Ratio</td>
</tr>
<tr>
<td>GHG</td>
<td>Greenhouse Gas</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge-Coupled Device</td>
</tr>
<tr>
<td>$E$</td>
<td>Modulus</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress</td>
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<tr>
<td>Symbol</td>
<td>Term</td>
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<tr>
<td>---------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Shear Stress</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Strain Ratio</td>
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<tr>
<td>$\Delta \beta$</td>
<td>Strain Increment Ratio</td>
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<tr>
<td>$\varepsilon_1$</td>
<td>Major Principal Strain</td>
</tr>
<tr>
<td>$\varepsilon_2$</td>
<td>Minor Principal Strain</td>
</tr>
<tr>
<td>$\Delta \varepsilon_1$</td>
<td>Incremental Major Principal Strain</td>
</tr>
<tr>
<td>$\Delta \varepsilon_2$</td>
<td>Incremental Minor Principal Strain</td>
</tr>
<tr>
<td>$w$</td>
<td>Deflection Function</td>
</tr>
<tr>
<td>$\sigma_{cr}$</td>
<td>Critical Stress</td>
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1.1 Motivation

Today, there is a need to reduce the weight of the vehicle to improve fuel efficiency and reduce global greenhouse gas (GHG) emissions. United States Environmental Protection Agency pointed out that 14 percent of global GHG emissions can be attributed to transportation sector in 2014 [1]. Due to the concern about the relationship between the amount of GHG emissions and the global warming, mandatory emission reduction targets have been set for new automobiles in Europe. By 2021, the emission target for new passenger car is 95 grams CO₂/km, corresponding to a 20 percent reduction, compared with the 2015 average emissions level of 119.5 grams per kilometre [2]. An excess emissions penalty will apply for manufacturer who fails to achieve the target. These legislations have led automotive industry to consider alternative lighter weight materials to replace the metal parts and thereby reduce the weight of the vehicle and the emissions. Fibre reinforced composites are increasingly used in automotive sector, because of superior mechanical properties and high strength to weight ratio. An example application of the usage of fibre reinforced composites in automotive industry can be found in BMW i3 model. The entire passenger cell is made out of carbon fibre reinforced plastic (CFRP), with a weight of 150 kg. The weight of this part is only half of the same structure made of steel. This corresponds to a 12.55% weight reduction for the 1,195 kg model [3, 4]. Golzar and Poorzeinolabedin [5] reported that a 20% weight reduction in automobile can yield fuel economy
Introduction

improvement of 12-14 percent. Thus, only by replacing the material for the passenger cell, BMW i3 model can save the emission by 7.5-8.8 percent.

Fibre reinforced composites are also increasingly replacing the metallic parts in aircraft industry. It is reported that as much as 50 percent of Boeing 787 Dreamliner’s primary structure is manufactured by composite [5-7]. Brady and Brady [8] pointed out that only 3 percent of the total weight reduction in Dreamliner can be attributed to the use of lighter weight fibre reinforced composite. It seems contradictive that such intensive usage of composite material can only yield a 3 percent of the weight reduction. This can be attributed to the lack of fundamental understanding in the failure of the composite materials. Manufacturers have to overdesign the structural parts to achieve the safe design and fibre reinforced composite is far from reaching its full potential [8-10]. For this reason, there is a need to develop a reliable and robust failure indicator for composite materials.

1.2 Research Objectives

One of the major impediments for the widespread usage of light-weight composite material in the automotive sector is a suitable mass production technique for this class of material system. Stamp forming process, regarded as a rapid production technique, is widely used in the transportation industry. During stamp forming, fracture and wrinkling are two major undesirable features in the formed products. To improve the quality and productivity as well as to reduce the process time and cost, it is crucial to be able to predict and eliminate wrinkling in the design stage. Though there are numerous studies carried out to investigate wrinkling initiation in sheet forming operations over
the last several decades, there is no mature indicator for the onset of wrinkling in sheet forming practice. This is because the initiation of wrinkling is influenced by several factors including material properties, blank geometry, boundary conditions and friction.

The aim of this thesis is to develop an indicator for predicting the onset of wrinkling for thermoplastic composite material. The wrinkling indicator is developed through the study of Yoshida tests. A range of experiments are carried out to benchmark the wrinkling behaviour of glass-fibre reinforced polypropylene (GRPP) composite and self-reinforced polypropylene (SRPP) material. Finite Element Analysis (FEA) results offer a more nuanced look on the underlying features of wrinkling initiation. A novel indicator based on the abrupt change in the slope of strain path is developed and validated in both experimental and FEA results. The validity of this indicator is explored in a series of the dome forming experiments involving composite materials and steel.

1.3 Thesis Structure

In the following chapter, an overview of the literature of the existing wrinkling indicators is given. The aim of the review is to provide a critical understanding of various existing wrinkling indicators. Chapters 3 introduces the materials used in this investigation and their manufacturing processes. It also outlines the procedures and the parameters for experimental plan and FEA in the investigation. Chapter 4 presents the experimental results which are set as benchmark for wrinkling behaviour. An indicator for onset of wrinkling is developed by analysing the experimental observations. In Chapter 5, this indicator is implemented in FEA and validated with experimental results.
Introduction

Then, dome forming studies are carried out experimentally and numerically to evaluate the feasibility and accuracy of the proposed indicator. In Chapter 6, an analytical approach is presented to predict the critical wrinkling stress and compared with experimental and simulation results. This approach explains the relationship between critical stress and material properties. Finally, in Chapter 7, conclusions and recommendations are drawn to summarize the investigation in this study.
Chapter 2  Literature Review

2.1 Introduction

This chapter presents an overview of the background knowledge related to the study. An overview of the similarities and differences between wrinkling and buckling phenomenon is first presented to define a clear boundary between these two failure modes. Next, the significance of eliminating wrinkling as well as developing a wrinkling indicator is outlined. Finally, a critical review on the existing wrinkling indicators will be given. This includes the introduction of these indicators and the examination of their applicability and limitation.

2.2 Buckling and Wrinkling

Buckling and wrinkling are two common types of instability, taking place when a structure is subjected to compressive loading. When a structure is being compressed, it can suddenly buckle into a curved shape after a certain threshold value of loading, leading to buckling or wrinkling instability. Typically both buckling and wrinkling are instability that occurs during loading, but the scales of the phenomenon are different. Buckling is a global phenomenon and the wavelength of the pattern is large and it appears over the entire structure. The occurrence of wrinkling, on the other hand, takes place at specific region rather than the entire body of the structure. The wavelength is small comparable to the size of the structure.
Figure 2.1 and Figure 2.2 show actual cases of buckling and wrinkling. Figure 2.1 illustrates a buckled steel column sections due to compressive load. Figure 2.2 presents wrinkles on thin-walled tubes due to the induced compressive stress during the bending process.

![Figure 2.1 Buckling of a Tubular Beam Column [11]](image1)

![Figure 2.2 Wrinkling of a Tubular Part [12]](image2)

### 2.3 The Necessity of Developing a Wrinkling Indicator

Wrinkling is an undesirable result in manufacturing, for it distorts the shape of the product, degrades mechanical properties and introduces difficulties in the following assembly process. The occurrence of wrinkling on visible part is unaesthetic and unacceptable in the manufacturing industries where final part appearance is important,
Literature Review

like the outer body panels of automobile. Currently, stamp forming is the most widely used rapid forming technique in sheet material manufacturing. Typically, wrinkling initiates either from under the binder (flange wrinkling) or in the side wall (frustum wrinkling) [13]. The occurrence of wrinkling can potentially damage the dies, leading to additional costs on die replacement or die maintenance [14]. To eliminate the defect in formed parts, trial-error and die tryout methodologies are often employed in the automobile industry. These processes are expensive, labour intensive and time-consuming. Therefore, the ability to predict and prevent the onset of wrinkling through analytical and simulative approach will optimize the production of defect free parts due to wrinkling.

2.4 Existing Wrinkling Indicators

2.4.1 Wrinkling Limit Diagram

A Wrinkling Limit Diagram (WLD) is the graphical representation of the principal strains over the entire specimen. It is drawn by plotting the minor principal strain along the abscissa and the corresponding major principal strain along the ordinate. An example for WLD is presented in Figure 2.3 where Wrinkling Limit Curve (WLC) separates the safe region from the wrinkled region. When the principal strain condition appears below the WLC, falling into wrinkled region, it indicates the onset of wrinkling. The slope of WLC is the reciprocal of strain ratio ($\beta$) which is defined as follow:

$$\beta = \frac{\text{minor principal strain} (\varepsilon_2)}{\text{major principal strain} (\varepsilon_1)}$$  2.4.1
**Literature Review**

In sheet metal forming practice, it is generally believed that when the strain ratio (SR) is lower than -2, the sheet thickens and wrinkling is likely to occur [15].

![Wrinkling Limit Diagram (WLD)](image)

**Figure 2.3 An example of Wrinkling Limit Diagram (WLD)**

There are numerous attempts that have been carried out to predict wrinkling occurrences with the help of WLD. One of the earliest attempts on WLD was carried out by Hassani and Neale [16]. They found that wrinkling initiates under conical cup deep drawing is when SR decreases below a critical value of -1.17. However, this critical slope is influenced by the geometry of the punch and the die as well as the forming parameters, so this metric is only valid for this specific test.

Kim, Son and Park [17, 18] examined the dependence of the wrinkling limit on the strain-hardening coefficient, the yield stress, the sheet thickness and the anisotropy parameter. The investigation was carried by comparing FEA simulation with Havranek's [19] experimental results on three typical sheet metal materials. They found that wrinkling occurs rapidly as hardening coefficient decreases, yield stress increases, thickness decreases and anisotropy parameter increases. However, there is a discrepancy
Literature Review

between the numerical results and Havranek's experimental tests [19]. Experimental results suggested that principal strain at critical condition (wrinkling limit) fall into a narrow linear band regardless of the material thickness, while simulation results showed that critical strain becomes larger as thickness increases. Their work focused only on three types of metal materials in a particular type of conical cup forming so the outcome may not be applicable to other class of material like composites and to other geometries.

Szacinski and Thomson [20] investigated the existence of a WLC by analysing strain behaviour at the onset of wrinkling in a representative industrial sink bowl forming test. Rectangular specimens made out of annealed 301 austenitic stainless-steel with a constant thickness of 0.9 mm were formed to depths of 50 mm, 150 mm and 190 mm. The gird marking technique facilitated the principal strain measurements from the final wrinkled part by tracing movement of the vertices of the quadrilateral, drawn on the initial un-deformed part. This methodology examined the localized strain condition in the final product, but failed to provide information of strain evolution during the loading process. The experimental results showed that wrinkling at the flange, corners of the walls and mid-part of the walls appeared at different values of SR of approximately -1.0, -0.5 and 0.5, respectively. Wrinkling limits are different at different regions on one part, which makes the prediction of the onset of wrinkling by using WLD unpractical. They pointed out that wrinkling might be indicated by the changes in strain path, but their experimental methodology was unable to take a nuanced look of the strain evolution during the deformation process.

Narayanasamy et al. [21-24] evaluated the effect of mechanical properties on the wrinkling initiation in conical and tractrix deep drawing. The sheet metal materials used
in these studies were commercially pure aluminium sheets annealed to different annealing treatments [21, 24], aluminium 5086 alloy sheet annealed at different temperatures [22] and interstitial-free steel sheet of different thickness [23]. They examined the blanks with different diameters drawn through conical and tractrix die using flat bottom punch. The grid measurements technique was employed for strain measurement and the deformation of the grid on the formed part facilitated the determination of localized strain distribution. To obtain a series of strain value before and during wrinkling formation, the blanks were partially drawn to at least six different depths until the wrinkling developed. The strain information obtained from each of the partially drawn specimen provided the information for intermediate process. The dependency of WLC was studied on these metal sheets for defining a safe working zone for manufacturing application. The experimental results showed that higher Young's modulus, higher yield stress, higher strain hardening value, higher anisotropy and larger thickness exhibited better resistance against wrinkling.

Li et al. [25] developed an analytical method to determine WLD for thin-walled tube with large diameter under different loading paths. Analytical wrinkling prediction model was based on the energy criterion associated with the information generated from simulative results. The analytical results were modified with the introduction of modification function, to make the final results closer to experimental observations. With the implementation of modification function, the WLD is capable of accurately predicting the wrinkling initiation in thin-walled tubular structure. However, the modification function developed in this study is only applicable to thin-walled tubular problems. The determination of the modification function is based on the theoretical
Literature Review

analysis of specimen geometries and boundary conditions, which make the employment of this method to general forming operations difficult.

Djavanroodi and Derogar [26] numerically and experimentally evaluated WLD and forming limit diagram (FLD) of hydroforming for Ti6Al4V titanium alloy and Al6061-T6 aluminium alloy sheets. They used the existing theoretical fracture limit curves and the SR of -2 for WLC to specify these two forms of failure in experimental and simulative results. The strain measurement technique employed in experimental work was the grid marking methodology, which only allows the strain information for the initial and final stage. The experimental and simulative results suggested that the WLC, with a SR of -2 can predict wrinkling phenomenon in this specific case. However, the examination was only carried out at post-wrinkling step due to the limitation of the strain measurement technique. There is no information available for the stage at or just before the wrinkling initiation.

Zanjani, Dervaric and Kalyanasundaram [27] investigated the wrinkling behaviour in 5005-H34 Aluminium alloy with different aspect ratios through a modified Yoshida test. A series of Yoshida specimens were extended uniaxially until the onset of wrinkling for constructing WLD. The strain ratio of -2 was used as the wrinkling limit in this work. It was observed that the strain condition at some nodes that in the wrinkling affected regions does not appear in the wrinkling region in WLD, elucidating that the commonly employed WLC (SR=-2) could not accurately predict the onset of wrinkling in these Yoshida specimens.

Zanjani and Kalyanasundaram [28] investigated the WLD of self-reinforced polypropylene woven composite material. The modified Yoshida samples were used to
study the onset of wrinkling in this work. To determine the wrinkling limit, the major principal strain was plotted as a function of the minor principal strains for all surface points at different elongation stages to construct WLD. It was reported that the initiation of the wrinkling could be predicted under Yoshida test is when SR decreases below a critical value. However, the critical SR for wrinkling limit was not given in this work, because this value depends on the geometrical parameters and the material properties so different test has a different critical value. The main contribution of this work was that it checked the validity of WLD in the prediction of the wrinkling for composite materials.

Bayraktar, Isac and Arnold [29] studied WLD numerically and experimentally on a series of modified Yoshida tests on circular Interstitial Steel blanks with various thicknesses. Schleich, Albiez, Papaioanu and Liewald [30] applied modified Yoshida tests to analyse wrinkling behaviour of Aluminium sheet alloys of different geometry. Both studies pointed out that defining a theoretical WLD with the consideration of every factor would be extremely hard. This is attributable to the complicated link between the geometrical parameters and the value of wrinkling limit, so the initiation and propagation of wrinkling depend on many factors. They also suggested that WLC can indicate the wrinkling behaviour in modified Yoshida tests with different geometries at wrinkled region, but the wrinkling limit curves used in their studies were not applicable to other forming operations.

In summary, the following issues can be identified when applying WLC to indicate the onset of wrinkling in general forming practices. Firstly, WLC cannot be used as a universal wrinkling indicator, since wrinkling limits are different in different forming operations and can vary with the change in material properties and specimen geometry.
More importantly, there is a lack of indicator of the point of time when wrinkling starts to occur. Most of current studies uses grid marking method to obtain the strain for formed part. This method only gives the strain information at the final stage without providing any information for the intermediate stages. Without adequate information available for the stage corresponding to the onset of wrinkling, the current studies in the literature lack the ability to predict the onset of wrinkling. Therefore, additional research effort is required to find an indicator which can accurately predict the onset of wrinkling for forming a wide range of production parts.

2.4.2 Energy Method

The energy method offers a direct way of finding critical load for the onset of instability in thin plates and shells under various boundary conditions [31]. Originally, energy method is developed for the critical stress estimation in buckling problems, but this approach has been implemented in predicting the onset of wrinkling. This section will give a critical review on the implementation of the energy method to wrinkling problems.

Geckeler [32] derived an expression based on the energy method to predict the critical stress for flange region wrinkling in deep-drawing process without a blank holder. This equation was improved by Senior [33] with the introduction of the blank holder force (BHF). During the drawing process, material in contact with the punch is drawn into the die while the flange region is subjected to a radial tensile stress and an induced compressive stress along tangential direction. This compressive stress is the prime cause of the flange region wrinkling. It was assumed that the amplitude of wrinkling
deflection is constant over the width of the flange, which is described by a single sine curve. A rectangular flange segment on flange region, which wrinkled into a half-wave sine shape, was selected to represent the unit deflection of the flange region and to simplify the usage of energy method. The critical stress for this segment can represent the critical stress for the entire flange. The energy conservation theory states that the critical condition is achieved when strain energy accumulated in the rectangular segment equals to the work done by the external loading. For the wrinkling at flange region, the sum of the bending energy and the restraining energy due to the lateral constraint (blank holder effect) accumulated in the segment should equal to the work done by the external loading calculated from the circumferential shortening of the flange segment. Thus, the critical wrinkling stress for flange region can be determined.

It was reported that the overall flange deflection function including the number and the amplitude of waves is crucial in the critical stress estimation, for it can affect the results of estimated energy and in turn influence the expression of the critical stress for the onset of wrinkling. For this reason, Senior [33] defined different surface deflection function for different boundary conditions and material characteristics for critical stress estimation.

However, Alexander [34] suggested that a good approximation for Senior’s approach was obtained only under narrow flange condition. This is attributable to the usage of one-dimensional buckling model in Senior’s approach, so that the amplitude of wave was considered to be constant over the width of the flange. For a wide flange, the amplitude of the flange wrinkle cannot be assumed to be constant over the width. For this reason, Senior’s approach might fail to provide accurate indication for the onset of flange wrinkling for deep drawing with wide flange. Yu and Johnson [35] improved
Literature Review

Senior’s approach by improving the deflection function for the flange wrinkling. In this study, the flange wrinkling under deep drawing process was simplified to an annular plate simply supported at inner and outer edges, subjected to a radial tensile stress along the inner edge to model the drawing effect and a normal constraint to simulate the blank-holder effect. The deflection function of the annular plate was expressed in the form of a two-dimensional buckling model. Zero deflection is defined along the inner edge, while the deflection shape of the outer edge of the annular plate is expressed in form of sine curve. The energy criterion with this modified deflection function was used for the critical stress estimation. The main contribution of this work is that it gives better definition of the deflection function for flange wrinkling under deep drawing problems, compared to Senior’s approach.

Morovvati et al. [36, 37] applied the energy method in the prediction of wrinkling in order to eliminate flange wrinkling by increasing the BHF. The study investigated the minimum required BHF to prevent the wrinkling in circular single [37] and two-layer (aluminium-stainless steel) sheet metal under the deep drawing process [36]. The critical condition was generated based on Senior’s approach, associated with the deflection function as well as the expressions for strain energy and external work derived by Kollar and Springer [38]. The results showed a good agreement between analytical predictions and experimental observations. The effects of material properties on the minimum required BHF to eliminate the wrinkling were also studied. It was demonstrated that wrinkling can be eliminated by changing the blank diameter, Young’s modulus and yield strength.
Agrawal, Reddy and Dixit [39, 40] investigated the minimum required BHF to achieve wrinkle free parts with different thickness under deep drawing. Deep drawing process was simplified to an annular plate simply supported at inner and outer edges. A radial tensile stress imposed on the inner edge and a normal constraint was applied over the surface to model the blank holder constraint. The deflection function of the annular plate was set to zero at the inner edge and expressed in sine curve along the outer edge. Then, the strain energy stored in the system, restraining energy of the blank holder and the work done by external loadings were calculated for the critical wrinkling stress estimation. The critical condition for the onset of wrinkling was determined by equating the sum of strain energy and restraining energy to the work done by external loadings. The predicted critical condition for the instability was validated with the published experimental results [33, 35, 41] and good agreement was observed. The study showed that a thicker specimen requires a higher BHF to eliminate the wrinkling.

Zheng et al. [42] carried out a series of deep drawing experiments on commercial AA6082-T6 aluminium alloy sheet with a thickness of 1.5mm to test the effect of draw ratios and blank-holding forces on the occurrence of flange wrinkling. The effect of draw ratio, which is defined as the diameter of the blank (170, 180 and 190mm) over the diameter of the punch (100mm), is also investigated. To examine the effects of process parameters on the onset of wrinkling, analytical models based on energy method were utilised in deep drawing of aluminium alloy. Bending energy, restraining energy and the work done by the external loading is estimated for a representative segment. This segment is selected from the flange region and assumed to have a half-wave sine shape. The critical wrinkling condition is achieved when the sum of bending and restraining energy equals to the external work. Two buckling modes (one-
Literature Review

dimensional and two-dimensional buckling mode) were examined for the estimation of the critical wrinkling condition for the flange region and the flange segment. The comparison with the experimental observations suggested that the model employing the two-dimensional buckling mode exhibits a better agreement than the one implementing with the one-dimensional deflection function. Both analytical and experimental results illustrated that draw ratio had a significant effect on the onset of wrinkling but the blank-holding force did not. Wrinkling is easier to initiate at a larger draw ratio (bigger blank).

Brosius and Mousavi [43] used macro structured tools to achieve a lubricant free deep drawing process. A macro-textured blank-holder surface comprising tangential grooves of circular cross-section was used in this deep drawing process. To eliminate the flange wrinkling in the final product, the energy method is employed to determine the critical condition. The critical condition obtained from analytical method was verified by the numerical and experimental analysis and only very small deviation was observed. The critical condition is believed as the state when the work done by external loading equals to the sum of bending, restraining and friction energy. Wrinkling was studied at a region near the outer edge of the flange and a region near the inner edge of the flange. The region near the outer edge of the flange showed a higher tangential compressive stress while a lower radial tensile stress than the region near the inner edge. This is attributed to that the region near the outer edge was only supported on one side and free in movement on the other side, while the inner region was constrained by the alternative bending introduced by two adjacent grooves. This makes the outer region of the blank becoming less stable than the other regions, so that wrinkling will initiate at the outer region under the deep drawing using macro structured blank-holder.
Literature Review

Kong et al. [44] used the energy method to predict the flange wrinkling under the spin forming of hemispherical part. The excessive circumferential compressive stress developed in the flange region is the prime cause of the flange wrinkling for this spinning of hemispherical part. To test the effect of feed ratio on flange wrinkling, this work employed the energy method in the prediction of the critical circumferential compressive stress. Feed ratio is defined as the ratio of the roller feed rate to the spindle speed. During the spin forming process, the flange region is free of any contact with the tool, so a theoretical model of annular plate which is subjected to in plane tensile stress along the inner and outer edges was introduced to describe the flange region wrinkling. The inner edge was assumed to be built-in and allowed neither movement nor deflection, while the outer edge was free to move. The deflection mode of the annular plate was assumed to be in form of single sine curve along the circumferential direction, but the amplitude was varying along the radial direction. Then, the bending energy and the work done by the forces acting on the middle plane of the sheet were estimated and the critical condition was achieved by equalizing the bending energy to the work. The theoretical critical circumferential compressive stress was validated with the experimental and simulative results and good agreement was observed. It was reported that the feed ratio is a crucial factor for flange wrinkling in spin forming of hemispherical part. Larger feed ratio gives the rise to the flange wrinkling, because it increases the circumferential stress distributed over the flange region.

Cao and Boyce [45] used the energy criterion in examining wrinkling behaviour on rectangular plate. The plate is simply supported along four edges, subjected to in-plane compression along a pair of edges. A lateral constraint is applied to constrain the normal movement of the flange. This proposed test was a simplified form of the flange region
for a general forming process. As the punch starts to deform the sheet, material is drawn into the cavity introducing a compressive stress within the part under blank holder. The radial tensile stress was not considered in the simplified test, for it is not the main driver of wrinkling. The lateral constraint, which constrains normal movement of the flange, models the effect of the blank holder. Various buckling modes were used for describing the deflection of the representation rectangular unit. The associated energy and work done by applied force were recorded for the critical stress estimation. Cao et al. [46, 47] further used this method in predicting the wrinkling behaviour in sheet metal cup forming. They pointed out that due to the usage of pure bending theory in the strain energy estimation, there is a small discrepancy between predicted and actual wavelength when the wavelength is less than six times of the sheet thickness.

Cao et al. [48-50] investigated the wrinkling on the side-wall in sheet metal forming processes. The validity of this analytical approach was examined in Yoshida test, square cup deep drawing and conical cup deep drawing and compared with the experimental observation. The energy method associated with the effective dimension was proposed for the critical stress estimation. In this study, effective dimension was the actual region undergoing compression, which is the driver of localized wrinkling. The shape, size and position of the effective dimension were determined from analysing the contour of compressive stress generated from the FEA results. For example, in Yoshida test, (a square sheet is subjected to tension along the diagonal direction), the tension along the loading direction induces a compressive stress along the perpendicular direction. This induced compressive stress is the prime source of wrinkling. According to the contour of compressive stress from simulation results, a rectangular region under compression was selected as the effective dimension. In this study, the Yoshida test was simplified to
Literature Review

A rectangular plate simply supported on two vertical edges and clamped at the other two horizontal edges. A uniform compressive stress was introduced on two vertical edges and tensile stress on the other two. The surface deflection function for this effective dimension for Yoshida test was assumed to be in form of double sine wave. For the square cup deep drawing, the effective dimension for side wall wrinkling was chosen as a rectangle under the actual compressive on the straight side-wall. The straight side-wall section is constrained by the straight edge of square punch and the die. The boundary condition of effective dimension was clamped on four edges and the deflection function was assumed to be in form of double sine curve with zero deflection along edges. In conical cup deep drawing, the effective region was considered as the annular curved sheet which represents the entire sidewall, with both inner and outer edge clamped. Again double sine curve was used for describing the deflection of the annular curved sheet and the energy and the work done by external force can be estimated from the deflection function and the boundary condition. According to the law of energy conservation, when the strain energy equals to the amount of work, wrinkling initiates and the stress causing the plate instability is the critical stress. This work introduced a novel method in the selection of effective region. Traditionally, the effective region was assumed to be the section which deformed into half-wave sine curve. The effective dimension in this study was determined through the study of the compressive stress contour obtained from FEA. By predicting the wrinkling initiation in the effective dimension, the critical load for wrinkling for the whole part can be calculated. The size shape and the location of the effective region seems to be the most crucial part in this novel approach, while the methodology provided in this work for the effective region identification was unclear. Furthermore, there was no justification on the selection of
the boundary conditions on the effective region, so that it seems to be challenge to apply this novel methodology to other cases.

This approach was checked by Shafaat, Abbasi, and Ketabchi [51]. They investigated the wrinkling phenomenon in sidewall area in the conical cup test for circular blanks with the thickness of about 0.77 mm and the diameter of 150 mm. This study was based on Cao et al.’s approach [48-50], but a new deflection function was used in describing the shape of the effective dimension. Again, the strain energy and work done by applied loading can be estimated from the deflection function and critical stress can thus be determined. It is reported that the prediction in critical stress based on the new deflection function is in better agreement with the experimental results than the Cao et al.’s approach whose deflection function was based on double sine wave [48]. In addition to the improved accuracy, the computational time decreased with the usage of this modified deflection function.

Wang and Cao [52] carried out a wrinkling analysis in thin-walled tube rotary draw bending process to examine the effects of bending angle, geometrical dimensions and material properties and thus to determine the minimum bending radius. The concept of effective dimension was implemented in study, which had been defined as the actual region undergoing compression in their previous work. A curved region in contact with the bending die and undergoing compression is selected as effective region, which is clamped at four edges and the deflection of the effective curved sheet is assumed to be in the form of double sine curve. The internal energy and the work done by the external force within the effective compressive area are estimated from the deflection function. The critical stress for the onset of wrinkling is when the internal energy of the wrinkled
Literature Review

Shell equals the work done by the external forces. It was found that the minimum allowable bending radius for creating wrinkling-free parts increases with a larger tube radius, a smaller thickness, a stiffer material and a lower strain hardening exponent.

Yang and Lin [12] introduced a new deflection function to describe the shape of wrinkled region based Wang and Cao’s work [52]. Instead of using double curved shape for the effective dimension, this study assumed that the curved sheet is deformed in to a sine shape along the longitudinal direction on the wrinkled section of the tube. It was reported that with the introduction of the new wave function, a simplified model for wrinkling prediction in thin-walled tube bending was achieved. A good agreement between the theoretical and experimental results is obtained; especially when the ratio of radius to wall thickness is larger than 20.

Although an extensive amount of research has been conducted in implementing energy criterion in predicating wrinkling initiation, the outcomes for calculating critical wrinkling stress are only applicable to the specific geometry. Even for the same forming operation, changing the geometrical parameters or the material properties slightly, can alter the sheet deflection function significantly and in turn make the expression for critical stress no longer applicable in this new condition. For example, with a lower BHF, the amplitude of the wrinkling will increase and in turn affect the value of energy and the expression for the critical stress.

The usefulness of the energy method in wrinkling prediction is that it offers a general idea of the overall wrinkling tendency and quantifies the effect of material properties, boundary conditions and geometrical parameters on the onset of wrinkling. However, it is challenging to determine an accurate deflection function to describe the shape of
Literature Review

Wrinkling and energy method is developed based on several assumptions, such as the thickness is uniform over the sheet; the stress through the thickness is uniform before wrinkling. For this reason, a discrepancy may occur between the theoretical predicted results and actual experiment observation.

2.5 Summary

This chapter summarizes literatures related to wrinkling. Firstly, a comparison between wrinkling and buckling phenomenon is presented. Then, this chapter highlights the need of a reliable and robust wrinkling indicator. Finally, a critical and extensive review on the existing wrinkling indicators is given, including wrinkling limit diagram (WLD) and analytical energy methods.

It is concluded that while an extensive amount of research has been done on the wrinkling prediction, there is no mature wrinkling indicator for the onset of wrinkling. WLC is the most intensively used one. Though several studies suggested that it is viable for complex forming operations, the wrinkling limit had to be determined case by case. The fundamental difficulty with WLC is that it can be different for different geometries and furthermore it can be different for different locations of the same part. It also fails to provide a clear indication of the exact point of time when the sheet first to wrinkle for localized wrinkled region. On the other hand the analytical approach gives the exact value of critical load and a clear relationship between the onset of wrinkling and the effective parameters. However, comparing with WLD, it is more difficult for energy approach to solve problems with complicated boundary conditions and irregular geometrical shapes.
Literature Review

This literature indicates that whilst considerable research has been carried out on wrinkling problems, a robust indicator that can be applied to a wide range production parts does not exist and will be the focus of this study.
Chapter 3   Materials and Methodology

3.1 Introduction

An overview of composite material and the manufacturing techniques of two woven thermoplastic composite materials studied in this thesis are firstly given. This is followed by an introduction of the experimental methodology of using the Yoshida test, including the experimental set up and specimen preparation. Finally, the details of FEA modelling are presented.

3.2 Woven Thermoplastic Composites

Fibre reinforced composite materials consist of high strength fibre reinforcement to carry the load and a matrix to transfer the load between fibres [53-56]. Normally, the woven composite material will exhibit better damage resistance than unwoven composite materials. Figure 3.1 compares three different weaving styles, which are commonly used in woven composites: a plain weave (1/1T) and two types of twill weave (2/1T, 2/2T).

![Figure 3.1 Commonly used weaving styles: plain weave (1/1T), twill weaves (2/1T, 2/2T) [57]](image-url)
Materials and Methodology

In a plain weave, each warp fibre passes alternately under and over each weft fibre, leading to a balanced structure. In a twill weave, each weft yarn floats across two or more warp yarn in a regular repeated manner. The balanced woven structure, such as the plain weave, 2/2 twill, 3/3 twill weave, minimizes the occurrence and the magnitude of warpage. Figure 3.2 compares the warpage between the balanced and unbalanced weave styles. It can be concluded that the warpage for balanced woven styles is less significant than the unbalanced ones. For this reason, balanced twill-weave seems to be optimal for manufacturing flat balanced woven composite material. The 2/2 twill-weave is more balanced than 2/1 twill structures, because equal amount of warp and weft yarn goes along the horizontal and vertical axis, offering similar mechanical properties along two directions [58].

![Figure 3.2 Single layer laminates from different weave styles showing increasing warpage with weave style unbalance. [57]](image)

Compared to plain weave (1/1T), the fabric woven in twill structure has a smoother surface and slightly higher mechanical properties because it has less interlacing of warp and weft fibre within a unit area, leading to a reduced fibre crimp [59, 60].

Based on the type of polymer used for matrix, fibre reinforced composite materials can be categorized into thermoset and thermoplastic composites. Thermoset products are typically produced by heating the uncured material within a mould, allowing the
**Materials and Methodology**

material to cure into its final shape [61, 62]. Due to the nature of the irreversible cross-linking of polymer chains during curing process, thermoset composites cannot be remoulded or reshaped and they are very difficult to recycle. However, this process introduces a highly linked three-dimensional molecular network offering thermoset matrices a better temperature resistance than thermoplastic composite material [63].

Thermoplastic composite material can be moulded, melted and remoulded without altering the chemical component, so they are easier to repair and recycle compared to thermoset composites [64, 65]. The majority of thermoplastic materials will not withstand a temperature of over 100°C over an extended period and only a few will withstand temperatures above 350°C [66]. For example, the melting temperature for commercial isotactic polypropylene (PP) ranges from 160 to 166 °C [67]; poly-ether-ether-ketone (PEEK) melts at around 340 °C [68]. Generally thermoplastic matrix composite materials are tougher and more ductile than thermosets, providing better impact resistance and damage tolerance. They are less dense than thermosets, making them an alternative for weight critical applications. Two different woven thermoplastic composite materials are used in the study (Curv® and Twintex®).

### 3.2.1 Curv®

Curv® is a self-reinforced polypropylene composite material with a fibre volume fraction of 55-65 percent [69]. The same polymer forms both the reinforcement and matrix phase, exhibiting a better recycling option compared to other thermoplastic composite materials which have two different materials for matrix and fiber [70]. In addition to the elevated recyclability, the weight of lightweight parts can be further
Materials and Methodology

reduced compared to the conventional fiber reinforced thermoplastic composites. The density of polypropylene reinforcement is 0.9g/cm³, which is much lower than that of glass (2.5-2.9g/cm³), carbon (1.7-1.9g/cm³) and basalt (2.7-3.0g/cm³) [71, 72]. For Curv® used in this study, the polypropylene tapes were woven in a balanced 2×2 twill-weave fabric structure [57, 73] as shown in Figure 3.3.

![Figure 3.3](image.png)

**Figure 3.3** (a) Twill-weave fabric Curv®; (b) representative unit cell of Curv® [74]

The processing of Curv® starts from making the polypropylene tapes, through co-extrusion and cold drawing. Peijs [64] developed a co-extrusion technique, which allowed a maximum melting temperature difference between the composite constituents to be 20-30°C, as shown in Figure 3.4 [57, 75, 76]. Through a continuous co-extrusion process, a PP homo-polymer tape was coated with co-polymer. Two types of polypropylene with different melting temperatures are co-extruded into tapes [75].
These tapes consist of an oriented polypropylene homo-polymer core and a polypropylene copolymer skin, as shown in Figure 3.4. A copolymer always melts at lower temperatures than the homo-polymer, owing to its molecular structure. Then, coextruded tape was further stretched through drawing oven, leading to a high-modulus, high-strength tape as shown in Figure 3.5. Over-drawn tape is preferred for the preform optimization of all-PP composite structures, because it offers better mechanical properties and has a reduced occurrence of internal void.

Figure 3.5 Coextrusion technology with additional stretching to produce high-strength tapes [57, 77].

Finally, a hot compaction technique is introduced to compact the woven structure into a composite sheet material under suitable temperature and pressure conditions [57, 72, 76,
Materials and Methodology

78, 79]. The fabric is subjected to a specific pressure depending on the thickness to prevent the thermal shrinkage. Typically, the compaction temperature is controlled between 140°C and 190°C [57, 72] and the temperature and pressure are kept for around 10 minutes before further raising the pressure for cooling. During this process, the polypropylene tape melts partially and the molten copolymer PP skin forms the matrix after solidification, as shown in Figure 3.6. The residual part of the tapes (homopolymer core) acts as the reinforcement in the final product of self-reinforced polypropylene composite material. The compaction temperature is crucial in the manufacturing of self-reinforced composite material. If temperatures are too low, the voids within the woven structure cannot be completely filled. On the other hand, if temperatures are too high, it will result in the reinforcement content being diminished.

![Figure 3.6 Principle sketch of hot compaction in the example of unidirectional arranged fibres [72]](image)

3.2.2 Twintex®

Twintex® is a continuous (woven) glass fibre reinforced polypropylene, woven in a balanced 2/2 twill-weave fabric pattern as shown in Figure 3.7 [57, 69, 80]. Glass fibre makes up 60 percent of the composite by weight and 35 percent by volume, giving a density of 1.485g/cm³.
Materials and Methodology

Figure 3.7 (a) Twill-weave fabric Twintex® with (b) representative unit cell [74]

Twintex® is formed by commingled yarn, which is the mixture of reinforcement fibre and thermoplastic filament, also named as hybrid yarn [81, 82] (Figure 3.8). Employing the usage of hybrid yarn in composite part simplifies the manufacturing process by eliminating the impregnation of matrix. This method requires a very short flow path for the molten PP filament to cover the void and form the matrix, leading to a fast impregnation (Figure 3.9). In addition, the hybrid yarn has the potential to produce part that has a homogeneous reinforcement and matrix distribution over the thickness. The volume fraction can also be easily adjusted by varying the number of reinforcement fibre and thermoplastic filament in hybrid yarn production [81-83].

Figure 3.8 Twintex® commingled glass/polypropylene yarns [69]
Materials and Methodology

Figure 3.9 Schematic diagram of a cross section of a (a) pre-consolidated and (b) consolidated Twintex® bundle [82]

One approach to mix reinforcing and matrix filaments is to use an air-jet texturing machine as shown in Figure 3.10. In this approach, mixing of the reinforcement and matrix filaments was carried out aerodynamically in an air-jet nozzle. Different types of air-jet nozzles are equipped for texturing and intermingling the filaments [81, 83].

![Commingled Yarn Diagram](image)

Figure 3.10 Schematic diagram of commingling process with the use of Air-Jet Texturing Machine [83]

Typically, the consolidation of pre-consolidated Twintex® woven fabric is carried out through compression mould, which consists of a pair of male-female moulds for temperature and pressure introduction [82]. Figure 3.11 presents a schematic diagram...
for a typical compression mould, commonly used for the manufacturing of thermoplastic matrix composite material. The pre-consolidated fabric is compressed between the male and female mould pair, which is pre-heated to a temperature above the melting temperature of polypropylene matrix material (180°C-230°C) by circulating oil or electricity. Pressure (0.1-3 MPa) is introduced with a hydraulic or pneumatic piston, according to the thickness of preconsolidated sheet and it is held during the cooling process [80, 82, 84].

![Figure 3.11 A schematic of a compression mould [82]](image)

### 3.3 Experimental Methodology

Yoshida tests were performed to determine the wrinkling behaviour of composite material [85]. The standard test specimen is a 100mm square piece, stretched at two diagonal corners to trigger wrinkling phenomenon [85-89]. The tensile elongation along vertical axis introduces compressive stress along the horizontal axis due to Poisson’s effect. This induced compressive stress causes the wrinkling initiation and propagation along the horizontal axis [90, 91]. All the tests were conducted using an INSTRON® 4505 testing frame, as shown in Figure 3.12. The Yoshida specimens were fixed
Materials and Methodology

diagonally, elongated uniaxially with a displacement rate of 1mm/min at the lower gripper.

![Experimental setup used in the Yoshida test](image)

The surface strain and deformation during the loading process was recorded, calculated and analysed by means of two sets of non-contact full field three-dimensional photogrammetric optical strain measurement system, namely ARAMIS® system developed by GOM, mbH [92, 93]. The current three-dimensional photogrammetry system uses triangular scheme by a pair of high speed, high resolution and digital CCD cameras to elucidate the evaluation of the displacement field during the loading. Each camera takes images of the specimen during the test. Then, the displacement evaluation and surfaces strain information in two-dimensional field can be obtained by processing these images with photogrammetric techniques. By correlating the two dimensional results from two cameras, a three-dimensional full-field contour of the surface strain and deformation distribution can be obtained. The system uses the gradient of displacement field to capture the evolution of strain during loading.
Materials and Methodology

Compared to the conventional strain measurement methodology, ARAMIS® system is able to provide real time full field strain results during the entire loading process. ARAMIS® 5M system measures strain at a maximum frame rate of 15 Hz, which allows the system to take an image and provide the strain information for every 0.0667 second. With respect to the precision and accuracy, ARAMIS® 5M system is capable of measuring strain that ranges from 200 με to more than $10^6$ με with error of no more than 100 με. A 200 με strain corresponds to a nominal strain for a 100mm tensile sample being stretched by 0.02mm. Two sets of ARAMIS® system, including ARAMIS® 5M system and ARAMIS® 1.3M system, are used for strain and displacement measurement in this study. Their specifications are summarised in Table 3.1.

Table 3.1 Specifications of the ARAMIS® 5M and 1.3M system [92, 93]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ARAMIS® 5M</th>
<th>ARAMIS® 1.3M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measuring volume (mm)</td>
<td>10 × 8 to 5000 × 4150</td>
<td>10 × 8 to 1700 × 1360</td>
</tr>
<tr>
<td>Camera resolution (pixels)</td>
<td>2448 × 2050</td>
<td>1280 × 1024</td>
</tr>
<tr>
<td>Shutter time (s)</td>
<td>0.0001 to 2</td>
<td>0.0001 to 2</td>
</tr>
<tr>
<td>Maximum frame rate (Hz)</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>Strain range</td>
<td>0.02% up to &gt;100%</td>
<td>0.05% up to &gt;100%</td>
</tr>
<tr>
<td>Strain accuracy</td>
<td>up to 0.01%</td>
<td>up to 0.02%</td>
</tr>
</tbody>
</table>

The ARAMIS® system provides a list of recommended default measuring volumes with appropriate configuration measures. Different default measuring volume of the ARAMIS® system can be achieved with different combination of the distance between the camera and the specimen, the angle and distance between two cameras, as listed in ARAMIS® user manual. For ARAMIS® 5M system, a default volume of 175 × 150mm was selected. The camera positioning and the dimensions used to obtain this volume are shown in Figure 3.13 (a). The actual camera angle, slider distance and measuring distance are 25°, 136mm and 395mm, respectively [94]. For ARAMIS®
Materials and Methodology

1.3M system, the default measuring volume was set to 175 × 140mm. It was found that small difference in the measuring volume between ARAMIS® systems does not influence the validity of the experimental data obtained. The camera angle, base distance and measuring distance are obtained from the ARAMIS® manual, which are 25°, 190mm and 410mm, respectively [92], as shown in Figure 3.13 (b). The camera position of ARAMIS® 5M is defined as slider distance, while ARAMIS® 1.3M uses base distance. The differences between these two types of distance are explained in Figure 3.13 (a) and (b), respectively. Shutter time is adjusted according to the expected test speed and recording rate, to avoid either underexposure or overexposure.

![Camera configuration](image)

Figure 3.13 Camera configuration [95] with dimensions shown for (a) 175*150mm measuring volume for ARAMIS® 5M system [94]; (b) 175*140mm measuring volume for ARAMIS® 1.3M system [92].

A high contrast stochastic pattern is required so that the ARAMIS® system can recognise the surface structure, allocates coordinates to each image pixels and computes the local displacement and strain. Before creating the pattern, the surfaces of the
Materials and Methodology

specimen were cleaned with isopropanol to remove the grease and oil. Next, a thin film of matte white body primer was sprayed over the surface and allowed to dry. The matte finish reduced the reflectivity of the surface. Finally, a stochastic black pattern (Figure 3.14) was created to facilitate the measurement of the local displacement and strain.

![Figure 3.14 An example of high contrast stochastic pattern](image)

The frame rate for both ARAMIS® systems was set as 1 frame per second, which means an image was taken in every 1/60 mm of elongation. The shutter time was adjusted depending on the lighting condition for each trial to avoid underexposed and overexposed conditions. Normally, the shutter time is set to around 20ms. The details of the experimental plan are illustrated in Table 3.2. Four repeats of each experiment was carried out.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Material</th>
<th>Fibre Orientation</th>
<th>Displacement Rate (mm/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Curv®</td>
<td>[0°/90°]</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Curv®</td>
<td>[45°/-45°]</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Twintex®</td>
<td>[0°/90°]</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Twintex®</td>
<td>[45°/-45°]</td>
<td>1</td>
</tr>
</tbody>
</table>

The standard Yoshida test specimen is a 100mm square piece, stretched at two diagonal corners, as shown in Figure 3.15 [85-89]. Traditionally the gripping width was 40 mm, but in this study it is reduced to 25mm. This reduced gripping width produces one major wrinkling mode for the entire sample.
Materials and Methodology

Figure 3.15 Schematic illustration of the Yoshida test

3.4 Finite Element Analysis

This section will introduce the formulation type, material modelling and the modelling parameters used to simulate the wrinkling behaviour. The Finite Element model developed in this work is implemented on LS-Dyna, using explicit formulation. The woven thermoplastic composite materials are modelled as orthotropic material meshed as shell element.

3.4.1 Explicit Formulation

The equation of motion of a deformable object with nodal displacements \( x \) is given as

\[
M \ddot{x} + K x = F_e
\]  

3.4.1

where, \( \ddot{x} \) is the nodal acceleration vector, \( M, K \) are the body’s mass and stiffness matrix respectively, and \( F_e \) is external body force and external load vector. The matrix equation of motion for the finite element model is given as [96]

38
Materials and Methodology

\[ R = M \ddot{x} + F_i - F_e \]  

where \( F_i \) denotes internal force and \( R \) is residual force vector. In explicit method, the displacement is a function of time, which means that the velocity and nodal acceleration as well as the mass and damping are considered in this scheme. The equation below expresses dynamic equilibrium condition at the instant of time [97, 98]

\[ M^t \ddot{x}^t + K^t x^t = F_e^t \]

where the term \((K^t x^t)\) represents the internal forces generated from the stiffness. By applying central difference integration rule, the velocities and displacements can be solved as

\[ \dot{u}^{t + \frac{\Delta t}{2}} = \left( M^t \right)^{-1} \left( F_e^t - K^t x^t \right) \Delta t^{t + \frac{\Delta t}{2}} + \dot{u}^{t - \frac{\Delta t}{2}} \]

\[ u^{t + \Delta t} = u^t + \dot{u}^{t + \frac{\Delta t}{2}} \Delta t^{t + \Delta t} \]

The equation of motion for the body is integrated with an explicit central difference integration rule

\[ \dot{u}^{t + \frac{\Delta t}{2}} = \dot{u}^{t - \frac{\Delta t}{2}} + \frac{(\Delta t^{t + \Delta t} + \Delta t^t)}{2} \ddot{u}^t \]

\[ u^{t + \Delta t} = u^t + \dot{u}^{t + \frac{\Delta t}{2}} \Delta t^{t + \Delta t} \]

where \( \dot{u} \) is velocity, \( \ddot{u} \) is the acceleration and \( t - \frac{\Delta t}{2} \) and \( t + \frac{\Delta t}{2} \) indicate the increment number and mid-increment number, respectively.
Materials and Methodology

\[ \ddot{u}^t = M^{-1} \cdot (P^t - I^t) \]  

3.4.8

where \( M \) is the mass matrix, \( P \) denotes the external applied load vector and \( I \) is the internal force vector. The main advantage of explicit formulations is that this scheme is easier to converge and requires a lower computational cost at large model size than implicit scheme. However, explicit scheme utilizes very small increments, commonly in microseconds, leading to millions of increments required for several-seconds loading processes.

3.4.2 Material Modeling

The sheet composite geometry is meshed as Belytschko-Leviathan shell element, which is developed based on Reissner-Mindlin kinematic assumption [99, 100]. The surface of each element is defined by a bi-linear nodal interpolation function, providing 5 degrees of freedom in local coordinate system, as shown in Figure 3.16.

![Figure 3.16 Presentation of integration point in shell formation in LS DYNA [100]](image)

The woven thermoplastic composite materials (Curv® and Twintex®) are characterised as elastic orthotropic material in this study for simulation in Yoshida test. The stress and strain relation for elastic orthotropic material is expressed as follow [53].

40
Materials and Methodology

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{bmatrix} = [S]\{\sigma\} = 
\begin{bmatrix}
1/E_1 & -v_{21}/E_2 & -v_{31}/E_3 & 0 & 0 & 0 \\
-v_{12}/E_1 & 1/E_2 & -v_{32}/E_3 & 0 & 0 & 0 \\
-v_{13}/E_1 & -v_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/G_{23} & 0 & 0 \\
0 & 0 & 0 & 0 & 1/G_{31} & 0 \\
0 & 0 & 0 & 0 & 0 & 1/G_{12}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix}
\]

where \([S]\) is the compliance matrix, the inverse of stiffness matrix \([C]\) \(([S] = [C]^{-1})\); \(E\) is the Young’s Modulus; \(v\) is the Poisson’s ratio; and \(G\) is the Shear Modulus.

The non-principal coordinates elastic constants, such as Curv® and Twintex® orientated at \(45^\circ/-45^\circ\), are related to nine independent elastic constants in principal coordinates and the orientation angle \((\theta)\). The transformation equations for stresses in xy-coordinate are shown in matrix form of

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yz} \\
\tau_{zx} \\
\tau_{xy}
\end{bmatrix} = [T]^{-1}\{\sigma\} = 
\begin{bmatrix}
c^2 & s^2 & 0 & 0 & 0 & 2cs \\
s^2 & c^2 & 0 & 0 & 0 & -2cs \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & c & -s & 0 \\
0 & 0 & 0 & s & c & 0 \\
-2cs & cs & 0 & 0 & c^2 - s^2
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix}
\]

where \(c = \cos(\theta), s = \sin(\theta)\) and \([T]\) is the transformation matrix.

3.4.3 Yoshida Test

Figure 3.17 shows a diagram of the parts modelled in the simulation, in which a square part is stretched uniaxially at two diagonal corners with the gripping width of 25mm. Both displacement and rotation at upper corner are fixed, while the lower corner is only allowed to move to negative \(y\)-direction at a displacement rate of 0.1mm/s. The
Materials and Methodology

Composite geometry is meshed with shell elements with a size of 2mm, resulting in 2,500 elements in 100mm square specimen. A convergence test is run for the model with a mesh size of 1mm, resulting in 10,000 elements. The change in out of plane displacement at central point is 1.4 percent with a fourfold number of elements. The computation time for the simulation with a finer mesh is fivefold longer than the 2mm meshing scheme. Based on the convergence test, it is found that 2mm square shell element can efficiently provide accurate result. Five integration points are defined throughout the thickness to capture the bending effect.

![Model Geometries and Mesh used in the Finite Element Simulation](image)

Figure 3.17 Model Geometries and Mesh used in the Finite Element Simulation

Considering the existence of manufacturing defects, wrinkling in the real world may initiate at lower axial elongation than in the simulative perfect condition. Due to the residual stress built up during consolidation process associated with the fact that the weaving structure cannot be perfect balanced, it is difficult to produce perfectly flat composite sheet [101]. The geometrical defects, for example the initial warpage, cannot be neglected in the design phase, for it may lead to an unsafe structure that wrinkles at a
Materials and Methodology

load lower than the estimated critical load. For this reason, it is crucial to consider the geometrical imperfection in simulation.

To implement geometrical imperfection, a concept of the ‘worst’ imperfection is introduced to estimate the critical load in the worst case scenario [102]. The specimen is modelled as perfect structure in worst imperfection study, with the imperfections directly applied in FEA model by introducing a small amount of force perpendicular to the sheet surface. [103-106].

The entire simulation is divided into two separate steps, referred to Worse Imperfection Introduction and Deformation, shown in Figure 3.18. In the Worse Imperfection Introduction step, fixed support is assigned to both upper and lower corners. The worst imperfection is introduced by applying a small amount of perpendicular force (0.05N) to the centre of the geometry. This force is sustained until the end of simulation. During the Deformation stage, a 0.1mm/s vertical displacement is assigned to the lower corner to trigger the out-of-place displacement. The time increment size is defined as 0.1s.

![Figure 3.18 Boundary Condition used in Simulation](image)

Figure 3.18 Boundary Condition used in Simulation
3.5 Summary

This chapter gives an overview of composite materials and the manufacturing process of two fibre reinforced woven thermoplastic composite materials (Curv® and Twintex®). Then, an introduction of the experimental methodology is presented, including the equipment set up and the specimen preparation. Two set of ARAMIS® system were used for all tests to provide data on both surfaces, which is of great importance for studying the wrinkling behaviour as well as constructing wrinkling indicator. Finally, this chapter highlights details of FEA modelling. The model is implemented on LS-Dyna, based on explicit formulation. Composite sheet is modelled as orthotropic material, using Belytschko-Leviathan shell element.
Chapter 4 Experimental Results

4.1 Introduction

This Chapter aims to study the wrinkling behaviour of selected composite materials through Yoshida tests experimentally, with a particular focus on the onset of wrinkling. The experimental design as explained in Chapter 3 is adopted for the study of wrinkling behaviour for Curv® and Twintex®.

4.2 Out-of-plane Displacement

Wrinkling behaviour is studied in Yoshida test with two types of woven thermoplastic composite materials and two different fibre orientations. Out-of-plane displacement as a function of elongation is plotted to facilitate the study of the onset of wrinkling. Wrinkling initiates when the out-of-plane displacement over the entire surface becomes non-uniform. Four points of interest are selected to study the out-of-plane displacement and the onset of wrinkling. Figure 4.1 illustrates the position of the points of interest on a typical contour of the out-of-plane displacement from the ARAMIS® system. The central region (node 1 and node 2) moves away from the ARAMIS® system in the front during experiment, exhibiting a small negative value for the out-of-plane displacement. The wing regions (node 4) move toward the camera, displaying a positive displacement. The regions (node 3) between the central and wing regions have a negligible out-of-plane displacement.
Experimental Results

Figure 4.1 Position of Points of Interests Marked in the Out-of-Plane Displacement Contour

The out-of-plane displacement as a function of vertical elongation at four points of interest for different material systems are plotted in Figure 4.2. The history of the out-of-plane displacement assists the identification of the onset of wrinkling. The elongation period representing the onset of wrinkling is highlighted in Figure 4.2, which is identified by the bifurcation of the curves for out-of-plane displacement at nodes of interest.
Experimental Results

![Graphs showing out-of-plane displacement vs. elongation](image)

**Figure 4.2** Evolution of out-of-plane displacement at points of interest of (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°] samples.

With a displacement rate of 1mm/min applied to the lower end, all specimens initially have a uniform negligible value for out-of-plane displacement. Curv® [0°/90°], Curv® [45°/-45°] and Twintex® [45°/-45°] samples start to wrinkle at different stages of elongation, whist there is no obvious change in out-of-plane displacement for Twintex® [0°/90°] samples. This Twintex® [0°/90°] sample fails at gripping region before wrinkling initiates indicating no wrinkling before failure.

Due to the loading rate used in the experimental methodology, the out-of-plane displacement changes slowly. For this reason, the wrinkling seems to initiate over a short period, so it is difficult to find a point that can exactly describe the onset of wrinkling. Thus, the onset of wrinkling in this study is defined over a short period of elongation. Different specimens wrinkle at different elongation periods, as summarized in Table 4.1.
Experimental Results

Table 4.1 Elongation Period during the Onset of Wrinkling

<table>
<thead>
<tr>
<th></th>
<th>Curv® [0°/90°]</th>
<th>Curv® [45°/-45°]</th>
<th>Twintex® [0°/90°]</th>
<th>Twintex® [45°/-45°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elongation (mm)</td>
<td>0.67-0.92</td>
<td>0.33-0.58</td>
<td>no wrinkle</td>
<td>0.42-0.67</td>
</tr>
</tbody>
</table>

Among the tested samples, Curv® [45°/-45°] specimen is the easiest to wrinkle. Wrinkling starts between 0.33 and 0.58mm of elongation; followed by Twintex® [45°/-45°] between 0.42 and 0.67mm. For these two samples, the young’s modulus along the loading approaches the value of the young’s modulus of the matrix. The matrix for these two samples is polypropylene and the similar onset of elongation period can be attributable to the usage of the same matrix material. The wrinkling tendencies of the samples clearly indicate that the young’s modulus is an influential parameter for the onset of wrinkling. The average normal strain at failure region of Twintex® [0°/90°] samples at failure was around 4%. This low strain to failure of these samples initiates the failure of the samples before wrinkling could occur. In addition to the influence of young’s modulus on wrinkling, the boundary conditions imposed on the warp and weft fibre bundles seem to influence the onset of wrinkling. For the fibre orientation of [45°/-45°], except for the warp and weft fibre bundle at corners that are clamped to INSTRON®, the remaining part of the sample is less constrained. For [0°/90°] fibre orientation, the warp fibres are fixed at two diagonal corners, while the weft fibres are free to move. This poses more constraint on warp fibres for the [0°/90°] samples.

4.3 Evolution of Strain at the Central Region (node 1)

The central region of a specimen is the key part in Yoshida test, as this region experiences higher state of compressive strain compared to wing regions. Since
wringling is instability due to compression, it is more likely to initiate at the central region, which undergoes higher strain compared to other regions. In this section, strain evaluation at the central region (node 1) for different material systems is investigated to understand the wrinkling behaviour, especially at the onset of wrinkling.

4.3.1 Strain Path

The evolution of strain paths up to an elongation of 1.5mm at central point on two faces of the sample is plotted to facilitate the study of principal strain behaviour at the onset of wrinkling. The strain path diagram is plotted as major principal strain over minor principal strain. Its localized reciprocal slope is the strain increment ratio (SIR) ($\Delta \beta = \Delta \varepsilon_2 / \Delta \varepsilon_1$), which will be used as metric for wrinkling indication in this study. The small fluctuation introduced by machine vibration and noise can significantly affect the localized slope. To eliminate the influence of systematic errors, Fourier curve fitting with power of 3 is applied to every set of data. A statistical measure R-square is used to measure the goodness of fitting. This statistic describes correlation between the original values and the fitted values, defined as the ratio of the sum of squares of the regression (SSR) and the total sum of squares (SST).

$$R - square = \frac{SSR}{SST}$$  \hspace{1cm} 4.3.1

where SSR and SST are expressed as follows;

$$SSR = \sum_{i=1}^{n} w_i (\hat{y}_i - \bar{y})^2$$  \hspace{1cm} 4.3.2
**Experimental Results**

\[ SST = \sum_{i=1}^{n} w_i(y_i - \bar{y})^2 \quad 4.3.3 \]

\( \hat{y}_i \) represents the fitted result; \( y_i \) is the original data and \( \bar{y} \) is the mean of the original data. R-square ranges between 0 and 1, with the value closer to 1 indicating that better fitting is obtained. In this study, R-square of 95 percent is used as a metric to ensure the quality of fitting. The fitted curve is only acceptable when its R-square is larger than 0.95.

The major-minor principal strain behaviour at the central point is plotted in Figure 4.3. The results for original raw data and the fitted curved are compared on two faces of the surface. The minor principal strain on one face of the surface becomes more negative after the onset on wrinkling, resulting in a more negative SR. On the other hand, the other face of the surface exhibits less negative minor principal strain. This behaviour can be attributable to the curvature induced by the wrinkled samples.
Experimental Results

Figure 4.3 Strain path at the central point of (a) Curv® [0°/90°]; (b) Curv® [-45°/45°]; (c) Twintex® [0°/90°]; (d) Twintex® [-45°/45°] samples.

For Curv® [0°/90°], Curv® [45°/-45°] and Twintex® [45°/-45°], the slopes of the strain path on faces remains constant before wrinkling starts. A sudden change in major-minor principal strain relation (strain ratio) is observed at the onset of wrinkling. On one face, the minor principal strain as well as the SR becomes more negative due to surface compression, while the other face experiences a less negative minor principal strain is observed. Twintex® [0°/90°] specimen fails before the onset of wrinkling. This is clear from Figure 4.3 (c), as there is no abrupt changed in strain path for both faces. These results indicate that it is possible to establish an indicator for wrinkling based on the evolution of the strain path.

This approach of using the evolution of the strain path to predict the onset of wrinkling is the fundamental contribution of this study. This approach is the first of its kind in predicting the onset of wrinkling.
**Experimental Results**

Different materials have different values for initial slope which is illustrated in Figure 4.3. The difference in initial slopes is the result of different mechanical properties. Larger Young’s modulus and smaller Poisson’s Ratio offers a steeper strain path, leading to a less negative SR.

During the period when the strain path evolves linearly, the sheet undergoes only in-plane deformation. Ideally the strain on two faces should behave in the same way before the onset of wrinkling, leading to a pair of coincided strain paths. The strain paths for Twintex® specimens have different slope on two faces. This can be attributable to the woven nature of Twintex®, leading to the difficulty of obtaining the strain information from the same fibre bundle for both faces. If the central point is located on a warp fiber bundle on one face, this point will be situated on a weft bundle on the opposite face. This issue is not observed on Curv® specimens. Due to the small size of the co-extruded tapes leading to small weaving structure, the strain result for Curv® specimen is the averaged value for the weaving units. For the Twintex® [45°/-45°] samples, the post wrinkling behaviour is different on different faces and this can be attributed to the woven nature of Twintex®.

**4.3.2 Strain Ratio**

The stain ratio ($\beta = \varepsilon_2 / \varepsilon_1$) is plotted as a function of elongation, as shown in Figure 4.4. The evolutions of SR for each material system are compared between two faces. To examine whether SR can be used as a satisfactory metric for wrinkling indication, the elongation period representing the onset of wrinkling is highlighted to verify whether the wrinkling indicated by the SR falls into the pre-defined elongation period.
Experimental Results

Figure 4.4 Strain ratio at the central point of (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°] samples.

It is observed that the values at the central point (node 1) for both faces have a similar SR and this value remains almost constant before the onset of wrinkling. After the onset of wrinkling, SR on one face becomes less negative. This is attributable to the onset of new tensile stress on this face caused by the surface curvature due to wrinkling, so that
Experimental Results

the compressive stress is reduced. On the opposite face, a more negative SR is obtained, since the surface curvature adds more compression to the central part of this face.

For the wrinkled samples, SR moves away from its initial value at different elongations, indicating the onset of wrinkling. These abrupt changes in SR are in good agreement to the elongation period obtained from the out-of-plane displacement. Curv® [45°/-45°] samples exhibit abrupt changes at the lowest deformation; followed by Twintex® [45°/-45°] and Curv® [0°/90°] samples. The figure for Twintex® [0°/90°] samples shows no obvious variation, because no wrinkling is observed.

However, in some samples, like Curv® [0°/90°] specimens, the changes are not abrupt, which may lead to a difficulty in the abrupt change quantification and identification. For this reason, establishing a wrinkling indicator based on the abrupt change in SR may fail to detect wrinkling.

4.3.3 Strain Increment Ratio

Strain increment ratio $\Delta \beta = \Delta \varepsilon_2 / \Delta \varepsilon_1$ is introduced to indicate the initiation of wrinkling. Here, $\Delta \varepsilon_2$ is the change in minor principal strain for one time increment and $\Delta \varepsilon_1$ is the change in major principal strain for the same period of time. The SIR is plotted as a function of elongation, as shown in Figure 4.5.
Experimental Results

Figure 4.5 Strain increment ratio at the central point of (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°] samples.

Before the onset of wrinkling, the SIR at the central node is unchanged and two faces exhibit a similar SIR. For Twintex® [0°/90°] samples, SIR appears to be constant over the loading process, elucidating no wrinkling occurs, because it fails before the onset of wrinkling. The initial SIR for Curv® [0°/90°], Curv® [45°/-45°] and Twintex® [45°/-45°] samples are -0.45, -0.6 and -0.95, respectively. The abrupt change in SIR is
Experimental Results

observed at different elongation for different material systems. All the abrupt change in SIR fall into the elongation period representing the onset of wrinkling, which implies that using the abrupt change in SIR is a viable metric for detecting the onset of wrinkling.

It is also observed that the overall evolutions of SIR are similar to those of SR, but the change in SIR is more pronounced than the change in SR when the sheet starts to wrinkle. This is attributable to the methods that SR and SIR are adopted to describe the strain path evolution. Figure 4.6 demonstrates the difference between SR and SIR in a typical strain path at the central node of Curv® [0°/90°] sample. The lower and upper bound of elongation period for the onset of wrinkling (from 0.67mm to 0.92mm) are selected for illustration the concept of SR and SIR.

![Figure 4.6 Method for calculating (a) strain ratio; (b) strain increment ratio](image)

SR (β) represents the **global reciprocal slope** of strain path ($\beta = \varepsilon_2 / \varepsilon_1$). For example, the major principal strain and minor principal strain at elongation of 0.67 mm are
Experimental Results

0.4145% and -0.1975%, respectively; hence, the SR at elongation of 0.67 mm is calculated as follows;

$$\beta = \frac{\varepsilon_2}{\varepsilon_1} = \frac{-0.1975\%}{0.4145\%} = -0.476$$

SIR captures the *localized reciprocal slope* during any stage in deformation ($\Delta\beta = \Delta\varepsilon_2 / \Delta\varepsilon_1$). For example, the SIR at elongation of 0.67 mm is calculated from principal strains at this stage (0.67 mm) and a stage before the lower bound (0.65mm). The major and minor principal strains at 0.65mm elongation are 0.4065% and -0.1935%, respectively. The SIR at the elongation of 0.67mm can be determined.

$$\Delta\beta = \frac{\Delta\varepsilon_2}{\Delta\varepsilon_1} = \frac{\varepsilon_{2,0.67} - \varepsilon_{2,0.65}}{\varepsilon_{1,0.67} - \varepsilon_{1,0.65}} = \frac{-0.1975\% - (-0.1935\%)}{0.4145\% - 0.4065\%} = -0.500$$

The values for SR and SIR at elongation of 0.67 and 0.92mm are listed in Table 4.2.

<table>
<thead>
<tr>
<th>Strain ratio</th>
<th>Strain increment ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower bound (0.67mm)</td>
<td>-0.476</td>
</tr>
<tr>
<td>Upper bound (0.92mm)</td>
<td>-0.487</td>
</tr>
</tbody>
</table>

It can be concluded from Table 4.2 that SIR is able to capture more subtle changes in the slope of strain path, compared to SR. The change in SIR during the onset of wrinkling is 26.2%, which is 11.4 times more than the SR variation (2.3%).

*This metric of using SIR to predict the onset of wrinkling is the fundamental contribution of the present work. This work clearly demonstrates that current common practise of using SR as a metric for predicting onset of wrinkling is not robust.* The work on representative industrial sink bowl forming test [20] was found
Experimental Results

that wrinkling at the flange, corners of the walls and mid-part of the walls initiated at different values of SR of approximately -1.0, -0.5 and 0.5, respectively. This observation clearly demonstrates that using SR as a metric to predict wrinkling behaviour is inadequate. It has been demonstrated in this work that the abrupt change in strain path provides a robust metric predicting the onset of wrinkling.

4.4 Evolution of Strain Increment Ratio at Four Points of Interest

The feasibility of using SIR as a metric to indicate the onset of wrinkling is tested at four nodes of interest on both faces. In this study, the comparison between two faces will be carried out to study the overall evolution of SIR, but the correlation of the nodes of interest between two faces will not be presented. Due to the woven nature of composite material, a point on the warp fibre bundle on one face may appear on the weft fibre bundle on the opposite face of the sample (especially on Twintex® samples). Under the Yoshida test, fibre bundle at different orientation exhibiting different strain behaviours, so that it is not fully meaningful to carry out a comparison between nodes of interest on different faces. In addition to the woven nature, the size and the arrangement of the facet cannot be adjusted to be exactly the same for two sets of ARAMIS® systems, so it is not possible to generate the result from exactly the same location but on different faces.

In this section, the comparison between the evolutions of SIR at four points of interest on two faces will be carried out. This helps to explore the similarities and differences of the overall strain evolution at the onset of wrinkling on two faces of the samples.
Experimental Results

4.4.1 Strain increment ratio

The SIR at four points of interest as a function of elongation is given in Figure 4.7. This face is subjected to an additional compressive stress after the onset of wrinkling due to the surface curvature, elucidating a more negative minor principal strain.

![Figure 4.7 Strain increment ratio at point of interest on the concave surface of (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°] samples.]
Experimental Results

Generally, the evolutions of SIR for all the nodes of interest in all samples follow a constant and similar slope before the onset of wrinkling. The initial SIR reflects the effective mechanical properties of each material system. With a higher effective young’s modulus or a smaller Poisson’s Ratio, the initial SIR will become less negative. For Curv® [0°/90°] and Curv® [45°/-45°] samples, the initial SIR for node 1 appears to be around -0.504 and -0.599, respectively.

The evolutions of SIR are constant at initial SIR until the onset of wrinkling. The onset of wrinkling is indicated by the abrupt change in SIR, elucidating the abrupt change in the slope of strain path. The SIR for node 1, 2 and 3 shows an abrupt decrease, because the additional compression caused by the wrinkling behaviour is added to the minor principal strain. The evolution of SIR for node 4 sees no abrupt change. This may attributable to that only a negligible amount of strain is developed in the nearby region, which fails to trigger the wrinkling instability.

During the post-wrinkling stages for wrinkled samples, SIR for node in wrinkled region (typically node 1, 2 and 3) continues to decrease to a more negative value. Then, SIR for different nodes stables at different values. Typically, the node 1 exhibits the most pronounced change in SIR during the onset and the growth of wrinkling compared to other nodes of interest. For example, the initial SIR for Curv® [0°/90°] samples at node 1, 2 and 3 are around -0.504, while the SIR after the abrupt change at node 1, 2 and 3 are stable at around -2.193, -1.799 and -1.220, respectively. The differences between the initial and final value of SIR for these three nodes are 1.689, 1.295 and 0.716 respectively. The result suggests the central region undergoes the highest state of compressive strain, has the most severe wrinkling and also exhibits the most
Experimental Results

pronounced change in SIR. There seems to be a clear relationship between the magnitude of the change in SIR and the wrinkling severity. Therefore, in addition to indicate the onset of wrinkling, it is also possible to use the abrupt change in SIR as a metric to examine the severity of wrinkling during the post-wrinkling process.

The point elucidating the onset of wrinkling for each node falls into the pre-defined elongation period of wrinkling initiation. Abrupt change for different nodes occurs at similar elongations but not at the exactly the same elongations. It seems that SIR at the node of interest exhibits the abrupt change in a sequential manner, indicating that different node starts to wrinkle at different elongation. For example, in Curv® [0°/90°] specimen, the abrupt drop for nodes 1 and 2 occurs at 0.7mm and 0.8mm, respectively; while the drop for node 3 started at 0.9 mm. This sequential drop may be attributable to the wrinkling propagation, for wrinkling should initiate at the central region which subjected to the highest state of compressive stress before growing to less stress-concentrated regions.

For Twintex® [45°/-45°] samples, SIR seems to become a slightly less negative after the abrupt decrease during the post-wrinkling process, while the SIR for Curv® samples continue to decrease to a more negative value. This may attributable to the different size of woven structure and different consolidating process, leading to different compressive strain behaviour. For this reason, evolutions of SIR may not necessarily follow the same path for different materials; however, it is clear that no matter in which material system, an abrupt change in SIR is observed when the samples start to wrinkle.

Figure 4.8 gives the information about the evolution of SIR at four points of interest on the opposite face of the sample. The minor principal stress on this face becomes less
Experimental Results

negative after the onset of wrinkling, because the induced compression is partially released due the onset of tensile effect caused by the surface curvature due to wrinkling.

Figure 4.8 Strain increment ratio at point of interest on the convex surface of (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°] samples.

For wrinkled samples, the evolutions of SIR remain constant before exhibiting an abrupt increase, indicating the onset of wrinkling. Unlike the evolutions of SIR on the opposite face becoming more negative after the abrupt drop, the abrupt increase leads to a less
Experimental Results

negative or even a positive SIR at large elongation. Due to the surface curvature introduced by wrinkling behaviour, the induced compression on this face of the sample is partially released, exhibiting a less negative minor principal strain and an abrupt rise in SIR. These results clearly illustrated that using the abrupt change in SIR for wrinkling indication can indicate the onset and the propagation of wrinkling on either face of the sample.

For the unwrinkled Twintex® [0°/90°] sample, the evolution of SIR does not exhibit any pronounced change, indicating no wrinkling starts before failure initiates. Owing to that it seems that using the abrupt change in SIR as a metric is capable of indicating wrinkling.

4.5 Summary

This chapter has studied the wrinkling behaviour on woven thermoplastic composite materials through Yoshida tests experimentally, with a particular focus on the onset of wrinkling. The elongation period for the onset of wrinkling is defined through analysing the out-of-plane displacement. It is found that the abrupt change in principal strain path can indicate the onset of wrinkling. SR and SIR are used as the metric to measure the abrupt change in strain path. Compared to SR, developing a wrinkling indicator based on SIR can capture more subtle change in strain path, offering an elevated robustness of the indication of the onset of wrinkling. This metric of using the abrupt change in SIR to predict the onset of wrinkling is the fundamental contribution of the present work.
Experimental Results
Chapter 5  Development and Validation of Wrinkling Indicator

5.1 Introduction

This chapter aims to develop an indicator based on the SIR and then test the usefulness of the proposed wrinkling indicator on the prediction of the onset of wrinkling in Yoshida tests. This indicator is programmed in MATLAB script, using the information extracted from FEA for predicting wrinkling. The modelling parameters as explained in Chapter 3 are adopted for simulating the wrinkling behaviour in Yoshida test. The predicted results are validated with the contour of out-of-plane displacement from experiments. The prime motivation of developing a wrinkling indicator is to detect and remove wrinkling for general forming practises, so that the validity of this indicator is also explored in a series of dome forming tests.

5.2 The Development of Wrinkling Indicator for the Implementation in Simulations

According to the experimental observations, the onset of wrinkling on either face of the samples can be identified by the abrupt change in the slope of the strain path. The metric of using SR or SIR has been tested and compared. It was concluded that compared to the SR based indicator, using the abrupt change in SIR as a metric provides a better indication of the abrupt change in slope of strain path. In this section, a wrinkling indicator based on SIR will be developed for the implementation in simulations.
**Development and Validation of Wrinkling Indicator**

SIR of every element at each time step is calculated from major and minor principal strain at this time step and one step before. For example, the value for time step N is calculated as follow,

\[
\Delta \beta_N = \frac{\Delta \varepsilon_2}{\Delta \varepsilon_1} = \frac{\varepsilon_{2,N} - \varepsilon_{2,N-1}}{\varepsilon_{1,N} - \varepsilon_{1,N-1}}
\]

where \( \varepsilon_1 \) and \( \varepsilon_2 \) are the major principal strain and minor principal strain, respectively. The time step ‘N’ ranges from the second to the last time step, because each SIR is calculated from two sequential time steps.

To capture the abrupt change in SIR at the onset of wrinkling, critical SIR (\( \Delta \beta_{cr} \)) is calculated element-wise and face-wise. If SIR at specific time exceeds the critical value, the wrinkling indicator will turn on. The critical SIR is developed from the initial SIR (\( \Delta \beta_{initial} \)) for each element. A factor of 1.1 is applied to initial SIR to quantify the allowable abrupt change, so wrinkling is suggested to initiate when the change in SIR is by either 10% larger or smaller than the initial SIR. The lower critical SIR (\( \Delta \beta_{cr,lower} \)) is calculated from equation 5.2.2, while the equation 5.2.3 gives the upper critical SIR (\( \Delta \beta_{cr,upper} \)). The lower critical SIR is more negative than the upper critical SIR.

\[
\Delta \beta_{cr,lower} = \Delta \beta_{initial} \times 1.1 \quad 5.2.2
\]

\[
\Delta \beta_{cr,upper} = \Delta \beta_{initial} / 1.1 \quad 5.2.3
\]

Initially, the evolution of SIR is in between of the lower and upper critical SIR, indicating that the sample does not experience wrinkling. Once the SIR drops below the lower critical SIR or rises over the upper critical SIR, the indicator at the corresponding
element at the specific stage will turn on. The relation 5.2.4 - 5.2.6 explain the method of using the critical SIR for wrinkling indication.

\[ \Delta \beta_N > \Delta \beta_{cr,lower} \quad \text{or} \quad \Delta \beta_{cr,upper} > \Delta \beta_N \]

Safe \hspace{1cm} 5.2.4

\[ \Delta \beta_N = \Delta \beta_{cr,lower} \quad \text{or} \quad \Delta \beta_{cr,upper} = \Delta \beta_N \]

Onset of Wrinkling \hspace{1cm} 5.2.5

\[ \Delta \beta_N < \Delta \beta_{cr,lower} \quad \text{or} \quad \Delta \beta_{cr,upper} < \Delta \beta_N \]

Wrinkle \hspace{1cm} 5.2.6

An example of the centre point for Curv® [0°/90°] samples is presented to illustrate the method adopted in the wrinkling indicator. The initial SIR for the face (namely face 1) imposed to additional compression after wrinkling is -0.5039. The initial SIR for the opposite face is -0.3725. Multiplying the factor of 1.1 to the initial SIR, the lower critical SIR on two faces is -0.5543 and -0.4098, respectively. Dividing the initial SIR by this factor, the upper critical SIR is given as -0.4581 and -0.3386, respectively. Figure 5.1 presents the evolution of SIR up to 1mm and the critical SIR at the centre point on two faces for Curv® [0°/90°] samples.
Development and Validation of Wrinkling Indicator

Figure 5.1 The evolution of SIR and the Critical SIR on (a) the face subjected to additional compression after wrinkling; (b) the opposite face at the centre point for Curv® [0°/90°] samples

The intersections between the evolution of SIR and the critical SIR indicate the onset of wrinkling. The predicted result is in good agreement to the experimental observation, because the intersections fall in the elongation period representing the onset of wrinkling. Furthermore, the intersections for two faces appear at similar elongation, so that the predicted result elucidates that wrinkling on two faces initiates around the same time. Thus, it can be concluded that using the critical SIR to measure the abrupt change in the slope of strain path has the potential to accurately predict the onset of wrinkling.

Another criterion is added to ensure that the indicator only reports the abrupt change due to wrinkling. The wrinkling affected region is subjected to in-plane loading before the onset of wrinkling, so the strain paths as well as the SIR on two faces of the sample should follow the same path. For this reason, the initial strain paths on two faces at the wrinkling affected region should be similar. On the other hand, when an element undergoes a change in deformation mode or is being bent, the strain paths on two faces may also exhibit abrupt change, but the initial strain paths on two faces have different slopes. These elements usually appear at the region next to the flange region or at the region in contact with the punch. When the tools start to deform the sample, out-of-plane deformation will be imposed these element. One face of the element is subject to more compressive stress, while a reduced compression or even tension is observed on the opposite face. This bending effect introduces different stress behaviour on two faces, leading to different initial strain paths on two faces of the shell element that undergoes change in deformation mode. Owing to that, to differentiate the abrupt change due to wrinkling from other sources, the allowable difference between the initial SIR on two
Development and Validation of Wrinkling Indicator

faces is set to be smaller than 20% of the averaged initial SIR. The tolerance is set to 20%, because this value is small enough to detect the element that is subjected to out-of-plane deformation (change in deformation mode). This value also allows small variations of initial SIR caused by small strain fluctuation for the element that only undergoes in-plane stresses.

The flow chart of the MATLAB script that contains the wrinkling indicator is presented in Figure 5.2. The outputs from simulation of major and minor principal strain for all elements are stored in a file and this information is read at the start of the MATLAB program. The script starts from imported files of the simulative results. Then, the values listed in Table 5.1 are calculated.

Table 5.1 The value calculated in the wrinkling indicator for one element

<table>
<thead>
<tr>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \beta^1_N(e) )</td>
<td>SIR on one face (face 1) at all time steps (N) for all elements (e)</td>
</tr>
<tr>
<td>( \Delta \beta^1_{\text{initial}}(e) )</td>
<td>Initial SIR on face 1 for all elements (e)</td>
</tr>
<tr>
<td>( \Delta \beta^1_{\text{cr,lower}}(e) )</td>
<td>“Lower critical SIR” on face 1 for all elements (e)</td>
</tr>
<tr>
<td>( \Delta \beta^1_{\text{cr,upper}}(e) )</td>
<td>“Upper critical SIR” on face 1 for all elements (e)</td>
</tr>
<tr>
<td>( \Delta \beta^2_N(e) )</td>
<td>SIR on the opposite face (face 2) at all time steps (N) for all elements (e)</td>
</tr>
<tr>
<td>( \Delta \beta^2_{\text{initial}}(e) )</td>
<td>Initial SIR on face 2 for all elements (e)</td>
</tr>
<tr>
<td>( \Delta \beta^2_{\text{cr,lower}}(e) )</td>
<td>“Lower critical SIR” on face 2 for all elements (e)</td>
</tr>
<tr>
<td>( \Delta \beta^2_{\text{cr,upper}}(e) )</td>
<td>“Upper critical SIR” on face 2 for all elements (e)</td>
</tr>
</tbody>
</table>
Development and Validation of Wrinkling Indicator

Figure 5.2 Flow Chart of Wrinkling Indicator Defined in this Study
Then, the value for \( e, N \) and \( \text{Light} \) is defined. ‘\( e \)’ is the number of the element, defined in form of numeric variable. This number is initially defined to 1, elucidating the first element. There are 2500 elements used in the simulation of Yoshida test, so ‘\( e \)’ ranges from 1 to 2500. ‘\( N \)’ is the time step, defined as a numeric variable. This value is initially defined to 2 because SIR is calculated from the current time step and one step before, so that SIR is only valid at time step starting from 2. There are 200 time steps used in simulation, so that the value for time step ‘\( N \)’ ranges from the second step to the 200\(^{th}\) step. \( \text{Light} \) is a logical matrix (Equation 5.2.7), which is initially \textit{OFF} for each element and for each time step. Only the value 0 and 1 are used in logical matrix. In this work, \textit{OFF} for \( \text{Light} \) is expressed as ‘0’, while \textit{ON} for \( \text{Light} \) means ‘1’. There are 2500 columns in \( \text{Light} \) matrix representing 2500 element in the Yoshida sample. 200 rows represent the 200 time step used in simulation. The first row corresponding to the first time step is defined in \( \text{Light} \) matrix, but this row will not be examined in the wrinkling indicator because the examined time step only ranges from 2 to 200. No wrinkling initiates at first time step (0.01mm of elongation), so that it is reasonable that the value for \( \text{Light} \) matrix at the first row remains \textit{OFF}. \textit{ON} for \( \text{Light} \) means that an abrupt change has been observed for the examined element in previous steps elucidating wrinkling has started, while \textit{OFF} for \( \text{Light} \) means that SIR for this element has not exhibited any abrupt change.

\[
\text{Light} = \begin{pmatrix}
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0
\end{pmatrix}
\]

Equation 5.2.7

To differentiate the abrupt change due to the onset of wrinkling from the abrupt change due to other sources, the difference between the initial SIR on two faces is examined at
Development and Validation of Wrinkling Indicator

each element. When the discrepancy between two values is larger than 20% of the averaged initial SIR for two faces, the abrupt change in SIR for this element cannot be attributed to the onset of wrinkling. In this case, the indicator will move to next element \((e=e+1)\); otherwise the indicator will move to wrinkling indication phase.

In the wrinkling indication phase, the values for time step ‘N’ and a universal logical variable \(Flag\) are firstly defined. Time step ‘N’ is defined to 2 when indicator starts to examine a new element, so that the indicator checks the abrupt change in SIR from the first valid value. \(Flag\) is a universal logical variable for the simplification of program, which is switched to \(OFF\) when the indicator starts to examine a new element. \(Flag\) turns \(ON\) after the first abrupt change in SIR, elucidating the onset of wrinkling. When the \(Flag\) is \(ON\), the \(Light\) matrix for this element can be directly switched to \(ON\) for the following time steps, without examining the abrupt change in SIR.

The abrupt change in SIR on two faces behaves differently, so the indications for the onset of wrinkling are defined through different methods. In this indicator, the face number (either 1 or 2) is shown as the superscript. Once the SIR \((\Delta \beta_N^1(e))\) at one element \((e)\) on face 1 at one time step \((N)\) becomes more negative than the lower critical SIR on face 1 \((\Delta \beta_{cr,lower}^1(e))\) and SIR at the same element on face 2 becomes less negative than the upper critical SIR for face 2 \((\Delta \beta_{cr,upper}^2(e))\), the universal variable \(Flag\) and the \(Light\) for this step for this element are switched to \(ON\). On the other hand, if the compression on face 1 is being partially released, \(Flag\) and \(Light\) will switch \(ON\) when the SIR on this face 1 becomes less negative than the upper critical SIR for face 1 \((\Delta \beta_{cr,upper}^1(e))\) and SIR on face 2 becomes more negative than the lower critical SIR for face 2 \((\Delta \beta_{cr,lower}^2(e))\). The criteria for two faces are summarized in Table 5.2.
Development and Validation of Wrinkling Indicator

Table 5.2 The Summarized Wrinkling Criteria for Concave and Convex surface

<table>
<thead>
<tr>
<th>Condition 1</th>
<th>Condition 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \beta_N^1(e) &lt; \Delta \beta_{cr,lower}^1(e) )</td>
<td>( \Delta \beta_N^1(e) &gt; \Delta \beta_{cr,upper}^1(e) )</td>
</tr>
<tr>
<td>( \Delta \beta_N^2(e) &gt; \Delta \beta_{cr,upper}^2(e) )</td>
<td>( \Delta \beta_N^2(e) &lt; \Delta \beta_{cr,lower}^2(e) )</td>
</tr>
</tbody>
</table>

Next, the value for ‘\( N \)’ and ‘\( e \)’ will be checked to see whether the time step or the element being examined is the last time step or the last element. If the current time step is not the last one (200) for this element, the program will move to examine the next time step (\( N=N+1 \)). Otherwise, the value for element will be checked. There are 2500 elements used for Yoshida sample. If the value for the number of element (\( e \)) does not equal to 2500, next element will be examined (\( e=e+1 \)). Otherwise, it is the end of the test.

5.3 Examination of the Wrinkling Indicator in Yoshida test

The proposed indicator is used to examine wrinkling behaviour of Yoshida test from simulations. The predicted results will be validated with the experimental observations. The predicted result of wrinkling formation and propagation through the proposed wrinkling indicator is presented in Figure 5.5-Figure 5.5. The criterion is checked at every element at every step, and the evaluation results are stored in Light matrix. If the value of Light matrix is ON for the specific step at one element, the element at this step will be highlighted.

Three states of interest are selected to facilitate the validation of predicted results to the experimental observations for wrinkled samples (Curv® [0°/90°], Curv® [45°/-45°] and Twintex® [45°/-45°] samples), including a pre-wrinkling stage at the lower bound for
Development and Validation of Wrinkling Indicator

the onset of wrinkling (Table 4.1) and two post-wrinkling stages at upper bound and at a
elongation of 1.2mm. For the unwrinkled Twintex® [0°/90°] samples, failure is
observed prior to the onset of wrinkling. Three states of interest are also checked for this
sample, including an elongation of 0.6mm, 0.9mm and 1.2mm (Figure 5.6).

The differences between the out-of-plane displacement at the central node and the
displacement at the node locating at the wing region for different samples at three
examined states are listed in Table 5.3.

Table 5.3 The differences between the out-of-plane displacement of the central node and the
displacement of the node locating at the wing region for different samples

<table>
<thead>
<tr>
<th>Sample Type</th>
<th>Lower Bound (mm)</th>
<th>Upper Bound (mm)</th>
<th>1.2mm Elongation (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curv® [0°/90°]</td>
<td>0.207</td>
<td>2.037</td>
<td>4.431</td>
</tr>
<tr>
<td>Curv® [45°/-45°]</td>
<td>0.232</td>
<td>1.724</td>
<td>8.176</td>
</tr>
<tr>
<td>Twintex® [0°/90°]</td>
<td>0.046</td>
<td>0.052</td>
<td>0.042</td>
</tr>
<tr>
<td>Twintex® [45°/-45°]</td>
<td>0.140</td>
<td>3.382</td>
<td>8.258</td>
</tr>
</tbody>
</table>

Figure 5.3 (a) Predicted Results from wrinkling indicator; (b) experiment results of the out-of-plane
displacement at three states of interest for Curv® [0°/90°] samples
Development and Validation of Wrinkling Indicator

<table>
<thead>
<tr>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>1.2mm Elongation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Figure 5.4 (a) Predicted Results from wrinkling indicator; (b) experiment results of the out-of-plane displacement at three states of interest for Curv\(^{\circledR} \left[45^\circ/-45^\circ\right]\) samples

<table>
<thead>
<tr>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>1.2mm Elongation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Figure 5.5 (a) Predicted Results from wrinkling indicator; (b) experiment results of the out-of-plane displacement at three states of interest for Twintex\(^{\circledR} \left[45^\circ/-45^\circ\right]\) samples
The experimental observation suggests that the composite sheet is flat at the stage of lower bound. The differences between the out-of-plane displacements at the central node and the node from wing region, as presented in Table 5.3, are negligible for all examined samples. Then, for wrinkled samples, the out-of-plane displacement becomes significant when the elongation reaches the upper bound, indicating the occurrence of wrinkling in experimental results. As for the predicted results, no element triggers the indicator before and at the lower bound. When the elongation approaches the upper bound, several elements located at the central region trigger the indicator. This suggests that the abrupt change in SIR is detected for these elements, indicating the occurrence of wrinkling at the central region of the Yoshida sample. For this reason, the onset of wrinkling in the predicted results also happens within the elongation period obtained from experimental results. This observation validates that the proposed indicator is capable of providing accurate indication of the onset of wrinkling.
Development and Validation of Wrinkling Indicator

Two post-wrinkling stage including the upper bound of the elongation period and the elongation of 1.2mm are checked to examine the capability of the indicator for predicting the post-wrinkling behaviour. The experimental results suggest that when the specimen is being further deformed, wrinkling becomes more severe. A larger difference between the out-of-plane displacements is obtained at 1.2 mm of elongation than at the upper bound, showing the propagation of wrinkling, as presented in Table 5.3. With respect to the predicted results, wrinkling grows from a few elements at the central region to a larger region. This reveals that the proposed indicator also can be used for describing the wrinkling propagation.

For Twintex® [0°/90°] samples, there is no wrinkling observed in the experiments before the onset of failure and the difference between the out-of-plane is negligible over the loading process. Predicted results also suggest no element triggers the wrinkling indicator, elucidating a good agreement between the predicted results and the experimental observations. This observation highlights that the proposed indicator can accurately predict the occurrence of wrinkling.

5.4 Examination of the Wrinkling Indicator in Dome Forming

Wrinkling is a common failure mechanism in sheet forming operations, especially in dome forming where the samples are not completely constrained at the flange region. The prime motivation of developing a wrinkling indicator is to be able to detect and in turn remove wrinkling in forming practises. In this section, the proposed wrinkling indicator is implemented in dome forming simulations on different samples to test the validity of the indicator in forming practise and to other class of material. Then, the
Development and Validation of Wrinkling Indicator

predicted results will be validated with the experimental observations. The schematic of dome forming model is given Figure 5.7.

![Diagram of dome forming model](image)

**Figure 5.7 Model Geometries of Dome Forming used in the Finite Element Simulation**

The geometries developed in the FE simulation include a blank, a punch, a die, and a blank-holder. The die stays stationary during forming and the blank holder can only move vertically. A 2kN BHF is applied to tighten the blank. The specimen is a full circular blank with a diameter of 180mm, which is formed to a depth of 50mm in FEA. The punch is placed just in contact with the blank and it is set to move down at a speed of 20mm/s. All the tool geometries are defined as rigid parts and master surfaces. The forming simulation is divided into two sequential steps, referred to BHF introduction and Forming. In the BHF introduction step, initial boundary conditions are assigned to simulate the blank-holder effect. In the Forming step, the punch is set to move down at a constant rate of 20mm/s. The material used in the simulation are two types of woven thermoplastic composite materials (Curv®, Twintex®) and metallic material (steel). The user-defined material model (UMAT) was implemented in FEA to model the
Development and Validation of Wrinkling Indicator

constitutive relationship for composite materials. The details of UMAT for Curv®, Twintex® are given in Appendix.

<table>
<thead>
<tr>
<th></th>
<th>Initial Stage</th>
<th>Just after onset</th>
<th>Post-Wrinkling</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>10 mm</td>
<td>40 mm</td>
<td>50 mm</td>
</tr>
<tr>
<td>(c)</td>
<td>10 mm</td>
<td>35 mm</td>
<td>50 mm</td>
</tr>
</tbody>
</table>

Figure 5.8 Wrinkling Indicator of two states of interest for (a) Curv®; (b) Twintex®; (c) Steel samples.

The predicted results are expressed in form of surface contour at initial stage, the stage just after wrinkling and a post-wrinkling stage, as presented in Figure 5.8. Initially, there is no wrinkling detected. Wrinkling starts at forming depth of 40 mm for Curv®,
**Development and Validation of Wrinkling Indicator**

35mm for Twintex® and 30mm for steel blank. Then, wrinkling grows as the forming depth increases. When the forming depth reaches 50mm, the predicted result clearly shows that there is a severe wrinkling occurs on the flange region. For composite samples, the top view of the shape of the deformed blank appears like a square with rounded corners. This is due to orthotropic nature of the composite materials caused by the fibre reinforcement.

Most of the wrinkling in dome forming initiates at the flange regions, which makes them difficult to observe because the tools (the blank-holder and the die) cover the flange region during the experiments. For this reason, the predicted results for dome forming are validated with the formed parts. In dome forming experiments, full circular blanks with diameter of 180 mm were used. A BHF of 2kN is used and all samples are formed at a constant punch rate of 20 mm/s to the depth of the onset of wrinkling suggested by simulation and to the depth of 50mm.

### 5.4.1 Dome Forming for Curv® Specimens

Figure 5.9 and Figure 5.10 present the predicted results and the formed part of the full circular Curv® samples. The samples are formed to the depth of 40mm and 50mm.
Development and Validation of Wrinkling Indicator

Figure 5.9 Comparison between Predicted Results and Experimental Results of Full Circular Curv® Specimens after Dome Forming (a) predicted result; (b) top view of the formed part; (c) side view; (d) detailed view of flange region with fibre orientation of [0°/90°]; (e) detailed view of flange region with fibre orientation of [45°/-45°] of the formed part at Forming Depth of 40 mm.
Figure 5.10 Comparison between Predicted Results and Experimental Results of Full Circular Curv® Specimens after Dome Forming (a) predicted result; (b) top view of the formed part; (c) side view; (d) detailed view of flange region with fibre orientation of [0°/90°]; (e) detailed view of flange region with fibre orientation of [45°/-45°] of the formed part at Forming Depth of 50mm.
Generally, the predicted results from wrinkling indicator are in good agreement to experimental observations. Both predicted results and experimental results suggest that wrinkling initiates at the flange region when the forming depth reaches 40mm. The severity of the wrinkling is non-uniform over the flange. Wrinkling is more significant at the region with the fibre orientation of [0°/90°]. The flange region with [0°/90°] fibre orientation tends to flow more into the die than the rest of the off-fibre directions. The flow of material offers additional compressive stress at the flange region for the region with the fibre orientation of [0°/90°], leading to a wrinkling.

When the sample being further deformed to a depth of 50mm, the wrinkling on the flange region becomes severe and tends to propagate to the region with the fibre orientation of [45°/-45°]. In addition to the wrinkling at flange region, some wrinkles appear at the side wall of the samples, which is also suggested by the predicted results.

The comparison between the predicted results using the proposed wrinkling indicator and the final parts of the dome forming experiments underlines that the proposed indicator is applicable to general forming operations.

5.4.2 Dome Forming for Twintex® Specimens

For the full circular Twintex® samples, simulation result suggests that wrinkling starts from a forming depth of 35mm. However, the experimental results suggest that Twintex® samples fail at the flange region prior to the onset of wrinkling, so that apart from being formed to 35mm and 50mm, the samples are also formed to the depth of 30mm to examine the failure developed in Twintex® samples.
No wrinkling is observed for the specimens formed to 30mm, while the fibre at the flange region fails before the onset of wrinkling at a forming depth of 35mm, as shown
in Figure 5.11. When the sample was formed to 50mm, the fibre failure then grows along the radial direction (Figure 5.11). This study focuses only on investigating the onset of wrinkling and failures, such as fiber fracture, are beyond the scope of this work.

5.4.3 Dome Forming for Steel Specimens

The validity of the proposed indicator is also checked in a sheet metal (steel) dome forming test, as shown in Figure 5.13. The full circular steel specimen is deformed to the depth of 30mm and 50mm.

Figure 5.12 Comparison between Predicted Results and Experimental Results of Full Circular Steel Specimen after Dome Forming at Forming Depth of 30 mm (a) predicted result; (b) top view of the formed part; (c) side view of the formed part.
A good agreement was observed in experimental and simulative results. The proposed indicator detects that wrinkling occurs on the flange region and side wall at the forming depth of 30mm, which is in agreement with the experimental observations. When the blank is further deformed, more elements at the flange region and on the side wall trigger the indicator, indicating the propagation of wrinkling. The experimental results also reveal that wrinkling at the flange region and the side wall becomes severer at the forming depth of 50mm compared to 30mm. Therefore, the dome forming on steel
samples points out that the proposed indicator is capable of predicting the onset and the growth of wrinkling for metal materials.

5.5 Summary

A wrinkling indicator based on SIR is developed and tested in this chapter. The abrupt change in SIR only takes place at the onset of wrinkling, but not necessarily at the wrinkling propagation. To recognize wrinkling at the onset and the growth, the indicator is defined in such a way that once the onset of wrinkling is detected, the wrinkling indicator will be turned ON for the following time steps. For this reason, this proposed wrinkling indicator can detect the onset and the growth of the wrinkling. The effectiveness of this indicator is validated through Yoshida tests and dome forming tests. This wrinkling indicator was programmed in MATLAB as a post processing tool from FEA simulation results. This provides a simulative indicator for onset of wrinkling. By comparing these results with experimental observations, it is found that the proposed indicator can predict the region where the wrinkling initiates in different material systems.
Development and Validation of Wrinkling Indicator
Chapter 6  Energy Approach of Predicting Wrinkling

6.1 Introduction

This chapter utilizes the analytical energy method to predict the onset of wrinkling in Yoshida test. Originally, the analytical energy method was developed to predict the critical buckling stress. This approach predicts the critical wrinkling load under specific boundary condition, which describes the overall wrinkling tendency and quantifies the effect of material properties, boundary conditions and surface deflection at the onset of wrinkling. In this chapter, this analytical approach is applied at an effective region to predict the wrinkling instability. The effective region is defined as the region under compression leading to the localized wrinkling. Since the effective region is the prime source of the localized wrinkling, the critical wrinkling stress for the effective region can represent the critical stress for the wrinkling affected region and for the entire sample.

This chapter will firstly introduce methodology used for the identification of the effective region and boundary conditions imposed on the effective region. Then, this method will be implemented in the Yoshida test to estimate the critical stress for different samples. Finally, a comparison between the predicted, experimental and simulative results will be presented to validate the usefulness and examine the limitations of this proposed energy approach.
6.2 The Significance of Using Energy Method in Wrinkling Prediction

Though there are numerous attempts in the implementation of energy method into the wrinkling prediction, the methodology used for selecting the wrinkling segment in these works leads to some difficulties in the critical stress estimation. In literatures, the typical effective wrinkling region was defined as the segment which is curved into a half-wave sine curve [33-37]. The width of the effective region is a function of the wrinkling expansion and the number of waves. In this thesis, a novel approach in the effective region selection is developed, which is based on the methodology introduced by Wang and Cao [48]. Wang and Cao suggested that the effective region is the region under compression leading to the wrinkling at localized region, which is determined from the simulative stress contour. However, in Wang and Cao’s work [48], there is no justification on the selection of the effective region and the determination of boundary condition of the effective region is unclear. In this chapter, FEA results are also employed in the selection of effective region, but a clear justification on the determination of effective regions and boundary conditions will be given.

Compared to the conventional methodology for effective region determination (assuming the segment is curved into specific shape), the calculation procedure for the critical wrinkling stress in this work is simplified. This is because using the results from FEA in effective region determination, the effective region determined in this approach has a constant dimension rather than being a function of the number of waves, so that it avoids using too many unknowns in the calculation. As a result, using energy approach associated with this improved effective region selection methodology leads to a faster
**Energy Approach** of Predicting Wrinkling

and simpler approach in the estimation of critical wrinkling stress, compared to the previous applications in wrinkling predication.

Utilizing energy approach in predicting wrinkling can also provide a direct relationship between the critical wrinkling stress and material properties, boundary conditions and geometrical parameters. Having an equation that quantifies the effect of these effective parameters on the onset of wrinkling is important in the investigation of wrinkling behaviour. As long as the compressive stress imposed on the effective region is lower than the estimated critical stress, this region and the entire structure will not exhibit any wrinkling.

In summary, the combined approach of analytical energy method and numerical method introduced in this chapter offers a faster and simpler approach in critical wrinkling stress estimation than the previous applications of energy method.

### 6.3 Effective Region in Yoshida test

Wang and Cao [48] suggested that the effective region is the dimension undergoing compression, which is determined from the contour of compressive stress from simulations. This work stated that the shape of the effective region for Yoshida test is rectangle, situated at the central of the sample. However, this work failed to justify the size, shape and location of the effective region and the boundary condition imposed on this selected region of interest. This section will explain and justify the methodology used for selecting the effective region and the boundary conditions for the region.
Energy Approach of Predicting Wrinkling

6.3.1 Size, Shape and Location of the Effective Region

The effective region as the driver of the localized wrinkling is the region under compression that leads to localized instability. Figure 6.1 shows the contours of simulative in-plane principal minor stress for Yoshida test at the elongation of 0.3mm, 0.67mm and 0.92mm for Curv® [0°/90°] samples. The state at elongation of 0.3mm and 0.67mm (lower-bound of elongation period) are pre-wrinkling states, while the elongation of 0.92 is the upper-bound of elongation period, representing the state just after the onset of wrinkling.

![Figure 6.1 Example contours of minor principal stress (unit: MPa) at elongation of (a) 0.3mm; (b) 0.67mm; (c) 0.92mm for Curv® [0°/90°] samples.](image)

Figure 6.1 suggests that the central part of the sample undergoes high state of in-plane minor principal stress, while the stress developed in the rest part of the sample is small.

In this work, a stress limit ($\sigma_{\text{limit}}$) is defined for the effective region selection. When the minor stress developed in one element is more negative than the stress limit, this element is imposed to adequate amount of compressive stress that may trigger the onset of wrinkling in later steps. The stress limit is determined from the most negative value of minor principal stress ($\sigma_{2,\text{min}}$). A factor of 0.2 is multiplied to this minimum minor
Energy Approach of Predicting Wrinkling

principal stress to have the stress limit. The equation for calculating the stress limit is given as follows;

\[ \sigma_{\text{limit}} = 0.2 \sigma_{2,\text{min}} \]  \hspace{1cm} 6.3.1

The region with the imposed minor principal stress ranging from stress limit to the minimum minor principal stress is defined as the effective region in this work. The examples of effective region for Curv® [0°/90°] samples at the elongation of 0.3mm, 0.5mm, 0.67mm, 0.92mm, 1.2mm and 1.5mm are presented in Figure 6.2.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figures/figure6_2.png}
\caption{Effective region for Curv® [0°/90°] samples at elongation of (a) 0.3mm; (b) 0.5mm; (c) 0.68mm; (d) 0.92; (e) 1.2mm; (f) 1.5 mm.}
\end{figure}

Figure 6.2 (a)-(d) suggest that the shape, size and the location of the effective region are almost the same at elongations before or just after the onset of wrinkling. For this
Energy Approach of Predicting Wrinkling

reason, using the stress limit is a feasible method for the effective region determination at any time steps before and just after the onset of wrinkling.

The shape and the size of the qualified region change slightly at post-wrinkling stages (Figure 6.2 (e), (f)), because the surface curvature due to wrinkling introduces additional stress to two faces of the samples. For this reason, the effective region changes after the onset of wrinkling. This also implies that using the effective region is only capable of predicting wrinkling up to the onset, while the post-wrinkling behaviour cannot be captured using the effective region determined from the pre-wrinkling states. Though it might be possible to describe the post-wrinkling behaviour by defining new effective region from post-wrinkling states, the growth and the propagation of wrinkling is beyond the scope of this study.

The effective regions for other samples are determined from the contour of minor principal stress at the pre-wrinkling step with an elongation of 0.3mm. The same method for selecting the effective region is implemented in Curv® [0°/90°], Curv® [45°/-45°], Twintex® [0°/90°] and Twintex® [45°/-45°] samples. The effective regions are shown in Figure 6.3.
Energy Approach of Predicting Wrinkling

Figure 6.3 Effective region and the approximated rectangular region for (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°] samples at elongation of 0.3mm.

In this study, the effective region is approximated by rectangular shapes as shown in Figure 6.3. Using this approximation may lead to a larger critical stress compared to the exact shape (rounded corner rectangle), because the additional material at the corners of rectangle shape offers a resistance to wrinkling. To test the effect of using rectangular region as a replacement, linear buckling analysis was carried out using implicit formulation implemented in ANSYS Workbench 16.1. Two different geometries with the same boundary conditions are used for examining the shape effects. The edges of the geometry are being simply supported and subjected to uniformed in-plane compressive loading to trigger the plate buckling. Figure 6.4 compares the load multipliers for different effective regions under the same boundary conditions.
Energy Approach of Predicting Wrinkling

According to Figure 6.4, the load multiplier for different geometries is 4.86 and 5.20, respectively, showing a discrepancy of 7%. For this reason, using the rectangular shape can provide an acceptable approximation for the critical stress estimation. It is clear that the rectangular effective region can withhold a higher state of stress before approaching the critical stress than rounded corner one. For this reason, using the rectangular effective region approximation may slightly over-estimate the onset of wrinkling. This approximation is implemented in other samples and the effective dimensions are listed in Table 6.1.

Table 6.1 The dimension of effective region for different samples

<table>
<thead>
<tr>
<th></th>
<th>Curv® [0°/90°]</th>
<th>Curv® [45°/-45°]</th>
<th>Twintex® [0°/90°]</th>
<th>Twintex® [45°/-45°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (mm)</td>
<td>48.1</td>
<td>56.6</td>
<td>36.8</td>
<td>59.4</td>
</tr>
<tr>
<td>b (mm)</td>
<td>76.4</td>
<td>73.6</td>
<td>70.7</td>
<td>99.0</td>
</tr>
</tbody>
</table>
6.3.2 Boundary Condition of the Effective Region

Then, the boundary condition of the effective region is determined. At the time step before and just after the onset of wrinkling, there is only a negligible amount of bending moment developed on the samples. This outlines that the effect of bending moment along the edges is ignorable, so that using the clamped support as the boundary for the edges is not appropriate. It is also clear that the edges are not allowed to move freely. For this reason, it seems that simply support is the most appropriate type of boundary condition along the edges of the effective region. The simply support boundary condition provides fixity in vertical displacements.

With respect to the loading, the induced compressive stress rather than the tensile stress is the prime cause of wrinkling in the effective region, so the effect of the tensile stress is neglected in the critical stress estimation. Though the tensile stress may change the shape of deflection and in turn influence the predicted critical stress, the error caused by ignoring the tensile stress is illustrated in Figure 6.5. The rectangular plate is simply supported on four edges and subjected to a compressive stress of 5MPa along two vertical edges. The model presented in Figure 6.5 (a) is imposed to a 15MPa tensile stress along two horizontal edges. A tensile stress of 15MPa is selected because the compressive stress should be induced by the tensile stress in Yoshida test, due to the Poisson’s effect. The Poisson’ ratio for the material used in simulation is 0.3.
Energy Approach of Predicting Wrinkling

![Figure 6.5 Buckling results (load multiplier) for rectangular effective region (a) with a tensile loading; (b) without tensile loading.](image)

The load multiplier for the rectangular plate with and without a tensile stress is 5.0164 and 5.2154 respectively, so that the critical buckling stress for the plate without a tensile stress is by 4% higher than the plate under tension. For this reason, it is reasonable to ignore the tensile effect for the effective region. Ignoring the tensile stress will make the critical stress estimation slightly higher than the exact value, so that this approximation again overestimates the wrinkling stress in Yoshida test.

Thus, the wrinkling in the Yoshida test is simplified to the onset of buckling of a rectangular plate being simply supported along four edges and subjected to a uniform compressive stress along one pair of edges ($x = 0$ and $x = a$), as shown in Figure 6.6.
**6.4 Critical Stress for Effective Region**

The analytical energy method at the effective region is utilized in the prediction of the onset of wrinkling in Yoshida test. The effective region under compression has been determined from the simulations. This section will outline the methodology for the critical stress estimation at the rectangular effective region.

**6.4.1 Expression for Critical Stress Estimation**

The deflection surface of the plate with four simply supported edges is described in form of double Fourier series [31], which is given by the following equations

\[
w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}
\]

where \(a_{mn}\) is the amplitude, \(a\) and \(b\) are the length and width of the plate as shown in Figure 6.6. The critical stress \((\sigma_{cr})\) is obtained when the strain energy \((U)\) stored in the
Energy Approach of Predicting Wrinkling

The critical stress for rectangular orthotropic plates simply supported on four edges is given by the equation as follows [107];

$$\sigma_{cr} = \frac{\pi^2}{t} \left[ D_{11} \left( \frac{m}{a} \right)^2 + 2(D_{12} + D_{66}) \left( \frac{n}{b} \right)^2 + D_{22} \left( \frac{a}{m} \right)^4 \right]$$

6.4.2

A buckling mode of one \((m = n = 1)\) is observed from the experiments of the Yoshida test. The critical stress can be simplified by the following equations;

$$\sigma_{cr} = \frac{\pi^2}{t} \left[ D_{11} \frac{1}{a^2} + 2(D_{12} + D_{66}) \frac{1}{b^2} + D_{22} \left( \frac{a^2}{b^4} \right) \right]$$

6.4.3

where \(D_{ij}\) is the bending stiffness matrix for orthotropic material, which are given as the following equations;

$$[D_{ij}] = \begin{bmatrix}
\frac{Q_{11} t^3}{12} & \frac{Q_{12} t^3}{12} & 0 \\
\frac{Q_{21} t^3}{12} & \frac{Q_{22} t^3}{12} & 0 \\
0 & 0 & \frac{Q_{66} t^3}{12}
\end{bmatrix}$$

6.4.4

where \(Q_{ij}\) is the stiffness for orthotropic materials, which are given by the equations as follows;

$$[Q_{ij}] = \begin{bmatrix}
\frac{E_1}{1 - \nu_{12} \nu_{21}} & \frac{v_{12} E_2}{1 - \nu_{12} \nu_{21}} & 0 \\
\frac{v_{12} E_2}{1 - \nu_{12} \nu_{21}} & \frac{E_2}{1 - \nu_{12} \nu_{21}} & 0 \\
0 & 0 & 2G_{12}
\end{bmatrix}$$

6.4.5

100
Curv® and Twintex® are balanced orthotropic materials, so Young’s Modulus and Poisson’s Ratio are the same along two directions ($E_1 = E_2, v_{12} = v_{21}$). Then, the stiffness matrix can be simplified by the following equations;

$$
\begin{bmatrix}
Q_{ij}
\end{bmatrix} = 
\begin{bmatrix}
\frac{E}{1-v^2} & \frac{vE}{1-v^2} & 0 \\
\frac{vE}{1-v^2} & \frac{E}{1-v^2} & 0 \\
0 & 0 & \frac{E}{1+v^2}
\end{bmatrix}
$$

6.4.6

Implementing the stiffness matrix ($D_{ij}, Q_{ij}$) into equation 6.4.3, the critical stress for the effective region is given from the equations given below;

$$
\sigma_{cr} = \frac{\pi^2 E}{12(1-v^2)} \left(\frac{t}{b}\right)^2 \left(\frac{b}{a} + \frac{a}{b}\right)^2
$$

6.4.7

The critical buckling stress for the effective region is influenced by material properties, thickness of the sample and the dimension of effective region. Higher Young’s modulus, Poisson’s ratio or a higher thickness leads to a higher critical buckling stress. The length of the vertical edges (b) of the effective region will also affect the wrinkling in Yoshida samples. This implies that in order to trigger wrinkling, not only the magnitude of the compressive stress is important, this stress need to act over a sufficient extended length as well. In addition to the length of the effective region, the shape of the effective region is also important, which is suggested by $\left(\frac{b}{a} + \frac{a}{b}\right)^2$. This term is known as plate buckling coefficient (K), which is given by the following equations;

$$
K = \left(\frac{b}{a} + \frac{a}{b}\right)^2
$$

6.4.8
Energy Approach of Predicting Wrinkling

Considering the ratio of width and length of the effective region to be $\phi$ ($\phi = a/b$). Then plate buckling coefficient can be expressed by the equations as follows;

$$K = \left(\frac{1}{\phi} + \phi\right)^2 = \left(\frac{1}{\phi^2} + 2 + \phi^2\right)$$  \hspace{1cm} 6.4.9

Figure 6.7 suggested that the lowest plate buckling coefficient is obtained when the width to length ratio ($\phi$) reaches 1. Therefore, when the deflection of the plate follows the buckling mode 1, the critical stress reaches the minimum value when the width equals to the length.

![Figure 6.7 Plate buckling coefficient at different width to length ratio for the effective region.](image)
**Energy Approach** of Predicting Wrinkling

### 6.4.2 Effective Engineering Constants along Non-Principal Coordinates

The values for effective engineering constants along principal and non-principal coordinates are required for the critical stress estimation. The effective constants along the non-principal axis are related to principal engineering constants and the orientation angle ($\theta$), which can be calculated from the equation given as follows:

\[
E_x = \left[ \frac{1}{E_1} c^4 + \left( \frac{1}{G_{12}} - \frac{2v_{12}}{E_1} \right) s^2 c^2 + \frac{1}{E_2} s^4 \right]^{-1} \quad 6.4.10
\]

\[
E_y = \left[ \frac{1}{E_1} s^4 + \left( \frac{1}{G_{12}} - \frac{2v_{12}}{E_1} \right) s^2 c^2 + \frac{1}{E_2} c^4 \right]^{-1} \quad 6.4.11
\]

\[
G_{xy} = \left[ \frac{1}{G_{12}} (s^4 + c^4) + 4 \left( \frac{1}{E_1} + \frac{1}{E_2} + \frac{2v_{12}}{E_1} - \frac{1}{2G_{12}} \right) s^2 c^2 \right]^{-1} \quad 6.4.12
\]

\[
v_{xy} = E_x \left[ \frac{v_{12}}{E_1} (s^4 + c^4) - \left( \frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}} \right) s^2 c^2 \right] \quad 6.4.13
\]

where $c = \cos \theta$, $s = \sin \theta$. The mechanical properties of Curv® and Twintex® with fibre orientation of $[0^\circ/90^\circ]$ and $[45^\circ/-45^\circ]$ are listed in Table 6.2.

**Table 6.2 Mechanical data for Curv® and Twintex® Along Different Orientation Angle [108, 109]**

<table>
<thead>
<tr>
<th></th>
<th>Curv® $[0^\circ/90^\circ]$</th>
<th>Curv® $[45^\circ/-45^\circ]$</th>
<th>Twintex® $[0^\circ/90^\circ]$</th>
<th>Twintex® $[-45^\circ/45^\circ]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile Modulus (MPa)</td>
<td>1345</td>
<td>1136</td>
<td>7866</td>
<td>1651</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.14</td>
<td>0.27</td>
<td>0.45</td>
<td>0.88</td>
</tr>
<tr>
<td>Shear Modulus (MPa)</td>
<td>446</td>
<td>590</td>
<td>438</td>
<td>2712</td>
</tr>
</tbody>
</table>

Then, the critical stress for the effective region can be calculated. For example, the critical instability for Curv® $[0^\circ/90^\circ]$ is calculated from the equations given below:

\[
\sigma_{cr} = \frac{\pi^2 E}{12(1 - \nu^2)} \left( \frac{t}{b} \right)^2 \left( \frac{b}{a} + \frac{a}{b} \right)^2
\]


**Energy Approach of Predicting Wrinkling**

\[
\sigma_{cr} = \frac{\pi^2 1345MPa}{12(1 - 0.14^2)} \left( \frac{1mm}{76.4mm} \right)^2 \left( \frac{76.4mm}{48.1mm} + \frac{48.1mm}{76.4mm} \right)^2 = 0.95MPa
\]

The critical stress for Curv® \([0^\circ/90^\circ]\) is recorded as -0.95MPa, and the negative sign is applied to show compressive effect. Similarly, substituting the values of effective elastic constants (Table 6.2) and the dimensions of effective region (Table 6.1) to analytical equation, the critical value for stress for each composite system can be determined, as listed in Table 6.3.

<table>
<thead>
<tr>
<th></th>
<th>Curv® ([0^\circ/90^\circ])</th>
<th>Curv® ([-45^\circ/45^\circ])</th>
<th>Twintex® ([0^\circ/90^\circ])</th>
<th>Twintex® ([45^\circ/-45^\circ])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical Stress (MPa)</td>
<td>-0.95</td>
<td>-0.80</td>
<td>-9.45</td>
<td>-3.5</td>
</tr>
</tbody>
</table>

### 6.5 Validation of Analytical Critical Stress

In this section, the estimated critical stress obtained from analytical approach will be validated with the experimental observations and simulative results to examine whether the proposed methodology can provide accurate indication on wrinkling. The experimental and simulative principal minor stress along the vertical edges of the effective region is averaged to provide a uniform compressive loading. The evolutions of averaged experimental and simulative minor stress at the corresponding location the vertical edge of the effective region in Yoshida samples are used to examine the validity of critical stress obtained from the theoretical approach that use energy method at region of interest. The results from simulation can be exported directly, while the minor principal stress from experimental results cannot be obtained directly. Section 6.5.1
Energy Approach of Predicting Wrinkling

presents the methodology used for calculating the principal minor stress from the strain based results.

6.5.1 Critical Stress from Experimental Results

The evolution of minor principal stress from experiment is obtained from the in-plane principal strain extracted from ARAMIS® system. The major and minor principal strain along the vertical edges of the effective region \((x = 0 \text{ and } x = a)\) over the loading process is exported from the simulation of Yoshida test. The strain based results are then transformed to the stress results using the stress strain relationship for orthotropic material (equation 6.5.1).

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = [Q]
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}/2
\end{bmatrix}
\]

6.5.1

In Yoshida test, the strain along x and y axis can be approximated by the minor and major principal strain, because there is only a negligible in plane shear strain developed. The stiffness matrix \([Q]\) for balanced orthotropic material is given in equation 6.4.6. The stress results at different time stage is calculated by substituting the engineering constants and the ARAMIS® strain results to the stress-strain relation. The compressive stress acting on the vertical edges for each stage is the average of minor principal stress at the nodes along the vertical edge.

6.5.2 Comparison between Analytical, Experimental and FEA Results

The analytical, experimental and FEA results are compared in Figure 6.8. The analytical result offer a critical wrinkling stress, below which (more negative) indicates the
Energy Approach of Predicting Wrinkling

occurrence of wrinkling. The experimental and simulative results are presented in form of the evolution of averaged minor principal stress acting on the vertical edge of the effective region.

![Comparison between Experimental, FEA Compressive Stress and Analytical Critical Stress](image)

**Figure 6.8 Comparison between Experimental, FEA Compressive Stress and Analytical Critical Stress for (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°]**

Overall, good agreement is obtained between the prediction of onset of wrinkling in analytical, experimental and FEA approaches. The intersections between the critical
Energy Approach of Predicting Wrinkling

wrinkling stress from analytical approach and the stress evolution of experimental and simulative results illustrate that the onset of wrinkling occurs when the minor principal stress developed in samples approaches the theoretical critical wrinkling stress. For wrinkled Curv® [0°/90°], Curv® [45°/-45°] and Twintex® [45°/-45°] samples, the intersections fall into the elongation region representing the onset of wrinkling observed in experiments. For Twintex® [0°/90°] samples, the compressive stress from the experimental and simulative result is much lower than the critical wrinkling stress estimated from the theoretical method, indicating no wrinkling initiated. These results indicate that the critical wrinkling stress estimated by the energy method using the effective region can indicate the onset of wrinkling.

6.6 Summary

In this chapter, the analytical energy method is implemented at an effective region for the critical wrinkling stress estimation. This effective region is the region under compression, which is the driver of localized wrinkling; however, there was no clear definition of the selection of the effective region in the previous studies. This work introduces a clear methodology for selecting the effective region for Yoshida test, and this method is likely to be applicable to other wrinkling problems. With a correct boundary condition assigned to the effective region, the equation for the critical wrinkling loading can be derived, which quantifies the effect of material properties, boundary conditions and surface deflection at the onset of wrinkling. Then, the usefulness of proposed method is validated with the experimental and simulative results and good agreement is obtained.
Energy Approach of Predicting Wrinkling
Chapter 7  Conclusion and Recommendations

7.1 Thesis Contribution to knowledge

The approach of using the evolution of the principal strain path to predict the onset of wrinkling is the fundamental contribution of this study. It was found that the abrupt change in strain path can indicate the onset of wrinkling. This approach is the first of its kind in predicting the onset of wrinkling.

SR and SIR are compared for being used as the metric to measure the abrupt change in strain path. This work clearly demonstrates that current common practise of using SR as a metric for predicting onset of wrinkling is not robust. The metric of using SIR to predict the onset of wrinkling can capture more subtle change in strain path, offering elevated robustness of the indication of the onset of wrinkling. This wrinkling indicator based on SIR is another contribution of the present work.

A fully automatic wrinkling indicator using the abrupt change in SIR is developed. This indicator is applicable to predict the onset of wrinkling and the wrinkling propagation in Yoshida test and dome forming operations. This indicator is also applicable to a variety of material systems, including but not limit to the composite materials and metallic materials.

The energy method was used at small wrinkling affected region for the critical wrinkling stress estimation and good agreement between the simulation and experiment was obtained. This effective region is defined as the region under compression from the simulative minor principal stress. The definition of effective region simplifies the usage
Conclusion and Recommendations

of energy method compared to conventional energy method application in the literatures. This work also clarifies a viable methodology for the selection of the size, shape, orientation and location of effective region, which was not clear in the previous studies.

7.2 Recommendations for future work

This thesis has programmed the wrinkling indicator based on SIR in MATLAB script, using the information extracted from FEA for predicting wrinkling. In the future, it would be valuable to develop a user defined loop to implement the indicator in FEA.

The validity of the wrinkling indicator based on SIR is only tested in Yoshida test and dome forming test. The material systems examined in this work include two fibre reinforced woven thermoplastic composite material and one metallic material. It seems to be valuable to further test this indicator to other forming operations and to other material systems.

The energy approach discussed in Chapter 6 is only applied to Yoshida test. Though a good agreement is obtained in this study, it is crucial to evaluate the validity of this approach to other tests. For the test with more complicated geometries, the rectangular approximation of the shape and the size of the effective region may not valid. In this case, a two-step implementation can be considered as an alternative for critical stress estimation. After the identification of the effective region from the primary simulation, the effective region (irregular) and its boundary conditions can be modelled in FEA to have the critical wrinkling stress.
Chapter 8  Bibliography


Bibliography


Bibliography


Bibliography


Bibliography


Appendix A – Evolution of Strain Path

The evolution of strain paths up to an elongation of 1.5mm at four nodes of interest are plotted as major principal strain over minor principal strain obtained from the face which is subjected to more compressive stress after the onset of wrinkling.

Figure 8.1 Strain path at point of interest on one face of the sample. (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°]
The evolution of strain paths at the other face is presented below. The strain path is plotted as major principal strain over minor principal strain up to an elongation of 1.5mm at four nodes of interest.

Figure 8.2 Strain path at point of interest on one face of the sample. (a) Curv® [0°/90°]; (b) Curv® [45°/-45°]; (c) Twintex® [0°/90°]; (d) Twintex® [45°/-45°]
Appendix B - UMAT script for Curv®

subroutine umat43(cm,eps,sig,eps,dr1, capa, etype,tt,temper,fail,crv,cma,qmat,elsize,idele)

C******************************************************************
C|Livermore Software Technology Corporation (LSTC)            |
C|------------------------------------------------------------|
C|Copyright 1987-2008 Livermore Software Tech. Corp            |
C|All rights reserved                                         |
C******************************************************************
C******************************************************************
C   User Defined Material Subroutine of RT SRPP (Curv, Propex) by Jae Nam
C    based on room temperature SRPP characterised by Nima A. Zanjani.
C    NB: No differentiation between elastic and plastic region 
C    (i.e. elastic part not defined).
C    Modified 19/05/2015
C
C Note to self:
C Units: kg, mm, ms, kN, GPa, kNmm, 0.92e-6 kg/mm3
C IVECT = 1, IORTHO = 1, strain through thickness direction NOT CONSIDERED!! Need to figure out elastic/plastic region?!
C THIS IS FOR 2D THIN SHELL with PLANE STRESS assumptions. Need to consider choice of shell elem in conjunction with umat.
C
C   include 'nlqparm'
C   include 'bk06.inc'
C   include 'iounits.inc'
C   dimension cm(*),eps(*),sig(*),hsv(*),crv(lq1,2,*),cma(*),qmat(3,3)
C   character*5 etype
C   logical fail
C
E1= 1300 * exp(-ABS(hsv(3))/14188.75)
E2= 1300 * exp(-ABS(hsv(4))/14188.75)

IF (ABS(hsv(3)).LT.(0.017)) THEN
  v12 = -1*(4* 77606  * ABS(hsv(3))**3 -
                3* 2976.9 * ABS(hsv(3))**2 + 
                2* 39.73  * ABS(hsv(3))**1 - 
                3* 0.45)
ELSE IF ((ABS(hsv(3)).LT.(0.07))
  THEN
    v12 = -1*(4* 77606  * ABS(hsv(3))**3 -
                3* 2976.9 * ABS(hsv(3))**2 + 
                2* 39.73  * ABS(hsv(3))**1 - 
                3* 0.45)
ELSE IF ((ABS(hsv(3)).LT.(0.07))
  THEN
    v12 = +0.14
ELSE
  v12 = +0.1
END IF
C
IF (ABS(hsv(4)).LT.(0.017)) THEN
  v21 = - 1*(4* 77606 * ABS(hsv(4))**3 - 1 * 3* 2976.9 * ABS(hsv(4))**2 + 2 * 39.73 * ABS(hsv(4))**1 - 3 * 0.45)
ELSE IF ((ABS(hsv(4)).LT.(0.07)).AND. (ABS(hsv(4)).GT.(0.017))) THEN
  v21 = +0.14
ELSE
  v21 = +0.1
END IF

G = 117.8 + 328.7 * EXP (-28.35*ABS(hsv(6)))
GC = G*capa

DETERMINE LAMINAR STIFFNESS

Q11 = E1/(1 - v12 * v21)
Q12 = (E1 * v21)/(1 - v12 * v21)
Q21 = (E2 * v12)/(1 - v12 * v21)
Q22 = E2/(1 - v12 * v21)

Compute stress (past stress + incremental stress)

sig(1) = sig(1) + Q11*eps(1) + Q12*eps(2)
.sig(2) = sig(2) + Q21*eps(1) + Q22*eps(2)
sig(3) = 0.0
.sig(4) = sig(4) + G*eps(4)
sig(5) = sig(5) + GC*eps(5)
sig(6) = sig(6) + GC*eps(6)

Compute local z strain based on in

eps(3) =

Update local strains

hsv(3) = hsv(3)+eps(1)
.hsv(4) = hsv(4)+eps(2)
hsv(5) = hsv(5)+eps(3)
.hsv(6) = hsv(6)+eps(4)

gc = capa*g
.q1 = abs(cm(1))*cm(2)/(1.0+cm(2))*(1.0-2.0*cm(2)))
.q3 =1/(q1+g2)
c eps(3)= q1*(eps(1)+eps(2))**q3
.c davg =(-eps(1)-eps(2)-eps(3))/3.
c p = davg*abs(cm(1))/(1.2.0*cm(2))
c sig(1)=sig(1)+p+g2*eps(1)+davg)
c sig(2)=sig(2)+p+g2*eps(2)+davg)
c sig(3)=0.0
.c sig(4)=sig(4)+g **eps(4)
c sig(5)=sig(5)+gc*eps(5)
c sig(6)=sig(6)+gc*eps(6)

OPEN (unit = 8, file = '/'short/x33/jyn660/umat.dat')
WRITE (8,*) 'umat'
WRITE (8,*) hsv(3), sig(1)

if (ncycle.eq.1) then
  call usermsg('mat43')
endif

return
end
subroutine umat43(cm,eps,sig,epsps,hsv,dr1,capa,etype,tt,
 1 temper,failel,crv,cmq,mat,elsiz,idele)
  
c******************************************************************
c|  Livermore Software Technology Corporation (LSTC)           |
  c|                                                                 |
  c|  Copyright 1987-2008 Livermore Software Tech. Corp         |
  c|  All rights reserved                                        |
c******************************************************************
  c User Defined Material Subroutine of GRPP (Twintex, Propex) by Davood Rahiminejad
  c
  c include 'nlqparm'
  c include 'bk06.inc'
  c include 'iounits.inc'
  c dimension cm(1),eps(1),sig(1),hsv(lq1,2,1),cmq(3,3)
  c character*5 etype
  c logical failel
  c
  c compute shear modulus, g
  c open(unit=11, file='/short/x33/lch660/TwintexRectangle/Sig1.dat')
  c open(unit=88, file='/short/x33/lch660/Analysis/Strain.dat')
  c open(unit=11, file='/short/x33/ddr660/Analysis/Strain4.dat')
  c
  c
g2=2697
  E= cm(1)
  Nu= cm(2)
  SM1= E/(1-Nu**2)
  SM2= E*Nu/(1-Nu**2)
  C1= cm(3)
  C2= cm(4)
  C3= cm(5)
  
  c hsv(4)=hsv(4)+eps(1)
  c hsv(5)=hsv(5)+eps(2)
  hsv(10)=hsv(10)+eps(4)
  c
  c sig(1)=sig(1)+ SM1 * eps(1)+ SM2 * eps(2)
  c sig(2)=sig(2)+ SM2 * eps(1)+ SM1 * eps(2)
  c sig(3)=0
  c
  c write(88,*) eps(1), Sig(1)
  c write(11,*), eps(4), hsv(4), g
  c10 format/
  c 1 *** Error element type 'a,' can not be,
  c 2 run with the current material model.)
  c return
  end