Squeezed-State Generation in Optical Bistability

by

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Statement of authorship

The contents of this thesis, except where indicated by references, are entirely my own work.

Deborah Maree Hope
February 1993
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Abstract

A experimental study of quantum fluctuations and squeezing was performed in a near-bistable cavity-atom system where atomic and cavity decay rates were comparable in magnitude. A dense well-collimated beam of atomic barium was directed at right angles through the fundamental mode of an optical cavity and the fluctuations of the cavity transmitted field were analysed. Squeezing of up to 18% was observed. After correction for the total quantum efficiency of the experimental configuration it could be inferred that squeezing of 50% below the quantum noise limit was generated by the coupled cavity-atom system.

In contrast to the experiments of previous workers squeezing was found in a new regime which produced a squeezed beam of significant power. Best squeezing was found on the upper branch of dispersive bistability. This power is large enough to permit the shape and orientation of the squeezing ellipse to be experimentally determined.

Experimental results were compared with the predictions of a microscopic quantum-statistical theory of squeezing without adiabatic elimination. The theory was extended to account for the spatial variation of the cavity field mode. The results of the experiments show good agreement with the complete theory.
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Chapter 1 : Introduction

1.1 Overview and motivation

One of the fundamental models of quantum optics concerns the interaction of a collection of two-level atoms with a single mode of an optical cavity. The cavity is driven by an external field resonant or near-resonant with the atomic transition. Dissipative effects are included via the spontaneous emission from the atoms out the sides of the cavity and the decay of the cavity field through the output mirror. If an additional mechanism were invoked to invert the atomic population, this model would describe a coherently-pumped single-mode laser. In the absence of inversion, it describes the system of optical bistability. The name refers to the two stable levels of the cavity transmission - low-intensity and high-intensity - which can exist for a single value of the pump laser intensity. Bistability is generated by collective effects among the atoms, which are coupled by the cavity field inside the driven resonator. This coupling can also lead to the generation of nonclassical states of light.

Einstein introduced the concept of the particle nature of light to account for the photoelectric effect (Einstein 1905). It was subsequently shown that many significant aspects of light-atom interaction could be fully described using theories which considered the interaction of quantised atoms with a classical wave, and neglected the quantum nature of the light (Mandel 1976). These 'semiclassical' treatments gave excellent quantitative agreement with the results of spectroscopic experiments in weak and strong fields, as well as with the results of experiments which examined nonlinear light-atom interactions. Later versions of the semiclassical models were able to explain the photoelectric effect itself (Wentzel 1926, Mandel et al. 1964, Lamb and Scully 1969).

The development of the laser in the early 1960s led to renewed interest in the study of quantum optics. Many features of the laser require a model in which the cavity electromagnetic field is fully quantised (Loudon 1973, Haken 1981). In general quantisation of the field is necessary when the stability, coherence, fluctuations and photon statistics of the light are the focus of interest.

Glauber (1963) introduced the quantum-mechanical representation of a light field which has a well-defined phase and amplitude, calling it a 'coherent state'. Unlike the classical field, the coherent state has fluctuations in the two quadratures of the electromagnetic field which are imposed by the Heisenberg uncertainty relation. The fluctuations are equal for both quadratures and minimise the uncertainty relation; coherent states are minimum uncertainty states. A phase-locked laser operating well above threshold has been shown to produce a beam that is a good approximation to a coherent state (Haken 1981). A measurement of the photon number of the laser beam will show a level of noise which cannot be reduced and is proportional to the square-root of the photon number. This noise level is known as the quantum noise limit.

The coherent state is a special case of the more general set of minimum uncertainty states known as 'squeezed states' (Stoler 1970, Yuen 1976, Walls 1983), for which the
fluctuations in one quadrature of the electromagnetic field are reduced below the level of the quantum noise limit. The fluctuations in the other quadrature phase are correspondingly raised, so that the Heisenberg uncertainty principle is always satisfied. Squeezed states are of great interest because they arise only in a quantum mechanical analysis. Experiments in the generation and detection of squeezed states can therefore be used to provide insight into the quantum-mechanical nature of light and of light-matter interactions.

Squeezed states are generated by nonlinear optical processes which correlate two photons. One such process is degenerate optical parametric oscillation, where the downconversion of a single pump photon in a crystal produces two photons of equal frequency. Another powerful technique is four-wave mixing, where two pump photons of frequency $\omega_p$ are mixed to give two photons of frequency $\omega_1 - \omega_2$, where $2\omega_p = \omega_1 + \omega_2$. Squeezing was first achieved experimentally via nondegenerate four-wave mixing in a beam of atomic sodium by Slusher et al. (1985).

It is well known that the dissipative process of spontaneous emission inherent in resonantly driven atomic systems degrades squeezing (Walls 1983). The largest amounts of quadrature squeezing to date have been generated in crystals (Wu et al. 1987, Sizmann et al. 1990) via the processes of optical parametric oscillation and second harmonic generation respectively. Nonetheless it is important to investigate quantum fluctuations and squeezing using two-level atoms. The fundamental processes underlying the interaction of a collection of two-level atoms with a narrowband electromagnetic field, both in free space and in a cavity, are believed to be well understood. We therefore have some confidence that squeezed-state generation in atomic media can be modelled from first principles using microscopic theories of atom-field interaction.

Theoretical treatments of squeezing in atomic media are numerous but prior to the present work only two experimental groups following Slusher et al. (1985) carried out experiments using atoms. Reasons for the small number of such experiments may include the technically demanding nature of atomic squeezing experiments, the limitations upon achievable squeezing imposed by spontaneous emission, and the fact that squeezing via atoms does not lend itself readily to practical applications.

Maeda et al. (1987) employed a vapour of atomic sodium in a forward four-wave mixing single-pass configuration to generate small amounts of squeezing. The analysis of the system was complicated by Doppler broadening and collisional interactions within the vapour. Reported in the same year was an experiment by a group at the University of Texas at Austin led by Professor H.J. Kimble (Orozco et al. 1987, Raizen et al. 1987). They generated squeezing using the interaction between a well-collimated beam of atomic sodium and a single mode of a high-finesse cavity. The experiment was performed in a regime where the decay rates of the atomic polarization and of the cavity mode were comparable in magnitude.

In this regime the optical cavity no longer simply acts to increase the interaction length through the medium but plays an integral role in the dynamics of the system. The coupled cavity-atom system has a new set of eigenfrequencies which differ from those of the atoms or the light taken separately. The strongly-coupled regime was predicted to be a particularly favourable configuration for squeezing in optical bistability. This prediction
was verified by the Austin group, who observed squeezing of 30% below the quantum noise limit.

The purpose of the work described in this thesis was to realise the ideal cavity-atom interaction in a regime where cavity and atomic decay rates are similar in size. We aimed to extend the work of Orozco et al. (1987) and Raizen et al. (1987), to resolve the discrepancies between theory and experiment which were encountered in this work, and to explore further the complex system of optical bistability.

1.2 Review of previous work

A classical electric field (Dirac 1927), may be written as

\[ \mathbf{E}(t) = \lambda (a_1 \cos \omega t + a_2 \sin \omega t) \]  

(1.1)

where \( \lambda \) contains the spatial structure of the field and is constant, and \( a_1 \) and \( a_2 \) are the quadrature phase amplitudes. When it is quantised the quadrature phase amplitudes are replaced by the corresponding quantum mechanical operators \( \mathbf{X}_1 \) and \( \mathbf{X}_2 \). These operators are conjugate observables which may be written in terms of the creation and destruction operators \( \mathbf{a}_f \) and \( \mathbf{a}_d \) of the quantised harmonic oscillator as

\[ \mathbf{X}_1 = \frac{(a + a^\dagger)}{2}, \quad \mathbf{X}_2 = \frac{(a - a^\dagger)}{2i} \]  

(1.2)

The boson commutation relation \( [a, a^\dagger] = 1 \) implies \( [\mathbf{X}_1, \mathbf{X}_2] = 1/(2i) \), and necessarily gives rise to an uncertainty relation (Loudon and Knight 1987)

\[ \langle \Delta \mathbf{X}_1^2 \rangle \langle \Delta \mathbf{X}_2^2 \rangle \geq \frac{1}{4} |\langle [\mathbf{X}_1, \mathbf{X}_2] \rangle|^2 = \frac{1}{16} \]  

(1.3)

where \( \langle \Delta x^2 \rangle \) denotes the variance of the operator \( x \). Minimum-uncertainty states such as coherent states and squeezed states satisfy the equality in Eq. (1.3) above for the variances of the quadrature phase amplitude operators.

In the work of Yuen (1976) 'squeezed states' were known as two-photon coherent states. The term 'squeezing' may have come about via the common pictorial representation of the electric field in a phasor diagram (\( \mathbf{X}_2 \) versus \( \mathbf{X}_1 \)), as shown in Fig. 1.2-1. The length of the arrow denotes the magnitude of the electric field amplitude and the shaded figure at the tip of the arrow gives the fluctuations in the field, or the uncertainty area. This is circular for the coherent state in Fig.1.2-1(a), and elliptical, or a 'squeezed circle', for the state in Fig.1.2-1(b). Detailed accounts of the properties of nonclassical states of light and overviews of experiments in squeezed-state generation may be found in Walls (1983), Loudon and Knight (1987), Yamamoto et al. (1987) and Bachor et al. (1991), and in the special issues J. Mod. Optics 34, No. 6 (1987), J. Opt Soc. Am. B 4, No. 10 (1987) and Appl. Phys. B 55, (1992).
Figure 1.2-1: Phasor diagram showing the fluctuations of an electromagnetic field for
(a) a coherent state
(b) a squeezed state
where $X_1, X_2$ are the conjugate quadrature phase amplitude operators. The length of the arrow denotes the magnitude of the electric field amplitude and the shaded figure at the tip of the arrow shows the uncertainty area.

Figure 1.2-2: Hysteresis curve of optical bistability. As cavity input intensity is increased cavity output intensity switches from a low value on the 'lower branch' to a high value on the 'upper branch' of bistability. Decreasing the input intensity down the upper branch eventually causes the system to switch back down to the lower branch. For a range of values of input intensity (the bistable region, shaded) there exist two stable values of output intensity.
Because of its conceptual simplicity, the system of optical bistability was one of the first systems to be analysed for its ability to squeeze light. When a nonlinear medium is placed between two mirrors, continuous variation of the pump power reveals a distinctive hysteresis and switching in the cavity transmission (Lugiato et al. 1984). The system switches from the lower to the upper branch and vice versa with a dependence on the history of the system (see Fig 1.2-2). Bistability exists in the absorptive regime, where the pump laser is resonant with both the atoms and the cavity mode, and in the dispersive regime, where the laser is detuned from the atomic resonance, the cavity resonance, or both. The first experimental observation of bistable behaviour was made in 1976 by Gibbs and McCall, who used a sodium cell within an optical cavity and operated in the dispersive regime. For an atomic medium the cooperativity $C$ plays the part of a threshold parameter for bistability. $C$ is given by

\[ C = \frac{\alpha_0 \ell F}{2\pi} \]  

where $\alpha_0 \ell$ is the optical depth on resonance of the atomic medium and $F$ is the finesse of the optical cavity. The cooperativity is a measure of the degree to which the atoms interact via the intracavity electric field. The condition $C > 4$ must be satisfied for the generation of absorptive bistability in a ring cavity driven by a plane wave.

The significance of this phenomenon for the design of optical switches and other forms of optical processors was quickly realised and bistable behaviour has been generated and studied in solid, liquid and gaseous media. The literature on optical bistability is too extensive to describe here, and the reader is referred to review articles by Lugiato (1984) and Carmichael (1986b). In this work we primarily draw upon the experiments which were performed with the aim of testing the 'classic' model of optical bistability; that is, a model where a homogeneously-broadened atomic medium interacts with a single mode of a high-finesse optical cavity. This system is one of the few in nonlinear optics that has allowed rigorous theories of light-atom interaction to be put to quantitative test. The definitive experiments of this type have been carried out by co-workers of Professor H.J. Kimble (Orozco 1987, Rosenberger et al. 1991).

Predictions of a small amount of squeezing in the regime of absorptive bistability and the good-cavity limit were made by Lugiato and Strini (1982). Analysis of the regime of dispersive bistability by Walls and Milburn (1982) also indicated that squeezing could be generated, but again the effect was very small. These results caused experimentalists to abandon consideration of the system of optical bistability as a potential squeezer.

It was later realised that the initial calculations, which only considered the squeezing in the internal cavity field, were not directly applicable to the field which emerged from the cavity. Work by Yurke (1984) and Collett and Gardiner (1984) on the connection between the light fluctuations in the internal and external cavity fields revealed that squeezing in the output field could be considerably larger. This finding prompted a reassessment of squeezing in optical bistability.

Collett and Walls (1985) predicted perfect squeezing at the pump frequency for the turning points in dispersive optical bistability. However, a macroscopic nonlinear polarizability model was employed which neglected the spontaneous emission and
absorption due to the atomic medium. Reid and Walls (1985) developed a microscopic analysis which was able to include these features, where the system was considered as an ensemble of N two-level atoms interacting with a single quantised cavity mode. This analysis indicated that losses due to the atoms had a significant impact upon the degree of squeezing which could be attained and upon the parameters of optimum squeezing. At the pump frequency squeezing could only become large for extremely high values of the cooperativity C.

The theory of Reid and Walls (1985) was derived with the assumption of the good-cavity limit. The good cavity (or high-Q) limit, where the decay rate $\gamma_\perp$ of the atoms is assumed much greater than the decay rate $\kappa$ of the cavity, is applicable to many experiments conducted in the optical regime (see Fig 1.2-3, and Table 1.2-1). In the case of pure radiative broadening $\gamma_\perp$ is given by $\gamma_\perp = \gamma_\parallel / 2$, where $\gamma_\parallel$ is the longitudinal decay rate of the atomic transition. The cavity half-linewidth $\kappa$ is defined by the choice of cavity parameters. It is inversely proportional to cavity length and finesse (Kimble et al. 1983);

$$\kappa = \frac{\pi c}{2LF} \quad (1.5)$$

where c is the speed of light, L is the length of the cavity, and F is the cavity finesse. Taking the limit $\gamma_\perp \gg \kappa$ or $\kappa \gg \gamma_\perp$ allows the rapidly-decaying variables to be eliminated in the adiabatic approximation, which assumes that the fast-decaying variables are 'slaved' to the dynamics of the slowly-decaying variables (Haken 1970).

Some theoretical work suggested that the bad-cavity, or low-Q, limit $(\kappa \gg \gamma_\perp)$ would generate the best squeezing in optical bistability (Carmichael 1986a). Practical limitations on the length of a cavity render this regime difficult to access experimentally in optical physics, as shown in Table 1.2-1. It was also noted that in some instances neither the good nor bad cavity limits were appropriate to the description of experiments (Carmichael 1986). The mathematical and physical complexity of the system is greatly increased if neither atomic nor cavity variables are adiabatically eliminated. It was found, however, by numerical solution and optimisation, that the full dynamical model allowed the possibility of excellent squeezing for readily attainable values of number density and cavity finesse (Reid et al. 1986).

For comparable decay rates, in the low intensity limit and the absorptive configuration, the cavity mode and atomic polarization may be treated as strongly coupled oscillators which transfer excitation between each other at a beat frequency $g\sqrt{N}$ (Carmichael 1986), where N is the number of atoms and g is the single-atom coupling constant, itself given by

$$g = \frac{\mu}{\hbar} \sqrt{\frac{\hbar \omega_0}{2e_0 V}} \quad (1.6)$$

where V is the mode volume of the cavity, $\omega_0$ is the resonant frequency of the atom and $\mu$ is the dipole matrix element of the transition. This phenomenon is known as 'vacuum-field Rabi splitting'. In a dressed-state picture for a single excitation of the mode the splitting may be represented as shown in Fig. 1.2-4, where the first excited state of the coupled cavity-atom system is shown to be split by the amount $g\sqrt{N}$. The quantity $g\sqrt{N}$ is a measure of the strength of the system coupling. (Note that the rates $\kappa$, $\gamma_\perp$, $\gamma_\parallel$, $g\sqrt{N}$ are in
Chapter 1

High-Q cavity ('good cavity')
\[ \kappa \ll \gamma_\perp \]

Intermediate regime
\[ \kappa \sim \gamma_\perp \]

Low-Q cavity ('bad cavity')
\[ \kappa \gg \gamma_\perp \]

**Figure 1.2-3:** Schematic representation of high-Q, low-Q and intermediate cavity-atom systems, for cavities with the same finesse F, but varying lengths L (atomic decay rate \( \gamma_\perp \) is the same for all cases). Diagram is not to scale; actual length changes required are in orders of magnitude.

**Table 1.2-1:** Ratio of atomic and cavity decay rates \( q \) for cavities with varied lengths \( L \), finesse \( F =150 \), and \( \gamma_\perp/(2\pi) = 5 \) MHz, as for atomic sodium. The table demonstrates that very short cavities are required in order to approach the bad-cavity regime.

<table>
<thead>
<tr>
<th>Cavity length L</th>
<th>Cavity decay rate ( \kappa = \frac{c}{2\pi 4LF} )</th>
<th>( q = \frac{\gamma_\perp}{\kappa} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 m</td>
<td>0.5 MHz</td>
<td>10</td>
</tr>
<tr>
<td>10 cm</td>
<td>5.0 MHz</td>
<td>1.0</td>
</tr>
<tr>
<td>1 cm</td>
<td>50 MHz</td>
<td>0.1</td>
</tr>
<tr>
<td>1 mm</td>
<td>500 MHz</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Figure 1.2-4: Coupling of N atoms to a single cavity mode for low photon number, with driving laser frequency $\omega_L$ equal to cavity and atomic frequencies $\omega_C$ and $\omega_0$. The first excited state is split by an amount $g\sqrt{N}$ (vacuum-field Rabi splitting). $g$ is the single-atom coupling coefficient. Photons are scattered at $\pm g\sqrt{N}$ about $\omega_L$. 

\[ \omega_0 = \omega_C = \omega_L \]

\[ \omega_L \]

\[ \omega_L + g\sqrt{N} \]

\[ \omega_L - g\sqrt{N} \]
angular frequency units throughout this work.)

For certain sets of parameters the cavity-atom system is predicted to generate squeezed light over a frequency range of the order of the cavity half-linewidth $\kappa$, with optimum squeezing located at an analysis frequency $g\sqrt{N}$. This was shown to be the case in the experiment of Raizen et al. (1987) and Orozco et al. (1987). The squeezing was observed on the lower branch of optical bistability. The experimental parameters of optimum squeezing were close to those derived from a numerical optimisation of the deterministic Maxwell-Bloch equations with quantum noise included as a small fluctuation upon the steady-state solutions. After correction for total quantum efficiency the observed optimum squeezing was 53%, compared to a predicted optimum squeezing of 78%. Orozco et al. (1987) suggested the discrepancy between the observed and predicted squeezing in their experiment may have been due to the spatial structure of the mode. The mode structure was not included in their theoretical treatment.

The Austin group achieved the matching of cavity and atomic decay rates and a strong cavity-atom coupling by the use of an optical cavity with finesse in the range 600-1200, and a length of 1mm or less. An appreciable atomic number density was required, (optical depth on resonance $0.3<\alpha_c<0.6$), so that the atomic cooperativity $C$ would be large enough to ensure strong coupling. This placed stringent demands on the collimation of the Na atomic beam.

The nonzero nuclear spin of sodium is responsible for hyperfine splitting in the energy levels of sodium, so the Na transition cannot be correctly modelled as a two-level atom. The well-known technique of optical pre-pumping (Abate 1974) can be used to force the atomic population into a near two-state transition. This technique was employed by Raizen et al. (1987) and Orozco et al. (1987) so that the experimental results could appropriately be compared with two-level theories. Use of the technique limited the search for optimum squeezing, however. The cavity input intensity, analysis frequency, cavity detuning and quadrature phase were varied in the search for optimum squeezing, but atomic detuning $\Delta$ was necessarily fixed, due to the constraints of the optical pre-pumping.

Prompted by the success of the experiment at the University of Texas, full quantum-statistical theories of optical bistability without adiabatic elimination were published independently by Castelli et al. (1988) and by Reid (1988). Both works presented analytic expressions for the spectrum of squeezing in the mixed absorptive/dispersive regime and the general cavity case. Castelli et al. showed plots of the variation in optimised squeezing with detuning parameters and with the ratio of atomic and cavity decay rates. Reid illustrated the inelastic scattering processes leading to squeezing in the low-Q, high-Q and intermediate regimes with dressed-state diagrams.

1.3 Outline of the present work

Within the constraints imposed by the availability of equipment, the experiment described in the present work was constructed to;
(i) satisfy the conditions of the cavity-atom model described at the beginning of this chapter, and

(ii) to optimise the squeezing for experimentally accessible parameters.

To realise the conditions of the model, an atomic medium was required with a transition that could be assumed to act as a two-level atom if driven near resonance by a narrowband laser (where 'narrowband' indicates that the laser beam has a linewidth much less than that of the atomic transition). Additional criteria were that:

1) The atomic transition should be located within the visible spectrum in the range 540-650 nm (that is, within the range of the standard dyes used in the laboratory).

2) The levels should be free of hyperfine structure

3) The transition should be from ground state to nondegenerate upper state, preferably J=0→J=1, to simplify the modelling of the experiment.

The atoms must be prepared in a well-collimated atomic beam, so that Doppler broadening and collisional interactions are negligible.

The pair of partially-transmitting high-reflectivity mirrors forming the cavity were chosen to satisfy the assumption of a single-mode single-ported cavity. Carmichael et al. (1983) pointed out that the cavity mirror reflectivity R is required to be approximately unity for theories which assume weak coupling to the external world via the mirrors. A convenient lower limit for the finesse is F=100, corresponding to R ~ 0.97. Taking a value of at least this magnitude simultaneously ensures that the single-mode criterion is satisfied. According to the definition of finesse as

\[ F = \frac{\text{FSR}}{\Delta \nu} \]  

(where FSR is the free spectral range of the cavity and \( \Delta \nu \) is the empty cavity transmission linewidth), a value of F=100 indicates that the distance in frequency between consecutive TEM\(_{00}\) modes of the cavity is 100 times greater than the cavity linewidth. The single-ported criterion may be approximately satisfied by choosing the input coupling mirror to have a transmissivity much less than that of the output mirror, so that the dominant decay route of the cavity mode is through the output coupler.

An initial study was carried out to determine the feasibility of the proposed experiment, given the possibility of using different atomic media. Using the equations derived by Reid (1988), a numerical search was conducted for the conditions of optimum squeezing. The chief difficulty of the study was the large size of the parameter space. Six independent parameters are involved. Squeezing in the regime of interest is sensitive to the ratio of the atomic and cavity decay rates \( q = \gamma J / \kappa \), and to the atomic cooperativity \( C \), the intracavity intensity \( I \), atomic detuning \( \Delta \), cavity detuning \( \phi \) and the analysis frequency \( \omega \).

Orozco et al. (1987), Castelli et al. (1988) and Raizen (1989) showed by numerical optimisation that in general squeezing improved for decreasing \( q \) and increasing \( C \) if all the other parameters were optimised. Thus the best situation for squeezing in this model
exists where neither the decay rates of the atomic polarization or the cavity mode can be neglected, but where $\kappa$ is still at least an order of magnitude greater than $\gamma_\perp$. i.e. $q = \gamma_\perp/\kappa < 0.1$. We therefore aimed to minimise $\gamma_\perp$ and to maximise $\kappa$.

$\gamma_\perp$ is fixed by the choice of atomic medium. The atomic medium most commonly employed in spectroscopic experiments is sodium. Na is readily evaporated to form an atomic beam, and the wavelength of the 590 nm transition is near the peak of the gain curve of the most stable dye used in tunable cw dye lasers, Rhodamine 6G. We note that $\gamma_\perp/(2\pi) = 5$ MHz for Na in the case of pure radiative broadening. Use of the technique of optical pre-pumping to produce a two-state transition requires two wavelength-tunable lasers. Having only one tunable ring-dye laser available to us, this solution was not feasible.

Kimble (1987) proposed that squeezing in the vacuum-field Rabi splitting regime could be considerably improved by using an atomic transition such as the $J=0\rightarrow1$ 556 nm transition of ytterbium, for which $\tau = 875 \pm 20$ ns (Gustavsson 1979), corresponding to $\gamma_\perp/(2\pi) = 0.1$ MHz. Ytterbium was eliminated as the atomic medium of choice in the present experiments after consideration of two further difficulties. Transit broadening becomes of concern when the time taken by the atoms to cross the cavity mode is much less than the decay time of the transition, such that an atom in the upper state is unlikely to decay in the interaction region. Also, since the commercial ring dye laser to be used had an rms linewidth of at best 1 MHz additional frequency stabilisation would be required to avoid marked instrumental broadening of the atomic line.

It was found that barium, despite the larger decay rate of its resonance transition at 553 nm, $(\gamma_\perp/(2\pi) = 10$ MHz), could in theory generate about 55% squeezing in a short cavity of finesse~100. Use of the well-understood J=0 to J=1 transition in $^{138}\text{Ba}$, which is free from hyperfine structure, eliminated the need for optical pre-pumping. Detailed spectroscopic studies of barium had been carried out in the Department of Physics at the ANU in previous years and a functioning Doppler-free barium beam was already in existence. It was therefore decided that the first experiments should be carried out using atomic barium.

For a given experimental configuration, an upper limit on $\kappa$ (as given in Eq. 1.5) was set by the implicit assumptions of the theory which require $F > 100$, and by the physical width of the atomic beam. The diameter of the barium atomic beam was known to be approximately 2 mm at the interaction region. To avoid degradation of the cavity mirrors by scattered barium particles a cavity length of 4 mm was chosen. Cooperativity $C$ is proportional to cavity finesse $F$ and optical depth on resonance of the medium ($\alpha_0 \ell$), as shown in Eq. (1.4). There is a trade-off between $\kappa$ and $C$ since both quantities must be maximised for best squeezing. The atomic number density can be used as an additional control parameter for $C$, however, to compensate for a low finesse. On the basis of the considerations above, the cavity mirrors were selected to give a finesse of approximately $F=150$.

In most studies of squeezing generated by the cavity-atom interaction the internal cavity field has been assumed to be a plane wave. In real experiments of this type the cavity modewill have a Gaussian transverse profile, in either a standing-wave or a ring cavity.
Chapter 1

The need for a realistic model of the short-cavity experiments prompted a study of the
effect of a spatially varying cavity mode on squeezing. Theoretical work was undertaken
in consultation with Dr Craig Savage and Dr David McClelland of the Department of
Physics and Theoretical Physics at the Australian National University, to investigate the
effect of the spatial mode structure on short-cavity squeezing. A full description of the
work was published in Physical Review A (Hope et al. 1990).

Reid's theory of squeezing in optical bistability without adiabatic elimination (Reid 1988)
was generalised to an arbitrary mode shape. The techniques used were those employed
by Xiao et al. (1987a) to take into account the effect on squeezing of the spatial mode
structure for a high-Q cavity. It was found that the optimum squeezing was always
reduced with respect to the plane-wave case, at a given value of the atomic cooperativity
C, for any type of spatially varying cavity mode. The reduction in optimum squeezing
was small for the standing wave, larger for the Gaussian mode in a ring interferometer,
and largest for the Gaussian mode in a standing wave cavity. Nonetheless the discrepancy
observed by Orozco et al. (1987) was only partly due to the spatial structure of the mode.
Predicted optimum squeezing for the case of a Gaussian mode in a standing-wave cavity
with the parameters reported by Orozco et al. (1987) was 70% (Hope et al. 1990).

The spatial structure analysis demonstrated that for the ratio of decay rates to be used in
the experiments (0.01<q<0.1), the structure of the cavity mode was unlikely to degrade
the squeezing by more than 10 percentage points. The Gaussian transverse mode structure
of the laser beam was impossible to modify, and the effect of the standing-wave structure
alone was found to be negligible compared to other likely sources of degradation such as
Doppler broadening, so that an attempt to set up a ring-cavity configuration would be of
little value. Thus the experiments were able to proceed as originally planned on the basis
of the plane wave ring-cavity theory.

Based on the initial theoretical investigations a series of experiments were carried out to
demonstrate squeezing and to find the optimum parameter range for squeezing empirically.
As can be seen from Fig. 1.2.5, a noise suppression of about 18% below the quantum
noise limit was observed.

1.4 Structure of this thesis

In this thesis these experimental results will be described and interpreted in detail. As a
first step, however, the theory of the cavity-atom interaction will be discussed in full. In
Chapter 2 the microscopic quantum model of the interaction between a single quantised
mode of a cavity and N two-level atoms is outlined, without elimination of either the
cavity or atomic decay rates. The theory is developed to include the spatial variation of
the mode, so as to account for the effect of Gaussian or standing-wave structures upon
the squeezing generated in the short-cavity regime. Chapter 3 contains a discussion of
the optimum parameter regimes for the generation of squeezing, as derived from numerical
optimisation of the full quantum theoretical model.

The experimental apparatus and procedures used are described in Chapter 4 and initial
experiments in optical bistability are analysed in Chapter 5. The purpose of this work was to characterise the atom-cavity system and to compare the semiclassical behaviour of the system with theoretical predictions, and with experiments performed by previous workers.

In Chapter 6 we report the results of experiments in squeezed-light generation in an cavity-atom system. Squeezed light was observed in a new parameter regime of atomic and cavity detunings, such that best squeezing was obtained on the upper branch of optical bistability. The squeezed light emerging from the cavity was found to have a significant intensity, allowing us to make a phasor map of the squeezing ellipse. Comparisons between theory and experiment and theory are made and discussed. A summary of the thesis and suggestions for further work are provided in Chapter 7.
Figure 1.2-5: Measurement of the quadrature noise of the output beam of the cavity-atom system for varying cavity detuning, where trace (i) shows the quantum noise limit and trace (ii) shows the phase-sensitive quadrature noise (from Hope et al. 1992)
Chapter 2: Theory

2.1 Overview

With few exceptions, analyses of the squeezing generated by the interaction between an ensemble of two-level atoms and a single mode of an optical cavity have taken the internal cavity mode as a plane wave. The sodium short-cavity experiments of Raizen et al. (1987) and Orozco et al. (1987) actually employed a Gaussian transverse mode in a Fabry-Perot interferometer. It was suggested that the discrepancy encountered between the amount of the observed and predicted squeezing could be partly attributed to the spatial variations of the mode (Orozco et al. 1987). It has been well-known for some time from theory (Drummond 1981, Ballagh et al. 1981) and experiment (Sandle and Gallagher 1981, Kimble et al. 1983) that a spatially varying mode structure has considerable impact upon the switching parameters for optical bistability. It has also been predicted that squeezing in this system in the high-Q limit will be appreciably reduced by a Gaussian mode structure (Xiao et al. 1987b). Hence plane-wave models become unsatisfactory once quantitative comparisons between theory and experiment are required.

Drummond (1981) described a general method for including the spatial variations of the cavity mode in theories of optical bistability. Following Drummond, Xiao et al. (1987a, 1987b) analysed the effect of Gaussian and standing wave structures upon quantum fluctuations and squeezing in the high-Q cavity, with adiabatic elimination of the atomic variables. The high-Q limit is not appropriate for the description of the squeezing generated in the experiments by Orozco et al. (1987) and Raizen et al. (1987) for which atomic and cavity decay rates were comparable in magnitude.

In this chapter the theory developed by Reid (1988), which does not incorporate the spatial structure of the mode, is extended to an arbitrary mode shape in the manner of Xiao et al. (1987a, 1987b). We study the effect of the mode structure on squeezing for cavity and atomic polarization damping rates $\kappa$, $\gamma_x$, of similar magnitude, and compare our theoretical results to the results of the squeezing experiments performed in the vacuum-field Rabi splitting regime. The effect of Gaussian and standing wave structures on the optimised squeezing in the low-Q or 'bad-cavity' limit ($\kappa >> \gamma_j$) is also investigated.

The following section reviews the method of solution of the quantum-statistical problem of the cavity-atom interaction. Section 2.3 gives the Fokker-Planck equation for a spatially varying mode, (Xiao et al. 1987a), from which we derive the linearised equations for the fluctuations without adiabatic elimination of atomic or cavity variables. The solution for the squeezing spectrum external to the cavity is obtained in Section 2.4. Optimised squeezing is calculated for a Gaussian mode in a ring cavity, a plane wave in a Fabry-Perot cavity, and a combination of these, and the results are given in Section 2.5. Section 2.6 contains a discussion of the agreement between the experimental findings of Orozco et al. (1987) and the calculations of the previous section. In Section 2.7 we examine the effect of a Gaussian transverse mode structure on the squeezing in the low-Q limit and compare with the squeezing obtainable in the high-Q limit for the same type of mode, previously calculated by Xiao et al. (1987a,1987b). A summary of the findings of the work is given in Section 2.8, and analytical expressions for the squeezing spectrum generated for a
travelling wave with transverse Gaussian profile are shown in the Appendix.

### 2.2 Review of solution technique

In this section the microscopic quantum statistical treatment of the cavity-atom interaction for the case of a plane wave in a ring cavity case is outlined, for the sake of completeness and understanding. In the absence of dissipative processes, the cavity-atom system may be identified as the system described fully in the Tavis-Cummings model (Tavis and Cummings 1968, Tavis and Cummings 1969). With the addition of dissipation the problem becomes much more complicated. Quantum-mechanical methods must be united with the methods of statistical physics to account for the effect of the fluctuations which are introduced by spontaneous emission, and for those which enter the cavity via the lossy mirror.

The great value of the microscopic treatment is that solutions for the system variables of interest may be derived from elementary principles; the disadvantage is the mathematical difficulty and the need for numerous approximations based on knowledge of system sizes and time scales. A flow diagram (Fig. 2.2-1) adapted from Graham (1973) provides an overview of the techniques and approximations used to obtain an analytic solution for the squeezing spectrum of the external cavity field. Detailed discussions of the background to the problem and the mathematical approaches employed may be found in a number of works (Haken 1970, Drummond and Walls 1980, Drummond 1980, Drummond and Walls 1981, Lugiato and Strini 1982, Carmichael et al. 1983, Reid and Walls 1985, Reid et al. 1986, Carmichael 1986a, 1986b, Reid and Walls 1986).

The system of optical bistability is described by a well-known Hamiltonian (Agarwal et al. 1978, Lugiato 1979, Drummond and Walls 1981 and Sargent et al. 1985) constructed of a sum over several separate terms,

\[ H = \sum_{k=1}^{s} H_k \]  

where the terms \( H_k \) are given below. The electromagnetic field within the cavity is modelled as a quantised harmonic oscillator with the Hamiltonian

\[ H_i = \hbar \omega_c a^\dagger a \]  

where a and \( a^\dagger \) are destruction and creation operators for the single cavity mode of resonant frequency \( \omega_c \). \( \sigma_i^z, \sigma_i^x \) are the Pauli spin operators for the ith two-level atom. The frequency of the homogeneously-broadened atomic transition is given by \( \omega_a \). The operators \( S^i = \Sigma \sigma^i, e^{ikr_i}, S = \Sigma \sigma^x, e^{ikr_i}, S^z = \Sigma \sigma^z \) are cooperative atomic operators, with the summations running over \( i=1 \) to \( N \) atoms. The atomic Hamiltonian is

\[ H_2 = \frac{1}{2} \hbar \omega_a S^z \]  

In the rotating-wave and electric-dipole approximations the cavity-atom interaction Hamiltonian is given by

\[ H_3 = i\hbar g (a^\dagger S - a S^i) \]
Hamiltonian including atoms, modes, reservoirs

Von Neumann equation for density operator of total system

Born-Markov approximation—eliminate reservoir variables.

Equation for reduced density operator including modes, atoms

c-number representation

Equation of motion for quasi-probability density $P$, with infinite-order derivatives.

Truncate to second-order using scaling arguments

Fokker-Planck equation

Ito stochastic differential equations

Set time derivatives to zero

Steady-state solutions

Linearize about steady-state solutions for small fluctuations

Differential equations in fluctuations

Squeezing spectrum of external field

**Figure 2.2-1:** Flow diagram sketching the procedure used to find the squeezing spectrum of the external field for the cavity-atom system, in the microscopic quantum-statistical treatment.
where \( g \) denotes the single-atom coupling coefficient defined in Eq. (1.6). The counter­
rotating terms in \( a^\dagger S^\dagger \) and \( aS \) have been dropped in the rotating-wave approximation,
being negligible at or near resonance unless the field intensity is very large. The coupling
of the (classical) driving laser field into the cavity is described by the Hamiltonian
\[
H_4 = i\hbar (a^\dagger e^{-i\omega_L t} - ae^{i\omega_L t})
\]
where the driving field frequency is denoted by \( \omega_L \) and has complex amplitude \( e \).

Further terms are added to the total Hamiltonian to account for atomic dissipation via spontaneous emission;
\[
H_5 = \sum_{i=1}^{N} \left( \Gamma \sigma_i^+ + \Gamma^\dagger \sigma_i^- + \Gamma \sigma_i^\dagger \sigma_i^z \right)
\]
where \( \Gamma, \Gamma^\dagger \) and \( \Gamma_c \) represent reservoirs for the photons lost from the system. The reservoir \( \Gamma_c \) describes the photons dissipated via the lossy mirrors, and the associated Hamiltonian is
\[
H_6 = a^\dagger \Gamma_c + a\Gamma_c^\dagger
\]

The concept of a reservoir or heatbath is a fundamental one of statistical physics. It was originally derived from the classical model of Brownian motion, which concerns the motion of larger particles (e.g. pollen grains) within a liquid. The pollen grains appear to undergo a jerky random motion as they are pushed by the much smaller particles of the fluid. The particle experiences a frictional force due to the fluid, of opposite sign to its velocity but proportional to it. The particle also experiences fluctuations in its motion due to the large number of 'kicks' of random force and direction given it by the liquid particles. The particle may be considered as a system, and the liquid constitutes a reservoir or heatbath which removes energy from the system and also acts on the system in a random fashion. In the case of the system of optical bistability, a reservoir is considered as consisting of an infinite number of quantised harmonic oscillators which are coupled to the atoms or to the field mode, producing a resultant fluctuating (stochastic) force on the system which vanishes when averaged over the ensemble or over time.

The equation of motion for the density operator for the cavity-atom system, including the reservoir operators, can be written as
\[
\frac{d\rho}{dt} = \left( \frac{1}{\hbar} \right) [H, \rho].
\]

It is necessary to eliminate the reservoir variables, to be left with only the system variables of interest. Some reasonable assumptions can be made about the nature of the reservoirs. A reservoir should be in thermal equilibrium before the coupling to the system is turned on and must remain in thermal equilibrium thereafter (valid if the reservoir is very large and weakly coupled to the system), and the internal decay times of the reservoir must be much smaller than the decay times of the system. These principles are the basis of the Born-Markov approximation, which allows the reservoir variables \( \Gamma \) to be projected out using standard techniques (Haken 1970, Louisell 1973).
The complete quantum-mechanical master equation for optical bistability then becomes:

\[
\frac{d\rho}{dt} = \frac{1}{i\hbar}[H_1 + H_2 + H_3 + H_4, \rho] + \frac{\partial\rho}{\partial t_{\text{light}}} + \frac{\partial\rho}{\partial t_{\text{atoms}}} \tag{2.9}
\]

where

\[
\frac{\partial\rho}{\partial t_{\text{light}}} = \kappa( [a\rho, a^\dagger] + [a^\dagger, a\rho] )
\]

\[
\frac{\partial\rho}{\partial t_{\text{atoms}}} = \frac{\gamma_s}{2} \sum_{i=1}^{N} ( [\sigma_i^+, \rho\sigma_i^-] + [\sigma_i^-, \rho\sigma_i^+] ) + \frac{\gamma_p}{4} \sum_{i=1}^{N} ( [\sigma_i^+, \rho\sigma_i^+] + [\sigma_i^-, \rho\sigma_i^-] )
\]

Time constants of the dissipative processes are introduced in the dissipative irreversible parts of the master equation \( \partial\rho/\partial t_{\text{light}}, \partial\rho/\partial t_{\text{atoms}} \). One of these, \( \gamma_p \), is the collisional dephasing rate of the atomic polarization, being related to the transverse and longitudinal atomic damping rates by \( \gamma_s = \frac{\gamma_p}{2} \). It is assumed that there are no thermal photons in the system.

Possessing the master equation (2.9), it remains to make a series of approximations and simplifications which will enable it to be solved; it generally cannot be solved as it stands. A common technique is to convert it to a Fokker-Planck equation for a quasi-probability distribution \( P \) defined in a classical phase space.

The Fokker-Planck equation (FPE) is a powerful and central tool of classical statistical physics. Returning to the example of the system of Brownian motion, we find that both the deterministic motion and the fluctuations of the pollen grains may be described with a Fokker-Planck equation which has the form

\[
\frac{\partial}{\partial t} f = \left[ \frac{\partial}{\partial v} \gamma v + \frac{Q}{2} \frac{\partial^2}{\partial v^2} \right] f \tag{2.10}
\]

where \( f(v,t) \) is a normalised probability distribution giving the probability of finding a particle at time \( t \) with a velocity \( v \) in the interval \( v, v+dv \). The probability distribution is derived from a large number of identical experiments in which a particle undergoes Brownian motion. The particle velocities are measured at some time \( t \) and the number of particles having a velocity \( v \) is established. \( \gamma v \) is the drift coefficient, where \( \gamma \) is the coefficient of friction per unit mass. \( Q \) is the diffusion coefficient, a measure for the strength of the fluctuations which is proportional to reservoir temperature. The drift and diffusion coefficients of the Fokker-Planck equation describe the motion of the system completely. All other quantities of interest (for example, correlation functions of the velocity at differing times or positions) may be derived from them.

In the quantum-mechanical problem of optical bistability, the probability distribution over \( (v,t) \) becomes a quasi-probability function (so-called as it is not necessarily positive) in a five-dimensional complex phase space. We wish to establish a one-to-one correspondence between the quantum mechanical operators and classical complex variables. We reproduce
from Haken (1975) a table summarising the correspondences introduced (Table 2.2-1).

Owing to the different orderings possible for classical variables as opposed to non-commuting operators, several distinct but related quasi-probability distributions may be defined; including the Wigner distribution, the Glauber-Sudarshan P-distribution, and the Q-representation. Different quasi-probability distributions are found to be suitable for the treatment of different problems. In optical bistability the usual representations lead to a Fokker-Planck equation without a positive-definite diffusion matrix. This is a signature of nonclassical effects such as photon antibunching and squeezing, but it means that standard techniques for the derivation of differential equations from the FPE cannot be applied directly. Drummond and Gardiner (1980) developed the positive-P representation, which permitted a Fokker-Planck equation with a positive-definite diffusion matrix to be found for the system of optical bistability.

The correspondence between quantum operators and c-numbers is taken to be

\[(S,S^\dagger,S^*,a,a^*) \leftrightarrow (\nu,\nu^\dagger,D,\alpha,\alpha^\dagger), \quad (2.11)\]

operators \quad c-numbers

where the density operator \(\rho\) is associated with the positive P quasi-probability distribution, \(P\) (Drummond and Gardiner 1980). (The complex variables \(\nu,\nu^\dagger\) (and \(\alpha,\alpha^\dagger\)) are not in general complex conjugates of one another.) The correspondence calculation leads to an equation of motion for the quasi-probability distribution \(P\) with infinite-order derivatives. For a system containing a large number \(N\) of atoms, truncation to second order in the approximation \(1/N \ll 1\) reduces the equation of motion to a Fokker-Planck equation, (Drummond 1980, Drummond and Walls 1981), with the form

\[
\frac{\partial P(\alpha,t)}{\partial t} \equiv \left[ \frac{\partial}{\partial \alpha_\mu} A_\mu(\alpha) + \frac{1}{2} \frac{\partial}{\partial \alpha_\mu} \frac{\partial}{\partial \alpha_\eta} D_{\mu\eta}(\alpha) \right] P(\alpha,t) \quad (2.12a)
\]

where \(\mu,\eta = 1,...,5\) and \(\alpha = (\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5) = (\alpha,\alpha^\dagger,\nu,\nu^\dagger,D)\). In full, the FPE is

\[
\frac{\partial P(\alpha,t)}{\partial t} = \left[ -\frac{\partial}{\partial \alpha} [\epsilon - \kappa^* \alpha + g\nu] - \frac{\partial}{\partial \alpha^\dagger} [\epsilon^* - \kappa \alpha^\dagger + g\nu^\dagger] \right] \\
- \left[ \frac{\partial}{\partial \nu} (g\alpha D - \gamma \nu) + \frac{\partial}{\partial \nu^\dagger} (g\alpha^\dagger D - \gamma^* \nu^\dagger) \right] \\
- \frac{\partial}{\partial D} [\gamma_s (D + N) - 2g(\nu^\dagger \alpha + \nu \alpha^\dagger)] + \gamma_s \frac{\partial^2}{\partial \nu \partial \nu^\dagger} (D + N) \\
+ \frac{1}{2} \left[ \frac{\partial^2}{\partial \nu^2} (2g\nu) + \frac{\partial^2}{\partial \nu^\dagger^2} (2g\nu^\dagger) \right] \\
+ \frac{1}{2} \frac{\partial^2}{\partial D^2} [2\gamma_s (D + N) - 4g(\nu^\dagger \alpha + \nu \alpha^\dagger)] \right] P(\alpha,t)
\]
Table 2.2-1. Correspondence of quantum mechanical and classical quantities®

<table>
<thead>
<tr>
<th>Quantum mechanical quantity</th>
<th>Classical quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density matrix</td>
<td>Quasi-distribution function</td>
</tr>
<tr>
<td>Operator</td>
<td>Variable and/or differentiation</td>
</tr>
<tr>
<td>Density matrix equation</td>
<td>Stochastic equation (generalised or ordinary Fokker-Planck equation)</td>
</tr>
<tr>
<td>Expectation values (traces)</td>
<td>Expectation values (averages over distribution functions)</td>
</tr>
<tr>
<td>Time-ordered correlation functions operators</td>
<td>Correlation functions of classical quantities</td>
</tr>
</tbody>
</table>

*From Haken (1975)*
where \( \kappa' = \kappa(1+i\phi) \), \( \gamma = \gamma(1+i\Delta) \), and

\[
\phi = \frac{\omega_e - \omega_i}{\kappa}, \quad \Delta = \frac{\omega_0 - \omega_L}{\gamma_L}
\]  

(2.12b)

are the normalised cavity and atomic detunings. The Fokker-Planck equation in the positive P-representation is known to be directly equivalent to a set of Ito stochastic differential equations (Drummond and Gardiner 1980, Gardiner 1983, Reid 1988),

\[
\dot{\alpha} = \varepsilon - \kappa(1+i\phi)\alpha + g\nu + \Gamma_\alpha(t)
\]

\[
\dot{\alpha}^* = \varepsilon^* - \kappa(1-i\phi)\alpha^* + g\nu^* + \Gamma_\alpha^*(t)
\]

\[
\dot{\nu} = -\gamma_\nu(1+i\Delta)\nu + g\alpha D + \Gamma_\nu(t)
\]

\[
\dot{\nu}^* = -\gamma_\nu(1-i\Delta)\nu^* + g\alpha^* D + \Gamma_\nu^*(t)
\]

\[
\dot{D} = -\gamma_D(D+N) - 2g(\nu^*\alpha + \nu\alpha^*) + \Gamma_D(t)
\]  

(2.13a)

The \( \Gamma(t) \) are zero-mean Gaussian noise terms governing the quantum fluctuations in the system. We note that in the absence of the fluctuating terms these equations are precisely the ordinary deterministic semiclassical Maxwell-Bloch equations. If adiabatic elimination of either the field or atomic variables is to be carried out it is generally performed at this stage of the calculation, by setting the relevant time-derivatives in Eqs. (2.13a) to zero and solving to reduce the dimensionality of the system (Haken 1970, Louisell 1973). The correlation properties of the fluctuation terms comprise elements of the diffusion matrix and may be calculated using methods of stochastic differential calculus. They are given by Reid (1988) as

\[
\langle \Gamma_\alpha(t)\Gamma_\alpha(t') \rangle = \left[ 2\gamma_\alpha(D+N) - 4g(\nu^*\alpha + \nu\alpha^*) \right]\delta(t-t')
\]

\[
\langle \Gamma_\nu(t)\Gamma_\nu(t') \rangle = 2g\alpha\nu\delta(t-t')
\]

\[
\langle \Gamma_{\nu^*}(t)\Gamma_{\nu^*}(t') \rangle = 2g\alpha^*\nu^*\delta(t-t')
\]

\[
\langle \Gamma_\nu(t)\Gamma_{\nu^*}(t') \rangle = \langle \Gamma_{\nu^*}(t)\Gamma_\nu(t') \rangle = \gamma_D(D+N)\delta(t-t')
\]

\[
\langle \Gamma_{\alpha^*}(t)\Gamma_\alpha(t') \rangle = 0
\]  

(2.13b)

Note that in the absence of thermal photons the correlations of the internal field fluctuations are zero. Together Eqs. (2.13) define drift and diffusion matrices \( \mathbf{A} \) and \( \mathbf{D} \) in terms of the intracavity system variables \( \alpha,\alpha^*,\nu,\nu^*,D \).

The stationary state means are obtained by setting time derivatives to zero in Eqs. (2.13), taking expectation values and factorising products in the semiclassical approximation. From the equations in the atomic variables \( \nu,\nu^*,D \) of Eq. (2.13) it follows directly that the steady-state solutions \( \alpha_0,\alpha_0^*,\nu_0,\nu_0^*,D_0 \) are related by
Substituting these into the field equations of Eq. (2.13) we find that

\[ \frac{Y}{I} = \left(1 + \frac{2C}{1 + \Delta^2 + I}\right)^2 + \left(\phi - \frac{2CA}{1 + \Delta^2 + I}\right)^2 \]  

This is the state equation of optical bistability for a plane wave in a ring cavity (Drummond 1981, Drummond and Walls 1981). In Eq. (2.15a) \( Y \) and \( I \) are proportional to the pump intensity and mean intracavity intensity respectively, and are normalised by \( n_0 \), the saturation photon number on resonance;

\[ Y = \frac{|e|^2}{\kappa^2 n_0}, \quad I = \frac{|\alpha|^2}{n_0}, \quad n_0 = \frac{\gamma \gamma_1}{4g^2} \]  

The atomic cooperativity \( C \) is given by

\[ C = \frac{g^2 N}{2\gamma_1 \kappa} \]  

It may readily be shown (Carmichael 1986b) that this expression is equivalent to the previous definition (Eq. 1.4) in terms of the cavity finesse and optical depth on resonance of the atomic medium.

For the case of a large number of atoms in the interaction region (\( N \gg 1 \)) and small fluctuations (an assumption which is valid away from the turning points of optical bistability) it is legitimate to linearise about the steady-state solutions in order to find the eigenvalues of the stochastic differential equations. The system variables may be written as the sum of the steady-state solution and a small fluctuation term as follows;

\[ \alpha = \alpha_0 + \delta \alpha, \quad \nu = \nu_0 + \delta \nu, \quad D = D_0 + \delta D \]  

and inserted into the coupled nonlinear equations (2.14) to obtain the differential matrix equation

\[ \frac{d \delta \alpha(t)}{dt} = A \delta \alpha(t) + B \varepsilon(t) \]  

to first order in the fluctuations, where

\[ (\delta \alpha(t))^T = (\delta \alpha(t), \delta \alpha(t), \delta \nu(t), \delta \nu'(t), \delta D(t)) \],

where \( B \) is related to the diffusion matrix \( D \) by \( B^T B = D \), and

\[ F(t) = [B \varepsilon(t)]^T = \left[ \Gamma_{\alpha}(t), \Gamma_{\nu}(t), \Gamma_{\nu'}(t), \Gamma_{\nu''}(t), \Gamma_{\nu'}(t) \right] \]

\( \varepsilon(t) \) is a zero-mean noise vector with the property \( <\varepsilon(t)\varepsilon^T(t')> = \delta(t-t') \), and \( A \) is the 5
The noise correlation properties are
\[ <F(t)F^T(t')> = B <\epsilon(t)\epsilon^T(t')> B^T = D \delta(t-t'). \] (2.17b)

The drift and diffusion matrices \(\Delta\) and \(\mathcal{D}\) are now defined in terms of the steady-state values of the system variables as

\[
\Delta = \begin{pmatrix}
\kappa(1+i\Delta) & 0 & -g & 0 & 0 \\
0 & \kappa(1-i\Delta) & 0 & -g & 0 \\
-gD_0 & 0 & \gamma_1(1+i\Delta) & 0 & -g\alpha_0 \\
0 & -gD_0 & 0 & \gamma_1(1-i\Delta) & -g\alpha_0^* \\
2gv_o & 2gv_0 & 2g\alpha_0 & 2g\alpha_0^* & \gamma_\parallel
\end{pmatrix}
\] (2.18)

\[
\mathcal{D} = \begin{pmatrix}
0 & d_{\alpha\alpha'} & 0 & 0 & 0 \\
d_{\alpha\alpha'} & 0 & 0 & 0 & 0 \\
0 & 0 & d_{\nu\nu'} & d_{\nu\nu'} & 0 \\
0 & 0 & d_{\alpha\alpha'} & d_{\alpha\alpha'}^* & 0 \\
0 & 0 & 0 & 0 & d_{\mathcal{D}\mathcal{D}}
\end{pmatrix} = BB^T
\]

where
\[
d_{\nu\nu'} = 2g\alpha_0 v_0 \\
d_{\alpha\alpha'} = (D_0 + N)\gamma_p \\
d_{\mathcal{D}\mathcal{D}} = 2\gamma_\parallel(D_0 + N) - 4g(v_0^*\alpha_0 + v_0\alpha_0^*) .
\]

The squeezing spectrum outside the cavity is (Collett and Gardiner 1984, Savage and Walls 1986, Reid 1988)
\[
V(X_\theta, \omega) = 1 = 2\kappa [S_{13}(\omega) + S_{21}(\omega) + e^{2i\theta}S_{11}(\omega) + e^{2i\theta}S_{22}(\omega)]
\] (2.19)

where \(\theta\) is the phase angle of the squeezing, \(X_\theta\) is the quadrature phase amplitude of the cavity transmitted field, and \(\omega\) is the analysis frequency defined with respect to the frequency of the pump laser (for which \(\omega=0\)). \(V\) is the spectral variance, where \(V=1\) for a coherent state and \(V=0\) for perfect squeezing.

The spectral matrix \(S(\omega)\) is defined as the Fourier transform of the two-time correlation function \(<\delta\alpha(\tau),\delta\alpha^T(\tau)>\) and has been related to the quantities of the intracavity field (Gardiner 1983) as shown;

\[
S(\omega) = (A - i\omega I)^{-1}\mathcal{D}(A^T + i\omega I)^{-1}
\] (2.20)

The squeezing spectrum of the cavity transmitted field is therefore known in terms of the drift and diffusion matrices, and in principle analytic expressions for the squeezing can be
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derived immediately. In practice the analytic inversion and multiplication of 5 x 5 matrices is extremely cumbersome. Orozco et al. (1987) performed all calculations of the squeezing spectrum and optimisation of the squeezing over the parameter space by numerical means. This precluded an investigation of the effect of the spatially varying structure of the mode upon squeezing in the $\kappa \sim \gamma_L$ regime, as the computer time required would have been prohibitively large.

Reid (1988) avoided this difficulty by taking the Fourier transform of the stochastic differential equations (2.25a). This procedure led to a simplification of the mathematics and allowed solutions for the squeezing spectrum without adiabatic elimination to be obtained. The method of solution in Fourier space is described in detail in Section 2.4 of this chapter.

2.3 Model and linearised equations for a spatially varying field mode

We now wish to examine the effect of a spatially varying field mode upon the squeezing created in the interaction between N two-level atoms and a mode of a single-ported optical cavity, where the cavity is driven by an external coherent electromagnetic field. To this end we adopt the definitions of Drummond (1981) and Xiao et al. (1987a, 1987b) and define the coupling coefficient $g_j$ between the cavity mode and atoms in a small section $j$ of the cavity as

$$g_j = \left( \frac{\mu^2 \omega_c}{2\hbar \epsilon_0} \right)^{1/2} |U(r_j)| \equiv g_j |U(r_j)|$$

(2.21)

where $\mu$ is the transition dipole moment, $\omega_c$ is the resonance frequency of the cavity and $|U(r_j)|$ is the normalised mode function.

The $j$th section of the optical cavity is assumed to be large enough to contain many atoms ($N_j \gg 1$) and sufficiently small for the electric field to be approximately constant in it. We take M such sections, whose shape and extent are determined when the spatial profile of the cavity mode is specified.

A master equation in the Markovian, electric-dipole and rotating-wave approximations has been derived for this system (Xiao et al. 1987a). In the notation of Reid (1988) it becomes

$$\frac{d\rho}{dt} = \frac{1}{i\hbar} \left[ \hbar \omega_c \{a^\dagger a, \rho\} + \frac{1}{2} \hbar \omega_a \sum_{j=1}^{M} \{S_j^x, \rho\} \right] + \sum_{j=1}^{M} \left( g_j \{a^\dagger S_j, \rho\} - g_j \{aS_j^\dagger, \rho\} \right)$$

$$+ \kappa \left( [a\rho, a^\dagger] + [a, a^\dagger \rho] \right) + \left( e^{i\omega_L t} \{a^\dagger, \rho\} - e^{-i\omega_L t} \{a, \rho\} \right)$$

$$+ \sum_{j=1}^{M} \left[ \frac{\gamma_j}{2} \sum_{i=1}^{N_j} \left( [\sigma_{ij}^x, \rho, \sigma_{ij}^\dagger] + [\sigma_{ij}^x, \sigma_{ij}^\dagger, \rho] \right) + \frac{\gamma_p}{4} \sum_{i=1}^{N_j} \left( [\sigma_{ij}^y, \sigma_{ij}^y, \rho] + [\sigma_{ij}^y, \sigma_{ij}^y, \rho] \right) \right]$$

(2.22)
where \( \sigma_{ij} \), \( \sigma_{ij}^* \) are the Pauli spin operators for the ith two-level atom in the jth section and the operators \( S_j^\dagger = \sum \sigma_i^* e^{ik_{ij}} \), \( S_j = \sum \sigma_i e^{ik_{ij}} \), \( S_j^* \) are cooperative operators for a section j, with the summations running over \( i=1 \) to \( N \) atoms. We again assume that the number of thermal photons in the reservoirs associated with atoms and field can be neglected.

A c-number generalised Fokker-Planck equation can be obtained from the master equation using well-established methods (Risken 1984). We take the correspondence between quantum operators and c-numbers to be

\[
(S_j, S_j^\dagger, S_j^*, a, a^\dagger) \leftrightarrow (v_j, v_j^\dagger, D_j, \alpha, \alpha^\dagger).
\]

After truncation to second order in the approximation \( 1/N_j \ll 1 \) (where \( N_j \) is the number of atoms in the jth block of the cavity) the Fokker-Planck equation is, (Xiao et al. 1987a, Drummond 1980)

\[
\frac{\partial P(\alpha, t)}{\partial t} = \left[ -\frac{\partial}{\partial \alpha} \left[ \epsilon - \kappa' \alpha + \sum_{j=1}^{M} g_j v_j \right] - \frac{\partial}{\partial \alpha^\dagger} \left[ \epsilon^* - \kappa' \alpha^\dagger + \sum_{j=1}^{M} g_j v_j^\dagger \right] \\
- \sum_{j=1}^{M} \frac{\partial}{\partial v_j} (g_j \alpha D_j - \gamma v_j) + \frac{\partial}{\partial v_j^\dagger} (g_j \alpha^\dagger D_j - \gamma^* v_j^\dagger) \\
- \sum_{j=1}^{M} \frac{\partial}{\partial D_j} [-\gamma_D (D_j + N_j) - 2g_j (v_j^\dagger \alpha + v_j \alpha^\dagger)] + \gamma_p \sum_{j=1}^{M} \frac{\partial^2}{\partial v_j \partial v_j^\dagger} (D_j + N_j) \\
+ \frac{1}{2} \sum_{j=1}^{M} \left[ \frac{\partial^2}{\partial v_j^2} (2g_j \alpha v_j) + \frac{\partial^2}{\partial v_j^\dagger v_j^\dagger} (2g_j \alpha^\dagger v_j^\dagger) \right] \\
+ \frac{1}{2} \sum_{j=1}^{M} \left[ \frac{\partial^2}{\partial D_j^2} [2\gamma (D_j + N_j) - 4g_j (v_j^\dagger \alpha + v_j \alpha^\dagger)] \right] P(\alpha, t)
\]

\[
= \left[ \frac{\partial}{\partial \alpha_{\mu}} A_{\mu}(\alpha) + \frac{1}{2} \frac{\partial}{\partial \alpha_{\mu}} \frac{\partial}{\partial \alpha_{\eta}} D_{\mu\eta}(\alpha) \right] P(\alpha, t)
\]

where \( \mu, \eta = 1, 2, 3, ..., 3M+2 \)

\[
\alpha = (\alpha, \alpha^\dagger; v_1, ..., v_M; v_1^\dagger, ..., v_M^\dagger; D_1, ..., D_M)
\]

\[
\kappa' = \kappa (1 + i \phi), \quad \gamma = \gamma (1 + i \Delta)
\]

and the normalised cavity and atomic detunings \( \phi \) and \( \Delta \) are as given previously in Eq. (2.12b).
The Ito stochastic differential equations corresponding to this Fokker-Planck equation are, (Xiao et al. 1987a)

\[ \dot{\alpha} = \varepsilon - \kappa(1 + i\phi)\alpha + \sum_{j=1}^{M} g_j v_j + \Gamma_\alpha(t) \]
\[ \dot{\alpha}^\dagger = \varepsilon^* - \kappa(1 - i\phi)\alpha^\dagger + \sum_{j=1}^{M} g_j v_j^\dagger + \Gamma_\alpha^\dagger(t) \]
\[ \dot{v}_j = -\gamma_\perp(1 + i\Delta)v_j + g_j \alpha D_j + \Gamma_v(t) \]
\[ \dot{v}_j^\dagger = -\gamma_\perp(1 - i\Delta)v_j^\dagger + g_j \alpha^\dagger D_j + \Gamma_v^\dagger(t) \]
\[ \dot{D}_j = -\gamma_\parallel(D_j + N_j) - 2g_j (v_j^\dagger \alpha + v_j \alpha^\dagger) + \Gamma_{D_j}(t) \]

(2.25a)

Note that the dimensionality of the problem is 3M+2. There are three atomic variables for each section j of the cavity, plus the field variables \( \alpha, \alpha^\dagger \) which extend over all cavity sections.

The \( \Gamma(t) \) are zero-mean Gaussian noise terms governing the quantum fluctuations in the system. The nonzero correlations of these are (Eqs. (9), Xiao et al. 1987a),

\[ \langle \Gamma_{D_j}(t)\Gamma_{D_j}(t') \rangle = \left[ 2\gamma_\parallel(D_j + N_j) - 4g_j(v_j^\dagger \alpha + v_j \alpha^\dagger) \right] \delta_{jj}\delta(t - t') \]
\[ \langle \Gamma_{v_i}(t)\Gamma_{v_j}(t') \rangle = 2g_j \alpha v_j \delta_{ij}\delta(t - t') \]
\[ \langle \Gamma_{v_i}^\dagger(t)\Gamma_{v_j}^\dagger(t') \rangle = 2g_j \alpha^\dagger v_j^\dagger \delta_{ij}\delta(t - t') \]
\[ \langle \Gamma_{v_i}^\dagger(t)\Gamma_{v_j}(t') \rangle = \langle \Gamma_{v_j}(t)\Gamma_{v_i}^\dagger(t') \rangle = \gamma_p(D_j + N_j)\delta_{ij}\delta(t - t') \]

(2.25b)

The stationary state means are obtained by from the equations in the atomic variables \( v, \ v^\dagger, D \) of Eq. (2.25a) as in Section 2.2 and become

\[ v_{0j} = \frac{g_j \alpha_0 D_j}{\gamma_\perp(1 + i\Delta)} \quad v_{0j}^* = \frac{g_j \alpha_0^* D_j}{\gamma_\perp(1 - i\Delta)} \quad D_{0j} = \frac{-N_j}{1 + \frac{1}{I_j} / (1 + \Delta^2)} \]

(2.26)

Substituting back into the field equations of Eq. (2.25a) we find the state equation of optical bistability for a spatially varying field mode (Xiao et al.1987a, Drummond 1981);
\[
\frac{Y}{I} = \left(1 + \sum_{j=1}^{N} \frac{2C_j}{1 + \Delta^2 + I_j}\right)^2 + \left(\phi - \sum_{j=1}^{N} \frac{C_j \Delta}{1 + \Delta^2 + I_j}\right)^2
\]  

(2.27a)

In Equation (2.27a) \(Y\) and \(I\) are given by expressions identical to the plane wave ring cavity case of Eq. (2.15b), but \(n_0\) (the saturation photon number on resonance) now includes the spatial variation of the mode (Xiao et al. 1987a);

\[
\gamma \gamma' \frac{V_{eff}}{4g_0^2 s} \]  

(2.27b)

\(V_{eff}\) is the effective mode volume for an interferometer of length \(L\), \(s\) is the ratio of absorber length \(l\) to cavity length \(L\), and \(V\) is the volume of the interaction region between atoms and field;

\[
V_{eff} = \frac{s^2}{\iiint \sqrt{\left|U(r)\right|^4} \, d^3 r}, \quad s = \frac{1}{L}
\]  

(2.28a)

The scaled photon number \(I_j\) for a single section \(j\) of the cavity is defined as in (2.27b), with \(n_0\) replaced by \(n_{0j}\) as shown below.

\[
I_j = \frac{\left|\alpha_0\right|^2}{n_{0j}}, \quad n_{0j} = \frac{\gamma \gamma' \frac{V_{eff}}{4g_j^2}}{\sqrt{\left|U(r_j)\right|^2}}
\]  

(2.28b)

The atomic cooperativity \(C_j\) in a single cavity section is given by

\[
C_j = \frac{g_j^2 N_j}{2\gamma' \kappa} = \frac{g_0^2 \rho |U(r_j)|^2 \Delta V_j}{2\gamma' \kappa} = \frac{|U(r_j)|^2 \Delta V_j}{s}
\]  

(2.28c)

where \(\Delta V_j\) is the volume of a section \(j\) containing \(N\) atoms with density \(\rho\) (constant for the entire sample), such that \(N_j = \rho \Delta V_j\).

The stochastic equations may be linearised about the stationary state means for small fluctuations. Writing

\[
\alpha = \alpha_0 + \delta \alpha, \quad v_j = v_{0j} + \delta v_j, \quad D_j = D_{0j} + \delta D_j
\]  

(2.29)

and inserting into the coupled nonlinear equations (2.25a) we obtain the matrix equation
\[ \frac{d\delta\alpha(t)}{dt} = A\delta\alpha(t) + B\varepsilon(t) \]  

(2.30)

to first order in the fluctuations, where

\[
(\delta\alpha(t))^T = (\delta\alpha(t), \delta\alpha(t), \delta v_1(t), \delta v_1^T(t), \delta D_1(t) \ldots \delta v_M(t), \delta v_M^T(t), \delta D_M(t)) ,
\]

\[
[B(t)]^T = \left( \Gamma_{\alpha}(t), \Gamma_{\alpha}(t), \Gamma_{v_1}(t), \Gamma_{v_1^T}(t), \Gamma_{D_1}(t) \ldots \Gamma_{v_M}(t), \Gamma_{v_M^T}(t), \Gamma_{D_M}(t) \right)
\]

\(\varepsilon(t)\) is a zero-mean noise vector with the property \(<\varepsilon(t)\varepsilon^T(t')> = I_5\delta(t-t')\) as before, and \(A\) is the drift matrix of order \(3M + 2\), as shown below.

\[
A = \begin{pmatrix}
\kappa(1+i\phi) & 0 & -g_1 & 0 & 0 & \ldots & -g_M & 0 & 0 \\
0 & \kappa(1-i\phi) & 0 & -g_1 & 0 & 0 & -g_M & 0 \\
-g_1D_{01} & 0 & \gamma(1+i\Delta) & 0 & -g_1\alpha_0 & \ldots & 0 & 0 & 0 \\
0 & -g_1D_{01} & 0 & \gamma(1-i\Delta) & -g_1\alpha_0 & \ldots & 0 & 0 & 0 \\
2g_1v_{01}^* & 2g_1v_{01} & 2g_1\alpha_0^* & 2g_1\alpha_0 & \gamma_{ll} & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-g_MD_{0M} & 0 & 0 & 0 & 0 & \ldots & \gamma(1+i\Delta) & 0 & -g_M\alpha_0 \\
0 & -g_MD_{0M} & 0 & 0 & 0 & 0 & \gamma(1-i\Delta) & -g_M\alpha_0^* & \gamma_{ll} \\
2g_Mv_{0M}^* & 2g_Mv_{0M} & 0 & 0 & 0 & \ldots & 2g_M\alpha_0^* & 2g_M\alpha_0 & \gamma_{ll}
\end{pmatrix}
\]

The correlations of the stochastic noise terms \(\Gamma(t)\) are given by (Reid 1988)

\[
B <\varepsilon(t)\varepsilon^T(t')> B^T = D\delta(t-t')
\]

Thus the diffusion matrix \(D\) becomes
\[ D = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & d_{V_1V_1} & d_{V_1V_1} & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & d_{V_1V_1} & d_{V_1V_1}^* & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & d_{D_1D_1} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & d_{V_MV_M} & d_{V_MV_M}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & d_{V_MV_M}^* & d_{V_MV_M}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & d_{D_MD_M} \end{pmatrix} \]

\[ d_{V_jV_j} = 2g_j \alpha_0 v_{0j} \]  \hfill (2.31b)

\[ d_{V_jV_j}^* = (D_{0j} + N_j) \gamma_p \]

\[ d_{D_jD_j} = 2\gamma_p (D_{0j} + N_j) - 4g_j (v_{0j}^* \alpha_0 + v_{0j} \alpha_0^*) \]

Independent atoms are uncorrelated, hence (direct) noise correlations between different sections of the cavity are zero.

### 2.4 Squeezing spectrum for a spatially varying field mode

A full analytical solution to the linearised equation in the fluctuations (Eq. 2.30) may be obtained by transforming the equation to Fourier space, in the manner of Reid (1988). For a stable stationary state it has been shown that

\[ 0 = (-\Delta + i\omega I) \delta \alpha (\omega) + B \varepsilon (\omega) \]  \hfill (Equation 26(a) of Reid 1988).  \hfill (2.32)

where we have defined

\[ \delta \alpha (\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \delta \alpha e^{i\omega t} dt , \]

\[ \varepsilon (\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \varepsilon (t) e^{i\omega t} dt \]

with

\[ \delta \alpha^T (\omega) = (\delta \alpha (\omega), \delta \alpha^T (\omega), \delta v_1 (\omega), \delta v_1^T (\omega), \delta D_1 (\omega), \delta v_M (\omega), \delta v_M^T (\omega), \delta D_M (\omega)) \]

and
In Fourier space the nonzero correlations of the quantum fluctuation terms are

\[ \mathbf{B}^H(\omega)e^{T(\omega')}\mathbf{B}^T = \mathbf{B}\mathbf{B}^T\delta(\omega + \omega') = \mathbf{D}\delta(\omega + \omega') \]  

(2.33)

Each set of equations in the atomic variables \( \delta v_j, \delta v_j^+ \) and \( \delta D_j \) for a single cavity block \( j \) is independent and has the same form, since no noise correlations or direct coupling terms exist between different blocks of the cavity. Each set of atomic equations can therefore be solved in just the same way as in the plane-wave case. The solutions giving the polarization \( \delta v_j \) in terms of the field variables \( \delta \alpha \) for the case of a spatially varying field mode are a straightforward extension of Equations (28), Reid (1988). The intracavity photon number \( I \) and the atomic cooperativity \( C \) of Equations (28), Reid (1988) are replaced by \( I_j \) and \( C_j \) respectively, as defined in Eqs. (2.27b).

Once \( \delta D(\omega) \) is eliminated we have

\[
\begin{align*}
g_j \delta v_j(\omega) &= -\kappa \gamma(\omega)\delta \alpha(\omega) - \kappa b_j(\omega)\delta \alpha^+(\omega) + F_{v_j}(\omega) \\
g_j \delta v_j^+(\omega) &= -\kappa \gamma^*(\omega)\delta \alpha^+(\omega) - \kappa b_j^*(\omega)\delta \alpha(\omega) + F_{v_j^+}(\omega)
\end{align*}
\]  

(2.34a)

where \( \bar{\omega} = \omega/\gamma_j, \Delta(\bar{\omega}) = \Delta - \bar{\omega}, \)

\[
\begin{align*}
b_j(\bar{\omega}) &= \frac{-2C_j}{\Pi_j(0)} \left[ \frac{I_j}{2[1 + i\Delta(\bar{\omega})]\Pi_j(\bar{\omega})} \left[ \frac{1}{1 + i\Delta} + \frac{1}{1 - i\Delta} \right] \right], \\
\Pi_j(\bar{\omega}) &= 1 - \frac{i\bar{\omega}}{2\bar{f}} + \frac{I_j}{2[1 + i\Delta(\bar{\omega})]} + \frac{I_j}{2[1 - i\Delta(-\bar{\omega})]}, \\
\gamma_j(\bar{\omega}) &= \gamma_{JR}(\bar{\omega}) + \gamma_{JL}(\bar{\omega}) = \frac{2C_j}{\Pi_j(0)} \left[ \frac{1}{1 + i\Delta(\bar{\omega})} \left[ 1 - \frac{I_j}{2(1 - i\Delta)\Pi_j(\bar{\omega})} - \frac{I_j}{2[1 + i\Delta(\bar{\omega})]\Pi_j(\bar{\omega})} \right] \right],
\end{align*}
\]

and \( \bar{f} = \gamma_j/(2\gamma_j) \), such that \( \bar{f} = 1 \) for perfect radiative damping.

The quantum noise terms are
\[ F_{v_j}(\omega) = \frac{g_j \Gamma_{v_j}(\omega)}{\gamma_j[1 + i\Delta(\omega)]} \left[ 1 - \frac{I_j}{2\Pi_j(\omega)[1 + i\Delta(\omega)]} \right] + \frac{g_j^2 \alpha_0 \Gamma_{D_j}(\omega)}{\gamma_j[1 + i\Delta(\omega)]} \left[ 1 - \frac{2g_j^3 \alpha_0^2 \Gamma_{v_j}(\omega)}{\gamma_j[1 + i\Delta(\omega)][1 - i\Delta(-\omega)]\Pi_j(\omega)} \right] \]  

\[ F_{v_j}(\omega) = \frac{g_j \Gamma_{v_j}(\omega)}{\gamma_j[1 - i\Delta(-\omega)]} \left[ 1 - \frac{I_j}{2\Pi_j(\omega)[-1 - i\Delta(-\omega)]} \right] + \frac{g_j^2 \alpha_0 \Gamma_{D_j}(\omega)}{\gamma_j[1 - i\Delta(-\omega)]} \left[ 1 - \frac{2g_j^3 \alpha_0^2 \Gamma_{v_j}(\omega)}{\gamma_j[1 + i\Delta(\omega)][1 - i\Delta(-\omega)]\Pi_j(\omega)} \right] \]  

and the correlations of these stochastic terms are

\[ \left< F_{v_j}(\omega)F_{v_j}(\omega') \right> = \kappa d_j(\omega) \delta(\omega + \omega') \]

\[ \left< F_{v_j}(\omega)F_{v_j}(\omega') \right> = \kappa d_j^*(\omega) \delta(\omega + \omega') \]  

\[ \left< F_{v_j}(\omega)F_{v_j}(\omega') \right> = \kappa \Lambda_j(\omega) \delta(\omega + \omega') \]  

where

\[ d_j(\omega) = -\frac{2C_j I_j f}{\Pi_j(0)[1 + i\Delta(\omega)][1 + i\Delta(-\omega)][1 + i\Delta(\omega)][1 + i\Delta(-\omega)]} \left[ 1 - \frac{I_j}{2\Pi_j(\omega)[1 + i\Delta(\omega)]} \right] \left[ 1 - \frac{I_j}{2\Pi_j(-\omega)[1 + i\Delta(-\omega)]} \right] \]

\[ -\frac{2C_j I_j^2(1 - f)}{\Pi_j(0)[1 + \Delta^2][1 + i\Delta(\omega)][1 + i\Delta(-\omega)][1 - i\Delta(\omega)][1 - i\Delta(-\omega)]\Pi_j(-\omega)} \left[ 1 - \frac{I_j}{2\Pi_j(\omega)[1 + i\Delta(\omega)]} \right] \]

\[ -\frac{2C_j I_j^2(1 - f)}{\Pi_j(0)[1 + \Delta^2][1 + i\Delta(-\omega)][1 + i\Delta(\omega)][1 - i\Delta(-\omega)][1 - i\Delta(\omega)]\Pi_j(\omega)} \left[ 1 - \frac{I_j}{2\Pi_j(-\omega)[1 + i\Delta(-\omega)]} \right] \]
\[ \frac{2C_j I_j^2}{\Pi_j(0)(1 + \Delta^2)\Pi_j(\omega)^2[1 + i\Delta(\omega)][1 + i\Delta(-\omega)]} \]

\[ -\frac{2C_j I_j^2}{4\Pi_j(0)(1 - i\Delta)\Pi_j(\omega)^2[1 + i\Delta(\omega)][1 + i\Delta(-\omega)]} \]

\[ \Lambda_j(\omega) = \frac{2C_j(1 - f) \left[ 1 - \frac{I_j}{2\Pi_j(\omega)[1 + i\Delta(\omega)]} \right]^2}{\Pi_j(0)(1 + \Delta^2)\Pi_j(\omega)^2[1 + i\Delta(\omega)]} + \frac{2C_j(1 - f) I_j^2}{4\Pi_j(0)(1 + \Delta^2)\Pi_j(\omega)^2[1 + i\Delta(\omega)][1 + i\Delta(-\omega)]} \]

Note that the second term of the equation for \( \gamma(\omega) \) is correct as written above. In the corresponding equation of Eqs. (28a), Reid (1988), the term \([1 + i\Delta(\omega)]^{-1}\) should be substituted for \([1 + i\Delta(\omega)]\) (Reid 1989).

According to the matrix equation (2.32) in Fourier space we have

\[ -\kappa (1 + i\phi(\omega)) \delta \alpha(\omega) + \sum_{j=1}^{M} g_j \delta \nu_j(\omega) + \Gamma(\omega) = 0 \]

(2.35)

\[ -\kappa (1 - i\phi(-\omega)) \delta \alpha^t(\omega) + \sum_{j=1}^{M} g_j \delta \nu_j^t(\omega) + \Gamma^t(\omega) = 0 \]

where \( \phi(\omega) = \phi - \omega/\kappa \).

Substituting the solutions (2.34a) for the atomic variables \( \delta \nu_j \) into the above we find that the final expressions for the field can be put into the form

\[ 0 = (-\Delta'(\omega) + i\omega I) \delta \alpha(\omega) + [B(\omega) e(\omega)]_R \]

where
\[ A'(\omega) = \kappa \begin{pmatrix} a'(\omega) & \sum_{j=1}^{M} b_j(\omega) \\ \sum_{j=1}^{M} b_j(-\omega) & a^*(-\omega) \end{pmatrix}, \quad a'(\omega) = 1 + \i \phi + \sum_{j=1}^{M} \gamma_j(\omega) \] (2.36)

\[ \delta \alpha_R = \begin{pmatrix} \delta \alpha(\omega) \\ \delta \alpha^*(\omega) \end{pmatrix}, \quad [B(\omega)\epsilon(\omega)]_R = \begin{pmatrix} \sum_{j=1}^{M} F_{\nu_j}(\omega) + \Gamma_{\alpha}(\omega) \\ \sum_{j=1}^{M} F_{\nu_j}^*(\omega) + \Gamma_{\alpha^*}(\omega) \end{pmatrix} \]

(The superscript "*" indicates that a function given by Reid (1988) has been modified to a summation over all regions \( j = 1 \ldots M \) of the optical cavity.) We thus have a 2x2 matrix equation, which may be solved without difficulty.

The frequency-dependent diffusion matrix \( D'(\omega) \) is given by the expression

\[ B(\omega)\epsilon(\omega)\epsilon(\omega')B^T(\omega') = B(\omega)B^T(\omega')\delta(\omega + \omega') = D(\omega)\delta(\omega + \omega') \] (2.37a)

(Eq.(30b) of Reid 1988), so that from Eqs. (2.34c) we have

\[ D'(\omega) = \kappa \begin{pmatrix} \sum_{j=1}^{M} d_j(\omega) & \sum_{j=1}^{M} \Lambda_j(\omega) \\ \sum_{j=1}^{M} \Lambda_j(\omega) & \sum_{j=1}^{M} d_j^*(\omega) \end{pmatrix} \]

Expressions for the squeezing spectrum external to the cavity \( V(X_q, w) \) and the spectral matrix \( S(\omega) \) were given in Eqs (2.19) and (2.20) respectively. It can be shown (Reid 1988) that the optimum squeezing over all local oscillator phase \( \theta \) is

\[ V_{opt}(X_\theta, \omega) = 1 + 2\kappa[S_{11}(\omega) + S_{12}(-\omega) - 2|S_{11}(\omega)|] \] (2.38)

where the optimal phase is given by

\[ \cos \theta_{opt} = -\frac{\text{Re}[S_{11}(\omega)]}{|S_{11}(\omega)|}, \quad \sin \theta_{opt} = -\frac{\text{Im}[S_{11}(\omega)]}{|S_{11}(\omega)|} \] (2.39)

\( S_{11}(\omega) = [S_{22}(\omega)]^*, \quad S_{12}(\omega) = S_{21}(-\omega) \). The normalised solution for the spectral matrix is (Reid 1988),
An analytic solution for the system we are considering may be obtained easily using this expression and the reduced frequency-dependent 2x2 drift and diffusion matrices of Equations (2.36) and (2.37). Taking the inverses of the matrices and performing the matrix multiplication we find that

\[ \kappa S(\omega) = \left[ \frac{A(\omega)}{\kappa} - i\omega \frac{1}{\kappa} \right]^{-1} D(\omega) \left[ \frac{A^T(-\omega)}{\kappa} + i\omega \frac{1}{\kappa} \right]^{-1} \]  

(2.40)

where all summations run over \( j = 1 \ldots M \) cavity sections, and \( S_{22}(\omega) = S_{11}^*(\omega), S_{12}(\omega) = S_{21}(-\omega) \). The squeezing spectra may be calculated from the matrix elements using Equation (2.38).

The solution above holds for a cavity mode of arbitrary spatial structure and is a direct extension of the plane-wave solution. The intermediate functions \( \gamma(\omega), \Lambda(\omega), d(\omega), b(\omega) \) in the plane-wave model have been interpreted as governing the effect of the internal field on the atoms (Reid et al. 1986, Reid 1988, Reid and Walls 1986). They describe the processes of absorption, dispersion, fluorescence, nonlinear coupling, and coupling of quantum fluctuations. In the spatially varying model they are replaced by a new set of atomic functions, the summations \( \sum \gamma_j(\omega), \sum \Lambda_j(\omega), \sum d_j(\omega), \sum b_j(\omega) \) for \( j = 1 \) to \( M \).
2.5 Results

We can calculate the spectrum of squeezing for a cavity mode of given spatial structure using the expression for the squeezing spectrum Eq. (2.38) and the definitions (2.28b), (2.28c) for the normalised intracavity intensity $I_j$ and the atomic cooperativity $C_j$ for each interferometer element (Drummond 1981, Xiao et al. 1987a, 1987b). The calculation may be performed analytically as in Xiao et al. (1987a) by taking the limit $M \rightarrow \infty$, $\Delta V_j \rightarrow 0$ and thereby changing all summations to integrations, or numerically, by directly summing over all cavity sections. Both approaches have been used in this work.

We consider a range of $q$ values, where $q$ is the ratio of the transverse atomic decay rate to the cavity decay rate, $q = \gamma_x / \kappa$. A travelling wave with a constant-intensity transverse profile will be called a plane wave for the sake of brevity. Similarly a travelling wave with a Gaussian transverse profile will be termed a Gaussian mode.

A. Gaussian mode in a ring cavity

For a travelling wave with Gaussian transverse profile, we divide a mode with Gaussian beam radius $w$ into $M$ cylindrical shells along the cavity axis (as shown in Fig. 2.5-1). Each shell has radius $r_j$ and thickness $\Delta r_j$ and contains $N_j$ atoms. From Xiao et al. (1987a) we have

$$|U(r_j)| = \left( \frac{2}{V} \right)^{1/2} \exp \left( -\frac{r_j^2}{w^2} \right), \quad V_{\text{eff}} = sV, \quad V = \text{cavity volume} = \pi w^2 L \quad (2.42)$$

In our numerical work $C_j$ and $I_j$ are computed for volume elements taken over the sample length in the $z$-direction, and out to twice the Gaussian beam radius $w$ in the radial direction (where we assume that the change in the mode radius over the interaction length is negligible). Substitution of these values into Eq. (2.41) yields the squeezing spectrum for a Gaussian mode.

Analytical expressions for the squeezing generated in a Gaussian mode have been derived using the methods of Xiao et al. (1987a, 1987b) and are given in the Appendix. These hold for the general cavity, without adiabatic elimination of atomic or cavity variables, for the case of pure radiative damping. To obtain these expressions we divide the cavity mode into cylindrical sections and take the statistical limits $M \rightarrow \infty$, $\Delta V_j \rightarrow 0$ such that the integral

$$\int \int \int \frac{1}{V} d^3 r$$

may be substituted for the summation
Figure 2.5-1: Schematic diagram of a Gaussian TEM$_{00}$ mode divided into cylindrical sections.
The new drift and diffusion matrix coefficients are shown in full in the Appendix. Although all types of spatially nonuniform mode can be treated using the numerical method, the analytical solution for the Gaussian mode in a ring cavity is valuable as a check on the numerical results. The results of the two treatments typically agree to better than 3% if 100 or more sections are taken over the mode in the numerical calculations.

Figures 2.5-2 to 2.5-4 show optimised squeezing plotted against the ratio of damping rates \( q \). The spectral variance \( V(X_\phi, \omega) \) is zero for perfect squeezing and unity for a coherent state. Optimum squeezing was found by numerically minimising \( V(X_\phi, \omega) \) over values of the normalised intracavity field \( I \), atomic and cavity detunings \( \Delta, \phi \), and spectral output frequency \( \omega \) for fixed \( q \) and \( C \). The geometrical simplex FORTRAN algorithm 'AMOEBA' (Press 1986) was chosen for the numerical optimisation procedure, being straightforward to implement. The numerical search was effectively bounded by the choice of the five four-dimensional initial values required for the algorithm. Negative values of \( I \) were explicitly disallowed. It is not possible to guarantee that the minimum values returned by the numerical algorithm are global rather than local minima of the function \( V(X_\phi, \omega) \). Nonetheless, through taking initial search values which covered a large parameter range, and comparing the value of \( V \) obtained for each \( q \) with preceding and following values for consistency, we developed high confidence in our results.

Figure 2.5-2 shows optimised squeezing against the ratio of polarization and cavity damping rates \( q \) in the regime of similar decay rates. Atomic cooperativity \( C \) was kept constant at a value of \( C=50 \). Previous work (Reid and Walls 1986, Xiao et al. 1987b, Reid 1988) has shown that in general increases in cooperativity allow improved squeezing for the high-Q, low-Q and intermediate regimes if other parameters are varied freely. The value \( C=50 \) was chosen to be large enough to show good squeezing, yet low enough to allow comparisons with experiment (Raizen et al. 1987, Orozco et al. 1987).

Figure 2.5-2 demonstrates that the optimum squeezing is decreased when the Gaussian mode structure is included, with respect to the plane-wave value for the same cooperativity \( C \). This is consistent with previous results for the high-Q limit (Xiao et al. 1987a, 1987b).

The plot of optimised squeezing for a Gaussian mode in a ring interferometer is divided into two distinct regions. In the flat section of the plot, the region from \( q = \gamma_\perp / \kappa \approx 0.3 \) to \( q = 1.0 \), the optimised squeezing is clearly independent of the ratio of damping rates \( q \). This is the regime approaching the high-Q limit. The squeezing in this regime is maximal for \( \omega = 0 \) (Savage and Walls 1986). In Figure 2.5-2 the values of the noise reduction for a plane wave and a Gaussian travelling wave for \( C=50 \) are in agreement with the results of Xiao et al. (1987b, Fig. 1) for the good-cavity limit. Squeezing generated from the splitting in the normal-mode structure of the coupled cavity-atom system certainly exists in the region \( \kappa = \gamma_\perp \) (Reid 1988), but the squeezing generated at the pump frequency via the dispersive optical bistability nonlinearity is large enough to predominate.
Figure 2.5-2: Optimised squeezing $V(X_\phi, \omega)$ against ratio of damping rates $q = \gamma_v / \kappa$ for (i) plane wave; (ii) Gaussian transverse mode in a ring cavity. Trace (ii) obtained using analytical solution. For both traces $C=50$, and $\Delta, I, \phi, \omega$ were varied for fixed $q$.

Figure 2.5-3: Optimised squeezing $V(X_\phi, \omega)$ against ratio of damping rates $q = \gamma_v / \kappa$ for (i) plane wave; (ii) standing wave. Trace (ii) obtained using numerical solution, $M_z=100$. For both traces $C=50$, and $\Delta, I, \phi, \omega$ were varied for fixed $q$. 
Figure 2.5-4: Optimised squeezing $V(X_q, \omega)$ against ratio of damping rates $q=\gamma_+/\kappa$ for (i) plane wave; (ii) standing wave with Gaussian transverse profile. Trace (ii) obtained using numerical solution, $M_x=10$, $M_y=20$. For both traces $C=50$, and $\Delta, I, \phi, \omega$ were varied for fixed $q$. 
In the region from \( q = 0.3 \) to low \( q \) values (\( q \rightarrow 0.001 \)) the value of the optimised squeezing is strongly dependent on the ratio of damping rates of the cavity mode and the atomic polarization, indicating that the dominant squeezing mechanism is the coupling of these two oscillators. An interesting new feature evident in this plot is that the effect of the mode structure also depends on \( q \gamma_j / \kappa \). The Gaussian mode structure has only a small impact upon the optimised squeezing for higher values of \( q \) in the vacuum-field Rabi splitting regime. The difference between the plane-wave and Gaussian mode results amounts to about one or two percentage points on a linear scale of squeezing. The degradation of optimised squeezing (with respect to the plane wave case) due to the radial mode structure increases as the cavity linewidth becomes large compared to the longitudinal atomic linewidth, from about 1% near \( q = 0.1 \) to 20% near \( q = 0.001 \). The best squeezing achievable for a Gaussian mode in a ring cavity for an atomic cooperativity of \( C = 50 \) is approximately 80% for \( \gamma_j / \kappa = 0.01 \). In the plane-wave case the optimised squeezing continues to improve with decreasing \( q \) past this point.

As described in Chapter 1, practical limitations on the length of an optical cavity indicate that the best way to achieve a decreased value of \( q \gamma_j / \kappa \) is to use a transition of smaller linewidth. The spatially varying model suggests that the influence of the Gaussian mode structure would tend to negate any improvements in squeezing for \( q \gamma_j / \kappa < 0.01 \). The mode structure therefore sets a limit on the degree of noise reduction below the quantum limit obtainable in this system for a given cooperativity \( C \).

### B. Plane wave in standing wave interferometer

To examine the effect of a standing-wave structure with plane-wave transverse profile we first note that each region of the cavity mode of length \( \lambda / 4 \) can be described by the mode function

\[
|U(r)| = \left( \frac{2}{V} \right)^{1/2} \cos \left( \frac{2\pi z}{\lambda} \right)
\]  

(2.43a)

Each \( \lambda / 4 \) region therefore makes the same contribution to the sum over atoms, and is divided into \( M \) small sections along the cavity axis. From Xiao et al. (1987a)

\[
V_{\text{eff}} = \frac{2sV}{3}
\]

(2.43b)

The effect of the standing wave structure upon squeezing is qualitatively the same as that of the Gaussian mode structure (as can be seen from Figure 2.5-3). The reduction in squeezing with respect to the plane-wave case is not so severe, however. For a large region from approximately \( q = 0.3 \) to \( q = 0.03 \) in the regime of vacuum-field Rabi splitting the effect is negligible. As in the Gaussian case the degradation of squeezing induced by the mode structure grows past \( q = 0.03 \), increasing to an effect of about 2% at \( q = 0.01 \) and 9% at \( q = 0.001 \). Maximum possible squeezing at \( C = 50 \) is about 82%. 
C. Gaussian mode in a standing wave interferometer

The mode function for a Gaussian mode in a Fabry-Perot cavity is given by

\[
|U(r_j)| = \left(\frac{4}{V}\right)^{1/2} \cos\left(\frac{2\pi z}{\lambda}\right) \exp\left(-\frac{r_j^2}{w^2}\right), \quad V_{\text{eff}} = \frac{2}{3} \pi L w^2
\]  

(2.44)

where \( w \) is the Gaussian beam radius.

The combined mode shape is responsible for a greater reduction of the optimal squeezing than the individual cases of a Gaussian mode and standing wave (shown for Figs. 2.5-2 and 2.5-3), but not more than that is suggested by simply adding the effects of the two. Fig. 2.5-4 shows optimised squeezing against \( q \) for the standing wave with a radial Gaussian structure. In general the qualitative characteristics of the graph are similar to those of the previous plots, but we also see the emergence of a flat region at the low \( q = \gamma_j/\kappa \) end. This corresponds to the low-Q or bad cavity regime. Squeezing in the low-Q cavity is generated by a strong coupling of photons produced at the Rabi sidepeaks by inelastic scattering processes (Reid et al. 1986, Reid 1988). The optimised squeezing in the low-Q region is better than the high-Q case but the deleterious effect of the mode structure is also greater.

The emergence of the low-Q limit as the regime of optimal squeezing, as the cavity linewidth becomes large with respect to the atomic linewidth is not in itself surprising. Ultimately as the decay rates of the two coupled oscillators (the cavity mode and atomic polarization) become dissimilar we must lose the vacuum-field Rabi oscillations giving rise to good squeezing in the intermediate region. The unexpected feature is that squeezing due to the low-Q cavity mechanism becomes optimal earlier (i.e. at higher values of the ratio \( \gamma_j/\kappa \)) for the spatially varying cavity mode than for the plane wave. We conclude that the mode structure is responsible for a rapid degradation of the squeezing with decreasing \( q \) once \( q = \gamma_j/\kappa \) becomes less than approximately 0.01, in the vacuum-field Rabi splitting regime.

Maximum achievable squeezing for the Gaussian mode in a standing wave interferometer at \( C=50 \) is limited to 70%. For \( q = 0.001 \) the plane-wave value for optimum squeezing is degraded by about 30%. Even at \( q=0.1 \), where the mode-induced degradation of squeezing is relatively small, we find a reduction from the plane-wave value, of about 9% for a spatially nonuniform mode of this type.

2.6 Comparison with experiment of previous workers

It is interesting to consider reported experimental data in the light of our work. In the experiment of Raizen et al. (1987) and Orozco et al. (1987), the cavity was a high-finesse Fabry-Perot interferometer supporting a single mode with a Gaussian transverse profile. Comparisons of observed squeezing and theoretical predictions from a plane-wave model modified by known loss factors revealed a discrepancy between theory and experiment.
The observed reduction below the quantum noise level was 30±4% for one cavity configuration. Upon including measured loss factors (Orozco et al. 1987) such as the quantum efficiency of the photodiode and mode-matching efficiency at the detector beam-splitter it was possible to infer the actual degree of squeezing, 53±9%. (The effect of the loss mechanisms may be accounted for in a phenomenological fashion. Orozco et al. 1987, Wu et al. 1987). Predicted squeezing from the plane-wave theory was 78%. The lack of agreement was attributed to the difficulty of optimising the squeezing over many experimental variables, and to the effect of the mode structure of the beam. The frequency of optimum squeezing agreed well with the (approximate) plane-wave predictions, however. The frequency \(\omega/2\pi = g\sqrt{N} - \sqrt{\kappa \gamma_0 C} = 265\pm30\) MHz was calculated from experimental parameters using Eq. (2.15c) and is in accord with the value \((\omega/2\pi)_{\text{exp}} = 280\) MHz. The experimental parameters of best squeezing are shown in Table 2.6-1, and compared with the predicted parameters of optimum squeezing from the plane-wave model.

We examine the effect of the longitudinal standing wave and transverse Gaussian mode structure by inserting the experimental parameters into the (numerical) model of squeezing in such a mode. Figure 2.6-1 shows the squeezing spectra predicted by the plane-wave and standing-wave-Gaussian models for the experimental parameters of Table 2.6-1, plotted against analysis frequency \(\omega\) normalised by cavity linewidth \(\kappa\). The results for the spatially varying mode display much more structure in the spectrum than the plane-wave curve. Towards higher frequencies \((\omega \to 2\kappa)\) the difference between the two spectra becomes negligible. The experimental results showing the frequency and magnitude of the optimal squeezing (corrected for losses) have also been added to the graph.

It is evident that agreement between the spatially varying model and the experimental data is rather poor as regards the frequency of optimum squeezing, but the theory which includes the mode structure gives better agreement for the magnitude of the optimum squeezing than does the plane-wave model. The spatially varying model predicts an actual degree of squeezing, without detection losses, of 46%. This result is consistent with the inferred experimental noise reduction of 53±9%.

We note that the normalised intracavity field was taken to be \(x = \alpha/(\sqrt{n_0}) = 25\pm5\). An uncertainty of 20% in the intracavity field leads to an uncertainty in the normalised intensity \(I\) of about 40%. The calculated squeezing is extremely sensitive to this parameter. The plots in Figure 2.6-2, where we have varied the parameter \(I = \alpha \alpha^*/n_0\) within experimental error bounds, provide some support for the spatially-varying model. This shows the squeezing spectra for the cases examined in the previous Figure, plotted for \(x = 20\), \((I = 400)\). \(\Delta, C\) and \(q\) are fixed at their previous values, while \(\phi\) is adjusted to optimise squeezing. We see that both the plane-wave and spatially varying models provide reasonable quantitative agreement with the experimental noise reduction below the quantum limit. Nevertheless the Gaussian mode- Standing-wave model is to be preferred since it gives better agreement with the experimental frequency and magnitude of optimum squeezing.

It has been demonstrated that the spatially varying model is consistent with results of the experiment performed by Orozco et al. within the experimental error. To make a significant distinction between the models the uncertainties need to be reduced, particularly for the intracavity field \(x\). It would also be interesting and useful to determine the noise reduction for a number of analysis frequencies with other parameters fixed; that is, determine the
Table 2.6-1. Comparison of optimum squeezing and corresponding parameters, for the plane wave theory and experimental data of Orozco et al. (1987).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Theoretical *(plane wave)</th>
<th>Experimentalb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic cooperativity C</td>
<td>52</td>
<td>52±8</td>
</tr>
<tr>
<td>Ratio of damping rates $\kappa/\gamma_l=1/(2q)$</td>
<td>13.5</td>
<td>13.5±0.5</td>
</tr>
<tr>
<td>Normalised atomic detuning $\Delta=(\omega_0-\omega_c)/\gamma_c$</td>
<td>-14.6</td>
<td>-14.6±0.2</td>
</tr>
<tr>
<td>Normalised intracavity field $x=(\alpha_0^*\alpha_0/n_0)^{1/2}$</td>
<td>16.4</td>
<td>25±5</td>
</tr>
<tr>
<td>Normalised cavity detuning $\phi=(\omega_0-\omega_c)/\kappa$</td>
<td>0.86</td>
<td>not given</td>
</tr>
<tr>
<td>Frequency of optimum squeezing</td>
<td>265±30MHzc</td>
<td>280MHz</td>
</tr>
<tr>
<td>Optimum squeezing (no detection losses)</td>
<td>78%</td>
<td>53±9%d</td>
</tr>
<tr>
<td>Optimum squeezing (with detection losses)</td>
<td>57%e</td>
<td>30±4%f</td>
</tr>
</tbody>
</table>

*a* Orozco et al. (1987). $C$ and $\kappa/\gamma_l$ fixed, $\Delta, \phi, x, \omega$ varied to optimize squeezing spectrum.  
*b* Orozco et al. (1987). Cavity configuration $b$, $C$, $\kappa/\gamma_l$ and $\Delta$ fixed, $\phi, x, \omega$ varied to optimize squeezing.  
*c* Orozco et al. (1987). Given by $(\omega_{opt}/2\pi)$, based on approximate prediction $\omega_{opt}=(g^2N)^{1/2}=\gamma_l(\mu C)^{1/2}$.  
*d* Inferred from observed squeezing value of $30\pm4\%$ using known loss factors, according to the methods of Orozco et al. (1987), Wu et al. (1987).  
*e* Obtained from theoretical optimum squeezing by including known loss factors.  
*f* Observed squeezing.
Figure 2.6-1: Comparison between plane-wave model, spatially-varying model, and experimental data. Squeezing spectrum $V(X, \omega)$ versus scaled offset frequency $\omega/\kappa$, parameters as in experiment of Orozco et al. (1987); $x=25, \Delta=-14.6, C=52, \mu=\kappa/\gamma=13.5$, $\phi$ optimised. Trace (i) plane wave. Trace (ii) Gaussian mode in standing-wave cavity, numerical solution, $M_r=50, M_z=50$. Point (iii) experimental data of Orozco et al. (1987); $V=0.47\pm0.09$, $\omega/\kappa=2.1$.

Figure 2.6-2: Comparison between plane-wave model, spatially-varying model, and experimental data, $I=(\alpha\alpha^+)/\hbar_0 = x^2$ varied. Squeezing spectrum $V(X, \omega)$ versus scaled offset frequency $\omega/\kappa$ for $x=20, \Delta=-14.6, C=52, \mu=\kappa/\gamma=13.5$, $\phi$ optimised. Trace (i) plane wave. Trace (ii) Gaussian mode in standing-wave cavity, $M_r=50, M_z=50$. Point (iii) experimental data of Orozco et al. (1987), $V=0.47\pm0.09$, $\omega/\kappa=2.1$. 
Note that the effect of the combined standing-wave and Gaussian mode structures is not necessarily as large as shown in Figures 2.6-1 and 2.6-2. In fact if the procedures of the experiment are imitated in the numerical optimisation for the Gaussian mode in a standing-wave cavity, that is, $C, q$ and $\Delta$ are fixed and $I, \phi$ and $\omega$ are varied freely, we obtain the results of Figure 2.6-3. In this figure we plot the spectrum of squeezing for a Gaussian mode in a standing-wave interferometer optimised as described above. The best squeezing for this mode structure is 73%, compared to 78% for the plane wave. Thus according to this model the optimum squeezing achievable in the experiment by Orozco et al. (1987) was not located. Another feature of interest is that the bandwidth of good squeezing narrows for the spatially nonuniform mode, relative to the plane wave.

2.7 Squeezing for a spatially varying field mode in a bad cavity

Reid (1988) has pointed out that the low-Q or bad cavity limit ($\kappa >> \gamma$) can easily be extracted from the equations for the general cavity. In the normalised solution for the spectral matrix (Equation 2.40) the term in $\omega / \kappa = i q \Omega / L$ will tend to zero as $q = \gamma / \kappa$ becomes very small. Thus we have

$$
\bar{S}(\omega) = \frac{1}{[A(\omega)]^T D(\omega) A(\omega)]^{-1}}
$$

from which the spectrum of squeezing $V(X_\omega, \omega)$ can be obtained.

Figure 2.7-1 is a graph of optimised squeezing for the low-Q limit against cooperativity $C$, for the plane-wave and Gaussian mode cases. Earlier writers (Carmichael 1986a, Reid et al. 1986) have shown that the bad cavity is capable of producing good squeezing at smaller values of $C$ than the high-Q cavity.

This has been attributed to the fact that the squeezing is generated at the Rabi sidepeaks (for high intensities approaching saturation), in the wings of the spectrum. Here it is possible to avoid the worst of the phase-insensitive fluorescence (Reid et al. 1986, Reid 1988). The good cavity produces best squeezing at the pump frequency (Reid 1988, Savage and Walls 1986) and its transmitted spectrum is located well within the central peak of the fluorescence for $\gamma_L >> \kappa$. Hence the trade-off between the driving intensity necessary to generate nonlinear coupling, and the squeezing-destructive spontaneous emission, is less favourable for squeezing in the high-Q cavity.

The discussion of the properties of the low-Q cavity, touched briefly above, suggests some aspects to consider with regard to the large degradation of squeezing in the low-Q limit for a spatially varying mode.

One obvious consequence of replacing the plane wave with a spatially varying mode is that for intensities approaching saturation in particular, there exists a distribution of
Figure 2.6-3: Optimised squeezing spectrum $V(X_\phi, \omega)$ versus scaled offset frequency $\omega/\kappa$; $C$, $\mu$, $\Delta$ as in experiment of Orozco et al. (1987). $C=52$, $\Delta=-14.6$, $\mu=\kappa/\gamma=13.5$; Trace (i) plane wave, $I=269$, $\phi=0.86$. Trace (ii) Gaussian mode in standing-wave cavity, $I=247$, $\phi=0.85$.

Figure 2.7-1: Optimised squeezing $V(X_\phi, \omega)$ versus cooperativity $C$ in low-Q limit, ($q=0$), for (i) plane wave (ii) Gaussian mode, analytical solution. For both traces $\Delta,I,\phi,\omega$ were varied for fixed $C$. 
dressed-atom level structures across the beam. On the edges of a Gaussian mode intensity will be insufficient to give rise to the dynamic Stark effect and the resonance fluorescence spectra for the atoms in this region will show the usual single-peaked Lorentzian at the driving frequency. In the centre of the mode the atoms are driven harder and the fluorescence spectra will display the characteristic three peaks of ac Stark splitting, a large peak at \( \omega = 0 \) and two smaller sidepeaks shifted by the Rabi frequency \( \Omega = \pm \gamma_s (\Delta^2 + 2\gamma)^{1/2} \) from the pump.

It could therefore be expected that all the features in the atomic functions \( \gamma(\theta), b(\theta), \Lambda(\theta), d(\theta) \) (mentioned in Section 2.3) sensitive to the Rabi frequency would be broadened and flattened in the new atomic functions for the spatially varying mode, which are summations \( \Sigma \gamma_j(\theta), \Sigma b_j(\theta), \ldots \), where \( j = 1 \) to \( M \), over the entire beam.

It is evident from the results presented in Figures 2.5-4 that the new atomic functions for a Gaussian mode are invariably worse in terms of squeezing generation than the original plane-wave functions. We suggest that the broadening of the atomic fluorescence, such that it extends over spectral regions previously unaffected by the spontaneous emission noise, may be one factor contributing to this effect.

### 2.8 Summary

The general quantum theory of the interaction between \( N \) two-level atoms and a single cavity mode, given by Reid (1988), has been extended to apply to the case of an arbitrary spatially varying field mode. Calculations were carried out for the cases of a Gaussian mode in a ring cavity, a plane wave in a standing-wave cavity and a Gaussian mode in a standing-wave cavity. In all these cases the best squeezing is smaller at a given value of the atomic cooperativity \( C \) than predicted in the plane-wave case. The reduction in optimum squeezing is small for the standing wave, larger for the Gaussian mode in a ring interferometer, and largest for the combination of the two. A similar decrease in squeezing was found in the case of the high-Q cavity in previous work (Drummond 1981).

A new feature of the degrading effect of a spatially varying field mode on the optimum squeezing in the regime of comparable cavity and atomic relaxation rates is its dependence on the ratio of relaxation rates \( q \). For a large region of ratios of the cavity and atomic damping rates in the vacuum-field Rabi splitting regime the effect of the mode structure on the squeezing is quite small. As the low-Q limit is approached, the degradation of squeezing with respect to the plane wave becomes markedly greater for all types of spatial variation studied.

Good agreement was found between the results of the model for a Gaussian mode in a standing-wave interferometer and the experimental results of Orozco et al. (1987) once one experimental parameter was permitted to vary within its error bounds. Our work has shown that in the presence of a spatially varying cavity mode there exists a well-defined value of \( q = \gamma / \kappa \) (for a given atomic cooperativity) which gives rise to optimal squeezing. This has important implications for further experiments in the short-cavity system. Squeezing is not invariably improved by causing the atomic linewidth to become much
smaller than the linewidth of the cavity, as the plane-wave model would suggest.

The optimal squeezing possible in the low-Q limit was shown to be much better than the squeezing in the good cavity limit for the same atomic cooperativity $C$, but the effect of a Gaussian mode structure on squeezing was more damaging in the bad cavity. Overall the best squeezing obtainable for a system of two-level atoms interacting with a single cavity mode was found in the regime of comparable cavity and atomic relaxation rates, where the squeezing mechanism is the vacuum-field Rabi splitting. This holds true even with the inclusion of a spatially varying field mode.
Chapter 3: Identification of regimes of optimum squeezing

3.1 Overview

The general quantum optical theory for the interaction of two-level atoms with a single mode light field in a cavity has been developed in a number of papers (Castelli et al. 1988, Reid 1988, Hope et al. 1990) and was described in the previous chapter. No adiabatic elimination of atom or field variables was performed. Squeezing spectra can therefore be calculated in both the good and bad cavity limits, as well as in the intermediate regime in which the atomic and cavity decay rates are on the same order, for a plane wave or a spatially-varying field mode. The model introduces six independent parameters, now shown together in full for convenient reference, in terms of quantities that can be calculated from tabulated values, or measured experimentally;

\[
\begin{align*}
\Delta &= (\omega_A - \omega_L) / \gamma_L ; & \phi &= (\omega_C - \omega_L) / \kappa ; \\
I &= |\alpha|^2 / n_0 ; & q &= \gamma_L / \kappa ; \\
C &= \alpha_0 F / (2\pi) ; & \omega &= \ldots \\
\end{align*}
\]

where \(\omega_L, \omega_A\) and \(\omega_C\) are the frequencies of the laser, the atomic transition and the cavity mode respectively. The decay rate of the atomic polarization is given by \(\gamma_L\). \(\kappa\) denotes the cavity decay rate, \(\Delta\) and \(\phi\) are the (dimensionless) atom-laser and cavity-laser detunings, respectively; \(I\) is the intracavity photon number normalised by the saturation photon number on resonance \(n_0\); \(C\), the atomic cooperativity, is determined by the optical depth on resonance \((\alpha_0\ell)\) and the cavity finesse; and finally \(\omega\) denotes the spectral output (detection) frequency.

The theory presented in Chapter 2 is a detailed and complete microscopic quantum model of the generation of squeezing in optical bistability for the case of the general cavity. The parameter space described by the theory is very large. This makes it difficult to devise simple qualitative explanations, which possess some predictive power, for the parameters of optimum squeezing. The aim of this chapter is to characterise the regimes of optimum squeezing as determined from numerical optimisation of the plane-wave ring cavity theory of Reid and from the spatially-varying model presented in Chapter 2 of this work; and also to draw together, and extend, the interpretive schemes of previous workers.

There are some straightforward ideas about squeezing which we employ in the attempt to construct physical explanations for squeezing in a cavity-atom system.

(i) Coherent states are generated in ideal stimulated one-photon processes, whereas squeezed states are generated in ideal stimulated two-photon processes for two photons in the same mode (Yuen 1976)

(ii) Squeezing in resonant and near-resonant media is significantly degraded by atomic loss processes (spontaneous emission and absorption), such that best squeezing will be observed in a spectral region that is distanced from the main fluorescence peaks (Reid et al. 1986, Reid and Walls 1985, Reid and Walls 1986).
(iii) For atomic squeezing in a cavity in the high-Q limit \((\gamma L >> \kappa)\) the dynamics of the atomic variables cannot be seen via the cavity transmission, and squeezing can be seen only at zero frequency. In the low-Q limit \((\kappa >> \gamma L)\), the atomic dynamics can be seen directly in the transmitted light and squeezing is broadband (Carmichael 1986a, Savage and Walls 1986).

In summary, to obtain substantial broadband squeezing in the cavity-atom system, we require a process that produces two correlated photons at a frequency away from the frequency of the pump laser, (and hence outside the bandwidth of the associated elastic scattering peaks), where \(\kappa >> \gamma L\).

Although the microscopic quantum theories and computations of Carmichael (1986a), Orozco et al. (1987), Raizen et al. (1987), Reid (1988), Castelli et al. (1988) and Raizen (1989) are in agreement, the interpretations they give for the mechanisms for squeezing and the prescriptions they give for squeezing in a cavity-atom system differ considerably. This can be attributed to the complexity of the system of optical bistability and also to the various methods used to examine and interpret it.

Collett and Walls (1985) used a macroscopic polarizability model for their early proposal for the generation of squeezed states of light in dispersive bistability. They predicted perfect squeezing at the switching point of bistability for the pump laser frequency \(\omega = 0\); based on the assumption that small fluctuations in one quadrature (phase) could lead to large changes of the other quadrature (amplitude) and that consequently the correlation between the quadratures is strongest at this point. Theirs was a macroscopic analysis, which did not include the crucial atomic loss processes of absorption and fluorescence.

Orozco et al. (1987) and Raizen et al. (1987) make no reference to the dynamics of bistability. In the cavity-atom system with \(\gamma L \sim \kappa\), good squeezing is predicted along the low transmission branch of optical bistability at detection frequencies proportional to \(g\sqrt{N}\), the vacuum-field Rabi splitting. There is an exchange of excitation between the atomic polarization and the cavity mode at this frequency. Orozco (1987) states that the lower branch is where atoms behave collectively, giving rise to an oscillatory exchange of excitation, and in consequence, squeezing. Castelli et al. (1988) state that squeezing on upper branch should emerge centered around the generalised Rabi frequency. Castelli et al. (1988) optimised over variables but did not present a consistent account of mechanisms leading to generation of large squeezing. Reid (1988) presented dressed-state diagrams for indicating mechanisms by which pairs of correlated photons might be generated but did not conduct numerical searches for optimum squeezing.

### 3.2 Numerical investigation of the parameter space

In Table 3.2-1 the parameters of optimum squeezing are compared for two regimes, using the plane-wave ring-cavity theory, with \(C = 50\), and with \(q\) varied from 0.01 to 0.1. For each value of \(q\) the spectral variance \(V\) was numerically minimised over \(I, \Delta, \phi\) and \(\omega/\kappa\) and the values of these parameters leading to the generation of best squeezing are shown. Table 3.2-1 presents essentially the same data as examined in Section 2.5 of
Table 3.2-1: Comparison of two regimes of optimum squeezing for the plane-wave ring-cavity model.

<table>
<thead>
<tr>
<th>q</th>
<th>g\sqrt{N} regime</th>
<th>Upper branch regime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>Δ</td>
</tr>
<tr>
<td>0.01</td>
<td>2590</td>
<td>37.9</td>
</tr>
<tr>
<td>0.02</td>
<td>806</td>
<td>23.4</td>
</tr>
<tr>
<td>0.03</td>
<td>410</td>
<td>18.0</td>
</tr>
<tr>
<td>0.04</td>
<td>255</td>
<td>15.0</td>
</tr>
<tr>
<td>0.05</td>
<td>175</td>
<td>13.0</td>
</tr>
<tr>
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<tr>
<td>0.07</td>
<td>100</td>
<td>10.5</td>
</tr>
<tr>
<td>0.08</td>
<td>80.2</td>
<td>9.6</td>
</tr>
<tr>
<td>0.09</td>
<td>66.0</td>
<td>8.9</td>
</tr>
<tr>
<td>0.10</td>
<td>55.8</td>
<td>8.4</td>
</tr>
</tbody>
</table>
Chapter 2, except that we now display the parameters of optimum squeezing.

The solution giving best squeezing is the one set out in the section of the table headed "gN regime". This section corresponds to the regime accessed experimentally by Orozco et al. (1987) and Raizen et al. (1987). The frequency of optimum squeezing \( \omega_{opt} \) is an important parameter because it is directly linked to the physical process which generates the pair of strongly-correlated photons. The optimal values of \( \omega/\kappa \) shown in Table 3.2.1 for the "gN regime" are close to the predicted values of gN, particularly for \( q<0.05 \). From Eq. 2.15c for pure radiative broadening it can be calculated that

\[
\frac{\omega}{\kappa} = \left( \frac{\kappa \gamma_N C}{2} \right)^{1/2} = (2qC)^{1/2} = (100q)^{1/2}, \text{ for } C = 50
\]

indicating that the 'coupled-mode' picture presented by these workers holds well for this regime.

Other workers have shown (Castelli et al. 1988, Raizen 1989), and the investigations of the present work confirm, that some rules of thumb for the parameters of optimum squeezing can be extracted from multi-dimensional numerical optimisation.

The first rule of thumb (Orozco et al. 1987, Castelli et al. 1988, Xiao et al. 1987b) is that for an increase in C the achievable squeezing also increases, provided all other variables are permitted to vary. The improvement in squeezing is roughly logarithmic so that an order of magnitude increase in the cooperativity is required for any substantial increase in squeezing. Fig. 3.2-1 corresponds to Fig. 2 from Orozco et al. (1987). It demonstrates the effect of the cooperativity upon squeezing and also shows that the results of Orozco et al. (1987) are duplicated by my programs.

The effect of the ratio of decay rates \( q \) is also shown in this figure. Note that \( q = \gamma / \kappa \) is equal to \( 1/(2\mu) \), where \( \mu = \kappa / \gamma \) is the equivalent variable used by Orozco et al. (1987), Raizen et al. (1987) and Castelli et al. (1988). This work uses 'q' for consistency with the analysis of Reid (1988). In general it is clear that decreasing \( q \), (increasing the cavity decay rate relative to the atomic decay rate) leads to better squeezing, as discussed in previous chapters. Points (squares) are marked on the curves \( \mu=5 \) and \( \mu=13.5 \) (equivalent to \( q=0.1 \), \( q=0.037 \) ) which correspond to the experimental configurations of the Austin group. The experimental configuration of this work corresponds most closely to the curve \( \mu=5 \) \( (q=0.08) \) for \( C=50-70 \). The relevant point is marked with a circle.

Numerical studies also revealed that the parameters \( I \) and \( \Delta \) are strongly associated in a proportional fashion. The situation where intracavity field \( (x = \sqrt{I}) \sim \Delta \) is very favourable for squeezing (Orozco et al. 1987, Castelli et al. 1988). Orozco et al. suggest that the field inside the cavity must be increased to this level to provide enough excitation for atomic processes of a four-wave mixing type to occur.

The appropriate cavity detuning \( \phi \) is generally located within a cavity linewidth of the cavity transmission peak. In dispersive bistability the peak of the cavity transmission is shifted some distance away from the empty cavity peak, defined as \( \phi=0 \), because the refractive index of the medium has been modified by the intracavity intensity. If \( I \sim \Delta^2 \) and \( \Delta^2 \gg 1 \) then it is clear from the state equation of optical bistability reproduced below from Eq. (2.15a),
Figure 3.2-1: Spectral variance $V$ versus atomic cooperativity $C$ for fixed $\mu = 1/(2q)$ as in Fig. 2 of Orozco et al. (1987). Values of atomic and cavity detunings, intracavity intensity and analysis frequency were chosen at each point to optimize squeezing. Squares; $C$, $\mu$ for experiment of Orozco et al.. Circle; $C$, $\mu$ for experiment of present work.
that $\phi_{opt} \sim (C / \Delta)$ for peak transmission. Figure 3.2-2 illustrates the shift and distortion of the cavity transmission function in this case.

It is also clear that in the $g\sqrt{N}$ regime the atom and cavity detunings are required to have opposite signs for the generation of squeezing ($\Delta \phi < 0$). If instead we set the condition $\Delta \phi > 0$ in the numerical optimisation, an alternative regime of squeezing was located in which the system was highly nonlinear but not bistable, generating good squeezing when the cavity was in a high transmission state. The frequency of optimum squeezing in this instance was not given by $g\sqrt{N}=(\kappa \gamma C)^{1/2}$, but took a significantly smaller value.

In dispersive bistability the peak transmission of the cavity is shifted by an amount which is proportional to and has the same sign as the term $(2C \kappa)/\Delta$, as shown in Fig. 3.2-2. If the atomic and cavity detunings are required to have opposite signs for good squeezing then this is equivalent to a requirement that the squeezing must arise on the lower branch or for low transmission, for a situation where the cavity detuning is swept. If $\Delta \phi > 0$ then it is possible for good squeezing to coincide with the peak cavity transmission. The regime of good squeezing for which $\Delta \phi > 0$ will from now on be termed the 'upper branch' regime.

The domains of bistability are illustrated in Fig 3.2-3, which plots the outer contour of the bistable region for three values of the cooperativity $C$ in the $\Delta-\phi$ plane, where the control variable is the input intensity (Agrawal and Carmichael 1979, Drummond and Walls 1981). In general the domain of bistable behaviour increases for increasing $C$, is symmetrical with respect to the product $\Delta \phi$, and is small for $\Delta \phi < 0$. By comparison of Table 3.2-1 and numerically-obtained solutions for the boundary of bistability it is found that in the $g\sqrt{N}$ regime for $C=50$ and $q < 0.04$, the system should not be bistable. For $q > 0.04$ it appears that the system is on the boundary of bistability for the optimum values of $\Delta \phi$ shown. This suggests that the generation of squeezing in this regime is not directly related to bistability.

For the upper branch regime, the values of the atomic detuning remain very similar over an order of magnitude of $q$ values. For $q=0.01$ to $q=0.1$ the optimum region of the bistability domain for best squeezing is the extreme tip of the contour, near $\phi=0$. This suggests that the value of $q$, although permitting larger squeezing for decreasing $q$, in this regime may be less important for good squeezing than the dynamics of bistability.

Table 3.2-2 shows the results of the same numerical optimisation procedure as carried out for Table 3.2-1 for the case of a Gaussian transverse mode in a standing-wave cavity. Squeezing in both regimes is degraded by the spatially-varying mode structure. Degradation of squeezing is greatest in the upper branch regime.
Figure 3.2-2: Shift and distortion of cavity transmission peak in dispersive bistability. For $\phi=0$ (empty cavity on resonance) the cavity transmission is symmetrical. For a range of atomic detunings $\Delta$ in a cavity containing atoms, switching and hysteresis is seen for changing cavity detuning $\phi$ (equivalent to changing cavity length). The cavity transmission is shifted from the empty-cavity value by approximately $C/\Delta$.

Figure 3.2-3: Contours of the region of bistable behaviour in the $\Delta-\phi$ plane, for $C=30$, 50, and 70.
Table 3.2-2: Comparison of two regimes of optimum squeezing for a Gaussian transverse mode in a standing-wave cavity.

<table>
<thead>
<tr>
<th>$g/N$ regime</th>
<th>$\Delta$</th>
<th>$\phi$</th>
<th>$\omega/\kappa$</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>2112</td>
<td>0.38</td>
<td>1.22</td>
<td>0.93</td>
</tr>
<tr>
<td>0.02</td>
<td>726</td>
<td>0.38</td>
<td>1.42</td>
<td>1.43</td>
</tr>
<tr>
<td>0.03</td>
<td>393</td>
<td>0.38</td>
<td>1.58</td>
<td>1.57</td>
</tr>
<tr>
<td>0.04</td>
<td>249</td>
<td>0.38</td>
<td>1.71</td>
<td>1.73</td>
</tr>
<tr>
<td>0.05</td>
<td>177</td>
<td>0.38</td>
<td>1.89</td>
<td>1.95</td>
</tr>
<tr>
<td>0.06</td>
<td>130</td>
<td>0.38</td>
<td>2.00</td>
<td>2.05</td>
</tr>
<tr>
<td>0.07</td>
<td>101</td>
<td>0.38</td>
<td>2.16</td>
<td>2.19</td>
</tr>
<tr>
<td>0.08</td>
<td>80.1</td>
<td>0.38</td>
<td>2.25</td>
<td>2.30</td>
</tr>
<tr>
<td>0.09</td>
<td>66.8</td>
<td>0.38</td>
<td>2.36</td>
<td>2.43</td>
</tr>
<tr>
<td>0.10</td>
<td>55.8</td>
<td>0.38</td>
<td>2.45</td>
<td>2.53</td>
</tr>
</tbody>
</table>
3.3 Eigenvalue spectra

The eigenvalues of the drift matrix $A$ determine the stability of the steady states of the system; they govern the response of a system to a perturbation. In the previous chapter it was assumed that the quantum noise of the cavity-atom system could be included as small fluctuations upon the steady-state solutions. This linearisation procedure is valid if the number of atoms is much greater than unity. The dynamics of the quantum noise can then be assumed to follow the deterministic dynamics of the system. The eigenvalues of the cavity-atom system for an arbitrary cavity decay rate in a plane-wave ring cavity and the low-intensity limit have been derived by Castelli et al. (1988), Reid (1988) and Raizen (1989).

In the low field limit ($x \to 0$) these equations reduce to simple forms and are shown here.

$$\lambda_{2,3} = \text{Re} \lambda_{2,3} + i \text{Im} \lambda_{2,3}, \quad \lambda_4 = \lambda_{2*}, \quad \lambda_5 = \lambda_{3*},$$

$$\text{Re} \lambda_{2,3} = -\frac{(\gamma + \kappa)}{2} \pm \frac{1}{2} \left[Z_R + (Z_R^2 + Z_I^2)^{1/2}\right]$$

$$\text{Im} \lambda_{2,3} = -\frac{(\omega_0 - \omega_c)}{2} \pm \frac{1}{2} \left[-Z_R + (Z_R^2 + Z_I^2)^{1/2}\right]$$

(3.2)

where

$$2Z_R = (\gamma - \kappa)^2 - 4g^2N - (\omega_0 - \omega_c)^2$$

$$2Z_I = 2(\gamma - \kappa)(\omega_0 - \omega_c)$$

where the eigenvalues $\lambda$ are given in units of the cavity decay rate $\kappa$. We see that there are two complex conjugate pairs, and that in the low field limit the fifth eigenvalue, corresponding to the atomic inversion, is decoupled from the other eigenvalues to give $\lambda_5 = -\gamma$ (purely real). Evidently the eigenvalues depend strongly on the matching of the cavity and atomic decay rates, the coupling rate $g\sqrt{N}$ and the mistuning of the empty cavity from the (unperturbed) atomic transition frequency.

In a realistic search for squeezing the limit $x \to 0$ must be relaxed and the behaviour of the eigenvalues for increasing intracavity field can best be demonstrated graphically. Much useful information may be derived from the 'eigenvalue spectra'. The technique of linearising about the eigenvalues of the system has much in common with conventional linear stability analysis (Liepholz 1970). In particular,

(i) If the real part of one or more eigenvalues becomes positive the solution is unstable. Thus the turning points of bistability can be detected on the eigenvalue diagrams (since one eigenvalue must vanish when the stability changes).

(ii) The imaginary parts of the eigenvalues can be said to give the oscillation frequencies of the system (the normal mode splitting) (Raizen et al. 1989).
(iii) The real parts give the corresponding linewidths of the normal modes of the coupled cavity-atom system.

In the absorptive case ($\Delta = \phi = 0$) the expressions (3.2) reduce to

$$\lambda_{2,3} = -\frac{\left(\gamma_\perp + \kappa\right)}{2} \pm \sqrt{\frac{\left(\kappa - \gamma_\perp\right)^2}{4} - g^2 N}$$

(3.3)

The eigenvalues have imaginary parts only for $|\kappa - \gamma_\perp| < 2g\sqrt{N}$, i.e. where the coupling is larger than the difference between the cavity and atomic decay rates. The real part shows linewidth averaging, where the new decay rate of the system is the mean of the uncoupled cavity and atomic decay rates. For $\kappa << \gamma_\perp$, subnatural linewidths can be observed. In our experiments $\kappa >> \gamma_\perp$, so that the linewidth of the coupled system would be given roughly by $\kappa/2$.

The absorptive regime will be discussed further, as a test case. For simplicity, and in order to compare our results with those of Raizen et al. (1987) and Raizen (1989) data sets to be examined will be taken only from the plane wave ring cavity case, in Table 3.2-1.

Fig 3.3-1(a) is a typical diagram for the absorptive case showing the imaginary parts of the eigenvalues in units of the cavity half-linewidth $\kappa$, for increasing intracavity field for $C=50$, $q=0.1$. There are two complex conjugate pairs, and one eigenvalue (the inversion) is purely real. The imaginary parts of the eigenvalues are degenerate at $x=0$ but split for greater intensities. One eigenvalue increases linearly, while the other tends to zero for large $x$. Raizen (1989) notes that the linearly increasing eigenvalue corresponds to the generalised Rabi splitting $\Omega = (2x^2 + \Delta^2)^{1/2}$; in the dressed-state picture transitions are occurring between sets of split upper levels (Reid 1988). The decreasing eigenvalue is said to correspond to the effective cavity detuning. Values of the intracavity field from $x = 1.25$ to $x = 10$ are non-physical solutions; they correspond to the unstable branch of absorptive bistability. This can be seen directly from Fig. 3.3-1(b), which shows the real parts of the eigenvalues. One eigenvalue becomes positive for this range of $x$ values.

Fig 3.3-1(c) shows the spectral variance $V$ for increasing intracavity field $x$, with the analysis frequency $\omega$ optimised at each point to minimise $V$. Evidently the non-physical values of $x$ also correspond to the values of optimum squeezing. Intuitively one would say that for the upper branch of absorptive optical bistability, where the atoms are bleached by the field and thereby decoupled from each other, that there would be little or no squeezing. It is clear from Fig. 3.3-1(c) that this is correct. For best squeezing in the absorptive case one must therefore concentrate on the lower branch.

Fig. 3.3-1(d) shows the squeezing spectrum calculated for $x = 1$ (a stable point). We predict from study of Fig. 3.3-1(a) that the spectrum should displays two peaks located symmetrically about the pump frequency, giving optimum squeezing at a coupling frequency $\omega/\kappa = 2$. This is indeed the case. (Not visible in Fig. 3.3-1(e), depicting the transmitted intensity spectrum $S_{12}(\omega)$, are two sidebands at the same frequency. They are two orders
Figure 3.3-1: Eigenvalues of the drift matrix $A$ in absorptive bistability for increasing normalised intracavity field $x$, and associated squeezing. $C=50$, $q=0.1$, $\Delta=\phi=0$. (a) Imaginary parts of eigenvalues. (b) Real parts of eigenvalues. (c) Spectral variance $V$ (with $\omega/\kappa$ and phase angle of squeezing $\theta$ optimised for best squeezing).
Figure 3.3-1: Spectrum of squeezing and intensity in absorptive bistability. C=50, q=0.1, I=4, Δ=ϕ =0. (d) Squeezing spectrum (phase angle of squeezing θ optimised for best squeezing). (e) Spectrum of transmitted intensity.

Figure 3.3-2: Dressed-state diagram showing vacuum-field Rabi splitting in absorptive regime.
of magnitude smaller than the main fluorescence and scattering peak at $\omega=0$.

In Fig. 3.3-2 a dressed-state diagram for the absorptive configuration is shown, corresponding to the case of a few excitations in the system. This diagram in essence conveys the same information as is provided in the expressions for the imaginary parts of the eigenvalues in the weak-field limit, above (Eq. 3.2). In the strongly-coupled case for the cavity-atom system ($g^N$ large, $\kappa_1 \sim 1$) the first excited state of the system is split by $2g^N$, the vacuum-field Rabi splitting. This leads to the possibility of inelastic scattering processes which take two laser pump photons and produce two photons scattered symmetrically about the pump frequency $\omega=0$ at the coupling frequency $g^N$.

The absorptive case, although simplest to analyse, is not satisfactory from the point of view of generating substantial amounts of squeezing. We therefore proceed to examine the dispersive configuration, where the pump laser is not tuned to either the atomic or empty-cavity resonances. The dispersive case is of greater interest for squeezing, on the basis of the discussion of atomic losses in the cavity-atom system above, and of the results of Tables 3.2-1 and 3.2-2.

### 3.4 The $g^N$ regime

Typical diagrams are presented for the $g^N$ regime (Fig. 3.4-1(a),(b)) showing the imaginary and real parts of the eigenvalues of $\Delta$ for increasing $x$. Parameters $\Delta$, $\phi$ were obtained from Table 3.2-1 for $q=0.05$. As in the absorptive case there are two complex conjugate pairs and one eigenvalue (the inversion) is purely real. None of the real parts of the eigenvalues become positive, indicating that there is no bistability for these parameters. Squeezing is optimum near the crossing of the different complex conjugate pairs, where the imaginary parts become degenerate (as shown in Fig. 3.4-1(c)).

Figures 3.4-1(d) and 3.4-1(e) show the squeezing spectrum and the intensity spectrum respectively for $q=0.05$, and $C=50$ (other parameters as above). The intensity spectrum is shows feature similar to the absorptive case, but the magnitude of the sidebands at $g^N$ with respect to the main scattering peak is far greater than for the resonant situation - the sidebands are readily visible on the same scale. The magnitude of the fluorescence at the main peak is greatly reduced for the dispersive case. Squeezing at the $g^N$ sidebands is correspondingly greatly enhanced, reaching 75%. Good squeezing is possible over a much larger range of $I$ than in the absorptive case; the 'window' of intensities for which squeezing exists is many times greater and does not overlap a region of system instability, as occurs for the absorptive regime.

This eigenvalue structure is characteristic of the case where the pump laser frequency is tuned midway between the uncoupled cavity and atomic frequencies. The dressed state picture is shown in Fig 3.4-2. As can be seen from the analytical expressions for the eigenvalues in the low-field limit, for coupling $g^N >> (\omega_0-\omega_c)^2 / 4$, $[S=(g^2 + (\omega_0-\omega_c)^2) / 4]^{1/2}$, the normal mode splitting is $g^N$ and best squeezing will be seen near that frequency, as for the absorptive case.
Figure 3.4-1: Eigenvalues of the drift matrix $\Delta$ in the $g\sqrt{N}$ regime for increasing normalised intracavity field $x$, and associated squeezing. $C=50$, $q=0.05$, $A$ and $\phi$ as in Table 3.2-1 for the $g\sqrt{N}$ regime. (a) Imaginary parts of eigenvalues. (b) Real parts of eigenvalues. (c) Spectral variance $V$ (with $\omega/\kappa$ and phase angle of squeezing $\theta$ optimised for best squeezing).
Figure 3.4-1: Spectrum of squeezing and intensity in the g\sqrt{N} regime. C=50, q=0.05, I=175, Δ =13.0, φ = -0.91 (parameters from Table 3.2-1 for the g\sqrt{N} regime). (d) Squeezing spectrum (phase angle of squeezing θ optimised for best squeezing). (e) Spectrum of transmitted intensity.

Figure 3.4-2: Dressed-state diagram showing vacuum-field Rabi splitting in dispersive regime, pump laser tuned midway between (unperturbed) cavity and atomic frequencies.
3.5 The upper branch regime

We now show a similar set of figures as in the previous section, for the case of $q=0.05$ in the upper branch regime. This regime was not examined for squeezing by Raizen (1989). Figures 3.5-1(a) and 3.5-1(b) are the imaginary and real parts of the eigenvalues plotted for increasing intracavity field. The most striking feature of the eigenvalue structure of these parameters is the region where one pair of imaginary complex conjugates becomes zero and a pair of degenerate real parts split; i.e. the eigenvalues become purely real. This region seems to correspond to best squeezing. Fig. 3.5-1(c) shows the squeezing for the same horizontal axis, with analysis frequency $\omega$ optimised at each point. The squeezing is almost flat across the entire region, at 66%. The real parts of the eigenvalues are all negative, indicating that the solution is stable for the region of good squeezing. We may thus exclude the possibility that a positive-slope instability is involved.

The structure shown in Figs. 3.5-1(a)-(b) is not difficult to achieve. Picking at random values of $\Delta$ between 10 and 35 and $\phi$ between 0.5 - 3.5 for fixed $q$ and $C = 50$ we always found a similar structure, always corresponding to best squeezing. The amount of squeezing was remarkably insensitive to the variation of the detunings.

It is now possible to make predictions about the appearance of the squeezing spectrum, from study of the imaginary parts of the eigenvalues. The associated squeezing spectrum (for a value of intracavity intensity chosen in the region) should show four distinct peaks, two close to the pump intensity ($\omega/\kappa<1$) and two some distance away at $\omega/\kappa=4$. Although two imaginary parts merge at $\omega=0$ it can be surmised that since fluorescence is also being generated due to elastic scattering processes at the pump frequency, the squeezing at $\omega=0$ will be degraded, leading to the appearance of two peaks around this position. Figs 3.5-1(d) and 3.5-1(e) bear out this analysis. The dominant feature of the intensity spectrum $S_{12}$ in Fig 3.5-1(e) is the scattering peak at $\omega=0$. This corresponds to the 'bite' taken out of the broad peak of width $2\kappa$ in the squeezing spectrum.

It could be speculated that in the good-cavity case the width of the scattering peak at $\omega=0$ would be greater than the width of the squeezing peak (typically on the order of the cavity bandwidth) and the squeezing they predicted would be obliterated by the atomic loss effects. Increasing the cavity decay rate (decreasing $q$) would have the effect that the squeezing peak could emerge past the atomic scattering peak and good squeezing would be permitted near $\omega=0$.

Since this eigenvalue structure appears to be a general feature of the dispersive configuration in a short cavity for $\Delta \phi>0$ we select the most general situation of $\omega_c \neq \omega_o \neq \omega_c$ to draw the associated dressed state picture in Fig 3.5-2. (See Reid (1988) Fig. 7e)

3.7 Observation of distinct regimes

The two regimes of optimum squeezing may be likened to two islands in the parameter space. We now consider the question of how the different 'islands' of best squeezing may be accessed experimentally, given that $C$ and $q$ are fixed. The parameters $I$, $\Delta$, and $\omega$
Figure 3.5-1: Eigenvalues of the drift matrix $\Delta$ in the upper branch regime for increasing normalised intracavity field $x$, and associated squeezing. $C=50$, $q=0.05$, $\Delta$ and $\phi$ as in Table 3.2-1 for the upper branch regime. (a) Imaginary parts of eigenvalues. (b) Real parts of eigenvalues. (c) Spectral variance $V$ (with $\omega/\kappa$ and phase angle of squeezing $\theta$ optimised for best squeezing).
Figure 3.5-1: Spectrum of squeezing and intensity in the upper branch regime C=50, q=0.05, I=352, Δ=24.1, Φ = 2.22 (parameters from Table 3.2-1 for the upper branch regime). (d) Squeezing spectrum (phase angle of squeezing θ optimised for best squeezing). (e) Spectrum of transmitted intensity.

Figure 3.5-2: Dressed-state diagram showing vacuum-field Rabi splitting in dispersive regime, pump laser tuned below between (unperturbed) cavity and atomic frequencies.
may all be varied, and it can be assumed in searching the available parameter space that for each I, Δ, and π the local oscillator phase and cavity detuning will also be freely varied to optimise squeezing. In the $g\sqrt{N}$ regime the optimum frequency for squeezing is close to

$$\omega_{\text{opt}} = g\sqrt{N} = \sqrt{\kappa \gamma_4 C}$$

(from Eq. (2.15c)). The analysis frequency of optimum squeezing increases as $\sqrt{C}$. In the real experimental situation, therefore, the cooperativity cannot be increased without bound, and neither can the decay rates $\kappa, \gamma_4$ be made arbitrarily high, since the bandwidth of the photodetectors must be taken into account. In general the response of silicon photodiodes rolls off rapidly for frequencies above 200 MHz.

A set of parameters appropriate to our experiment ($\gamma_4/(2\pi) = 10$ MHz, $\kappa/(2\pi) = 125$ MHz and $C = 50$) lead to an optimum analysis frequency $\omega/(2\pi)_{\text{opt}} = 350$ MHz. This suggests that if $C$ is large and other parameters are varied freely, squeezing may be observed in the upper branch regime rather than the $g\sqrt{N}$ regime; the coupling frequency $g\sqrt{N}$ will be out of the range of the detectors.
Chapter 4: Experimental apparatus and techniques

This chapter contains a description of the experimental arrangement and techniques. Four major experimental runs were carried out. Progressive improvements were made in the total detection efficiency over the course of the experiments. Variation in the mirror spacing and cavity finesse between experimental runs were responsible for variations in the cavity decay rate $\kappa$, and thus in the $q$ parameter. There was also some variation in the degree of intensity stabilisation, mode-matching and cavity stability between experiments. Otherwise the essential experimental configurations and operating conditions were not altered. In Chapter 6 the data to be analysed will be taken from all four experimental runs. Different procedures were used to search the large parameter space of the cavity-atom system in successive experiments, so that different aspects of the system dynamics were revealed. Values and procedures which were typical of the experiment are discussed in this chapter.

4.1 Overview of experimental arrangement

The experimental arrangement is shown in Figure 4.1-1. A Spectra-Physics 380D ring dye laser typically generated 300 mW of light at the wavelength of the barium resonance transition, 553 nm. The laser was pumped by $\sim 7$ W from an argon ion laser (Spectra-Physics 2020 or Spectra-Physics 2030) operated in broadband mode. In order to access the green wavelengths it was necessary to employ the chemically unstable dye Rhodamine 110 (also known as Rhodamine 560). This dye is degraded by oxidation and by pumping, so that approximately three full days of laser use were available before a dye change was required. The frequency jitter of the ring dye laser light was restricted to less than 1 MHz rms by the commercial active frequency stabilisation system.

An acousto-optic modulator (AOM) served to isolate the laser from cavity feedback and also played a part in the intensity stabilisation system (INTSTAB), which reduces the intensity noise of the laser beam from approximately 10% p-p to less than 2.5% p-p. The zeroth-order beam from the AOM was directed to a wavemeter, which allowed the laser wavelength to be set to within $\pm 0.001$ nm of wavelength of the atomic transition. The first-order diffracted beam from the modulator illuminated the approximately single-ported optical cavity located inside the vacuum chamber. Lenses L1 and L2 matched the signal beam to the TEM$_{00}$ mode of the cavity. The cavity, comprised of mirrors M1 and M2, had typical finesse 140±10, a throughput on resonance $T_0$ of 1.3 %, and length 4.3 mm. Both mirrors were mounted upon piezoelectric stacks so that the cavity could be tuned through resonance.

Polarizers POL1 and POL2 were used to control the power onto the cavity (with a maximum available power of 35 mW). POL2 was set such that the linear polarization of the signal beam was horizontal. Three beam-folding mirrors located before the polarizers were placed to satisfy mode-matching requirements on the optical path length, and could be aligned to adjust the height and angle of the beam into the cavity. The beamsplitter BS1 directed approximately 2 mW of the mode-matched beam into the local oscillator.
Figure 4.1-1: Schematic layout of the experiment. Guide to abbreviations; mirrors M1 and M2, acousto-optic modulator AOM, intensity stabiliser INTSTAB, chopper CH, lenses L1, L2, beamsplitters BS1, BS2, BS3, local oscillator LO, polarizers POL1, POL2, POL3, POL4, photodetectors PD1, PD2, PD3 and photomultiplier PMT. A barium beam with a maximum optical depth of 3.5 is operated in a diffusion-pumped vacuum chamber with background pressure $10^6$ torr. The atoms cross the chamber at right angles to the cavity axis.
(LO), which is then recombined at 50-50 beamsplitter BS3 with the cavity output to form a balanced homodyne difference detection system. Mode-matching on the detector beamsplitter BS3 was facilitated by ensuring that the LO and signal beams traversed an equal pathlength. The polarization of the LO beam was defined as horizontal by polarizer POL4. The intensity of the LO beam was adjusted by polarizer POL3. Two lenses were inserted into the LO beam to compensate for the focusing effects of the cavity mirror substrates.

Typically 10-15 dBm common mode subtraction was achieved in the homodyne detector. Changing the phase of the local oscillator with the scanning galvoplate allowed the quadratures of the cavity output field to be interrogated, consecutively. The beams from the two output ports of BS3 were focused on to photodiodes PD2 and PD3 by anti-reflection (AR) coated lenses, and the ac component of the signal from the homodyne detection system was displayed on a spectrum analyser before being stored on computer via the data acquisition system.

During an experimental run, the cavity input and output powers and the DC levels of the photodetectors PD2 and PD3 were recorded synchronously with the spectrum analyser traces. A glass plate (BS2) directed ~5% of the signal beam on a calibrated photodiode (PD1, UDT PIN6D) in order to monitor the cavity input power. A small fraction of the cavity output beam was reflected from the second surface of BS3, despite the AR coating of the surface. This weak beam was directed to photomultiplier PMT, to serve as a monitor of cavity output power.

A pellicle beamsplitter split off 10% of the main beam before mode-matching lenses L1 and L2 for use as intensity and fluorescence monitor beams. The monitor beam was modulated at about 70 Hz by chopper CH so that lock-in amplifiers (EG&G 5104, 5204) could be used to discriminate against ambient light. A small portion of the beam (not shown), attenuated and focused, was directed vertically downwards through the interaction region between the cavity mirrors on to a photodiode (PD2, HP4022). This served as the absorption monitor. The remainder (about 2 mW) passed at right angles through the glass fitting on the vacuum system, into which the atomic beam is directed. Fluorescence is easily observable for optical depths $\alpha_l$ as small as $\alpha_l=0.1$. It was possible to adjust laser frequency in steps of between 10 MHz to 1 GHz while visually checking the fluorescence, using a laser frequency tuner near the system.

4.2 The atomic beam

The nonlinear medium was provided by a dense well-collimated beam of natural barium directed through the optical cavity at right angles to the cavity axis. Use of the simple, well-understood J=0 to J=1 transition in $^{138}\text{Ba}$ avoided complications with hyperfine structure, thus eliminating the need for optical pre-pumping.

The construction of a high-intensity, well-collimated atomic beam was essential to the success of the experiment. Optical depths in excess of $\alpha_l = 0.6$ were required in order to achieve a nonlinear cavity-atom interaction strong enough to allow bistable behaviour and
squeezing. Good collimation of the atomic beam was necessary to prevent the closely-spaced cavity mirrors from becoming coated with barium. It was also significant because the presence of Doppler broadening is known to degrade squeezing.

The high melting point (710° C) and low vapour pressure of barium render these requirements difficult to achieve. Previous experiments in this laboratory had indicated that a conventional resistively-heated effusive source was unable to produce such a dense narrowly-directed beam of barium. It was therefore decided to employ the principle of supersonic flow through a converging nozzle, as initially suggested by Kantrowitz and Grey (1951) and first demonstrated by Kistiakowsky and Slichter (1951). A source of this type can generate a high-intensity atomic beam with a substantially narrowed velocity distribution, as the initial random thermal motions of the atoms are transformed into directed mass motion of a gas via expansion through the nozzle. Heating of the oven to the temperatures required (~1000 C) was achieved using an electron bombardment technique. Electrons from the filament were accelerated on to the oven, which was held at a large positive high voltage.

The vacuum system constructed for the experiment consisted of a stainless steel O-ring sealed diffusion-pumped system backed by a pair of oil-sealed rotary pumps. As shown in Fig. 4.2-1, the main system was divided into two sections: the atomic beam chamber and the chamber containing the optical cavity. These were separated by a metal plate containing two apertures. The apertures were a few millimetres in diameter only, such that in the free molecular flow regime the pressures of the sections were essentially independent (differential pumping). The metal plate also acted to prevent barium and carbon dust from entering the clean optics chamber. The atomic beam chamber and the optics chamber were each connected to a diffusion pump via stainless steel bellows of a metre in length. The purpose of the bellows was to prevent the vibrations generated by the backing pumps from being transmitted to the optics chamber.

The atomic beam chamber was evacuated by a diffusion pump (Varian 0184) with a nominal pumping speed of 1000 L/s, giving an ultimate pressure of $2 \times 10^{-6}$ torr. The optics chamber was maintained at a pressure of $5 \times 10^{-6}$ torr by a smaller diffusion pump (Dynavac OD600) with a nominal pumping speed of 600 L/s. High vacuum pressure measurements were made by two Penning ionization gauges; one was mounted on the oven chamber and the other on the optics chamber. Rough vacuum and diffusion pump foreline pressures were monitored by two thermocouple gauges (Veeco TG-70). The ultimate pressure of the atomic beam chamber was best whilst the atomic beam was in operation; the getter properties of the reactive barium were found to improve the vacuum.

A schematic diagram of the atomic beam apparatus and its heating circuit is shown in Figure 4.2-2. The stainless steel (Sandvik 253 MA) oven was cylindrical in shape, with a diameter of 25 mm and a length of 60 mm. The back section with a mounting rod is attached by means of a screw thread of large pitch (2 mm) to the main body of the oven in order to allow disassembly without damage to the oven. A metal to metal seal in the form of a curved section pressing on a plane conical surface proved reliable and allowed the reuse of the components. With proper tensioning and with positioning of the filament near the nozzle of the oven, no significant leakage of barium through this seal was observed. The mounting rod was clamped to a second rod (50 mm long, diameter 3 mm)
Figure 4.2.1: Schematic diagram of the atomic beam and optic chambers.
Figure 4.2-2: Schematic diagram of Ba oven, oven mount and electron bombardment heating circuit.
which was firmly mounted to the support structure. By bending the thin rod with a two
dimensional translator the oven position could be aligned with the skimmer even during
the operation of the apparatus. This alignment allowed us to compensate for thermal
distortions of the hot apparatus and was important for the optimisation of the beam
density.

The oven was surrounded by a seven-coil spiral filament of tantalum wire. The diameter
of the wire was 0.5 mm and the length was 1.06 m. Heated by a current of 10 A at a
voltage of 40 V by two Delta Elektronika SM3540 power supplies linked in series, the
filament acted as the thermal source of electrons. These were accelerated towards the
oven which was held at a potential of +300 V or more by a Hewlett-Packard high-voltage
power supply. An electron current of 1.5 A was required to heat the oven to temperatures
in the region of 1000 °C, as measured by an optical pyrometer. The entire beam apparatus
was water-cooled via tubing running within the walls of the vacuum system and through
brass jackets on the outside of the chamber.

The oven had an exit aperture of 1 mm diameter. In order to prevent clogging, the exit
opening was only 1 mm deep, expanding to a 6 mm diameter aperture leading to the
barium chamber. No clogging was observed when the barium used was kept in storage
under argon, rather than oil.

It was possible to reduce the spread of transverse velocities in the atomic beam by means
of a skimmer (a sharp-edged conical collimator). The skimmer removed the outer streamlines
with high transverse velocity components, leaving a central section that was highly
collimated. The positioning and dimensions of the skimmer were found to be critical to
achieving the required Ba densities. We used a skimmer 25 mm downstream from the
oven with an opening of 1.5 mm formed by a sharp leading edge tilted at 15 degrees to
the axis of the beam. Figure 4.2-3 is a scaled diagram showing the atomic beam passing
through the series of three collimators to the interaction region. The chamber separation
plate was 83 mm downstream from the skimmer opening, and the final aperture of 2 mm
diameter (bored into the end of a cylindrical tube which could be inserted between the
cavity mounts) was located 80 mm from the plate. The interaction region was 12 mm
from the final aperture, and the atomic beam diameter was calculated as 2.3 mm at this
point.

The performance of the atomic beam was characterised spectroscopically. A weak laser
beam (power less than 0.5 μW) was tuned through the resonance transition of $^{138}$Ba at
553.3 nm and absorption spectra were recorded via the absorption probe beam was
passing vertically downwards through the interaction region. Absorption profiles for
natural barium are shown in Fig. 4.2-4. Trace (a) shows an experimental absorption
profile with a maximum depth on resonance of 70% for the transition of interest, $^{138}$Ba.
When the corresponding optically thin trace was extracted from the optically thick
experimental scan of Fig. 4.2-4(a) by taking the natural logarithm of the profile, the
FWHM proved to be $23\pm3$ MHz. Assuming no collisions, the Lorentzian component of
the profile can be taken as the natural linewidth, 19.5 MHz, and the Gaussian component
due to Doppler broadening is therefore 10.5 MHz.

A theoretical simulation of the absorption by a weighted sum of pure Lorentzian profiles
Chamber separation plate 2 mm diam. aperture

Converging nozzle with 1 mm diam. aperture

Barium chamber

25.0 mm

Atomic beam

Interaction region

Probe laser beam

Figure 4.2.3: Scale diagram of Ba oven, atomic beam and collimators.
Figure 4.2-4: Absorption profiles for natural barium (a) experimental, and (b) theoretical, for a pure Lorentzian fit.
is presented in trace (b). The additional absorption peaks upon the high-frequency side of the transition of interest are due to the presence of other isotopes of barium, the natural abundance of $^{138}\text{Ba}$ being 71.9%. Figure 4.2-5 and Table 4.2-1 show the energy level scheme of $^{138}\text{Ba}$, and the isotope abundances and shifts of natural barium, respectively. To avoid pumping the high-frequency transitions and thereby generating additional spontaneous emission noise we took care to tune the laser frequency below the frequency of the $^{138}\text{Ba}$ transition.

The measured residual Doppler linewidth is in good agreement with a maximum theoretical residual Doppler width of 12 MHz calculated using the assumption of an effusive source at a temperature of 1050°C, and knowledge of the system geometry. (Since it has been demonstrated theoretically and experimentally that a supersonic beam has an intrinsically smaller velocity spread than an effusive source, the effusive source approximation should give a reasonable upper limit for the Doppler width.) The atomic detunings used during the experiments (greater than 150 MHz) ensured that we operated well outside the residual Doppler profile of the line.

4.3 The optical cavity

The optical cavity was formed by two partially-transmitting high reflectance, low loss mirrors. The input coupler $M_1$ was a Newport supercavity mirror, with a nominal total loss (T+A) of 185 ppm = 0.185%, and a radius of curvature of 1 m. The output coupler $M_2$ had a 3.6 % transmittance at 553 nm and a radius of curvature of 2 m. (Schematic diagrams of the cavity configuration and the transmission function are shown in Fig. 4.3-1.) The mirrors were selected to form a highly asymmetric cavity, such that the dominant cavity decay route was through the output coupler. The finesse $F$ of the cavity was generally measured as $140\pm10$. In fact, over the course of the experiments, $F$ decreased from about 148 to 135. This was attributed to gradual degradation of the mirrors by scattered Ba particles; a slight discoloration of the surface became visible. Throughput on resonance for the empty cavity (defined as $\phi=0$) ranged from 1.1% to 1.3% depending on the mode-matching efficiency and mirror losses. The cavity length was set to approximately 4 mm by mechanical adjustment of the mirror mounts, and the cavity beam waist was calculated as $w_0=97\ \mu\text{m}$ for the radii of curvature and mirror separation given above (Kogelnik and Li 1966). Both mirrors were mounted on piezoelectric crystal stacks. A high-voltage ramp was applied to one piezoelectric stack to scan the cavity length by a wavelength or more. The other piezoelectric stack was connected to a variable DC high voltage supply (Delta Electronika E 0300-0.1) so that the cavity could be manually tuned to resonance. This stack could also be used to correct for large-scale thermal and mechanical drift.

The optical cavity could be characterised as a spherical near-planar resonator, for which the cavity length is much less than the radii of curvature of the mirrors. The resonant modes of such a cavity are nondegenerate, and so the higher-order transverse modes were revealed in the transmission when the cavity length was varied. These modes were largely suppressed by matching the wavefront curvature and beam waist of the laser
Figure 4.2-5: Schematic diagram of the energy levels of barium involved in the experiments. For a linearly polarized pump laser, only the $\Delta M_J = 0$ Zeeman transition (shown) is driven.

Table 4.2-1: The most abundant isotopes of natural barium, shown with abundances, hyperfine splittings and isotope shifts measured in MHz from the $6s\ 6p^1P_1$ energy level of $^{138}$Ba.

<table>
<thead>
<tr>
<th>Isotopes</th>
<th>Abundance</th>
<th>Hyperfine splittings</th>
<th>Isotope shifts (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{134}$Ba</td>
<td>2.4 %</td>
<td>—</td>
<td>143</td>
</tr>
<tr>
<td>$^{135}$Ba</td>
<td>6.5 %</td>
<td>$F = 5/2$</td>
<td>122</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F = 3/2$</td>
<td>327</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F = 1/2$</td>
<td>547</td>
</tr>
<tr>
<td>$^{136}$Ba</td>
<td>7.8 %</td>
<td>—</td>
<td>129</td>
</tr>
<tr>
<td>$^{137}$Ba</td>
<td>11.2 %</td>
<td>$F = 5/2$</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F = 3/2$</td>
<td>275</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F = 1/2$</td>
<td>552</td>
</tr>
<tr>
<td>$^{138}$Ba</td>
<td>71.9 %</td>
<td>—</td>
<td>0</td>
</tr>
</tbody>
</table>
**Figure 4.3-1:** Schematic diagram of the transmission of a 4.3 mm long, spherical near-planar resonator. The off-axis modes are typically suppressed to less than 10% of the TEM₀₀ mode in power by cavity alignment and mode-matching.
signal beam to the corresponding curvature and waist defined by the optical cavity. The efficiency of mode-matching was typically 80 - 90% in power. It was primarily limited by the shape of the beam obtained from the ring-dye laser and by the distortions of the beam introduced by its passage through many optical components.

Fig. 4.3-2 shows a scale diagram, looking from the source of the atomic beam, of the optical cavity and mounts. The cavity mounts, base and supports were machined from aluminium for ease of working. Use of one material minimised the problem of differential rates of thermal expansion and contraction. The cavity mirror mounts were of a conventional pivoted-plate type, and tilting adjustments were made with spherical-tipped micrometers for smooth motion. The mirror mounts could be moved along a slot in the plane of the laser signal beam, but no motion perpendicular to this was possible. The piezoelectric stacks were glued to smaller aluminium plates (insulated by strips of Mylar) which could be removed, replaced, and adjusted in height as a separate unit. The mirrors themselves were screwed into disk-shaped aluminium mounts which were glued to the piezoelectric stacks. The 0.5 mm lip on the front of the disk mounts proved effective in reducing degradation of the mirrors by scattered barium. Angular adjustments of the cavity mirrors could be made with the vacuum chamber sealed using rotary feedthroughs slotting into the micrometer heads, but in general it was convenient to align the beam into the cavity using the three folding mirrors.

Correct placement of the cavity mirrors about the atomic beam was critical to prevent coating and degradation of the mirror surfaces by barium. It was achieved by viewing the aperture of 2 mm diameter located before the cavity (Fig 4.2-3) through a telescope which was aligned to look down the path of the atomic beam to the oven aperture. The cavity mount positions were set by eye such that the mirror surfaces were 1 mm away from the edges of the 2 mm aperture. This procedure gave a cavity of approximate length 4 mm, spaced evenly about the atomic beam. The cavity length was measured optically, by determining the free spectral range of the cavity. In general FSR ~ 35 GHz; hence for a finesse of 140 the cavity linewidth (FWHM) was 250 MHz.

Cavity escape efficiency is an important parameter in the determination of the total quantum efficiency of the system, by which the squeezing obtained experimentally is related to the theoretically predicted amount of squeezing. It is a measure of the cavity losses and of the degree of single-endedness of the cavity, and is given by

$$\rho = \frac{F}{\pi} \left(1 - \sqrt{1 - T_2}\right)$$

(4.1)

where \( F \) is the cavity finesse and \( T_2 \) is the transmittance of the output mirror. For \( F=140 \) and \( T_2=0.036 \), escape efficiency \( \rho = 0.81 \).

4.4 The squeezing detector

The phase-sensitive balanced homodyne detector consisted of a 50-50 beamsplitter sending two output beams of equal power to two identical photodiodes (EG&G FFD-060). The
Figure 4.3-2: Schematic diagram (side view) of optical cavity and mirror mounts.
detector beamsplitter BS3 was coated for low loss at 553 nm. The measured loss of the beamsplitter was 1.5 % in power. The laser beams are focused on to the small active area of the photodiodes (1 mm²) using anti-reflection coated lenses of focal lengths 50-100 mm, and the light reflected from the surface of the silicon photodiodes was re-focused onto the diodes using tightly curved mirrors. The bias voltage applied to the photodiodes was 70 V, and the windows of the photodiodes were removed to reduce absorption and scattering losses. The ac and dc components of the signal are separated by the photodetector circuitry (shown in Fig. 4.4-1) and the ac components from each detector are subtracted using a 180 degree splitter/combiner (Mini-Circuits ZFSCJ-2-1). The ac components of the subtracted output of the photodiodes pass through a 60 dB Trontech amplifier and are displayed on a spectrum analyser (HP 71000).

Other parameters contributing to the total detection efficiency of the squeezing experiment were the photodiode quantum efficiency $\alpha$, the mode-matching efficiency $\eta$ at the beamsplitter BS3 and the efficiency of propagation $T_p$ through the optical components positioned between the cavity and the photodiodes.

The quantum efficiency of the photodiodes (EG&G FFD-060) was measured by measuring the dc current generated per milliwatt of light on the photodiodes at the wavelength of interest, 553 nm. EG&G Vactec (1990) give the quantum efficiency as

$$q.e. = 124 \times \frac{S_R}{\lambda}$$

(4.2)

where $S_R$ is the radiometric sensitivity, (the ratio of the photocurrent generated by the photodiode to the energy of the incident light in units of A/W) and $\lambda$ is the wavelength of the light in microns. The power of the light was measured by an NBS-traceable powermeter (Newport 815-SL) accurate to within ±5 %. The quantum efficiency of the photodiodes used in our experiment was found to be 53 %, and was raised to 65 % by the method of refocusing the reflected light. The quantum efficiency of the photodiodes was measured independently by colleagues at the CSIRO Measurement Laboratory (Linfield NSW), and was verified to be 52 % without refocusing. It is possible that the relatively poor performance of the photodiodes at the wavelengths of interest could be improved by deliberate selection of photodiodes for high efficiency in the green at the time of purchase.

The efficiency of mode matching of signal and LO beams on the detector beamsplitter BS3 was measured as the dc visibility of the interference fringes formed upon the beamsplitter with LO phase varied and with the LO signal attenuated so as to match the signal beam in power (~0.2 mW). The mode-matching efficiency depended upon the quality of the laser mode, the pathlength difference between the cavity signal beams and LO beams and the beam polarizations. Typical visibilities achieved were in the range 80-90%. One significant difficulty was that the optical cavity acted as a mode-cleaner for the signal beam. The mode quality of the cavity output beam was much better than that of the LO beam, and the overlap at BS3 was consequently degraded.

Losses as the light transmitted from the cavity propagated through various components - the coated vacuum system output window, the beamsplitter BS3 and the lenses before the photodiodes - were measured individually and the total propagation efficiency $T_p$ was found to be 88 %.
Figure 4.4.1: Circuit diagram for squeezing detector.
The analysis frequency $v$ selected on the spectrum was restricted within the band 50 MHz to 250 MHz, the lower limit being due to technical and laser frequency noise, and the upper limit to amplifier roll-off. Within this band the choice of $v$ was further restricted to regions free of residual external RF pickup and excess noise due to the presence of longitudinal modes of the argon ion pump laser. Figure 4.4-2 shows a broadband frequency scan from 50 to 200 MHz. The upper trace shows the spectrum obtained from the balanced homodyne detector, while the lower trace shows the spectrum due to the electronics alone, with incident light blocked. The amount of quantum noise above electronic noise was in the range 1.5-2.0 dB for the Trontech amplifier. It is evident that most of the large coherent signals are of optical origin. Such sources made it impossible to compile a complete squeezing spectrum. The resulting standard quantum noise level in the quiet 'window' regions was checked against the noise generated by a spectrally-filtered white light source and found to be consistent with the white light noise to within 0.05 dB. We also verified that no further noise was added by the cavity in the absence of atoms, by blocking the atomic beam with the travelling aperture, with all other experimental conditions preserved.

Because of the chosen cavity configuration and the fact that squeezing was observed upon the upper branch of bistability, the cavity output signal beam was not negligible in power with respect to the LO beam. Typical values for the cavity output beam at the cavity peak transmission and the LO beam power respectively, were 0.15 mW and 2.0 mW. When LO phase was varied with the rotating galvoplate, readily detectible interference fringes were produced upon the beams leaving the two output ports of the beamsplitter. The amplitude of the fringes was of course proportional to cavity transmission, so that it varied across the cavity scan. One outcome of this was to impose more stringent requirements upon the matching of the power of the output beams of BS3. It was essential that the dc parts of the signal be well-matched, or the differenced noise signal would show the same modulation as the input signal beams.

The information obtained from the interference fringes has considerable value. Comparison of the phase of the cavity signal beam with the phase of the modulation upon the quantum noise could be used to determine the orientation and tilt of the squeezing uncertainty area. This information had not been previously obtained in an atomic quadrature squeezing experiment, and will be discussed in detail in a later chapter.

### 4.5 Experimental Procedure

The perpendicular alignment of the short-cavity axis, the atomic beam and the signal laser beam with respect to each other was a lengthy and cumbersome procedure, but it was a critical element in the experiment. The interaction region was defined by the position of the skimmer (which was fixed) and by the positioning of the cavity mounts along the plane of the atomic beam path (which was set by the slots in the base of the mounts). All other characteristics, such as oven position, mirror height and laser beam position, could be varied for fine-tuning.
Figure 4.4-2: Spectrum analyser scan across broad frequency range 10-200 MHz. Top trace, light on balanced homodyne detector. Bottom trace, light blocked (electronic noise only).
Initial alignment tasks were performed with the vacuum chamber open. The optical cavity mirrors were selected, cleaned and screwed securely into their cavity mounts. The vertical height of the cavity axis with respect to the atomic beam path was approximately set by ensuring that the cavity signal beam crossed the final aperture (2 mm diameter) at its midsection, as judged by looking down the telescope. The final aperture was kept aligned with the skimmer and separation plate aperture. The mirror separation was set to a few millimetres as described previously and the cavity length was determined optically. The initial alignment of the cavity was carried out at this stage, using the micrometer controls on the cavity mounts. The barium oven was then loaded with 7-8 g of natural barium pellets. The vacuum chamber was closed and allowed to pump down overnight to a pressure of approximately $10^{-5}$ torr.

At this point a current of a few amperes was allowed to flow through the filament, in order to bake out the oven chamber. The current was gradually increased to the operating conditions of 10-13 A at 60V. The water cooling to the oven chamber walls and oven adjustment mechanism was turned on midway through this process. The alignment of the 1 mm diameter oven aperture with the fixed (2 mm diameter) skimmer hole was then performed using the oven adjustment feedthroughs, since at these conditions the filament provided sufficient light for the oven aperture to be seen through the skimmer. Once pressures less than $10^{-5}$ torr were achieved for the high temperatures generated by the filament alone, the oven high-voltage supply was set to +300 V and a small electron bombardment current was produced (0.02 A). Many neutral species can be ionized by an electric field of 300 V. Subsequent formation of a plasma and generation of arcing behaviour can eventually destroy the filament. It was found that the oven chamber pressure had to be less than $10^{-5}$ torr to prevent this phenomenon. At electron bombardment currents of 0.2 A for 300 V, a beam of barium was obtained which was sufficient to produce fluorescence via the cavity signal beam.

A single-pass beam directed vertically downwards through the glass fitting acted as the fluorescence monitor. A highly-attenuated beam (0.5 $\mu$W) parallel to it was directed down through the interaction region between the cavity mode and the atomic beam, and acted as the absorption monitor. The absorption monitor beam was aligned to be perpendicular to the top and bottom chamber windows, as shown by reflections. Once peak absorption reached 10-20% the fluorescence monitor beam was aligned such that the absorption and fluorescence peaks coincided to within ±10 MHz (the limit of resolution of the laser frequency tuner).

The signal beam was optimised in height with respect to the atomic beam by maximising the observed fluorescence. The cavity was then realigned and mode-matched. If possible only the folding mirrors outside the cavity were used so that the mirror position would be shifted as little as possible, but if necessary the output coupler was adjusted using the rotary feedthroughs. The cavity signal beam was judged to be satisfactorily perpendicular to the atomic beam and optical monitor laser beams if both the entrance and exit beam spots on the chamber windows passed within a few millimetres of the window centres. Given the length of the optics chamber (80 cm) and the narrow apertures through the piezoelectric stacks (6 mm), this restriction was sufficient to ensure that the Doppler
broadening due to misalignment of the cavity beam axis with respect to the atomic beam was less than 10 MHz.

Once the cavity signal beam was aligned the local oscillator beam could be combined with the output cavity signal beam (detuned from atomic resonance by several hundred MHz) on the detector beamsplitter BS3. After the overlap was achieved and the visibility was measured as described in Section 4.3, polarizer POL3 was rotated to increase LO beam power to 2 mW. This operation slightly decreased the beam overlap, which could be regained by maximising the noise modulation fringes on the spectrum analyser.

Having achieved a good beam overlap and the same power in each output arm of the beamsplitter, the common-mode subtraction of the homodyne detector was measured. When the beams from both output ports of the beamsplitter were subtracted by the 180 degree splitter, these peaks were too large to be completely eliminated by the 10-20 dB subtraction; thus it was necessary to select quiet window regions in the noise spectrum between 60 and 200 MHz to search for squeezing. Several frequencies were chosen and checked for residual classical noise by ensuring that the noise from each detector combined in quadrature for the balanced homodyne detector.

The experiment proper commenced by setting the oven temperature to a value which gave 80-85\% peak absorption ($\alpha=2.5-3.0$), thus setting the cooperativity $C$ to a value greater than 50. Once bistability was observed, the search for squeezing began. The quantities which could be varied continuously were the input intensity $Y$, atomic detuning $\Delta$, and the cavity detuning $\phi$.

The input intensity $Y$ was measured by a UDT PIN-6D photodiode which received a pick-off from the cavity signal beam reflected by a glass plate immediately before the input window. It was calibrated by inserting an NBS-traceable power meter (Newport 818-SL) into the beam for each new setting of the input power.

The atomic detuning was varied with a laser frequency tuner (which sent a low voltage signal to the laser external ramp input) in steps of between 10 MHz and 1 GHz, whilst observing the fluorescence from the fluorescence monitor beam. Line centre was defined as the fluorescence maximum. Once set to a particular value (often 300 or 600 MHz) the atomic detuning was checked for evidence of laser frequency drift between squeezing measurements by returning to the atomic line-centre. In general the laser frequency remained stable for periods up to 45 minutes. We note that $\Delta$ could not be varied in the experiment of Orozco et al. (1987) and Raizen et al. (1987) since their optical pre-pumping laser beam was shifted by a fixed frequency from the cavity signal beam by an AOM, and locked to a single frequency using an atomic fluorescence signal. Any change in $\Delta$ would upset this arrangement.

The cavity detuning $\phi$ was varied by changing the length of the cavity with the piezoelectric stacks. One stack was connected to a low-noise 300 V supply and used to correct manually for large scale motions and drift of the cavity. The other was supplied with a ramp of 2-4 V from the computer, which was sufficient to scan across the cavity linewidth.
It was difficult to hold cavity length constant by hand so most squeezing measurements were made by scanning across the cavity linewidth. The results of such a measurement were easy to interpret and relate to the theory since the entire cavity transmission profile was revealed. The cavity transmission was monitored using the weak beam reflected from the second surface of the beamsplitter. This beam was selected by a sharp-edged mirror, allowed to traverse a distance of 2 m, and apertured in front of a photomultiplier tube used to detect it, in order to separate the beam from scattered light also produced by the beamsplitter.

A disadvantage of the practice of scanning the cavity length was that since the cavity detuning and LO phase were varied simultaneously, best squeezing could be missed. LO phase scan rate was adjusted to scan in 4 s through approximately $10\pi$ (5 cycles) over a full cavity scan of approximately 3 cavity linewidths (or $5-6\kappa$). Cavity drift and jitter meant that it was not possible in this experiment to define the position $\phi=0$, corresponding to the empty cavity on resonance. As explained in Chapter 3, in the regime of dispersive bistability the peak cavity transmission is shifted from $\phi=0$ by up to seven cavity half-linewidths for $\gamma\Delta>200$ MHz. To obtain the reference point $\phi=0$ it would have been necessary to chop the atomic beam, or to keep the cavity completely stable by some form of active stabilisation. However, with knowledge of $C$, $Y$, and $\Delta$ the cavity detuning could be calculated from the state equation of optical bistability for a Gaussian transverse mode in a standing wave cavity.
Chapter 5: Bistability in atomic barium

In the classic model of optical bistability a near-resonant coherent pump drives an optical cavity which contains a collection of homogeneously broadened two-level atoms. This is the model which has proved most amenable to direct quantitative comparison between theory and experiment (Sandle and Gallagher 1981, Weyer et al. 1983, Orozco 1987, Rosenberger et al. 1991). Our aim in this chapter is to compare the experimentally-determined steady-state characteristics of the cavity-atom system with the standard model. In contrast to previous workers, however, our major interest is in the regime where the properties of the medium are almost entirely governed by the dispersive part of the atomic susceptibility, rather than the absorptive or mixed absorptive-dispersive regime. Best squeezing in our experiment was encountered in the dispersive configuration. Earlier workers have in general used sodium in atomic bistability experiments. To our knowledge no other bistability experiment has been performed in a barium atomic beam. The problems of using the sodium D lines as a two level atom are well understood. Choice of barium as the two level medium raises some new questions; for instance, the correct treatment of the presence of other isotopes of Ba with hyperfine structure, and the impact of transit broadening for barium, for which the atomic weight is much greater than that of sodium.

Our aim in this chapter is to test the validity of some of the underlying assumptions of the theory, to verify that we can quantitatively relate the normalised variables in the state equation to measured quantities, and to become familiar with the scale of the uncertainties associated with the measurement of the important variables, by analysis and experiment. It is also of interest to consider the extent and impact of various broadening mechanisms upon the steady-state conditions of bistability. Previously we have made the approximation of pure radiative broadening $\gamma_x = \gamma_y / 2$. In reality, as indicated in Chapter 4, there is a measurable Doppler component to the spectral width of the atomic line, and the Lorentzian component is not entirely due to the natural linewidth.

5.1 Absorptive and dispersive bistability

In absorptive bistability the driving laser is set on resonance with the optical cavity and with the atoms. The absorption coefficient is intensity dependent, and can be approximately written as

$$\alpha l = \frac{\alpha_0 l}{1 + I}$$ (5.1)

For sufficient cooperativity (that is, for sufficient nonlinearity and feedback) bistable behaviour can occur. As the intracavity power increases the absorption coefficient saturates, causing the intracavity power to increase further until there is a discontinuous transition to the upper branch, where the medium is bleached. As the pump power is reduced the atomic medium continues in effect to see a high-finesse cavity, and the switch returning the system to the lower branch occurs for a smaller value of incident power. There is therefore a dependence upon the history of the system, or hysteresis.

In the dispersive case an intensity-dependent saturable refractive index exists for the
atomic medium, with the form
\[ n = n_0 + \Delta n, \text{ where } \Delta n = \frac{\Delta I}{(1 + I)(1 + \Delta^2)} \]  

An increase in intracavity power \( I \) causes a decrease in refractive index. The cavity resonance frequency is caused to change towards the frequency of the driving laser field, leading to another increase in intracavity power. Ultimately the cavity is pulled on to the laser frequency. This high transmission state corresponds to upper branch.

Schematic diagrams of the hysteresis cycle of bistability are shown in Figures 5.1-1(a) and (b). The switching powers \( I_1, Y_1 \) and \( I_2, Y_2 \) are defined in the diagrams. (Note that the intracavity power is generally known as \( X \) in the literature of optical bistability, but for the sake of consistency with the usage of Reid (1988), we continue to use \( I \) to represent the intracavity intensity normalised by the saturation intensity on resonance.) The upper branch approaches the line \( I = Y \) for absorptive bistability since the medium effectively becomes transparent as shown in Fig. 5.1-1(a), but has some lesser slope for \( \Lambda, \phi \neq 0 \), as shown in Fig. 5.1-1(b). The size of the hysteresis loop is always a maximum for absorptive bistability.

In the absorptive case the normalised atomic detuning \( \Delta \) and cavity detuning \( \phi \) are set to zero, leaving a function which is dependent only on the cooperativity \( C \) and the normalised input and output powers \( Y \) and \( I \). The state equation for optical bistability in the plane wave ring-cavity case (Eq. 2.22) reduces to
\[ \frac{Y}{I} = \left(1 + \frac{2C}{1 + I}\right)^2 \]  

Previous workers have concentrated on this regime, and after a decade of work have achieved excellent quantitative agreement with a modified theory for the switching powers as a function of cooperativity \( C \) (Rosenberger et al. 1991, and references therein). The cooperativity \( C \) was varied by increasing or decreasing the oven temperature and thus varying the atomic number density, and the input and output powers were measured with calibrated photodiodes at each setting of \( C \). Knowledge of the cavity characteristics allowed the measured switching powers to be converted to the normalised parameters \( I \) and \( Y \). It was found necessary to extend the theory to the case of a Gaussian mode in a standing-wave cavity and to include the effects of transit and residual Doppler broadening. Once these modifications were made absolute agreement was claimed, with uncertainties in \( Y \) and \( I \) of less than 12% (Rosenberger et al. 1991).

Numerous technical difficulties are encountered in the absorptive experiment. The laser frequency must be finely controlled to be held on atomic resonance, and the optical cavity must be held on resonance with the pump laser beam. Cavity stability is limited by thermal (oven heating) and mechanical (water-cooling, backing vacuum pumps) vibrations of the cavity, and by the electrical noise upon the voltage supplied to the piezoelectric stacks. The cooperativity parameter \( C \) is increased by raising the oven temperature to increase the atomic number density. At each point the apparatus must be permitted to stabilise thermally. If many points are required the experiment becomes time-consuming. The longer the time taken, the greater the likelihood that the pump laser beam will drift in
Figure 5.1-1: Schematic diagrams illustrating (a) absorptive bistability and (b) dispersive bistability, where $Y$ is the normalised incident intensity and $X$ is the normalised transmitted intensity. Note that the slope of the upper branch is a maximum in the absorptive case, as is the size of the hysteresis loop. The diagrams also define the switching points $I_1, I_2, Y_1, Y_2$. 

\[ \Delta, \phi = 0 \]

\[ \Delta, \phi \neq 0 \]
frequency and power, and that the cavity will require realigning and mode-matching. This activity may change the throughput of the cavity, thus changing the normalised input-output power parameters I and Y. Absorptive bistability in barium was certainly observed during our work, and experiments were attempted, but the inconsistency of the results frustrated quantitative analysis.

With the apparatus available to us, a preferable procedure involved fixing the cavity input power and scanning the cavity length with a piezoelectric stack, rather than fixing the cavity length and scanning the input power. This frequency-tuning procedure yields cavity transmission functions of distinctive asymmetry, as demonstrated by Sandle (1980) and Sandle and Gallagher (1981). If the cavity length is scanned in both directions the characteristic hysteresis loop is revealed by this procedure, as shown in the experimental plot of Fig. 5.1-2. An atomic vapour was employed in the experiment of Sandle and Gallagher and large discrepancies between experiment and theory were reported, although the overall phenomenology of bistability was described adequately by the theory. Doppler broadening would be expected to play a large part in modifying the expected critical cooperativity and switching powers in such a system. Our work, using a well-collimated atomic beam, could be expected to show greater agreement with the theory.

5.2 Model and state equation

The standard model of optical bistability, where a single plane-wave cavity mode interacts with a homogeneously-broadened atomic medium in a high-finesse ring cavity, has been considerably refined and elaborated by interaction with experiment in recent years (Rosenberger et al. 1991 and references therein). Various assumptions have been examined and relaxed separately and together to discover the effect of a spatially-varying mode structure, Doppler broadening, transit broadening, standing-wave cavity structure, a confocal cavity (leading to a set of degenerate cavity modes), absorptive and scattering losses in the cavity mirrors; imperfect mode-matching, and so on.

One of the primary assumptions made in order to obtain a neat and tractable analytic solution to a demanding problem was the so-called 'mean-field' limit (Bonifacio and Lugiato 1976, 1977). This limit requires $\alpha l \to 0$, $T \to 0$: that is to say, the cavity mirror transmissivity $T$ is required to be low (a high-finesse cavity) and the atomic density is also assumed to be low enough so that in one round trip of the optical cavity the field amplitude remains unaltered. Lugiato (1984) examined the effect of relaxing the mean field limit. He compared the exact stationary solutions graphically to the mean-field limit solution, and demonstrated that for atomic densities as high as $\alpha l=10$, and transmissivity $T=0.1$, the exact solution was close to the solution derived in the mean field limit. This is so particularly for the dispersive regime, where the effect of absorption along the cavity round-trip is minimal. In our experiments typical values are $\alpha l=3$, and $T=0.02$, so that the mean-field condition is preserved.

We present the state equation for a Gaussian transverse mode structure and a standing-wave cavity, but will assume for the time being that residual Doppler broadening is negligible. The state equation becomes
Figure 5.1-2: Hysteresis loop in dispersive bistability for cooperativity $C > 50$, atomic detuning 200 MHz, where cavity length was swept forwards and backwards. Lines showing the switching points were drawn in to aid clarity.
\[ Y = I \left[ (1+2C\chi)^2 + (\Theta - 2C\Delta\chi)^2 \right] \]  
\hspace{1cm} (5.4)

where for a Gaussian transverse mode in a standing-wave cavity

\[ \chi = \frac{3}{2I} \ln \left[ \frac{1}{2} \left( 1 + \frac{1}{2} \left[ 1 + \frac{8I}{3(1+\Delta)^2} \right] \right) \right] \]  
\hspace{1cm} (5.5)

where I and Y are related to experimentally obtainable quantities by

\[ Y = \frac{3P_0 T_0}{\pi w_0^2 I_s T_2}, \]  
\hspace{1cm} (5.6)
\[ I = \frac{3P_i}{\pi w_0^2 I_s T_2} \]

with \( P_i \) and \( P_o \) the cavity input and output powers respectively, \( T_0 \) is the throughput on resonance of the empty cavity, and \( T_2 \) is the transmissivity of the output mirror; \( w_0 \) is the cavity beam waist, (defined at the 1/e point of the Gaussian mode), and both I and Y are normalised by the saturation intensity on resonance, \( I_s \). Note that the definitions (5.6) differ from earlier formulations by the same authors (see Kimble et al. 1983).

As shown by Rosenberger et al. (1991) \( I_s \) is given by

\[ I_s = \frac{4\pi^2 \hbar \gamma_{\perp}}{3\lambda^2} \]  
\hspace{1cm} (5.7)

and we may readily compute the value for the \(^{138}\)Ba transition at 553.5nm, having \( \gamma_{\perp} = 9.75 \text{ MHz} \) in the case of pure radiative broadening, as \( I_s = 146 \text{ Wm}^2 \). The corresponding value for atomic sodium is \( I_s = 64 \text{ Wm}^2 \). Thus for a similar optical cavity, the switching powers for Ba are required to be more than twice as high as those for Na.

Eq. (5.5) defining \( \chi \) was derived for the case of a Gaussian transverse mode in a standing-wave cavity by integrating over the mode, as described in Chapter 2, and was initially derived by Drummond (1981). In this work we adopt the normalisation of Drummond, who defines I, Y and C in such a way that convergence in the dispersive limit is obtained for all types of optical cavity. I and Y are defined with respect to the power averaged over the mode area (\( \pi w_0^2 \)), while the terms in \( (1/T_2) \) and \( (T_0/T_2) \) account for absorptive and scattering losses in the cavity mirrors, and for the cavity asymmetry; (mirrors 1 and 2 may have different transmissivities).

Cooperativity C is defined for the standing wave cavity with a Gaussian transverse mode as (Eq. 1.4)

\[ C = \frac{\alpha_\perp \ell F}{2\pi} \]
Experimentally, the true small-signal absorption experienced by the cavity mode is not available. In our system a monitor beam is passed through the interaction region at right angles to the cavity axis, giving a measurement of the optical depth $\alpha_n l$. Since both the absorption monitor beam and the cavity mode were aligned for each experiment centrally through the atomic beam such that fluorescence was maximised simultaneously, we take $\alpha_0 l = \alpha_n l$.

Figure 5.2-1 shows an absorption scan taken while the dispersive bistability experiment was in progress, fitted by a Voigt profile with a 10 MHz Gaussian component and a Lorentzian fraction of 21 MHz, taking into account the abundance of the other isotopes of barium. The atomic number density was determined to be $N = 3 \times 10^{16} \text{m}^{-3}$ from the fitted curve, giving $\alpha_0 l = 3.1$ from the equation (Sandle and Gallagher 1981)

$$ \alpha l = \frac{Nl\pi c^2 \gamma}{\omega_0^2 \gamma_\perp^*} \quad (5.8) $$

where $l$ is the length of the atomic medium, $\gamma$ is the natural linewidth of the transition, $\gamma_\perp^*$ is the fitted dephasing rate of the atomic polarization and $\omega_0$ is the angular frequency of the transition. The frequency calibration of the experimental profile was determined by taking the detuning between the $^{138}$Ba peak and the second peak on the high-frequency side of the main transition to be 129 MHz. A degree of nonlinearity in the laser frequency scan is responsible for the lack of overlap of the experimental and theoretical isotope peaks near 300 MHz.

The cavity finesse $F$ is measured experimentally as the ratio of the cavity free spectral range to the FWHM of the empty cavity transmission function. A typical transmission function is shown in Fig. 5.2-2. It may be observed directly from the figure that the finesse of the optical cavity is greater than 100. For cavities with finesse greater than 200-300 it becomes difficult to measure the finesse directly, and modulation and ringing techniques must be employed. For our cavity of finesse ~140 the direct method is satisfactory. The FWHM of the peak can be measured within ±5% using the horizontal expansion feature of a digital oscilloscope, so that typically the finesse can be measured within ±7% overall.

An optical cavity can be completely characterised by determination of the finesse $F$ and the throughput on resonance $T_0$. They are related by

$$ F = \frac{\pi \sqrt{R_{\text{eff}}}}{1 - R_{\text{eff}}} = \pi \left[ \frac{T_0}{T_1 T_2} \right]^{1/2} \quad (5.9) $$

$$ R_{\text{eff}} = \sqrt{R_1 R_2} $$

where all these quantities have been previously defined. The second definition of $F$ (valid for $(R_{\text{eff}})^{1/2} << 1$) is more useful in general since the empty cavity throughput on resonance, $T_0$, and mirror transmissivities $T_1$, $T_2$, can be measured accurately, whereas the
Figure 5.2-1: Absorption trace taken during dispersive bistability experiment, fitted by a theoretical Voigt profile with a 10 MHz Gaussian component and a Lorentzian fraction of 21 MHz.
corresponding reflectivities and absorption coefficients are not known. \( T_0 \) can be calculated by (Siegman 1986):

\[
T_0 = \frac{T_1 T_2}{(1 - R_{\text{eff}})^2}
\]  

Since \( T_1 + R_1 + A_1 = 1 \) and \( T_2 + R_2 + A_2 = 1 \) it is convenient to include the absorption terms in the equations above via

\[
R_{\text{eff}} = \sqrt{R_1 R_2} = \left[ (1 - T_1 - A_1)(1 - T_2 - A_2) \right]^{1/2}
\]

\[
= \left[ 1 - (T_1 + A_1)(1 - (T_2 + A_2)) + (T_1 + A_1)(T_2 + A_2) \right]^{1/2}
\]

This is given by

\[
R_{\text{eff}} = [1 - (T_2 + A_2)]^{1/2}
\]

for the case of a highly asymmetric cavity where \((T_1 + A_1) \ll (T_2 + A_2)\), as is true for our experiment.

\( T_0 \) is known to depend on the efficiency of mode-matching into the cavity. If mode-matching is imperfect then for a non-confocal resonator some of the power incident on the cavity is lost into higher order transverse modes, rather than being transferred completely into the desired TEM\(_{00}\) Gaussian mode. This can be seen directly from the transmission function obtained by scanning the cavity length, as shown in Fig. 5.2-2. An approximate measure of the mode-matching efficiency may be obtained by taking the ratio of the power in the TEM\(_{00}\) mode to the summed power in all other modes. This calculation indicates that the mode-matching efficiency for the dispersive bistability experiment was 86 %. We correct for this factor by dividing the measured throughput on resonance, 1.1 %, by 0.86 to obtain \( T_0 = 1.3 % \). This is in agreement with the throughput measured at times when mode-matching efficiency was close to 100%.

Although our mode-matching was always imperfect, and consequently the closest higher order modes were always excited to some degree, the nearest transverse mode was approximately 1.5 GHz from the fundamental mode. This distance was far greater than the detunings and Rabi frequencies used in the experiments. The cavity free spectral range was typically 35 GHz. The single cavity mode assumption can be taken as valid in these circumstances. The FWHM of the cavity resonance, 250-300 MHz, is large enough so that despite mechanical instabilities of the cavity mount as compared to the laser frequency stability of 1 MHz rms, that the cavity could remain on resonance for periods of up to 15 s.

Straightforward calculations indicate that transit broadening is unlikely to be as significant a factor in bistability experiments using Ba as when using Na. An oven temperature of \( T=1050 \) C corresponds to a most probable velocity of \( u_p=400 \) m/s (assuming a effusive thermal source), where the equation for the most probable velocity is \( u_p = (2kT / m)^{1/2} \), \( m \) being the atomic mass. The large atomic mass of barium compared to that of sodium...
Figure 5.2-2: Experimental cavity transmission, with piezoelectric stack scanned through one cavity free spectral range of 35 GHz (the two narrow peaks on the RH side are simply the result of the sawtooth voltage ramp flyback). The imperfectly-suppressed higher order modes are visible on the left (high-frequency) side of the fundamental cavity modes. Mode-matching efficiency was calculated as 86 % in power.
(137.34 compared to 22.99) is responsible for the relatively low calculated velocity, despite the higher oven temperature necessary to generate a beam of barium. The transit time across a cavity mode diameter of 180 µm is 450 ns, equivalent to 27 transverse atomic lifetimes. This value corresponds to a linewidth contribution of the order of \( \frac{1}{(450 \text{ ns}}) = 2 \text{ Mrads/s} \), much less than \( \gamma_B = 119 \text{ Mrads/s} \). We may thus neglect the effect of transit broadening in this system.

We can also assume that for our system the variation in diameter of the mode along the cavity axis is negligible, since the near-planar conformation of the spherical mirrors ensure that the beam radius at the mirrors is approximately equal to the waist at the centre of the cavity. The expression for the cavity mode waist for two mirrors with radii of curvature given by \( R_z_1 \) and \( R_z_2 \) separated by length \( L \) is (Corney, p. 366),

\[
\psi_0 = \left( \frac{\lambda}{\pi} \right)^2 \left\{ \frac{L(R_z_1-L)(R_z_2-L)}{(R_z_1+R_z_2-2L)^2} \right\}
\]  

(5.13)

The corresponding expression for the beam radius at \( M_1 \) is

\[
\psi_1 = \left( \frac{\lambda}{\pi} \right)^2 \left( \frac{R_z_1 L}{(R_z_1+R_z_2-L)} \right) \left( \frac{R_z_2-L}{R_z_1-L} \right)
\]  

(5.14)

(The beam radius at mirror 2, \( w_2 \), is obtained by reversing the indices in the expression above). Using these expressions it can be computed directly that for a cavity of length 4.3 mm with mirrors of radii of curvature \( R_z_1=1m \) and \( R_z_2 = 2m \), \( w_0=97.1 \text{ µm} \), \( w_1=97.2 \text{ µm} \) and \( w_2=97.1 \text{ µm} \). Evidently the variation in diameter of the cavity mode as it crosses the atomic beam can be neglected for our experiment. The same conclusion could be arrived at more directly by noting that in general for the case of \( L<<R_z_1,R_z_2 \) the equations above both reduce to

\[
w^2 = \left( \frac{\lambda}{\pi} \right)^2 \frac{LR_z_1R_z_2}{R_z_1+R_z_2}
\]  

(5.15)

so that the mode radius is constant along the entire cavity axis. An accurate estimate of the cavity mode waist is important because it affects the overall scaling of the values of \( I \) and \( Y \) as \( 2(\Delta w_0/w_0) \), where \( \Delta w_0 \) is the uncertainty in \( w_0 \). We see in Fig. 5.2-2 that the second transverse mode is approximately 10% of the fundamental mode in power. It has been shown (Anderson 1984) that the presence of power in this mode indicates a fault in the focusing of the laser beam; the size and position of the laser beam waist is incorrectly matched to the cavity.

### 5.3. Experiment

The experimental arrangement was essentially that of the squeezing experiment, except that the phase-sensitive detection system was replaced by a power meter (Newport 818-SL)
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at the cavity output (see Fig 5.3-1).

The following values were measured before and after the experiment. The cavity was a spherical near-planar Fabry-Perot resonator, with a mirror spacing of 4.3 ± 0.1 mm. The free spectral range of a cavity of this length was 35 ± 0.8 GHz. The input coupler had transmissivity T = 0.01% and a radius of curvature of 1 m. The output coupler had T = 3.6 % and radius of curvature 2 m. The finesse of the cavity was measured to be 136 ± 9, and the throughput on resonance was (1.1 ± 0.2)%. The cavity beam waist size was calculated from the cavity length and known radii of curvature to be 97 ± 3 μm.

A beamsplitter in front of the cavity directed a fraction of the input beam to a PIN-6D photodiode as input intensity monitor, and cavity output power was measured by a Newport powermeter (815 SL). Both signals were sent to the analogue data acquisition system. Total transmission through the beamsplitter and the vacuum chamber front window was 87 ± 8 %.

The atomic beam had diameter 2.3 mm at the interaction region, calculated from knowledge of the collimation geometry. The (chopped) absorption monitor beam passed vertically downwards through the atomic beam within the interaction region and was captured on an HP photodiode and sent to a lock-in amplifier for discrimination against ambient light. Absorption measurements were made at the start, middle and end of the experiment. Optical depths of α₀ℓ = 3.1 were measured during the experiment, as explained in section 5.2.

The input and output powers were initially calibrated for the empty cavity using the Newport powermeter and the ADA signal gains were not adjusted subsequently. The atomic detuning was set using a laser frequency controller calibrated in units of 10 MHz. The frequency controller was calibrated against a Tropel 300 MHz Fabry-Perot etalon, and the uncertainty in setting a laser frequency difference of 300 MHz was found to be 8%. The optical cavity length was scanned by a small amount using a 2 V ramp from the computer, for a fixed atomic density and atomic detuning. A 4-second scan over the cavity transmission profile was acquired.

At the outset of the experiment the finesse and throughput were measured for the optical cavity, with the laser detuned from the spectral line. Calibration measurements were then performed using the Newport powermeters. Four different input powers were employed, ranging over a factor of two, from 16 mW to 7.5 mW. The cooperativity C was C=65±8, and the atomic detunings set at each fixed input power were 0, 100, 200, 300, 600 and 900 MHz. The laser was always detuned to the low frequency side of the atomic transition, in order to avoid any complications which might arise in pumping transitions of other isotopes.

5.4 Results

There was a striking consistency and similarity between the four sets of results, of which one set (for 10.7 mW in) is shown in this section. At each atomic detuning, the shape of
Figure 5.3-1: Schematic diagram of experimental arrangement for experiment in dispersive optical bistability. The arrangement is essentially a simplified version of the squeezing experiment, and all abbreviations employed were defined for Fig. 4.1-1. The phase-sensitive detection system was not necessary and was replaced with a single photodetector at the cavity output. Cavity input and output intensities were recorded on the data acquisition system.
the cavity transmission was distinctive and varied little over the range of input powers used, except for an overall scaling in power. The range of powers in terms of the normalised variable $Y$ was 1650 to 3500. If we consider the switching powers in absorptive bistability in the plane wave ring cavity case and the standing wave Gaussian cases (Figs 5.4-1) we see that for both cases, for $C = 65$, these values lie within the bistable regime.

One set of results, for the input power $10.7 \pm 0.3 \text{ mW}$, are compared with the plane-wave ring cavity theory in Fig 5.4-2, and with the standing-wave Gaussian mode theory in Fig. 5.4-3. The experimental results are shown on the left side of the page. Experimental cavity output powers were converted to the normalised variable $I$ via Eq. (5.6). They are compared to $I = f (C, \Delta, \phi, Y)$, calculated by numerical solution of the cubic state equation Eq. (5.2) or Eq (5.6). The vertical axis $I$ is absolute. The horizontal axis is scaled in terms of the cavity half-linewidth $\kappa$, with the scale given in the theoretical plots. The absolute value of the cavity detuning $\phi = (\omega_L - \omega_C)/\kappa$ was not known because the cavity had a tendency to drift in length. The theoretical plots are given in the same units, so that there is an absolute comparison between theory and experiment. The range, as distinct from the scale, of the cavity detunings was chosen such that the cavity transmission peaks for the theoretical plots were aligned with the peaks in the experimental graphs (in order to aid visual comparison).

The agreement of the plane-wave ring-cavity theory with the experimental results appears best for the cases of the 600 MHz and 900 MHz detunings i.e. towards the dispersive limit. For a detuning of 900 MHz the cavity transmission function is almost identical to that expected for the empty cavity; the output $I$ is equal to the input $Y$, the function is nearly symmetrical and the FWHM of the peak is equal to the empty cavity linewidth $2\kappa$.

The agreement of the spatially varying field mode theory also appears excellent for these atomic detunings. The cavity profiles calculated from both the plane wave ring cavity and standing wave Gaussian mode theories appear effectively identical. This is not surprising, since the variables $I$ and $Y$ were normalised in such a way as to unify the results for all types of cavity and wave in the dispersive limit (Drummond 1981).

Examining the agreement of the theories with experimental observation for the cases of 100 MHz, 200 MHz and 300 MHz, we note that the boundary of bistability in the experiment seems to be encountered at 300 MHz. The plane wave theory shows switching at this atomic detuning, but the cavity transmission profile from the standing wave Gaussian theory, although steep, is single-valued everywhere. An increase in the cooperativity parameter $C$ within the experimental error bounds of 12% would ensure that the spatially varying theory also showed switching at this detuning.

At the atomic detuning 200 MHz both theories give a good qualitative and quantitative description of the experimental results. The plane wave theory appears a little better, but the suggested modification of $C$ would be likely to change the shape of the cavity profile at the peak, where the standing wave Gaussian theory seems less convincing. At 100 MHz both theories overestimate the maximum cavity power output by a factor of a third or more, although the shape of the transmission functions is well-described. An increase in $C$ or an increase in the dephasing rate of the atomic polarization $\gamma_1$, (which normalises the atomic detunings) would both have the effect of scaling down the cavity transmission
Figure 5.4-1: Switching points of the normalised incident intensity $Y$ versus cooperativity $C$, determined from the state equation of optical bistability for (a) plane wave in a ring cavity (b) Gaussian mode in a standing wave cavity.
Figure 5.4-2: Absolute comparison of experimental data (LH pictures) and model simulations assuming a plane wave in a ring cavity, (RH pictures), for an experiment in dispersive bistability. Cooperativity $C$ was constant at 65, input power was constant at 10.7 mW, and atomic detuning was varied from 0 to 900 MHz as indicated on the diagrams. Each plot presents normalised cavity transmitted intensity versus cavity detuning $\phi$, varied by scanning the cavity length.
Figure 5.4-2: continued
Figure 5.4-3: Absolute comparison of experimental data (LH pictures) and model simulations assuming a Gaussian transverse mode in a standing-wave cavity, (RH pictures), for an experiment in dispersive bistability. Cooperativity $C$ was constant at 65, input intensity was constant at 10.7 mW, and atomic detuning was varied from 0 to 900 MHz as indicated on the diagrams. Each plot presents normalised cavity transmitted intensity versus cavity detuning $\phi$, varied by scanning the cavity length. No theoretical solution existed for the case of $\Delta = 0$, $C = 65$ in the standing wave Gaussian case.
$\gamma I \Delta = 300 \text{ MHz}$

$\gamma I \Delta = 600 \text{ MHz}$

$\gamma I \Delta = 900 \text{ MHz}$

Figure 5.4-3: continued.
to match the experimental results.

Both theories fail badly at zero detuning. The standing wave Gaussian theory has no upper branch solution for $C=65$, $Y=2300$, $\Delta=0$, so no theoretical profile was plotted. Although the plane wave solution appears superficially plausible, it gives a ratio of maximum cavity output to cavity input of 87%, whereas experimentally this ratio is 62%. We rewrite the state equation for $\Delta, \phi = 0$ as

$$C = \frac{I}{2} \left( \frac{\sqrt{Y}}{I} - 1 \right)$$

and discover that for the experimental values of $I,Y$, believed to be correct within 15%, a value of $C=193$ is required to give the observed degree of absorption. This values is far outside experimental error bounds for $C$. If we rewrite the corresponding standing wave Gaussian equation to solve in terms of $C$ we find that for all four sets of data ($Y$ varied by a factor of two) $C$ is required to be $42 \pm 6$. This suggests that the absorptive and dispersive cases must be treated separately, and that the absorptive cooperativity parameter is modified by effects which are not important in the dispersive regime. The most likely explanation is that the amount of Doppler broadening present in our experiment (10 MHz, compared to the natural linewidth of 19.5 MHz for the barium resonance transition), substantially reduces the effective cooperativity in the absorptive regime. The dispersive regime is dominated by the Lorentzian wings of the line, and the Lorentzian component is essentially the natural linewidth in this experiment.

5.5 Conclusion

We conclude from the analysis above that care must be taken in treating the absorptive case where the amount of residual Doppler broadening is as great as it is in our experiment. The steady-state solutions in the dispersive regime do not seem to be affected greatly by the inhomogeneous broadening, as the value of $C=65$ estimated from the maximum small-signal absorption and the cavity finesse gives a reasonable fit to experimental results for both the plane-wave and standing-wave Gaussian theories. The theories converge for large atomic detunings, and give similar results. A small rescaling of $C$ or $\gamma_\perp$ seems to be all that is required to match the results of plane-wave ring-cavity and standing-wave Gaussian-mode calculations.
Chapter 6: Squeezed-state generation in a cavity-atom system

This chapter presents results of the experiments in squeezed-state generation in the system of optical bistability. Representative results are shown, discussed, and compared with the results of the microscopic quantum plane wave ring cavity theory of squeezing discussed in detail in Chapter 2 and Chapter 3. Experimental results are also compared with those of previous workers performing short-cavity atomic squeezing experiments, and discussed with respect to the theoretical predictions of Chapter 3. The large coherent amplitude of the squeezed beam permitted the phase of the squeezing with respect to the cavity beam phase to be determined, thus allowing the orientation of the squeezing uncertainty area to be determined and compared to theory.

6.1 Experiment

In the search for squeezing the experimental parameters available to be varied are the cooperativity $C$, the atomic detuning $\Delta$, the cavity detuning $\phi$, the incident field $Y$, and the spectrum analyzer frequency $v$. The cooperativity could be varied only by changing the oven temperature. It was considered better to fix this value, since it was known from theory that a change in $C$ would be likely to modify the optimum value of all other parameters. In addition, although high values of optical depth $\alpha_{o}$ (up to 3.5) could be achieved with our atomic beam, and the electron bombardment system allowed rapid changes of temperature, this would lead to more rapid depletion of the barium load and to more rapid degradation of the mirror surfaces by scattered barium particles. Hence a value of 80-90% peak absorption was usually chosen to give $C \sim 50-80$.

In contrast to the experiments of the Austin group, whose atomic detuning was locked at a pre-set value decided upon by reference to the model, the atomic detuning in our experiment could be varied readily over a range of 1 MHz to 1 GHz using a laser frequency controller. Cavity detuning could also be varied over an arbitrarily large range. Cavity input power $P_{c}$ was variable over a range from 1 mW to 20 mW, using the crossed polarizers before the cavity. With due care taken, varying the rotation of the polarizer did not misalign the mode-matching into the cavity, but this variable was still less easy to modify than the atomic and cavity detunings. As described in Chapter 4, several values of detection frequency were chosen in regions of the rf spectrum free of classical noise, and these were investigated sequentially.

After the initial experiments it was found that a good starting point for a search for phase-sensitive noise and squeezing was to set the initial parameter values to an atomic detuning of 300 MHz on the low frequency side of the atomic transition, an input power of about 10 mW, an analysis frequency of between 100-150 MHz, and to tune the cavity across the (shifted) cavity resonance. The oven temperature was then raised to give the optimum peak absorption (as seen on the absorption monitor beam) of approximately 85%; this generally resulted in the observation of phase sensitive noise. The phase of the local oscillator beam was varied continuously using the galvo-plate so as to show 5-10 fringes of phase-sensitive noise on the spectrum analyzer for the 4-second cavity
transmission scan. Once phase sensitive noise was observed parameters \( \Delta, \omega, \) and \( I \) (as a function of input intensity \( Y \)) were varied about their initial values so as to optimise squeezing, and to observe the dependence of the squeezing upon the parameters.

When the experiment was operated in a regime where the cavity showed bistable response, we found noise suppression of up to 10% on the upper branch of the bistability curve. Subsequent optimisation of the experimental parameters for best noise suppression led to a regime of high optical depth, high input power and large atomic detuning. In general these conditions the cavity did not in general display explicit bistability, only some jitter and switching due to fluctuations in the cavity length. Measurements carried out across the available range of detection frequencies revealed that the squeezing was broadband in character, being observed between 95 and 155 MHz.

The accurate determination of the quantum noise level was the crucial element in the measurement of the squeezing. In all our experiments the quantum noise level was measured by blocking the cavity output beam, giving a noise spectrum which was due to the local oscillator alone. This measurement was usually taken between two measurements of the noise spectrum of the combined LO and cavity beams, and within 1-2 minutes of these. It is implicitly assumed that the power of the LO did not change between the measurement of the quantum noise level and the measurement of the squeezing trace. This assumption can be tested by calculating the visibility of the fringes for given cavity beam and LO powers, using the equations (Born and Wolf 1970),

\[
I_{\text{max}} = I_{\text{cav}} + I_{\text{LO}} + 2\sqrt{I_{\text{cav}}I_{\text{LO}}} \\
I_{\text{min}} = I_{\text{cav}} + I_{\text{LO}} - 2\sqrt{I_{\text{cav}}I_{\text{LO}}} 
\]

\((6.1)\)

\(I_{\text{max}} \) and \( I_{\text{min}} \) can be measured directly from the analogue data acquisition traces corresponding to each noise trace, as can \( I_{\text{cav}} \) and \( I_{\text{LO}} \), using calibration traces taken during the experiment. A term for the efficiency of mode-matching must of course be included in this calculation, as a proportionality constant beside the cross term.

Conversion of the measured squeezing in units of power (dBm) to a linear scale and correction for the amount of electronic noise is facilitated by introduction of intermediate variables \( r \) and \( s \), defined in terms of the electronic noise power \( N_e \), the shot noise power \( N_{\text{LO}} \) and the noise power attributable to the squeezing \( N_{\text{sq}} \):

\[
r = 10\log\left(\frac{N_e + N_{\text{sq}}}{N_e}\right) \\
s = 10\log\left(\frac{N_e + N_{\text{LO}}}{N_e}\right) 
\]

\((6.2)\)

Quantities actually measured are \( N_e \) (with all light sources blocked), \( N_e + N_{\text{LO}} \) (the sum of the electronic noise and the quantum noise due to the local oscillator alone, with the cavity beam blocked) and \( N_e + N_{\text{sq}} \) (the sum of the electronic noise, and the noise due to the
interference of the local oscillator and the squeezed cavity beam). r and s are therefore known, and the relative noise level $\phi$. (in dBm) of fluctuations below the quantum noise limit can be determined by

$$\phi = 10 \log \left( \frac{N_{sq}}{N_{LO}} \right) = 10 \log \left( \frac{10^{r/10} - 1}{10^{s/10} - 1} \right)$$

(6.3)

On a linear scale we define the noise enhancement or reduction about the quantum noise level $R_\pm(\omega)$ as

$$R_\pm(\omega) = 10^{\phi/10}$$

(6.4)

The measured noise reduction is related to the spectrum of squeezing via the measured losses defined in Chapter 4 by (Wu et al. 1987, Orozco et al. 1987)

$$R_\pm(\omega) = 1 + \frac{\rho T_p \alpha}{\eta^2 S_\pm(\omega)}$$

(6.5)

The shot noise limit was given by the signal level when the cavity output was blocked. Any reduction below this level corresponds to squeezing. We note, however, that depending on the cavity detuning, there may be significant power in the cavity output. The quantum noise corresponding to this power, $q_{\text{noise}_{\text{AV}}}$, should be added in quadrature to the local oscillator quantum noise $q_{\text{noise}_{\text{LO}}}$ to obtain the actual shot noise level, $q_{\text{noise}}$; that is, the squeezing measured when taking the local oscillator noise alone to be the shot noise level represents a minimum estimate of the actual squeezing. The formula for the calculation of the actual shot noise level is;

$$q^2_{\text{noise}} = q^2_{\text{noise}_{\text{LO}}} + q^2_{\text{noise}_{\text{AV}}}$$

$$= q^2_{\text{noise}_{\text{LO}}} \left( 1 + I_{\text{AV}} / I_{\text{LO}} \right)$$

(6.6)

### 6.2 Observation of squeezing

Three sets of data are presented here, referred to as cases 1, 2 and 3, which show the variety of regimes for which phase-sensitive quantum noise was observed and give an indication of the way in which modelling of the system was carried out (see Figs. 6.2-1, 6.2-2, 6.2-3). In each case, the noise-power traces were taken at a fixed rf detection frequency with a spectrum analyser resolution bandwidth of 300 kHz and a video bandwidth of 10 Hz. The corresponding cavity output is shown below each rf trace. The experimental parameters corresponding to each case are summarised in Table 6.2-1. The experimental data in these cases is not corrected for electronic noise and the cavity beam contribution, as in these initial pictures we emphasise the location of the phase sensitive noise and squeezing with respect to the steady-state response of the cavity, and with the respect to the significant parameters. The simpler plane-wave ring-cavity model was used for these preliminary comparisons. Both the location and size of the noise (traces c) and the steady-state response (traces d) are in good qualitative agreement with the experiment.
Figure 6.2-1. Case 1. (a) Experimental results obtained for an atomic detuning of +200 MHz, cavity input power 34 mW, cooperativity C=50 and detection frequency 250 MHz. The local oscillator phase is scanned at a constant rate and the cavity length is varied to sweep through resonance. Trace (i) Phase sensitive noise on the low transmission side of the cavity, but no squeezing; trace (ii) quantum noise limit, obtained by blocking the cavity output signal for the same experimental conditions. (b) Experimental cavity output power. (c) Modelling of the experiment for q=0.08, C=50, Δ=20.5, 2πν/κ = 3.0, φ scanned through 3 cavity linewidths, I optimised for best squeezing. Spectral variance V=1 for a coherent state and V=0 for perfect squeezing (d) Cavity output power, for model parameters as in (c). The horizontal axis shows the detuning of the cavity from the empty-cavity resonance at zero, in units of cavity half-linewidth κ.
Figure 6.2-2: Case 2. All axes, units and experimental procedure are as in Fig. 6.3-1. (a) Experimental results, for an atomic detuning of +200 MHz, cavity input power 10.6 mW, cooperativity C=80, detection frequency 146.5 MHz. Trace (i) Phase sensitive noise on the upper branch, with some squeezing; trace (ii) quantum noise limit. (b) Experimental cavity output power. (c) Modelling of the experiment; q=0.08, C=80, Δ=20.5, 2πν/κ =1.2, φ scanned through 5 cavity linewidths, I optimised. (d) Cavity output power, for model parameters as in (c).
Figure 6.2-3. Case 3. All axes, units and experimental procedure are as in Fig. 6.3-1. (a) Experimental results obtained for an atomic detuning of +300 MHz, cavity input power 13 mW, cooperativity C=57, detection frequency 150.5 MHz. Trace (i) Phase sensitive noise observable across the whole cavity transmission, showing squeezing of 18 ± 3%; trace (ii) quantum noise limit. (b) Experimental cavity output power. (c) Modelling of the experiment with q=0.08, C=57, Δ=33, $2\pi\nu/\kappa=0.69$, $\phi$ scanned through 3 cavity linewidths, I optimised. (d) Cavity output power, for model parameters as in (c).
Table 6.2-1: Experimentally-measured parameters for data presented in Figs. 6.2-1, 6.2-2, and 6.2-3. In all cases $q = 0.08$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detection frequency (MHz)</td>
<td>250</td>
<td>146.5</td>
<td>150.5</td>
</tr>
<tr>
<td>Atom-laser detuning (MHz)</td>
<td>+200</td>
<td>+200</td>
<td>+300</td>
</tr>
<tr>
<td>Input power (mW)</td>
<td>34</td>
<td>10.6</td>
<td>13</td>
</tr>
<tr>
<td>Cooperativity</td>
<td>50</td>
<td>80</td>
<td>57</td>
</tr>
</tbody>
</table>
In case 1 the experimental results were obtained for an atomic detuning of $+200$ MHz, a large cavity input power of 34 mW, cooperativity $C=50$ and detection frequency 250 MHz. During the scan (4 seconds) the phase of the local oscillator was changed at a constant rate while the cavity detuning was scanned slowly across the cavity transmission peak. This trace was taken in a deliberate attempt to achieve the conditions of the regime accessed by Orozco et al. (1987) and Raizen et al. (1987). In this experiment the quantity of quantum noise above amplifier noise had been improved by replacing the amplifier with one of a superior type so that rf frequencies above 200 MHz could be analysed for squeezing.

As customary, the local oscillator phase was scanned at a constant rate and the cavity length was varied to sweep through resonance. The horizontal axis shows the detuning of the cavity from the empty-cavity resonance at zero, in units of cavity half-linewidth $\kappa$. The top trace (Fig. 6.2-1a) shows the noise power measured with a spectrum analyzer with a resolution bandwidth of 300 kHz and a video bandwidth of 10 Hz. The noise power is compared with the standard quantum noise limit (which was determined by a consecutive scan under identical conditions with the output of the cavity blocked). Trace (i) of (a) shows phase sensitive noise on the low transmission side of the cavity, becoming large close to the point where the cavity changes to the high-transmission state, but no squeezing. For high cavity transmission some excess classical noise seems to exist, as the noise of the cavity signal does not meet the quantum noise line. There is no phase sensitive modulation of the noise for high cavity transmission.; when the cavity switches on, the fluctuations are greatly reduced and phase-sensitive noise is no longer found. No squeezing was observed during the course these experiments for the ‘lower-branch’ regime. In general excess noise appeared to be present near the point of transition to the upper state where the noise modulation was greatest.

Figure 6.2-1(c) shows the modelling of the experiment for $q=0.08$, $C=50$, $\Delta=20.5$, and $2\pi\nu/\kappa = 3.0$; where cavity detuning $\phi$ was scanned through 3 cavity linewidths, and intracavity intensity $I$ was optimised for best squeezing. Trace (d) shows the modelled cavity output power, for the same parameters as trace (c).

In Case 2 (Fig. 6.2-2) experimental results were obtained for an atomic detuning of $+200$ MHz, cavity input power 10.6 mW, cooperativity $C=80$, and a detection frequency of 146.5 MHz. Trace (i) reveals phase sensitive noise on the upper branch, with some squeezing. The largest amounts of phase sensitive noise and squeezing occur on the upper branch away from the switching point, with no significant phase noise on the low transmission side. Trace (ii) gives the quantum noise limit. Trace (b), the experimental cavity output power, shows that the cavity is clearly bistable with switching observable in the output. In Case 2, atom-laser detuning was the same as for Case 1. Cooperativity was increased (from 55 to 80), detection frequency was reduced to 146.5 MHz and the cavity input power was reduced to 10.6 mW. Modelling of the experiment in Fig. 6.2-2(c) was carried out using the parameters $q=0.08$, $C=80$, $\Delta=20.5$, $2\pi\nu/\kappa = 1.2$, with $\phi$ scanned through 5 cavity linewidths, and $I$ optimised. Fig. 6.2-2(d) gives the theoretical cavity output power, for model parameters as in (c).

Experimental data for Case 3 are shown in Fig. 6.2-3. In this particular case the detection
frequency is 150.5 MHz, cooperativity C = 57, and the normalised atom-laser detuning \( \Delta = +300 \) MHz. The cavity input power was 13 mW. The noise power shown in Fig.6.2-3(a) is clearly phase dependent. For these experimental parameters squeezing is observable for cavity detunings of 0 - 6 \( \kappa \). Trace (i) shows phase sensitive noise observable across the whole cavity transmission, showing squeezing of \((18 \pm 3)\%\). Phase sensitive noise is now seen across the cavity scan. Trace (ii) shows the quantum noise limit. The cavity transmission is shown in Fig. 6.2-3(b). The cavity was still highly nonlinear but not bistable; some instability can be observed on the steep left hand side of the peak. The power in the cavity output beam at the peak of the cavity transmission was 0.17 mW, giving a significant coherent amplitude to the squeezed beam.

In Fig 6.2-3(c) modelling of the experiment is demonstrated, with \( q = 0.08 \), \( C = 57 \), \( \Delta = 33 \), \( 2\pi\nu/\kappa = 0.69 \), \( \phi \) scanned through 3 cavity linewidths, and I optimised for best squeezing. (d) Cavity output power, for model parameters as in (c). We note that the cooperativity \( C = 57 \) was similar to that of Case 1, but with laser-atom detuning increased to 300 MHz, and the detection frequency of 150.5 MHz and the input power (13 mW) were similar to the conditions of Case 2.

6.3 Analysis of a single trace

In the following, we shall concentrate on Case 3, which generated the best squeezing in our experiment. The noise suppression observed was about 0.2 dB. The quantum noise trace was corrected for the power in the cavity output field (0.17 mW at peak transmission compared to 2 mW in the local oscillator beam). The electronic noise level was 1.6 dB below the standard quantum level (1 VNU). This noise was treated as uncorrelated classical noise. Fig. 6.3-4 shows the data presented in Fig.6.2-3 corrected for the electronic noise, on a linear VNU (Vacuum Noise Unit) scale. The quantum noise level (1VNU) and the noise level corresponding to a suppression of 20\% (0.8 VNU) are shown as dashed lines. The minima consistently reach 0.8 VNU and the observed noise suppression can be conservatively quoted as 18 ±3\%.

As a consequence of losses only a fraction of the noise suppression inside the cavity is actually measured in our apparatus. The efficiency factors were determined separately. The escape efficiency of the cavity (Wu et al. 1987) which depends on the losses of the cavity mirrors, was measured to be 0.88. The propagation efficiency from cavity to the detectors was also 0.88, and the efficiency of the mode-matching in the homodyne detector was determined as 0.70. The quantum efficiency of the photodetectors at 553.5 nm was measured to be 0.65. Note that the typical relative decrease in quantum efficiency of silicon photodiodes from peak efficiencies to the green wavelengths is 15 \%. The total detection efficiency can be estimated as 0.35 ± 0.05. By making a correction for the electronic noise contribution and including all losses the noise suppression generated by the cavity-atom system was estimated to be (50 ± 10) \%. This corresponds well with the theoretical results shown in Fig. 6.2-3 (c-d).
Figure 6.3-1: (a) Squeezing and quantum noise traces for the parameters of Figure 6.2-3, corrected for the non-negligible power (0.17 mW) of the squeezed cavity beam. (b) Observed dc interference fringes on one photodetector.
6.4 Comparison with the experiment of previous workers

The notable differences between the present experiment and the experiment of Orozco et al. (1987), both in terms of the initial configurations and parameters are summarised in Table 6.4-1 and below.

(i) The atomic decay rate for our experiment is greater by a factor of two, making $q = \gamma_{\perp}/\kappa$ twice as large, such that the vacuum-field Rabi splitting frequency $(\sqrt{N}/(2\pi)) = \kappa (2Cq)^{1/2}/(2\pi)$ is also greater (330 MHz compared to 260 MHz).

(ii) Our detectors roll off completely by 200 MHz, so we would not be able to locate an optimum squeezing centred on 330 MHz, although we might be able to observe the wings of the spectrum. We observe optimum squeezing at $\omega \sim \kappa$ rather than at $\omega \sim 3\kappa$.

(iii) The atomic detunings employed in the present experiment are greater by factors of two to four.

(iv) It appears that we observe a greater proportion of the achievable squeezing - our squeezing may be more robust with respect to classical noise, Doppler broadening, cavity jitter and so on, as well as with respect to variations in key parameters.

(v) In the Austin experiment the atomic and cavity detunings had opposite signs. In our experiment they have the same sign.

(vi) In the Austin experiment squeezing is encountered on the lower branch only. Our experiment - squeezing on the upper branch. It is interesting to note that in the experiment of Orozco et al. (1987) and Raizen et al. (1987) phase sensitive noise was seen upon the upper branch over wide regions of the spectrum, but no squeezing was encountered there. It could be surmised that the input intensity used was too great to observe squeezing in the 'upper branch regime', and excessive spontaneous emission was created (see the theoretical plots in Figs 6.2-1 to 6.2-3).

6.5 Mapping the uncertainty area

The degree to which the light beam is squeezed was determined by measuring how far the phase sensitive quantum noise dipped below the standard quantum limit whilst the phase of the local oscillator beam was scanned. This measurement can show that the uncertainty area of the light is not a coherent state circle (Loudon and Knight 1987), but it cannot provide any information on the orientation of the squeezing ellipse with respect to the quadrature axes. To measure the orientation the phase of the squeezed beam with respect to the local oscillator must also be known. This means that a calibrated differential phase shifter must be used, as used by Levenson et al. (1985) and Milburn et al. (1987), or that the squeezed light beam must be intense enough to produce detectable interference fringes when mixed with the local oscillator. In the atomic squeezing experiments
### Table 6.4-1: Comparison of the present experiment with that of Orozco et al. (1987).

<table>
<thead>
<tr>
<th></th>
<th>Orozco et al. *</th>
<th>Hope et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finesse F</td>
<td>660</td>
<td>150</td>
</tr>
<tr>
<td>Optical depth ( \alpha_0 \ell )</td>
<td>0.6</td>
<td>1.0-3.5</td>
</tr>
<tr>
<td>Cooperativity C = ( \alpha_0 \ell F/(2\pi) )</td>
<td>52</td>
<td>45-80</td>
</tr>
<tr>
<td>Cavity length L</td>
<td>0.83 mm</td>
<td>4.3 mm</td>
</tr>
<tr>
<td>Cavity decay rate ( \kappa/(2\pi) )</td>
<td>134 MHz</td>
<td>124 MHz</td>
</tr>
<tr>
<td>Atomic decay rate ( \gamma/(2\pi) )</td>
<td>10 MHz</td>
<td>19.5 MHz</td>
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<tr>
<td>Ratio of decay rates ( q = \gamma_|/\kappa )</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>Atom detuning ( \Delta = (\omega_0 - \omega_|)/\gamma_| )</td>
<td>-14.6 ( \gamma_| )</td>
<td>30 - 60 ( \gamma_| = 300 - 600 \text{ MHz} )</td>
</tr>
<tr>
<td>Cavity detuning ( \phi = (\omega_c - \omega_|)/\kappa )</td>
<td>0.7 - 0.8( \kappa )</td>
<td>1.5 - 2.0( \kappa ) (calculated)</td>
</tr>
<tr>
<td>( (g\sqrt{N})/(2\pi) = \kappa(2Cq)^{1/2}/(2\pi) )</td>
<td>265 MHz</td>
<td>330 MHz</td>
</tr>
<tr>
<td>Observed frequency of best squeezing</td>
<td>280 MHz</td>
<td>120 - 150 MHz</td>
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<tr>
<td>Best squeezing observed</td>
<td>30 ± 4 %</td>
<td>18 ± 3 %</td>
</tr>
<tr>
<td>Inferred squeezing</td>
<td>53 ± 9 %</td>
<td>50 ± 10 %</td>
</tr>
<tr>
<td>Predicted squeezing#</td>
<td>78 %</td>
<td>45 %</td>
</tr>
</tbody>
</table>

* cavity configuration b
# plane wave theory
performed to date knowledge of the absolute phase difference was not available, or the output state produced by these experiments was a squeezed vacuum, so that it was not possible to map the uncertainty area.

Our system allowed us a direct measurement of the phase $\phi_{LO}$ of the local oscillator at any one instant. The power of the squeezed light was sufficient to produce interference fringes of good visibility with the local oscillator. These fringes could be detected by monitoring the photocurrent from one of the two photodiodes in the homodyne detector. Fig. 6.4-1(b) displays the dc photocurrent from one of the two photodetectors. Interference fringes with a maximum fringe contrast of 0.4 were observed. Using this trace, and the corresponding trace for the other detector, the relative phase $\theta$ between the squeezed beam and the local oscillator can be uniquely determined, allowing the noise power $V(\phi_{LO})$ to be reconstructed.

The shape of $V(\phi_{LO})$ and the angle $\phi_{\text{opt}}$ for maximum noise suppression can be best visualised by plotting the noise power in a polar diagram $V(\phi_{LO}) \cos(\phi_{LO})$ versus $V(\phi_{LO}) \sin(\phi_{LO})$. Fig. 6.5-1 shows such a plot for both the trace shown in Fig. 6.2-3 (dots) and the noise rescaled with the efficiency factor of 0.35 (diamonds). It should be noted that this diagram is not a direct representation of the quadratures $X_1$ and $X_2$. $V(\phi_{LO})$ represents the measured variance for a specific angle of the local oscillator. It is linked to the fluctuations $\delta X_1$ and $\delta X_2$ by (Levenson 1987)

$$V(\phi_{LO}) = \delta X_{12} \cos^2(\phi_{LO} - \phi_{\text{opt}}) + \delta X_{12} \sin^2(\phi_{LO} - \phi_{\text{opt}})$$ (6.7)

For a coherent state, $\delta X_1 = \delta X_2 = 1$. $V(\phi_{LO})$ is a constant, represented by a solid-line circle in the phasor diagram. For a squeezed minimum uncertainty state $\delta X_1 = e^\gamma$, $\delta X_2 = e^{-\gamma}$, $\delta X_1 \delta X_2 = 1$; and $V(\phi_{LO})$ is phase dependent. An example for $\delta X_1 = 0.5$, $\delta X_2 = 2$ and $\phi_{\text{opt}} = 38^\circ$ is given in Fig. 6.5-1 (solid line). It has the shape of a 'bowtie' or elliptical lemniscate (Loudon 1989) rotated counterclockwise by $\phi_{\text{opt}}$ with respect to the horizontal axis, which corresponds to the amplitude quadrature.

From this plot it can clearly be seen that the noise suppression is well below the standard quantum limit. The curves are symmetric in regard to an axis of minimum noise, rotated by 38 degrees to the horizontal. The squeezed light is neither an amplitude nor a phase-squeezed state. The rescaled experimental data agree well with this simple picture, indicating that the system generates a minimum uncertainty state. The corresponding output from our theoretical model is shown in Fig. 6.5-2. The structure, symmetry and orientation of the uncertainty area are in good agreement with the measurement: thus we are satisfied that the model gives a good qualitative description of our system.

Levenson et al. (1985a) reported observing squeezing in a combined amplitude/phase quadrature for the simple case of the nonlinear Kerr effect. Fluctuations of amplitude are correlated to phase fluctuations through a refractive index which depends quadratically on the field amplitude. A full quantum analysis (Levenson et al. 1985a, 1985b) predicted best noise suppression at 45° to the horizontal axis in the limit of small squeezing, and was supported by their experimental results. The more complex atom-cavity system contains both an intensity-dependent refractive index and a nonlinear absorption term,
Figure 6.5-1: Phasor diagrams of $V(\phi_{LO})\cos(\phi_{LO})$ versus $V(\phi_{LO})\sin(\phi_{LO})$. The solid circle represents the uncertainty area of a coherent state. Plots are shown for the experimental trace of Fig. 6.2-3 (dots) and the noise rescaled with the efficiency factor of 0.35 (diamonds). One full period of $\phi_{LO}$ around the peak of the cavity transmission is selected. In this diagram an amplitude-squeezed state has a minimum projection along the vertical axis. The measured uncertainty area is rotated by $38\pm 5^\circ$ from such a state and has the shape of a bowtie. An example for a squeezed minimum-uncertainty state is also given for $\delta X_1 = 0.5$, $\delta X_2 = 2$ and $\phi_{opt}$ chosen as $38^\circ$ (solid line with a bowtie shape).
Figure 6.5-2. Modelled phasor plot, corresponding to the parameters of Fig. 6.2-3. A full period of the phase angle of squeezing $q$ is selected at the peak of the cavity transmission. $q=0.08$, $C=57$, $\Delta=33$, $2\pi v/\kappa=0.69$, $\phi=1.9$, $I=905$. 
coupling the amplitude and phase of the light. As a result, we would anticipate noise suppression for a range of quadrature combinations, depending on the parameters. Further investigation is required into these aspects of the system.
Chapter 7: Conclusion

The work described in this thesis fell into two main sections;

(i) the theoretical work, in which the general quantum theory of optical bistability without adiabatic elimination given by Reid (1988) was extended to apply to the case of an arbitrary spatially varying field mode and compared with the results of previous workers.

(ii) the experimental work, in which phase sensitive noise and squeezing were generated in the interaction between a beam of atomic barium and a single mode of an optical cavity.

7.1 Summary of the work

Best squeezing was invariably found to be smaller for a given cooperativity $C$ than predicted for the plane-wave case, when the effect of a spatially varying mode upon squeezing in the short-cavity regime was calculated. The degradation of squeezing was found to be small for the standing wave, larger for the Gaussian mode in a ring interferometer, and largest for the Gaussian transverse mode in a standing-wave cavity. The estimated decrease in squeezing was consistent with the decrease predicted for the high-Q cavity by previous workers (Xiao et al. 1987a, b).

An aspect of the degrading effect of a spatially varying field mode on the optimum squeezing peculiar to the short-cavity regime was the dependence of the degrading effect upon the ratio of decay rates $q$. The effect of the mode structure on the squeezing was minimal for a large region of values of $q$ in this regime. As the bad-cavity limit was approached, the degradation of squeezing with respect to the plane increased considerably for all types of spatially varying mode analysed.

The present work demonstrated that in the presence of a spatially varying cavity mode there exists a particular value of $q = \gamma_j/\kappa$ (for a given cooperativity $C$) which gives rise to best squeezing. For $C = 50$ this was approximately $q = 0.01$. This finding has significant implications for further experiments in the strongly-coupled cavity-atom system. The plane-wave model indicated that squeezing was generally improved by moving towards the bad-cavity regime (Castelli et al. 1988), but this is not correct for the realistic situation of a Gaussian transverse mode.

The experimental results of Orozco et al. (1987) were compared with the results of the theory for a Gaussian mode in a standing-wave interferometer. Good agreement was found once the intracavity intensity $I$ (one of the most poorly determined quantities of the experiment) was allowed to vary within error bounds. It was also found that some, but not all, of the discrepancy between theory and the experiment of Orozco et al. as to the amount of optimum squeezing could be accounted for by the degrading effect of the spatially varying mode. 78% optimum squeezing was predicted by the plane wave theory, as compared to 73% by the theory for the spatially varying mode; the value
obtained experimentally was 53 %, indicating that there were losses in the experiment which could not be explained by extending the theory to include a Gaussian mode in a standing wave interferometer.

In general the best squeezing achievable for a system of two-level atoms interacting with a single cavity mode was found in the regime of comparable cavity and atomic relaxation rates, in agreement with the theoretical work of Raizen (1989). This fact held true even with the inclusion of a spatially varying field mode. The optimal squeezing possible in the low-Q limit was shown to be considerably affected by the spatially varying mode structure, but was still much better than the squeezing in the good cavity limit for the same atomic cooperativity C. The effect of a Gaussian mode structure on squeezing caused the largest decrease in the bad cavity case, however.

Our aim was to devise an experiment which could realistically be compared with the theoretical model, which should give a complete description of quantum fluctuations and squeezing in the interaction of a single cavity mode with an ensemble of two-level atoms. Detailed theoretical studies had shown that a situation where the atomic decay rate $\gamma_4$ is comparable to the cavity response rate $\kappa$ was best suited for squeezing experiments. We therefore chose an experimental arrangement similar to the one employed by Orozco et al. (1987) for observing squeezing in a short cavity. Barium was selected as the atomic medium since it provided an almost ideal two-level system for the resonance transition of the most abundant isotope $^{138}$Ba, free of hyperfine splitting and Zeeman degeneracies.

In the experimental configuration for squeezing a dense, well-collimated atomic beam of natural barium was directed between the mirrors of a 4.3 mm long single-port cavity. The cavity was illuminated at the resonant wavelength of the atomic medium, 553.5 nm. The input beam detuning laser was varied and the cavity was scanned through resonance over a few cavity linewidths. The output beam was detected by a balanced heterodyne detector, allowing measurement of the fluctuations in the range 60 to 170 MHz for the two different quadratures of the output field.

When the experiment was operated in a regime where the cavity-atom system showed bistability, we found phase sensitive noise and squeezing of up to 10 % on the upper branch of the bistability curve. Optimisation of the experimental parameters for best noise suppression led to a regime of high optical depth, high input power and large atomic detuning. Under these conditions the cavity did not display explicit bistability, only an asymmetry in the transmission curve. Best squeezing was observed to be 18 ± 3 %. By including the total detection efficiency of our experiment, the noise suppression generated by the atom-cavity system was estimated to be 50 ± 10 %. The nonlinearity resulting in squeezing appeared to be very robust and noise suppression was observed for a wide range of parameters.

Measurements carried out across the available range of rf detection frequencies revealed that the squeezing was broadband in character, but the optimum analysis frequency was not given by the vacuum-field Rabi splitting $g\sqrt{N}$. In addition, the laser-atom detunings for optimum squeezing were much larger than predicted, and the squeezing was observed upon the upper branch of dispersive bistability and for peak cavity transmission when the
system was not bistable. These facts were surprising in the light of the previous experiments and analysis by Raizen et al. (1987), and Orozco et al. (1987), and prompted us to return to the model.

By optimising the parameters $I$, $\Delta$, $\phi$, and $\omega$ to give best squeezing, with values of cooperativity and $q=\gamma_c/\kappa$ appropriate to the experiments being considered, for the cases of $\Delta \phi < 0$ and $\Delta \phi > 0$, we identified two distinct regimes for the generation of squeezing. One regime led to large squeezing (>70%) when the cavity was nonresonant and atomic detunings were small (10 - 20 $\gamma_c$). The other showed good squeezing (60%) near peak transmission of the cavity at large atomic detunings (30-60 $\gamma_c$) for the same values of $q$ and $C$. The first regime gives rise to squeezing at the coupling frequency $\omega_{opt}=g\sqrt{N}$, whereas the second regime has best squeezing for significantly lower values of $\omega$. We infer that the different regimes arise as a consequence of competing mechanisms within the strongly-coupled cavity-atom system.

The parameters of the first regime correspond to those used in the experiment of Raizen et al. (1987) and Orozco et al. (1987) and the parameters for the latter regime were realised in the present experiment. In both cases we found good agreement between theory and experiment. We note that for both cases the cooperativity (or bistability parameter) is approximately 50, and that the ratio of decay rates $q$ differs only by a factor of two; nevertheless the behaviour of the system is significantly different. The fact that similar experiments gave rise to qualitatively different outcomes demonstrates the sensitivity of the short-cavity configuration to the variation of key parameters.

In both of the identified regimes it was demonstrated experimentally that squeezing could be obtained when the system was bistable. For a bistable system the best squeezing in the Orozco experiment was found on the lower branch of the bistability curve, in clear contrast to our case where best squeezing occurred on the upper branch.

A novel feature of our experiment in relation to other atomic squeezing experiments is the significant power of the squeezed beam (0.17 mW). The squeezed beam generated in the Orozco experiment had a low power (of the order of a few $\mu$W). The difference is partly due to the fact that best squeezing in the Orozco experiment was found on the lower branch of the bistability curve, but is also due to the chosen cavity configuration. The power of the squeezed beam in our experiment was such that detectable fringes were produced by the interference of the squeezed cavity beam and the local oscillator beam. These interference fringes were recorded at the same time as the phase sensitive noise, and these data taken together allowed us to make a full reconstruction of the shape and tilt of the uncertainty area of the squeezed beam to be made. In previous atomic squeezing experiments the output beam produced was a squeezed vacuum or a beam of very low power, or knowledge of the absolute phase difference was not available, so that it was not possible to determine the shape and orientation of the uncertainty area.

A theoretical simulation of the complete quantum optical behaviour of our experiment was developed from the microscopic quantum-statistical theory of optical bistability. Results for the case of a plane wave, ring cavity model were used to simulate the experiment by evaluating as a function of cavity detuning the spectral variance, equivalent to the measured
noise power, and the intracavity photon number, proportional to the cavity transmission. This calculation predicted a best noise suppression of 50-60 % with the maximum occurring near the peak of the cavity transmission. This was in good agreement with our observations. Both the nonlinear behaviour of the system and the predicted degree of squeezing agreed well with the experiment. As in the experiment the best noise suppression occurred close to the peak cavity transmission and was not restricted to the critical point of bistability.

7.2 Conclusions drawn

We have presented here experimental observations of the squeezing generated by a closely-coupled cavity-atom system. This system can produce a squeezed state of low intensity or of significant intensity. The squeezed quadrature is a combination of the amplitude and phase quadratures.

Our observations are in good agreement with a complete quantum optical theory based on the coupling of two level atoms to a single mode of an optical cavity. To describe the conditions of the experiment five parameters must be chosen. A search for the best squeezing was carried out in this five-dimensional parameter space. The search using the experiment clearly led us to a regime with high density, high cavity input intensity, and large detunings (the dispersive regime), where the optimum cavity detuning had the same sign as the atom-laser detuning. This regime corresponded to a regime in the model which is associated with the upper branch of dispersive bistability.

The model can distinguish between two separate regimes for squeezing; the $g\sqrt{N}$ regime (observed by Orozco et al. 1987) and the regime for upper branch bistability, observed in the present work. An attempt was made to interpret these findings and to provide a simplified explanation for the noise suppression. The first typical feature of this regime is an intensity-dependent refractive index which effectively shifts the atom-cavity system into resonance with the driving field. The importance of the nonlinear refractive index is further supported by the similarity of our results with squeezing produced by the nonlinear Kerr effect.

This observation leads to the somewhat simplified picture that the nonlinear refractive index is the key property of the cavity-atom system. This squeezing occurs due to a coupling of the phase and the intensity of the light inside the cavity, induced by the nonlinear Kerr effect due to the atoms. This interpretation is further supported by the similarity of our observations, in particular the shape and orientation of the plot of the variance, to the earlier observation by Levenson et al. (1985) of squeezing in a nonlinear quartz fibre. However this should be regarded as an analogy only - it emphasises the dominance of the dispersive effect but neglects the full complexity of the atomic system.

The fact that similar experiments give rise to qualitatively different outcomes (as in cases 1, 2, and 3 of Chapter 6) demonstrates the sensitivity of the short-cavity configuration to the variation of key parameters. This highlights the flexibility and variety available in an experiment using atoms, and provides the potential to tailor the properties of the squeezed
light to the given application. Potential applications such as interferometry, absorption spectroscopy or the excitation of atoms impose different requirements upon the quadrature of the squeezing. For example, interferometric applications require 'phase squeezed' vacuum states, whereas atomic excitation needs a bright source of adjustable orientation. It might well be possible to use the flexibility of atoms, a flexibility which is not available in nonresonant systems such as optical parametric oscillators, second harmonic generators or Kerr media, to optimise the squeezed light for certain applications. More work is required to assess the potential of these applications, and it is possible that investigation of the regime of dispersive bistability would amply repay further experimental effort into the generation of squeezed states.
The full expressions for the drift and diffusion coefficients for a Gaussian mode in a ring cavity, with pure radiative damping ($f=1$), are shown below. In computation it is necessary to avoid the point $\omega=0$, as the solutions to the integrals $\int b_j(\omega), \int \gamma_j(\omega), \int d_j(\omega)$ for $j=1..M$, have a part which tends to $\infty$ as $\omega\to0$, although a finite limit exists for the functions as a whole.

It is known (Reid 1988) that $\omega=0$ corresponds to the good-cavity case, $\kappa<<\gamma_1$, and the full analytic solution for the drift and diffusion matrix elements in a spatially varying mode in this case has been given in Xiao et al. (1987a). It is not possible to obtain the high-Q limit solutions in a direct analytic fashion from the equations presented here. For our purposes this restriction is not significant, as we generally wish to investigate the regime with $\kappa\sim\gamma_1$, where the squeezing is generated through a normal-mode coupling between the cavity mode and the atomic polarization. Here the optimum squeezing is found at a considerable detuning from the pump frequency, at $\omega/(2\pi)=(\gamma \mu C)^{1/2}$, and the squeezing at $\omega=0$ typically amounts to less than $1\%$.

The analytical results permit a saving in computation time when optimising the squeezing spectrum $V(X_\omega,\omega)$ for a spatially varying mode over the many parameters of this system.

\[
\Pi_j(\omega) = \frac{1}{2} - \frac{i\omega}{2} + \frac{I_j}{2[1+i\Delta(\omega)]} + \frac{I_j}{2[1-i\Delta(-\omega)]} = A + BI_j, \quad A = 1 - \frac{i\omega}{2},
\]

\[
B = \frac{1}{2} \left[ \frac{1}{1+i\Delta(\omega)} + \frac{1}{1-i\Delta(-\omega)} \right]
\]

\[
b(\omega) = \frac{-Cx_j}{1+i\Delta(\omega)} \left[ \frac{1}{1+i\Delta} + \frac{1}{1-i\Delta(-\omega)} \right] \left[ \frac{1}{(1+\Delta^2)B-A} \right]
\times \left[ \frac{A}{B} \ln \left( 1 - \frac{1}{x_2} \right) - (1+\Delta^2) \ln \left( 1 - \frac{1}{x_1} \right) \right]
\]

\[
\gamma(\omega) = \frac{-Cx_j}{1+i\Delta(\omega)} \left[ 2\ln(1-\frac{1}{x_1}) + \frac{1}{(1+\Delta^2)B-A} \right] \left[ \frac{1}{1-i\Delta} + \frac{1}{1+i\Delta(\omega)} \right]
\times \left[ \frac{A}{B} \ln \left( 1 - \frac{1}{x_2} \right) - (1+\Delta^2) \ln \left( 1 - \frac{1}{x_1} \right) \right]
\]
$\Lambda(\omega) = \frac{-8\lambda^2 c}{3c_i(1 + \Delta)(1 + i\Delta(\omega))}\left[\ln\left(\frac{c_4}{\sum_{i=1}^{4} c_i}\right) + \sum_{i=1}^{4} G(x_i)\right]$

$+ \text{Re}\left(\frac{8\lambda^2 c}{(1 + i\Delta)[1 + i\Delta(-\omega)]1 + i\Delta(\omega)]^2} \times \right.$

$\left\{ \frac{1}{c_i[1 + i\Delta(\omega)]}\left[ -1 - \frac{c_2}{3c_i} \ln\left(\frac{c_4}{\sum_{i=1}^{4} c_i}\right) + \sum_{i=1}^{4} J(x_i) \right] \right.$

$+ \left[ \frac{x_1}{A^* - B^*(1 + \Delta)} \right] x_3 \ln\left[\left(1 - \frac{1}{x_2}\right)^{-1} - x_2^2 \ln\left(1 - \frac{1}{x_1}\right) - \frac{x_1}{2IB^*}\right] \right\}$

where $x_1 = \left(\frac{1 + \Delta^2}{2I}\right), x_2 = -\left(\frac{A}{2IB}\right), x_3 = -\left(\frac{A}{2IB}\right)^*$, $c_1 = \frac{8\lambda^2 |B|^2}{1 + \Delta^2}$, $c_2 = 4I^2 \left(\frac{A^*B + B^*A}{1 + \Delta^2} + |B|^2\right), c_3 = 2I\left(A^*B + B^*A + \frac{|A|^2}{1 + \Delta^2}\right)$, $c_4 = |A|^2$

are the roots and polynomial coefficients respectively of

\[
\left(1 + \frac{2Ix}{1 + \Delta}\right)(A + 2IBx)(A^* + 2IB^*x)
\]

and

$G(x_i) = (c_3 - 2c_2x_i)\Psi(x_i), J(x_i) = \left[ \frac{Ix_i}{1 + i\Delta(\omega)} \left(\frac{2c_2^2}{3c_1} - c_3\right) + \frac{c_2c_3}{3c_1} - c_4 \right] \Psi(x_i)$,

$\Psi(x_i) = \frac{1}{3c_1x_i^2 + 2c_2x_i + c_3} \ln\left(\frac{-x_i}{1 - x_i}\right), i = 1, 2, 3$. 

\[
d(\omega) = \frac{2\text{IC}}{[1 + i\Delta][1 + i\Delta(\omega)][1 + i\Delta(-\omega)]} \left[ 2\times_1^2 \left\{ \ln \left( 1 - \frac{1}{x_1} \right) + \frac{1}{x_1} \right\} ight. \\
- \left[ \frac{1}{1 + i\Delta(\omega)} \right] \left[ \frac{1}{B - A / (1 + \Delta^2)} \right] \left[ x_2^2 \ln \left( \frac{x_2}{x_2 - 1} \right) + x_1^2 \ln \left( 1 - \frac{1}{x_1} \right) + \frac{x_1}{B} \right] \\
- \left[ \frac{1}{1 + i\Delta(-\omega)} \right] \left[ \frac{1}{B* - A* / (1 + \Delta^2)} \right] \left[ x_3^2 \ln \left( \frac{x_3}{x_3 - 1} \right) + x_1^2 \ln \left( 1 - \frac{1}{x_1} \right) + \frac{x_1}{B*} \right] \\
+ \frac{2}{c_1[1 + i\Delta(\omega)[1 + i\Delta(-\omega)]]} \left[ -1 - \frac{c_2}{3c_1} \ln \left( \frac{c_4}{\sum_{i=1}^4 c_i} \right) + \frac{3}{i} J(x_i) \right] \right] \\
+ \frac{4\text{IC}}{3c_1x_1[1 + i\Delta(\omega)][1 + i\Delta(-\omega)]} \left[ \ln \left( \frac{c_4}{\sum_{i=1}^4 c_i} \right) + \sum_{i=1}^3 G(x_i) \right] \\
+ \frac{4\text{I}^2\text{C}}{c_1(1 - i\Delta)[1 + i\Delta(\omega)]^2[1 + i\Delta(-\omega)]^2} \left[ -1 - \frac{c_2}{3c_1} \ln \left( \frac{c_4}{\sum_{i=1}^4 c_i} \right) + \frac{3}{i} J(x_i) \right]
\]
References


References


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References


