A STATISTICAL APPROACH TO THE ANALYSIS
OF GEOTECTONIC ELEMENTS

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A thesis submitted for the
degree of Doctor of Philosophy
Australian National University
1) **STATEMENT OF ORIGINALITY**

Except where reference is made to the literature in the usual manner, the entirety of this thesis is solely my own work and creation. Dr. R.E. Miles of the Statistics Department, Research School of Social Sciences, has participated jointly with me on some aspects related to random polygons. This work is referenced in the body of the thesis in the customary manner, and is included in the appendices, wherein I have attempted to make clear the nature of the partitioning of this work between Dr. Miles and myself.

2) **SCIENCE AND THIS THESIS**

The scientist is, first, an observer. Whether his study is of the universe or microbiology, he seeks to record the physical properties and behaviour of some aspects of nature. The geologist seeks to observe the planet Earth. His aim is to record the structure, and the physical and chemical behaviour of the rocks on the Earth's surface. The aim of this thesis is to observe the structure (the physical shape and arrangement) of the Earth's continents and oceans on a very large scale, obtaining perhaps the sort of view that an astronomer on a distant planet might obtain.

The scientist is, second, a describer. He seeks to describe, to quantify, to organize, to generalize his observations. The object of such a description is to aid in the comparison and communication of observation. The geologist often seeks to classify and order his observations, to aid in communicating the structure and history of the Earth's surface to other geologists. In the more physical of sciences, it is common to use quantitative descriptions. Whether mathematical or numerical, these descriptions are highly useful in communicating certain properties of the observation. This thesis attempts to obtain a quantitative description of shape, arrangement, and physical inter-relationships of some features of the Earth's crust.
The scientist is, third, a predictor. He proposes hypotheses, the consequences of which can be evaluated and tested by further observation. Having observed, and then described, he uses the description to formulate a model of predictable behaviour. The model may be either deterministic or stochastic and may predict exact numerical consequences or only vague trends. By prediction I do not only mean the forecasting of future events (in the time sense), but the forecasting of the results of experiments not yet made, or of descriptions not yet shaped from apparently formless data. Such is scientific deduction. It is the aim of this thesis to deduce from a description of the Earth's surface, some of the mechanisms which may have produced its present form.

3) THE PERSON OF THIS THESIS

Classically, and in most European countries today, theses are traditionally written in the first person. Despairing of the egotistical and pompous overtones that this style sometimes produces, British and New World countries have, in recent years, favoured the third person or impersonal style. Unfortunately, the third person in English is often awkward, and to ensure clarity, must inevitably be more verbose, and sometimes less precise than the equivalent in the first person.

After much consideration of the relative merits of the two styles, weighted against the strength of tradition, I have decided somewhat reluctantly against the first person. Therefore all parts of this thesis except for this Foreword and the Acknowledgements are written in the third person. To those who find annoying the repeated use of such phrases as "the present writer" and "in this work", I offer my sincere apologies, and I hope that I have made clear which "has been done" refer to my own work, and which refer to those who have worked before me.
4) AN INVITATION TO CRITICISM

A certain amount of the material presented in this thesis is new to geology both in approach and formulation. Therefore there is a higher-than-average risk that some of this work will be found, in later years, to be primitive, naive or simply incorrect. Recognizing this danger, I would like to encourage anyone who reads the following pages to examine the contents critically and, in this way to improve and build upon the rudimentary foundation I have attempted. I would consider my efforts worthwhile should even the whole of my formulations be rejected, if in the process, useful quantitative descriptions and descriptions of geotectonic geometries are eventually obtained.
ABSTRACT

Many authors in the past have commented on geometrical regularities in the arrangement of point, line and areal features on the Earth's surface. In order to obtain a more quantitative view of such patterns, a study of the arrangement of tectonic elements can be approached by geometrical statistics.

Random models for geometrical elements on a sphere are considered; in particular, the length distribution of "lineaments" of random origin is shown to be similar to a gamma distribution and probability density functions for the Poisson and Voronoi random polygons are derived by Monte-Carlo methods. Length, azimuth and intersection angle distributions for tectonic lineaments on the continent deviate significantly from random. These deviations are systematic in that preferred lengths and intersection angles are evident. Azimuth distributions for the various continents are sufficiently similar to suggest the magnitude of relative continental rotations.

The polygonal nature of Africa closely resembles the Voronoi polygon random model, and the intersection angle data from the continents more closely resembles this model as opposed to the Poisson polygons.

From the detailed observations of continental lineaments it is concluded that no continuous global pattern exists at the present, but rather, a continuous pattern is evident if continents are rotated to Paleozoic positions. The plate-tectonic concept is thus reinforced and it is clear that there can be little uncoupling of the crust and upper mantle. The statistical methods developed can be applied to oceanic as well as continental tectonic elements.
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PART I

INTRODUCTION

This thesis has two principal objectives which are approximately of equal importance. The first is the development and presentation of mathematical and statistical techniques for the analysis of linear and polygonal seismic elements. The second is the application of these techniques to the geometry of the Earth's surface.

There can be no doubt that the tectonic geometry of the Earth reflects in some way the forces and mechanisms that have, in the past, created the presently observed distribution of continents and oceans. For this reason, any analysis of tectonic elements on a global scale cannot be disconnected from theories of continental drift and creation of ocean basins.

It cannot be expected that the analysis will "solve" the problems of continental drift and the origin of ocean basins. The application of semi-quantitative techniques to global geometry included in this work is intended to show the gross relationships between the tectonic structures of the continents and oceans, and thereby add to the evidence for or against continental drift. A generalized statistical approach such as this cannot act as a diagnostic test of various drift reconstructions, although it could give general support to some configurations more than others. In addition, it is hoped that the quantitatively observed regularities in the Earth's surface, if any, will yield information, or at least, restrictions on the scale and nature of the forces controlling tectonic evolution.

1) HISTORICAL AND RECENT CONCEPTS OF CONTINENTAL DRIFT

a) Early concepts of continental drift

Obviously, a clear conception of the nature of the earth's crust could not be obtained before large portions of the globe were explored and mapped. This dates all "continental drift" theories as early as about 1500.
PART I
INTRODUCTION

1) NATURE AND SCOPE OF THE WORK

This thesis has two principal objectives which are approximately of equal importance. The first is the development and presentation of mathematical and statistical techniques for the analysis of linear and polygonal tectonic elements. The second is the application of these techniques to the geometry of the Earth's surface.

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2) HISTORICAL AND RECENT CONCEPTS OF CONTINENTAL DRIFT

a) Early concepts of continental drift

Obviously, a clear conception of the nature of the earth's crust could not be obtained before large portions of the globe were explored and mapped. This dates all "continental drift" theories as later than about 1600.
Suggestions (such as by du Toit, 1937) that Francis Bacon in his Novum Organum (1620) (translation by Anderson, 1960), contemplated the separation of South America and Africa appear to be unfounded (Rupke, 1970). It would seem very likely that the apparent jig-saw fit did not go un-noticed, but none of the well-known seventeenth century natural scientists commented on it directly. Placet (1668), and the brilliant early geologist Compt de Buffon in 1749 and 1778 (see Buffon, 1814) considered the origin of the Atlantic by the subsidence of the mythical Atlantis. The earliest known reference which approaches remotely, more recent concepts of continental drift if Lilienthal (1756) who pointed out the Atlantic fit. Friedrich von Humboldt also remarked on the apparent fit (von Humboldt, 1803) and explained the Atlantic as an erosional channel (due to the Flood).

The first really geological approach was that by Snider (1858) who envisaged a single primeaval continent splitting catastrophically along fracture zones. (A hypothesis which still receives attention today). As evidence he used the coastal fit, and geological and paleontological agreements. He apparently drew the first map showing a continental reconstruction. Fisher (1882) envisaged crustal movement to fill the gap left by the birth of the moon from crustal material. This lunar concept remained in the literature well into the twentieth century, as reviewed by du Toit, but has been fairly conclusively disproved by recent geochemical studies of lunar rocks (Ringwood, 1970).

Global expansion as a mechanism for the breakup was first suggested by Owen (1857), and Fisher (1882) reports that R. Mantovani published similar views.

Taylor (1910) and Wegener (1912) are the two main authors who elaborated the continental drift concept and began the systematic accumulation of geological evidence in its favour. Apart from Rupke (1970), previously mentioned, another review of pre-1900 continental drift is given by Carozzi (1970).
b) Modern continental drift

For fifty years after the publication of Wegener's theory, (Wegener, 1912a,b,1915,1920,1922,1968) debate of its merits raged rather fiercely, and although circumstantial evidence was mounting rapidly, the lack, or apparent lack, of a physical mechanism was sufficient criticism for many to reject the idea until fairly recently, even though Fisher (1881 and 1889) postulated convection cells rising at the mid-Atlantic ridge and plunging beneath continents. It was not until Holmes (1931) and Griggs (1939) reiterated the convection hypothesis much later, that it gained appreciation as a plausible mechanism for continental displacement. This, combined with the growing geophysical evidence, of paleomagnetism, ocean-floor magnetic lineaments, and seismology, have converted most scientists, in principle, to accept the drift concept (with some exceptions, such as, Meyerhoff, 1969).

The question of driving mechanism is far from settled. The most popular modern mechanism is thermal convection (Venning-Meinesz, 1962). In addition there are supporters of global expansion (Halm, 1935; Carey, 1958; Egyed, 1957; von Hilgenberg, 1966).

An excellent review of the development of modern concepts of continental drift is given by Harland (1969).

One of the most recent off-shoots of classical "drift" is the concept of "plate tectonics". Current theories, (McKenzie and Parker, 1967; Le Pichon, 1968; Morgan, 1969; Isacks et. al., 1968) consider the Earth's surface as a series of rigid plates. All movements are confined to the boundaries of these plates; certain boundaries (mid-ocean ridges) represent zones of production of new oceanic crust, and other boundaries (trenches, island-arcs) are "subduction zones" where material is being re-absorbed into the mantle. Other boundaries are transform faults (Wilson, 1965) where lateral translation of plates occurs.

The concept is more descriptive than genetic, and it is more or less independent of the driving mechanism. Convectionists would see the amounts of crustal creation
on spreading ridges (Vine and Mathews, 1966) as equal to the crustal destruction occurring in trenches. Expansionists envisage an imbalance in favour of crustal creation.

Both convection and expansion remain as plausible mechanisms and, of course, they are not mutually exclusive alternatives. The main objection to expansion is the apparent lack of a sufficient source of energy.

Plate tectonics remains, then, as a useful description of crustal movements and geometry, irrespective of origin.

c) Previous studies of the geometry of global tectonics

Coincident with the development of theories of continental drift, there has been an increasing tendency to analyse at least qualitatively, the geometrical arrangement of crustal tectonic elements on a global scale. The object has been to determine regularities in the time-space arrangement of linear and areal tectonic features, and from these regularities, to develop hypotheses of gross crustal evolution.

One of the first of these geometrical tectonic approaches was by Taylor (1810). Although covering only a small portion of the globe, his pattern of rifts and wrench faults for the North Atlantic Continents differs little from present interpretations. His conclusion that the pattern was due to lunar capture in the Cretaceous seems fairly unlikely, however. Hobbs (1911) found consistent repeating patterns of lineaments.

Brock (1957) gives a good review of the subject of geometrical analysis of lineament patterns and contributes to the subject himself.

Brock's work is qualitative and rather disorganized and, because it covers so many aspects of tectonics, it is difficult to summarize. He discusses in detail the polygonal nature of African tectonics, and further proposes a polygonal tessellation for most of the continents. He examined arcuate structures and found preferred radii and suggested that centres of groups of related arcs could "be related to geological or topographical features". Of great
circles he says: (Brock, 1956, p.165)

Summarising our observations on circles, the following appear to be the outstanding generalisations:

1. Major fractures, fracture zones and other linear structures in the earth's crust are aligned on great circles.

2. The great circles are in groups, each of which has a pair of vertices (or nodes) common to the circles of the group.

3. The equator of each group is structurally significant.

4. The small circles parallel with the equators, each having a "latitude" with respect to the equator of an aliquot part of a quadrant, are structurally significant, and are commonly related to vertices of other groups of circles.

5. The network of great and small circles obtained from alignments is a fundamental pattern in the earth's structural history, dating back to pre-Cambrian times.

6. The network is related to all known structures, (and probably unknown ones).

7. Any given circle exercises a variety of types of control along its course.

8. The control of any circle is intermittent both spatially and temporally.

The multiplicity of structural circles in this intensive network over the face of the globe leads inevitably to a concept of a crust composed of relatively small prisms or plates bounded by vertical planes of weakness. The principal forces holding this assemblage together can only be gravity and friction.

In his opinion, the major great-circle lineaments meet at a suite of "major vertices", of which he lists twenty, and thirty-eight "vertices of second order". His vertices do not show strict regularity, but indicate a preference for angular spacings of $36^\circ$, $45^\circ$, $72^\circ$, and $90^\circ$. (Brock, 1956, Table IV). He introduced the concept of the "structural ellipse", an elliptical envelope surrounding a structural province, and demonstrated that certain axial lengths predominate. In a later work Brock (1957) examined lineaments more specifically. He delineated regular continental and global lineament grids, with particular reference to a fundamental set of great circles and vertices. He concluded that continental displacement is an untenable hypothesis, since lineament patterns on different continents appear to be related through great circle patterns, and continental lineaments are constantly re-activated and transect age boundaries.
Hills (1956) related the lineaments of Australia to a theoretical shear pattern deduced by Venning-Meinesz (1947). (The shear pattern was calculated by assuming a large shift in geographic pole, for which there is little evidence.) A more extended comparison made by Carr (1966) finds agreement with the Venning-Meinesz grid pattern and the conclusion of continental permanence is again stated.

Boutakoff (1952) recognized a lineament pattern with symmetry about the axis of rotation and proposed that it represented a set of conjugate shears resulting from the deformation of the Earth from a spherical to an ellipsoidal shape. Using evidence from lineaments and strike-slip faults Carey (1957) and Tanner (1963) proposed an equatorial shear zone with a relative westward motion of the northern hemisphere. Rance (1967, 1968, 1969) has found evidence for a pattern which matches a theoretical torsional shear.

Fleeting world-wide acclaim was given to Rouse and Bisque (1968) who proposed a series of small circle belts of "major linear structure system", and like Brock define some nineteen vertices or "multiple intersections". They relate the small circles to planes tangent to the core and propose core-mantle slip as the origin for the stress planes. They claim also, correlation with geoid and magnetic anomalies. (Bisque and Rouse, 1968).

Some of their vertices are similar to those of Brock (1957), whereas others are notably different. No attempt is made to substantiate their contention that an inordinate or statistically unlikely number of linear elements occur near their proposed small circles.

Torsional, axially-symmetric shear patterns in major lineaments were also noted by Sonder (1938) and re-iterated by Hupé (1958). To add further confusion, Chang (1969) relates modern tectonic patterns to tetrahedral symmetry about a set of "geologic poles". Dolitskiy (1967) presents evidence for an axial symmetry for linear features but with a pattern differing substantially from Sonder (1938) and Hupé (1958).
Rickard (1967) has observed a regular polygonal pattern in global lineaments with a consistency of area and side-length for the polygons. The origin of this polygonalization was assigned by Rickard to a tensional regime resulting from global expansion.

The two most significant contributions to the subject of lineament tectonics are by Moody and Hill (1956) and Coode (1968). Moody and Hill analysed wrench faults and lineaments and related them to two global shear patterns. On a semi-theoretical foundation, they predicted eight main directions of faulting for a given principle stress direction. From an examination of the major strike-ship faults on the globe, they concluded that "the shear pattern may have resulted from stresses which are oriented essentially meridionally and have been acting in nearly the same direction throughout most of crustal history" (Moody and Hill, 1956, p.1207). The pattern of wrench faults which they deduced is symmetric with respect to the geometric axis, with dextral faults occurring at 15°E and 60°E of north, and sinistral faults at 30°E and 75°E, the reverse occurring west of north. Like Brock (1956) they conclude that, since the pattern seems to have been relatively constant throughout time, immobility of the continents is required. In a later paper by Moody (1966) evidence is presented for two orthogonal compressional shear sets generated by meridional and equatorial compression (i.e. global contraction). Continental displacement is allowed along strike-slip faults, "with little, if any rotation". (They are apparently unaware of the geometrical result that any displacement on a sphere can be represented as a single rotation).

Coode (1966) is the only author to have treated lineaments mathematically. By treating tectonic features, such as mountain systems, as linear $\delta$-functions, he calculated spherical harmonic coefficients up to twelfth-order.

The results show that seismically active features have a dominant frequency content of order 5, and older, aseismic features show lower order harmonics. This work
is important in that it shows quantitatively that linear features have a marked regularity (otherwise no particular harmonic will dominate). Coode interpreted the difference in harmonic content of active and inactive features as an indication of a recent change in convection cell arrangements in the mantle. He also presented evidence for symmetry about the Atlantic and correlation with the geoid and magnetic anomalies.

Wise (1968) presents an interesting discussion of many of the problems of lineament analysis and interpretation. While recognizing the subjective aspects of lineaments, "Topographic linear analysis for underlying fracture control is a subtle mixture of science, art, and self-delusion" (Wise, 1968, p.175), he give evidence, in the form of distribution roses, for organized patterns of lineaments over extremely large areas of the Earth's surface. Additionally he points out the hereditary nature of lineaments; their continual reactivation through time, and their relative permanence, as also noted by Moody and Hill (1956) and Brock (1956).

In summary, then, many authors agree that there are regularities and preferred orientations of tectonic elements on a global scale. They agree that ancient lineaments are constantly reactivated, and that oceanic and continental lineaments in general form a coherent set. There is complete disagreement, however, in the nature of the regularities, the orientation of preferred directions, and the reality, position, number and significance of "geological" vertices. It is the purpose of this work to attempt to place the analysis of tectonic elements on a more quantitative statistical basis in order to describe whatever regularities or symmetries may be present and to determine whether the geometry of the Earth's surface departs significantly from randomness.

Moody and Hill (1956, p.1242) have, in a sense, commissioned this work by the words "Statistical analyses of the strikes of structural and topographic linears across the surface of the earth should support or refute the hypotheses presented here."
Previous analyses have concentrated on the azimuths and angular relationships between lineaments. In this work it is proposed to examine in addition the lengths, areas and relative positions of tectonic elements.

3) THE STATISTICAL APPROACH TO TECTONIC ANALYSIS

a) Introduction

It is common in nature that regularities are obscured by irregularities, and indeed many natural phenomena are purely irregular or random (Mann, 1970). For the analysis of tectonic elements one must therefore establish first an appropriate model for the expected random arrangement of elements, those which have no bearing on any regular deep-seated process. The observations can then be compared quantitatively to this model and regular deviations noted. The next few sections describe in more detail how this may be accomplished.

b) The concept of randomness

A series of "events" whose occurrence cannot be predicted exactly is said to be random. Events which can be predicted are called deterministic. (An "event" is the occurrence of a physical object or measurable numerical quantity at a point or series of points in time or space (or both) of any dimensions.) A model which describes a series of random events is called a stochastic model (Girault, 1966). Most of classical physics, for instance, is described by deterministic models, for example, given the force applied to a body of given weight, we can calculate exactly its velocity and hence predict its arrival at a particular place exactly. Natural events, however, are invariably best described by stochastic properties. If we, for instance, drop a handful of sand on the floor the exact path of any one grain cannot be predicted, although the generalized location of the resulting pile can.

c) Statistical inference and its application

The fact that a series of events is known to be random (a stochastic process) does not mean that it is entirely unpredictable; it means rather that it is not entirely
predictable. Having observed the nature of the process one can determine (at least approximately) a probability density function for the process, and from this make quantitative inferences. For example, having observed that the heights of a group of people are approximately normally distributed with mean \( h \), one can predict with some certainty just how many exceed \( h \) by an amount \( x \) etc. Another related use for stochastic models is to quantitatively isolate regularities in an otherwise stochastic process. (The separation of "signal" from "noise"). One determines, either experimentally or theoretically, the stochastic model for the "noise", the events which can be expected to occur purely by chance. The observed events are compared with theoretical models and any consistent deviations may be quantitatively assessed and an origin (whether deterministic or again stochastic) determined.

As an illustration, consider the flow of traffic on a street. If the traffic flow is very low, and unimpeded by signals or other external influences, the arrival of a car within any particular short interval of time is purely by chance and each interval is equally likely to have an event (the arrival of a car). Such a situation is described as Poisson process and will be described in more detail later. Knowing this basic random model, one could possibly deduce from observation at a particular point in the road whether or not there are control signals upstream from the observation point. Any regularly repeating deviations from the expected uniformly random model will indicate possible controls. It would still be impossible to predict the arrival of a car (an event); nevertheless, by noting the regularities one could predict with calculated assurance periods of relatively dense and sparse traffic.

Statistical inference is normally applied somewhat inversely; it endeavours to disprove hypotheses so as to give weight to an alternate hypothesis. If one can disprove or at least discredit all hypotheses except one alternative, by inference, this hypothesis is correct. Hypotheses are considered acceptable if they cannot be shown to be statistically unlikely. Usually some level of likelihood (significance level) is chosen as the threshold of
acceptibility, hence the use of 95% and 99% "confidence" etc.

The use of statistical models in geology has increased rapidly in the last few years (Mann, 1970). Statistical techniques are in widespread use in stratigraphic analysis, and many other fields (Harbaugh and Merriam, 1968). Recently, stochastic models for magnetic reversals have been proposed (Cox, 1970) and have been used for delineating long period regularities in paleomagnetic history (Crain and Crain, 1970). The application in this work to the problem of tectonic geometry consists of three stages: first, to develop plausible stochastic models and expected frequency distributions for various geometric elements on a sphere which occur by uniformly random chance; second, to observe the global pattern of tectonic elements and compare it with the random model; third, to discuss the geological and tectonic significance of any regular deviations of the observed patterns from the hypothetical random pattern.

4) THE USE OF COMPUTERS IN THIS WORK

As previously outlined, one of the major faults of previous geometrical tectonic studies has been their lack of quantitativeness or a rigorous statistical foundation. Much of this problem is due to the scarcity of statistical methods for such geometric analyses. Another consideration is the difficulty of performing quantitative analyses, either due to the sheer bulk of computation required, or the complexity of the algebra generated by problems in spherical trigonometry. The only practical way of overcoming these difficulties is through the use of modern computing facilities, and therefore extensive use of such devices has been made in this thesis.

The machines available to the author for this work were the Australian National University IBM-360 Model-50 (with accessory tape drives and disc packs), and the CSIRO Division of Computing Research, CDC-3600 (with tape, disc, and drum peripherals, and two Calcomp plotters).

The computer usage can be classified into four main applications: problems in spherical geometry, statistical analyses, Monte-Carlo simulations and display. A brief outline of these applications is given below; details of
all the computer programs are given in Appendix I.

a) Problems in spherical geometry

In this application the computer was used as a calculating machine, to perform manipulations of points and lines on a globe. A suite of subprograms was developed to perform such routine calculations as conversions from geographic to spherical and Cartesian coordinate systems, the length of great circle and small circle segments, angles of intersection of lineaments, radius and centre of curvature of small circles defined by three points, rotations of points about specified axes, etc.

b) Statistical analysis

This group of programs includes those which estimate the parameters of various statistical distributions from an observed sample, collect and print histograms of observations, perform $\chi^2$ tests, and determine confidence intervals. Some details of the methods (where they are new or not widely known) are given in Part II, and program documentation is given in Appendix I.

c) Monte-Carlo simulations

The phrase "Monte-Carlo simulation" refers rather broadly to the computer or mechanical modelling of real or imaginary systems through the controlled use of random number selection. (Naylor et al., 1966). The purpose of such simulations in this work was to determine approximations to probability density functions which cannot be derived explicitly.

For instance, if one wishes to determine the probability of scoring three consecutive "heads" in coin-tossing, one could flip a coin a large number of times and record the results, or, better still, simulate a large number of flips by random number generation on a computer. Both would be Monte-Carlo simulations, the name deriving from the famous gambling casino. (Of course, in this trivial example the true probability of 1/8 could have been calculated. Such is not always the case.)

Monte-Carlo techniques were employed to find the frequency distributions of the number of sides, perimeter
and area of two classes of random polygon (the Poisson polygons), those due to the intersection of random lines in the plane, and the so-called Voronoi polygons, or cell-model (Gilbert, 1962). The results and application of these methods are given in Parts II and III, computer programs in Appendix I. Techniques of random number generation on a computer are described by Smith (1968, p.9-10).

d) Display

The fourth main computer application in this work is for the display of data. Coast lines and continental shelf outlines were digitized using a D-MAC automatic coordinate digitizer at the Division of Computing Research, CSIRO, Canberra. The resulting outlines could then be plotted on variously selected projections and rotated to hypothetical positions. Thus in addition to the programs already mentioned, a number were written for map projection conversion, continental rotation, and map plotting.

For reasons of economy, the bulk of the computing was processed through the ANU IBM-360 computer. Appendix I therefore contains a program for the routine plotting at CSIRO of magnetic tapes of coordinates produced from programs run in the University establishment.
PART II

STATISTICAL METHODS FOR GEOTECTONIC ANALYSIS

1. INTRODUCTION

The aim of this part is to bring together some well-known, some little-known, and a few new results developed herein of geometrical probability theory which have possible bearing on the statistical analysis of the tectonic features observable on the earth and other planets. Whereas the primary object of Part I is the presentation of the results and mathematical methods involved, some examples of applications are included and, in particular, the areas of possible application are discussed. Particular emphasis is given to tests for "randomness", since the non-random arrangement of tectonic elements implies an organization in the mode of origin, such as convection currents, or a global stress pattern. Qualitative analysis of the systematic deviations of these elements from randomness could provide information about the mechanism involved. Such analysis requires the comparison of assemblages of observations with predicted frequency density functions. These tests are normally performed using the "chi-squared" statistic (Argand).

It is easy to grasp an intuitive concept of randomness of points in a space of any dimensions, but such intuition fails for multidimensional elements, such as, lines, planes and spheres, even in three-dimensional space. Hence, in this, the term "random" must be defined carefully in geometrical statistics and tests for randomness refer only to the particular model considered. A rejection of the hypothesis of randomness of geometrical elements is therefore a statement that the observed distribution cannot be readily explained by the specified random process. For example, there are at least three ways of defining classes of "random" chords of a circle, each having different means (Kendall & Moran, 1963, 2-15). The physical restrictions of applying geometrical statistics to tectonics are usually sufficient to determine a suitable random model from amongst the possible choices.
1) INTRODUCTION

The aim of this part is to bring together some well-known, some little known, and a few new results developed herein of geometrical probability theory which have a possible bearing on the statistical analysis of the tectonic features observable on the earth and other planets. Whereas the primary object of Part II is the presentation of the results and mathematical methods involved, some examples of applications are included and, in particular, the areas of possible application are discussed. Particular emphasis is given to tests for "randomness", since the non-random arrangement of tectonic elements implies an organization in the mode of origin, such as convection currents, or a global stress pattern. Quantitative analysis of the systematic deviations of these elements from randomness could provide information about the mechanisms involved. Such analysis requires the comparison of assemblages of observations with predicted frequency density functions. These tests are normally performed using the "chi-squared" statistic (Argentiero et al. 1969).

It is easy to grasp an intuitive concept of "randomness" of points in a space of any dimensions, but such intuition fails for multidimensional elements, such as, lines, planes and spheres, even in three dimensional space. Because of this, the term "random" must be defined carefully in geometrical statistics and tests for randomness refer only to the particular model considered. A rejection of an hypothesis of randomness of geometrical elements is therefore a statement that the observed distribution cannot be readily explained by the specified random process. As an example, there are at least three ways of defining classes of "random" chords of a circle, each having different mean. (Kendall & Moran, 1963, 9-10). The physical restrictions of applying geometrical statistics to tectonics are usually sufficient to determine a suitable random model from amongst the possible choices.
2) NOTATION

By way of notation, probability density functions (density functions, or densities) will be denoted by lower case letters, the probability distribution function (distribution function, distribution or cumulative distribution) of the same random variable by the corresponding upper case letter. Random variables are denoted also by lower case letters, and parameters by lower case Greek letters. For example, a normal density function with mean, $\mu$, and standard deviation, $\sigma$, would be denoted by $n(\mu, \sigma)$ and the corresponding cumulative distribution by $N(\mu, \sigma)$. Exceptions will be made where previous convention has given general usage to a particular symbol for a density. As is common, the phrase "the probability that $x$ is greater than or equal to $g$" will be contracted to $P(x \geq g)$, etc. On occasions it may be helpful to include the random variable in the notation for the density, hence $n(x; \mu, \sigma)$. The expectation (or mean value) of a random variable $x$ will be denoted by $E(x)$.

3) RANDOMLY DISTRIBUTED POINTS

a) Points in a line

The occurrence of uniformly random event is governed by the well-known Poisson density, given by

$$p(n; \lambda) = \frac{\lambda^n e^{-\lambda}}{n!} \quad \text{n=0, 1, 2 ...} \quad (1)$$

The classic example to which such a model applies is that of a telephone switchboard where calls can be assumed to arrive with uniform randomness, there being on the average $\lambda$ calls per unit time. The probability of exactly $n$ calls occurring in a unit time is then given by (1). The time interval, $t$, between successive calls has the exponential density.

$$f(t; \lambda) = \lambda e^{-\lambda t} \quad t>0 \quad (2)$$

If $\lambda$ is invariant with time, reference is made to a "Poisson process of constant intensity", otherwise the process is of variable intensity or, in the time connotation, non-stationary (Girault, 1966, p.8).
b) Points in a plane or volume

For the case of points occurring in a plane area, or in a volume, eq. (1) still holds; \( \lambda \) is the average number of points per unit area, or volume. The distance between nearest neighbour points is no longer exponentially distributed, however. In the planar case, the expression \( 2\lambda \pi r^2 \) will have the \( \chi^2 \) distribution with two degrees of freedom (Kendall & Moran, 1962, p. 39), whereas for three dimensions \( \frac{8}{3} \lambda \pi r^3 \) has that distribution (Kendall & Moran, 1962, p. 48). The \( \chi^2 \) distribution with \( \gamma \) degrees of freedom is given by

\[
\chi^2(x; \gamma) = \frac{1}{2^{\gamma/2} \Gamma(\gamma/2)} \chi^{(\gamma-2)/2} e^{-(x/2)\chi} (3)
\]

Where \( \Gamma(z) \) denotes the tabulated gamma function.

Many of the possible tests for randomness of points in a plane were presented and applied by Kretz (1969) to the distribution of crystals in a thin section, and will not be reiterated here. Such methods are equally applicable to the study of geological "points" (such as volcanoes) distributed over large areas. The problem of the three-dimensional distribution of points does not arise in the geotectonics of planetary surfaces.

c) Random points on a sphere

Most of the results and methods useful in the plane can be extended for application to a spherical surface. The obvious difference is that the area of the spherical surface is finite, thus some of the relations only hold exactly in the limiting case of an infinite number of points, or a sphere of infinite radius. The nearest neighbour distribution, for example, holds only if the mean inter-point distance is small compared with the radius, that is, the density of points is high. No matter how few points, however, it is still true that points falling in an area of fixed size will be governed by the Poisson density (eq. 1).

Application:

As an example, consider the spatial distribution of so-called Martian "oases", the rather nebulous dark spots
from which the "canals" radiate. There is, of course, great debate on the reality of these markings, but this is not the topic for discussion at hand. The locations of some 142 oases taken from the map of Slipher (1964) are shown in Fig. 1. They appear to be approximately uniformly distributed up to 60° of latitude. The absence of spots at higher latitudes may be an observational effect, so only the range of 60°S to 60°N was considered. The nearest neighbour to each point was calculated, giving the histogram of Fig. 2a. Observed and expected frequencies are shown in Table I.

The observed χ² statistic is 13.32 which is less than the 95 percentile point of the χ² distribution with 8 degrees of freedom (14.07), thus on the basis of this test the hypothesis of randomness cannot be rejected. It is apparent that there are "too few" oases closely spaced, although this trend is not sufficiently marked to satisfactorily reject the hypothesis.

Further, to be random in the Poisson sense, equal areas should contain on the average equal numbers of points. A χ² test similar to the above on counts of N-S and E-W strips of the Martian surface showed no significant deviation from uniformity.

The final test employed was designed to test for the Poisson distribution of point counts within areas of fixed size. One hundred small circles of diameter 10 degrees were placed randomly in the area, resulting in the histogram of Fig. 2b. Observed and expected frequencies are given in Table II.

This χ² statistic of 8.62 compares unfavourably with the 95 percentile of the χ² distribution with 3 degrees of freedom (7.81) so that the hypothesis of randomness must be rejected. The oases therefore are fairly uniformly distributed (same number per unit area) but are tending towards a regularity in spacing, significantly differing from randomness. The histogram in Fig. 2b shows this, in that there is a lack of both very low and very high numbers of points in the counting circles.
Fig. 1. Martian "oases" after Slipher (1964).
Fig. 2.  

a) Frequency of distance to nearest neighbour of Martian Oases, horizontal scale is angular distance in degrees.

b) Frequency of number of Oases within 10° circle.
<table>
<thead>
<tr>
<th>range (radians)</th>
<th>frequency observed (o)</th>
<th>frequency expected (e)</th>
<th>((o-e)^2/e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-.01</td>
<td>31</td>
<td>38.0</td>
<td>1.29</td>
</tr>
<tr>
<td>.01-.02</td>
<td>34</td>
<td>27.1</td>
<td>1.76</td>
</tr>
<tr>
<td>.02-.03</td>
<td>19</td>
<td>20.8</td>
<td>0.16</td>
</tr>
<tr>
<td>.03-.04</td>
<td>24</td>
<td>15.2</td>
<td>5.09</td>
</tr>
<tr>
<td>.04-.05</td>
<td>9</td>
<td>10.7</td>
<td>0.27</td>
</tr>
<tr>
<td>.05-.06</td>
<td>5</td>
<td>7.6</td>
<td>0.89</td>
</tr>
<tr>
<td>.06-.07</td>
<td>8</td>
<td>6.0</td>
<td>0.67</td>
</tr>
<tr>
<td>.07-.09</td>
<td>8</td>
<td>7.2</td>
<td>0.09</td>
</tr>
<tr>
<td>.09-.00</td>
<td>4</td>
<td>9.4</td>
<td>3.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>13.32</td>
</tr>
<tr>
<td>number of points in circle</td>
<td>frequencies</td>
<td>(o-e)^2/e</td>
<td></td>
</tr>
<tr>
<td>--------------------------</td>
<td>-------------</td>
<td>-----------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>observed(o)</td>
<td>expected(e)</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>20</td>
<td>24.7</td>
<td>0.89</td>
</tr>
<tr>
<td>1</td>
<td>31</td>
<td>34.5</td>
<td>0.36</td>
</tr>
<tr>
<td>2</td>
<td>37</td>
<td>24.2</td>
<td>6.66</td>
</tr>
<tr>
<td>3 &amp; 4</td>
<td>12</td>
<td>15.3</td>
<td>0.71</td>
</tr>
</tbody>
</table>

8.62
This observed regularity may indicate that these spots are real tectonic features related to global forces, but the evidence is indeed flimsy. More importantly it is a good example of a non-random set of point "tectonic" features, and exhibits how one also can calculate numerical measures of its non-randomness (such as the \(\chi^2\) statistic) for comparison with other observed tectonic features.

It is perhaps worthy of note that the distribution of lunar craters cannot be treated by such a simple model, since each impact actually represents a disc rather than a point, and obliterates entirely or partially former craters (Marcus, 1964, 1966, a,b,c).

4) THE ANALYSIS OF LINEAR ELEMENTS

a) Random lines in a plane

A line of infinite extent in the X-Y plane may be defined by two parameters \(p\) and \(\theta\) so that the equation of a general line is

\[
p = x \cos \theta + y \sin \theta \quad (\infty < p < -\infty, \ 0 < \theta < \pi)
\]

(4)

If \(p\) is allowed to assume a uniformly random value between plus and minus infinity, and \(\theta\) random between 0 and \(\pi\), a set of random lines is produced (\(p\) can be thought of as a distance from an arbitrary origin). Fig. 3 shows an example of part of such a set. Since there is an obvious correspondence between these random lines and random points in an area, or rather in infinite strip of width \(\pi\), many of the results of the previous section apply: the distance from an arbitrary origin to a line will be uniformly random distributed between 0 and infinity; the azimuths of the lines are also uniformly distributed. Other results are given by Miles (1964) and may be summarized as follows. The mutual intersection angles of the lines (taken as the least of the two complementary angles) have the probability density function \(\sin \theta\) \((0 < \theta < \pi/2)\). This is a fairly important result, because at first glance, one might expect a uniform distribution, whereas in fact an intersection near 90° is much more likely
Fig. 3. Poisson random lines.
than one near $0^\circ$ (Fig. 4).

The number of lines intersecting any convex figure of perimeter $s$ has a Poisson density with mean $T S/\pi$ where $T$ is the mean number of lines per unit length. This is an additional useful test for randomness, similar to the Poisson density test used for points in an area.

b) Random great circles on a sphere

Most of the results of the previous section apply directly to the case of great circles on a sphere. The particular great circle is characterized by two angles between $0$ and $\pi$ which assume uniformly random values independently of each other. Two exactly equivalent intersections of two great circles will occur at conjugate points on the sphere. For this reason measurements need only be made on one hemisphere. Otherwise the distribution of intersection angles and number of lines intersecting convex figures is the same.

c) The length of linear features

The analysis of "lineaments" as an aid to interpreting regional tectonics has been widely used. Most of the available definitions of the word "lineament" such as in Dictionary of Geological Terms (1957) or Dennis (1967) require a topographic alignment known to be structurally controlled. It is obvious, however, that a linear structural feature need not have a consistent topographic expression, and further that linear arrangements of entirely unrelated features may occur due to the chance alignment of more or less random geological events. For this work the term lineament is defined broadly as any approximate rectilinear alignment of geological, tectonic or topographic features of known or unknown origin, whether known to be co-related or not.

Lines in a plane as previously studied can be defined by their position and orientation. A lineament however, being a line segment, has a third measurable parameter, its length. Since the length distribution of a set of lineaments may reflect gross tectonics it is
Fig. 4. Probability density (upper curve) and distribution (lower curve) for intersection angles of random lines.
important to consider possible "random" distributions, or more correctly, stochastic models for the distribution of lineament lengths.

In determining the appropriate stochastic model the various factors which influence the decision to draw a lineament on a geological or topographic map, to interrupt or continue the line, etc. must be considered. The assemblage of lineaments which an individual determines from a map is the result of an interaction between the processes of visual perception, the scale, resolution and accuracy of the map, and the actual physical location of the points and distinct areas on the map. It is impossible to separate these psychological and geometrical effects, so that the formulation of a mathematical model which parallels this process is very difficult. Solesz (1962) has considered some of the perceptive aspects of this problem. Intuitively, both very long and very short lineaments will be uncommon; there will be none of negative length, and the mean will exceed the (single) mode. One family of such positively skewed densities is the gamma density (also called the Pearson Type III) (Kendall & Stuart, 1961, vol. 2, p.64) given by

\[ g(x; \alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta} \]  

(5)

1) Evidence for the gamma density

In order to study theoretically the problem of the length distribution of lineaments, the situation must be idealized. A closely related one dimensional problem concerns the lengths of dense areas of traffic in random flow. Under certain conditions a Chi-squared density is predicted. (Oliver, 1962). As this is a special case of the gamma density, it does not seem unreasonable to use this more general density for the more general case of lineaments.

The Chi-squared and gamma densities have rather general use in describing the "life-time" of random processes. For example, the time of passage of N cars in Poisson-random
traffic has the Chi-squared density with 2N degrees of freedom.

The gamma density was observed empirically in another model which, although still idealized, more closely approaches the geological situation. Uniformly random points (1000) on a 10" page were generated by computer. A number of subjects were then asked to draw the observed "lineaments". Although there was great variation in the total number of lineaments marked, the mean length, and relative skewness, all would be adequately described (in the $\chi^2$ sense) by one of the families of gamma densities. An example of a subject's lineaments is shown in Fig. 5 and a histogram and fitted gamma curve in Fig. 6.

The testing of observed lineaments for randomness against a gamma density is obviously a very conservative test. It is quite possible for lineaments of non-stochastic origin, due to a single physical phenomenon, to show a gamma density, but on the other hand truly random lineaments due to disorganized sources will be highly unlikely to deviate significantly from a gamma density. Thus the test is sufficient but not necessary; while a significant deviation from a gamma density is evidence of non-randomness, the converse is not necessarily true.

There are several ways such a situation could occur. The non-randomness might be quantitatively so small as to be unobservable in the relatively small sample available for examination. Secondly, it is possible but unlikely in nature, to have variables which are pseudo-random, that is, appear to have a random distribution, but are in reality completely determinative events.

ii) Properties of the gamma density

Because of its potential usefulness, several aspects of the gamma density will be considered in detail, particular reference being given to parameter estimation and related numerical problems.
Fig. 5. An example of a subject's lineaments.
The moments of the density, about the origin are given by

\[ \mu'_k = \beta^k \frac{\Gamma(\alpha+k)}{\Gamma(\alpha)} \]  \hspace{1cm} (6)

so that \( \mu'_1 = \beta \alpha \) and \( \mu'_2 = \beta^2 (\alpha+1) \alpha \)

thus the mean and variance are given by \( \mu = \beta \alpha \) and \( \sigma^2 = \beta^2 \alpha \)

The mode of the density is at \( \alpha = (\alpha-1) \beta \).

The exponential density is a special case of (5) where \( \alpha = 1 \), and the gamma density reduced to a \( \chi^2 \) density when \( \beta = 2 \) and \( \alpha \) is a half-integer.

The estimation of the parameters \( \alpha \) and \( \beta \) (estimates denoted by \( \hat{\alpha} \) and \( \hat{\beta} \)) is not as straightforward as with, say, a normal density. The "method of moments" gives

\[
\hat{\alpha} = \frac{(m'_1)^2}{m'_2 - (m'_1)^2} \quad \quad \hat{\beta} = \frac{m'_2 - (m'_1)^2}{m'_1} \]  \hspace{1cm} (7)

where \( m'_k \) is the \( k \)'th sample moment given by

\[ m'_k = \frac{\Sigma X_i}{N} \]  \hspace{1cm} (8)

These are useful estimators, but they are known to be inefficient (Kendall and Stuart, 1961, Vol. 2, p.67).

The maximum likelihood estimators are, of course, more desirable both from the point of view of efficiency, and their useful properties concerned with \( \chi^2 \) tests of goodness of fit (Fisher, 1924). The maximum likelihood estimators cannot be given explicitly in terms of elementary functions.

The likelihood equations are

\[
N \ln \beta - \frac{N}{\alpha} \ln \Gamma(\alpha) + \Sigma \ln x_i = 0 \quad \quad a
\]

\[
-\frac{N}{\alpha} + \frac{\Sigma x_i}{\beta^2} = 0 \quad \quad b
\]  \hspace{1cm} (9)
Fig. 6. Histogram of lineaments of Fig. 5. Fitted gamma curve dotted.
Substituting the simple equation (9b) into (9a) gives the single non-linear equation

\[ \ln \alpha - \frac{d}{d \alpha} \ln \Gamma(\alpha) - \ln \bar{x} + \ln x = 0 \] (10)

where

\[ \bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \]

and \[ \ln \bar{x} = \frac{\sum_{i=1}^{N} \ln x_i}{N} \]

One of the difficulties in solving (10) is the evaluation of the logarithmic derivative of the gamma function. A rapidly converging series expansion is given by Morse and Feshbach (1953, p.422).

\[ f(\alpha) = \frac{d}{d \alpha} \ln \Gamma(\alpha) = -\gamma - \frac{1}{\alpha} = \sum_{i=1}^{\alpha} \left( \frac{1}{i} - \frac{1}{i+\alpha} \right) \] (11)

where \( \gamma = 0.5772157 \)

The following recursion relation is also helpful

\[ f(\alpha) = \frac{1}{\alpha} f(\alpha+1) \] (12)

Equation (10) may then be solved on a computer using one of the several techniques available for nonlinear equations. (see for example Ralston, 1965, pp.318-347). The "initial guess" required by most iterative solutions is provided by the method of moments estimators, eq. (7). In addition the writer has found that a simple "trial and error" procedure centred about the method of moments estimators will rapidly locate the maximum likelihood estimators with sufficient accuracy for most purposes.

iii) The gamma distribution

The cumulative distribution of the gamma density is given by

\[ G(X;\alpha,\beta) = \frac{1}{\beta^{\alpha}} \int_{0}^{x} w^{\alpha-1} e^{-w/\beta} dw \] (13)
The function \( I(u, \rho) \), "the incomplete gamma function", has been tabulated by Pearson (1951) and is defined by

\[
I(u, \rho) = \int_0^{u/\rho + 1} e^{-w} w^\rho \, dw / \Gamma(\rho + 1)
\]  
(14)

It is related to the gamma distribution by

\[
G(x; \alpha, \beta) = I(x, \alpha - 1)
\]  
(15)

The use of tables is somewhat bothersome due to its tabulation for values of \( u \), rather than \( x \). The integral may be evaluated fairly easily by a numerical procedure, thus by computer. This is particularly important where goodness of fit tests are to be made. Two useful expressions are

\[
G(x; \alpha, \beta) = \frac{w^{\alpha+1}}{\Gamma(\alpha+2)} \left(1 - \frac{w^{\alpha+1}}{1! \, \alpha+2} + \frac{w^2}{2! \, \alpha+3} - \frac{w^3}{3! \, \alpha+4} + \ldots\right)
\]  
(16)

where \( \alpha = \alpha - 1 \)  
(Jahnke and Emde, 1945, p.22)

\[
W = x/\beta
\]

and

\[
G(x; \alpha, \beta) = 1 - e^{-w} \frac{1}{\Gamma(\alpha)} \frac{w^{\alpha+1}}{w + 1 - \alpha} \frac{w + 2 - \alpha}{w + 3 - \alpha}
\]  
(17)

(Abramowitz and Stegun, 1964, p.263)

Equation (16) is useful for relatively low values of \( x \) and, with a little care in the program organization, it will give four figure accuracy up to the 99th percentile over a wide range of \( \alpha \) and \( \beta \). Equation (17) is preferable where great accuracy in the tail of the distribution is desired.

iv) The generalized gamma density

It is possible that a more general form of the gamma density may be required for the analysis of "durational"
features such as lineaments, thus the properties of the "generalized gamma density" are given below:

\[ g(x; \alpha, \beta, \theta) = \frac{\theta}{\beta^\alpha \Gamma(\alpha)} \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\theta^\alpha} \]  

(18)

This obviously reduced to e.g. (5) when \( \theta = 1 \)

The moments are given by

\[ \mu_k' = \frac{\beta^\theta}{\Gamma\left(\frac{\alpha+k}{\theta}\right)} \frac{\alpha^k}{\Gamma\left(\frac{\alpha}{\theta}\right)} \]  

(19)

The likelihood equations are

\[ \frac{\alpha \beta}{\theta} - \frac{1}{N} \sum_{i=1}^{N} x_i^\theta = 0 \]

\[ \ln \frac{\beta}{\theta} - \frac{d}{d\alpha} \ln \Gamma\left(\frac{\alpha}{\theta}\right) \sum_{i=1}^{N} \ln x_i = 0 \]  

\[ \frac{\beta}{\theta^2} + \frac{\alpha \beta \ln \beta}{\theta^3} - \beta \frac{d}{d\alpha} \ln \Gamma\left(\frac{\alpha}{\theta}\right) - \sum_{i=1}^{N} \frac{x_i}{N} = 0 \]  

(20)

These equations will be very difficult to solve under most circumstances, but there would appear, also, to be no simple expression for the method of moments estimators, so that the numerical solution to eq. (20) is the best approach, if it is necessary to use this density function.

v) An example of the application of the gamma density

Shreve (1966) concluded that stream systems developed in the absence of geological control are topologically random. Moreover, a gamma density with \( \alpha = 2 \) was suggested for the distribution of internal link-lengths in natural stream systems. Fig. 7 shows a histogram of some 258 link-lengths measured on 1:2400 scale maps of eastern Kentucky taken from Shreve (1969, p.401). Using the maximum likelihood techniques presented here, estimates for \( \alpha \) and \( \beta \) for a fitted gamma curve were 2.32 and 0.0226 respectively.
The unweighted statistic is 14.64 for degrees of freedom which indicates only slightly less than 2.5% of samples of this size taken from the density will show a statistic as high as that observed. This deviation is only marginally acceptable, however, if gamma density with $w = 2$ is about twice as unlikely as $w = 1, 2, 3, 6$, and log-normal and exponential densities fit even more poorly. These calculations do support the gamma density, although Shreve's suggestion of $w = 2$ may not be substantiated.

Although the $x^2$ statistic for the gamma density is considerably lower than for all other commonly used densities, it is still higher than one might normally desire. In seeking an explanation for this, it may be noted that the fitted gamma curve tends to be consistently too high at the ends of the curve and too low in the middle, especially around 0.62-2.68 miles. The addition of a small normal distribution component centered about 2.67 mi. greatly lowers the chi-square statistic to 13.47. It was realized that the area chosen for the study is not truly one of strictly biological control, so that effects of a factor such as a mean spacing of about 6.67 mi. is being

\[ \text{Fig. 7. Histogram of interior link lengths (after Shreve, 1969).} \]
The calculated $\chi^2$ statistic is 24.64 (13 degrees of freedom) which indicates that slightly less than 2.5% of samples of this size taken from this density will show a deviation as high as that observed. This deviation is only marginally acceptable, however, a gamma density with $\alpha = 2$ is about twice as unlikely ($\chi^2 = 27.78$) and log-normal and exponential densities fit even more poorly. These calculations do support the gamma density, although Shreve's suggestion of $\alpha = 2$ may not be substantiated.

Although the $\chi^2$ statistic for the gamma density is considerably lower than for all other commonly used densities, it is still higher than one might normally desire. In seeking an explanation for this, it may be noted that the fitted gamma curve tends to be consistently too high at the ends of the curve and too low in the middle, especially around 0.06-0.08 miles. The addition of a small normal distribution component centered about 0.07 mi. greatly reduces the $\chi^2$ statistic to 13.67 (in one trial). It seems possible that the area chosen for the study is not truly lacking in geological control, so that effects of a joint pattern with a mean spacing of about 0.07 mi. is being observed faintly beneath the random noise.

5) DISTRIBUTION OF POLYGONAL ELEMENTS.

a) Introduction.

In the previous section, the division of the plane into a set of non-overlapping polygons by a number of random lines was discussed. Intrinsic to this model is the concept of random fracture by transcurrent planes. Such a set will be referred to as the Poisson polygons (Miles 1970).

A different, but related, formulation yields the Voronoi or cell-model polygons (Miles, 1970; Meijering, 1953). This random model is appropriate to problems involving growth about random centres, or contraction cracking, e.g. basalt columns. Both these models are discussed in detail through the use of stochastic simulations in the following sections.

b) Polygons formed by random lines in a plane.

The set of random lines described in the Section 4b delineate an assemblage of polygons referred to as the Poisson polygons. Of particular interest are the density functions for the number of sides $n$, the lengths of the sides $l$, the
perimeters, the "in circle" (inscribed circle of largest diameter) d, and the area a, of the polygons. The major references to what has been done in this field are Goudsmit (1945) and Miles (1964a, b, 1970) and the following results are known:

The density of d is exponential with mean 1/τ.
The density of 2τs/π is exponential with mean 1.
The expectation values of some of the variables are as follows:-

\[ E(n) = 4 \]
\[ E(s) = \frac{2\pi}{\tau} \]
\[ E(a) = \frac{\pi}{\tau^2} \]
\[ E(n^2) = \frac{(\pi^2 + 24)}{2} \]
\[ E(s^2) = \frac{\pi^2 (\pi^2 + 4)}{2 \tau^2} \]
\[ E(a^2) = \frac{\pi^4}{2\tau^4} \]
\[ E(a^3) = \frac{4\pi^7}{7\tau^6} \]

(Miles 1964a, p.904)

These variables are not independently distributed, for example, the number of sides of large polygons will in the average be higher than that of small polygons. Thus the "cross expectations" are also important

\[ E(sn) = \frac{\pi (\pi^2 + 8)}{2\tau} \]
\[ E(an) = \frac{\pi^3}{2\tau^2} \]
\[ E(as) = \frac{\pi^4}{2\tau^3} \]
\[ E(a^2n) = \frac{\pi^4 (8\pi^2 - 21)}{21\tau^4} \]
\[ E(a^2s) = \frac{8\pi^7}{21\tau^5} \]
\[ E(a^{m-1}s) = \frac{2\tau E(a^m)}{m} \]

(Miles 1964, p.904)

Although the exact density of none of the variables is known, the probability that \( n = 3 \) has been calculated to be 0.3551 (Miles 1964a, p.903) and the density of 2τs/π for the class of k-sided polygons is \( \chi^2 \) with 2(k - 2) degrees of freedom. Thus if the expected frequencies of the number of sides were known to be \( p_3, p_4, p_5, \ldots \) etc., then the density of 2τs/π would be

\[ f(s) = \sum_{i=1}^{\infty} p_k \chi^2 \quad 2(k - 2) \]

(Certain tests may be applied with only the knowledge of the means and other expectations, but it is obviously of great...
interest to obtain the densities more explicitly. The writer has obtained values of $P_R$ by a Monte-Carlo method which is described in a later section and in Appendix II. (Crain & Miles, 1971).

These values are shown in Table III. Figure 8 shows the density function of $2\pi s/\tau$ derived from eq. 23. Figure 9 shows a histogram of the frequencies of areas of polygons, obtained by the same method. This knowledge then allows for the $\chi^2$-testing of observed distributions against the predicted distributions. It is worth noting that a set of polygons can only be considered "random" (in the sense of being formed by random lines) if all the related distributions are in sufficiently close agreement. To test for one attribute, e.g. the distribution of the number of sides, is insufficient. Conversely, significant disagreement of any one of the associated distributions is sufficient to reject the random model.

c) Polygons formed by random great circles on a sphere.

The fundamental difference between random great circles and random lines, is that with the former the area under consideration is finite, whereas with the latter it is infinite. For instance, it is ridiculous to talk of the distribution of the areas of the figures formed by two random lines in a plane, since all these figures will be infinite in extent. On the other hand, two great circles form four finite figures (2 pairs). In fact, it is absurd to consider any finite number of lines in a plane, rather one discusses an infinite number with average intensity $\tau$. On the sphere all the expectation values vary with the number (finite) of great circles. There is an obvious correspondence, however. If $N$ great circles are randomly placed on a unit sphere one would encounter $2N$ intersections in a random traverse of the globe. Although rigourous proof is difficult, all the results of the previous section can be applied to the sphere if $N$ is sufficiently large, by replacing $\tau$ with $2N$. For small $N$ this is not true, for example the mean number of sides of the spherical polygons is not 4 but rather

$$E_N(n) = \frac{4N(N-1)}{N^2 - N + 2}$$  \hspace{1cm} (24)
Fig. 8. Histogram of perimeter of Poisson polygons.
Fig. 9. Histogram of area of Poisson polygons.
TABLE III

P_n FOR THE POISSON POLYGONS

<table>
<thead>
<tr>
<th>n</th>
<th>Frequency</th>
<th>P_n</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>36732</td>
<td>.3551*</td>
</tr>
<tr>
<td>4</td>
<td>37857</td>
<td>.3779</td>
</tr>
<tr>
<td>5</td>
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<td>.1918</td>
</tr>
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<td>6</td>
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<td>.0592</td>
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<tr>
<td>7</td>
<td>1233</td>
<td>.0132</td>
</tr>
<tr>
<td>8</td>
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<td>.00196</td>
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</tr>
<tr>
<td>10</td>
<td>3</td>
<td>.000038</td>
</tr>
</tbody>
</table>

* Known value all others estimated by Monte-Carlo methods.
Similarly

\[ E_N(n) = \frac{4\pi N}{N^2 - N + 2} \quad \text{and} \quad E_N(a) = \frac{4\pi}{N^2 - N + 2} \]  

As \( N \) increases these expressions soon approach the planar values, for example \( E_{100}(N) = 3.992 \).

One very important additional property of all these distributions is that they are invariant, if through a random (or non-random) process some of the lines, or parts of lines are covered, removed, or "thickened" (Miles, 1964a, p. 905). Those polygons which are still observed to be complete will have the same statistical properties as the entire set. This is of obvious importance in tectonic analyses where the preserved record of tectonic lineaments is fragmentary. It is equally valid, in the statistical sense, to extend great circle segments to full closure and perform tests on the resulting hypothetical polygons.

d) The Voronoi polygons

If two dimensional 'crystals' are allowed to grow uniformly about Poisson-random centres in the plane until mutual contacts prevent further growth, the resulting non-overlapping polygons are known as the Voronoi tesselation (Coxeter, 1961; Miles, 1970). Figure 10 shows a typical realization of a portion of such a tesselation.

These random polygons are of interest as stochastic models in many sciences including metallurgy (Gilbert, 1962; Meijering, 1953) cell biology (Lewis, 1946), communications (Shannon, 1949), astrophysics (Kiang, 1966), sterology (Miles, 1971), zoology (Hamilton, 1971) and in geology (Smalley, 1966).

Of interest are the frequency distributions of the number of sides (neighbours) \( n \), perimeter \( s \), and area \( a \). Theoretical results are scarce; only the mean values of \( n \), \( s \) and \( a \), and a numerical result for the expectation of \( a^2 \) are known. They are as follows (Miles, 1970):

\[ E(n) = 6, \quad E(s) = \frac{4}{\rho}, \quad E(a) = \frac{1}{\rho}, \quad E(a^2) = \frac{1.28}{\rho^2} \]  

where \( \rho \) is the intensity of the Poisson point process. The probabilities of occurrence of the various-sided polygons, which will be denoted by \( \rho^1, \rho^2, \rho^3, \ldots \), are unknown, as are the higher order moments and probability distributions of \( s \) and \( a \).
Fig. 10. A typical realization of the Voroni polygons.

Fig. 12. Histogram of area of Voroni polygons.
TABLE IV

PROBABILITIES OF SIDES OF VORONOI POLYGONS

<table>
<thead>
<tr>
<th>n</th>
<th>Frequency</th>
<th>( p_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>125</td>
<td>.011</td>
</tr>
<tr>
<td>4</td>
<td>1215</td>
<td>.110</td>
</tr>
<tr>
<td>5</td>
<td>2846</td>
<td>.259</td>
</tr>
<tr>
<td>6</td>
<td>3172</td>
<td>.288</td>
</tr>
<tr>
<td>7</td>
<td>2266</td>
<td>.206</td>
</tr>
<tr>
<td>8</td>
<td>953</td>
<td>.087</td>
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<tr>
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<td>321</td>
<td>.029</td>
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<td>10</td>
<td>85</td>
<td>.0077</td>
</tr>
<tr>
<td>11</td>
<td>15</td>
<td>.0014</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>.0002</td>
</tr>
</tbody>
</table>

Table IV shows the frequency of occurrence of the various-sided polygons. Probabilities are calculated as the ratio of the observed frequencies to the total number of polygons. The distributions are shown in Figures 8 and 12.
To determine these unknown probabilities, distributions and higher moments, a Monte-Carlo simulation similar to that used for the Poisson polygons was performed. A general outline of the Monte-Carlo technique is given in a following section, and further details with computer programs are given in Appendix I and II. Sample estimates of the higher order moments based on 11,000 independent trials are as follows:

\[
\begin{align*}
E(n^2) &= 37.0 \\
E(n^3) &= 249 \\
E(n^4) &= 1720 \\
E(\rho s^2/16) &= 1.06 \\
E(\rho^3/s^3/64) &= 1.17 \\
E(\rho^2 a^2) &= 1.24 \\
E(\rho^3 a^3) &= 1.78
\end{align*}
\]

Table IV shows the frequency of occurrence and deduced probabilities of the various - sided polygons. Perimeter and area distributions are shown in figures 11 and 12. These distributions are far more confined than the previously described random line configurations. Although the simulation performed here uses a smaller sample size, the results are sufficient to test hypotheses against the distributions.

The Poisson polygons represent to some extent random fracture under compression, whereas the Voronoi polygons can be considered to model random fracture under extension.

Even with certain statistical restrictions, such as homogeneity and isotropy, there are an infinity of random tessellations which can be generated from Poisson-random points in a plane. The two models discussed here have the most obvious and direct applications to the statistical analysis of planetary surfaces. For this reason, other tessellations of this inexhaustible series will not be discussed until applications are apparent.
Fig. 11. Histogram of perimeter of Voronoi polygons.
e) Monte-Carlo simulations of random polygons.

"No substantial part of the Universe is so simple that it can be grasped and controlled without abstraction. Abstraction consists in replacing the part of the universe under consideration by a model of similar but simpler structure. Models are thus a central necessity of scientific procedure."

(Rosenblueth and Wiener, 1945, p. 316)

As previously stated, the object of a Monte-Carlo simulation is to obtain, through random number generation, approximations to probability density functions, which are not otherwise obtainable. This section is designed to outline the procedures used in this work for the Poisson and Voronoi polygons; details of the procedure, program listings and complete results (some of which have wider application than this work) are given in Appendices I and II.

The procedure for modeling the Poisson polygons contains four basic steps. First, random lines are generated within a circle. A computer pseudo-random number generator (Smith, 1960) was used to produce Poisson-random points in the rectangle $0 \leq \theta \leq \pi$, $0 \leq p \leq 1$ (representing a circle of radius 1). These points are converted to their equivalent lines and stored.

Second, using an appropriate algorithm, each rectangle is in turn located and its vertices determined and recorded.

Third, the area and perimeter of each is calculated.

Fourth, weighting factors are applied to the recorded polygons to eliminate edge-effects due to the finite boundary of the area (Miles, 1971c). This correction is necessary because the boundary preferentially cuts the larger (many-sided) polygons.

Ideally, the simulation will improve with the number of generated lines. Unfortunately, practical considerations put various restrictions on the feasible simulations.

In order to obtain any reasonable rate of generation, all the intersection points of the lines must be stored in core simultaneously. This restricts the maximum number of lines in one simulation to about 120. This number is
not sufficiently high to eliminate edge-effects, but it is sufficient to ensure that edge-effect corrections can be made accurately.

A total of 45 simulations were performed, generating a total of 100,495 polygons, which have been stored on magnetic tape for future reference. In addition 100,000 of a related variety of polygon were produced to obtain better estimates of the frequencies of the higher-sided polygons (Appendix II).

For the Voronoi polygons a different form of simulation was possible which allowed for the generation of separate independent polygons (compared with above where there are 45 independent simulations in each of which the individual polygons are inter-dependent). The procedure was, first, to generate a small number (about 30) Poisson-random points within a square. The right bisectors of neighbouring points are calculated and stored, providing a set of polygons, of which the most central is an independent Voronoi polygon, (Appendix II). This polygon is searched and recorded using the same search procedure as for the Poisson polygons.

6) Summary Remarks.

This part of the thesis has examined the various stochastic representations for random points, and great circles on a sphere. Naturally, the majority of these were based on Poisson statistics.

Evidence was presented to favour the use of gamma densities as a conservative test for the randomness in the lengths of a population of lineaments. The methods for estimating the parameters of this distribution and various associated calculations were described in some detail.

In addition to a summary of known geometrical statistical methods, the structure and results of Monte-Carlo simulations of two classes of random polygon were presented. Approximations to these previously unknown distributions were obtained, for possible application to tectonic geometry.

The object of the next part is to apply some of these results to the tectonic geometry of the Earth.
PART III

THE STATISTICAL GEOMETRY OF THE GLOBE

1. Introduction

Going back several centuries of exploration and study, most of the Earth's exposed surface has been mapped geologically. More recent study may reveal the nature of the ocean-floors, if only topographically. The amount of information contained in a tectonic map of a continent is so immense that the interpretation of all the details is extremely difficult.

A geological or tectonic map is a simplified description of the Earth's surface and can be considered to be a huge collection of inter-related points, lines, and areas of differing size and colour (type). The observer of such a colourful display is invariably struck by the observation that this collection of points, lines, and areas is in some way connected. There are always "trades", "correlations", "patterns". These apparent patterns are information, and the writer's aim is to describe and extract this information in a quantitative manner.

Such a problem belongs properly to the field of "pattern recognition" (Marr, 1971). Ideally it should be possible to convert a tectonic map into an array of numbers, and by applying pattern recognition techniques to extract the information content. With the present state of knowledge, full realization of this idea is not possible. The writer has, therefore, concentrated on an abstraction of this total picture.

One aspect of a map is its linear features. These areas that are sufficiently aligned and straight that their position and orientation may be adequately described by a straight line (or great circle) segments. In addition, some of the geological domains of the Earth can be idealized as finite-sided polygons. This thesis concentrates on studying the possible tectonics in that abstraction of the Earth's surface consisting of its lineaments and polygonal areas.
PART III.
THE STATISTICAL GEOMETRY OF THE EARTH.

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A geological or tectonic map is a simplified description of the Earth's surface and can be considered to be a huge collection of inter-related points, lines, and areas of differing sizes and colours (type). The observer of such a colourful display is invariably struck by the observation that this collection of points, lines, and areas is in some way non-random; there are always 'trends', 'correlations', 'patterns'. These apparent patterns are information or 'signal' buried in geological noise. It is the writer's aim to describe and extract this information in a quantitative manner.

Such a problem belongs properly to the field of "pattern recognition" (Corcoran, 1971). Ideally it should be possible to convert a tectonic map into an array of numbers, and by applying pattern recognition techniques to extract the information content. With the present state of knowledge, full realization of this ideal is not possible. The writer has, therefore, concentrated on an abstraction of this total picture.

One aspect of a map is its linear features, those areas that are sufficiently elongate and straight that their position and orientation may be adequately described by a straight line (or great circle) segment. In addition, some of the geological domains of the Earth can be idealized as finite-sided polygons. This thesis concentrates on studying the possible patterns in that abstraction of the Earth's surface consisting of its lineaments and polygonal areas.
2) Sources of Information and Methods of Acquisition.

a) Sources of Continental Information.

The primary source of continental information has been the tectonic maps of the various continents published in several countries. Tectonic maps at a scale of 1:5m, printed at various times are available for Africa, Australia, North America, India, Europe and Asia. No tectonic map for South America is available at present. Whenever possible the most recent editions were used as the primary source, with additional information being gathered where appropriate from older editions.

For South America, most of the data was obtained from the most recent geological map, with additional tectonic information from (Zambrano, and Urien, 1970). Geological maps provided a secondary source for the other continents. These maps are of various scales, are based on differing tectonic-cartographic principles, and are of varying quality and reliability, depending on the information and its availability to the map-makers. The writer hopes that by referring to many sources for each continent, these effects have been minimized.

The maps employed for the data acquisition are listed separately following the References and no attempt has been made to refer to particular maps in the text.

b) Sources of Oceanic Information.

The study of the oceanic lineaments and other tectonic features is basically a study of bathymetry, thus bathymetric maps were the primary source of information. The writer was fortunate in being able to obtain from Scripps Institute a series of detailed bathymetric maps covering the North Pacific Ocean.

Ocean ridges and fracture zones are perhaps best defined by their seismicity, so that recent world seismicity maps proved useful.
c) Data Acquisition.

To define a lineament, the latitude and longitude of each end of the feature were measured from the map. This was done with as much accuracy as possible using multipoint dividers to interpolate between marked geographical coordinate lines. Measurements were recorded usually to within 0.1° and if possible within 0.01°. In cases where the map projection was particularly distorted, it was found advisable to record a mid-point on the lineament, and subsequently calculate its radius of curvature. This was used to eliminate arcuate lineaments, which although worthy of study, were regrettably beyond the scope of this work due to time considerations. In a detailed study made of Africa, the lineaments were additionally classified according to subjective criteria of reliability. It was subsequently found, and will be shown in the later presentation of the data, that this information was unnecessary and the procedure was eliminated.

As might be imagined, the process is slow and tedious, each lineament requiring 5 - 10 minutes to record accurately. An additional problem is the extremely large size of some of the maps, so that the data gatherer is often obliged to spend a great deal of time crawling about on the floor in hallways and foyers, this being the only flat area of suitable size. Coincident with this procedure was a real danger of physical injury both to the writer and the map, from the feet of passing herds of undergraduates. In some instances large sheets of tracing paper were used upon which to delineate the lineaments, and subsequent measurements were performed on smaller pieces which could be managed on a reasonable-sized table.

The method, therefore, does not recommend itself well. However, automatic or semi-automatic data collection, through the use of electronic digitizers is made inconvenient also, by the large physical size of the maps, and by the vast array of differing projections employed by the various cartographers.
Manual data collection from source material of this nature is thus the only practical solution.

Once collected, the lineaments were recorded on coding sheets and punched into standard 80 column cards for computer data processing.

As will be described later, a preliminary case-study was performed on the polygons of Africa. In collecting this data there is the problem of keeping track of all the vertices of the polygon, which often have to be readjusted until an apparently good fit of the vertices to the natural polygon is obtained. It was found that it was most convenient to use coloured markers whose geographic positions were measured and recorded as with the lineaments. (The markers used were, in fact, jelly-beans whose various colours were useful for classification purposes, and whose nourishment provided occasional encouragement). As before, an accuracy of at least 0.1° was maintained.

d) The Problem of Subjectivity.

The process of drawing lineaments is subjective. With each lineament there must be a series of decisions to determine where it ends, where do two lineaments become one, is it a 'lineament' at all. There are, no doubt, a small group of unquestionable lineaments; in general, however, different people, given the same source, will draw differing sets of lineaments. Also, since one must decide what is and is not a lineament, it is obviously possible to preferentially delete certain lineaments in order to favour a preconceived hypothesis. This 'hypothesis bias' was minimized in this work by the data acquisition techniques used. No running tally of the results was maintained, so that until the data processing stage, the writer had no idea of the shape of the orientation or length histograms. This eliminated any conscious or unconscious tendency to bolster apparent trends or 'look for' lineaments to fill apparent gaps in the distribution.

The effect of the subjective decisions on each lineament will only be important if they are biased. Supposing that
there is such a thing as a 'true' position of a lineament, then random errors in decision as to its end points will only contribute random scatter to the data. This has the effect of reducing the chance of observing regularities in the data and hence reduces the chance of discovering false regularities.

As the results of the African detail study show, these problems are not as serious as the writer first imagined. The population of lineaments which the writer classified as uncertain have substantially the same distribution as those classified as definite. Thus the results appear to be unbiased by subjectivity.

Many authors in the past have commented on the possible subjective bias of lineaments (Wise, 1968). It is a particularly critical matter if only the orientation of the lineament is being studied. In that case the decision as to whether a lineament is one or two is important, since an attempt is being made to characterize the population of lineament by one variable only. Other workers (F.S.Hills, pers.comm.) have attempted to weight the orientation data according to the length of the lineament. Again there is the danger of attempting to describe a two dimensional object as a point, and an implicit assumption concerning the correlation between length and orientation.

The recording of the orientation and length of each lineament as done in this work, is designed to describe the lineaments as a bivariate population, thus reducing the importance of decision errors whether biased or random.

In spite of these precautions, the writer recognizes that a certain subjective element is an unavoidable part of an analysis of this kind, as in many fields of the natural sciences.

e) Data Processing.

The accumulated lineament data was computer processed to obtain the following distribution: lineament lengths, azimuths, intersection angles and potential intersection angles. (The distribution of potential intersection angles refers to the suite of intersections obtained by extending the lineaments into complete great circles).
For more detailed study of Africa additional distributions were accumulated, taking into account a subjective weighting factor, and separate distributions for each type of lineament.

The mathematics required for this processing is given in detail with the program descriptions in Appendix I.

Secondarily, the length distribution histograms were processed to obtain maximum-likelihood estimates of gamma distribution parameters, using the methods described in earlier chapters. Details of these two processing programs are also given in Appendix I along with the long list of required subroutines.
3) THE AZIMUTHAL DISTRIBUTION OF LINEAMENTS

The azimuth of a lineament is the angle it makes with true north. This is an imperfect definition, for the obvious reason that the azimuth of a great circle segment varies continuously along its length; this variation is significant for many of the long structures included in this study. The azimuth computed for each lineament is of its mid-point.

It is clear also, that the azimuth of a great circle segment becomes increasingly meaningless as the poles are approached, putting severe restrictions on the interpretation of azimuth distributions for polar regions.

In spite of these restrictions, the circular frequency histogram of azimuths of lineaments for a large region is an excellent way of summarizing the directional information. Random lineaments will be equally likely to fall in any given direction, so that the appropriate random model for comparison is the uniform circular distribution.

a) Africa

If one considers for the moment that each lineament is of equal importance, without reference to the nature of its expression, then the frequency histogram for Africa is given by fig. 13. In this and subsequent diagrams the frequency histogram has been normalized to 100% to ease comparisons.

The salient features of the distribution are that the general shape is elliptical with its major axis trending generally in a northwest-southeast direction. Prominent maxima in the distribution occur at azimuths 110°, 126°, and 145° with less prominent peaks at 2°, 36°, 60° and 78°.
Fig. 13. Azimuth histogram for African lineaments.
A $\chi^2$ test demonstrates that this distribution cannot be reasonably considered to be uniform-random. The observed $\chi^2$ statistic is 66.7 (35 degrees of freedom) while the lower limit of the 95% critical region is 50.3. The mean azimuth (not a particularly significant parameter) is $124^\circ$.

The distribution of figure 14 takes into account the relative strength of expression of each lineament. This was accomplished by giving each a weighting factor between 0 and 10 as a subjective indication of the confidence that the writer felt in the tectonic significance of the lineament. Poorly expressed linear features of unknown origin would receive a weight of 1 to 3, whereas well documented structures, such as major strike-slip faults would receive maximum weight, and so on.

It is clear that this weighting has had little influence on the distributions and the statistics verify this observation. The major peaks are at $108^\circ$, $126^\circ$ and $148^\circ$ with minor maxima at $2^\circ$, $22^\circ$, $58^\circ$, $82^\circ$ and $168^\circ$. The $\chi^2$ statistic for circular uniformity is similar to the previous at 70.4 for 35 degrees of freedom. The mean azimuth is $122^\circ$.

One could go further and propose that the two samples are from identical population distributions. There are several ways to perform such a test; for example those given by Freund (1962, p. 290-291). One simple method is to propose the hypothesis that the distribution observed for the unweighted lineaments is the "true" distribution. The weighted histogram can then be compared to this hypothetical density using the standard goodness-of-fit test. In this case the $\chi^2$ statistic is a very low 13.1 compared to the critical region of less than 21.6. So the hypothesis that these two samples are from identical populations cannot be rejected. The writer also performed the Mann-Whitney test (Freund, 1962, p. 291), which is far more time consuming and similar results were obtained.
TABLE V

Test for Uniform Randomness of African Lineament Azimuths

<table>
<thead>
<tr>
<th>Class</th>
<th>Sample</th>
<th>$\chi^2$</th>
<th>d.f.</th>
<th>95% Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>unwtd.</td>
<td>456</td>
<td>66.7</td>
<td>35</td>
<td>50.3</td>
</tr>
<tr>
<td>wtd.</td>
<td>456</td>
<td>70.4</td>
<td>35</td>
<td>50.3</td>
</tr>
<tr>
<td>faults</td>
<td>162</td>
<td>69.9</td>
<td>35</td>
<td>50.3</td>
</tr>
<tr>
<td>rivers</td>
<td>144</td>
<td>30.6</td>
<td>17</td>
<td>27.6</td>
</tr>
<tr>
<td>s. trends</td>
<td>100</td>
<td>38.7</td>
<td>17</td>
<td>27.6</td>
</tr>
<tr>
<td>others</td>
<td>42</td>
<td>11.8</td>
<td>7</td>
<td>14.1</td>
</tr>
</tbody>
</table>

TABLE VI

$\chi^2$ - Test for Identity of Distribution

<table>
<thead>
<tr>
<th>Class</th>
<th>$\chi^2$</th>
<th>d.f.</th>
<th>95% Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>wtd.</td>
<td>13.1</td>
<td>35</td>
<td>21.6</td>
</tr>
<tr>
<td>faults</td>
<td>14.5</td>
<td>35</td>
<td>21.6</td>
</tr>
<tr>
<td>rivers</td>
<td>6.4</td>
<td>17</td>
<td>8.7</td>
</tr>
<tr>
<td>s. trends</td>
<td>7.4</td>
<td>17</td>
<td>8.7</td>
</tr>
<tr>
<td>others</td>
<td>3.5</td>
<td>7</td>
<td>2.1</td>
</tr>
</tbody>
</table>
Lineaments of similar tectonic expression may be of widely differing origin, and lineaments which result from similar forces and events may, for various reasons, have varying expression. It is therefore important to consider whether lineaments of similar tectonic expression show azimuth distributions differing from the total view. African lineaments were classified according to the headings "faults", "rivers", "structural trends" and "others". Figures 15 through 18 show the separate azimuth distributions for these classifications. It is visually apparent that the distributions are, with the exception of "others", quite similar. Tables V and VI summarize their statistics. All distributions but "others" can be clearly distinguished from the uniform random distribution as shown by the high $\chi^2$ statistics. The smaller sample size of "rivers" and "structural trends" required the use of a larger counting window; hence, the lower degrees of freedom. (Similarly for "others", where the sample size is very small.)

The distributions in general can be considered to have come from the same population, as shown by the results in Table VI. The few lineaments classified as "others" fall into the middle ground where one cannot conclusively distinguish them from the uniform distributions. The class of "others", which represents about 9% of the lineaments, therefore shows hints of being merely random "noise", but the sample size is rather small for confirmation.

Table VII summarizes the arrangement of maxima on the various distributions, demonstrating again their similarity, although in a subjective manner. Figure 19 displays the geographical arrangement of African lineaments.

b) South America

The azimuth distribution for South America is shown in figure 20. The maxima in the histogram are more
Fig. 14. Azimuth histogram for African lineaments with weighting factors.
Fig. 15. Azimuth histogram for African faults.
Fig. 16. Azimuth histogram for African rivers.
Fig. 17. Azimuth histogram for African structural trends.
Fig. 18. Azimuth histogram for other African lineaments.
Fig. 19. African lineaments.
Fig. 20. Azimuth histogram for South American lineaments.
### TABLE VII

Maxima of African Azimuths

<table>
<thead>
<tr>
<th>Class</th>
<th>Major</th>
<th>Minor</th>
</tr>
</thead>
<tbody>
<tr>
<td>unwtd.</td>
<td>110° 126° 145°</td>
<td>2° 36° 60° 70°</td>
</tr>
<tr>
<td>wtd.</td>
<td>108° 126° 148°</td>
<td>2° 22° 58° 82° 168°</td>
</tr>
<tr>
<td>faults</td>
<td>110° 154° 20° 55° 82° 92° 168°</td>
<td></td>
</tr>
<tr>
<td>rivers</td>
<td>108° 127° 146°</td>
<td>18° 48° 82° 92° 166°</td>
</tr>
<tr>
<td>s. trends</td>
<td>112° 128° 138°</td>
<td>58° 96° 168°</td>
</tr>
<tr>
<td>others</td>
<td>112°? 122°?</td>
<td>2° 38° 75° 160°?</td>
</tr>
</tbody>
</table>

The table shows the maxima of African azimuths classified by different categories. The maxima are given in degrees, with major and minor maxima listed separately. The results indicate a well-defined trend towards a north-south orientation, with certain similarities and differences observed among the categories.
pronounced than for Africa and occur at 63°, 92°, 134° and 168°, with a minor peak at 31°. Certain similarities to the African distributions become apparent by rotating the diagram about 50° counterclockwise. Further discussion of these similarities will be given in a later section. The \( \chi^2 \) test for uniformity indicates a highly non-uniform distribution with a statistic of 112.9 (35 degrees of freedom). Such information is summarized for all the continents in Table VIII. Geographic distribution is shown in figure 21.

c) Eurasia

The azimuth distribution for Eurasia (figure 22) shows a rather dramatic maximum in a north-south direction. This is due to the coincidence of the large number of northerly trending structures in the Urals, and again in the Indonesia-Pacific Area. Other major maxima are at 106° and 133°. Minor peaks occur at 33°, 48° and 82°. There is a certain lack of symmetry to the Eurasian distribution which may indicate a composite continent in accordance with some paleomagnetic evidence (McIlhinney, pers. comm.). Geographic distribution is shown in figure 23.

d) North America

Figure 24, the azimuth distribution for North America, exhibits a remarkable symmetry about true north. Doubtless, it is this North American pattern which encouraged Moody and Hill to seek similar patterns on other continents. Most common azimuths are at 45°, 86°, 118° and 156° with minor maxima at 0°, 26°, 72° and 138°. The distribution has about the same deviation from uniformity as Africa, with a \( \chi^2 \) statistic of 69.7. Mean azimuth is 94.1°. Lineament map for North America is figure 25.
<table>
<thead>
<tr>
<th>Continent</th>
<th>$\chi^2$</th>
<th>Degrees of Freedom</th>
<th>95% Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>South America</td>
<td>112.9</td>
<td>35</td>
<td>50.3</td>
</tr>
<tr>
<td>Africa</td>
<td>66.7</td>
<td>35</td>
<td>50.3</td>
</tr>
<tr>
<td>Eurasia</td>
<td>69.5</td>
<td>35</td>
<td>50.3</td>
</tr>
<tr>
<td>North America</td>
<td>69.7</td>
<td>35</td>
<td>50.3</td>
</tr>
<tr>
<td>Australia</td>
<td>86.4</td>
<td>17</td>
<td>27.6</td>
</tr>
<tr>
<td>India</td>
<td>23.6</td>
<td>6</td>
<td>12.6</td>
</tr>
<tr>
<td>Arabia</td>
<td>17.3</td>
<td>8</td>
<td>15.1</td>
</tr>
</tbody>
</table>
Fig. 21. South American lineaments.
Fig. 22. Azimuth histogram for Eurasian lineaments.
Fig. 23. Eurasian lineaments.
Fig. 24. Azimuth histogram for North American lineaments.
Fig. 25. North American lineaments.
e) Australia

Australia exhibits a remarkable, almost bi-directional lineament pattern (figure 26). A large number of lineaments trend at about 55° and a slightly lesser number near 0°. An intermediate direction of 32° is also in evidence. Minor peaks occur at 120° and 160°. This non-uniformity yields a $\chi^2$ statistic of 86.4 and not unexpectedly a mean azimuth in the vicinity of 52°.

f) India

The sample size for Indian lineaments is so small (37) that the azimuth distribution (figure 27) is of little value, except to show that two directions predominate; 48° and 128°. With the small sample it was only possible to obtain 6 degrees of freedom in calculating the $\chi^2$ comparison to the uniform distribution. The statistic obtained was 23.6. Lineament map is shown in figure 28.

g) Arabia

As with India, only a small sample is available for the Arabian subcontinent. The 53 lineaments are shown in figure 29 and their azimuth distribution in figure 30. $\chi^2$ statistic is 17.3 (6 degrees of freedom) and prominent azimuths are perhaps at 0°, 55° and 112°.

4) INTERCONTINENTAL RELATIONSHIP OF AZIMUTH DISTRIBUTION

All the azimuth distributions have certain similarities: they are more or less symmetrical about at least one direction and flanking the long axis of the diagrams are usually two maxima separated by about 60°, and the directions parallel and perpendicular to the long axis are often minor maxima. The patterns, however, show no more than a casual relationship to absolute geographical coordinates, except that the long axis of the distributions tend to be more often near 90°.
Fig. 26. Azimuth histogram for Australian lineaments.
Fig. 27. Azimuth histogram for Indian lineaments.
It is important to consider the apparent rotations required to bring the similarities in the distributions into coincidence. To some degree this could be done by inspection, but it is best done by a numerical procedure. To this end, the similarity of juxtaposed azimuth distributions was measured by calculating the correlation coefficient at each of a series of two degree rotations, thereby locating the point of maximum correlation between the two.

Of particular interest, of course, are the Africa-South America and Europe-North America relationships. Maximum correlation between Africa and South America occurs at 42° (rotating South America eastwards) and for Europe-North America, the result is 32° in the same sense. The maximum correlation between Eurasia and Africa occurs at a rotation of only 2° (rotating Africa westwards). The "best fit" of the Australian and African azimuths occurs by rotating Australia 66° westward, and remains high through 90°. The India-Africa maximum correlation is at 14° (India rotating eastward) and for Arabia-Africa at 6° (Arabia rotating westward). Table IX summarizes this information.

The sense and magnitude of these rotations compare well with suggested continental reconstruction patterns (eg. Bullard, 1965 and McIlhinny and Luck, 1970). The rotation angles found by Bullard (1965) were for North America versus Europe 38.1° and for South America versus Africa 56.3°. In both cases, the rotation implied by the lineament patterns is somewhat less than the purely geometrical rotation. Considering the aforementioned restrictions on the interpretation of azimuth patterns, they show surprisingly good agreement.

Depending on the particular reconstruction, rotations for Australia vary from about 50° to 90°, the Arabia-Africa rotation is usually quoted in the range of 4° to 12° (Ivanhoe, 1967), while the exact place of India is a
Fig. 29. Azimuth histogram for Arabian lineaments.
### TABLE IX
**CORRELATIONS BETWEEN AZIMUTH DISTRIBUTIONS**

<table>
<thead>
<tr>
<th>Continents</th>
<th>Rotation at max. correlation</th>
<th>Correlation coeff. (max.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa-South America</td>
<td>$42^\circ$ E</td>
<td>.78</td>
</tr>
<tr>
<td>Africa-India</td>
<td>$14^\circ$ E</td>
<td>.62</td>
</tr>
<tr>
<td>Africa-Australia</td>
<td>$66^\circ$ W</td>
<td>.77</td>
</tr>
<tr>
<td>Africa-Eurasia</td>
<td>$2^\circ$ E</td>
<td>.81</td>
</tr>
<tr>
<td>Eurasia-North America</td>
<td>$32^\circ$ E</td>
<td>.74</td>
</tr>
<tr>
<td>Africa-Arabia</td>
<td>$6^\circ$ W</td>
<td>.69</td>
</tr>
</tbody>
</table>

### TABLE X
**INTERSECTION ANGLES**

<table>
<thead>
<tr>
<th>Continents</th>
<th>Sample</th>
<th>$\chi^2$</th>
<th>Degree of Freedom</th>
<th>95% Limit Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. America</td>
<td>54</td>
<td>12.2</td>
<td>6</td>
<td>12.6 25 55</td>
</tr>
<tr>
<td>S. America</td>
<td>60</td>
<td>2.0</td>
<td>6</td>
<td>12.6 35 60</td>
</tr>
<tr>
<td>Eurasia</td>
<td>78</td>
<td>6.2</td>
<td>6</td>
<td>12.6 15 45 65</td>
</tr>
<tr>
<td>Africa</td>
<td>170</td>
<td>14.5</td>
<td>7</td>
<td>14.1 25 55</td>
</tr>
<tr>
<td>Australia</td>
<td>34</td>
<td>6.3</td>
<td>4</td>
<td>9.5 25 55 85 5</td>
</tr>
<tr>
<td>India</td>
<td>14</td>
<td>3.6</td>
<td>2</td>
<td>6.0 15 45 65</td>
</tr>
<tr>
<td>Arabia</td>
<td>22</td>
<td>4.8</td>
<td>2</td>
<td>6.0 25 50</td>
</tr>
</tbody>
</table>
matter of great variation. Figure 30 shows the azimuth
distributions of the continents in rotated positions
based on the Bullard (1965) and McIlhinny and Luck (1970)
fits.

Figure 31 compares the geographical distribution
for Africa and South America in a reconstructed position
and figure 32 shows the equivalent for North America
against Europe. A remarkable degree of continuity in
the pattern is evident on inspection.

5) INTERSECTION ANGLES OF CONTINENTAL LINEAMENTS

If the observed lineaments are of non-random
origin, one might expect to find a certain regularity in
their two dimensional arrangement, such as preferred
intersection angles. Figures 33 through 39 show the
frequency histograms of the intersection angles of linea-
ments on the various continents. Below each histogram is
a chart showing the deviation of the observed frequency
from the Poisson-line model where the expected probability
density is given in Part II, Section 4a. Only about 30%
of the lineaments actually intersect each other, so that
sample sizes become rather small. Table X shows the
sample sizes, maxima, and \( \chi^2 \) test against the Poisson-
random model.

All the distributions deviate somewhat from the
random model and show a fair degree of similarity. The
small sample sizes, however, make detailed comparisons
meaningless and in most cases are too small to produce
definitive \( \chi^2 \) statistics. It was shown in an earlier
section that there are valid arguments for extending
all great circle segments to completion and studying the
resultant intersections. Such intersections are termed
"potential intersections" and are shown in figures 40
through 46. The upper histogram in each figure is of
the potential intersections with the curve for Poisson
Fig. 30. Azimuth histogram for rotated continents.
Fig. 31. Lineaments of Africa and South America in reconstructed position.
Fig. 32. Lineaments of North America & Europe in reconstructed position. (Greenland is not drawn)
Fig. 33. Histogram of intersection angles of lineaments of Africa.
Fig. 34. Histogram of intersection angles of lineaments of South America.
Fig. 35. Histogram of intersection angles of lineaments of Eurasia.
Fig. 36. Histogram of intersection angles of lineaments of North America.
Fig. 37. Histogram of intersection angles of lineaments of Australia.
Fig. 38. Histogram of intersection angles of lineaments of India.
Fig. 39. Histogram of intersection angles of lineaments of Arabia.
Fig. 40. Histogram of potential intersection angles of lineaments of Africa.
Fig. 41. Histogram of potential intersection angles of lineaments of South America.
Fig. 42. Histogram of potential intersection angles of lineaments of Eurasia.
Fig. 43. Histogram of potential intersection angles of lineaments of North America.
Fig. 44. Histogram of potential intersection angles of lineaments of Australia.
Fig. 45. Histogram of potential intersection angles of lineaments of India.
Fig. 46. Histogram of potential intersection angles of lineaments of Arabia.
randomness superimposed. The lower figure charts the deviation from the random model. All continents deviate significantly as shown by the $X^2$ statistics of Table XI and show a marked similarity to each other.

The number of potential intersections created in this manner will, of course, be enormous, since $n$ lineaments extended around the globe will produce $2(1 + 2 + 3 + \ldots + (n - 2) + (n - 1))$ intersections (approximately equal to $n^2$). Thus, in continents such as Africa, with over 400 lineaments, there would be over 160,000 such intersections. This is a large computational task even on a computer. To alleviate this problem, the lineaments were subdivided by geographical area into smaller groups of about 50 lineaments. This served the dual purpose of reducing the computation required, and enhancing the possible significance of the procedure by considering only the intersections of geographically intimate lineaments.

In all cases there is an excess of lower angle intersections and a marked depletion of intersections greater than $60^\circ$. Typically, there are major peaks at $15^\circ$, $22^\circ$ and $30^\circ$ with less pronounced maxima at $5^\circ$, about $45^\circ$ and $60^\circ$. It would seem likely that such regularity in spacing of preferred angles is tectonically significant.

The only apparent deviation from the pattern described above occurs with Arabia which shows many intersection angles between $45^\circ$ and $60^\circ$.

6) THE LENGTH DISTRIBUTION OF LINEAMENTS

In Part II, Section 4, some of the expectations concerning the lengths of random linear features were discussed. It was argued that one might reasonably expect one of the family of gamma densities to describe the length distribution of random lineaments.

The great circle arc-length of each lineament was calculated, histograms accumulated and maximum likelihood
TABLE XI

POTENTIAL INTERSECTION ANGLES

<table>
<thead>
<tr>
<th>Continent</th>
<th>Sample Size</th>
<th>$\chi^2$</th>
<th>95% Limit</th>
<th>Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. America</td>
<td>4712</td>
<td>76.5</td>
<td>60.46</td>
<td>5 15 25 32 48</td>
</tr>
<tr>
<td>S. America</td>
<td>5983</td>
<td>82.3</td>
<td>76.5</td>
<td>7 12 20 34 60?</td>
</tr>
<tr>
<td>Eurasia</td>
<td>8339</td>
<td>80.6</td>
<td>515 25 32 48</td>
<td></td>
</tr>
<tr>
<td>Africa</td>
<td>10610</td>
<td>78.2</td>
<td>76.5</td>
<td>8 15 23 38 60?</td>
</tr>
<tr>
<td>Australia</td>
<td>3486</td>
<td>79.4</td>
<td>515 23 38 60?</td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>597</td>
<td>101.7</td>
<td>6 15 25 30 40</td>
<td></td>
</tr>
<tr>
<td>Arabia</td>
<td>1178</td>
<td>76.5</td>
<td>6 18 23 33 48 60?</td>
<td></td>
</tr>
</tbody>
</table>

The length distribution of lineaments will be affected by many "style" more than other measures, thus the differences for South America can be attributed to the fact that most of the information for this continent was obtained from geological maps, whereas tectonic maps formed the primary source for the other continents. All of the histograms clearly show a similarity to their various likelihood fitted gamma densities. The mean lineament length for the separate continents vary from above .05 to .15 earth radii harmonic means are in the vicinity of .06 earth radii. Variance of the distributions fall between .602 and .813.

The relative closeness of fit is compared by $\chi^2$ statistic in Table XI. All except Arabia differ significantly from the best-fit gamma distribution. The lower portion of each of figures 47 through 51 show the deviations.
estimates of gamma distribution parameters calculated using the methods of Part II, Section 4-c-ii.

Figures 47 through 53 show the resulting histograms for the continents. Table XII summarizes the statistics of the distributions. Of the continents with large sample size, South America stands out from the others. The parameter $a$, describes the rise of the gamma curve and the $\beta$ parameter controls the decline of the right tail of the distribution. Thus, South America increases less steeply and decreases more slowly than the others. This produces a higher mean length and higher variance. The harmonic means (usually a better measure of central tendency in skewed distributions) are all approximately equal, however.

The length distribution of lineaments will be affected by map "style" more than other measures; thus the differences for South America can be attributed to the fact that most of the information for this continent was obtained from geological maps, whereas tectonic maps formed the primary source for the other continents.

All of the histograms clearly show a similarity to their maximum likelihood fitted gamma densities. The mean lineament length for the separate continents vary from about .06 to .10 earth radii; harmonic means are in the vicinity of .06 earth radii. Variance of the distributions are between .003 and .013.

The relative closeness of fit is compared by $\chi^2$ statistics in Table XIII. All except Arabia differ significantly from the best-fit gamma distribution. The lower portion of each of figures 47 through 53 shows the deviation.

Table XIV summarizes the main positive deviation of the length distribution, showing the pattern which is apparent in the diagrams. Particularly important would
Fig. 47. Histogram of lengths of lineaments of Africa. (length in Earth radii)
Fig. 48. Histogram of lengths of lineaments of South America. (length in Earth radii)
Fig. 49. Histogram of lengths of lineaments of Eurasia. (length in Earth radii)
Fig. 50. Histogram of lengths of lineaments of North America. (length in Earth radii)
Fig. 51. Histogram of lengths of lineaments of Australia.
(length in Earth radii)

Fig. 52. Histogram of lengths of lineaments of India.
(length in Earth radii)
Fig. 53. Histogram of lengths of lineaments of Arabia. (length in Earth radii)
<table>
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### TABLE XIV

**MAXIMA OF LENGTH DISTRIBUTION**

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<td>.05</td>
<td>.08</td>
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<td>.22</td>
</tr>
</tbody>
</table>

It is clear from the foregoing chapters that the distributions of various tectonic elements on continents cannot be considered to closely follow a Poisson-random model. In order to test hypotheses concerning the pattern of tectonic elements in a particular region against a global model, it would be useful to have generalized probability density functions, even if only approximate. The subject of this section is to present some generalized probability density functions for continental lineament densities, intersection angles and lengths. The densities have been approximated by inspection of cumulative histograms of all the continents. There is no rational method of testing the "reality" of such an approximation, so each time a $x^2$ test was performed to ensure that the observed histograms could be reasonably expected from the predicted density. There are an infinite number of densities for which this would be true.

Figure 34 shows the composite length distribution for all continents, each has been rotated to its position of nearest correlation, approximating Africa to be equidistant. The result is a distribution which is highly symmetrical.
appear to be lineament lengths of .08, .16 and .24 earth radii. The very long lineaments tend to be centered at about 0.44. This agrees well with the findings of Rickard (1967, figure 1) who found a multimodal distribution of lineament lengths with peaks at .18, .23 and .47 earth radii.

Whereas, there may be no relationship between the present earth radius and the pattern in the length distribution, it is clear that there are significant preferred lengths, and these lengths form a pattern which cannot be explained by purely random processes.

7) ESTIMATION OF PROBABILITY DENSITY FUNCTIONS OF CONTINENTAL LINEAMENTS

It is clear from the foregoing chapters that the distributions of various tectonic elements on continents cannot be considered to closely follow a Poisson-random model. In order to test hypotheses concerning the pattern of tectonic elements in a particular region against a global model, it would be useful to have generalized probability density functions, even if only approximate. The object of this section is to present some generalized probability density functions for continental lineament azimuths, intersection angles and lengths. The densities have been approximated by inspection of cumulative histograms of all the continents. There is no rational method of testing the "reality" of such an approximation. In each case a $\chi^2$ test was performed to ensure that the observed histogram could be reasonably expected from the proposed density. There are an infinite number of densities for which this would be true.

Figure 54 shows the composite azimuth distribution for all continents. Each has been rotated to its position of maximum correlation, considering Africa to be immobile. The result is a distribution which is highly symmetrical
Fig. 54. Total azimuth histogram for all continents.
in appearance and has maxima at $32^\circ$, $58^\circ$, $88^\circ$, $127^\circ$ and $150^\circ$, with a lesser peak at $2^\circ$. The apparent $30^\circ$ spacing is very suggestive and a possible probability density function is shown in figure 55.

Figure 56 is a cumulative histogram of lineament intersection angles for all continents. As previously noted, angles of $30^\circ$ and $60^\circ$ dominate. Figure 57 shows an idealized probability density function for continental intersection angles.

The cumulative total histogram for potential intersections is shown in figure 58. The regularly spaced preferred intersections observed in the individual plots are emphasized here. Figure 59 shows one possible idealized density function for potential intersection angles.

Figure 60 represents the total lineament length distribution. The best-fit gamma distribution has parameters $\alpha = 2.03$ and $\beta = .040$. The proposed ideal density function consists of this best fit curve with additional small normal distribution components centered at $.08$, $.16$, $.24$ and $.48$ earth radii (figure 61).

Other statistics are: mean length $.082$ earth radii, variance $.0048$ and harmonic mean $.063$. If the distribution is standardized (converted to unit mean) the parameters are $\alpha = 2.03$ and $\beta = 0.492$. The fact that these are nearly "round numbers" is probably of no particular significance.

8) APPLICATION TO OCEANIC LINEAMENTS: An Example

The tectonics of the ocean bottoms is almost entirely deduced from bathymetry and geophysical anomalies. Because of this, the detailed information available for the continents cannot be duplicated for the oceans. The methods presented in this thesis can be applied equally well to this type of information.
Fig. 55. Possible probability density function of azimuths of lineaments.
Fig. 56. Total intersection histogram for all continents.
Fig. 57. Possible probability density function for intersection angles.
Fig. 58. Total potential intersection angle histogram for all continents.
Fig. 59. Possible probability density function for potential intersection angles.
Fig. 60. Total lineament length distribution for all continents. (length in Earth radii)
Fig. 61. Possible probability density function of lineament lengths. (Earth radii)
As an example, consider the lineaments of the North Pacific Ocean. Figure 62 shows their azimuth distribution. The pattern is clearly similar to the continents and appears to be rotated about 20° westward with respect to North America.

Figures 63 and 64 show the intersection angle and potential intersection angle distributions. The same preferred angles appear on these diagrams as on the continents, with a stronger than usual maximum at 60°. Finally, figure 65 shows the length distribution. Gamma distribution parameters are $\alpha = 2.46$ and $\beta = 0.42$. The longer preferred lengths are at 0.18, 0.22, 0.32 and 0.44 earth radii. This is again similar in many respects to the lengths of continental lineaments. Mean length is higher, being 0.20 earth radii.

9) APPLICATION OF THE VORONOI POLYGONS - AFRICA

In Part II, Section 5, stochastic models to describe the distribution of polygonal elements were described. Of application to tectonics are two models - the Poisson polygons, to represent random compressional failure and the Voronoi polygons representing random extensional failure (and nucleation about random centres).

The polygonal outlines of the tectonic elements of Africa are well documented. Figure 66 shows one possible polygonalization of Africa based on the tectonic map of Africa. The result, of course, is a rather subjective polygonalization. The method of quantifying the coordinates of the polygons was described in a previous section. Figure 67 shows a histogram of the number of sides of the polygons, with the Voronoi expected frequencies (from Table IV) superimposed. The mean number of sides is 5.4. The absence of triangles and relatively large numbers of many-sided areas completely rules out the Poisson polygons as a model. The $\chi^2$ statistic for goodness-of-fit is 5.6 with 3 degrees of
Fig. 62. Azimuth distribution of North Pacific lineaments.
Fig. 63. Intersection angle histogram of North Pacific Ocean.
Fig. 64. Potential intersection angle histogram of North Pacific Ocean.
Fig. 65. Histogram of lineament length, North Pacific Ocean.
Fig. 66. A polygonalization of Africa.
Fig. 67. Histogram of the number of sides of African polygons.

Fig. 68. Area histogram of African polygons.
freedom. Thus, the Voronoi model cannot be rejected on the basis of the number of sides. Figure 68 compares the expected and observed area distribution of the African polygons. The agreement here is even better with a $\chi^2$ statistic of 4.39 for 3 degrees of freedom. Similarly, the Voronoi model cannot be rejected on the basis of the perimeter distribution (figure 103) which yields a $\chi^2$ statistic of 4.82. Standardization of the curves was performed by converting to a unit mean area.

Recalling the expected area and perimeter distribution for Poisson polygons (figures 8 and 9), it is clear that this model must be rejected in favour of the Voronoi model. It should be emphasized that the azimuth distribution of the lineaments of Africa showed positive signs of non-uniformity. Thus, although the Voronoi polygons model the shape and size distribution of the African elements, the polygons appear to have preferred orientations.

8) THE POLYGONALIZATION OF THE EARTH'S SURFACE

Although the writer has not attempted an explicit polygonalization of the continents as was done for Africa, certain of the observations made concerning the linear elements can be studied in the context of random polygons.

For each of the continents the intersection angle distributions were compared to the expected Poisson polygon frequencies. In all cases the agreement was very poor. In general there was an excess of small intersection angles and a deficiency of larger angles.

In order to consider the Voronoi polygons, recall that in defining the "intersection angle" of two lineaments, the minimum interior angle was taken so that, by definition, the angle cannot exceed $90^\circ$. In the case of the Voronoi polygons, the lines do not "intersect", but meet at nodes. The intersection angle distribution of the extended sides of the Voronoi polygons is not known explicitly, but certain
restrictions can be placed on it. It is clear from the six-sided nature of these polygons that the most common intersection angle will be around $120^\circ$ and thus, the maximum of the external intersection angle distribution will be at $60^\circ$. It is also true that angles of $0^\circ$ and $180^\circ$ will have zero probability, thus the intersection angle distributions will be zero in the origin. Such a distribution will therefore be higher for angles greater than $60^\circ$ than the corresponding Poisson curve and will inevitably fit these observations more closely. Figure 69 shows an approximate intersection angle density function for the Voronoi polygons based on the above reasoning, combined with results from the Monte-Carlo simulation, with the generalized probability density function for continental intersections superimposed. The lower curve shows the relative deviation. It is inevitable that there will be a series of fairly regularly spaced maxima in the observed intersection angle distributions, due to the preferred orientations visible in the azimuth distribution. This will result in the preferred intersection of $30^\circ$ and its multiples and submultiples which appear in the lower portion of figure 69.

In future work the writer hopes to pursue further the study of the statistical geometry of tectonic regions in terms of areas and perimeters for all continents and oceans in the manner of the preceding study of Africa. Unfortunately there are insufficient resources to incorporate such work in this thesis.

It is appropriate to this section to note that the areas of the continents themselves are relatively consistent in size and thus, much more closely resemble a small sample drawn from the Voronoi polygons rather than the Poisson polygons.
Fig. 69. Approximate intersection angle density function for Voronoi polygons compared with continental observations.
PART IV

SUMMARY AND CONCLUSIONS

The separation of conclusions and summary of observations is, in the writer's opinion, important, especially in the present arrangement of this chapter. The consequent stated conclusions which follow are naturally based on the observations, but do not always bear a simple one-to-one correspondence to the sequence of observations.

a. The examples of distributions of line segments produced under conditions of rather general randomness can be well represented by the family of gamma distributions.

b. Monte-Carlo estimates of the frequency of random polygons caused by Poisson random lines have been determined and were shown in Tables IX and Figures 9 and 10.

c. Monte-Carlo estimates of the frequency of random vertical polygons have been determined and shown in Table X and Figures 11 and 12.

d. The azimuth distributions of groups of occurrences from Africa are statistically indistinguishable and are apparently independent of ventures of topographic exposition. The observation samples from the distinct types which consist of straight segments of finite length, trends and coarse faults can be considered to have come from the same population. Similarly, a subjective migration factor applied to the occurrences has no effect on the distribution (Figures 11 to 18, Tables V to VIII).

e. The azimuth distributions of all continents and many median of the ocean floor studied, showed a normal and constant variation and cannot be considered to be uniformly distributed (Table VIII).
1) SUMMARY OF OBSERVATIONS

The separation of conclusions and summary of observations is, in the writer's opinion, important, resulting in the present arrangement of this chapter. The sequentially lettered conclusions which follow are naturally based on the observations, but do not always bear a unique one-to-one correspondence to the sequence of observations.

a) The length distributions of line segments produced under conditions of rather general randomness can be well represented by the family of gamma distributions.

b) Monte-Carlo estimates of the frequencies of random polygons caused by Poisson random lines have been determined and were shown in Table III and figures 8 and 9.

c) Monte-Carlo estimates of the frequencies of random Voronoi polygons have been determined and shown in Table IV and figures 11 and 12.

d) The azimuth distributions of groups of lineaments from Africa are statistically indistinguishable and are apparently independent of tectonic or topographic expression. The lineament samples from the distinct groups which consist of straight segments of rivers, structural trends and large faults can be considered to have come from the same population. Similarly, a subjective weighting factor applied to the lineaments has no effect on their distribution (figures 13 to 19, Tables V and VI).

e) The azimuth distributions of all continents and those portions of the ocean floor studied, showed preferred orientations and cannot be considered to be uniformly distributed (Table VIII).
f) Most azimuth distributions show two dominant directions, separated by about 60° and less prominent maxima at 30° intervals. Each continent has its own peculiarities, however. Specifically, the African distribution is more uniform than most, North and South America are very similar and both have pronounced symmetry, Australia is dominantly bimodal, and the Eurasian distribution is more irregular and shows signs of being a mixture of two populations (figure 30).

g) The intercontinental similarities between lineament azimuth distributions are strong enough to suggest relative rotations between the distributions. Numerical methods were used to estimate these rotations and they are in good agreement with rotations based on continent-fitting by geometrical and geological criteria. Apparent rotations based on lineament patterns were shown in Table IX.

h) Lineament patterns displayed on classical continental reconstructions form an apparently continuous pattern.

i) Intersection and potential intersection angle distributions from the various regions deviate from a Poisson random model, and show similarities to each other. Their deviations from the model show consistent preferences for angles of about 7°, 15°, 22°, 30°, 45° and 60°, superimposed on an overall trend exhibiting an excess of angles greater than 45° (figures 33 to 46).

j) The histograms of the lengths of lineaments closely resemble gamma densities, but deviate significantly from such distributions in a regular manner. The mean value of the α parameter for all continents is 2.03 and β is 0.0402. Mean lineament length is .082 earth radii. Separate histograms show some preferred lineament lengths, particularly at .08, .16, .24 and .48 earth radii and their maxima are consistent from continent to continent (figures 47 to 53, Tables XII to XIV).
k) The geometrical statistics of the North Pacific Ocean are similar to the continents. The azimuth distribution compares favourably in outline with that for North America and shows a relative rotation of about 20° (Pacific rotated westwards). Pacific intersection angle distributions relate well to the equivalent continental distributions, but show a higher than average maximum at 60°. Lineament length statistics are also similar and preferred lengths are .18, .22, .32 and .44. The oceanic lineaments are, on the average, larger than on the continents (figures 62 to 65).

l) The Voronoi polygons form a statistically reasonable model for the side, perimeter and area distributions of the African polygons. Mean number of sides of African polygons is 5.4. The polygons, however, are not randomly oriented (figures 66 to 68).

m) The angular relationships between linear elements on the earth's surface correspond to that expected from random Voronoi polygons rather than to that expected from Poisson random polygons. The arrangement of lineaments and polygons on the globe cannot be considered to be uniformly random from the point of view of either model, as there are significant preferred intersection angles (figure 69).

2) CONCLUSIONS

a) The probability density function of line segments whose existence is wholly stochastically dependent on a uniform Poisson point process is the gamma density.

b) Smooth lines through the histograms of figures 8 and 9 are close approximations to the probability density functions of the perimeter and area of random Poisson polygons. The discrete probabilities of the various-sided polygons are closely approximated by the figures of Table III.
c) Smooth lines through the histograms of figures 11 and 12 are close approximations to the probability density functions of the perimeter and area of random Voronoi polygons. The discrete probabilities of the various-sided polygons are closely approximated by the figures of Table IV.

d) In studying terrestrial lineaments, the researcher is justified in considering all lineaments to be of equal weight, without regard to their tectonic or topographic expression:

e) The azimuth and intersection angle distributions of all continents are similar, indicating a similarity in the origin of these patterns. The patterns are apparently contiguous when continents are placed in their supposed Paleozoic position. This would suggest that the observed lineament pattern pre-dates the Cretaceous. Obviously, this conflicts with the knowledge that many of the observed lineaments are of post Cretaceous "age" or at least have been active or re-activated since the Cretaceous. This paradox will be considered later in the conclusions.

f) The statistical methods suggested in the thesis are of potential use in determining continental rotations, should other evidence be lacking or inconclusive.

g) The preferred intersection angles show a regularity which can only be explained by a consistent pattern of global forces acting over a long period of time.

h) The lineaments of the earth's surface show preferred lengths of .08, .16, .24 and .48 earth radii. It is logical to conclude that these lengths reflect the sizes of various orders of internal global processes. There is insufficient evidence to further refine this rather vague conclusion at this time.
i) The lineament patterns for the oceans are similar to those for the continents and the continents are rotated with respect to the oceans.

j) The African polygons are sufficiently similar to the Voronoi polygons to suggest that their existence is due to an agglomerative process about random centres, or fracture under extension (or both). Failure by shear under compression cannot account for the observed pattern.

k) The above conclusion appears also to apply to the earth's surface as a whole. The geometrical arrangement of the surface features on the earth more closely resemble that expected from an agglomerative or extension failure process, than from that expected by shear or compressional failure.

l) The lineament patterns of oceans and continents do not form a continuous pattern in their present positions. Within a continent-sized area, however, the patterns are uniform. This supports the current theories that the earth's surface deforms as a series of more or less rigid plates, which deform little within themselves.

m) The consistency of geometrical tectonic patterns from continent to continent, and through time boundaries creates the aforementioned paradox. Because of limited statistics, some earlier workers concluded that a single fracture pattern extended over the whole globe and that the same pattern has been active throughout the Phanerozoic. This forced some to adopt a position contrary to continental displacement (eg. Moody and Hill, 1956; Tanner, 1963), in spite of the weight of circumstantial evidence in favour of the form contiguity of certain continents. The statistics of this work change the nature of the paradox somewhat. It is now clear that there is no globally continuous continental
tectonic pattern, but rather a tectonic pattern for each plate. But still, the patterns appear to be of ancient origin and are presently still active. This would suggest that the dynamic mechanism for creating the pattern (or responsible for keeping it active) moves with the continents. Continental displacement is therefore a deep-seated process. This conclusion agrees with the more recent concepts of ocean-floor spreading, whereby continents are no longer required to "drift" through the upper mantle. The movement of mantle (or mantle forces) with the continents may not be true for continents with extreme rotations, such as Australia, where one could suggest that its rather different pattern is due to a rotation between an old and a new pattern, thereby accidently reinforcing the two obvious preferred directions. This speculation is unsupported by good age relationship data on the lineaments.

n) In many ways the above conclusions support an expanding earth model and certainly rule out the static continent concept. The plate-tectonic model as presently formulated is supported by much of the evidence and is not irreconcilable with any of it. The question of whether the rate of crustal production on ridges matches crustal destruction in trenches is unresolved by this study.

o) The broad generalizations of Rouse and Bisque (1968) concerning small circle tectonic patterns are unsupported and appear to be due to chance alignment of broken pieces of a regular pattern. Because the study of the oceans is incomplete, a regular global pattern for all oceans cannot yet be ruled out.
ACKNOWLEDGEMENTS

A submission for the degree of Doctor of Philosophy must necessarily be a piece of original research performed by the candidate. Such a work cannot be done, however, without the assistance of a large number of people, to whom I am very grateful.

Dr. M.J. Rickard has directly supervised my work and has provided assistance of incalculable value to my research. He allowed me great freedom to pursue lines of investigation which must at times have appeared fruitless, and I am most grateful for his patience. Many of the best attributes of this thesis have arisen through his criticism, and many of the deficiencies, no doubt, have arisen from occasions when I did not follow his advice.

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APPENDIX I

PROGRAM DESCRIPTIONS
Call:

CALL QWT(T,X,R,X,Y,Z)

T = longitude in radians
Y = colatitude in radians
Z = radius vector
X = Cartesian coordinates

Orientation:
- X = 0° 90°
- Y = 90° 0°
- Z = 90° 0°

Method:

f = \( X^2 + Y^2 - K^2 \)
f = \( \cos^{-1} \left( \frac{Z}{K} \right) \)
f = \( \tan^{-1} \left( \frac{X}{Y} \right) \)

with appropriate corrections for the proper quadrant.
Purpose: Conversion of Cartesian coordinates to spherical coordinates

Call:

```
CALL CATS(T,F,R,X,Y,Z)
```

- **T** = longitude in radians \( \theta \)
- **F** = colatitude in radians \( \phi \)
- **R** = radius vector \( Y \)
- **X** = )
- **Y** = ) Cartesian coordinates
- **Z** = )

orientation: \( X = 0^\circ, 90^\circ \)
\( Y = 90^\circ, 270^\circ \)
\( Z = 0^\circ, 180^\circ \)

Method:

\[
\begin{align*}
  r &= X^2 + Y^2 + Z^2 \\
  \phi &= \cos^{-1}(Z/Y) \\
  \theta &= \tan^{-1}(Y/X)
\end{align*}
\]

with appropriate corrections for the proper quadrant
ENTRY POINT  STAC

Purpose:  Conversion of spherical coordinates to Cartesian coordinates

Call:

CALL STAC(T,F,R,X,Y,θ)

Method:

\[ X = Y \sin \phi \cos \theta \]
\[ Y = Y \sin \phi \sin \theta \]
\[ Z = Y \cos \phi \]
SUBROUTINE CATS(T,F,R,X,Y,Z)
DATA PI/3.141592/
C CARTESIAN TO SPHERICAL POLARS
C T AND F IN RADIANS
RETURN TO SPHERICAL POLAR
105 R=SQRT(R)
F=ARCCOS(ABS(Z/R))
104 IF(Y)Z,402,404
402 IF(Z)400,401,403
400 IF(F)360,401,402
360 F=360-F
RETURN
401 F=1.5708
RETURN
C NEWTON-RAPHSON
402 IF(X)501,502,503
501 F=1.5708
RETURN
502 F=1.5708
RETURN
503 IF(Y)510,511,512
510 ADD=1.5708
RETURN
511 ADD=-1.5708
RETURN
600 T=ATAN(ABS(Y/X))
T=T*FACT+ADD
RETURN
100 R=0.
RETURN
C SPHERICAL POLARS TO CARTESIAN
C T AND F ARE IN RADIANS
ENTRY STAC(T,F,R,X,Y,Z)
X=R*COS(F)*COS(T)
Y=R*SIN(F)*COS(T)
Z=R*SIN(T)
RETURN
END
LATS

Purpose: lat. + long. to spherical polars

Call: LATS (A,IA,B,IB,T,F)

A : latitude in radians
IA : N or S
B : longitude in radians
IB : E or W
T : longitude in radians (θ)
F : colatitude in radians (φ)

Method:
θ = B if East longitude
θ = 2π - B if West longitude
θ = π/2 - A if North latitude
θ = π/2 + A if South latitude

ENTRY POINT

STAL

Purpose: spherical polars to lat & long

Call: CALL STAL (A,IA,B,IB,T,F)

same as above.

Method:
B = θ, E if θ < π
B = 2π - θ, W if θ > π
A = π/2 - φ, N if φ < π/2
A = φ - π/2, S if φ > π/2
SUBROUTINE LATS(A, IA, B, IB, I, F)
DATA IEF, IEN, IEW, IES/ 'E', 'N', 'W', 'S'/
C A=LATITUDE
C B=LONGITUDE
C F=COLATITUDE
C ALL ANGLES IN RADIANS
C PI=3.1415927
IF(I=1)GO TO 101,100,101
100 T=1
GO TO 105
101 T=2*PI-B
105 IF(IA-IEN)107,106,107
106 F=PI/2.-A
GO TO 108
107 F=PI/2.+A
108 RETURN
C SPHERICAL POLARS TO LAT AND LONG
ENTRY STA(LA,IA,LA,IA,T,E)
PI=3.1415927
IF(T-PI)209,209,201
200 B=1
IF(IEE)GO TO 205
201 B=2.*PI-T
IF(IEW)GO TO 205
205 IF(T-P1/2.)206,205,207
207 A=F-PI/2.
IA=IES
RETURN
206 A=PI/2.-F
IA=IEN
RETURN
END
DEG (FUNCTION)

Purpose: radians to degrees

Call: \( D = \text{DEG}(R) \)

\( R: \) radians  \( D: \) degrees

Method: \( R = \frac{D \times \pi}{180} \)
FUNCTION DEG(A)
DEG=A*57.295780
C RADIANS TO DEGREES
RETURN
END
RAD (FUNCTION)

Purpose: degrees to radians

Call: $R = \text{RAD}(D)$

$R$: radians  $D$: degrees

Method: $R = \frac{D \times \pi}{180}$
FUNCTION RAD(A)
RAD = A / 57.295760
RETURN
END
D&T

(FUNCTION)

Purpose: dot product of two 3-vectors (Cartesian)

Call: D&T(A,B)
A, B are 3-vectors (DIMENSION A(3), B(3) )
C is resultant dot product C = A.B

Method: \[ C = a_1b_1 + a_2b_2 + a_3b_3 \]
FUNCTION DOT(A, B)
C DOT PRODUCT
DIMENSION A(3), B(3)
DOT = A(1) * B(1) + A(2) * B(2) + A(3) * B(3)
RETURN
END

BEGIN

DOT = a1*b1 + a2*b2 + a3*b3

RETURN
CRSS

Purpose: cross product of two 3-vectors

Call: LALL CROSS (A, B, C)
C = A x B  (DIMENSION (A(3), B(3), C(3)) )

Method:

\[ C_1 = a_2 b_3 - a_3 b_2 \]
\[ C_2 = a_3 b_1 - a_1 b_3 \]
\[ C_3 = a_1 b_2 - a_2 b_1 \]
SUBROUTINE CROSS(A,B,C)
DIMENSION A(3),B(3),C(3)
C CROSS PRODUCT OF TWO 3-VECTORS
C C=A X B
C C(1)=A(2)*B(3)-A(3)*B(2)
C C(2)=A(3)*B(1)-A(1)*B(3)
C C(3)=A(1)*B(2)-A(2)*B(1)
RETURN
END

BEGIN
C1 = a2 b3 - a3 b2
C2 = a3 b1 - a1 b3
C3 = a1 b2 - a2 b1
RETURN
VAB (FUNCTION)

Purpose: absolute value of 3-vector

Call: A = VAB(B)

B: is 3-vector (DIMENSION B(3))
A: is absolute value of B

Method: \[
A = /B/ = \sqrt{b_1^2 + b_2^2 + b_3^2}
\]
FUNCTION \textsc{vab}(\textsc{a})
\begin{align*}
\text{DIMENSION } \textsc{a}(3) \\
\text{C ABSOLUTE VALUE OF 3-VECTOR} \\
\textsc{vab} &= \text{Sqrt}(\textsc{a}(1) * \textsc{a}(1) + \textsc{a}(2) * \textsc{a}(2) + \textsc{a}(3) * \textsc{a}(3)) \\
\text{RETURN} \\
\text{END}
\end{align*}
NORM

Purpose: to normalize a 3-vector

Call: CALL NORM(A)

A: a 3-vector (DIMENSION A(3) )

Method: \[ a_1 = \frac{a_1}{|A|} \quad a_2 = \frac{a_2}{|A|} \quad a_3 = \frac{a_3}{|A|} \]

Error Message: if \(|A|\) equals zero, message "NULL VECTOR NORM" is printed.
SUBROUTINE NORM(A)
DIMENSION A(3)
D=A(1)*A(1)+A(2)*A(2)+A(3)*A(3)
IF(D).GT.100.0,100,200
200 D=SQR(D)
A(1)=A(1)/D
A(2)=A(2)/D
A(3)=A(3)/D
RETURN
100 WRITE(3,201)
201 FORMAT(17H NULL VECTOR NORM)
RETURN
END
DET (FUNCTION)

Purpose: to calculate the determinant of a 3 x 3 array

Call: D = DET(A)

where A is a 3 x 3 array

Method: D = a_{11}(a_{22}a_{33} - a_{23}a_{32})

+ a_{12}(a_{23}a_{31} - a_{21}a_{33})

+ a_{13}(a_{21}a_{32} - a_{22}a_{31})
FUNCTION DET(A)
DIMENSION A(3,3)
D=A(1,1)*A(2,3)*A(3,2)-A(2,1)*A(3,3)
E=A(1,2)*A(2,3)*A(3,1)-A(2,1)*A(3,2)
F=A(1,3)*A(2,1)*A(3,2)-A(2,2)*A(3,1)
DET=D+E+F
RETURN
END
**SMIT**

**Purpose:**
to calculate the x-y coordinates of a point on a Schmidt net, given the spherical coordinates of the point.

**Call:**
SMIT (T,F,X,Y,ITB,R)

- T = colatitude of point ($\theta$)
- F = longitude of point ($\phi$)
- R = desired radius of net
- X = )coordinates of point (returned)
- Y = )

**Method:**

$$x = \sqrt{2} R \sin (\phi/2) \sin \theta$$

$$y = \sqrt{2} R \sin (\phi/2) \cos \theta$$

**ITB = Hemisphere indicator**

- 1 = upper
- 2 = lower
SUBROUTINE SM1(F,X,Y,II,IB,K,IS,FS)
C STEREOGRAPHIC PROJECTION
DATA PI,R2,3.141592,1.4142136/
IF(F-PI/2.)100,100,200
100 IT=1
X=R2*RSIN(F/2.)*SIN(T)
Y=R2*RSIN(F/2.)*COS(T)
RETURN
200 IT=2.
C=PI-F
XR=R2*CRSIN(G/2.)
X=RR*RSIN(T)
Y=RR*RCOS(T)
RETURN
END
HIST

Purpose: to accumulate and print histograms

Call: HIST(I,VL,VH,VI)
   I: histogram identification number (1-6)
   VL: lower limit of histogram
   VH: upper limit of histogram
   VI: interval width

This Call initializes the histogram. VI should be such that the number of intervals is less than 100.

Entry Points: HISTPM (I,V,IW) - adds a value to the histogram
   I: as above
   V: value to be added to histogram
   IW: value to be added IW times

HISTPT (I) - prints out histogram

Error messages:

"HISTOGRAM I HAS NOT BEEN INITIALIZED"

This message will appear if a call to HISTPM(I) is not proceeded by a call to HIST to initialize the histogram.
ENTRY: HIST

SET ROW ELEMENTS BLANK
SET COUNTER FOR C% HISTOGRAM TO ZERO

STORE UPPER AND LOWER LIMITS AND WINDOW WIDTH (UM, VL, VI)

CALCULATE NUMBER OF WINDOWS (NV(L))

IF NV(L) > 100 THEN

SET INITIALIZATION FLAG

RETURN

ENTRY: HISTFM

DO THIS HISTOGRAM IS READY INITIALIZED?

YES

PRINT MESSAGE

NO

RETURN
2.

VLim = 0
VI = increase
VL = lower limit

Calculate # of pts. in first window
Print first row of histogram
Add to total

Calculate # of pts. in next window
Print next row
Add to total

All rows finished?

Print total, scale

Return
SUBROUTINE HIST(L, VLOW, VHIGH, VINT)
   DIMENSION HIST(6, 100), IRCH(100), VL(6), VF(6), VI(6), NV(6), INIT(6)
   DIMENSION XPT(100)
   DATA INIT/0.0, 0.0, 0.0, 0.0, 0.0, 0.0/ 
   DATA LAST, 1, 1.0/ 
   DO 40 L = 1, 100
   IRCH(L) = 100
   30 IF(SUM( == 0)
      VL(L) = VLOW
      VF(L) = VHIGH
      VI(L) = VINT
      NV(L) = (VHIGH-VLOW)/VINT+2.0001
      IF(NV(L)-100) = 60, 60, 61
   61 NV(L) = 100 
   60 INIT(L) = 999
   RETURN
   ENTRY HISTP(L, V, I)
   IF(INIT(L)-V, 62, 63
   63 WRITE(6, 400)
   400 FORMAT(1H, 'HISTORAM', I4, ' HAS NOT BEEN INITIALIZED'
   RETURN
   62 NV = NV(L)
   VL = VL(L)
   VF = VF(L)
   VI = VI(L)
   IF(V-V) = 40, 40, 41
   41 HIST(L, NV) = HIST(L, NV)+I 
   RETURN
   40 IF(V-VW) = 43, 43, 42
   42 HIST(L, L) = HIST(L, L)+I 
   RETURN
   41 VW = (V-VL)/V 
   IV = V+2* 
   HIST(L, IV) = HIST(L, IV)+I 
   RETURN
   ENTRY HISTPT(L)
   MAX = C
   ITOT = C
   NV = NV(L)
   DO 44 I = 1, NV
      HIST(L, I) = MAX)-MAX)+44, 44, 100
   100 MAX = HIST(L, I)
   44 CONTINUE
   IF(MAX-100) = 33, 33, 32
   32 PMAX = MAX
   SCALE = MAX/100 
   GO TO 34
   33 SCALE = 1 
   34 VLIM = C
   VIM = VI(L)
   VL = VL(L)
   ILEN = ILEN(L, I)/SCALE
   IF(ILEN) = 79, 79, 8 
   79 DO 61 J = 1, ILEN
   61 IRCH(J) = LAST
   79 WRITE(3, 101) VLIM, IRCH, HIST(L, I)
   101 FORMAT(1H, 1J, 9, 2X, LOCAL1, IX, I4)
   ITOT = ITOT+HIST(L, I)
   VLIM = VLIM-V 
   DO 70 I = 2, NV
      VLIM = VLIM-V 
   DO 86 K = 1, 100
   86 IRCH(K) = 100
   ILEN = ILEN(L, I)/SCALE
   ITOT = ITOT+HIST(L, I)
   IF(ILEN) = 70, 70, 77
   77 DO 47 J = 1, ILEN
   47 IRCH(J) = LAST
   77 WRITE(3, 101) VLIM, IRCH, HIST(L, I)
   WRITE(3, 100) SCALE, ITOT
   WRITE(3, 106) SCALE FACTOR, F1C.5/J10, 'TOTAL SAMPLE', I6)
49. IROW(1)=13
RETURN
ENTRY CHISQ(L,XPI,N1,N2,X2,NDF)
X2=0
NDF=1
EXPT=0
FHIST=0
DO 500 I=1,N2
FHIST=FHIST+FHIST(L,I)
EXPT=EXPT+XPT(I)
IF(EXPT-5.0)<0.050,503,507
503 NDF=NDF+1
X2=X2+(FHIST-EXPT)*2/EXPT
EXPT=0
FHIST=0
500 CONTINUE
RETURN
END

Recursion relation: f(A) = 1 + f(A-1)

Reference: Morse & Feshback, 1953, p.422.

Error messages:

If A is less than or equal to zero, the message
"NEGATIVE ARGUMENT IN DLGA" is printed, followed by the
value of the offending argument.

DLGA is set equal to zero.
DLGA (FUNCTION)

Purpose: to calculate the logarithmic derivative of the gamma function \( \frac{\partial \ln \Gamma(A)}{\partial A} \)

Call: \( G = \text{DLGA}(A) \)

Method: Series expansion

\[
f(A) = \ln \Gamma(A) = -0.5772157 - \frac{1}{A} + \sum_{i=1}^{\infty} \left( \frac{1}{i} - \frac{1}{i+A} \right)
\]

Recursion relation: \( f(A) = \frac{1}{A-1} + F(A-1) \)

Reference: Morse & Feshback, 1953, p.422.

Error messages:

If A is less than or equal to zero, the message "NEGATIVE ARGUMENT IN DLGA" is printed, followed by the value of the offending argument.

DLGA is set equal to zero.
FUNCTION DLGA(A)
DATA 6.5772157/
IF(A>100,109,101
101 Z=A
ADD=0.
104 IF(Z>1.)102,102,103
103 ADD=ADD+1./Z
Z=Z-1.
GO TO 104
102 N=3000.×Z
N=N+1
S1=0.
S2=0.
DO 106 1=1,N
FI=1
S1=S1+1./FI
106 S2=S2+1./FI+Z
AN=AN+1./Z+S2
AP=ADD+S1
DLGA=AP-AN
RETURN
100 WRITE(3,200)A
200 FORMAT('NEGATIVE ARGUMENT IN DLGA 'E16.7)
DLGA=0.
RETURN
END
ORP

Purpose: To calculate orthographic projection in coordination with ORTHO.

Call: CALL $RP(XT,YT,ZT,XS,YS,TS,FS)

$RP

XT) true Cartesian coordinates of point to be projected
YT) spherical coordinates of centre of projection.
ZT)
TS)
FS)
XS) X, Y coordinates of projected point on unit sphere in orthographic projection.
YS)

Method:

$S = 2\pi - TS
\phi_S = \pi - FS
X_S = X_s \cos \theta_s + Y_s \sin \theta_s
Y = -X_s \sin \theta_s + Y_s \cos \theta_s
Y_S + Z + \sin \phi_S = Y_s \cos \phi_S

Reference: (Szava-Kovats, 1964, 59).

Subprograms required: COS, SIN

Error messages: None.
SUBROUTINE ORP(YT,XT,ZT,XM,YM,RS,GS)
DATA PI/3.1415926/
TS=2.*PI-KS
FS=PI-GS
XM=XT*COS(TS)+YT*SIN(TS)
Y=(-1.)*XT*SIN(TS)+YT*COS(TS)
YM=ZT*SIN(FS)+YT*COS(FS)
RETURN
END

BEGIN

θ' = 2·π - θ
φ' = π - φ
XM=XT*COS(θ')
   +YT*SIN(θ')
Y=-XT*SIN(θ')
   +YT*COS(θ')
YM=ZT*SIN(φ')
   +YT*COS(φ')

RETURN
Purpose: to find radius and centre of circle passing through 3 points on a sphere.

Call: CALL CIRCL (TT,FF,CENT,ANG)
TT: Array (3) of θ's of 3 points
FF: Array (3) of φ's of 3 points
CENT: calculated centre of circle (θ,F)
ANG: Angular radius (angle subtended at centre of sphere)

All angles in radians.

Requires: STAC, CRSS, NRM, DT

Method:

a) Convert three points to Cartesian coordinates.
b) Obtain 2 vectors A, B which join any two points.
c) Find cross product vector C
d) Normalize C
e) Normalized C₁ and C₂ are θ and φ of centre when converted to sphericals.
f) Find angle between centre vector, and one radius vector to any one of the points - this is the angular radius of the circle.
BEGIN

CONVERT 3 PTS TO CARTESIANS
\(x_1, y_1, z_1\)
\(x_2, y_2, z_2\)
\(x_3, y_3, z_3\)

FORM 2 VECTORS IN PLANE OF CIRCLE
\(a = (x_1-x_3, y_1-y_3, z_1-z_3)\)
\(b = (x_2-x_3, y_2-y_3, z_2-z_3)\)

\(\varepsilon = a \times b\) Normalise \(\varepsilon\)

CONVERT CIRCLE TO SPHERICALS
Yielding centre \(\Omega\) of circle
\(\Theta, \Phi\)

\(d = x_1, y_1, z_1\)
NORMALIZE \(d\)
\(\text{ANG} = \cos^{-1}(d \cdot \varepsilon)\)

\(\text{ANG} > \pi/2\) YES
\(\text{ANG} = \pi - \text{ANG}\)

RETURN
SUBROUTINE CIRCLE(XI, XI, CENTER, ANG)
DIMENSION A(3), B(3), C(3), XI, XI, FF(3), CENTER(3), PR(3)
DIMENSION XX(3), YY(3)
DIMENSION ZZ(3), AA(3)

C TO FIND CENTRE AND RADIUS OF CIRCLE

10 CALL STAC(TI(1),XI(1),PR(1),XX(1),YY(1),ZZ(1))

C CONVERT TO CARTESIAN

A(1)=XX(1)-XX(2)
A(2)=YY(1)-YY(2)
A(3)=ZZ(1)-ZZ(2)
B(1)=XX(2)-XX(3)
B(2)=YY(2)-YY(3)
B(3)=ZZ(2)-ZZ(3)

CALL CROSS(A,B,C)

CALL NORM(A)

CALL CATS(TC,FC,RC,C(1),C(2),C(3))

CENTER(1)=TC
CENTER(2)=FC
CENTER(3)=1

D(1)=XX(1)
D(2)=YY(1)
D(3)=ZZ(1)

CALL NORM(D)

ANG=ARCOS(DOT(D,C))
PI=3.141592

IF(ANG-P1/2.)*336.337,337

337 ANG=P1-ANG

CENTER(1)=CENTER(1)+PI

IF(CENTER(1)-2.*PI1338,338,339

339 CENTER(1)=CENTER(1)+PI

340 CENTER(2)=PI-CENTER(2)

346 CONTINUE

RETURN

END
LENGTH

Purpose: to find the great circle length between two points on a sphere.

Call:

CALL LENGTH (T1,F1,T2,F2,ALEN)

T1,F1; 6,¢ of first point  ) radians
T2,F2; 6,¢ of second point  ) radians
ALEN:  calculated distance in radians

Requires: STAC, NORM, D¢T

Method:

a) Convert 61,¢1; 62,¢2 to Cartesian A, B

b) d = arcos \[ \frac{A \cdot B}{|A||B|} \]
BEGIN

CONVERT \( \theta_1, \phi_1 \) to \( a \)

CONVERT \( \theta_2, \phi_2 \) to \( b \)

\[ b = \cos \left( \frac{\theta - \phi}{2\|a\|} \right) \]

RETURN
SUBROUTINE LENGTH(T,F,U,G,D)
C GREAT CIRCLE LENGTH BETWEEN TWO POINTS (IN RADIANS)
DIMENSION A(3), B(3)

CALL STAC(T,F,R,A(1),A(2),A(3))
CALL STAC(U,G,R,B(1),B(2),B(3))
CALL NORM(A)
CALL NORM(B)
D=ARCCOS(DOT(A,B))
RETURN
END

ALLEN: Calculated length

Requires: STAC, NORM, DOT.

Methods:

a) Convert $\theta$, $\phi$ to $x_1$, $y_1$, $z_1$.

b) $x = \text{RIN}(\text{ANG})$

c) $\theta = \text{RIN}(\text{ANG})$

d) form vector

$\mathbf{a} = (x_1-x_2)(y_3-y_2) - (y_1-y_2)(x_3-x_2)$

$\mathbf{b} = (x_1-x_2)(y_3-y_2) - (y_1-y_2)(x_3-x_2)$

e) $\cos D = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$

By ALLEN = TD
ARCLEN

Purpose: small circle arc length between two points on a circle of given radius and centre.

Call: ARCLEN(T1,F1,T2,F2,TC,FC,ANG,ALEN)
T1,T2,F1,F2 as before
ANG, TC, FC, centre of circle
ALEN: calculated length

Requires: STAC, NRM, D*T.

Method:

a) Convert $\theta_1, \phi_1$ to $X_1, Y_1, Z_1$,
$b) r = \sin(\text{ANG})$
$c) d = \cos(\text{ANG})$
$\theta_2, \phi_2$ to $X_2, Y_2, Z_2$,
$\theta_c, \phi_c$ to $X_3, Y_3, Z_3$,
$d) \text{form vectors}$
$A: (X_1-X_3), (Y_1-Y_3), (Z_1-Z_3)$
$B: (X_2-X_3), (Y_2-Y_3), (Z_2-Z_3)$
$e) \beta = \cos^{-1}\left(\frac{A \cdot B}{|A| |B|}\right)$
$f) \text{ALEN} = r\beta$
BEGIN

\( \theta_1, \theta_2 \rightarrow x_1, y_1, z_1 \)
\( \theta_3, \theta_4 \rightarrow x_2, y_2, z_2 \)
\( \text{Radius (RSS)} = \sum_{i=1}^{n} \sqrt{(x_i - x_{\text{avg}})^2 + (y_i - y_{\text{avg}})^2} \)

\( D = \cos(\theta_\text{avg}) \)
\( \theta_2, \theta_3 \rightarrow x_3, y_3, z_3 \)
\( \Delta = x_1 - x_3, (y_1 - y_3), (z_1 - z_3) \)
\( \beta = (x_2 - x_3, (y_2 - y_3), (z_2 - z_3)) \)

Normalize \( \Delta \) and \( \beta \)
\( \beta = \text{Lor}^{-1}(\Delta, \beta) \)
\( \text{LENGTH} = \sqrt{\text{RSS} \times \beta} \)

RETURN
SUBROUTINE ARCLEN(T1,F1,T2,F2,TC,FC,ANG,ALEN)
C ARC
LENGTH GIVEN END RADII, CENTRE
DIMENSION X(3),Y(3),Z(3),A(3),P(3)
R=1.
RSS=5IN(ANG)
CALL STAC(T1,F1,R,X(1),Y(1),Z(1))
CALL STAC(T2,F2,R,X(2),Y(2),Z(2))
D=COS(ANG)
CALL STAC(FC,F1,D,X(3),Y(3),Z(3))
M(1)=X(1)+X(3)
M(2)=Y(1)+Y(3)
M(3)=Z(1)+Z(3)
M(4)=X(2)+X(3)
M(5)=Y(2)+Y(3)
M(6)=Z(2)+Z(3)
CALL NORM(A)
CALL NORM(B)
BETA=ARCCS(DTT(A,3))
RETURN
END
GAMD

Purpose: to calculate value of the gamma density

Call: 
G = GAMD(A, B, X)
A = "α" parameter of gamma density
B = "β" parameter of gamma density
X = value of random variable

Method: 
\[ GAMD(A, B, X) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \frac{x^{\alpha-1}}{\beta} e^{-x/\beta} \]
FUNCTION GAMD(A, B, X)
   IF (X) 100, 101, 101
  101   CALL GAMMA(A, B, IER)
      V = X/B
   IF (V < 100) 88, 89, 89
  89   GAMD = 0.0
      RETURN
  88   F = (B ** A) * R
      GAM = X ** (A-1.) * EXP(-1.*V)/F
      GAMD = GAM
      RETURN
  100   GAMD = 0.
      GAMD = GAM
      RETURN
END

BEGIN

GAMD = 0.

YES

X = 0 ?

NO

R = R(A)
V = X/B

YES

V too big ?

NO

GAMD = X ** (A-1) - V / R

RETURN
GAMINC

Purpose: to calculate the incomplete gamma function
\[ G(X,\alpha) = \frac{1}{\Gamma(\alpha)} \int_{1}^{X} t^{\alpha-1} e^{-t} dt \]

Call: \[ G = \text{GAMINC}(X, A) \]
\[ X = x \text{ in above equation} \]
\[ A = \alpha \text{ in above equation} \]

Method: Continued fraction expansion (Abramowitz and Stegun, 1964, p.263)

Program based on CSIRO -Q4 CSIR GAMINC
FUNCTION GAMINC(X, ALPH)
C INCOMPLETE GAMMA FUNCTION
SML=1.0E-30
10 IF (X-SML)110,10,10
10 GALPH=Gamma(ALPH)
1000 GAMINC=1.-EXP(-1.*X)
RETURN
1001 IF (X-1.)20,40,40
20 PROG=X**ALPH/(ALPH*GALMPH)
SUM=PROG
ALPH1=ALPH-1.
DO 30 J=1,11
30 PROG=-X*(ALPH1+F)*PROG/(F*(ALPH+F))
30 SUM=SUM+PROG
GAMINC=SUM
50 TO 100
40 NALPH=ALPH
RESAL=ALPH-NALPH
IF (RESAL)50,50,60
50 RESAL=RESAL+1.
NALPH=NALPH-1
GRAM=Gamma(RESAL)
GAMINC=1.-EXP(-X))/GRAM
50 TO 80
60 GAMINC=1./X
NAIL=40
NAIL1=NAIL+1
ALPH1=ALPH1-RESAL
DO 70 J=1,NAIL
70 GAMINC=1./X+(ALPH1-J)/(GAMINC*(NAIL1-J)+1.)
GAMINC=GAMINC*EXP(-X)*X**RESAL
GAMINC=1.-GAMINC/GRAM
IF (NALPH)100,100,80
80 PROG=1./ALPH
SUM=1.
ALPH1=ALPH+1.
DO 90 J=1,NALPH
90 PROG=PROG*(ALPH1-J)/X
90 SUM=SUM+PROG
GAMINC=GAMINC-EXP(-X)*X**ALPH*SUM/GALMPH
100 IF (GAMINC)110,110,120
110 GAMINC=0.
120 RETURN
END
GAMMA

(FUNCTION)

Purpose: To obtain the gamma function of a number \((\Gamma(a))\)

Method: Calls SSP subroutine GAMMA

Call: \(G = \text{GAMMA}(A)\)

Uses SSP subroutine GMMMA
FUNCTION GAMMA(YY)
CALL GAMMA(YY,X,IER)
GAMMA=X
RETURN
END
GAMINT

Purpose: To calculate the integral from $x_b$ to $x_t$ of the gamma density with parameters $\alpha, \beta$.

Method: 

$$G = \frac{1}{\Gamma(\alpha)\beta^\alpha} \int_{x_b}^{x_t} t^{\alpha-1} e^{t/\beta} \, dt$$

Uses incomplete gamma function subroutine GAMINC.

Call:

CALL GAMINT (P,Q,XB,XT,YY)

P : $\alpha$
Q : $\beta$
XB : $x_b$
XT : $x_t$
YY : result

Method: calls incomplete gamma function GAMINC twice.
SUBROUTINE GAMINT(P, Q, XB, XT, YY)
X = XB / Q
AL = P
Y1 = GAMINC(X, AL)
X = XT / Q
Y2 = GAMINC(X, AL)
YY = Y2 - Y1
RETURN
END
GAMAX

Purpose:
To find the maximum likely-hood estimators of the parameters $\alpha, \beta$ of a gamma density from a set of observations.

Call:
CALL GAMAX(B#, XB, XLB, A, B, D)

B# : first guess at $\beta$
XB : mean value of variable x
XLB: mean log of variable x
A : returned max. likelyhood estimator of $\alpha$
B : returned max. likelyhood estimator of $\beta$
D : minimum accuracy of $\alpha$ (returned)

Method:
Trial and error solution of simultaneous equations

\begin{align*}
\beta &= \frac{x}{\alpha} \tag{1} \\
\log \alpha &= \frac{\beta}{\alpha} \Gamma(\alpha) - \log x \tag{2}
\end{align*}

Subprograms required:
FUNCTION DLGA (Logarithm derivative of $\Gamma$ function)

Error messages:
Should no zero of eq.(2) be found in the range $\alpha_0/3$ to $>\alpha_0/3$ the message "NOT.IN.RANGE" is printed. Estimators returned in this case are erroneous.
BEGIN

\[ x_0 = \sqrt{b_0} \]
\[ t_0 = x_0 / 10 \]
\[ A_0 = x_0 / 3 \]

\[ A = A_0 + D \]
\[ \beta = x / A \]
\[ c = 0.5 \Gamma(A) \]
\[ \text{RES} = \log x - G - \log B \]

NO

RES

A \times x_0 / 3 ?

YES

PRINT "NOT IN RANGE"

RETURN

\[ A = A - (D / 10) \]
\[ D = D / 10 \]

NO

FOUR ITERATIONS COMPLETE?

YES

RETURN
SUBROUTINE GAMAX(BO, X3, XLB, A, B, D)
A0=X3/B0
D=A0/30.
A=A0/3.
106=1
DO 797 I=1,50
A=A+D
B=X3/A
G=D*6A(A)
K=XLB-G-ALOG(B).
792 FOKMAT(14.,22X,3E12,5)
797 CONTINUE
WRITE(3,1502)
1502 FOKMAT('OUT OF RANGE!')
RETURN
1500 I06=I06+1
IF(I06-4)1506,1503,1503
1526 A=A-(D+D/5. )
D=D/10.
GO TO 1504
1503 RETURN
END

I1,71: Calculated points of intersection of great
circles
I18; 1. if the segments intersect
0. if the segments do not intersect
ANG: Calculated angle of intersection

Method:
Let \( q \) be a vector normal to first great circle.
Let \( b \) be a vector normal to second great circle.
Then \( q \cdot a \times b \) will be zero at intersection.
Convert \( q \) to spherical to geom II.
Angle of intersection \( \phi = \arccos \left( q \cdot a \times b \right) \).
If the segments intersect, distance from point of intersection to end points will be less than
distances between end points.
INSECT

Purpose:
To find the angle of intersection of two great-circles and determine if two great circle segments intersect.

Call:
Call Insect (T1,F1,T2,F2,T3,F3,T4,F4,T1,F1,IYN,ANG)

T1,F1: Spherical coordinates of end points of first great circle segment (in radians)
T2,F2: Spherical coordinates of end points of second great circle segment (in radians)
T3,F3: Spherical coordinates of end points of second great circle segment (in radians)
T4,F4: Great circle coordinates of intersection of great circles
IYN: = 1 if the segments intersect
= 0 if the segments do not intersect
ANG: Calculated angle of intersection

Requires:
STAC, CROSS, CATS, DOT, NORM, LENGTH

Method:
Let a be a vector normal to first great circle
b be a vector normal to second great circle
Then c = a x b will be along intersection
Convert c to sphericals to get T1, F1
Angle of intersection σ = arcos (a . b)
If the segments intersect, distances from point of intersection to end points will be less than distances between end points.
BEGIN

θ₁, θ₁ → a
θ₂, θ₂ → b
α = a x b

θ₃, θ₃ → α
θ₄, θ₄ → b
δ = a x b

ε = ε x δ
α = -ε
θ₅, θ₅ ← ε
θ₆, θ₆ ← α

ANGLE:
\cos^{-1}(\frac{\epsilon \cdot \delta}{|\epsilon| |\delta|})

IF N = 1 or 2

YES INTERSECT

IF N = 0

RETURN
SUBROUTINE INSECT(T1,F1,T2,F2,T3,F3,T4,F4,FI,F1,IYN,ANG)
DIMENSION T1(2),FI(2),A(3),B(3),C(3),D(3),E(3)
RIT=COO1
PI=3.141592
R=1
CALL STAC(T1,F1,R,A(1),A(2),A(3))
CALL STAC(T2,F2,R,3(1),B(2),B(3))
CALL CROSS(A,B,C)
CALL STAC(T3,F3,R,A(1),A(2),A(3))
CALL STAC(T4,F4,R,B(1),B(2),B(3))
CALL CROSS(A,B,D)
A(1)=-1.*F(1)
A(2)=-1.*F(2)
A(3)=-1.*F(3)
CALL CATS(T1(1),FI(1),R,E(1),F(2),E(3))
CALL CATS(T1(2),FI(2),R,A(1),A(2),A(3))
CALL NORM(C)
CALL NORM(D)
ANG=ARCCOSDOT(C,D))
IF(ANG-PI/2.)151,151,16C
160 ANG=PI-ANG
151 CALL LENGTH(T1,F1,T2,F2,S1)
CALL LENGTH(T2,F2,T3,F3,S2)
DO 666 II=1,2
CALL LENGTH(T1,F1,II(II),FI(II),S)
666 IF(S-S1-BIT)160,670,666
670 CALL LENGTH(T1,F1,II(II),FI(II),S)
671 CALL LENGTH(T1,F2,II(II),FI(II),S)
672 CALL LENGTH(T1,F4,II(II),FI(II),S)
673 IYN=II
GO TO 680
666 CONTINUE
IYN=C
680 CONTINUE
50 RETURN
END
GTCRL

Purpose: to calculate the coordinates of a series of points along a great circle segment, given the end points.

Call:

CALL GTCRL(T1,F1,T2,F2,DIST,TT,FF,NPTS)

\[ \begin{align*}
T1 & \} \text{spherical coordinates of one end point (radians)} \\
F1 & \} (\theta, \phi) \\
T2 & \} \text{spherical coordinates of other end point (radians)} \\
F2 & \} (\theta, \phi) \\
\text{DIST: angle between two points (radians)} & \text{if known, otherwise = 0.0} \\
TT & \} \text{output arrays of up to one hundred points along the great circle segment (radians)} \\
FF & \} \\
NPTS & \} \text{output parameter giving number of points in array.}
\end{align*} \]

Method: The interval is divided into segments of approximately 0.1 degrees or 100 segments whichever is the smaller number. Points are calculated using the rotation subroutine ROT by rotating the first point about an axis normal to the great circle.

Subprograms required:

LENGTH,STAC,CRSS,NRM,LATS,RFT

Error messages: None.
SUBROUTINE GTCRL(T1,F1,T2,F2,DIST,TT,FF,NPTS)
C SUBROUTINE TO GENERATE POINTS ON A GREAT CIRCLE
DIMENSION TT(100),FF(100),A(3),B(3),N(3),V(3)
REAL N
DATA DIV/0.004363327/
R=1.
IF (DIST)100,10C,101
100 CALL LENGTH (T1,F1,T2,F2,DIST)
101 NPTS=DIST/DIV+2.
CALL STAC(T1,F1,R,A(1),A(2),A(3))
CALL STAC(T2,F2,R,B(1),B(2),B(3))
CALL CROSS(A,B,N)
CALL NORM(N)
IF(NPTS-100)102,102,110
110 NPTS=100
DIV=DIST/100.
102 TT(1)=T1
FF(1)=F1
IF(NPTS)=T2
FF(NPTS)=F2
D=0.
NP=NPTS-1
CALL CATS(TA,FA,RA,N(1),N(2),N(3))
DO 70 K=2,NP
D=D+DIV
IF(K-2)1241,1240,1241
1240 CALL ROT(T1,F1,TA,FA,D,TP,FP,1)
GO TO 1242
1241 CALL ROT(T1,F1,TA,FA,D,TP,FP,2)
70 CONTINUE
RETURN
END
ROT

Purpose: To rotate a point $\theta,\phi$ about an axis $\theta_a,\phi_a$ through an angle $\delta$.

Call: CALL R\$T (T,F,TA,FA,DEL,TN,FN,IN)

T ) spherical coordinates of point
F ) (in radians) $(\theta,\phi)$
TA} spherical coordinates of axis of
FA} rotation (radians)
DEL: angle of rotation (radians)
TN) output new spherical coordinates of
FN} rotated point (radians)
IN) function indicator: IN = 1 if this is the first call with this particular axis of rotation.

IN = 2 axis of rotation the same as last call.

Method: The axis of rotation $\theta_a,\phi_a$ is converted to a vector $\mathbf{a}$. Vectors $\mathbf{b}$ and $\mathbf{c}$ are set up on orthogonal dextral coordinate system.

The point $\theta,\phi$ is converted to Cartesian coordinate in the $X,Y,Z$ system (as in STAC)

\[
x_1 = x_1a + y_1a + z_1a
\]
\[
y_1 = y_1b + y_1b + z_1b
\]
\[
z_1 = x_1c + y_1c + z_1c
\]

The point $x_1,y_1,z_1$ is then rotated about through the angle $\delta$ to give

\[
x'' = x_1
\]
\[
y'' = -z_1 \sin(\delta) + y_1 \cos \delta
\]
\[
z'' = z_1 \cos \delta + y_1 \sin \delta
\]

The $(x'',y'',z'')$ point in the $a,b,c$ coordinates are then reconverted to the $x,y,z$ system giving

\[
x_\eta = x''a_1 + y''b_1 + z''c_1
\]
\[
y_\eta = x''a_2 + y''b_2 + z''c_2
\]
\[
z_\eta = x''a_3 + y''b_3 + z''c_3
\]

$(x_\eta,y_\eta,z_\eta)$ is then converted to the new spherical coordinates $(\theta_\eta,\phi_\eta)$

Reference: (Osgood and Graustein, 1921, p.595).

Subprograms required: STAC,N\$RM,CR\$SS,LATS
BEGIN

YES

IS THIS THE FIRST INCOME ABOUT THIS AXIS?

NO

\[ \theta, \phi \rightarrow x, y, z \]

CONVERT \(x, y, z\) TO \(a, b, c\) COORDINATE SYSTEM TO GET \(x', y', z'\)

ROTATE \((x', y', z')\) ABOUT \(a\) THROUGH ANGLE \(b\), GIVING \((x'', y'', z'')\)

CONVERT \((x'', y'', z'')\) TO \(x, y, z\) COORDINATES TO GIVE \((x_0, y_0, z_0)\)

CONVET \(x_0, y_0, z_0\) TO SPHERICAL COORDINATES

RETURN

\[ \theta = \theta_0, \phi = \phi_0 \]

\[ t = \frac{1}{2} \]

\[ e = a \times b \]
SUBROUTINE ROT(T,F,TA,FA,DEL,IN,FN,IN)
DIMENSION A(3),B(3),C(3)
S = 1.
IF(IN=1)1800,1700,1800
1700 CALL STAC(TA,FA,S,A(1),A(2),A(3))
B(1)=1.
B(2)=1.
B(3)=(-1.)*(A(1)+A(2))/A(3)
CALL NO-M(9)
CALL CROSS(A,B,C)
1800 IF(DEL)+1921,1802,1802
1802 TN=T
FN=F
RETURN
1801 CALL STAC(T,T,F,S,X,Y,Z)
XP=A*A(1)+Y*A(2)+Z*A(3)
YP=X*A(1)+Y*B(2)+Z*B(3)
ZP=X*C(1)+Y*C(2)+Z*C(3)
YPF=ZP*COS(DEL)+YP*COS(DEL)
ZPF=ZP*SIN(DEL)+YP*SIN(DEL)
XP=XPF
YN=XPF
ZN=XPF
RETURN
END

Method:
The relation is considered in two stages:
1) A great-circle rotation of the first point (T,F) to coincide with (0,0,0).

Then the angle of rotation a = \theta = \tan^{-1} (y \sqrt{a})
The rotation vector \vec{u} = \tan^{-1} (y \sqrt{a})

2) Rotation about the first point to bring the second pair into coincidence

\theta = \tan^{-1} (y \sqrt{a})

The resultant of two rotations \theta_1, \theta_2 is

\theta_1 + \theta_2 = \theta = \tan^{-1} (y \sqrt{a})

Resulting angle of rotation is therefore the same for both pairs at (0,0,0) and (T,F).
Purpose:

To find the centre of rotation and angle of rotation between two continents given two matching points on opposite coast lines.

Call:

CALL TWOPT(T1,F1,T2,F2,TA,FA,TB,FB,TC,FC,ANG)

T1 ) spherical coordinates of one point on first continent
T2 ) spherical coordinates of second point on first continent
TA ) spherical coordinates of one point on second continent
FA ) second continent
TB ) spherical coordinates of second point on second continent
FB ) on second continent
TC ) spherical coordinate of axis of rotation
FC )

ANG : angle of rotation

all angles in radians

Method:

The rotation is considered in two stages:

1) A great circle rotation to bring \((8_1,\phi_1)\) to coincide with \((8a,\phi a)\)

Set \(a = (\theta_1,\phi_1)\)

\(b = (\theta a,\phi a)\)

Then the axis of rotation is \(\vec{r} = a \times b\)

The angle of rotation \(\delta\) is \(\arccos (a \cdot b)\)

Thus rotation vector \(\Omega = \tan^{-1} (\delta/2)\vec{r}\)

2) A rotation about the first point to bring the second pair into coincidence

\(d = (\theta b,\phi b)\)

\(c^1 = \) the rotated position of \(c = (\theta_2,\phi_2)\)

Centre of rotation is \(b\)

Angle of rotation is \(\cos \delta^1 = (b-c^1) \cdot (b-d)\)

Thus second rotation is \(\Omega_2 = \tan^{-1} (\delta^1/2)b\)

The resultant of two rotations \(\Omega_1,\Omega_2\) is

\[\Omega^* = \frac{\Omega_1 + \Omega_2 + \Omega_1 \times \Omega_2}{1 - \Omega_1 \cdot \Omega_2}\]

Resulting angle of rotation is therefore

\[2 \arctan \left| \frac{\Omega^*}{\Omega^*} \right|\]

Resulting axis of rotation is \(\frac{\Omega^*}{|\Omega^*|}\)

Subprograms required: STAC, LATS, DPHI, CROSS, NORM, ARCCOS

Error messages: None.
BEGIN

T1,F1 → a
T2,F2 → b
T3,F3 → c
c
T4,F4 → d

\( \beta = a \times b \)

\( \beta_1' = \cos^{-1} (2 \cdot b) \)

\( \alpha_1 = \tan \theta_0 / 2 \cdot \beta_1 \)

\( \alpha_1' = \beta \cdot \beta_1' \)

\( \varepsilon = \alpha_1 \times \varepsilon \)

\( \varepsilon' = ((1 - \alpha_1^2) \varepsilon + 2 \cdot \alpha_1 \cdot \varepsilon) / (1 + \alpha_1^2) \)

\( \bar{x} = \frac{b - d}{2} \)

\( y = b - e \)

\( \theta_2 = \cos^{-1} (x \cdot y) \)

\( \alpha_2 = \frac{b}{\tan \theta_2 / 2} \)

\( \varepsilon = \alpha_1 \times \varepsilon \)

\( \beta = (\alpha_2, \alpha_2', \varepsilon) / (1 - \alpha_2^2 - \alpha_2') \)

ANGLE OF ROT =
2 \cdot \tan^{-1} |2| 

\( \ell = \text{norm } \beta \)

\( \ell \) is axis of rotation.

RETURN
SUBROUTINE TWOPT(I1,F1,T2,F2,TA,FA,TB,FB,T,F,ANG)

Y(3)

DIMENSION D(3)

R=1
CALL STAC(T1,F1,R,A(1),A(2),A(3))
CALL STAC(T2,F2,R,C(1),C(2),C(3))
CALL STAC(TA,FA,R,B(1),B(2),B(3))
CALL STAC(TB,FB,R,D(1),D(2),D(3))
CALL NORM(A)
CALL NORM(C)
CALL NORM(D)

CALL NORM(D)

1400 FORMAT(1H4,4E16.8)
WRITE(3,1400)A
WRITE(3,1400)C
WRITE(3,1400)D
CALL CROSS (A,B,OHM1)
CALL NORM(OHM1)
CALL SENSE(OHM1,A,B,SIGN)
WRITE(3,1400)OHM1
A1=ARCOS(DOT(A,B))*SIGN
E=TAN((A(2)/2))

DO 30 I=1,3
OHM1(I)=OHM1(I)*F
WRITE(3,1400)OHM1
D=OHM1(1)**2+OHM1(2)**2+OHM1(3)**2
CALL CROSS (OHM1,C,E)
DO 31 I=1,3
CP(I)=((1.-OH)*C(I)+2.*DOT(OHM1,C)*OHM1(I)+2.*E(I))/(1.+OH)
WRITE(3,1400)CP
X(I)=B(I)-D(I)
Y(I)=B(I)-CP(I)

CALL NORM(X)
CALL NORM(Y)
CALL SENSE(B,CP,D,SIGN)
A2=ARCOS(DOT(X,Y))*SIGN
E=TAN((A2/2))

DO 33 I=1,3
OHM2(I)=B(I)*F
WRITE(3,1400)OHM2
CALL CROSS(OHM2,OHM1,E)
DO 34 I=1,3
OHM(I)=(OHM1(I)+OHM2(I)+E(I))/(1.-DOT(OHM1,OHM2))
1ANTZ=SQRT(OHM1(1)**2+OHM(2)**2+OHM(3)**2)
CALL NORM(OHM)
WRITE(3,1400)OHM
CALL CATS(T,F,R,OHM(1),OHM(2),OHM(3))
T2=ATANT(ZANTZ)
ANG=T2*2
CALL SENSE(OHM,A,B,SIGN)
ANG=ANG*SIGN

901 RETURN
END
ORTHO

Purpose: To calculate points on an oblique orthographic projection.

Call: CALL ORTHO(TT,FF,XS,YS,ITB,Q,TS,FS)

TT ) spherical coordinates of the point
FF ) spherical coordinates of the point
XS ) output x,y coordinates in orthographic projection on unit sphere
YS ) projection on unit sphere
ITB: 1 point is in lower hemisphere with respect to centre of projection
2 point is in upper hemisphere
Q : dummy variable = 1.0
TS ) spherical coordinates of centre of projection (radians)
FS ) projection (radians)

Method: Convert point to Cartesian coordinates (X,Y,Z)

Calculate Xs, Ys in orthographic projection using subroutine ORP

Determine if point is in upper or lower view by testing whether angle between centre of projection and point is less than or greater than 90°.

Subprograms required:

$RP, STAC, N$RM, D$T, ARC$S

Error messages: None.
BEGIN

CONVERT θ, φ to x, y, z
q = (x, y, z)

x_s, y_s = orthographic projection of x, y, z

Centre of projection = ζ
angle = \cos^{-1}(q \cdot \zeta)

IF upper hemisphere

YES

angle > 90°?

RETURN

IF lower hemisphere

RETURN

NO
SUBROUTINE ORTHO(TT, FF, XS, YS, ITB, Q, TS, FS)
C OBLIQUE ORTHOGRAPHIC PROJECTION
DATA PI/3.1415926/
DIMENSION A(3), B(3), C(3)
CALL STAC(TT, FF, Q, X, Y, Z)
CALL ORP(X, Y, Z, XS, YS, TS, FS)
A(1)=X
B(1)=Y
C(1)=Z
CALL NORM(A)
CALL NORM(B)
IF (ABS(T)-1.) 12, 12, 13
12 ANG=ARCUS(T)
 10 T=1
 70 IF (ANG=PI) 14, 1, 15
 14 ANG=PI
 10 T=70
 15 ANG=0.
70 IF (ANG=PI/2) 101, 101, 100
101 ITB=1
RETURN
100 ITB=2
RETURN
END
CARE

Purpose: A subroutine used in the plotting of points on stereographic and orthographic projection which takes care of the overhead required to sort out upper and lower hemisphere points, to call in the plot routine whenever a point sequence changes hemisphere, and to call in the plot routine when all points have been generated.

Call: CALL CARE(IN, ITB, XS, YS)

IN : Input mode; IN=1 add one point to the present arrays
    IN=2 finish plotting all partially filled arrays and initialize routine

ITB : hemisphere indicator ITB=1 upper hemi.
      ITB=2 lower hemi.

XS, YS: Cartesian coordinates of point on unit circle

Method: If the hemisphere indicator is the same as the previous point handled, the point is stored in the appropriate array. If it differs, the previous array is plotted and the point added to the other array. Calling with IN=2 causes both unfilled arrays to be plotted.

Subprograms required: CPL$T

Error messages: None.
1

PLOT LOWER HEMISPHERE

PLOT UPPER HEMISPHERE

UNSET FLAG
N1=0
N2>0

RETURN
SUBROUTINE CARE(IN, ITB, XS, YS)
DIMENSION X1(1000), Y1(1000), X2(1000), Y2(1000)
DATA INIT, N1, N2/1, 0, 0/
GO TO (10, 20), IN
10 IF(INIT-1) 12, 11, 12
11 INIT=2
40 GO TO (13, 14), ITB
13 N1=N1+1
X1(N1)=XS+5.
Y1(N1)=YS+5.
ITBP=ITB
IF(N1=1000) 1206, 1201, 1200
1201 CALL CPlot(N1, 0, 1, X1, Y1)
N1=0
1200 RETURN
14 N2=N2+1
Y=YS=(-1.)+12.
Y2(N2)=Y+5.
ITBP=ITB
IF(N2=1000) 1306, 1301, 1300
1301 CALL CPlot(N2, 0, 1, X2, Y2)
N2=0
1300 RETURN
12 IF(ITBP-ITB) 41, 40, 41
41 GO TO (50, 51), ITBP
50 CALL CPlot(N1, 0, 1, X1, Y1)
N1=0
51 CALL CPlot(N2, 0, 1, X2, Y2)
N2=0
141 GO TO (43, 44), ITB
43 N1=N1+1
X1(N1)=XS+5.
Y1(N1)=YS+5.
ITBP=ITB
IF(N1=1000) 1206, 1201, 1200
44 N2=N2+1
Y=YS=(-1.)+12.
X2(N2)=XS+5.
Y2(N2)=Y+5.
ITBP=ITB
IF(N2=1000) 1306, 1301, 1300
20 CALL CPlot(N1, 0, 1, X1, Y1)
CALL CPlot(N2, 0, 1, X2, Y2)
INIT=1
N1=0
N2=0
RETURN
END
SENSE

Purpose: To determine the sense of a rotation.

Call: SENSE(OHM,A,B,SIGN)

OHM - vector of rotation
A - vector representation of first point
B - vector representation of second point
SIGN - sense of rotation

Method: If \( \Omega_3 (a_1b_2 - a_2b_1) \) is negative, sense is negative set \( \text{SIGN} = -1 \).
If positive set \( \text{SIGN} = 1 \).

Subroutines required: None.
SUBROUTINE SENSE (AXIS, A, B, SIGN)
DIMENSION AXIS(3), A(3), B(3)
DET = A(1) * B(2) - A(2) * B(1)
DET = DET * AXIS(3)
IF (DET) 100, 100, 101
100 SIGN = (-1)
RETURN
101 SIGN = 1
RETURN
END
R360 (FUNCTION)

Purpose: To reduce a given angle to the range 0 - 360°.

Call: A = R360(A)

Method: Successive addition of 360° if angle is negative
         Successive subtraction if angle is positive.

Subroutine required: None.
FUNCTION R360(ANY)
DATA PI2 /6.2831847/
ANG=ANY
100 IF(ANG)160,206,300
110 ANG=ANG+PI2
200 IF(ANG)160,206,300
201 ANG=ANG-PI2
GO TO 300
300 RETURN
END

BEGIN

ADD 2*Pi TO ANGLE

IF ANGLE > 2*Pi
SUBTRACT 2*Pi FROM ANGLE

IF ANGLE < 0
ANGLE = 0;

RETURN
LINPLT

Purpose: To read and plot lineaments on a map

Call: LINPLT(ILIN,CNT,ANG,TS,FS)

ILIN = 0 if no lineaments
CNT = centre (θ,φ) of rotation of continent as in main program
ANG = angle of rotation
TS,FS = centre of projection of map.

Method: Lineaments are read under format
I6,3(F5.2,A1,F5.2,A1),37X,I1

Input coordinates are in lat. and long. in degrees.
These are converted to spherical coordinates and the great circle is drawn between them.

Subroutines required:
RAD,LATS,GTCRL,ROT,ORTHO,CARE
SUBROUTINE LIMPLT(LIN,CNT,ANG,TS,FS)
C SUBROUTINE TO PLOT LINE PATTERN

DIMENSION TL(100),FL(100),XX(3),IXX(1),YY(1),IYY(1),I=1,3
1001 I=1.
107 READ(1,106)IFAKE
111 FORMAT(A1)
WRITE(6,1700)(XX(I),IXX(I),YY(I),IYY(I),I=1,3),IEND
1200 FORMAT(IH,3(5.2,A1,1X,F5.2,A1,1X))
112 X1=RA0(XX(1))
Y1=RA0(YY(1))
X2=RA0(XX(3))
Y2=RA0(IYY(3))
100 FORMAT(IH,3(F5.2,A1,F5.2,A1),37X,11)
CALL LATS(X1,XX(1),Y1,YY(1),I,F1)
CALL LATS(X2,XX(3),Y2,IYY(3),I2,F2)
D=0.9
1201 FORMAT(IH,4(F12.5))
CALL GETCL(TL,F1,F2,D,TL,FL,NPTS)
DO 60 K=1,NPTS
TX=TL(K)
FX=FL(K)
ANP=ANG
C1=CNT(1)
C2=CNT(2)
CALL ROT(TX,FX,C1,C2,ANP,TP,FP,K)
60 Q=5.
DO 62 K=1,NPTS
TX=TL(K)
FX=FL(K)
CALL ORTHO(TX,FX,XS,YS,ITB,G,TS,FS)
62 CALL CAFE(1,ITB,XS,YS)
CALL CAFE(2,ITB,XS,YS)
GO TO 111
1000 RETURN
END
CPL&T

Purpose: to write tapes of x-y coordinates for plotting on the CSIRO Calcomp plotter

Call: CALL CPL&T(IA,IB,IC,XA,YA)

IA = number of points to be plotted
IB = mode indicator
   IB = 0  line joining points
   IB = positive points and line to be drawn, value of IB indicates type of point to be marked negative, points only
IC = height of symbols at points
XA,YA = arrays of x,y coordinates of points

Method: After error checking, data is written on 7-track tape. Flow chart gives details.

Diagnostics: Various error messages as necessary. Refer to listing.

Entry points: STRPLT

STRPLT(XW,YW,XSC,YSC)
An initialization routine which must precede other plot calls.
XW,YW = origin of coordinate system
XSC,YSC= scale factors in X and Y directions.

ENDPLOT

ENDPLOT(IA)
Places on indicator on tape indicating the end of a plot, and places ENDFILE mark on tape.
ENTRY: CPOLOT

IA > 1000?

WRITE A, B, C ON TAPE

RETURN

PRINT MESSAGE SET IA:1000

WRITE A, B, C ON TAPE

RETURN

I0?

IC?

WRITE A, B, C ON TAPE

RETURN

|IB| < 5?

|IC| = 9?

PRINT ERROR MESSAGE SET IB=0 IC=1

PRINT ERROR MESSAGE IC=1

WRITE A, B, C ON TAPE

PRINT ERROR MESSAGE IB=1

WRITE A, B, C ON TAPE

RETURN

I0?

IC?

WRITE A, B, C ON TAPE

RETURN

PRINT ERROR MESSAGE SET IC=1
ENTRY START

WRITE xo,yo XSCALE YSCALE ON TRUE

SET FLAG

RETURN

ENTRY ENOPLT

WRITE NEGATIVE VALUE OF A ON TAE

PRINT MESSAGE

RETURN
C PROGRAM FOR CREATING MAPE FOR PLOTTING AT CERO
C 1 R. G. RICH 1969
SUBROUTINE GPLOT(A1,IA,IC,XA,YA)
DIMENSION XA(1000),YA(1000)
IA=1
JA=1
IP=17
(131-132) 621,400,201
691 WRITE(10,692)
692 FORMAT(39H **GPLOT ERROR--GPLOT NOT INITIALIZED/
151H **A CALL TO GPLOT MUST PRECEeed THE PLOTTING JOB ABORTED/
101H RETURN)
693 FORMAT(13H **ARRAY LENGTH NEGATIVE, SET TO ZERO)
702 IA=7
X=0.
Y=0.
A=0.
C=0.
WRITE(10,2111)A,IA,C
WRITE(10,2111)X,Y
WRITE(10,1110)A,B,C
WRITE(10,1117)X,Y
RETURN
700 IF(IA-1000)21,21,22
18 MUST BE LESS THAN 6 OR EQUAL TO +OR-9
20 WRITE(10,1001)IA,IB,IC
100 FORMAT(73H **GPLOT ERROR--ARRAY LENGTH GREATER THAN 1000, ASSUME L/
101H LENGTH EQUALS 1000/1H ,313)
IA=100)
21 IF(IA)22,23,23
22 WRITE(10,1011)IA,IB,IC
101 FORMAT(22H END OF PLOT REQUESTED/1H ,316)
A=IA
B=IB
C=IC
WRITE(10,2111)A,IA,C
C END OF PLOT RETURN
RETURN
23 IF(IA)24,25,24
24 IF(IA-10)5,27,27,26
26 IF(IA-109)127,27,26
28 WRITE(10,1021)IA,IB,IC
102 FORMAT(73H **GPLOT ERROR--INVALID POINT MODE INDICATOR, ASSUME TB=
103H /1H ,316)
10=0
60 TO 25
27 IF(IA)30,30,25
30 WRITE(10,1031)IA,IB,IC
103 FORMAT(1004 **GPLOT ERROR--NEGATIVE OR ZERO CHARACTER HEIGHT, ASSUME /
104H IL=1/1H ,316)
IC=1
60 TO 25
27 IF(IA)31,31,32
31 WRITE(10,1031)IA,IB,IC
C
12

i
?

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l
.

\( \text{ENTRY ENOPLT}(1A) \)
\( \text{A}=1A \)
\( \text{B}=0. \)
\( \text{C}=0. \)

\text{IF(A)200,209,201}
\text{WRITE(10,205)}
\text{205 FORMAT(11H END OF JOB)}
\text{WRITE(1P,2111)A,B,C}
\text{END FILE IP}
\text{RETURN}
\text{201 WRITE(10,204)1A}
\text{204 FORMAT(14H END OF PLOT ,13)}
\text{A}=A*(-1.)
\text{207 WRITE(1P,2111)A,B,C}
\text{RETURN}
\text{ENTRY STRPLT(XN,YN,XSC,YSC)}
\( \text{IP}=17 \)
\( \text{IO}=3 \)
\text{WRITE(IP,2111)XN,YN,XSC,YSC}
\text{WRITE(10,208)XN,YN,XSC,YSC}
\text{208 FORMAT(18H PLOT INITIALIZATION/6H OF IGN,2F10.5,6HSCALES,2F10.5)}
\( \text{UNIT}=999 \)
\text{RETURN}
\text{END}

\text{WRITE}(1P,2111)A,B,C
\text{211 FORMAT(8F12.5)}
\text{WRITE(10,111)A,B,C}
\text{111 FORMAT(14H,3F10.5)}
\text{WRITE(1P,112)}(X(A(I)),I=1,1A)
\text{112 FORMAT(24A4)}
\text{RETURN}

C CHARACTER STRING PLOTTED

25 \text{A}=1A
\text{B}=IP
\text{C}=1C
\text{WRITE(IP,2111)A,B,C}
\text{WRITE(IP,2111)(XX(A(I)),YX(A(I)),I=1,1A)
\text{WRITE(10,111)A,B,C}
\text{117 FORMAT(14H,10F10.5)}
\text{C LINE PLOTTED}
\text{ENTRY ENOPLT}(1A)
\text{A}=1A
\text{B}=0. \)
\( \text{C}=0. \)

\text{IF(A)200,209,201}
\text{WRITE(10,205)}
\text{205 FORMAT(11H END OF JOB)}
\text{WRITE(1P,2111)A,B,C}
\text{END FILE IP}
\text{RETURN}
\text{201 WRITE(10,204)1A}
\text{204 FORMAT(14H END OF PLOT ,13)}
\text{A}=A*(-1.)
\text{207 WRITE(1P,2111)A,B,C}
\text{RETURN}
\text{ENTRY STRPLT(XN,YN,XSC,YSC)}
\( \text{IP}=17 \)
\( \text{IO}=3 \)
\text{WRITE(IP,2111)XN,YN,XSC,YSC}
\text{WRITE(10,208)XN,YN,XSC,YSC}
\text{208 FORMAT(18H PLOT INITIALIZATION/6H OF IGN,2F10.5,6HSCALES,2F10.5)}
\( \text{UNIT}=999 \)
\text{RETURN}
\text{END}

\text{WRITE}(1P,2111)A,B,C
\text{211 FORMAT(8F12.5)}
\text{WRITE(10,111)A,B,C}
\text{111 FORMAT(14H,3F10.5)}
\text{WRITE(1P,112)}(X(A(I)),I=1,1A)
\text{112 FORMAT(24A4)}
\text{RETURN}

C CHARACTER STRING PLOTTED

25 \text{A}=1A
\text{B}=IP
\text{C}=1C
\text{WRITE(IP,2111)A,B,C}
\text{WRITE(IP,2111)(XX(A(I)),YX(A(I)),I=1,1A)
\text{WRITE(10,111)A,B,C}
\text{117 FORMAT(14H,10F10.5)}
\text{C LINE PLOTTED}
\text{ENTRY ENOPLT}(1A)
\text{A}=1A
\text{B}=0. \)
\( \text{C}=0. \)
PART II

MAIN PROGRAMS
PROGRAM TO ANALYSE LINEAMENTS

Purpose:
To calculate and display azimuth, length, intersection angle and potential intersection angle distributions for lineaments, given latitude and longitude at their end points.

Method:
Lengths, angles, etc., are calculated using the three-dimensional geometry subprograms previously described.

Subroutines Required:
HIST, LATS, LENGTH, STAC, NORM, DOT, RAD, DEG, INSECT.

Input Data Cards:
Card 1
Col: 1 - 10 (F10.0)
Tolerance angle in degrees:
Where mid-points of lineaments are given, radius of curvature is calculated. Lineaments whose radius is less than the tolerance angle are rejected as straight and considered to be arcs.

Data Deck
One card per lineament
Col. 1 - 6
Code number
Cols. 8 - 11
Latitude of end points (to nearest 1/100°)
Col. 12
N or S latitude
Cols. 13 - 17
Longitude of end points
Col. 18
E or W longitude
Col. 20 - 23, Col. 24, Col. 25 - 29, Col. 30
Same as above for mid point
Col. 32 - 35, Col. 36, Col. 37 - 41, Col. 42
Same as above for second end point

Col. 52
Weighting factor of 1 to 9

Col. 80
= 1 for last card
LINEAMENT ANALYSIS

BEGIN

INITIALIZE VARIABLES

READ TO LAND.

INITIALIZE HISTOGRAMS

READ A LINEAMENT

LAST CARD?

CONVERT TO SPHERICAL
CALCULATE LAMBDA
PRINT AND TO HISTOGRAM

CONVERT TO CARTESIAN
CALCULATE AZIMUTH
ADD TO HISTOGRAM
ADD TO SUMS

PRINT HISTOGRAMS, MEANS

CALCULATE INTERSECTION ANGLES IN GROUPS OF 50

DO LINEMENTS INTERSECT?

ADD TO POT INT ANGLE HISTOGRAM

ADD TO INT ANGLE HISTOGRAM

FINISHED?

STOP
C. PROGRAM TO ANALYZE LINES SUPPORTED

DIMENSION M(3), XX(3), YY(3), IT(3), FF(3), X1(3), Y1(3), X3(3), Y3(3)

DIMENSION T(l000)

MO=1
PCF=3

READ(1,1311) T

1311 FORMAT(* E, D)

CALL HIST(T, M, 5, 0, 1)

AVO=0.

CALL HIST(T, M, 5, 0, 1)

AVO=0.

AVO=0.

SAVO=0.

READ(L,1200), IC, IOG, IXX(1), IYX(1), IYI(1), I=1, 3, ICONT, IOG,

TAGE, ISCL, ISCL, TG, NAME, IEND

IF(NL>10) GO TO 114, IEND

112 DO 3 I=1, 3

XX(I)=M(I)

YY(I)=M(I)

6 CALL LENGTH(1X1(1), IXX(I), IYX(I), IYI(I), IT(I), FF(I), PM(I))

132 CALL ISCL(1H + 1, L, F6.1, A6.1, F6.1, A6.3)

133 FORMAT(IH, F6.2)

43 CALL LENGTH(1X1(1), FF(I), T(I), FF(I), A6)

AVD=0.3, ISCL, ISCL, TW, A6, NAME, IEND

125 FORMAT(IH, 16, I, 6, F6.1, A6.1, I, 1, 3, 1, 1, 3, 1, 3, 1, 3, 1, 3, 5, 1, 2, A4)

IF(NL<100) GO TO 2661, 2661, 2666

2661 A=M(1)

CC=M(1)

D0(DN=1) FF(3)

10 IF(DN=10) GO TO 2661, 2666

2665 NL=NC-1

2662 CONTINUE

A=A+AD

F=F+1.

A=A+AF10+AD

F=F+F11.

CALL HIST(1, 10, 1)

CALL HISTP(2, 30, 10)
VAR'I = 1.
VAR(S) = 1.
VAR(3) = 1.

    CALL ST4C(T1(3),FF(3),I,L,A(1),A(2),A(3))

770  CALL SM(4,N,1-VSAI(2))
    FNR=FF(1)-..,01
    CALL SM(T1(1)+PA(1),1.,8(2),E(3))

771  UC1AI(E-DD-1-VSAI(2))
    CALL VNM(0)
    CALL NVM(0)
    AZM=406,3000(0,1)
    DAZM=DEG(AZM)
    AZM=AZM
    CALL HISTRN(3,DAZM,1)
    CAVD=CAV+GOSD(AZ)
    SAVD=SAV+SDT(AZ)
    SAVD=SAV+SDT(AZ)@F10
    CHAV=CAV+CSSD(AZ)@F10
    GO TO 311

43  IFAC=IFAC+1
    WRITE(15)icode,TXT(1),TXT(2),YX(1),YXY(1),[E=1,3],ICONT,1,OC,
    IGL,ISCL,1SSCL,1T,FF,LENN,FL(2)

1000  DD 1301 I=1,4
1901  CALL HISTPT(1)
    AVL=AV/FF
    AVL=AV/FF
    AVL=ATAN(SAVD/CAVD)
    AVL=ATAN(SAVD/GOSD)
    CAVD=DEG(AVD)
    CAVD=DEG(AVD)
    WRITE(15)AV,YX,AVD,AVD
137  FORMAT(5H AVERAGE, LENGTH,F7.3,4X,15WEIGHTED AV LEN,F7.3/
    120+AV AVERAGE AZ,MLT,7,F7.3,4X,16WEIGHTED AV AZ,F7.3)
    IF(ARC1501,531,502)
502  AV=0.
    FN=0.
    WR=1.
    CAVD=0.
    SAVD=0.
    GDV=0.
    CHAV=0.
    WRITE(15)109
108  FORMAT(' PROCESSION OF ARG'S1
    FEND 15
    CALL HIST(1,1,21,1,21)
    CALL HIST(2,1,21,1,21)
    CALL HIST(3,1,21,1,21)
    CALL HIST(4,1,21,1,21)
    DO 400 1=1,26
    READ(15)ICODE,[XX(J),YY(J),YY(J),J=1,3],ICONT,1,OC,
    IGL,ISCL,1SSCL,1T,AGE1,AGE2,NAME ,T,T,FF,LENN,RAD1
I ...}.

CALL MISTEP1, AV, FN
336 CALL MISTEP1(1)
AV=AV/FN
335 CALL MISTEP1, AV=AV/FN

208 FORMAT(10**1, AVERAGE LENGTH', P7, 2, 4X, 'WEIGHTED AV_LEN', P7, 2/
10, 'AVERAGE RADIUS', P6, 3, 4X, 'WEIGHTED AV ради', P6, 3), CSE, 1}

501 WRITE(3, 259)
1206 WRITE(1, 2709)

2707 FORMAT('INTERSECTION ANALYSIS', 16)

2701 CALL HISTI(1, 20, 2)
32 6500 IXN=1, NXP
HNL=HNL/IXN=20
HEFT=HEFT+HNL
IF(HEFT.LT. 0) STOP
6502 NLX=NLX-1
GO TO 6503

6503 CONTINUE
1601=HEF+1
1602=NLX+1

DO 2702 1=1601, ITOP
V1=1+1
1116=ITOP-1
DO 2702 J=NS, NLX
2702 J=10I(J) 10I(J) 10I(J)
CALL INSET(AAI, J, BCI, J, CCI, J, DDI, J, EAI, J, EBJ, J, CCI, J, DDI, J, WJ
L, 1, 1, 1, ANGELE, 4)
CALL MISTEP1, ANS, 1)
CALL HISTPT(2, ANGO, LG)  
I (XY) 2770, 2727, 2727
2772 CALL HISTPT(1, ANGO, LG)  
CALL HISTPT(2, ANGO, LG)  
1773 FORMAT(4, 1F7.4, 1L)  
1974 FORMAT(4, 1F7.4, 1L)  
1975 FORMAT(4, 1F7.4, 1L)  
1976 FORMAT(4, 1F7.4, 1L)  
2772 OPEN(1, 2773)  
2773 FORMAT(* POTENTIAL INTERSECTIONS *)  
CALL HISTPT(1)  
CALL HISTPT(2)  
2776 FORMAT(* POTENTIAL INTERSECTIONS -- WEIGHTED *)  
CALL HISTPT(2)  
CALL HISTPT(2)  
2796 FORMAT(* ACTUAL INTERSECTIONS *)  
CALL HISTPT(3)  
WRITE(*, 2776)  
2775 FORMAT(* ACTUAL INTERSECTIONS -- WEIGHTED *)  
CALL HISTPT(4)  
6500 CONTINUE  
STEP
136 FORMAT(16, 1X, 3IF4.2, 1X, 1X, 1X, 1X, 1X, 1X, 1X, 1X, 1X, 1X, 1X, 1X)  
END

Cols. 1 - 6  
Number of data points or data windows (NN)

Cols. 12  
1 = point mode  
2 = histogram mode

Cols. 13 - 22  
Lowest value for histogram

Cols. 23 - 32  
Highest value for histogram

Cols. 33 - 41  
Width of histogram windows

Data Deck  
Data values or histogram window heights 20  
per card (40 cards max) until NN points are entered.
Repeats Card 2 and Data Decks until NN groups  
have been entered.

Subroutines Required:  
HIST, GAMMA, GAMMAP
PROGRAM TO FIT GAMMA DISTRIBUTIONS

Purpose:
To fit a gamma distribution to a set of observations.

Method:
Maximum likelihood estimation of parameters as described in Part II of body of this thesis.

Input Data Cards:
Card 1
Cols. 1 - 6
Number of groups of data to be treated (NJ)

Card 2
Cols. 1 - 6
Number of data points or data windows (NN)
Col. 12
1 = point mode
2 = histogram mode
Cols. 13 - 22
Lowest value for histogram
Cols. 23 - 32
Highest value for histogram
Cols. 33 - 42
Width of histogram windows

Data Deck
Data values or histogram window heights 20 per card (4 cols. each) until NN points are entered.
Repeat Card 2 and Data Decks until NJ groups have been entered.

Subroutines Required:
HIST, GAMAX, GAMINT
Begin
READ # OF GROUPS (N)
READ NO OF POINTS OR WINDOWS, HISTOGRAM CONSTANTS, MODE

MODE

HISTOGRAM MODE
SET SUM = 0
$S = \sum x, x > 0$
READ WINDOW HEIGHTS
FOR EACH WINDOW CREATE EQUIVALENT NO OF POINTS
ADD POINTS TO HISTOGRAM
$SUM x, x > 0$

POINT MODE
SET $S = \sum x, x > 0$
READ DATA POINTS
ADD POINTS TO HISTOGRAM
$SUM x, x > 0$

CALCULATE
$\hat{\varphi} = \frac{S \cdot \ln x}{2\cdot \ln x}$
METHOD OF MOMENTS ESTIMATORS
PRINT ANSWER
1

OBTAIN ALL ESTIMATORS FROM GANX

PRINT ESTIMATES, HISTOGRAMS

CALCULATE THEORETICAL CURVE AND \( \chi^2 \) STATISTIC

CALCULATE STANDARDIZED DISTRIBUTION

PRINT

2

FINISHED

STOP

NO

YES
DIMENSION X(1000),FX(50),LZ(50),XPT(100)
DIMENSION N1(100)
READ(1,120)NJ
DO 120 KK=1,NJ
120 FORMAT(16)
121 FORMAT(20F4.0)
READ(1,129)NN,IM,VL,VH,V1
129 FORMAT(2I9,3F14.5)
CALL HIST(1,VL,VH,V1)
127 FORMAT(20F4)
WRITE(1,501)
501 WRITE(1,502)
502 FORMAT(' ',HISTOGRAM MODE '/////////')
SX=0.
SX2=0.
SLX=1.
READ(1,127)(N1(I),I=1,NN)
1109 FORMAT(1H,2814)
N=0
DO 1110 I=1,NN
F1=1
XX=(F1*VI)-(VI/2.)+V1
N1=NI(I)
IF(N1)1102,1102,1103
1103 DO 1101 K=1,N1
N=N+1
CALL HISTFM(1,XX,1)
SX=SX+XX
SX2= SX2+XX*XX
X(K)=XX
1101 SLX=SLX+ALOG(XX)
1102 FN=N
37 CONTINUE
750 IOUT=1
751 X=X*(FX-1.)
SIG2=(SX2/(FN-1.))-X**2
FM1=FX/FN
FM2=SX2/FN
A=FM1**(2)/(FM2-FM1)**2
B=FM1**(2)/FM1
WRITE(3,200)SX,SX2,XB,SIG2,AD,BO,XLB
200 FORMAT(' ',SX,'E16.8',' SX2 ','E16.8',' MEAN 'E16.8',' VARIANCE ')
10  METHOD OF MOMENTS ESTIMATORS
20  ALPHA= 'E16.8' BETA= 'E15.8'
30  CALL GAMAX(BD, XB, XL, A, B, D)
40  WRITE(*,1200) A, B, D
50  1200 FORMAT(* MAXIMUM LIKELIHOOD ESTIMATORS--ALPHA= 'E16.8' BETA= 'E15.8')
60  CALL HISTPT(1)
70  A(I)=A
80  B(I)=B
90  XPT(I)=0.
100  CONTINUE
110  XL=0.
120  XU=VL+VI
130  DO 1500 K=1,30
140  CALL GAMINT(A(I),B(I),XL,XU,Y)
150  XL=XL+VI
160  XU=XU+VI
170  Y=Y+YN
180  CONTINUE
190  WRITE(*,1501) Y
200  1501 FORMAT('H',F10.5)
210  N2=27
220  CALL CHISO(1,XPT,N1,N2,X2,NFR)
230  WRITE(*,1520) X2,NFR
240  1520 FORMAT('CHI-SQUARE STATISTIC -- F10.5,16',DEG OF FREEDOM')
250  IF((IOUT-1678,505,678)
260  505 WRITE(*,504)
270  SX=0.
280  SX2=0.
290  SL=0.
300  VL=0.
310  VH=9.9
320  CONTINUE
330  CALL HIST(L,VL,VH,VI)
340  504 FORMAT('STANDARDIZATION '///)
350  IOUT=2
360  DO 133 I=1,N
370  X(I)=X(I)/AS
380  XX=X(I)
390  CALL HISTFM(1,XX,1)
400  SX=SX+XX
410  SX2=SX2+XX*XX
420  133 SL=SLX+ALUG(XX)
430  GO TO 751
440  678 CONTINUE
450  STOP
460  END
PROGRAM TO PLOT CONTINENTS

Purpose:
To draw orthographic projections of coast-lines or continental shelves in natural or rotated positions. Lineaments may also be drawn.

Method:
From the given parameters, oblique orthographic projections are calculated for the digitized points or the continental outlines (subroutine ORTHO). Continental rotations are performed by subroutine ROT previously described. Plotting is done through subroutine CPLOT.

Input Data Cards:
Card 1
Cols. 1 - 10
Co-latitude of centre of desired projection (in degrees)
Cols. 11 - 20
Longitude of centre of desired projection
Cols. 21 - 30
Spacing of latitude lines to be drawn
Cols. 31 - 40
Spacing of longitude lines

Card 2
Cols. 1 - 6
Number of continents or segments to be plotted

Card 3
Cols. 1 - 32
Name of continent
Cols. 33 - 42
Co-latitude of centre of rotation in degrees
Cols. 43 - 52
Longitude of centre of rotation
Cols. 53 - 62
Angle of rotation
Col. 67
0 = no lineaments to be plotted

Data Deck
Five data points per card each point consisting of a co-latitude followed by longitude in decimal degrees in fields of .8 columns.
Blank or zero co-ordinates are ignored.
Pen is lifted on encountering a co-latitude of 999.
Continent ends on encountering a co-latitude of 888.
Lineament deck follows with cards in same format as for lineament analysis program. Last card has 1 in column 80.
Repeat card 3 and data decks for each continent.

Subroutines Called:
CPLOT, RAD, DEG, ROT, ORTHO, CARE, LINPLT.
C PROGRAM TO PLOT CONTINENTS
DIMENSION XX(5), YY(5), CENT(2), NAME(8), X1(1000), Y1(1000)
READ(1), ITLS, TS, BLAT, DLong
1 FORMAT(4E10.5)
TS = ADJS(TS)
R$= ADJS(R$)
Q$= R$
SY = 5.
S$= 5.
AN$= 0.
DO$= 3.141592/180.
40 CALL STPLT(-1.,-1.,1.,1.)
DO 44 J=1,2
DO 43 K=1,361
AN$= AN$+D$
Y1(K) = DE8IN(AN$)+SY
44 CALL CPLT(361,0,1,X1,Y1)
45 S$= S$+12.
46 READ(1,20) NCONT
201 FORMAT(16)
DO 600 UK=1,NCONT
40 CALL STPLT(-1.,-1.,1.,1.)
DO 44 J=1,2
DO 43 K=1,361
AN$= AN$+D$
Y1(K) = DE8IN(AN$)+SY
44 CALL CPLT(361,0,1,X1,Y1)
45 S$= S$+12.
46 READ(1,20) NCONT
201 FORMAT(16)
DO 600 UK=1,NCONT
READ(1,20) NAME, CENT, ANG, ILIN
202 FORMAT(84,3E10.2,4,15)
CENT(1) = RAD(CENT(1))
CENT(2) = RAD(CENT(2))
ANG = RAD(ANG)

T$= 1.

CALL ROT(T$, F$, CENT(1), CENT(2), ANG, IT$, FF, 1)
WRITE(3, 203) NAME, CENT, ANG, ILIN
203 FORMAT(16)
DO 49 K=1,5
IF(X(K)-599.)101,100,101
101 IF(X(K))1270,1270,1270
1271 IF(Y(K))1272,1274,1272
1270 CONTINUE
1272 CONTINUE
49 CONTINUE

CALL ROT(T$, F$, YY(K))
T$ = RAD(YY(K))

CALL ROT(T$, F$, CENT(1), CENT(2), ANG, IT$, FF, 2)
CALL ORTHU(T$, FF, XS,YS, IT6, 0, TS, FS)
204 FORMAT(108, 2)
46 CALL CARET(1, IT6, XS, YS)
GO TO 46
100 CALL CARET(2, IT6, XS, YS)
CALL LINPLT(ILIN,CENT, ANG, TS, FS)
IF(Y(K))-888.)49,690,49
49 WRITE(3, 50)
50 FORMAT(16)
GO TO 50
600 CONTINUE

C DRAWING OF LAT AND LONG LINES
CL = DLAT
ILAT=(180./DLAT)-0.2
30 80 K=1,ILAT
CALL RAD(CL)
N1 = 0
N2 = 0
CLN = 0.
DO 81 J = 1, 360
CLX = RAD(CLN)
CALL OTHO(CLX, CLL, XS, YS, ITH, Q, TS, FS)
CALL CAPE(1, ITH, XS, YS)
81 CLN = CLN + 1.
DO 91 CALL CAPE(2, ITH, XS, YS)
91 CL = CL + 1.
END

---

**CALL CAPE(1), XS, YS**

END

---

**CALL CAPE(2), ITH, XS, YS**

END

---

**CALL ENDPLT(1)**

**CALL ENDPLT(-1)**

---

**CALL RAD, DEG, TWD**

---

**Subroutines Required**

---

**Purpose**

- **Radians**
- **Degrees**
- **TWD**

---

**Input**

- **CL**
- **J**

---

**Output**

- **XS, YS**
- **ITH, Q, TS, FS**

---

**Method**

- **OPI**
- **OPI + 0.1**

---

**Example**

- **DO 91 J = 1, 180**
- **CALL RAD(CL)**
- **CALL OTHO(CLX, CLL, XS, YS, ITH, Q, TS, FS)**
- **CALL CAPE(1, ITH, XS, YS)**
- **91 CL = CL + 1.**
- **CALL CAPE(2, ITH, XS, YS)**
- **CALL ENDPLT(1)**
- **CALL ENDPLT(-1)**

---

**Notes**

- **Longitude of same point**
- **Col. 31 - 30**
- **Co-latitude of second point on first continent**
- **Col. 31 - 40**
- **Longitude of same**
- **Col. 41 - 50, 51 - 60, 61 - 70, 71 - 80**
- **Repeat for two points on second continent**
- **Repeat similar cards as desired**
- **End with blank card**

---

**Subroutines Required**

- **RAD, DEG, TWD**
PROGRAM TO GET CENTRES OF ROTATION

Purpose:

To calculate centres and angles of rotation between continents.

Method:

Calls subroutine TWOPT previously described.

Input Data Cards:

Card 1

Cols. 1 - 10
   Co-latitude of point on first continent
Cols. 11 - 20
   Longitude of same point
Cols. 21 - 30
   Co-latitude of second point on first continent
Cols. 31 - 40
   Longitude of same
Cols. 41 - 50, 51 - 60, 61 - 70, 71 - 80
   Repeat for two points on second continent
Repeat similar cards as desired.
End with blank card.

Subroutines Required:

RAD, DEG, TWOPT
BEGIN

READ TWO POINTS FROM
BACK CONT.

CONVERT POINTS
to RADIANS

CALL THROT
TO GET ROTATION
ANGLE, CENTRE

PRINT
RESULTS

YES

ANY
CARDS LEFT?

NO

STOP
C PROGRAM TO GET CENTRES AND ANGLES OF ROTATION
DIMENSION TF(8), G(8)

100 READ(1,10)TF
10 FORMAT(8F10.5)
101 CONTINUE
GO TO 70
70 TF(I)=RAD(TF(I))
CALL TANCP(TF(1),TF(2),TF(3),TF(4),TF(5),TF(6),TF(7),TF(8),TC,FC,ANG)
WRITE(3,21)G
TC=DEG(TC)
FC=DEG(FC)
ANG=DEG(ANG)
WRITE(3,22)TC,FC,ANG
GO TO 100
1000 CALL EXIT

21 FORMAT(140,4(2F8.2,2X))
22 FORMAT(1H,74X,2F8.2,2X,F8.2)
20 FORMAT('1 CONTINENT ONE',1X
3 CENTRE 1/ THETA1 PHI1 THETA2 PHI2 ANGLE')
3 THETA2 PHI2 THETA PHI ANGLE
END
PROGRAM TO CALCULATE POISSON POLYGONS

Purpose:
To generate using a Monte-Carlo method random lines in a circle and display histograms of sides, areas and perimeters of resulting polygons.

Method:
See Appendix II.

Flow Chart:
Also in Appendix II.

Input Data Cards:
Card 1

Col. 1 - 2
Number of simulations to be performed

Col. 3 - 14
Starting random number (must be odd)
PROGRAM TO CALCULATE FREQUENCY DISTRIBUTIONS OF POLYGONS DUE TO RANDOM LINES IN A PLANE

1. CRAYN  MAY 1969

COMMON A,B,XP,YP,NPTS
COMMON XI(120),YI(120),YSAVE(130),XSAVE(130)
DIMENSION A(120),B(120),XP(21),YP(21)
DIMENSION XI(120),YI(120),SIF(21)
DIMENSION XSAVE(130),YSAVE(130)

PI=3.1415926

RR=1.
TAU=50.
WRITE(3,1001)
1001 FORMAT('1PROG STARTS ////////')

SMN=0.
SMN2=0.
SMN3=0.
SMN4=0.
SMP=0.
SMP2=0.
SMP3=0.
SMP4=0.
SMS=0.
SMS2=0.
SMS3=0.
SMS4=0.
SMA=0.
SMA2=0.
SMA3=0.
SMA4=0.

GENenerate RANDOM LINES
NPOLY=0
READ(1,1)I,X,Y
WRITE(3,4)I,X,Y
4 FORMAT(1H,I10,T2)
1 FORMAT(I10,I2)
CALL RANDU(I,X,Y,F)
DO 1760 IWW=1,I1WW
NCNT=0
CALL HIST(1,0.,20.,1.)
CALL HIST(2,-10.,5.,5.)
CALL HIST(3,0.,9.7.,5.)
CALL HIST(4,0.,2.3,.025)
READ(1,2)NPTS
2 FORMAT(I16)
WRITE(3,3)NPTS
3 FORMAT('NO OF POINTS ',I6)
DO 200 I=1,NPTS
IX=IY
CALL RANDU(I,X,Y,F)
R=F*RR
IX=IY
CALL RANDU(I,X,Y,F)
ANG=F*PI*2.
A(I)=TAN(PI/2.+ANG)
B(I)=R/SIN(ANG)
200 CONTINUE
**C**

```plaintext
WRITE(3,900)
900 FORMAT(1H )
WRITE(3,992)IX
992 FORMAT(' LAST RANDOM NUMBER ',I12)
778 FORMAT(1H ,5616-8)
779 FORMAT(1H ,1016)
DO 4215 KX=2,NPTS
KTT=KX-1
DO 4316 KY=1,KTT
CALL INTER(KX,KY,X,Y)
XXI(KX,KY)=X
YYI(KX,KY)=Y
YYI(KX,KY)=YYI(KX,KY)
XXI(KX,KY)=XXI(KX,KY)
4316 CONTINUE
4215 CONTINUE
DO 300 I=1,NPTS
WRITE(3,901)I
901 FORMAT(' LINE NO ',I4)
AI=A(I)
BI=B(I)
DO 301 J=1,NPTS
IF(I-J)304,301,304
304 XI(J)=XXI(I,J)
YI(J)=YYI(I,J)
301 CONTINUE
XI(1)=0.
YI(1)=100.
C SORT VERTICES IN ASCENDING ORDER OF Y
CALL SORT(XI,YI)
NPM=NPTS-1
C START FINDING POLYGONS
DO 400 J=1,NPM
IF(INCIR(XI(J),YI(J)))400,400,401
401 CONTINUE
402 XP(1)=XI(J)
YP(1)=YI(J)
C FIRST VERTEX FOUND
600 CALL FIND(A1,B1,1,XP(1),YP(1),A2,B2,JI)
I=I
JS=JI
C FIND CROSS LINE
C DETERMINE DIRECTION OF SEARCH FOR RH POLYGON
439 IF(A1)440,440,441
441 IF(A2)442,442,443
442 IUD=1
INU=1
AS=A1
UP=1.
BS=B1
GO TO 446
C UP LINE 1
443 IF(A2-A1)445,445,444
444 IUD=1
INU=2
AS=A2
BS=B2
```

**C**
UP=1.
GO TO 446
C UP LINE 2
445 IUD=1
INU=1
AS=A1
BS=B1
UP=1.
GO TO 446
C UP LINE 1
440 IF(A2<448,448,447 IUD
=1
INU=2
AS=A2
BS=B2
UP=1.
GO TO 446
C UP LINE 2
448 IF(A1-A2)<449,449,450 IUD
=2
INU=2
AS=A2
BS=B2
UP=-1.
GO TO 446
C DOWN LINE 2
450 IUD=2
INU=1
AS=A1
BS=B1
UP=-1.
C DOWN LINE 1
446 CALL SEC(A1,B1,A2,B2,I,JJ,XP(1),YP(1),IUD,INU,X,Y)
C SECOND POINT FOUND
IF(INCIR(X,Y))451,451,452
452 XP(2)=X
YP(2)=Y
N=2
GO TO (1631,1632),INU
1631 I=I
GO TO 1633
1632 I=J
1633 CONTINUE
560 CALL FIND(A1,B1,A2,B2,I,JJ,XP(N),YP(N),X,Y)
C FIND CROSS LINE
1499 CALL NEXT(A1,B1,UP,A2,B2,I,JJ,XP(N),YP(N),X,Y,UP)
UP=UP2
C FIND NEXT POINT
IF(INCIR(X,Y))451,451,453
453 N=N+1
IF(N<20)1641,1640,1640
1640 CONTINUE
GO TO 451
1641 XP(N)=X
YP(N)=Y
C IS POLYGON CLOSED?
IF(X-XP(1))554,700,554
700 IF(Y-YP(1))554,701,554
C POLY NOT FINISHED
701 N=N-1
5550 NPOLY=N POLY+1
NCNT=NCNT+1
C POLY IS CLOSED
105 FORMAT(1H,216,15F6.3))
F=N
FX=FN+.1
CALL HISTFM(1,FX,1)
SMN=SMN+FN
SMN2=SMN2+FN\textsuperscript{2}
SMN3=SMN3+FN\textsuperscript{3}
SMN4=SMN4+FN\textsuperscript{4}
CALL AREA(N,AB)
AR=AB*TAU*TAU/PI
AG=AR
SMA=SMA+AB
SMA2=SMA2+AB\textsuperscript{2}
SMA3=SMA3+AB\textsuperscript{3}
SMA4=SMA4+AB\textsuperscript{4}
CALL HISTFM(4,AR,1)
CALL PERIM(N,S,SID)
SS=S\textsuperscript{2}+AB
SS2=SS2+SS\textsuperscript{2}
SS3=SS3+SS\textsuperscript{3}
SS4=SS4+SS\textsuperscript{4}
CALL HISTPT(I KC)
NCNT=0
GO TO 400
C GO TO NEXT POLYGON
4760 WRITE(3,4761)IW
4761 FORMAT(1H,9,1E16.8)
CALL EXIT
END
SUBROUTINE SORT(XI,YI)
COMMON A(3),XP,YP,NPTS
DIMENSION XI(120),YI(120),XM(21),YP(21)
DIMENSION XI(1000),YI(1000)
NM=NPTS-1
DO 30 I=1,NM
IXCH=0
IN=NPTS-1
IF(YI(J)-YI(J+1))30,30,31
31 YS=YI(J)
YS=XS(J)
YI(J)=YI(J+1)
XI(J)=XI(J+1)
YI(J+1)=YS
XI(J+1)=XS
IXCH=IXCH+1
30 CONTINUE
IF(IXCH)20,40,20
20 CONTINUE
RETURN
END

SUBROUTINE INTER(L,M,X,Y)
DOUBLE PRECISION A1,A2,B1,B2,XO,YD
COMMON A,B,XP,YP,NPTS
COMMON XI(120),YI(120),XM(21),YP(21)
DIMENSION A(120),B(120),XM(21),YP(21)
IF(L-M)371,371,372
372 I=M
J=L
GO TO 373
371 I=M
J=M
373 CONTINUE
A1=A(I)
A2=A(J)
B1=B(I)
B2=B(J)
XO=(B1-B2)/(A2-A1)
RETURN
END

SUBROUTINE FIND(A1,B1,II,X,Y,AP,B2,JJ)
COMMON A,B,XP,YP,NPTS
COMMON XI(120),YI(120),XM(21),YP(21)
DIMENSION A(120),B(120),XM(21),YP(21)
DO 60 T=1,NPTS
51 IF(X-XI(T))60,61,61
50 CONTINUE
A2=0.
B2=1000.
JJ=1
RETURN
63 A2=A(I)
B2=B(I)
JJ=I
RETURN
END
SUBROUTINE PERIM(N,S,SID)
COMMON A,Y,XP,YP,NPTS
DIMENSION XX(120,120),YY(120,120)
DIMENSION A(120),B(120),XP(21),YP(21)
SID(1)=SQRT((XP(1)-XP(1+1))**2+(YP(1)-YP(1+1))**2)
S=S+SID(I)
SID(N)=SQRT((XP(1)-XP(N))**2+(YP(1)-YP(N))**2)
S=S+SID(N)
RETURN
END

SUBROUTINE FINDRT(I,J,X,Y,ICX)
COMMON A,Y,XP,YP,NPTS
COMMON XX(120,120),YY(120,120)
DIMENSION A(120),B(120),XP(21),YP(21)
XX=XX(I,J)
YY=YY(I,J)
ICX=0
DO 600 I=1,NPTS
IF(I-I1)601,600,601
601 IF(I-I1)602,600,603
602 DO 603 I=1,JX=X
IF(I-I1)604,600,603
603 IF(I-I1)604,600,600
604 IF(I-I1)605,700,605
605 ICX=1.
CALL FIND(E,F,JJ,X,Y,A2,B2,ICX)
RETURN
600 X=100.
Y=100.
RETURN
END
SUBROUTINE AREA(N,AA)
COMMON A,0,XP,YP,NPTS
COMMON XXI(120,120),YYI(120,120)
DIMENSION A(120),B(120),XP(21),YP(21)
AA=0.
NT=N-2.
X1=XP(1)
Y1=YP(1)
DO 66 I=1,NT
X2=XP(I+1)
Y2=YP(I+1)
X3=XP(I+2)
Y3=YP(I+2)
F=SQRT((X2-X3)**2+(Y2-Y3)**2)
G=SQRT((XI-X2)**2+(Y1-Y2)**2)
S=0.*G*(D+D+E)
AS=S*(S-C)*(S-D)*(S-E)
IF(AA)790,790,791
790 AA=AA+AS
791 GO TO 66
66 AA=AA+AS
RETURN
END
FUNCTION INCIR(X,Y)
RETURN
200 INCIR=0
RETURN
END

GENERATE using a Monte-Carlo method random
Voronoi polygons and display histograms of sides,
areas and perimeters of the polygons.

SUBROUTINE SEC(A1,B1,A2,B2,I,J,XX,YY,IUD,INU,X,Y)
COMMON A,B,XP,YP,PTS
COMMON XX(120,120),YY(120,120)
DIMENSION A(120),B(120),XP(21),YP(21)
GO TO (100,200),INU
100 C=A1
D=A1
I=1
GO TO 101
200 C=A2
D=A2
JJ=J
I=1
101 GO TO (103,203),IUD
103 DET=1
GO TO 106
203 DET=-1
105 CONTINUE
CALL NEAR(C,D,II,JJ,XX,YY,DET,X,Y)
RETURN
END

SUBROUTINE NEXT(A1,31,UP,A2,02,II,JJ,XX,YY,X,Y,UP1)
COMMON A,B,XP,YP,PTS
COMMON XX(120,120),YY(120,120)
DIMENSION A(120),B(120),XP(21),YP(21)
IF(A1)190,101,101
100 ANG=ATAN(ABS(A1))
ALFI=cos(ANG)*UP
ALFI=sin(ANG)*UP
GO TO 107
101 ANG=ATAN(A1)
ALFI=cos(ANG)*UP
ALFI=sin(ANG)*UP
107 IF(A2)>200,201,201
200 ANG=ATAN(ABS(A2))
ALFI=cos(ANG)
ALFI=sin(ANG)
GO TO 207
201 ANG=ATAN(A2)
ALFI=cos(ANG)
ALFI=sin(ANG)
207 DET=ALFI*BET2-BET1*ALFI2
DET=DET*(-1)
IF(DET)300,300,301
300 UP1=1
GO TO 102
301 UP1=1
302 CONTINUE
CALL NEAR(A2,B2,II,JJ,XX,YY,DET,X,Y)
RETURN
END
PROGRAM FOR THE VORONOI POLYGONS

Purpose:
To generate using a Monte-Carlo method random Voronoi polygons and display histograms of sides, areas and perimeters of the polygons.

Method:
In Appendix II

Input Data Cards:
Card 1
Col. 1 - 12
Starting random number (odd)
Col. 13 - 17
Number of points to be generated for each polygon
Col. 18 - 27
Number of polygons to be generated
C PROGRAM FOR VORONOI POLYGONS

COMMON XI(35,35), YI(35,35), A(35), B(35), XG(35), YG(35)
COMM N NPTS

DIMENSION XP(35), YP(35), SIDES(35)
READ(1, 1) IX, NPTS, NTIMES

1 FORMAT(I12, I5, 110)

RHO=NPTS
RHO=RHO/4.
SQR=SQRT(RHO)
W=0.
WX=20.
WW=9.9
Www=0.1
L=1
CALL HIST(L, W, WX, 1.)
L=2
CALL HIST(L, W, 4.9, 0.05)
L=3
CALL HIST(L, W, 4.9, 0.05)
L=4
CALL HIST(L, W, 4.9, 0.05)
CALL RANDU(IH, IY, F)
IX=IY
WRITE(3, 100) IX, NPTS
100 FORMAT(1H, I12, I5)
NIT=0

DO 7000 IT=1, NTIMES
NIT=NIT+1
IF(NIT<200)2355, 2356, 2357__
2356 CONTINUE
2357 WRITE(3, 2358) IT

2358 FORMAT(1H, I6, ' POLYGONS GENERATED '

DO 50 I=1, NPTS
CALL RANDU(IX, IY, X)
IX=IY
CALL RANDU(IX, IY, Y)
A(I)=(XP(I)/YP(I))*(-1.)
B(I)=YP(I)/2.-A(I)*XP(I)/2.
50 XP(I)=2.*((X-.5)
51 YP(I)=2.*((Y-.5)
101 FORMAT(1H, 10F12.7)
DO 51 J=1, NPTS
55 CALL INTER(I, J, XXX, YYY)
53 CONTINUE
52 CONTINUE
55 CONTINUE

DO 62 J=1, NPTS
L=J
DO 63 I=L, NPTS
D(1)=(-1.):
D(2)=(-1.):
D(3)=(-1.):
D(4)=(-1.):
D(5)=(-1.):
D(6)=(-1.):
D(7)=(-1.):
D(8)=(-1.):

IF(I-J)65,63,65
65 XI(I,J)=XI(J,I)
   YI(I,J)=YI(J,I)
63 CONTINUE
XI(J,J)=100.
   YI(J,J)=100.
   DMIN=100.
   DO 71 I=1,NPTS
      DIST=XP(I)**2+YP(I)**2
      IF(DIST-DMIN)73,71,71
73 DMIN=DIST
   IMIN=I
71 CONTINUE
   DMIN=100.
   DO 72 I=1,NPTS
      IF(I-IMIN)170,72,170
170 DIST=XI(IMIN,I)**2+YI(IMIN,I)**2
      IF(DIST-DMIN)171,71,71
171 DMIN=DIST
      JMIN=I
72 CONTINUE
      XI(I)=XI(IMIN,JMIN)
      YI(I)=YI(IMIN,JMIN)
   FORMAT(1H ,2F10.8,2I4)
   I=IMIN
   J=JMIN
   V1=XP(I)
   V2=YP(I)
   W1=XP(J)
   W2=YP(J)
   CR0S=V1*W2-V2*W1
   IF(CROS)80,80,81
   IF(A(I))91,91,90
   IF(A(I))94,94,95
   IF(A(I))92,92,93
   GO TO 96
90 IF(A(I))94,94,95
94 IUP=2
   GO TO 96
95 IUP=1
96 CONTINUE
   FORMAT(1H ,2I5)
   AA=XG(1)
   BB=YG(1)
   CALL NEXTPT(IL,IUP,AA,BB,AAA,BBB,ILX)
   XG(2)=AAA
   YG(2)=BBB
   N=2
105 FORMAT(1H ,2E14.8,16)
   IL=ILX
   N=N+1
IF(N-20)1220,1220,1221
1221 NX=NX+1
GO TO 7000
1220 IF(B(IL))191,191,190
191 IF(A(IL))192,192,193
192 IUP=1
GO TO 196
193 IUP=2
GO TO 196
190 IF(A(IL))194,194,195
194 IUP=2
GO TO 196
195 IUP=1
196 CONTINUE
NM=N-1
AA=XG(NM)
BB=YG(NM)
CALL NEXTPT(IL,IUP,AA,BB,AAA,BBB,ILX)
XG(N)=AAA
YG(N)=BBB
IF(XG(N)-XG(1))200,201,200
201 IF(YG(N)-YG(1))200,202,200
202 CONTINUE
N=N-1
CALL PERIM(N,S,SIDES)
CALL AREA(N,AB)
AA=AB*RHO
SS=S*0.25*SQR
FN=N
CALL HISTFM(1,FN,1)
CALL HISTFM(2,SS,1)
CALL HISTFM(3,AA,1)
DO 772 KW=1,N
AL=SIDES(KW)*SQR*1.666667
CALL HISTFM(4,AL,1)
772 CONTINUE
106 FORMAT(1H,16,10F10.6)
7000 CONTINUE
WRITE(3,2019)NX
DO 774 KW=1,4
WRITE(3,775)
775 FORMAT(1H1)
CALL HISTPT(KW)
774 CONTINUE
2019 FORMAT(1H,16)
WRITE(3,991X)
99 FORMAT(1H 'LAST RANDOM NUMBER',112)
CALL EXIT
END
SUBROUTINE NEXTPT(IIL, IUP, X, Y, XX, YY, ILX)
COMMON XI(35, 35), YI(35, 35), A(35), B(35), XG(35), YG(35)
COMMON NPTS
GO TO (1, 2), IUP

1 YMIN=1000.
DO 60 I=1, NPTS
61 IF( I-IL)61, 60, 61
62 D=YI( I, IL)-Y
IF(YMIN=D)60, 60, 65
65 YMIN=D
ILX=I
IK=1
60 CONTINUE
IF(IK=1)101, 200, 101

200 XX=XI(ILX, IL)
YY=YI(ILX, IL)
RETURN

2 YMIN=1000.
DO 70 I=1, NPTS
71 IF( I-IL)71, 70, 71
72 D=Y-YI(I, IL)
IF(YMIN=D)70, 70, 75
75 YMIN=D
ILX=I
IK=1
70 CONTINUE
XX=XI(ILX, IL)
YY=YI(ILX, IL)
700 IF(IK=1)101, 100, 101
101 WRITE(3, 202)
202 FORMAT(1 NO POINT FOUND BY NEXTPT*)
100 RETURN
END
APPENDIX II

MONTE-CARLO SIMULATIONS

This appendix contains the text of two papers which concern the Monte-Carlo simulation of random polygons and are closely connected with the thesis material.

The first, "Monte-Carlo estimates of the distributions of random polygons determined by random lines in a plane", has been written jointly by the writer and Dr. H.E. Miller. Initials have been used to denote the authorship of the various sections of the paper. The text included here is of a preliminary draft which is now being extensively revised. However, the basic material remains unchanged.

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Monte-Carlo Simulation of the Random Voronoi Polygons:

Preliminary Results.

Ian K. Crain

Geology Department

Australian National University.

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Of interest are the frequency distribution of the number of sides (neighbours), $n$, perimeter, $s$, and area, $a$. Theoretical results are scarce; known are the mean values of $n$, $s$, and $a$, and a numerical result for the expectation of $a^2$. They are as follows (Miles, 1971; Crain, 1973):

$$s(n) = n^2 - 2n + 2$$
$$a(n) = n$$
$$a^2 = 3.5$$

where $p$ is the intensity of the Poisson point process. The probabilities of occurrence of the various-sided polygons, which I will denote by $p_1$, $p_2$, $p_3$, $\ldots$, are unknown as are the higher order moments and probability distributions of $s$ and $a$.

I am presently proceeding with a Monte-Carlo generation to estimate the unknown moments and approximate the probability distribution functions. The object of this communication is to present preliminary results of this work.

11,082 random Voronoi polygons have been generated by a Monte-Carlo procedure which is to be detailed elsewhere (Crain, 1973). Briefly, a finite rectangle containing Poisson random points is generated and the details of the central polygon are examined. The process of examining only the central polygon eliminates the edge effect problem.
Monte-Carlo simulation of the random Voronoi polygons: Preliminary results.

If 'crystals' (two-dimensional) are allowed to grow uniformly about Poisson-random centres in the plane until mutual contacts prevent further growth, the resulting coverage of the plane by non-overlapping polygons is known as the Voronoi tessellation (Coxeter, 1961; Miles, 1970). Figure 1 shows a typical realization of a portion of such a tessellation.

These random polygons are of interest as stochastic models in many sciences including metallurgy (Gilbert, 1962; Meijering, 1953), cell biology (Lewis, 1946), communications (Shannon, 1949), astrophysics (Kiang, 1966), stereology (Miles, 1971), zoology (Miles, 1971, pers. comm.) and in geology (Smalley, 1966; Crain, 1971; Crain, 1972).

Of interest are the frequency distribution of the number of sides (neighbours), n, perimeter, s, and area, a. Theoretical results are scarce; known are the mean values of n, s, a, and a numerical result for the expectation of a^2. They are as follows (Miles, 1970): E(n)=6, E(s)=4/p^{1/2}, E(a)=1/p, E(a^2)=1.28/p^2 where p is the intensity of the Poisson point process. The probabilities of occurrence of the various-sided polygons, which I will denote by p_1, p_2, p_3, ..., are unknown as are the higher order moments and probability distributions of s and a.

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11,000 random Voronoi polygons have been generated by a Monte-Carlo procedure which is to be detailed elsewhere (Crain, 1972). Briefly, a finite rectangle containing Poisson random points is generated and the details of the most central polygon are examined. The process of examining only the central polygon eliminates the edge effect problem.
which is common in analysing finite realizations of planar processes (Miles, 1971b). Once the polygons are generated, the side-tracing technique employed is identical to that used in previous work on the polygons due to random lines in the plane. (Crain and Miles, 1971). The Monte-Carlo routine was implemented on the IBM 360-50 computer at the Australian National University.

Sample estimates of higher-order moments are:

\[ E(n^2) = 37.8, \quad E(n^3) = 24.9, \quad E(n^4) = 1720 \]

\[ E(p_{2^2}/16) = 1.06, \quad E(p_{3^2}/s^3/64) = 1.17 \]

\[ E(p_{2a^2}) = 1.24, \quad E(p_{3a^3}) = 1.78 \]

Table I shows the frequency of occurrence of the various-sided polygons for 11,000 independent experiments, and sample estimates of \( p_3, p_4, p_5, \ldots \). Figure 2 shows a histogram of observed perimeters and Figure 3 the areas for 5000 independent experiments. (a subset of the total 11,000).

Ultimately, it is hoped to generate 100,000 Voronoi polygons to better refine the estimates of the densities presented here. As this may take some time, preliminary results are now presented which should be of considerable use in hypothesis testing in various disciplines.

I would like to thank Dr. R.E. Miles for interesting me in this problem and his assistance on the theoretical aspects.
TABLE I: Sample frequencies of sides of random Voronoi polygons.

<table>
<thead>
<tr>
<th>n</th>
<th>Frequency</th>
<th>$p_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>125</td>
<td>0.011</td>
</tr>
<tr>
<td>4</td>
<td>1215</td>
<td>0.110</td>
</tr>
<tr>
<td>5</td>
<td>2846</td>
<td>0.259</td>
</tr>
<tr>
<td>6</td>
<td>3172</td>
<td>0.288</td>
</tr>
<tr>
<td>7</td>
<td>2266</td>
<td>0.206</td>
</tr>
<tr>
<td>8</td>
<td>953</td>
<td>0.087</td>
</tr>
<tr>
<td>9</td>
<td>321</td>
<td>0.029</td>
</tr>
<tr>
<td>10</td>
<td>85</td>
<td>0.0077</td>
</tr>
<tr>
<td>11</td>
<td>15</td>
<td>0.0014</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

11000
References:


SMALLEY, I.J. (1966) Geol. Mag., 103 110.
Fig. 1. Typical realization of the Voronoi polygons.

Fig. 3. Histogram of area of Voronoi polygons.
Fig. 2. Histogram of perimeter of Voronoi polygons.
Monte Carlo estimates of the distribution of random polygons determined by random lines in a plane

by I.K. Crain and R.E. Miles

A ROUGH SURVEY OF THE PROBABILITY ASPECTS (by R.E.M.)

§1. L and P

Suppose a line \( \lambda \) in the plane \( E^2 \) is specified by the polar coordinates \((p, \theta)\) of the foot of the perpendicular from the origin \( O \). The range of \((p, \theta)\) is the strip \( T: 0 \leq p < \infty, 0 \leq \theta < 2\pi \). The Poisson point process of intensity \( \tau/\pi \) in \( T \) induces the Poisson line process \( L \) of intensity in \( E^2 \). The lines of \( L \) serve to partition \( E^2 \) into an aggregate \( P \) of random polygons. For a review of the interesting properties of \( L \) and \( P \), see Miles (Proc. Nat. Acad. Sci., 1964).

Fairly immediate properties are

(i) \( L \) is strictly stationary, i.e. stochastically invariant with respect to arbitrary translations in \( E^2 \). Since, moreover, \( L \) is clearly isotropic, \( L \) is stochastically invariant with respect to arbitrary Euclidean motions in \( E^2 \).

(ii) The points of intersection of an arbitrary line \( \lambda \) with \( L \) constitute a linear Poisson point process of intensity \( 2\tau/\pi \), the associated angles of intersection being mutually independent with common p.d.f.

\[ \frac{1}{2} \sin \theta \quad (0 \leq \theta \leq \pi) . \]

(iii) The number \( M \) of lines of \( L \) intersecting an arbitrary convex figure of perimeter \( S \) has a Poisson distribution with mean value \( \tau S/\pi \). Furthermore, given that \( M = m \), the \( m \) lines are independently and identically distributed.

Now turn to \( P \). Important characteristics of a convex polygon are \( N \), the number of sides (or vertices); \( S \), the perimeter; \( A \), the area; and \( D \), the in-circle diameter.
Let $Z$ represent a finite set of such characteristics and $Q(q)$ represent the disc in $E^2$ with centre $O$ and radius $q$. If $R_q(Z)$ represents the empirical d.f. of $Z$ for those polygons of $P$ lying completely within $Q(q)$, then $F_q(Z)$ as $q \to \infty$. (see Miles (Ph.D., 1961)). In other words, $Z$ for $P$ has a well-defined \textit{ergodic distribution}, specified by $F(Z)$. Many of the known properties of $F(Z)$ are given in §3 below. It is the main purpose of this paper to estimate the unknown properties of $F(N,S,A)$ by a Monte Carlo simulation.

§2. The fundamental importance of $L$

There follows a heuristic discussion of why $L$ enters naturally as a limit when (sufficiently independent) isotropic uniform random curves are superposed in $E^2$. A rigorous theory has yet to be developed, but it is clear that such a theory will parallel that of Goldman (Ann. Math. Statist., 1967) for the superposition of point processes in $E^2$.

Suppose $C$ is a smooth curve in $E^2$ of uniformly bounded curvature $\kappa$ and length $\lambda < \infty$. Suppose $X$ is a domain of $E^2$, and denote its boundary by $\partial X$. We assume $0 < |X| < \infty$. To fix ideas, the linear dimensions of $X$ are assumed to be much greater than $\lambda$. We define the random image $\tilde{C}$ of $C$ in $X$ to be the curve obtained by random Euclidean transformation of $C$, as follows:

(i) the centroid of $C$ $\to$ a uniform random point $\tilde{P}$ of $X$.

(ii) the orientation of $\tilde{C}$ about $\tilde{P}$ is independently uniform.

[Notice that this is the probability distribution induced by the kinematic density of integral geometry Santaló ("Integral Geometry", 1953)].

Let $y$ be a fixed point of $X$, such that the distance of each point of $\partial X$ from $y$ exceeds $\lambda'$, where $\lambda' > \lambda$; and suppose $Q(\varepsilon)$ is the closed disc with centre $y$ and radius $\varepsilon$. These conditions ensure that edge effects do not influence the intersection properties of $Q(\varepsilon)$ and $\tilde{C}$. Thus, it is easily proved that
Suppose in what follows that it is given that \( Q(\varepsilon) \neq \emptyset \).

Let \((\varepsilon \rho - \delta, \theta)\) be the polar coordinate with respect to \( y \) of parallel lines almost surely uniquely defined by the properties that \( Q(\varepsilon) \cap C \) lies between or on the two lines, and that \( \delta \) is minimal. Then, as \( \varepsilon \to 0 \),

(i) \( \delta \to 0 \) (by the bounded curvature);

(ii) the distribution of \((p, \theta)\) is uniform on \((0,1) \times (0,2\pi)\) (by the uniformity of the stochastic construction).

Now let \( \tilde{C}_1, \ldots, \tilde{C}_n \) be \( n \) independent replicas of \( \tilde{C} \).

Suppose \( n \to \infty \) and \( \varepsilon \to 0 \) in such a way that

\[
2n\varepsilon \lambda / |X| \to \rho, \quad \text{constant.}
\]

Then, in the usual way, the distribution of the number of curve intercepts with \( Q(\varepsilon) \) tends to Poisson \((\rho)\). In fact, the \((p, \theta)\)-values of the curve intercepts with \( Q(\varepsilon) \) correspond asymptotically to a Poisson process of intensity \( \rho \) on \((0,1) \times (0,2\pi)\). That is, the local realisation of \( \tilde{C}_1, \ldots, \tilde{C}_n \) for large \( n \) is approximately \( L \) with \( \tau = n\lambda / |X| \). Extending slightly, the asymptotic local realisation for non-identical \( \tilde{C}_1, \ldots, \tilde{C}_n \) is \( L \) with \( \tau = \sum \lambda_i / |X| \) (mild sufficient conditions are \( 0 < l_i < k_i < k \)). Clearly the same property holds for dependent \( \tilde{C}_i \), provided they are "sufficiently independent".

Even more general instances of curve superpositions yielding \( L \) limits may be described, but at this point we prefer to end the discussion, trusting that the fundamental importance of \( L \), and consequently \( P \), at least in one respect, has been demonstrated.
§3. Known relevant exact distributional properties of $P$

The only known absolute distribution is that of $D$:

$$f(D) = \tau e^{-\tau D} \quad (D \geq 0) \quad \text{(exponential)}$$

The only other known distribution is the conditional distribution of $S$ given $N$:

$$f(S \mid N) = \left(\frac{\tau}{\pi}\right)^{N-2} s^{-3} e^{\frac{\pi}{\tau} s} / (N-3)! \quad (S > 0) \quad (\Gamma(N-2, \frac{\pi}{\tau}))$$

"Left Tails" are as follows:

$$P(N=3) = 2 - \pi^2 / 6 = 0.3551$$

$$f(S) = \left(12 - \pi^2\right) \tau / 6\pi + O(\tau^2 S) \quad (0 < S << \tau^{-1})$$

$$f(A) = \frac{1}{c \tau A^2} + O(\tau^2) \quad (0 < A << \tau^{-2})$$

where

$$c = \frac{1}{3\tau} \int_0^\pi \int_0^{\pi - \phi} 2 \sin \phi \sin \psi \sin(\phi + \psi)^{1/2} \, d\psi \, d\phi.$$

Moments:

$$E(N) = 4 \quad E(S) = 2\pi / \tau \quad E(A) = \pi / \tau^2$$

$$E(N^2) = (\pi^2 + 24) / 2$$

$$E(SN) = \pi (\pi^2 + 8) / 2\tau \quad E(S^2) = \pi^2 (\pi^2 + 4) / 2\tau^2$$

$$E(AN) = \pi^3 / 2\tau^2 \quad E(AS) = \pi^4 / 2\tau^3 \quad E(A^2) = \pi^4 / 2\tau^4$$

$$E(NA^2) = \pi^4 (8\pi^2 - 21) / 21\tau^4 \quad E(SA^2) = 8\pi^7 / 21\tau^5 \quad E(A^3) = 4\pi^7 / 7\tau^6$$

Inter-moment relations:

$$E(SA^{m-1}) = 2\tau E(A^m) / m \quad (m = 1, 2, \ldots)$$

$$E(N+i+j+2k-3)^i s^j A^k = (\tau / \pi)^j E(S^{i+j} A^k) \quad (i, k = 0, 1, \ldots; j = 1, 2, \ldots).$$
§4. Underlying theory of experimental method

The Monte Carlo procedure consisted of generating 45 independent realisations of $L$ with $\tau = 50$ within $Q(l)$. Thus the total number of polygons generated was of the order of $45 \times 2,500 = 100,000$. The stochastic construction of $L$ within $Q(l)$ is straightforward:

(i) sample Poisson (100) variate, $M$ say;

(ii) generate $M$ independent identically distributed random secants in $Q(l)$ according to the uniform distribution on $(0,1) \times (0,2\pi)$.

For further details, see below. Rough design considerations were as follows. A disc is the best shape for the containing region, in view of the simple $(p,\theta)$ specification of $L$. $\tau = 50$ was chosen in consideration of computer storage restrictions. 100 replications (each of the order of 10 minutes duration) allowed results of fair precision with reasonably narrow confidence bands.

At this point it seems worthwhile to stress that this study was exclusively directed towards the probability distributions of the fundamental (in the integral geometry sense) characteristics $N, S, A$. Other variables, such as the minimum width, maximum width (= diameter) radius of smallest disc containing the polygon, interior angles, etc. are not only of less fundamental importance, but are generally speaking much more difficult to compute. Thus they have been ignored in this study. However, the coordinates of all the vertices, together with the perimeter and area of each polygon, 100,000 of each have been stored on tape. This will allow fairly easy investigation of other aspects of $P$, should a future need arise.

The actual computations for each realisation were of two types:

I. "Obvious".

$N, S, A$ for each of the polygons contained completely within $Q(l)$ were computed. From these values, natural estimates of the individual and joint distribution of $N, S, A$
were made. Since partial exact results relating to these distributions are known (§3), it was possible to slightly improve these estimates - see below.

II. "Trick".

Suppose $s$ is a segment of a line of $L$ between adjacent vertices. The removal of $s$ from the system causes the two polygons with $s$ as common side to coalesce into one larger convex polygon.

Write $P^*$ for the class of polygons obtained in this way.
If $f(N, S, A)$ is the p.f. (i.e. combined p.m.f./p.d.f.) for $P$ then the p.f. of $P^*$ is (see Miles (1961))

\[ f^*(N, S, A) = S f(N, S, A) / E(S) = \frac{TS}{2\pi} f(N, S, A) / 2\pi. \]

In particular,

(a) $f^*(S) = \frac{TS}{2\pi} f(S) / 2\pi$.

Again,

\[ f^*(N) = \int f^*(N, S) \, dS \]

\[ = (\frac{\tau}{2\pi}) \int S f(N, S) \, dS \]

\[ = (\frac{\tau}{2\pi}) f(N) \int S f(S, N) \, dS \]

\[ = (\frac{\tau}{2\pi}) f(N) \frac{(\frac{\tau}{\pi})^{N-2}}{(N-3)!} \int S^{-2} \frac{\tau}{\pi} e^{-S} \, dS \]

\[ = \frac{N-2}{2} f(N). \]

In a large area, there are roughly twice as many members of $P^*$ than of $P$. Hence, taking (2), (3) into account, it is clear that counting $P^*$ allows a more efficient estimation of $f(N)$, $f(S)$ for the less frequent larger $N, S$ values.
Outline of the method.

The computer program to generate random lines, locate the vertices of the polygons thus created, and calculate the perimeter and area of these figures is an exercise in logical, rather than mathematical programming. The actual mathematics of computation involved is trivial, whereas, the logic flow is somewhat complex.

Reference to the flow diagram of fig.1 may aid in understanding the following brief outline of the procedure.

The first step is the generation of, M. Poisson-random points in the rectangle \( 0 \leq \theta \leq \pi, 0 \leq p \leq 1 \). This is equivalent to the generation of M random lines within a circle of unit radius. The slope and intercept of these lines are then calculated and placed in internal core storage. If a line is characterized by the point \((\theta, p)\), its slope \(a\) and \(y\) - intercept, \(b\), are given by

\[
a = \tan (\theta + \pi) \\
b = p \sec (\pi - \theta)
\]

Next, all the points of mutual intersection of the lines are calculated by the simultaneous solution of

\[
\begin{align*}
y_{ij} &= a_{i} x_{ij} + b_{i} & i = 1, M & i \neq j \\
y_{ij} &= a_{j} x_{ij} + b_{j} & j = 1, M \\
\end{align*}
\]

giving

\[
x_{ij} = \frac{b_{i} - b_{j}}{a_{i} - a_{j}} & i \neq j \\
y_{ij} = \frac{a_{j} b_{i} - a_{i} b_{j}}{a_{j} - a_{i}}
\]

The resulting points are stored in an array for future reference.

The actual search for polygons commences with the first of the generated lines, referred to for convenience in the sequel as the "Primary" line. The lowest point of intersection which is in the unit circle on the primary line is located.
At any given intersection there are four possible polygons which could be examined. For definitiveness, the polygon which lies entirely to the right of the point is chosen. In this manner each polygon would be determined twice. To eliminate this duality, the polygon is only counted when the primary line is "lower" than the intersecting line. For example, the polygon corresponding to the intersection of lines "three and six" would be counted, whereas that corresponding to lines "six and three" would not. The appropriate polygon is determined by an examination of the orientation of the two lines passing through the point. Polygons are delineated in the clockwise direction, so that the search to locate the second point may proceed in either the upward or downward direction on either the primary line or the cross line (fig. 2). Having determined the sense of the search and the line along which the search is to proceed, the nearest point on this line is then located. Thus the second vertex of the polygon is determined.

The cross line through the second vertex is identified, the sense of the search determined and the next vertex located. In order to ensure clockwise searching, the lines are considered as being directed. The line from which the search is proceeding has known direction cosines \((\alpha, \beta)\). The equation of the intersecting line is known so that its direction cosines \((\gamma, \delta)\) are known except for sign. The sign is determined by insisting that the determinant

\[
\begin{vmatrix}
\alpha & \beta \\
\gamma & \delta
\end{vmatrix}
\]

is positive. Thus the search direction for the next vertex is determined.

From the third vertex onwards, the newly calculated point is compared with the first vertex. If they are identical, the search has closed upon itself, and the entire polygon has been calculated. While this search is proceeding, the number of sides are counted, and vertices stored. At the closure of the polygon the perimeter, area and other measures may then be calculated.
Given a triangle with sides $a$, $b$, $c$ the area is given by

$$A = \sqrt{S(S-a)(S-b)(S-c)} \quad (4)$$

where $S = \frac{(a + b + c)}{2}$

The area of an $N$-gon is determined by subdividing it into $N-2$ triangles and using equation (4).

If, however, any of the determined vertices lie outside the unit circle, the polygon is rejected. After either such an "open" or a truly "closed" polygon has been determined and mensurated, the process is repeated for the next higher intersection on the primary line.

The occurrence of an intersection point outside the circle indicates that all valid polygons on the primary line have been determined. The next of the originally generated lines then becomes the primary line and is examined in a similar manner until all polygons are evaluated. Fig. 3 shows the sequence of events for a simple case of four lines.

In order to more efficiently evaluate the p.d.f. of $N$ for high $N$, a set of $P^*$ polygons was also created. This was performed independently of the "normal" polygons and the slightly altered logic is not shown in Figure 1. The basic change is that succeeding the location of the first vertex, the primary line is artificially shifted outside the unit circle, so that it cannot be "found" by the search procedure. After the polygon is located, the primary line is re-instated and the next polygon is located as before.

**Program Design**

The program was modular in design for ease in coding and testing. The main program carries out the gross logic, subroutines being used for repetitious operations. The primary subroutines were used to find the next vertex given the equation of the line and sense of search; to identify the cross line, given the first line and point of intersection; and to locate the second vertex. Minor subroutines were used to find the intersection of two lines, to determine whether a point fell within the unit circle, to calculate areas and perimeters, and to accumulate and print histograms.
The output of the program consisted of a continuous listing of the number of sides, vertices, perimeter and area of each polygon. These parameters were also recorded on magnetic tape for archival purposes. Histograms of the number of sides, perimeter, and area were printed after each 1000 polygons, along with the mean sample values of $N$, $S$, $A$, $N^2$, $S^2$, $A^2$ etc. The language used was FORTRAN IV and approximately 430 statements were required. The simulation was performed on the IBM 360 - 50 computer at the Australian National University Computer Centre. Random number generation was done using a routine from the IBM Scientific Subroutine Package.

In order to perform the simulation with reasonable efficiency, a great deal of core storage was employed. In order to deal with an average of 100 lines approximately 180 K bytes of storage were required. Compile and "linkedit" time was approximately 1 minute and polygons were measured at a rate of about 200 per minute of run time. It is obvious that in spite of the precautions taken to optimize the run time at the expense of core storage, the procedure is still rather slow. The 200,000 polygons generated thus represents about 15 hours of computer operation.

Accuracy and Sources of Error

A computer of the type used has, in theory, an accuracy of eight significant figures. Some of this accuracy may be lost, however, through roundoff error in numerical calculation, such as by the repetitious addition of small quantities to a large number, or through the subtraction of nearly equal quantities. This roundoff error will be introduced in the calculation of the coordinates of the vertices and in the area and perimeter calculations. Two facts suggest, however, that such error will be unimportant. First, there are no repetitious calculations, and the point-of-intersection calculations are such that at least 5 significant figures will be maintained, except in the highly unlikely event of "parallel" lines. Second, roundoff error will be unbiased, so that its effect on histograms and mean values will be negligible. As an additional precaution double-precision arithmetic was employed in critical calculations.
Although it is possible that polygons have had sides, in effect, added or subtracted due to errors in the vertices, this effect has not been observed in any of the polygons that have been examined in detail during the early testing of the program. The incidence of this "counting" error must be very small. Error introduced through roundoff is considered, therefore, to be several orders of magnitude less than the sampling error.

One instance of a "two-sided" polygon has been observed, that is, the second and third vertices were identical to 8 significant figures, so that it was, in effect, a triangle of "zero" area. Due to their larger size (in general), it is highly unlikely that any quadrilaterals were misidentified as triangles.

The simulation of an infinite process by a finite realization introduces an error or "edge effect" due to the elimination of the incomplete polygons intersected by the boundary. This edge effect was found to be significant and was corrected using the methods described by Miles (1971b).

Summary of Results (by I.K.C. and R.E.M.)

Table I summarizes the estimate of the probabilities $p_n$ of the various sided polygons.

Figure 4 shows a histogram of the distribution of perimeters $S$. Figure 5 depicts the area $(A)$ distribution.

Table II shows sample estimate of some of the higher order moments.
Fig. 1. Flow diagram for program to delineate random polygons.
Fig. 2. Possible search direction.
Fig. 3. Search sequence for case of four lines.
Fig. 4. Histogram of perimeters of Poisson polygons.
Fig. 5. Histogram of area of Poisson polygons.


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* Known value all others estimated by Monte-Carlo methods.
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