MECHANISM STUDIES OF DEUTERON
STRIPPING REACTIONS AT LOW ENERGIES

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PREFACE

This thesis describes a series of experiments carried out in the Department of Nuclear Physics at the Australian National University using the 12 MeV tandem Van de Graaff accelerator.

The work discussed in Chapter 3 and the study of the $^{208}_{\text{Pb}}(d,p)^{209}_{\text{Pb}}$ reaction were shared approximately equally with Dr. G.M. Crawley and Dr. B.V.N. Rao under the supervision of Dr. G.M. Crawley.

The laboratory work for the $^{124}_{\text{Sn}}(d,p)^{125}_{\text{Sn}}$ study was shared with Dr. P.J. Dallimore. The planning, data reduction and theoretical analysis was carried out by the author under the supervision of Dr. Dallimore.

Some help with the data collection for Chapter 5 was given by various people while the planning, data reduction and theoretical analysis were done by the author under the supervision of Dr. B.A. Robson of the Department of Theoretical Physics and Dr. Dallimore.

The main computer programs used in the theoretical analysis were written by Drs. Robson and Dallimore.

Some of the work reported has appeared in the following publications:

2) 'Spectroscopic Factors from $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ at 8.00 MeV' [with G.M. Crawley and B.V.N. Rao] Nuclear Physics, A112 (1968) 223.

3) 'Deuteron Induced Reactions in $^6\text{Li}$, $^9\text{Be}$ and $^{10}\text{B}$ at Bombarding Energies of 4.5 to 6.0 MeV' [with G.M. Crawley, B.V.N. Rao and B.A. Robson] Nuclear Physics, A147 (1970) 65.

Two experiments unrelated to the topic of this thesis and which are not described here are reported in:


'Experimental Study of Low Lying Levels and Decay Structure in the Nucleus $^{62}\text{Cu}$' [with W.F. Davidson, M.R. Najam, P.J. Dallimore and J. Hellström] Accepted for publication in Nuclear Physics.

It is a pleasure to thank Dr. G.M. Crawley for his patient and enthusiastic supervision during the early stages of the work and Dr. P.J. Dallimore, my supervisor after Dr. Crawley's departure to Michigan State University, for his help and advice and detailed reading of the thesis.

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No part of this thesis has been submitted for a degree at any other University.

\[\text{Powell}\]

D.L. Powell

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The (d,p) reaction has been studied at tandem energies on light and heavy nuclei and several features of the reaction mechanism observed.

Spectroscopic factors from the reactions $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ at $E_d = 8.0$ MeV and $^{124}\text{Sn}(d,p)^{125}\text{Sn}$ at $E_d = 5.1$ and 12.0 MeV have been obtained by comparison with DWBA calculations.

A study of the effects on the spectroscopic factors caused by varying the parameters in the calculations shows that the spectroscopic factors depend quite significantly on variations in spin orbit coupling in the bound state potential, geometry of the bound state potential, and on the inclusion of finite range and non-locality corrections. This dependence is the same in both energy regions, but the choice of optical model parameters used to generate the distorted waves is found to be far less critical below the Coulomb barrier. Therefore sub-Coulomb deuteron stripping reactions should yield a more accurate spectroscopic factor than stripping above the Coulomb barrier.

The analysis of energy averaged cross sections of (d,p) reactions in light nuclei using a combination of the DWBA and Wolfenstein, Hauser and Feshbach theories has resulted in satisfactory agreement of spectroscopic factors with calculations based on the shell model in the lp shell.
Some techniques have been developed to improve the resolution of solid state particle detectors used in the data collection for the (d,p) reaction studies and have also been applied to the measurement of energy level positions in $^{27}\text{Al}$ and $^{89}\text{Y}$ using inelastic proton scattering.
CHAPTER 1
INTRODUCTION

Nuclear reactions tend to be divided into two separate categories in order to facilitate their theoretical analysis. Namely, direct reactions which are assumed to involve the minimum interaction between the colliding particles, characterized by a short reaction time approximately equal to the time taken for a particle to traverse a nuclear diameter (~ $10^{-22}$ sec); and 'compound nucleus' reactions as postulated by Bohr [Bo 36] involving the maximum interaction between the colliding particles with a correspondingly longer reaction time (~ $10^{-18}$ sec).

Although the distinction is made, it is not always clear that either description is predominant in a particular reaction. When a particle which may be a nucleon, or a group of nucleons, collides with a nucleus at low energies, several different processes may occur. The particle may be elastically scattered, inelastically scattered leaving the nucleus in an excited state or if the particle is a group of nucleons a re-arrangement such as a (d,p) reaction may take place. All of these possibilities can be described by both the direct reaction and compound nucleus mechanisms. A graphical description due to Weisskopf is shown in figure 1.1.
Fig. 1.1: Different reaction modes according to Weisskopf [We 61]. The top part of the figure shows how the different processes can be imagined to take place, whereas the bottom part shows the time scale for the different processes.
The relative probability for the occurrence of the two mechanisms is determined by the type and energy of the incident particle and by the target nucleus. For example, deuteron stripping reactions are predominantly direct above approximately 15 MeV, whereas at lower energies the smaller number of channels open to the decay of the compound nucleus allows this mode of reaction as well as the direct mechanism to contribute significantly to the cross section in any one channel. For low energy deuterons incident on a heavy target nucleus the mechanism is expected to be primarily direct because formation of the compound nucleus is inhibited by both the low barrier penetration of the incident particle and the large number of open channels which restricts the cross section for decay into a particular channel. The latter example applies particularly to stripping reactions when the incident energy is below the Coulomb barrier as discussed in Chapter 4.

At low energies with low mass targets the situation is more complicated. The limited number of open exit channels introduces the difficulty of having significant compound nucleus decay into channels which are normally assumed exclusive to direct reactions. The incident particle may be re-emitted into the entrance channel yielding compound elastic scattering, which cannot be accounted for using the normal optical model analysis [Ho 63, see below, section 2.2] or protons can be emitted into (d,p) channels to which direct reaction theory is normally applied.
The theoretical analysis of deuteron stripping reactions has been accomplished mainly by the use of the distorted wave Born approximation (DWBA) theory [To 61, Sa 65]. It has been applied to nuclear spectroscopy and structure studies and, although a number of simplifying assumptions are made in the theory, a wealth of experimental proof testifies to its success both in determining transferred angular momentum values and predicting absolute cross sections. The cross sections contain the nuclear structure information through the spectroscopic factor (S) which is a measure of the extent to which the final state is described by a single particle outside an inert core [Ma 60].

This thesis describes some experiments undertaken to test the DWBA theory when applied to deuteron stripping reactions at low incident energies. In recent years, the approximations inherent in the DWBA have been investigated and improved upon to yield a direct reaction theory, which when applied to reactions where the conditions of the approximations are reasonably well met, gives good agreement with the experimental data. Reactions where the approximations are not so easily justified have also received a fair amount of attention. The object of such experiments is to test the applicability of the theory, the accuracy of the approximations, and the validity of applying the theory when other reaction mechanisms are contributing significantly to the yield.

The experiments discussed in Chapter 4 were designed to test the approximations for deuteron stripping reactions when the
incident energy is below the Coulomb barrier. Such reactions should occur well outside the nucleus with a considerably reduced distortion of the wave functions of the initial and final states by the nuclear potential allowing a more accurate determination of spectroscopic factors [Go 65].

In the absence of a comprehensive theory for nuclear reaction analysis several attempts have been made to account simultaneously for compound nucleus and direct reaction contributions. One method has been to subtract a variable isotropic component from the observed differential cross section until the fit is optimized [Be 56]. This method is not reliable because the compound nucleus cross section is usually not isotropic. More recently the theory of Wolfenstein, Hauser and Feshbach (HF) [Wo 51, Ha 52] has been used to estimate compound nucleus contributions to elastic scattering and reactions [Ho 67, Co 69, Da 69]. When coupled to optical model or distorted wave predictions good fits were obtained to the observed cross sections.

Of course this division into direct and compound nucleus processes is a simple approximation. One can more realistically imagine particles interacting with nucleons so that the reaction times vary from that of a direct reaction to that of a compound nucleus reaction depending on how many nucleons are disturbed in each interaction.

As usual the simplest assumption is made initially and the validity is tested by comparison with experiment.
A case in which both mechanisms could be competing is the (d,p) reaction on light nuclei at low bombarding energies. Some reactions of this type are studied in Chapter 5 using a combination of HF and DWBA theories in the theoretical analysis.

To provide a convenient point of reference a brief description of the theories used is contained in Chapter 2 with emphasis on the approximations required to derive the cross sections. The work described in Chapter 3 is essentially the experimental techniques developed and employed in order to obtain the measurements required in the investigations of the later chapters.
2.1 THE OPTICAL MODEL OF ELASTIC SCATTERING

The optical model \cite{Ho 63} has been developed from an early suggestion by Fernbach, Serber and Taylor \cite{Fe 49}, that the elastic scattering of nucleons from nuclei would be analogous to the scattering of a wave by a refracting and absorbing sphere. Quantum mechanically this amounts to solving the Schrödinger equation describing the scattering of a particle by a complex potential. Recently the model has been extended to include composite particles such as deuterons, tritons, helions (\(^3\)He) and alpha particles \cite{Ho 63, Ho 66, Ho 68}.

2.1.1 THE OPTICAL MODEL CROSS SECTION

The solution of the Schrödinger equation

\[ \psi = \psi + \frac{2m}{\hbar^2} (E - V(r)) \psi = 0 \quad (2.1.1) \]

is required subject to the conditions of an incident plane wave and an outgoing spherical wave. The phenomenological optical potential \( V(r) \) has the form

\[ V(r) = V_C(r) + Uf(r) + iWg(r) + (V_{so} + iW_{so}) h(r) \quad (2.1.2) \]

where \( V_C(r) \) is the Coulomb potential, \( U \) and \( W \) are the depths of the
real and imaginary parts of the central potential, $U_{so}$ and $W_{so}$ are the depths of the real and imaginary parts of the spin dependent potential and $\hat{s}\cdot\vec{z}$ is the spin orbit operator. The form factors $f(r)$, $g(r)$ and $h(r)$ describe the radial variation of the terms in the potential. The Saxon-Woods form

$$f(r) = [1 + \exp\left(\frac{r}{a} - R\right)]^{-1}$$

is usually assumed for the real part of the central potential. The radius $R = r_0 A^{1/3}$ accounts for the variation of the potential with mass. The constant $r_0 (\approx 1.25 \text{ fm})$ is the nuclear radius parameter.

The form factor $g(r)$ for the imaginary part of the central potential is normally either the Saxon-Woods form representing volume absorption, or its derivative $\left(\frac{df(r)}{dr}\right)$ for surface absorption. It is not clear just what is the best form for the imaginary part, but theoretical arguments support the use of surface absorption for deuteron scattering, especially at low energies [Ho 66]. Several forms of the spin orbit potential have been used all containing derivatives of the central potential. The Thomas form

$$h(r) = C \frac{1}{r} \frac{df(r)}{dr}$$

has been employed throughout the present work, where $C$ is included so that the spin orbit potential is expressed in MeV.

To solve equation (2.1.1) the total wave function is expanded into spatial, angular and spin dependent parts.
where \( \chi_s^{\mu} \) is a spin function, \( \langle \ell s \lambda \mu | j m \rangle \) is a Clebsch-Gordan coefficient and \( Y^\lambda_\ell (\theta, \phi) \) is a spherical harmonic. Substitution of equations (2.1.2) and (2.1.3) and subsequent integration yields equations for the radial wave functions \( U_{j \ell} (r) \) corresponding to the eigenvalues of the spin orbit operator. They may be integrated out from the origin and matched in the region beyond the nuclear field to the appropriate asymptotic wave functions. The matching fixes the complex phases \( (\delta_\ell) \) which give the elements of the scattering matrix

\[
S_{\ell} = \exp (2i \delta_{\ell})
\]

from which the elastic scattering cross section \( \sigma_{SE} \) and total reaction cross section \( \sigma_A \) are obtained.

2.2 THE EFFECT OF THE COMPOUND NUCLEUS IN OPTICAL MODEL ANALYSIS

At low incident energies, excitation functions exhibit resonances due to isolated levels in the compound nucleus. As the energy is increased into the region of overlapping levels, Ericson fluctuations [Er 60] will be observed if the energy spread of the incident beam is less than, or of the order of the mean level spacing. The scattering cross section will show characteristic variations which cannot be described by an optical model.

The re-emitting of a particle into the entrance channel cannot be distinguished experimentally from the shape elastic
scattering. The optical model theory described in the previous section calculates a cross section $\sigma_{SE}$ which should be compared with the shape elastic scattering. The total cross section is

$$\sigma_T = \sigma_{SE} + \sigma_A \quad (2.2.1)$$

The absorption cross section $\sigma_A$ consists of the compound elastic and reaction cross sections,

$$\sigma_A = \sigma_{CE} + \sigma_R \quad (2.2.2)$$

The optical model gives $\sigma_{SE}$ and $\sigma_A$, while experimental measurements give $(\sigma_{SE} + \sigma_{CE})$ and $\sigma_R$. It is therefore impossible to make a direct comparison between the optical model and experiment unless the compound elastic cross section can be estimated.

The theory of Wolfenstein-Hauser-Feshbach (HF) can be applied to the calculation of compound nucleus cross sections provided the measured cross section represents the average behaviour of the compound nucleus in the region of excitation considered. It is meaningless to apply the HF theory to data taken with a beam whose energy spread is small compared with the mean level width ($\Gamma$) and mean level spacing ($D$) in the compound nucleus. In medium to heavy nuclei this is taken care of by the natural energy spread of the beam, whereas in light nuclei, where $\Gamma$ and $D$ may be ~ 100 keV or more, it is necessary to measure cross sections over an energy range and take an average. This can be done by either fitting a curve to the excitation
function or by taking an arithmetic mean of the measured angular distributions. The optical model cross section is then calculated at the average energy.

2.3 THE WOLFENSTEIN-HAUSER-FESHBACH CROSS SECTION

The differential cross section for a nuclear reaction initiated in the reaction channel $a$ with channel spin $j$ and angular momentum $\ell$, and corresponding magnetic quantum numbers $\mu$ and $m$ is given by Preston [Pr 62, Formula 16-36]

$$
\frac{d\sigma}{d\Omega} (\theta, \phi) = \frac{\pi}{k^2} \left| \sum_{\ell', j'} U_{a', j', \mu', \ell', m'}; a j \mu \ell m \gamma_{\ell'} (\theta, \phi) \right|^2
$$

(2.3.1)

where $U$ is the collision matrix. The primed quantities refer to the exit channel $a'$. For convenience the $z$-axis is defined as parallel to the beam direction giving $m = 0$ in equation (2.3.1).

In order to obtain the HF expression from equation (2.3.1) a number of assumptions have to be made with regard to the conditions of the reaction:

1) the interference between different partial waves in both the entrance and exit channels can be ignored if there are no definite phase relations between the contributions from various $\ell$-values. The modulus square operation in (2.3.1) can then be performed on each term separately.

2) it is assumed that compound nucleus states with total angular momentum $J$ and parity $P$ are numerous within the energy interval
considered so that whatever J is formed from a channel spin j and orbital angular momentum & the reaction may proceed.

3) the neglect of interference terms between outgoing waves arising from compound states with different values of J or P implies that once again the modulus square operation applied to the collision matrix U can be performed under the summation sign in (2.3.1). These interference terms cancel out when the cross section is averaged over many compound states for each value of J and P, because the outgoing waves have random phases. These conditions require that measured cross sections be averaged over an appropriate energy interval.

4) this also allows the separation of the collision matrix $U^J$ into two parts which describe the independent formation and decay of the compound nucleus. This is the independence hypothesis or Bohr assumption, which was strongly supported by the experiment of Ghoshal [Gh 50] comparing yields of reaction products from the same compound system produced through different channels.

5) the reciprocity theorem [Bl 62, p.136] finally allows replacement of the collision matrix in terms of transmission coefficients $T^J_{\alpha l j}$:

$$|U_{\alpha'\ell'j',\alpha lj}|^2 = \frac{\sum T^J_{\alpha'\ell'j'} T^J_{\alpha lj}}{T^J_{\alpha''\ell''j''}}$$  \hspace{1cm} (2.3.2)

where the geometrical properties of the collision matrix U with
regard to formation of compound nucleus states of total angular momentum $J$ and magnetic quantum numbers $M$, and their subsequent decay, are separated from $U$ [Pr 62, equation 16-40b]

$$U_{\alpha'j',\mu'\ell'm',\alpha j\mu\ell0} = \sum_{JM} (j\ell\mu m|JM)(j'\ell'\mu'm'|JM) U^J_{\alpha'j',\alpha j\ell} . \quad (2.3.3)$$

where the sum over $J$ includes a sum over parities $P$. The sum over $M$ is not performed since the $z$-component of angular momentum is conserved throughout the reaction and consequently $M$ must be replaced by $\mu$ and to be consistent with the notation of Hauser and Feshbach $\mu$ is replaced by $m$. The transmission coefficients $T_\alpha$ and $T_{\alpha'}$ allow the formation and decay of the compound nucleus through channels $\alpha$ and $\alpha'$, respectively. Thus the inverse reactions have to be studied to obtain the decay probabilities $T_{\alpha'}/\sum_{\alpha''} T_{\alpha''}$. The sum in the denominator of equation (2.3.2) has to be extended over all open channels. The double primed quantities were introduced to have independent summation indices.

The processes of disintegration of the compound nucleus by $\gamma$-ray emission or many particle break-up have been neglected. The most important of the above assumptions is the independence hypothesis. The independent formation and decay depends on the assumptions either that only a single $J$ contributes or that $T_J^{\alpha}$ is independent of $J$.

Taking into consideration formalae (2.3.1-3) the HF compound nucleus cross section becomes
\[
\left[ \frac{d\sigma}{d\Omega}(\theta) \right]_{\alpha\alpha'} = \sum_{JpJ'J'p'} \frac{(2J+1)}{(2I+1)(2I'+1)} A(j|j'|J|\theta) \frac{T^J_{\alpha j} T^J_{\alpha' j'}}{\sum_{\alpha''} T^J_{\alpha'' j'}}
\] (2.3.4)

where \(A(j|j'|J|\theta) = \sum |(j|0m|Jm)|^2 |(j'|m'm'-m'|Jm)|^2 |\gamma_{Jm'}|^2\). (2.3.5)

### 2.4 THE WIDTH FLUCTUATION CORRECTION

The assumption of independence of formation and decay used in deriving the cross section in the HF theory enables the term containing the transmission coefficients to be written

\[
\left\langle \frac{T^J_{\alpha j} T^J_{\alpha' j'}}{\sum_{\alpha''} T^J_{\alpha'' j'}} \right\rangle = \frac{\left\langle T^J_{\alpha j} \right\rangle \left\langle T^J_{\alpha' j'} \right\rangle}{\sum_{\alpha''} \left\langle T^J_{\alpha'' j''} \right\rangle} = \frac{T^J_{\alpha j} T^J_{\alpha' j'}}{\sum_{\alpha''} T^J_{\alpha'' j'}}
\] (2.4.1)

There is, however, some evidence [La 57, Mo 61, Mo 69, Tu 65, Ga 66] that this expression is not sufficiently accurate because of fluctuations in the widths of the compound nucleus states. By assuming a linear relationship between the level width \(\Gamma_\alpha\) and the transmission coefficient \(T_\alpha\), i.e.

\[
T_\alpha = \frac{2\pi \left\langle \Gamma_\alpha \right\rangle}{\left\langle D_\alpha \right\rangle}
\] (2.4.2)

the inaccuracy in the HF expression due to fluctuating level widths can be removed by multiplying by the correction factor

\[
W_{\alpha\alpha'} = \left\langle \frac{\Gamma_\alpha \Gamma_{\alpha'}}{\Gamma} \right\rangle \left/ \left\langle \frac{\Gamma_\alpha \Gamma_{\alpha'}}{\Gamma} \right\rangle \right. \left/ \left\langle \Gamma \right\rangle \right.
\] (2.4.3)
Hodgson [Ho 67a] evaluated the factor $W^{\alpha\alpha'}$, under the assumption of a Porter-Thomas distribution for the level widths $\Gamma^\alpha$, as

$$W^{\alpha\alpha'} = (1 + 2\delta^{\alpha\alpha'}) \left( \sum_{\alpha''} T^{\alpha''} \right) \int_0^\infty \left( 1 + 2xT^\alpha \right)^{-1} \left( 1 + 2xT^{\alpha'} \right)^{-1} \times \pi^{\alpha''} \left( 1 + 2xT^{\alpha''} \right)^{-1/2} dx,$$

(2.4.4)

where $x = \frac{\Gamma^\alpha}{\langle \Gamma \rangle}$ and $\delta^{\alpha\alpha'}$ is the usual Kronecker symbol.

2.5 **The Reduction Factor**

The imaginary part of the optical model potential accounts for the fraction of the incident flux removed from the shape elastic channel.

Explicitly, the absorption cross section

$$\sigma_a = \frac{\pi}{k^2} \sum_{\ell} \left( 2\ell + 1 \right) \left( 1 - |\langle S_{\alpha\ell} \rangle|^2 \right)$$

(2.5.1)

and the cross section for formation of the compound nucleus,

$$\sigma_c = \frac{\pi}{k^2} \sum_{\ell} \left( 2\ell + 1 \right) T_{\alpha\ell},$$

(2.5.2)

will differ from each other if part of the absorbed flux goes into the direct reaction process. Since the optical model potentials are used in the HF calculation without considering that direct reactions also take place in the reaction channels, the calculated compound nucleus cross sections are generally too high. The difference may be described by a reduction factor $R$ such that
However, a reduction of the right hand side of (2.5.1) can be obtained in many ways and in general reduction factors \( R_{\alpha \ell} \) depending on the reaction channel and orbital angular momentum should be introduced in order to obtain the correct transmission coefficients

\[
T^c_{\alpha \ell} = R_{\alpha \ell} (1 - |\langle S_{\alpha \ell} \rangle|^2) .
\]  

Unfortunately, at present it does not seem possible to determine the correct factors \( R_{\alpha \ell} \) either theoretically or experimentally. In this situation the simple assumption is made that the reduction factors are the same for all orbital angular momenta

\[
R_{\alpha \ell} = R .
\]  

Inspection of equation (2.3.4) shows that the reduction of all transmission coefficients by a common factor \( R \) leads to the reduction of the differential cross section by the same factor. For this reason, the HF calculations have been performed ignoring the factor \( R \) in equation (2.5.4), but including it as a factor applied to the resulting cross section.

2.6 DISTORTED WAVE THEORY

2.6.1 THE DISTORTED WAVE BORN APPROXIMATION (DWBA)

For the stripping reaction \( A(a,b)B \) the transition amplitude is given by [Sa 65]
\[ t = \langle \psi_\beta e^{i k \cdot r_\beta} | V_\beta | \psi_\alpha^{(+)} \rangle \]

(2.6.1)

\[ = \langle \psi_\beta (-) | V_\alpha | \psi_\alpha e^{i k \cdot r_\alpha} \rangle , \]

where \( \alpha \) denotes the \( a + A \) channel and \( \beta \) the \( b + B \) channel, and
\[ \psi_\alpha = \psi_a \psi_A, E_\alpha = E_a + E_A. \]

The subscript \( \alpha \) on \( \psi_\alpha^{(+)} \) signifies the solution with incoming waves in the \( \alpha \) channel, but outgoing waves in all open channels. Asymptotically (i.e. beyond the range of interactions) it has the form
\[ \psi_\alpha^{(+)} \rightarrow (a,a) \text{ elastic} + (a,a') \text{ inelastic} + (a,b) \text{ re-arrangement, etc.} \]

(2.6.2)

That is \( \psi_\alpha^{(+)} \) is the total wave function describing the whole system.

In order to evaluate the transition amplitude (2.6.1) certain approximations have to be made. The interaction \( V_\beta \) in equation (2.6.1) is divided into two parts \( V_\beta \) and \( U_\beta \), where \( U_\beta \) is the optical model potential in the exit channel with corresponding optical model wave functions \( \chi_\beta \). The DWBA follows from the observation that elastic scattering is usually the dominant process so that other parts of the wave function in equation (2.6.2) may be ignored. The total wave function in the entrance channel is approximated by the elastic scattering wave function and in practice this is usually obtained from the optical potential which reproduces the elastic scattering cross section. It is then assumed that this wave function may be
extrapolated into the interior region of strong interaction where a and A approach closely. That is

$$\psi_\alpha^{(+)} = \psi_\alpha \chi_\alpha^{(+)} (r_\alpha).$$  \hspace{1cm} (2.6.3)$$

The optical model wave function in the exit channel $\chi_\beta$ is also taken from the corresponding elastic scattering and the transition amplitude becomes

$$t_{(DW)} = \langle \psi_\beta \chi_\beta^{(-)} | V_\beta - U_\beta | \psi_\alpha \chi_\alpha^{(+)} \rangle,$$  \hspace{1cm} (2.6.4)$$

where $U_\beta$ is the optical potential in the $\beta$ channel.

The choice of optical model wave functions which reproduce the elastic scattering in the entrance and exit channels is one which raises some questions. In (d,p) reactions the main contribution to the cross section comes from wave functions near the nuclear surface, which may not be well described by extrapolation of the asymptotic wave functions obtained from the appropriate elastic scattering [De 69, Ba 68]. The importance of the approximation will depend on the reaction considered. For example low energy, low Q-value deuteron stripping reactions should occur at the surface, because of the binding energy of the neutron to the target [Wi 58]. These reactions should be less sensitive to the choice of distorted waves, especially if the incident deuteron energy is below the Coulomb barrier, since the polarization of the deuteron in the Coulomb field of the nucleus causes the stripping of the neutron to take place outside the nuclear radius.
When the above conditions do not apply, it might be more reasonable to vary the optical parameters away from those which fit the elastic scattering data in order to get a wave function which best describes the reaction data.

2.6.2 THE EFFECTIVE INTERACTION

The distorted wave amplitude for the reaction $A(a,b)B$ can be written in the form

$$t = J \int \frac{dr}{r_{aA}} \int \frac{dr}{r_{bB}} \chi_a^{(-)}(k, r) \chi_b^{(+)bB} \langle Bb | V | Aa \rangle \chi_{aA}^{(-)}(k, r_a), \quad (2.6.5)$$

where $r_{a\beta}$ refers to the separation of the centres of mass of $a$ and $\beta$ and $k_a$ and $k_b$ are relative momenta of the incoming particle $a$ and outgoing particle $b$. $\langle Bb | V | Aa \rangle$ is the matrix element of the interaction $V$ taken between the internal states of the colliding pairs. It plays the role of an effective interaction for the transition between the elastic scattering states described by the distorted waves $\chi_a^{(+)}$ and $\chi_b^{(-)}$.

Now in $A(a,b)B$, $a = b + x$ and $B = A + x$ so that in equation (2.6.4) the interaction $V$ becomes

$$V_{bB} - U_{bB} = V_{bx} + (V_{bA} - U_{bB}). \quad (2.6.6)$$

It is customary to take $V_{bx}$ as the important interaction term for stripping. However, even though the remaining terms are assumed to cancel, it is unknown to what degree this applies. Some attempts have
been made to include the residual interaction \((V_{bA} - U_{bB})\) in the DWBA calculation [Pe 64a, Kr 67, Mc 70]. The effect appears to be small, however more calculations and analysis of experiments are required before the exact significance of the approximation can be established.

2.6.3 THE ZERO RANGE APPROXIMATION

The transition amplitude given by equation (2.6.5) involves a six dimension integral which can be reduced to three dimensions by using the zero range approximation. The product of the interaction \(V_{pn}\) and the deuteron internal wave function \(\phi_d\) is assumed to be a delta function so that the vectors \(r_{-A}\) and \(r_{-B}\) are parallel. The physical meaning is that the proton is emitted at the same point at which the neutron is absorbed.

The deuteron is usually assumed to be in an S state and the tensor force is neglected. Calculations including the D state and a tensor force indicate that these are reasonable approximations [De 70].

The nuclear matrix element contains the product

\[
D(r_{pn}) = V_{pn} \langle r_{pn} \rangle \phi_d(r_{pn}) . \tag{2.6.7}
\]

The zero range approximation consists of putting

\[
D(r_{pn}) \approx D_0 \delta(r_{pn}) . \tag{2.6.8}
\]

If it is further assumed that \(V_{pn}\) is of zero range then

\[
D_0^2 \approx 1.0 \times 10^4 \text{ MeV}^2 \text{ fm}^3 . \tag{2.6.9}
\]
The wave function \( \phi_d \) which results when \( V_{pn} \) is assumed to have zero range has the correct asymptotic form but the wrong normalization. An improved estimation of \( D_0^2 \) is given by the Hulthén wave function [Sa 65] as

\[
D_0^2 \approx 1.65 \times 10^4 \text{ MeV}^2 \text{ fm}^3 ,
\]  

(2.6.10)

which, according to Satchler [Sa 65] agrees with the normalization obtained from n-p scattering and effective range theory.

2.6.4 THE LOCAL ENERGY APPROXIMATION

The finite range of the deuteron can be taken into account without resorting to a complete DWBA calculation by using the 'local energy approximation' (LEA) [Bu 64, Be 64, Pe 64]. The method yields a multiplicative factor to be included in the radial integral to the zero range calculation. The factor, \( \Lambda(r) \) is given by,

\[
\Lambda(r) = 1 - \frac{[V_d(r) - V_n(r) - V_p(r) - B_d]}{(\beta^2/\alpha^2) B_d}
\]  

(2.6.11)

where \( V_d, V_n \) and \( V_p \) are the optical model potentials for the deuteron, neutron and proton used in the DWBA calculation. \( B_d \) is the binding energy of the deuteron and \( \alpha \) and \( \beta \) are parameters which define the deuteron radius and neutron-proton separation in the deuteron. For stripping reactions

\[
\beta^{-1} = 1/7 \alpha^{-1} .
\]
2.6.5 THE BOUND STATE WAVE FUNCTION

The wave function for the neutron bound to the target to form the final nucleus is taken as being proportional to the shell model single particle wave function for the orbit \((n\ell j)\) so that

\[ R_{\ell j}(r) = \sqrt{S_{\ell j}} \, u_{n\ell j}(r) \tag{2.6.12} \]

where \(S_{\ell j}\) is the spectroscopic factor.

If there is negligible re-arrangement of the core when the extra nucleon is added then the overlap between the target wave function \(\psi_a\) and the residual nucleus wave function \(\psi_B\) defines the bound state wave function and the separation energy defines the corresponding energy eigenvalue. The well for the capture of the neutron is usually taken to have the same shape as the optical potential which describes the elastic scattering of nucleons from the target, and the depth is adjusted to give the required binding energy.

2.6.6 NON-LOCALITY EFFECTS

The optical model and shell model potentials are believed to be non-local. In the optical model analysis of scattering, part of this non-locality is taken care of by the dependence of the potential on the bombarding energy. Another important effect of the non-locality is the reduction of the wave function in the nuclear interior. This reduction can be represented by a damping factor obtained from the local energy approximation [Bu 64, Be 64, Pe 64] as
Here \( \mu \) is the reduced mass of the particle, \( U(r) \) is the 'equivalent' local potential and the energy dependence of nucleon and deuteron optical model potentials gives values of \( \beta = 0.85 \) and \( \beta = 0.54 \) respectively. The constant \( c \) is unity for scattering wave functions. The same reduction occurs for shell model bound state wave functions; if a radial function \( u(r) \) is calculated in a local potential \( U(r) \) then \( u'(r) \) for the same binding energy in the equivalent non-local potential is given by, \( u'(r) = H(r) \ u(r) \). In this case \( c \) is obtained by demanding the new wave function remains normalized and since \( u'(r) \) is reduced in the interior the tail of \( u'(r) \) is increased relative to that of \( u(r) \). Therefore, the non-locality of the bound state potential should be important for reactions occurring beyond the nuclear surface.

2.6.7 THE STRIPPING CROSS SECTION IN THE DWBA THEORY

If, in the stripping reaction \( A(a,b)B \), the spins of the particles are denoted by \( J_A, J_B, S_a \) and \( S_b \) and their corresponding \( z \)-components \( M_A, M_B, m_a \) and \( m_b \) then the transition amplitude (2.6.5) can be written

\[
T = \langle J_B \ M_B, \ S_b \ m_b ; \ k_b | V | J_A \ M_A, \ S_a \ m_a ; \ k_a \rangle \\
= J \int \int \phi_{aA}^{(-)*} (k_a, r_{aA}) \langle J_B \ M_B, \ S_b \ m_b | V | J_A \ M_A, \ S_a \ m_a \rangle \phi_{bB}^{(+)} (k_b, r_{bB}), \quad (2.6.14)
\]
where \( J \) is the Jacobian of the transformation to the relative variables \( r_{aA} \) and \( r_{bB} \), and \( k_a \) and \( k_b \) are relative momenta of the incoming particle \( a \) and the outgoing particle \( b \). The \( \phi(\pm) \) are outgoing \((+\)) and incoming \((-\)) distorted waves. They can be expanded in partial waves as

\[
\phi_{aA}^{(-)} (k_{aA}^-, r_{aA}) = \frac{4\pi}{k_{aA}^a r_{aA}^a} \sum_{L_a M_a} Y_{L_z}^{*(a)} (\hat{r}_{aA}) Y_{L_z}^{a} (\hat{k}_{aA}) \chi_{L_a}^{a} (k_a, r_a)
\]

\[
\phi_{bB}^{(+)} (k_{bB}^+, r_{bB}) = \frac{4\pi}{k_{bB}^b r_{bB}^b} \sum_{L_b M_b} Y_{L_z}^{*(b)} (\hat{r}_{bB}) Y_{L_z}^{b} (\hat{k}_{bB}) \chi_{L_b}^{b} (k_b, r_b)
\]

where \( \hat{r} \) and \( \hat{k} \) represent the polar co-ordinates of the appropriate position and wave vectors. Each partial distorted wave \( \chi_{L_i}^{(i)} (k_i, r_i) \) is a solution of a radial Schrödinger equation with a central potential \( U_i(r) \),

\[
\left( \frac{d^2}{dr^2} + \frac{2}{r} \frac{n_i}{r} - \frac{2\mu_i}{\hbar} U_i(r) - \frac{L(L+1)}{r^2} \right) \chi_{L_i}^{(i)} (k, r) = 0 \tag{2.6.16}
\]

The centre of mass kinetic energy is \( \hbar^2 k^2/2\mu_i \), where \( \mu_i \) is the reduced mass and \( \eta_i \) is the Coulomb parameter \( Z_1 Z_2 e^2/\hbar \). The effective interaction is now expanded with appropriate Clebsch-Gordan coefficients as

\[
J \langle J_B | M_B, S_b, m_b | V | J_A, M_A, S_a, m_a \rangle = \sum_{j,l,s} \langle J_A, jM_A, M_B - M_A | J_B, M_B \rangle \langle \ell \ell s_m, m_a - m_b | jM_B - M_A \rangle \langle S_a S_b m_a - m_b | S m_a - m_b \rangle \tag{2.6.17}
\]

\[
\times (-)^{S_b - m_b} i^{-l} A_{\ell_sj, (B_b, A_a)} f_{\ell_sj, m} (r_{bB}, r_{aA})
\]
where $j$ is the transferred total angular momentum and, $i = J_B - J_A$, 
$p = S_a - S_b$, $j = p + S$, $m = M_B + m_b - M_A - m_a$.

The coefficient $A$ includes the strength of the interaction $V$ and a spectroscopic amplitude (which depends upon the internal nuclear structure) arising from the overlap of wave functions. The separation into the coefficient $A$ and form factor $f$ is for convenience in programming since calculations of different types of direct reactions only differ in the form factor and normalization contained in $A$.

To evaluate the form factor for the stripping reaction $A(a,b)B$ where $a = b + x$, and $B = A + x$ equation (2.6.17) is written as

$$
\langle J_B M_B, S_b m_b | V | J_A M_A, S_a m_a \rangle = \int \psi^*_{J_B M_B} (\xi, \tau_x \sigma_x) \psi^*_{S_b m_b} (\sigma_B) 
$$

$$
V(\tau_{bx}) \psi_{J_A M_A} (\xi) \psi_{S_a m_a} (\tau_{bx} \sigma_B \sigma_x) \ d\xi d\sigma_B d\sigma_x ,
$$

(2.6.18)

where $\xi$ represents the internal co-ordinates of the target, $\sigma$ the internal co-ordinates (e.g. spin) of particle $i$, $V_{bx}$ is assumed central and $b$ and $x$ are in an $S$ state of relative motion within $a$. The integral over $\xi$ may be carried out formally and expressed in terms of angular momentum states of the transferred particle $x$

$$
\int \psi^*_{J_B M_B} (\xi, \tau_x \sigma_x) \psi_{J_A M_A} (\xi) \ d\xi = \sum_{j \ell s m s} \langle J_A j m_A M_B - M_A | J_B M_B \rangle 
$$

$$
\times \langle \ell s m | j M_a - M_A \rangle _{BA} (\ell s j) u_\ell (\tau_x) [(i^\ell Y^m_\ell (\tau_x)]^* \psi_\mu (\sigma_x)^* .
$$

(2.6.19)
$f(\xi sj)$ is the fractional parentage coefficient; it indicates the extent to which the residual nucleus 'looks like', the target in state $|J MA_A\rangle$ plus particle $x$ bound with momenta $j$, $l$. $\sigma_x$ represents any internal variables for particle $x$ (e.g. intrinsic spin).

The spectroscopic factor $S$ is given by

$$S(\xi sj) = n[f(\xi sj)]^2$$  \hspace{1cm} (2.6.20)

where $n$ is the number of identical particles $x$ within $B$.

A similar expansion for the particle $a$ where $b$ and $x$ are in an $S$ state of relative motion can be obtained by integrating over their internal variables

$$\int \psi^*_S b_m b (\sigma_b) \psi^*_S \mu (\sigma_x) \psi_S a_m a (r_{b \xi}, \sigma_b, \sigma_x) \, d\sigma_x \, d\sigma_b$$

$$= a(S) \phi_a (r_{bx}) \langle S_b S m_b | S_a m_a \rangle$$  \hspace{1cm} (2.6.21)

where $\int \phi_a (r) \, r^2 \, dr = 1$. $A(s)$ is analogous to $f(\xi sj)$ above and for deuteron stripping is equal to 1.

Inserting equations (2.6.19) and (2.6.21) in equation (2.6.18) yields

$$A_{\xi sj} (B, A) \ f_{\xi sj, m} (r_{bB}, r_{aA}) = J \sqrt{\frac{2S_a + 1}{2S + 1}} f(\xi sj)$$

$$\times V(r_{b \xi}) \phi_a (r_{b \xi}) \, u_{\xi} (r_{\xi A}) \, Y_l^m (r_{\xi A})^*.$$  \hspace{1cm} (2.6.22)

Now applying the zero range approximation gives the form factor
\[ F_{\ell sj}(r) = u_{\ell}(r) \quad (2.6.23) \]

and the coefficient

\[ A_{\ell sj} = \sqrt{\frac{2S+1}{2S+1}} \int \ell sj D_0. \quad (2.6.24) \]

The radial function \( u_{\ell}(r) \) may, in general, be expanded in terms of the radial eigenfunction for the particle \( x \) moving in some central potential (i.e. in shell model orbitals). This is

\[ u_{\ell}(r) = \sum_{N} C_N u_{N\ell}(r) \quad (2.6.25) \]

where \( N \) is the principal quantum number. Normally it is assumed only one orbital contributes and \( C_N = 1 \).

Matrix elements for transfer of definite angular momenta \( \ell sj \) are defined as

\[
\sqrt{2\ell+1} \beta\ell^m_{Sj} = \frac{(i^{-\ell})}{\sqrt{4\pi}} \frac{k^2_b}{m_B} \frac{m_A}{m_B} \int dr_a a \int dr_b b \phi^{(+)}_{bB}(k_b, r_B) f_{\ell sj, m}(r_B, r_a) \times \phi^{(+)}_{aA}(k_a, r_a). \quad (2.6.26)
\]

The expression (2.6.26) reduces to

\[
\beta\ell^m_{Sj}(\theta) = (-)^m \beta_{Sj}^{\ell-m}(\theta) = \sum_{L_b L_a} r_{L_b L_a}^a \rho_{L_b}^m(\theta) f_{L_b L_a}^\ell \quad (2.6.27)
\]

in the zero range approximation, where the partial wave expansions of the distorted waves have been substituted; the \( z \) axis has been chosen as the beam direction and \( \theta \) is the angle between \( k_a \) and \( k_b \).
In equation (2.6.27) \( m > 0 \) and \( \Gamma \) is given by
\[
\Gamma_{L_b L_a}^m = i \frac{L_a - L_b - \ell}{(2L_b + 1)} \frac{(L_b - m)!}{(L_b + m)!} \langle L_b \ell 00 | L_a 0 \rangle \langle L_b \ell m, -m | L_a 0 \rangle.
\]

(2.6.28)

The \( P^m_{L} (e) \) are associated Legendre functions, and the radial integral is
\[
f_{L_b L_a}^L = \frac{m_a k_{b}}{m_b k_{a}} \int \chi_{L_b}^{(b)} (r) \left\{ \frac{m_A}{m_B} K_B (r) \right\} \chi_{L_a}^{(a)} (k_{a} r) \, dr.
\]

(2.6.29)

The occurrence of the 'parity conserving' Clebsch-Gordan coefficient \( \langle L_b \ell 00 | L_a 0 \rangle \) ensures that only even values of \( L_a + L_b + \ell \) contribute; that is \((-)^{\ell} \) is the parity change in the transition.

The DWBA computer codes calculate the matrix elements \( \beta^m \).

The differential cross section
\[
\frac{d\sigma}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi \hbar^2)} \frac{k_b}{k_a} \frac{\Sigma_{M_B M_a m_a}}{(2J_A + 1)(2S_a + 1)} \frac{|T|^2}{(2J_b + 1)} \frac{1}{k_b k_a} \Sigma_{l s_j} \frac{2m_j}{m} \frac{2m_j}{m} \beta^m_{S_j}^2.
\]

(2.6.30)

then becomes
\[
\frac{d\sigma}{d\Omega} (\theta) = \frac{\mu_a \mu_b}{\pi \hbar^4} \left( \frac{m_B}{m_A} \right)^4 \frac{(2J_b + 1)}{(2J_A + 1)} \frac{1}{k_b k_a} \Sigma_{l s_j} \frac{|A_{l s_j}|^2}{m} \frac{|\beta^m_{S_j}|^2}{m}.
\]

(2.6.31)

The factor
\[
\sigma_{l s_j} (\theta) = \left[ \frac{m_B^5 m_a m_a 9.268}{M_A (m_a + m_a)(m_b + m_B) k_b^3 k_a} \right] \Sigma_{m} \frac{|\beta^m_{S_j}|^2}{m}.
\]

(2.6.32)

is calculated for a given choice of form factor then
\[
\frac{d\sigma}{d\Omega} = \frac{2J_b + 1}{2J_A + 1} \Sigma_{l s_j} \left[ \frac{|A_{l s_j}|^2}{(2S_a + 1) 5.093 \times 10^3} \right] \sigma_{l s_j} (\theta) \text{mb/sr}.
\]

(2.6.33)
This normalization is so chosen that for deuteron stripping, if $F_{l\ell sj}(r)$ is taken to be the radial wave function of the captured nucleon (and assumed independent of $j$), the quantity

$$\sum_j |A_{l\ell sj}|^2 /[ (2S_a + 1) \cdot 5.093 \times 10^3 ]$$

is just the spectroscopic factor $S_{l\ell}$.

Thus the cross section for a (d,p) reaction in the zero range DWBA theory is

$$\frac{d\sigma}{d\Omega} = \frac{2J_B + 1}{2J_A + 1} \sum_{l\ell sj} S_{l\ell sj} \sigma_{l\ell sj}(0) \text{ mb/sr} . \quad (2.6.34)$$

As discussed in section 2.6.3 multiplication by 1.65 should compensate for the use of the zero range approximation. More recently finite range calculations by Delic and Robson [De 70, Ro 70] using a Reid potential and the corresponding deuteron wave function give a value of $D_0^2 = 1.45 \times 10^4 \text{ MeV}^2 \text{ fm}^3$. However, $1.65 \times 10^4$ was used in the present work.
CHAPTER 3

EXPERIMENTAL METHODS AND DETERMINATION OF ENERGY LEVELS WITH HIGH RESOLUTION SOLID STATE COUNTERS

This chapter describes the experimental details which are common to the work presented in Chapters 4 and 5. An experiment to develop techniques for improving the resolution obtained with solid state, charged particle detectors is discussed and an application of the techniques in determining energy levels in $^{27}$Al and $^{89}$Y is presented.

3.1 EXPERIMENTAL METHODS

In the experiments described in the following chapters, particles were detected by either commercial solid state counters or after analysis in a Buechner broad range spectrograph by photographic plates.

The ANU 12 MeV Tandem Van de Graaff accelerator provided the beams which were magnetically analysed by a 90 degree magnet.

3.1.1 THE SCATTERING CHAMBER

The targets and counters were mounted in a 51 cm diameter scattering chamber which the beam entered through a narrow collimation system optically aligned with the chamber centre. A view of a typical set-up is shown in fig. 3.1.1. The beam entered the scattering
Fig. 3.1.1: The target chamber including a typical set-up used in the \((p,p')\) measurements and \((d,p)\) studies.
chamber from the left and the scattered particles passed through anti-scattering slits before being detected in either surface barrier or Li-drifted silicon counters for which the solid angles were defined by rectangular apertures mounted in the counter blocks. The angular resolution was approximately 0.5 degrees and Rutherford scattering has shown the error in angle settings to be less than 0.2 degrees.

The charge was collected in a Faraday cup using magnetic and electric suppression of the secondary electrons, and integrated in an Elcor A3093 current integrator. An array of up to eight solid state detectors was used to detect the scattered particles.

3.1.2 ELECTRONICS

The pulses from the detectors were amplified in charge sensitive preamplifiers and main amplifiers. The combination of biased amplifiers and RC main amplifiers enabled the regions of interest to be expanded over the full range of an analogue to digital converter (ADC) before being stored in an IBM 1800 computer. A schematic diagram of the electronics is shown in fig. 3.1.2.

Each amplified pulse from a particular counter provided two pulses which were fed to two Intertechnique CA13 ADCs. One was a linear pulse (ADC1), of height proportional to the energy of the detected particles and the other was a logic pulse (ADC2), with its height dependent on the particular counter in which it was detected.
Figure 3.1.2.
A coincidence requirement (200 nsec) between the two ADCs assured that only pulses due to the same particle were accepted. The memory of the computer was divided into eight blocks of 512 channels. If two pulses arrived at the ADCs in coincidence, an address from ADC2 indicated in which of the eight blocks the address from ADC1 was to be stored. The addresses from ADC1 were stored as spectra in 512 channels.

When ADC1 and ADC2 are converting pulses to digital numbers, other pulses arriving during this interval will be lost. To estimate the fraction of lost pulses, a source of clock pulses (100/s) was used to feed two scalers. One of the scalers was gated to ignore clock pulses during the 'dead time' of the ADCs. The other scaler counted all clock pulses. All spectra were corrected for 'dead time' by multiplying by the ratio of the two scalers. The 'dead time' was kept to a tolerable level by the use of biased amplifiers, which excluded pulses of no interest and by reducing the beam when collecting data at angles where the cross section was high.

3.1.3 THE BUECHNER SPECTROGRAPH

The ANU Buechner spectrograph is similar in design to the original instrument constructed by Browne and Buechner [Br 56], the principal difference being that the radius of the pole pieces is 65 cm compared to 50 cm, thus increasing the range of particle
energies which can be simultaneously observed. A full description is
given by Scarr [Sc 66]. The spectrograph has a useful focal plane
length of 107 cm. The particles are deflected through approximately
90 degrees; they may be detected in the focal plane either with
photographic plates or with a position sensitive detector. The
maximum solid angle subtended decreases with distance along the focal
plane, from $6 \times 10^{-4}$ sr for the lowest energy particles to $3.5 \times 10^{-4}$
sr for the highest energy particles.

The magnetic field is measured and stabilized using a
nuclear magnetic resonance system. In principle a resolution of
$E/\Delta E > 1000$ can be obtained, although in practice this figure is
seldom achieved because of effects due to finite target thickness,
kinematic broadening, and beam spot size. The resolution is extremely
sensitive to this third effect, because the vertical dimension of the
beam at the target determines the spread about the image point of the
analyzing magnet.

The detection angle of the magnet, with respect to the beam
direction, can be varied from zero to 153 degrees.

The associated target chamber has an inside diameter of
13.7 cm. A solid state counter mounted on the outside wall of the
chamber views the target through a thin mylar window, providing a
monitor on the condition of the target.
3.2 **HIGH RESOLUTION FROM SOLID STATE PARTICLE COUNTERS**

For experiments requiring the optimum resolution of charged particle spectra, magnetic spectrographs generally are employed. However, as was pointed out in section 3.1.3, quite often the resolution is not determined by the method of detection. For instance, when studying low yield reactions, target thickness will certainly contribute, kinematic broadening could be an important factor, and the beam spot size will certainly be a limitation because the need of high beam current will not allow tight collimation at the entrance to the scattering chamber. Under these conditions it is no longer practical to use a spectrograph, and, if the outgoing particle energy is not too high, solid state counters are used to detect the reaction products.

The sacrifice in resolution may be compensated for by some distinct advantages associated with particle detection by solid state counters, such as the instantaneous accessibility of the data, the linear calibration and the wide energy range of particles accepted. Moreover, it is possible to achieve considerably better resolution by introducing some slight modifications to the usual solid state counter detection system.

### 3.2.1 THE CONTRIBUTIONS TO PEAK BROADENING

The overall energy resolution in a charged particle experiment is made up of a number of contributions.

1) The statistics of the electron-hole production process. The
energy required to form an electron-hole pair in silicon is 3.66 eV, and assuming the Fano factor of 0.15 obtained for electrons [Me 65] also pertains in the case of protons, the statistical limit on the resolution would be ~ 6 keV for 10 MeV protons. This is a negligible contribution since typically a resolution of 30 to 50 keV is accepted and an optimum of ~ 11 keV has been achieved, see Andersson-Lindstroem [An 67].

2) The energy resolution of the beam.

3) Kinematic broadening, i.e. broadening of the peaks due to the finite width of the detector slit in the reaction plane, which enables the detector to see particles scattered over a range of angles. The broadening depends on the energy variation with angle of the scattered particle, being larger for heavier outgoing particles and lighter target nuclei.

4) Target thickness.

5) The stability of the electronics. This is not usually a problem, although precautions have to be taken in case gain shifts occur.

6) Electronic noise from the preamplifier and amplifier system, including the effect of cable and detector capacitance at the input to the preamplifier.

7) Detector noise from the reverse current in the diode.

8) Pileup of pulses both from positively charged particles and from low energy electrons stripped from atoms in the target.

9) Charge collection effects in the detector.
The effects 2), 3) and 4) are controllable in principle. The effects 6), 7), 8) and 9) can be minimized by employing some simple techniques, the development of which is discussed in the next section, resulting in vastly improved resolution.

3.2.2 DETECTOR TESTS

The test runs were made using an Ortec surface barrier counter (Model SBCJ-050-1000) with a specified noise of 14 keV FWHM. A thin gold target on a carbon backing was bombarded with a 10 MeV proton beam. The gold-carbon separation acted as an internal calibration and the resolution was then measured from the width of the gold peak. A pulser connected at the input of the preamplifier monitored the electronic noise. All slits were kept small enough to eliminate kinematic broadening. Cabling to the preamplifier both inside and outside the chamber, was kept as short as possible to reduce the effect of stray capacitance on the electronic noise. An Ortec 109 FET preamplifier was coupled to an Ortec 410 main amplifier and the signals were passed through a biased amplifier and pulse stretcher before being analyzed in an Intertechnique CA13 analogue to digital converter and stored in an IBM 1800 computer. The resolution of both the pulser peak and the gold peak was found to improve quite rapidly when the time constants of the 410 amplifier were increased to about 1 μsec, and then more slowly as they were further increased. This reflects the dependence of the capacitive noise on the shaping
time constants. The best resolution was finally obtained using RC shaping with equal time constants of 2.0 μsec. However, under normal experimental conditions pile-up effects would probably necessitate the use of smaller time constants.

Pile-up of high energy charged particles in the detector can be improved by reducing the beam intensity or by the use of more sophisticated electronics. It was found that for the thin targets used in the present experiments, this was unnecessary. However, another source of pile-up, which is not always recognized, is the large number of atomic electrons stripped off the target atoms. As these electrons are mostly of very low energies, the maximum energy transfer being 25 keV in the case of 12 MeV protons, they produce a noise-like spectrum in the detector. The intensity of these electrons is so high that the peak resolution is considerably worsened. The inclusion of a low magnetic field in front of the collimator slit can completely eliminate this effect. Initially a 'horseshoe' magnet of approximately 500 gauss was fixed on the counter block with the field direction vertical (fig. 3.2.1). It is necessary to have a field sufficiently small to negligibly effect the detection of the nuclear particles, but large enough to prevent the electrons from reaching the counter.

The final permanent system uses small bar magnets, of ~ 500 gauss field strength, which allow counter blocks to be placed at 10 degree intervals in the scattering chamber (see fig. 3.1.1).
Fig. 3.2.1: The counter block including the small magnet and cooling tubes used in the detector resolution tests.
Both the resolution and the peak to valley ratio were improved when the detector was overbiased and, in general it was run at 300 V bias rather than the 225 V required for full depletion. This presumably reflects the importance of higher charge collection efficiency in obtaining the best resolution. The influence of trapping centres on the charge collection is strongest in regions of low electric field strength, because of the reduced charge carrier velocity. For a partially depleted detector, as was used in this case, this region occurs at the back of the detector. The application of high voltages reduces the region of low electric field strength, increasing the charge collection efficiency. The use of a fully depleted counter would result in improved resolution because of this effect.

In order to reduce the intrinsic noise of the detector and to allow high bias voltages to be applied, the detector was cooled to about -70 degrees centigrade using a dry ice and alcohol eutectic mixture. The counter was mounted in a brass block thermally insulated by lucite and cooled alcohol was circulated through the block (fig. 3.2.1). This gave rapid and efficient cooling of the detector, reducing the detector current from ~ 0.9 μA to less than 0.05 μA in about five minutes. The detector resolution decreased from a previous best value of 24 keV to 16.5 keV FWHM. At this stage the electronic noise was measured as 7 keV.

The final cooling system circulates refrigerated alcohol (-100° C) from the cold trap on the diffusion pump at the high energy
end of the tandem accelerator. Warm alcohol from a reservoir can be pumped through prior to breaking vacuum, preventing condensation of vapours onto the counter surface. A circular pipe inside the chamber provides cooling for any number of counters (fig. 3.1.1).

The results of all the tests are summarized in table 3.2.1.

### TABLE 3.2.1

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3.3 ENERGY LEVEL DETERMINATION IN $^{27}$Al AND $^{89}$Y

The techniques discussed in the previous section were applied to the measurement of energy level positions in $^{27}$Al and $^{89}$Y. The energy levels of $^{89}$Y have been the subject of a number of investigations [Aw 66, Sh 67, St 67] but there still remain inconsistencies in the region up to 4 MeV of excitation.

3.3.1 CALIBRATION: LEVELS IN $^{27}$Al

In order to determine the positions of the levels in $^{89}$Y with accuracy, it was necessary to calibrate the energy scale. Protons scattered from a thin $^{27}$Al target provided such a calibration since the energies of the low lying levels of this nucleus are known to an accuracy of better than 1 keV. Spectra from the $^{27}$Al target were taken before each $^{89}$Y run at incident energies of 10 and 12 MeV.

All the spectra were analyzed using a program Mikimaus 4,* which makes a least squares fit of a cubic curve to the background, subtracts the background, and finds the centroid of each peak. The program then generates both a linear and a quadratic calibration curve from known peaks and calculates the Q-value corresponding to any number of 'unknown' peaks in this or following spectra.

Examination of the calibration spectra indicated a number of new levels especially in the region between 7.5 and 8.5 MeV. A

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* Mikimaus 4 is derived from Mikimaus 3 which was made available by G. Berzins and J. Kolota of the Cyclotron Laboratory, Michigan State University, East Lansing, U.S.A.
spectrum of protons from $^{27}$Al is shown in fig. 3.3.1. Only the levels above 2 MeV are shown so as to allow greater expansion of the region containing the unknown levels. Levels previously suspected and confirmed in this experiment are indicated by an asterisk while the new levels are marked by arrows.

The spectrum also contains peaks corresponding to $\alpha$-particles from the $^{27}$Al($p,\alpha$)$^{24}$Mg reaction but these were easily identified by their greater width and the kinematic shift that they show with change of angle.

The energy calibration below 7.5 MeV of excitation was readily obtained using levels in $^{27}$Al whose energies are well determined. The levels used for the calibration are given in table 3.3.1. In order to determine the energies of levels above 7.5 MeV the calibration was first extrapolated to a region above the $^{26}$Mg plus proton threshold (8.271 MeV). A number of resonances have been observed in the $^{26}$Mg($p,\gamma$) reaction [En 67] and two of these at 292 and 338 keV were close to the energies of three levels in the inelastic spectrum obtained from the extrapolated calibration. Although one of the levels was consistently within about 25 keV of an energy corresponding to the 292 keV resonance, each of the three levels was matched to each of the two resonances and a complete new calibration was determined. The standard deviations for both a linear and a quadratic calibration curve were always smaller if one of the levels was set equal to 8.552 MeV corresponding to the 292 keV resonance.
Fig. 3.3.1: Spectrum of 12 MeV protons scattered at 70 degrees from a thin aluminium target. The broader states marked α are from the $^{27}\text{Al}(p,\alpha)^{24}\text{Mg}$ reaction. The levels marked with arrows have not been previously observed: those marked with an asterisk were previously uncertain. All energies shown are in MeV.
Now it is possible that the \((p,\gamma)\) reaction and the inelastic scattering process excite different states in \(^{27}\text{Al}\). In fact, the resonance at 392 keV does not appear to correspond to any strong level excited in \((p,p')\). Therefore, the matching of the level in the \((p,p')\) spectrum with the 292 keV resonance in \(^{26}\text{Mg}(p,\gamma)\) may be accidental. There is thus the possibility of a systematic error in the energies of the levels above 7.5 MeV which increases with excitation energy and in the worst case is estimated to be about 30 keV. One check which suggests that such a systematic effect is not present is the agreement of the level observed at 8.194 MeV with a previously observed level at 8.200 MeV \([\text{En 67}]\), although no error is assigned to this latter energy.

The average energies of levels above 4 MeV obtained from about six different angles are shown in table 3.3.1, together with the mean deviation. The previously known levels from Endt and van der Leun are given for comparison. In all cases, except for unresolved doublets, the energies match within the quoted errors. It should also be noted that six of the levels marked as 'new' in table 3.3.1 between 6.5 and 8 MeV are within 30 keV of levels observed recently in inelastic scattering at 17.5 MeV \([\text{Cr 68}]\). The lowest lying new level is the lower member of a close doublet at about 6.5 MeV with an energy of 6.514 MeV ± 5 keV. There is also evidence from the consistent broadening of the level at 7.674 MeV that this is also a close doublet with a separation of less than 20 keV. The previously suspected states at 6.119 and 7.231 MeV show up clearly at all angles and, in
fact, the 7.231 MeV state is one of the levels above 3 MeV most strongly excited in this reaction.

TABLE 3.3.1

STATES IN $^{27}$Al FROM $^{27}$Al(p,p')$^{27}$Al

All energies are in keV

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* Levels observed by Crawley and Garvey (1968) at 6.94, 7.23, 7.44, 7.66, 7.79 and 7.99 MeV.
† Probably close doublet with about 20 keV separation.
‡ Probably doublet with energies 6514 and 6547 keV.
3.3.2 LEVELS IN $^{89}$Y

The yttrium target was prepared by vacuum evaporation of the metal (100% $^{89}$Y) onto a carbon backing to obtain a thickness of 100 μg/cm$^2$ of metal. The main impurities in the target were carbon, oxygen and fluorine. This last impurity has excited states in the region of interest which can be identified by their kinematic shift with angle.

Spectra were obtained at 10 MeV and later 12 MeV at about six angles from 40 to 140 degrees. A spectrum at 85 degrees is shown in fig. 3.3.2. The states at 1.5 and 0.91 MeV are obscured by impurity peaks, indicating the necessity of running at a large number of angles. The energy levels up to 4 MeV are given in table 3.3.2 together with previously published values. Again the errors in the present results reflect the standard deviation of the mean of the energies obtained at different angles. In addition to these random errors in determining the peak position, a small systematic error is possible because of the uncertainty in the laboratory angle of the counter. Using the $^{27}$Al states as a calibration therefore involves an error because of the different masses of $^{27}$Al and $^{89}$Y. In the worst case this may introduce an uncertainty of about 5 keV into the level position. Since this effect is angle dependent it should lead to a systematic change of excitation energy with angle. No such systematic effects were observed.

The present values agree with the recent results on the levels up to 2 MeV of Long and Fox [Lo 68]. A doublet was observed
near to 3.0 MeV, and a second, close, doublet seen at around 3.7 MeV has not been reported previously. The levels at 3.851 and 4.226 MeV indicated in parentheses were observed at only a few angles and with poor statistics, and their energy is not well determined.

The more recent work by Hinrichsen et al. [Hi 68] on the reaction $^{89}\text{Y}(p,p')^{89}\text{Y}$ using a magnetic spectrograph has confirmed these levels as well as adding many more which were not observed in the present experiment (see table 3.3.2).

3.3.3 CONCLUSIONS

Care in reducing pile-up effects and electronic noise and the use of a simple cooling system allows a resolution of less than 20 keV to be obtained with a solid state counter. This has proved to be a useful technique for determining the positions of nuclear energy levels. The energy levels of $^{89}\text{Y}$ have been investigated up to 4 MeV and some new levels have been found. Many new levels in $^{27}\text{Al}$ have been observed up to the region of the proton threshold.
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* Unresolved multiplets.
CHAPTER 4

SUB-COULOMB STRIPPING

4.1 INTRODUCTION

In Chapter 2 the various approximations of the distorted wave theory were outlined. Several authors [Da 63, Go 65, Go 65a, Br 66, Da 66, Go 67] have suggested that the effects of these approximations are far less important when the incident deuteron energy is low compared with the Coulomb barrier of the target nucleus.

Figure 4.1.1 pictorially illustrates the physical mechanism involved in sub-Coulomb stripping. Classically, one can consider the incident deuteron and outgoing proton as following Rutherford scattering trajectories in the Coulomb field of the nucleus with stripping taking place in the vicinity of the turning point of the deuteron orbit, where the probability of capture of the neutron is greatest. This is shown in fig. 4.1.1a. On this model one would expect the angular distribution to peak at large angles corresponding to closer distances of approach and therefore higher probability for capture of the neutron. The distorted wave stripping amplitude in the zero range approximation is directly proportional to a radial overlap integral of the incident deuteron and outgoing proton scattering wave function $\psi_d$ and $\psi_p$ with the bound state neutron wave function (fig. 4.1.1b). As illustrated the bound state neutron wave
Fig. 4.1.1: Pictorial representation of Coulomb stripping. 
$B_C$ is the Coulomb barrier as seen by the incident deuteron (or outgoing proton), and $T$ represent the DWBA zero range stripping amplitude. In the classical representation, both incident deuteron and outgoing proton follow Rutherford scattering trajectories with stripping of the neutron taking place in the vicinity of the classical turning point of the deuteron orbit [taken from Po 67].
function falls off exponentially outside the nucleus whereas the
deuteron and proton wave functions fall off exponentially inside
their respective classical point of closest approach. Hence one
would expect the stripping amplitude to be small for low energy
deuterons, and the major contribution to the overlap integral to
arise from the region between the distance of closest approach and
the nuclear radius. This is more likely if the proton energy is also
well below the Coulomb barrier. In such a case, nuclear distortions
of $\psi_d$ and $\psi_p$ are small, so that the stripping angular distributions
should be insensitive to the choice of optical potentials for $\psi_d$ and
$\psi_p$.

For the same reasons, the neglect of the optical potential
$U_{BB}$ in equation (2.6.4) should be more reasonable.

The effect of the zero range approximation and polarization
of the deuteron in the Coulomb field has been included in distorted
wave calculations by Gibson and Kerman [Gi 66]. By taking the
reaction $^{209}\text{Bi}(d,p)^{210}\text{Bi}$ and calculating the cross section for
varying incident deuteron energy and Q-value they found that
polarization had negligible effect on the cross section and that the
effect of finite range varied with Q-value from negligible, at zero or
negative Q, to ~ 16 per cent for $Q = 6$ MeV. For all calculations they
required that the energy of entrance and exit channels were below
their respective Coulomb barriers.

Goldfarb and Parry [Go 68] claim that if the zero range
calculation is corrected with the LEA the cross section for sub-Coulomb stripping is increased by ~ 7 per cent.

The representation of the captured neutron by a single particle wave function is questionable and has been discussed by several authors [Pi 65, U1 68]. The approximation is most valid for closed shell nuclei (see section 2.6.5). When the target is not doubly magic, and odd particles are present which complicate the final state, it has been suggested [Yn 63, Sh 64] that if the bound particle is to be described by a single particle wave function then the use of an effective binding energy which is not quite the separation energy is required. Outside the nucleus, however, the separation energy must be used and since this is the region of importance in sub-Coulomb stripping the uncertainty in the binding energy of the transferred neutron will be removed. The effective binding energy is usually taken as the separation energy which is only really valid outside the nucleus, which tends to support the use of low energy, deuteron stripping.

However, the tail of the bound state wave function contributes most to the overlap integral as shown in figure 4.1.1. Hence the cross section and therefore the spectroscopic factor should be sensitive to the choice of bound state potential. The advantages claimed for the sub-Coulomb stripping, which are attributed to the occurrence of the reaction outside the nucleus, might well be offset by the uncertainty in the parameters which determine the bound state.
The effect of the use of non-local interactions for the distorted waves has been investigated by Jeans et al. [Je 69], who studied \(^{208}\text{Pb}(d,p)\) at 8.0 to 18.7 MeV. Negligible changes in cross section were found at 8.0 MeV while changes of \(-13\) per cent were observed at 18.7 MeV, which agreed with the work of Muehllehner et al. [Mu 67]. Furthermore, compound nucleus and heavy particle stripping contributions should also be small.

Both the shape of sub-Coulomb stripping angular distributions and lack of sensitivity to the choice of optical potentials discussed above have been verified experimentally by several authors [Er 62, Do 65, Br 66, Po 67].

### 4.2 \(^{208}\text{Pb}(d,p)^{209}\text{Pb}\) AT \(E_d = 8.0\) MeV

An experimental test of the theories discussed in the previous section is provided by the reaction \(^{208}\text{Pb}(d,p)^{209}\text{Pb}\). The requirements are:

1) that the spectroscopic factors be well established or calculable from theory; the previous experimental work on the spectrum of \(^{209}\text{Pb}\) [Mu 67, Do 65, Do 67, St 56, Mu 62] as well as the stability of \(^{208}\text{Pb}\) seen in alpha decay systematics indicate that the states strongly excited in the \((d,p)\) reaction are essentially single particle in character and hence should have spectroscopic factors close to unity.

2) the target nucleus should be a simple core so as to allow
confident calculation of the bound state wave function; this should be the case since $^{208}$Pb appears to be a good closed shell nucleus.

3) both the incoming and outgoing projectiles must have energies well below the Coulomb barrier; this is ensured by the high Coulomb barrier and low ground state Q-value (1.71 MeV) for $^{208}$Pb(d,p)$^{209}$Pb for deuteron energies below 9 MeV.

4.2.1 EXPERIMENTAL METHODS AND RESULTS

An 8.00 MeV deuteron beam from the ANU tandem was used to bombard an isotopic target (99.3%) of $^{208}$Pb, prepared by evaporation of the metal onto a thin carbon backing. The reaction protons were analyzed in a Buechner broad range spectrograph. To facilitate the proton counting a thin aluminium foil was used to prevent elastically scattered deuterons from entering the emulsion. To check the errors in scanning the tracks, the same group was scanned by different observers and the differences were always less than 2 per cent. The scattering chamber was modified to hold a solid state counter at a fixed angle of 150 degrees. This monitored the condition of the target and ensured accurate relative normalization. An absolute normalization was carried out in a 51 cm scattering chamber using a solid state counter to measure the ground state cross section at a number of angles. A monitor counter was used here also and the absolute measurements were made by comparison with Rutherford scattering of the elastic deuterons.
Fig. 4.2.1.1: Angular distributions for the first seven states in $^{209}\text{Pb}$ from the $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ reaction. The errors shown are relative errors only. The solid curve drawn for the ground state is an example of a DWBA fit.
at forward angles. The errors in the absolute cross sections were thus limited to statistics and to uncertainties in the background subtraction and were about ± 5 per cent for the ground state.

Angular distributions for the ground state (9/2^+), 1.56 MeV (5/2^+), 2.02 MeV (1/2^+), 2.47 MeV (7/2^+) and 2.52 MeV (3/2^+) states are shown in fig. 4.2.1.1.

The relative errors on these points arising from statistics, background and scanning uncertainties are between 3 and 5 per cent for all angles greater than 100 degrees. Angular distributions were also measured for the two weakly excited states at 0.77 MeV (11/2^+) and 1.44 MeV (15/2^-) but since the errors are much larger for these states, they are not used in the later analysis.

All the angular distributions showed the backward peaking characteristic of sub-Coulomb stripping. In an attempt to observe any difference in shape with different \( \ell \) values, the ratio of the cross sections relative to the \( \ell = 0, 2.02 \text{ MeV} \ (1/2^+) \) state are shown in fig. 4.2.1.2. The angular distributions for both of the states reached by \( \ell = 0 \) and \( \ell = 2 \) transfer have the same shape as the \( (1/2^+) \) state from 150 degrees to 60 degrees but for both of the states reached by \( \ell = 4 \) transfer the differential cross sections decrease faster at angles forward of 110 degrees. The consistency of the ratios at a large number of angles gives confidence in the accuracy of the measurements.
Fig. 4.2.1.2: Ratio of the cross sections for the four strongest groups relative to the 2.02 MeV (1/2⁺) state.
4.2.2 THEORY

In order to test the usefulness of the sub-Coulomb stripping process as a means of obtaining accurate spectroscopic factors, a distorted wave code DRC [Gi 64] with zero range approximation, was used to calculate cross sections for the five strongest states. The sensitivity of the absolute cross sections to the choice of optical parameters in both the proton and deuteron channels was first investigated. Generally, changes in the proton parameters were less important than changes in the deuteron parameters and the sensitivity decreased for higher excited states. Variation of both the real and imaginary potentials by ± 20 per cent made a difference of less than 1 per cent to the spectroscopic factors. The geometry had a greater effect, but, again, changes of ± 20 per cent in all cases caused changes of less than 5 per cent in the absolute cross sections provided the radius was not made much greater than the bound state value. The potentials used in all the following calculations are shown in table 4.2.1. A real deuteron potential depth of 100 MeV was used since this now seems to be more widely accepted [Ho 66, Mu 67].

| TABLE 4.2.1 |
| PROTON AND DEUTERON OPTICAL POTENTIALS USED IN DWBA CALCULATIONS |

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<tr>
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<th>$V_0$</th>
<th>$r_0$</th>
<th>$a_0$</th>
<th>$W_0$</th>
<th>$W_D$</th>
<th>$r'_0$</th>
<th>$a'_0$</th>
<th>$V_{so}$</th>
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<td>proton</td>
<td>59 MeV</td>
<td>7.4 fm</td>
<td>0.65 fm</td>
<td>0 MeV</td>
<td>19.5 MeV</td>
<td>7.4 fm</td>
<td>0.70 fm</td>
<td>0 MeV</td>
</tr>
<tr>
<td>deuteron</td>
<td>100 MeV</td>
<td>7.29 fm</td>
<td>0.88 fm</td>
<td>8.5 MeV</td>
<td>0 MeV</td>
<td>7.29 fm</td>
<td>0.88 fm</td>
<td>0 MeV</td>
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</table>
The bound state was calculated using a Woods-Saxon potential shape with a spin orbit term of the form

\[ V_{so} = CV_N \left( \frac{1}{2} \lambda^2 \pi^2 \sigma \frac{1}{r} \frac{d}{dr} f(r) \right) \]  

(4.2.1)

where \( V_N \) is the depth of the Woods-Saxon well,

\[ f(r) = \left(1 + \exp\left(\frac{r - r_{QA}^{1/3}}{a}\right)\right)^{-1} \]  

(4.2.2)

and \( \lambda^2 \) is the pion Compton wave length. The value of \( V_N \) is adjusted to give the correct binding energies for all the states. The variation in the spectroscopic factor for various \( \ell \) transfers was minimized with the constant \( C \) equal to about 0.5 and this value was used in all the following calculations. The neglect of finite range effects in the DRC code was compensated for approximately by dividing the spectroscopic factors obtained from the program by 1.65 (see section 2.6.7).

4.2.3 DISCUSSION OF RESULTS

The fits to the angular distributions are shown in fig. 4.2.1.1. The values of the theoretical cross sections and hence the spectroscopic factors were found to be dependent on the choice of geometry for the bound state well. This dependence is strikingly shown in fig. 4.2.3.1 where the spectroscopic factors are plotted for different values of the radius and diffuseness. Although there is a tendency for the spectroscopic factors to show less variation for a
Fig. 4.2.3.1: Variation of spectroscopic factors for various geometries of the bound state potential well.
diffuseness $a_n = 0.6$ fm, all can give spectroscopic factors of unity if the radius is allowed to vary. Figure 4.2.3.1 also shows that variations in $S$ are greater for larger $\lambda$ values. A comparison of the spectroscopic factors with other results is shown in table 4.2.2.

Table 4.2.2
Spectroscopic Factors for States in $^{209}$Pb

<table>
<thead>
<tr>
<th>State</th>
<th>ANU, $E_d = 8$ MeV</th>
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<th>Dost et al.</th>
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<td></td>
<td>$(R_N = 7.25$ fm, $a_N = 0.6$ fm)</td>
<td>$E_d = 25$ MeV, $\theta = 135^\circ$</td>
<td>$E_d = 8$ MeV, $\theta = 135^\circ$</td>
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<tr>
<td>0.0 (9/2$^+$)</td>
<td>0.99</td>
<td>0.67</td>
<td>0.88</td>
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<tr>
<td>0.79 (11/2$^+$)</td>
<td>0.94</td>
<td>1.52</td>
<td></td>
</tr>
<tr>
<td>1.44 (15/2$^+$)</td>
<td>1.13</td>
<td>1.38</td>
<td></td>
</tr>
<tr>
<td>1.56 (7/2$^+$)</td>
<td>1.10</td>
<td>1.00</td>
<td>1.06</td>
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<tr>
<td>2.02 (1/2$^+$)</td>
<td>1.05</td>
<td>0.93</td>
<td>0.97</td>
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<tr>
<td>2.47 (7/2$^+$)</td>
<td>0.96</td>
<td>1.17</td>
<td>1.04</td>
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<td>2.52 (3/2$^+$)</td>
<td>1.00</td>
<td>1.17</td>
<td>0.99</td>
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</table>

Following the lead of Greenlees et al. [Gr 66], a single parameter, the root mean square (rms) radius was derived from the Woods-Saxon shape and this parameter was calculated for a number of different geometries which gave unit $S$ for each of the states. The results are shown in fig. 4.2.3.2. The point is made that while the radius and diffuseness of the neutron well can vary widely and still give $S = 1$, the spread in values of the rms radius is much smaller.
Fig. 4.2.3.2: Root mean square radius as a function of radius and diffuseness of the neutron well for $S = 1$. 
(The increased variation of the rms radius if \( a \) is made very large or very small is also shown in fig. 4.2.3.2.) However, the average value from all the points shown in fig. 4.2.3.2 is 6.06 fermi with a standard deviation of 0.15 fermi. This suggests that, in a sub-Coulomb stripping experiment, the product of the absolute spectroscopic factor and the rms radius is the useful quantity which can be measured. If an independent measurement of either of these quantities is available, then the other can be determined.

4.2.4 THE GEOMETRY OF THE BOUND STATE POTENTIAL

More recently several different approaches have been taken in an attempt to establish the parameters of the bound state potential for the \(^{208}\text{Pb}(d,p)^{209}\text{Pb}\) reaction. From shell model considerations one expects the low lying states of \(^{209}\text{Pb}\) to have spectroscopic factors which are very close to unity. However, as was discussed in the previous section and shown in fig. 4.2.3.2 the condition can be achieved by quite a large range of the radius and diffuseness. Apparently further limitations are required. Dost et al. [Do 67] have attempted to fit, with a single set of neutron parameters, the low lying levels of \(^{207}\text{Pb}\) and \(^{209}\text{Pb}\) and also the stripping cross sections for 8 MeV incident deuterons, whilst requiring that the spectroscopic factors be unity. The parameters varied were the potential radius \( r_n \), diffuseness \( a_n \), spin orbit potential depth \( V_{so} \), and the non-locality range of the neutron potential. The best energy level positions were not
obtained for the set of parameters which best fitted the stripping cross sections. Nevertheless the values used for $r_n$, $a_m$ and $V_{so}$, were in quite good agreement with the results of neutron elastic scattering from $^{209}$Bi at 7.0 MeV, namely $r_n$ between 1.15 and 1.3 fm, $a_n = 0.65$ fm, and $V_{so} = 7$ MeV. This corresponds to $R_n$ in the range 6.8 to 7.7 fm in fig. 4.2.3.1 for which a large variation in $S$ is still allowed.

In a method essentially equivalent to that discussed in relation to fig. 4.2.3.2 Jeans et al. [Je 69] have taken the mean square radius of nuclear matter $\langle r^2 \rangle_m$ obtained from the optical model analysis of elastic scattering by Greenlees et al. [Gr 66] and calculated the potential radius using the relation

$$\langle r^2 \rangle_U = \langle r^2 \rangle_m + \langle r^2 \rangle_d , \quad (4.2.3)$$

where $\langle r^2 \rangle_d$ is the mean square radius of the spin and isospin independent part of the nucleon-nucleon potential and is taken as 2.24 fm$^2$ [Gr 67]. The potential radius $r_U$ can then be used to define the parameters of the Saxon-Woods well; for small values of $a_n/r_n A^{1/3}$,

$$\langle r^2 \rangle_U = \frac{7}{5} \pi^2 a_n^2 + \frac{3}{5} r_n^2 A^{2/3} . \quad (4.2.4)$$

Use of equations (4.2.3) and (4.2.4) reduces the variation of $S$ with $r_n$ (or $a_n$), if $\langle r^2 \rangle_U$ is held constant. The extent to which this method will limit the range in $r_n$ and $a_n$, thereby allowing a better
determination of $S$, will depend on the accuracy with which $\langle r^2 \rangle_m$ can be extracted from experimental measurements. Greenlees et al. give the value $5.84 \pm 0.3$ fm for the matter radius of $^{208}$Pb and hence $\langle r^2 \rangle_U = 36.35 \pm 3.5$ fm$^2$, which is not sufficiently restrictive since for $a_n = 0.65$ fm this allows values of $r_n = 1.13$ to $1.27$ fm. Clearly a more accurate measurement of $\langle r^2 \rangle_m$ is required in order to limit the geometry of the bound state well.

The most reliable measurement so far has been obtained from pion scattering. Auerbach [Au 68] has determined $\langle r^2 \rangle_m^{1/2} = 5.4 \pm 0.1$ fm from the optical model analysis of $(\pi^\pm - ^{208}$Pb) scattering data [Ab 56]. If this value of $\langle r^2 \rangle_m$ is substituted in equation (4.2.3) the corresponding value of $\langle r^2 \rangle_U$ gives $r_n = 1.09$ to $1.14$ for $a_n = 0.65$ when $\langle r^2 \rangle_d = 2.24$ fm$^2$ is used. Calculations with this geometry for the bound state potential yield spectroscopic factors of approximately 2.5 for the low lying states of $^{209}$Pb. The error lies in the value of $\langle r^2 \rangle_d$ used to obtain the Saxon-Woods parameters. The error did not show up in the suggestion by Jeans et al. discussed above. They arrive at a value of $\langle r^2 \rangle_U$ by taking $\langle r^2 \rangle_m$ and $\langle r^2 \rangle_d$ obtained from elastic scattering parameters by Greenlees, which amounts to using the $\langle r^2 \rangle_U$ of Greenlees obtained from the optical model analysis.

The preceding discussion of the neutron bound state has not included any consideration of the possible non-locality of the shell model potential in the lead region. Jeans et al. [Je 69] have
employed the Perey form of LEA (sections 2.6.4-6) for both the real and spin orbit potentials and found increases in the calculated cross sections of from 17 per cent for the $S_{1/2}$ state to 31 per cent for the $J_{15/2}$ state.

This increase is attributed to the fact that the LEA depresses the wave function within the nucleus and therefore must increase it outside in order to preserve the overall normalization. Ulrici and Hering [Ul 68] have recently compared LEA with a more complete treatment based on the work of Krell [Kr 67] and found remarkable agreement between the two methods. They also conclude that non-locality of the bound state potential is particularly important in reactions which occur outside the nuclear surface such as sub-Coulomb stripping.

The DWBA program, DRC used in the analysis discussed above does not contain options for including the local energy approximation (see sections 2.6.4, 2.6.6) to compensate for the neglect of finite range or correct for non-locality in the optical model or neutron bound state potentials. Since the completion of the analysis the DWBA code Julie has become available, which does include such options the effects of which were investigated on the present data.

The inclusion of non-locality in the bound state only, was to decrease the spectroscopic factors by $\approx 17$ per cent for the $\ell_n = 4$, $\approx 15$ per cent for $\ell_n = 2$ and $\approx 10$ per cent for $\ell_n = 0$ transitions. The inclusion of non-locality in all potentials and use
of the LEA for finite range resulted in a further decrease of \( \approx 5 \) per cent for \( l_n = 4 \) and \( \approx 3 \) per cent for \( l_n = 2 \) and \( l_n = 0 \) transitions.

4.2.5 CONCLUSION

The spectroscopic factors derived from the DWBA analysis of deuteron stripping reactions at energies below the Coulomb barrier are virtually independent of the nuclear part of the distorting potentials, but it is difficult to obtain absolute spectroscopic factors because of their sensitivity to the geometry of the bound state potential. Small variations in the neutron bound state radius for a fixed diffuseness have resulted in large changes in the spectroscopic factor. However, the sensitivity to the root mean square radius derived from the Woods-Saxon form factor was found to be much lower, suggesting that the product of the spectroscopic factor and root mean square radius can be accurately determined from sub-Coulomb stripping experiments. Thus if the root mean square radius is accurately known a more reliable spectroscopic factor could be obtained. The difficulty is in obtaining accurate root mean square radii which are calculated from a knowledge of nuclear matter radii and the radius of the spin and isospin independent part of the nucleon-nucleon potential. The former can be accurately measured but the latter is not known with sufficient accuracy to allow extraction of absolute spectroscopic factors by this method.
The inclusion of finite range and non-locality corrections to the DWBA calculations via the LEA did not affect the shape of the angular distributions but did increase the magnitudes of the cross sections thereby decreasing the spectroscopic factors by up to 21 per cent depending on the transferred angular momentum.

If sub-Coulomb deuteron stripping is to be proposed as an improvement over deuteron stripping above the Coulomb barrier for extracting spectroscopic factors, it is necessary to show that the dependence of the spectroscopic factors on the bound state geometry is at least as critical at the higher energy. The inclusion of finite range and non-locality corrections using the LEA is becoming an accepted step in the DWBA analysis of direct reactions and will have to be shown to have the same effect above and below the Coulomb barrier so that any uncertainty in the approximation, such as the non-locality range, will not depend on the energy of the reaction.

4.3 THE \(^{124}\text{Sn}(d,p)^{125}\text{Sn}\) REACTION

As a further study of the theories of section 4.1, and following on the conclusions resulting from the analysis of sub-Coulomb stripping on \(^{208}\text{Pb}\) [section 4.2.5], an investigation of the reaction \(^{124}\text{Sn}(d,p)^{125}\text{Sn}\) with incident deuteron energies above and below the Coulomb barrier was carried out. The ground state Q-value for the reaction is 3.506 MeV [Ne 64] and the low lying states up to approximately 2 MeV are sufficiently resolved
to allow the measurement of angular distributions for the first six levels, providing data for a test of sub-Coulomb stripping over a range of positive Q-values.

The (d,p) reaction at 15.0 MeV has been employed previously by Schneid et al. [Sc 67] in a shell model study of the nuclear structure of the tin isotopes. In tin the protons form a closed shell (Z = 50), making the neutron spectrum relatively simple, while the large number of stable isotopes enable a reaction such as (d,p) to be studied on a range of nuclei so that trends can be observed as the 50-82 neutron shell is filling.

Since the only data on spectroscopic factors for the reaction is that of Schneid et al. at 15.0 MeV, the measurements in the present work were made at 5.1 MeV and also above the Coulomb barrier at 12.0 MeV to provide a consistent check on the sub-Coulomb process.

The high positive Q-value introduces a difficulty in choosing the incident deuteron energy. The entrance and exit channel energies have to be sufficiently below the Coulomb barrier to minimize nuclear distortions while keeping the deuteron energy high enough to ensure a reasonable count rate.

To facilitate the choice of bombarding energy, an excitation function of the reaction $^{124}\text{Sn}(d,p)^{125}\text{Sn}$ leading to the strongly excited state at 210 keV was measured in intervals of 100 keV from 4.8 to 6.1 MeV. Rough calculations give the Coulomb barrier of tin for
deuterons and protons as ~ 9 MeV and therefore the Q-value of 3.506 MeV restricts bombarding energies to around 5 MeV. The choice of 5.1 MeV was made after studying the excitation function shown in fig. 4.3.1.

4.3.1 EXPERIMENTAL PROCEDURE

A deuteron beam from the ANU tandem was used to bombard an isotopic target (94.74%) of $^{124}$Sn, prepared by evaporation onto a thin (~ 20 µg) carbon backing. The reaction products were detected in solid state counters using the techniques described in Chapter 3.

4.3.2 EXPERIMENTAL RESULTS

Typical spectra are shown in fig.4.3.2.1. The first excited state in $^{125}$Sn is at 0.026 MeV and was not resolved from the ground state. The spectra taken with deuterons of 5.1 MeV contain several unidentified peaks. Because of the low cross sections for sub-Coulomb stripping, other reactions with low yields which are normally not a problem can produce peaks in the spectra which are comparable to the proton groups of interest.

One possibility is that the peaks observed in the present experiment are due to reactions produced in the silicon counters by elastically scattered deuterons from $^{124}$Sn, i.e.

\[ ^{124}\text{Sn} + d \rightarrow d + ^{124}\text{Sn} \]
\[ d + ^{28}\text{Si} \rightarrow p + ^{29}\text{Si} + 6.251 \text{ MeV} \]
\[ d + ^{28}\text{Si} \rightarrow \alpha + ^{26}\text{Al} + 1.4 \text{ MeV} \]
Fig. 4.3.1: Excitation functions in the sub-Coulomb region for \(^{124}\text{Sn}(d,p)^{125}\text{Sn}\) 0.210 MeV, \(3s_{1/2}\) state at 110 and 120 degrees.
Fig. 4.3.2.1a: Spectrum of $^{125}\text{Sn}$ from $^{124}\text{Sn}(d,p)^{125}\text{Sn}$ at $E_d = 5.1$ MeV.
Fig. 4.3.2.1b: Spectrum of $^{125}\text{Sn}$ from $^{124}\text{Sn}(d,p)^{125}\text{Sn}$ at $E_d = 12.0$ MeV.
the slightly broader widths of the impurity peaks compared with the
proton groups from tin would lend support to this argument because
the width is partly due to energy straggling in the target which
depends on the mass and charge of the detected particle.
However the level schemes for both final nuclei, $^{29}$Si and $^{26}$Al, did
not account for the positions of the unidentified peaks. Another
possible reaction is:

\[
^{124}\text{Sn} + d \rightarrow n + ^{125}\text{Sb} + 4.98\text{ MeV} \\
n + ^{28}\text{Si} \rightarrow P + ^{28}\text{Al} - 3.86\text{ MeV} \\
n + ^{28}\text{Si} \rightarrow \alpha + ^{25}\text{Mg} - 2.655\text{ MeV} .
\]

However, once again the level schemes did not compare with
the peak positions and in this case the peak widths would have been
narrower than those observed. The conclusion is that the peaks are
not due to reactions in the silicon counters and since angular
distributions for the $^{124}\text{Sn}(d,p)^{125}\text{Sn}$ reaction at 5.1 MeV are slowly
varying with angle the omission of peaks obscured by the impurities
only slightly affected the results and no further effort was put into
their identification.

The reduction of the data to angular distributions was
achieved by summing the peaks and subtracting a linear background.
The statistical error for the number of counts in a peak was
calculated from the expression,
\[
\text{No. of Counts under a Peak} + 2 \over \text{No. of Background Counts under a Peak} \times 100
\]

(4.3.2.1)

The data taken at 5.1 MeV yielded angular distributions for the following transitions: \(^{124}\text{Sn}(d,p)^{125}\text{Sn}\) (g.s. + 0.026 MeV), (0.210 MeV), (0.936 MeV), (1.257 MeV) and (1.540 MeV). The energy levels have been taken from the work of Nealy and Sheline [Ne 64]. The angular distributions are shown in fig. 4.3.2.2 where the errors on the points are statistical only.

As stated earlier the ground state and first excited state were not resolved, but since the ground state transition is weaker than the transition to the first excited state [Ne 64] the angular distribution of the sum of the two transitions was extracted from the spectra and analysed initially ignoring the ground state. Data forward of 90 degrees was difficult to obtain, but sufficient points were measured in the angular distributions for the sum of the ground and first excited states, and the 0.210 MeV state to show the backward peaking typical of sub-Coulomb stripping. Indeed the difficulty in measuring data points at forward angles, although mainly due to background, reflects the decrease in cross section with angle, which is characteristic of sub-Coulomb stripping. The 'two-point' angular distribution for the transition to the 0.936 MeV level is a result of the low strength of the (d,p) reaction to this state allowing complete obscurity by contaminant groups at many angles.
Fig. 4.3.2.2: Angular distributions for five low lying states of $^{125}\text{Sn}$ from the reaction $^{124}\text{Sn}(d,p)^{125}\text{Sn}$ at $E_d = 5.1$ MeV. The solid curves are DWBA fits to the data.
Angular distributions measured with 12 MeV deuterons leading to the ground state plus 0.026 MeV, 0.210 MeV, 0.936 MeV, 1.257 MeV, 1.362 MeV and 1.540 MeV levels in $^{125}$Sn are shown in fig. 4.3.2.3. The errors are approximately 4 per cent and were obtained from repeated points. Where the error exceeds 4 per cent it was due to background subtraction and the statistical error given by equation (4.3.2.1) is shown.

4.3.3 ABSOLUTE CROSS SECTIONS

The absolute cross sections were calculated using the formula,

$$\frac{d\sigma}{d\Omega} = \frac{n}{d\Omega} \frac{\cos \theta_t}{\rho \Delta x} \frac{Ae}{Q N_0}$$  \hspace{1cm} (4.3.2.1)

where

- $n = \text{the number of counts in the peak considered,}$
- $\theta_t = \text{the angle of the target to the beam direction,}$
- $A = \text{the effective atomic weight of the element of interest in the target,}$
- $e = \text{the charge on the electron,}$
- $d\Omega = \text{the solid angle of the detectors,}$
- $Q = \text{the quantity of charge passed through the target,}$
- $N_0 = \text{Avogadro's number,}$
- $\rho \Delta x = \text{the target thickness in units of mass per unit area.}$

All of the quantities in equation (4.3.2.1) have small errors compared to $\rho \Delta x d\Omega$ and therefore the error in the absolute
Fig. 4.3.2.3: Angular distributions for the first seven states of $^{125}$Sn from the reaction $^{124}$Sn(d,p)$^{125}$Sn at $E_d = 12.0$ MeV. The solid curves are DWBA fits to the data.
cross section can be taken as the error in the product of target thickness and solid angle. The quantity $\rho \Delta x d \Omega$ was measured by comparing the yield obtained in the scattering of 3 MeV deuterons from the $^{124}$Sn target with the Rutherford scattering cross section. The measurement was made for eight angles for each detector and the average value and standard deviation was taken as the product $\rho \Delta x d \Omega$ and its error. The target thickness was estimated to be $\approx 220 \mu g/cm^2$ and the error in the product was $\pm 3$ per cent. The error in the absolute cross section can confidently be taken as $\approx 5$ per cent.

4.3.4 DWBA ANALYSIS

An analysis of the data in terms of the DWBA theory was achieved with the computer code Julie. The program employs the zero range approximation and has options to include the LEA approximation for finite range of the deuteron and non-locality in the potentials generating the distorted waves as well as the bound state potential (see sections 2.6.3, 2.6.4, 2.6.6). A spin-orbit term of the Thomas form,

$$V_{so}(r) = -V_R \frac{\text{VSOR}}{45.2} \frac{1}{r} \frac{d}{dr} f(X_R),$$  \hspace{1cm} (4.3.4.1)

when

$$f(X_i) = 1 + \exp \left( \frac{r - R_{0i} A^{1/3}}{a_i} \right)^{-1},$$

was included in the deuteron, proton and neutron potentials.

The optical model parameters used by Schneid et al. [Sc 67] in fitting (d,p) data for the tin isotopes at $E_d = 15$ MeV were taken
as a starting point in the present analysis. The 12 MeV data were fitted first in an attempt to verify the tentative $\lambda_n$ and $J^+$ assignments of Schneid et al.

The spins and parities of the ground and first two excited states are known from $\beta$- and $\gamma$-ray studies and from inelastic proton scattering to be $11/2^-$, $3/2^+$ and $1/2^+$ respectively [Go 52, Du 56, Bu 57, Yu 60, Ne 64] thus providing a starting point for the analysis. The procedure adopted was to fit the angular distribution for the 0.210 MeV, $3s_{1/2}$ state and use the same parameters to fit the other data. The fits were obtained by varying the potential depths only and maintaining the same geometry as used by Schneid et al.

The angular distribution for the ground state - 0.026 MeV doublet was fitted using a program to optimize the fit of a sum of $2d_{3/2}$ and $1h_{11/2}$ calculations to the experimental points by varying the two spectroscopic factors.

The final fits were obtained with the potentials D1 for the deuteron and P1 for the proton, listed in table 4.3.1, and are shown in fig. 4.3.2.3 where the DWBA calculations include a non-local range of $\beta = 0.8$ and the LEA correction for the finite range of the deuteron.

The angular momentum assignments are shown in table 4.3.2. The DWBA fits to the ground state doublet and 0.210 state verify the assignments mentioned above. The 0.936 MeV state has been described by Cohen and Price [Co 61] as a non-single-particle state, but in the
TABLE 4.3.1

<table>
<thead>
<tr>
<th>Particle</th>
<th>Potential</th>
<th>( V ) (MeV)</th>
<th>( R_V ) (fm)</th>
<th>( a_V ) (fm)</th>
<th>( W ) (MeV)</th>
<th>( r_W ) (fm)</th>
<th>( a_W ) (fm)</th>
<th>( r_c )</th>
<th>VSOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deuteron</td>
<td>D1</td>
<td>120</td>
<td>1.15</td>
<td>0.81</td>
<td>54</td>
<td>1.34</td>
<td>0.68</td>
<td>1.115</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>120</td>
<td>1.15</td>
<td>0.81</td>
<td>64</td>
<td>1.34</td>
<td>0.68</td>
<td>1.115</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>D3</td>
<td>80</td>
<td>1.15</td>
<td>0.81</td>
<td>64</td>
<td>1.34</td>
<td>0.68</td>
<td>1.115</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>D4</td>
<td>100</td>
<td>1.15</td>
<td>0.81</td>
<td>64</td>
<td>1.34</td>
<td>0.68</td>
<td>1.115</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>D5</td>
<td>50</td>
<td>1.15</td>
<td>0.81</td>
<td>64</td>
<td>1.34</td>
<td>0.68</td>
<td>1.115</td>
<td>10</td>
</tr>
<tr>
<td>Proton</td>
<td>P1</td>
<td>49</td>
<td>1.25</td>
<td>0.65</td>
<td>54</td>
<td>1.25</td>
<td>0.47</td>
<td>1.25</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>39</td>
<td>1.25</td>
<td>0.65</td>
<td>54</td>
<td>1.25</td>
<td>0.47</td>
<td>1.25</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>P3</td>
<td>69</td>
<td>1.25</td>
<td>0.65</td>
<td>54</td>
<td>1.25</td>
<td>0.47</td>
<td>1.25</td>
<td>10</td>
</tr>
<tr>
<td>Neutron</td>
<td>N1</td>
<td>Adjusted</td>
<td>1.25</td>
<td>0.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

The level at 1.362 MeV has been previously observed by Nealy and Sheline [Ne 69] but was not resolved in the study by Schneid et al. The fit shown to the data for this state is calculated for a

Note to Table 4.3.1: A factor of four is not included in \( \frac{df}{dx} \) in the program so that the usual \( W_d \) values have to be multiplied by four for comparison with \( W \) of Table 4.3.1.
The $1g_{7/2}$ transition which gave the best comparison with the experimental angular distribution. However, the assignment of $7/2^+$ to the 1.362 MeV level is to be regarded as tentative because of the uncertainty in the forward angle data due to the low cross section for this transition.

The 1.257 and 1.540 MeV levels have been tentatively assigned $J^\pi$ values of $5/2^+$ by Schneid et al. and this value has been used in both calculations shown in fig. 4.3.2.3. The theoretical predictions for $d_{3/2}$ transitions to both states differ from the $d_{5/2}$ in magnitude only allowing no discrimination between either of the possible values and therefore $5/2^+$ must be taken as a tentative assignment.

<table>
<thead>
<tr>
<th>$E$ (MeV)</th>
<th>$l_n$</th>
<th>$J^\pi$</th>
<th>$S$ (5.1 MeV)</th>
<th>$S$ (12.0 MeV)</th>
<th>$S_{\text{zero range}}$ (5.1 MeV)</th>
<th>$S_{\text{zero range}}$ (12.0 MeV)</th>
<th>$S_{\text{Schneid et al.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.026</td>
<td>$5/2$</td>
<td>$11/2^-$</td>
<td>0.267</td>
<td>0.262</td>
<td>0.336</td>
<td>0.300</td>
<td>0.340</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>$11/2^-$</td>
<td>0.272</td>
<td></td>
<td></td>
<td></td>
<td>0.315</td>
</tr>
<tr>
<td>0.026</td>
<td>2</td>
<td>$3/2^+$</td>
<td>0.164</td>
<td></td>
<td></td>
<td></td>
<td>0.212</td>
</tr>
<tr>
<td>0.210</td>
<td>0</td>
<td>$1/2^+$</td>
<td>0.148</td>
<td>0.144</td>
<td>0.186</td>
<td>0.168</td>
<td>0.250</td>
</tr>
<tr>
<td>0.936</td>
<td>2</td>
<td>$(5/2^+)$</td>
<td>0.007</td>
<td>0.012</td>
<td>0.008</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td>1.260</td>
<td>2</td>
<td>$(5/2^+)$</td>
<td>0.026</td>
<td>0.028</td>
<td>0.032</td>
<td>0.034</td>
<td>0.039</td>
</tr>
<tr>
<td>1.360</td>
<td>4</td>
<td>$(7/2^+)$</td>
<td></td>
<td>0.024</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.540</td>
<td>2</td>
<td>$(5/2^+)$</td>
<td>0.013</td>
<td>0.016</td>
<td>0.016</td>
<td>0.021</td>
<td>0.023</td>
</tr>
</tbody>
</table>
The potentials were then used in the analysis of the 5.1 MeV data together with angular momentum assignments obtained from the 12.0 MeV data. The results of the analysis are shown in fig. 4.3.2.2 and table 4.3.2. The table includes the spectroscopic factors obtained with and without the LEA corrections so as to provide a comparison with the values of Schneid et al. which were obtained with the zero range Julie code employing a lower cut-off of 6.7 fm in the radial integrals.

The spectroscopic factors derived from the higher energy data were obtained by fitting the DWBA curve to the experimental points and finding a minimum in $\chi^2$. The values obtained were not significantly different from those derived from fitting forward angle points up to 90 degrees.

The results for both energies are in close agreement and compare well with the work of Schneid et al.

It was not possible to obtain separate spectroscopic factors for the ground state and first excited state at 5.1 MeV because of the lack of structure in the angular distribution. Schneid et al. assume the ground state contribution to be much weaker than that of the 0.026 MeV level and extract a spectroscopic factor for the 0.026 MeV level by comparing the DWBA calculation for the transition to that level with the experimental angular distribution of the doublet. The present values of 0.336 measured at 5.1 MeV and 0.300 at 12 MeV agree with their value of 0.34 at 15.0 MeV using the same method.
In order to check the consistency of the spectroscopic factors determined by fitting two DWBA calculations to the ground state - 0.026 MeV doublet at 12 MeV, calculations at 5.1 MeV were carried out using the spectroscopic factors obtained at the higher energy. The result is shown in fig. 4.3.2.4 where the agreement is well within the uncertainties and supports the values obtained at 12 MeV.

4.3.5 COMPARISON OF STRIPPING ABOVE AND BELOW THE COULOMB BARRIER

4.3.5.1 The Effect of the Optical Model Parameters

It is well known that ambiguities exist in the deuteron optical potentials so that several discrete sets of parameters give equally good fits to the same elastic scattering data [Ha 64, Ba 64]. Each of these potentials generates a wave function which can be used in the DWBA analysis of appropriate stripping reactions. The recent work of van der Merwe and Heymann on $^{208}$Pb(d,d)$^{208}$Pb and $^{208}$Pb(d,p)$^{209}$Pb [Va 69] verifies that spectroscopic factors obtained from DWBA analysis of the (d,p) reaction above the Coulomb barrier can vary by up to 100 per cent using the different optical potentials which fit the elastic scattering data whereas below the Coulomb barrier the variation is less than 5 per cent.

No search for ambiguities was attempted here, but the parameters were varied over a wide range to investigate the sensitivity of the cross section for sub-Coulomb stripping to the
Fig. 4.3.2.4: The DWBA calculation for the ground state \((1h_{11/2})\) and 0.026 MeV \((2d_{3/2})\) state at 5.1 MeV using the spectroscopic factors obtained at 12.0 MeV.
nuclear distortion. Figure 4.3.3.1 demonstrates the lack of sensitivity to the optical potentials for $^{124}\text{Sn}(d,p)^{125}\text{Sn}$ leading to the 0.210 MeV, $\ell_n = 0$ state which has a Q-value of 3.296 MeV and should have the largest dependence on the optical model parameters of all the states studied. The curves are shown for up to 100 per cent variations in well depths so as to include the possibility of ambiguous elastic scattering potentials. The same calculations were made for the transitions to the 0.026 and 1.540 MeV levels. The variation in cross section with the parameters was largest for the 0.210 MeV state being always less than 10 per cent for angles larger than 110 degrees. Changes in the real well depth for the proton had the largest effect on the shape of the curve emphasizing the need to ensure that the outgoing protons are sufficiently below the Coulomb barrier in a sub-Coulomb stripping experiment.

4.3.5.2 The Geometry of the Bound State Potential

Following on the results of section 4.2.3 the effect on the spectroscopic factor of varying the bound state potential geometry for stripping above and below the Coulomb barrier was investigated. The results are shown in fig. 4.3.3.2 where the cross section has been plotted for three values of $r_n$ and with $a_n = 0.65$ fm for the 0.210 MeV, $\ell = 0$ and 1.540 MeV, $\ell = 2$ transitions. The open circles are for 5.1 MeV incident deuterons and the full circles are for 12.0 MeV. The $r_n = 1.25$ fm point at 12.0 MeV has been normalized to the same value as the 5.1 MeV cross section to allow easy comparison. The calculations have been done for the ground
Fig. 4.3.3.1: DWBA calculations for the 0.210 MeV $3S_{1/2}$ state using optical model potentials of table 4.3.1.
Fig. 4.3.3.2: The variation of the peak cross section with bound state radius for a diffuseness of $a_n = 0.65$ fm.
state $\ell = 5$ transitions with similar results. The conclusion is that the variation of the spectroscopic factor with the geometry of the bound state potential is the same for incident deuteron energies above and below the Coulomb barrier.

4.3.5.3 Spin Orbit Coupling in the Bound State Potential

It has been suggested that spin orbit coupling in the bound state potential affects the cross section and, therefore, the spectroscopic factor thereby preventing the extraction of absolute spectroscopic factors from sub-Coulomb stripping data [Do 67].

The effect of varying the spin orbit strength for both energies was investigated and the results are shown graphically in fig. 4.3.3.3. The variation in the cross section for the $\ell_n = 2$ transition to the 1.54 MeV level for three values of the spin orbit strength is shown. Once again the effect is equal for energies below and above the Coulomb barrier. The same calculations were also done for the ground state $\ell = 5$ with similar results.

4.3.5.4 Inclusion of LEA

The local energy approximation appears to be a good approximation for finite range and non-locality in potentials. The effect of non-locality in the bound state potential is expected to be more important in reactions which occur outside the nuclear surface, as do deuteron stripping reactions. Ulrici and Hering suggest that the effect will be particularly important for sub-Coulomb stripping since the major contribution to the overlap
Fig. 4.3.3.3: The variation of the peak cross section with spin orbit strength in the bound state potential for the 1.54 MeV $\ell = 2$ transition. A similar dependence was observed for the ground state $\ell = 5$ transition.
integral comes from a region outside the nucleus. An investigation of the effect of including the LEA in calculations for stripping reactions above and below the Coulomb barrier is shown in fig. 4.3.3.4. The percentage change in the cross section for both 12 and 5.1 MeV were found to be approximately the same when:
1) the non-locality of the bound state was corrected for using the LEA with $\ell = 0.8$.
2) LEA corrections for non-locality in the bound state potential and for finite range were included.
3) LEA corrections for non-locality in all potentials and for finite range were included.

Figure 4.3.3.4 shows the results of calculations for the 1.54 MeV $\ell = 2$ transition. Similar results were obtained for the ground state $\ell = 5$ transition.

4.3.6 CONCLUSIONS

The study of the $^{124}$Sn(d,p)$^{125}$Sn reaction with 12 MeV deuterons has confirmed orbital angular momentum values for five low lying states of $^{125}$Sn and suggested values for two other states. These values have been used in the analysis of data taken with 5.1 MeV deuterons in order to compare the extraction of spectroscopic factors from deuteron stripping reactions with incident energies above and below the Coulomb barrier. The spectroscopic factors obtained at both energies are in close agreement and compare well with measurements from a previous work at 15.0 MeV.
Fig. 4.3.3.4: The variation of the peak cross section with the inclusion of various LEA corrections. The curves shown are for the 1.54 MeV $\ell = 2$ transition. Similar results were obtained for the 0.210 MeV, $\ell = 0$ and ground state, $\ell = 5$ transitions.
A comparison of the dependence of the spectroscopic factors to various assumptions in the calculations at both energies was carried out. Wide variations in the optical model parameters used in calculating the distorted waves have negligible effect on the spectroscopic factors derived from the sub-Coulomb stripping data, whereas at the higher energy such variations result in large discrepancies in the spectroscopic factors.

The dependence of the spectroscopic factor on the geometry of the bound state potential, the inclusion of spin orbit coupling in the bound state potential, and the inclusion of the local energy approximation for non-locality in the optical and bound state potentials and for the finite range of the deuteron has been found to be the same for both 5.1 and 12.0 MeV incident deuterons for \( \ell = 0 \), \( \ell = 2 \) and \( \ell = 5 \) transitions.

The conclusion is that more reliable absolute and relative spectroscopic factors can be obtained from stripping reactions when the incident deuteron energy is below the Coulomb barrier because of the lack of sensitivity to the parameters of the distorting potentials even for Q-values up to 3.5 MeV. The low cross sections for sub-Coulomb stripping reactions makes the collection of data slow but one reliable value of the cross section beyond 120 degrees should be sufficient to extract a spectroscopic factor, compared with a complete angular distribution for energies above the Coulomb barrier.
CHAPTER 5

THE (d,p) REACTION IN SOME 1p SHELL NUCLEI

5.1 INTRODUCTION

The excitation functions for deuteron induced reactions in light nuclei at energies up to approximately 20 MeV excitation in the compound nucleus usually exhibit isolated resonances, becoming increasingly structureless with increase in energy, reflecting the increasing fraction of direct reaction mechanism in the exit channel. At higher energies, fluctuations observed in the excitation functions are due to broad overlapping levels in the compound nucleus. Examples of this are shown in figures 5.1.1-3.

The angular distributions on the other hand are generally forward peaked, especially for the (d,p) channels, indicating a significant direct reaction contribution. Indeed, most experimenters to date have assumed only a direct interaction mechanism and have analyzed their data using the Butler stripping theory [Bu 57a].

The DWBA theory of direct interactions (discussed in section 2.6) describes the forward angle data of deuteron stripping reactions with light nuclei reasonably well, but invariably underestimates the large angle cross section as is borne out by the DWBA fits to the data shown in figures 5.1.4,5.

In most cases, the contribution of other reaction mechanisms to the forward angle cross section is smaller than the
Fig. 5.1.1: Excitation functions for $^{9}\text{Be}(d,d_{0})^{9}\text{Be}$. 
Fig. 5.1.2: Excitation functions for the $^9$Be(d,p$_0$)$^{10}$Be (g.s.) reaction.
Fig. 5.1.3: Excitation functions for the $^9$Be(d,p$_1$)$^{10}$Be (3.37 MeV) reaction.
Fig. 5.1.4: DWBA fits to the $^{15}$N(d,n)$^{16}$O reaction taken from the paper of Fuchs et al. [Fu 67] indicating the possible presence of a significant compound nucleus mechanism in a reaction presumed direct.
Fig. 5.1.5: DWBA fits to $^{10}\text{B}(d,p)^{11}\text{B}$ at $E_d = 12$ MeV taken from the paper by Schiffer et al. [Sc 67a].
expected error in the extracted spectroscopic factor, which is due to uncertainties in optical model parameters and in shell model calculations, and can be ignored. However, when the spectroscopic factor for a final state is small a comparison of the cross section in the DWBA theory with experiment to obtain the spectroscopic factor can result in a large error unless other reaction mechanisms are small contributors. When they are significant, analysis with a combination of theories may help to explain the apparent disagreement between theory and experiment.

Several recent studies of deuteron induced reactions in $^{12}$C [Ho 67, Co 69], $^{14}$N [Po 69] and $^{16}$O [Di 68, Co 69a] at bombarding energies in the range 1 to 10 MeV have been described satisfactorily using the optical model, distorted wave (DWBA) and Wolfenstein-Hauser-Feshbach (HF) theories (see Chapter 2). The (d,p) reaction has been the main one studied so far allowing a comparison of the spectroscopic factors obtained from the reaction theories with those derived from shell model calculations. In practice, the (d,p) calculations are an incoherent sum of a direct component obtained with the conventional DWBA method and a compound nucleus background estimated using the HF statistical theory, including the width fluctuation factor [La 57] and reduced by a fraction R to allow for that portion of the incident flux which proceeds without compound nucleus formation (see section 2.5). In general, the cross section at forward angles is largely determined by the direct component and
the large angle cross section then determines the reduction factor R.
The spectroscopic factors required to describe the data agree well
with those deduced from shell model calculations. This, perhaps, is
surprising in view of the doubtful validity of the various reaction
theories when applied to such nuclei.

In order to apply the theoretical analysis, an average over
the compound nucleus fluctuations must be made. As shown in
figures 5.1.1-3, the excitation functions appear to be due to broad
(~ 1 MeV) overlapping levels. An estimate of the level spacing in
the compound nuclei $^8$Be, $^{11}$B and $^{12}$C was obtained from the curves
shown in figure 5.1.6. Ericson [Er 59] pointed out that if $N(E)$ is
the number of excited states with energy less than $E$, then in many
cases a plot of $\log N(E)$ versus $E$ gives a straight line, that is

$$\log N(E) = \frac{1}{T} E \quad . \quad (5.1.1)$$

The plot for each compound nucleus is shown in figure 5.1.6.
The straight line drawn through the points has been extrapolated to
the relevant excitation energy and the corresponding mean level
spacing ($D$) calculated from the curve.

The values obtained are shown on the figure. They are at
least upper limits on the true value and as such are sufficient for
the present purpose. In principle, a cross section averaged over a
few MeV is desirable. However, this requires energy dependent
parameters and energy averaging in the theoretical analysis since the
Fig. 5.1.6: Plots of log N(E) versus E for the three compound nuclei $^8$Be, $^{11}$B and $^{12}$C. The straight lines have been extrapolated to the excitation energy reached in the present experiments and the mean level spacing estimated.
cross sections are not sufficiently linear with energy over such a large interval for the values at the mean energy to be accurate enough to represent the whole interval. For these reasons the cross sections have been averaged over 1.0 to 1.5 MeV in the present work.

5.2 EXPERIMENTAL PROCEDURE

The $^6$Li, $^9$Be and $^{10}$B targets were prepared by evaporation of the enriched isotopes on to ~ 20 µg/cm$^2$ carbon substrates. The $^6$Li target was transferred from the evaporator to the scattering chamber in a vacuum lock in order to limit oxygen contamination. A better quality target was obtained by placing a thin film of Formvar$^+$ over the surface on to which the $^6$Li metal was to be evaporated. The target thickness for all these isotopes was approximately 100 µg/cm$^2$.

Charged particle spectra were collected simultaneously in an array comprising up to five detection systems. The particles were stopped in silicon surface barrier counters and, after amplification, the signals were analyzed and stored in an IBM computer. Tantalum slits in front of the detectors defined solid angles of $\approx 3 \times 10^{-4}$ sr. Some measurements were repeated with different counters to check that all offered the same solid angle; the results agreed to within 5 per cent. The target condition was monitored by a counter set at

---

$^+$ Formvar is the trade name for polyvinyl formal.
150 degrees and deterioration was found to be negligible. Dead-time was restricted to a few per cent by decreasing the beam intensity for the forward angle measurements. Typical spectra are shown in figure 5.2.1.

The same detectors were used for the $^6$Li and $^9$Be targets. The better resolution in the $^9$Be spectra was due to the counters being cooled and to the use of a small magnet mounted in front of the detector slits to sweep away electrons scattered from the target, as described in section 3.2.

The higher resolution obtained in this way allowed the elastically scattered deuterons from $^9$Be to be separated from contaminant groups. The $^{10}$B spectra were taken with 2000 µm counters because of the high Q-value for the ground state (d,p) reaction. The energy resolution of 70 keV was determined mainly by the counters and was adequate for separating the first few proton groups, but not for extracting the elastically scattered deuterons.

5.3 DATA REDUCTION

As can be seen from figure 5.2.1 the background in the $^6$Li + d and $^9$Be + d data could be assumed linear and was subtracted from the area of the peaks without introducing large errors. The $^{10}$B + d spectrum has considerable background due to the $^{10}$B(d,α)$^8$Be and to subsequent alpha particle decay of the final nucleus, $^8$Be. The areas of the peaks in the $^{10}$B spectra were obtained using a
Fig. 5.2.1: Typical particle spectra using surface barrier detectors.
program Mikimaus 4 which makes a least squares fit of a cubic curve
to the background, sums the peak and subtracts the background.

The errors in the relative cross sections are estimated to be about 5 per cent, obtained from the scatter of repeated points, except where this is exceeded by the statistical error due to weaker transitions or uncertainty in the subtraction of background from $^{12}\text{C}(d,p_0)^{13}\text{C}$ and $^{16}\text{O}(d,p_0)^{17}\text{O}$ reactions. All other sources of error were considered insignificant in these cases.

5.4 ABSOLUTE CROSS SECTIONS

The $^6\text{Li}$ cross sections were normalized to the absolute data of Meyer et al. [Me 63], which agree with the recent measurements of Dürr et al. [Du 68]. The absolute cross section measurements of Chohan and Hussain [Ch 65] for the $^{10}\text{B}(d,p_0)^{11}\text{B}$ reaction at 5.1 MeV were used to normalize the $^{10}\text{B}$ results.

The absolute cross sections for the $^9\text{Be} + d$ reactions were measured at 5.25 MeV incident deuteron energy.

The expression for calculating the cross section from experimentally measured quantities is:

$$ \frac{d\sigma}{d\Omega} = \frac{n \cos \theta_t A e}{d\Omega / \rho \Delta x Q N_0} , $$

(5.4.1)

where $n$ = the number of counts in the peak considered,

$\theta_t$ = the angle of the target to the beam direction,
A = the effective atomic weight of the element of interest in the target,
e = the charge of the electron,
\( d\Omega \) = the solid angle of the detectors,
Q = the quantity of charge passed through the target,
\( N_0 \) = Avogadro's number,
\( \rho \Delta x \) = the target thickness in units of mass per unit area.

All of these quantities, except e, \( N_0 \), and A have uncertainties in their measurement, which combine to give a total error in the measured cross section. Each of these will be considered in turn.

a) The number of counts in the peak, \( n \); this was taken to be the statistical error, allowing for subtraction of background, and in this case was \( \approx 1 \) per cent.
b) The target angle, \( \theta_t \); the angle of the target holder could be set to within 0.5 degrees, which gives an error of \( \approx 1 \) per cent.
c) The solid angle, \( d\Omega \); the error in the solid angle involves the error in the area of the defining slit, which was measured with a microscope to 1 per cent, and the error in the distance of the slit from the target centre which was also approximately 1 per cent. The error in the solid angle is then given by the sums of the two fractional errors, i.e. 2 per cent.
d) The charge passed through the target, Q; the accuracy of the Elcor current integrator is quoted as 1 per cent by the manufacturers.
e) The target thickness, $\rho \Delta x$; in any absolute cross section measurement this is the most difficult to obtain and contributes by far the largest error to the total. The target thickness was measured by the following method. The position on the energy scale of a resonance at 4.808 MeV in $^{12}$C + p was observed with and without the incident protons passing through the $^9$Be target and the comparison of the energy shift with range and stopping power tables [Wi 66] gave the target thickness. The $^{12}$C target was conveniently provided by the carbon substrate on to which the $^9$Be had been evaporated. The $^9$Be could then be inserted by rotation of the target ladder. In this way the two resonances were plotted simultaneously by alternate rotations of the target, thereby eradicating the possible error due to the beam energy.

The results of the measurement are shown in figure 5.4.1 for two angles of detection; the 90 degree data gave the most accurate estimate of the energy loss which was equivalent to a target thickness of $114 \pm 9 \mu g/cm^2$. This error is only the error in estimating the shift of the resonance. Two other sources of error exist in this measurement; one is the error in the stopping power tables, which the authors claim to be less than 5 per cent; the second is due to the presence of oxygen in the target. During the target thickness measurement a carbon foil taken from the same batch as the substrate for the $^9$Be target, was mounted on the same target holder so that the elastically
Fig. 5.4.1: The resonance in $^{12}$C + p at 4.808 MeV used to measure the thickness of the $^9$Be target. The crosses denote the energy shifted resonance due to the $^9$Be foil.
scattered protons from $^9$Be and $^{12}$C could be identified quickly by comparison of the scattering from both targets. The intensity of peaks in the $^{12}$C + p spectrum indicated a negligible amount of oxygen and therefore it was assumed that any oxygen present in the $^9$Be + p spectrum was associated with the $^9$Be. The amount of this contaminant was calculated from the known $^{16}$O(p,p)$^{16}$O cross section at 5.25 MeV and the energy shift was corrected for by the corresponding energy loss. This gave an additional error of 6 per cent in the target thickness.

The total error in the absolute cross section measurement is then given by the square root of the sums of the squares of the separate errors, yielding ± 10 per cent. Therefore allowing 2 per cent for unknown contributions, an accuracy on the absolute cross section of ± 12 per cent is obtained.

5.5 EXPERIMENTAL RESULTS

Angular distributions were obtained for the reactions

$^6$Li(d,$d_0$)$^6$Li (g.s.), $^6$Li(d,$p_0$)$^7$Li (g.s.), $^6$Li(d,$p_1$)$^7$Li (0.48 MeV state), $^9$Be(d,$d_0$)$^9$Be (g.s.), $^9$Be(d,$p_0$)$^{10}$Be (g.s.), $^9$Be(d,$p_1$)$^{10}$Be (3.37 MeV state), $^{10}$B(d,$p_0$)$^{11}$B (g.s.), $^{10}$B(d,$p_1$)$^{11}$B (2.12 MeV state), $^{10}$B(d,$p_2$)$^{11}$B (4.44 MeV state) and $^{10}$B(d,$p_3$)$^{11}$B (5.02 MeV state).

The measurements were carried out at bombarding energies of 4.5 to 5.5 MeV for $^6$Li and $^{10}$B and 4.5 to 6.0 MeV for $^9$Be in steps of 0.25 and lab angles 10 degrees to 160 degrees usually at five
degree intervals. The results are shown in figures 5.5.1-5 which also includes the energy averaged cross sections; it is seen that fluctuations about the mean values are small.

5.6 THEORETICAL ANALYSIS

The angular distributions were averaged over the energy intervals 4.5 to 5.5 MeV ($\bar{E}_d = 5.0$ MeV) for the $^6$Li + d and $^{10}$B + d reactions and 4.5 to 6.0 MeV ($\bar{E}_d = 5.25$ MeV) for the $^9$Be + d reactions in order to study the average behaviour of these data. The HF statistical theory including the width fluctuation factor and reduced by a fraction $R$ was used to estimate the compound nucleus contribution to each reaction. The shape elastic and direct reaction cross sections were calculated with the optical model and DWBA theory and were added incoherently to the corresponding compound nucleus component to give total cross sections.

The optical model potentials were assumed to be of the form

$$U(r) = C - Vg(V) - iWf(W) + S\frac{1}{r}\frac{dg(S)S}{dr}$$

where

$$g(i) = \{1 + \exp[(r - r_1A^{1/3})/a_1]\}^{-1},$$

$$f(i) = 4[g(i)]^2 \exp[(r - r_1A^{1/3})/a_1].$$

C is the Coulomb potential for a uniform charge distribution of radius $R_C = 1.3 A^{1/3}$ and $A$ is the mass number of the nucleus. For simplicity only the central parts of these potentials were used in
5.5.4: Angular distributions for the $^{10}\text{B}(d,p)^{11}\text{B}$ (2.12 MeV state) and $^{10}\text{B}(d,p)^{11}\text{B}$ (5.02 MeV state) reactions in mb/sr (centre of mass).

Fig. 5.4: Angular distributions for the $^{10}\text{B}(d,p)^{11}\text{B}$ (2.12 MeV state) and $^{10}\text{B}(d,p)^{11}\text{B}$ (5.02 MeV state) reactions in mb/sr (centre of mass).
Fig. 5.5.5: Angular distributions for the $^6\text{Li}(d,d)^6\text{Li}$ and $^9\text{Be}(d,d)^9\text{Be}$ elastic scattering in mb/sr (centre of mass).
the HF calculations, and for a given final particle the same potential was employed for each channel irrespective of the excitation energy of the residual nucleus.

5.7 FITTING PROCEDURE

The following procedure was adopted in order to obtain the reduction factor $R$ from the measurements. Optical model potentials for all open channels of the compound system were employed in the calculation of transmission coefficients for the Hauser-Feshbach theory. The potentials were obtained from optical model fits to the appropriate elastic scattering and are listed with their references in table 5.1. It is not possible to obtain triton, helion, alpha particle and neutron potentials which correspond exactly to the particle energy of each channel to which they were applied. However this is not a serious problem since small variations of optical model parameters in a particular channel have a significant effect on the cross section in that channel only. The theory requires the energies, spins and parities of all possible final states, which in the present case were obtained from Lauritsen and Ajzenberg-Selove (La 62]. The analysis was carried out by applying a fitting procedure to all data. The value of

$$\chi^2 = \Sigma \left( \frac{\sigma_{\text{exp}} - \sigma_{\text{DI}} - R\sigma_{\text{HF}}}{\sigma_{\text{exp}}^2} \right)^2$$

was minimized to obtain a best fit of the theoretical expression
\[ \sigma_{\text{th}} = S\sigma_{\text{DI}} + R\sigma_{\text{HF}} \]  

(5.7.2)

to the measured differential cross section \( \sigma_{\text{exp}} + \delta\sigma_{\text{exp}} \). The Hauser-Feshbach cross section \( \sigma_{\text{HF}} \) is calculated from equation (2.3.4), \( \sigma_{\text{DI}} \) is the direct interaction cross section calculated from the distorted wave theory equation (2.6.34), and \( S \) is the spectroscopic factor.

Because of the uncertainty surrounding the value of \( R \), the spectroscopic factors calculated from the shell model by Cohen and Kurath [Co 67] were used in equations (5.7.1-2) and only \( R \) was allowed to vary. The fits to the data were slightly improved when both \( R \) and \( S \) were varied.

For the elastic scattering data \( \sigma_{\text{DI}} \) is the shape elastic scattering cross section calculated from the optical model.

5.8 \((d,p)\) REACTIONS

The DWBA calculations were performed in the zero range approximation, corrections being made for finite range [Di 65] and non-local [Hj 65] effects. These cross sections were multiplied by the appropriate theoretical spectroscopic factors [Co 67] and a factor of 1.65 to correct for the normalization of the deuteron wave function [Sa 65].

The initial deuteron parameters were based upon a potential found by Satchler [Sa 66] and employed previously to describe deuteron induced reactions in \(^{12}\text{C}\) [Co 69] and \(^{16}\text{O}\) [Co 69a]. A
similar set of parameters has also been used successfully by Schiffer et al. [Sc 67a] in their study of the (d,p) reaction in the 1p shell at 12 MeV. The optical model parameters finally adopted are shown in table 5.1. The geometry of the deuteron potential was kept identical with that previously used for $^{12}$C [Co 69] and only the well depths $V$ and $W$ were allowed to vary to obtain fits to the data. It was found that rather larger depths are required to describe the present measurements. No search was made for other types of potentials or ambiguities in the parameters. The proton parameters were kept constant and close to conventional values for all of the calculations. The neutron bound state wave functions were obtained using potential N2 with the well depth being varied to give the correct asymptotic forms.

Figure 5.8.1 exhibits the results for the parameters of table 5.1 and the spectroscopic and reduction factors of table 5.2. It is seen that theory gives a good overall fit to the energy averaged data. The $^6$Li and $^9$Be (d,p) reactions show typical $\ell = 1$ stripping patterns at forward angles which are satisfactorily described by the DWBA calculations using shell model spectroscopic factors. In most cases, the large angle measurements require a significant compound nucleus contribution which is well accounted for by the HF cross sections reduced by a factor of 0.1 to 0.2.

The $^{10}$B(d,p)$^{11}$B angular distributions are the most interesting since the various transitions show very different amounts
Fig. 5.8.1: Energy averaged angular distributions for the $^6\text{Li}(d,p)^7\text{Li}$ (g.s. and 0.48 MeV state), $^9\text{Be}(d,p)^{10}\text{Be}$ (g.s. and 3.37 MeV state) and the $^{10}\text{B}(d,p)^{11}\text{B}$ (g.s., 2.12 MeV, 4.44 MeV and 5.02 MeV states) reactions at mean deuteron bombarding energies of 5.00, 5.25 and 5.00 MeV respectively, compared with the compound nucleus contributions (CN) from the HF calculations, the direct reaction contributions DI from the DWBA calculations and the totals $T = CN + DI$. The optical model parameters of table 5.1 and the reduction factors and spectroscopic factors of table 5.2 have been used.
TABLE 5.1
OPTICAL MODEL POTENTIALS

<table>
<thead>
<tr>
<th>Particle Potential</th>
<th>V (MeV)</th>
<th>r_V (fm)</th>
<th>a_V (fm)</th>
<th>W (MeV)</th>
<th>r_W (fm)</th>
<th>a_W (fm)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>deuteron D1</td>
<td>160</td>
<td>0.90</td>
<td>0.90</td>
<td>12</td>
<td>2.10</td>
<td>0.50</td>
<td>[Co 69]</td>
</tr>
<tr>
<td>D2</td>
<td>170</td>
<td>0.90</td>
<td>0.90</td>
<td>12</td>
<td>2.10</td>
<td>0.50</td>
<td>[Co 69a]</td>
</tr>
<tr>
<td>D3</td>
<td>150</td>
<td>0.90</td>
<td>0.90</td>
<td>12</td>
<td>2.10</td>
<td>0.50</td>
<td>Sa 66</td>
</tr>
<tr>
<td>D4</td>
<td>150</td>
<td>0.85</td>
<td>0.83</td>
<td>12</td>
<td>1.66</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>proton P1</td>
<td>49</td>
<td>1.25</td>
<td>0.65</td>
<td>7</td>
<td>1.25</td>
<td>0.47</td>
<td>[Ho 67]</td>
</tr>
<tr>
<td>neutron N1</td>
<td>45</td>
<td>1.32</td>
<td>0.66</td>
<td>9</td>
<td>1.26</td>
<td>0.48</td>
<td>[Ho 67]</td>
</tr>
<tr>
<td>neutron N2</td>
<td>varied</td>
<td>1.25</td>
<td>0.65</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>triton T1</td>
<td>142</td>
<td>1.16</td>
<td>0.78</td>
<td>12</td>
<td>1.88</td>
<td>0.61</td>
<td>[Ea 67]</td>
</tr>
<tr>
<td>helion H1</td>
<td>142</td>
<td>1.16</td>
<td>0.78</td>
<td>12</td>
<td>1.88</td>
<td>0.61</td>
<td>[Ea 67]</td>
</tr>
<tr>
<td>alpha particle A1</td>
<td>80</td>
<td>2.10</td>
<td>0.55</td>
<td>4</td>
<td>2.10</td>
<td>0.30</td>
<td>[Ca 64]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Particle Potential</th>
<th>S (MeV.fm²)</th>
<th>r_S (fm)</th>
<th>a_S (fm)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>deuteron D1-D4</td>
<td>15</td>
<td>1.20</td>
<td>0.90</td>
<td>[Co 69, Co 69a, Sa 66]</td>
</tr>
<tr>
<td>proton P1</td>
<td>16</td>
<td>1.25</td>
<td>0.65</td>
<td>[Ho 67]</td>
</tr>
<tr>
<td>neutron N2</td>
<td>16</td>
<td>1.25</td>
<td>0.65</td>
<td>[Ho 67]</td>
</tr>
</tbody>
</table>

of direct and compound nucleus contributions. The ground state transition is similar to the $^6\text{Li}$ and $^9\text{Be}$ results although, because of the larger Q-value the stripping peak is less prominent and in spite of the increased number of open channels the contribution from the compound nucleus mechanism is more significant especially at forward angles. In the case of the 4.44 MeV level, the
TABLE 5.2

SPECTROSCOPIC AND REDUCTION FACTORS

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Level (MeV)</th>
<th>Q (MeV)</th>
<th>Jπ</th>
<th>Deuteron Potential</th>
<th>S_{1/2}</th>
<th>S_{3/2}</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>6Li(d,p)7Li</td>
<td>0.00</td>
<td>5.03</td>
<td>3/2⁻</td>
<td>D1</td>
<td>0.289</td>
<td>0.431</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>0.48</td>
<td>4.55</td>
<td>1/2⁻</td>
<td>D1</td>
<td>0.039</td>
<td>0.855</td>
<td>0.118</td>
</tr>
<tr>
<td>9Be(d,p)10Be</td>
<td>0.00</td>
<td>4.59</td>
<td>0⁺</td>
<td>D2</td>
<td>0.000</td>
<td>2.356</td>
<td>0.173</td>
</tr>
<tr>
<td></td>
<td>3.37</td>
<td>1.22</td>
<td>2⁺</td>
<td>D3</td>
<td>0.226</td>
<td>0.048</td>
<td>0.166</td>
</tr>
<tr>
<td>10B(d,p)11B</td>
<td>0.00</td>
<td>9.23</td>
<td>3/2⁻</td>
<td>D2</td>
<td>0.000</td>
<td>1.094</td>
<td>0.194</td>
</tr>
<tr>
<td></td>
<td>2.12</td>
<td>7.11</td>
<td>1/2⁻</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>4.44</td>
<td>4.79</td>
<td>5/2⁻</td>
<td>D2</td>
<td>0.039</td>
<td>0.096</td>
<td>0.203</td>
</tr>
<tr>
<td></td>
<td>5.02</td>
<td>4.21</td>
<td>3/2⁻</td>
<td>-</td>
<td>0.000</td>
<td>0.005</td>
<td>0.180</td>
</tr>
<tr>
<td>6Li(d,d)6Li</td>
<td>0.00</td>
<td>0.00</td>
<td>1⁺</td>
<td>D1</td>
<td>-</td>
<td>-</td>
<td>0.100</td>
</tr>
<tr>
<td>9Be(d,d)9Be</td>
<td>0.00</td>
<td>0.00</td>
<td>3/2⁻</td>
<td>D4</td>
<td>-</td>
<td>-</td>
<td>0.295</td>
</tr>
</tbody>
</table>

Spectroscopic factors are small and the compound nucleus component is enhanced on account of the higher spin of the residual nucleus so that the stripping peak has almost disappeared. The reduction factors required for these two transitions are about the same although the contributions from the various reaction mechanisms are quite different. The theoretical spectroscopic factors are zero and \( \approx 0.01 \) for the 2.12 and 5.02 MeV transitions respectively so that no direct components were calculated in these cases. The contributions estimated by the HF theory for the same reduction factors as the other transitions are comparable to the measured cross sections. One should not expect to describe the compound nucleus component in
detail since the statistical theory is of doubtful validity for such light nuclei. Moreover, small contributions from other reaction mechanisms may be present.

As stated earlier the fits to the angular distributions were slightly improved when both $S$ and $R$ were allowed to vary in equations (5.7.1-2). Table 5.3 shows the percentage change in $\chi^2$ from a best fit value to the theoretical value and the resulting change in $R$ to optimize the fit. In general an increase in $S$ is compensated for by a corresponding decrease in $R$ and vice versa. The large change in $R$ for the $^6$Li(d,p)$^7$Li (0.48) reaction reflects the small compound nucleus contribution and the large change in $S$ for the $^{10}$B(d,p)$^{11}$B (4.44) reaction is a result of the high compound nucleus cross section in this case.

**TABLE 5.3**

<table>
<thead>
<tr>
<th>Final State</th>
<th>$S_{\text{Theory}}$</th>
<th>$S_{\text{Best Fit}}$</th>
<th>$%\Delta S$</th>
<th>$%\Delta R$</th>
<th>$\chi^2$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^7$Li (g.s.)</td>
<td>-12%</td>
<td>+30%</td>
<td>$S_{\text{Theory}}$</td>
<td>5.6</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>$^7$Li (0.48)</td>
<td>+3%</td>
<td>-60%</td>
<td>$S_{\text{Theory}}$</td>
<td>3.6</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>$^{10}$Be (g.s.)</td>
<td>-15%</td>
<td>+33%</td>
<td>$S_{\text{Theory}}$</td>
<td>2.2</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>$^{10}$Be (3.37)</td>
<td>-11%</td>
<td>+5%</td>
<td>$S_{\text{Theory}}$</td>
<td>10.03</td>
<td>9.02</td>
<td></td>
</tr>
<tr>
<td>$^{11}$B (g.s.)</td>
<td>+2%</td>
<td>-5%</td>
<td>$S_{\text{Theory}}$</td>
<td>1.79</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>$^{11}$B (4.44)</td>
<td>+30%</td>
<td>-10%</td>
<td>$S_{\text{Theory}}$</td>
<td>8.4</td>
<td>5.6</td>
<td></td>
</tr>
</tbody>
</table>
In the analysis by Schiffer et al. [Sc 67a] of $^7\text{B}(d,p)^{11}\text{B}$ at $E_d = 12.0$ MeV the spectroscopic factor for the transition to the 4.44 MeV level was approximately twice the value obtained by Cohen and Kurath from the shell model. They explain the difference as being due to the relatively low spectroscopic factor (0.135) causing the theoretical cross section to be sensitive to the calculation. The discrepancy could be explained by the large compound nucleus contribution observed in the present analysis. Although at 12.0 MeV this reaction would normally be considered to proceed via a direct reaction mechanism, the DWBA theory underestimates the cross section at large angles (figure 5.1.5) suggesting the presence of a compound nucleus component.

5.9 ELASTIC SCATTERING DATA

A satisfactory description of the energy averaged ($\bar{E}_d = 5.0$ MeV) $^6\text{Li}(d,d_0)^6\text{Li}$ data was not obtained although the optical model parameters were varied over a wide range of values. For parameters close to those used to fit the corresponding $(d,p)$ measurements, the theoretical curves exhibit much more structure at backward angles. A best fit to the elastic scattering data required a reduction factor $\approx 0.7$ which is inconsistent with the values found for the $(d,p)$ reactions and deuteron scattering from neighbouring nuclei. There is considerable experimental evidence in support of a cluster structure for $^6\text{Li}$ consisting of an alpha particle and a deuteron
[Mo 60, An 60, Mo 63, 01 64, Ba 67]. The presence of deuterons at the surface could give rise to an exchange mechanism and enhance the elastic scattering cross section at large angles. The fitting procedure would try to compensate for this by increasing the compound elastic scattering through a higher reduction factor.

Figure 5.9.1 shows the theoretical prediction for the same deuteron parameters and reduction factors as were employed for the \( (d,p) \) calculations and it is seen that the curve grossly underestimates the cross section at large angles.

The energy averaged \( (E_d = 5.25 \text{ MeV}) \) elastic scattering results for \(^9\text{Be}\) are shown in figure 5.9.1. It was found that the deuteron parameters used to describe the corresponding \((d,p)\) reactions do not fit the elastic scattering data very well. A better fit was obtained for potential D4 of table 5.1 although this potential gives an inferior description of the \((d,p)\) reactions. This type of discrepancy implies inadequacies in either the optical model or the DWBA theory which have been discussed previously [Ba 68, De 69]. The same parameters cannot be expected to optimize the deuteron wave function both asymptotically for the elastic scattering data and within an internal region where the main contribution to the \((d,p)\) cross section occurs, especially for large Q-value reactions.
by $^6\text{Li}$ and $^9\text{Be}$ at mean deuteron bombarding energies of 5.00 and 5.25 MeV respectively, compared with the compound elastic contributions (CE) from the HF calculations, the shape elastic contributions (SE) from the optical model calculations and the totals ($T = CE + SE$). The optical model parameters of table 5.1 and the reduction factors of table 5.2 have been used.
5.10 CONCLUSIONS

In general the optical model, DWBA and HF theories provide a satisfactory overall description of the present (d,p) data when spectroscopic factors obtained from shell model calculations and reduction factors of 0.1 and 0.2 are used. The deuteron potential previously employed to describe similar data for $^{12}$C, $^{14}$N and $^{16}$O targets provided a useful starting point for the present analysis. However, rather larger well depths (for fixed geometrical factors) are required to fit the $^6$Li, $^9$Be and $^{10}$B (d,p) reactions. These deeper potentials appear to be partly a reflection of the larger Q-values of the reactions. Approximately equal reduction factors were found for each transition although the contributions from the various reaction mechanisms are quite different in some cases. This suggests that the HF theory gives a good estimate of the non-direct reaction contributions although it does not provide a detailed description of the cross sections.

It was not possible to describe the $^6$Li(d,d$^0$) measurements in a consistent manner, probably because an exchange mechanism is taking place which enhances the cross section at large angles. A fit was obtained to the $^9$Be(d,d$^0$) results but only by employing a deuteron potential with rather different parameters than those used to describe the corresponding (d,p) angular distributions. This implies that the theoretical description is not strictly valid, and that the different parameters are required to partially compensate for deficiencies in the models.
A study of low energy \((d,p)\) reactions has been carried out on both light and heavy nuclei. Some features of the mechanism of the deuteron stripping reaction have been investigated in relation to extracting spectroscopic factors and accounting for different reaction mechanisms.

Sub-Coulomb stripping on heavy nuclei has been compared with stripping above the Coulomb barrier. For the two energy regions the analysis using the DWBA theory has yielded spectroscopic factors which agree within the uncertainties of the calculations and measurements.

The parameters affecting the extraction of spectroscopic factors have been investigated with the conclusion that the lower energy data allows a more reliable value to be obtained. This is because of the relative independence on the choice of optical model parameters for the distorted waves when the incident deuteron and emitted proton energies are below the Coulomb barrier. Although the magnitude of the cross section is sensitive to other parameters in the calculations the dependence has been shown to be essentially equal at both energies for a range of transferred angular momentum values.
The lack of structure in the region of the maximum of the angular distribution, together with the insensitivity of the cross section to the parameters for the distorted waves used in the DWBA theory indicate that the sub-Coulomb deuteron stripping reaction is a useful method of obtaining spectroscopic factors. One reliable measurement at a large angle and a minimum of theoretical analysis would yield a spectroscopic factor more accurately than more extensive measurements at an energy above the Coulomb barrier.

Low energy deuteron stripping reactions on light nuclei have been analyzed in terms of the DWBA and HF theories in an attempt to account for the competition of the direct reaction and compound nucleus mechanisms. The agreement between experiment and theory is surprisingly good. The spectroscopic factors derived from the fitting of the combination of the two theories over the complete angular range are in good agreement with shell model calculations in the lp shell.

The inclusion of the compound nucleus component in the analysis of (d,p) reactions for obtaining spectroscopic factors is not always necessary because of the dominance of the direct reaction mechanism at the forward angles. However in some cases when the Q-value is large and the final state has a large angular momentum, the compound nucleus component is significant at all angles and if ignored could result in large errors in the extracted spectroscopic factor.
The analysis of low energy deuteron elastic scattering from $^9$Be has not been achieved with the same optical model parameters as used in the DWBA analysis of the (d,p) reaction on $^9$Be indicating a deficiency in the DWBA theory pertaining to the choice of wave functions in the entrance and exit channels.

The elastic scattering of low energy deuterons from $^6$Li has not been satisfactorily described by the optical model and it is suggested that an exchange mechanism is contributing significantly at large angles.
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