STUDIES OF TENSOR POLARIZATION AND ELASTIC SCATTERING IN FEW-NUCLEON SYSTEMS

by

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A Thesis submitted in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY
(Nuclear Physics)

At the
Australian National University, March, 1967.
PREFACE

The experiments presented in this thesis were all performed using the Tandem Van de Graaff electrostatic accelerator in the Department of Nuclear Physics of the Australian National University.

The work discussed in chapters 1-2 and 5-6 was carried out in collaboration with Drs. G. G. Ohlsen and P. G. Young. The work described in chapter 3 was performed in collaboration with Dr. P. G. Young and the experiment described in chapter 4 was carried out by myself with the help in running the accelerator provided by Dr. G. U. Din and Mr. H. Cords. During the first two years my supervision was conducted by Drs. G. G. Ohlsen and P. G. Young. For the last year the sole supervisor was Professor E. W. Titterton.

The work described in this thesis has been reported in the following publications:

"Tensor Polarization in alpha-Deuteron Scattering"
P.G. Young, G.G. Ohlsen and M. Ivanovich,
Nuclear Physics A90(1967)41.

"Tensor Polarization of Deuterons from p-d Elastic Scattering"
P.G. Young, M. Ivanovich and G.G. Ohlsen,
(ii)

"Tensor Polarization in p-d Elastic Scattering"
P.G. Young and M. Ivanovich,

"Tensor Polarization in the $^9$Be(p,d)$^8$Be Reaction"
M. Ivanovich, H. Cords and G.U. Din,
Nuclear Physics (to be published A94(1967)).

"Alpha-Triton Elastic Scattering"
G.G. Ohlsen, M. Ivanovich and P.G. Young,
Phys. - to be published).

"The Scattering of Protons, Deuterons and $^3$He by
Tritium"
M. Ivanovich, G.G. Ohlsen and P.G. Young

Thanks are due to Dr. G. G. Ohlsen for his super­
vision and guidance during my first year at the A.N.U.
It is a pleasure to thank Dr. P.G. Young for his continued
guidance and friendship he has given me not only while he
was at the A.N.U. but also after he left us. I am also
grateful to Professor E. W. Titterton for his support and
encouragement. I am grateful to the Australian National
University for the award of the Research Scholarship.
Thanks are due to Dr. Garry Crawley for useful discussion
on my first draft of the thesis. Thanks are also due to the
tandem technical and workshop staff, in particular to Mr. T. A. Brinkley and Mr. G. P. Clarkson, to the former for eager help I could always count on and to the latter for providing most of the drawings for this thesis. Finally, I would like to thank Miss, Terry Sampson for typing my manuscript which must have been a major task considering my bad hand-writing.

No part of this thesis has been submitted for a degree at any other university.

M. IVANOVICH
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INTRODUCTION

In recent years there have been considerable advances in the techniques used for the investigation of nuclear properties in low energy Nuclear Physics. The result of this work has been accumulation of great wealth of experimental data on nuclear energy levels and on the interactions between nuclei. This has led to the development of nuclear models which have proved to be successful in interpreting much of the experimental material.

In the field of nuclear reactions and scattering processes spatial orientation may be introduced into the system whenever the direction of the beam of incident particles is defined. Hence in many cases anisotropic effects associated with the emitted or scattered particles can be expected. By observing these effects it is possible to deduce information about the mechanism of the nuclear reaction or scattering process in question. The ambiguities and uncertainties which arise in the interpretation of angular distributions of such processes can be reduced and even eliminated in some cases, by studying the polarization of their products.

The measurement of the spin polarization of particles resulting from nuclear reactions or scattering has been almost entirely restricted to spin-$\frac{1}{2}$ particles. The usual
method of determining the polarization of such a beam of particles is by elastic scattering with a left-right azimuthal observation. Such polarization measurements serve as a supplement to differential cross section measurements which are usually relatively insensitive to the spin dependence of the nuclear interactions. Hence, polarization experiments with particles of spin greater than $1/2$, (e.g. deuterons) should be of even greater interest owing to the fact that the spin dependence of their interaction with nuclei is correspondingly more complex.

In both the spin-$\frac{1}{2}$ and the spin-one case, polarization experiments provide information on the non-central interactions. However, in addition to the usual central and spin-orbit interactions used for nucleons in optical model considerations, additional tensor coupling is allowed for spin one particles (Sa 60a, Sa 60b). Nevertheless, very little is known about the importance of either vector or tensor non-central forces in deuteron scattering or reactions at present.

Relatively few experiments have been done involving polarized deuterons at lower energies. Double scattering measurements near 6 MeV (Al 60, Gu 53, Ne 64) present results in the form of asymmetries between left, right and up azimuthal positions. Polarization has been detected by
using a reaction with an unknown analyzing power (Ba 60, La 61, Po 61a, Ve 66) and polarization parameters have been measured using a known analyzer (Po 61b, Mc 64b, Se 64, Mc 64b, Da 65). Additional experiments have been done using a polarized ion source at about 22 MeV (Be 63, Ar 66).

The experiments to be described here involve oriented recoil deuterons obtained from elastic scattering of protons and alphas from a deuterated polyethelene target and oriented deuterons from the $^9\text{Be}(p,d)^8\text{Be}$ reaction. Only the components of the second-rank tensor were measured here although in order to describe fully the deuteron spin orientation both vector and second-rank tensor components are required. The $^3\text{He}(d,p)^4\text{He}$ reaction was used as a polarization analyzer with known analyzing power. All measurements were done in the tandem Van de Graaff accelerator energy range (between 3 and 11 MeV) and at forward laboratory angles. They extend the $\alpha$-$d$ elastic scattering measurements of McIntyre and Haeberli (Mc 64b) to large c.m. angles and the measurements of Darden and Froelich for the $^9\text{Be}(p,d)^8\text{Be}$ reaction to energies up to 10 MeV.

In the case of $\alpha$-$d$ scattering the purpose of the investigation was two fold since the results can be used in two different ways. Firstly, the knowledge of polarization parameters in the $\alpha$-$d$ scattering would make $^4\text{He}$ a useful
analyzer in deuteron double scattering experiments. Secondly, these measurements can be used in an investigation of the α-d scattering process and the energy levels of the compound nucleus $^6$Li (through a phase shift analysis of the scattering). In the case of the p-d scattering and the $^9$Be(p,d)$^8$Be reaction, the measurements were made with the purpose of establishing the importance of non-central interactions in the two processes. In particular, the knowledge of deuteron spin orientation parameters in direct nuclear interactions can provide information as to how important the tensor coupling terms are in optical model considerations.

Other experiments described in this thesis involve systems composed of from four to seven nucleons. The work was undertaken in an effort to provide data which may be generally useful in acquiring further knowledge on the energy level structure of some of the very light nuclei. An energy level search over limited excitation regions in $^4$He, $^5$He, $^6$Li and $^7$Li nuclei was conducted by elastically scattering protons, deuterons, $^3$He- and $^4$He-particles from tritium gas.

The theory of the description and measurement of the polarization of spin-one particles is discussed in chapter 1. In particular the analyzing power of the $^3$He(d,p)$^4$He reaction is considered. Chapter 2 offers a discussion on
the deuteron polarization measurements for α-d elastic scattering. A detailed description of the apparatus and the method used in the experiments is provided. This is followed by a presentation of the α-d polarization data and a discussion of the results. Chapters 3 and 4 present the results of the measurements of the deuteron tensor moments obtained for the p-d elastic scattering and for the $^9$Be(p,d)$^8$Be reaction, respectively. A summary of the present knowledge of the $^7$Li level scheme up to 8 MeV is given in chapter 5 followed by a full description of the experimental method used in the elastic scattering measurements and a discussion of sources of experimental errors. The $^4$He-T elastic scattering measurements are presented and a phase shift analysis of the angular distribution data is described briefly. Finally, in chapter 6, the data obtained in short experiments on elastic scattering of p-T, d-T and $^3$He-T are presented.
1.1 Description of Spin-One Polarization

The description of the polarization of the deuteron, as applied in the present measurements, was first developed by Lakin (La 55) and subsequently with a different formalism by Stapp (St 55) (see Mc 65, Appendix C). A convenient set of operators first given by Lakin includes the unit matrix, three linear combinations of the spin-one operators, and five second-rank tensor moments formed from linear combination of products of the spin-one operators. The advantages of this particular representation are that these irreducible operators transform in spin space just as the spherical harmonics transform in coordinate space, and further that the second-scattered intensity may be simply expressed in terms of their expectation values, \( \langle T_{qk} \rangle \).

A wave function can only describe a fully polarized state (referred to as a "pure state"). The pure state spin wave function for a particle with arbitrary spin \( s \) can be written:
2.

\[ \chi = \sum_m a_m \chi^{(m)} \]

where \( \chi^{(m)} \) are the \( 2s + 1 \) possible states with magnetic quantum number \( m \), and \( a_m \) are the complex amplitudes of these states.

On the other hand, an unpolarized or partially polarized beam of particles cannot be described by a single wave function and is referred to as a "mixed state". A mixed state can be written as an incoherent superposition of pure states and is conveniently described by the density matrix (La 58, p. 35) the elements of which are defined by (Fa 57, equation 3.4):

\[ \rho_{ij} = \sum_{n=1}^{N} p(n) a_i(n)^* a_j(n) \]

where \( N \) is the number of pure states present, \( a_i(n) \) the amplitudes in the expression of the \( n \)th pure state, and \( p(n) \) the weight of the \( n \)th pure state, with \( \sum p(n) = 1 \). Thus the incoherent superposition is carried out in the formation of the density matrix. Three important characteristics of the density matrix \( \rho \) which follow from its definition are:
3.

(1) \( \text{Tr} \, \rho = 1 \)

(2) \( \rho \) is hermitian

(3) \( \langle 0 \rangle = \text{Tr} (\rho 0) \)

where the symbol 0 is an arbitrary operator, and \( \langle 0 \rangle \) its expectation value. The density matrix gives complete information on the orientation state of a system of particles with arbitrary spin. A \( 2s + 1 \) by \( 2s + 1 \) hermitian matrix with unit trace, therefore involving \( (2s + 1)^2 - 1 \) real parameters, is required.

An alternate method of presenting the same information by the use of spin tensor moments was given by Fano (Fa 57). These parameters were introduced through their relation to the density matrix. The coefficients in the expansion of the density matrix in terms of a complete set of orthogonal basic matrices or operators, completely determine the density matrix and therefore may be used as parameters to specify the spin orientation state of the system. The spin tensor moments are coefficients of this type. A set of \( (2s + 1)^2 \) basic matrices or operators \( U_i \) is chosen with the orthogonality condition
4.

\[ \text{Tr}(U_i U_k^*) = (2s + 1) \delta_{i,k} \]

(\(^*\) indicates the hermitian conjugate) and the density matrix expanded as follows:

\[ \rho = \sum_i a_i U_i^+ \]

Multiplying through by \(U_k\), then taking the trace and applying the orthogonality condition, the coefficients in the expansion are found to be just the expectation values of the \(U_i\)'s. The basic expansion of the density matrix is thus:

\[ \rho = \frac{1}{(2s+1)} \sum_k \langle U_k \rangle U_k^+ \quad (1.1) \]

where the expansion coefficients are the expectation values of the spin tensor moments.

In order to apply this formalism to the case of particles of spin one, a set of nine orthogonal operators must be chosen to serve as a basis for the expansion of the density matrix. The expectation value of these basic operators will then provide a complete description of the spin state of the particles. The following operators \(U_k (=T_{qk})\) are used here (La 55):
The symbols $S_x$, $S_y$ and $S_z$ are the angular momentum matrices for spin one as given by Schiff in equation 24.15 (Sc 55). The expectation value of $T_{00}$ is related to the beam intensity and does not contain information on the spin states involved. In fact $T_{00}$ specifies the normalization, i.e., the trace of $\rho$. In general, particles with spin $S$ will have spin tensor moments of rank up to and including $2S$. The first- and second-rank moments are referred to as the vector and tensor polarization components respectively.

From the conjugation property (Sa 60a)

$$\langle T_{q-k} \rangle = (-)^k \langle T_{qk} \rangle^*$$  \hspace{1cm} (1.3)

it is apparent that eight real numbers are required to
describe the most general spin-one polarization state.
The application of parity and time-reversal restrictions
reduces this number to four. The above follows from
the general formula (Si 53, equation 3.2) which relates
incoming and outgoing spin tensor moments.

If a right-handed coordinate system is chosen
with y axis in the direction $k_{in} \times k_{out}$ and the z
axis in the direction $k_{out}$, where $k_{in}$ and $k_{out}$ indicate
the incoming and outgoing beam directions, the following
restrictions apply:

1. $\langle T_{qk} \rangle$ is real if $q$ is even
2. $\langle T_{qk} \rangle$ is imaginary if $q$ is odd  (1.4)
3. $\langle T_{q0} \rangle = 0$ if $q$ is odd

Thus 1.3 and 1.4 reduce the number of real parameters
characterizing the spin state of an outgoing deuteron
beam from a reaction (or scattering) to $i\langle T_{11} \rangle$, $\langle T_{20} \rangle$,
$\langle T_{21} \rangle$ and $\langle T_{22} \rangle$. According to Satchler (Sa 60a) the
theoretical limits imposed on the values of these para-
meters are
The vector polarization $i \langle T_{11} \rangle$ is proportional to the expectation value of the component of the spin operator in the $y$ direction of the above right-handed coordinate system. The three tensor polarization components involve quadratic combinations of components of the spin operator and hence are not so easy to picture. However, $\langle T_{20} \rangle$ can be thought of as a measure of the average spin projection on the $z$ axis; i.e.

$$\langle T_{20} \rangle = \frac{1}{\sqrt{2}} (|a|^2 + |c|^2 - 2|b|^2)$$

for a spinor of the form $(a, b, c)$.

1.2 Measurement of Spin-One Polarization

The first of the two methods for the measurement of deuteron spin polarization described here involves double scattering. Deuterons produced either in a reaction or elastic scattering are scattered off a second target and the azimuthal angular distribution is measured. The cross section for deuteron scattering may be written

$$\sqrt{\frac{3}{2}} \leq i \langle T_{11} \rangle \leq \sqrt{\frac{3}{2}}$$

$$- \sqrt{\frac{3}{2}} \leq \langle T_{20} \rangle \leq \frac{1}{\sqrt{2}}$$

$$- \frac{\sqrt{3}}{2} \leq < T_{21} > \leq \frac{\sqrt{3}}{2}$$

$$- \frac{\sqrt{3}}{2} \leq < T_{22} > \leq \frac{\sqrt{3}}{2}$$
\[
\sigma(\theta_2, \phi) = \sigma(\theta_2) \left[ 1 + \langle T_{20} \rangle_1 \langle T_{20} \rangle_2 + 2 \langle iT_{11} \rangle_1 \langle iT_{11} \rangle_2 \\
- \langle T_{21} \rangle_1 \langle T_{21} \rangle_2 \cos \phi + 2 \langle T_{22} \rangle_1 \langle T_{22} \rangle_2 \cos 2\phi \right]
\]

where \( \phi \) is the azimuthal angle between the normals to the two scattering (or reaction and scattering) planes; \( \langle T_{qk} \rangle_1 \) refers to the tensor moments after scattering (or reaction) of an unpolarized beam at an angle \( \theta_1 \) by the first target; and \( \langle T_{qk} \rangle_2 \) is the same quantity for the second target at an angle \( \theta_2 \).

One difficulty with this method is that there is no analyzer available for which elastically scattered deuteron polarization has been measured or where the corresponding phase shifts are known sufficiently well to predict this polarization. Furthermore, the differential cross section given by equation 1.6 when integrated over all \( \phi \) is not equal to the cross section for unpolarized incident deuterons. In fact, \( \langle T_{20} \rangle \) determines this difference. This means that in order to measure \( \langle T_{20} \rangle \), a comparison must be made between the cross section for unpolarized and polarized incident beams. Finally, the usual double scattering is not sufficient
to separate the $\langle iT_{11} \rangle$ and $\langle T_{21} \rangle$ parts of the $\cos \phi$ term in equation 1.6. To do this, it is necessary to perform second scattering of two differently polarized beams, one of which has been appreciably changed by the action of a large magnetic field between the first and second targets*. The use of polarized ion sources in which case the deuteron spin axis and polarization can be oriented essentially arbitrarily is an alternate method of separating the parts in the $\cos \phi$ term of equation 1.6.

Another method of analyzing deuteron polarization is by means of a reaction. In principle, any reaction with large enough analyzing power can serve the purpose providing its analyzing power is known experimentally or can be calculated. Galonsky et al. (Ga 59) observed, and Goldfarb (Go 59) calculated, that the reactions $^T(d, n)^4\text{He}$ and $^3\text{He}(d, p)^4\text{He}$, when proceeding through the respective $J^\pi = \frac{3}{2}^+$ resonances, serve as efficient detectors of deuteron polarization.

*In a double scattering experiment, magnetic deflection between the scattering causes a transformation among the various $\langle T_{2k} \rangle$ components of polarization (Bu 60).
The experimental evidence for the feasibility of the $^3\text{He}(d,p)^4\text{He}$ reaction as a deuteron polarization analyzer can be summarized in the following way. A pronounced resonance is observed at a laboratory energy of about 425 keV (Bo 52, Ya 53, Ku 55, Po 58) and the angular distribution of the protons is isotropic up to 800 keV. Resonance parameter fits to the cross section indicate that the resonance occurs with $J^\pi = \frac{3}{2}^+$, formed by s-wave deuterons and emitting d-wave protons. Energies of the observed protons are of the order of 17 MeV since the reaction has a Q-value of the 18.354 MeV (La 66). The full width at half-maximum of the resonance is approximately 500 keV so that the incident deuteron beam need not be well-defined in energy.

Assuming only one matrix element is involved it follows from the general equation given by Welton (We 63, equation 141) that the even-rank spin tensor moments of the reaction products are determined by the even-rank moments of the incident beam, whereas the odd-rank outgoing moments are determined by the odd-rank incoming moments. Since the differential cross section is a tensor of zero order it is regarded as an even-rank
m1. moment, i.e. it is a measure of the second rank (tensor) moments of the incoming deuteron beam. In order to obtain information on the deuteron vector polarization through this reaction the vector polarization of the outgoing protons must be measured (Se 62).

The above remarks apply to the mirror T(d,n)⁴He reaction as well. (Its resonance occurs at a lab energy of 107 keV and the Q-value is 17.590 MeV (Le 66).)

In the experiments described in the following text deuteron tensor polarization was measured by slowing oriented deuterons to an energy that would ensure predominance of the s-wave \( J^\pi = \frac{3^+}{2} \) resonance, and observing the deviation from isotropy of the protons from the resulting \( ^3\text{He}(d,p)^4\text{He} \) reaction. However, the possibility of the presence of other channels in the energy region up to 1 MeV warrants the question as to the extent of the effect of such channels on the analyzing power of the reaction. This important point is discussed in some detail in the following section.
1.3 The Analyzing Power of the $^3$He(d,p)$^4$He Reaction

The analyzing power of the $^3$He(d,p)$^4$He reaction for oriented deuterons, depends on the assumption that this reaction is taking place exclusively through a $J^\pi = \frac{3}{2}^+$ level with incident s-wave deuterons and outgoing d-wave protons. A discussion on this assumption is given below.

As previously stated the observed angular distribution of the protons is isotropic up to 800 keV. States which lead to isotropic angular distributions are those with either $J = \frac{1}{2}$ or $\ell = 0$. Thus, considering the spins of the incident particle ($s_1 = 1$) and the target nucleus ($s_2 = \frac{1}{2}$) the initial states must be either doublets (channel spin $\frac{1}{2}$) or quartets (channel spin $\frac{3}{2}$). The final states, however, can be achieved through only one channel spin available ($s' = \frac{1}{2}$), so they will be doublets. For initial channel spin $\frac{3}{2}$, the transitions yielding isotropy are $^4S_{3/2} \rightarrow ^2D_{3/2}$, $^4D_{1/2} \rightarrow ^2S_{1/2}$, and $^4P_{1/2} \rightarrow ^2P_{1/2}$; for initial channel spin $\frac{1}{2}$, the transitions are $^2S_{1/2} \rightarrow ^2S_{1/2}$ and $^2P_{1/2} \rightarrow ^2P_{1/2}$. Only two values of the total angular momentum are possible in these transitions, and they are $J = \frac{3}{2}$ and $J = \frac{1}{2}$. 

Since the observed isotropy excludes any possibility of mixing of any of the above transitions there are only two feasible cases: one is that only one of the above transitions is involved and the other is that a single transition with initial channel spin $\frac{3}{2}$ and a single transition with initial channel spin $\frac{1}{2}$ are involved. The above is supported by the good fits to the resonance in the $^3\text{He}(d,p)^4\text{He}$ total cross section obtained by assuming that a single $J = \frac{3}{2}$ level in $^5\text{Li}$ is involved (Bo 52, Ya 53, Ku 55, Po 58). Furthermore, the value of the maximum cross section at the resonance is inconsistent with an assignment of $J = \frac{1}{2}$. Thus, it has been concluded that the major contribution to the cross section is due to a $^4S_{3/2} \rightarrow ^2D_{3/2}$ transition. However, a contribution from either a $^2S_{1/2} \rightarrow ^2S_{1/2}$ transition or a $^2P_{1/2} \rightarrow ^2P_{1/2}$ transition (but not both) is not excluded.

McIntyre (Mc 65) estimated the effect of the s-wave $J^\pi = \frac{1}{2}^+$ channel by evaluating the general equation relating differential cross section to incoming tensor moments (We 63, equation 146) assuming the
contribution of both $J^\pi = \frac{3}{2}^+$ and $J^\pi = \frac{1}{2}^+$ s-wave reaction matrix elements. The result is given in the following form:

$$\sigma(\theta, \phi) = \frac{x^2}{6} \left\{ R_{3/2} \left\{ \frac{1}{2} \right\}^2 + \frac{1}{2} \left[ R_{1/2} \left\{ \frac{1}{2} \right\}^2 + \left( R_{3/2}^* R_{1/2} + R_{1/2}^* R_{3/2} \right) \right] \left\{ \frac{1}{2} \frac{\sqrt{2}}{3} \cos 2\theta - 1 \right\} < T_{20} > + \frac{\sqrt{3}}{2} \sin \theta \cos \phi < T_{21} > - \frac{\sqrt{3}}{2} \sin^2 \theta \cos 2\phi < T_{22} > \right\}$$

(1.7)

where $x$ is the reduced de Broglie wavelength of the incident deuteron and the $R_j$'s are the reaction matrix elements for the total angular momentum involved. The orientation parameters are referred to a right handed coordinate system similar to one described in section 1.1. The angle $\theta$ is the c.m. reaction angle, the azimuthal angle $\phi$ is the angle between the $y$ axis of the incident orientation parameters and the direction $k_{\text{in}} \times k_{\text{out}}$, where $k_{\text{in}}$ is the direction of the incident deuterons and $k_{\text{out}}$ is the direction of the outgoing protons. Equation 1.7 can be rewritten as

$$\sigma(\theta, \phi) = \sigma_0 \left\{ \frac{\sqrt{2}}{4} (3\cos^2 \theta - 1) < T_{20} > + \frac{\sqrt{3}}{2} \sin \theta \cos \phi < T_{21} > - \frac{\sqrt{3}}{2} \sin^2 \theta \cos 2\phi < T_{22} > \right\}$$

(1.8)

where the unpolarized c.m. differential cross section
\[ \sigma_o, \text{ and the factor } f \text{ are given by} \]
\[ \sigma_o = \frac{\hbar}{6} \left[ |R_{3/2}|^2 + \frac{1}{2} |R_{1/2}|^2 \right] \]
\[ f = \frac{|R_{3/2}|^2 + R_{3/2}^* R_{1/2} + R_{1/2}^* R_{3/2}|}{|R_{3/2}|^2 + \frac{1}{2} |R_{1/2}|^2} \quad (1.9) \]

Equations 1.7 and 1.9 reveal interesting characteristics: the unpolarized cross section does not contain an interference term; the vector polarization does not contribute to the polarized cross section; and all the terms containing \( \langle T_{2k} \rangle \) are multiplied by the same factor \( f \) defined in equation 1.9. In the absence of a \( J^\pi = \frac{1}{2}^+ \) contribution \( f \) has the value one and \( \sigma_o = \frac{\hbar}{6} |R_{3/2}|^2 \) as expected for a single channel case. Thus, the magnitude of the factor \( f \) gives a measure of the effect of a \( J^\pi = \frac{1}{2}^+ \) reaction matrix element on the analyzing power of the \(^3\text{He}(d,p)^4\text{He} \) reaction.

In order to estimate the magnitude of the \( J^\pi = \frac{1}{2}^+ \) contribution McIntyre (Mc 65) used results of the measurements obtained at the Carnegie Institution of Washington (Br 66) with polarized deuterons from a polarized ion source. Using polarized deuterons the
\[ \theta = 0^\circ - \theta = 90^\circ \text{ anisotropy of protons from the } \text{He}(d,p) \text{He reaction was measured as a function of bombarding energy and was found to have a maximum value of 1.25 instead of 1.27 as was expected.} \] 
(The ion source produced deuterons with only one non-zero second-rank moment, namely \( \langle T_{20} \rangle = -\frac{1}{3\sqrt{2}} \).) This observation led to the assumption that the reaction was not pure \( J^\pi = \frac{3}{2}^+ \) but that the deviation from the expected anisotropy was entirely due to a \( J^\pi = \frac{1}{2}^+ \) contribution. The result of these calculations yielded values of the factor \( f \) as a function of energy obtained from equation 1.8 using the measured values of the anisotropy. The conclusion drawn from the above calculations is that, for the mean deuteron energy of about 500 keV, an upper limit of the effect of \( J^\pi = \frac{1}{2}^+ \) channel on the measured tensor moments is about 15%. This implies that the values of deuteron tensor moments measured using the \( \text{He}(d,p) \text{He reaction can be uniformly low by 15% at deuteron energy of 500 keV if a pure } J^\pi = \frac{3}{2}^+ \text{ channel is assumed.} \]

The results of McIntyre's calculations (Mc 65) using measured \( \theta = 0^\circ - \theta = 90^\circ \) unpolarized
anisotropy of 1.04 (Bo 52) for a p-wave contribution at 1140 keV, indicate a few percent effect on the polarized cross section, the effect decreasing towards lower energies due to the rapidly decreasing p-wave penetrability with energy. Furthermore, the above results are confirmed by the measurements made at the Carnegie Institution of Washington (Br 66) where the proton anisotropy from the $^3\text{He}(d,p)^4\text{He}$ reaction was measured as a function of deuteron energy with the deuteron orientation axis both parallel and perpendicular to the line of flight. As was expected, for purely s-wave deuterons, the angular distribution rotated 90° with the deuteron orientation axis up to the energies as high as 1200 keV. A significant p-wave contribution would presumably not permit this.

In conclusion, the above evidence indicates that in the case of the $^3\text{He}(d,p)^4\text{He}$ reaction analyzer equation 1.8 can be regarded valid up to deuteron bombarding energy of at least 1 MeV. From the available data (Br 66) the value of the factor $f$ in equation 1.8 can be estimated to be about 0.85 for the mean deuteron energy of about 500 keV. A systematic error in the
factor $f$ of $5\%$ is expected to introduce an uncertainty of the same order in the measured values of $\langle T_{2k} \rangle$.

The effect of a p-wave contribution, however, is expected to be insignificant providing the average deuteron bombarding energy is kept below 1 MeV.
CHAPTER 2

DEUTERON TENSOR POLARIZATION IN ALPHA-DEUTERON ELASTIC SCATTERING.

2.1 Introduction

Deuteron-alpha elastic scattering has been of interest for many years both from the point of view of obtaining information on energy levels in $^6$Li and from the viewpoint of its potential usefulness in the production or analysis of polarized deuteron beams. In both cases a knowledge of the elastic scattering phase shifts is highly desirable. However, when only cross section data are available, phase shift analyses are frequently ambiguous, particularly at energies above a few MeV where the number of parameters to be determined is quite large. The orientation parameters provide additional constraints on the phase shifts and hence help to resolve these ambiguities. In this chapter measurements of the second-rank tensor polarization of deuterons from deuteron-alpha scattering are presented.

The d-$\alpha$ elastic scattering differential cross
section has been measured by a number of investigators for bombarding energies below about 10 MeV (Gu 47, Bl 49, Al 51, Bu 52b, La 53, Ga 55a, St 62, Se 64a, Oh 64). A sharp resonance is observed at a bombarding energy of 1.07 MeV which has been found to correspond to a $3^+$ level in $^6\text{Li}$ at an excitation energy of 2.184 MeV (La 53, Ga 55b). Additional levels with $J^\pi$ of $2^+$ and $1^+$, and excitation energies of 4.57 and 5.60 MeV respectively have been proposed to explain the broad anomaly in the cross section from 3 to 6 MeV bombarding energy (Ga 55b, Se 64a, Mc 65).

In earlier phase shift analyses information was obtained on the d-α phase shifts by Galonsky and McEllistrem (Ga 55b) in obtaining resonance theory fits to cross section data below 4.6 MeV deuteron energy and by Gammel, Hill and Thaler (Ga 60) in applying a model of the d-α interaction to cross section measurements at 8 and 10.3 MeV. In the case of the latter analysis, some of the cross section data (Bu 52b) used are in substantial disagreement with more recent measurements (St 62, Se 64a, Oh 64).
McIntyre and Haeberli (Mc 64a) employed their tensor polarization measurements (Mc 64b) and existing differential cross section data (Bl 49, Ga 55b, St 62, Oh 64, Se 64b) to obtain a set of phase shifts spanning the deuteron energy range 2 to 10 MeV. Their analysis failed to confirm the existence of a p-wave resonance at excitation energy 7 to 9 MeV in $^6\text{Li}$, which was indicated in an earlier phase shift analysis by Senhouse and Tombrello (Se 64b). This result is not too surprising as only cross section data were used in the latter analysis, and at higher energies a large number of parameters had to be determined from relatively limited experimental information. As it will be shown in section 2.4 at some energies and angles the polarization parameters computed from the two sets of phase shifts differ significantly.

Several measurements of the tensor polarization in d-$\alpha$ scattering have been made previously at deuteron energies between 1 and 11 MeV. Pondrom and Daughtry (Po 61b) obtained data around 1 MeV deuteron energy for $\theta_{\text{c.m.}} = 45^\circ$. Seiler et al. (Se 64) measured
deuteron tensor polarization between 4 and 7.5 MeV for $\theta_{\text{c.m.}} = 85.5^\circ$ and McIntyre and Haeberli (Mc 64b) measured the tensor moments between 2 and 9 MeV for four c.m. angles between 65$^\circ$ and 120.2$^\circ$. In addition, information has been obtained on the vector component of polarization by Trier et al. (Tr 65) at deuteron energies between 4 and 11 MeV from three c.m. angles between 65$^\circ$ and 104$^\circ$. The latter data were obtained using a polarized ion source. A very recent measurement of both vector and tensor polarization reported by Arvieux et al. (Ar 66) performed with deuterons from a polarized ion source on a $^4$He target in the energy region about 20 MeV at c.m. angles up to 120$^\circ$, has shown that helium does not appear to be very efficient analyzer of deuterons in that region i.e. polarization is smaller than at lower energies.

The above measurements of the tensor polarization parameters were made by accelerating deuterons onto a $^4$He gas target, and no data were obtained at c.m. angles greater than 120.2$^\circ$. To extend the measurements to larger angles (up to 180$^\circ$ c.m.) most of the results presented here were obtained by bombarding
a deuterated polyethylene target with alpha particles and observing the tensor polarization of the recoiling deuterons. Four measurements were made in this manner at each of the lab. angles $0^\circ$, $22.5^\circ$ and $30^\circ$ (corresponding to c.m. angles $180^\circ$, $135^\circ$ and $120^\circ$ respectively) for alpha energies between 4.4 and 11.1 MeV. Additional measurements were made at c.m. angles $66.5^\circ$ (three points) and $85.8^\circ$ (one point) by accelerating deuterons onto a $^4$He gas target and using lab scattering angles of $45.5^\circ$ and $60^\circ$. These latter measurements overlap data obtained previously at the University of Wisconsin (Se 64, Mc 64b) and were performed primarily for comparison with the earlier work. Good agreement was obtained.

2.2 Apparatus

2.2a Description of the Experimental Arrangement

The arrangement used for the measurements is shown in figure 2.1. After energy analysis in a 90° magnet and collimation, a beam of particles from the A.N.U. tandem accelerator was allowed to strike the first target which was located centrally in a 10.5 cm
FIGURE 2.1

A schematic diagram showing the experimental arrangement used in the measurements. The directions indicated by $k_0$, $k_d$ and $k_p$ refer to the bombarding particle, the emitted deuteron from the first target, and the protons from the $^3\text{He}(d,p)^4\text{He}$ reaction, respectively.
diameter scattering chamber. The incoming beam was defined by a 2.4 mm circular aperture in a 0.5 mm thick tantalum disc, located 42 cm from the centre of the scattering chamber. For the gas target measurements a beam collimator of 1.5 mm diameter was used. A similar disc, used as an anti-scattering baffle and having a 3.2 mm diameter aperture, was located 32 cm behind the collimator.

A horizontal cross-sectional view of the experimental assembly is shown in figure 2.2. Details of the counter array are omitted in this diagram. A vertical cross-sectional view is shown in figure 2.3. The main items in both figures are appropriately labelled.

The scattering chamber was mounted on the lower table as indicated in figure 2.3. The counter support and the $^3$He gas cell support were rigidly attached to the upper table, which was free to rotate about the scattering chamber. Thus, the geometry of the detectors with respect to the $^3$He gas target remained fixed for any first scattering (or reaction) angle. This was a necessary feature required by the technique employing runs with gold as a first target to cancel geometry effects.
FIGURE 2.2

A horizontal cross-sectional view of the apparatus. Details of the counter collimators and support have been omitted. The numbered parts are (1) beam collimator and antiscattering baffle, (2) $\text{CD}_2$ target foil, (3) beam stop, (4) Havar foil, (5) defining slit, (6) $^3\text{He}$ cell, (7) CsI crystal, (8) lucite light pipe, and (9) photomultiplier tube.
FIGURE 2.3

A vertical cross-sectional view of the apparatus. The annular CsI crystal in $\theta = 0^\circ$, $\phi = 0^\circ$ position was used for d-$^4$He scattering only as mentioned in the text. The numbered parts are (1) scattering chamber (2) CD$_2$ target foil, (3) assembly used in rotating the CD$_2$ target, (4) gold target foil, (5) $^3$He cell, (6) square CsI crystal, (7) annular CsI crystal, (8) lucite light pipe, and (9) photomultiplier tubes.
The scattering chamber was constructed from mild steel with an inside diameter of 10.5 cm and an internal height of 21 cm. Fixed windows, 2 cm high, were located at 15° intervals around both sides of the chamber, the windows on either side being offset so that the angle of observation could be varied in increments of 7.5°. Each window had an angular acceptance of about 8.6°. A 6 micron Havar foil* covered the windows and, during the measurements, all windows except the one in use were covered with a thin lead shield to eliminate background arising from the $^{14}\text{N}(d,p)^{15}\text{N}$ reaction taking place in air.

After leaving the scattering chamber through the 6 micron Havar foil deuterons produced in the first target penetrated about 8 mm of air and the suitably chosen slowing Mylar foil before entering the $^3\text{He}$ cell through a second 6 micron Havar foil. The cell consisted of a 2.5 cm diameter hemisphere which was pressed from 100 micron thick aluminium foil. Slits of various angular acceptances were located immediately in front of the cell.

* Hamilton Watch Co., Lancaster, Pennsylvania, U.S.A.
Protons formed in the $^3\text{He}(d,p)^4\text{He}$ reaction have a lab energy of approximately 17 MeV for a deuter-on energy of 800 keV. Before reaching the four Csl scintillation crystals located at the lab angles $\theta = 0^\circ$, $\phi = 0^\circ$ and $\theta = 52.55^\circ$, $\phi = 0^\circ$, $90^\circ$ and $180^\circ$, protons lost approximately 1 MeV of energy passing through the aluminium hemisphere and 9 cm of air. Consequently, in order to keep the neutron background to a minimum, the Csl crystals were polished down to a thickness of 1.6 mm (which is sufficient to completely stop 16 MeV protons).

The crystals used were 2.5 x 2.5 cm square and were bonded with epoxy resin to 7 mm long lucite light pipes which were similarly attached to 5 cm diameter Dumont photomultiplier tubes. All crystals were covered by a thin light-tight foil. Tantalum collimators with 1.9 cm square holes in the centre were mounted directly over the crystals defining a polar angle of $\pm 6.2^\circ$ subtended with respect to the centre of the $^3\text{He}$ cell.

For the gas measurements an annular crystal, already described by Young (Yo 65), having a mean
polar angle of 16.9° was centred in the $\theta = 0°$, $\phi = 0°$ position. The annular geometry was used to avoid detecting direct protons from the $^{14}\text{N}(d,p)^{15}\text{N}$ reaction in the first target due to a slight impurity of the $^4\text{He}$ gas. The annular detector subtended a polar angle of $\pm 1.5°$ with respect to the centre of the $^3\text{He}$ cell. The resolution of all the counters under experimental conditions was about 8%.

2.2b Targets and Absorber Foils.

The targets used in the measurements to be described were a $^4\text{He}$ gas target, a $\text{CD}_2$ rotating-foil target, a gold foil target, and a $^3\text{He}$ gas target. The $^4\text{He}$ gas target was used in the d-$^4\text{He}$ measurements and was similar in design to that described by Brown et al. (Br 63). The deuterons entered and left the target through a 6 $\mu$m Havar foil. The diameter of the target was 7.6 mm, and it was operated at pressure of 12 atm. A rectangular $1.6 \times 8$ mm slit was attached to the target 7 mm from its centre which, together with one located in front of the $^3\text{He}$ cell,
defined the volume of the $^4\text{He}$ target viewed by the $^3\text{He}$ cell.

For the measurements on α-d scattering a rotating deuterated-polyethylene target was used. The assembly is shown in figure 2.3 and has been described by Young (Yo 65). The CD$_2$ target* foil, 1.6 mg/cm$^2$ thick, was positioned so that the normal to its surface was parallel to the incident particle beam. To permit the use of beams of sufficient intensity, a thin film of aluminium (~0.01 mg/cm$^2$) was evaporated onto both sides of the target foil, and it was rotated at about 100 r.p.m. With rotation diameter of 2.4 cm, no serious deterioration of the target occurred with beam currents up to 0.09 μa at an energy of 11 MeV for alpha beams. It may be added that, aside from increasing the energy spread slightly, the aluminium had no effect on the measurements since the $^{27}\text{Al(a,d)}^{29}\text{Si}$ reaction has a large negative Q-value. By moving the assembly vertically the gold-foil target 4.5 mg.cm$^3$ thick could be brought into position when required. Deuteron beam currents of about 1 μa were used for d-Au runs.

* U.S. Nuclear Corporation, Burbank, California, U.S.A.
Details of the $^3$He gas target assembly are given in figure 2.4. A 6 micron Havar foil separated the gas volume of the cell from the atmosphere. Deuterons from the first target entered through this foil and protons from the $^3$He(d,p)$^4$He reaction emerged into the atmosphere through the hemisphere pressed from the 100 micron aluminium foil. The gas target was assembled by attaching the aluminium cell with a Duppont adhesive (46951) to the piece labelled 1 in the figure 2.4. The 6 micron Havar foil was similarly attached to the opposite side of the piece, and both were clamped as indicated.

At an operating pressure of 5.5 atm, the cell was sufficiently large to stop 900 keV deuterons. Rectangular slits which were located immediately in front of the cell defined an angular acceptance of approximately $\pm 2.7^\circ$ for all measurements except those made at energies above 6 MeV and at $\theta_{\text{lab}} = 0^\circ$. In the former case an angular acceptance of $\pm 1.4^\circ$ was used. A circular slit of angular acceptance $\pm 4.1^\circ$ was used for the $\theta_{\text{lab}} = 0^\circ$ measurements.

In order to keep the mean deuteron energy below 1200 keV when these particles entered the $^3$He cell, Mylar foils of various thicknesses were placed
FIGURE 2.4

Details of the $^{3}$He gas target assembly. The numbered parts are (1) centre supporting piece to which the aluminium hemisphere and Havar foil were bonded, (2) aluminium hemisphere, and (3) filling lead. The two outer pieces were used to clamp the hemisphere and Havar foil to opposite sides of part (1).
between the two targets. Wolfenstein's (Wo 49) calculations for 7 MeV protons stopped in a foil, indicated that the slowing down process should result in a negligible change in the proton polarization (about one part in $10^5$). Thus, it was reasonable to assume that the slowing foils would not change the deuteron polarization appreciably. Furthermore, the large spread in angle due to multiple scattering does not have any effect on the angular distribution of the protons from the $^3\text{He}(d,p)^4\text{He}$ reaction since in a reaction induced by s-waves the angular distribution of the reaction products is independent of the incident beam direction.

The deuteron energy variation with scattering angle was quite severe (particularly at higher energies). This energy spread not only results in lower proton yields from the $^3\text{He}(d,p)^4\text{He}$ reaction but also introduces a spurious asymmetry into the yields because the effective centre of the $^3\text{He}$ cell is shifted towards the small angle side of the cell. Additional Mylar foils of appropriate thickness, therefore, were placed over the $^3\text{He}$ cell slit in such a way that the higher energy deuterons traversed a greater distance in the foil than the lower energy deuterons, thus tending
to partially compensate for the energy variation across the slit. To minimize broadening of the angular acceptance due to multiple scattering the Mylar slowing foils were attached directly to the slit of the $^3$He cell over the compensating foils when the latter were used.

With the slit system used for the $^4$He gas target measurements, the energy spread of the bombarding deuterons across the volume of the target viewed by the $^3$He cell was approximately 100 keV for a deuteron energy of 5 MeV. For the $CD_2$ measurements the energy lost by the alpha particles in traversing the target varied from about 1.5 MeV at bombarding energy of 4.5 MeV to about 0.75 MeV at 11.5 MeV. However, the energy spreads present in the measurements were substantially reduced by the technique employed in the experiment. In particular, with the alpha bombarding energy set such that the mean deuteron energy in the $^3$He cell was about 550 keV, alpha particles which penetrated to the rear part of the $CD_2$ target before scattering produced deuterons of insufficient energy to pass through the slowing foils and thus resulted in no proton yield from the $^3$He cell. The computer program described in section 2.3d was used to compute the mean energy of the bombarding alphas and to estimate the effective energy spread. The
32.

results indicate that the energy spread varied from about 600 keV for the measurements at $E_\alpha = 4.45\ MeV$ to about 200 keV for the measurements at $11.1\ MeV$.

Required thicknesses of the compensating foils for a given energy-angle combination were calculated from range-energy relationships and they included the effects of the Havar foils and the air gap between the $^3\text{He}$ cell and the scattering chamber. However, the calculations ignored the deuteron energy spread due to thickness of the first target.

The energy variation across the $^3\text{He}$ cell increases with energy. Thus at higher energies for some measurements the small slit system ($\pm 1.4^0$) was used. Furthermore, for measurements above 5 MeV the slit was divided vertically into four equal sections and compensating foils of the thickness increasing by a regular amount towards the small angle side of the slit were used. This technique, avoiding the use of thicker compensating foils over one half of the slit, provided a smoother and smaller deuteron energy variation across the slit. Slowing and compensating Mylar foils of up to 70 and $2.25\ \text{mg/cm}^2$ thick respectively were used.
2.3 Experimental Procedure

2.3a Data Collection

A block diagram of the electronics used in the polarization measurements is shown in figure 2.5. Signals from each of the four photomultiplier tubes were amplified with a Franklin double-delay-line preamplifier-amplifier system. The outputs of the four amplifiers were mixed by a simple diode circuit and the resulting signal fed into an R.I.D.L. 400-channel analyzer. The memory of the analyzer was split into four 100-channel sections. Simultaneously, the outputs of each of the four amplifiers were fed into a separate Cosmic single channel analyzer. Pulses from the Cosmic outputs were used to route the signal pulses into the appropriate 100-channel quadrant of the analyzer. The same pulses were also used to generate gate pulses for the analyzer. The gating was required to prevent the accumulation of low energy signals below the discrimination levels, since these signals were not accompanied by routing pulses. To ensure that routing and memory of the system were working properly, pulses from each discriminator were scaled.

The scattering chamber, already described in
FIGURE 2.5

Block diagram of the electronics used in the polarization measurements.
section 2.2a, was electrically insulated to allow monitoring of the incident beam current. For the measurements made at $\theta_{1\text{ab}} = 0^\circ$, the beam stop shown in figure 2.2 was replaced by an 11 micron Havar foil plus such additional foil as was necessary to stop the incident alpha beam.

It should be pointed out, however, that by operating four counters simultaneously, the actual polarization measurements were independent of current integration. Nevertheless, a relative accuracy in the current integration of a few percent was required in the yield measurements with the purpose of setting the bombarding energy at which each polarization measurement was to be made (described in section 2.3b).

There were two types of measurements made. One was a polarization measurement with $^4\text{He}$ or $\text{CD}_2$ as first targets and was done for various bombarding energies of the incident beam and absorber foils determined as described in section 2.3b. The purpose of the second type of measurements was to normalize the yields of the four detectors in order to cancel, to first order, factors such as detector geometry. These
measurements utilized the unoriented beam of deuterons obtained by scattering 5.35 MeV deuterons from a 4.5 mg/cm² gold foil. The normalization technique is discussed in section 2.3c.

A d-Au run at one of the three lab angles 30°, 37.5° or 45° was made. In four runs, a total of about 35,000 counts in each counter was collected. Then the bombarding energy of the incident beam of particles was set as described in the following section. This was followed by a measurement of the proton yield from oriented deuterons in a series of approximately one-hour runs with a minimum total between 1000 and 3000 counts in each counter. Runs with gold as the first target were repeated regularly.

The counting rate for the measurements with the CD₂ target varied from about 40 to 130 counts per hour. There were two methods used for background measurements and they gave equivalent results. One type of background runs was taken by replacing the ³He in the hemispherical cell with ⁴He and the other by inserting a thin foil in front of the ³He cell of sufficient thickness to stop the deuterons from the first target.
Some selected spectra from the polarization measurements are shown in figure 2.6 together with a typical spectrum obtained from d-Au runs. The most severe background encountered with the CD₂ target was in detector 1 at $\theta = 0^\circ$, $\phi = 0^\circ$ position, and was less than 5% of the observed yields at $\theta_{lab} = 0^\circ$ for an alpha bombarding beam. At all other energy-angle combinations the background was found to be independent of the alpha beam and was ~ 1 count per hour in each counter.

The counting rate for the $^4$He gas target varied between 100 and 300 counts per hour. There were two components of background measured in two different ways. Background due to neutrons was measured as in the case of the solid target above, by inserting a stopping foil in front of the $^3$He cell. However, the second component of background resulted from deuterons doubly scattered by the Havar foil on the $^4$He gas target. This was measured by replacing the $^4$He in the first target with hydrogen. Since deuterons scattered from hydrogen cannot be observed at angles greater than $30^\circ$ because of the
FIGURE 2.6
Selected pulse-height analyzer spectra.
**α-D ELASTIC SCATTERING**  
**ALPHA ENERGY 11.1 MeV**  
**LAB ANGLE 0°**

**d-Au ELASTIC SCATTERING**  
**DEUTERON ENERGY 5.35 MeV**  
**LAB ANGLE 30°**
kinematics of the scattering, this technique permitted measurement of the yield from foil-scattered deuterons under the same conditions of foil bulging, etc., as were present in the data runs. The total background generally amounted to about 10% of the observed yields and in the worst case was about 18%.

At the end of each run the memory of the 400-channel analyzer, containing the four pulse-height spectra, was printed out on a tape and yields were obtained by summing the appropriate peaks with a desk calculator. Calculation of the first order deuteron tensor moments from these raw data is described in section 2.3c.

2.3b Determination of the Bombarding Energy.

The analysis of deuteron tensor polarization with the $^3$He(d,p)$^4$He reaction depends on the assumption that only one reaction channel is involved, namely that associated with the 425 keV resonance. This implies that equation 1.8 can be correctly used to determine polarization parameters only if the deuterons incident on the $^3$He cell are predominantly s-wave, i.e. below
approximately 1.2 MeV in energy. Thus, in the measurements described here, the accelerator energy was varied while the mean energy of the scattered deuterons at the entrance of the $^3$He cell was kept under 1.2 MeV, by using absorber foils as described in section 2.2b.

To determine the appropriate bombarding energy for each slowing foil-angle combination used, the proton yield from the $^3$He(d,p)$^4$He reaction was measured as a function of incident beam energy. These measurements were then compared with theoretical yield curves calculated for each experimental arrangement, and a bombarding energy consistent with the s-wave requirement was selected.

In order to increase the counting rate, a large counter was placed in contact with the $^3$He cell for the yield measurements. It had the same design as did the CsI detectors used for the polarization measurements, except that it had an extended lucite light pipe (which enabled the crystal to reach the $^3$He cell from the $\theta = 0^\circ$, $\phi = 0^\circ$ position in the detector frame). The counting rate with this detector
was about a factor of 4 larger than the combined
counting rate of the four CsI detectors, and thus
speeded up the bombarding energy setting procedure.

The theoretical yield curves were obtained
by numerically integrating over the first target
thickness, the relevant volume of the $^3$He cell, and
the area of the detector. The calculations were made
with a CDC-3600 computer (later, programs were converted
to IBM-360 computer), taking into account the energy
loss of particles traversing the targets, the various
foils, variation of cross sections with energy and
angle, c.m. to lab conversion, and the exact shape and
location of the $^3$He cell and CsI detector.

This procedure resulted in the bombarding
energy being set at a point corresponding to 40 to
60% of the maximum yield. In general, the larger the
deuteron energy spread across the cell, the lower down
on the yield curve the energy had to be set. The
bombarding energies were generally set such that the
mean deuteron energy in the $^3$He cell was approximately
550 keV for $\alpha$-$d$ scattering measurements. It may be
added that the calculations indicated that almost all
the deuterons in the cell were below 1.2 MeV in energy.

A similar procedure was followed in setting the deuteron bombarding energy for the d-Au normalizing runs. The energy variation with angle is negligible in this case and thus the bombarding energy was adjusted so that all deuterons had an energy of about 800 keV at the entrance of the $^3$He cell. For these runs, a slowing Mylar foil of 17.5 mg/cm$^2$ thickness and lab angles of 30°, 37.5° and 45° were used. These arrangements resulted in the bombarding energy set at about 5.35 MeV which could be reproduced well throughout the entire measurements. The differential cross section measurements of Igo et al. (Ig 61) with $E_d$=11.8 MeV indicate that the d-Au scattering cross section is Rutherford up to an angle of 50°. This, then, supports the assumption of unoriented scattered deuterons from Au at $E_d$ = 5.35 MeV and lab angles of 30°, 37.5° and 45°.

2.3c Calculation of Polarization from Counter Yields

In order to calculate the three second-rank tensor moments of the deuterons from the first target
incident on the $^3\text{He}$ gas, yields in the four counters detecting $^3\text{He}(d,p)^4\text{He}$ protons are compared. The comparison is carried out by evaluating the expression for the $^3\text{He}(d,p)^4\text{He}$ cross section for polarized incident deuterons (equation 1.8) at each of the counter positions by substituting the appropriate values of $\theta$ and $\phi$. The resulting four equations can be solved for $\langle T_{20} \rangle$, $\langle T_{21} \rangle$, $\langle T_{22} \rangle$ and the unpolarized cross section $\sigma_o$. If we denote corresponding polarized cross section in detectors by

$$
\sigma_1 = \sigma(0^0,0^0) \quad \sigma_2 = \sigma(54.7^0,0^0) \\
\sigma_3 = (54.7^0,90^0) \quad \sigma_4 = (54.7^0,180^0)
$$

the results are

$$
\sigma_o = \frac{1}{4}(\sigma_2 + 2\sigma_3 + \sigma_4) \quad \langle T_{20} \rangle = \frac{\sqrt{2}}{f} \left(1 - \frac{\sigma_1}{\sigma_o}\right) \\
\langle T_{21} \rangle = \frac{\sqrt{6}}{4f} \frac{\sigma_2 - \sigma_4}{\sigma_o} \quad \langle T_{22} \rangle = \frac{\sqrt{3}}{4f} \frac{2\sigma_3 - \sigma_2 - \sigma_4}{\sigma_o}
$$

Since the differential cross section for unpolarized deuterons is expressed as a linear combination of polarized differential cross sections at different counter positions, the expressions for the three tensor moments involve only ratios of linear combinations of polarized cross sections. Therefore, if the counters subtended
identical solid angles from the $^3$He target, the yields of each of the counters could be used directly as the polarized differential cross section in equation 2.1 for the tensor moments since all factors relating yield to cross sections would cancel.

Since in fact the counters were not identical, the yields had to be normalized in order to cancel the differences in solid angle. This normalization was accomplished by making use of the $^3$He(d,p)$^4$He protons resulting from incident s-wave unpolarized deuterons from d-Au elastic scattering as described in section 2.3a. Yield from a counter with $^4$He, CD$_2$ (or $^9$Be) as first targets was normalized by dividing it by the yield from that counter during a run with gold as the first target.

Ignoring the effects of finite geometry, the following expression for the ratio of the two yields in a detector is correct to first order (Yo 65):

$$R(\theta, \phi) = c \left\{ 1 - \frac{\sqrt{2}}{4} \langle T_{20} \rangle (3\cos^2 \theta - 1) - \sqrt{3} \langle T_{21} \rangle \sin \theta \cos \theta \cos \phi 
+ \frac{\sqrt{3}}{2} \langle T_{22} \rangle \sin^2 \theta \cos 2\phi \right\}$$

(2.2)

where c is a constant for all detectors and involves the number of bombarding particles, number of the first-
target nuclei per unit volume, thickness of the first target, and lab cross section for the first scattering (or reaction) and d-Au scattering. For the annular detector an integration over $\phi$ (see equation 2.4) had to be performed resulting in elimination of the last two terms in equation 2.2. Thus, the three second-rank tensor moments were calculated from the following equations which were obtained in the same way as equations in (2.1) using the ratios rather than polarized differential cross sections in each detector

$$\langle T_{20} \rangle = \frac{1.4142}{f} \left[ 1 - \frac{4R_1}{R_2 + 2R_3 + R_4} \right]$$

$$\langle T_{21} \rangle = \frac{2.4495}{f} \left[ \frac{R_4 - R_2}{R_2 + 2R_3 + R_4} \right]$$

$$\langle T_{22} \rangle = \frac{1.7321}{f} \left[ \frac{4R_3}{R_2 + 2R_3 + R_3} - 1 \right]$$

(2.3)

where the subscripts 1, 2, 3 and 4 refer to the detectors as labelled in figure 2.1.

To obtain the final values of the $\langle T_{2k} \rangle$, corrections for finite geometry and known experimental asymmetries were made using an iterative procedure. In particular, using the values calculated from equation 2.3, the yield from each detector was computed by
numerically integrating over the experimental geometry, as will be described in more detail in the following section. The $\langle T_{2k} \rangle$ were then varied until the experimental data were reproduced. The magnitude of the differences between the first order and final values of $\langle T_{2k} \rangle$ is discussed for individual cases in subsequent chapters.

2.3d Experimental Uncertainties and Corrections

The effect of the finite extent of the four detectors and $^3$He cell can be seen by an examination of equation 1.8. This equation contains the trigonometric functions $(3 \cos^2 \theta - 1)$, $\sin \theta \cos \theta \cos \phi$ and $\sin^2 \theta \cos 2\phi$, which are to be evaluated at the $^3$He($d$,p)$^4$He reaction polar and azimuthal angles corresponding to the positions of the detectors. Since in fact the detectors as well as the $^3$He target are finite, a range of angles is viewed and hence a suitable mean value of these functions must be calculated.

Although the normalization technique described in the previous section eliminates spurious asymmetries in first order approximation, an additional source of false asymmetries arises from the difference in energy
and intensity variations of the incident deuterons across the entrance of the cell for the measurement and normalization runs. The effect results in a shift of the effective centre of the $^3$He cell.

The coordinate system referred to here is a right-handed system, where $z$ axis is parallel to the lab direction of the emitted deuterons, $k_d$ (figure 2.1). The $y$ axis is defined by $k_o \times k_d$; i.e. perpendicular to the scattering (or reaction) plane. The mean deuteron energy at the entrance of the $^3$He cell is generally not quite the same for the two cases, and this can cause a shift of the effective centre in the $z$-direction. The energy spread due to the first target thickness is much smaller in the case of the gold target which can shift the effective centre in both $x$- and $z$-direction from the geometrical centre of the $^3$He cell. An additional reason for a shift of the effective centre of the $^3$He cell is due to a difference in the point of origination of deuterons for the measurement and normalization runs. This is possible both because of distortion of the first target (especially in the case of CD$_2$ target) and because of differing focusing conditions prevailing for different incident particle beams.
In order to correct the first order tensor moments for above effects (with exception of the two latter effects) a numerical integration was performed with a CDC-3600 computer. (However, for the $^9\text{Be}(p,d)^8\text{Be}$ measurements the program was converted for use with an IBM 360 computer.) The integration of the form

$$Y = nN_1 N_2 D \int dt_1 dt_2 \sigma(\theta_{\text{lab}}) d\Omega_1 J(E_d, \theta) \sigma(E_d, \theta, \phi) d\Omega_2$$

(2.4)

was performed over the appropriate region of the $^3\text{He}$ cell. The four square crystal surfaces were divided into 4 area increments each, and the annulus (when used) was divided into 16 increments. Symbols in equation 2.4 are: $n$ is the number of bombarding particles; $N_1$ and $N_2$ are the number of the first and $^3\text{He}$ target nuclei per unit volume, respectively; $t_1$ and $t_2$ are respective thickness of the targets; $D$ is the detection efficiency of the detectors; $\sigma(\theta_{\text{lab}})$ is the lab cross section for the first scattering (or reaction); $J(E_d, \theta)$ is the c.m. to lab Jacobian for the $^3\text{He}(d,p)^4\text{He}$ reaction cross section; $\sigma(E_d, \theta, \phi)$ is the c.m. cross section for the $^3\text{He}(d,p)^4\text{He}$ reaction given by equation 1.8 with $\sigma_0(E_d)$ being a function of deuteron energy; and
finally $d\Omega_1$ and $d\Omega_2$ are the lab solid angles.

Because of the thick targets used, the first target was divided into 30 equal thickness increments and the subsequent integration over the $^3$He cell and the four detectors was carried out for each deuteron energy which varied with the increment of origination in the target. Having calculated the deuteron energy corresponding to a given thickness increment, the coordinate system of the $<T_{2k}>$ was rotated appropriately for scattering into any volume increment of the $^3$He cell, and the position angles of each volume increment were determined accordingly. The deuteron energy at the centre of the volume increment was determined from the range-energy relationships of Havar, Mylar and $^3$He gas, taking into account the compensating foils. A similar yield calculation was performed for the gold target with $<T_{2k}>$ set to zero. Thus calculated normalized yields were obtained. Using equation 1.8 for the polarized cross section of the $^3$He(d,p)$^4$He reaction, the corrections $d<T_{2k}>$ necessary to make calculated and measured normalized yields equal were computed. Finally, the required adjustment of initial $<T_{2k}>$ was carried out. The above procedure produced final tensor moments corrected for asymmetries due to all finite geometry.
effects.

A possible source of systematic error in the $\langle T_{2k} \rangle$ measurements resulted from uncertainty in the factor $f$ of equation 1.8. The values used in calculations performed in the above program were taken from the results of Brown et al. (Br 66) and were in the range 0.85 to 0.90, depending on the case involved. The uncertainty in the value of $f$ was estimated to be about 5% which resulted in a similar uncertainty in $\langle T_{2k} \rangle$.

Additional uncertainties in the measurements resulted from uncertainties in the counter yields. A sufficient number of runs were taken at each datum point to determine the counter yields to a statistical accuracy of between 2% and 4%. The yields of the normalization runs were determined to a statistical accuracy of about 1%. Assuming an uncertainty of 2.5% in the normalized yields, the resulting uncertainties when these numbers are combined to determine the tensor moments are approximately $\pm 0.050$ for $\langle T_{20} \rangle$, $\pm 0.025$ for $\langle T_{21} \rangle$ and $\pm 0.035$ for $\langle T_{22} \rangle$. The exact values depend on the magnitude of the tensor moments. The larger uncertainty
in $\langle T_{20} \rangle$ results from the choice of angles at which the detectors were placed. These numbers generally characterize the statistical uncertainties of the results presented in the following chapters, although uncertainties different from these by a factor of two may be found. The statistical uncertainties associated with each measurement were calculated from equation 1.8 in the manner described in appendix A. Uncertainties in yields due to background in various cases were presented in section 2.3a and appropriate corrections were applied when necessary.

2.4 Results

The final values of the $\langle T_{2k} \rangle$ for $\alpha$-$d$ elastic scattering are presented with the statistical errors in tables 2.1 and 2.2. In table 2.1 the tensor moments are referred to a coordinate system with the $z$ axis in the lab direction of the recoiling or scattered deuteron beam. The results given in table 2.2 are referred to a coordinate system with the $z$ axis along the initial deuteron direction in the $d$-$\alpha$ c.m. system. In both tables the $y$ axis is as defined in figure 2.1. To obtain the values listed in table 2.2, the $z$ axis of the tensor
 TABLE 2.1

Measured values of the tensor polarization of deuterons from $\alpha$-d elastic scattering. The tensors are referred to a coordinate system with the z axis in the lab direction of the recoil (or scattered) deuterons. The y axis is the direction $k_o \times k_d$ where $k_o$ and $k_d$ indicate the initial beam and final deuteron directions in the c.m. system of the first target (see figure 2.1).
### Table 2.1

<table>
<thead>
<tr>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>$\theta_{\text{lab}}$ (deg)</th>
<th>$\theta_{\text{c.m.}}$ (deg)</th>
<th>Incident Particle</th>
<th>$&lt;T_{20}&gt;$</th>
<th>$&lt;T_{21}&gt;$</th>
<th>$&lt;T_{22}&gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.20</td>
<td>45.5</td>
<td>66.5</td>
<td>d</td>
<td>0.162 ± 0.053</td>
<td>-0.093 ± 0.035</td>
<td>-0.492 ± 0.039</td>
</tr>
<tr>
<td>4.99</td>
<td>45.5</td>
<td>66.5</td>
<td>d</td>
<td>0.086 ± 0.059</td>
<td>0.044 ± 0.037</td>
<td>-0.281 ± 0.043</td>
</tr>
<tr>
<td>5.71</td>
<td>45.5</td>
<td>66.5</td>
<td>d</td>
<td>0.057 ± 0.061</td>
<td>-0.094 ± 0.035</td>
<td>0.035 ± 0.043</td>
</tr>
<tr>
<td>5.23</td>
<td>60</td>
<td>85.8</td>
<td>d</td>
<td>0.473 ± 0.057</td>
<td>-0.036 ± 0.033</td>
<td>0.574 ± 0.043</td>
</tr>
<tr>
<td>4.45</td>
<td>30</td>
<td>120</td>
<td>a</td>
<td>-0.005 ± 0.076</td>
<td>-0.060 ± 0.040</td>
<td>-0.007 ± 0.049</td>
</tr>
<tr>
<td>5.30</td>
<td>30</td>
<td>120</td>
<td>a</td>
<td>0.007 ± 0.069</td>
<td>0.052 ± 0.037</td>
<td>-0.018 ± 0.045</td>
</tr>
<tr>
<td>7.80</td>
<td>30</td>
<td>120</td>
<td>a</td>
<td>0.119 ± 0.077</td>
<td>0.214 ± 0.046</td>
<td>-0.443 ± 0.052</td>
</tr>
<tr>
<td>9.15</td>
<td>30</td>
<td>120</td>
<td>a</td>
<td>0.129 ± 0.062</td>
<td>0.197 ± 0.037</td>
<td>-0.377 ± 0.042</td>
</tr>
<tr>
<td>4.65</td>
<td>22.5</td>
<td>135</td>
<td>a</td>
<td>0.029 ± 0.081</td>
<td>-0.014 ± 0.042</td>
<td>-0.037 ± 0.052</td>
</tr>
<tr>
<td>6.95</td>
<td>22.5</td>
<td>135</td>
<td>a</td>
<td>0.045 ± 0.097</td>
<td>0.331 ± 0.052</td>
<td>-0.119 ± 0.062</td>
</tr>
<tr>
<td>8.40</td>
<td>22.5</td>
<td>135</td>
<td>a</td>
<td>0.305 ± 0.058</td>
<td>0.276 ± 0.036</td>
<td>-0.356 ± 0.042</td>
</tr>
<tr>
<td>10.70</td>
<td>22.5</td>
<td>135</td>
<td>a</td>
<td>0.018 ± 0.066</td>
<td>-0.002 ± 0.037</td>
<td>-0.336 ± 0.042</td>
</tr>
<tr>
<td>7.20</td>
<td>0</td>
<td>180*</td>
<td>a</td>
<td>0.460 ± 0.064</td>
<td>-0.001 ± 0.040</td>
<td>0.052 ± 0.052</td>
</tr>
<tr>
<td>8.15</td>
<td>0</td>
<td>180*</td>
<td>a</td>
<td>0.578 ± 0.053</td>
<td>-0.037 ± 0.038</td>
<td>0.025 ± 0.044</td>
</tr>
<tr>
<td>9.90</td>
<td>0</td>
<td>180*</td>
<td>a</td>
<td>0.366 ± 0.052</td>
<td>-0.088 ± 0.032</td>
<td>0.037 ± 0.033</td>
</tr>
<tr>
<td>11.10</td>
<td>0</td>
<td>180*</td>
<td>a</td>
<td>0.315 ± 0.077</td>
<td>-0.037 ± 0.045</td>
<td>-0.016 ± 0.055</td>
</tr>
</tbody>
</table>

*The mean c.m. angle for the $\theta_{\text{lab}} = 0^\circ$ measurements is 175°.*
TABLE 2.2

Rotated values of the tensor polarization of deuterons from α-d elastic scattering. The tensors are referred to a coordinate system with the z axis in the direction $k_0$ and with the y axis in the direction $k_0 \times k_d$ where $k_0$ and $k_d$ indicate the initial beam and final deuteron directions in the c.m. system of the first target (see figure 2.1).
<table>
<thead>
<tr>
<th>$E_{lab}$ (MeV)</th>
<th>$\theta_{lab}$ (deg)</th>
<th>$\theta_{c.m.}$ (deg)</th>
<th>Incident Particle</th>
<th>$\langle T_{20} \rangle$</th>
<th>$\langle T_{21} \rangle$</th>
<th>$\langle T_{22} \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.20</td>
<td>45.5</td>
<td>66.5</td>
<td>d</td>
<td>-0.383 ± 0.051</td>
<td>-0.343 ± 0.038</td>
<td>-0.270 ± 0.038</td>
</tr>
<tr>
<td>4.99</td>
<td>45.5</td>
<td>66.5</td>
<td>d</td>
<td>-0.100 ± 0.054</td>
<td>-0.195 ± 0.042</td>
<td>-0.204 ± 0.041</td>
</tr>
<tr>
<td>5.71</td>
<td>45.5</td>
<td>66.5</td>
<td>d</td>
<td>-0.080 ± 0.052</td>
<td>-0.016 ± 0.043</td>
<td>0.091 ± 0.041</td>
</tr>
<tr>
<td>5.23</td>
<td>60</td>
<td>85.8</td>
<td>d</td>
<td>0.430 ± 0.053</td>
<td>0.016 ± 0.039</td>
<td>0.592 ± 0.040</td>
</tr>
<tr>
<td>4.45</td>
<td>30</td>
<td>120</td>
<td>a</td>
<td>0.059 ± 0.066</td>
<td>-0.029 ± 0.050</td>
<td>-0.033 ± 0.047</td>
</tr>
<tr>
<td>5.30</td>
<td>30</td>
<td>120</td>
<td>a</td>
<td>-0.014 ± 0.069</td>
<td>0.075 ± 0.037</td>
<td>0.018 ± 0.045</td>
</tr>
<tr>
<td>7.80</td>
<td>30</td>
<td>120</td>
<td>a</td>
<td>-0.288 ± 0.071</td>
<td>0.362 ± 0.052</td>
<td>-0.277 ± 0.051</td>
</tr>
<tr>
<td>9.15</td>
<td>30</td>
<td>120</td>
<td>a</td>
<td>-0.243 ± 0.057</td>
<td>0.330 ± 0.042</td>
<td>-0.225 ± 0.042</td>
</tr>
<tr>
<td>4.65</td>
<td>22.5</td>
<td>135</td>
<td>a</td>
<td>0.027 ± 0.074</td>
<td>0.016 ± 0.050</td>
<td>-0.037 ± 0.051</td>
</tr>
<tr>
<td>6.95</td>
<td>22.5</td>
<td>135</td>
<td>a</td>
<td>-0.273 ± 0.089</td>
<td>0.296 ± 0.060</td>
<td>0.011 ± 0.061</td>
</tr>
<tr>
<td>8.40</td>
<td>22.5</td>
<td>135</td>
<td>a</td>
<td>-0.065 ± 0.055</td>
<td>0.453 ± 0.039</td>
<td>-0.205 ± 0.041</td>
</tr>
<tr>
<td>10.70</td>
<td>22.5</td>
<td>135</td>
<td>a</td>
<td>-0.044 ± 0.061</td>
<td>0.125 ± 0.041</td>
<td>-0.310 ± 0.042</td>
</tr>
<tr>
<td>7.20</td>
<td>0</td>
<td>180</td>
<td>a</td>
<td>0.460 ± 0.064</td>
<td>-0.001 ± 0.040</td>
<td>0.052 ± 0.052</td>
</tr>
<tr>
<td>8.15</td>
<td>0</td>
<td>180</td>
<td>a</td>
<td>0.578 ± 0.053</td>
<td>-0.037 ± 0.038</td>
<td>0.025 ± 0.044</td>
</tr>
<tr>
<td>9.90</td>
<td>0</td>
<td>180</td>
<td>a</td>
<td>0.366 ± 0.052</td>
<td>-0.088 ± 0.032</td>
<td>0.037 ± 0.039</td>
</tr>
<tr>
<td>11.10</td>
<td>0</td>
<td>180</td>
<td>a</td>
<td>0.315 ± 0.077</td>
<td>-0.037 ± 0.045</td>
<td>-0.016 ± 0.056</td>
</tr>
</tbody>
</table>

$^*$The mean c.m. angle for the $\theta_{lab} = 0^\circ$ measurements is $175^\circ$. 
moments of table 2.1 was rotated about the y axis through an Euler angle (Sa 60a) \( \beta = - (180^\circ - \theta_{\text{lab}}) \) for the measurements with an alpha bombarding beam and through \( \beta = - \theta_{\text{lab}} \) for the measurements with a deuteron bombarding beam (see appendix B).

A check on the accuracy of the results as well as on the presence of any serious spurious asymmetry is obtained with \( \theta_{\text{lab}} = 0^\circ \) measurements, because at this angle both \( \langle T_{21} \rangle \) and \( \langle T_{22} \rangle \) are identically zero. From table 2.1, only one of the eight points differs from zero by more than one standard deviation.

In figures 2.7 and 2.8 the results obtained with a deuteron bombarding beam (closed circles) are compared to a portion of the data obtained by McIntyre and Haebel (Mc 64b, Mc 65) at lab angles 45° and 60° (diamond-shaped symbols). The \( \langle T_{2k} \rangle \) in the diagrams are referred to the same coordinate system as in table 2.1. The error bars represent statistical errors only. The greatest disagreement between the two sets of experimental data occurs with \( \langle T_{22} \rangle \) at \( \theta_{\text{lab}} = 60^\circ \) (figure 2.8), where the present experiment datum point appears to be about 20% greater than the earlier
FIGURE 2.7

The deuteron tensor polarization measured at lab angle 45.5° by bombarding a \(^4\)He gas target with deuterons. The closed circles indicate data from the present experiment; the diamond-shaped symbols are from reference Mc 65. The solid and dashed curves were computed from the phase shifts in references Mc 65 and Se 64a, respectively.
\( ^4\text{He}(d, d)^4\text{He} \)

\[ \theta_{\text{LAB}} = 45.5^\circ \]

\[ \theta_{\text{CM.}} = 66.5^\circ \]

\[ \langle T_{20} \rangle \]

\[ \langle T_{21} \rangle \]

\[ \langle T_{22} \rangle \]

DEUTERON ENERGY (MeV)
FIGURE 2.8

The deuteron tensor polarization measured at lab angle $60^\circ$ by bombarding a $^4\text{He}$ gas target with deuterons. The closed circles indicate data from the present experiment; the diamond-shaped symbols are from reference Mc 65. The solid and dashed curves were computed from the phase shifts given in references Mc 65 and Se 64a, respectively.
measurements. However, this is understandable in view of the fact that in the latter data a value of unity was used for factor f in equation 1.8. This implies that McIntyre and Haeberli results are expected to be of the order of 10-15% less in magnitude than the present results. An adjustment of this order generally improves the agreement between the two sets of data. The curves shown in the figures were calculated from the phase shifts of McIntyre and Haeberli (Mc 64a, Mc 65) (solid curves) and Senhouse and Tombrello (Se 64b, Se 64a) broken curves (see appendix C). In the case of the former, the diamond-shaped symbols represent part of the data used in obtaining the phase shifts. It should also be pointed out that to make calculations, smooth curves were fitted to the phase shifts given in references Mc 65 and Se 64a.

In figures 2.9 - 2.11 the results obtained in the present experiment with an alpha bombarding beam are given along with the computed curves. The diamond-shaped symbols in figure 2.9 represent data obtained by McIntyre and Haeberli at a deuteron energy of 4.85 MeV. These data were taken at a lab angle of
FIGURE 2.9

The deuteron tensor polarization measured at lab angle 30° by bombarding a CD₂ target with alphas. The closed circles indicate data from the present experiment; the diamond-shaped symbols are from reference Mc 65. The solid and dashed curves were computed from the phase shifts given in references Mc 65 and Se 64a, respectively.
FIGURE 2.10

The deuteron tensor polarization measured at lab angle $22.5^\circ$ by bombarding a CD$_2$ target with alphas. The solid and dashed curves were computed from the phase shifts given in references Mc 65 and Se 64a, respectively.
\[ D(a,d)^4\text{He} \]

\[ \theta_{\text{LAB}} = 22.5^\circ \]
\[ \theta_{\text{C.M.}} = 135^\circ \]

\[ \langle T_{20} \rangle \]
\[ \langle T_{21} \rangle \]
\[ \langle T_{22} \rangle \]

\[ \text{ALPHA ENERGY (MeV)} \]
FIGURE 2.11

The deuteron tensor polarization measured at lab angle 0° by bombarding a CD₂ target with alphas. The solid and dashed curves were computed from the phase shifts given in references Mc 65 and Se 64a, respectively.
90° (θ_{c.m.} = 120.2°) and have been rotated through β = +60° for comparison in the present coordinate system. The experimental data in figures 2.9 - 2.11 generally support the phase shifts of McIntyre and Haeberli although some quantitative disagreement does exist. The values of <T_{20}> computed from the phase shifts of Senhouse and Tombrello are systematically too large at the lower bombarding energies.

2.5 Discussion and Conclusion

Although the normalization technique with gold runs described in section 2.3c eliminated spurious asymmetries in the first order approximation, additional sources of false asymmetries discussed in section 2.3d do exist and can be summarized as follows: changes in the origin of the deuterons between the measurement and normalization runs; changes in energy and intensity of the incident deuterons across the entrance of the ³He cell for the two types of runs; differences in the mean deuteron energy at the entrance of the cell for the two cases; and the smaller energy spread due to the first target thickness in the case of the gold target. These effects and the angular dependence of the α-d and
d-Au cross sections were fully taken into account in the iterative procedure used to determine the $<T_{2k}>$. The typical magnitude of difference between the first order and final values were 0.010 for $<T_{21}>$ and $<T_{22}>$ and 0.060 for $<T_{20}>$. These values correspond to a shift in the effective centre of the $^3$He cell by about 5% of the target width in the positive z direction and about 1% the target width in the x direction. If it is assumed that the correction of the first order values is accurate to only 50%, a maximum error of ± 0.035 is obtained for $<T_{20}>$ and ± 0.015 for $<T_{21}>$ and $<T_{22}>$. Taking into account an estimated systematic error in the factor f (equation 1.8) of 5% there is an additional 5% uncertainty in the final values of $<T_{2k}>$.

From the quite large values of tensor polarization observed in $\alpha$-d elastic scattering at several energy-angle combinations it may be concluded that the analysis of deuteron polarization by elastic scattering from $^4$He is feasible. Fairly large polarizations found in several regions vary slowly with energy (which is a desirable property for an analyzer). The moment $<T_{21}>$
is essentially zero (or small) for most of the measurements. This feature indicates an interesting effect on the results of a double scattering experiment. Namely, if the vector polarization in α-d scattering is known, unknown deuteron vector polarization can be determined by a simple left-right asymmetry measurement. However, an unknown \( T_{21} \) cannot be determined. Finally, qualitative agreement between the experimental data presented in the previous section and parameters calculated from the phase shifts of references Mc 64a and Mc 65 confirm the validity of this set of phase shifts. Although, a further adjustment of the phase shifts would be required to improve the quantitative agreement, it is indicated by the results that the \( 2^+ \) and \( 1^+ \) assignment (Mc 64a, Mc 65) for the \( ^6 \text{Li} \) levels using the above phase shifts in the considered energy region is correct.
55.

CHAPTER 3
DEUTERON TENSOR POLARIZATION IN PROTON-DEUTERON ELASTIC SCATTERING.

3.1 INTRODUCTION

The elastic scattering of nucleons from deuterons offers a convenient method for studying the important three-nucleon system. Measurements of the nucleon on deuteron polarization can be particularly useful in determining the importance of spin-orbit or tensor terms in the interaction. In addition, p-d and n-d scattering provides an approach to the extraction of information on the neutron-neutron force law using the three-body system as the simplest system available.

Satisfactory fits to n-d and p-d angular distribution data, based on the resonating group method, have been obtained by using velocity-independent central forces only and by assuming that the deuteron is not distorted in the interaction (Bu 41, Bu 52a, Ch 53, Ma 53, Bo 55, Ca 55, Se 57). Such forces, however, result in predictions of zero polarization for both the nucleon and the recoil deuteron. This is contrary to the published data (El 62, Wa 63, Co 64, Co 65, Ha 65), which indicate the presence of small but non-zero values of the nucleon polarization at energies below 10 MeV and increasingly larger values at higher energies and particularly at backward scattering angles.
Both spin-orbit and tensor terms could lead to nucleon and deuteron polarization if included in the numerical calculations. A resonating group formalism which included tensor (but not spin-orbit) forces has been published (Br 58) but numerical results have not yet appeared. The problem has also been formulated with a spin-orbit (but not tensor) force (Bo 61) but again no numerical results have appeared. A more approximate calculation, in which the exchange of the two like nucleons is not taken into account, has been published (De 59, De 62). Although this calculation predicts large values of tensor moments and does take into account, to some extent, the distortion of the deuteron in the scattering and includes a tensor force, the model is a simple one and is expected to give only qualitative results (De 59). Finally, a calculation using resonating group formalism and neglecting both tensor and spin-orbit terms, but taking into account the d-state admixture in the deuteron wave function, has been performed (Ra 66). Although the non-zero deuteron tensor moments was a significant result of the numerical calculation, the agreement with the experimental data presented here is rather poor, especially that taken at a lab angle of 45°. The poor agreement between the calculation and the data indicates the necessity for the inclusion of spin-orbit and
tensor forces in the above model (Ra 66).

Published data on the deuteron polarization in nucleon-deuteron scattering are somewhat limited. At a proton energy of 22 MeV, non-zero values of some of the deuteron tensor polarization parameters have been found for c.m. angles between 60° and 120° (Be 63).

In the present experiment second-rank tensor polarization moments of the recoil deuterons from p-d elastic scattering have been measured at five energies between 3.4 and 9.8 MeV for θ_{lab} = 22.5° (θ_{c.m.} = 135°), at six energies between 4 and 9.8 MeV for θ_{lab} = 30° (θ_{c.m.} = 120°), and at three energies between 6 and 9 MeV for θ_{lab} = 45° (θ_{c.m.} = 90°).

3.2 Experimental Details and Results

With exception of a few experimental details the apparatus and method used for the measurements of deuteron tensor polarization in p-d elastic scattering was identical to the one described for the α-d elastic scattering measurements in the previous chapter.

The rotating CD₂ target foil, 1.6 mg/cm² thick, was positioned so that the normal to its surface was parallel to the outgoing deuterons. The rotation diameter
employed was 1.9 c.m. Thus, no serious deterioration of the target occurred with proton beam currents up to 0.25 μa at $E_p = 4$ MeV. For the higher energy points taken at 45° lab angle the proton beam current was increased to about 0.8 μa. In order to minimise the effect of the deuteron energy variation with scattering angle at proton energies above 6 MeV (which was quite severe) the slit in front of the $^3$He cell had an acceptance angle of ± 1.4°. For the same reason the slit was divided vertically into four equal sections and compensating foils of the thickness increasing by a regular amount towards the small angle side of the slit were used. The resulting energy spread varied from 150 keV at $E_p = 3.5$ MeV to about 50 keV at 9.8 meV proton energy. The mean deuteron energy in the $^3$He cell was approximately 450 keV. The counting rate with the CD$_2$ target in this case varied from about 70 to 900 counts per hour. The background corrections amounted to only 2% for the worst cases.

The final values of the $<T_{2k}>$ for p-d elastic scattering are presented with the statistical errors in tables 3.1 and 3.2. In table 3.1 the tensor moments are referred to a coordinate system with the z axis in the lab direction of the recoiling deuteron beam. In table 3.2 the tensor
Measured values of the tensor polarization of deuterons from p-d elastic scattering. The tensors are referred to a coordinate system with the z-axis in the lab direction of the recoiling deuterons $k_d$. The y-axis is in the direction $k_o \times k_d$ (see figure 2.1).
<table>
<thead>
<tr>
<th>$E_{lab}$ \ (MeV)</th>
<th>$\theta_{inh}$ \ (deg)</th>
<th>$\theta_{c.m.}$ \ (deg)</th>
<th>$\langle T_{20} \rangle$</th>
<th>$\langle T_{21} \rangle$</th>
<th>$\langle T_{22} \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.40</td>
<td>22.5</td>
<td>135</td>
<td>0.094 ± 0.037</td>
<td>0.038 ± 0.020</td>
<td>-0.011 ± 0.025</td>
</tr>
<tr>
<td>4.74</td>
<td>22.5</td>
<td>135</td>
<td>0.052 ± 0.040</td>
<td>-0.044 ± 0.021</td>
<td>-0.055 ± 0.026</td>
</tr>
<tr>
<td>5.79</td>
<td>22.5</td>
<td>135</td>
<td>0.008 ± 0.047</td>
<td>-0.018 ± 0.025</td>
<td>-0.068 ± 0.030</td>
</tr>
<tr>
<td>7.39</td>
<td>22.5</td>
<td>135</td>
<td>0.054 ± 0.054</td>
<td>-0.014 ± 0.030</td>
<td>-0.086 ± 0.036</td>
</tr>
<tr>
<td>9.79</td>
<td>22.5</td>
<td>135</td>
<td>0.071 ± 0.062</td>
<td>-0.052 ± 0.034</td>
<td>-0.026 ± 0.041</td>
</tr>
<tr>
<td>4.06</td>
<td>30</td>
<td>120</td>
<td>0.020 ± 0.048</td>
<td>-0.032 ± 0.030</td>
<td>-0.071 ± 0.037</td>
</tr>
<tr>
<td>5.06</td>
<td>30</td>
<td>120</td>
<td>-0.001 ± 0.052</td>
<td>-0.001 ± 0.033</td>
<td>-0.114 ± 0.038</td>
</tr>
<tr>
<td>5.84</td>
<td>30</td>
<td>120</td>
<td>0.049 ± 0.052</td>
<td>-0.056 ± 0.033</td>
<td>-0.062 ± 0.039</td>
</tr>
<tr>
<td>6.64</td>
<td>30</td>
<td>120</td>
<td>0.017 ± 0.055</td>
<td>0.023 ± 0.030</td>
<td>-0.072 ± 0.036</td>
</tr>
<tr>
<td>8.12</td>
<td>30</td>
<td>120</td>
<td>-0.085 ± 0.058</td>
<td>-0.045 ± 0.029</td>
<td>-0.047 ± 0.036</td>
</tr>
<tr>
<td>9.78</td>
<td>30</td>
<td>120</td>
<td>0.057 ± 0.072</td>
<td>0.043 ± 0.039</td>
<td>-0.013 ± 0.047</td>
</tr>
<tr>
<td>6.10</td>
<td>45</td>
<td>90</td>
<td>-0.129 ± 0.055</td>
<td>-0.004 ± 0.031</td>
<td>-0.016 ± 0.038</td>
</tr>
<tr>
<td>7.59</td>
<td>45</td>
<td>90</td>
<td>-0.152 ± 0.055</td>
<td>-0.009 ± 0.031</td>
<td>-0.032 ± 0.039</td>
</tr>
<tr>
<td>8.69</td>
<td>45</td>
<td>90</td>
<td>-0.078 ± 0.055</td>
<td>0.010 ± 0.032</td>
<td>-0.048 ± 0.039</td>
</tr>
</tbody>
</table>

TABLE 3.1
**TABLE 3.2**

Rotated values of the tensor polarization of deuterons from p-d elastic scattering. The tensors are referred to a coordinate system with the z axis in the lab direction of the bombarding proton beam $k_0$. The y axis is in the direction $k_0 \times k_d$ (see figure 2.1).
<table>
<thead>
<tr>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>$\theta_{\text{lab}}$ (deg)</th>
<th>$\theta_{\text{c.m.}}$ (deg)</th>
<th>$&lt;T_{20}&gt;$</th>
<th>$&lt;T_{21}&gt;$</th>
<th>$&lt;T_{22}&gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4</td>
<td>22.5</td>
<td>135</td>
<td>0.038 ± 0.034</td>
<td>-0.072 ± 0.023</td>
<td>0.012 ± 0.024</td>
</tr>
<tr>
<td>4.74</td>
<td>22.5</td>
<td>135</td>
<td>-0.008 ± 0.036</td>
<td>-0.073 ± 0.025</td>
<td>-0.031 ± 0.026</td>
</tr>
<tr>
<td>5.79</td>
<td>22.5</td>
<td>135</td>
<td>-0.022 ± 0.043</td>
<td>-0.040 ± 0.029</td>
<td>-0.056 ± 0.030</td>
</tr>
<tr>
<td>7.39</td>
<td>22.5</td>
<td>135</td>
<td>0.014 ± 0.050</td>
<td>-0.064 ± 0.034</td>
<td>-0.070 ± 0.035</td>
</tr>
<tr>
<td>9.79</td>
<td>22.5</td>
<td>135</td>
<td>0.006 ± 0.057</td>
<td>-0.077 ± 0.039</td>
<td>0.000 ± 0.040</td>
</tr>
<tr>
<td>4.06</td>
<td>30</td>
<td>120</td>
<td>0.026 ± 0.046</td>
<td>-0.025 ± 0.034</td>
<td>-0.073 ± 0.036</td>
</tr>
<tr>
<td>5.06</td>
<td>30</td>
<td>120</td>
<td>-0.034 ± 0.048</td>
<td>-0.048 ± 0.036</td>
<td>-0.100 ± 0.037</td>
</tr>
<tr>
<td>5.84</td>
<td>30</td>
<td>120</td>
<td>0.072 ± 0.048</td>
<td>-0.025 ± 0.036</td>
<td>-0.071 ± 0.038</td>
</tr>
<tr>
<td>6.64</td>
<td>30</td>
<td>120</td>
<td>0.013 ± 0.048</td>
<td>-0.029 ± 0.036</td>
<td>-0.070 ± 0.035</td>
</tr>
<tr>
<td>8.12</td>
<td>30</td>
<td>120</td>
<td>-0.116 ± 0.049</td>
<td>0.002 ± 0.037</td>
<td>-0.034 ± 0.035</td>
</tr>
<tr>
<td>9.78</td>
<td>30</td>
<td>120</td>
<td>0.078 ± 0.063</td>
<td>-0.014 ± 0.047</td>
<td>-0.021 ± 0.046</td>
</tr>
<tr>
<td>6.10</td>
<td>45</td>
<td>90</td>
<td>-0.047 ± 0.047</td>
<td>0.071 ± 0.038</td>
<td>-0.049 ± 0.037</td>
</tr>
<tr>
<td>7.59</td>
<td>45</td>
<td>90</td>
<td>-0.069 ± 0.067</td>
<td>0.076 ± 0.039</td>
<td>-0.066 ± 0.037</td>
</tr>
<tr>
<td>8.69</td>
<td>45</td>
<td>90</td>
<td>-0.037 ± 0.048</td>
<td>0.024 ± 0.039</td>
<td>-0.065 ± 0.038</td>
</tr>
</tbody>
</table>
moments are referred to a coordinate system with the z axis along the initial proton direction in the lab system. In both tables y axis is defined by $k_o \times k_d$ as in figure 2.1. To obtain the values listed in table 3.2, the z axis of the tensor moments in table 3.1 was rotated about the y axis through on Euler angle $(Sa 60a) \beta = -\theta_{lab}$. Data from table 3.1 taken at lab angles $22.5^\circ$, $30^\circ$ and $45^\circ$ are shown in figures 3.1 - 3.3.

As can be seen in the figures, the measured values of the $\langle T_{2k} \rangle$ are generally small although some non-zero values of polarization are indicated. In the measurements the most consistent deviation from zero polarization occurs with $\langle T_{22} \rangle$ which is found to be of the order of $-0.075$ at $E_p = 7$ MeV.

3.3 Discussion and Conclusion

In determining the $\langle T_{2k} \rangle$ from equation 1.8, a value for the factor $f$ was estimated from the results of Brown et al. (Br 66) to be 0.89. This value corresponds to a mean deuteron energy in the $^3$He cell of 450 keV and is thought to be accurate to about 5%. Corrections were made to the $\langle T_{2k} \rangle$ for finite geometry and known experimental asymmetries by the iterative procedure described previously (section 2.3d). In the worst case the corrections for
FIGURE 3.1

The deuteron tensor polarization measured at lab angle 22.5° (θ_{c.m.} = 135°) by bombarding a CD₂ target with protons.
P-D ELASTIC SCATTERING
LABORATORY ANGLE 22.5°

PROTON ENERGY (MeV)

\[ \langle T_{20} \rangle \]

\[ \langle T_{21} \rangle \]

\[ \langle T_{22} \rangle \]
FIGURE 3.2

The deuteron tensor polarization measured at lab angle $30^\circ \ (\theta_{c.m.} = 120^\circ)$ by bombarding a CD$_2$ target with protons.
P-D ELASTIC SCATTERING
LABORATORY ANGLE 30°

\[ \langle T_{20} \rangle \]

\[ \langle T_{21} \rangle \]

\[ \langle T_{22} \rangle \]

PROTON ENERGY (MeV)
FIGURE 3.3

The deuteron tensor polarization measured at lab angle $45^\circ$ ($\theta_{c.m.} = 90^\circ$) by bombarding a CD$_2$ target with protons.
P-D ELASTIC SCATTERING
LABORATORY ANGLE 45°

\[\langle T_{20} \rangle\]

\[\langle T_{21} \rangle\]

\[\langle T_{22} \rangle\]

PROTON ENERGY (MeV)
\langle T_{21} \rangle \text{ and } \langle T_{22} \rangle \text{ amounted to about 30\% of the statistical errors. For } \langle T_{20} \rangle \text{ corrections of the order of 0.035 were required.}

A rather rough comparison of the present measurements to the values calculated by Delves and Brown (De 59) can be carried out. Their calculation for neutron energy of 3.25 MeV predicts the rather large value of -0.3 for \langle T_{20} \rangle \text{ at 90}^\circ \text{ c.m.}, \text{ almost zero for 120}^\circ \text{ c.m.} \text{ and about +0.1 at } 135^\circ \text{ c.m. The latter compares well with the present measurement taken at 3.4 MeV proton energy but not at higher energies. The value of } \langle T_{22} \rangle \text{ is predicted to be zero (or very small) at all angles, and } \langle T_{21} \rangle \text{ takes on rather large values of the order of +0.15 at the three c.m. angles. Although there is some qualitative agreement between the experimental } \langle T_{20} \rangle \text{ and the predictions of Delves and Brown model, there is little or no agreement in the case of } \langle T_{21} \rangle \text{ and } \langle T_{22} \rangle \text{. However, not much significance should be attached to this comparison because different bombarding energies were involved in the present measurements and the above calculations.}

On the other hand, the predictions of Ramachandran's (Ra 66) calculations, mentioned in section 3.1, are in reasonable qualitative agreement with the present data.
taken at 120° c.m. The agreement is within the statistical error bars from 4 up to 8 MeV proton bombarding energy but discrepancies become larger above 8 MeV. The theoretical values of $<T_{20}>$ at 90° c.m. are much smaller than the data. Furthermore, for these calculations, in addition to $<T_{21}>$ and $<T_{22}>$ for the same frame of reference as described in section 3.2 for table 3.1, the vector polarization $<T_{1k}>$ of the recoiling deuteron is also zero. This is expected to be the case because of the absence of higher order phase shifts from non-central terms in the low energy nucleon-nucleon interaction (Ra 66).

From the non-zero values of $<T_{2k}>$ at some energy-angle combinations, it is clear that the present data lead to the same qualitative conclusion as do the nucleon polarization that the inclusion of spin-orbit or tensor forces is necessary to describe low energy nucleon-deuteron scattering. However, the effects of such forces are relatively small.
CHAPTER 4

DEUTERON TENSOR POLARIZATION IN THE $^9$Be$(p,d)^8$Be REACTION.

4.1 Introduction

Polarization in direct reactions (stripping and pickup reactions) has become an increasingly useful method for studying the properties of nuclei. While angular distributions can be used to obtain orbital angular momenta from direct-interaction theory, polarization measurements can determine the total angular momentum of the states involved in these reactions. Furthermore, polarization measurements on the products of stripping and pickup reactions are of considerable importance for establishing the validity of current theories (Sa 58, Vy 59, Go 60) of such reactions and determining the parameters involved in these theories.

Of particular interest to the study of direct reactions is the use of the distorted wave theory (Li 50, Ge 53, Br 59). This serves as an improvement over the use of the earlier plane wave Born approximation which ignores the distortion of the incoming and outgoing waves by the target and residual nuclei, respectively. If no spin-dependent interactions (spin-orbit coupling or tensor forces) are assumed to be present in the distorted wave
Born approximation (DWBA) description of (p,d) reactions, only vector polarization of the outgoing deuterons will result (Go 60). Nevertheless, although the optical model includes a spin-orbit interaction this is usually ignored (To 59) on the grounds that its effect is relatively small. However, the importance of the spin-dependent interactions is indicated by the observation of substantial polarizations in the elastic scattering of nucleons from nuclei. This is also supported by the observed magnitude of the nucleon polarization produced in deuteron stripping reactions which exceeds the maximum value predicted by the theory when spin-dependent interactions are neglected (Go 61). If, on the other hand, the spin-dependent interactions are included in the DWBA description of the (p,d) reaction the outgoing deuterons will exhibit second-rank tensor polarization in addition to vector polarization. The above arguments indicate that the tensor polarization of deuterons from a (p,d) reaction may provide a useful method for investigation of the spin-dependent part of the interactions within the model chosen to describe the reaction.

Differential cross section measurements (Ha 51, Ra 56, We 56, Re 61, Ya 64, Mo 65) on the $^9\text{Be}(p,d)^8\text{Be}$
reaction \((Q = 0.559 \text{ MeV})\) show quite strongly the direct nature of this reaction process. Aside from gradual narrowing of the forward peak with increasing bombarding energy, the shape and magnitude of the angular distributions up to about \(60^\circ\) do not change significantly over the proton energy range extending from 2.2 to about 10 MeV.

Two earlier measurements of the vector polarization of deuterons from the \(^9\text{Be}(p,d)^{8}\text{Be}\) reaction have been reported. Lambert et al. (La 61) initiated \(D(d,p)^3\text{H}\) with the deuterons from the \(^9\text{Be}(p,d)^{8}\text{Be}\) reaction and measured deuteron vector polarization from the asymmetry of the protons from the former reaction. Using stripping theory without spin-dependent interactions, the second-rank tensor polarization was ignored. Values of the vector polarization \(P_d\) of the order of +10% (Basel convention*) were obtained for deuterons emitted at lab angles \(30^\circ\) and \(90^\circ\) for incident proton energy of 3 MeV. Verbinski and Bokhari (Ve 66) carried out a similar experiment, using the \(^{12}\text{C}(d,p)^{13}\text{C}\) reaction as the analyzer, at a proton energy of 5 MeV and obtained values of \(P_d\) of the order of 5% at lab angles \(30^\circ\) and \(45^\circ\), and a value of

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*The outgoing polarization is considered positive if measured in the direction of \(\mathbf{k}_O \times \mathbf{k}_d\), where \(\mathbf{k}_O\) and \(\mathbf{k}_d\) are propagation vectors as shown in figure 2.1.
about -3\% at a lab angle 60^\circ. For both of these experiments the validity of the results depended on the absence of the deuteron second-rank tensor polarization.

In order to establish the importance of spin-dependent interactions in the (p,d) process Darden and Froelich (Da 65) measured tensor moments <T_{2k}> of the deuterons from the ^9Be(p,d)^8Be reaction. The measurements were carried out for proton bombarding energies of 2.5 and 3.7 MeV at ten-degree intervals for six lab emission angles between 0^\circ and 50^\circ. They found that the <T_{2k}> were generally negative with absolute magnitudes of the order of 0.2 or less. Their preliminary calculations of the <T_{2k}> using DWBA with spin-orbit terms in the distorting potentials agreed in sign with the measured values but were consistently smaller than these by at least an order of magnitude.

With the purpose of extending the above measurements to higher energies (up to 10 MeV) the present experiment was carried out at the four incident proton energies 4.91, 6.90, 8.27 and 9.80 MeV. Values of <T_{2k}> were obtained at lab angles 22.5^\circ, 30^\circ, 45^\circ and 60^\circ. In addition, zero degree measurements were made at the two lower energies.
4.2 Experimental Details and Results

With the exception of a few experimental details the apparatus used for the measurements of deuteron tensor polarization in the $^9\text{Be}(p,d)^8\text{Be}$ reaction was identical to the one described in chapter 2.

A stainless steel target holder sliding through the bottom lid of the scattering chamber was used to raise beryllium and gold targets into position as required. To obtain reasonable yields, a 4.6 mg/cm$^2$ thick beryllium foil* was used. Considerable energy spread in the outgoing deuterons is introduced by the use of thick foils because the deuteron energy loss in the foil exceeds that of the proton. This energy spread was minimized by proper choice of the angle between the normal to the plane of the beryllium foil and the direction of the incident proton beam. The rectangular slit in front of the $^3\text{He}$ cell had an acceptance angle of $\pm 2.7^\circ$ for all measurements except those made at $\theta_{\text{lab}} = 0^\circ$ where a circular slit of angular acceptance $\pm 4.1^\circ$ was used. The effect of the deuteron energy spread across the $^3\text{He}$ cell slit is not so serious in this case because the deuteron energy varies slowly with angle.

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* The Brush Beryllium Co. Cleveland, Ohio, U.S.A.
(up to 45°). Thus covering half of the large slit was necessary only at higher energies and larger angles. The resulting energy spread varied from 120 keV at $E_p = 4.9$ MeV to about 50 keV at 9.8 MeV. The mean deuteron energy in the $^3\text{He}$ cell was approximately 490 keV. The method of monitoring the beam current for the $\theta_{\text{lab}} = 0^\circ$ measurement described in chapter 2 could not be used in this case because emitted deuterons would have been stopped as well as the proton beam. Instead the proton beam was monitored from the first target support. The counting rate of protons from the $^3\text{He}(d,p)^4\text{He}$ analyzing reaction varied between 80 and 500 counts per hour. The background corrections amounted to only 2% for the worst cases.

The final values of the $\langle T_{2k} \rangle$ for the $^9\text{Be}(p,d)^8\text{Be}$ reaction are presented with statistical errors in tables 4.1 and 4.2. In table 4.1 the tensor moments are referred to a coordinate system with the z axis in the lab direction of the emitted deuteron beam. On the other hand, in table 4.2 the tensor moments are referred to a coordinate system with the z axis in the lab direction of the incident proton beam. In both tables the y axis is defined by $k_o \times k_d$ as in figure 2.1. To obtain the values listed in table 4.2, the z axis of the tensor moments of table 4.1 was rotated
Measured values of the tensor polarization of deuterons from the $^9\text{Be}(p,d)^8\text{Be}$ reaction. The tensors are referred to a coordinate system with the $z$ axis in the lab direction of the reaction deuterons $k_d$. The $y$ axis is in the direction $k_o \times k_d$ (see figure 2.1).
<table>
<thead>
<tr>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>$\theta_{\text{lab}}$ (deg)</th>
<th>$\theta_{\text{c.m.}}$ (deg)</th>
<th>$&lt;T_{20}&gt;$</th>
<th>$&lt;T_{21}&gt;$</th>
<th>$&lt;T_{22}&gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.91</td>
<td>0</td>
<td>0</td>
<td>-0.106 ± 0.046</td>
<td>0.029 ± 0.026</td>
<td>-0.039 ± 0.033</td>
</tr>
<tr>
<td>4.91</td>
<td>22.5</td>
<td>26</td>
<td>-0.135 ± 0.059</td>
<td>0.092 ± 0.030</td>
<td>-0.040 ± 0.036</td>
</tr>
<tr>
<td>4.91</td>
<td>30</td>
<td>34.5</td>
<td>-0.036 ± 0.056</td>
<td>-0.168 ± 0.030</td>
<td>-0.121 ± 0.036</td>
</tr>
<tr>
<td>4.91</td>
<td>45</td>
<td>51.5</td>
<td>-0.277 ± 0.064</td>
<td>-0.203 ± 0.031</td>
<td>-0.157 ± 0.039</td>
</tr>
<tr>
<td>4.91</td>
<td>60</td>
<td>68</td>
<td>-0.441 ± 0.075</td>
<td>-0.159 ± 0.035</td>
<td>-0.156 ± 0.043</td>
</tr>
<tr>
<td>6.90</td>
<td>0</td>
<td>0</td>
<td>0.100 ± 0.065</td>
<td>0.001 ± 0.032</td>
<td>0.030 ± 0.039</td>
</tr>
<tr>
<td>6.90</td>
<td>22.5</td>
<td>26</td>
<td>-0.012 ± 0.060</td>
<td>0.107 ± 0.032</td>
<td>-0.155 ± 0.040</td>
</tr>
<tr>
<td>6.90</td>
<td>30</td>
<td>34.5</td>
<td>0.064 ± 0.054</td>
<td>-0.036 ± 0.029</td>
<td>-0.068 ± 0.036</td>
</tr>
<tr>
<td>6.90</td>
<td>45</td>
<td>51.5</td>
<td>-0.013 ± 0.056</td>
<td>0.181 ± 0.029</td>
<td>-0.049 ± 0.036</td>
</tr>
<tr>
<td>6.90</td>
<td>60</td>
<td>68</td>
<td>-0.307 ± 0.066</td>
<td>0.414 ± 0.031</td>
<td>-0.002 ± 0.038</td>
</tr>
<tr>
<td>8.27</td>
<td>22.5</td>
<td>26</td>
<td>0.006 ± 0.059</td>
<td>0.083 ± 0.031</td>
<td>-0.028 ± 0.038</td>
</tr>
<tr>
<td>8.27</td>
<td>30</td>
<td>34.5</td>
<td>-0.006 ± 0.058</td>
<td>0.024 ± 0.030</td>
<td>-0.035 ± 0.037</td>
</tr>
<tr>
<td>8.27</td>
<td>45</td>
<td>51.5</td>
<td>-0.021 ± 0.060</td>
<td>0.241 ± 0.030</td>
<td>0.090 ± 0.038</td>
</tr>
<tr>
<td>8.27</td>
<td>60</td>
<td>68</td>
<td>-0.122 ± 0.051</td>
<td>0.363 ± 0.024</td>
<td>0.100 ± 0.031</td>
</tr>
<tr>
<td>9.80</td>
<td>22.5</td>
<td>26</td>
<td>-0.018 ± 0.067</td>
<td>0.185 ± 0.034</td>
<td>0.060 ± 0.042</td>
</tr>
<tr>
<td>9.80</td>
<td>30</td>
<td>34.5</td>
<td>-0.131 ± 0.077</td>
<td>-0.121 ± 0.038</td>
<td>-0.030 ± 0.047</td>
</tr>
<tr>
<td>9.80</td>
<td>45</td>
<td>51.5</td>
<td>-0.162 ± 0.073</td>
<td>0.045 ± 0.035</td>
<td>0.092 ± 0.043</td>
</tr>
<tr>
<td>9.80</td>
<td>60</td>
<td>68</td>
<td>-0.309 ± 0.083</td>
<td>0.164 ± 0.038</td>
<td>0.128 ± 0.047</td>
</tr>
</tbody>
</table>

+ The mean c.m. angle for the $\theta_{\text{lab}} = 0^\circ$ measurements is about $2.5^\circ$. 

\[ \text{TABLE 4.1} \]
Rotated values of the tensor polarization of deuterons from the $^9$Be(p,d)$^8$Be reaction. The tensors are referred to a coordinate system with the z axis in the lab direction of the bombarding proton beam $k_0$. The y axis is in the direction $k_0 \times k_d$ (see figure 2.1).
<table>
<thead>
<tr>
<th>E_{lab} (MeV)</th>
<th>\theta_{lab} (deg)</th>
<th>\theta_{c.m.} (deg)</th>
<th>\langle T_{20} \rangle</th>
<th>\langle T_{21} \rangle</th>
<th>\langle T_{22} \rangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.91</td>
<td>0</td>
<td>0</td>
<td>-0.106 ± 0.046</td>
<td>0.029 ± 0.026</td>
<td>-0.039 ± 0.033</td>
</tr>
<tr>
<td>4.91</td>
<td>22.5</td>
<td>26</td>
<td>-0.033 ± 0.060</td>
<td>0.110 ± 0.031</td>
<td>-0.082 ± 0.025</td>
</tr>
<tr>
<td>4.91</td>
<td>30</td>
<td>34.5</td>
<td>-0.238 ± 0.063</td>
<td>-0.117 ± 0.024</td>
<td>-0.038 ± 0.025</td>
</tr>
<tr>
<td>4.91</td>
<td>45</td>
<td>51.5</td>
<td>-0.413 ± 0.068</td>
<td>0.091 ± 0.025</td>
<td>-0.101 ± 0.026</td>
</tr>
<tr>
<td>4.91</td>
<td>60</td>
<td>68</td>
<td>-0.257 ± 0.069</td>
<td>0.246 ± 0.030</td>
<td>-0.232 ± 0.028</td>
</tr>
<tr>
<td>6.90</td>
<td>0</td>
<td>0</td>
<td>0.100 ± 0.065</td>
<td>0.001 ± 0.032</td>
<td>0.030 ± 0.039</td>
</tr>
<tr>
<td>6.90</td>
<td>22.5</td>
<td>26</td>
<td>0.055 ± 0.056</td>
<td>0.026 ± 0.028</td>
<td>-0.182 ± 0.026</td>
</tr>
<tr>
<td>6.90</td>
<td>30</td>
<td>34.5</td>
<td>-0.019 ± 0.055</td>
<td>-0.081 ± 0.031</td>
<td>-0.034 ± 0.025</td>
</tr>
<tr>
<td>6.90</td>
<td>45</td>
<td>51.5</td>
<td>0.189 ± 0.048</td>
<td>-0.016 ± 0.023</td>
<td>-0.131 ± 0.025</td>
</tr>
<tr>
<td>6.90</td>
<td>60</td>
<td>68</td>
<td>0.476 ± 0.039</td>
<td>-0.045 ± 0.025</td>
<td>-0.321 ± 0.026</td>
</tr>
<tr>
<td>8.27</td>
<td>22.5</td>
<td>26</td>
<td>0.071 ± 0.056</td>
<td>0.047 ± 0.025</td>
<td>-0.054 ± 0.026</td>
</tr>
<tr>
<td>8.27</td>
<td>30</td>
<td>34.5</td>
<td>0.010 ± 0.053</td>
<td>0.000 ± 0.024</td>
<td>-0.042 ± 0.026</td>
</tr>
<tr>
<td>8.27</td>
<td>45</td>
<td>51.5</td>
<td>0.345 ± 0.046</td>
<td>0.058 ± 0.024</td>
<td>-0.060 ± 0.027</td>
</tr>
<tr>
<td>8.27</td>
<td>60</td>
<td>68</td>
<td>0.492 ± 0.040</td>
<td>-0.073 ± 0.018</td>
<td>-0.150 ± 0.022</td>
</tr>
<tr>
<td>9.80</td>
<td>22.5</td>
<td>26</td>
<td>0.157 ± 0.058</td>
<td>0.159 ± 0.028</td>
<td>-0.012 ± 0.030</td>
</tr>
<tr>
<td>9.80</td>
<td>30</td>
<td>34.5</td>
<td>-0.220 ± 0.080</td>
<td>-0.004 ± 0.032</td>
<td>0.007 ± 0.033</td>
</tr>
<tr>
<td>9.80</td>
<td>45</td>
<td>51.5</td>
<td>0.071 ± 0.063</td>
<td>0.145 ± 0.035</td>
<td>-0.003 ± 0.031</td>
</tr>
<tr>
<td>9.80</td>
<td>60</td>
<td>68</td>
<td>0.330 ± 0.055</td>
<td>0.138 ± 0.032</td>
<td>-0.133 ± 0.033</td>
</tr>
</tbody>
</table>

* The mean c.m. angle for the \theta_{lab} = 0° measurements is about 2.5°.
about the y axis through an Euler angle \( \beta = -\theta_{\text{lab}} \) (see appendix B).

A check on the presence of any spurious asymmetry is provided by \( \theta_{\text{lab}} = 0^\circ \) measurements since at this lab angle both \( \langle T_{21} \rangle \) and \( \langle T_{22} \rangle \) vanish. It is evident from table 4.1 that both measurements differ from zero by not much more than their statistical errors. This is an encouraging result as to the accuracy of the experiment.

Results obtained for average proton energies 4.91, 6.90, 8.27 and 9.80 MeV are shown in figures 4.1 to 4.4 respectively. The \( \langle T_{2k} \rangle \) have been plotted as a function of lab emission angle, \( \theta_{\text{lab}} \), and refer to the coordinate system of table 4.1. Only statistical uncertainties are indicated by the error bars in the figures.

From figures 4.1 to 4.4 it is evident that the tensor polarization of deuterons from the \( ^9\text{Be}(p,d)\,^8\text{Be} \) reaction is considerably different from zero. The largest value of \( \langle T_{20} \rangle \) observed was \(-0.441\), which corresponds to lab angle 60° and proton energy 4.91 MeV. \( \langle T_{21} \rangle \) exhibits some large positive values the largest being 0.414 at lab angle 60° for the proton energy of 6.90 MeV. The largest \( \langle T_{22} \rangle \) value of \(-0.157\) was measured at lab angles 45° and 60° for proton energy 4.91 MeV. The similarity
FIGURE 4.1

The deuteron tensor polarization measured by bombarding a Be foil with protons at the bombard­
ing energy of 4.91 MeV.
$^9\text{Be}(p,d)^8\text{Be}$

$E_p = 4.91$ MeV

\[ \langle T_{22} \rangle \]

\[ \langle T_{21} \rangle \]

\[ \langle T_{20} \rangle \]

$\theta_{\text{LAB}}$ (degrees)

[Data points with error bars]
FIGURE 4.2

The deuteron tensor polarization measured by bombarding a Be foil with protons at the bombarding energy of 6.90 MeV.
\( ^9 \text{Be}(p,d)^8 \text{Be} \)  
\( E_p = 6.90 \text{ MeV} \)

\[ \langle T_{22} \rangle \]

\[ \langle T_{21} \rangle \]

\[ \langle T_{20} \rangle \]

\( \theta_{\text{LAB}} \) (degrees)
FIGURE 4.3

The deuteron tensor polarization measured by bombarding a Be foil with protons at the bombarding energy of 8.27 MeV.
$^9$Be(p,d)$^8$Be

$E_p = 8.27$ MeV

$\langle T_{22} \rangle$, $\langle T_{21} \rangle$, $\langle T_{20} \rangle$ vs $\theta_{LAB}$ (degrees)
FIGURE 4.4

The deuteron tensor polarization measured by bombarding a Be foil with protons at the bombarding energy of 9.80 MeV.
in the behaviour of polarization at different incident proton energies is not surprising in view of the similarity of the angular distribution data at different energies. The largest differences occur for the lab angle 60°. This might be expected since this angle is almost off the forward peak in the angular distributions and compound nucleus effects may be expected to become relatively more important for this case.

4.3 Discussion and Conclusion

The largest correction of the first order values of \(<T_{20}>\) obtained by the iterative procedure described in section 2.3d, taking into account finite geometry effects and known asymmetries, was 0.035. The respective correction for \(<T_{21}>\) and \(<T_{22}>\) was 0.025. In addition to the already discussed uncertainties present in the experiment, systematic uncertainty in the values of \(<T_{2k}>\) of 5% is attributed to the uncertainty in the value of the factor f of the same order of magnitude. An average value of f of 0.88 was used corresponding to an average deuteron energy in the \(^3\)He cell of 490 keV (Br 66).

The results of Beckner et al. (Be 61) obtained from deuteron spectra from elastic scattering of deuterons off a thin Be foil at 5.2 MeV, indicated the presence of a small contribution from the sequential decay process
From these data it can be estimated that the deuterons entering the $^3\text{He}$ cell in the present experiment contain a contribution from the sequential decay deuterons of the order of 4% in the energy region considered. This may give rise to an additional uncertainty in the value of $\langle T_{2k} \rangle$ of the same order.

Poor quantitative agreement between the data of Darden and Froelich (Da 65) and a DWBA calculation performed using only a spin-orbit coupling term in the optical potential of the proton and deuteron reported by them, raises a number of possibilities as to the reason for the discrepancy. The analysis has shown that varying the depth of the central potential and using volume absorption instead of surface absorption for the deuteron does not improve the agreement significantly. Nor does the use of a complex spin-orbit term improve the agreement. There are two other possibilities not taken into account in their analysis. One is the failure to include tensor interaction in the calculations. The other is a possibly erroneous assumption that $^9\text{Be}$ is well described by a $p_{3/2}$ neutron moving in the field of a spherically symmetric $^8\text{Be}$ core. They suggested that the inclusion of deformation effects in the $^9\text{Be}$ nucleus may alter the predictions of the DWBA appreciably.
An optical model calculation using the present data (Ro 66) is in progress. Beside including the usual spin-orbit term (see for example reference Da 65) the analysis takes into account two of the three possible tensor interactions (Sa 60b).

It is believed that the inclusion of tensor interactions will give larger values of the tensor polarization than appear possible with only spin-orbit interaction. If it can be shown that these tensor interactions are important then it may be possible to give a better description of the j-dependence of stripping and pickup reactions which at the present time is not understood.

In conclusion, the relatively large values of the \( \langle T_{2k} \rangle \) parameters in the \(^9\)Be(p,d)\(^8\)Be reaction indicate that spin-dependent interactions are present. The data presented here together with those of Darden and Froelich should be of considerable use to the efforts concerned with the spin-dependent interactions in direct reactions.
CHAPTER 5

ELASTIC SCATTERING OF $^4$He-PARTICLES FROM TRITIUM.

5.1 Introduction

A summary of the present knowledge concerning the level structure of $^7$Li and $^7$Be up to 8 MeV excitation energy is shown in figure 5.1 (La 66).

The level structure of $^7$Li and $^7$Be has been predicted theoretically from a number of nuclear models (Zn 53, Kn 56, So 57, Pe 60, Ta 61, KI 61, Cl 62, Ch 63). The results of intermediate coupling calculations by Inglis (In 53), and separately by Kurath (Ku 56) predict the following order (energy increasing) for the odd parity states (in LS notation): $^2P_{3/2}$, $^2P_{1/2}$, $^2F_{7/2}$, $^2F_{5/2}$ and $^4P_{5/2}$. The first three of these have been known for some time. However, until recently, the only established $J^\pi = 5/2^-$ levels were found at 7.47 MeV in $^7$Li and 7.18 MeV in $^7$Be. These states have long been thought to be the upper members of the $^2F$ doublets, and the lowest level of the $^4P$ multiplet was thought to lie at a somewhat higher energy. The lower levels of the $^2F$ multiplet have been considered to be the known states at 4.63 MeV in $^7$Li and 4.55 MeV in $^7$Be. If the states at 7.47, 7.18 and 4.63 4.55 MeV comprise the $^2F$ doublets.
FIGURE 5.1
Level diagram for $^7$Li and $^7$Be.
the splitting is about 6 times the splitting of the $^2P$ doublets which are well established as the ground and first excited states. Thus, Meshkov and Ufford (Me 56) suggested that the levels at 7.47 and 7.18 MeV in the two mirror nuclei, were actually members of the $^4P$ multiplet and that the $^2F_{5/2}$ levels were at lower energies. Further considerations of the nucleon and $\alpha$-particle reduced widths of these levels led French and Fujii (Fr 57) to a similar conclusion. Additional experimental evidence ensured that the interpretation of the 7.47 and 7.18 MeV levels as being $^4P_{5/2}$ levels is indeed correct (Jo 54, Ma 56, Ma 57, Le 57, Ha 60).

The success of the predictions of the above levels, made the apparent absence of the $F_{5/2}$ state quite disturbing.

The assignment of positive parity to the 6.5 MeV state in $^7$Be observed by Marion et al. (Ma 56) was motivated by the existence of odd Legendre polynomial terms in the angular distribution of the $^6$Li$(p,\alpha)^3$He reaction. An S-state for this level with $J^\pi = \frac{3}{2}^+$ was suggested. However, evidence against such a state in this energy region comes from the work done on the $^6$Li$(p,\gamma)^7$Be reaction by Warren et al. (Wa 56) who found that this reaction is not resonant in this region and
shows no indication of $E1$ capture. Furthermore, a careful analysis of the $^6\text{Li}(p,p)^6\text{Li}$ and $^6\text{Li}(p,\alpha)^3\text{He}$ data up to 1.5 MeV proton energy does not show a level around 6.5 MeV in $^7\text{Be}$ (Mc 63). Extension of the analysis to higher proton energies up to 3 MeV indicated that a $^1\!\!_2^+$ level may lie above the $^4P_{5/2}$ state. However, by taking proton bombarding energy higher up in $^6\text{Li}(p,p)^6\text{Li}$ scattering, Harrison and Whitehead (Ha 63) found a level above $^4P_{5/2}$ state but the analysis of the data favoured the $^4P$ configuration for the state rather than one of positive-parity proposed by McCray (Mc 63). This excluded the possibility of an $S$-state at about 6.5 MeV.

On the other hand, evidence for the presence of the $^2F_{5/2}$ state in $^7\text{Li}$ at about 6.5 MeV comes from an analysis of the intensities of proton groups from the $^7\text{Li}(p,p')^7\text{Li}^*$ reaction (Ma 57). Moreover, it was suggested (Le 57) that the positive-parity state played no significant part in the above inelastic scattering and instead evidence was produced that in fact the state at 6.5 MeV was a $5/2^-$ state. Chesterfield and Spicer (Ch 63) also found that their rotational model could not predict
a positive-parity state below 9 MeV excitation energy. Finally, no positive-parity states with appreciable \( \alpha \)-particle width were observed in the range of excitation energy covered by Tombrello and Parker (To 63a) in scattering of \( ^3 \text{He} \)-particles from \( ^4 \text{He} \) and indeed the existence of a 6.5 MeV \( 5/2^- \) level in \( ^7 \text{Be} \) was confirmed.

The aim of the present measurements of \( ^4 \text{He}-T \) elastic scattering cross section was to confirm the existence of the two F-states in \( ^7 \text{Li} \). The data consists of angular distributions at alpha lab energies 5.07, 6.64, 8.20, 9.76 and 10.92 MeV and of excitation functions (110-keV steps over the energy range 5 to 11 MeV) at c.m. angles 35.09°, 65.29°, 90°, 125°, 140° and 150°.

### 5.2 Apparatus

Apparatus used in making the measurements described in this and the following chapter is shown in figures 5.2 and 5.3. Beams of protons, deuterons, doubly charged \( ^3 \text{He}^- \) and \( ^4 \text{He}^- \)-particles were provided by the A.N.U. tandem accelerator. Beam currents of about 0.05 to -1.0 \( \mu \text{A} \) were used. The beam was energy analysed by a 90° magnet whose calibration was known to 0.1%. Energy stability was estimated to have been 5 keV or less. After
FIGURE 5.2

Vertical cross section of the scattering chamber. The numbered parts are (1) collimator tube, (2) gas target, (3) nickel windows, (4) suppressor magnet, (5) Faraday cup, (6) O-ring seal, (7) Teflon bearing rings and (8) drive sprocket.
FIGURE 5.3

Horizontal cross section of the scattering chamber. The numbered parts are (1) collimator tube, (2) gas target, (3) Mylar foil, (4) nickel windows, (5) detector slit assembly, (6) solid state counter and (7) Faraday cup assembly.
energy analysis, the beam was deflected by a 30° magnet and passed through a pair of "tracking slits". The current in the deflection magnet was regulated by the beam current on the tracking slits, requiring the beams to pass through the centre of the slits. The beam, then, passed through a magnetic quadrupole lens and finally, before striking the gas target entrance foil, through the tantalum collimators 0.5 mm thick with 1.5 mm aperture diameters spaced 20 cm apart. A tantalum anti-scattering baffle with a 2.2 cm aperture was placed 10 cm behind each collimator disc. The beam collimating system was situated in a brass tube inserted into the entrance part of the scattering chamber.

Tritium gas was contained in a target assembly similar in design to that described by Corelli et al. (Co 57). The beam entered the 7.5 cm diameter and 2.5 cm high target through a 0.6 μm nickel foil* and left it through another 1.2 mm nickel foil at the exit port of the target assembly. A 1.3 cm high window extending from 12.5° to 167.5° with respect to the incident beam

*For the 4He-T measurements the foils were 0.25 μm and 0.6 μm respectively. This was to reduce the spread and loss of the 4He-particle energy.
direction was located on both sides of the gas target. The scattered and recoil particles passed out of the target assembly through a 6.25 \( \mu \text{m} \) aluminized Mylar foil which was satisfactorily attached to the stainless steel frame by an ordinary epoxy resin (Dupont adhesive 46951). The thickness of the foil was equivalent to 1.51 mg/cm\(^2\) of aluminium.

The beam was collected by a Faraday cup whose acceptance half-angle was about 4°. The Faraday cup was equipped with both magnetic and electrostatic suppression to avoid spurious effects (e.g. collection of electrons produced by the beam passing through the foils and striking the collimators; loss of secondary electrons from the tantalum beam stop in the cup). A suppression potential of -300 volt was used.

The scattered particles were collimated by a rectangular slit system consisting of nominally identical slits with dimensions 3 x 5 mm together with an anti-scattering baffle with an aperture of 5 x 7.5 mm midway between. The slits were made of precision steel shim stock* 0.23 mm thick. Ortec surface barrier detectors

*L.S. Starret Co., Athol, Massachusetts, U.S.A.
with depletion depths of about 500 μm at full bias, were placed immediately behind the rear slit in each system. The angular resolution of the two systems was 2°.

The aluminium scattering chamber which contained the entire experimental set-up had an inside diameter of 45 cm and an inside height of 29 cm. The two detector systems were situated on the independently rotatable lids of the chamber and were used simultaneously throughout the measurements. The angles of the detectors could be set to an accuracy of better than ± 0.1° by rotation of the lids.

The gas target was filled with tritium at about 25 mm Hg pressure from an uranium oven specially prepared for tritium storage (see appendix D for full description). Pressure in the target cell was measured to an absolute accuracy of ± 0.05 mm by means of a Wallace and Tiernan FA-145 aneroid manometer, which was calibrated with a dibutyl phthalate oil manometer. Two calibrations carried out during the course of the measurements showed no appreciable shift in calibration. The pressure gauge was connected to the gas cell via a filling system built of 1.5 mm inside bore copper tubing.
and Hoke needle valves. The system allowed pumping on the target assembly with a roughing pump, linking of the target with the scattering chamber pumping system for simultaneous evacuation, as well as connection between the uranium oven and the target via the Wallace and Tiernan gauge.

5.3 Experimental Procedure and Errors

Pulses from the detectors were amplified with Ortec double delay line amplifier systems. Windows were placed around particle groups and counts were accumulated on fast A.N.U. scalers. Counting rates were kept below 1000 counts per second to make dead time losses negligible. Spectra from both counters were also accumulated on an R.I.D.L. kicksorter. The pulses were routed in such a way that each spectrum was stored in a 200-channel block of the 400 channel analyzer. The average dead time factor found was about 2 to 5% and appropriate dead time correction was applied to the data.

An Elcor Model A-309 current integrator was used to measure the collected charge and to terminate the runs when a desired amount of charge was collected. The integrator was calibrated frequently during the experi-
ment with a constant current source, known to about 0.03%. A Philips electronic timer accurate to 0.1% was used in the calibrations. Drift in the integrator between calibrations was typically about 0.2%.

Pressure and temperature were recorded at regular intervals. Variations in the pressure reading, which were of the order of 0.5 mm Hg, are attributed to temperature changes in the target as well as small leakages through foils and foil seals. Hydrogen exchange represented the greatest problem in determining the net pressure due to tritium present in the target cell. The hydrogen impurity was monitored by the spectra accumulated on the kicksorter at 10 MeV bombarding energy and a fixed lab angle where proton and triton groups were well resolved. Pressure calibration curves were also plotted from proton-group yields and measured cross section for elastic scattering of protons, deuterons, $^3$He- and $^4$He-particles from a hydrogen gas target at various energy-angle pairs. The estimated error in the measured cross section was calculated from the expression derived in appendix D (equation D1) and it was estimated to be about 4%.

The cross section of the scattering was computed from measured yields using the relation (Si 59)
\[ \sigma(\theta) = \frac{Y \sin \theta}{nNG} \]  

(5.1)

where \( \sigma(\theta) \) is the lab cross section, \( Y \) is the measured yield, \( n \) is the total number of bombarding particles determined from the charge collected, \( N \) is the number of target nuclei per unit volume determined from the measured pressure and temperature of the tritium gas, using the ideal gas law, and \( \theta \) is the lab scattering angle. The factor \( G \) is the geometry factor of the detection system and is given by (Si 59)

\[ G = G_{oo} \left[ 1 + \Delta_0 + \frac{\sigma}{\sigma} \Delta_1 + \frac{\sigma''}{\sigma} \Delta_2 + \ldots \right] \]  

(5.2)

where

\[ G_{oo} = \frac{4b_1 b_2 \ell}{R_0 h} \]  

(5.3)

and

\[ \Delta_0 = \frac{1}{3} \frac{\cos^2 \theta b_2^2}{\sin^2 \theta R_0^2} - \frac{1}{2h^2} (b_1^2 + b_2^2) - \frac{1\ell^2}{8R_0^2} \]

with the higher order terms being neglected. In equation 5.3 the quantities are: front and rear slit widths, \( b_1 = b_2 = 3 \text{ mm} \), respectively; rear slit height, \( \ell = 5 \text{ mm} \); distance from centre of target to rear slit, \( R_0 \) = 185 mm; and distance between slits, \( h = 132 \text{ mm} \). An error of less than 0.05% in the worst case was introduced by ignoring
the higher terms in equation 5.2.

Calculation of cross section from equation 5.1 was performed on an I.B.M. 1620 computer. The program included computation of the yield from various particle-peaks in the spectra, correction to the data for the background and for the partial pressure arising from the hydrogen impurity in the tritium target, computation of the cross section from the yields, conversion of the lab cross section and scattering angles to the c.m. system, adjustment of the collected charge and pressure measurement for calibration errors, as well as computation of the incident particle energy at the centre of the gas target for each energy used.

The energy losses in the nickel entrance foil and in the tritium gas to the centre of the gas target were computed from an expression of the form

$$\frac{dE}{dx} = a(\ln E + b)/E$$

(5.4)

where E is the energy of the incident particle and a and b were obtained from Whaler (Wh 58). The energy loss varied from 130 to 80 keV for $^4\text{He}$-particles to less than 50 keV for protons and deuterons. The energy resolution at the lowest energies used for the $^4\text{He}$-particles...
and protons was estimated to about 100 and 40 keV respectively.

An estimate of the error involved in the cross section measurements described here can be given if individual effects contributing to uncertainties in the measurements are considered separately.

One error in charge collection was due to multiple scattering of the incident beam in the nickel entrance and exit foils and, to a lesser extent, in the tritium gas. The two constrictions which limited charge collection were the magnet in the Faraday cup (half-angle 4.3° with respect to the centre of the gas target) and the exit port of the gas target (half-angle 2.1° with respect to the nickel entrance foil). Young (Yo 65) estimated for d-α scattering case that the effect of multiple scattering was negligible above 5 MeV deuteron energy. The error in charge integration, is estimated to have been about 0.3%. This error results from uncertainty in the calibration of the integrator (0.1%) and from drift in the integrator itself (0.2%).

An error of 0.3% is assigned to the geometry factor G, due to the limited accuracy (3 μm) in measurements of the slit dimensions. The combined effects of error in the angle setting and deviation of the beam from the
centre of the scattering chamber were estimated by Young (Yo 65) to be about 1% (from left-right scattering measurements).

The error resulting from the measurement of the tritium gas pressure is about 0.2%. This estimate is based on a 0.1% uncertainty in the visual reading of the pressure gauge and a 0.1% uncertainty in its calibration. However, the error resulting from the corrections due to the hydrogen impurity in the gas target is estimated to be as high as 4%.

The measured temperature of the gas target was usually about 1°C above the room temperature. The uncertainty in the measurement, estimated to be 0.3%, resulted from the relatively poor heat conductivity of the stainless steel bottom plate of the gas target. The effect of the heating along the path of the beam through the gas has been investigated by Young (Yo 65) for deuterons through helium and the expected error was found to be 1.3% at 4 MeV and 0.6% at 10 MeV with measured d-α cross section being too low by these amounts. The latter effect has been ignored in correcting the measured cross section.

Effects of the slit-edge scattering are manifested
by a continuous distribution of low energy particles, resulting from the penetration at the slit edges of particles originating anywhere along the line source. The error due to this effect was estimated to be about 0.5%.

Multiple scattering by the Mylar foil and the gas result in an estimated error of about 1%. This estimate is based on proton-proton scattering checks (Yo 65).

The overall systematic error in the cross section measurements described here can be estimated by combining individual errors described above. The arithmetic total amounts to 8.4% and the r.m.s. total to 4.4%.

5.4 Results and Discussion

In addition to the $^4\text{He}-T$ measurements, excitation curves of $^4\text{He}-^3\text{He}$ elastic scattering at c.m. angles 65.29°, 90° and 125° were obtained between 5.8 and 7.9 MeV bombarding energy in order to check the present energy scale against that of Tombrello and Parker (To 63a). That was done primarily because in the present experiment the beam was introduced through a foil thus causing an energy shift and spread. A very good agreement was found between the two sets of data in the energy region around the $^2\text{F}_{7/2}$ level in $^7\text{Be}$. 
$^4$He-T elastic scattering excitation curves are shown together with the $^4$He-$^3$He elastic scattering excitation curves in figure 5.4. Solid circles represent the former and crosses represent the latter data. The $^4$He-T curves show a pronounced anomaly for bombarding energies near 5.1 MeV corresponding to a level in $^7$Li at 4.6 MeV. This resonance appears as a dip in the excitation function at 65.29° c.m. angle and as a peak at other angles. Also in evidence is another structure near 9.7 MeV which corresponds to a broad level in $^7$Li at approximately 6.6 MeV. The identification of the higher resonance as coming through the same $\ell$-partial wave as that of the lower resonance is indicated by the striking similarity of its energy variation at each angle. Indeed, the set of phase shifts obtained for angular distributions indicates an F-wave contribution to both levels observed. Furthermore, a visual comparison between the present data and those of reference (To 63a) indicates that the levels are the mirror levels of those in $^7$Be. Consequently, there is little doubt that the two levels are $\ell = 3$ with spin and parity assignments $7/2^-$ and $5/2^-$. The phase shift analysis programme has been written using partial waves through $\ell = 3$. The programme works
FIGURE 5.4

Excitation functions from elastic scattering of $^4\text{He}$ from tritium (circles). Also three excitation functions from elastic scattering of $^4\text{He}$ from $^3\text{He}$ (crosses). The differential cross section is in c.m. system.
CROSS SECTION (mb/ster)

**He-^7^He ELASTIC SCATTERING**

**He-^3^He ELASTIC SCATTERING**

\[ \theta_{CM} = 35.1^\circ \]

\[ \sigma(\theta) \times 2 \]

\[ \theta_{CM} = 150.0^\circ \]

\[ \theta_{CM} = 65.3^\circ \]

\[ \theta_{CM} = 140.0^\circ \]

\[ \theta_{CM} = 90.0^\circ \]

\[ \theta_{CM} = 125.0^\circ \]

ALPHA ENERGY (MeV)
basically as follows: first, an initial set of phase shifts is read in. These initial phase shifts are varied simultaneously each one being incremented by an amount proportional to the partial derivative of $x^2$ with respect to that phase shift, where $x^2$ is given by

$$x^2 = \sum_{i=1}^{N} \left[ \frac{\sigma_{\text{exp}}(\theta_i) - \sigma_{\text{calc}}(\theta_i)}{\sigma_{\text{calc}}(\theta_i)} \right]^2$$

where $\sigma_{\text{exp}}$ = measured cross section at a given angle $\theta_i$, $\sigma_{\text{calc}}$ = calculated cross section for a set of phase shifts as described in appendix E, and $N$ = number of points on the angular distribution. In other words, phase shifts are adjusted so that each phase shift changes in the direction of decreasing $x^2$.

After each variation of the phase shifts, the new value of $x^2$ is compared with the old value, and the procedure is repeated until $x^2$ goes through a minimum. When this happens, the whole procedure is begun again, the new phase shifts as initial values, with the amount by which each phase shift is incremented reduced by some factor. The process is terminated when the increment is less than some pre-determined amount. At each energy for a set of phase shifts giving a minimum value of $x^2$ an angular distribution was calculated. To obtain these fits, the phase shifts of Tombrello and Parker (To 63a)
from the \(^3\text{He}-^4\text{He}\) scattering were used as initial values. The fits are presented along with the experimental points in figure 5.5 and they appear to be very good. The final values of the seven parameters used in the fits are presented in table 5.1.

Barnard et al. (Ba 64a) repeated the work of Tombrello and Parker (To 63a) in the energy region 2.5 to 5.7 MeV and obtained two sets of phase shifts from the measured cross sections. The two sets differ principally in the sign of the splitting of the p-wave phase shifts. The set which has \(\delta_1^+\) larger (closer to 180°) than \(\delta_1^-\) was preferred on the basis of the continuity of phase shifts as a function of energy. This p-wave splitting is opposite to that used by Miller and Phillips (Mi 58) and by Tombrello and Parker (To 63a). It disagrees with the present results too, but this is not surprising since the phase shifts of reference (To 63a) were used as a starting set.

No resonance parameters have been extracted so far. However, a single level parametrization of \(\delta_3^-\) and \(\delta_3^+\) should be attempted (see Appendix E). Presumably there is negligible effect on the \(^4\text{He}-\text{T}\) cross section by the \(^4\text{P}_{5/2}\) level at 7.47 MeV, since none was found in the case of \(^3\text{He}-^4\text{He}\) scattering (To 63a).
Angular distributions from elastic scattering of $^4$He from tritium. Solid circles represent data from the $\alpha$-group and crosses represent data from the recoil triton group. Solid lines represent the fit to the data given by the phase shifts summarized in the table.
<table>
<thead>
<tr>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>5.07</th>
<th>6.64</th>
<th>8.20</th>
<th>9.76</th>
<th>10.92</th>
</tr>
</thead>
<tbody>
<tr>
<td>phase shift</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>-48.21</td>
<td>-57.12</td>
<td>-64.59</td>
<td>-56.33</td>
<td>-65.60</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>150.18</td>
<td>144.86</td>
<td>135.20</td>
<td>133.49</td>
<td>124.11</td>
</tr>
<tr>
<td>$\delta_1^+$</td>
<td>153.59</td>
<td>146.71</td>
<td>140.10</td>
<td>138.36</td>
<td>129.91</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-1.12</td>
<td>-2.20</td>
<td>-3.65</td>
<td>-5.32</td>
<td>-6.83</td>
</tr>
<tr>
<td>$\delta_2^+$</td>
<td>-1.12</td>
<td>-2.20</td>
<td>-3.65</td>
<td>-5.32</td>
<td>-6.83</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>3.26</td>
<td>5.07</td>
<td>21.60</td>
<td>76.27</td>
<td>121.52</td>
</tr>
<tr>
<td>$\delta_3^+$</td>
<td>78.08</td>
<td>173.41</td>
<td>173.80</td>
<td>171.78</td>
<td>173.97</td>
</tr>
</tbody>
</table>

Average deviation in cross sections observed and calculated %

|       | 4.46 | 4.39 | 2.18 | 3.49 | 4.71 |

The phase shifts derived from the angular distributions.
CHAPTER 6

LEVEL SEARCH IN \(^4\)He, \(^5\)He and \(^6\)Li COMPOUND NUCLEI.

6.1 P-T Elastic Scattering

The greatest theoretical and experimental effort in the study of light nuclei \((A = 3, 4, 5)\) up to now has been dedicated to the nuclear system with \(A = 4\), that is the problem of the existence of some excited levels of \(^4\)He, and of the \(^4\)H, \(^4\)Li and \(^4\)n nuclei. Five classes of experiments which might reveal the existence of \(^4\)He levels are inelastic scattering of nucleons on \(^4\)He; inelastic scattering of electrons on \(^4\)He; deuteron break-up on \(^3\)He and \(T\); elastic scattering of nucleons on \(^3\)He and \(T\); \((p,n)\) reactions such as \(T(p,n)\) \(^3\)He; as well as reactions like \(^3\)He\((T,d)\) \(^4\)He and \(^7\)Li\((p,\alpha)\) \(^4\)He with \(^4\)He as recoil nucleus. Up to now the direct or indirect experimental research on the excited levels of \(^4\)He has provided some evidence for the existence of levels in the following ranges of the excitation energy; 20, 21-22, 24 and 30 MeV (Ar 65).

One of the two body reactions involving two protons and two neutrons is \(p + T\), the study of which may afford information about the excited states of \(^4\)He nucleus. Data for elastic scattering of protons by tritium near the threshold of the \(T(p,n)\) \(^3\)He reaction and extending from 0.3 to 3.5 MeV proton bombarding energy have been obtained.
by a number of investigators (He 49, En 54, Ja 59, Ba 63, We 64, Ba 64). The phase shift analysis of the data between 0.1 and 0.76 MeV by Werntz (We 64) and up to 2 MeV by Meyerhof and McElearney (Me 65a) gives supporting evidence for a $0^+$ excited state of the $^4$He nucleus at about 20.1 MeV. Conclusions from the point-to-point phase shift analysis performed by Balashko et al. (Ba 64) are in good agreement with the work in reference (Me 65a). Other evidence for this state comes from Werntz (We 62), Young and Ohlsen (Yo 64) and Donovan et al. (Do 64) in analysis of neutron spectra from the $T(d,pn)T$ reaction. Furthermore, in a careful study of the reaction $T(d,n)^4$He* Poppe et al. (Po 63) observed this resonance at 20 MeV. It is interesting that it appears in the stripping reaction $(d,n)$ in a very light nucleus. More recent evidence on this state comes from the work reported by Ohlsen et al. (Oh 66) and by Stokes et al. (St 66).

Several states have been proposed (Ba 54, VI 55) for excited states above the already mentioned $0^+$ state at 20 MeV but none of the experimental data till recently could be taken as conclusive evidence. However, from the particle-particle correlation in the three-body system resulting from the bombarding of deuterium with $^3$He beam, Parker et al. (Pa 65) and Zürmühle (Zu 65) identified a
level at 19.94 MeV as the $0^+$ level, and the second level at 21.2 MeV observed to decay by both proton and neutron emission. No $J^\pi$ assignment was given. Vlasov and Samallov (VI 64) reviewed experimental evidence on the excited states of $^4$He and proposed the following levels with respective $J^\pi$ assignments in brackets: 20 MeV ($0^+$), 22 MeV ($2^-$) and 24 MeV ($1^-$). The conclusions reached by Meyerhof (Me 65b) are similar to these in reference VI 64: 20.3 MeV ($0^+$), 22.5 MeV ($2^-$) and 26 MeV ($1^-$). Another peak at about 30 MeV has been seen via the $^6$Li($\pi^+,2p)^4$He reaction (Ta 66) but no $J^\pi$ assignment is given. The two states at 22 and 24 MeV can be compared to the levels in $^4$H and $^4$Li discussed by Tombrello (To 66).

The most recent review article on $A = 4$ by Argan et al. (Ar 65) however, does not consider the evidence conclusive enough for the levels above 20 MeV. They propose the following isotopic scheme for the $A = 4$ group: the 20 MeV level has $T = 0$ or indefinite isotopic spin. The $^4$He level at 22 MeV has an isotopic spin of $T = 0$. This level together with the one at 24 MeV should represent the splitting $P_{3/2} - P_{1/2}$ foreseen by the shell model and due to the spin orbit interaction. The triplet $^4$H-$^4$He*$-^4$Li exists at about 24 MeV with $T = 1$ although this is not sufficiently demonstrated experimentally. A
quintuplet $T = 2$ exists at about 30 MeV but there is some evidence of only $^4\text{Li}$ and $^4\text{He}$.

In order to investigate excited states of $^4\text{He}$ in the energy range between 22.3 and 27.9 MeV, protons have been elastically scattered from tritium. Excitation curves at c.m. angles 39.1°, 90° and 120.8° in steps of 100 keV have been obtained. The data is shown in figure 6.1. The p-T curves show no pronounced anomaly for the proton bombarding energies between 3.2 and 11 MeV. The cross section is smooth by varying between 200 and 300 mb/sr at the forward c.m. angle and decreases slowly from 80 to 10 mb/sr for the c.m. angles 90° and 120.8°.

The conclusion from the above results is that there are no excited states in the $^4\text{He}$ nucleus in the excitation region between 22 and 27 MeV which can be readily observed by p-T elastic scattering. It could be argued, however, that since the lifetime of the $^4\text{He}$ nucleus in its excited states is very short (of the order of $10^{-20}$ sec) the width of the states is very large and the resonant effects would not be very pronounced. Thus, it should not be surprising if the levels are not observed in all experiments (VI 64).

From the review articles (VI 64, Ar 65) it is evident that the present experimental and theoretical
Excitation functions from elastic scattering of protons from tritium. The differential cross section is in c.m. system.
93.
situation with the problem of the nuclear levels in $A = 4$
group is rather uncertain. The authors (Ar 65) express
their concern about the contradictory knowledge (and the
lack of it) of as simple a nuclear structure as $^4\text{He}$.
They suggest that to throw further light on the problem,
two types of experiments should be carried out: high energy
techniques should be employed; and two-dimensional analysis
experiments for a three-body final state should be used.
Improved resolution in detection of the products of reac-
tions should be of great advantage.

6.2 D-T Elastic Scattering
In their consideration of the $^5\text{He}$ excited states,
Galonsky and Johnson (Ga 56) identified the only known
even-parity state of $^5\text{He}$ nucleus, the $\frac{3}{2}^+$ state at 16.7
MeV excitation, with the state having a $1S_{1/2}$ hole in
the $^6\text{Li}$ ground state. Higher up even-parity states were
then expected to be $1S_{1/2}$ hole in various excited states of
$^6\text{Li}$. Thus, a state at about 19 MeV in $^5\text{He}$ is expected
considering the fact that the first excited state of $^6\text{Li}$
is a $3^+$ state at 2.2 MeV above the ground state. Blanchard
and Winter (Bl 57), discussing possible existence of $^5\text{H}$,
have speculated on the existence of a broad excited state
in $^5\text{He}$ a few MeV above the 16.7 MeV state. This broad
level was imagined as the triton moving in the diffuse domain of the deuteron. Furthermore, Inglis (In 62) lists two "near threshold" states in the $^5\text{He}$ nucleus. One is a sharp level at 16.7 MeV within only 70 keV of the threshold for breakup into $\text{T} + \text{d}$ and the other at about 20 MeV and 280 keV above the threshold for breakup into $^3\text{He} + 2n$.

The nucleon plus $^4\text{He}$ form of the five-nucleon scattering system has received detailed theoretical and experimental attention. Many searches for an excited state around 20 MeV in $^5\text{He}$ and $^5\text{Li}$ have failed to find one (Ga 56, Sh 63, Be 65). However, Bame and Perry (Ba 57), Stewart et al. (St 60), and Tombrello et al. (To 65) found evidence for the existence of such a state in the data obtained from $\text{T}(d,n)^4\text{He}$, $^3\text{He}(d,p)^4\text{He}$ and $^3\text{He}(d,d)^3\text{He}$, respectively. Rosen (Ro 59, Ro 60) reported a very broad anomaly at 90° c.m. angle for elastic scattering of deuterons from both $\text{T}$ and $^3\text{He}$. This broad anomaly was found to correspond in energy to the peaks in the total neutron cross section obtained by integrating differential cross section over $4\pi$ for $\text{T}(d,n)^4\text{He}$ and $^3\text{He}(d,p)^4\text{He}$ reactions (Ro 59). Thus, these resonances were believed to be due to the presence of one or a number of excited states in $^5\text{He}$ and $^5\text{Li}$ at about 20 MeV (Ro 59).
The purpose of this work was to investigate the excitation region between 18 and 23 MeV in search of excited states of $^5$He nucleus. In the course of this work excitation functions between 3.5 and 11 MeV deuteron bombarding energy at ten c.m. angles from d-T elastic scattering have been obtained and are shown in figure 6.2. Excitation functions at c.m. angles of 29.9°, 45.3°, 54.7°, 60.7°, 73.2° and 90° corresponding to lab angles 18°, 27.3°, 33.3°, 37°, 45° and 56.2° respectively, were obtained from the deuteron group. Excitation functions at c.m. angles of 106°, 113.5°, 125.3° and 144° corresponding to lab angles 37°, 33.3°, 27.3° and 18° respectively, were obtained from the recoil triton group. In general, the bombarding energy was varied in 200 keV steps. Steps of 100 keV were taken at 29.9°, 90° and 144° c.m. angles from 4 to 11 MeV. In addition, 50 keV steps were taken at 54.7°, 73.2°, 90° and 113.5° c.m. angles between 3.3 and 4.2 MeV bombarding energy. An angular distribution at 6.98 MeV deuteron bombarding energy taken between lab angles 15° and 85°, is shown in figure 6.3. Three excitation functions obtained concurrently from the alpha group from the T(d,n)$^4$He reaction are shown in figure 6.4. These were taken at lab
**FIGURE 6.2**

Excitation functions from elastic scattering of deuterons from tritium. The differential cross section is in c.m. system.
D-T ELASTIC SCATTERING

CROSS SECTION (mb/ster)

\( \theta_{CM} = 29.9^\circ \)
\( \theta_{CM} = 45.3^\circ \)
\( \theta_{CM} = 54.7^\circ \)
\( \theta_{CM} = 60.7^\circ \)
\( \theta_{CM} = 90.0^\circ \)
\( \theta_{CM} = 73.2^\circ \)
\( \theta_{CM} = 106.0^\circ \)
\( \theta_{CM} = 113.5^\circ \)
\( \theta_{CM} = 125.3^\circ \)

DEUTERON ENERGY (MeV)
FIGURE 6.3

Angular distribution for elastic scattering of deuterons from tritium. The differential cross section and scattering angle are in the c.m. system.
D-T ELASTIC SCATTERING

DEUTERON ENERGY
6.98 MeV

CROSS SECTION (mb/sr)

CENTRE OF MASS ANGLE (DEGREES)
FIGURE 6.4

Excitation functions from the T(d,α)n reaction. The differential cross section is in the c.m. system.
$d + T \rightarrow n + ^4He + 17.59 \text{ MeV}$

**α Laboratory Angle**

- 18°
- 27.3°
- 37°

**Cross Section** (mb/sr)

**Deuteron Energy (MeV)**
angles 18°, 27.3° and 37°. Bombarding energy was varied in steps of 200 keV with an exception of $\theta_{\text{lab}} = 18°$ where the steps of 100 keV were taken.

Excitation functions from d-T elastic scattering with exception of the two taken at c.m. angles of 125.3° and 144°, all show a very broad anomaly whose position can be best located at c.m. angles of 73.2° and 90° as about 6.7 MeV bombarding energy. A qualitative analysis of the above d-T data is given only. First of all, the disappearance of the anomaly in the excitation function taken at $\theta_{\text{c.m.}} = 125.3°$ indicates the presence of d-waves in the d-T scattering because $P_2(\cos\theta)$ vanishes at that c.m. angle. The excitation function taken at $\theta_{\text{c.m.}} = 54.7°$ (which is another zero of the same order Legendre polynomial $P_2(\cos\theta)$) shows a small peak at about 6.7 MeV indicating a contribution from partial waves other than $\ell = 2$. Trying to establish which partial wave is contributing to the above picture, one can argue the following way. A P-wave is not acceptable because the peak is quite pronounced at $\theta_{\text{c.m.}} = 90°$ at which $P_1(\cos\theta)$ has a zero; one would therefore expect a decrease in the cross section (at least), which does not occur. An f-wave can be dismissed in a similar manner from the curves taken at c.m. angles of 29.9° and 90°. However, even $\ell$ from 0 0
to higher than 4 partial waves cannot be discarded with any degree of certainty. In fact there is another puzzle in evidence with the curve taken at $\theta_{\text{c.m.}} = 144^\circ$. Having established the presence of d-waves one would expect a peak due to d-waves at $\theta_{\text{c.m.}} = 144^\circ$ even in the absence of $\ell = 4$ partial waves at that angle (which is near a zero of $P_4(\cos \theta)$). But there is none and this indicates some destructive interference between various partial waves at that angle. At other zeros of $P_4(\cos \theta)$ the peak is in evidence (especially at $73.2^\circ$ and $90^\circ$).

From the above arguments the evidence is inconclusive as to whether it is a pure d-wave involved in the formation of the 20 MeV level in $^5\text{He}$. The indication is that although d-wave is predominant there are other even-\ell partial waves involved. Another possibility is that several levels are overlapping in the considered energy region, giving an impression of a broad state. If, however, for the sake of argument, it is assumed that the observed anomaly is formed by d-wave deuterons one can speculate on the $J^\pi$ assignment in the following manner: since there is a choice of two possible incoming channel spins of $3/2$ and $1/2$, there are four possible compound nucleus states: $\frac{1^+}{2}, \frac{3^+}{2}, \frac{5^+}{2}, \frac{7^+}{2}$. Taking into account the
available \(^3\)He\((d,p)^4\)He and \(T(d,n)^4\)He data (Ba 57, St 60), a tentative spin assignment of either 3/2\(^+\) or 5/2\(^+\) can be made. This is in agreement with Tombrello et al. (To 65) in the case of their broad excited state in \(^5\)Li obtained from \(^d^3\)He elastic scattering.

The above situation is not clarified by the excitation functions obtained at back c.m. angles from the \(T(d,n)^4\)He reaction. These curves display no structure and all decrease as the deuteron bombarding energy increases.

The D-T elastic scattering angular distribution obtained at the deuteron bombarding energy of 6.98 MeV, displays two minima, one on either side of 90° c.m. angle, and a small maximum is found at about \(\theta_{c.m.} = 95°\). The process does not appear to be symmetric about 90° c.m. The increasing cross section at the back angles is interpreted as being due to an inverse stripping process as in p-D and n-D scattering (Ro 60).

In view of the fact that there is no theoretical study of the d-T scattering in literature it is difficult to assign a definite \(J^\pi\) value to the broad anomaly observed here at about 20.5 MeV in \(^5\)He. A phase shift analysis of the elastic scattering, although complex, would be of great value to throw some light on this question. However, it is unlikely definitive results could be obtained unless polarization data were available (since this is a spin 3/2 system).
6.3 $^3$He-T Elastic Scattering

The $^6$Li nucleus is of interest since it is the only member of the isotopic triplet $^6$He-$^6$Li-$^6$Be which displays numerous energy levels up to an excitation energy of about 16 MeV (La 66). The other two members of the triplet display only one level each at 1.5 and 1.8 MeV in $^6$Be and $^6$He respectively (La 66). One of the possible experiments which may afford information about the excited states in the six nucleon system in the excitation energy region above 15 MeV, is elastic scattering of particles of mass-3 from nuclei of mass-3. Unfortunately, little work has been done on these systems. Bransden and Hamilton (Br 60a, Br 60b) compared their resonating group calculations to some existing data (see Br 60a) at 29 MeV for $^3$He-$^3$He elastic scattering and found poor agreement. In order to find out if this occurred at lower energies as well, Tombrello and Bacher (To 63b) obtained data from $^3$He-$^3$He elastic scattering between 3 and 12 MeV bombarding energy and compared the data to their resonating group calculations also finding poor agreement. It appears that the only other data available in the literature are angular distributions at 19 and 25 MeV from both $^3$He-T and $^3$He-$^3$He elastic scattering reported by Rosen (Ro 59, Ro 60) and $^3$He-T elastic scattering data for bombarding energies
between 12 and 15 MeV reported by Leland et al. (Le 65). Thus, it was considered of interest to investigate the excitation region between 17 and 21 MeV in $^6$Li nucleus by elastic scattering of $^3$He-particles from tritium.

The data presented here consist of three excitation functions at c.m. angles of 40°, 90° and 140° corresponding to lab angles of 20°, 45° and 20° respectively. The latter two excitation functions were obtained from the triton recoil group. The curves are shown in figure 6.5. In addition, four angular distributions from $^3$He-T elastic scattering were obtained at $^3$He bombarding energies 5, 7, 9 and 11 MeV and are presented in figure 6.6. For the purpose of comparison two angular distributions at 9 and 11 MeV were obtained from $^3$He-$^3$He elastic scattering. The latter are shown as solid lines in figure 6.6. In addition, points from the $^3$He-$^3$He elastic scattering are shown in figure 6.7.

Excitation functions are completely structureless; i.e. there are no excited states of $^6$Li nucleus which can be observed by $^3$He-T elastic scattering in the excitation region considered.

Angular distributions, however, do have some interesting features. The large cross section in the forward direction is a feature which indicates a direct reaction mechanism (or pronounced Coulomb scattering).
FIGURE 6.5

Excitation function from elastic scattering of $^3$He from tritium. The differential cross section is in the c.m. system.
$^3$He — T. ELASTIC SCATTERING

\[ \theta_{\text{C.M.}} = 40.0^\circ \]

\[ \theta_{\text{C.M.}} = 90.0^\circ \]

\[ \theta_{\text{C.M.}} = 140.0^\circ \]
FIGURE 6.6

Angular distributions from elastic scattering of $^3$He from tritium. Solid lines represent a visual fit of points obtained from the scattering of $^3$He from $^3$He angular distributions for the purpose of comparison. The differential cross section and scattering angle are both in the c.m. system.
$^3\text{He} - T$ ELASTIC SCATTERING

$^3\text{He}$ ENERGY
5.00 MeV

$^3\text{He}$ ENERGY
7.00 MeV
$\sigma(\theta) \times 2$

$^3\text{He}$ ENERGY
9.00 MeV

$^3\text{He}$ ENERGY
11.00 MeV

CROSS SECTION (mb/sr)

CENTRE OF MASS ANGLE (DEGREES)
Angular distributions from elastic scattering of $^3\text{He}$ from $^3\text{He}$. The differential cross section and scattering angle are both in the c.m. system.
$^3$He – $^3$He ELASTIC SCATTERING

$^3$He ENERGY
9.00 MeV

$^3$He ENERGY
11.00 MeV

CENTRE OF MASS ANGLE (DEGREES)
Whereas $^3$He-$^3$He scattering shows only one minimum for bombarding energies 5 and 7 MeV, they display two minima for the data taken at 9 and 11 MeV. The only minimum for 5 and 7 MeV cases is found at about $90^\circ$ c.m. angle. The first minimum for both 9 and 11 MeV angular distributions is at about $\theta_{c.m.} = 75^\circ$, and the second minimum is at about $\theta_{c.m.} = 125^\circ$ and $140^\circ$ respectively. Intermediate maxima between the two minima are found about c.m. angles of $105^\circ$ and $120^\circ$ respectively. The general characteristic of these angular distributions is that they are not symmetric about $\theta_{c.m.} = 90^\circ$ and cross section increases slowly at backward angles beyond the last minimum. The asymmetry of these curves can be interpreted as being due to the presence of exchange forces in $^3$He-$^3$He scattering (Ro 60a). Large cross sections at backward angles is explained by the presence of the charge exchange forces as above, as well as increased compound nucleus formation. In comparison the $^3$He-$^3$He scattering angular distributions do not show the same shape and are symmetric about $90^\circ$ because identical particles are involved.
CONCLUSION

From the observed large values of tensor polarization in $^a$-d elastic scattering at several energy-angle combinations it may be concluded that the analysis of deuteron polarization by elastic scattering from $^4$He is feasible. The validity of the set of phase shifts reported by McIntyre and Haeberli (Mc 64a, Mc 65) has been established by qualitative agreement between the experimental data and calculated parameters from these phase shifts. Although, a further adjustment of the phase shifts is required to improve the quantitative agreement, it is indicated by the results that the $2^+$ and $1^+$ (Mc 64a, Mc 65) for the $^6$Li levels using the above phase shifts in the considered energy region is correct.

From the non-zero values of the $\langle T_{2k} \rangle$ obtained in p-d elastic scattering, it is clear that the present data lead to the same qualitative conclusion as do the nucleon polarization: the inclusion of spin-orbit or tensor forces is necessary to describe low energy nucleon-deuteron scattering although the effects of such forces are relatively small. However, further measurements of the vector polarization component for the deuterons from the scattering are desirable to establish the importance of the spin-orbit term in the deuteron potential in particular.
The relatively large values of the $\langle T_{2k} \rangle$ parameters in the $^9\text{Be}(p,d)^9\text{Be}$ reaction indicate that the spin-dependent interactions are present. The present data together with those of Darden and Froelich (Da 65) should be of considerable use to the efforts concerned with the spin-dependent interactions in direct reactions.

The excitation functions obtained from the elastic scattering of $^4\text{He}$-particles from tritium have revealed two anomalies corresponding to two excited states at about 4.6 and 6.5 MeV in $^7\text{Li}$ compound nucleus. These excited states have been identified as mirror states of the ones reported by Tombrello and Parker (To 63a) in $^7\text{Be}$ nucleus and have been given a tentative assignment of $7/2^-$ and $5/2^-$ respectively. A phase shift analysis limited to angular distribution data has shown a large contribution from $\ell = 3$ partial wave. However, no level parametrization has been carried out so far. Accumulation of further elastic scattering data in the form of excitation functions is regarded necessary in order to carry out a full phase shift analysis with the purpose of extracting the level parameters.

Data obtained in the form of excitation functions from elastic scattering of protons and $^3\text{He}$-particles from tritium target displayed no structure thus indicating that no excited states in $^4\text{He}$ and $^6\text{Li}$ compound nuclei could be
seen in the energy region considered by the elastic scattering process.

From the d-T elastic scattering data it is evident that there is a broad anomaly present at about 20 MeV in $^5\text{He}$ compound nucleus. From a qualitative analysis of the data, the indication is that although d-wave is predominant there are other even-$\ell$ partial waves involved. Another possibility is that several levels are overlapping in the considered energy region, giving an impression of a broad state. A phase shift analysis of the elastic scattering, although complex, would be of great value to throw some light on this question. However, it is unlikely definitive results could be obtained unless polarization data were available (since this is a spin 3/2 system).
APPENDIX A

STATISTICAL ERROR CALCULATIONS

We denote the ratio of the polarized yield to that of the normalizing yield in any of the four detectors by

\[ R_i = \frac{Y_i}{Y_{G_i}} \text{ where } i = 1, 2, 3, 4. \]

Then the expression for statistical error for any of the three parameters \( \langle T_{2k} \rangle \) is given by

\[ \Delta \langle T_{2k} \rangle = \sqrt{\sum_{i=1}^{4} \left( \frac{\Delta(T_{2k})}{\Delta R_i} \Delta R_i \right)^2} \quad (A.1) \]

where

\[ (\Delta R_i)^2 = R_i^2 \left( \frac{1}{Y_i} + \frac{1}{Y_{G_i}} \right) \quad (A.2) \]

Evaluating \( \frac{\Delta(T_{2k})}{\Delta R_i} \) for each parameter from equations 2.3c and using equation A.2 the following expression for the statistical error in the three parameters can be obtained

\[ \Delta(T_{20}) = \frac{\sqrt{2}}{4f} R_1 \sqrt{16 \left( \frac{1}{Y_1} + \frac{1}{Y_{G1}} \right) + \left( \frac{1}{Y_2} + \frac{1}{Y_{G2}} \right) + } \]
\[ \left( \frac{1}{Y_3} + \frac{1}{Y_{G3}} \right) + \left( \frac{1}{Y_4} + \frac{1}{Y_{G4}} \right) \]

\[ \Delta(T_{21}) = \frac{\sqrt{6}}{16f} \sqrt{R_2^2(R_2-R_4-4)^2 \left( \frac{1}{Y_2} + \frac{1}{Y_{G2}} \right) + 4R_3^2(R_2-R_4)^2 \times } \]
\[ \left( \frac{1}{Y_3} + \frac{1}{Y_{G3}} \right) + R_4^2(R_2-R_4+4)^2 \left( \frac{1}{Y_4} + \frac{1}{Y_{G4}} \right) \]
\[ \Delta (T_{22}) = \frac{\sqrt{3}}{4f} \sqrt{\frac{R_3^2}{Y_2} \left( \frac{1}{Y_2} + \frac{1}{Y_{G2}} \right) + 4(2-R_3)^2 \left( \frac{1}{Y_3} + \frac{1}{Y_{G3}} \right) + \frac{R_3^2}{Y_4} \left( \frac{1}{Y_4} + \frac{1}{Y_{G4}} \right)} \]
APPENDIX B

ROTATION OF THE SPIN TENSOR MOMENTS

If the coordinate system in which the spin orientation of a beam of deuterons is described is rotated by the Euler angles, $\alpha$, $\beta$, and $\gamma$, the tensor moments change in the following manner (Sa 60a):

$$\langle T_{qk} \rangle' = \sum_{k} D_{qk}^{q'}(\alpha, \beta, \gamma) \langle T_{qk} \rangle$$  \hspace{1cm} (B.1)

where the primed moments describe the orientation in the new coordinate system.

The formulae relating the second-rank tensor moments (primed) in a coordinate system rotated about the $y$ axis by an angle $\beta$ and those in the original coordinate system are the following:

$$\langle T_{20} \rangle' = \frac{1}{2}(3\cos^2\beta-1)\langle T_{20} \rangle - \frac{3}{2}\sin\beta \cos\beta \langle T_{21} \rangle + \frac{3}{8}\sin^2\beta \langle T_{22} \rangle$$

$$\langle T_{21} \rangle' = \frac{3}{2}\sin\beta \cos\beta \langle T_{20} \rangle + (2\cos^2\beta-1)\langle T_{21} \rangle - \sin\beta \cos\beta \langle T_{22} \rangle$$

$$\langle T_{22} \rangle' = \frac{3}{8}\sin^2\beta \langle T_{20} \rangle + \sin\beta \cos\beta \langle T_{21} \rangle + \frac{1}{2}(\cos^2\beta+1)\langle T_{22} \rangle$$  \hspace{1cm} (B.2)

Rotating the coordinate system by an angle $\beta$ is equivalent to rotating the deuteron spin by an angle $-\beta$. 
APPENDIX C

CALCULATION OF POLARIZATION PARAMETERS FROM PHASE SHIFTS.

The outgoing wave in elastic scattering of a polarized beam is in a mixed state and can be represented as a linear combination of the pure states $\chi_{m_s}^s$ (Po 59)

$$\psi(m_s) = \sum_{m_s'} \langle m_s | M | m_s' \rangle \chi_{m_s'}^s,$$  

(C.1)

where $\psi(m_s)$ is the outgoing wave; $\chi_{m_s}^s$ is the eigenfunction of the Schrödinger wave equation with the operators $S_z$; $m_s$ is the eigenvalue of the spin projection of the polarized beam in the initial state; $m_s'$ is the projection on the $z$ axis of the spins; and $\langle m_s | M | m_s' \rangle$ is the transition amplitude from state $m_s$ to state $m_s'$.

By considering equation C.1, the expectation value for the tensor moments, $T_{\lambda\kappa}$, of a beam described in section 1.1, initially in the $\mu$th substate can be written as

$$\langle \mu | T | \mu \rangle = \sum_{\beta} \langle \beta | M^+ | \mu \rangle <\beta | T_{\gamma} \langle \mu | M | \gamma \rangle | \gamma \rangle$$

$$= \sum_{\beta\gamma} \langle \beta | M^+ | \mu \rangle <\mu | M | \gamma \rangle <\beta | T_{\gamma} | \gamma \rangle$$

in the Dirac bra and ket notation. For a beam that is essentially unpolarized, it is now necessary to average over the values of $\mu$ which gives

$$\langle T \rangle = \frac{1}{\text{Tr}_\rho} \sum_{\beta\gamma} T_{\beta\gamma}$$  

(C.2)

where
\[ \rho_{\beta\gamma} = \frac{1}{2S_{i+1}} \sum_{\mu} M^+_{\beta\mu} M_{\mu\gamma} \]  

(C.3)

The \( \beta\gamma \) element of the density matrix and \( s_i = 1 \), the initial spin state.

The value for \( M \) is \((\text{Se 64a})\)

\[
M = \begin{pmatrix}
  a & -g & e \\
  f & b & -f \\
  e & g & a
\end{pmatrix}
\]

where

\[ a = R + \ell \left( e^{\frac{1}{2}i\alpha_\ell} \right) \frac{e^{\frac{1}{2}i\alpha_\ell}}{\ell(\ell+1)} \]

\[ e^{\frac{1}{2}i\alpha_\ell} \left[ (\ell+1)U_{\ell,\ell}^\ell + (2\ell+1)U_{\ell,\ell}^\ell + (\ell-1)U_{\ell,\ell}^{\ell-2}(2\ell+1) \right] + e^{\frac{1}{2}i\alpha_\ell} \left[ (\ell+2)(\ell+1) \right] \]

\[ e^{\frac{1}{2}i\alpha_\ell} + p \left[ (\ell+1)(\ell+2) \right] \frac{1}{2} U_{\ell,\ell+2}^{\ell+1} + e^{\frac{1}{2}i\alpha_\ell - 2} p_{\ell-2} \left[ (\ell-1) \frac{1}{2} U_{\ell,\ell-2}^{\ell-1} \right] \]

\[ b = R + \ell \left( e^{\frac{1}{2}i\alpha_\ell} \right) \frac{e^{\frac{1}{2}i\alpha_\ell}}{2i} \]

\[ e^{\frac{1}{2}i\alpha_\ell} \left[ (\ell+1)U_{\ell,\ell}^\ell + (2\ell+1)U_{\ell,\ell}^\ell - (2\ell+1) \right] - e^{\frac{1}{2}i\alpha_\ell + 2} p_{\ell+2} \left[ (\ell+1)(\ell+2) \right] \frac{1}{2} U_{\ell,\ell+2}^{\ell+1} - e^{\frac{1}{2}i\alpha_\ell - 2} p_{\ell-2} \left[ (\ell-1) \frac{1}{2} U_{\ell,\ell-2}^{\ell-1} \right] \]

\[ g = \frac{\sqrt{2}}{\sin \theta} \left( e^{\frac{1}{2}i\alpha_\ell} \right) \frac{e^{\frac{1}{2}i\alpha_\ell}}{\ell(\ell+1)} \]

\[ e^{\frac{1}{2}i\alpha_\ell} \left[ (\ell+1)U_{\ell,\ell}^\ell + (2\ell+1)U_{\ell,\ell}^\ell - (2\ell+1) \right] - e^{\frac{1}{2}i\alpha_\ell + 2} p_{\ell+2} \left[ \frac{\ell+1}{\ell+2} \right] U_{\ell,\ell+2}^{\ell+1} + e^{\frac{1}{2}i\alpha_\ell - 2} p_{\ell-2} \left[ \frac{\ell-1}{\ell-2} \right] U_{\ell,\ell-2}^{\ell-1} \]

\[ f = \frac{\sqrt{2}}{\sin \theta} \left( e^{\frac{1}{2}i\alpha_\ell} \right) \frac{e^{\frac{1}{2}i\alpha_\ell}}{2i} \]

\[ e^{\frac{1}{2}i\alpha_\ell} \left[ U_{\ell,\ell}^\ell - U_{\ell,\ell}^{\ell-1} \right] + e^{\frac{1}{2}i\alpha_\ell + 2} p_{\ell+2} \left[ \frac{\ell+1}{\ell+2} \right] U_{\ell,\ell+2}^{\ell+1} - e^{\frac{1}{2}i\alpha_\ell - 2} p_{\ell-2} \left[ \frac{\ell-1}{\ell-2} \right] U_{\ell,\ell-2}^{\ell-1} \]
$$\frac{1}{\sqrt{2}} \sin^{2} \theta = \frac{1}{2 \sqrt{2}} \left( \frac{e^{\frac{1}{2} i \alpha_{\ell}}}{\ell (\ell + 1)} \right) \left[ U_{\ell, \ell}^{\ell+1} - (2 \ell + 1) U_{\ell, \ell}^{\ell} + (\ell + 1) U_{\ell, \ell}^{\ell-1} \right] +$$

$$\frac{1}{2 \sqrt{2}} \left[ \frac{e^{\frac{1}{2} i \alpha_{\ell}}}{\ell (\ell + 2)} \right]^{\frac{1}{2}} \left[ U_{\ell, \ell + 2}^{\ell + 1} + \frac{1}{2 \ell} \frac{d}{d \ell} (\cos^2 \theta - 1) \right]$$

The following quantities were used in the above expressions:

- $R = \text{Rutherford amplitude} = -\frac{1}{2} \ln \csc^2 (\theta / 2) \exp \left[ i \ln \csc^2 (\theta / 2) \right]$

- $\alpha_{\ell} = \text{arctan} \left( \frac{n + n/2 + \ldots + n/2}{\ell} \right)$ where $n = zz' e^2 / h\nu$

- $P_{\ell} (\cos \theta) = \frac{1}{2 \ell} \frac{d}{d \ell} (\cos^2 \theta - 1)$ and $\theta$ is the c.m. scattering angle. The phase shifts are related to the elements of the collision matrix $U_{\ell, \ell'}^{J}$ in the following way:

$$U_{\ell, \ell'}^{J} = \delta_{\ell \ell'} \exp (2 i \delta_{e}) \quad (C.4)$$

for all elements except those for $J = 1^+$. These elements are (Se 64b)

- $U_{0, 0}^{1} = \cos^2 e \exp (2 i \delta_{e}^1) + \sin^2 e \exp (2 i \delta_{e}^2) \quad (C.5)$

- $U_{2, 2}^{1} = \sin^2 e \exp (2 i \delta_{e}^1) + \cos^2 e \exp (2 i \delta_{e}^2) \quad (C.6)$

- $U_{0, 2}^{1} = \frac{1}{2} \sin (2 e) \exp (2 i \delta_{e}^1) \quad (C.7)$

where $\delta_{e}^1$ and $\delta_{e}^2$ are the eigen-phase shifts and $e$ is the mixing parameter.

The following expressions for the tensor parameters result:
\[\text{Tr}_\rho = \frac{2}{3} \left[ |a|^2 + \frac{1}{2} |b|^2 + |g|^2 + |f|^2 + |e|^2 \right] \]  
(C.8)

\[\langle T_{20} \rangle = \left( \frac{\sqrt{2}}{3} / \text{Tr}_\rho \right) \left[ |a|^2 + |f|^2 + |e|^2 - 2 |g|^2 - |b|^2 \right] \]  
(C.10)

\[i \langle T_{11} \rangle = \left( \frac{\sqrt{2}}{3} / \text{Tr}_\rho \right) \left[ \text{im}(bf^*) + \text{im}(ge^*) + \text{im}(ag^*) \right] \]  
(C.9)

\[\langle T_{21} \rangle = - \left( \frac{\sqrt{2}}{3} / \text{Tr}_\rho \right) \left[ \text{Re}(bf^*) + \text{Re}(e^*g) - \text{Re}(ag^*) \right] \]  
(C.11)

\[\langle T_{22} \rangle = \left( \frac{1}{3} / \text{Tr}_\rho \right) \left[ 2 \text{Re}(ae^*) - |f|^2 \right] \]  
(C.12)

\[\langle T_{10} \rangle = 0 \]  
(C.13)

It should be pointed out that invariance under time reversal was neglected in reference Ga 55b which resulted in the wrong signs of the off-diagonal elements of their collision matrix (Se 64a).
APPENDIX D
TRITIUM HANDLING AND CALCULATION OF ERROR DUE TO HYDROGEN IMPURITY

To prepare the UT$_3$ oven for storage of tritium, 7 gm of Uranium chips were cleaned with dilute nitric acid and distilled water to remove the oxide layer. The wet uranium was placed in an oven constructed by welding a plug in the bottom of a stainless steel tube 10 cm long and 0.6 cm inside diameter. The volume over the uranium chips was packed with glass-wool and the upper seal was placed in position for welding. The seal consisted of a 0.6 cm plug welded to a 0.3 cm diameter stainless steel tube which was in turn hard soldered to a Hoke valve. Helium was then flown through the valve and the plug was heliarc welded while the bottom of the oven was submerged in water. Water was removed from the uranium by pumping on the oven until 25 µm vacuum was achieved.

The oven was connected to a vacuum and gas handling system. Hydrogen was introduced into the system and the oven was heated. At about 300°C the reaction began and hydrogen was supplied as long as it was taken up by the uranium. Near 400°C the hydrogen was given off and the system was pumped to 25 µm and allowed to cool. This cycle was completed three times to activate the uranium. After considerable pumping time tritium was admitted to the cell
and absorbed by uranium to form UT₃ compound. At room temperature the equilibrium pressure in contact with uranium is below 25 μm. The tritium was removed from the target cell at room temperature. Since uranium sinters above 400°C caution was taken to prevent overheating.

The following is a derivation of an expression for error in the elastic scattering from tritium cross section due to the uncertainty in the hydrogen impurity pressure.

Denote pressure due to hydrogen, tritium and total pressure in the cell by Pₜ, Pₜ and Pₜ. Also denote the triton and proton groups and combined counting in the spectra by Yₜ, Yₜ and Yₜ respectively. Then

\[ Yₜ = kPₜ \quad \text{and} \quad Yₜ = Yₜ - Yₜ = Yₜ - kPₜ. \]

The corresponding cross section will then be given by

\[ \sigmaₜ = \frac{CYₜ}{Pₜ} = \frac{CYₜ}{Pₜ-Pₜ} \quad \text{or} \quad \sigmaₜ = \frac{C(Yₜ-kPₜ)}{Pₜ-Pₜ} \]

where C and k are some constants of proportionality. Then from above

\[ d\sigmaₜ = CdPₜ \frac{(Yₜ-kPₜ)-k(Pₜ-Pₜ)}{(Pₜ-Pₜ)^2} \]

now \[ k = \frac{Yₜ}{Pₜ} \quad \text{and} \quad C = \frac{\sigmaₜ(Pₜ-Pₜ)}{Yₜ} \]

\[ \frac{d\sigmaₜ}{\sigmaₜ} = \frac{dPₜ}{Pₜ-Pₜ} \left( \frac{Yₜ-Yₜ}{Yₜ-Pₜ} - \frac{Yₜ}{Yₜ} \cdot \frac{Pₜ-Pₜ}{Pₜ} \right) \]
or

$$\frac{d\sigma_T}{\sigma_T} = \frac{dP_H}{P_\text{tot} - P_H} - \frac{dP_H}{P_H} \frac{Y_H}{Y_T} \quad (D.1)$$

since $Y_T = Y_\text{tot} - Y_H$.

From the quantities in (D.1) a corresponding error in the cross section measurement can be calculated. Thus for 1 mm of hydrogen present in a 25 mm tritium target an approximate estimate of the error involved is 4%.
APPENDIX E

CROSS SECTION CALCULATION AND SINGLE LEVEL PARAMETRIZATION

The expression for the differential cross section for the elastic scattering of spin-$\frac{1}{2}$ particles from a spin-zero target has been given previously by Critchfield and Dodder (Cr 49).

$$\sigma(\theta) = |f_c|^2 + |f_1|^2$$  \hspace{1cm} (E.1)

where

$$f_c(\theta) = \frac{n}{2k} \csc^2(\theta) \exp\left[\ln\ln\csc^2\left(\frac{\theta}{2}\right)\right]$$

$$\sum_{\ell=0}^{\infty} \frac{1}{k} e^{2i\alpha_\ell} P_\ell(\cos\theta) \left[\left(\ell+1\right)e^{i\delta_\ell^+} \sin\delta_\ell^+ + le^{i\delta_\ell^-} \sin\delta_\ell^- \right]$$

and

$$f_1(\theta) = \frac{1}{k} \sum_{\ell=1}^{\infty} e^{2i\alpha_\ell} \sin\theta \frac{dP_\ell(\cos\theta)}{d(\cos\theta)} \left[\delta_\ell^- \sin\delta_\ell^+ - e^{i\delta_\ell^+} \sin\delta_\ell^- \right]$$

in the expressions $\theta$ is the c.m. scattering angle, $k$ is the wave number, $n = Z_1 Z_2 e^2/\hbar v$, and $v$ is the relative velocity of the two particles. The phase shifts for total angular momentum $j = \ell + \frac{1}{2}$ and $j = \ell - \frac{1}{2}$ are denoted by $\delta_\ell^+$ and $\delta_\ell^-$, respectively. The $\alpha$'s are related to the usual Coulomb phase shifts, $\alpha_\ell = \sigma_\ell^- - \sigma_0$.

Assuming the $^4P_{5/2}$ level has an extremely small effect on the $\alpha+T$ elastic scattering (To 63a), a single level parametrization of $\delta_3^-$ can be applied

$$\delta_3^- = -\tan^{-1}\left[\frac{F_3}{G_3}\right] \rho = kR + \tan^{-1}\left[\frac{(K/A_3)^2}{E_\lambda + \Delta_\lambda - E}\right] \rho = kR$$  \hspace{1cm} (E.2)

where
\[ \Delta_{\lambda} = \frac{\gamma^{2}_{\lambda}}{R} \left[ \frac{1}{A^{2}_{3}} (F_{3} \frac{\partial F_{3}}{\partial \rho} + G_{3} \frac{\partial G_{3}}{\partial \rho}) + \frac{3}{3} \right] = kR \]

and

\[ A^{2}_{3} = F^{2}_{3} + G^{2}_{3} \]

E is the energy in the c.m. system, and \( F_{3} \) and \( G_{3} \) are the regular and irregular Coulomb wave functions. \( R \) is the nuclear radius, and \( \gamma^{2}_{\lambda} \) is the reduced width.

The following parameters have to be obtained for a fit in equation E.2: nuclear radius \( R \), reduced width \( \gamma^{2}_{\lambda} \), resonance energy \( E_{\text{res}} \), excitation energy \( E_{\chi} \), and ratio to the Wigner limit \( \frac{\theta^{2}_{\alpha}}{\gamma^{2}_{\lambda}} = \frac{\gamma^{2}_{\lambda}}{(3\hbar^{2}/2\mu R)} \). The quantity \( \mu \) is the reduced mass, and \( E_{\text{res}} \) is defined as that lab energy for which \( E_{\lambda} + \Delta_{\lambda} - E \) goes to zero.
117.

REFERENCES


(Ba 64a) A.C.L. Barnard, C.M. Jones and G.C. Phillips, Nucl. Phys. 50(1964)629.

(Ba 64b) Iu. G.Balashko et al., Yzvestia Akad, Nauk. SSSR 28(1964)1124; Proc. Int. Conf. of Nuclear Physics Paris (July 1964).


(Be 63) R. Beurtey, et al., Compt. Rend. 257(1963)1267.


(Br 59) see for example: G.E. Brown, Rev. Mod. Phys. 31 (1959) 893.


(Br 60c) J.E. Brolley et al., Phys. Rev. 117 (1960) 1307.


119.
(Bu 41) R.A. Buckingham and H.S.W. Massey, Proc. Roy. Soc. A179 (1941) 123.
(Ch 53) R.S. Christian and J.L. Gammel, Phys. Rev. 91 (1953) 100.
(Cr 49) C.L. Critchfield and D.C. Dodder, Phys. Rev. 76 (1949) 602.
(En 54) M.E. Ennis and A. Hammendinger, Phys. Rev. 95(1954)772.
(Fa 57) U. Fano, Rev. Mod. Phys. 29(1957)74.
(Ga 55b) A. Galonsky and M.T. Ellistrem, Phys. Rev. 98(1955)590.
(Go 60) L.J.B. Goldfarb and R.C. Johnson, Nucl. Phys. 18(1960)353.
121.

(Ha 51) J.A. Harvey, Phys. Rev. 82(1951)298.
(Ha 60) E.W. Hamburger and J.R. Cameron, Phys. Rev. 117 (1960)781.
(In 53) D.R. Inglis, Rev. Mod. Phys. 25(1953)390.
(La 55) W. Lakin, Phys. Rev. 98(1955)139.
122.

(La 58) L.D. Landau and E.M. Lifshitz "Quantum Mechanics"


(Li 50) B.A. Lippmann and J. Schwinger, Phys. Rev. 79(1950)469.

(Ma 53) H.S.W. Massey, Progr. Nucl. Phys. 3(1953)235.


(Mc 63) J.A. McCray, Phys. Rev. 130(1963)2034.

(Mc 64a) L.C. McIntyre and W. Haeberli, Bull. Am. Phys. Soc. 9(1964)627.


123.


(Mo 65) S. Morita et al., Nucl. Phys. 66(1965)17.

(Ne 64) Yu A. Nemilov and L.A. Pobedonostsev, Soviet Phys. JETP 18(1964)76.

(Oh 64) G.G. Ohlsen and P.G. Young, Nucl. Phys. 52(1964)134.


124.
(Ro 59) L. Rosen, "Lectures in IV Summer Meeting of Nuclear Physicists" Yugoslavia (1959), Published by Federal Nuclear Energy Commission of F.P.R. Yugoslavia, 171.
(Ro 66) B. Robson (private communication)
(Sa 58) G. R. Satchler, Nucl. Phys. 6(1958)543.
(Sa 60a) G.R. Satchler, Oak Ridge National Laboratory Report O.R.N.L. 2861(1960)
(Sc 55) L.l. Schiff, "Quantum Mechanics" (McGraw-Hill, N.Y. 1965) 2nd, Ed.
(Se 64) F. Seiler, S.E. Darden, L.C. McIntyre and W.G. Weitkamp, Nucl. Phys. 53(1964)65.
(Se 64a) L.S. Senhouse, Jr., Ph.D. Thesis, California Institute of Technology (1964).
(Se 64b) L.S. Senhouse, Jr., and T.A. Tombrello, Nucl. Phys. 57(1964)624.
(Si 53) A. Simon, Phys. Rev. 92(1953)1050.
(So 57) J.M. Soper, Phil. Mag. 2(1957)1219.
(St 52) W. R. Stratton et al., Phys. Rev. 88(1952)257.
(To 59) W. Tobocman, Phys. Rev. 115(1959)98.
(To 63a) T.A. Tombrello and P.D. Parker, Phys. Rev. 130(1963)1112.
(To 63b) T.A. Tombrello and A.D. Bacher, Phys. Rev. 130(1963)1108.
(Ve 66) V.V. Verbinski and M.S. Bokhari, Phys. Rev. 143(1966)688.
(Vl 64) N.A. Vlasov and L.N. Samoilov, Atomnia Energia 17
(1964)3; (UCRL-Transl. 1183, Feb. 1965).
(Vy 59) G.L. Vysotskii and A.G. Sitenko, English Transl.
(We 56) G. Weber, L.W. Davis and J.B. Marion, Phys. Rev. 104
(1956)1307.
(We 62) C. Werntz, Phys. Rev. 128(1962)1336.
(We 63) T.A. Welton, "Fast Neutron Physics Part II", edited
(We 64) C. Werntz, Phys. Rev. 133(1964)819.
(Wo 49) L. Wolfenstein, Phys. Rev. 75(1949)1664.
(Ya 53) J.L. Yarnell, R.H. Lovberg and W.R. Stratton, Phys.
Rev. 90(1953)292.
(Yo 64) P.G. Young and G.G. Ohlsen Phys. Letters 8(1964)